Overview of the cascade calculations for π^0 photoproduction from complex nuclei.

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Main goal: The development of a Monte Carlo algorithm for the interpretation of the inelastic π^0 background embedded in the "main signal" of the PrimEx data.

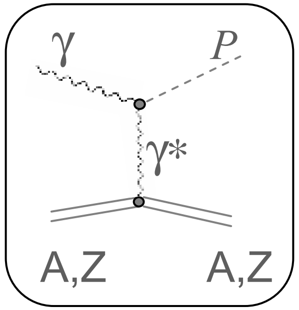
Outline:

- Introduction (photoproduction from complex nuclei)
- Summary of the Monte Carlo calculations
- The elementary meson photoproduction (π^0, η)
- π^0 – nucleus Final State Interactions
- Momentum distribution of the bound nucleons for ^{12}C
- Sensitivity of the NI cross section with the cascade inputs
- Normalization of the NI cross section and A_{eff} factor for ^{12}C
- π^0 photoproduction from ^{12}C and ^{208}Pb (PrimEx kinematics)
- Working example: $\eta \longrightarrow \gamma\gamma$ decay width revisited ($k \sim$ 9 GeV)
- Conclusions and final remarks

➢Introduction (photoproduction from complex nuclei)

Coulomb production (Primakoff effect)

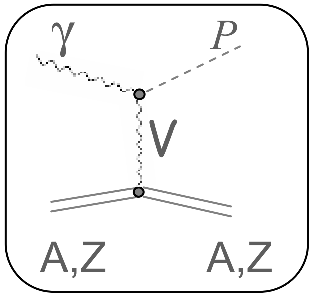
$$= \Gamma_{P\to\gamma\gamma}\frac{8\pi\alpha Z^2}{m_P^3}\frac{\beta^3 k^4}{Q^4}\left|\tilde{F}_{em}(Q)\right|^2 \sin^2\theta$$



$$\frac{d\sigma}{d\Omega} = \overbrace{\frac{d\sigma_P}{d\Omega}} + \underbrace{\frac{d\sigma_{NC}}{d\Omega}} + \underbrace{\frac{d\sigma_{NI}}{d\Omega}} + 2\sqrt{\frac{d\sigma_P}{d\Omega}\frac{d\sigma_{NC}}{d\Omega}}\cos\phi$$

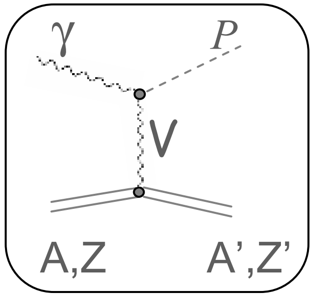
Nuclear Coherent production

$$= A^2 L^2 \left|\tilde{F}_N(Q)\right|^2 \sin^2\theta$$



Nuclear Incoherent production

$$= A_{eff}\, f(Q)\left(\frac{d\sigma_N}{d\Omega}\right)$$



T. E. Rodrigues *et al.*, Phys. Rev. C **71**, 051603 (R) (2005)
T. E. Rodrigues *et al.*, B.J.P. 36, 4B, 1366 (2006)

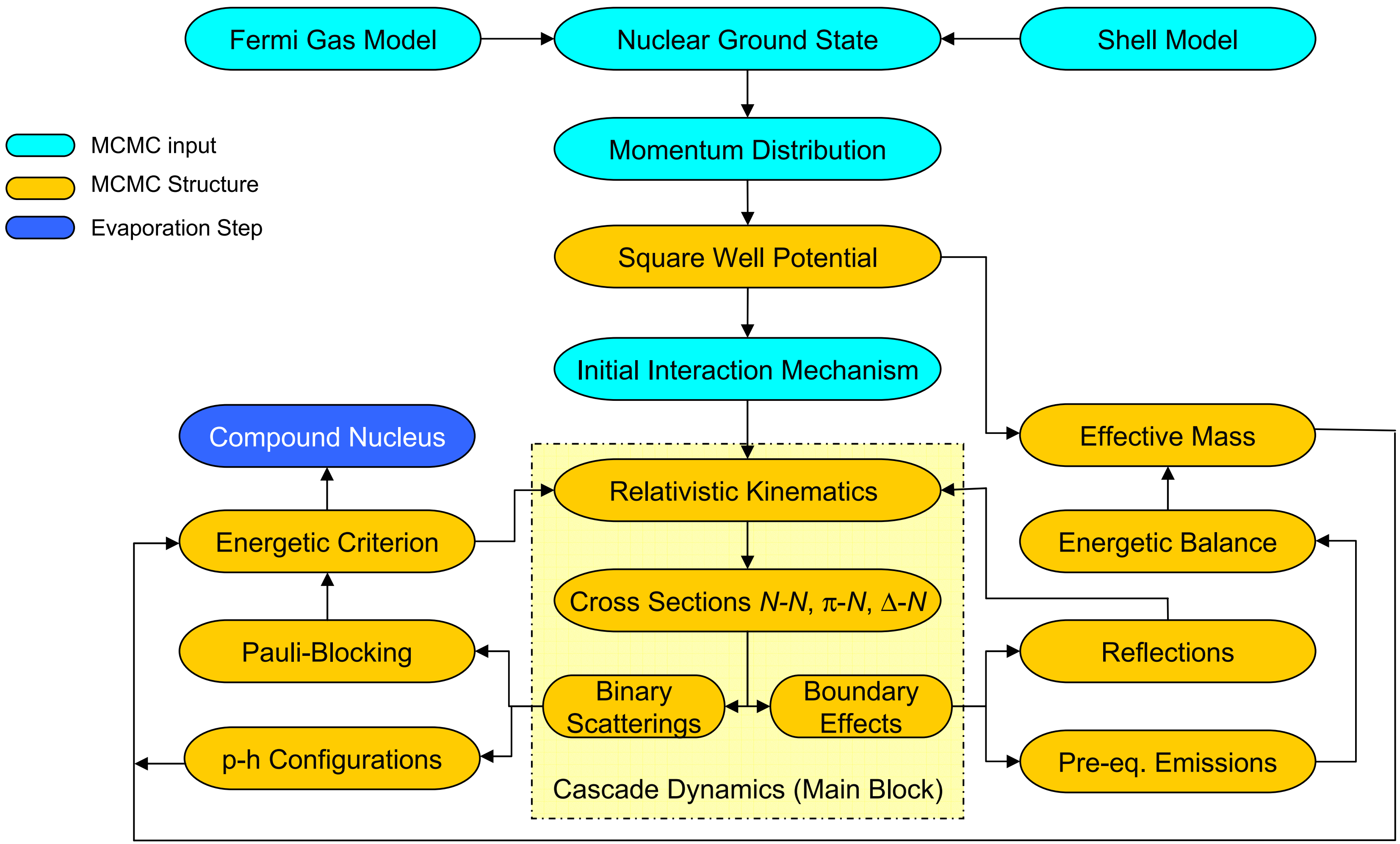
➢Summary of the Monte Carlo calculations

	$\frac{d\sigma_P}{d\Omega}$	$\frac{d\sigma_{NC}}{d\Omega}$	$\frac{d\sigma_{NI}}{d\Omega}$
relativistic recoil of the nucleus	✓	✓	
Electromagnetic and nuclear form factors	✓	✓	
shell-model/Fermi gas momentum distribution of the nucleons (1)		✓	✓
elementary operator (ρ, ω and b1 exchange - VMD) (1)			✓
Pauli-blocking mechanism (relativistic kinematics) for low Q^2 (1)			✓
FSI of the produced mesons via the MCMC model (1-3)		✓	✓
multiple meson-nucleon scatterings (1)		✓	✓

(1) T. E. Rodrigues *et al.*, Phys. Rev. C 71, 051603 (R) (2005)
(2) T. E. Rodrigues *et al.*, Phys. Rev. C 69, 064611 (2004)
(3) M. G. Gonçalves *et al.*, Phys. Lett. B 406, 1 (1997)

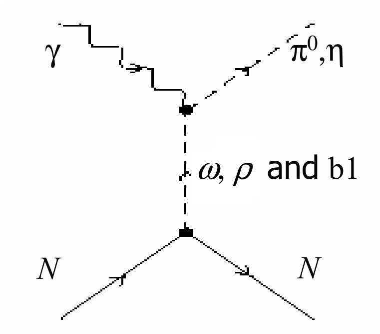
➢ Summary of the Monte Carlo calculations (NC/NI cross sections)

Overview of the MCMC plus Evaporation model (Flow Diagram)



➢The elementary meson photoproduction

Cross sections calculated in terms of *t*-channel helicity amplitudes (F_i)



$$\gamma(k)+N(p_1)\to P(p)+N(p_2)$$

$$s=(k+p_1)^2$$

$$t=(k-p)^2$$

$$\frac{d\sigma_N}{dt}\cong\frac{1}{32\pi}\Bigg\{F_2^2+\frac{F_3^2}{4m_N^2}-\left[t+\left(\frac{\mu^2}{2k}\right)^2\right]\left(F_4^2+\frac{F_1^2}{4m_N^2}+\frac{F_3^2}{16m_N^4}+\frac{F_1F_3}{2m_N p\sqrt{s}}\right)$$
$$-\frac{\left[t+\left(\frac{\mu^2}{2k}\right)^2\right]}{p\sqrt{s}}\left(F_2-\frac{F_3}{2m_N}\right)\left(\frac{F_1}{2}-\frac{\sqrt{s}}{24p}F_2+\frac{4p-5\sqrt{s}}{16m_N p}F_3\right)\Bigg\}\ (*)$$

(*) **A. Gasparian and S. Gevorkyan, *Theoretical part of PrimEx*, 2004**

➢ The elementary meson photoproduction

Helicity amplitudes calculated using a **Regge Model** with cuts (*)

natural parity (ω, ρ exchange)	F_1, F_2 : nucleon helicity non-flip
F_2, F_4 : unnatural parity (b1 exchange)	F_3, F_4 : nucleon helicity flip
$F_1 = \frac{\sqrt{2}}{m}\gamma_1\xi(t)\alpha(t)(\alpha(t)+1)(\alpha(t)+2)\left(\frac{s}{s_0}\right)^{\alpha(t)-1}$ $F_2 = t\eta_{b1}\alpha_{b1}(t)a_{b1}$ $F_3 = \frac{2\sqrt{2}}{m}t\gamma_3\xi(t)\alpha(t)(\alpha(t)+1)(\alpha(t)+2)\left(\frac{s}{s_0}\right)^{\alpha(t)-1}$ $F_1^{cut} = \frac{\sqrt{2}}{m}\gamma_1^{cut}\xi(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1}$ F_1, F_3 : $\frac{e^{at}}{\ln\frac{s}{s_0}}$ $F_3^{cut} = 2\sqrt{2}\gamma_3^{cut}\xi(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1}\frac{e^{at}}{\ln\frac{s}{s_0}}$ $F_1^C = -2\frac{m}{t}0.0543\Gamma_{\gamma\gamma}(eV)\lvert F(q)\rvert^2$	$s = 2mk + m^2$ $u = 2m^2 + 2\mu^2 - t - s$ $k_{cm} = \sqrt{\frac{m^4 + s^2 - 2m^2 s}{4s - 2m^2}}$ $\Delta_{cm} = \frac{\mu^2}{2k_{cm}}$, $p_{cm} = \sqrt{\frac{s + m^2 - \mu^2}{2\sqrt{s}} - m^2}$ $\xi(t) = \frac{1 - e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}$ $\eta_{b1}(t) = (\alpha_{b1}(t)+1)(\alpha_{b1}(t)+2)(\alpha_{b1}(t)+3)\times$ $\xi_{b1}(t)\left(\frac{s-u}{2s_0}\right)^{\alpha_{b1}(t)-1}$

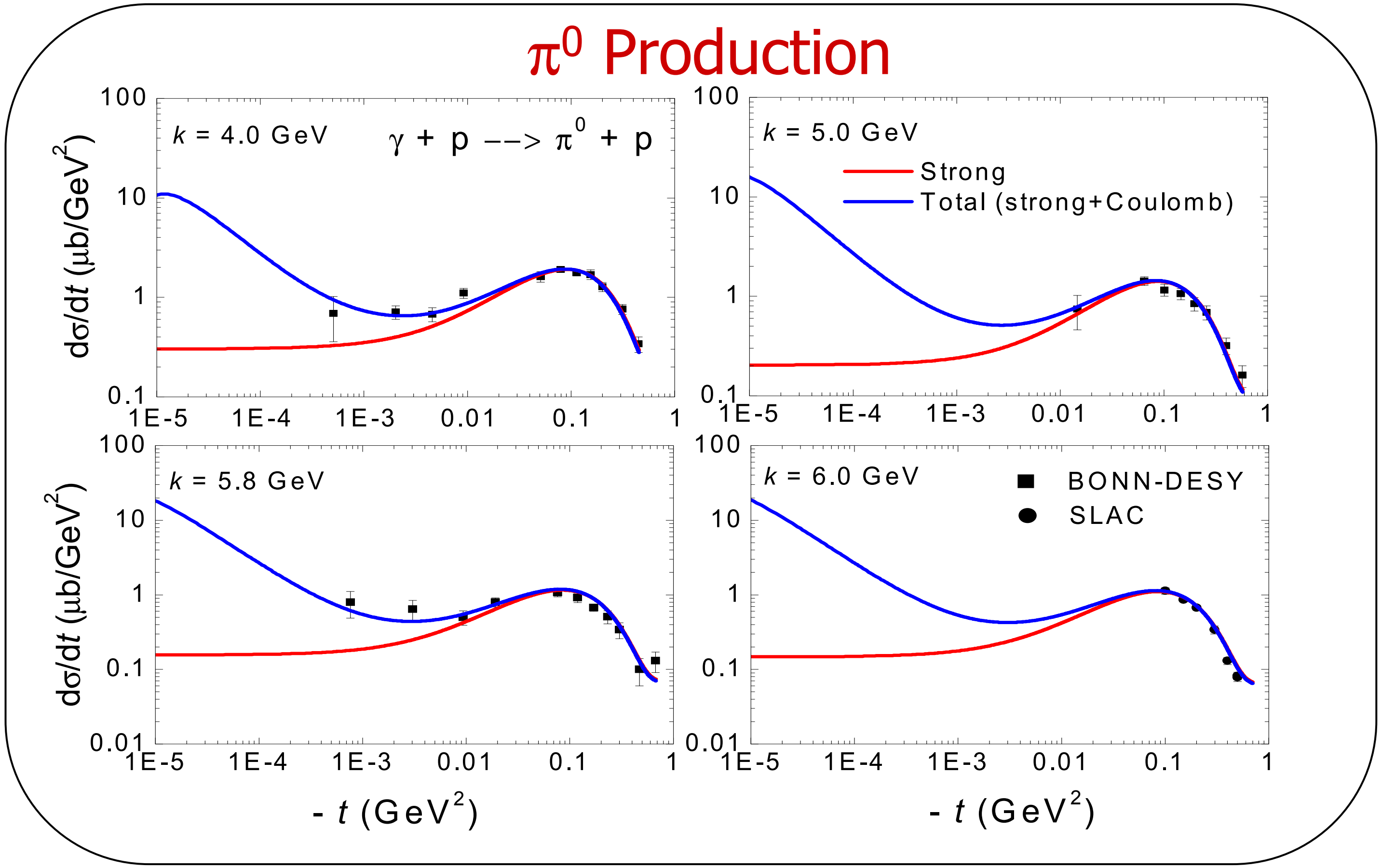
(*) M. Braunschweig *et al.*, Nucl. Phys. B 20, (1970) 191.

➢The elementary meson photoproduction

Fitting parameters and approximations	
π^0 photoproduction	η photoproduction
$F_1 \rightarrow F_1^{\rho} + F_1^{\omega} + F_1^{cut} + F_1^{C}$ $F_2, F_4 \rightarrow 0$ $F_3 \rightarrow F_3^{cut}$ $\alpha_{\rho,\omega}(t) = 0.45 + 0.9t$ $s_0 = 1.0\,\mathrm{GeV}^2$ $\gamma_1^{\rho} + \gamma_1^{\omega} = 115.4\sqrt{\mu b}, \gamma_1^{cut} = 55.2\sqrt{\mu b}$ $\gamma_3^{cut} = 13.5\frac{\sqrt{\mu b}}{GeV}$ $a = 1.20\,\mathrm{GeV}^{-2}$	$F_1 \rightarrow F_1^{\rho} + F_1^{\omega} + F_1^{cut} + F_1^{C}$ $F_3 \rightarrow F_3^{\rho} + F_3^{\omega} + F_3^{cut} + 2mF_2^{b1}$ $F_4 \rightarrow 0$ $\alpha_{\rho,\omega}(t) = 0.39 + 1.0t, \alpha_{b1}(t) = -0.342 + 0.9t$ $s_0 = 1.0\,\mathrm{GeV}^2$ $\gamma_1^{\rho} + \gamma_1^{\omega} = -42.09\sqrt{\mu b}, \gamma_1^{cut} = -149.06\sqrt{\mu b}$ $\gamma_3^{\rho} + \gamma_3^{\omega} = 23.89\frac{\sqrt{\mu b}}{GeV}, \gamma_3^{cut} = 7.595\frac{\sqrt{\mu b}}{GeV}$ $a = 1.976\,\mathrm{GeV}^{-2}, a_{b1} = 69.05\frac{\sqrt{\mu b}}{GeV}$

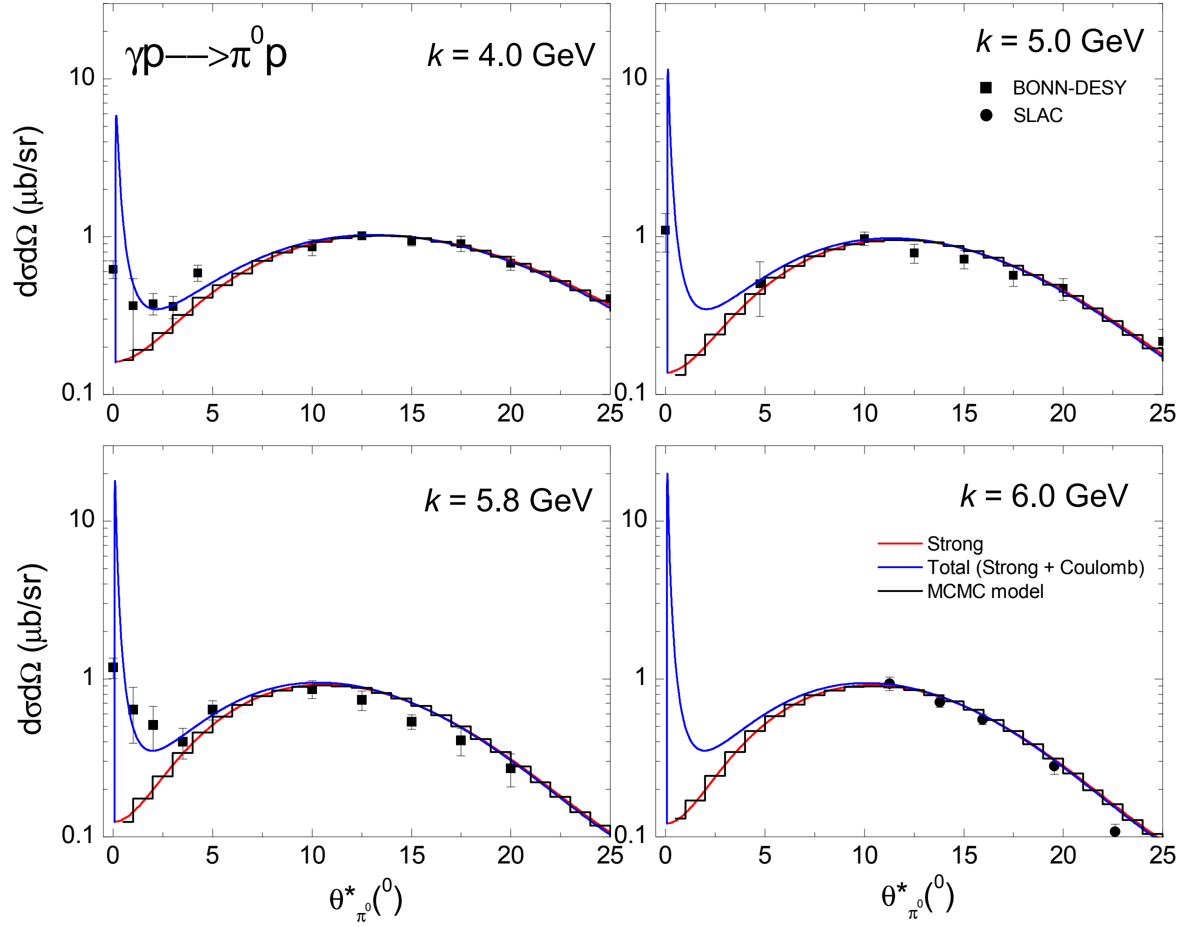
➢ The elementary meson photoproduction

Checking the Regge Model with the best fit parameters



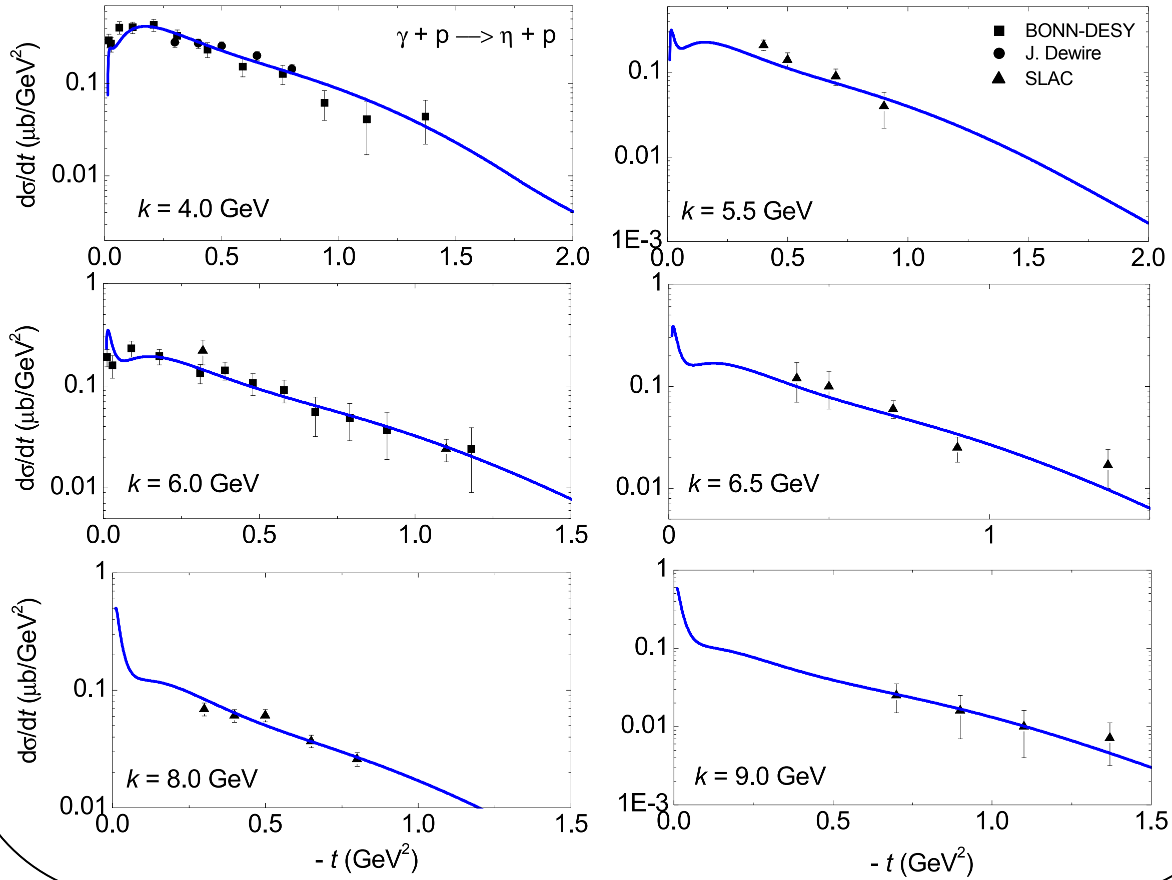
➢ The elementary π^0 photoproduction as a function of $\theta^*_{\pi 0}$

The cascade input for the NI cross section includes only the strong part (red line)



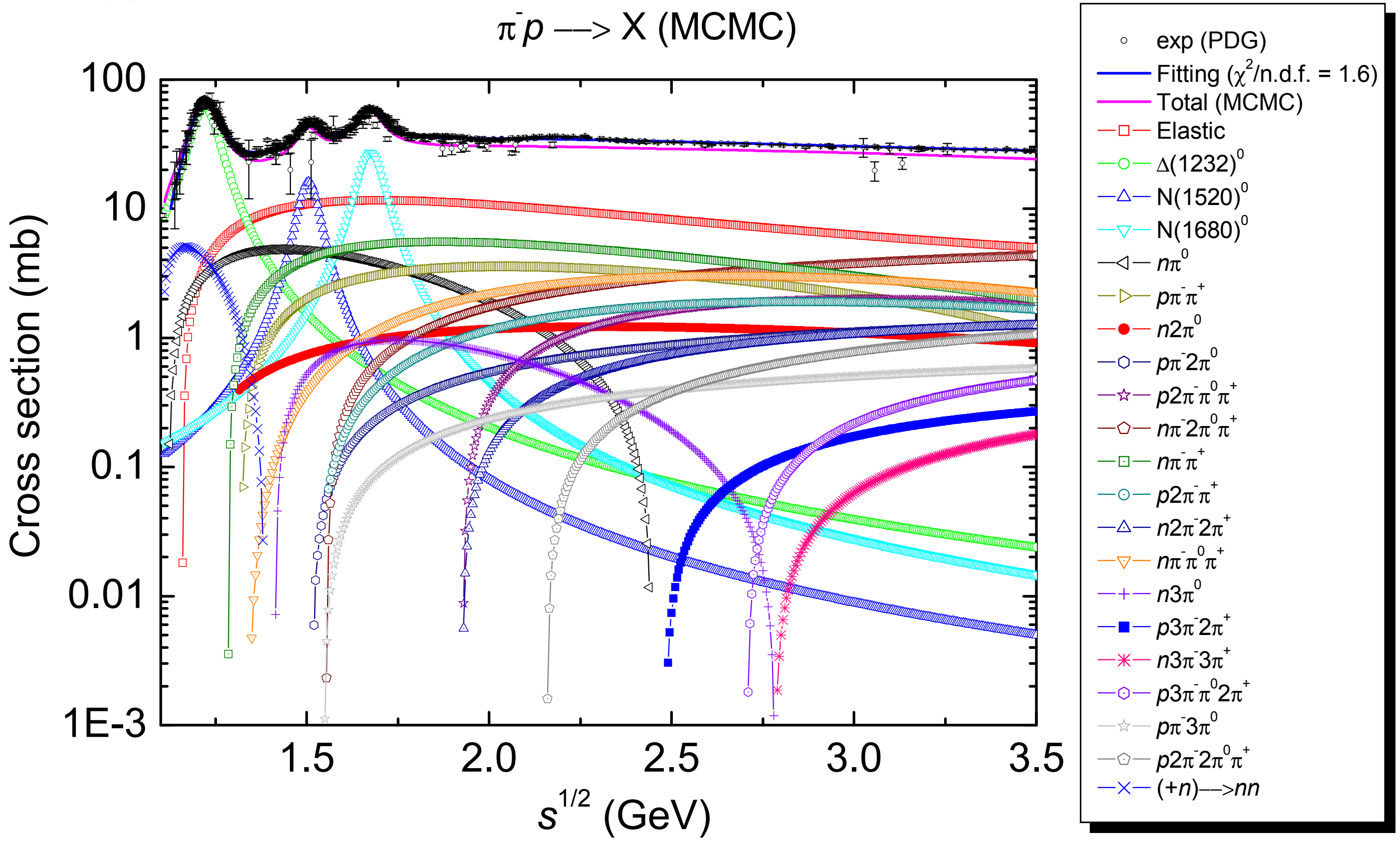
➢The elementary meson photoproduction

η Production



➢π^0-nucleus Final State Interactions

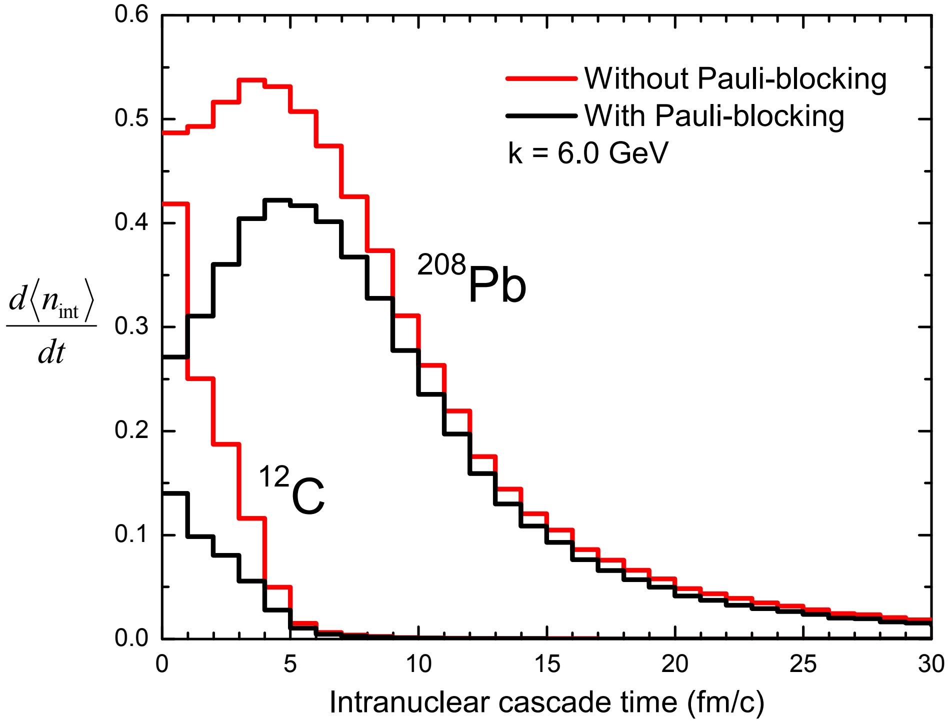
Typical πN channels included in the MCMC cascade (*)



(*) **T. E. Rodrigues *et al.*, Phys. Rev. C 71, 051603 (R) (2005).**

➢π^0-nucleus Final State Interactions

Time derivative of the average number of πN interactions



Average number of πN binary scatterings for a typical cascade at 6 GeV:

$$\langle n_{\mathrm{int}}\rangle_{^{208}Pb} = \int_t \frac{d\langle n_{\mathrm{int}}\rangle_{^{208}Pb}}{dt} dt \approx 5.5$$

$$\langle n_{\mathrm{int}}\rangle_{^{12}C} = \int_t \frac{d\langle n_{\mathrm{int}}\rangle_{^{12}C}}{dt} dt \approx 0.4$$

➢Momentum distribution of the bound nucleons for ^{12}C:

General ideas:

1) ^{12}C is a very well studied nuclei and we must include realistic parameterizations in order to make the analysis of the incoherent production mechanism less model dependent.
2) The PrimEx experiment consists of the ultimate challenge for the available models as it contains the most accurate data obtained at forward angles so far.
3) Kinematics is a crucial step in this analysis and even for low momentum transfer it would be useful to include relativistic kinematics, which is considered in the present Monte Carlo cascade.
4) The proton knock-out data from ^{12}C - obtained via quasi-elastic electron scattering - allows an empirical verification of the spectral function and, consequently, represents the most convenient constraint to describe the ^{12}C nuclear structure.

➢Momentum distribution of the bound nucleons for ^{12}C:

The following table summarizes the 1*p* and 1*s* proton knockout data used in this analysis

	Dataset	ΔE_m (MeV)	T_p (MeV)	S_{1p}/S_{1s}	Kinematics	Q^2 (GeV/c)^2
1*p*-knockout	Saclay 76 [1]	15-22	87	2.19(13)	perpendicular	0.16
	Saclay 81 [2]	15-22	99	2.28(7)	parallel	0.09-0.32
	Saclay 81 [2]	15-22	99	2.31(6)	perpendicular	0.09-0.32
	Tokyo 76 [3]	6-30	159	2.16(10)	perpendicular	0.29
	NIKHEF 88 [4]	G.S.	70	2.20(4)	parallel	0.02-0.26
1*s*-knockout	Saclay 76 [1]	30-50	87	0.84(2)	perpendicular	0.16
	Tokyo 76 [3]	21-66	136	1.19(5)	perpendicular	0.29
	NIKHEF 88 [4]	30-39	70	0.047(2)	parallel	0.02-0.26

[1]: NPA 262, 461 (1976)
[2]: NPA 375, 381 (1982)
[3]: NPA 268, 381 (1976)
[4]: NPA 480, 547 (1988)

➢Momentum distribution of the bound nucleons for ^{12}C:

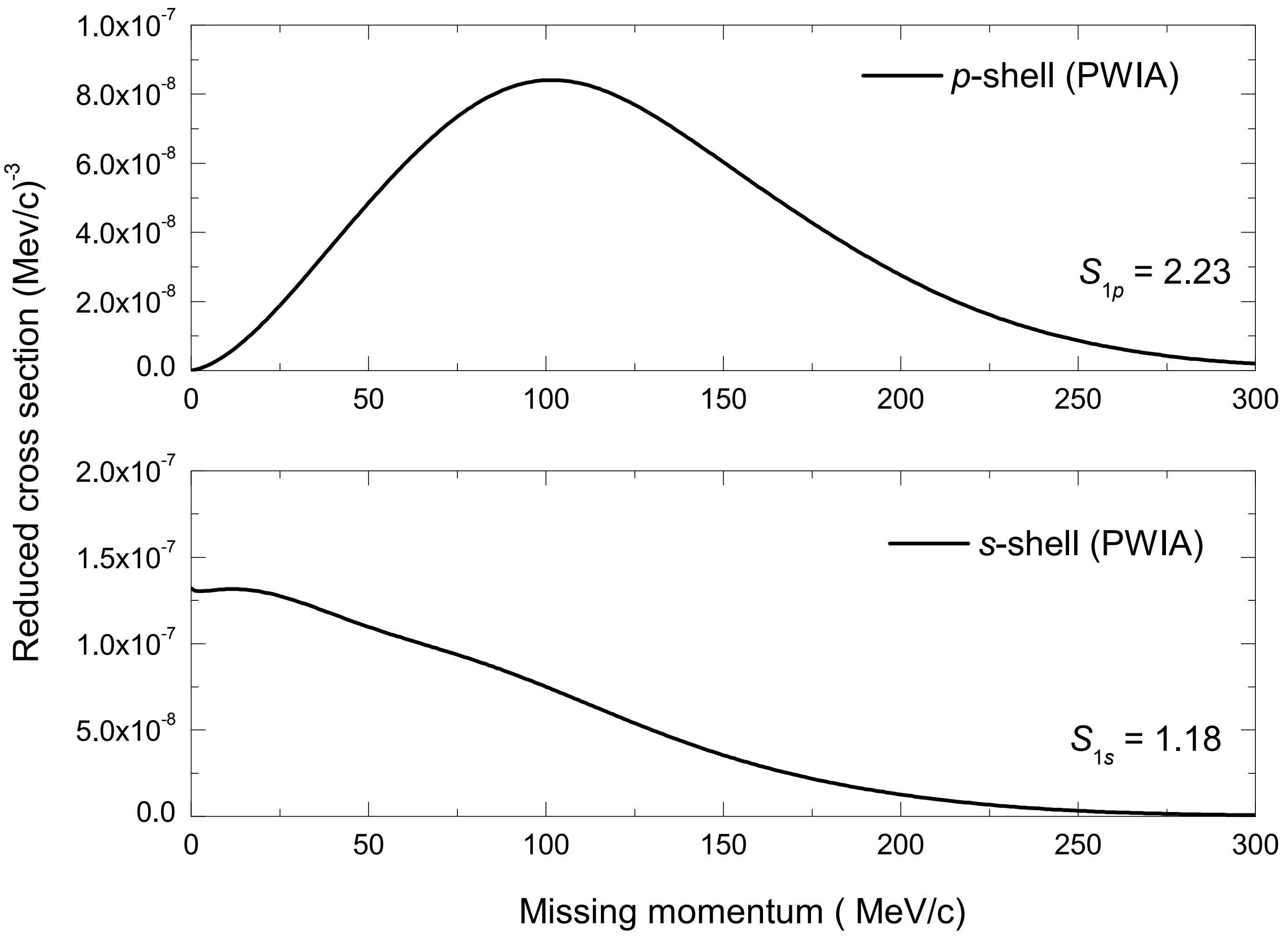
General strategy:

1) The reduced cross section (spectral function integrated in missing energies) represents the initial momentum distribution of the nucleons taking into account the FSI of the emitted proton and electron on their way out of the nucleus.
2) For the purposes of the PrimEx analysis, we have to consider the true momentum distribution (PWIA), since the distorted distributions (real data) depend upon the range of momentum transfer and also to the reaction mechanisms. The quasi-particle concept seems to depend on the momentum transfer and for this reason I have used only the data obtained for low Q^2 in order to be consistent with the PrimEx kinematics. See ref.[5] for a complete review about momentum distribution and nuclear transparency for ^{12}C. (I also refer you to fig. 8 from ref.[5])

[5]: PRC 61, 04325 (2000)

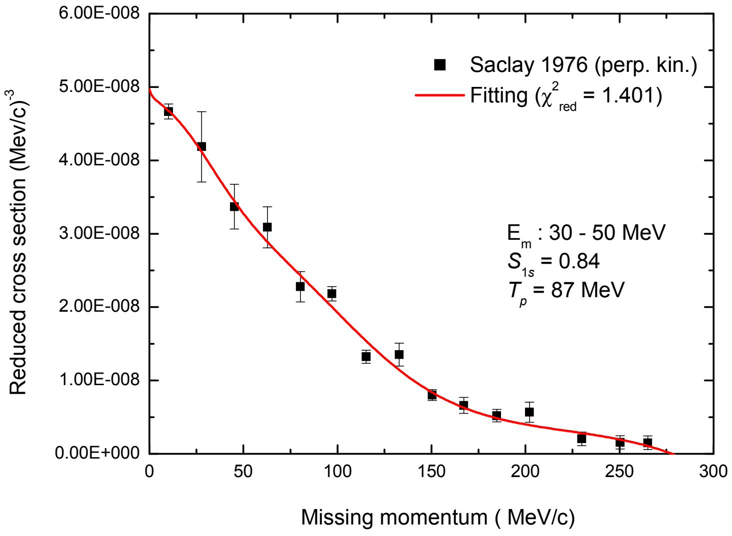
➢Momentum distribution of the bound nucleons for ^{12}C:

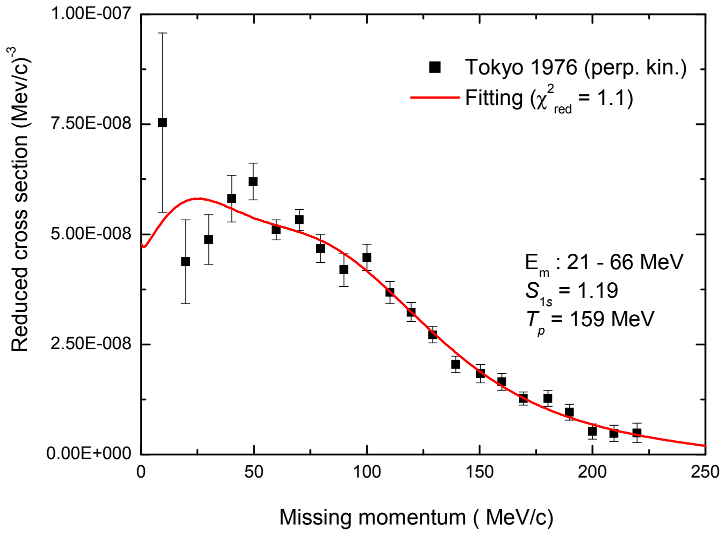
Plane Wave Impulse Approximation (true momentum distribution – ref.[5])

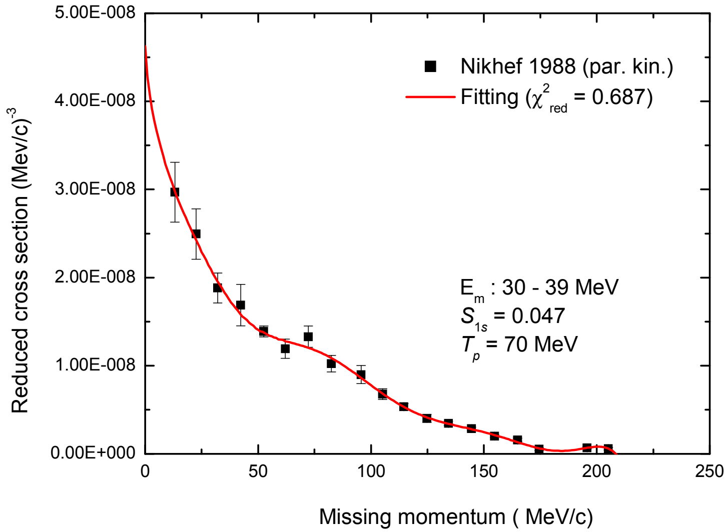


➢Momentum distribution of the bound nucleons for ^{12}C:

1s-knockout data used in this analysis

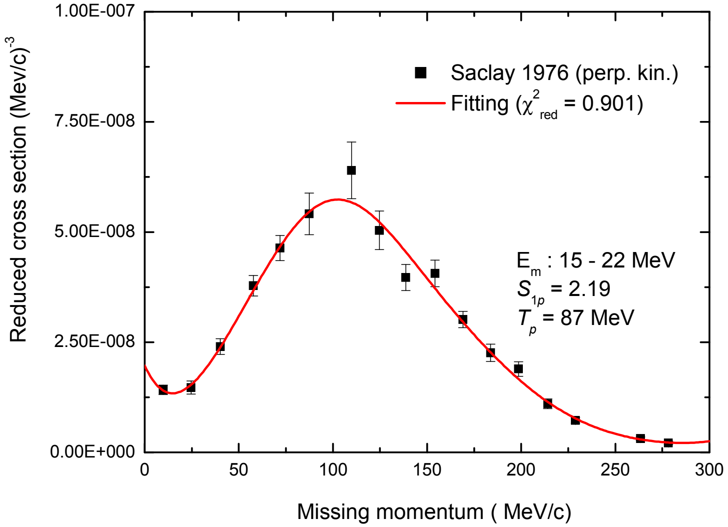


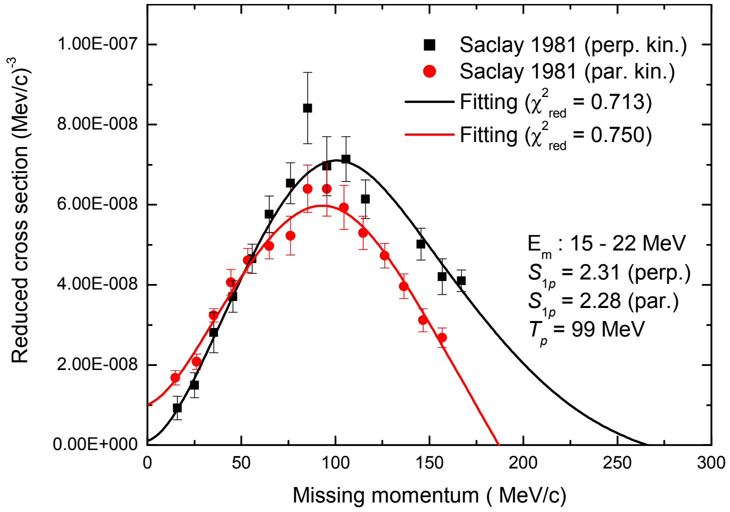


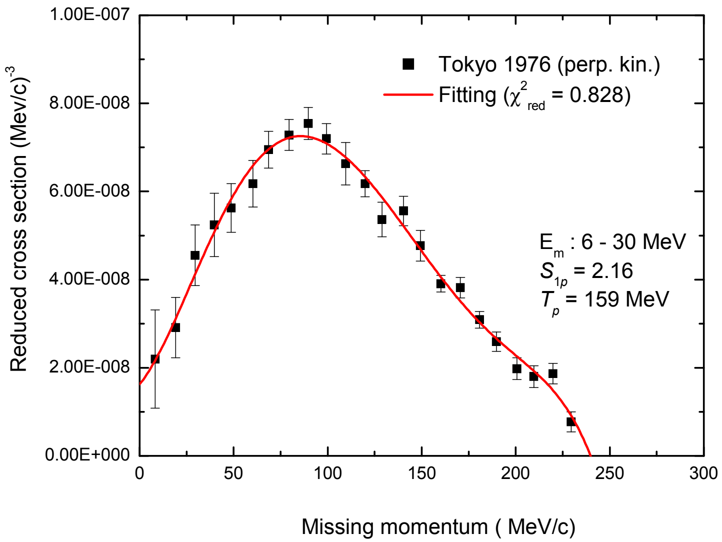


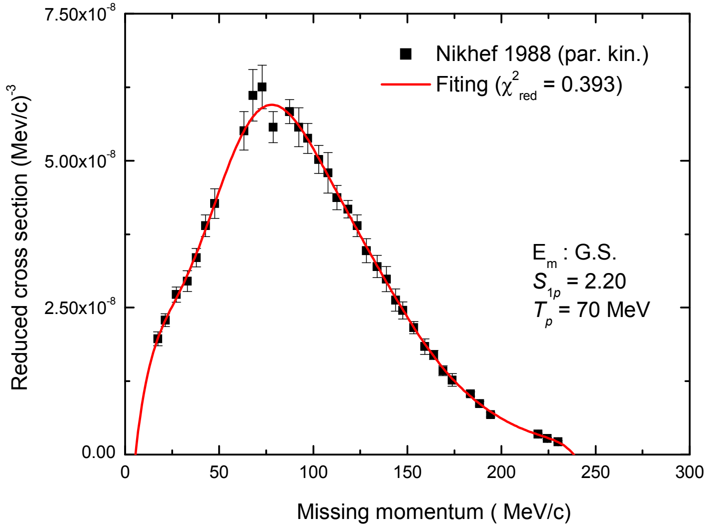
➢Momentum distribution of the bound nucleons for ^{12}C:

1*p*-knockout data used in this analysis









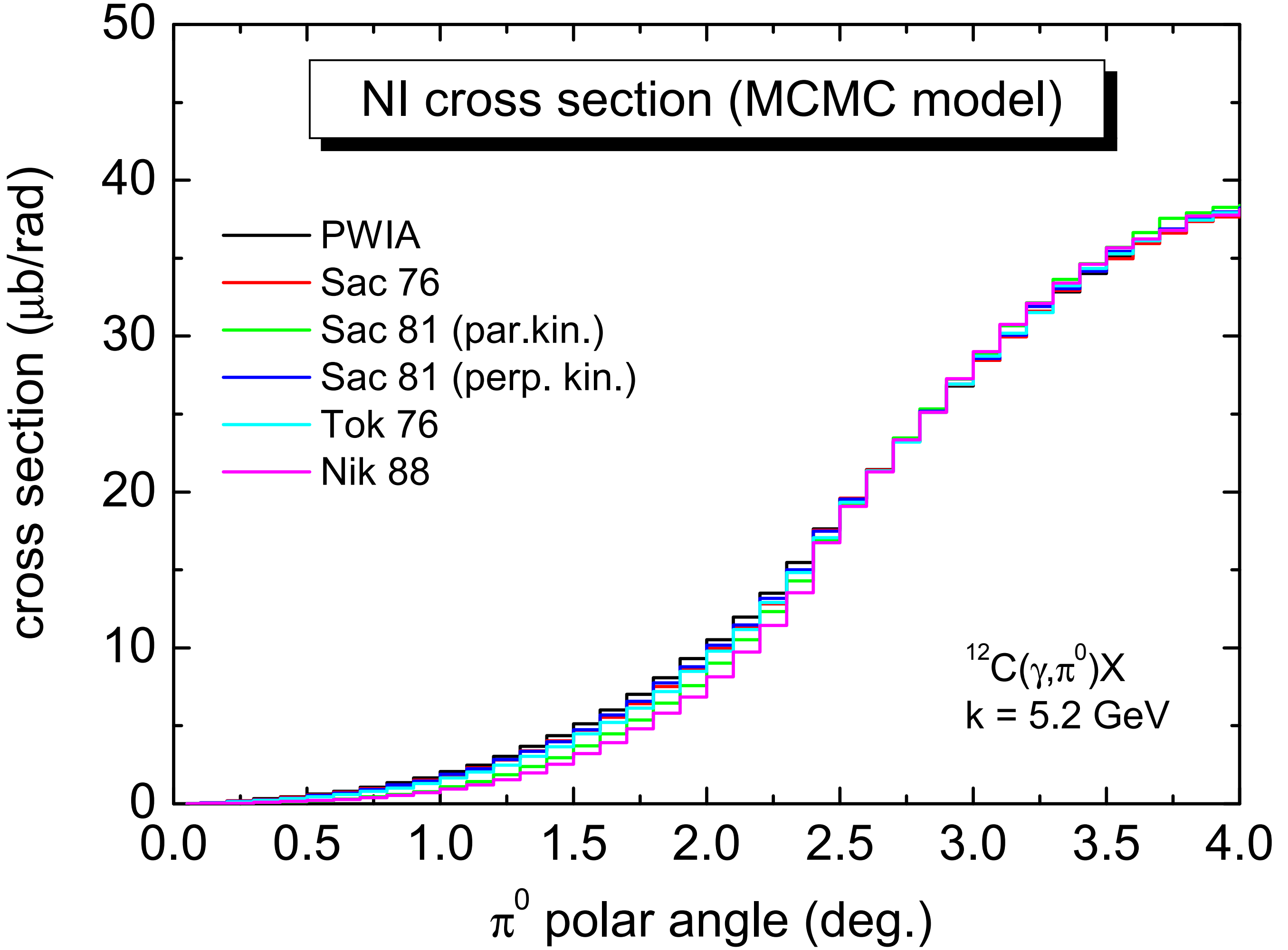
➢Momentum distribution of the bound nucleons for ^{12}C:

1*s* and 1*p*-proton knockout parameterizations used for the Momentum Distribution

	s-shell	*p*-shell	cascade run
Parameterization	PWIA	PWIA	1 (reference run)
	Saclay 76 [1]	Saclay 76 [1]	2
	Saclay 76 [1]	Saclay 81 [2] (perp. kin.)	3
	Saclay 76 [1]	Saclay 81 [2] (par. kin.)	4
	Tokyo 76 [3]	Tokyo 76 [3]	5
	NIKHEF 88 [4]	NIKHEF 88 [4]	6

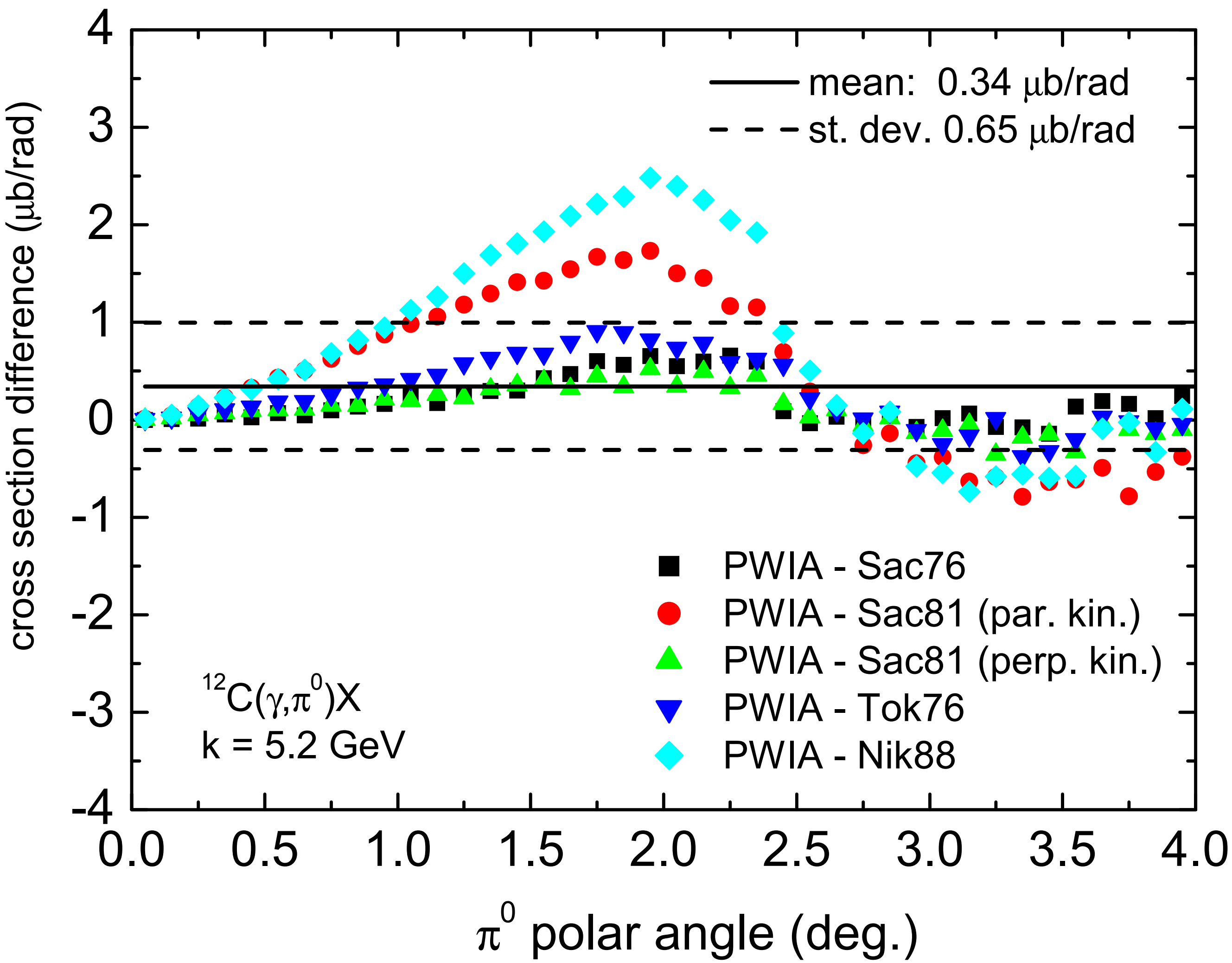
➢Sensitivity of the NI cross section with cascade inputs

Ni cross section versus momentum distribution



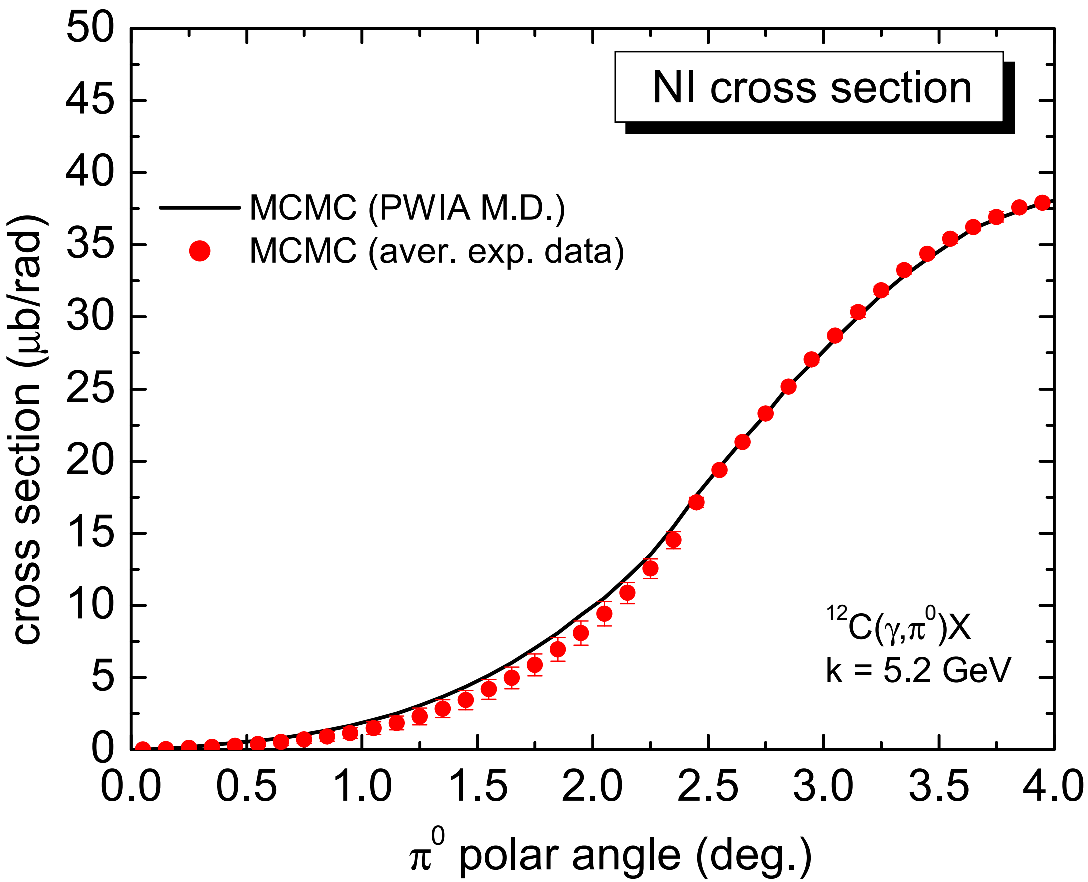
➢Sensitivity of the NI cross section with cascade inputs

Absolute cross section difference versus momentum distribution



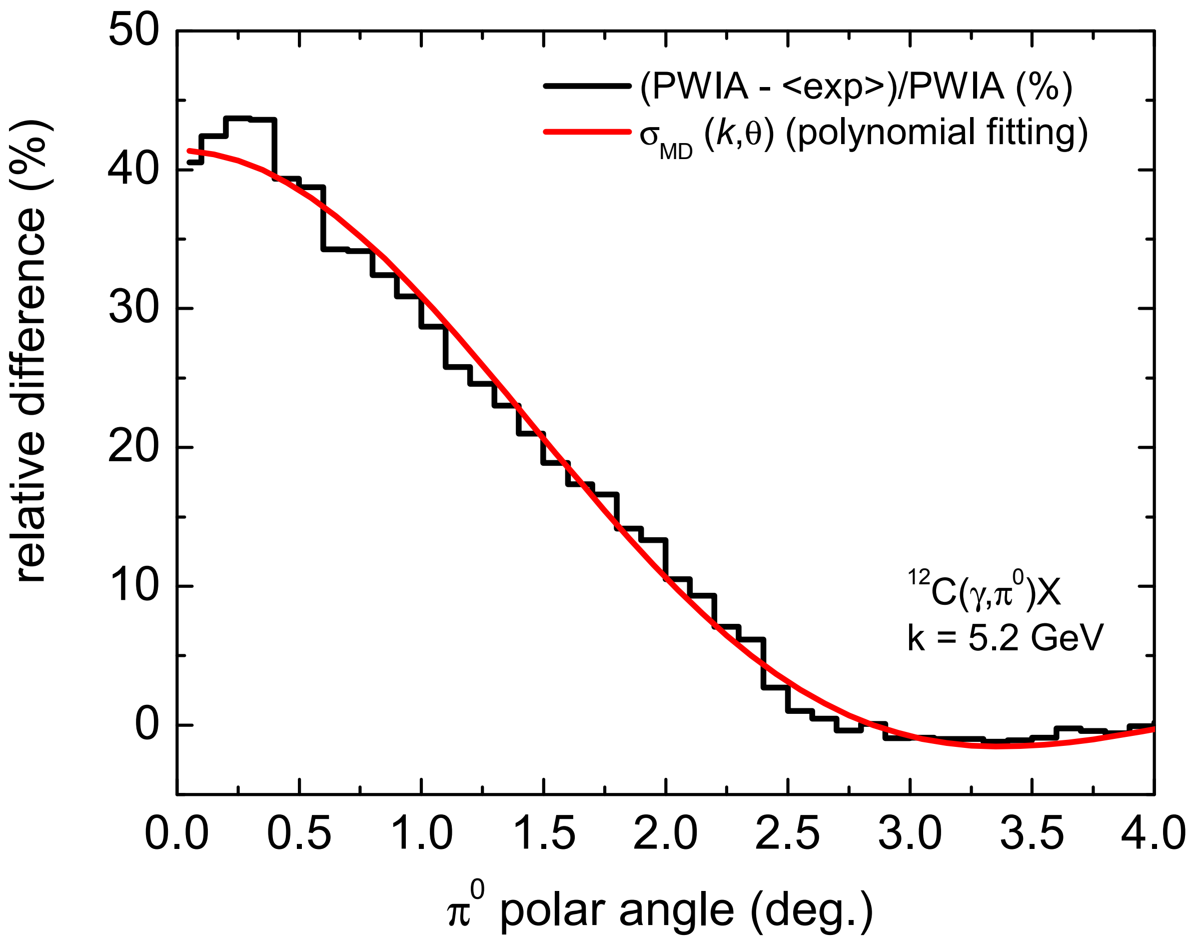
➢Sensitivity of the NI cross section with cascade inputs

Estimating the systematic error associated with the PWIA: comparing the PWIA result (ref. run) with the average of the results obtained with the available experimental data (run 2 to 6)



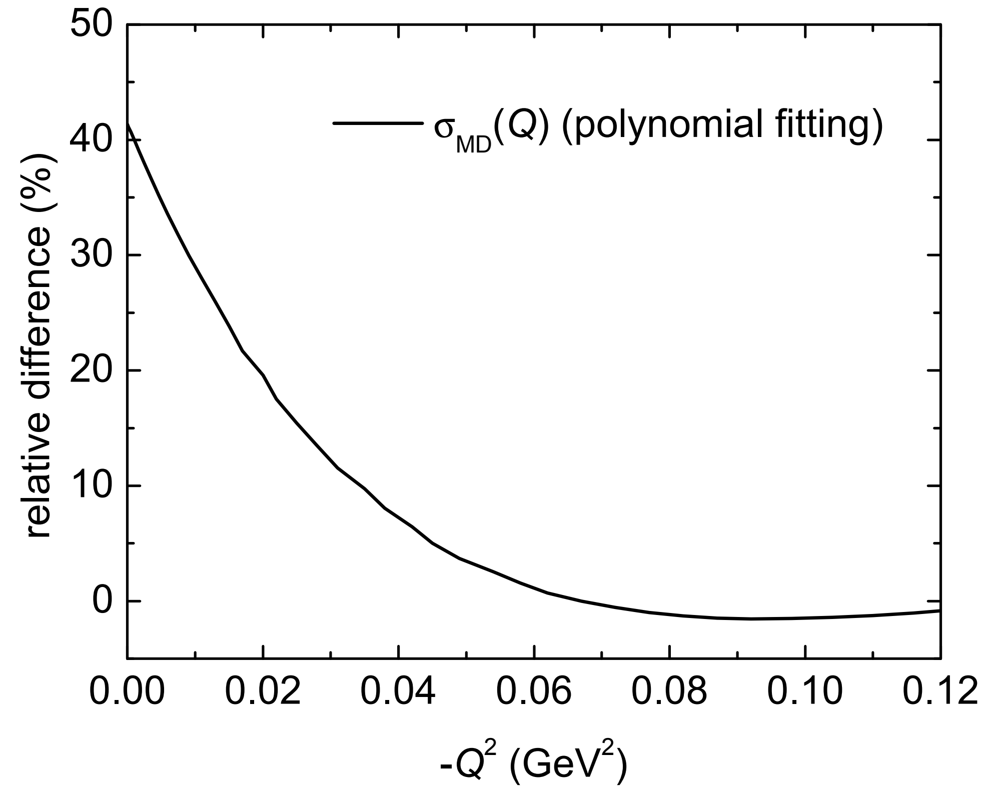
➢Sensitivity of the NI cross section with cascade inputs

Extracting the relative difference (%) associated with the MD: $\sigma_{MD}(k,\theta)$



➢Sensitivity of the NI cross section with cascade inputs

Extracting the relative difference (%) associated with the MD as a function of Q^2

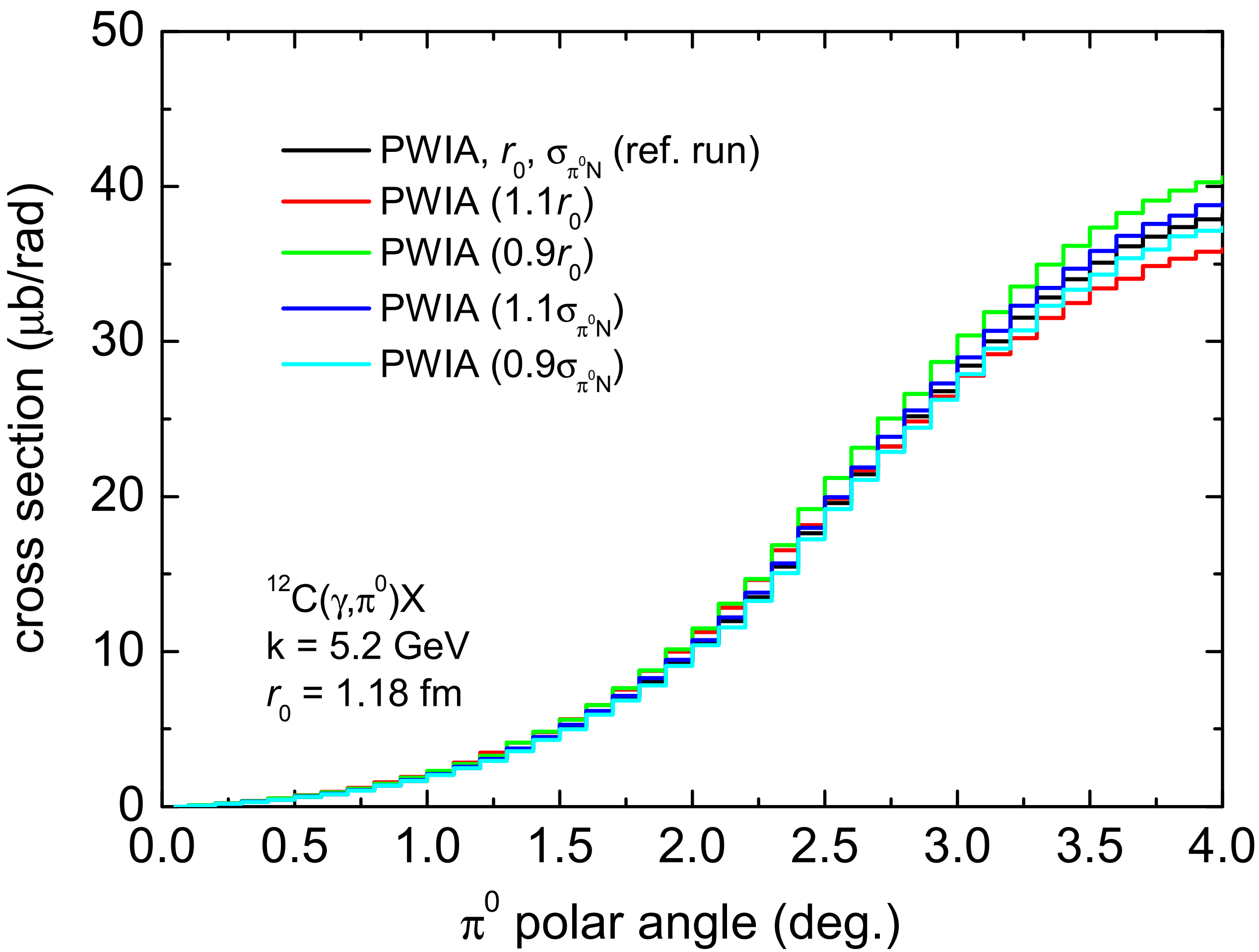


$$\sigma_{MD}(Q) = \sum_{i=0}^{8} a_i |Q|^i$$

$$a_i = \begin{pmatrix} 41.391307 \\ -1.414722 \times 10^3 \\ 1.797025 \times 10^4 \\ -1.152304 \times 10^5 \\ 4.217095 \times 10^5 \\ -9.145046 \times 10^5 \\ 1.160193 \times 10^6 \\ -7.946722 \times 10^5 \\ 2.267359 \times 10^5 \end{pmatrix}.$$

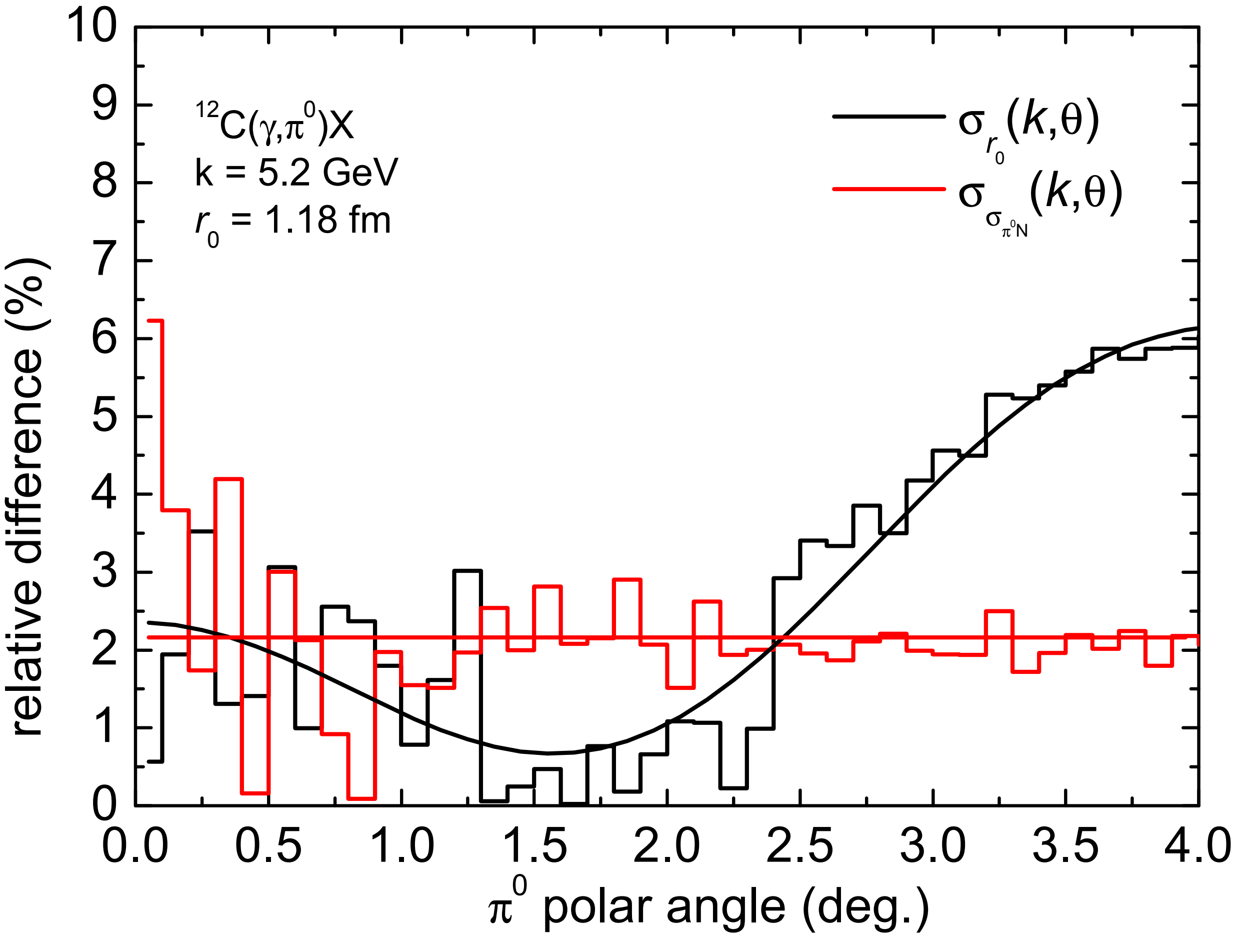
➢Sensitivity of the NI cross section with cascade inputs

NI cross section versus other cascade parameters (r_0 and total $\pi^0 N$ cross section)



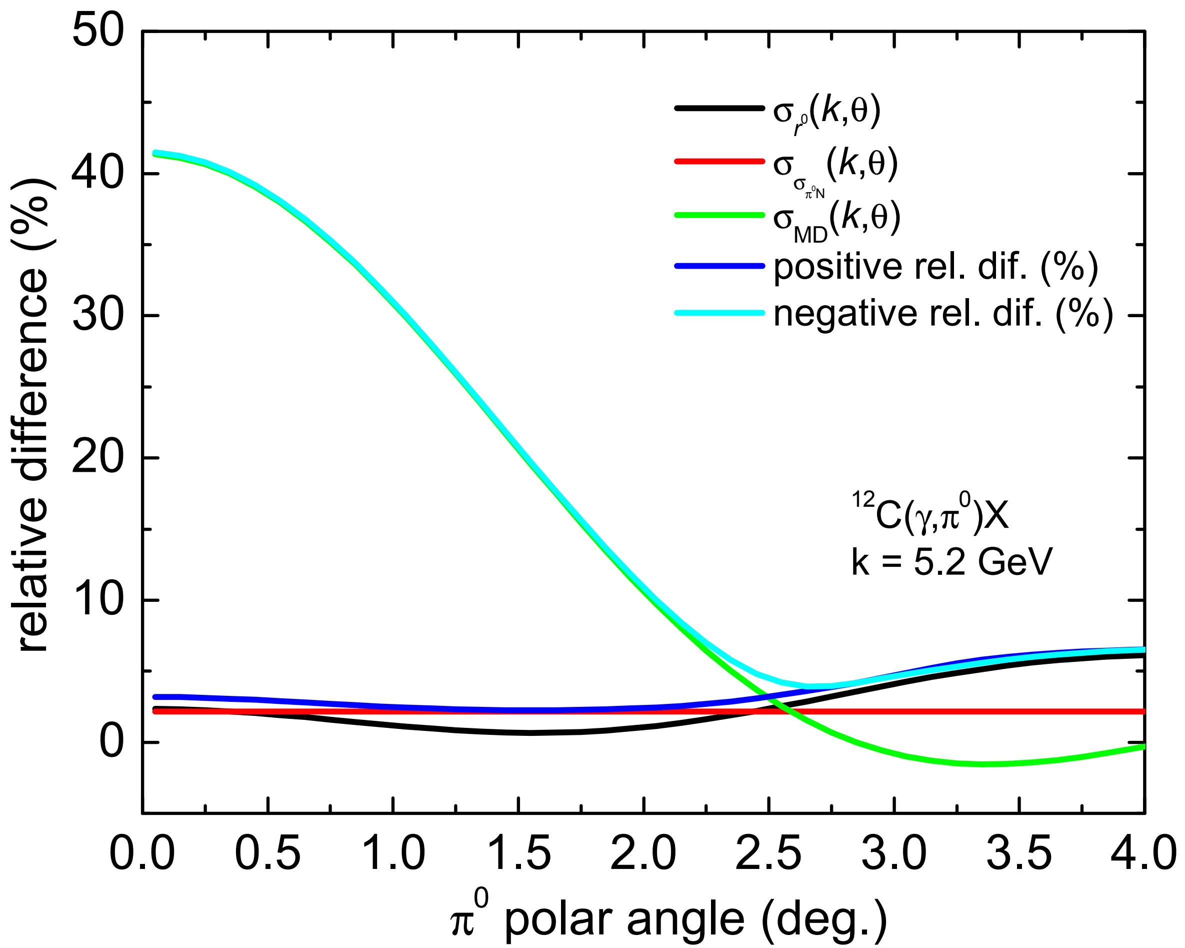
➢Sensitivity of the NI cross section with cascade inputs

Extracting the relative difference of the NI cross section with a 10% variation of the cascade parameters r_0 and $\sigma_{\pi 0N}$: σ_{r0} (k,θ) and $\sigma_{\sigma\pi 0N}(k,\theta)$



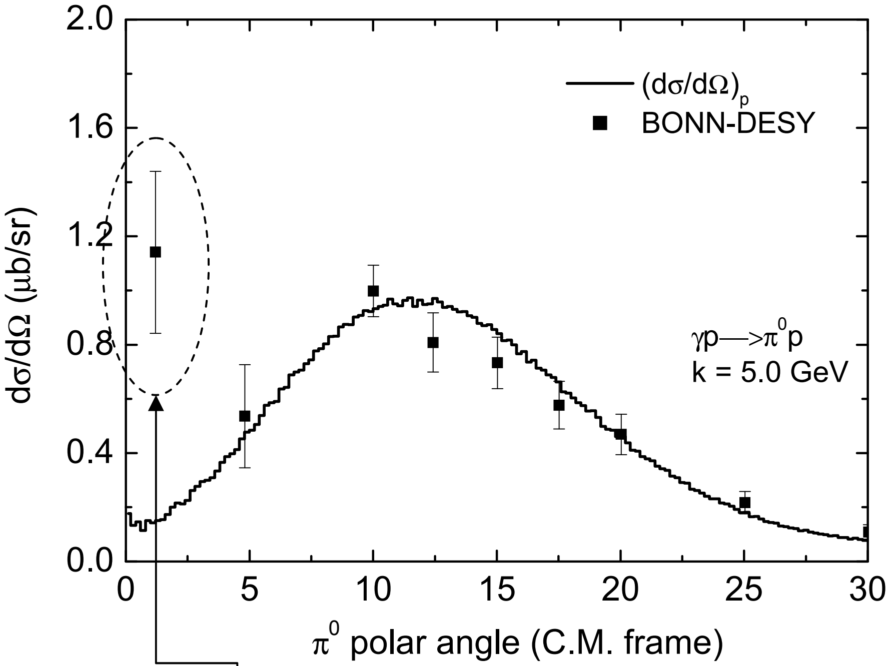
➢Sensitivity of the NI cross section with cascade inputs

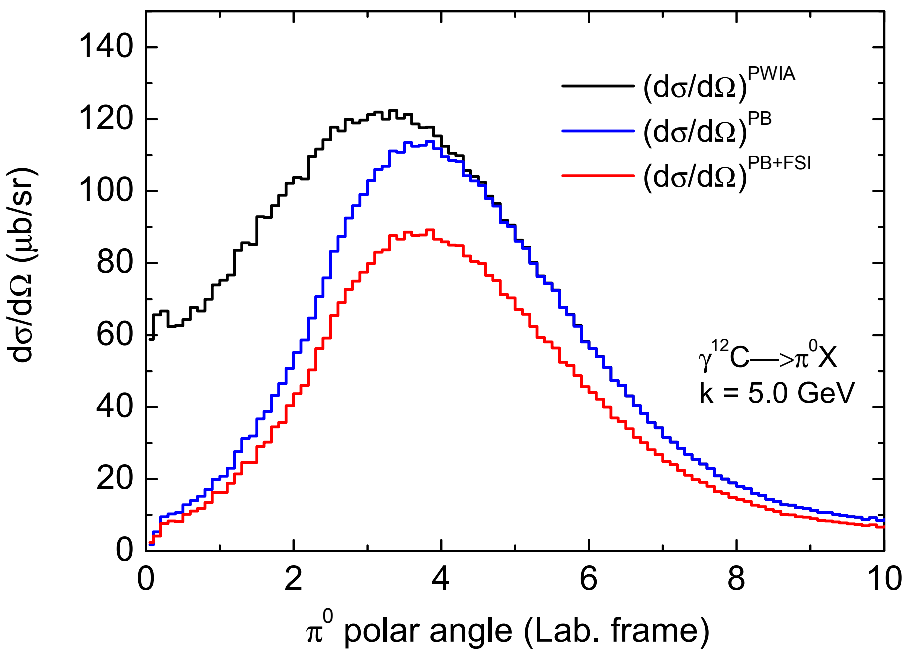
Relative difference budget



➢ Normalization of the NI cross section and A_{eff} factor for ^{12}C

Working example: NI cross section at 5.0 GeV





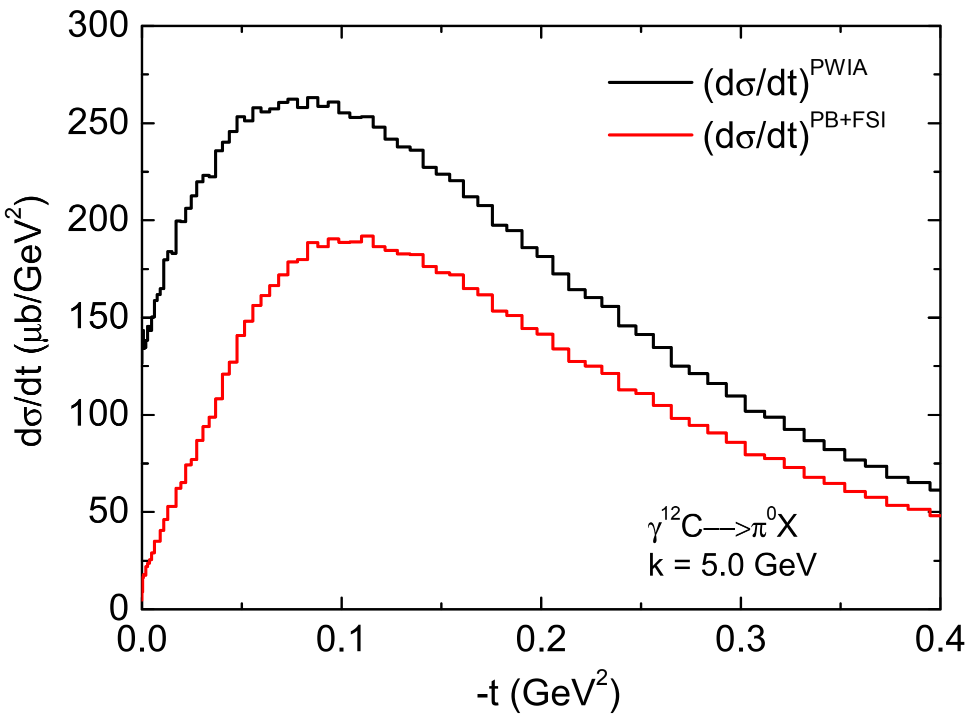
Elementary Primakoff peak

$$\frac{\int \left(\frac{d\sigma}{d\Omega}\right)^{\text{PWIA}} d\Omega}{12 \times \int \left(\frac{d\sigma}{d\Omega_*}\right)_{\text{p}} d\Omega_*} = 1.003$$

➢ Normalization of the NI cross section and A_{eff} factor for ^{12}C

$$\sigma_t(\gamma A) = \int \frac{d\sigma_{\gamma A}}{dt} dt = A_{eff}\sigma_t(\gamma N)$$

Comparing the MCMC model with the experimental data of incoherent π^0 photoproduction from complex nuclei (PRL 28, 1344 (1972)). The data were integrated in the range $0.1 \le |t| \le 0.25$ GeV^2



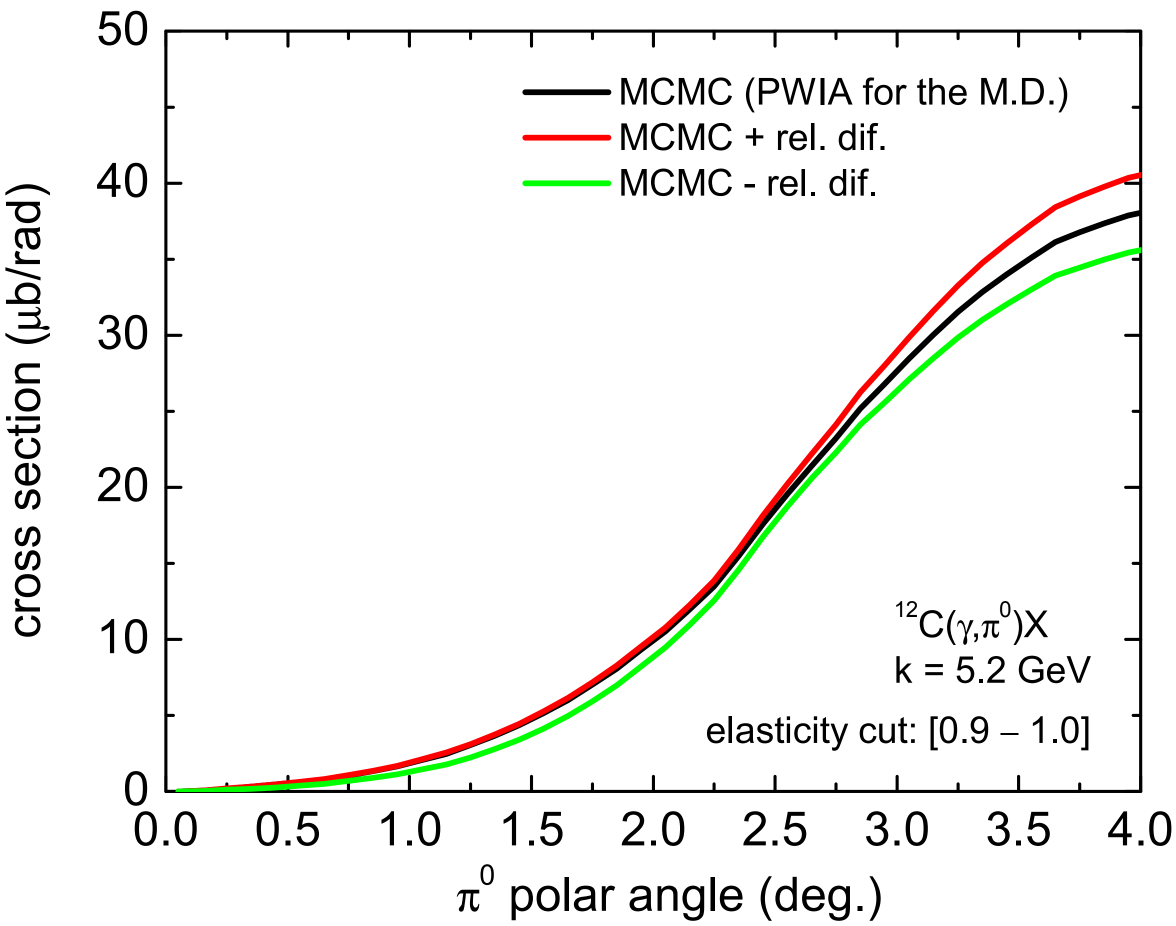
$$A_{eff}(^{12}C)_{\mathrm{MCMC}} = 12 \times \frac{\int_{0.1}^{0.25} \left(\frac{d\sigma}{dt}\right)^{\mathrm{PB+FSI}} dt}{\int_{0.1}^{0.25} \left(\frac{d\sigma}{dt}\right)^{\mathrm{PWIA}} dt} = 9.25$$

$$A_{eff}(^{12}C)_{\exp} = 9.10 \pm 0.71, k = 4.6\ \mathrm{GeV}$$

$$A_{eff}(^{12}C)_{\exp} = 9.00 \pm 0.68, k = 6.4\ \mathrm{GeV}$$

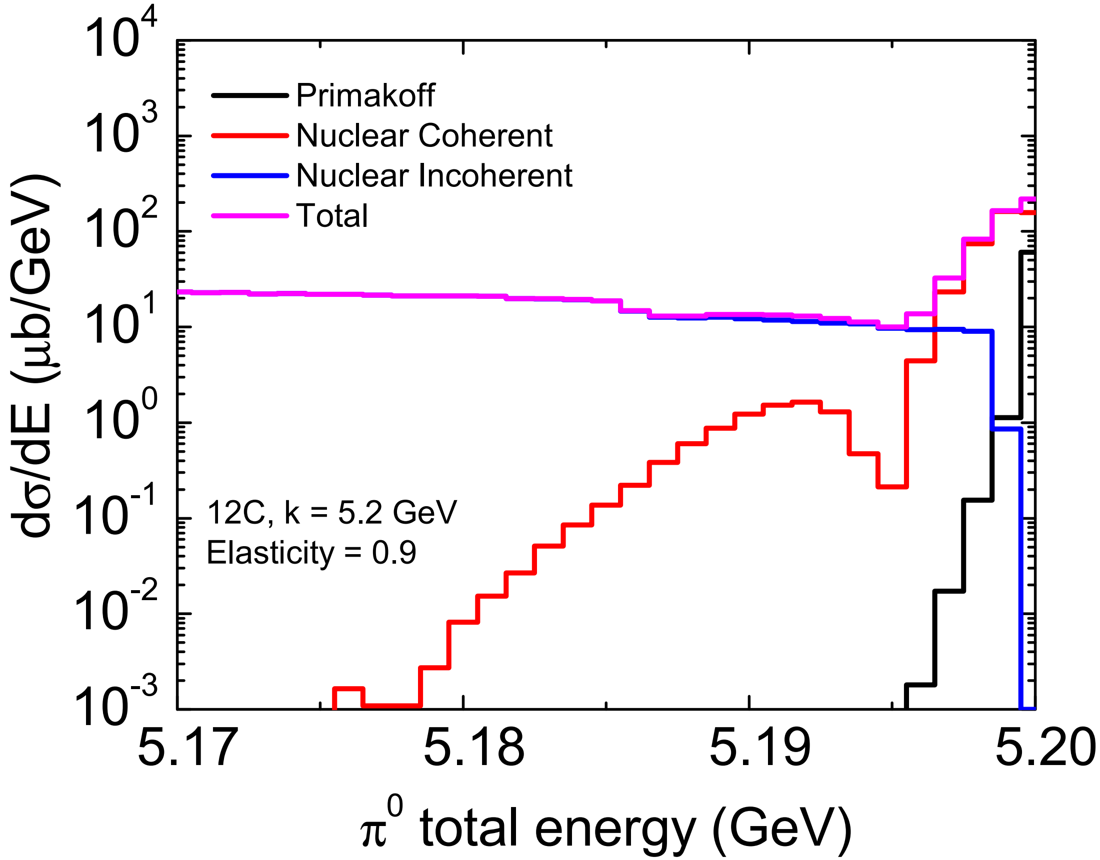
➢ π^0 photoproduction from ^{12}C (PrimEx kinematics)

NI differential cross section and the relative difference between the reference run and the others



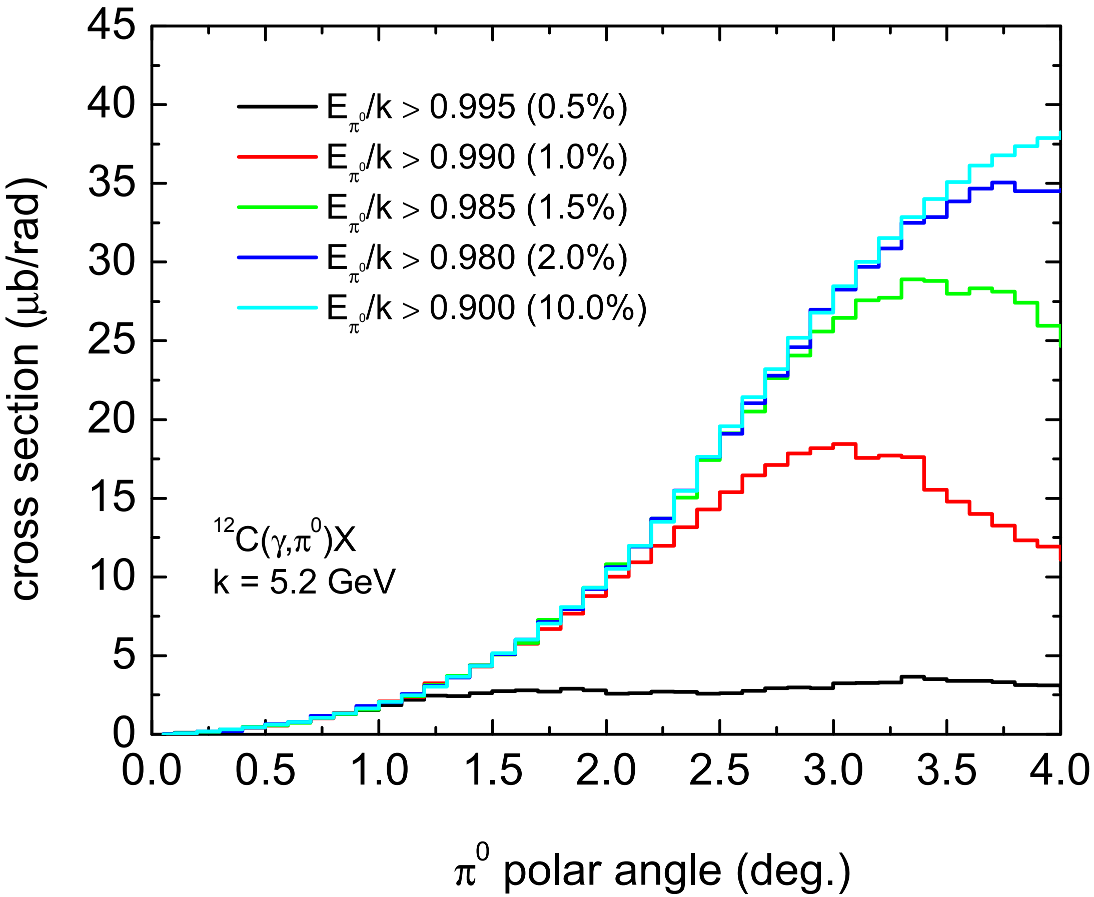
➢π^0 photoproduction from ^{12}C (PrimEx kinematics)

π^0 energy spectrum for the Primakoff, NC and NI components



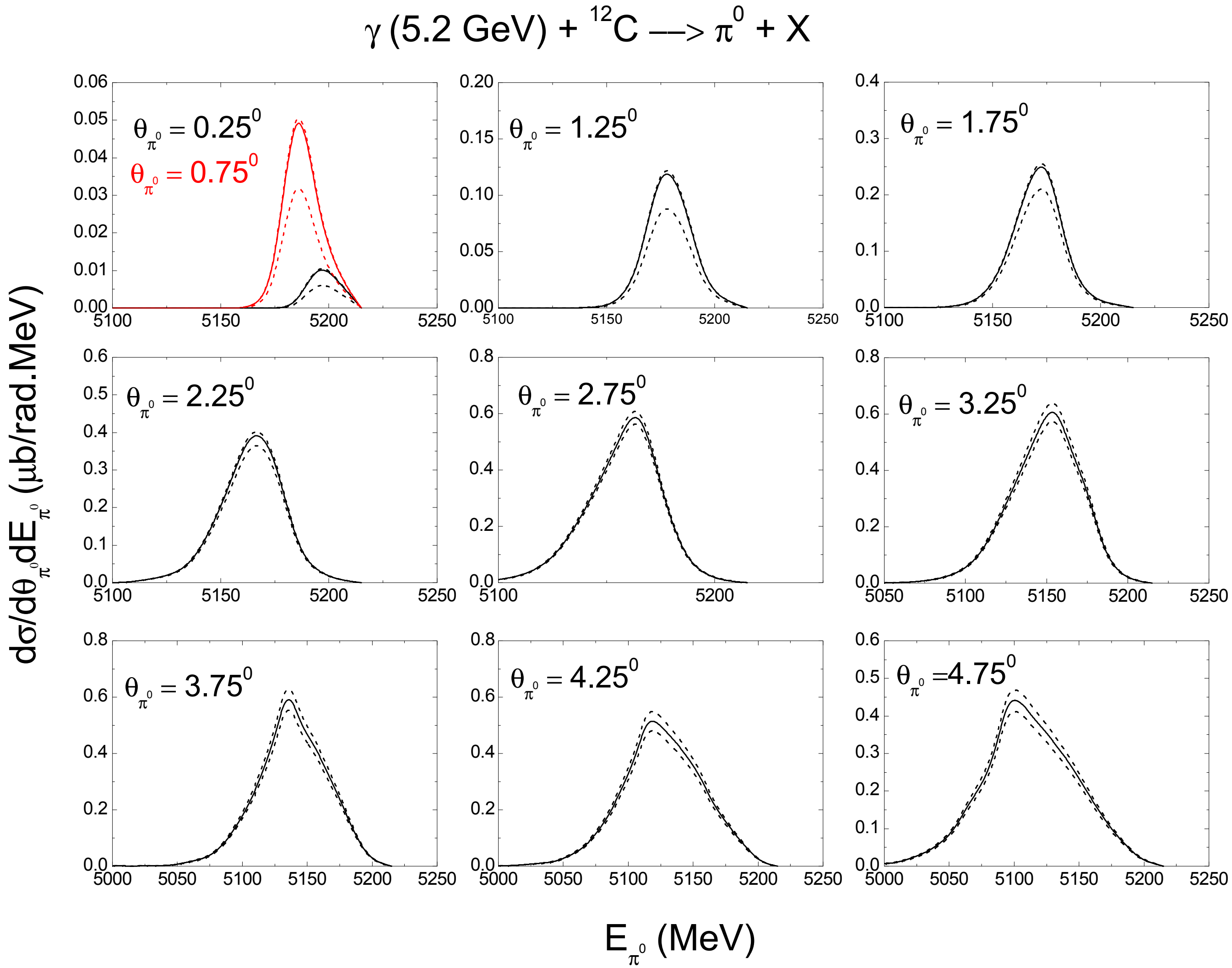
➢π^0 photoproduction from ^{12}C (PrimEx kinematics)

Effects of cuts on π^0 energy (sensitivity of the NI cross section with the two-photon cluster energy resolution)



➢π^0 photoproduction from ^{12}C (PrimEx kinematics)

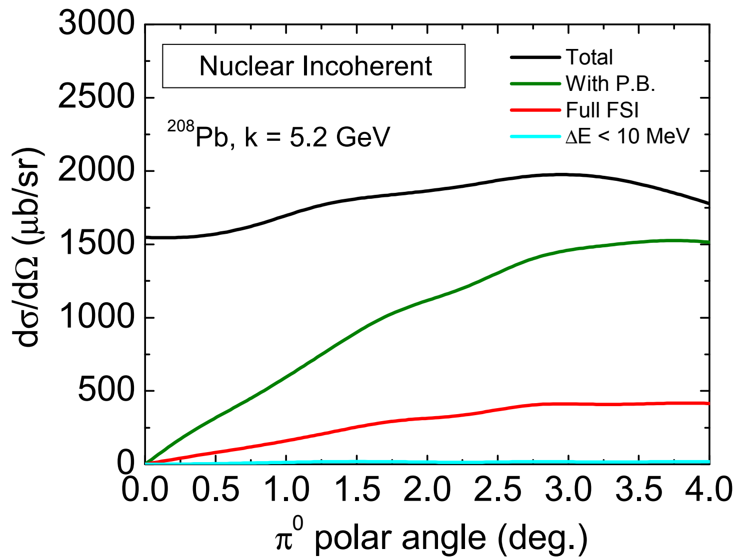
NI double differential cross section with systematic error (dashed lines)



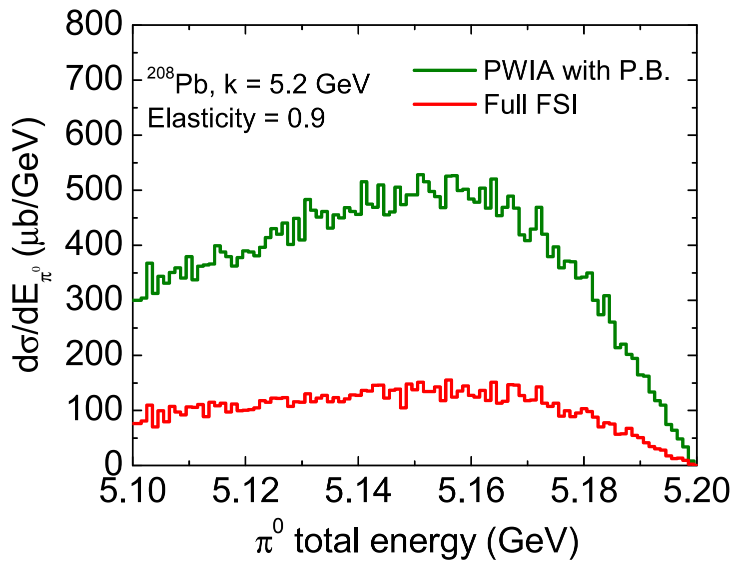
➢π^0 photoproduction from ^{208}Pb (PrimEx kinematics)

NI mechanism

π^0 angular distributions

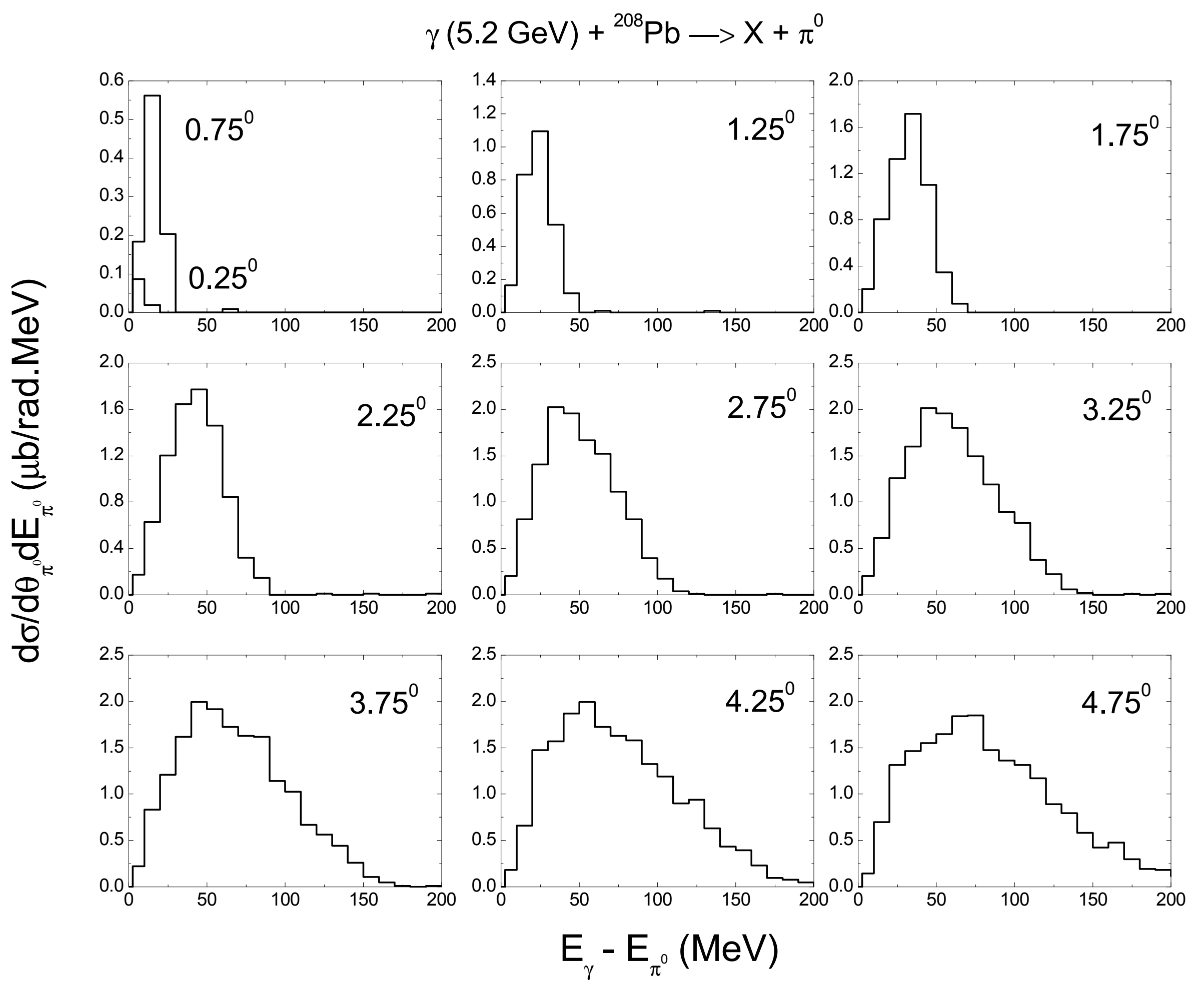


π^0 energy spectrum



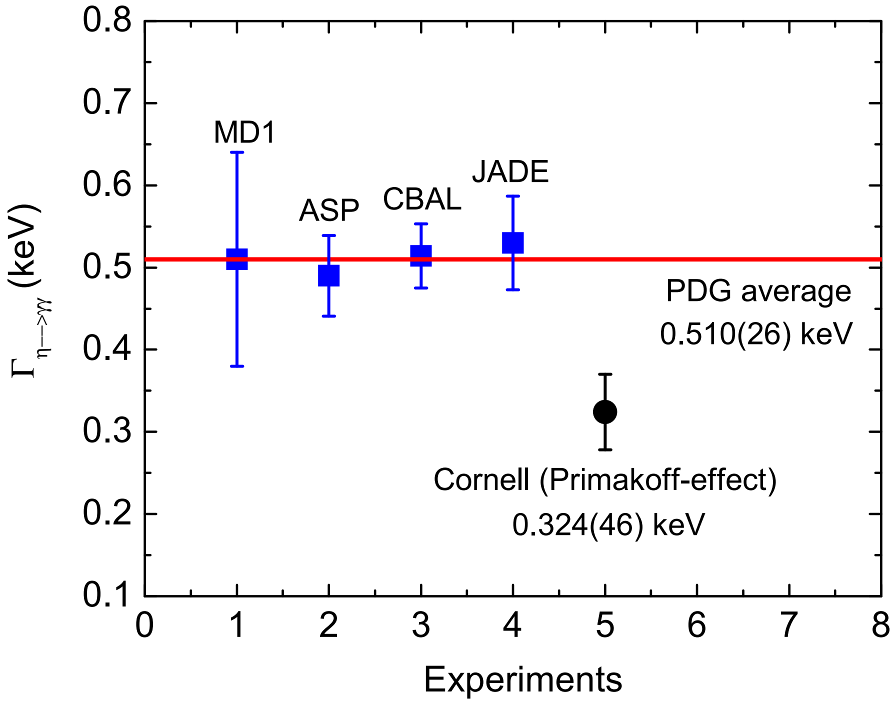
➢π^0 photoproduction from ^{208}Pb (PrimEx kinematics)

Double differential cross section (NI mechanism)

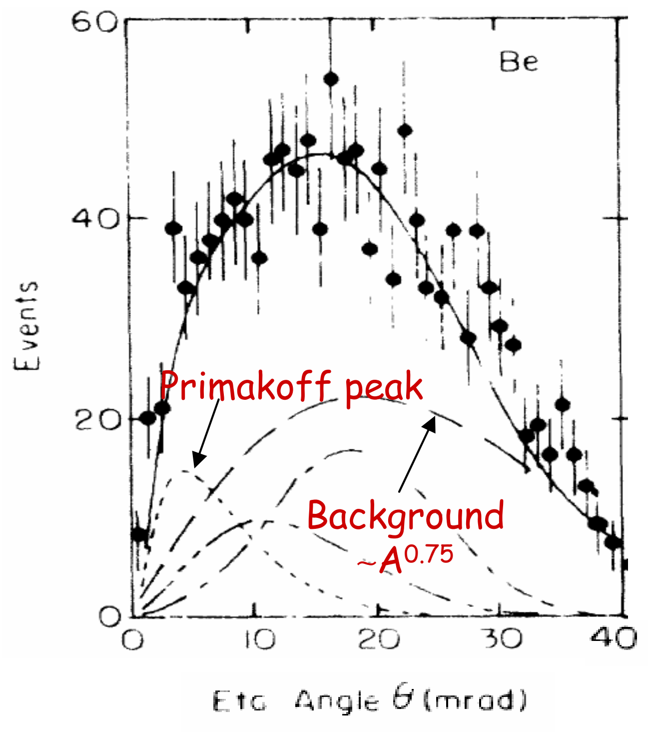


➢Working example: η —> γγ decay width revisited

Experimental scenario for the decay width



Typical yield curve at E_B = 9 GeV (Cornell)



MD1: S. E. Baru *et al.,* Z. Phys. C **48**, 581 (1990)
ASP: N. A. Roe *et al.*, Phys. Rev. D **41**, 17 (1990)
CBAL: D. A. Williams *et al.*, Phys. Rev. D **38**, 1365 (1988)
JADE: W. Bartel *et al.*, Phys. Lett. **160B**, 421 (1985)
Cornell: A. Browman *et al.*, Phys. Rev. Lett. **32**, 1067 (1974)

Possible cause for discrepancy:
Inadequate treatment of the nuclear incoherent contribution (inelastic background)

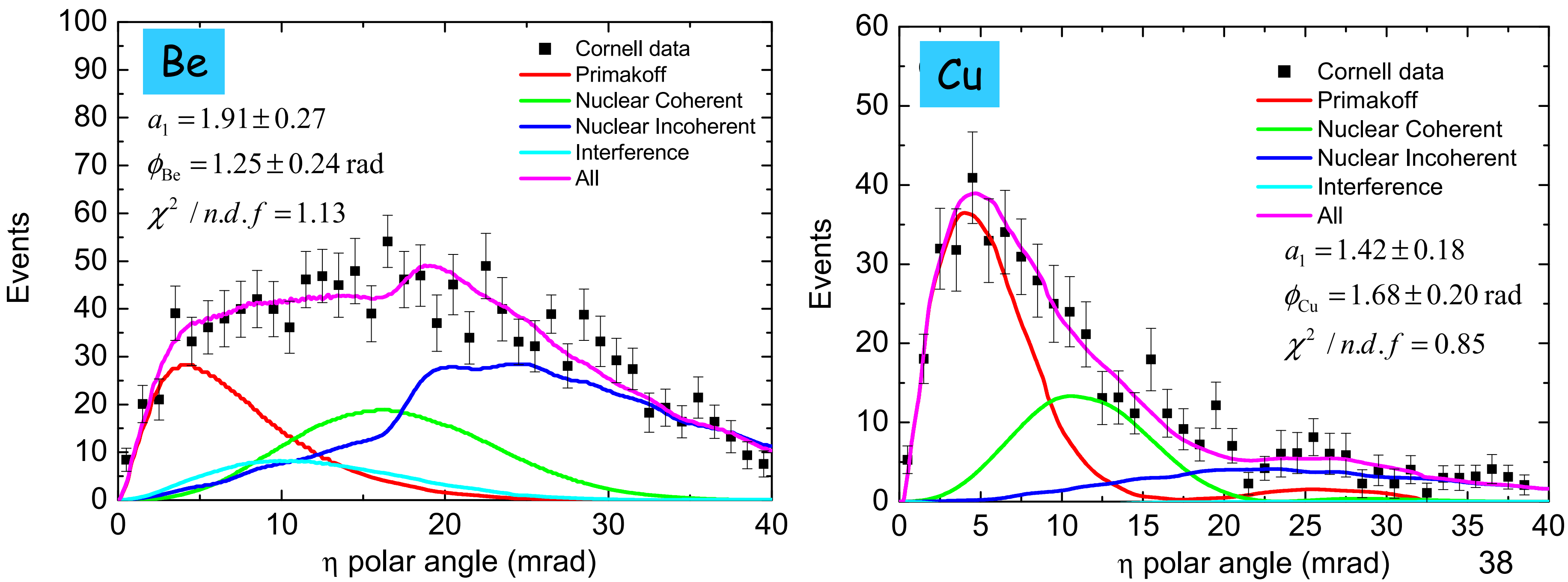
➢ Working example: η ––> γγ decay width revisited

Proposed fitting for the η yield

$$n(\theta) = a_1 S_P(\theta) + a_2 S_{NC}(\theta) + a_3 S_{NI}(\theta) + 2\sqrt{a_1 a_2}\cos(\phi)\sqrt{S_P(\theta) S_{NC}(\theta)}$$

Cornell parameterization

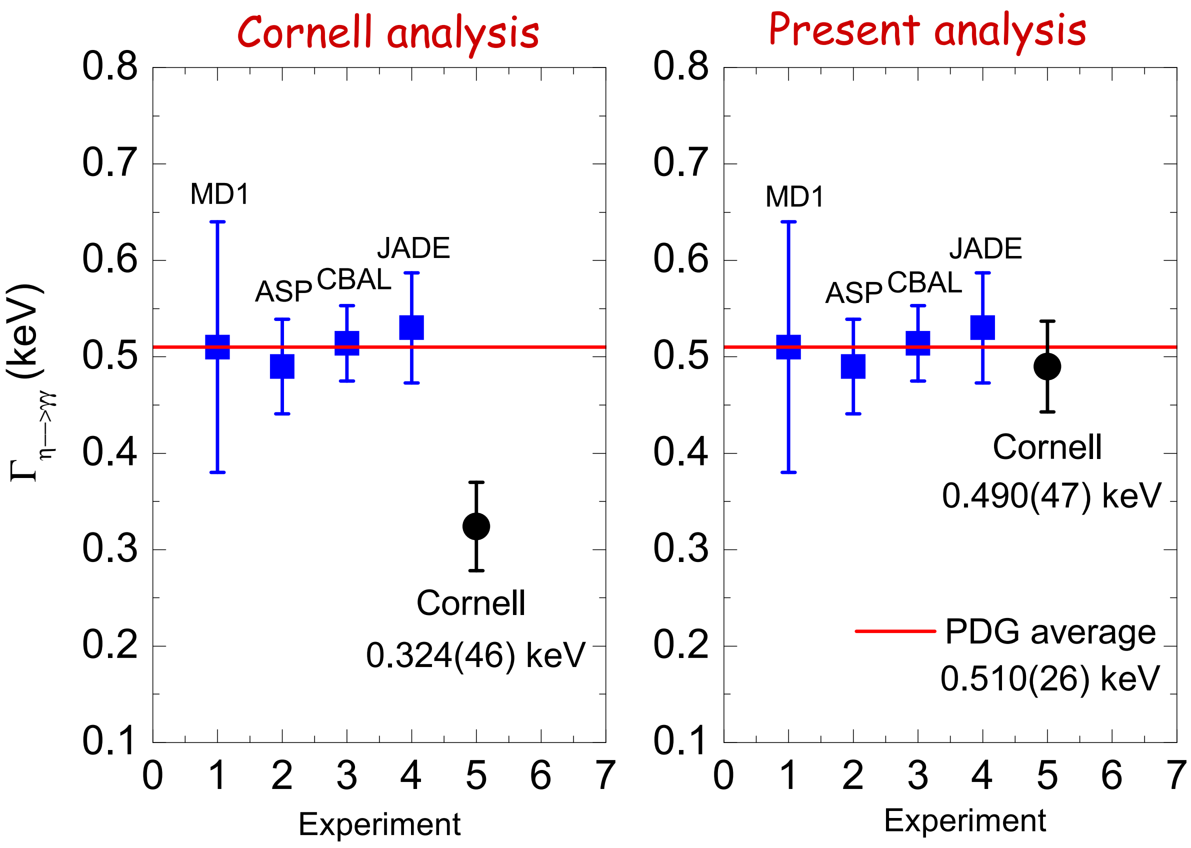
MCMC model



➢Working example: η ––> γγ decay width revisited

$$\Gamma^{\mathrm{Be(Cu)}}_{\eta\to\gamma\gamma} = a_1^{\mathrm{Be(Cu)}} \times \Gamma^{\mathrm{Cornell}}_{\eta\to\gamma\gamma}$$

Combining Be and Cu results:



➢ Conclusions and final remarks

✓The MCMC intranuclear cascade is a sophisticated framework to address the nuclear matter effects within wide ranges of target masses and incident energies.

✓The elementary photoproduction operator for π^0 and η reproduce quite accurately the proton data.

✓The MCMC model has been successfully applied to describe meson-nucleus FSI in both NC and NI mechanisms (absorption and re-scattering).

✓The A_{eff} factor obtained for π^0 production from ^{12}C within the PrimEx energy range (A_{eff} = 9.25) is consistent with the only dataset available at these energies obtained at Cornell (PRL 28, 1344 (1972))

➢Conclusions and final remarks

✓The systematic error of the MCMC cascade due to the M.D. has a maximum value (~40%) at **extreme forward angles**. This result is a consequence of the Pauli-blocking mechanism at low momentum transfer and the variations of the momentum distributions, basically of the p-shell, for large missing momentum (slide 19). The absolute value of the cross sections, however, do not differ by more than **1.0 μb/rad** below approximately one degree (slide 22), which is a good conclusion for the case of PrimEx.

✓The results of the NI differential cross section using all the available data for the MD **are consistent** with the result using PWIA within one sigma (slide 23).

✓The NI cross sections are **strongly dependent** on **cuts on the π^0 energy** (slide 33). This suggests that the **double-differential cross section** are used instead of the single differential in order to account for the effects of energy losses during the construction of the event generator.

➢Conclusions and final remarks

✓The shape factors for the NC and NI components from Be and Cu for E_B = 9 GeV reproduce very accurately the Cornell data. The inelastic background can be solely attributed to the NI component, which is clearly anisotropic.

✓The final result for the $\eta \longrightarrow \gamma\gamma$ decay width obtained by our fitting is consistent with the PDG average and is more than 50% higher than the previous value obtained at Cornell using the same dataset.

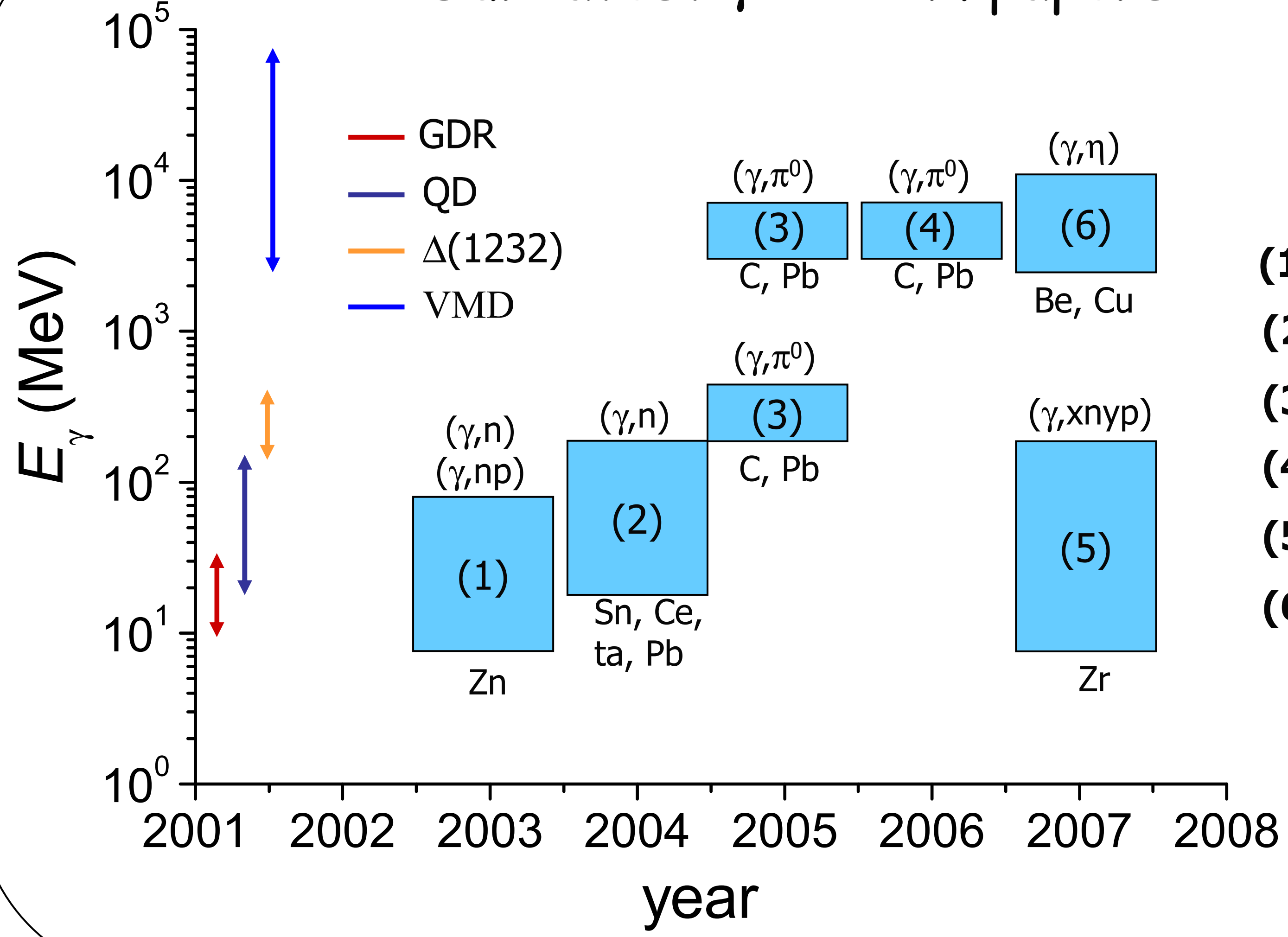
✓Our next step (mid-term project):

⇨ inclusion of vector mesons ρ and ω into the cascade

Single differential cross sections for ^{12}C from 4.5 to 5.8 GeV in 100 MeV steps are posted at: http://axpfep1.if.usp.br/~tulio/PrimEx/12C/new_results/single_diff/
Double differential cross sections for ^{12}C from 4.5 to 5.8 GeV in 100 MeV steps for pion angles up to 5 degrees in steps of 0.5 degree are posted at:
http://axpfep1.if.usp.br/~tulio/PrimEx/12C/new_results/double_diff/

A typical on-demand business (low manpower)

Our latest γA —> X papers



(1) PRC 68, 014618

(2) PRC 69, 064611

(3) PRC 71, 051603

(4) BJP 36, v4b

(5) PRC 75, 014605

(6) Submitted to PRL

Suggestions are very welcome!