Phase-shift angle for the Coulomb-Nuclear Coherent interference term for π^0 photoproduction in PrimEx

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Motivation:

- To provide theoretical estimates for the P-NC phase shift for π^0 photoproduction including pion-nucleus FSI

- To investigate the validity of fitting the same phase-shift for Carbon and Lead.

Outline of the calculation (main approximations/hypothesis):

- The Coulomb phase (ϕ_C) is taken as zero ($ImF_C \sim 0$). Pion-nucleus FSI are neglected for the Coulomb part.
- The strong phase (ϕ_{NC}) is written as a sum of two phases:
  - ϕ_0: phase angle due to a coherent sum of spin non-flip single nucleon amplitudes (photoproduction), and
  - ϕ_{int}: phase-shift due to pion-nucleus FSI (re-scattering via the elastic channel).
- The value of ϕ_{int} is determined by the product of an elementary quantity ϕ_{elast} and the probability of elastic $\pi^0 N$ scattering (calculated in the framework of the cascade model).
- ϕ_0 is connected with ω and ρ meson trajectories via signature factors (Regge model).
- ϕ_{elast} is related with P and P′ trajectories (vacuum poles) and signature factors suitable for the elastic $\pi^0 N$ scattering.

Calculation of the phase-shift (ϕ)

$$F_C = |F_C| e^{i\phi_C}, \; F_{NC} = |F_{NC}| e^{i\phi_{NC}}$$

$$\frac{d\sigma_{\text{interf}}}{d\Omega} = 2|F_C||F_{NC}|\cos(\phi_C - \phi_{NC}) \cong 2|F_C||F_{NC}|\cos(\phi)$$

$$\phi \cong \phi_{NC} \cong \phi_0 + \phi_{\text{int}}$$

Phase due to π^0 photoproduction (strong)

Phase due to π^0-nucleus FSI

$$\phi_{\text{int}} \cong \phi_{elast}\underbrace{\left(P_1 + 2P_2 + 3P_3\right)}$$

Phase due to π^0N elastic scattering

P_n: Probability for n elastic scattering (MCMC model)

Calculation of the phase-shift (ϕ_0)

π^0 photoproduction amplitude expanded in t-channel helicity amplitudes (*):

$$\frac{d\sigma_N}{dt} \cong \frac{1}{32\pi}\left\{ F_2^2 + \frac{F_3^2}{4m_N^2} - \left[t + \left(\frac{\mu^2}{2k} \right)^2 \right] \left(F_4^2 + \frac{F_1^2}{4m_N^2} + \frac{F_3^2}{16m_N^4} + \frac{F_1 F_3}{2m_N p\sqrt{s}} \right) \right\}$$

Coherent π^0 photoproduction from nuclei (spin non-flip amplitude only)

$$\therefore F_{NC} \propto F_1$$

F_1 amplitude calculated in terms of a Regge model (ρ and ω exchange) with cuts (**)

$$F_1 \rightarrow F_1^{\rho,\omega} + F_1^{cut}$$

(*) A. Gasparian and S. Gevorkyan, *Theoretical part of PrimEx*, 2004
(**) M. Braunschweig et al., Nucl. Phys. B 20, 191 (1970).

Calculation of the phase-shift (ϕ_0)

$$F_1^{\rho,\omega} = \frac{\sqrt{2}}{m}\gamma_1\xi(t)\alpha(t)\big(\alpha(t)+1\big)\big(\alpha(t)+2\big)\left(\frac{s}{s_0}\right)^{\alpha(t)-1}$$

$$F_1^{cut} = \frac{\sqrt{2}}{m}\gamma_1^{cut}\xi(0)\left(\frac{s}{s_0}\right)^{\alpha(0)-1}\frac{e^{at}}{\ln\frac{s}{s_0}}$$

The Regge trajectory and signature factor were taken as (*):

$$\alpha(t) = 0.45 + 0.9t$$

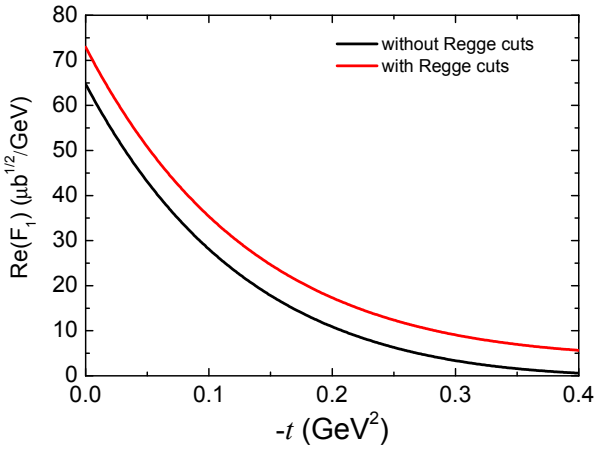
$$\xi(t) = \frac{1-e^{-i\pi\alpha(t)}}{\sin(\pi\alpha(t))}$$

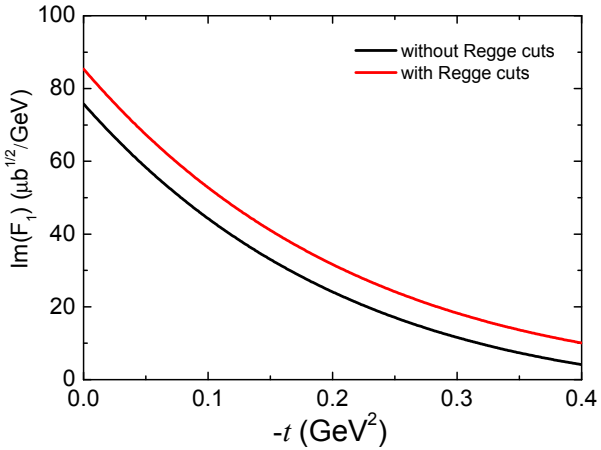
$$\therefore \phi_0 = \tan^{-1}\left(\frac{\mathrm{Im}\,F_s}{\mathrm{Re}\,F_s}\right) = \tan^{-1}\left(\frac{\mathrm{Im}\,F_1}{\mathrm{Re}\,F_1}\right) = \tan^{-1}\left(\frac{\mathrm{Im}\,\xi(0)}{\mathrm{Re}\,\xi(0)}\right) = 0.864\ \mathrm{rad}$$

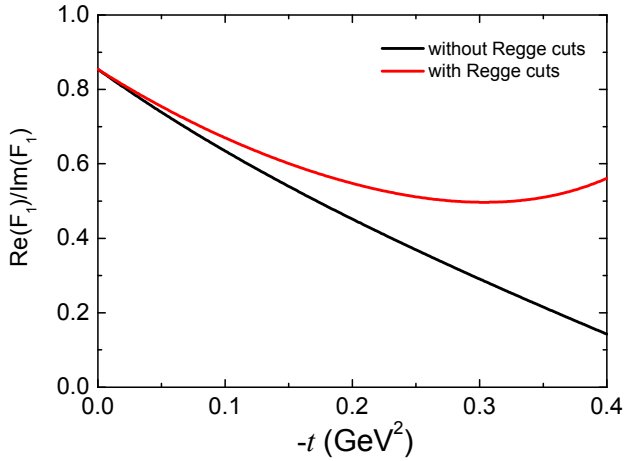
(*) M. Braunschweig et al., Nucl. Phys. B 20, 191 (1970).

Calculation of the phase-shift (ϕ_0)

$\mathrm{Re}\,F_1/\mathrm{Im}F_1$ with and without Regge cuts

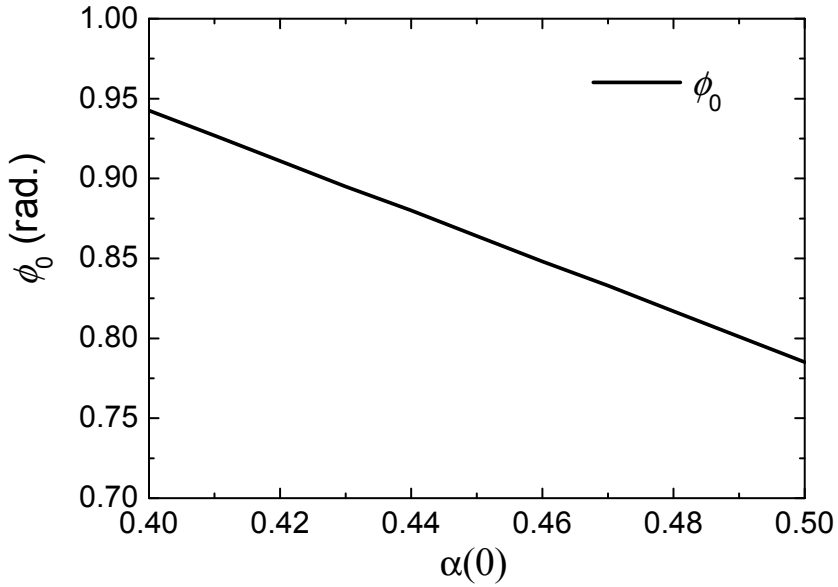






Calculation of the phase-shift (ϕ_0)

ϕ_0 versus the intercept of Regge trajectory $\alpha(0)$



Assuming $\alpha(0) = 0.45 \pm 0.02$, we have

$\phi_0 = 0.864 \pm 0.031$ rad

Calculation of the phase-shift (ϕ_{int})

To calculate ϕ_{int} we need the elementary phase ϕ_{elast} due to elastic pion-nucleon scattering

The differential cross section for πN elastic scattering is given by(*):

$$\frac{d\sigma(s,t=0)}{dt} = \frac{1}{\pi s}\left(\frac{m_N}{4k}\right)^2 \left|A'\right|^2$$

$$A' = C_0 e^{C_1 t}\alpha(\alpha+1)\left(\frac{E_L}{E_0}\right)^{\alpha}\xi(0), \quad \xi(t) = -\left(\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right)$$

$$\therefore A' \propto \frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha} = \frac{e^{\frac{i\pi\alpha}{2}} + e^{\frac{-i\pi\alpha}{2}}}{\sin\pi\alpha} = 2\cot(\pi\alpha)e^{\frac{-i\pi\alpha}{2}} \rightarrow \phi_{elast} = \frac{-\pi\alpha}{2}$$

(*) W. Rarita et al., Phys. Rev. 165, 1615 (1968).

Calculation of the phase-shift (ϕ_{int})

Two different approaches were used to calculate ϕ_{elast}:

Approach # 1: using an effective Regge trajectory from Ref. (*):

$$\phi_{elast} = \frac{-\pi\alpha_{eff}(0)}{2},\ \alpha_{eff}(0) = 0.968 \pm 0.030 \rightarrow \phi_{elast} = -1.521 \pm 0.025\ \mathbf{rad}$$

Approach # 2: estimating ϕ_{elast} by the ratio between the real and imaginary parts of the forward scattering amplitude A':

$$\phi_{elast} = \tan^{-1}\left(\frac{\mathrm{Im}\,A'}{\mathrm{Re}\,A'}\right),\ \frac{\mathrm{Re}\,A'}{\mathrm{Im}\,A'} = -0.2 \pm 0.2 \rightarrow \phi_{elast} = -1.37 \pm 0.19\ \mathbf{rad}$$

Assumed uncertainty

(*) K. J. Foley et al., Phys. Rev. Lett. 15, 45 (1965).

Calculation of the phase-shift (ϕ_{int})

Probabilities for $\pi^0 N$ elastic scatterings

Main features of the extended version of the MCMC model

• π^0 events ($E_{\pi 0} \sim k$) are generated within the nucleus constraining the angular distribution with the undistorted strong FF. This procedure was employed to provide accurate estimates for the pion polar angle AFTER successive elastic scatterings.

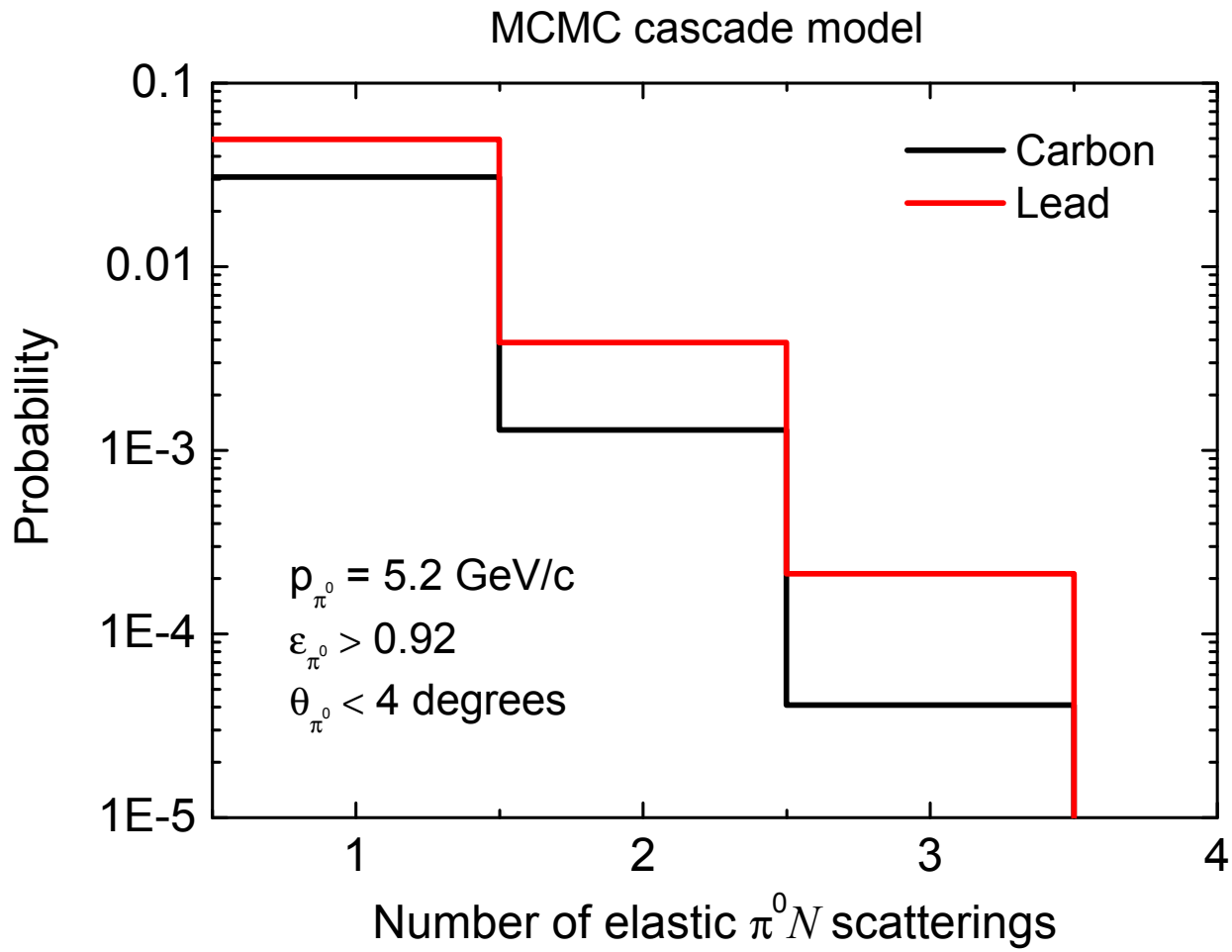
• each elastic $\pi^0 N$ scattering satisfying the Pauli-principle is collected.

• the interaction probabilities for single, double and triple pion-nucleon elastic scattering are calculated assuming a pion elasticity greater than 0.92 and using a polar angle cut of 4 degrees. These cuts are included to provide realistic kinematical constraints for the PrimEx detection. Pions that undergo to elastic scatterings with much lower elasticities (lower than 0.92) are not suppose to be within the elastic peak and will not interfere with the Coulomb term.

• the elastic channel takes into account the elastic cross section (~0.2 σ_{tot}) and the realistic angular distributions from $\pi^{+/-}$p –—> $\pi^{+/-}$p systematics.

Calculation of the phase-shift (ϕ_{int})

Probabilities for $\pi^0 N$ elastic scatterings



Calculation of the phase-shift (ϕ)

Representative values for the phase-shift

$$|\phi| = |\phi_0 + \phi_{elast}(P_1 + 2P_2 + 3P_3)|$$

Regge Trajectory $\alpha_{\rho,\omega}(0)$	$\|\Phi_0\|$ (rad)	C $\|\Phi\|$ (rad)	Pb $\|\Phi\|$ (rad)	C $\|\Phi\|$ (rad)	Pb $\|\Phi\|$ (rad)	$\Delta = \|\Phi\|^{C} - \|\Phi\|^{Pb}$ (Model 1)	$\Delta = \|\Phi\|^{C} - \|\Phi\|^{Pb}$ (Model 2)
		$\alpha_{eff}(0) = 0.968 \pm 0.016$ (Model 1)		ReA'/ImA' = -0.2 ± 0.2 (Model 2)			
0.40	0.942	0.892(31)	0.855(31)	0.897(32)	0.863(33)	0.037	0.033
0.41	0.927	0.876(31)	0.839(31)	0.881(32)	0.848(33)	0.037	0.033
0.42	0.911	0.860(31)	0.823(31)	0.865(32)	0.832(33)	0.037	0.033
0.43	0.895	0.845(31)	0.808(31)	0.849(32)	0.816(33)	0.037	0.033
0.44	0.880	0.829(31)	0.792(31)	0.834(32)	0.801(33)	0.037	0.033
0.45	0.864	0.813(31)	0.776(31)	0.818(32)	0.785(33)	0.037	0.033
0.46	0.848	0.797(31)	0.761(31)	0.802(32)	0.769(33)	0.037	0.033
0.47	0.833	0.782(31)	0.745(31)	0.787(32)	0.753(33)	0.037	0.033
0.48	0.817	0.766(31)	0.729(31)	0.771(32)	0.738(33)	0.037	0.033
0.49	0.801	0.750(31)	0.714(31)	0.755(32)	0.722(33)	0.037	0.033
0.50	0.785	0.735(31)	0.698(31)	0.739(32)	0.706(33)	0.037	0.033

How to perform a constrained fitting of the data

Main hypothesis: If the magnitude of the statistical error of the phase-shift (obtained from the fitting) is larger than Δ, which seems to be reasonable since $\Delta/|\phi| \sim 5\ \%$, we can eliminate one fitting parameter and fit the same phase-shift for Carbon and Lead. In other words, we do not have enough statistics to "see" any difference in the phase-shifts, assuming that this difference is small

The notation from now on is:

Functions of θ_π (known quantities):

$$\left(\frac{d\sigma}{d\theta}\right)_P^{Carbon} \equiv P_1, \quad \left(\frac{d\sigma}{d\theta}\right)_P^{Lead} \equiv P_2$$

$$\left(\frac{d\sigma}{d\theta}\right)_{NC}^{Carbon} \equiv N_1, \quad \left(\frac{d\sigma}{d\theta}\right)_{NC}^{Lead} \equiv N_2$$

$$\left(\frac{d\sigma}{d\theta}\right)_{NI}^{Carbon} \equiv B_1, \quad \left(\frac{d\sigma}{d\theta}\right)_{NI}^{Lead} \equiv B_2$$

$$\left(\frac{d\sigma}{d\theta}\right)_{INT}^{Carbon} \equiv I_1, \quad \left(\frac{d\sigma}{d\theta}\right)_{INT}^{Lead} \equiv I_2$$

Fitting parameters (unknown quantities):

$$\Gamma(\pi^0 \to \gamma\gamma) \equiv a, \quad a_{NC}^{Carbon} \equiv a_1, \quad a_{NC}^{Lead} \equiv a_2$$

$$a_{NI}^{Carbon} \equiv b_1, \quad a_{NI}^{Lead} \equiv b_2$$

$$|\phi|^{Carbon} \cong |\phi|^{Lead} \equiv |\phi|$$

Cross section data:

$$\left(\frac{d\sigma}{d\theta}\right)_{Total}^{Carbon} \equiv y_1, \quad \left(\frac{d\sigma}{d\theta}\right)_{Total}^{Lead} \equiv y_2$$

How to perform a constrained fitting of the data

First, lets begin with a 7 parameter fitting:

$$y_1^{(i)} = aP_1^{(i)} + a_1 N_1^{(i)} + c_1 I_1^{(i)} + b_1 B_1^{(i)}$$

$$y_2^{(j)} = aP_2^{(j)} + a_2 N_2^{(j)} + c_2 I_2^{(j)} + b_2 B_2^{(j)}$$

i / j —> Carbon/Lead

$$c_1 = 2\sqrt{aa_1}\cos(\phi) \qquad c_2 = 2\sqrt{aa_2}\cos(\phi)$$

$$\therefore \begin{pmatrix} y_1^{(i=0)} \\ \vdots \\ y_1^{(i=n)} \\ y_2^{(j=0)} \\ \vdots \\ y_2^{(j=m)} \end{pmatrix} = \begin{pmatrix} P_1^{(i=0)} & N_1^{(i=0)} & 0 & I_1^{(i=0)} & 0 & B_1^{(i=0)} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_1^{(i=n)} & N_1^{(i=n)} & 0 & I_1^{(i=n)} & 0 & B_1^{(i=n)} & 0 \\ P_2^{(j=0)} & 0 & N_2^{(j=0)} & 0 & I_2^{(j=0)} & 0 & B_2^{(j=0)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_2^{(j=m)} & 0 & N_2^{(j=m)} & 0 & I_2^{(j=m)} & 0 & B_2^{(j=m)} \end{pmatrix} \begin{pmatrix} a \\ a_1 \\ a_2 \\ c_1 \\ c_2 \\ b_1 \\ b_2 \end{pmatrix}$$

How to perform a constrained fitting of the data

To perform a six parameter fitting, we call $\cos\phi = x$ and use the relationship between c_1 and c_2 as follows:

$$y_1 = \tilde{a}P_1 + \tilde{a}_1 N_1 + 2\sqrt{\tilde{a}\tilde{a}_1}\,\tilde{x} I_1 + \tilde{b}_1 B_1$$
$$y_2 = \tilde{a}P_2 + \tilde{a}_2 N_2 + 2\sqrt{\tilde{a}\tilde{a}_2}\,\tilde{x} I_2 + \tilde{b}_2 B_2 \qquad (1)$$

Expanding the new parameters (~ values) to first order, we have:

$$\tilde{a} = a_{(0)} + \delta a, \quad \tilde{a}_1 = a_{1(0)} + \delta a_1, \quad \tilde{a}_2 = a_{2(0)} + \delta a_2$$
$$\tilde{b}_1 = b_{1(0)} + \delta b_1, \quad \tilde{b}_2 = b_{2(0)} + \delta b_2, \quad \tilde{x} = x_{(0)} + \delta x \qquad (2)$$

The parameters labeled with (0) represent our first guess obtained in the 7 parameter fitting (previous slide). The value of $x_{(0)}$ can be taken either using $c_{1(0)}$ or $c_{2(0)}$. The final result should not depend upon using $c_{1(0)}$ or $c_{2(0)}$!

$$x_{(0)} = \frac{c_{1(0)}}{2\sqrt{a_{(0)}a_{1(0)}}} \quad \text{or} \quad x_{(0)} = \frac{c_{2(0)}}{2\sqrt{a_{(0)}a_{2(0)}}}$$

How to perform a constrained fitting of the data

The next step is to insert the set of equations (2) into (1) and collect terms to first order of δ*:

$$y_1 = a_{(0)}P_1 + a_{1(0)}N_1 + 2x_{(0)}\sqrt{a_{(0)}a_{1(0)}}I_1 + b_{1(0)}B_1$$
$$+\left(P_1 + \frac{a_{(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}}\right)\delta a + \left(N_1 + \frac{a_{1(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}}\right)\delta a_1 + 2\sqrt{a_{(0)}a_{1(0)}}I_1\delta x + B_1\delta b_1$$

$$y_2 = a_{(0)}P_2 + a_{2(0)}N_2 + 2x_{(0)}\sqrt{a_{(0)}a_{2(0)}}I_2 + b_{2(0)}B_2$$
$$+\left(P_2 + \frac{a_{(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}}\right)\delta a + \left(N_2 + \frac{a_{2(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}}\right)\delta a_2 + 2\sqrt{a_{(0)}a_{2(0)}}I_2\delta x + B_2\delta b_2$$

Known quantities

New parameters

$$(*)\quad \sqrt{\tilde{m}\tilde{n}} = \sqrt{(m_0+\delta m)(n_0+\delta n)} = \sqrt{m_0n_0 + m_0\delta n + n_0\delta m + O(\delta^2)} \cong \sqrt{m_0n_0} + \frac{m_0}{2\sqrt{m_0n_0}}\delta n + \frac{n_0}{2\sqrt{m_0n_0}}\delta m$$

How to perform a constrained fitting of the data

The fitting can now be performed subtracting the data vectors y by the known quantities, such that:

$$y_1' = y_1 - \left(a_{(0)}P_1 + a_{1(0)}N_1 + 2x_{(0)}\sqrt{a_{(0)}a_{1(0)}}I_1 + b_{1(0)}B_1\right) =$$

$$\left(P_1 + \frac{a_{1(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}}\right)\delta a + \left(N_1 + \frac{a_{(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}}\right)\delta a_1 + 2\sqrt{a_{(0)}a_{1(0)}}I_1\delta x + B_1\delta b_1$$

$$y_2' = y_2 - \left(a_{(0)}P_2 + a_{2(0)}N_2 + 2x_{(0)}\sqrt{a_{(0)}a_{2(0)}}I_2 + b_{2(0)}B_2\right) =$$

$$\left(P_2 + \frac{a_{2(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}}\right)\delta a + \left(N_2 + \frac{a_{(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}}\right)\delta a_2 + 2\sqrt{a_{(0)}a_{2(0)}}I_2\delta x + B_2\delta b_2$$

How to perform a constrained fitting of the PrimEx data

The last step is to perform successive fittings for the δ parameters. At each fitting, the (~) values are updated and another set of δ's are obtained. The procedure finishes when the δ's are much lower than the desired precision of (~) values. The resulting (~) values are the best fit values for equations (1) assuming the same phase-shift for Carbon and Lead.

The fitting Matrix is now written as

$$Y' = X'\beta$$

$$X' = \begin{pmatrix} P_1 + \frac{a_{1(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}} & N_1 + \frac{a_{(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}} & 0 & 2\sqrt{a_{(0)}a_{1(0)}}I_1 & B_1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_1 + \frac{a_{1(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}} & N_1 + \frac{a_{(0)}x_{(0)}I_1}{\sqrt{a_{(0)}a_{1(0)}}} & 0 & 2\sqrt{a_{(0)}a_{1(0)}}I_1 & B_1 & 0 \\ P_2 + \frac{a_{2(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}} & 0 & N_2 + \frac{a_{(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}} & 2\sqrt{a_{(0)}a_{2(0)}}I_2 & 0 & B_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_2 + \frac{a_{2(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}} & 0 & N_2 + \frac{a_{(0)}x_{(0)}I_2}{\sqrt{a_{(0)}a_{2(0)}}} & 2\sqrt{a_{(0)}a_{2(0)}}I_2 & 0 & B_2 \end{pmatrix}$$

$$\beta = \begin{pmatrix} \delta a \\ \delta a_1 \\ \delta a_2 \\ \delta x \\ \delta b_1 \\ \delta b_2 \end{pmatrix}$$

Conclusions and final remarks

- The Coulomb-Nuclear coherent phase-shift depends almost exclusively on the intercept of ω and ρ exchange trajectory (Regge model). This parameter is obtained by fitting the proton data and, consequently, should be treated as a free parameter in PrimEx.

- The π^0 - nucleus FSI do not modify substantially the initial phase angle due to photoproduction. The results for Carbon and Lead are consistent with each other within approximately 5 %. This finding is closely related with pion elasticity constraints applied in PrimEx.

- The phase shift difference between Carbon and Lead (less than 5 %) supports the idea of performing a simultaneous fitting constraining the fitted parameters with just one phase-shift. This procedure will reduce the number of fitted parameters and also the correlation between the Coulomb and Nuclear Coherent terms for a given nucleus.

- This calculation is preliminary, suggestions are very welcome!