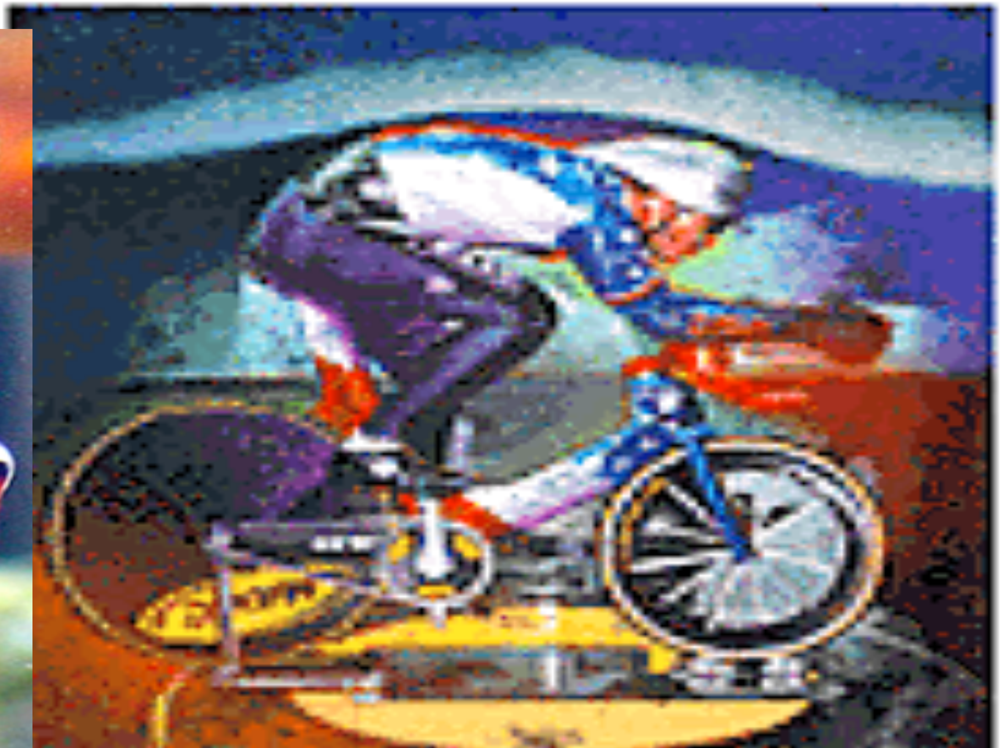
Two dimensional supersonic nonlinear Schrödinger flow past corner

Gennady A. El[1], Anatoly M. Kamchatnov[2] ,
Vadim V. Khodorovskii[1], Eder S. Annibale[3], Arnaldo Gammal[3],

[1]Loughborough University, UK
[2]Institute of Spectrocopy, Russian Academy of Science
[3]Universidade de São Paulo, São Paulo, Brazil



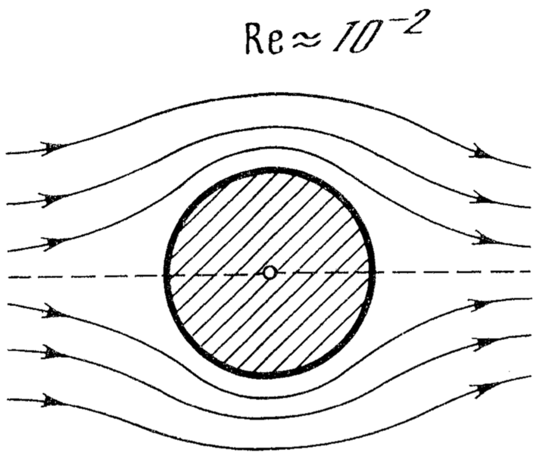


Fig. 9.1. Laminar flow around a cylinder for small Re

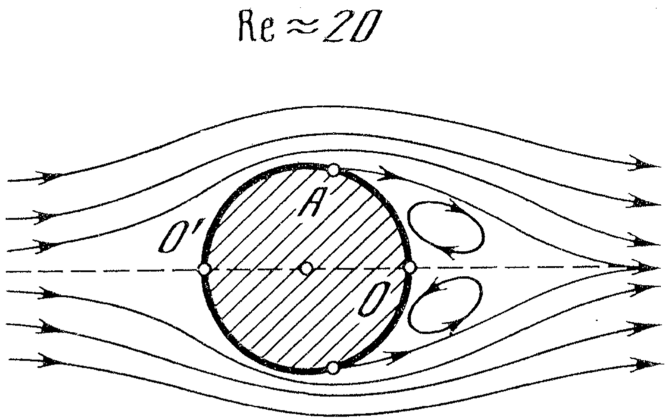


Fig. 9.2. Steady flow past a cylinder with two vortices

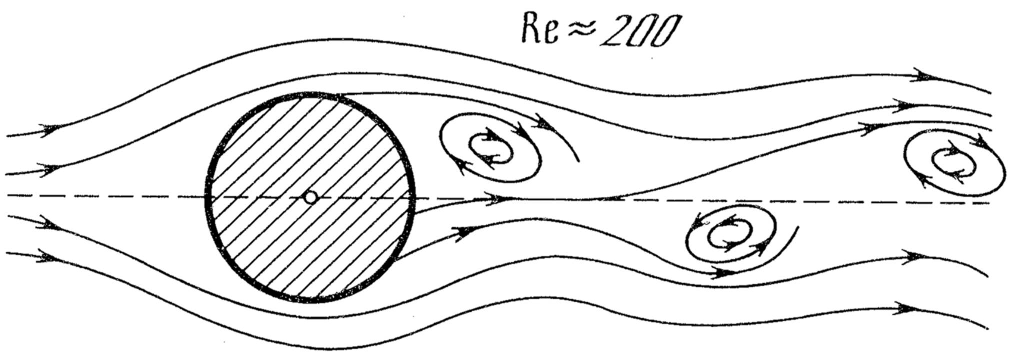


Fig. 9.3. Illustrating a Karman street

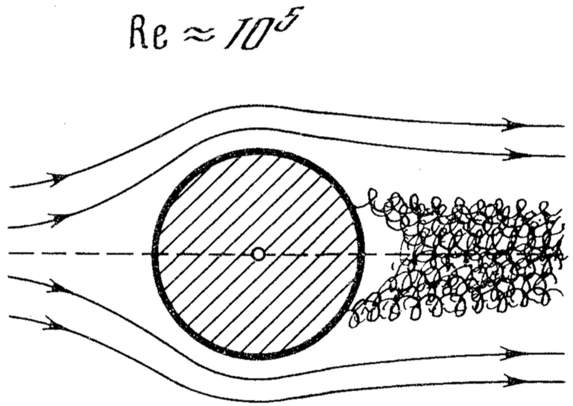


Fig. 9.4. The flow with a fully developed turbulent wake

Classical Fluids with viscosity

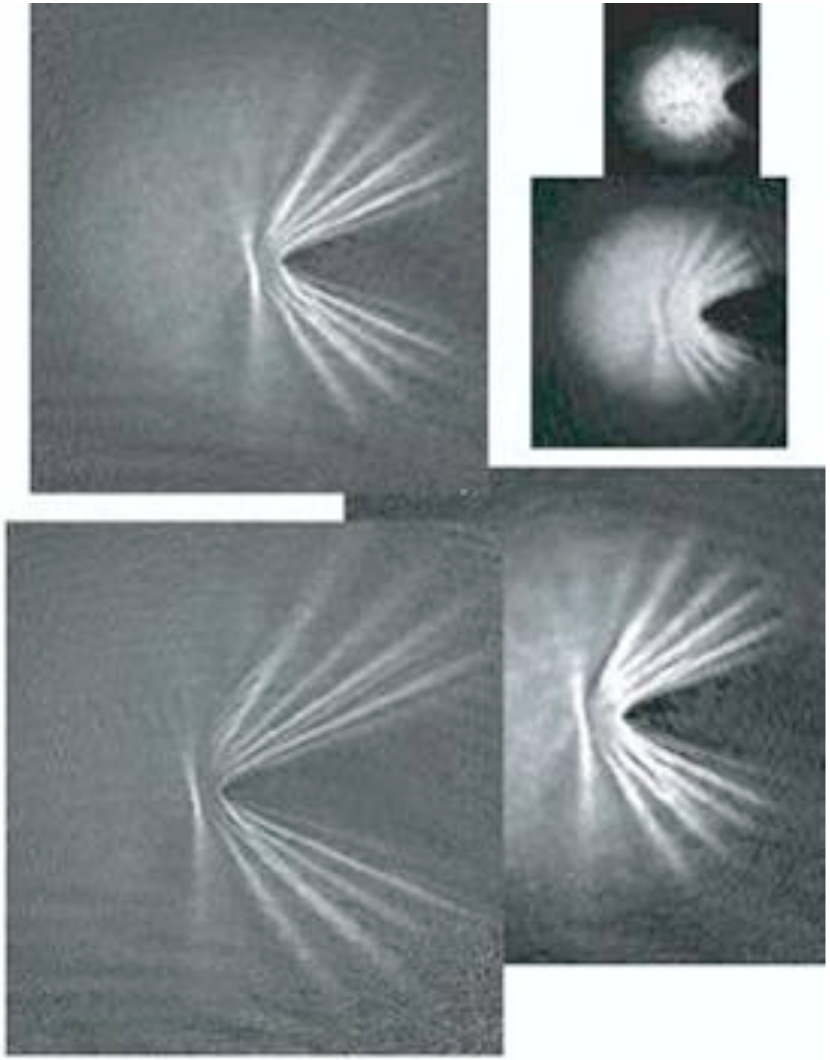
$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{v}) = 0$$ continuity

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{1}{\rho}\left\{\nabla p + \eta \nabla^2 \mathbf{v} + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla.\mathbf{v})\right\}$$

momentum conservation

Navier(1827) & Stokes (1845)
with viscosity η
2nd viscosity ζ

Quantum Fluids-
BEC



Shocks in the flow of BEC past an obstacle

JILA 2005

moving through a
er than the phase
o which the source
vestigated in this
etic waves emitted
nduced by a strong
dium [6,7], to the
oving at supersonic
perfluid [8], and in
emitted by a boat
]. In this Letter we
ensity perturbation
BEC) which flows
n the experimental
Modulo a Galilean
g source in a sta-
the one of a uni-
with a stationary
rmed by letting a
inst the localized

and placed in the vicinity of the trapped condensate, and remains in this position during the experiment. Images of the BEC density profile after different expansion times t_{exp} are then taken by means of destructive absorption imaging. Two examples are shown in Fig. 1. The field of view is centered in the region around the defect in order to observe

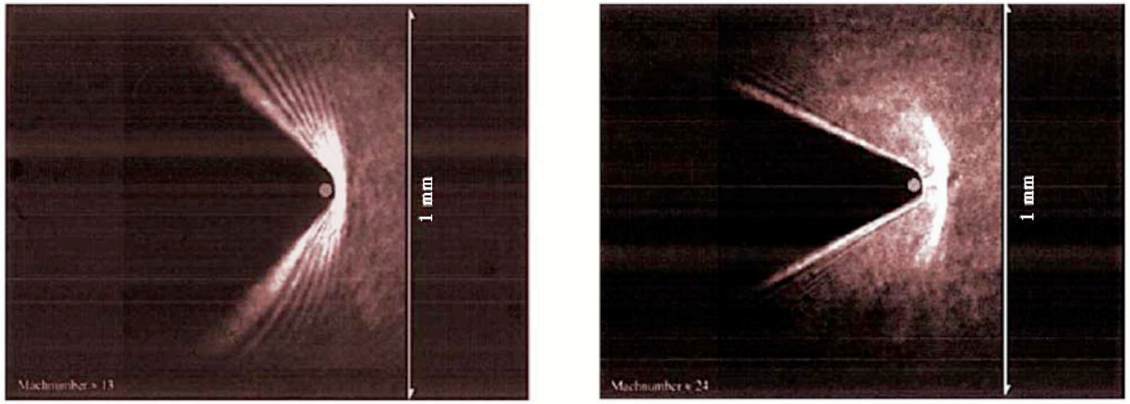


FIG. 1 (color online). Experimental [10] density profiles (integrated along z) of a BEC hitting an obstacle at supersonic velocities $v/c_s = 13$ (a) and 24 (b). The angles of the conical wave fronts are $\sin(\theta) = 0.73$ and $\sin(\theta) = 0.43$, respectively. The condensate flow is from the right to the left.

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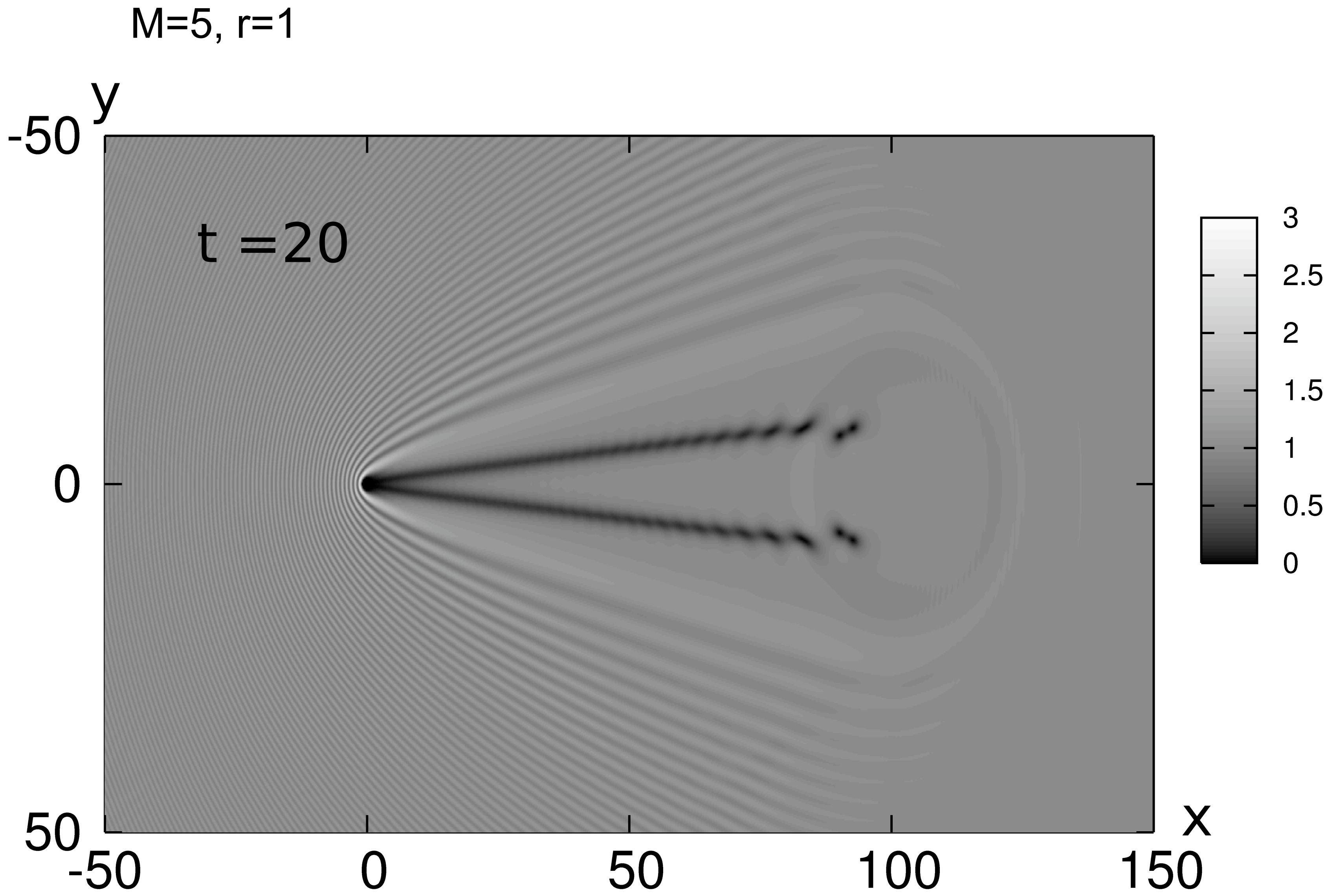
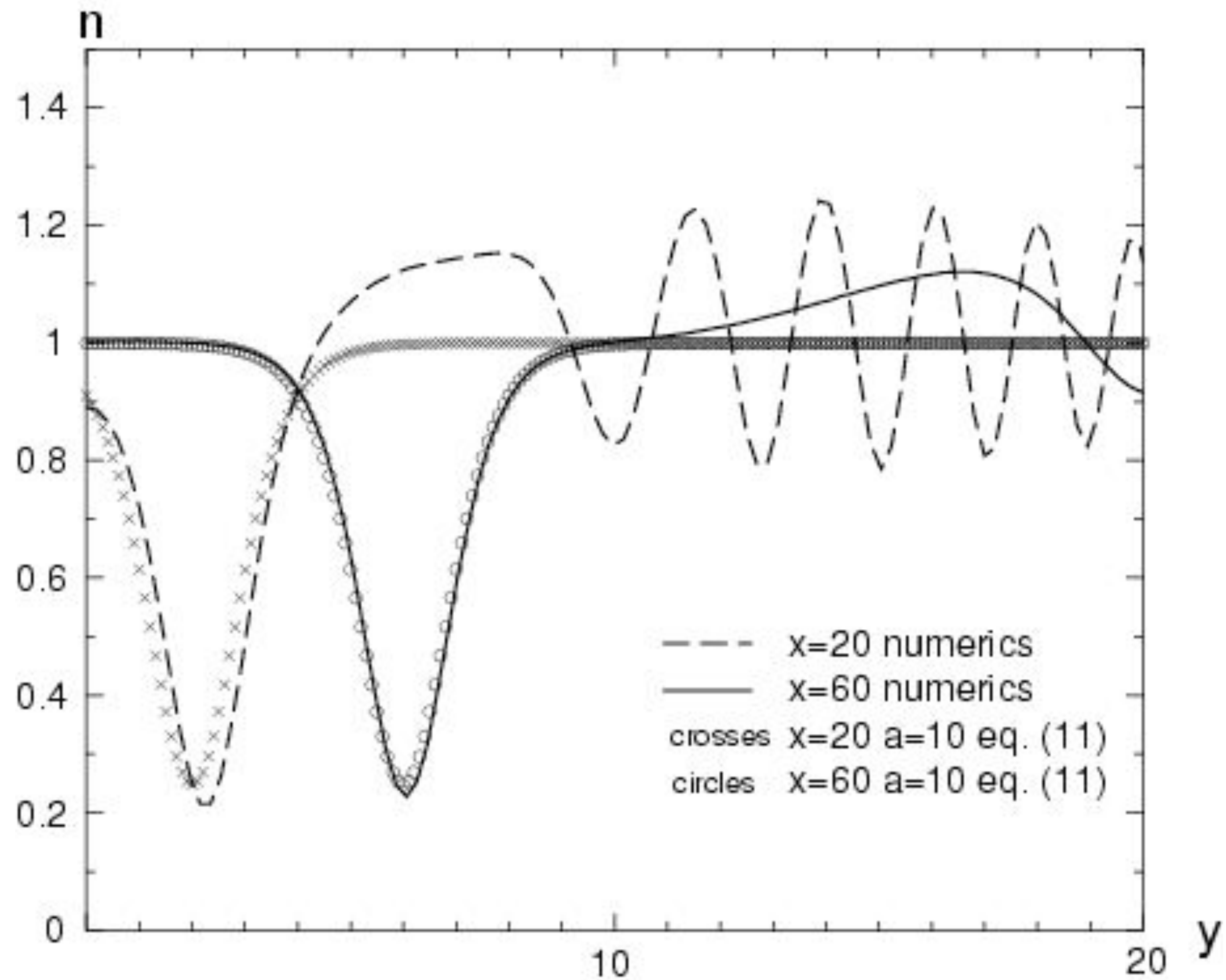
Carusotto et.al, PRL(2006)

We consider point obstacles

Frisch et al PRL1992, subsonic
Winiecki et al, PRL 1999 supersonic-> “vortex street”



Cutting in x we see dark solitons



G.El, A.G., A.M.Kamchatnov
PRL (2006)

We consider now extended obstacles like a corner (wedge)

Gross-Pitaevskii equation

Dynamics of a dilute condensate is described by the Gross-Pitaevskii equation ~1961

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + |\psi|^2\psi$$

in dimensionless units.

Gross-Pitaevskii Eq. in hydrodynamic form for potential flow $\nabla \times \mathrm{u} = 0$

$$\frac{\partial n}{\partial t} + \nabla . (n\mathrm{u}) = 0$$

$$\frac{\partial \mathrm{u}}{\partial t} + (\mathrm{u}.\nabla)\mathrm{u} + \nabla n + \nabla\left[\frac{(\nabla n)^2}{8n^2} - \frac{\nabla^2 n}{4n}\right] = 0$$

And sound velocity for uniform solution is

$$c_s = \sqrt{n}$$

No viscosity

quantum pressure term

With boundary conditions at infinity

$$n \to 1, \qquad \mathbf{u} \to (M, 0) \quad \text{as} \quad |\mathbf{r}| \to \infty$$

and impenetrability condition at body surface *S*

$$\mathbf{u}.\mathbf{N}|_S = 0,$$

Now we consider in the hydrodynamic form a stationary system of equations for the density *n(x,y)* and two components of the velocity field **u**= *(u(x,y),v(x,y))*

$$(nu)_x + (nv)_y = 0$$

$$uu_x + vu_y + n_x + \left(\frac{n_x^2 + n_y^2}{8n^2} - \frac{n_{xx} + n_{yy}}{4n} \right)_x = 0$$

$$uv_x + vv_y + n_y + \left(\frac{n_x^2 + n_y^2}{8n^2} - \frac{n_{xx} + n_{yy}}{4n} \right)_y = 0$$

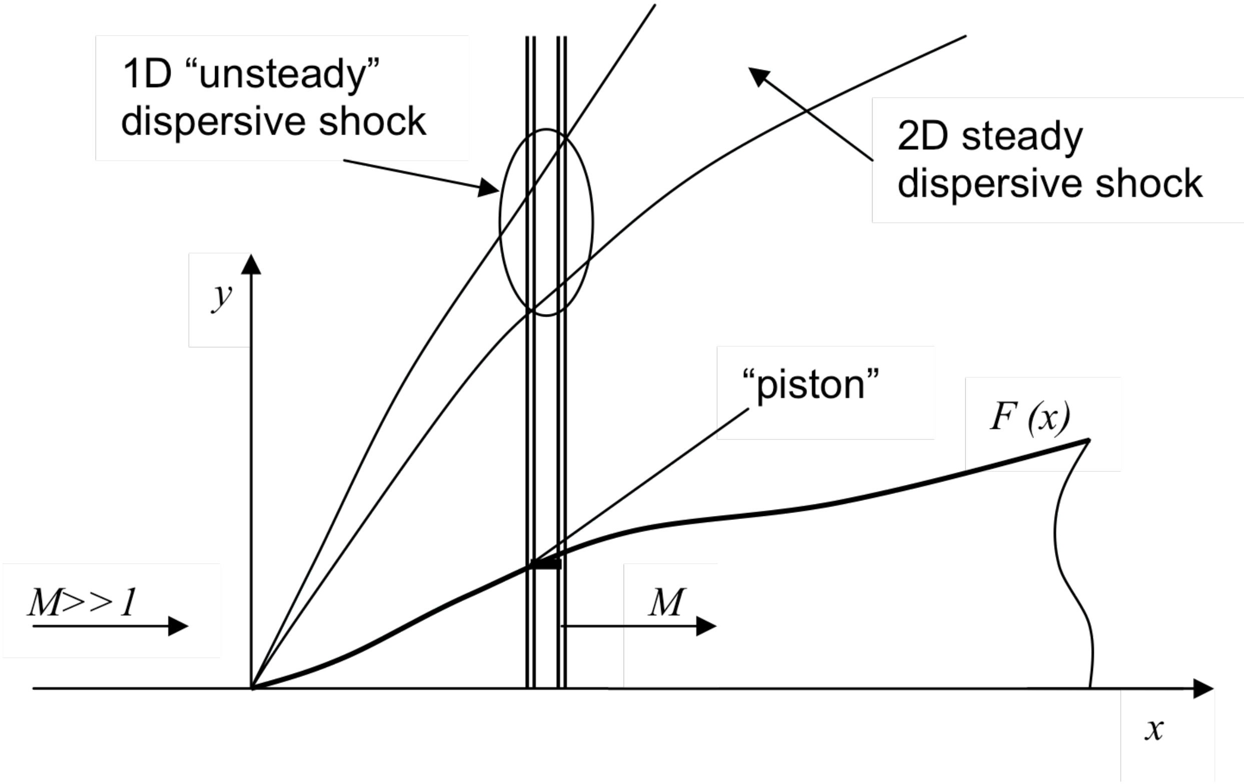
$$u_y - v_x = 0$$

Supersonic flow $M >> 1$

Stationary 2D NLS can be shown to asymptotically reduce to a 1D NLS

$$i\Psi_T + \frac{1}{2}\Psi_{YY} - |\Psi|^2 \Psi = 0$$

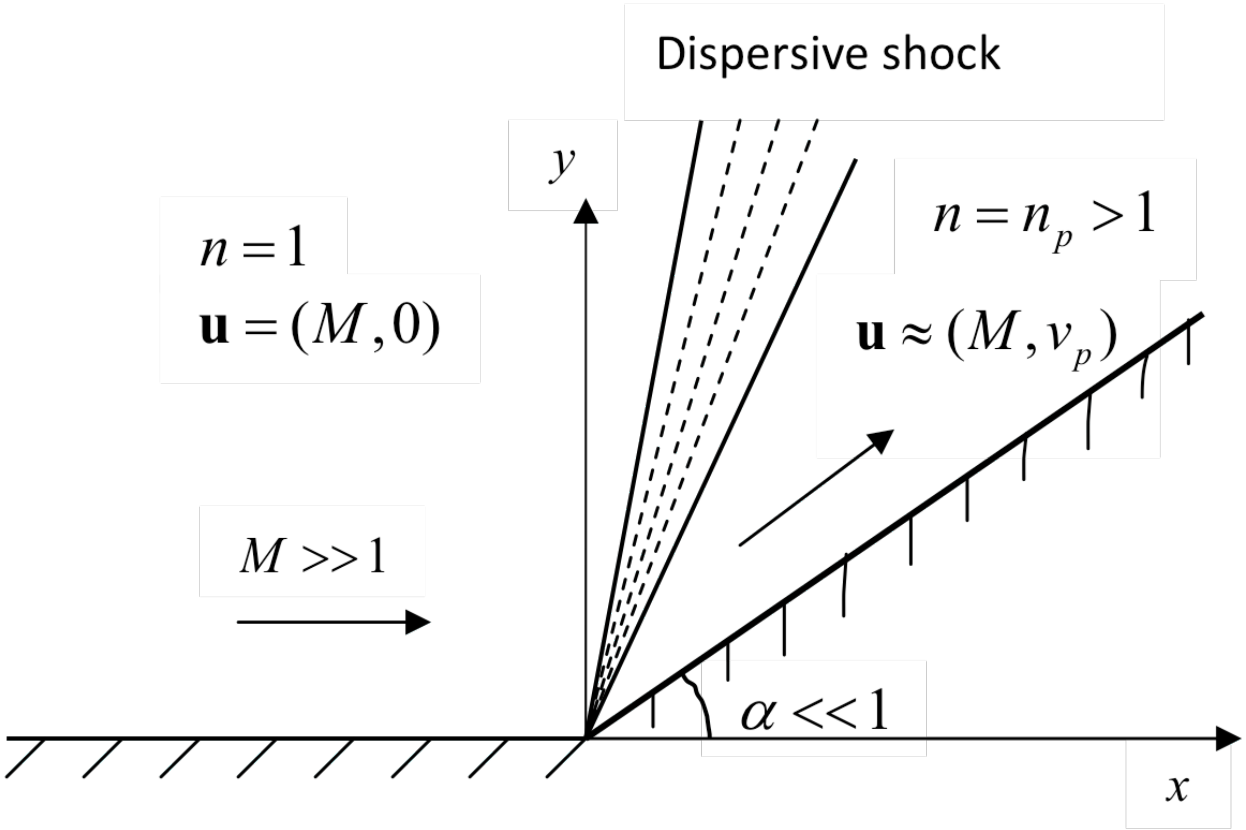
Where *T=x/M* and *Y=y*



Piston analogy in the problem of flow in dispersive shock flow of dispersive fluid past body

The theory of DSWs is based on the study of a certain nonlinear free-boundary problem for the modulation (Whitham) equations—the so-called Gurevich-Pitaevskii problem (1973).

A.V. Gurevich and L.P. Pitaesvkii, Sov. Phys. JETP, 38,291 (1974)



Dispersive shock
y
n = 1
u = (M, 0)
n = n_p > 1
u ≈ (M, v_p)
M >> 1
α << 1
x

Analytical theory of shocks

The region of oscillations is presented as a modulated periodic wave:

$$n(Y,T) = \frac{1}{4}(\lambda_4 - \lambda_3 - \lambda_2 - \lambda_1) +$$
$$(\lambda_4 - \lambda_3)(\lambda_2 - \lambda_1) sn^2(\sqrt{(\lambda_4 - \lambda_3)(\lambda_2 - \lambda_1)}\theta, m)$$

where

$$\theta = Y - UT - \theta_0 \qquad m = \frac{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_4)}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_4)}.$$

$U = \frac{1}{2}\sum \lambda_i$ (phase velocity)

The parameters $\lambda_i = \lambda_i(Y,T),\ i = 1,2,3,4$ change slowly along the shock. Their evolution is described by the Whitham modulational equations

$$\frac{\partial \lambda_i}{\partial T} + V_i(\lambda)\frac{\partial \lambda_i}{\partial Y} = 0$$

With characteristic velocities

$$V_i(\lambda) = \left(1 - \frac{L}{\partial_i L}\partial_i\right)U \qquad \partial_i = \frac{\partial}{\partial \lambda_i},$$

and wavelength $$L = \frac{2K(m)}{\sqrt{(\lambda_4-\lambda_2)(\lambda_3-\lambda_1)}}$$

For the corner, the relevant modulation solution has the form of a centered characteristic fan with

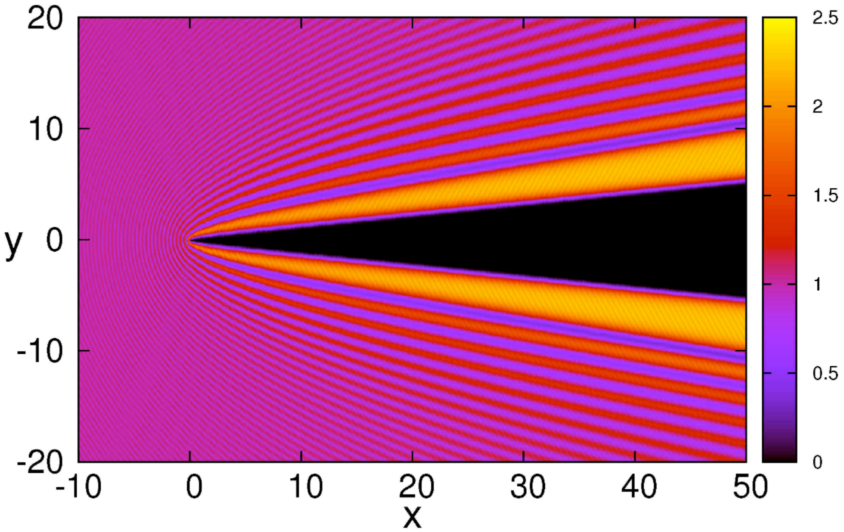
$$\lambda_1 = -1,\ \lambda_2 = 1, \quad \lambda_4 = 1 + \alpha M$$

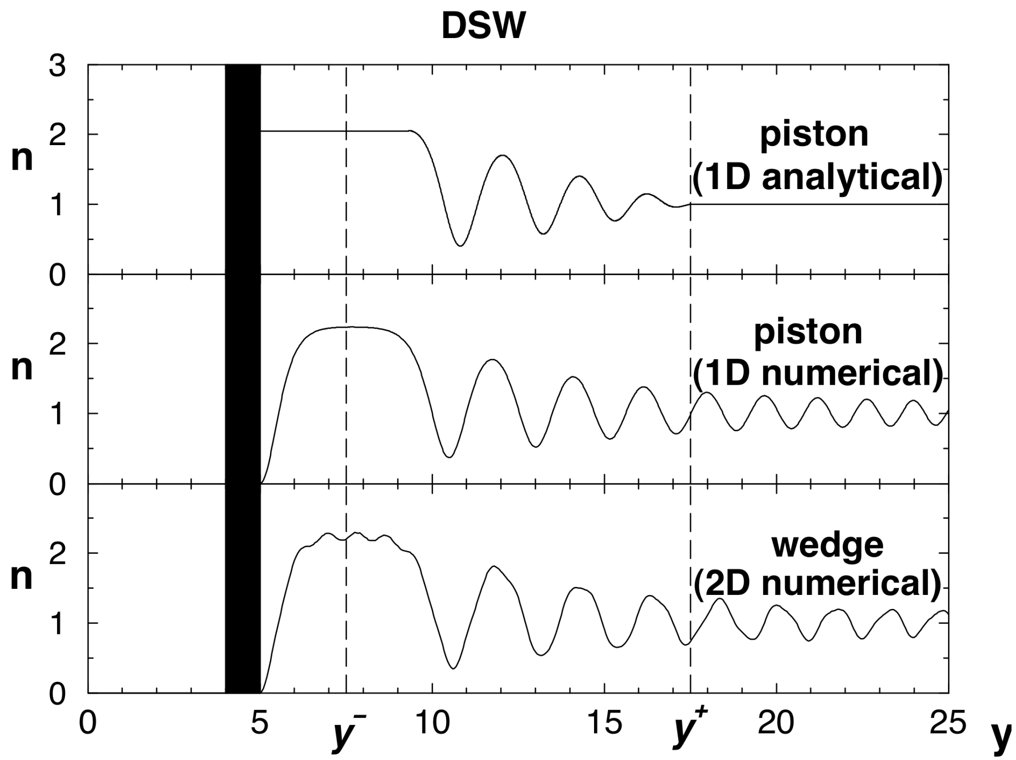
$$\frac{Y}{T} = V_3(-1, 1, \lambda_3, 1 + \alpha M)$$

which explicitly takes the form

$$\frac{Y}{T} = \frac{1}{2}(\lambda_3 + 1 + \alpha M) - \frac{(1 + \alpha M - \lambda_3)(\lambda_3 - 1)K(m)}{(\lambda_3 - 1)K(m) - \alpha M E(m)}$$

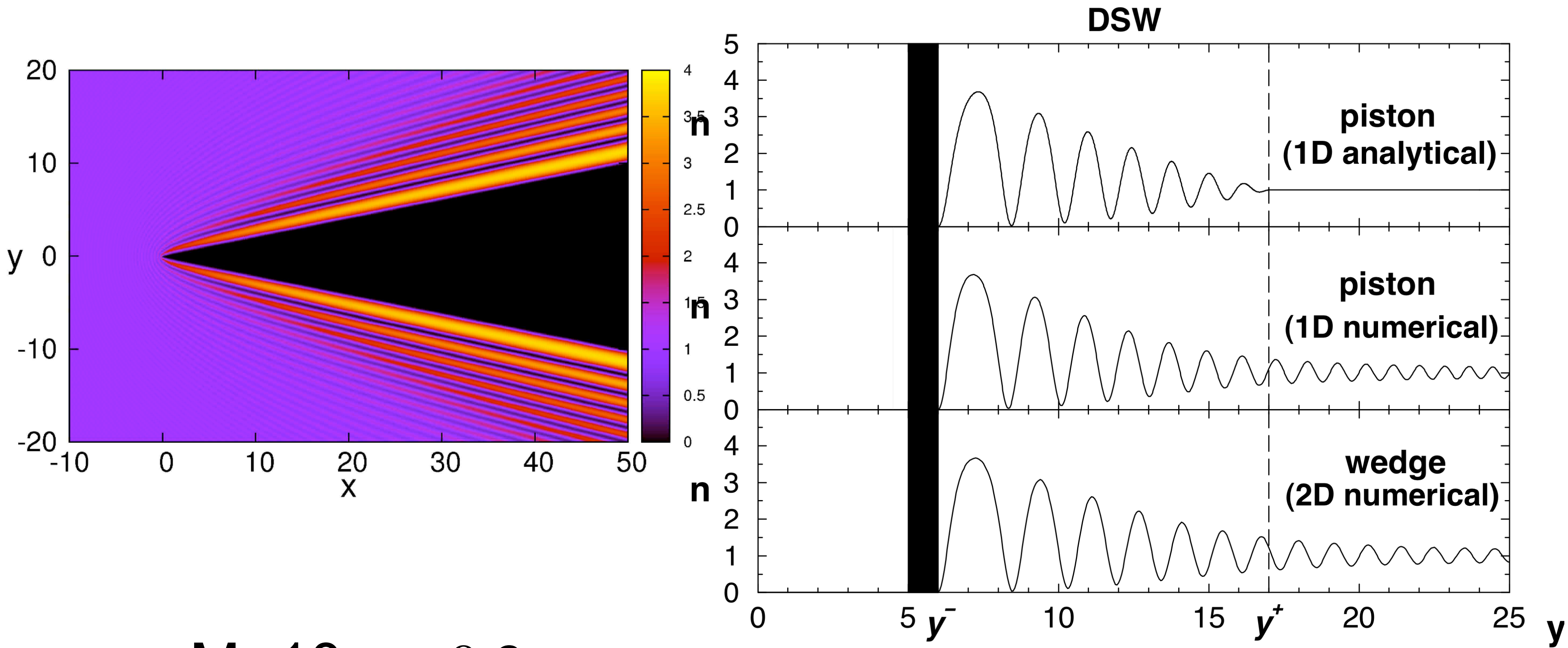
$$m = \frac{2(1 + \alpha M - \lambda_3)}{\alpha M(\lambda_3 + 1)}$$





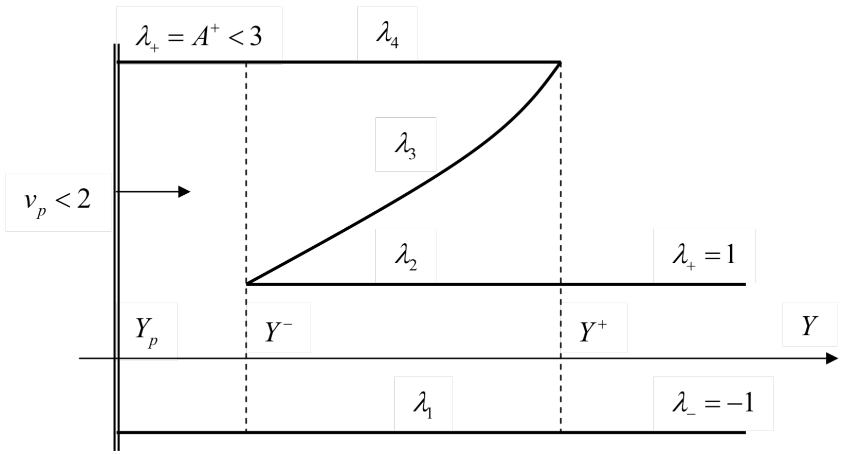
M=10, α=0.1

Piston 1D see M. Ablowitz et al, PRL 2008.

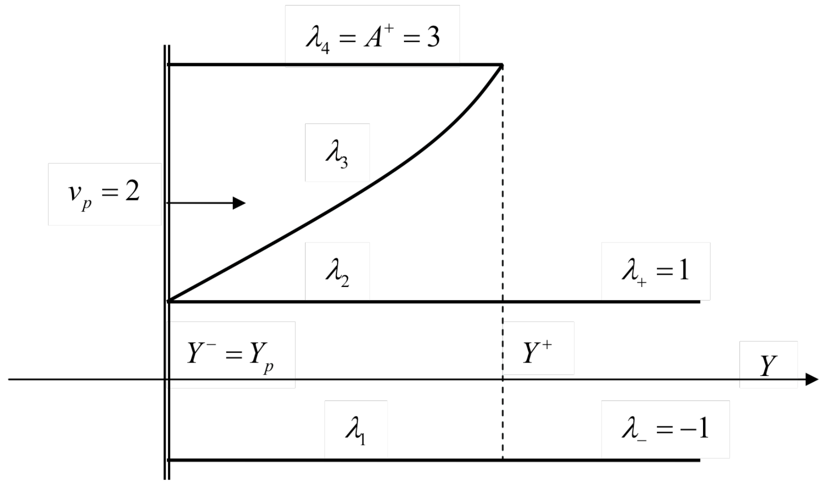


y
x
n
DSW
piston
(1D analytical)
piston
(1D numerical)
wedge
(2D numerical)
y⁻
y⁺
y

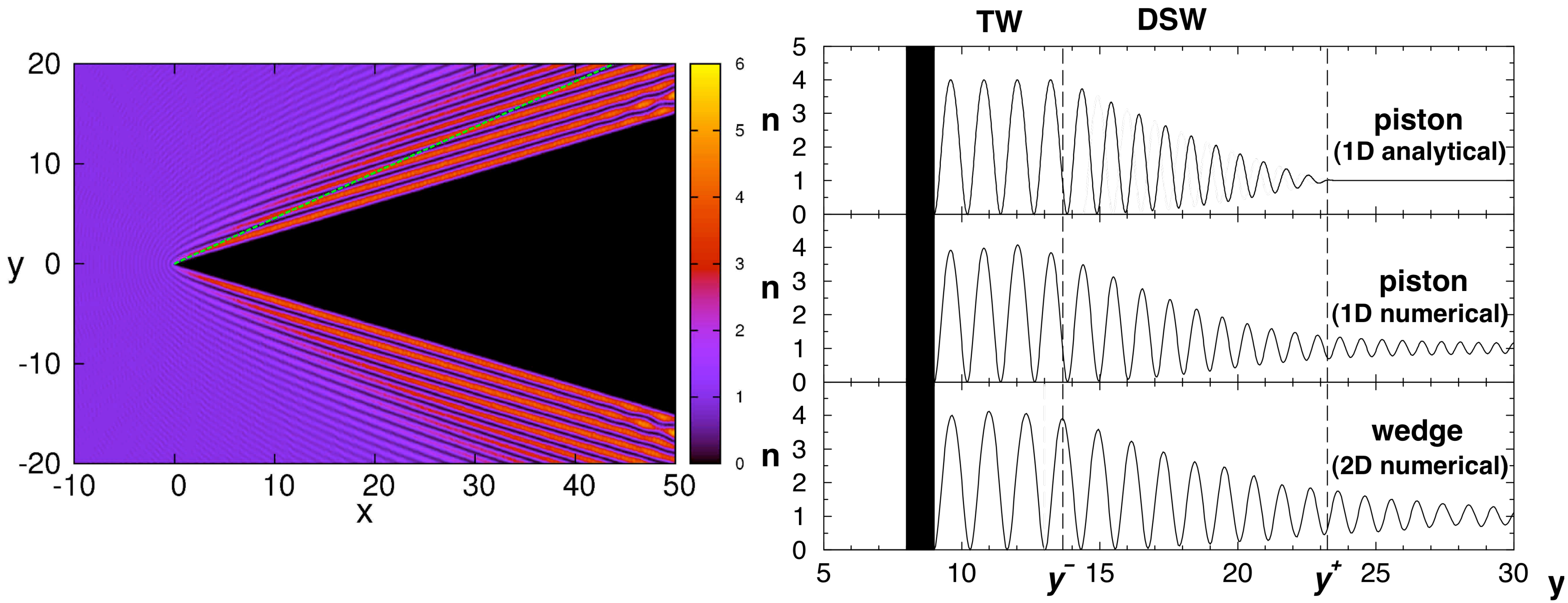
M=10, α=0.2



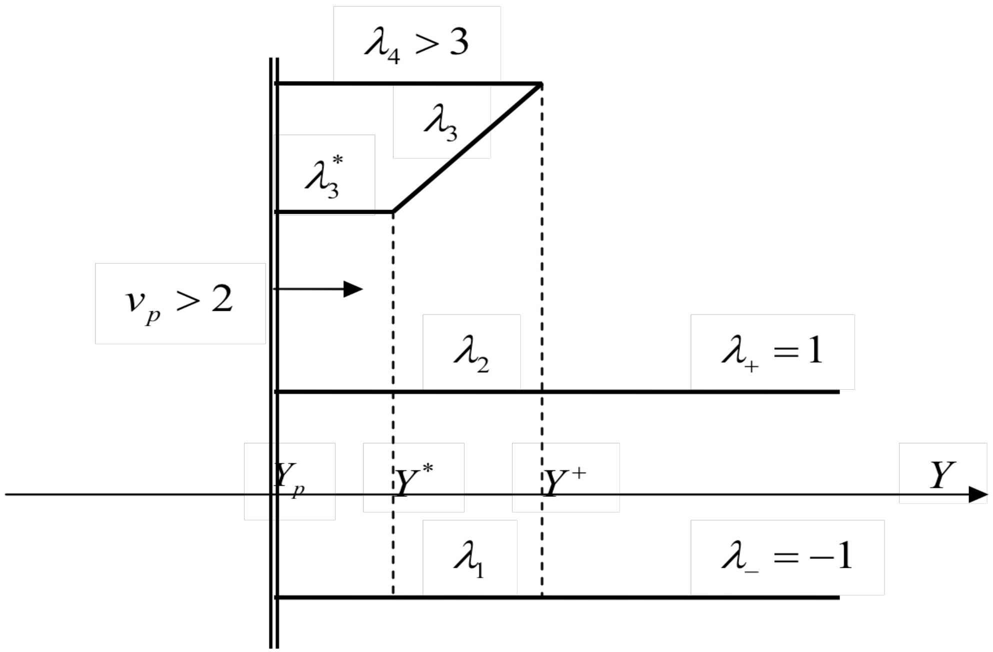
$\lambda_+ = A^+ < 3$
λ_4
λ_3
$v_p < 2$
λ_2
$\lambda_+ = 1$
Y_p
Y^-
Y^+
Y
λ_1
$\lambda_- = -1$



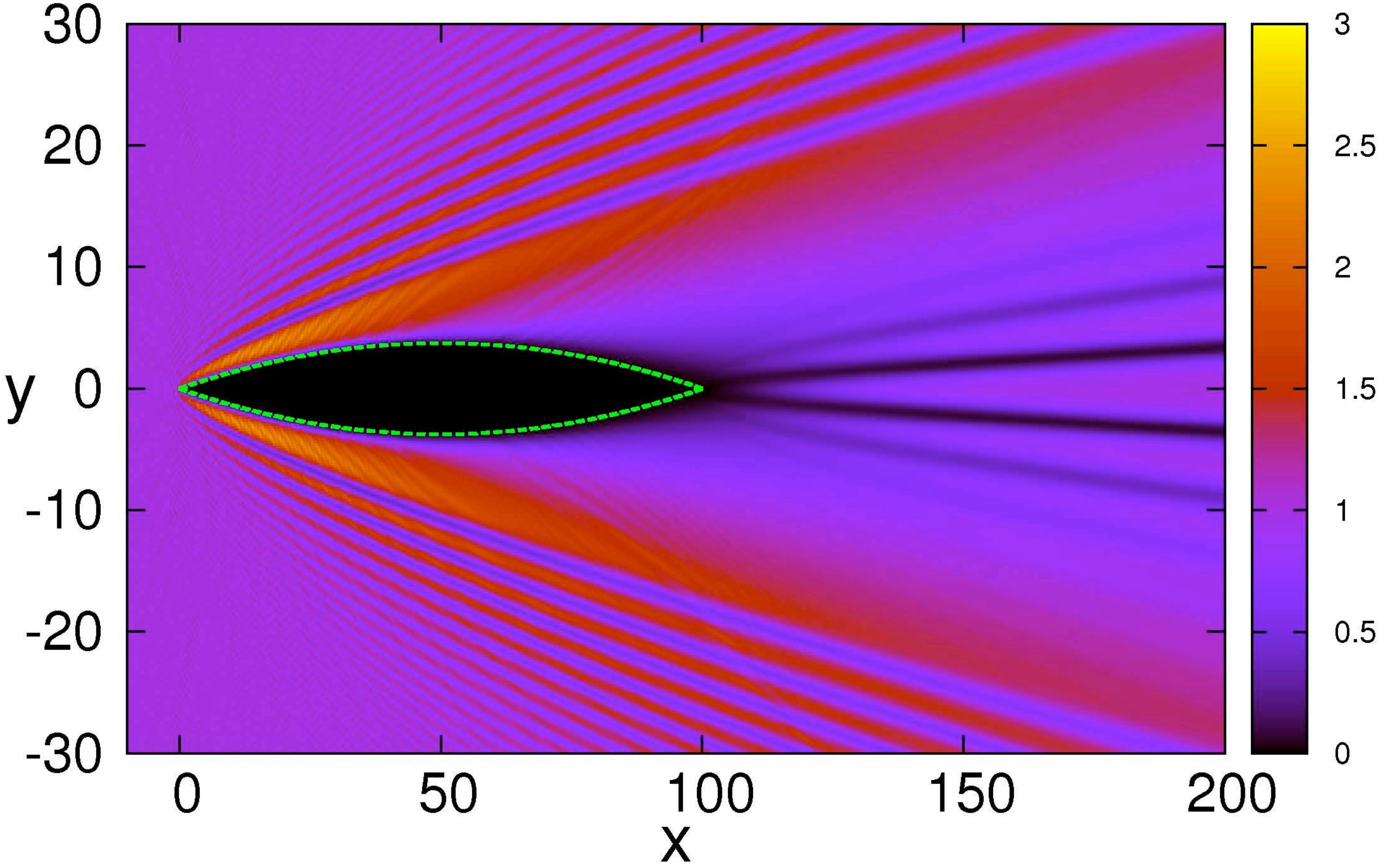
$\lambda_4 = A^+ = 3$
λ_3
$v_p = 2$
λ_2
$\lambda_+ = 1$
$Y^- = Y_p$
Y^+
Y
λ_1
$\lambda_- = -1$


20
10
y 0
-10
-20
-10
0
10
20
30
40
50
x
6
5
4
3
2
1
0
TW
DSW
n
piston
(1D analytical)
piston
(1D numerical)
wedge
(2D numerical)
5
10
y⁻ 15
20
y⁺ 25
30
y

M=10, α=0.3



$\lambda_4 > 3$
λ_3
λ_3^*
$v_p > 2$
λ_2
$\lambda_+ = 1$
Y_p
Y^*
Y^+
Y
λ_1
$\lambda_- = -1$



Extended wing

See arXiv:0906.2394

Conclusions

-Depending on corner aperture different patterns arise for supersonic flow past a corner.

-Problem can be viewed as a Gurevich-Pitaevskii problem and is tractable through Whitham modulation theory

-Remarkable agreement of theory and numerical simulations of 1D NLS stationary 2D NLS

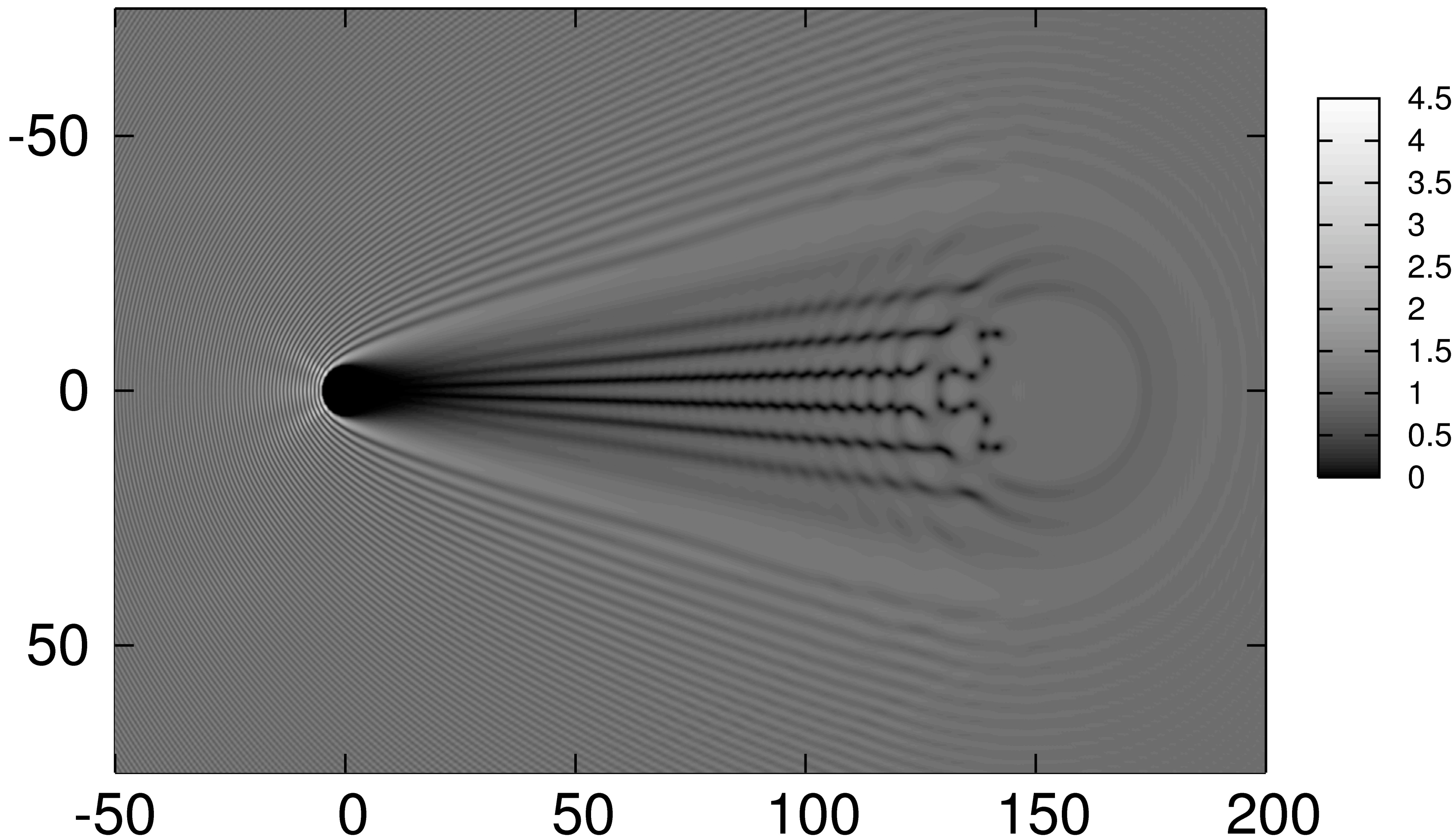
-Transition wave appearance for $\alpha M>2$.

-Results can also be applicable to more general forms of slender obstacles as a wing.


Thank You !

Increasing the radius -> more solitons!

M=5, r=5



Increasing radius generate more dark solitons!