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Dynamics of Bose-Einstein condensed atoms with attractive two-body interaction and three-body dissipation *

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The criterion proposed by Deissler and Kaneko [1] to diagnose spatiotemporal chaos in non-linear Schrödinger equation (NLSE), is applied to a model that describes the grow and collapse of Bose-Einstein condensed atoms. The time-dependent NLSE is intended to describe a system of trapped ultra-cold atoms with attractive two-body interaction. It includes two non-conservative terms: a linear one, corresponding to atomic feeding, and a quintic one, corresponding to three-body dissipation. Our dynamical analysis of the model presents signature of chaotic behavior for certain class of parameters. The chaotic behavior depends on the relative magnitude of the two non-conservative terms.

In ref. [2], the complex dynamics accompanying BEC of ^{7}Li atoms, was observed in the time evolution of the number of atoms in the condensate. It was verified the high sensibility of the numerical accuracy with the change of parameters: when a repulsive three-body interaction (quintic term in the NLSE) was considered, the numerical results were more stable for the condensate [2]. It was also verified that the numerical precision decreases very fast by increasing the modulus of the strength of an attractive three-body interaction. These preliminary results lead us to the suspicion of a possible chaotic behavior of the time-dependent NLSE with trapped atoms.

So, in our investigation, we use the criterion given in ref. [1] to investigate numerically the onset of chaotic behavior of the time-dependent NLSE considered in ref. [3]. The model is for a trapped gas with attractive two-body interaction. Two non-conservative terms are present to take into account the decrease of the density due to three-body recombination (parametrized by ξ), and the feeding process of the condensate from the nonequilibrium thermal cloud (parametrized by γ). The NLSE considered is the mean-field approximation to the quantum many-body problem of a dilute gas, in which the average interparticle distance is much larger than the absolute value of the scattering length. One should also observe that the validity of such approximation is limited to wave-lengths much larger than the average interparticle distance.

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In dimensionless units, as given in [2], the $s-$wave radial NLSE can be written as

$$i\frac{\partial\Phi}{d\tau} = \left[-\frac{d^2}{dx^2}+\frac{1}{4}x^2-\frac{|\Phi|^2}{x^2}-2i\xi\frac{|\Phi|^4}{x^4}+i\frac{\gamma}{2}\right]\Phi, \quad (1)$$

where x is related to the physical radius $\vec{r}$ by $x \equiv \sqrt{2m\omega/\hbar}|\vec{r}|$, $\tau \equiv \omega t$ is the dimensionless time variable, with ω the frequency of the harmonic trap interaction. ξ is the dissipation parameter, originated from three-body collisions, and γ is a parameter related to the feeding of atoms from the thermal cloud (ref. [3] gives an estimative for such parameters). $\Phi \equiv \Phi(x,\tau)$ is related to the physical wave-function $\Psi(\vec{r},t)$ by $\Phi(x,\tau) \equiv \sqrt{8\pi N(t)|a|}|\vec{r}|\Psi(\vec{r},t)$, where $N(t)$ is the number of atoms and a is the two-body scattering length (here, assumed to be negative). Using these definitions, $\Psi(\vec{r},t)$ is normalized to one and $\Phi(x,\tau)$ is normalized to the reduced number of atoms $n(\tau) \equiv 2N(t)|a|\sqrt{2m\omega/\hbar}$:

$$\int_0^\infty dx|\Phi(x,\tau)|^2 = n(\tau). \quad (2)$$

One particular interesting observable, to further analyze the dynamical behavior of eq. 1, is the mean-square-radius. We define this observable in dimensionless units by $X(\tau) \equiv \sqrt{\langle x^2(\tau)\rangle}$. In figure 1, we plot $dX/d\tau$ as a function of $X(\tau)$, for a set of values of the parameter γ (γ =0.01, 0.05, 0.1). In all cases, the wave-functions were evoluted up to $\omega t = 1000$ and the strength of the three-body dissipative interaction is kept fixed at $\xi = 0.001$. In figure 1, we observe that a complex dynamical structure starts to appear as the value of the parameter γ increases. For $\tau = 0$ (with $N = 0.75$ times the critical number of atoms for the collapse), $\sqrt{\langle x^2(\tau)\rangle}$ is close to 1.52 and $\frac{d}{d\tau}\sqrt{\langle x^2(\tau)\rangle}$ is zero, in all the cases. Initially, for $\gamma = 0.01$, the radius decreases to a center near 1.34 with zero derivative, then it starts to oscillate with larger radius, but keeping the center fixed. A similar behavior is found for $\gamma < 0.01$. For larger values of γ, the center of the oscillation in X grows up to the point it reaches an attractor at very large radius. In case of γ=0.1, for example, the plot clearly resembles a chaotic behavior with a strange attractor, which is around $X(\tau) \sim 23$. We have observed a fast transition in the pattern of the trajectory, when comparing the results obtained for γ=0.01 and γ=0.012, in a similar plot. This gives an indication of the existence of a critical range of values for the parameter γ for the transition from order to chaos. A clear experimental signature of the onset of chaos will be a non-continuous increasing of the radius up to very large values, compared to some typical ground-state value, as we found for $\gamma > 0.02$.

In order to characterize the dynamical behavior of the equation, we consider an useful criterion defined by Deissler and Kaneko [1] to diagnose spatiotemporal chaos in NLSE. The criterion relies on the determination of the time evolution of a function ζ, defined by the integral of the square modulus of the difference between wave-functions with nearby initial conditions:

$$\zeta(\tau) = \left(\int_0^L |\delta\Phi(x,\tau)|^2 dx\right)^{1/2}. \quad (3)$$

The average slope of this function, when plotted as a function of time, gives the largest Lyapunov exponent. In [1], it was studied the complex quintic Ginzburg-Landau equation and showed that, for an appropriate choice of the parameters the system could present a chaotic behavior. We consider the same function ζ, defined in eq. (3), to characterize the chaotic behavior of eq. (1), for several combinations of the parameters γ and ξ. In figure 2, we display the results for the time evolution of ζ, obtained for the same parameters γ shown in figure 1, with fixed ξ. The chaotic behavior, identified by the exponential increase of ζ, is clearly observed when $\gamma = 0.1$ and 0.05.

In conclusion, as shown in figures 1 and 2, the NLSE (1) used for the description of the dynamics of the Bose condensed wave-function in atomic traps with attractive interactions, for certain class of parameters (as, for example, the parameters considered in [3], ξ =0.001 and γ=0.1), presents a chaotic behavior. Such chaotic behavior starts to disappear as one decreases the feeding parameter γ or increases the three-body dissipative parameter ξ.

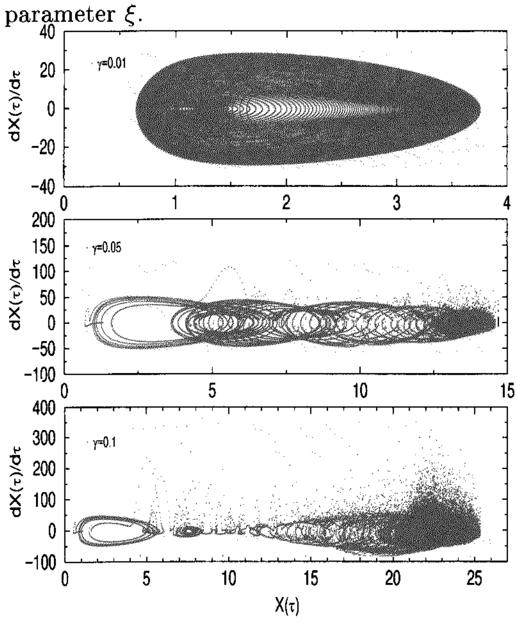

Figure 1. Phase-space plots for the mean-square-radius $X(\tau) \equiv \sqrt{\langle x^2(\tau)\rangle}$, in dimensionless units, for a set of values of the feeding parameter γ. In all the cases, the wave-functions were evolved until $\omega t = 1000$ and $\xi = 0.001$.

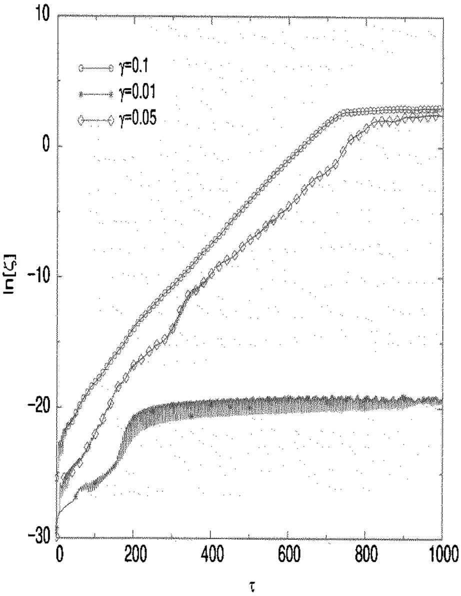

Figure 2. Logarithmic plot of the separation between two nearby states, as given by eq. (3), for several values of the parameter γ, shown in the figure. ξ is maintained fixed to 0.001.

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