Unit E
FIELD AND POTENTIAL

Trevor Sandford
Henbury School, Bristol

Suggested time allocation: four weeks

PLAN OF THE UNIT

Section E1
The uniform electric field

forces between +ve and −ve charges ► **Forces on charges**

Uniform field between parallel plates

$g = F/m$
Millikan experiment
energy $= Fd$
p.d. = energy/charge (Unit B) ► **$E = F/Q$ ($= V/d$ for a uniform field)**

Patterns of fields

Flame probe

meaning of p.d.;
choice of zero (Unit B) ► **Potentials in a field; equipotentials**

$E = -\Delta V/\Delta x$

Parallel plates: Q, V, A, $1/d$; effect of medium

use of coulombmeter to measure charge; $C = Q/V$ (Unit B) ► **Reed switch and coulombmeter experiments**

$$\sigma = \varepsilon_0 E, \; C = \varepsilon_0 \frac{A}{d}, \; \varepsilon_r$$ ► Unit J, 'Electromagnetic waves'

Applications

Section E2
Gravitational field and potential

energy = force × distance ► **Gravitational potential difference in a uniform field**

gravitation ► $$g = -\frac{F}{m}, \quad F = -G\frac{m_1 m_2}{r^2},$$

$$g = -\frac{GM}{r^2}$$

Newtonian dynamics ► **Data used to justify** $g \propto -\dfrac{1}{r^2}$

energy = area under force–distance graph (Unit A) ► $\dfrac{1}{r^2}$ **field** $\Rightarrow \dfrac{1}{r}$ **potential** $\left(-\dfrac{GM}{r}\right)$

continued

vectors (Unit A) ▶ **Vector field, force; field = – potential gradient**

$\frac{1}{r}$ **potential hill, well** ▶ nuclear atom: Unit F, 'Radioactivity and the nuclear atom' electrons in atoms: Unit L, 'Waves, particles, and atoms'

dynamics ▶ **Centripetal acceleration; orbits**

Section E3
The electrical inverse-square law

flame probe: Section E1 ▶ $\frac{1}{r}$ **potential** $\Leftrightarrow \frac{1}{r^2}$ **field**

$$V = \frac{kQ}{r};\ E = \frac{kQ}{r^2}$$

$$k = 9 \times 10^9\ \mathrm{N\,m^2\,C^{-2}}$$

$$k = 1/4\pi\varepsilon_0$$

Sizes of electrical and gravitational forces

Field of dipole, array of charges ▶ materials: Unit A, 'Materials and mechanics'

INTRODUCTION

Students will probably have met the notion of a field (magnetic, gravitational, or electric) before. Here it is more clearly defined and the important concept of potential is introduced. Using the analogy between gravitational and electric fields the relationships between force, field, energy, and potential can be drawn out. The emphasis is on a sound *physical* understanding, and applications cover a wide range, from atoms to the Solar System.

Concepts emerge, as elsewhere in the course, from an experimental base wherever possible. A minimum of mathematical terminology and techniques is employed; the emphasis is always on grappling with the difficult physical concepts.

Starting with forces on charges in a uniform field, these Sections develop the idea of potential difference towards that of potential and its gradient. The inverse-square field is introduced with gravity, where it may be more familiar. The link between $\frac{1}{r^2}$ fields and $\frac{1}{r}$ potentials is forged here and then used in the analogous electrical case for charged spheres.

Practical applications concerning gravity include the orbits of satellites, which can be tackled after circular motion is understood. Electrical applications range from industrial processes and problems to the microscopic world of atomic structure.

THE PLACE OF THE UNIT IN THE COURSE

Essential topics which must have been studied *before* this Unit include the following:

Basic Newtonian dynamics (GCSE and Unit A);

Potential difference (GCSE and Unit B);

Charge and capacitors (GCSE and Unit B).

The work on circular motion ties up a little with simple harmonic motion from Unit D, 'Oscillations and waves' which provides a useful break between the electrical work of Units B and C and Section E1, though it would be possible to complete almost all of Unit E before Unit D.

Other topics which use ideas developed in this Unit include:

Forces between ions (Section A2);

The nuclear atom (Unit F, 'Radioactivity and the nuclear atom');

Electrons in the atom (Unit L, 'Waves, particles, and atoms').

The Unit also has its own 'end-points'. A theoretical one is the appreciation of the analogy between electrical and gravitational fields and the links between the key concepts of force, energy, field, and potential. Another is an appreciation that such abstract ideas have practical applications, ranging from the orbits of spacecraft round the Earth to an understanding of electrons within atoms, from photocopying and printing to painting and prospecting.

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS

SECTION E1
THE UNIFORM ELECTRIC FIELD

Forces on charges

'Field and potential' – what is a 'field'? What fields have we already met and studied – gravitational, magnetic, electrical ...? What does a field do? What sorts of things does it affect? (For example, gravity affects masses, magnetism affects magnets or currents ....) These are the sorts of question which can open up discussion leading towards the idea that a field exists where we can observe a *force* on something. Depending on what that something is, then, we say a different type of field exists. If students take kindly to this sort of debate, it can form a useful introduction. Discussion can continue around the following demonstration which may have been met in an earlier course, but can be introduced as a puzzle to be explained.

DEMONSTRATION
E1 Forces on a charged ball between charged plates

ITEM NO.	ITEM
14	e.h.t. power supply
65	2 metal plates with insulating handles
57L	table tennis ball coated with colloidal graphite
1153	nylon sewing thread
1101	sensitive galvanometer (*e.g.*, internal light beam)
51G	polythene strip
503–506	2 retort stand bases, rods, bosses and clamps
1000	leads

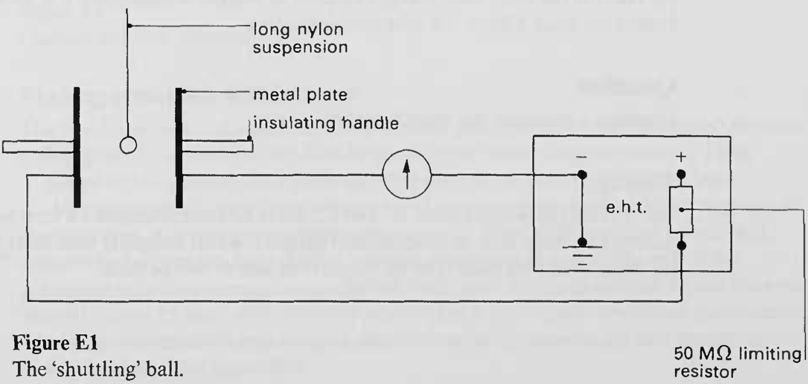


Figure E1
The 'shuttling' ball.

The galvanometer may at first be left out of the circuit and the p.d. increased to about 4 kV, with the plates about 0.1 m apart. The ball, on making contact with one or other of the plates, will 'shuttle' back and forth continuously.

Students may suggest that:
– forces between like and unlike charges are causing the effect;
– the direction of the force depends on the charge on the ball;
– the ball is 'ferrying' charge between the plates.

This is a good point to revise the idea that current is simply 'charge in motion' and that potential difference (here provided by the power supply) is what causes the charge to move.

Induced charges

The effect may begin spontaneously with the ball initially not touching either plate. The idea of induced charge (that is, charge separation) on an object with no net charge may be mentioned, and examples (for instance a charged comb picking up paper or attracting a stream of water) may be shown in passing.

The main thrust, however, should be towards establishing the idea that the ball moves because there is a *force* on it, and that this force arises because the ball carries an *electric charge*. So we say that an electric field exists where electric charges experience a force.

What factors affect this force?

Students may suggest, or have noticed, that the p.d., V, affects the force on the ball and they can also see the effect of varying the separation, d, between the plates.

Quantitative results are not expected at this stage as it is probably only the frequency of oscillation, or perhaps the current, which is readily observed rather than the force on the ball. There is no clear evidence yet that decreasing d, say, does increase this force. Nor is the effect of the ball's own charge observed as this too is changed if V is altered. These factors are dealt with in the next demonstration.

Question

Question 1 discusses the 'shuttling ball'.

Timing

The first two demonstrations, E1 and E2, serve to focus attention on vital concepts rather than being ends in themselves. Therefore, whilst adequate time must be allowed for discussion, they need take no longer than one or two periods.

DETECTING AN ELECTRIC FIELD

'How can we detect where there is a force on a charged object, and how can we see how it varies from place to place?' The answers to these questions may be found, using some sort of field detector, basically a charged object on which the electric force has some observable effect.

DEMONSTRATION

E2 Using a charged foil strip as a field detector

ITEM NO.	ITEM
1025	1 pair capacitor plates
30	2 slotted bases
51M	2 square polythene tiles
51G	polythene strip
	foil (see note below)
1153	adhesive tape
1153	razor blade or scissors
14	e.h.t. power supply
1000	leads
	means of projection (optional)

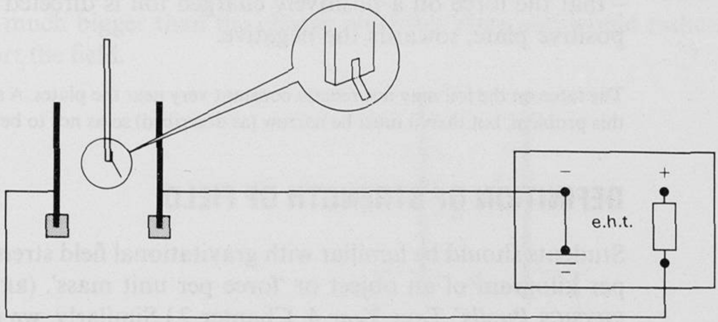


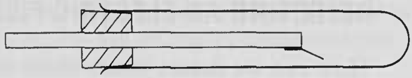
Figure E2
Charged foil field detector.

Making a suitable foil detector

The best foil to use is aluminized plastic film (25 gauge), for example 'Melanex'. (Normal cooking foil is too thick.) It needs to be made rather narrow (approximately 1 mm × 10 mm of foil exposed) and stuck carefully onto the polythene strip with tape.

Very good results have been obtained with the very thin aluminium leaf supplied for electroscopes. A rather larger strip (approximately 8 mm × 40 mm) may be cut with scissors by first sandwiching the foil between two sheets of paper. The end of the polythene strip is moistened using a licked finger and the damp surface offered towards the end (about 15 mm) of the foil strip whereupon it sticks quite firmly and permanently. The larger movement is more clearly seen but the foil is more fragile and should be stored carefully. (See figure E3.)

Figure E3
Storage of foil detector.



Set up the plates a few centimetres apart. The foil is charged by contact with the positive plate and the angle at which it hangs gives an indication of the force on it. The p.d. between the plates can be adjusted to give a convenient angle, say 45°. For large groups projection on to a screen may be a help, but students must be close enough to see the foil being moved about in the space between the plates.

The foil may first be used to show
– that there *is* a field in the region between the plates, and that it even extends somewhat outside this region,
– that increasing the p.d. whilst holding the charged foil steady increases the force on it,
– that decreasing the separation of the plates has the same effect,
– that the angle at which the foil hangs, and therefore the force on it and so the field, is almost constant over much of the region between the plates (shown by moving the detector in 3 directions mutually at right angles, parallel and perpendicular to the plates),
– that the force on a positively charged foil is directed away from the positive plate, towards the negative.

The force on the foil may not remain constant very near the plates. A shorter foil avoids this problem, but then it must be narrow (as described) so as not to be too stiff.

DEFINITION OF STRENGTH OF FIELD

Students should be familiar with gravitational field strength as the force per kilogram of an object or 'force per unit mass'. (REVISED NUFFIELD PHYSICS *Pupils' Text Year* 4, Chapter 2.) Similarly, we can now define the strength of an electric field as the 'force per unit charge' or $E = \frac{F}{Q}$, where E represents the electric field strength, F the force, and Q the charge on which it acts. Note here that field, like force, is a *vector* quantity, possessing both magnitude and direction. Its units are clearly $\mathrm{N\,C^{-1}}$, and students should get used to this unit as the fundamental unit for field since it reinforces the definition.

The foil experiment shows that between parallel plates the electric field strength is uniform and affected by the p.d. between and separation of the plates.

A more strict definition

A strict definition regards field as a limit:

$$E = \lim_{Q \to 0} \frac{F}{Q}$$

If the field is due to a charged conductor, the 'test' charge can cause movement of charge in the conductor, thereby altering the field itself. This effect becomes less as Q falls to zero. Students might be given some hint of this problem, though teachers must be careful not to introduce further confusion: a large test charge does give a true measure of the field it is testing, but this field might be quite different if the test charge were removed. The situation might be compared with that of using a 'bad' (that is, low resistance) voltmeter to measure p.d. in a circuit. If the original field were due to charges which could *not* move, then the test charge, Q, could have any magnitude without affecting the field due to those charges.

Problems in using F/Q to measure fields

There are two clearly *practical* objections to using $\frac{F}{Q}$ as a measure of the field: F and Q are both very small and therefore difficult to measure accurately. A larger, more measurable force on the foil might be gained by giving it a much greater charge, say 1 coulomb. But this would be very much bigger than the charge on either plate and would radically distort the field.

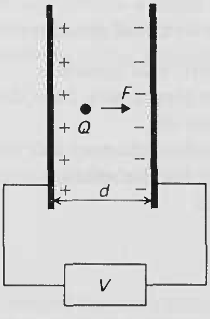


Figure E4
Relationship between F, Q, V, and d.

The way out of this problem relies on looking again at the relationships between F, Q, V, and d (figure E4). The ratio $\frac{V}{d}$ is equivalent to $\frac{F}{Q}$ for a uniform field. In this limited but very common context then we have a very simple method of measuring the field strength – with a voltmeter and a ruler – which avoids all the practical problems of measuring F and Q. Stress that $E = \frac{V}{d}$ is a special case for uniform fields only, but $E = \frac{F}{Q}$ is always true. (Strictly speaking, perhaps we should write $E = -\frac{V}{d}$. At this stage students must know that the field is in the direction of the force on a *positive* charge.)

Questions

Question 2 discusses the foil used above.
Question 3 shows the equivalence of $\mathrm{N\,C^{-1}}$ and $\mathrm{V\,m^{-1}}$ as field strength units.
Question 4 gives practice in calculating field strengths.

PATTERNS OF ELECTRIC FIELD FOR DIFFERENT GEOMETRIES

In the same way that iron filings align themselves to 'show up' a magnetic field, small poorly conducting particles may be used to reveal the shapes of electric fields.

DEMONSTRATION
E3 Patterns of electric field for different geometries

ITEM NO.	ITEM
	either
14	e.h.t. power supply
	or
60/1	Van de Graaff generator
149	electric field apparatus
1501	bare copper wire, 2 mm diameter, for electrodes
1153	castor oil
1153	semolina (chopped hair will do)
1156	1,1,1-trichloroethane
1000	leads

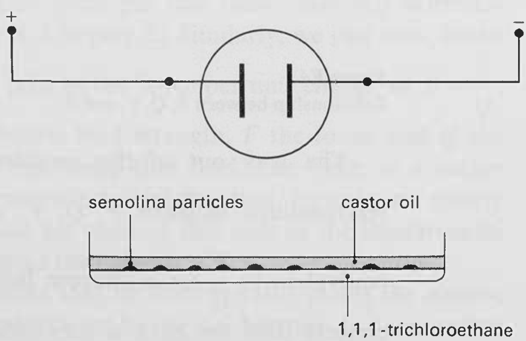


Figure E5
Using semolina to show electric fields.

Semolina powder floats on the surface of 1,1,1-trichloroethane. (Tetrachloromethane (carbon tetrachloride) has a toxic vapour and must not be used for this or any other experiment in the open laboratory. 1,1,1-trichloroethane is considered safer.) Put a small amount (about $\frac{1}{2}$–$\frac{3}{4}$ teaspoonful) of oil in the Petri dish, along with a sprinkling of semolina and enough trichloroethane to about half cover the electrodes, which may be of thick (2 mm) copper wire or simply welding rods. Stir the mixture and allow it to settle for about 3 minutes, when the semolina becomes quite mobile in the interface between the trichloroethane and the castor oil (which prevents the other liquid from evaporating). Semolina in oil alone gives satisfactory results, though since it is then less mobile and tends to sink, higher p.d.s may be needed and a Van de Graaff generator is required. Also, instead of semolina, the hair from a clean dry paintbrush cut to less than 1 mm in length, or grass seed, make acceptable substitutes.

A p.d. of a few kV from the e.h.t. supply is sufficient to cause the semolina particles to orientate themselves in accordance with the electric field between the electrodes. A little agitation or stirring may be necessary. When the pattern is clear the p.d. may be switched off. Different shaped electrodes may be used to show both uniform, radial, and other configurations of field in two dimensions. Overhead projection makes the effects most clearly visible, though heating may cause the liquid to evaporate, and after 5–10 minutes or more the oil may begin to conduct, giving spurious results. Otherwise the demonstration gives good results and rewards the little trouble it requires.

Field lines and induced charges

The semolina particles form into lines which follow the direction of the electric field. Since 'field lines' are just one way of representing a field we do not stress them. They can lead to misunderstanding (see Appendix V); here they are just a useful way of showing the direction and shape of a field. Nor do we deal here with induced charges and polarization, or attempt to explain why the experiment works. Interested students could follow this up for themselves.

Questions

Questions 5 and 6 involve estimating the shapes and strengths of electric fields.

FIELD AND POTENTIAL

Teaching sequence The next discussion based around demonstrations continues the argument towards the relationship between field strength and potential gradient, after which follows the first of the Section's class experiments (E5) on plotting equipotentials. If practical work is required at an earlier stage, the work based on exploring the charge on parallel plates (Experiments E6 and E7) could be taken first, then returning here to complete the Section.

The strategy here is to move from $E = \frac{V}{d}$ for the uniform field towards the more general $E = -\frac{\Delta V}{\Delta x}$ for any region of any field where changes in V and x can be measured. The flame probe is introduced as a tool to measure V.

Measuring electric fields 'in space'

The discussion might proceed as follows:

'So far we have seen that $\frac{V}{d}$ may be used as an alternative to $\frac{F}{Q}$ as a measure of the strength of the uniform field between charged parallel plates. We know that there is a certain potential difference between the plates; however, we know very little about the space between them. Could we somehow measure a p.d. between isolated points in space and thereby deduce the field (if it is uniform) between those points?' This is what we set out to do using the flame probe, which will not only introduce the concept of potential and its variation with distance but also give more meaning to the abstract talk about field as an effect 'in space'.

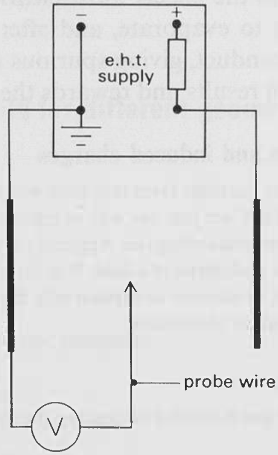


Figure E6
A p.d. probe.

The circuit of figure E6 will not work. If the wire probe is near, say, the positive plate, electrons will be drawn up the wire to its tip. This collection of negative charge will disturb the field. Recalling the strict definition of field, perhaps it can be seen that this problem is minimized if the charge on the tip is small, and indeed solved if it can be completely neutralized. This may be achieved by surrounding it with a collection of ions, which will carry away any excess charge acquired by the tip; this is most readily arranged in practice using a small gas flame. Thus the 'flame probe' may be introduced as a device to measure potentials within the field.

DEMONSTRATION

E4 Measuring potentials in a uniform field using a flame probe

ITEM NO.	ITEM
51A	gold leaf electroscope
51J	hook for electroscope
14	e.h.t. power supply
94A	lamp, holder, and stand
27	transformer
52K	crocodile clip
1025	pair of capacitor plates
51M	2 square polythene tiles
30	2 slotted bases
51G	polythene strip
501	metre rule (or Perspex rod)
1501	PVC covered copper wire
1155	hypodermic syringe, 1 cm^3
1155	25 gauge hypodermic needle
1155	PVC tubing, 2 m, 6.5 mm bore
1153	adhesive tape
1153	razor blade
552	Hoffman clip
503–6	retort stand base, rod, boss, and clamp

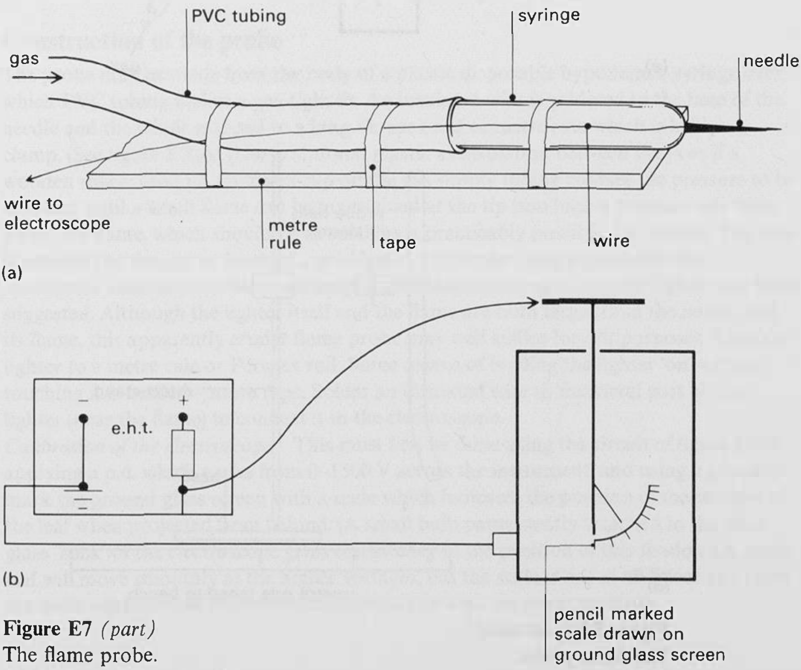
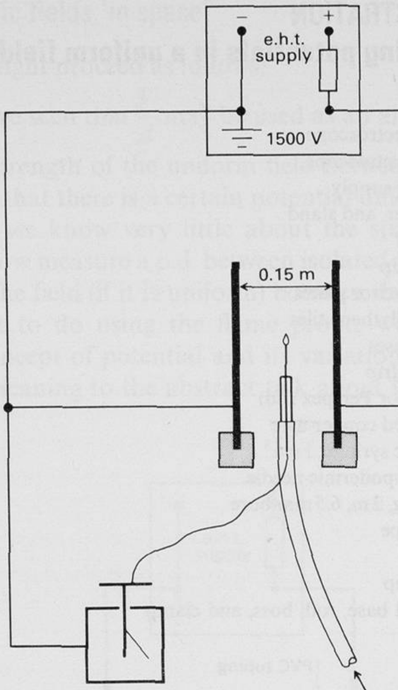
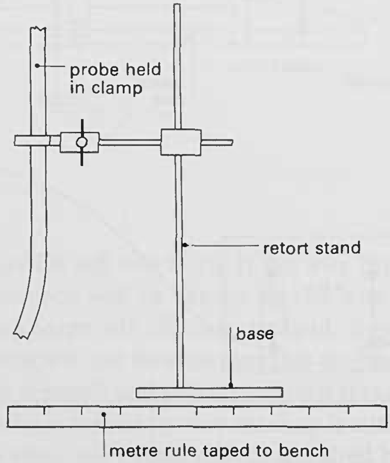


Figure E7 *(part)*
The flame probe.
(a) Construction of probe.
(b) Calibration of electroscope.

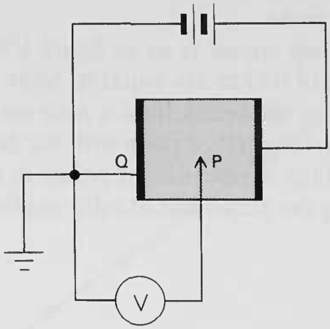


(c)



(d)

Figure E7 *(continued)*
The flame probe.
(c) Measuring potentials.
(d) Measuring the movement of the probe.



(e)

Figure E7 *(continued)*
The flame probe.
(e) 'Equivalent electrical circuit'.

Questions

Questions revising the potential divider and the idea of potential difference may help in leading towards the concept of potential at this point. Question 7 discusses the flame probe experiment.

Construction of the probe

The probe may be made from the body of a plastic disposable hypodermic syringe over which PVC tubing makes a gas-tight fit. An insulated wire is soldered to the base of the needle and the whole is taped to a long Perspex rod or metre rule which is held in a clamp. (See figure E7(a).) (Use additional plastic as insulation between the jaws if a wooden rule is used.) A Hoffman clip on the gas supply tubing enables the pressure to be adjusted until a small flame can be maintained at the tip (too high a pressure will 'blow away' the flame, which should be as small as is practicably possible, but stable). The wire is attached to the cap or hook of a gold leaf electroscope using a crocodile clip.

Alternative construction A much simpler construction using a cigarette lighter has been suggested. Although the lighter itself and the flame are both larger than the needle and its flame, this apparently cruder flame probe may well suffice for our purposes. Tape the lighter to a metre rule or Perspex rod. Some means of holding the lighter 'on' without touching it is needed – more tape. Solder an insulated wire to the metal part of the lighter (near the flame) to connect it to the electroscope.

Calibration of the electroscope This must first be done using the circuit of figure E7(b), applying a p.d. which varies from 0–1500 V across the instrument, and using a pencil to mark the ground glass screen with a scale which indicates the position of the shadow of the leaf when projected from behind. (A small bulb permanently attached to the clear glass 'back' of the electroscope gives consistency in the position of this shadow.) A good leaf will move smoothly at the higher voltages, but the scale is not at all linear and there are quite considerable uncertainties, particularly with the lower readings.

Use of the probe

The complete circuit is as in figure E7(c). A p.d. of 1500 V and plate separation of 0.15 m are suitable. Slide the retort stand along a metre rule taped to the bench from a zero mark determined by touching the probe on to the earthed plate with the flame extinguished – figure E7(d). Take readings of potential at points in the space, in particular along a line joining the plates and at right angles to them.

Potential

The circuit of figure E7(e) may be recalled from Unit B, 'Currents, circuits, and charge', and seen to be equivalent to the present arrangement. We are effectively going to measure the potential difference between point P and point Q which is *earthed*, and therefore at our chosen *zero* of potential. So, as in Units B and C, we can talk about the 'potential at P' rather than the more long-winded 'potential difference between P and 0 V': the potential at P is, say, 500 volts if there is a p.d. of 500 V between P and Q. (Potential can therefore be negative or positive, though it is clearly a *scalar*, not a vector, having, like energy or mass, no sense of direction. In this experiment the potentials will all be positive.)

Start without the flame by first touching the probe on to each plate, and showing that 0 V or 1500 V is indicated. With the probe about half way between the plates only a very small deflection is obtained until the flame is lit. Then move the lighted probe around a surface parallel to both plates: the potential should be unchanged. (The 'biofeedback' obtained by watching the electroscope whilst doing this is probably the best means of achieving the correct movement!) Explore the potential outside the immediate vicinity of the plates. Last and most important, measure the variation of potential with distance along a line perpendicular to the plates. Draw a graph of potential against distance. (It is convenient, though not necessary, to arrange for a potential gradient of 100 V per centimetre.) See figure E8.

Two important concepts emerge from the experiment: *potential gradient* and its relation to field strength, and *equipotentials*.

End effects Measurements of potential with the flame probe very near (about 1 cm from) the plates will yield values almost equal to those on the plates themselves. The ionized air around the flame effectively increases the size of the probe tip making electrical contact with the plate. So there is considerable inaccuracy here, as well as in the indication of the electroscope; nevertheless the graph is almost linear over most of the space.

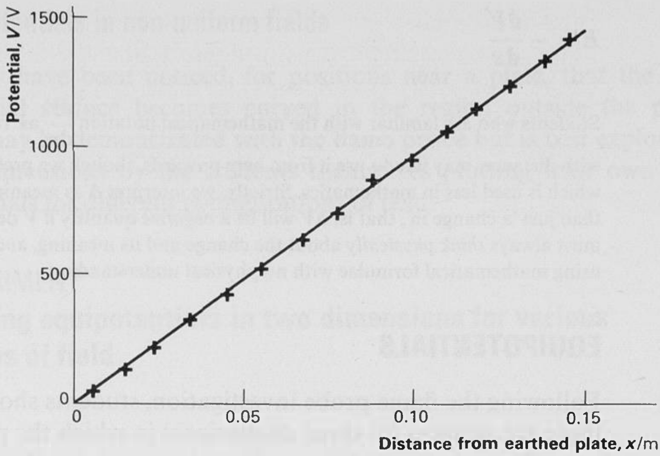


Figure E8
Potential against distance in a uniform field.

Field and potential gradient

Perhaps during the demonstration and certainly on plotting the graph, students will see the linear variation of potential with distance. They can measure the gradient of their graph $\frac{\Delta V}{\Delta x}$ and see that it is nearly equal in magnitude to the ratio of the overall p.d. to the total separation, *i.e.* $\frac{V}{d}$, which should be recognized as equal to the field strength, E, between the plates. Now it can be seen that potential difference/separation gives the electric field strength between the plates, and that p.d. and separation can refer to *any* two points on a perpendicular line joining the plates – not only points on the plates themselves.

This suggests the equation $E = \frac{\Delta V}{\Delta x}$ or, in words, 'field strength equals potential gradient'. Further discussion, based perhaps around question 8, indicates that $E = -\frac{\Delta V}{\Delta x}$ or 'field strength is the negative of potential gradient' is a truer statement and one important enough to commit to memory.

Questions

Question 8 is mentioned above; question 9 is about potentials and electric fields in an electronic tube.

$$E = -\frac{dV}{dx}$$

Students who are familiar with the mathematical notation $\frac{dV}{dx}$ as 'rate of change of V with distance' may like to use it from here onwards, though we prefer the Δ notation, which is used less in mathematics. Strictly, we interpret Δ as meaning 'a gain in', rather than just 'a change in', that is, ΔV will be a *negative* quantity if V decreases. Still, we must always *think physically* about the change and its meaning, and beware of students using mathematical formulae with no physical understanding.

EQUIPOTENTIALS

Following the flame probe investigation, students should be aware that there are surfaces (in three dimensions) in which the potential does not vary. These are called equipotentials, which for the uniform field are equally spaced parallel planes in the region between the plates. Equipotentials every 250 V, say, may be drawn in cross-section to represent the field as in figure E9, or indeed at any (arbitrary) chosen interval. Clearly the potential gradient over any small region can be worked out from these; their spacing is some visual measure of the field strength.

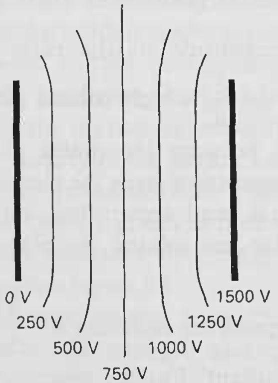


Figure E9
Equipotentials in a uniform field.

The analogy with contour lines on a map may be fruitful here and useful later on. Closely-spaced contours indicate a steep slope and therefore large downhill forces. Also the direction of the force (and therefore the field, in the electrical case) can be seen to be at right angles to the equipotentials (that is, 'down the slope').

Equipotentials in non-uniform fields

It may have been noticed, for positions near a plate, that the equipotential surface becomes curved in the region outside the plates. This may be demonstrated with the flame probe but is best explored in two dimensions by the students themselves plotting their own equipotentials for different configurations of field.

EXPERIMENT

E5 Plotting equipotentials in two dimensions for various shapes of field

ITEM NO.	ITEM
	either
1033	cell holder with 4 cells
	or
59	l.t. variable voltage supply
1064	low voltage smoothing unit
	either
1511	oscilloscope
	or
1507	voltmeter
	pencil or ball-point pen
1153	copper or aluminium sheet, 0.1–0.5 mm thick, 10 mm × 100 mm strips
1155	conducting paper ('teledeltos') cut into approximately A5 or A6 sheets
1155	conducting putty (see notes)
1153	staples and stapler or bulldog clips
1153	carbon paper and white paper, or 'no-carbon-required' paper
1153	drawing board or hardboard
	electrically conductive paint, *e.g.* 'Silverdag' (if available)
1000	leads

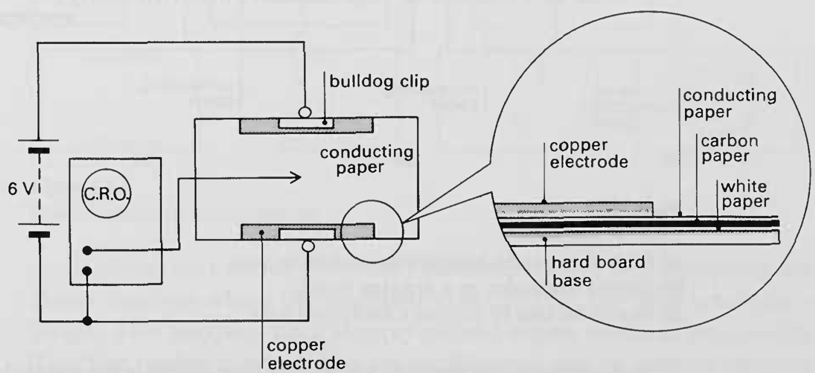


Figure E10
Plotting equipotentials on conducting paper.

A potential difference of 6 V (any p.d. up to about 12 V will do) is set up between two electrodes in good electrical contact with the conducting paper. Potentials are measured using the voltmeter or, preferably, an oscilloscope. A conducting probe can be touched on to the conducting paper surface and the potential at that point read off the oscilloscope. By moving the probe to different positions, points at equal potential are marked on paper beneath and subsequently joined up to reveal equipotentials at chosen intervals.

Good electrical contact with the conducting paper is essential, especially at the edge of the electrode. Colloidal silver paint (*e.g.* 'Silverdag', from RS Components) may be used to paint electrodes directly on to the paper but since this is expensive a means of re-using the arrangement is recommended. A drawing board base is used and a wire may be stapled to the painted electrode at one point only; the contact is 'touched up' using Silverdag, Aquadag, or conducting putty. This is particularly useful if irregular shapes are to be used (figure E11(a)). Using carbon paper or an equivalent, marks are made on white paper beneath rather than on the conducting paper itself, so that many permanent records can be taken from the same set of painted electrodes.

Copper or aluminium electrodes are cheaper. These can be the strips supplied for bending in materials kits or cut from copper sheet used to make electrodes for electrolysis. All electrodes must be cleaned with emery paper before use. If a drawing board base is used the electrodes must be carefully stapled every 2–3 cm to maintain good contact, but they may be placed anywhere on the conducting paper (figure E11(b)). With a hardboard base, bulldog clips holding down the electrodes ensure very good contact but generally restrict the electrodes to the edge of the paper (figure E11(c)). This, however, is the cheapest, simplest method.

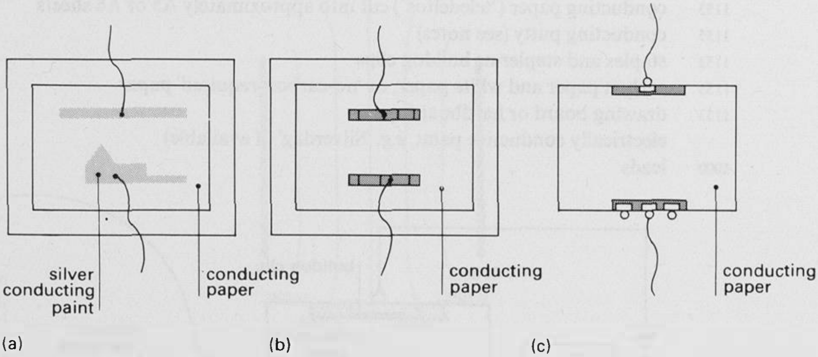


Figure E11
Types of electrodes.
(a) Painted electrodes using silver conducting paint.
(b) Stapled electrodes on a drawing board.
(c) Electrodes held by clips on a hardboard base.

Poor contact at any electrode will produce irregularities in the potential gradient nearby. A thin layer of conducting putty has a resistance very much lower than the paper for most arrangements and ensures better contact if placed under an electrode. A strong drawing pin makes a good 'point' electrode if the rim of its head grips the paper well enough and contact can be made underneath the board. The shaft of a pin is so narrow that it produces a very large potential gradient nearby making equipotential plotting very difficult but providing a very useful teaching point. A clean coin, or a bulldog clip works well.

Probes may be made from graphite pencils, used ball-point pens (either all-metal or with a wire soldered to the metal tip), test probes from a meter, or insulated stiff wire (figure E12). Metallized paper used by some computer printers may replace teledeltos paper.

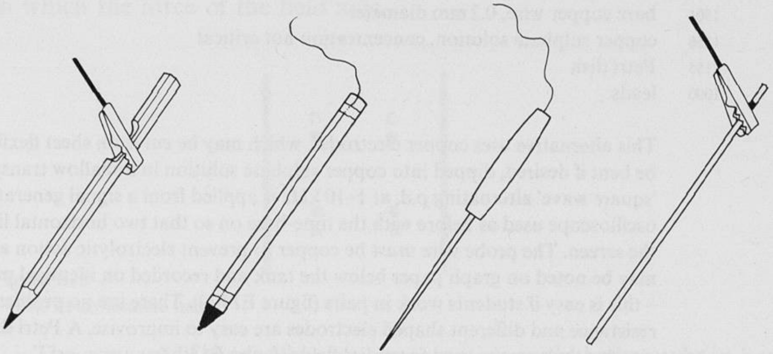


Figure E12
Types of probe.

Students should plot equipotentials at, say, 1 V intervals, for parallel electrodes and at least one other configuration: some suggestions are given in figure E13. From their equipotential patterns they should construct some field lines at right angles and see how the potential gradient varies with distance along these lines, perhaps plotting a graph of this. It is instructive to see how nearly uniform a field between quite small parallel electrodes really is. (This ties in with the computer program 'EFIELD', which might be used now or later.)

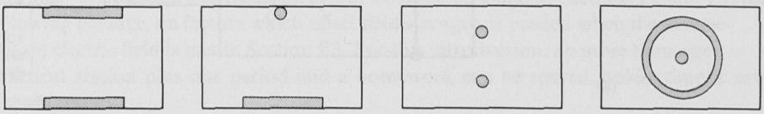


Figure E13
Possible electrode configurations.

Students may notice that if they sketch in several field lines these are closer together where the potential gradient (*i.e.*, the field strength) is larger. This happens near sharply curved edges, corners, and points. Two free probes connected to the oscilloscope can be used to measure the potential difference between *any* two points on the conducting paper and hence deduce the potential gradient.

ALTERNATIVE EXPERIMENT

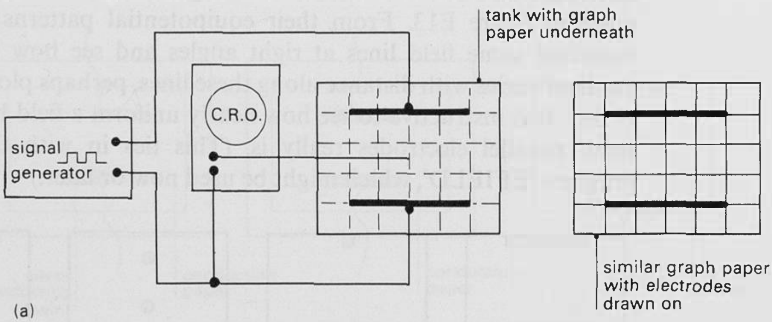
E5a Equipotentials in an electrolytic tank

ITEM NO.	ITEM
1109	signal generator
1511	oscilloscope
1153	rectangular transparent Perspex tank or 'lunchbox'
1153	copper sheet for electrodes
52K	crocodile clips
1501	bare copper wire, 0.2 mm diameter
1156	copper sulphate solution, concentration not critical
1155	Petri dish
1000	leads

This alternative uses copper electrodes, which may be cut from sheet flexible enough to be bent if desired, dipped into copper sulphate solution in a shallow transparent tank. A 'square wave' alternating p.d. at 1–10 kHz is applied from a signal generator and an oscilloscope used as before with the time-base on so that two horizontal lines appear on the screen. The probe wire must be copper to prevent electrolytic action and its position may be noted on graph paper below the tank and recorded on identical paper alongside – this is easy if students work in pairs (figure E14(a)). There are no problems with contact resistance and different shaped electrodes are easy to improvise. A Petri dish or larger circular tank can be used for radial fields (figure E14(b)).

An overhead projector can be used, as in Experiment E3, for a qualitative demonstration, provided students can see the oscilloscope screen as well.

Two probes may be used to measure potential difference, as in Experiment E5.



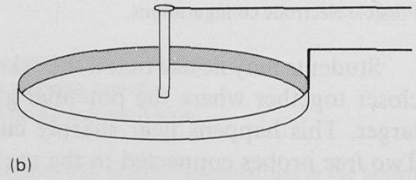


Figure E14
Plotting equipotentials in an electrolytic tank.

Direction of electric field vector

Probes at A and B (figure E15) indicate a certain potential difference. The same p.d. exists between A and C, but the distance AC is greater than AB. Hence the potential gradient and therefore the field strength is less in this direction. So the electric field has components in all directions (except perhaps AD and into the paper) but we say its direction is from A to B. Like the 'fall line' of a slope, this is the direction in which the force of the field acts.

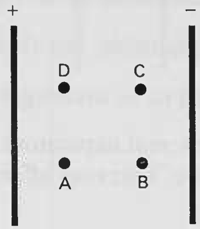


Figure E15
Points in an electric field.

The concept of the field being conservative could be brought in here by a very slight adaptation of the argument in Section E2 (pages 325–6). Also, the gravitational analogy can be exploited further using a rubber sheet to simulate potential gradients, probably for uniform fields at this stage.

Questions

Questions 10 and 11 are about equipotentials and fields.

Timing, sequence

Section E1 should take under 2 weeks. It should not be rushed, as the ideas are difficult and fundamental, but time must be allowed for the work on gravity and circular motion. The ideas of potential and its gradient will be taken up again in Sections E2 and E3; the following passage, on factors which affect field strength, is needed when the inverse-square electric field is met in Section E3. For this introduction, no more than one practical session plus one period and a homework can be spared, unless time is saved elsewhere.

FACTORS AFFECTING THE FIELD STRENGTH BETWEEN PARALLEL PLATES

The electric field between the plates is clearly associated with the charge stored on them. What factors affect how much charge can be stored, and how does this charge affect the field? These questions can lead to reasonable suggestions on the basis of students' present knowledge followed by empirical tests.

V? Parallel plates look, and perhaps act, like a capacitor; it is hardly necessary to test $Q \propto V$, though it can easily be done if Q can be measured. A reed switch or coulombmeter may be suggested.

A? Larger plates can be thought of as smaller plates in parallel and we know, from Unit B, that charge stored adds up in this situation. So $Q \propto A$ is not difficult to justify, and could be easily checked.

d? A smaller separation certainly increases the field strength, for fixed V. Perhaps this arises from more charge being crammed on to the same plate; maybe the closeness of an oppositely charged plate assists this process. $Q \propto \frac{1}{d}$ is plausible, but then so is $Q \propto \frac{1}{d^2}$ for instance, so this one really does need to be investigated.

The medium? Few real capacitors have air between the plates. Many have plastic or paper. This may affect the charge stored, all other factors being constant.

Any other suggestions the students make can be argued through. Although tests on all four factors above are easily made, it is suggested that only the latter pair need be done if time is short.

EXPERIMENT

E6 Investigating factors affecting the charge on parallel plates using a coulombmeter

ITEM NO.	ITEM
1512	coulombmeter, 100 nC
65	2 metal plates with insulating handles
14	e.h.t. power supply
51G	polythene strip for use as insulating handle
503–5	2 retort stand bases, rods, and bosses
501	metre rule
1000	leads

Coulombmeter See demonstration B16, Spooning charge (page 128 of this *Guide*) for a discussion of this instrument – use of high impedance voltmeter, or electrometer/d.c. amplifier.

One of the pair of plates is connected to earth. The other is charged by a flying lead from the e.h.t. positive, the negative being earthed. The flying lead *must* then be removed.

The charge stored is measured by bringing up the coulombmeter so that the probe rod touches this plate. The ratio of capacitances should be such that nearly all ($\approx 99\,\%$) of the charge is given to the coulombmeter. (Students can check this later when they know $C = \varepsilon_0 \frac{A}{d}$ and the value of ε_0; at this stage, discharging the coulombmeter and touching the plate a second time confirms that very little charge is left on it.)

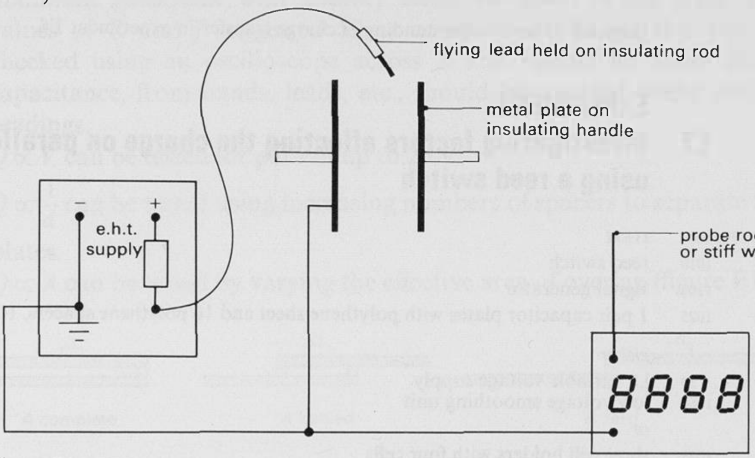


Figure E16
Using a coulombmeter to measure Q.

The effect of a medium (polythene or paper) can be tested by placing a sheet between the plates and repeating the experiment.

The effect of separation is tested using $V \approx 500$ V and $d \approx 5$–25 mm. Large separations may also be tried and certainly Q should be measured for an 'isolated' plate ($d \approx 0.5$ m or more).

Stray capacitance and fluctuations in the coulombmeter reading may complicate matters. Hands well away and fresh batteries (if appropriate) may help, but students should consider the range of uncertainty in their readings, representing these as error bars on a graph. For instance, how accurately repeatable is any given reading; does the coulombmeter faithfully return to zero when shorted; does it absorb all the charge on the plate?

Students should plot a graph of Q against $\frac{1}{d}$ and think carefully about the meaning of the very definite intercept. (Charge stored when d is large.)

This apparatus may also be used to test $Q \propto V$ for p.d.s up to 500 V and $d \approx 10$ mm.

$Q \propto A$ is less easy to check unless plates of various sizes are available.

Experiment E6, as described above, could be done as a demonstration.

Alternatively, it can be done as a class experiment in combination with experiment E7 which uses the reed switch as a charge measuring device, and may be more accurate than this one. All the suggested relationships can be tested in experiment E7.

Question 12 tests understanding of charge transfer in experiment E6.

EXPERIMENT
E7 Investigating factors affecting the charge on parallel plates using a reed switch

ITEM NO.	ITEM
1010	reed switch
1109	signal generator
1025	1 pair capacitor plates with polythene sheet and 16 polythene spacers, 10 × 10 × 1.5 mm
	either
59	l.t. variable voltage supply
1064	low voltage smoothing unit
	or
1033	three cell holders with four cells
1507	voltmeter, 100 V and 10 V
1101	sensitive galvanometer (*e.g.* internal light beam)
1017	resistance substitution box
1511	oscilloscope
501	metre rule
32	mass, 1 kg
1000	leads

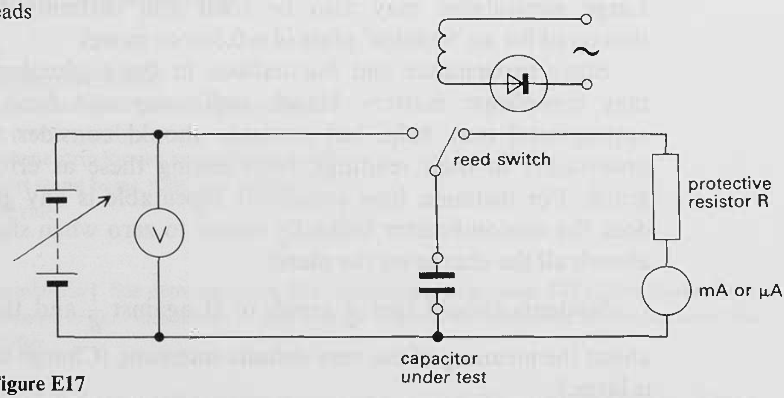


Figure E17
Using a reed switch to measure Q.

The charge stored, Q, is calculated from the signal generator frequency f, and the apparently steady discharge current I using $Q = \frac{I}{f}$. f should be about 400 Hz and R about 100 kΩ. A light beam galvanometer should be used if possible, though a digital meter can be used if it has better than 1 μA resolution. The capacitor plates must not touch whilst the reed switch is working, nor should the switch's operating p.d. be exceeded. The output p.d. of the signal generator should also be the minimum consistent with audibly clean vibration of the reed. The values of R and f suggested allow complete discharge: this can be checked using an oscilloscope across R and raising its value. Stray capacitance, from hands, leads, etc., should be avoided whilst taking readings.

$Q \propto V$ can be tested for p.d.s of up to 25 V.

$Q \propto \frac{1}{d}$ can be tested using increasing numbers of spacers to separate the plates.

$Q \propto A$ can be tested by varying the effective area of overlap (figure E18).

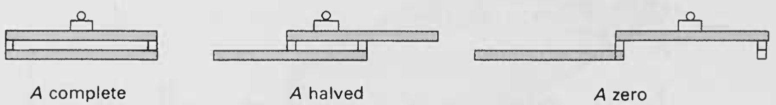


Figure E18
Changing the area of overlap.

The effect of the medium can be tested using polythene and other materials of equal thickness in place of an air gap.

Question 13 is about the reed switch experiment.

Summary of results from Experiments E6 and E7

Graphs such as those of figure E19 can be drawn. A definite intercept indicates that some charge is stored even when a plate is isolated ($d = \infty$), or not overlapping with another plate ($A = 0$). The capacitance of an isolated object is not discussed here, nor can we properly account for the effect of the bench, connecting wires, etc. Q clearly varies linearly with both A and $\frac{1}{d}$ and in the absence of the above effects would be proportional to both, as well as to V.

We may combine these into one relationship:

$$Q \propto \frac{AV}{d} \quad \text{or} \quad Q = \varepsilon_0 \frac{AV}{d}$$

where ε_0 ('epsilon nought') is the appropriate constant for air (strictly, for a vacuum). The effect of the medium is considered in a moment.

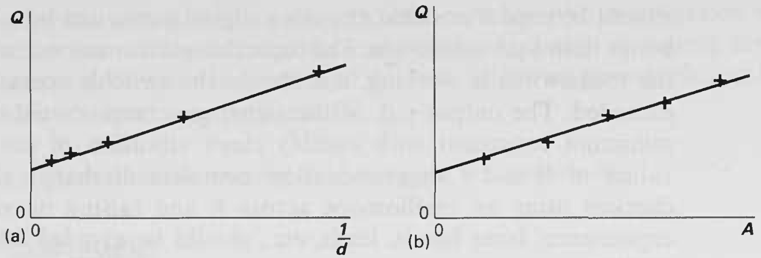


Figure E19
Graphs
(a) Q against $1/d$.
(b) Q against A.

Field and charge density Rearranging the above equation gives

$$\frac{Q}{A}=\varepsilon_0\frac{V}{d}$$

$\frac{V}{d}$ is identified as the field strength E, and $\frac{Q}{A}$ as the surface density of charge, σ ('sigma').

Hence

$$\sigma=\varepsilon_0 E$$

The value of ε_0 can be obtained from the gradient of the best straight line obtained (for example, the slope of the Q against $\frac{1}{d}$ graph will be $\varepsilon_0 AV$). The accepted value is $\varepsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$ (units should also be derived).

Teachers may wonder what effect a decision to assign a value to c, the speed of light *in vacuo*, making it rather than the metre one of the base units of the system of measurement would have on the validity of this 'experiment'. Since we also assign the value $4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$ to μ_0, the permeability of free space, we have implied a value $\varepsilon_0 = 1/c^2\mu_0$. In a sense then, experiments 'to measure ε_0' become experiments to check our metre rules. (See, for example, *Phys. Educ.*, **19**, 215, 1984.)

Meaning of ε_0 The accepted term 'permittivity of free space' suggests that ε_0 is some property of a vacuum. Here we understand it as defining a fundamental relationship between the charge density on a plate and the field around it, and it is a universal constant in the sense that this relationship is independent of other conditions (for example, material of plates, actual values of σ, E, etc.).

Effect of medium The medium increases the charge stored for a given field strength $\left(\frac{V}{d}\right)$ by some factor. Since it is a simple ratio it has no units and is referred to as the relative permittivity, ε_r. Our equation above then becomes

$$\sigma = \varepsilon_r \varepsilon_0 E$$

For air ε_r is almost unity, for plastics about 2–3, for water about 80. Students might be interested to find out why ε_r varies, and perhaps read about 'polarization' and induced charges. Measurement of ε_r for water proves rather tricky. A neat way of doing it for paper uses two sheets of foil sandwiched between pages of a book (figure E20).

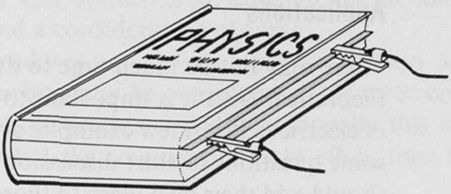


Figure E20
Measuring ε_r for paper.

Capacitance Another rearrangement of the relationship between Q, A, V, and d is

$$\frac{Q}{V} = \varepsilon_0 \frac{A}{d}$$

$\frac{Q}{V}$ stands out as the capacitance C of the arrangement. Including the effect of the medium we then have

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

This is a useful expression which enables the capacitance of the plates in either experiment to be estimated (neglecting the extra effects due to charge stored where either A or $1/d$ is zero). In this context also the units of ε_0 appear in the more palatable, but equivalent form of $\mathrm{F\,m^{-1}}$.

$$\mathrm{F\,m^{-1}} = \frac{\text{farad}}{\text{metre}} = \frac{\text{coulomb}}{\text{volt} \times \text{metre}} = \frac{\text{coulomb} \times \text{coulomb}}{\text{joule} \times \text{metre}}$$

$$= \frac{\text{coulomb}^2}{\text{newton} \times \text{metre} \times \text{metre}} = \mathrm{C^2\,N^{-1}\,m^{-2}}.$$

Home experiment

Home experiment EH1 challenges students to construct, at home, from readily obtainable materials (for example, cooking foil and paper), a capacitor of about the size of a matchbox. A prize might be given for the highest capacitance to volume ratio. The capacitance can be measured using the reed switch or a reactive bridge circuit; students should use the relationships developed in this Section to estimate the capacitance beforehand.

Questions

Question 14 is about a parallel plate arrangement.
Question 15 is about capacitor construction.
Question 16 tests understanding of the parallel plate relationships.
Questions 17 and 18 are review questions.

Applications

There may not be much time to dwell on these in class, but in a largely theoretical Unit it is important to mention the breadth of applications of electric fields. A few examples are included in the *Students' guide* with some questions to start discussion and thinking. Teachers and students should add their own ideas to those presented. Topics for reading might include the following:

1 The Xerographic process.
2 The 'ink jet' printer.
3 Electrostatic generators.
4 Electrostatic dust precipitator.
5 The capacitor microphone.
6 The electrostatic loudspeaker.
7 Electrostatic paint and crop spraying.
8 Thunderstorms.
9 A capacitor 'tiltmeter'.
10 Polarization, dielectrics, and design of capacitors.
11 Field plotting and design of cathode ray tube electrodes.
12 Problems caused by 'static' in everyday life.
13 Generation and use of electricity by fish.
14 Electrostatic prospecting.
15 Ionizers and their effects.
16 Electrets and their uses.

SECTION E2

GRAVITATIONAL FIELD AND POTENTIAL

INTRODUCTION

The approach here is to develop the idea of gravitational potential difference and its relationships to energy and field in the context of the uniform field at the Earth's surface. Then the gravitational inverse-square law is introduced and is shown to describe the way the field strength of the Earth varies with distance. Gravitational potential difference is explored in this context and leads to the definition of gravitational potential and a consideration of energy.

A separate part introduces, or revises, circular motion so that satellites and planetary orbits may be considered. This piece could be studied before the rest of the Section (indeed, historically this was the order Newton followed) but the treatment should allow time for the difficult concepts of gravity in a $\frac{1}{r^2}$ context to be understood.

The Section emphasizes physical thinking rather than a potentially more concise mathematical approach, in the belief that such thinking is essential to the work of a physicist at any level. The computer is used as a versatile calculating and graphical tool.

Students have arguably more experience of gravity than of electricity and may even have met the inverse-square law in their pre-A-level studies. Beginning with the uniform field helps strengthen the links between electricity and gravity which are heavily used in Section E3, for the inverse-square field.

A UNIFORM GRAVITATIONAL FIELD

The definition of gravitational field strength (g) as the force on unit mass ($N\,kg^{-1}$) and a way to measure it should emerge readily, and a short journey around the laboratory with a kilogram on a force meter shows the field to be uniform there.

Gravitational potential energy

The field strength around the kilogram is still the same if it is raised, say, by 1 m. But its gravitational potential energy has increased. Objects of different masses (students standing on a bench for instance) clearly gain different amounts of energy when raised by 1 m, though one kilogram of each gains just the same energy (about 10 J).

Gravitational potential difference

In this or some other, preferably visual, way the usefulness of the idea of potential energy per kilogram can be brought out. Marks at 1 m intervals on the wall could be labelled in steps of $10\,\mathrm{J\,kg^{-1}}$ and some quick calculations made of energy changes for moving different objects. Broadening the discussion, marks at 10 m or 100 m vertical intervals around the countryside can be seen to give contours, as on a map. Since these link points where a mass has the same potential energy they can be called gravitational *equipotentials*.

Some electrical examples might be considered to reinforce the analogy with gravity and the idea of a zero for gravitational potential energy could be discussed (the laboratory floor? sea level?). This is a useful place to introduce *negative* potential energy, particularly if a basement or lower floor is available.

Summary

Gravitational potential difference between two points is the change in gravitational potential energy when 1 kilogram moves between the two points.

Questions

Questions 19 and 20 introduce changes in gravitational potential energy and the concept of gravitational potential difference.
Question 21 uses the idea.

THE INVERSE-SQUARE LAW FOR GRAVITATIONAL FIELDS

Visual aid

It is useful to represent the Earth by some spherical object (for example, a football) during the following discussions, and to try to illustrate the examples visually.

Newton's inverse-square law for the gravitational force, F, between two masses, m_1 and m_2, a distance r apart will be a new idea for some, a reminder for others.

$$F = -G\frac{m_1 m_2}{r^2}$$

G is a universal constant (value 6.67×10^{-11} N m^2 kg^{-2}), and the negative sign indicates a vector *inwards, i.e.*, an attractive force. If $m_1 = M$, a large mass (for example, the Earth) and $m_2 = m$, a smaller 'test' mass nearby, then since

$$g = \frac{F}{m}$$

this leads to

$$g = -\frac{GM}{r^2}$$

as the expression for the field strength of a mass M.

Teachers may be surprised to see negative signs in the expressions for F and g. This is consistent with the treatment in Unit A, where attractive forces between molecules were considered negative and repulsive forces positive. Of course, it is more important for students to understand whether forces are attractive or repulsive, whether potential energy decreases or increases for movement in a given direction, than to learn rules about sign conventions. See also the note about signs on page 319.

Background to Newton's work

This is not part of the course, but students might be interested in the unfolding of the inverse-square law and the contributions of Galileo, Kepler, and others: it is a marvellous example of the process of building and refining a model. The puzzle of how 'action at a distance' could occur remained intriguing to Newton, but his mathematical scheme, which so accurately described the motions of heavenly bodies, went so far to fulfil Kepler's vision of 'a celestial physics based on causes' that we see here perhaps the beginnings of modern science.

Reading

BRONOWSKI *The ascent of Man.* Chapter 7.
COHEN 'Newton's discovery of gravity'. *Scientific American.*
KOESTLER *The sleepwalkers.*
ROGERS *Physics for the inquiring mind.*

Measurement of *G*

This is painstaking and difficult, because of the small size of the constant. Reference to Cavendish's experiment of 1798 may be made and students may watch a modern version on video, taking results from the screen to find their own value of G. This, however, is not an essential part of the Unit.

Television and video

'The determination of the Newtonian constant of gravitation' in the Granada television series *Experiment: Physics.*

Question 22 is based on Cavendish's experiment.

Testing the inverse-square law using spaceflight data

Newton's explanation of the motion of the Moon was a triumph for the inverse-square law. Nowadays, scientists are so confident that it holds over a vast range of distances to a high degree of accuracy that it is used to plan the trajectories of spacecraft. Students can use data from one such spaceflight to verify that the inverse-square law does apply.

Questions

From the data in question 23 students deduce the acceleration of the spacecraft and equate this to the gravitational field strength. ($a = \frac{\text{force}}{\text{mass}}$, from Newton's Second Law; and $g = \frac{\text{force}}{\text{mass}}$, from its definition.) This equivalence may need to be stressed beforehand.

In question 24, by taking a series of pairs of points, g can be found at different values of r. A graph of g against $\frac{1}{r^2}$ gives a straight line (figure E21). Note that both acceleration and field strength have negative values.

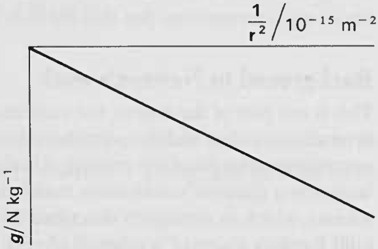


Figure E21
$g \propto -1/r^2$.

GRAVITATIONAL POTENTIAL DIFFERENCE, ΔV_g, AND GRAVITATIONAL POTENTIAL, V_g

Timing, structure, and sequence of this work

This work is central to the Section and will take a few lessons based either on the computer program 'GFIELD', and/or on the series of structured learning questions (25–29) in the *Students' guide*. Students without access to a computer must not be disadvantaged and all the results they need are reproduced in the questions. Those with a computer should use it to perform the tedious tasks such as repetitive calculations; the students themselves, or with the teacher, have to decide *what* calculations to do and *which* graphs to plot, so they need to be thinking about the physics all the time.

First the means of calculating ΔV_g from a g against r graph (by adding areas of strips under it) is introduced. Then the computer may be used to explore how accurate the method may be (question 25). The energy needed to launch spacecraft may be calculated (question 26). Values of ΔV_g for escape from various positions (question 27) lead to the concept of gravitational potential V_g (question 28) and a precise expression for it (question 29). Then data from an actual spaceflight are used to confirm this expression (question 30).

Calculating energy changes from force–distance graphs

Students should remember that change in energy = force × distance, and that this can be applied to a uniform field, where the force is constant. If the force does vary then the energy can be deduced from the area under a force–distance graph (as for a spring in Unit A, 'Materials and mechanics'; see figures E22 and E23).

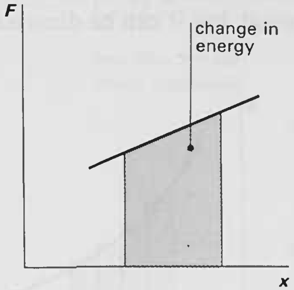


Figure E22

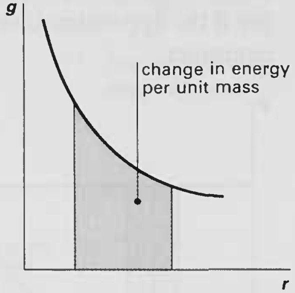


Figure E23

Since field strength, g, is force per unit mass, then the area under a g against r graph will be the change in energy per unit mass. This was previously introduced as the gravitational potential difference to which we now give the symbol ΔV_g (anticipating the later use of V_g for gravitational potential).

Integration

Mathematicians may suggest that integrating an expression for field strength with respect to r between specified limits will yield a value for ΔV_g. This can be done now or later (Appendix VI) but care must be taken to sort out the signs and limits to get the correct answer. We proceed with a longer but more physical argument which we believe will be of greater benefit to understanding. As it involves areas under graphs and (later) gradients, students who are not familiar with computing these might be given practice beforehand.

Sign of g

The computer program, and the argument here, use a g against r graph *above* the r axis, *i.e.*, depicting g as a *positive* quantity. We feel that this simplifies the consideration of adding up areas *under* the graph. In any case, at first, we are interested only in gravitational potential *difference*, ΔV_g and not in the sign of potential. When this is introduced later, students may be reminded that g is negative (indicating an attractive field) as indeed it must be to give negative potential.

Calculating ΔV_g from a g against r graph

Figure E24(a) shows that, for a uniform field, $\Delta V_g = g\Delta r$.

Where g varies with r, as in figure E24(b), we can consider small steps, Δr, over which g remains roughly constant. The sum of the areas of all the strips $g\Delta r$ as shown approximates to the gravitational potential difference between two limits and is almost equal to the area under the graph between those limits. Adding up such areas is a tedious job if the approximation is to be good, but it can be done easily using a computer.

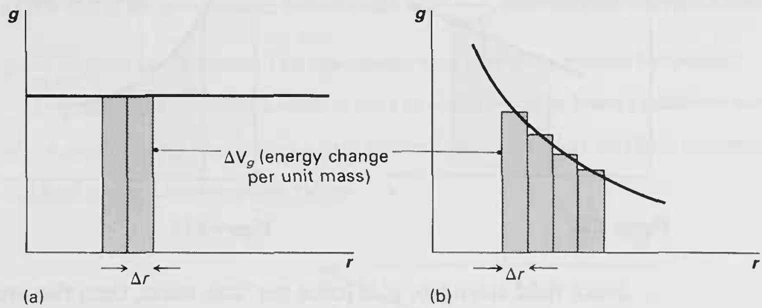


Figure E24
ΔV_g from field–distance graphs.

Using the computer program 'GFIELD'

Students are now ready to use the computer program, and to work through the questions based on it. (More information about the program itself is given in the *Notes* accompanying the package 'Software for Nuffield Advanced Physics', Longman's Micro Software.)

Questions

Question 25 shows how calculations of gravitational potential difference between the Earth's surface and a distance of 50×10^6 m are used to try out different step sizes, or values of Δr (figure E25).

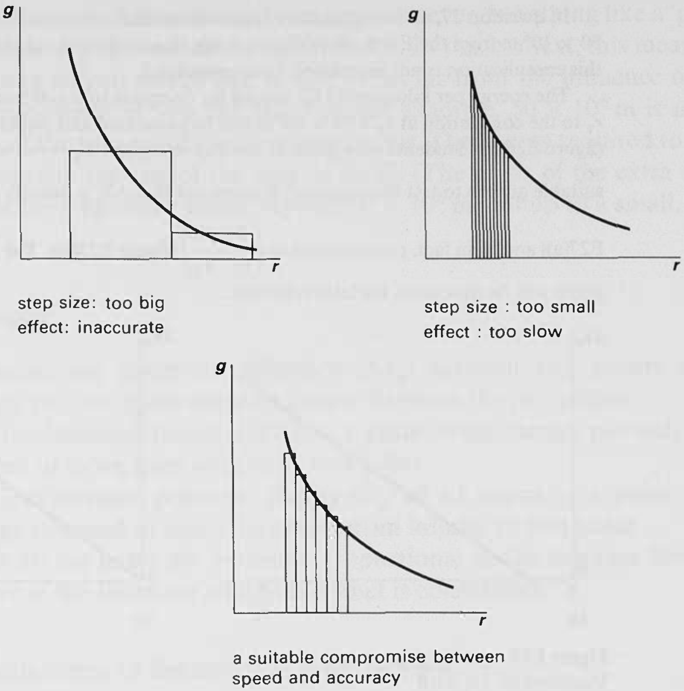


Figure E25
Effect of step size on calculation of ΔV_g.

Question 26 shows how the launch of satellites from a fixed value of r_1 (R, the radius of the Earth) to variable distance r_2 can be explored (figure E26(a)).

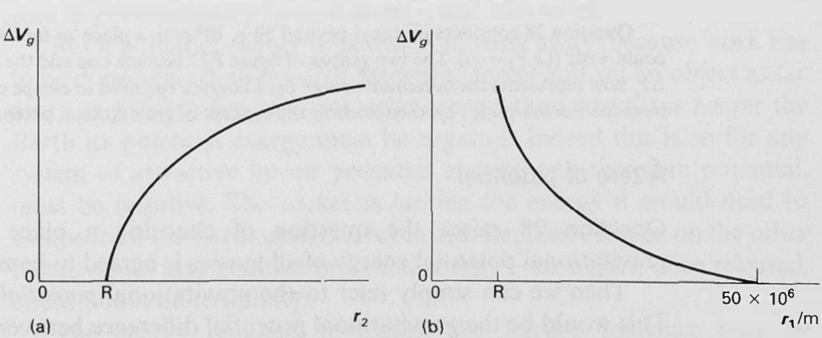


Figure E26
Potential energy change per kilogram for journeys:
(a) from the Earth's surface (R) to variable distances r_2;
(b) from variable r_1 to 50×10^6 m from the Earth's centre.

In question 27, an interplanetary space convention is to be held on a satellite 50×10^6 m from the Earth. (Strictly speaking, this satellite should be 'massless', though this complication is not mentioned in the question.)

The energy per kilogram (ΔV_g) needed for delegates to travel from various distances r_1 to the convention at $r_2 = 50 \times 10^6$ m can be calculated and displayed graphically (figure E26(b)). Students may guess at the way in which ΔV_g varies with r_1 and plot suitable graphs to test their guesses. It turns out that ΔV_g is linearly related to $\frac{1}{r_1}$ (figure E27(a)) and is, in fact, proportional to $\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ (figure E27(b)). The gradient of either graph can be measured, for later reference.

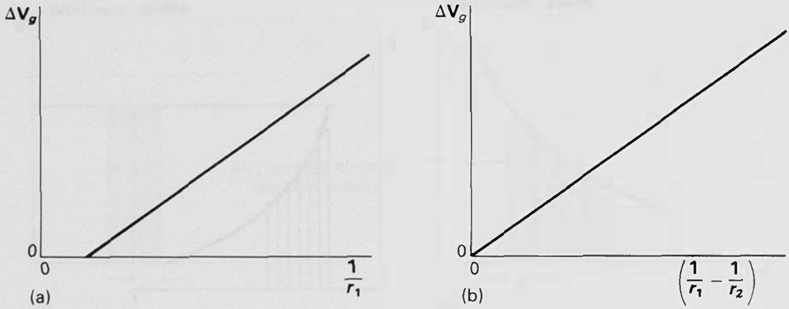


Figure E27
Variation of ΔV_g with
(a) $\frac{1}{r_1}$
(b) $\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Question 28 considers journeys beyond 50×10^6 m to a place 'as far away as one could wish' (*i.e.* $r_2 = \infty$). The two graphs of figure E27 become one and the same, and ΔV_g now represents the minimum energy per kilogram required to escape completely from the Earth's pull. This then leads to the concept of gravitational potential.

A zero of potential

Question 28 raises the question of choosing a place where the gravitational potential energy of all masses is agreed to be zero.

Then we can simply refer to the gravitational *potential* at a point. This would be the gravitational potential difference between that point and our chosen zero, that is the energy required to bring 1 kilogram from our zero to the point in question. (Remind students of earlier discussions about electrical potential and electrical potential difference.)

We might regard the Earth's surface (sea level) as a suitable zero. However, in dealing with situations involving other planets as well it is most helpful to regard some neutral, far away point as our zero. The only place which is sufficiently neutral, however, is nothing like a 'point' at all but a 'place' which we call 'infinity'. In this context, this means 'as far away as you would like to go', or 'as far from the influence of any massive object as you can be'. For our example, 50×10^6 m is nearly 90 % of the way to infinity in energy terms. The energy required to push 1 kilogram 'the rest of the way' is small. (The areas of the extra strips under the g against r graph beyond 50×10^6 m add up to a small, finite limit.)

Summary

Gravitational potential difference (ΔV_g) between two points is the energy per unit mass needed to move between the two points.

Gravitational potential (V_g) at a point is the energy per unit mass needed to move from infinity to that point.

Gravitational potential energy (E_p) of an object at a point is the energy required to move the object from infinity to that point.

Both the latter are inversely proportional to the distance from the centre of the Earth (or whatever planet is considered).

Consequences of the convention for zero potential

Up to now in dealing with ΔV_g we have been concerned only with *changes* in gravitational potential, worrying little about the signs of those changes. We can, and indeed should, always refer to the physical situation: moving away from the Earth 'requires' energy, so constitutes a *gain* in gravitational potential energy, and vice versa.

But if potential energy is gained in moving away (because work has to be done against an inwards attracting force) and yet an object as far away as possible has zero potential energy, then when it is nearer the Earth its potential energy must be *negative*. Indeed this is so for any system of attractive forces: potential energy, and therefore potential, must be negative. The rocket is *lacking* the energy it would need to escape from the Earth until fuel is burned. Repulsive forces, on the other hand, would give positive potential energy – an object, once released, could 'shoot off to infinity'.

The energy required to escape the Earth's influence may be calculated, but first a precise expression for the gravitational potential at a point must be established.

Questions

In question 29, the expression for ΔV_g obtained from the graphs of question 27 is compared with that found from mathematical integration (Appendix VI) and seen to be identical. It follows that the gravitational potential at distance r from mass M is given by:

$$V_g = -\frac{GM}{r}$$

In question 30 data from an actual spaceflight confirm the above expression for V_g.

FIELD AND POTENTIAL GRADIENT

Since graphs of gravitational potential against distance can now be drawn it is useful to remind students of the relationship between field strength and potential obtained for the uniform electric field in Section E1.

$$E = -\frac{\Delta V}{\Delta x}$$

The corresponding relationship for gravity is

$$g = -\frac{\Delta V_g}{\Delta r}$$

This has been met before in the uniform field situation ($\Delta V_g = -g\Delta r$) but it can also be applied to the inverse-square field if we consider small enough steps Δr. In the limit, as Δr becomes infinitesimal, we can write:

$$g = -\frac{dV_g}{dr}$$

using $\frac{dV_g}{dr}$ to represent the rate of change of potential with distance or 'potential gradient'. The field strength can therefore be found easily from a V_g against r graph by measuring the gradient at any point. Similarly, the precise relationship for electricity $\left(E = -\frac{dV}{dr}\right)$ applies *however* the field varies.

Students should note here again that $g = -\frac{dV_g}{dr}$ holds for all values of r, and that this simplifies to $g = -\frac{\Delta V_g}{\Delta r}$ if the field is uniform. However, it is always a mistake to find g from a V_g against r graph by dividing the value of V_g at a point by the value of r (although this happens to give the same result for $\frac{1}{r}$ graphs).

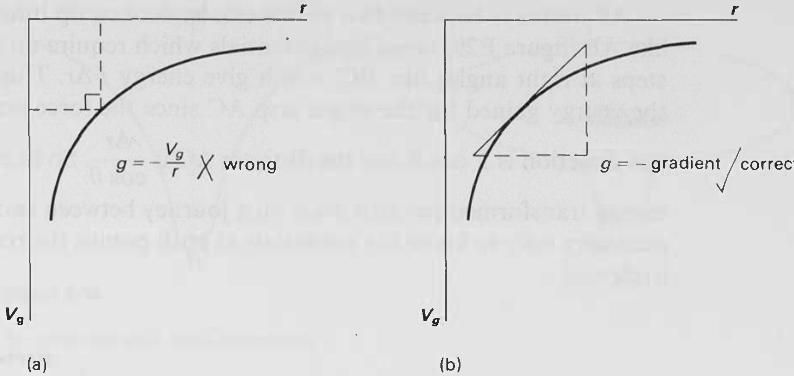


Figure E28
(a) Wrong, (b) right way to find field from a potential–distance graph.

Questions

Question 31 is mainly about escaping the Earth's pull.
Question 32 involves the *total* potential of the Earth–Moon system.

Formulae and relationships

A set of gravitational formulae and the links between them is given in the *Students' guide*, figure E21. This is worth looking at now so that students become familiar with the formulae and understand the links between them.

Equipotentials around the Earth

It is also worth looking at the drawing of equipotentials around the Earth (*Students' guide*, figure E18). This brings out visually the $\frac{1}{r}$ variation in potential contrasting with the uniform potential gradient of a uniform field (for example, the contours of question 19).

A conservative field

We have assumed that the kinetic energy 'lost' by a space probe on the way up may be fully regained on its return, regardless of the route taken. Perhaps this needs to be justified.

All journeys between two points can be broken up into small steps like AB (figure E29) *along* equipotentials which require no energy, and steps at right angles like BC, which give energy $F\Delta r$. This is equal to the energy gained for the single step AC since the force component in this direction is $F \cos \theta$, but the distance AC is $\dfrac{\Delta r}{\cos \theta}$. So to calculate the energy transformed per unit mass on a journey between two points it is necessary only to know the potentials at both points: the route taken is irrelevant.

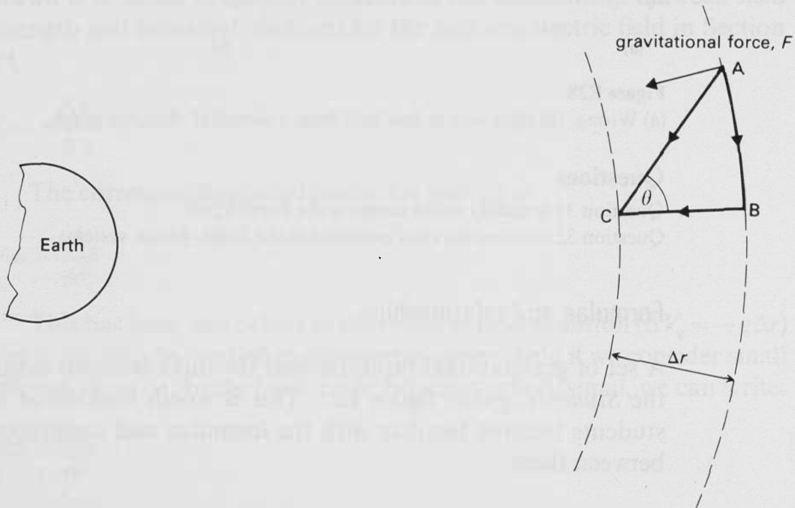


Figure E29
A conservative field.

The argument of course assumes that no other forces (such as friction) act on the mass. Also the force does not have to be inverse-square for it to work: any radial field would be conservative.

A model potential well (or hill)

Drawings of equipotentials are useful, two-dimensional visual models, but a tangible model is perhaps more useful still. A graph of potential against distance has a $\frac{1}{r}$ profile. Extending this to another dimension gives the $\frac{1}{r}$ potential well or upturned 'Chinese hat' (figure E30).

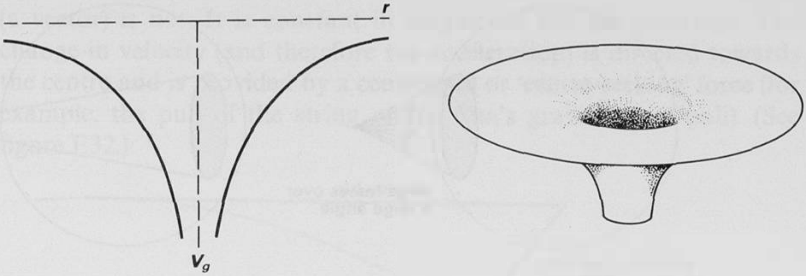


Figure E30

A $\frac{1}{r}$ potential well: the 'Chinese hat'.

Its profile is part of a curve whose depth varies as $\frac{1}{r}$ where r is the distance from the centre. The potential energy of a marble rolling in the well is proportional to its depth, and therefore to $-\frac{1}{r}$. Therefore the marble models the behaviour of an object moving in a gravitational field (at least in two dimensions). Paths of comets near the Sun are easily simulated; to demonstrate elliptical planetary orbits requires more skill – spin and friction are clearly complications. Students should realize that the *slope* of the well at any point indicates the field strength there.

The energy of the system is clearly negative, the marble being 'trapped' in the well, lacking the energy required to escape. This anticipates the use of the well to model the hydrogen atom in Unit L, 'Waves, particles, and atoms'. Positive potentials are easily simulated by inverting the well, turning it into a hill. This will be used to model the paths of α-particles near a nucleus in Unit F, 'Radioactivity and the nuclear atom'.

Home experiment EH2 gives directions for making a $1/r$ shaped hill or well from paper. A more flexible model can be made using a thin rubber sheet fixed to a circular frame and pulled down at a point. Positive potentials (useful later for electricity) can be obtained by pushing up and the combined potential of two bodies (for example, the Earth and Moon) can be easily simulated.

Field outside a solid sphere

The inverse-square law relates to point masses, but we have seen it work close to the Earth, which is large and approximately spherical. This is far from obvious for a good deal of the Earth is closer than the centre (though pulling at a wide angle) and a good deal is further away (though pulling more directly) (figure E31).

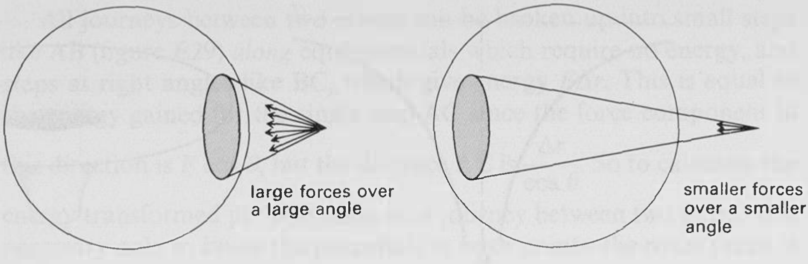


Figure E31
Effect of a large sphere.

When all the contributions to the field are added it turns out, that, if there is spherical symmetry, the overall effect is the same as if all the mass were concentrated at the centre. This result, first derived by Newton himself, is important since it applies to all inverse-square fields. It is assumed throughout this and the next Section.

Details of the integration are not important here, though it appears in some textbooks, and some students might like to see it.

Field inside a sphere

This is an interesting, if theoretical, problem for gravity but one of vital importance when electric fields are considered. Question 33 uses σ for the 'surface density' which can apply equally well to mass or charge. A few other small changes convert the question to the electrical situation. As written, though, it leads on to other interesting 'underground' ideas and applications (questions 34 and 35).

Circular motion

REVISED NUFFIELD PHYSICS *Year 5* contains a fairly full treatment of this topic, so it will be revision for many. A physical understanding of the ideas is vital and a wide range of applications should be discussed. The importance of orbital motion in leading towards Newton's Law of gravitation may have emerged from reading; Newton's triumph was to show that the most general orbit under an inverse-square law is an ellipse. For many purposes circular motion is a good approximation, and the mathematics is certainly simpler.

CENTRIPETAL FORCE

'What force keeps the Earth in orbit around the Sun?' 'What force causes a car to turn a corner?' 'What force causes a ball on a string to move in a circle, not a straight line?' Such questions can introduce the discussion. In all these cases the speed may be constant but the velocity (a vector) is not. It is constant in magnitude but not direction. The change in velocity (and therefore the acceleration) is directed towards the centre and is provided by a centripetal or 'centre-seeking' force (for example, the pull of the string or the Sun's gravitational pull). (See figure E32.)

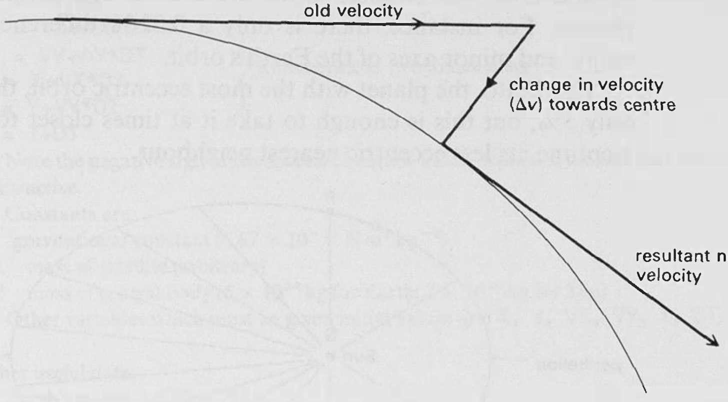


Figure E32
Constant speed, changing velocity.

Centrifugal force

This concept is only valid when the situation is viewed from a rotating frame of reference and whilst used by the layman and some engineers is only likely to confuse students; it is best avoided. If it does arise, students can be asked 'on what object does it act?' for the key point is that any object moving in a circle requires an overall *inwards* force, not an outwards one.

Question 36 applies Newton's Laws to circular motion.

Formulae for centripetal acceleration and force

Question 37 leads to the $\frac{mv^2}{r}$ formula for centripetal force in stages; the last of these (part **d**) is a more familiar derivation. Since for gravity a negative sign was used to denote an attractive, inward force $\left(\text{for example, } F = -G\frac{m_1 m_2}{r^2}\right)$, the same convention is used here $\left(\text{centripetal force} = -\frac{mv^2}{r}\right)$. This convention may not be used elsewhere, but students should always be aware of the direction in which a force acts when using mathematics to solve a physical problem.

Questions

Question 38 is about geostationary satellites.
Question 39 is about energy of satellites.
Question 40 links Kepler's Third Law of planetary motion with the inverse-square law.

Elliptical orbits

Kepler, from the observations of Brahe, deduced that the orbits of planets were elliptical, with the Sun at one focus. Newton later demonstrated that this was the expected orbit if the force was inverse-square. However, circular orbits are a close approximation for most planets. For instance, there is only a 0.02 % difference between the major and minor axes of the Earth's orbit.

For Pluto, the planet with the most eccentric orbit, this difference is only 3%, but this is enough to take it at times closer to the Sun than Neptune, its less eccentric nearest neighbour.

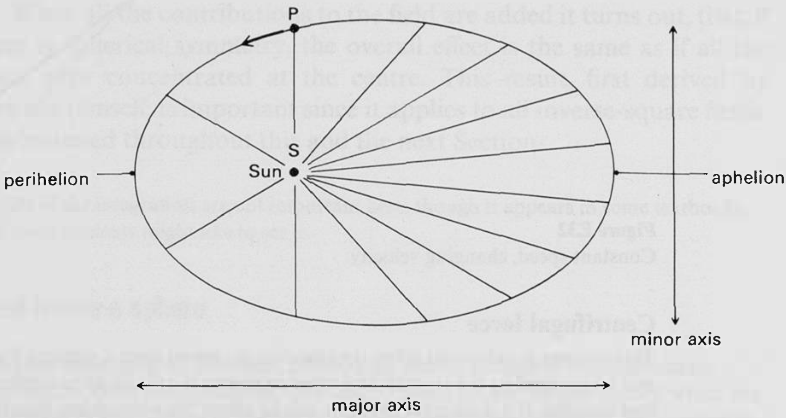


Figure E33
An elliptical orbit.

Since the total energy of a planet (potential and kinetic) is constant, a gain in P.E. caused by moving further away must be offset by a loss in K.E. Hence a planet moves more slowly near aphelion than perihelion (figure E33). This is expressed by Kepler's equal area Law (the line PS traces out equal areas in equal times).

Question 39 will have shown that the total energy is negative, as in any bound system (for example, the electron in the hydrogen atom: Unit L, 'Waves, particles, and atoms').

Computer

Dynamic modelling system
Satellite orbits and space probe trajectories can be modelled using the system. The ingredients needed to solve such a problem are as follows:
Newton's Law of Gravitation
Newton's Second Law } each in two dimensions
Kinematics }

A suitable set of equations is:

```
R = SQR(X^2+Y^2)        displacement
F = -G*M1*M2/R^2        gravitation
FX = F*X/R             } x, y components of force†
FY = F*Y/R             }
AX = FX/M1             } Newton II
AY = FY/M1             }
VX = VX+AX*DT          }
VY = VY+AY*DT          } kinematics, in two dimensions
X = X+VX*DT            }
Y = Y+VY*DT            }
T = T+DT
```

Note the negative sign in the second equation which expresses the fact that the force is *attractive.*

Constants are:

G gravitational constant ($6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$)

M1 mass of satellite (arbitrary)

M2 mass of central body (6×10^{24} kg for Earth; 2×10^{30} kg for Sun)

Other variables which must be given initial values are: X, Y, VX, VY, T, DT.

Other useful data:

1 Earth's radius $= 6.37 \times 10^6$ m.

2 A satellite launched tangentially at the Earth's surface with a speed of $8000\,\mathrm{m\,s^{-1}}$ will go into approximately circular orbit with a period of about 90 minutes. (Suitable starting values are X $= 6.4 \times 10^6$ m; Y $= 0$; VX $= 0$; VY $= 8000\,\mathrm{m\,s^{-1}}$; T $= 0$, DT $= 300$ s.)

3 Radius of Earth's orbit around Sun $= 150 \times 10^9$ m.

$$\text{Speed of Earth in its orbit} = \frac{2\pi \times 150 \times 10^9\,\mathrm{m}}{365 \times 24 \times 60 \times 60\,\mathrm{s}} \approx 3 \times 10^4\,\mathrm{m\,s^{-1}}.$$

By changing only one equation (and of course the starting values) this set of equations can be used to model the behaviour of an alpha particle near a nucleus, so it is well worth saving for use later. For more information see the booklet *Dynamic modelling system.*

†x and y components of force:
$F_x = F\cos\theta$; $F_y = F\sin\theta$; $\cos\theta = x/r$; $\sin\theta = y/r$ so $F_x = Fx/r$; $F_y = Fy/r$ (figure E34)

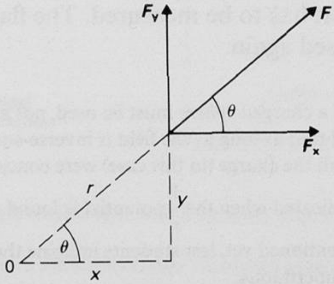


Figure E34

SECTION E3
THE ELECTRICAL INVERSE-SQUARE LAW

Having started with the uniform electric field between parallel plates and then moved on to the gravitational field and potential near a mass, the Unit ends with the electric field and potential around a point charge. The relationship between field strength and potential has already been developed: in particular, students should know that a $\frac{1}{r^2}$ field has a $\frac{1}{r}$ potential. Experimental tests of Coulomb's Law are often tricky; it is easier to explore the potential around a charged sphere using the flame probe. The potential varies as $\frac{1}{r}$; therefore the field strength must be inverse-square.

The importance of forces between charges

If this course were not concerned at all with atomic physics, we could stop at the uniform electric field. However, in this and other Units (F and L) we explore further the size, structure, and energies of atoms. In doing so we treat protons and electrons as point charges; therefore we must know how the electric field and potential around them vary.

POTENTIAL NEAR A CHARGED SPHERE

The shape of the field around a point electrode may be recalled from Experiment E3. Students may see that equipotentials will be spherical around a point charge but to explore the variation of potential with distance it has to be measured. The flame probe used in Experiment E4 can be used again.

In practice a charged *sphere* must be used, not a *point* charge. In the last Section it was established that as long as the field *is* inverse-square, the effect outside the sphere is the same as if all the charge (in this case) were concentrated at the centre. This assumption will be vindicated when the $\frac{1}{r}$ potential is found and the $\frac{1}{r^2}$ field deduced, but is perhaps best not mentioned yet, lest students imagine the argument to be circular or the test of potential superfluous.

DEMONSTRATION

E8a Investigating the variation of potential around a charged sphere using a flame probe

ITEM NO.	ITEM
1053	plastic football (see below for means of support)
10Y	colloidal graphite
14	e.h.t. power supply
51A	gold leaf electroscope
94A	lamp, holder, and stand
27	transformer
52K	crocodile clip
	flame probe (see experiment E4)
501	metre rule
1000	leads

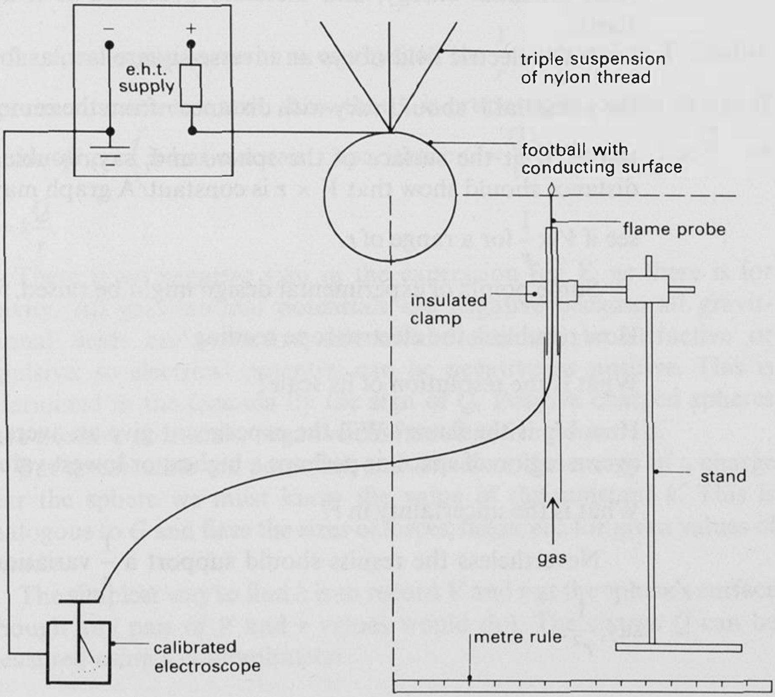


Figure E35
Using a flame probe to explore potential near a charged sphere.

The flame probe is set up and the electroscope calibrated as in experiment E4. Here the electroscope indicates the p.d. between the tip and distant walls, floor, bench, and ceiling which are at earth potential. The ball should be suspended as far as possible from these surfaces using a triple nylon suspension. It is painted with colloidal graphite or covered *smoothly* with aluminium foil. (The radius, about 0.1 m, can be deduced using string or a shadow method.) A metal screw in the valve socket can provide both support and electrical contact. A potential of $+1500$ V applied to the sphere is suitable.

The potential at the surface of the sphere can be measured by touching with the *unlit* probe. A ruler scale allows the potential to be measured at various distances from the centre, with the flame lit. (Take care not to melt the ball when taking close measurements.) A vertical flame reduces lateral uncertainty in the probe's position.

First the probe can be used to show that equipotentials are concentric spheres. It is easiest to do this by watching the electroscope and moving the probe so that the reading remains steady. As for gravity, there is no force component along the spherical surface so no change of energy (see 'A conservative field' page 325). The potential increases as we get closer to the sphere, as a positive test charge must be pushed against a repulsive force so gaining energy. Compare gravity, where a mass is attracted and loses potential energy, and therefore potential, as it approaches the Earth.

If the electric field obeys an inverse-square law, as for gravity, then the potential V should vary with distance r from the centre as $\frac{1}{r}$. A quick test of V at the surface of the sphere and, say, double or treble this distance should show that $V \times r$ is constant. A graph may be plotted to see if $V \propto \frac{1}{r}$ for a range of r.

Some points of experimental design might be raised, for example:

How reliable is the electroscope reading?

What is the resolution of its scale?

How big is the flame? Will the experiment give an average value for V over a region of space or perhaps a highest or lowest value?

What is the uncertainty in r?

Nevertheless the results should support a $\frac{1}{r}$ variation rather than, say, $\frac{1}{r^2}$.

THE ELECTRICAL INVERSE-SQUARE LAW

A $\frac{1}{r^2}$ gravitational field had a $\frac{1}{r}$ potential. Here the electrical potential varies as $\frac{1}{r}$ so it is reasonable to assume that the electric field strength varies as $\frac{1}{r^2}$. This will be tested later.

If this is so then it was reasonable to use a charged sphere in place of a point charge since for $\frac{1}{r^2}$ fields the effect outside a sphere will be the same as if all its charge were concentrated at the centre (as stated for gravity in Section E2).

The electrical force constant

For gravity, we have the formulae for field strength and potential:

$$g = -\frac{GM}{r^2}, \quad V_g = -\frac{GM}{r}$$

So far, for electricity, we have established the $\frac{1}{r}$ variation in V. Earlier experiments, or a test now, show that V is proportional to the charge Q on the sphere $\left(\text{it has constant capacitance } C = \frac{Q}{V}\right)$. Hence $V \propto \frac{Q}{r}$ or $V = k\frac{Q}{r}$.

There is no negative sign in the expression for V, as there is for gravity. All gravitational potentials are negative because all gravitational fields are attractive. Electrical fields can be attractive or repulsive; so electrical potential can be negative or positive. This is determined in the formula by the sign of Q. Positive charged spheres have positive potentials, negative charges negative potentials.

But to calculate the potential, and therefore the energy of a charge near the sphere we must know the value of the constant k. This is analogous to G and fixes the sizes of forces, fields, etc. for given values of Q and r.

The simplest way to find k is to record V and r at the sphere's surface (though any pair of V and r values would do). The charge Q can be measured using a coulombmeter.

DEMONSTRATION

E8b Measurement of the constant k in $V = k\dfrac{Q}{r}$

Additional apparatus required:

ITEM NO.	ITEM
1512	coulombmeter, 100 nC, with probe rod or stiff wire
51G	polythene strip for use as insulating handle

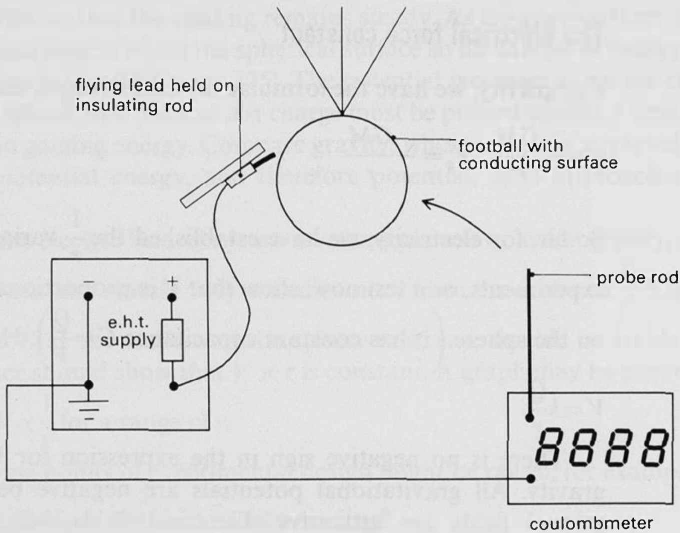


Figure E36
Measuring the value of k.

Using a method similar to that of experiment E6, the sphere is charged by touching with a flying lead to a potential of about 1000 V. This gives a charge of nearly 10^{-8} C for a ball of 0.1 m radius. The charge is measured by bringing up the coulombmeter so that the probe rod touches the ball. (1000 V on the ball and about 1 V across the coulombmeter shows that charge transfer is complete to within about 0.1 %.)

An accurate measurement gives k as 8.98×10^9 units. 9×10^9 is good enough for our purposes. The units can be deduced from $k = \dfrac{Vr}{Q}$.

$$\frac{\text{volt} \times \text{metre}}{\text{coulomb}} = \frac{\text{joule} \times \text{metre}}{\text{coulomb} \times \text{coulomb}} = \frac{\text{newton} \times \text{metre} \times \text{metre}}{\text{coulomb} \times \text{coulomb}}$$
$$= \mathrm{N\,m^2\,C^{-2}}$$

An astute student may spot these as the reciprocal of the units of ε_0.

THE ANALOGY BETWEEN ELECTRICITY AND GRAVITY

Now we are ready to complete the expressions for electrical field strength and force:

	Force	Field strength	Potential
Gravity	$-\dfrac{Gm_1m_2}{r^2}$	$-\dfrac{GM}{r^2}$	$-\dfrac{GM}{r}$
Electricity			$+\dfrac{kQ}{r}$

Comparing the expressions we now know for potential, it is a good guess that the electric field strength will be $\dfrac{kQ}{r^2}$, and the force $k\dfrac{Q_1Q_2}{r^2}$, if the analogy holds good. More rigorously we can apply the relationship $E = -\dfrac{\mathrm{d}V}{\mathrm{d}r}$, which does not depend on the force law involved, to confirm the expression for E. Then using $F = QE$, we have $k\dfrac{Q_1Q_2}{r^2}$ as the force between two charges Q_1 and Q_2.

Signs

If teachers have been consistent up to now, students should not be confused by the negative signs in the expressions for gravitational force, field, and potential, and their absence in the corresponding expressions for electricity. For gravity only one kind of field (an attractive one) is possible with inwards forces (represented by negative force and field expressions), and negative potential and potential energy. The electric field is directed away from a positive charge, Q_1 (*i.e.* it is positive), and the potential is also positive. So another positive charge, Q_2, in the vicinity experiences an outwards, repulsive force $\left(k\dfrac{Q_1Q_2}{r^2}\right)$ and has positive potential energy $\left(k\dfrac{Q_1Q_2}{r}\right)$. However, a negative charge, $-Q_2$, would be attracted $\left(\text{by a force } -k\dfrac{Q_1Q_2}{r^2}\right)$ and the overall system would have negative potential energy $\left(-k\dfrac{Q_1Q_2}{r}\right)$. The expressions are summarized in the *Students' guide* (page 302). They do not have to be memorized, but a physical understanding of their forms and relationships is of course needed.

Understanding of analogy

Students should be assured that the use of a formal analogy does not mean that gravitational and electrical forces are in any other way related. Physicists continue to search for some deeper theory which includes both, without success. Nevertheless the similarity in the mathematics that describe them is extremely useful for once we have solved a problem in one field, we can transfer the result directly to the other, without having to do the mathematics all over again. Such analogies are quite common in physics. For instance between the energy of a stretched spring ($\frac{1}{2}Fx$) and that stored in a capacitor ($\frac{1}{2}VQ$); current carried by charged particles in a wire and traffic carried by occupied vehicles on a road; flow of 'heat', of water, of current, and of magnetic 'flux'.

Constants

The difference in magnitude between the two constants
$G \approx 7 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ and $k \approx 9 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$
is worth remarking on now, as it has important consequences which will be referred to later.

Question 41 is about 'flux' and the inverse-square law.

COULOMB'S LAW

The electric force between charges Q_1 and Q_2 at separation r is

$$F = k\frac{Q_1 Q_2}{r^2}, \quad k \approx 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$$

This is regarded as one of the fundamental laws of physics, like Newton's Law for Gravity, but unlike others, such as Hooke's, or Ohm's, which, though simple and extremely useful, fall very far short of universal application. In discussion of what makes a *fundamental* law, factors mentioned may include

- generality, having no known exceptions
- power to explain many other things
- simplicity, lacking special conditions, particular qualifications
- having no deeper explanation of which it is only a special case.

Still the law attempts only to explain *how* charges behave, not *why*.

Testing Coulomb's Law

In 1785 Coulomb created a quantitative basis for electrostatics by establishing this Law (Cavendish's experiment for gravity was done in 1798). Students have approached the force law via the $\frac{1}{r}$ potential, so testing the $\frac{1}{r^2}$ force law here is confirmation rather than discovery. Such tests are subject to considerable inaccuracy. One admires Coulomb for his painstaking efforts and students may respond to a challenge to their practical expertise. The experiments are tricky but reasonable results have been obtained even by groups of 18 on a foggy winter morning.

EXPERIMENT
E9 Experiments to test the inverse-square law for electric forces

ITEM NO.	ITEM
51D	2 metallized polystyrene balls
51E	nylon thread for suspension
51L	2 proof planes
94A	lamp, holder, and stand
27	transformer
	either
14	e.h.t. power supply,
	or
60/1	Van de Graaff generator
	or
51K, I, M	electrophorus plate, 'rubber', and tile
503–6	retort stand base, rod, boss, and clamp
	graph paper
1153	glue (*Durafix or Evo-stik 863*)
1153	adhesive tape
1504	balance, resolution 10 mg
1515	hair dryer
1000	leads
	additional apparatus for parts b, c, d as specified below

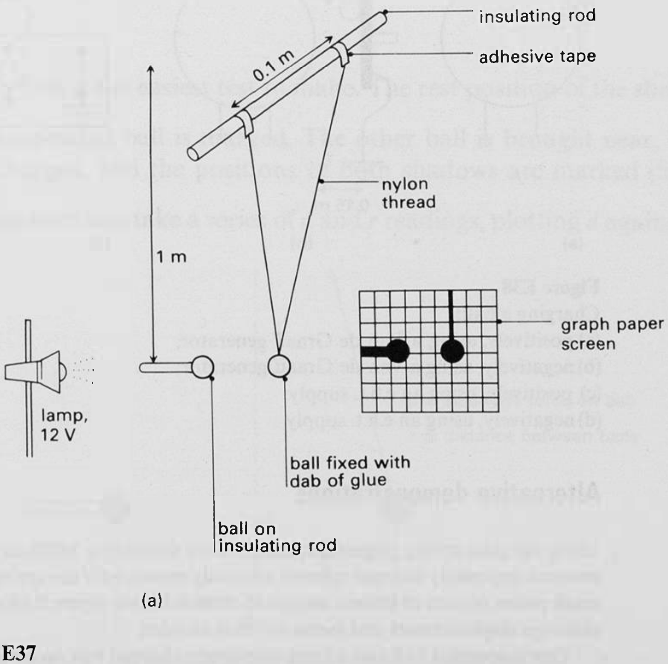


Figure E37
Testing Coulomb's Law.

One of the small balls is glued to an insulating rod (the small proof plane or the barrel of a used ball-point pen will do); the other is glued to a nylon 'trapeze' up to 1 m long, which is stable yet sensitive. All the apparatus should be clean, dry, and free of grease. (Use care in setting up and keep a hair dryer to hand throughout.)

A projection system allows measurements to be made without touching the balls. A distant lamp and close screen eliminates scale factors, or the distances can be magnified if desired.

The spheres are charged using the Van de Graaff dome (see figure E38 for methods of + and − charging) or the e.h.t. supply (with one terminal earthed). The former method gives a larger but less consistent charge.

The experiments assume knowledge (from Units A and D) that sideways displacement of a suspended object (if small) is proportional to the sideways force on it. This can be demonstrated using a rather massive pendulum and a newton meter if necessary.

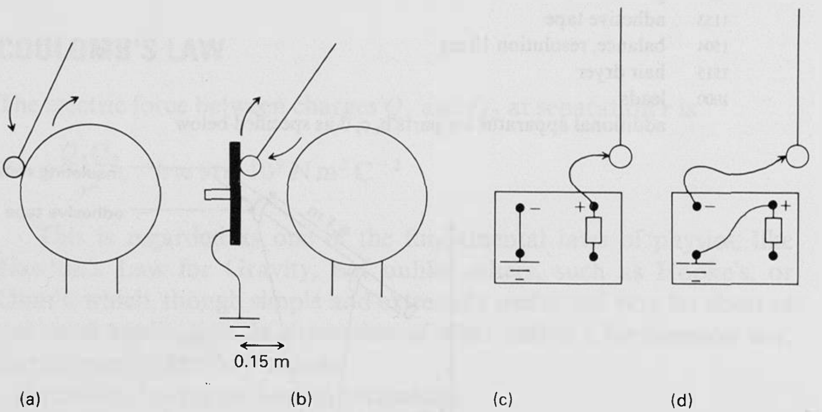


Figure E38
Charging a ball:
(a) positively, using a Van de Graaff generator;
(b) negatively, using a Van de Graaff generator;
(c) positively, using an e.h.t. supply;
(d) negatively, using an e.h.t. supply.

Alternative demonstrations

Using the glass spring suspension designed for simulating Millikan's experiment, forces between oppositely charged spheres are easily measured if the spring is calibrated (using small pieces of card of known weight to stretch it), see figure E39(a). Difficulties over sideways displacements and forces are thus avoided.

One suspended ball and a fixed, oppositely charged ball on an insulated stand on a top pan balance is another alternative, see figure E39(b).

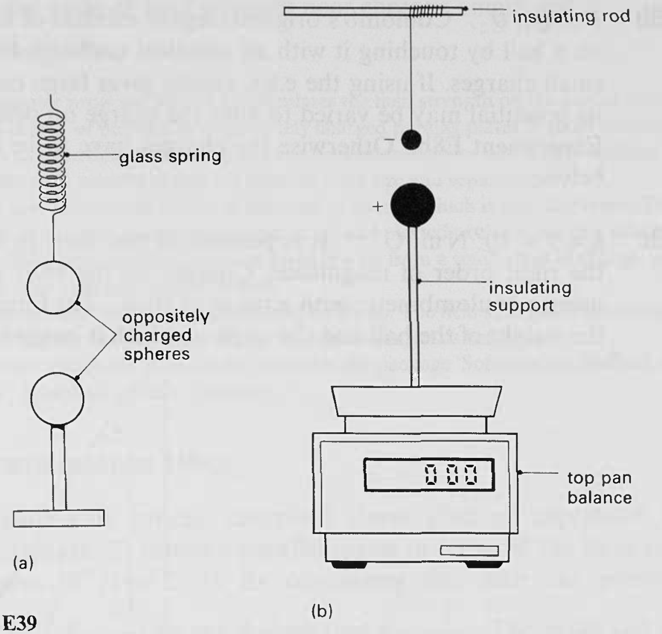


Figure E39
Alternative Coulomb's Law demonstrations.

E9a $F \propto \frac{1}{r^2}$ This is the easiest test to make. The rest position of the shadow of the suspended ball is marked. The other ball is brought near, both being charged, and the positions of both shadows are marked (figure E40). Students can take a series of d and r readings, plotting d against $\frac{1}{r^2}$.

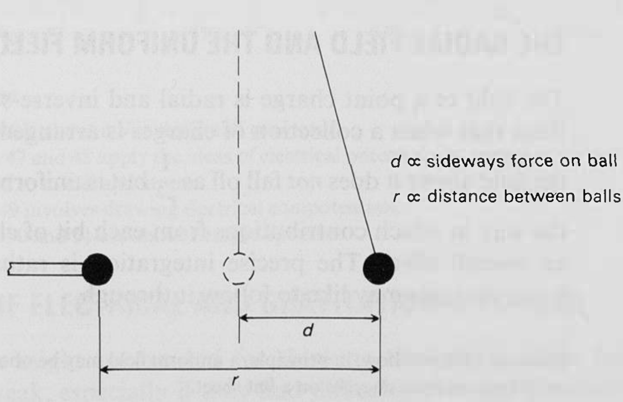


Figure E40
Deflection of a suspended ball.

E9b $F \propto Q_1, Q_2$ Coulomb's original elegant method of halving the charge on a ball by touching it with an identical uncharged ball leads to very small charges. If using the e.h.t. supply gives large enough deflections, its potential may be varied to alter the charge on one ball ($Q \propto V$ from Experiment E8b). Otherwise the charges have to be measured as in **c** below.

E9c $k \approx 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ It is possible, if not easy, to obtain a result of the right order of magnitude. Charges on the balls can be measured using a coulombmeter with a range of 10 nC. The force is deduced from the weight of the ball and the angle at which it hangs (figure E41).

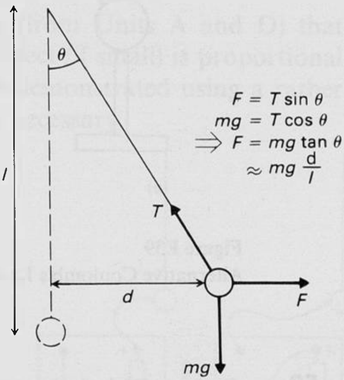


Figure E41
Calculating the force on a displaced ball.

Questions

Question 42 gives results for the above experiment as a series of photographs.
Question 43 discusses why the experiments often fail.
Question 44 is about forces between charged spheres.

THE RADIAL FIELD AND THE UNIFORM FIELD

The field of a point charge is radial and inverse-square. It seems odd, then, that when a collection of charges is arranged on a large flat plate the field above it does *not* fall off as $\frac{1}{r^2}$ but is uniform. This is the result of the way in which contributions from each bit of charge add up to give an overall effect. The precise integration is rather involved, though some students may like to follow it through.

Question 45 shows how, in principle, a uniform field may be obtained by adding contributions from charges on a flat sheet.

Appendix VI gives the full integration and shows that the constant $9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ is, in fact, $1/4\pi\varepsilon_0$.

Computer plots of field strength near charged single and parallel plates

The computer program 'EFIELD' calculates the field strength on the axis of a circular sheet of charge or between two oppositely charged parallel plates. It plots the field strength as a function of distance and can be used to explore how nearly uniform the field is between parallel plates; the effect of plate size and separation.

The user chooses the radius of the sheet of charge, which is then displayed. The field strength at positions on the axis is calculated and plotted or displayed as a table. The transition between an inverse-square force law far from a small sheet of charge, and the uniform field near a large sheet is seen.

Alternatively, two parallel plates may be set up. The field strength is calculated at points along the line joining their centres and displayed as before.

Further details are given in the *Notes* for the package 'Software for Nuffield Advanced Physics', Longman's Micro Software.

The force constant $1/4\pi\varepsilon_0$

The adding up process described above gives an expression for the field strength, E, between parallel plates in terms of the force constant k ($\approx 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$). By comparing this with our previous expression $\left(E = \dfrac{\sigma}{\varepsilon_0}\right)$ we can deduce that $k = \dfrac{1}{4\pi\varepsilon_0}$. The value and units of the constant can be checked; it takes its place among other 'universal' constants in determining the sizes of forces between all charges everywhere (as G does for the forces between masses). The electrical formulae and relationships can now be 'updated' by replacing $9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ by $1/4\pi\varepsilon_0$:

Force	Field	Potential	Potential energy
$F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{r^2}$	$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$	$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$	$E_p = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{r}$

Questions

Question 46 is about a Van de Graaff generator.
Questions 47 and 48 apply the ideas of electrical potential and energy in an inverse-square field to nuclei and atoms.
Question 49 involves drawing electrical equipotentials.
Questions 50 and 51 are about charged spheres.

SIZES OF ELECTRICAL AND GRAVITATIONAL FORCES

Students might be excused for thinking that electrostatic forces are rather weak, especially if they had difficulty over the Coulomb's Law experiments. However, a comparison of the constants in Coulomb's and

Newton's Laws reveals a striking contrast: $\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ whilst $G \approx 7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. Electrical forces on objects carrying unit charge will be enormously greater than gravitational forces between objects of unit mass at the same separation. A few comparisons might illustrate this.

Question 52 shows that the ratio F_e/F_g for an electron and proton in a hydrogen atom is of the order of 10^{39}. For two electrons it will be even greater, for two protons only about 2000 times less. So within atoms electricity is definitely the dominant force.

The charges in the Coulomb's Law tests were about 10^{-8} C; the masses were about 0.001 kg (1 g), and the distances 0.1 m at most. A comparison of electric and gravitational forces here gives a ratio of about 10^{10}.

$$\frac{F_e}{F_g} = \frac{QQ/4\pi\varepsilon_0 r^2}{G\,mm/r^2} = \frac{1}{4\pi\varepsilon_0 G}\frac{Q^2}{m^2} = \frac{9 \times 10^9}{7 \times 10^{-11}}\frac{(10^{-8})^2\,\mathrm{N}}{(10^{-3})^2\,\mathrm{N}} \approx 10^{10}$$

Note that the separation does not matter. The *ratio* F_e/F_g is the same at any distance.

Further estimates show that 10^{-8} C represents about 6×10^{10} electrons and if we have, say, 6×10^{21} atoms in the ball, this means that only 1 atom in 10^{11} or so in the ball is charged. But even this charge leaked away quickly unless we were very careful.

This is why electrical forces between charged objects are small: the balance of + and − charge can only be disturbed very slightly. Although very large forces exist between individual ions in a grain of salt, say, the almost perfect balance of + and − charges ensures that the whole grain is neutral and exerts very little force on other grains.

Question 53 argues from the field of a dipole towards the effect of a neutral array of charges as in a crystal.

If we take more massive neutral objects the gravitational force begins to dominate so that electrical forces are irrelevant when it comes to satellites and planetary orbits. It should be noted, though, that the range of both electrical and gravitational fields is infinite as both obey an inverse-square law.

Questions

Questions 54 and 55 test understanding of the similarities and differences between electricity and gravity.

Reading

ASIMOV *The collapsing Universe: the story of black holes.* Uses obsolete units, but Chapters 1 and 2 on gravity are interesting.
CALDER *The key to the Universe.* Takes the story from quarks to black holes.
DAVIES *The forces of nature.* Chapters 1 and 4 give an especially readable survey.
FEYNMAN, LEIGHTON, and SANDS *The Feynman lectures on physics* Volume 1. Pages 2 and 3 on the balance of positive and negative charges.

THE KNOWN INTERACTIONS

Of the four kinds of force known, electrical and gravitational are two. Question 52 introduced the 'strong' nuclear force which, with the 'weak' nuclear force, operates over very short ranges within nuclei ($\gtrsim 10^{-15}$ m). The tremendous energy associated with interactions involving these forces can be released in nuclear fission (Unit F, 'Radioactivity and the nuclear atom', and Unit G, 'Energy sources').

Students who ask about magnetic forces can be told that these arise between charges in motion (that is, currents) and that they follow as a result of transforming Coulomb's Law to a moving frame of reference.

Everyday 'contact' forces, pushes and pulls in springs, friction, and forces in fluids can be attributed to the interaction of atoms via the electric fields of the electrons which surround them (Unit A).

Much has been achieved in 'boiling down' forces to these four and one of the chief preoccupations of contemporary physics is to find some deeper unifying theory which links them all. So far progress has been made with linking electricity and the weak force. Many more questions remain unanswered: for example why is there only one kind of mass but two kinds of charge, why does every electron have just the same amount of charge, and so on?

Students may wonder why electrons, pulled by such enormous forces, do not collapse into the nuclei they surround. This is one question we can begin to answer, at least in a simple way, in Unit L, 'Waves, particles, and atoms'.

Reading

'The particles and forces of nature' in the Reader *Particles, imaging, and nuclei* deals with the four known forces, the particles associated with them, and attempts to produce a unified theory.

Unit F
RADIOACTIVITY AND THE NUCLEAR ATOM

Paul Jordan and Peter Harvey
Highfields School, Wolverhampton

Suggested time allocation: four weeks

PLAN OF THE UNIT

	Section F1 **The Rutherford model of the atom**	
GCSE ►	**Properties of α, β, and γ radiation**	
	α-particle scattering	
Coulomb's Law (Unit E) ►	**The nuclear atom**	► the nucleus: Section F3
	Section F2 **Exponential decay**	
	Half-life	
	Chance, randomness	► Unit K, 'Energy and entropy'
numerical methods (Units B, D) ►	**Exponential growth and decay**	► Unit G, 'Energy sources' ► the Boltzmann factor: Unit K, 'Energy and entropy'

Section F3
The nucleus

chemistry ▶ **Periodic Table**

collisions (Unit A) ▶ **Neutrons**

Proton number, *Z*

Nucleon number, *A*

Isotopes and their uses

Changes in *Z, A* on α and β decay

Ionization by collision ▶ Unit L, 'Waves, particles, and atoms'

potential in attractive field is negative (Unit E) ▶ **Electron binding energy**

chemistry ▶ **Ionization energy**

Nuclear binding energy

Mass and energy

Fusion and fission ▶ Unit G, 'Energy sources'

INTRODUCTION

The major aims of the Unit are:

1 to examine the properties of ionizing radiations;

2 to investigate the ionization of atoms;

3 to provide a satisfactory understanding of radioactive decay and of exponential change;

4 to develop an understanding of the nature of the nucleus;

5 to provide knowledge of the energy changes associated with nuclear reactions required for any consideration of the principles of nuclear energy generation.

Such a study of radioactivity provides much of the experimental evidence upon which the modern understanding of the structure of atoms is built and provides essential knowledge for citizens of the uses and the dangers of radioactivity. Such knowledge is likely to become of increasing importance for our civilization.

The first Section examines the properties of the three radiations and considers their energies. It proceeds to develop the Rutherford model of the nuclear atom. In the second Section the concern is with the process of decay and with the measurement of half-life. This is followed by a consideration of the equation for decay and its implications. In the third Section, the story of the nucleus is taken up again with specific reference to some questions which were previously left open; for example, where in the atom are the electrons to be found? what holds the nucleus together?

Before considering the development of the Rutherford model from the scattering experiment of Geiger and Marsden, students will require access to additional background information. This is conveniently acquired if students are asked to find answers to some specific questions whilst they are doing experiments F1 to F4. Individual students may be set specific tasks, such as those suggested on pages 359–360, make their own notes on their reading and be asked to report back to the group as a whole. This might occupy a single homework since the answers are easy to find from standard texts or from the brief history given in the *Students' guide* for this Unit.

Teachers may find it necessary to re-order the sequence of the experiments in this Unit in order to suit the availability of the equipment.

Teachers should know that Revised Nuffield Advanced Chemistry, Topic 4, 'Atomic structure' deals with the nuclear atom, the neutron, isotopes, and ionization energy.

THE PLACE OF THE UNIT IN THE COURSE

A theme running through the course is the attempt to explain large-scale happenings in terms of small-scale behaviour. Within this theme, understanding about the nature of the atom plays a big part. It requires students to fit together ideas from Unit E, 'Field and potential' and Unit B, 'Currents, circuits, and charge' into new patterns to explain new things.

It also provides ideas essential for later work and, in particular, Unit G, 'Energy sources' and Unit L, 'Waves, particles, and atoms'.

LIST OF SUGGESTED EXPERIMENTS AND DEMONSTRATIONS

SECTION F1

THE RUTHERFORD MODEL OF THE ATOM

Using radioactive sources in schools

The attention of teachers is drawn to DES Administrative Memorandum 2/76 *The use of ionizing radiations in educational establishments* and the accompanying *Notes for guidance.* This memorandum gives the regulations governing the type, quantity, and use of radioactive sources in schools. Of particular significance for first-year sixth forms is the regulation forbidding students under the age of 16 to use radioactive sources other than the naturally occurring isotopes of potassium, uranium, and thorium.

An introductory passage to the students' laboratory notes for this Unit draws their attention to sensible precautions to take when handling sealed sources or naturally occurring radioactive substances. It also suggests that any experiment involving uranium or thorium salts should be set up in a spill tray lined with absorbent paper. There are also some notes on the use of Geiger–Müller tubes.

Experiments on alpha, beta, and gamma radiations

The work provides an opportunity for reporting back and the results of each experiment will be required by all. The reading tasks (pages 359–360) may also be undertaken at the same time as these experiments.

Questions

Questions 1 to 7 concern these experiments and it is suggested that these could be assigned as the experiments are completed.

A number of the experiments in this Unit occur in earlier courses but, of course, as demonstrations. For example, the Revised Nuffield Physics course, *Year 5*, includes:

Radiations and counters (Demonstration 83);
Experiments with α-particles: range and stopping (Demonstration 84);
Experiments with cloud chambers (diffusion and expansion types) (Demonstration 86 and Experiment 87);
The deflection of beta particles in a magnetic field (Demonstration 88);
The decay of protactinium (Demonstration 89).

Reading

Students can find useful information about the experiments in:

AKRILL, *et al. Physics.*
CARO, *et al. Modern Physics.*
DUNCAN *Advanced physics; fields, waves and atoms.*
LEWIS and WENHAM *Radioactivity.*
WENHAM, *et al. Physics: concepts and models.*

THE PROPERTIES OF ALPHA, BETA, AND GAMMA RADIATIONS (EXPERIMENTS F1 TO F4)

These four experiments lead to the following conclusions:

i beta particles carry negative charge;

ii alpha particles are strongly ionizing and have energies in the order of MeV;

iii beta particles ionize much less strongly and therefore are more penetrating than alpha particles;

iv the intensity of gamma radiation is subject to an inverse-square law of distance.

EXPERIMENT
F1 The deflection of beta radiation in a magnetic field

ITEM NO.	ITEM
130/1	scaler
130/3	GM tube holder
130/5	thin window GM tube
195/2	beta particle source
196	source holder
92I	mild steel yoke
92B	2 Magnadur magnets
503–6	2 retort stand bases, bosses, rods, and clamps
1M	2 lead blocks
92D	plotting compass

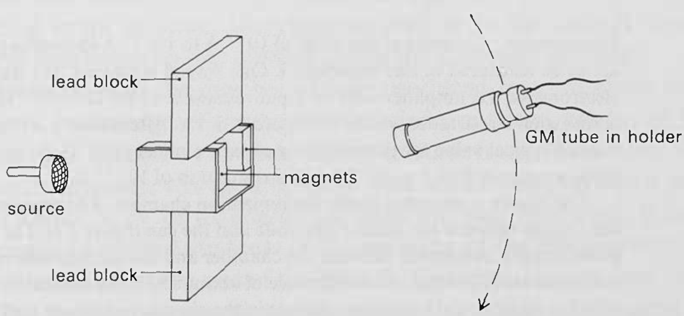


Figure F1
Arrangement of apparatus for the magnetic deflection of beta particles (seen from above).

Figure F1 shows the arrangement. Only the GM tube holder, not the tube, may be gripped by a clamp. The other clamp can be used to hold the source. The lead blocks will serve to prevent radiation reaching the GM tube directly. A ratemeter may be substituted for a scaler.

This experiment is essentially the same as Demonstration 88 in the Revised Nuffield Physics *Pupils' text* and *Teachers' guide, Year 5*. Brief instructions are given in the students' laboratory notes.

This simple experiment may be better suited to slow or inexperienced students.

An understanding of how magnetic deflection gives evidence for the sign of the charge carried by the particles will be helpful in discussing the nature of alpha radiation, which is essential in the development of the Rutherford model of the atom.

Having done this experiment, students should know that:

i beta particles are charged, and

ii the direction of the deflection suggests that the charge is negative.

Questions

Questions 1 and 2 are relevant to this experiment.

EXPERIMENT

F2 Measuring the ionization current to find the number of ion pairs produced by an alpha particle, and an estimation of the energy of the alpha particles emitted

ITEM NO.	ITEM
1516	picoammeter (see note below)
1108	ionization chamber
195/3	pure alpha source
14	e.h.t. power supply
196	source holder
1000	leads

Picoammeter Currents of the order of 10^{-9} A to 10^{-11} A (depending on the source used) are to be measured in this experiment. One way of achieving this sensitivity is to use the electrometer/d.c. amplifier with an input resistance of $10^9\ \Omega$ or $10^{11}\ \Omega$, since the p.d. across such a resistance will be of the order of 1 V. Alternatively, a digital meter could be used. A typical value for the resistance of such a meter is $10^7\ \Omega$. So on its 200 mV setting it has a range of 0 to 2×10^{-8} A and a resolution of 10^{-11} A.

The source is mounted inside the ionization chamber. The picoammeter measures the current between the central electrode and the can (figure F2). The output of the power supply, connected between the chamber and the earthed side of the meter, is increased until, at a potential difference of about 750 V, the current flowing in the meter, which is equal to the ionization current in the chamber, is steady and increasing no further.

If a 4×10^3 Bq (0.1 μCi) source is used then a range of 0–10^{-11} A may be needed. Teachers should try this beforehand and determine the ionization current caused by the source used. When the p.d. across the chamber is increased, there is a current surge before the constant current is achieved.

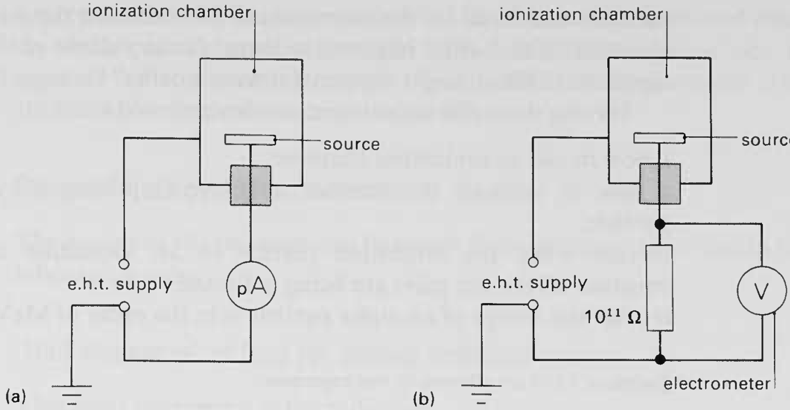


Figure F2
Measuring ionization current:
(a) with a picoammeter;
(b) by measuring p.d. across a very high resistance.

The idea is to measure the ionization current produced by alpha particles from the alpha source. As the charge on an ion is of the order of the charge on an electron, the number of ions produced per second can be estimated from the ionization current. (Note that although each ionizing event produces a pair of charged particles, for example, a single charged positive ion and an electron, the total charge transported across the ionization chamber by the pair is equal to one single charge. Each of the oppositely charged particles makes only part of the journey across the chamber – they travel in opposite directions from the ionizing event to either the chamber wall or to the central electrode. This is equivalent to one particle doing the complete journey.)

Estimation of the number of ions produced by each alpha particle requires knowledge of the number of alpha particles produced by the source per second. This can be estimated from the stated source strength. Students will need to know that, on average, an alpha particle loses about 30 eV of its energy for each ion pair produced. It may be necessary to remind students of the meaning of the electronvolt, which was introduced at the end of the Unit B, 'Currents, circuits, and charge'.

The experiment can give evidence for the energy of a typical alpha particle. This will be needed when estimating how close such a particle might approach a nucleus in a head-on collision. It also brings out the meaning of the unit of activity, the becquerel (Bq) and its relation to the earlier unit, the curie (Ci), as well as illustrating the great ionizing power and the short range of alpha particles. Nevertheless, there are several sources of uncertainty in the experiment. For example, do all the alpha particles produced by the source cause ionization in the air within the chamber? If not, what happens to them? Is the volume of the chamber significant? What might happen if it were smaller? Or bigger?

Having done this experiment, students should know:

i how to use an ionization chamber;
ii how to estimate the number of ion-pairs produced by an alpha particle;
iii that when the ionization current in an ionization chamber is constant all the ion pairs are being collected;
iv that the energy of an alpha particle is in the order of MeV.

Questions 3 to *6* are relevant to this experiment.

EXPERIMENT
F3 The penetrating power of alpha, beta, and gamma rays

ITEM NO.	ITEM
195/1–3	alpha, beta, and gamma sources
130/1	scaler
196	source holder
130/3, 5	GM tube and holder
507	stopwatch or clock
503–6	2 retort stand bases, rods, bosses, and clamps
501	metre rule
1052	set of absorbers (see below)
1155	Vernier callipers or micrometer screw gauge

Item 1052, the collection of absorbers, should include cigarette paper, thin glass and Perspex sheets, aluminium cooking foil, aluminium sheets of various thicknesses sufficient to cover the range 1 to 5 mm, lead sheet or blocks to cover the range of thickness from 5 to 20 mm.

This experiment has a number of parts (listed below) and details are given in the students' laboratory notes.

F3a Range of alpha particles in air

The energy of the alpha particles can be found from the graph of range against energy given in the laboratory notes. These notes also give the air equivalent of the windows of the MX 168 and MX 168/01 GM tubes (30 and 17 mm respectively).

An alternative method uses the ionization chamber fitted with a gauze lid and a central rod that reaches nearly to the top of the chamber. With a p.d. of about 500 V across the chamber (from e.h.t.

supply with 50 MΩ limiting resistor) the ionization current caused when an alpha particle source is within about 5 cm of the gauze can be detected by a picoammeter (*i.e.*, digital meter, or electrometer with $10^{10}\ \Omega$ input resistor – see experiment F2).

F3b Range of beta particles in aluminium

The energy of the particles can be found from the graph provided in the laboratory notes.

F3c 'Half-thickness' of lead for gamma radiation

This gives the energy of the radiation.

F3d The relation between the thickness of the absorber and the radiation transmitted for gamma and beta radiation

Students draw a graph of radiation transmitted against absorber thickness. For gamma radiation this curve is exponential in form. This graph and the data could be used in Section F2 as an example in a discussion of exponential behaviour.

F3e The relation between intensity and distance for gamma radiation

This provides practice in handling data in order to formulate a relationship (here an inverse-square law). Students should read the discussion of zero error in their laboratory notes.

A ratemeter can be substituted for a scaler in parts **b**, **c**, and **d**.

Parts **d** and **e** illustrate some of the factors involved in controlling dosage from radiation.

Before starting these experiments remind students of the need to deal with the background count.

These experiments will provide students with knowledge of:

i the range of alpha particles in air and the energy of each alpha particle;

ii the range of beta particles in aluminium and the maximum energy of the beta particles;

iii the meaning of 'half-thickness' and the energy of the gamma radiation;

iv the exponential relationship between absorber thickness and count rate for gamma radiation;

v the way in which the intensity of gamma radiation falls off with distance from the source (inverse-square law).

Questions

Questions 3 to 5 are relevant to experiment F3. Question 7 is about precautions to be taken when handling radioactive material.

EXPERIMENT
F4 Photographic detection of radiation

ITEM NO.	ITEM
16	radium source
	either
1155	dental X-ray film
	or
	bromide paper with developer and fixer

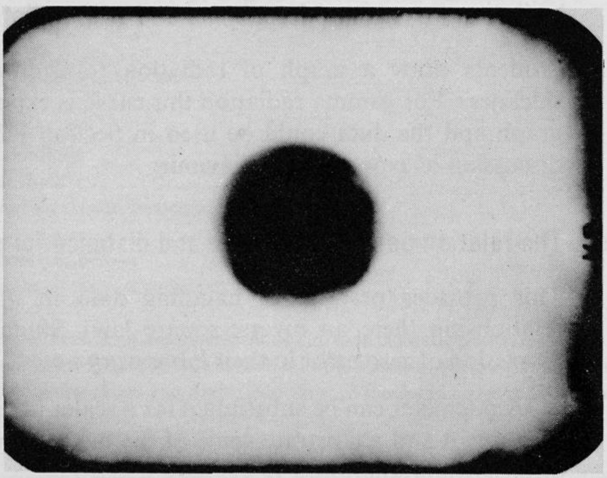


Figure F3
The fogging of dental film by radium; source of strength 2×10^5 Bq (approximately).
Mark Jeffery.

Dental X-ray film is more convenient to use than photographic paper. In one form the film is in a pod which also contains sachets of developer and fixer, so that no darkroom is needed for either exposure or processing. Another (less expensive) form of X-ray film consists of small, individually wrapped plates. This film contains a yellow filter on top of the emulsion which means that it can be processed in subdued light. Both types of film can be exposed by placing the source on top of the film packet for 20 to 30 minutes (the packet may contain a lead foil behind the film – the source must of course be on the opposite side to this). Most of the effect is due to beta radiation which penetrates the wrapping. Fast bromide paper can also be used ('hard' paper is better than 'soft'). If the source is placed face down on the emulsion side of the paper (in the dark, of course), exposure times should again be 20–30 minutes; the effect is almost entirely due to alpha particles. If the bromide paper is wrapped in black paper, a much longer exposure time will be needed – hours or even days.

Reading tasks

As mentioned in the Introduction to this Unit, students will need additional background information before discussing the Geiger and Marsden scattering experiment and Rutherford's interpretation of the results. A single homework might suffice for this, the students being asked to find answers to specific questions whilst doing Experiments F1 to F4; they may be directed to the brief history in the *Students' guide* (page 383) and to the references which follow.

Questions

a *In 1904, J. J. Thomson, the discoverer of the electron, suggested a model for the atom which is frequently called the 'plum-pudding model'. Describe this model and indicate how it incorporated the facts known at the time.*

The students' reports or notes should give a brief description of the model and at least some of the evidence incorporated: for example, atoms are neutral but since they contain negative charges they must also contain equal positive charge; electrons obtained from all sources and by all methods are alike; few atoms disintegrate naturally so there must be a balance of forces within most atoms; radiations from radioactive elements appear to pass through matter without deflection.

b *How was radioactivity discovered and how were the radiations sorted out into the three different types?*

The replies should indicate that absorption in matter suggested three radiations: one which was readily absorbed; one which passed through several millimetres of aluminium; and one which was quite difficult to absorb. The first two of these were deflected in electric and magnetic fields but in opposite directions; the last was undeflected in such fields and was very penetrating in the way that Röntgen's X-rays were penetrating. See Rutherford and Soddy, *The cause and nature of radioactivity* in Phil. Mag. S 6. Vol. 4. No. 2, Sept. 1902; Rutherford, *The magnetic and electric deviation of the easily absorbed rays from radium*, Phil. Mag. S 6. Vol. 5. No. 26, Feb. 1903. Both papers are reprinted in WRIGHT *Classical scientific papers – physics.*

c *What is the evidence for the identification of*
i an alpha particle with a helium ion, and
ii a beta particle with an electron?

i The experiment of Rutherford and Royds provides the evidence for this identification although estimates of the specific charge had already indicated that this was so; it may be worth reminding students of the right angle between the tracks produced when an alpha particle collides with a helium atom (Revised Nuffield Physics *Year 5*, and Section A4 of this course).
ii The similarities between the beta particles and the electrons produced in vacuum tubes and especially the possession of the same specific charge.

d *(An additional question) How does a Geiger–Müller (GM) tube work and what are its limitations?*

One might expect reference to the initial ionizing event and the consequent 'avalanche' of ions causing a pulse; to the threshold and operating voltages, and the 'dead-time'.

References

Reading section of *Students' guide*, page 383, 'Radioactivity and the nuclear atom: a brief history'.
AKRILL, BENNET, and MILLAR *Physics.*
BENNET *Electricity and modern physics.*
BOLTON *Patterns in physics.*
CARO, McDONELL, and SPICER *Modern physics.*
DUNCAN *Advanced physics: Fields, waves, and atoms.*
LEWIS *Electrons and atoms.*
LEWIS and WENHAM *Radioactivity.*
ROGERS *Physics for the inquiring mind.*
WENHAM, DORLING, SNELL, and TAYLOR *Physics: concepts and models.*
WRIGHT *Classical scientific papers – physics.*

THE DEVELOPMENT OF RUTHERFORD'S NUCLEAR MODEL

The discussion might follow the following lines. 'What was the Thomson model of the atom?' 'If one accepts this model, what might be expected to happen to an alpha particle which is fired towards a metal foil?' (Some scattering through small angles would be quite likely but large-angle scattering, which could only result from many chance encounters with atoms, would be improbable.)

Now Geiger had already shown (1910) that the vast majority of the deflections were by only a few degrees when alpha particles fell on such a foil. This could be explained by assuming that the deflection of a single alpha particle was the resultant of a large number of very small deflections caused as the particle passed through successive individual atoms of the foil. In this case theory suggests that the most probable number scattered through a particular angle depends on $\sqrt{\text{thickness}}$; Geiger's results confirmed this.

But scattering through quite large angles, even over 90°, was also observed. Results collected later by Geiger and Marsden for scattering through particular angles by gold foil are given in the *Students' guide* (page 373) and it can be seen that the number of alpha particles scattered through the large angles, although small, is significant. For these cases, the experiments showed that the number scattered at a particular angle depends directly on the thickness of the foil. Thomson's model could not account for this.

Rutherford, in whose laboratory Hans Geiger was working, realized that this scattering through large angles required an enormous force to be exerted on the incoming alpha particle as it travelled through the foil. He suggested that something like the force of repulsion or attraction between the charge on the particle and a charge equivalent to about 100 electron charges concentrated into a tiny space at the centre of a gold atom could turn the alpha particle back where it came from.

'What would be the shape of the path followed by the alpha particle if the force were **1** one of attraction or **2** one of repulsion?' (For **1** the path is rather like the nearly hyperbolic path followed by many comets as they fall in towards and then recede from the Sun, and for **2** it is a hyperbolic and symmetrical path in which the particle approaches the charge but does not pass round it. Teachers might like to introduce the 'Chinese hat model' at this point. Roll a ball towards the hat when it is in the inverted position and the path models that of the comet; turn the hat the other way up and the path models that of the alpha particle.)

Rutherford reasoned that such a 'nucleus' would be very small compared with the size of the atom (so that close encounters by alpha particles from the incident stream would be rare events), and massive. (Why? Alpha particles themselves have a small mass and are travelling very fast: any obstacle of comparable mass would be brushed aside or knocked on.)

The force responsible might well be the electric force between the two charges. Perhaps thinking, as we have done, about the motion of comets, Rutherford was at first inclined to think that an attractive force was responsible. This, however, didn't seem very likely. (Why? A negatively charged nucleus still leaves the problem of the positive charge which must neutralize the negative charges known to be present.) However, the theoretical analysis is the same in both cases; only the sign has to be changed.

'How does an electrical force depend on the distance between the charges?' (Inverse-square.) 'What is the force law?' $\left(F = k\dfrac{Q_2 Q_2}{r^2}\right)$.

Rutherford, assuming that this *was* the force which determined the scattering behaviour of the alpha particles, then developed the mathematics which could be applied. This led him to an equation which could be tested experimentally. He asked Geiger and Marsden to undertake this critical experiment. His account of his nuclear model was published in 1911 and the experimental results of Geiger and Marsden which followed a year later confirmed the hypothesis.

Both Thomson's and Rutherford's models of the atom predicted scattering in ways which could be tested experimentally. Rutherford's model to account for the unexpected large-angle scattering of some alpha particles may well have been derived from the analogy with the behaviour of a comet in the gravitational field of the Sun. It is of interest to see if we can take the gravitational model already used and develop it to provide some laboratory check on the way in which the inverse-square law is applied in the Geiger and Marsden experiment.

We might try using the repulsion between two magnets instead, as suggested in Revised Nuffield Physics *Year 5*, Demonstration 90X. But this cannot be made quantitative because the force between the magnets does not follow an inverse-square law. An electrostatic model (Demonstration 90Y) using a Van de Graaff generator would be suitable, but very difficult to control in a repeatable way. Instead, we intend to develop the gravitational hill model already mentioned.

'What happens to the alpha particle as it approaches a nucleus?' (It slows down, losing kinetic energy and gaining potential energy.)

A ball, given some kinetic energy and aimed at the hill, behaves in this way and might offer the model we are looking for.

'What shape must the hill have?' To answer this, consider how the potential energy of the alpha particle depends on its distance from the nucleus (the force varies as $1/r^2$, so the potential energy varies as $1/r$).

The hill must be constructed so that the potential energy of a ball rolling on it is proportional to $1/r$. The height must vary as 1/(distance from the centre). See figure F4.

At this stage teachers may wish to ask a student to verify that the hill to be used conforms to this requirement.

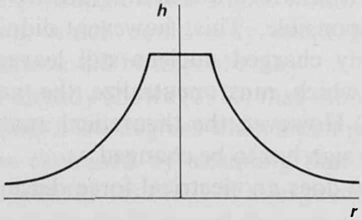


Figure F4
Section through alpha scattering analogue: $h \propto 1/r$.

We are now in a position to use the hill in a qualitative way as an aid to thought.

DEMONSTRATION
F5 Qualitative test of the gravitational hill model

ITEM NO.	ITEM
1028	alpha particle scattering analogue
1153	drawing board

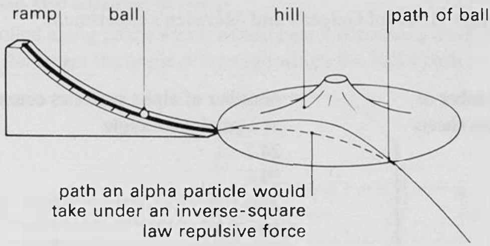


Figure F5
Gravitational analogue of alpha particle scattering.

Use of the hill model

Allow the ball to roll down the ramp and on to the hill. To avoid a bounce at the lip of the hill, it may be necessary to place the bottom of the ramp against the edge of the hill. First, roll the ball directly along a radius of the hill to establish the kinetic–potential–kinetic energy interchange. Then release the ball from the top of the ramp positioned for a deflection of about 30 °. Without moving the ramp, release the ball from a lower starting point to show that the deflection depends on the speed. Finally, repeat with the ramp positioned for a deflection of about 15 ° to show how the deflection depends on the aim.

The analogue enables us to study the path of a single ball rolling on the hill. Geiger and Marsden were unable to study the motion of a single alpha particle. Instead they allowed a random hail of alpha particles to fall on the many atoms in a foil and observed how many were detected at various angles to the incident beam (figure F6).

On the hill, a ball turns through a bigger angle if it runs more slowly. Geiger and Marsden set their detector at a fixed position and slowed their alpha particles down by inserting mica sheets in the incident beam. Table F1 summarizes their observations. It is evident that the slower the speed of the alpha particles, the greater is the number scattered through the set angle.

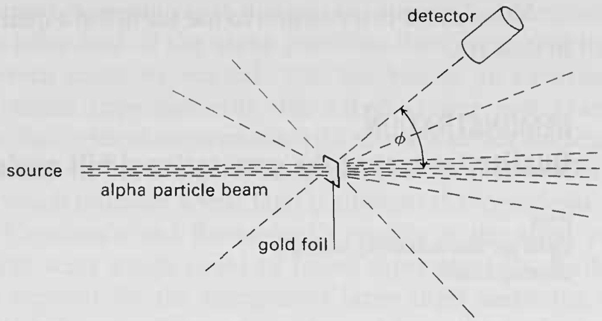


Figure F6
Rough sketch of Geiger's and Marsden's experiment.

Number of mica sheets		Number of alpha particles counted per minute at a particular angle
0		24.7
1		29.0
2		33.4
3		44
4		81
5	speed	101
6	decreasing	255

Table F1
From table VII of GEIGER, H. and MARSDEN, E. 'The laws of deflexion of alpha particles through angles'. Phil. Mag. **25**, 6th series, *1913, pp. 604–623.*

A quantitative test?

It is easy enough with the hill to measure the scattering angle, ϕ, for various values of the aiming error (impact parameter), p. But comparing these results with the results of a scattering experiment with alpha particles is not straightforward. There are two difficulties. With alpha particles one can neither choose nor measure p; and alpha scattering is three-dimensional, whereas with the hill the scattering is restricted to the plane of the table. The steps needed to compare the two sets of results, in order to test whether alpha particle scattering is governed by an inverse-square law, are, briefly:

i the alpha particle results are manipulated so that a graph can be plotted showing how the number scattered at angles greater than ϕ depends on ϕ;

ii measurements of ϕ for various values of p on the hill are used to plot a graph of p^2 against ϕ. If the hill is a good model, these two graphs should have the same shape – but some scaling is necessary before they can be compared directly.

The argument is complex and is presented here chiefly for teachers, though it may be worth pursuing with at least some students, as an example of how a model can be tested. Even without the full argument it is worth demonstrating with the hill that the larger the aiming error, the smaller the scattering angle. The fact that in an alpha particle experiment using foils many atoms thick, only a small fraction of particles are scattered at large angles suggests that very few have small impact parameters: the nucleus is very small.

The relation between the number of particles scattered at more than a certain angle and the aiming error *p*

When the ball is rolled along paths which would pass a distance p from the centre of the hill, the smaller p, the larger the angle ϕ through which the ball's path is turned.

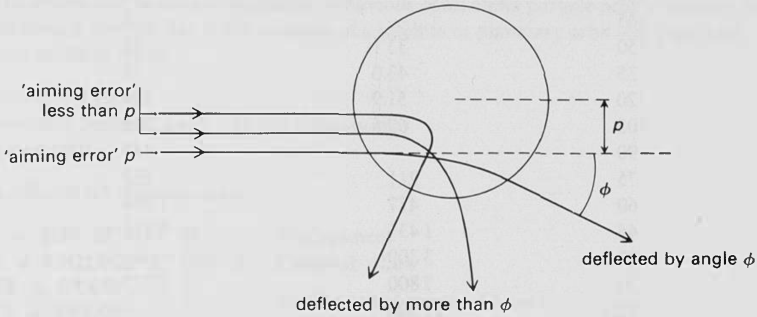


Figure F7

Figure F7 shows the paths of balls on the hill as p is varied. If the deflection of the path is ϕ when the 'aiming error' is p, an aiming error less than p will give a deflection which is greater than ϕ. The path of a ball is essentially two-dimensional, but an alpha particle can go above and below, as well as to the side of a nucleus. Figure F8 illustrates alpha particle paths in three dimensions.

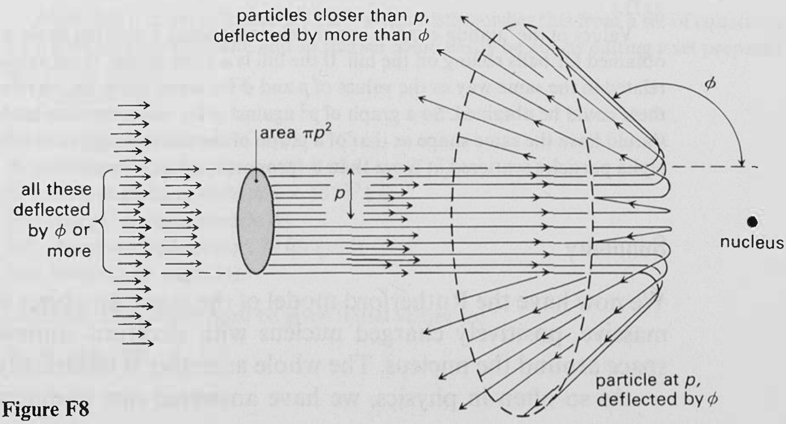


Figure F8

In the three-dimensional situation it is still true that a particle with aiming error less than p will be deflected through an angle greater than ϕ. These particles will be those that fall within an *area* of radius p. (For the hill, those scattered by more than ϕ fall within a *width* $2p$ on either side of the centre.) The area of the circle of radius p is πp^2. If the incoming hail of particles allows equal numbers to fall everywhere, the number passing through area πp^2 will be proportional to p^2.

So the number of particles scattered at more than ϕ is proportional to p^2.

Geiger and Marsden actually measured the number of particles arriving at a particular angle, ϕ. Their results can be treated to find the number scattered by more than ϕ and these are shown in table F2.

Angle ϕ in degrees	**Number of particles detected in fixed time at angle ϕ (Geiger and Marsden)**	**Numbers proportional to number of particles scattered at more than angle ϕ**
180	—	0
165	—	8
150	33.1	32
135	43.0	79
120	51.9	154
105	69.5	266
90	—	448
75	211	767
60	477	1 384
45	1 435	2 811
37.5	3 300	—
30	7 800	7 725
22.5	27 300	—
15	132 000	45 800

Table F2

This table is the end result of fairly complex calculations which involve, among other things, the geometry of the particular experiment. (For details see Appendices A and B of Nuffield Advanced Physics *Teachers' guide Unit 5*, Atomic structure. 1st edn, Longman 1971.)

Values of the 'aiming error' p and the corresponding scattering angle ϕ can be obtained for balls rolling on the hill. If the hill is a good model, these values should be related in the same way as the values of p and ϕ for alpha particles are related – if only these could be obtained. So a graph of p^2 against ϕ for measurements made with the hill should have the same shape as that of a graph of the numbers (given in table F2) of alpha particles scattered at more than ϕ (proportional to p^2) against ϕ.

Summary

We now have the Rutherford model of the atom: an object with a small, massive, positively charged nucleus with electrons somewhere in the space around the nucleus. The whole assembly is electrically neutral.

As so often in physics, we have answered one question, about the structure of the atom, but many more arise immediately. An obvious one concerns the size of the nucleus compared with the atom. This is considered in questions 8 and 9.

Questions

Questions 10 to 13 deal with alpha particle scattering and the inverse-square law.

Film

'The Rutherford model of the atom' is a filmed reconstruction of Geiger and Marsden's experiment using modern apparatus.

Computer

Dynamic modelling system

The system can be used to model the behaviour of an alpha particle near a nucleus. The problem is very similar to the problem of a satellite or planetary orbit (see page 330). This problem needs:

Coulomb's Law, Newton's Second Law, Kinematics } in two dimensions

A suitable set of equations is:

`R = SQR (X^2+Y^2)`	displacement
`F = K*Q1*Q2*E^2/R^2`	Coulomb's Law
`FX = F*X/R` `FY = F*Y/R`	x and y components of force†
`AX = FX/M` `AY = FY/M`	Newton II

`VX = VX+AX*DT`
`VY = VY+AY*DT`
`X = X+VX*DT`
`Y = Y+VY*DT`
	kinematics in two dimensions

`T = T+DT`	

Note that it is only Coulomb's Law which distinguishes this from a set of equations for a gravitational problem, and so this set could easily be got by editing a set prepared and stored earlier.

Constants are:

`E` charge on electron (1.6×10^{-19} C)
`M` Coulomb's Law constant (9×10^9 N m^2 C^{-2})
`K` mass of alpha particle (6.6×10^{-27} kg)
`Q1` charge on alpha particle (2)
`Q2` charge on nucleus (*e.g.*, 79 for gold)

†see footnote on page 331.

Other variables which must be given initial values are

`X, Y, VX, VY, T, DT`

Note that a typical alpha particle energy is 5 MeV which corresponds to a speed of $\sqrt{2 \times 5 \times 1.6 \times 10^{-13}\,\text{J}/6.6 \times 10^{-27}\,\text{kg}} = 1.6 \times 10^{7}\,\text{m s}^{-1}$.

Try VX $= 1.6 \times 10^{7}$ m s^{-1}; VY $= 0$; X $= -1 \times 10^{-13}$ m; and an initial y-displacement of, say, 0.5×10^{-13} m. A suitable time increment is 10^{-21} s. Explore the effect of varying the aiming error (Y), the alpha particle's energy (and so VX), and the nuclear charge (Q2).

For more information see the booklet *Dynamic modelling system.*

Other programs

Several special purpose programs, including J. Campbell's 'Simulation of alpha scattering', and Software Associates 'ALPHA', plot the trajectory of an alpha particle near a nucleus.

There are others (for example R. Beare's 'Alphafoil') which simulate the Geiger and Marsden experiment. J. Harris's 'Particle scattering' focuses on modelling and contrasts the scattering produced by a hard sphere and by a force which obeys an inverse-square law.

SECTION F2

EXPONENTIAL DECAY

RADIOACTIVE DECAY AND HALF-LIFE

Experiments F6 and F7 enable students to follow the decay of a radioactive substance and to measure conveniently short half-lives. A further experiment, F8 on the throwing of dice illustrates a chance-controlled behaviour which has important similarities with the behaviour observed in a radioactive decay.

Questions 15 to *18* are concerned with decay and half-life.

EXPERIMENT
F6 Decay and recovery of protactinium

ITEM NO.	ITEM
130/1	scaler
130/3	GM tube holder
130/5	thin end-window GM tube
1155	small polythene bottle
1156	uranyl(VI) nitrate
1156	concentrated hydrochloric acid
1156	2-methylbutan-3-one (isopropyl methyl ketone)
503–6	retort stand base, rod, boss, and clamp
507	stopwatch or clock
	tray lined with absorbent paper

Details for students appear in the laboratory notes. These assume that the solutions are provided made up and sealed in the polythene bottle.

Contents of the bottle

The plastic bottle, nearly full, contains equal volumes of 2-methylbutan-3-one and acidified uranyl(VI) nitrate solution. Each layer must be as deep as the width of the GM tube window. Each 10 cm^3 of the acidified solution is made from 1 g of uranyl(VI) nitrate dissolved in 3 cm^3 of water, to which 7 cm^3 of concentrated hydrochloric acid is then added. See *School Science Review*, **45**, 157, pp. 597–605 (1964); Revised Nuffield Physics *Teachers' guide Year 5*, page 143. Chemical eye protection should be used when making up the solution, and the bottle should be labelled TOXIC, CORROSIVE, with the appropriate hazard symbols. The bottle may be a cheap, chain-store article, if the cork washer and plastic cap are protected by a sheet of thin polythene screwed under the cap. This also prevents leakage. A better-quality bottle, with a snap-on polythene cap, will leak less easily, but the walls must be thin or many beta particles will be stopped by them and the count rates will be undesirably low. Most glass bottles are too thick, and polystyrene ones are attacked by the contents.

In this experiment, ^{234}Pa is extracted from a solution containing its parent ^{234}Th and 'grandparent' ^{238}U, with which it is in equilibrium. It is extracted by the 2-methylbutan-3-one and the decay of the ^{234}Pa can be followed. It is also possible to follow the recovery of ^{234}Pa in the aqueous solution from which some was removed.

The decay scheme is:

	^{238}U	$\xrightarrow{\alpha}$	^{234}Th	$\xrightarrow{\beta}$	^{234}Pa	$\xrightarrow{\beta}$	^{234}U
Half-life	4.5×10^9 years		24.1 days	(low-energy)	72 seconds	(2.32 MeV)	2.5×10^5 years

The organic reagent removes about 95 per cent of the protactinium, some uranium, but no thorium from the aqueous solution of the uranium salt. The counter does not detect either the alpha particles from the ^{238}U or the weak beta particles (≈ 0.2 MeV) from ^{234}Th, but records only the energetic beta radiation from ^{234}Pa.

The experiment should be performed within or directly above the tray with its absorbent lining in case of spillage.

It is advisable to check the activity of the solution each year; some teachers report that more consistent results are obtained if fresh solution is used each year.

Before starting, teachers should ensure that students know the significance of background counts.

The bottle is shaken for 10 to 15 seconds and placed beside the GM tube so that the window of the tube is opposite the top half of the bottle (figure F9). As soon as the layers have separated, the scaler is started and a 10-second count taken every half minute.

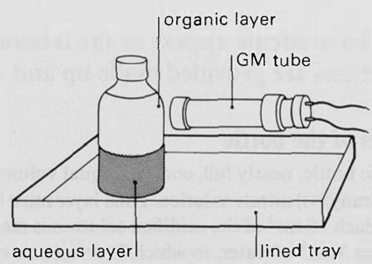


Figure F9
The decay of protactinium.

To observe the recovery, repeat with the GM tube window opposite the lower aqueous layer. To avoid any risk of contaminating the GM tube, it should not be allowed to come into contact with the bottle.

EXPERIMENT
F7 The decay of radon and the determination of its half-life

ITEM NO.	ITEM
1516	picoammeter (see below)
1108	ionization chamber
1066	radon generator
1033	2 cell holders with four cells
1000	leads

Details for students appear in their laboratory notes.

Picoammeter A current of about 10^{-11} A is to be measured in this experiment. As in experiment F2, this can be done by using an electrometer (very high impedance voltmeter) to measure the p.d. across an appropriate high resistance – in this case $10^{11}\ \Omega$.

Some ionization chambers are supplied both with solid lids and with gauze-covered lids: the solid lid is required here.

The ionization chamber has a probe rod running up its centre. This rod is connected to the electrometer input. The 12 V supply is connected between the wall of the ionization chamber and the earthed electrometer input.

For safety, the chamber should be connected to the radon generator by two tubes, so that air circulates between them and neither radon nor powder is puffed into the room when the bottle is squeezed.

Check the generator bottle each year before students handle it as some bottles have been known to split and to spill their contents.

If the bottle does split and powder is spilled, the area should be *gently* ventilated to remove the radon gas. A person wearing disposable plastic gloves, eye protection, laboratory coat, and (if possible) a disposable face mask, can then safely collect the bulk of the powder using two sheets of paper, and transfer it to a bottle for future use. The remaining powder can then be wiped up with damp tissues or filter paper. The contaminated tissues, disposable gloves, and mask should be sealed in a polythene bag and placed with other solid refuse.

Any bottles over ten years old should be *discarded*, and the powder transferred to a new bottle.

The decay scheme is:

$$^{232}\mathrm{Th} \xrightarrow{\alpha} {}^{228}\mathrm{Ra} \xrightarrow{\beta} {}^{228}\mathrm{Ac} \xrightarrow{\beta} {}^{228}\mathrm{Th} \xrightarrow{\alpha} {}^{224}\mathrm{Ra} \xrightarrow{\alpha} {}^{220}\mathrm{Rn} \xrightarrow{\alpha} {}^{216}\mathrm{Po}$$

	^{232}Th	^{228}Ra	^{228}Ac	^{228}Th	^{224}Ra	^{220}Rn	^{216}Po
Half-life	1.4×10^{10} years	6.7 years	1.1 hours	1.91 years	3.6 days	52 seconds	0.16 second

Note that the ^{220}Rn decays into ^{216}Po, which itself decays, also by emitting an alpha particle, with a very short half-life. This and the other decay products do not affect the result appreciably since they have half-lives which are very different from that of ^{220}Rn.

The thorium hydroxide is kept inside a soft polythene bottle fitted with a filter in the neck, or in a dustproof bag within the bottle, and the radon builds up to its equilibrium concentration inside. One gentle squeeze is usually enough to pass sufficient radon into the ionization chamber to give an ionization current of 10^{-11} A. This current diminishes exponentially with time as the radon decays. Meanwhile the concentration is building up again in the bottle, taking several minutes to regain equilibrium.

The experiment tells the same story as experiment F6, though in this case, the recovery of radon is not easy to follow. The experiments together have the value of emphasizing that the exponential decay pattern is common to all radioactive decays, while the rate of decay differs from material to material.

Having completed these experiments, students should know:

i that background counts need to be considered when determining count rates;
ii the general shape of the decay curve and of the growth or recovery curve;
iii how to determine half-life from a graph of activity against time;
iv that the decay of a radioactive isotope is part of a chain of events;
v that, when activity is low, unreliable results may be obtained because of the random nature of radioactivity decay.

Television or video

'The determination of radioactive half-life' (programme 6 in the Granada TV series *Experiment: physics*) could supplement experiments F6 and F7. It concerns the decay of an isotope of indium, and involves students in taking measurements from a filmed experiment.

EXPERIMENT
F8 A radioactive decay analogue

ITEM NO.	ITEM
1155	100 dice
	graph paper
	tray (*optional*)

Details appear in the students' laboratory notes, together with an explanation of why the experiment may be thought worth while.

The dice need to be kept in a box, so that they may be shaken around before being cast out onto the table or tray. Each time those that fall with, say, five pips uppermost are removed, being supposed to have 'decayed' in the 'time interval' represented by one throw of the 'active' dice. Those left are then returned to the box for another throw. Up to ten throws may be needed before the supply of dice is almost exhausted. The 'half-life' of a die is between three and four throws. Graphs of the number of dice remaining and of the number of dice eliminated are plotted against 'time', that is, against the number of throws.

Figure F10 illustrates the results that may be expected, idealized a good deal. Fluctuations are substantial. They can be reduced by repeating the exercise several times and averaging.

Students should be asked to check to see whether the graph is exponential in form, to estimate and to calculate the 'half-life'.

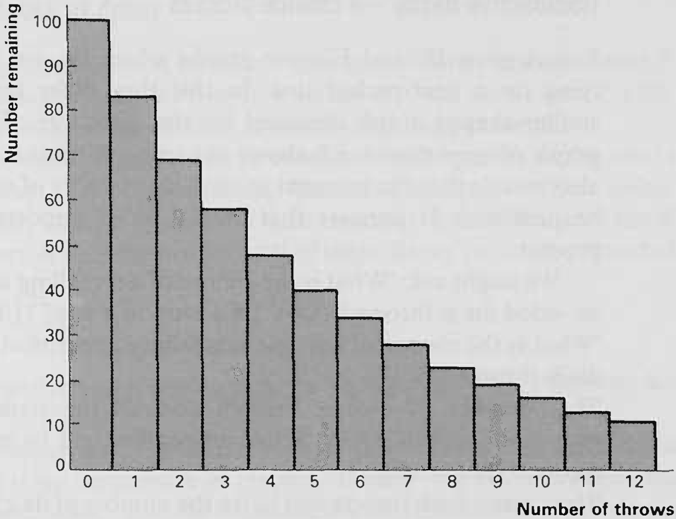


Figure F10
Decay of dice.

Our analogue has the advantage that dice are strongly linked in a student's mind with random events. Indeed, 'like the throw of a die' is what 'random' means to most people. Other analogues, such as those using ball-bearing balls rolling down a slope, give smoother exponential curves, but it is usually less clear how a *constant* chance of decay is built into the analogue.

Devices employing water running out of tubes are *not* recommended because they involve no random element.

Having completed this experiment, the student should know:

i that there are similarities between radioactive decay and the dice-throwing game, suggesting that radioactive decay is also a chance process;

ii that given a constant chance of decay, the variation of decay rate with time is exponential in form; and

iii that, when large numbers are involved, chance events lead to smooth decay rate–time graphs.

Randomness

The concept of randomness occurs again later in the course, especially in Unit K, 'Energy and entropy', and also in Unit L, 'Waves, particles, and atoms'.

Questions

Question 14 is about random events; questions 15 to 18 about radioactive decay, background count, and half-life.

Radioactive decay – a chance process

Experiments F6 and F7 give graphs which do not have their points lying on a near-perfect line. In this they differ markedly from the similar-shaped graph obtained for the discharge of a capacitor. The graph of experiment F8 shows the same variation. That experiment also reveals that the moment in time of the decay of any one die cannot be predicted. It appears that chance is an important factor in this process.

We might ask: 'What is the chance of a six falling uppermost when a six-sided die is thrown?' (1/6). 'Of a four, or a five?' (1/6).
'What is the chance of any one side falling uppermost when a ten-sided die is thrown?' (1/10).
'If a number of dice is thrown and all the sixes removed, what proportion is left?' (5/6). 'What proportion will be left after two such throws?' (5/6 × 5/6).
'How many such throws will halve the number of dice?' 'How many will reduce the number surviving to a quarter?'

Throws	Proportion remaining	Number remaining, starting with 100
1	$5/6 = 0.833$	83
2	$5/6 \times 5/6 = 0.694$	69
3	$5/6 \times 5/6 \times 5/6 = 0.579$	58
4	$5/6 \times 5/6 \times 5/6 \times 5/6 = 0.482$	48 (just under half)
5	$5/6 \times 5/6 \times 5/6 \times 5/6 \times 5/6 = 0.402$	40
6	0.335	33
7	0.279	28
8	0.232	23

Table F3

The proportion left after two 'half-lives' is *not* zero and this provides an opportunity to to clarify the meaning of the term.

These points may be plotted on to the graph obtained by the students in experiment F8. This will emphasize how the reality (with small numbers) differs from the theoretical.

'If one of the dice were marked and observed during the dice-throwing experiment, when would it decay?' (You can't tell; it could decay at the first or any other throw.) All that can be said is that the proportion that is left after each throw is approximately 5/6 and that the proportion that has decayed is about 1/6. This could be tried experimentally by marking one of the dice if necessary.

The chance of decay

In one minute, about half the ^{220}Rn nuclei decay in experiment F7. In one second, a constant small fraction of them decay (*not* 1/120). The rate of decay is the number of nuclei decaying per second.

For dice, at each throw, a constant small fraction of the total tends to 'decay'. The 'rate of decay' has no meaning for dice, unless one imagines throws to represent time intervals. At one throw of the dice, if there are N dice thrown and ΔN of them 'decay', it is to be expected that

$$\Delta N = -\frac{1}{6}N$$

the negative sign showing that the 'decayed' dice are removed, decreasing N.

Nobody, so far as we know, casts dice to decide the fate of a nucleus. There is no 'succession of throws', at each of which a nucleus may, or may not, decay. But the experiments on radioactive decay do suggest that a constant fraction of the nuclei present decay in any fixed time interval. That suggests writing $\lambda\Delta t$ for the chance that a nucleus will decay in a time interval Δt. The decay constant, λ, is the chance of decay per unit time (unit s^{-1}). Then if there are N atoms at the start of an interval Δt, on average $N\lambda\Delta t$ of them will decay in that interval. $\lambda\Delta t$ replaces the constant 1/6 appropriate to dice. Thus

$$\Delta N = -\lambda \Delta t N$$

or

$$\Delta N/\Delta t = -\lambda N$$

This is like an equation for the discharge of a capacitor (Unit B, 'Currents, circuits, and charge'):

$$\Delta Q/\Delta t = -Q/RC$$

The time interval must be small enough for N (or Q) not to change much within Δt; that is, $\Delta N/N$ ($\Delta Q/Q$) must be small. Even so, the equation does not deserve the full equals sign; $\approx$ might be fairer. (Of course, the decay of charge from a capacitor is not subject to the statistical variations which are so obvious in the case of radioactive decay.)

Like the decay of charge on a capacitor, the equation $\Delta N/\Delta t = -\lambda N$ leads to a dwindling away at a decreasing rate, as a numerical example now shows.

Using the equation $\Delta N/\Delta t = -\lambda N$ to produce a decay curve

The numerical example which follows takes a straightforward case of decay and produces graphs of the number of atoms remaining against time, and of the activity (the number of atoms decaying per second) against time. This latter graph is important because most measurements taken from radioactive materials are in terms of activity rather than numbers of atoms remaining.

Students may be given the initial data, shown how the table of results might be arranged, and how the first lines are worked out. They should then be able to complete the table for themselves. This might provide an opportunity to discuss the use of calculators in this and other work.

Let there be $N = 10\,000$ nuclei as yet unchanged into another type of nucleus and let $\lambda = 0.1\ \mathrm{s}^{-1}$. If $\Delta t = 2$ s, the number decaying in the first 2 s interval is 2000, leaving 8000 unchanged. In the next time interval of 2 s the number decaying is smaller, but it is the same fraction of the remaining 8000; that is, 1600 decay. Table F4 shows the example worked out for several successive intervals of 2 s.

Time in seconds	Number at beginning	Number decaying in 2 seconds	Number left after interval	Rate of decay $\Delta N/\Delta t$ per second
0	10000	2000	8000	1000
2	8000	1600	6400	800
4	6400	1280	5120	640
6	5120	1024	4096	512
8	4096	819	3277	410
10	3277	655	2622	328
12	2622	524	2098	262
14	2098	420	1678	210
16	1678	335	1343	168
18	1343	269	1074	134
20	1074	215	859	108
22	859	172	687	86
24	687	137	550	69

Table F4
Numerical example of decay.

It would be better to use a smaller time interval than 2 s, but the calculation would become tedious. Note that random fluctuations are ignored, though one could expect the 2000 decays in the first interval to fluctuate by perhaps 50.

The two graphs (of number of atoms remaining against time, figure F11; and of activity against time, figure F12) have a very similar shape. Indeed, the values of $\Delta N/\Delta t$ are each 0.1 of the corresponding values of N, and if the scale for $\Delta N/\Delta t$ is ten times larger than that for N, the graphs are identical. But, *of course* they are the same; that is what the equation says:

$$\Delta N/\Delta t = -0.1N$$

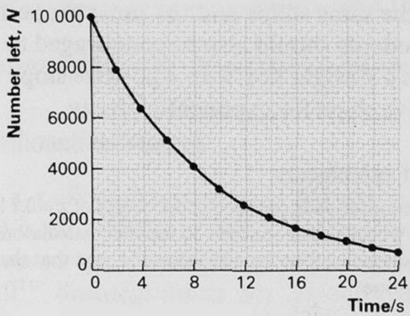


Figure F11
N against t for calculated decay of 10 000 atoms.

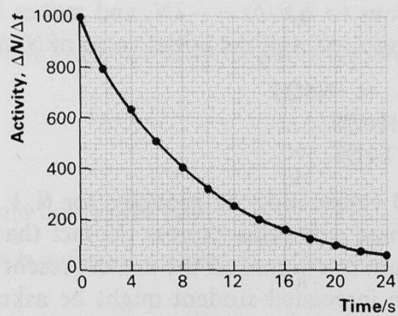


Figure F12
$\Delta N/\Delta t$ against t for a calculated decay of 10 000 atoms.

Subsequent discussion might consider whether or not a real decay curve has this shape.

Two properties of the curve can be checked:
i constant half-life (by testing two or three portions of the curve), and
ii constant ratio (by testing whether the activity falls by the same ratio in equal intervals of time).

For example, if we take a time interval of 4 s:

the activity at 0 s = 1000

ratio 1000/640 = 1.6

at 4 s = 640

ratio 640/410 = 1.6

at 8 s = 410

ratio 410/260 = 1.6

at 12 s = 260

These tests should be applied to the graphs obtained in experiments F6 and F7. Alternatively, the results from those experiments could be scaled to those of the activity curve in the example. If the curves fit they have the same shape and the same properties.

Students should also be encouraged to plot a graph of $\ln N$ against t. This is a straight line with a positive slope for exponential growth and a negative slope for exponential decay.

Use of calculators

In the example discussed above the choice of $\lambda = 0.1$ leads to very simple arithmetic. It may be worth asking students to use their calculators to show that the general form of the curves is the same for any value of λ, and that the larger λ is, the more steeply the curves slope.

Dynamic modelling system

The dynamic modelling system can of course be used to plot or tabulate solutions to $\Delta N/\Delta t = -\lambda N$, and makes it easy to explore the effect of varying λ, Δt, and the initial value of N. A suitable set of equations is:

```
DN = -L*N*DT
N = N+DN
T = T+DT
```

Initial values must be provided for N, L, T, and DT.

These equations express the fact that in each time interval (DT) a constant fraction of all the nuclei present will decay.

An interested student might be asked to write a set of equations expressing the fact that the chance of any *individual nucleus* decaying in one time interval is constant (*i.e.*, to write equations that model the dice analogue of radioactive decay). A possible set of equations is:

```
RANDOMIZE
D = 0
FOR I = 1 TO N
IF RND(1) <L THEN D = D+1
NEXT I
N = N-D
T = T+1
```

The variables N (number to nuclei), L (chance of decay), and T (time) must be given initial values.

It is easy to show how the decay curve becomes smoother if a higher initial value for N is used. (But note that the program gets quite slow for $N > 1000$, because so many random numbers have to be generated and tested.)

For more information see the booklet *Dynamic modelling system.*

The activity of radioactive sources

The activity of a radioactive source is quoted in *becquerel,* where 1 becquerel (Bq) is an activity of one disintegration per second. The earlier unit known as the curie (Ci) was the activity of one gram of radium (3.7×10^{10} disintegrations per second). Sources in use in schools may well have their activities given in μCi and will require re-labelling. The activity of a 5 μCi source is 185×10^3 Bq.

Questions

Questions 19 to 23 involve testing for exponential growth and decay in a variety of situations.

Summary

The students should now know that:

1 Radioactive decay is a chance process.

2 The chance of decay is constant with time.

3 The half-life of a radioactive isotope is the time for half of the nuclei present to decay.

4 The properties of random decay are best displayed if large numbers of events are involved.

5 The chance of a nucleus decaying in 1 second is known as the decay constant (λ). This is independent of temperature, pressure, and other physical conditions.

6 The equation $\Delta N/\Delta t = -\lambda N$ applies to the process of random decay.

7 The experimental data for radioactive decay and the graphs derived from them display the constant ratio property and give a value for half-life. This provides a way of recognizing this type of decay.

8 The unit in which activity is measured is the becquerel (Bq).

THE EXPONENTIAL FUNCTION

The form of the curve for the decay of radioactive nuclei or for the decay of charge on a capacitor shows that the changes are exponential; that is, that the rate of change of something is proportional to how much of that something there presently happens to be. In the two cases referred to, the change was a decay. But the description can apply to such other changes as the growth of bacteria or the spread of an epidemic. These concern growth, not decay, and it is growth we are going to look at in our search for a mathematical equation for exponential change.

The problem is to find a solution for the differential equation used in the numerical example. This yielded a graph of the equation $\Delta N/\Delta t = -0.1N$. 'What would the graph look like if the constant were to be changed from -0.1 to -1.0?' (The general shape would be the same, but the slope would be steeper.)

'What would a graph of $\Delta N/\Delta t = +0.1N$ look like?' (It would grow and grow rather than dwindle away. The bigger N became, the faster it would grow.)

To find a mathematical solution it is best to keep things as simple as possible. This is why we have changed the constant to $+1.0$. We are therefore looking for a solution of the equation $\Delta N/\Delta t = +1.0N$.

Mathematics – the exponential

This is one of the occasions on which we suggest that mathematical ideas can and should be taught within physics teaching. There are several things to be learned here. The suggested approach will, we hope, reveal more of what kind of thing a differential equation is, and what is involved in finding solutions of such equations. This is worth while because of the great importance of differential equations in all sorts of applied and pure science. In the process, the idea of a derivative should also become clearer.

Further, the function studied here – the exponential – is of wide interest and importance. Indeed, if one allows complex exponents, a large fraction of the functions of interest in science can be cast into exponential form. We do not suggest going so far, but the understanding of the exponential function with real exponents may well help later learning in this area.

Within the course, the exponential function has a significant role. It may already have been mentioned in connection with capacitor discharge. There are more examples of exponential growth in Unit G, 'Energy sources'. The exponential also appears in the Boltzmann factor $e^{-E/kT}$ in Unit K, 'Energy and entropy'.

We suggest a graph-drawing approach which is supported by a series of learning questions (24 to 27) in the *Students' guide*. Although not the only possible approach, this provides a detailed numerical look at 'how the thing goes'. It will enable students to work by themselves with little assistance. It may even be that no direct teaching, apart from a concluding summary, will be needed. What follows here is a summary of the development in the *Students' guide*, 'About exponential changes' (pages 404 to 409).

The graphical approach to the exponential function

The approach is in two steps. First the form $N = a^t$ is seen to have the necessary property of increasing by equal multiples when t increases in equal steps. This can be brought out with a numerical example or, more formally, in the following way.

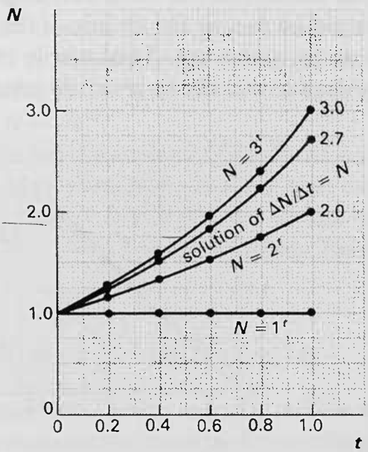


Figure F13
Plots of $N = 1^t$, $N = 2^t$, $N = 3^t$, and solution of $\Delta N/\Delta t = (1.0)N$.

It is convenient to plot graphs of $N = 1^t$, $N = 2^t$, and $N = 3^t$ with values of t from 0 to 1, in 0.2 steps, using either the y^x key on a calculator or tables of logarithms (see figure F13). The graphs can be inspected for a constant-ratio property – do, say, 2^0, $2^{0.2}$, $2^{0.4}$, $2^{0.6}$, etc., increase in constant ratio?

The link with the addition of exponents upon multiplication can be brought out: what, for example, is the value of $2^{3.2}$ and of $2^3 \times 2^{0.2}$?

The values can be found either from the graph or by using the y^x key of a calculator.

More formally, the exponents of the series a^t, a^{2t}, a^{3t}, etc., increase in equal steps t. But $a^{2t} = a^t \times a^t$; $a^{3t} = a^t \times a^t \times a^t = a^{2t} \times a^t$. Each term is larger than the one before by the factor a^t.

Construction of a solution to $\Delta N/\Delta t = 1.0N$

On the same axes as the graphs above, a step-by-step graphical solution to the equation $\Delta N/\Delta t = 1.0N$ can be constructed. Taking time intervals Δt of 0.1, and starting with $N = 1.0$, the solution is constructed by drawing segments of the graph one after the other as short, straight lines with slopes given by the equation. Figure F14 illustrates the process. The technique is just like that used in Unit B, 'Currents, circuits, and charge', and in Unit D, 'Oscillations and waves' for the discharge of a capacitor and for plotting displacement as a function of time.

In the simplest method, shown in figure F14, the slope of the next segment is decided by the previous value of N. This value is always too low, so the curve is not so steep as it ought to be. The proper value of N to use would be that in the middle of the segment to be drawn, but that value is as yet unknown. This simple method gives a graph which, if carefully drawn, will rise to $N = 2.59$ instead of $N = e = 2.718 \ldots$ at $t = 1$.

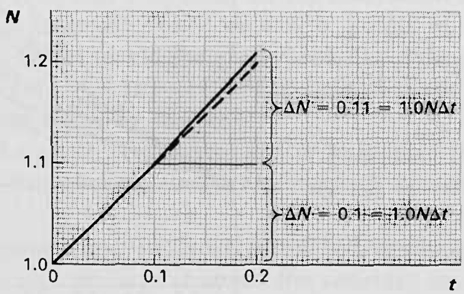


Figure F14
First two steps of a graphical solution of $\Delta N/\Delta t = (1.0)N$.

We see then, that at $t = 1.0$, the solution rises to a value of nearly 2.7, where $N = 2^t$ has risen to the value of 2.0 and $N = 3^t$ to the value 3.0. It may now seem reasonable to propose the form $N = 2.7^t$ for the graphical solution, and write this as $N = e^t$, where e is a number close to 2.7, that can be found more accurately by more accurate versions of the graphical solution process, or its equivalent.

To show that $N = e^{\lambda t}$ may be the form of a solution of $\Delta N/\Delta t = \lambda N$, a graphical solution of $\Delta N/\Delta t = 0.7N$ might be constructed. It is close to the curve $N=2^t$. Calculators, or the first graphical solution, show that $e^{0.7}$ is nearly 2.0. Thus when $\lambda=0.7$, the solution seems to be $N=(e^{\lambda})^t$, or $N=e^{\lambda t}$.

Summary

The growth equation $\Delta N/\Delta t=\lambda N$ has a solution of the form $N=a^t$, where a is some number. This form has the required property of yielding a 'constant-ratio' curve; one which increases N by a fixed factor when t increases in equal steps.

When $\lambda=1.0$ and $N=1.0$ at $t=0$, the value of a seems to be near 2.7, more accurately 2.718..., where this is the never-ending number e. This number e is the value of a for which the slope of $N=a^t$ is equal to 1.0 at $t=0$, $N=1.0$.

The graphical solution provides a rough estimate of e. More accurate methods are available, but all amount to doing arithmetic to find the value of N at $t=1.0$.

It might seem that to cope with $\Delta N/\Delta t=\lambda N$, one would need a table of values of the number a in $N=a^t$ appropriate to each value of λ. This is not so. The required value of a is just e^{λ}, and the solution (for $N=1.0$ at $t=0$) is $N=e^{\lambda t}$. A negative sign takes care of the case of decay. If $\Delta N/\Delta t=-\lambda N$, the solution is $N=e^{-\lambda t}$. This curve falls and falls, being the same as $N=1/e^{\lambda t}$.

In general, N is not 1.0 at $t=0$. This turns out to be easy to cope with. If $N=N_0$ at $t=1$, then N/N_0 is 1.0 at that time. If $\Delta N/\Delta t=-\lambda N$, then also $\Delta(N/N_0)/\Delta t=-\lambda(N/N_0)$. Here N has been scaled down by the factor N_0. So $N/N_0=e^{-\lambda t}$ is the solution. This may also be written $N=N_0\,e^{-\lambda t}$.

Testing curves for the exponential property

The arguments suggested above have stressed the 'equal-ratio' property that distinguishes exponential changes from others. The possible use of logarithms in the plotting of $N=a^t$ should suggest testing for exponential change of a quantity by plotting its logarithm against time (or whatever other variable is concerned). Students should try one or two such tests on data provided in, for example, questions 21 to 23. There will be further opportunity in Unit G, 'Energy sources'.

HALF-LIFE AND THE DECAY CONSTANT

The half-life of a radioactive isotope and the decay constant (λ) for that isotope are simply related.

When $N = \frac{1}{2}N_0$, $t = t_{\frac{1}{2}}$ (the half-life).

We may write $\frac{1}{2}N_0 = N_0 e^{-\lambda t_{\frac{1}{2}}}$

or $2 = e^{\lambda t_{\frac{1}{2}}}$

Whence $\ln 2 = \lambda t_{\frac{1}{2}}$

So that $t_{\frac{1}{2}} = \ln 2/\lambda = 0.693/\lambda$

Question 28 uses the concepts of decay constant and half-life.

RADIOACTIVE DECAY AND RECOVERY

Figure F15 shows the familiar decay curve for a radioactive substance in which the number of atoms is plotted against time. At $t = 0$, the number of atoms of isotope X is N_0. As X decays, the daughter isotope Y grows. 'If Y is stable (*i.e.*, does not decay) what will the growth curve look like?' (The broken line in figure F15.) 'Suppose there were no atoms of Y to start with. How many will there be after a very long time?' (Nearly N_0). 'How fast will the number of atoms of Y rise to begin with?' (As fast as the number of atoms of X falls.)

If the daughter Y is stable, the number of atoms of Y present is equal to the number of atoms of X which have decayed. This is $(N_0 - N)$. Since

$$N = N_0 e^{-\lambda t}, \qquad (N_0 - N) = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t})$$

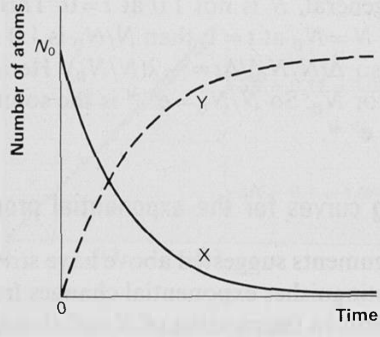


Figure F15

Now suppose that Y is also radioactive. Students should be able to explain why the number of atoms of Y falls exponentially (broken line in figure F16) beyond t_x where there are hardly any atoms of X left. (Almost no more Y atoms are being produced.) Initially, the number of atoms of Y rises almost as fast the number of X atoms falls, since there are very few Y atoms yet, and the number decaying is small, if the half-life of Y is not too short.

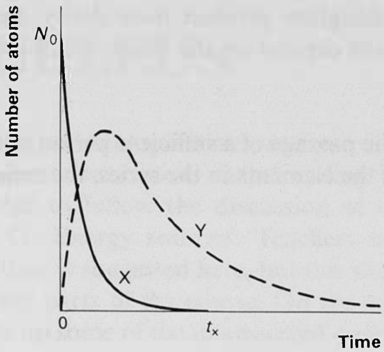


Figure F16
The parent only is present when $t=0$. The daughter Y decays with a half-life five times that of the parent X.

The experiment with protactinium (experiment F6) is an application of this. The parent of protactinium, ^{234}Th, has a half-life of about two weeks, so its decay will not appreciably reduce the amount that is present during a single laboratory session. Its rate of production of protactinium is essentially steady. But the protactinium that is produced decays with a short half-life. In the presence of the parent ^{234}Th, the amount of the daughter ^{234}Pa present builds up until as many protactinium nuclei decay in a second as are produced in a second. As the rate of production is nearly steady, so is the amount present, in the long run.

Note also that the ^{234}Th comes from ^{238}U, with a half-life of over 10^9 years. The sample is more than a few weeks old, so the amount of ^{234}Th has also stabilized. The number of protactinium decays recorded in a second, from the solution containing also ^{238}U and ^{234}Th, when this is steady, is equal to the number of uranium decays in a second.

Dynamic modelling system

The use of the dynamic modelling system suggested on page 378 to show the decay of radioactive nuclei could easily be extended to show the growth of daughter nuclei, and, if they are radioactive, their decay as well.

Summary

Students should now know that:

1 The growth curve of a daughter product which does not itself decay is given by $N=N_0(1-e^{-\lambda t})$.

2 If the daughter product does decay, the shape of the growth–decay curve will depend on the relative half-lives of the daughter and parent nuclei.

3 After the passage of a sufficient period of time, dependent upon the half-lives of the elements in the series, the series will attain equilibrium.

SECTION F3

THE NUCLEUS

This Section raises a series of questions, and provides partial answers. The aim is to give students some understanding of the nucleus and sufficient knowledge to follow the discussion of nuclear power and reactors in Unit G, 'Energy sources'. Teachers may be tempted to provide far more than is suggested here, but this should not be done at the expense of other parts of the course. On the other hand, students may wish to follow up some of the unanswered questions themselves.

Links with chemistry teaching about models of atoms

Some of the content of this Section will also be taught in chemistry (for example Revised Nuffield Advanced Chemistry, Topic 4). This is a good place to bring out the overlap in interests of the two subjects, and perhaps also their differing uses of the same ideas. Chemists will be more inclined to use the ideas to cope with a wide variety of problems, such as those of bonding. This may be reflected, in teaching, in a tendency of chemists to present the ideas as briefly as possible, so as to have time to go on and use them. Physicists may be more interested in using the ideas to probe further into the nature of atoms.

Neither party ought to cast doubt on the value of the other's activities. But chemists should be aware of the danger of accepting as a 'fact' some model or picture of an atom which is in need of further critical examination and development. And physicists should beware of having too limited a perspective; of failing to see the broad relevance of an atomic model for other people's problems.

These considerations suggest that this is a very good stage for a little examination, amongst students who are studying both physics and chemistry, of questions about which of the things being said are facts, which are fair inferences from some model, and which go beyond what a particular model or evidence can support.

The Rutherford model – questions raised

Once Rutherford's model had been established it provided answers to some of the questions which could not be answered by previous models. However, it did not provide all of the answers and it raised some more questions. The evidence collected is in line with the atom having a small, massive, charged nucleus. But at this point it offers no indication of how the nucleus is made or if it has constituent parts. And it offers no explanation as to the place of electrons in the atom, other than to suggest that they exist in the space surrounding the nucleus.

To try to answer some of these questions it is necessary to consider the elements. The fact that each element is different suggests that the atoms of any one element are all the same, but different in some way from the atoms of every other element. If an atom is made up of a nucleus plus electrons, perhaps the differences between atoms can be understood in terms of how they are built up from these parts.

Students should be familiar with the chemical concept of periodicity and the Periodic Table – whether or not they are taking chemistry at A-level. A serial number, Z, can be assigned to each element according to its position in the Periodic Table. Because, at this stage, we have no evidence about the significance of Z – other than that it is a serial number – it is convenient to use its old name 'atomic number' for the time being. While Z goes up regularly in steps of one unit, the mass number A rises a little irregularly, but roughly in steps of two. This suggests that there is more to the nucleus than might be apparent at first sight. As the nucleus is charged, does the charge rise with Z or with A? Are the chemical properties dependent upon the charge?

The charge on the nucleus from the scattering experiment

The alpha scattering experiment can be used to measure how much charge the nucleus has, although the experiment is difficult to do, and the calculation is not easy. The bigger the charge on the nucleus, the larger the angle through which it turns an alpha particle coming at it with a certain 'aiming error' (impact parameter). For an impact parameter p, the angle of deflection ϕ is given by $\cot \phi/2 = 2p/b$, where b depends on the charge on the nucleus and the alpha particle's kinetic energy (see, for example, Nuffield Advanced Physics, *Teachers' guide Unit 5*, Atomic Structure. 1st edn. Longman, 1971. Appendix A). Of course p cannot be controlled in an experiment with alpha particles, and theoretical analysis leads to a more complicated formula for the fractional number scattered at any angle when alpha particles are fired at a metal foil containing many nuclei. By counting just how many particles actually are swung through a certain angle (figure F17), it is possible to tell roughly how many electron charges the nucleus has.

The result of the theoretical analysis is that the fractional number detected at an angle ϕ is given by the equation

$$\frac{q^2 Q^2 nt \text{ (detector area)}}{(4\pi\varepsilon_0 \times \frac{1}{2}mu^2)^2 \ \ 16R^2 \sin^4(\phi/2)}$$

where

q = charge on alpha particle
Q = charge on nucleus
n = number of nuclei per unit volume
t = thickness of the foil
m = mass of alpha particle
u = initial velocity of alpha particle
R = distance from foil to the detector

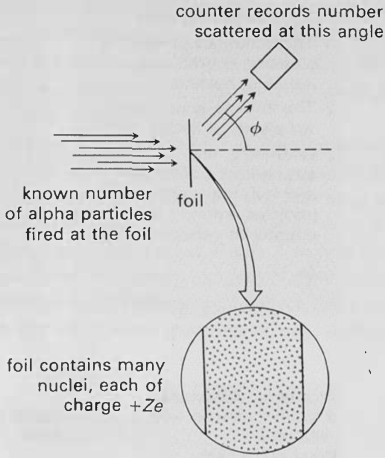


Figure F17
Measurement of Z from alpha particle scattering.

Students may be shown this result, though there is no intention that they should either know the formula or follow the steps in the derivation (which is given in full in Appendix A of Nuffield Advanced Physics, *Teachers' guide Unit 5*). Figure F18 is an attempt to show what went into the argument. It may be possible in this way to show students that such a derivation is possible and the steps that go into it, without laying out the argument in detail. It may help to emphasize that a gulf sometimes has to be bridged before experimental evidence can be interpreted in terms of a physical model.

The equation indicates that the number of alpha particles detected is proportional to the square of the charge on the nucleus. Thus the number of electronic charges in a nucleus can be determined. The results obtained are shown in table F5. The first result for gold is that from Geiger and Marsden (1913); the other results are from Chadwick (1920).

Element	Mass number	Atomic number	Z from scattering
gold	197	79	99 ± 20
gold	197	79	77.4 ± 1.0
silver	108	47	46.3 ± 0.7
copper	63.5	29	29.3 ± 0.5

Table F5
Mass number, atomic number, and experimentally determined value of Z.

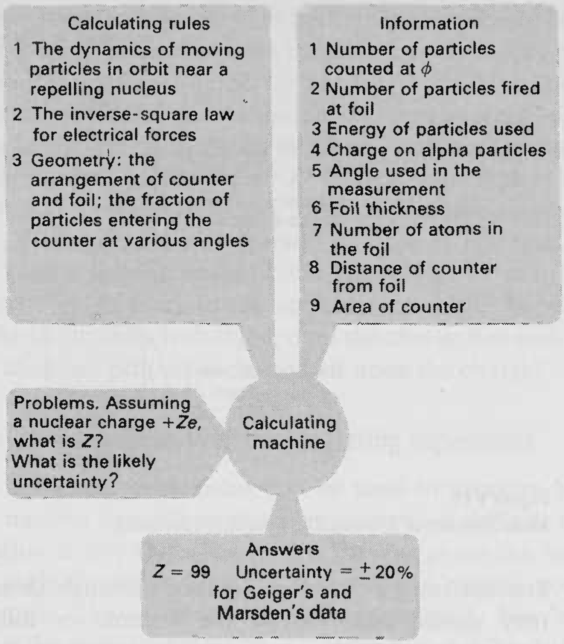


Figure F18

These results show that the charge on the nucleus is equal to the Periodic Table serial number, *i.e.*, the atomic number, Z, and not the mass number, A.

The model has been used here to answer an important question; but in so doing many other questions have been raised. The model may need modification in the light of the experimental evidence which has been generated and the new questions which now need answering.

Background for teachers: evidence for nuclear charge as the periodic serial number

There are several pieces of evidence which are all too difficult to present in detail to students at this stage.

1 The absolute and relative amounts of alpha scattering. The theory shows that the scattering depends on Z^2, and Geiger and Marsden compared the amount of scattering from different elements. See pages 174–7 of WRIGHT *Classical scientific papers – physics*, or page 160, CONN and TURNER, *The evolution of the nuclear atom.* Because mass number A is nearly equal to twice the atomic number Z, this test does not show whether the charge is equal to Z or to A, but only that it varies in proportion to either. But the absolute proportion of particles scattered at some angle does give a measure of Z. See *Classical scientific papers – physics*, p. 180, or *The evolution of the nuclear atom*, p. 162.

Some years later Chadwick made more precise measurements than Geiger and Marsden (see table F5).

2 Evidence from X-ray wavelengths. Moseley, in two papers reprinted in *Classical scientific papers – physics*, reports the remarkable work in which he measured X-ray

wavelengths for many elements. Even without a theory it can be seen that the measurements support Z rather than A as a fundamental measure of the number of charged building-bricks in the atom. In 1913, of course, Bohr's theory had recently been published, and so Moseley was able to use it to suggest a plot of $\sqrt{\text{frequency}}$ against atomic number, and to obtain straight-line graphs.
3 Amount of X-ray scattering. Barkla showed that the amount of scattering of X-rays, which was thought to depend on how many electrons the atoms had, was consistent with the number of electrons per atom being about half the mass number, A. If atoms are neutral, this also supports a nuclear charge of Z rather than A.
4 The wavelengths of X-ray absorption edges. When X-rays of varying wavelengths pass through materials, each element has a sharp increase of absorption at a characteristic wavelength, when inner electrons absorb energy. In effect, this evidence bears the same relation to Moseley's as the absorption spectrum of, say, sodium bears to its emission spectrum.

Some unanswered questions

The picture that has emerged of an atom containing a small, massive, charged nucleus raises several questions:

1 How is a nucleus of mass number A and charge Z made up?
2 What happens to a nucleus that decays by emitting alpha, beta, or gamma rays?
3 What can be said about the number, energy, and arrangement of the electrons in an atom?
4 Are there electrons in the nucleus?

How is a nucleus of mass number A and charge Z made up?

The questions which now arise are:

i Which part of the nucleus carries the positive charge?
ii What is the rest of the nucleus made from?
iii What keeps the particles within the nucleus confined in such a small space?

The following are suggestions as to the way in which the nucleus might be constructed:

a A protons, $A - Z$ electrons
b Z protons, $A - Z$ something else.

'Do these agree with the known facts?' (Consider both charge and mass.)

The possibility of a neutral particle – perhaps a proton–electron combination – was suggested by Rutherford. 'Why would it be hard to detect?' (No charge, therefore little ionization). From 1914 to 1932, some people thought that there might be such neutrons, but no one observed them. Then Chadwick, who had spent much time looking for neutrons, found them. It was known that when alpha particles bombarded beryllium, a very penetrating radiation was emitted – figure F19(a). Chadwick found that if this radiation fell on material which contains plenty of hydrogen (for example wax), fast protons (hydrogen nuclei) emerged – figure F19(b).

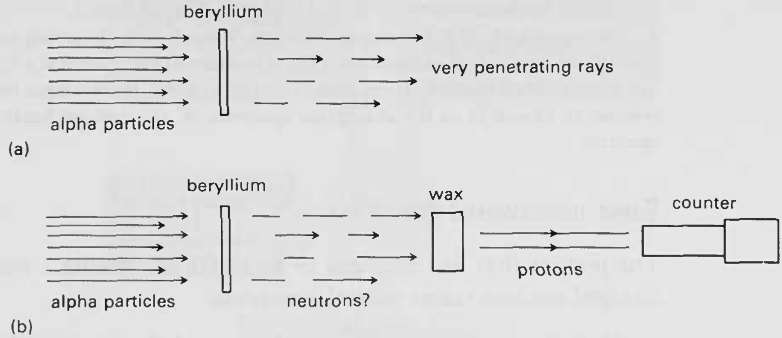


Figure F19
Production and identification of neutrons.

From the speed of the protons, Chadwick used considerations of dynamics to obtain the mass that neutrons would have to have, if they were the penetrating radiation. He showed that the mass of a neutron would be about that of a proton. Other arguments eliminated the other obvious possibility – that the radiation was gamma rays. (A reference back to the discussion of momentum and energy transfer in collisions in Unit A, 'Materials and mechanics', would be appropriate here.)

Thus the neutron has charge zero and mass approximately the same as a proton. This mass is approximately 1800 times the mass of an electron. The discovery of the neutron adds much in favour of a model of the nucleus made up of Z protons and $(A - Z)$ neutrons. And it is now appropriate to use the modern terms for Z and A: proton number and nucleon number.

Radioisotopes and their uses

Since its discovery in 1932 the neutron has played an increasingly important role in pure and applied science. An important example is the use of neutrons from reactors (or 'piles') to produce radioactive isotopes which are used in a wide variety of fields. Many examples are given in the article 'Radioisotopes' in the Reader *Particles, imaging, and nuclei.*

Students are expected to be familiar with the conventional representation, for example $^{12}_{6}C$, $^{14}_{6}C$.

Questions

Question 29 introduces the unified atomic mass unit u.
Questions 30 to 33 are about isotopes and their uses.

What happens to a nucleus that decays by emitting alpha, beta, or gamma rays?

In this course we consider only alpha decay, beta decay, and gamma emission. Students should know the changes in proton number and nucleon number involved in alpha and beta decay:

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

$${}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;A}Y + {}^{\;\,0}_{-1}\beta$$

They should also know that gamma radiation is not particulate, but is the very short-wavelength electromagnetic radiation emitted when a nucleus adjusts its energy levels. It may help to compare this radiation with the light emitted when an atom adjusts its electronic energy levels. There is no change in Z or A when a gamma ray is emitted.

Optical spectroscopy gives information about atomic energy levels, as will be seen in Unit L, 'Waves, particles, and atoms'. Similarly, nuclear spectroscopy gives information about nuclear energy levels. But these are not part of the course.

The nature of gamma emission may be easier to explain after the discussion of ionization energy and nuclear binding energy later in this Section.

Questions

Questions 39 to 41 are about radioactive transformations.

What can be said about the number, energy, and arrangement of electrons in an atom?

Our picture of the atom is now one with a nucleus of Z protons and $A-Z$ neutrons, with Z electrons outside the nucleus. So far, little information about the electrons has been gathered and the high-energy alpha particles do not seem to have told us much about them.

Information about the nucleus came from bombarding it with an alpha particle. The mass of the alpha particle was comparable with the mass of the target nucleus, and this suggests that to gain information about electrons they should be bombarded by other particles of comparable mass, that is, other electrons.

The energies of the bombarding electrons need only be in the region of 10–20 eV and this represents a considerable difference from the MeV alpha particles used previously.

Are there electrons in the nucleus?

In Unit L, 'Waves, particles, and atoms', it will be shown that free electrons cannot exist inside a nucleus. To cram them down into so small a space requires so short a de Broglie wavelength, and, from $\lambda = h/mv$, so large a momentum, that the electrons would fly out again at once. At this stage, it may be best to say that theory gives reasons for rejecting a protons-plus-free-electrons picture of the nucleus, and that a sketch of the theory will come later.

What happens when atoms are bombarded by low-energy electrons?

Students should know that the radioactive sources they have been using cause ionization, and that this is how the radiation is detected (*e.g.* in cloud chambers and in Geiger–Müller tubes). They should also know that air can be ionized by bombardment with much lower-energy electrons; for example, from a hot wire, or by ions from a flame. See, for example, Revised Nuffield Physics *Teachers' guide Year 5*, Demonstration 80. A coil of wire (say a tight coil made from about 0.3 m of 0.4 mm nichrome) heated by passing a current (say from the l.t. variable voltage supply) will also discharge the electroscope.

The energy required to remove an electron from an atom can be determined by controlling the energy of the electrons bombarding the atoms of a gas at a low pressure; the p.d. accelerating the electrons being increased until their energy is just sufficient to cause ionization.

Such experiments can be done in specially constructed apparatus, or in gas-filled 'valves' (thyratrons). The latter are becoming difficult to obtain: they are obsolete in electronics, having been replaced by solid-state devices.

OPTIONAL DEMONSTRATION

F9 Ionization by electron collision

a Using thyratrons

i Xenon
ii Argon

ITEM NO.	ITEM
1049	thyratrons and thyratron base
59	l.t. variable voltage supply
1064	low voltage smoothing unit
	either
1005	multirange meter, 25 V range
	or
1507	voltmeter, 100 V
27	transformer
1040	clip component holder
1000	leads
	i xenon thyratron EN91 or 2021
1507	milliammeter, 10 mA
	either
1151	protective resistor, 1000 Ω
	or
1017	resistance substitution box
	ii argon thyratron 884
1507	milliammeter, 100 mA
1151	protective resistor, 100 Ω

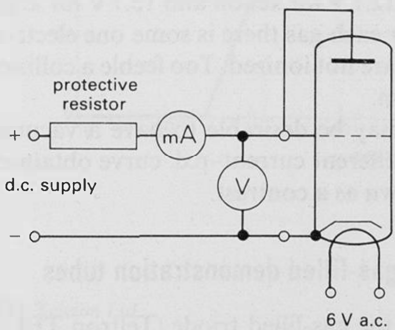


Figure F20
Thyratron circuit.

The grid and anode are connected together, the tube being used as a diode. The protective resistor is essential to prevent damage to the tubes.

Electrons from a hot filament can be accelerated by a potential difference, and the larger the p.d., the higher is the kinetic energy of the electrons. A diode containing gas as well as a hot filament would pass a current of electrons and then, if ions were formed by collision between electrons and gas atoms, there would be an extra ion current. So the experiment is to raise the p.d. across the tube slowly, looking for a rise in current due to the extra ions.

Figure F21 shows the result. At some fairly definite p.d., the current suddenly rises sharply. The gas atoms are ionized by collisions with electrons in the tube only when the electrons have at least the energy corresponding to the p.d. at which the current first starts to climb more steeply.

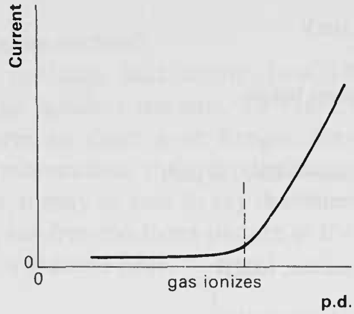


Figure F21
Graph of current against accelerating p.d. for a thyratron.

The gases ionize at different electron energies, the critical p.d.s being about 12.1 V for xenon and 15.7 V for argon.

For each gas there is some one electron energy below which the gas atoms are not ionized. Too feeble a collision will not chip an electron off an atom.

It may be desirable to have a vacuum diode available so that the quite different current–p.d. curve obtained with no gas in the tube may be shown as a contrast.

b Using gas-filled demonstration tubes

Either the gas-filled triode (Teltron TEL 532) or the critical potentials tube (TEL 533) could be used. Both contain helium at low pressure.

Follow the manufacturer's instructions as to appropriate power supply, meters, etc. If the potentially dangerous h.t. supply is used, connections to it must be made with leads having *shrouded* 4 mm plugs (*e.g.*, RS Components stock number 489–037).

i Gas-filled triode

When the filament is heated the current should show a sudden increase when the accelerating p.d. across the tube reaches about 25 V.

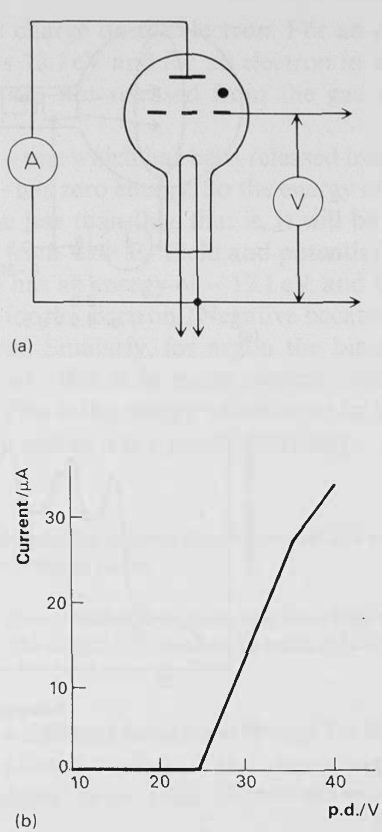
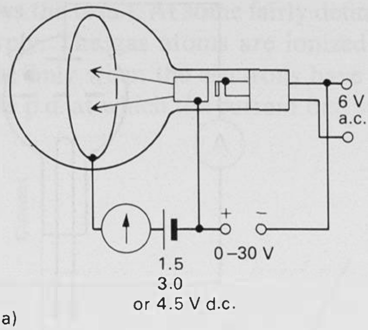


Figure F22
Gas-filled triode:
(a) circuit;
(b) current–p.d. graph.
Based on: Data sheet 532. *Teltron Ltd.*

ii Critical potentials tube

This tube also gives evidence for excitation at electron energies below the ionization potential. The important outcome now is the large effect at ionization. (See figure F23.)

The ionization potential of helium is 24.6 V. Again it may be desirable to show the current–p.d. characteristic of a vacuum tube (*e.g.*, planar diode TEL 520) to emphasize the role of the low-pressure gas in these demonstrations.



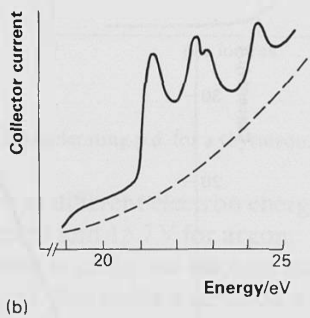


Figure F23
Critical potentials tube:
(a) circuit;
(b) collector current and electron energy.
Based on: Data sheet 533. *Teltron Ltd.*

Television or video

The programme 'Measurement of ionization potential' made by Granada TV as part of the series *Experiment: physics* could be used at this point as an alternative to the demonstrations suggested above.

Chemistry

Ionization is an important matter for chemists; see for example, REVISED NUFFIELD ADVANCED CHEMISTRY, Topic 4.

Ionization and electronic binding energies

Experiments with gases show that the gas atoms are ionized at a particular accelerating p.d., releasing more electrons into the tube, and this is shown by the rise in current. These extra electrons are 'liberated' from the atom and the energy required to liberate them is equal to the accelerating p.d. × charge on the electron. For an electron in a xenon atom this energy is 12.1 eV and for an electron in an argon atom it is 15.7 eV. Electrons are not released from the gas atoms below these energies.

A free electron – one which has been released from the atom but has no kinetic energy – has zero energy. So the energy of an electron bound to the atom will be less than this, that is, it will be negative. The idea should be familiar from Unit E, 'Field and potential'. Thus the electron in the xenon atom has an energy of −12.1 eV, and this is referred to as the *binding energy* for the electron. (Negative because it needs energy to bring it up to zero.) Similarly, for argon the binding energy for the electron is −15.7 eV. But it is more conventional to use the term *ionization energy*. This is the energy which must be *added* to the system to free the electron, and so it is a *positive* quantity.

Questions

Questions 34 and 35 give practice in using the electronvolt as a unit of energy, and conversion from electronvolts to joules.

More precise evidence about ionization energies, and also about excitation energies, is given by spectroscopy. The subject will be taken up again in Unit L, 'Waves, particles, and atoms'.

Each element has a different ionization energy for the first electron and this value can be plotted against Z, the atomic number. The graph is printed in the *Students' guide*, page 378.

Points that might be brought out from the graph

1 Ionization energies are in the range 4 to 24 eV and may typically be described as of the order of 10 eV.

2 The least reactive elements have large ionization energies and the most reactive have small ionization energies.

3 Each element has its own different ionization energy.

4 Next to each noble gas (He, Ne, Ar, ...) is a highly reactive element (Li, Na, K, ...).

Questions

Questions 36 and 37 are about ionization energies and the sizes of atoms and ions.

The fact that an energy of about 10 eV is needed to remove an electron from an atom allows a rough estimate to be made of the atom's size (*i.e.* the distance of the electron from the nucleus). The result, about 10^{-10} m, agrees well with evidence for the size of an atom from Unit A, 'Materials and mechanics' and shows that the electrons lie well outside the nucleus. The estimate is rough because it ignores the electrons' kinetic energy.

Question 38 asks students to make this estimation.

Conclusion

Electrons are bound to the atom and have typical binding energies of -10 eV. The electrons are held in the atom because of the electric attraction between the positively charged nucleus and the negatively charged electron.

HOW ARE NEUTRONS AND PROTONS HELD IN THE NUCLEUS?

The aims of this part of the course are modest. The strong nuclear force is touched on briefly and some discussion of nuclear binding energies is an essential prelude to any consideration of nuclear power. The equation $\Delta E = c^2 \Delta m$ is introduced in an *ad hoc* manner and without justification. It is the basis for our calculations of nuclear binding energies.

As protons are positively charged, the electric force between protons in a nucleus will be a repulsive one. Whatever attractive force holds the nuclear particles together must be strong enough to overcome the repulsive force between protons. (A simple calculation shows that the gravitational force between two protons is too weak by a factor of about 10^{-36}.) 'Strong nuclear force' is an appropriate term.

Question 42 is concerned with this calculation.

Other information about the strong nuclear force and its range comes from scattering experiments and from knowledge of nuclear binding energies.

In Rutherford scattering experiments 5 MeV alpha particles might typically be used. Such alpha particles can approach to within about 5×10^{-14} m of the nucleus (question 11). The results of such scattering experiments are well accounted for by Coulomb's Law: there is no evidence for another force between nuclear particles at this distance. On the other hand, later experiments with energetic protons show that a new force begins to affect them at a distance of 2 or 3×10^{-15} m from the nucleus.

Information about nuclear radii comes from electron scattering experiments (electron energies of several hundred MeV are used). It turns out that the density of nuclear matter is more or less constant, which means that each individual nuclear particle occupies about the same space whether it is in a light or in a massive nucleus. This evidence all suggests that the force between nucleons is strongly attractive at distances of 1 or 2×10^{-15} m, but falls off rapidly beyond this. It is a 'short range' force. To prevent the collapse of nuclear material the force must become repulsive at less than 1×10^{-15} m.

The graph of total nuclear binding energy against nucleon number (*Students' guide*, figure F10) shows that the more nucleons there are in the nucleus, the more negative the binding energy becomes: more energy is needed to take apart a big nucleus than a small one (just as more energy is needed to evaporate a large drop of water than a small one). The fact that the graph is more or less linear is further evidence for the short-range nature of the strong nuclear force. (If every nucleon interacted with every other nucleon in the nucleus we would expect the binding energy to depend approximately on (number of nucleons)^2, whereas, in fact it seems as if each one only interacts with a small number of near neighbours.)

Background for teachers

Although we do not expect students to know more about the strong nuclear force than that it is strong, independent of charge, and short range, teachers may find it useful to have more information themselves. GRIFFITH and TEBBUTT *Notes for guidance for the nuclear physics option* is a useful source.

But the total binding energy does not change quite linearly with A, and this has very important consequences. Figure F11 of the *Students' guide* (page 380) shows the probably more familiar graph of average binding energy per nucleon against A. The significance of the negative sign should be emphasized throughout: energy is required to release a nucleon from the nucleus against the attractive nuclear force.

Points to be brought out from the curve

1 For most values of A the average binding energy per nucleon is about -8 MeV, which is the slope of the graph of binding energy against nucleon number.

2 The lowest value, *i.e.* the most stable nucleus, is at ^{56}Fe.

3 Some nuclei, most notably ^{4}He, lie in pronounced troughs in the curve. This suggests that this nucleus is particularly stable and offers an explanation of the importance of the alpha particle in radioactive decay.

Question 43 is based on the curve of average nuclear binding energy per nucleon.

Background for teachers on nuclear binding energies

Teachers may know that the binding energy of a nucleus is fairly well accounted for by the 'semi-empirical formula':

$$-\mathrm{BE} = aA - bA^{2/3} - c(Z^2/A^{1/3}) - d(N-Z)^2/A$$

where a, b, c, and d are all positive constants.

The first term simply expresses the fact that the more nucleons there are in the nucleus, the more tightly bound they are; the second is a 'surface energy term' – the surface area depends on $A^{2/3}$, and nucleons at the surface are less tightly bound; the third term expresses the Coulomb repulsion between the protons. The fact that stable nuclei tend to have neutron number $N \approx Z$ is accounted for by the last term. See for example GRIFFITH and TEBBUTT *Notes for guidance for the nuclear physics option.*

Note that many sources treat binding energy as a *positive* quantity. To be consistent with other parts of this course we regard it as negative.

How do we know these values for nuclear binding energy?

By this time we hope that students are asking how physicists know the values of nuclear binding energies. We can offer no theoretical basis for $\Delta E = c^2 \Delta m$ and its significance is often woefully misunderstood. If it were not the basis for the calculation of nuclear binding energies, it would not be part of this course.

It is wrong to talk about mass being converted into energy. Einstein's achievement (it is part of his special relativity theory of 1905) was in uniting the two apparently unrelated conservation principles: conservation of mass and conservation of energy. This can be observed when an electron is accelerated to a speed approaching that of light. It is continually gaining energy by being in an electric field. Since it cannot travel faster than light, the 'ultimate speed', its speed cannot go on increasing indefinitely: its mass increases, and can rise without limit. This mass increase has to be allowed for in the design of particle accelerators. Einstein told us that mass and energy are measurements of the same thing. $\Delta E = c^2 \Delta m$ tells us the exchange rate between, say, kilograms and joules. $\Delta E = c^2 \Delta m$ applies not only to moving bodies and kinetic energy, but to all forms of energy.

It would be wrong to let students go away thinking that the equation $\Delta E = c^2 \Delta m$ applies only to nuclear physics. It is always true: the mass of any body or system of bodies is a measure of its (their) energy. So, for example, the mass of a charged dry cell is larger than the mass of the same cell when discharged; the mass of a spring increases when it is stretched; the mass of a heap of gunpowder is more than the mass of the reaction products (at the same temperature); the mass of the Earth is *less* than the sum of the masses of its individual atoms when they are far apart (because the total has less energy than the isolated constituent parts).

Questions

Questions 44 and 45 give practice in using $\Delta E = c^2 \Delta m$ in non-nuclear problems.

Reading

We offer no derivation of $\Delta E = c^2 \Delta m$. More important than trying to follow a derivation is an appreciation of what the relationship says and does not say. Among references which teachers may find useful are:

ROGERS *Physics for the inquiring mind.* Chapter 26.
DAVIES *The forces of nature.* Pages 31–33.
CARO, McDONNELL, and SPICER *Modern physics.* Section 6.5.

Calculation of nuclear binding energy: an example

The rest masses of the neutron, the proton, and the electron are known with considerable accuracy. The values, in unified atomic mass units, are:

mass of proton, $m_p = 1.007\,3$ u

mass of neutron, $m_n = 1.008\,7$ u

mass of electron, $m_e = 0.000\,5$ u

(1 u $= 1.66 \times 10^{-27}$ kg, one-twelfth of the mass of an atom of ^{12}C)

The total mass of the constituent parts of the atom $^{A}_{Z}$X is

$$Z m_p + (A - Z) m_n + Z m_e$$

The results of the calculation are always found to be *greater* than the accurately measured mass of the atom (*e.g.*, from a mass spectrometer). For example, the mass of an atom of ^{4}He is 4.002 6 u, whereas the total mass of two protons plus two neutrons plus two electrons is 4.033 0 u. There is a mass loss of 0.030 4 u or $0.050\,5 \times 10^{-27}$ kg. From $\Delta E = c^2 \Delta m$ this corresponds to an energy loss of 4.54×10^{-12} J or 28.4 MeV. Thus the nuclear binding energy of ^{4}He is -28.4 MeV, corresponding to an average binding energy per nucleon of -7.1 MeV, as shown in figure F11 of the *Students' guide.*

Questions

Question 46 asks students to go through this calculation for ^4He.
Question 29 gives practice with the unified atomic mass unit, u.

The direct conversion between u and MeV is 1 u = 931 MeV.
A book of data such as Tennent, *Science data book*, or the Revised Nuffield *Book of data* is invaluable for providing data for such calculations.

INDUCED FISSION

The average binding energy per nucleon is less negative for the massive nuclei (*e.g.* U) than for nuclei in the middle of the range (*e.g.* Fe). If the nucleons in one of the massive nuclei could be redistributed among two or more less massive nuclei then some energy would be released in the process. Although fission is energetically possible, it rarely happens spontaneously. But it can be induced, as for example when ^{235}U captures a 'slow' neutron* to form an excited ^{236}U nucleus which then splits up:

$$^{235}U + n \rightarrow {}^{236}U \rightarrow X + Y + xn + Q$$

In this reaction X and Y are the fission fragments: Ba and Kr are typical examples but there are many possibilities. Fission is usually asymmetric, that is, X and Y are different. Two or three neutrons are released, which can in turn trigger the fission of further ^{235}U nuclei in a chain reaction. Q represents the energy released; it is of the order of 200 MeV per nucleus split (which is equivalent to 8×10^{13} J per kg of ^{235}U). Most of this energy appears as kinetic energy of the fission fragments.

FUSION

If two nuclei from the lighter end of the curve of average binding energy per nucleon could combine into a more massive nucleus, then some energy should be released as the product nucleus will be lower on the curve. (The extreme conditions required for fusion are the subject of one of the 'Energy options' in Unit G, 'Energy sources'.)

For example, the fusion reaction

$$^{3}H + {}^{2}H \rightarrow {}^{4}He + {}^{1}n + Q$$

gives a net energy release of 17.5 MeV (2.8×10^{-12} J). Although this is

*A slow (or thermal) neutron is one with energy equal to the average thermal energy of the atoms of the medium (*i.e.*, about 0.025 eV at 20 °C).

less than the 200 MeV released by fission of a uranium nucleus, the energy release per unit mass of 'fuel' should be considered.

Fission and fusion are dealt with in more detail in Unit G, 'Energy sources', which includes questions dealing with the energy released.

Reading

The background article 'Our nuclear history' in the Reader *Particles, imaging, and nuclei* discusses fusion as the Sun's source of energy.

Summary

Using the graph of average binding energy per nucleon against A it is possible to calculate the energy which could be released in fission and fusion processes.

THE FOUR INTERACTIONS

Much has been omitted (models of the nucleus, energetics of alpha and beta decay, other modes of decay) and many questions left unanswered (which nuclei will be unstable and how will they decay...). But there is no time for more about the nucleus in this course. Instead one might end the Unit by offering students a very broad view and telling them that they have now met three of the four forces or interactions of physics: gravitational, electromagnetic, and strong nuclear. (The fourth, the weak nuclear interaction, governs beta decay.) One of the main goals of theoretical physics is unification. Magnetism and electricity were united by the work of Faraday and Maxwell, and finally Einstein, who showed how the magnetic effects which arise when charges are in motion can be explained by relativity theory. Recent experiments with high-energy particles (1982) support a theory which brings together the electrical and weak nuclear interaction. The aim of grand unification theories (GUTS) is to bring all four interactions together.

Further reading

PROJECT PHYSICS Text Unit 6, *The nucleus*, especially chapters 23 (Probing the nucleus) and 24 (Nuclear energy; nuclear forces) is a very good source for students who want to read more about these topics.

The article 'The particles and forces of nature' in the Reader *Particles, imaging, and nuclei* deals with the four forces and with attempts to produce a unified theory.

The article 'Lasers probe the atomic nucleus' in the Reader *Particles, imaging, and nuclei* describes a current area in nuclear research.

Rutherford Appleton Laboratory Monographs are short pamphlets dealing, mostly, with particle physics. Among those so far published which have some relevance to this Unit are *Atoms, particles, leptons and quarks, Experimental particle physics, The universe and Man.*

Unit G
ENERGY SOURCES

Maurice Tebbutt
Faculty of Education, University of Birmingham

Contributors
Michael Carrick, North Bromsgrove High School;
Bev Cox, Streetly School, Sutton Coldfield;
John Gardner, Shenley Court School, Birmingham;
Tom Gregory, Codsall School, Staffordshire;
John Minister,Bishop Walsh School, Sutton Coldfield;
Richard Spiby, King Edward VI Camp Hill School for Boys, Birmingham

Suggested time allocation: three weeks

PLAN OF THE UNIT

Section G1
The background to energy supply and demand

GCSE	▶	**Conversion and conservation of energy**	
GCSE	▶	**Efficiency** **Second Law of Thermodynamics**	▶ Unit K, 'Energy and entropy'
exponentials (Unit F)	▶	**Patterns in supply and demand for fuels**	

Section G2
Power from nuclear fission

nuclear masses (Unit F)	▶	**Origin of the energy**	
electrical potentials (Unit E)	▶	**Obtaining a chain reaction**	▶ nuclear power stations: Section G4
collisions (Unit A)	▶	**Moderation**	

Section G3
Conserving fuel in the home

electrical resistance (Unit B) ► **Thermal transfer of energy and thermal resistance**

Surface resistance

Cavity resistance

Double glazing, cavities, and cavity insulation ► domestic energy conservation: Section G4

Section G4
Energy options

Nuclear power

Solar energy

Fossil fuels

Other sources of internal energy

Water power

Conservation

Storage

INTRODUCTION

This Unit provides students with the physics background to energy supply and demand. There is no shortage of material on this topic. Some material, like *Science in society* and *Siscon in schools*, has been written for schools, and some is intended for a more general audience. While there is bound to be a little overlap between this Unit and existing material, our intention is to provide a somewhat different focus – specifically, the particular contribution that physics can make to the debate on energy supply and management. Thus the emphasis is on the basic physics which applies to many aspects of energy sources and the numerical data which are involved.

It is not possible, however, to ignore areas such as economics in which physicists are normally not involved. Here, more variables are involved, they are less controllable, and the situations are altogether more problematic than is usually the case in physics.

The content of the Unit is, therefore, somewhat unusual in that it ventures into other disciplines. The style adopted in this Unit is also different from that of most others in the course, since it must reflect the uncertainties inherent in much of the content.

Energy supply is a continuing problem likely to be subject to rapid change. This has both advantages and disadvantages. The Unit's concern with a 'live' issue should help motivation, but students often object if materials seem out of date. However, data can be kept reasonably up to date by revising the background book *Energy sources: data, references, and readings*, and the questions which are asked will continue to focus on important aspects of the problem.

THE STRUCTURE OF THE UNIT

Section G1: The background to energy supply and demand

Basic ideas

Conversion and conservation of energy

Efficiency and the need for fuels

Second Law of Thermodynamics

Personal energy

Patterns of growth in fuel consumption

Resources and their lifetimes

Detail

Contribution of different fuels

Balance of supply/demand for different nations

Difference in *per capita* consumption for different nations

What fuels are used for

Characteristics of fuels

Undesirable consequences of fuel use

Factors influencing further growth

Section G2: Power from nuclear fission

Energy from fission

calculation from nuclear masses

origin from potential

Triggered by extra neutron

Chain reaction

Critical mass

Nuclear cross-section

Isotopic composition of uranium

Neutron energy required for fission

Moderation and control

Enrichment

Section G3: Conserving fuel in the home

Analogy between thermal flow of energy and electric current

Thermal resistance

Thermal resistance coefficient

Surface resistance

Factors affecting surface resistance

Calculating temperatures in thermal transfer systems

Heating buildings:
—thermal lag
—ventilation

Section G4: Energy options

Nuclear power

A Nuclear power stations
B Nuclear fusion
C Nuclear safety

Solar energy

D Passive systems
E Active systems
F Photovoltaics
G Biomass
H Wind energy

Fossil fuels

I Power stations
J Alternative sources of fossil fuels

Other internal energy sources

K Geothermal
L Heat pumps
M Combustion of refuse

Water power

N Hydro-power
O Wave energy
P Tidal energy

Conservation

Q Domestic
R Industrial
S Transport

Storage

T Storage

Table G1

It is difficult to describe the material in the Unit in an entirely linear sequence. Table G1 outlines the content and shows some of its structure. The more detailed comments below should be read in conjunction with this table.

All students should study Sections G1, G2, and G3. It has been assumed that Sections G1 and G2 will be taught using the questions in the *Students' guide* as seems appropriate. Section G1 provides the basis for most of the work of the Unit. It is in two parts: 'Basic ideas' and 'Detail'. The principles of nuclear fission are dealt with in Section G2. This is a compulsory section since any student of A-level physics in the latter part of the twentieth century must surely come to grips with these principles. No value judgment about nuclear power is implied in this, and the application of these principles to electric power generation takes its place with other energy options in Section G4.

The core syllabus for physics at A-level includes thermal conduction. Section G3 sets this in the practical context of conserving fuel in the home. While this Section could be taught in class, it has been written so that the students could complete it on their own, perhaps as homework.

Each student will then study one or two of the energy options in Section G4. This will provide practice in the technique of reading to find information and reporting to the rest of the group, and also ensure that a reasonably balanced view is obtained of this very broad area. Because some students find reading for learning difficult, some guidance is given in the *Students' guide*.

THE PLACE OF THE UNIT IN THE COURSE

There are a number of reasons why this Unit may be appropriately placed towards the end of the first year of the course. Since it deals with a particularly relevant topic it can be regarded as an intermediate end-point. The Unit can also be taught piecemeal, which may be useful at the latter end of the summer term when work in schools tends to be disrupted. It could, for example, be taught in parallel with other Units, either as a whole or in part; or Section G4 might be scheduled for homework or as a vacation task. This last suggestion has *not* been assumed in estimating the time to be spent on the Unit.

The Unit involves practice or extension of the ideas of flow of charge met first in Unit B, 'Currents, circuits, and charge', exponential change (Unit B and Unit F, 'Radioactivity and the nuclear atom'), and nuclear structure (Unit F); and it provides a basis for work on internal energy met again in Unit K, 'Energy and entropy'.

Time allocation

The topic 'Energy sources' raises enough issues for a complete course. But within the context of an A-level physics course three weeks seems an appropriate allocation.

It could be divided like this:

Section G1 The background to energy supply and demand. Rather more than one week + homework.

Section G2 Power from nuclear fission. About half a week + homework.

Section G3 Conserving fuel in the home. No more than one double period + more than the Section's share of homework time.

Section G4 Energy options. Less than half a week + homework.

Reporting back. Half a week.

These suggestions are very approximate because the flexibility of the Unit will allow the proportion of homework time to be increased if this is useful. The allocation for Section G3 assumes that students will do most of the work on their own as suggested above. Alternatively, more class-teaching time will need to be found.

On the other hand, if students were to use the summer holiday for the reading on energy options, some class time could be saved.

Both *Students'* and *Teachers' guides* for Section G1 are rather longer in proportion to the whole Unit than the Section time allocation would justify. This is because a number of ideas which are likely to be unfamiliar to students and teachers are introduced; Sections G2 and G3, however, deal with more familiar topics and are relatively short.

BIBLIOGRAPHY

Teachers may need two different kinds of information to help in teaching the Unit – either general or specific. There are so many sources of general material that the problem is likely to be that of making a choice. Paperback books are likely contenders on grounds of cost and of being up-to-date, though even this cannot be guaranteed.

Useful ones to start on might be:

RAMAGE *Energy: a guidebook.*
McMULLEN, MORGAN, and MURRAY *Energy resources.*
CHAPMAN *Fuel's paradise.*
FOLEY *The energy question.*

Specific information may also be needed on nuclear power, thermal conduction, and the energy options.

Nuclear power is covered in the first two references and also in:

GRIFFITH and TEBBUTT *Notes for guidance for the nuclear physics option.*

Information on the particular approach to thermal conduction used in this Unit is not easy to obtain in one source, but is to be found, in part, in some books on building technology or heating system design, such as:

BASSETT and PRITCHARD *Heating*.

This book's approach to the problem of heating buildings contains a great deal of physics and it would be a very useful source of some of the material in Section G3. A search of the appropriate section of a school, college, or local public library may, however, be more productive of sources of similar information than an attempt to obtain any particular book that might be recommended here.

Specific articles on the 'Energy options' are included in the background books *Energy sources: data, references, and readings* and *Energy options: a reader*. The former also contains specific references and guidance for both teachers and students in locating suitable material.

Full bibliographic information about the sources listed above is given on page 505 of this *Teachers' guide*.

TEACHING ENERGY

Energy is rightly regarded as an important idea. In physics it is a highly abstract concept which has little connection with direct experience and which must draw on a number of contributory concepts if it is to be understood. 'Energy' is also used in everyday language, and is something that the general public is increasingly aware of. This is partly to do with its links with fuel (from which it is usually not distinguished) and fuel supplies; partly because of its everyday use; and partly because of the increasing use of energy as a unifying concept in elementary science courses.

The problems produced by introducing the concept at an early stage in school curricula have been dealt with in detail in the following:

WARREN *The teaching of physics*.
WARREN 'The nature of energy', *Eur. J. Sci. Educ.*
WARREN 'Energy and its carriers: a critical analysis', *Phys. Educ.*

Attempting to teach the concept of energy before it can be built on a framework of supporting concepts may run the risk of allowing (even encouraging) misconceptions and the need to unlearn these later. Alternatively, leaving the teaching of the concept until it can be taught logically would deny most of the population access to it. There is no doubt that energy should form part of an A-level course in physics, as should, we believe, a wider discussion of the issues concerned with energy supply, demand, and management. Whatever approach is used, clearly those students who take this course should be helped to have as clear an understanding of the concept as possible. Teachers may find the references listed above useful here.

The volume of discussion about the teaching of energy, including the teaching of 'heat', suggests a distinct lack of agreement about how this should be done. In addition to the references above teachers may wish to consult the following in order to appreciate the detailed arguments:

OGBORN 'Dialogue concerning two old sciences'.
SCHMID 'Energy and its carriers'.
SUMMERS 'Teaching heat: an analysis of misconceptions'.
WARREN 'The teaching of the concept of heat'.

We cannot reiterate all the arguments here but we offer some suggestions which may be helpful in dealing with various points, some arising in the references listed and some arising only in this Unit.

Energy is an abstract but very useful concept

When any well-defined system changes from one definite state to another, its energy changes by the same amount, no matter how the change of state came about. We can say how to measure the change of energy by choosing one particular way of changing the state: most commonly we keep it thermally isolated (adiabatic change) and then the energy change is the work performed. Strictly, we can only speak of energy *changes*, but it is useful to speak of the energy of a system, as long as we do not alter the implied zero value. Because energy is conserved it is a useful book-keeping concept enabling us to keep track of energy transfers from one part of the system to another.

An important part of the energy of a system is its *internal energy*. For example, a bullet has kinetic energy because it is moving; it has potential energy because it is above the surface of the Earth; and it has internal energy because it is hot. (The internal energy is simply the sum of the kinetic and potential energies of all the individual particles of the bullet.) If the bullet is suddenly stopped it loses kinetic energy. Some of this kinetic energy is transferred to internal energy of the bullet and the bullet gets hotter. As internal energy is transferred to the surroundings again the bullet cools.

Mistakes can arise from not considering the *whole* system. Thus to speak of the 'chemical energy of a lump of coal' is to ignore the oxygen which is needed to burn it. While it would be cumbersome to say 'the chemical energy of the fuel-plus-oxygen system' every time, the point should not be lost sight of.

Energy is not an objective substance, nor is it a fluid

The uncritical use of the idea of energy flow could imply some notion that energy is a substance; of course it is not. Nevertheless there are some circumstances in which it is useful and sensible to think of energy 'flowing'. We do not speak of energy flowing from potential to kinetic as a ball falls. But it is reasonable to think of energy flowing between parts of a system. If one part loses energy and another gains it, through some interface, it makes sense to think of a flow across that interface. Thus we do think of energy from a radiator entering a room, and then leaking out through walls and windows to the outside. For the idea of flow to be useful the situation has to be one in which energy is lost by one part of a system to an immediately adjacent one.

Thus the idea of flow is a useful generalizable concept, and there are other areas of physics where it is used, although no objective fluid substance actually flows. Examples in this course occur in Unit B, 'Currents, circuits, and charge' and Unit H, 'Magnetic fields and a.c.' For example, flux (whose very name means flow), is a useful concept to physicists and engineers in electromagnetism.

See also the discussion of flow in the article 'Electromechanical similarities' in the Reader *Physics in engineering and technology.*

Forms of energy

In earlier courses phrases such as 'chemical energy' or 'electrical energy' will have been used, as well as potential energy and kinetic energy. One difficulty with 'chemical energy' has already been mentioned. Another is apparent from a consideration of the meaning of the term. Consider a chemical reaction, for example burning a fuel in air, and heating the surroundings. In such a chemical reaction there is a difference between the potential energy of the bonds in the reactant molecules and those in the product molecules. In the example chosen this potential energy decreases, the internal energy of the surroundings and the product molecules (*i.e.*, their kinetic energy and the potential energy due to forces *between* molecules) increases. Similarly, a careful analysis of other terms, for example 'nuclear energy', reveals that what is involved can be described in terms of changes in potential and kinetic energy and, ultimately, internal energy. While the notion of many different forms of energy has its uses, it may be worth while, from time to time, to raise the issue of their reducibility to potential and kinetic energy.

'Heat' is not a 'substance' but a process

We commonly speak of the heat in a body, meaning its internal energy, but scientific usage of the word 'heat' has changed. A simple way to look at the matter is to regard the statement of the first law of thermodynamics

$$\Delta U = \Delta Q + \Delta W$$

as saying that the internal energy, ΔU, can be changed in two ways: by working, ΔW, and by thermal transfer of energy across a temperature difference, ΔQ. Calculations of energy transferred by either of these processes are changes in internal energy, so it is wrong to use 'work' and 'heat' to mean amounts of energy *in* something. Our solution is to use the terms 'working' or 'doing work', and 'heating' or 'thermal transfer' to describe processes and to avoid the use of 'heat' as a noun.

This still requires care since 'heating' has a range of colloquial meanings, including 'making hotter', and also the process of thermal transfer of energy in situations when there is no temperature rise, such as melting a block of ice, or maintaining the temperature of a house. There is, of course, in all these cases, thermal transfer of energy due to a difference in temperature. For these reasons it is clearer to use 'thermal transfer' for all cases where energy flow is the result of a temperature difference; but 'heating' may be less clumsy and will be satisfactory as long as its full meaning is appreciated.

Language difficulties

Energy, heat, and work all have colloquial meanings which can easily interfere with the precise meanings appropriate to physics, and some perseverance is needed to instil the latter. Much of the discussion above relates to this point, and students may also be told that what they have been used to calling 'heat' in everyday conversation is more properly called 'internal energy' in scientific speech and writing.

Hidden difficulties

An example is question 1 of this Unit which is intended as revision of a familiar situation, leading to the conservation of energy and extension to the need for fuel. While at first sight it may seem an innocent question of a familiar type, a closer examination brings up some difficulties. For example, once the steady state is reached, the potential energy of the (mass + Earth) system is decreasing, but the rotational kinetic energy of the generator is constant. Does it make sense to speak of the former being transformed into the latter? This issue has been raised in the question, but it is left to teachers to emphasize the question's main purpose rather than raising too many doubts and difficulties which may destroy confidence.

KEEPING UP-TO-DATE

Some of the information in this Unit (for example figure G4 in the *Students' guide*) is inevitably out-of-date even before it is published. We hope that the Department of Energy will continue to publish that figure, in updated form, and that teachers will be able to obtain supplies sufficiently regularly to keep these data in the *Students' guide* up to date. Other information will need to be brought up-to-date too, and it is intended to revise the background book *Energy sources: data, references, and readings* from time to time to make sure it contains current information and statistics, and a list of references to new books, articles, and teaching aids.

As time goes on schools will no doubt build up a library of relevant books and articles. Since some suitable examples will be expensive and perhaps difficult for schools to come by, most of *Energy sources: data, references, and readings*, and the whole of *Energy options: a reader* contain a selection of articles which will help to make a start. When *Energy sources: data, references, and readings* is republished we hope that as well as current data it will contain new articles. It is not intended to republish the Reader *Energy options*.

SECTION G1

THE BACKGROUND TO ENERGY SUPPLY AND DEMAND

The scope of the topic and its treatment

As will be seen from table G1 the Unit as a whole, and this Section in particular, involve the bringing together of many ideas. Although some may appear to be familiar to students, many may prove difficult. While there is little conventional experimental work in the Unit the approach throughout has been to apply the techniques of physics to an important practical problem. Like most practical problems, the problem of energy supply and demand has many ramifications, and it will not be possible to explore all the implications. It will also be necessary to treat some questions rather lightly, while maintaining their seriousness of purpose. They are intended to raise issues, rather than to explore them in depth. Where it has been possible to draw attention to complications without making questions too cumbersome this has been done, but it has not always been possible. Every student need not answer every question, and teachers are encouraged to omit or share out introductory and practice questions as the needs and strengths of their students dictate.

In this Section a number of topics are introduced fairly briefly in the first part and then taken up in more detail in the second part – the spiral approach.

Energy is conserved; Sankey diagrams

Students are reminded that the conservation of energy is a basic principle of physics; the need for fuels arises because energy spreads out and becomes less useful (question 1). Sankey diagrams are a convenient way of representing energy transfers. The idea of flow is useful in this context, but must not be used uncritically.

Efficiency; personal energy

The key concept of this part is efficiency. It is applied to various energy converters, including the human body, in order to pave the way for such later work as the thermodynamic limitation on efficiency in this Unit and in Unit K, 'Energy and entropy'; the need for fuels to replace the energy which has become ‘spread out’ and hence less useful; and the dependence on fossil fuels of the developed countries in particular.

Questions

Questions 3, 4, and 5 all provide opportunities for students to perform simple calculations and to become familiar with units. In addition, questions 3 and 5 aim to familiarize students with the amounts of energy involved in physical activities and in domestic appliances. The idea of ‘energy slave’, introduced in question 5, is a useful way of highlighting our dependence on fuels while providing a different slant on the purposes indicated above.

Teachers (though probably not students) may be surprised to find that the mechanical efficiency of the body is as high as about 35 per cent (question 3). If it were treated as a simple heat engine operating between 37 °C and, say, 15 °C, its efficiency would be $[(310–288)/310] \times 100\,\% \approx 7\,\%$. The body, which is an *open system*, is much more efficient than an ideal heat engine working between these temperatures.

Fuels: energy converters; primary, end-use, and functional energy; energy units

Energy is conserved but the supply of fuel is finite. It is therefore important to consider the efficiency of a variety of energy transfer processes. Table G1 in the *Students' guide* shows that efficiency may range from 90 per cent for processes which involve mechanical change, but do not involve thermal flow of energy from hot to cold (for example, turbine, electric motor, or generator) down to as low as 10 per cent for a steam engine. This last case is an example of those processes which inevitably involve thermal flow of energy from hot to cold, which put limits on the proportion of the total energy which can be transferred to do useful work for us. But if our aim is to heat a building or some water, then the process of burning fuel may be very efficient. Note that the figures given in table G1 in the *Students' guide* for the efficiencies of various boilers are only less than 100 per cent because not all the energy is transferred to internal energy of *that part of the system we are interested in*; some heats the flue gases and is lost up the chimney. But given that all the energy released in the chemical reaction when fuel is burned is transferred to internal energy the process could be regarded as 100 per cent efficient.

(The formula, efficiency $= (T_1 - T_2)/T_1$ is simply quoted. It will be discussed in Unit K, ‘Energy and entropy’.)

Home experiment

Home experiment GH1, The cost of a ‘cuppa’, in which students compare the amount of fuel used, the cost, and the efficiency of using electricity and gas to boil water, is relevant at this point.

Learning the language and reading the signs

The last part of this Unit (Section G4) is designed to provide students with practice in reading and reporting back. Therefore some of the emphasis in this first Section is on introducing them to terms and units which may be used in articles, and giving practice in interpreting data published in the form of graphs, tables, or diagrams.

Units of measurement are a major problem, as the *Students' guide* explains. This Unit does not use SI exclusively. Articles on energy do not tend to use SI and students might be discouraged from reading if they suddenly came across the plethora of units which are used. Instead, an attempt has been made to explain the systems which are used and to provide a conversion table. (Table G4 in the *Students guide.*)

A second difficulty with articles about energy is the use of terms which are not precisely defined and which are used in different ways on different occasions. To specify a definition and stick to it would have been an easy route to consistency within this Unit but again, would not have helped students in reading articles. The work on primary and secondary energy and fuels, end-use, and functional energy (and question 5 on energy slaves) is intended to be helpful in itself, and also to make the point that many similar and loosely defined terms, which it has not been possible to consider in such detail, are used in this topic.

Questions

Question 4 gives practice with various energy units. Question 6 is about efficiency; question 7 is about primary, end-use, and functional energy: both are intended to provide practice in interpreting data presented in complex diagrammatic forms. For this reason an official publication has been used unmodified. Some parts of question 6 have been deliberately framed so that they cannot be answered precisely from the diagram and some parts of the diagram are not entirely clear. The intention is for students to interpret the information as well as they can, or to find supplementary data if necessary.

Changes in fuel supply; making predictions; is there a crisis?

The intention of this part is to revise and extend students' knowledge of exponential change in determining whether growth in fuel consumption is exponential or not. They should also see the implications of two simple growth patterns (exponential growth or steady consumption) for the lifetime of the World's probable or proved reserves of fossil fuels.

Many factors make it difficult to obtain even an elementary view of present and future fuel consumption patterns. Some of these are mentioned in the *Students' guide*, but this topic is particularly dependent on variations in the data. For instance, total World fuel consumption was effectively constant for the three years up to 1983. Some commentators suggest that this was due only to economic recession and that demand will eventually resume exponential growth as it did after the 1973 Middle East war. Others suggest that consumption will actually fall from now on.

This part of the *Students' guide* has been written to try to avoid the need for major changes in the teaching whatever pattern of consumption becomes established. Since the emphasis is not on students predicting future consumption in detail, but rather on seeing what current patterns are and might imply for the future, only a limited amount of up-dating of factual data, rather than of detailed predictions, should be necessary.

Questions

Question 8 asks students to determine whether World population growth is exponential. The doubling-time for population growth is used in question 17.

Question 9 makes the point that, during exponential growth, as much resource is used in each doubling-time as has been used in the whole time hitherto. Hence the resource is used up at an increasing rate.

Questions 10 and 11 are revision questions on the nature of exponentials and may be omitted if not required. Question 11 establishes the relation $t_D = 0.693/g$, where t_D is the doubling-time and g is the growth constant. This relation should already be familiar, in terms of half-life and decay constant from Unit F, 'Radioactivity and the nuclear atom'.

Question 12 introduces terms to do with reserves and involves working with tables of data to calculate lifetimes of resources assuming constant consumption.

Question 13 is similar, but more complicated, since it assumes continued growth in consumption at the average growth rate of the last ten years. This question is much more manageable if the 'Dynamic modelling system' is available. If students have had some experience of this already they should be well able to set up the appropriate equations. Questions 14 and 15 consider what happens to growth as resources become more difficult to obtain. Question 14 develops the bacterial colony analogy of question 9, while question 15 is concerned with fuels. The 'Dynamic modelling system' is useful here, too, but the equations are rather harder to set up.

Useful sets of equations are:

For exponential growth (question 13)

```
DC = C*G*DT
C = C+DC
R = R-C*DT
T = T+DT
```

Initial values are needed for LC, G, DT, R, T

For consumption with limited resources (question 15)

```
DC = C*G*DT*(R-2*A)/R
C = C+DC
A = A+C*DT
T = T+DT
```

Initial values are needed for C, G, DT, R, A, T

Notes

1 The factor (R−2*A)/R is arranged to cause the growth in consumption (rate) to become negative when half the total resource has been used up. This fraction is entirely hypothetical and the effect of other fractions could be investigated.

2 An initial value will need to be set for A. This can be done by examining historical consumption figures or from a knowledge of current consumption and average doubling-times.

For each set of equations the meaning of the symbols is:

C current consumption (rate)
DC increment in C
A total resource consumed to date
R current or total value of the resource
G rate of growth of consumption
T time
DT time increment

Question 16 may appeal to the students' sense of humour, but we hope that its serious purpose – highlighting the difficulties of prediction – will be apparent, and that it will not prove offensive.

Questions 17 and 18 are ultimately to do with food production. Question 17 concentrates on the availability of land for food production if population growth continues exponentially, and is intended to draw attention to this in preparation for the next topic. Food production in the developed world is heavily dependent on fossil fuels – partly for machinery and partly for artificial fertilizers. Question 18 concentrates on this.

MORE ABOUT FUEL USE

The second part of this Section provides more detail about fuel use and the growth of fuel demand and hence develops some of the ideas in the first part. The Section has been arranged like this because introducing all the detail at once might make the material difficult to comprehend. Although the text only scratches the surface it should enable students to read articles on energy options with reasonable understanding.

Distribution

The first part of this Section examined fuel use on a World scale. This part of the Section makes a more detailed examination.

One way is to examine the large and increasing contributions made to global fuel supplies by fossil fuels.

A global perspective has been adopted to try to counter the unduly insular attitudes which are sometimes taken. However, national energy policies will be determined in part by the degree of each country's own self-sufficiency in fuels.

There remains the large disparity in *per capita* fuel use between the developed and the less developed countries. The link between fuel use and industrial activity is often demonstrated by diagrams, such as figure G12 (*Students' guide*) which shows a substantial correlation between *per capita* fuel use and gross domestic product *per capita.* Some commentators object to such diagrams on the grounds that they show bias in using parameters which more properly represent the activity of industrial societies than non-industrial ones. To resolve this and many similar issues would require more time and expertise than is available, but raising the issues may encourage students to be generally more critical of these and similar data.

In spite of the link with industrial activity, a substantial proportion of the U.K.'s fuel use is in fact devoted to space heating, resulting partly from our northern geographical location. This provides a justification, in addition to the core syllabus requirement, for the approach taken to thermal conduction in Section G3; and also needs to be borne in mind for a number of the energy options which feature in Section G4.

Some characteristics of fuels

This part prepares the way more overtly for the energy options by adding another dimension to the increasingly complex picture of fuel supply. By now students will have realized that it may be difficult enough to satisfy future demand for fuel. Other characteristics, such as energy density, transportability, time, and feasibility will make a difficult task worse.

Undesirable consequences of fuel use

It would be difficult to produce such a relatively short Unit on energy sources, which would be universally accepted as balanced. While it would, therefore, be wrong to neglect any mention of the undesirable consequences of fuel use, to do much more than mention them presents problems. The difficulty is well illustrated by the example of thermal pollution. The details of the Earth's energy balance are quite complex because of the variety of causes of reflection, absorption and re-radiation, together with absorption of energy in biomass. A reasonable treatment of radiation would have required the introduction of Stefan's Law. Only then can the effect of fuel consumption on the Earth's temperature be predicted. Students with a good mathematical background would cope adequately with the derivation and use of the relation

$$\frac{\Delta T}{T}=\frac{1}{4}\frac{\Delta E}{E}$$

but this would be an added complication for others. It is for these reasons that only a little has been made of thermal pollution and similar topics. However the points mentioned above may find some application later, since most of the energy options in Section G4 involve a study item concerned with environmental factors. It should be emphasized at this point that thermal pollution results not only from combustion of fuels but from any form of energy 'consumption'.

Similarly, detailed treatment of risks is beyond the scope of this Unit but discussions of *Students' guide* figures G17 and G18 will raise the issues sufficiently. Figure G17 shows how the frequency of accidents of various kinds in the U.S.A. involving more than some number of fatalities X, varies with X. Thus an explosion causing more than 1000 fatalities has a frequency of 10^{-2} per year; that is, on average one such accident might be expected every 100 years. Figure G18 uses some similar data to point up an apparently similar slope to all the graphs. A major difficulty with all such graphs is the scarcity of data relating to the energy industry. A number of points about risks are made in the *Students' guide* which may help discussion. A related point which is not made in the *Students' guide* is that all risks apply to the future, and that we should think about the welfare of the generations to come. There are risks for these generations associated with our use of nuclear power, for instance. But if we choose to use up a substantial proportion of the oil or coal resources now, then future generations will have to live with the consequences of a decision they have not been party to.

Questions

Some of these questions are intended to provide the basis for discussion. It is not necessary for students to do every question, as long as they meet the issues in discussion. Thus, questions 19, 20, and 22 could be shared out with each student doing only one question, unless there is time to spare. The discussion will then be concerned with the following:

Question 19 discusses the effect on the energy policy of some nations of imbalance between fuel supply and demand.

Question 20 is intended to encourage students to examine figure G12 in the *Students' guide* closely and to provide more opportunities for calculation.

Question 22 shows that substantial savings by the developed world will buy some time but not very much. The fact that savings might be made in fuel use for space heating provides some justification for Section G3.

All students ought to do questions 21, 23 and either 24 or 25 since they provide more opportunities for calculation and also emphasize energy density, time, and transportability. Question 25 makes use of the Gas Laws. Question 26 gives practice in estimation.

MORE DETAIL ABOUT GROWTH

The strategy here is to reflect the increasing complexity of the picture of fuel supply and demand, having established some basic ideas in the earlier part of the Section. There is not enough time to develop the picture very far but the implications of two 'scenarios' can be examined.

One of these is of continuing exponential growth of population. If *per capita* fuel consumption remains the same, growth of fuel consumption will clearly be exponential. Question 27 deals with this.

The second 'scenario' assumes that the *per capita* fuel consumption of every citizen of the World rises to somewhere near the current levels in the developed nations. Even if population does not change, demand for fuel will increase to calculable extents as shown in question 28.

Question 29 deals with a situation which is closer to what is likely to happen – rising population together with increasing expectations. This combination predicts a further reduction in fossil fuel resources. Question 30 provides a contrast to this by indicating the vast resources of energy available from the Sun.

Questions

Questions 27, 28, and 30 form a group which could be shared among students. Question 29 depends on both 27 and 28 and could be used in class to extend the discussion of those questions.

There are, of course, many other factors which could affect future fuel consumption (see the references in the background book *Energy sources: data, references and readings*). For example, growth of population might follow a more complicated pattern than it has been possible to assume in this Unit. If World population reaches a peak before the middle of the twenty-first century, then long-term predictions based on assumptions of exponential growth will obviously be incorrect. The implications for the growth which will occur in the medium term will, however, be clear, and students should be able to understand the more complicated predictions.

Table G6 in the *Students' guide* may help to provide a focus for discussion to round off this Section and look forward to the remaining Sections in this Unit. It shows the variety of factors which influence 'energy planning'. Leave students to study it now, and come back to it later, after only a brief introduction.

SECTION G2

POWER FROM NUCLEAR FISSION

This Section links closely with Section F3, 'The nucleus'. Examination of figure G19 in the *Students' guide* (average binding energy per nucleon against nucleon number) to see the possibility of energy release on fission provides an overlap between the Units and should help students to make a rapid transition. The quantity of energy which is released on fission can be calculated using nuclear masses. What is not obvious is that this is something of a circular process since most nuclear masses were determined by measurements of the energies involved in nuclear reactions. Moreover, the physical source of the energy is not apparent from such a calculation. It is for this reason that question 31 requires a parallel calculation of the initial potential energy of the fission fragments which is transformed to kinetic energy as the charged fragments repel each other.

MAKING FISSION WORK

It is one thing to obtain energy from the fission of a single nucleus, but quite another to extract power on a commercial scale. To do this a number of conditions must be satisfied simultaneously, and it is this complexity, together with the difficulty of some of the ideas involved, which may cause problems.

Critical mass

The twin needs for many fissions per second, and for neutrons to trigger the fissions, are met when a nuclear chain reaction is established. In turn this requires a critical mass of fuel.

Nuclear cross-section

The idea of nuclear cross-section is introduced to explain first that fission occurs either with energetic neutrons and ^{238}U (utilized in fast reactors – see Section G4, part A, Nuclear power stations), or with ^{235}U and neutrons which have very low kinetic energies – so-called 'thermal' neutrons (utilized in thermal reactors, the subject of the remainder of this Section).

Absorption of neutrons, predominantly by ^{238}U, competes with the fission reaction and this is the second feature illustrated by means of nuclear cross-sections. The third element of this rather complicated picture is the very small proportion of ^{235}U in either natural or artificially-enriched uranium metal. Students are taken through this argument step by step by means of two learning questions, 34 and 40.

Moderation and control

The process of moderation involves instructive applications of dynamics and may be useful as revision of earlier work.

Control of the chain reaction also makes use of the concept of nuclear cross-section but requires little extension of the work already done on moderation.

THE SCOPE OF THIS SECTION

As in many parts of this Unit, it has been necessary to omit detail. One such point is that control would be very difficult to achieve if all the neutrons were emitted at the instant of fission. In fact, enough of them are sufficiently delayed to make control possible.

Similarly, although the deliberate enrichment of the uranium, and the need for reprocessing to remove fission products which may absorb neutrons, are mentioned in the *Students' guide*, the increasing amount of fission which is produced in ^{239}Pu, resulting from

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \rightarrow ^{239}_{94}Pu + 2\,^{0}_{-1}e$$

is not. These points are made to emphasize that this Section is only intended to provide a brief treatment of nuclear energy: there are opportunities for students to examine nuclear energy in more detail, as well as other energy options, in Section G4.

Questions

Question 31 compares the physical origin of the energy from fission with its mere calculation using nuclear masses.

Questions 32 and 34 are learning questions: question 32 tries to give an idea of the meaning of nuclear cross-section and how it might be measured, while question 34 uses data on cross-sections to determine the neutron energy required for most effective reactor operation. Question 33 is about neutron energies.

Questions 35 to 39 are to do with moderation and control. Question 35 has links with Unit F, 'Radioactivity and the nuclear atom'; questions 33 and 36 to 38 use ideas from Unit A, 'Materials and mechanics'. All students should do questions 37 and 39 but questions 36 and 38 might be better reserved for students with good mathematical skills.

Question 40 involves calculations on a power reactor.

SECTION G3

CONSERVING FUEL IN THE HOME

The word 'conserving' in the title of this Section is used in the everyday sense of 'not wasting' and therefore refers to fuels rather than to energy. Since about 40 per cent of the U.K.'s fuel consumption is for space heating, there is obvious potential for making savings. It can be argued with some force that using fuel more effectively is equivalent to discovering a new source of fuel equal in size to the quantity of fuel saved. In this Section thermal conduction is treated in the practical context of home heating.

The Section also provides opportunities to revise ideas of flow which were introduced in Unit B, 'Currents, circuits, and charge', by making use of the analogy between thermal and electrical conduction. Electrical conduction can be said to involve a current (or flow of charge) which arises when a potential difference is established across a length of conductor (that is, when an electric field exists). Analogously thermal conduction involves a flow of internal energy which arises when a temperature difference is established across a length of conductor.

The features of the analogy are as shown in table G2.

Feature of the analogy	Electrical conduction	Thermal conduction
Variable 1	potential difference V(volt)	temperature difference $\Delta\theta$ (kelvin)
Variable 2	current I(ampere)	rate of flow of energy ϕ(watt)
Relationship: (variable 1) = constant × (variable 2)	$V=IR$	$\Delta\theta=\phi\mathscr{R}$
Proportionality constant	Electrical resistance, R ($\mathrm{V\,A^{-1}}$ or Ω)	Thermal resistance, $\mathscr{R}$ ($\mathrm{K\,W^{-1}}$)
Determinants of resistance	$R=\rho\dfrac{l}{A}$	$\mathscr{R}=\dfrac{1}{k}\dfrac{l}{A}$*
	ρ is resistivity of the material (Ωm)	k is thermal conductivity of the material ($\mathrm{W\,m^{-1}\,K^{-1}}$)

*Note this is only true in practice if the length l is small compared with the area A.

Table G2
Analogy between electrical and thermal conduction.

It will be clear that the equations which describe the two situations must be similar in form since they describe the essential feature of the two systems – that of flow. Other common features are the ideas of resistance and conductivity.

But of course the systems are not identical. The units obviously cannot be the same. Moreover the things that are said to flow (charge and energy) are not only not the same but are not of the same kind. It is worth emphasizing the point about models and flow which was made on page 415 of this *Teachers' guide*.

Teaching strategies

The *Students' guide* for this Section places the questions within the text rather than in one block at the end of the Unit. This is done to allow teachers to choose between (at least) two strategies for teaching the material.

Strategy A
The Section may be taught like any other, using the material in the *Students' guide* to support the teaching in class and using questions in class or for homework as seems appropriate. If this strategy is adopted the following example serves only as a brief guide to teachers of the main points of the Section.

Strategy B
The example which follows may be used in class to introduce the main ideas of the Section while accepting that students will not understand the work in detail. The students may then work through the text in the *Students' guide* on their own, either in class or at home, in order to develop their understanding.

Stage 1 Introducing the equations:

$$\mathscr{R} = \frac{l}{kA} \qquad \text{and} \qquad \Delta\theta = \phi\mathscr{R}$$

The equations will be approached by analogy with electric current, but since the students will subsequently be working through the *Students' guide* there is no need to treat the analogy in great detail at this stage.

Stage 2 Calculation on a single-glazed window produces a surprising result
Consider a room at 20 °C when the outside temperature is 0 °C.

Calculate the rate of flow of energy through a single-glazed window of the dimensions 2.0 m × 1.0 m, and 6 mm thick, where the thermal conductivity of glass, $k = 1.0\,\mathrm{W\,m^{-1}\,K^{-1}}$.

Thermal resistance of the glass

$$\mathcal{R} = \frac{l}{kA} = \frac{6 \times 10^{-3}\,\mathrm{m}}{1\,\mathrm{W\,m^{-1}\,K^{-1}} \times 2\,\mathrm{m} \times 1\,\mathrm{m}} = 3 \times 10^{-3}\,\mathrm{K\,W^{-1}}$$

$$\phi = \frac{\Delta\theta}{\mathcal{R}} = \frac{20\,\mathrm{K}}{3 \times 10^{-3}\,\mathrm{K\,W^{-1}}} = 6.7\,\mathrm{kW}$$

This is clearly much too high.

Stage 3 Introducing surface resistance

Examination of the equations shows that the calculated value of $\mathcal{R}$ must be too low.

We have assumed that the window glass separates a body of air at a uniform temperature, θ_1, from another body of air, also at uniform temperature, θ_2, outside. This is too simple a picture. The glass surfaces are not at θ_1 and θ_2. The situation is rather complex but can be treated as a boundary or surface layer of air on each side of the glass. Energy from the bulk of air in the room must pass through these two layers as well as the glass. They present extra thermal resistances 'in series' with the glass and there is a temperature difference across each layer. As the calculation below will show, the 'surface resistances' are actually greater than the thermal resistance of the glass, and most of the temperature drop between interior and exterior is across the surface layers, not across the glass.

These surface layers are of variable thickness and have diffuse boundaries. However, for practical purposes, it is possible to regard the layers as presenting ordinary thermal resistances, providing that the conditions are similar to those of a typical domestic situation, for example, having limited temperature differences.

Resistances of unit area of conductor are referred to in this Section as *resistance coefficients* (but they are also known as unit thermal resistances).

Typical values of surface resistance coefficients (*i.e.*, surface resistances for one square metre) are

$0.13\,\mathrm{m^2\,K\,W^{-1}}$ for internal surfaces

$0.06\,\mathrm{m^2\,K\,W^{-1}}$ for external surfaces.

The power loss through the window can now be recalculated. For 1 m^2 of glass

$$\mathscr{R} = \frac{l}{kA} = \frac{6 \times 10^{-3}\,\mathrm{m}}{1\,\mathrm{W\,m^{-1}\,K^{-1}} \times 1\,\mathrm{m} \times 1\,\mathrm{m}} = 6 \times 10^{-3}\,\mathrm{K\,W^{-1}}$$

This resistance and the internal and external surface resistances act in series and their values therefore add to give the total thermal resistance of one square metre of window as

$$0.06 + 0.006 + 0.13 = 0.196\,\mathrm{K\,W^{-1}}$$

and the power loss through each square metre is

$$\phi = \frac{\Delta\theta}{\mathscr{R}} = \frac{20\,\mathrm{K}}{0.196\,\mathrm{K\,W^{-1}}} \approx 102\,\mathrm{W}$$

Hence the total power loss through the whole window with area 2 m^2 is 2 × 102 W = 204 W. This is much more reasonable.

Home experiment

Home experiment GH2, Surface layer, is relevant at this point.

Stage 4 Using the analogy to calculate temperatures

An electrical analogue of one square metre of window is shown in figure G1.

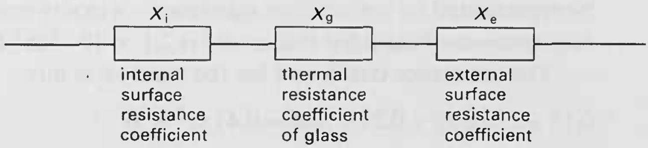


Figure G1

The resistances act in series and have a total temperature difference of 20 °C across them when the thermal energy flow is 102 W. A calculation analogous to the electrical one gives temperature differences proportional to the resistances

i.e. $$\frac{\Delta\theta_e}{0.06} = \frac{\Delta\theta_g}{0.006} = \frac{\Delta\theta_i}{0.13} = \frac{20}{0.196}$$

giving $\Delta\theta_e = 6.1\,°\mathrm{C}$
$\Delta\theta_g = 0.6\,°\mathrm{C}$
$\Delta\theta_i = 13.3\,°\mathrm{C}$

The reason for the error in the original calculation of power loss is now clear: the temperature difference across the glass is much less than 20 °C.

Stage 5 Double glazing

Now suppose that the window is double-glazed with a cavity 20 mm wide, filled with air which is assumed to act just as a normal conductor, having conductivity $k = 0.024\ \mathrm{W\,m^{-1}\,K^{-1}}$. (This is, in fact, an invalid assumption, as is shown later.)

The resistance coefficient of the cavity is

$$\frac{l}{kA} = \frac{0.020\ \mathrm{m}}{0.024\ \mathrm{W\ m^{-1}\ K^{-1}} \times 1\ \mathrm{m^2}} = 0.83\ \mathrm{m^2\ K\ W^{-1}}$$

The total resistance coefficient of the window (cavity plus two pieces of glass plus external and internal surfaces) is therefore

$$0.83 + 2(0.006) + 0.06 + 0.13 = 1.032\ \mathrm{m^2\,K\,W^{-1}}$$

The power loss per square metre is

$$\frac{\Delta\theta}{\mathcal{R}} = \frac{20\ \mathrm{K}}{1.032\ \mathrm{K\,W^{-1}}} = 19.4\ \mathrm{W}$$

and hence for the $2\ \mathrm{m^2}$ window it is 39 W.

Even double-glazing firms do not claim an 80 per cent reduction! In fact, the cavity does not act as a simple slab of conductor. Nor does it act simply as two internal surfaces, since radiation and convection have a substantial influence as the *Students' guide* shows.

For the moment it is enough for students to know that a cavity can be represented by yet another resistance – a cavity resistance. The cavity resistance coefficient for this cavity is $2.1 \times 10^{-1}\ \mathrm{m^2\,K\,W^{-1}}$.

The resistance coefficient for the window is now

$$0.13 + 2(0.006) + 0.21 + 0.06 = 0.41\ \mathrm{m^2\,K\,W^{-1}}$$

and the power loss per square metre $= 20\ \mathrm{K}/0.41\ \mathrm{K\,W^{-1}} = 49\ \mathrm{W}$.

Hence, for the whole window the power loss is an altogether more reasonable 98 W.

After this introduction students should now be able to work on the *Students' guide* which deals with this same material but with the following slight changes:

a there is more material in the *Students' guide*, for example, work on ventilation;

b there is more detail, for example spelling out the effects of convection and radiation on energy flow through a cavity;

c the treatment is slightly changed in order to be a little different from this example *e.g.*, the use of the relation $\mathcal{R} = X/A$ where X is the thermal resistance coefficient of a material, that is to say, the thermal resistance of unit area. (We choose to use X rather than the more familiar U-value often used by heating engineers, so that the discussion can be in terms of resistance which is an idea students are used to. The thermal resistance coefficient, X, is in fact the reciprocal of the U-value. X is, in fact, sometimes used to compare the effectiveness of different kinds of roof insulation, for example.)

Questions

Questions 41 to 45 deal with the electrical analogy and give practice in calculating thermal resistance and power loss.

Questions 46 to 48 deal with thermal resistance coefficient X – the thermal resistance of unit area. The relationships $\mathcal{R} = X/A$ and $X = l/k$ are introduced. It is important for students to grasp that the thermal resistance of a large area of wall, window, etc., is less than that of a small one.

Questions 49 and 50 are about surface resistance and surface resistance coefficient.

Questions 51 to 57 deal with cavities: cavity walls and double glazing.

Finally, questions 58 to 60 are about the overall heating of buildings: new factors including thermal capacity of the building, ventilation, and the power output of the occupants of a room are introduced.

SECTION G4
ENERGY OPTIONS

In this Section students study a selection of energy options. Some of these options are actually energy sources while others are major factors associated with energy sources, such as conservation and storage.

Students should learn about these topics by reading and help others by reporting their findings. Organizing their knowledge in order to report back effectively will, of course, also help them to learn.

In order to help students to get started some 'study items' (mostly questions, but some instructions, or hints) have been included for each option. Although we have made these 'study items' reasonably comprehensive (so that conscientious students should be successful), students should be encouraged to go beyond them.

The background books *Energy sources: data, references, and readings* and *Energy options: a reader* contain some 'starter' articles and references to suitable sources of information. Many books on energy topics are now available, but a sufficiently large library, even of paperback books, will be expensive and take time to build up. Spreading this part of the course over a few weeks, perhaps while other Units are being taught, will give students the opportunity to use public libraries (even if they are initially reluctant to do so).

Some suggestions are made in the *Students' guide* about reporting back (verbal report; written report; slides or other illustrations; poster session). These will be familiar to teachers, but are mentioned since students may be working on their own, perhaps during the holidays, and written help may be useful. The suggestions will, of course, have to be interpreted for the circumstances of each school.

A simulation, such as the 'Power Station Game', either the original IEE version or the shorter one from *Science in society* could form a good finale to the Unit (details of both of these are in the background book *Energy sources: data, references, and readings*). The time needed could be saved by reporting back in the form of duplicated notes or a poster session.

Seven energy options are listed in the *Students' guide*; each is divided into a number of topics.

The individual topics are of different lengths and the amount of work involved is unpredictable; a good student will find a large amount of information on an apparently 'thin' topic. Two topics might be the norm, but teachers will be able to give advice about the number to be investigated, in the light of their knowledge of their students and the topics which are being considered.

The energy options (and topics) are:

Nuclear power (nuclear power stations; nuclear fusion; nuclear safety).

Solar energy (passive solar devices; active solar devices; photovoltaic devices; biomass; wind energy).

Fossil fuels (power stations; alternative sources of fossil fuels).

Other sources of internal energy (geothermal; heat pumps; combustion of refuse).

Water power (hydro-power; wave energy; tidal energy).

Conservation (domestic; industrial; transport).

Storage

ANSWERS TO QUESTIONS

UNIT A
Materials and mechanics

1 The graph is shown in figure Q1.

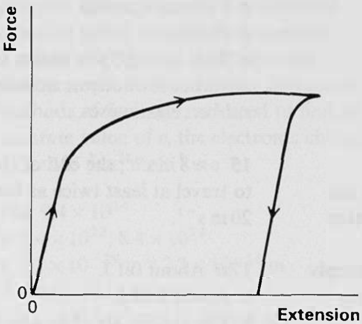


Figure Q1

2 Question **1** asked for a translation from words to a graph: this one asks students to reverse the process. Being able to 'read' a graph is an important skill to acquire. The four substances are: polythene strip, copper or aluminium wire, rubber cord, and steel or cast iron wire (this last one is too curved to be a thin glass rod).

3 'Tough' is the opposite of 'brittle'; 'plastic' and 'ductile' are almost the same; 'strong' refers to a high breaking stress. (See table Q1.)

4a *ii*
b The person will oscillate up and down (might even bounce). The oscillations are likely to be heavily damped and die away quickly. The final equilibrium position will be as in part **a**.

5a $100\,\mathrm{N\,m^{-1}}$
b 0.02 m; $50\,\mathrm{N\,m^{-1}}$
c 0.005 m; $200\,\mathrm{N\,m^{-1}}$
d Halved; doubled.

6a $10^6\,\mathrm{N\,m^{-2}}$
b 0.33
c 10 N. This is very thin wire — about 0.12 mm in diameter.
d About 3.6 mm for a one tonne car.
e 0.02 m
f Assuming that the rubber band is 1 mm × 3 mm when stretched to three times its original length (which it isn't), the force is 6 N.
g The interpretation is wrong in the sense that a steel wire would not survive such a strain.

7a The force is doubled. $F = 0.8$ N.
b*i* 0.16 N. *ii* Over 10 N. It is not likely that Hooke's Law is applicable over such a range.
c Doing the straight-forward calculation would suggest a mass of 2000 kg. But the wire is unlikely to stand this sort of stress. So the answer is 'There's no knowing'!

	ductile	elastic	plastic	strong	tough	brittle
rubber	✗	✓	✗	✓	✓	✗
copper wire	✓	for tiny strains	✓		fairly	fairly ✗
biscuit	✗	✗	✗	✗	✗	✓
glass	✗	✓	✗	✓	✗	✓ (with cracks)
etc.						

Table Q1

8a Glass is stiffer than Perspex; it requires more force per unit cross-sectional area to stretch it the same amount.
b It is easier to flex wood across the grain – in the low Young modulus direction.
c Wood.

9a 10^{11} to 10^{12} N m^{-2}
b 100 to 1000 times greater.
c 3.4 mm

10a M L T^{-2}
b M T^{-2}
c T^{-1}
d T^{-1} *i.e.*, frequency.
e $1/2\pi$ is purely a number. It does not represent a mass, length, or any other physical quantity, as k does.
f [stress] = M L^{-1} T^{-2}; strain is simply the ratio of two lengths, so it has no dimensions; [the Young modulus] = M L^{-1} T^{-2}

11a 100 N
b 10 N
c 50 N
d 10 J
e $\frac{1}{2}Fx$
f Less. The area under the force–extension graph (*i.e.* between the curve and the extension axis) is less than it would be if the force were proportional to extension.

12a No. The tension in the elastic cord is continually decreasing.
b About 0.4 m s^{-1}, kinetic energy about 0.08 J.
c Using $\frac{1}{2}Fx$, the energy stored is about 0.11 J.
d The calculation of energy stored gives a larger value than the actual kinetic energy found, partly because some energy is dissipated in friction and partly because, for rubber, the force is not proportional to the extension.
e, f If the force, F, were proportional to extension, x, so that $F = kx$, then the energy $\frac{1}{2}Fx$ would be $\frac{1}{2}kx^2$. The graph of potential energy against extension would be a parabola.

13a 1 N
b*i* 2 N to the left.
ii 2.5 m s^{-2} to the left.
c*i* 0.2 J
ii 0.25 J
d 0.05 J

14a Tensile stress $= F/A$
b Tensile strain $= x$/original length
c $\frac{1}{2}$ (stress × strain)
d Rubber.
e Steel springs; yew bows; tendon: shock absorber for body or athletic ability; rubber: elastic uses.

15 $v \approx 8$ m s^{-1}; the end of the wire is likely to travel at least twice as fast, *i.e.*, about 20 m s^{-1}.

17a*i* About 0.1 J.
ii About 0.07 J.
b The graphs are shown in figure Q2.

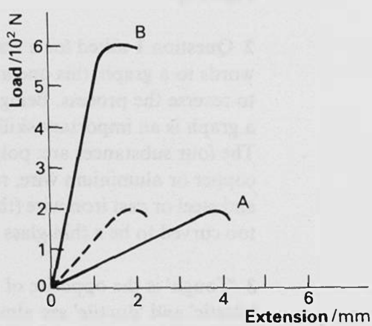


Figure Q2

c The longer cable can stretch more than a shorter one before breaking, and can store more energy. The longer one is therefore less likely to snap when there are sudden jerks.

18b 2.4×10^{-9} m
c 10 %; 1 %; 30 %
d The layer might be more than one molecule thick.
e About 10.
f About 10^{19}.

19a 6000:1
b 5×10^{-9} m
c Smaller.

20a Na: 22.99 g; Cl: 35.45 g.
b NaCl: 58.44 g.
c 6×10^{23}
d The charge per mole of electrons (or any other singly charged particles, for example hydrogen ions) is a universal constant called the *Faraday constant*, $F = 9.65 \times 10^4$ C mol^{-1}. Electrolysis measurement of F and X-ray diffraction methods of finding L are used to find an accurate value of e, the electronic charge.
e 3×10^{-5} m^3 mol^{-1}

21a 9.4×10^{24}
b 8.4×10^{28}; 8.4×10^{22}
c 1.2×10^{-29} m^3; 2.3×10^{-10} m
d 1.04×10^{-5} m^3 mol^{-1};
1.00×10^{-5} m^3 mol^{-1} Atoms are about the same size despite the large difference in mass.

22a Taking one grain as a cube of side 10^{-5} m its volume will be 10^{-15} m^3 and its mass about 2×10^{-12} kg. So 1 mole of grains is 6×10^{12} kg.
b About 10^{17} atoms, *i.e.*, 10^{11} million.
c A tyre lasts about 3×10^4 km during which about 0.01 m thickness wears away. This gives an answer of several thousand kilometres.

23a 3
b Each bubble attracts other bubbles, and the result is that they cluster as closely as possible. The bubbles are round, and attract other bubbles equally in any direction, whereas, if the forces tended to act in certain special directions, some arrangement like **1** or **4** might result.
c By analogy, it might be true that the attractive forces between atoms in such metals were equal in all directions.
d Because the sizes of the Na^+ and Cl^- ions are very different.

24a 7.12×10^{-6} m^3
b*i* 8.68×10^{-30} m^3
ii 8.20×10^{23} mol^{-1}
iii This is an upper limit for L because the spherical copper atoms cannot fill all the space in a crystal of copper.
c*i* 16.6×10^{-30} m^3
ii 4.30×10^{23} mol^{-1}
iii This is a low value for L because the assumption of a 'square' array over-estimates the volume taken up by each atom.
d 6.15×10^{23} mol^{-1}

25a There are no forces between the spheres until they touch. The repulsive force then becomes very, very large; there is a discontinuity.
b r_0 is the diameter of either sphere.
c Suppose the balls obey Hooke's Law with a large force constant (the larger the force constant, the steeper the line). (See figure Q3.)

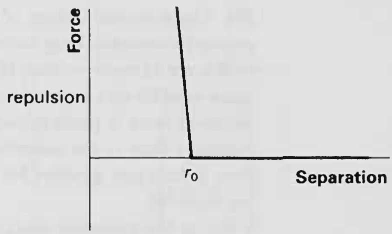


Figure Q3

d The potential energy curve will be parabolic. (See figure Q4.)

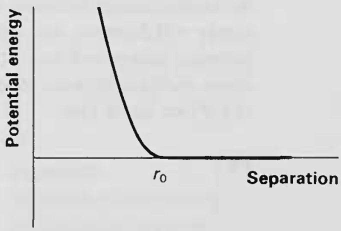


Figure Q4

26a, b, c The graphs are shown in figure Q5.
d 0A is the equilibrium separation. 0B indicates the maximum separation between atoms before the substance yields.
e 0A is about 2.5×10^{-10} m.

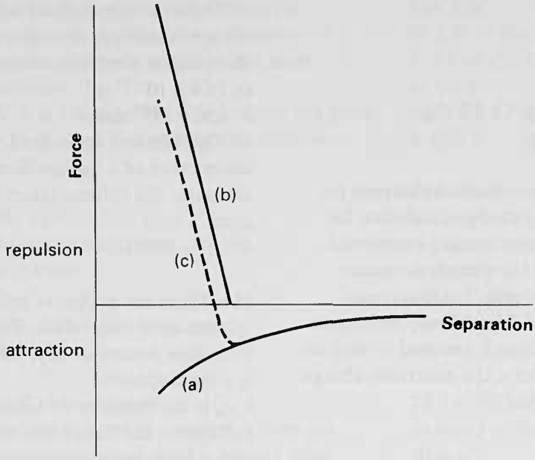


Figure Q5

27a The potential energy of the two atoms decreases as they move together with a net attractive force. 0Q gives the mean equilibrium separation, r_0. As the repulsive force is predominant for closer distances than r_0 the potential energy rises. It becomes positive for separations less than 0P.
b 0Q on the potential energy–separation graph = 0A on the force–separation graph.
c The sum of potential and kinetic energies must be constant if there is no external source or sink of energy. So as the kinetic energy increases the potential energy will decrease, and vice versa. The potential energy will vary by ΔE as the atoms oscillate between separations X and Y (see figure Q6).

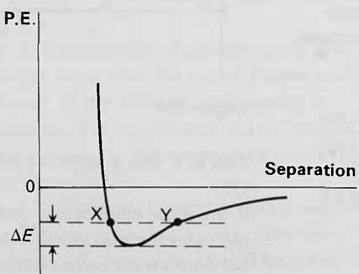


Figure Q6

d The depth of the potential well at Q gives the binding energy, E (see figure Q7).

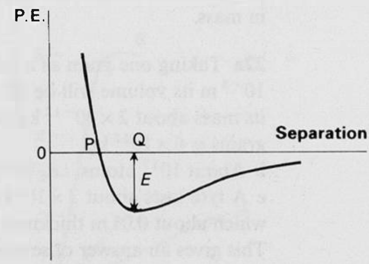


Figure Q7

28a A
b B
c B
d B

29a 2 N
b $400\,\mathrm{N\,m^{-1}}$

30a nr_0
b $n\Delta r$
c $\Delta r/r_0$
d $\mathrm{N\,m^{-1}}$
e $mk\Delta r$
f $1/r_0^2$
g $k\Delta r/r_0^2$
h k/r_0
i $60\,\mathrm{N\,m^{-1}}$

31a $x\sqrt{n}$
b xn
c About 9; since strain is $(xn - x\sqrt{n})/x\sqrt{n}$.

32 The Summary for Section A2 gives a brief description of these terms under the heading 'Structure of solids'. Several text books have good explanations too.

33a Metals bend because the crystal grains can slide over each other. The size of the grains is important in determining the mechanical properties of a material. There are dislocations too.
b Toffee will shatter if it is cold. Glasses do not have a sharp melting-point (unlike crystalline solids) but just become more viscous as they cool, hence the disorderliness and lack of regular crystal structure.
c Unlike many crystalline solids, glass-like substances do not have a sudden change of density when they melt, nor do they absorb energy (latent heat).
d Plastic flow occurs in metals where only one atom has to move at a time, *i.e.*, where there are dislocations in the grains which can move easily. (See question **34**.)

34 '.. in the next picture the dislocation has moved on by one atom. (See figure Q8.) In each succeeding picture the dislocation moves on by one atom until it reaches the edge.'

35 See, for example, Akrill, Bennet, and Millar *Physics*, Chapter 5.

36a A
b B
c A is high carbon steel; B is mild steel.
d The introduction of foreign atoms makes the material harder; there is less chance of slip. Many foreign atoms will cause the substance to become hard and brittle – no slip possible (A). With fewer, there is still some hardening and also slip, so some ductile effect (B).

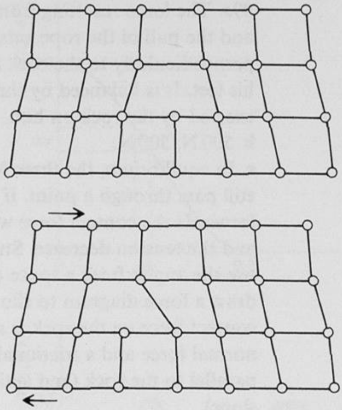


Figure Q8

37a The diagrams are shown in figure Q9.

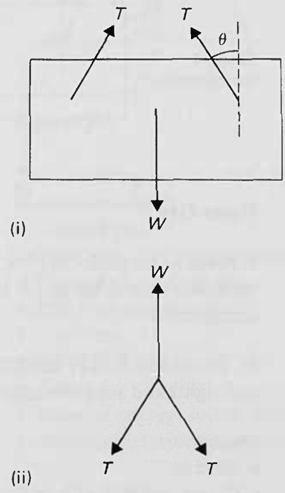


Figure Q9
(i) Forces on the picture.
(ii) Forces on the hook.

b The tension $T = W/2\cos\theta$. As $\theta \to 0$ so $T \to W/2$. The tension is less in a long cord than a short one.

38a $F = N\sin\theta$
b $W = N\cos\theta$
c $F = W\tan\theta$
d $F\cos\theta = W\sin\theta$

39a The force resulting from his weight and the pull of the rope acts perpendicularly to the rock face through his feet. It is balanced by the normal force exerted by the rock on him.
b 500 N; 500 N.
c In equilibrium, the three forces must still pass through a point. If he leans forwards the contact force will increase and the tension decrease. Students should use the angles from a space diagram to draw a force diagram to show this. The contact force *on* the rock is made up of a normal force and a frictional force parallel to the rock (and acting down the slope).

40a Hinge 2 is the top one.
b See figure Q10.

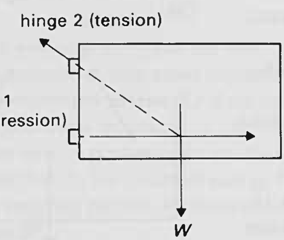


Figure Q10

c Force at hinge 2 = 1414 N, at 45° to vertical. Force at hinge 1 = 1000 N, horizontal.

41 Forces are 9.7 kN and 12.3 kN at left- and righthand supports respectively.

42a mg
b Greater.
i $T = mg \cos\theta$
ii $F_e = mg \sin\theta$
c For small angles $\sin\theta \approx \tan\theta \approx \theta$, and $\theta \propto d$.

43a*i* 400 N
ii 346 N
iii 346 N to the right.
b*i* 200 N
ii 173 N
iii 173 N to the right; 100 N upwards.
c*i* 600 N
ii 520 N
iii 520 N to the right; 100 N upwards.
d*i* 795 N
ii 521 N
iii 521 N to the right; 200 N *downwards*.

44a*i* The strut is in compression.
ii The tie is in tension.
iii Compression force in strut is 14 kN; tension force in tie is 10 kN.
b See figure Q11.

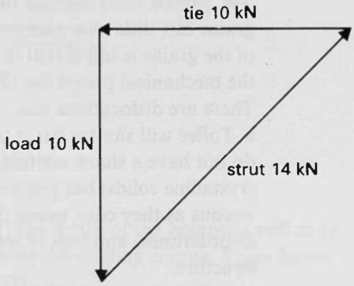


Figure Q11

c Strut.

45a BC and CD are struts (compression); AB and AC are ties (tension).
b BC and AC.
c They prevent the structure bending.
d Ties: BC, CD, AC, CE.
Struts: AB, DE, BD.

46 See, for example, Gordon *The new science of strong materials* for information on cracks, structure of wood, and composite materials.

47a The weight of the earth above the pipe forces it into an oval shape. The breaks occur where the greatest (tension) stresses are.
b X is at a point of maximum compression rather than tension. Concrete is strong in compression and weak in tension.
c The concrete surface is under compression from the plastic. So when the pipe is buried the concrete remains in reduced compression or only low tension. The plastic cover also has to stretch.

d Concrete is porous, the plastic prevents leakage. Surface damage is also stopped.

48a 4:1
b 1:1

49a Glass fibre has as great a tensile strength as steel. Concrete and wood are very bulky materials to hold large loads.
b Steel: elastic, flexible, high Young modulus, slightly malleable and ductile, brittle fracture; can be machined, melts.
Glass fibre: will give in matrix only slightly, rigid, no grain, composite material; cast in mould.
Concrete: weak matrix, brittle, strong in compression, composite material; cast in mould.
Wood: grained, strength with grain, weak in compression (empty cells collapse).
What effect do cracks have? Fatigue?
c Think about: volume available, use, availability, appearance, corrosion, inflammable, insulating/conducting, ease and cost of manufacture in appropriate shape, installation cost, etc.

50a $8\,\mathrm{m\,s^{-1}}$
b $6\,\mathrm{m\,s^{-1}}$
c 1.4 J
d $1.4\,\mathrm{kg\,m\,s^{-1}}$
e 35 N

51a The centre of mass of the system of man and truck never moves as there are no unbalanced external forces. While the man runs, the truck moves to keep the system's centre of mass in the same place. When the man stops, the truck does also.
b $mv/(M+m)$
c 0
d $mvt/(M+m)$

52a 6000 N s
b $6000\,\mathrm{kg\,m\,s^{-1}}$
c About $10\,\mathrm{m\,s^{-1}}$.
d The force at any instant is proportional to the slope of the graph of velocity against time. The graph is shown in figure Q12 (i).

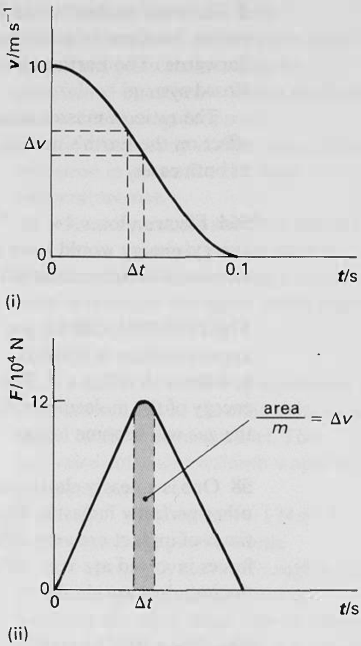


Figure Q12

53a $+0.08\,\mathrm{kg\,m\,s^{-1}}$; $-0.12\,\mathrm{kg\,m\,s^{-1}}$ (positive to right).
b $-0.04\,\mathrm{kg\,m\,s^{-1}}$
c $0.096\,\mathrm{kg\,m\,s^{-1}}$ in magnitude.
d 0.096 N s in magnitude.
e 0.96 N in magnitude.
f $-0.08\,\mathrm{m\,s^{-1}}$
g 0.0165 J; 0.024 J
h 0.0016 J
i Thermal energy, sound, and perhaps permanent deformation of vehicles.

54a The Earth.
b The pull of the Earth on the book, the push of the table on the book. The force pairs of Newton's Third Law are the two gravitational forces (Earth on book, book on Earth), and the two contact forces (book on table, table on book). Newton's Third Law does not require that the two forces acting on the book be equal. If they were not the book would accelerate (Newton 1).
c The Earth moves up towards the body with equal and opposite momentum.

d The train pushes on the Earth which moves 'backwards' as the train moves 'forwards'. The Earth and train make the closed system.

The ratio of masses ensures that the effect on the Earth's motion is negligible in both cases.

56d Electron loses 4×10^{-4} of its kinetic energy; energy would have to be measured to better than 0.04 %.

57a 7499:7501; 0.03 %; yes, the approximation is justified.
b $400\,\mathrm{m\,s^{-1}}$; $402\,\mathrm{m\,s^{-1}}$. The kinetic energy of the molecules will increase, *i.e.*, the gas will become hotter.

58 One is a nearly elastic collision, the other perfectly inelastic. The areas and times of impact are very different so the forces involved are very different even if the impulses are similar.

60a $2.89 \times 10^{-2}\,\mathrm{kg\,mol^{-1}}$
b $8.29\,\mathrm{J\,mol^{-1}\,K^{-1}}$; a more accurate value is $8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$.
c 2.08×10^{25} molecules.
d $3 \times 10^{-9}\,\mathrm{m}$; about 10 times diameter of an atom.
e $1.38 \times 10^{-23}\,\mathrm{J\,kg^{-1}}$

61a 42 moles
b 70 kg
c $6.0\,\mathrm{m^3}$

62a $c_x/2l$
b $2mc_x$
c mc_x^2/l
d mc_x^2/l
e $Nm\overline{c_x^2}/l$
f $\frac{1}{3}Nm\overline{c^2}/l^3$
g $\rho = Nm/l^3$; $\frac{1}{3}\rho\overline{c^2}$
h $\overline{c^2} = 2.5 \times 10^5\,\mathrm{m^2\,s^{-2}}$; $c_{\mathrm{r.m.s.}} = 500\,\mathrm{m\,s^{-1}}$

63a $350\,\mathrm{m\,s^{-1}}$
b $17\,\mathrm{m\,s^{-1}}$ to left.
c $3.8 \times 10^{-21}\,\mathrm{J}$
d $390\,\mathrm{m\,s^{-1}}$
e $390\,\mathrm{m\,s^{-1}}$. Root mean square speed is the name given to the speed calculated in part **d**. For the large number of particles in a gas the r.m.s. speed is 1.09 times the mean speed.
f $8.5 \times 10^{-25}\,\mathrm{kg\,m\,s^{-1}}$ to the left.
g Parts **b** and **f** only.

65a $5.65 \times 10^{-21}\,\mathrm{J}$
b $4.71 \times 10^{-26}\,\mathrm{kg}$
c $5.65 \times 10^{-21}\,\mathrm{J}$
d $1300\,\mathrm{m\,s^{-1}}$

66a Using $\frac{1}{2}MV^2 = \frac{1}{2}mv^2$ gives $400\,\mathrm{m\,s^{-1}}$ (to one significant figure, which is all that the data given justifies).
b Using $c_{\mathrm{r.m.s.}} = (3p/\rho)^{\frac{1}{2}}$ gives the same value.

67a $210\,\mathrm{m\,s^{-1}}$
b 2.4 ms
c 110 km; 1.2×10^{12}
d $9.2 \times 10^{-8}\,\mathrm{m}$ *i.e.*, approximately $10^{-7}\,\mathrm{m}$.
e $1.5 \times 10^{-10}\,\mathrm{m}$
f Yes. The assumption for the theoretical calculation is that all molecules are of the same diameter. This is approximately true.

68a $1.7 \times 10^6\,\mathrm{s}$ (nearly three weeks).
b Convection currents in a room cause circulation as will any draughts.

69a pA
b $pA\Delta l$
c $p\Delta V$
d $p\Delta V$
e No frictional losses at piston; change happens slowly.
f When air is compressed the work done on it increases its internal energy; when it expands it does work, its internal energy is reduced.

UNIT B
Currents, circuits, and charge

1a 0.16 C
b 10^{18} particles.
c 6×10^{22} particles per metre length.
d $1.67 \times 10^{-5}\,\mathrm{m}$
e $1.0 \times 10^{-6}\,\mathrm{m\,s^{-1}}$

f The algebraic answers to **a**–**e** become: It, It/Q, nA, It/QnA, I/QnA

The speed at which anything moves obviously does not depend on the length of time for which its motion has been observed.

g Reducing n increases v; fewer particles have to move faster.

Reducing Q raises v; each carries less charge, so they must move faster to carry the same total charge in a given time.

A molar solution is strong, but can be made, and contains 6×10^{23} solute molecules in one litre, and so 6×10^{26} molecules in one cubic metre. If every molecule provides an ion, there will be 6×10^{26} ions in each cubic metre. Water molecules are not by any means all dissociated into ions in very pure water while, say, paraffin would contain almost no ions.

2a Unless there is some evidence that the junction acquires a net charge, all charge flowing in flows out.

b No.

c Yes.

d Not if both are copper.

e See answer to a.

3a A possible estimate can be reached by considering the time for which the cell could light a torch bulb (say 5 hours) carrying a current of, say, 300 mA.

b Estimate mass and cruising velocity and calculate $\frac{1}{2}mv^2$. An allowance for inefficiency is necessary (typically the overall efficiency of conversion of internal energy into mechanical energy is about 30 per cent). An all-electric train will have a much higher efficiency *by itself*, but the overall inefficiency is transferred back to the power station.

c With no losses, about 110 seconds.

d*i* 1000 A *ii* 100 A *iii* $(10)^2 = 100$ times greater. *iv* 100 times longer.

4 B

5a An estimate of mass equivalent to 1 kg of water and a temperature rise of 80 K gives 4.7 A (assuming a 240 V supply).

b*i* $0.076\,\mathrm{kg\,s^{-1}}$. A lower limit.

ii If the water inlet temperature drops the output temperature can only be maintained by either reducing the flow rate or increasing the power.

Likewise a drop in flow rate requires a reduction in power for the same temperature rise.

Inlet temperature and flow sensors can thus in principle initiate control actions. A full control strategy would have to consider the upper limits imposed on power and flow rate.

6a If a p.d. is in some way equivalent to pressure in a liquid then the units of each ought to show some parallel. The equivalent of joule/coulomb would be joule/(metre)^3.

$$1\,\mathrm{J\,m^{-3}} = 1(\mathrm{N\,m})\,\mathrm{m^{-3}} = 1\,\mathrm{N\,m^{-2}}$$

i.e., 1 pascal, the pressure unit.

If a charge, Q, moves through a p.d. of V, the energy transformed is VQ. Similarly the work done when a volume of fluid, V, is driven through a pressure difference p is pV.

b Some points to bring out:

Apart from overheads, payment is for fuel used in power stations. Kirchhoff's First Law has to hold.

The 'return path' for water flow into a house tends to be ignored: there are of course many, and we think of water as being 'consumed'.

At any instant the current flow into a house is equal to the current leaving – whether it is a.c. or d.c. is thus irrelevant. The *energy* flow is related to the outward *and* return conductors independent of current direction – it is directed *out* of the power station. (An interesting parallel is with a motor driving some machinery by means of a pulley on a drive belt.)

7 The resistance of (a) is larger than the resistance of (b) and both seem to be linear components, obeying Ohm's Law. (c) might be a diode, as its resistance changes when the p.d. is reversed. (d) could be a filament lamp, for its resistance increases as the current increases. Initially, (d) conducts better than any of the others. One could also compare the forward and reverse resistances of (c) with the resistances of (a) and (b).

8 B

9a Only graph (b) obeys Ohm's Law as stated. Graph (a) is linear, but there is current when there is zero applied p.d. (perhaps the component included a source of p.d.). (c) shows a constant current, not a constant rate of change of current with change of p.d. Graph (d) is non-linear, though it might be said to obey Ohm's Law for p.d.s in one direction only.

b Yes. A graph of current against p.d. is a straight line through the origin. Alternatively, equal increases in current go with equal increases in voltage. The resistance at 2.0 mA, and at all other currents given, is $14.5 \times 10^3\,\Omega$.

c The answers here are partly matters of opinion. Certainly Ohm's Law is not applicable to all objects that conduct, but it does compactly describe the behaviour of some. Suggestion **3** avoids trouble by defining it away; sometimes physicists do this, but in this particular case they usually speak of 'ohmic' or 'linear' materials. **4** is wrong, for Ohm's Law requires the current–voltage graph to pass through the origin and a small section of the curve, though nearly straight, will not project back to the origin. Whether Ohm's Law is important and general enough to count as a law is a matter of opinion. It does not rank with those 'Laws of Physics' to which we know no exception (give an example).

10a 2 A

b 4 A

c 6 A

d 2 Ω

e The key step is the application of Kirchhoff's First Law to the junction of R_1 and R_2, giving $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$

11a*i* The resistance of A increases with applied p.d.

ii The resistance of B is constant.

b For 2 A, $V_A = 5$ V and $V_B = 10$ V so the combined resistance is $\dfrac{(5+10)\,\text{V}}{2\,\text{A}} = 7.5\,\Omega$.

c If they are in parallel, the p.d. is the same across each. A p.d. of 5 V produces $I_A = 2$ A and $I_B = 1$ A, *i.e.*, a total current of 3 A.

d The resistance of a parallel combination is less than the resistance of the smallest component. The parallel combination of A and B has a minimum resistance when that of A is as low as possible, *i.e.*, at a low value of p.d.

12b The *proportional* change in resistance of the whole circuit which the rheostat can make is insufficient to bring about the current change necessary for this variation in p.d. across the lamp.

13a Their resistivities rise in constant ratio.

b The logarithm to base ten of the resistivity.

c More than 10^{20} km, over ten million light years; further from us than the nearest galaxy.

14 Notice that the question says *might* apply, not *does* apply.

a X will if any do; one might argue for Y on the grounds that even a tiny current could represent a flow of electrons.

b Z

c Y or Z because resistivity falls with rise in temperature.

d X because resistivity rises with temperature.

e Y, identifying X with metals and Z with insulators.

f Z

g Y, excluding graphite.

h Z

i X

15a 1.6 A

b 2*R*

16b The coefficients of ($\delta l/l$) and ($\delta r/r$), the *proportional* changes in length and radius, in the expression for $\delta R/R$, are the same as the *powers* of l and r in the formula for R. This result is general and is particularly useful in thinking about accuracy of measuring and experimental uncertainty.
c $8.3\,\Omega$
d The change in resistance is proportional to strain.

17a $I/A = V/AR$
b $I/A = V/\rho l$
c $5.9 \times 10^7\,\mathrm{A\,m^{-2}}$; current in a $1\,\mathrm{mm}^2$ wire is 59 A.
d $v = 3.7 \times 10^{26}/n$
e 8.5×10^{28} atoms, or electrons, per cubic metre.
f About $4\,\mathrm{mm\,s^{-1}}$.

18a A is connected electrically to B, B is not connected to D (unless by a very high resistance). A may or may not be connected to C (or D).
b The effective resistance between A and B has 1.5 V across it when a current of (4.5/1000) A passes, and is thus just over $330\,\Omega$.
c The effective resistance between A and B is just over $300\,\Omega$, in agreement with **a** and **b**.
d If there were resistors connected between A and C or between B and D, the meter would read more than 18 mA (but **a** suggested that B and D were not connected).
e If A and C were connected, the meter would give a reading when the switch was open. So they are not. The reading of 18 mA when the switch is closed confirms earlier suggestions that the circuit is as in figure Q13.

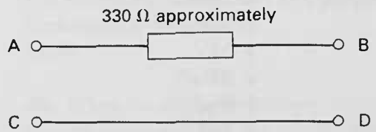


Figure Q13

19a (a)
b (b)

20a $11\,000\,\Omega$
b 0.27 mA
c Three-quarters of the length of CD.
d The lamp resistance ($15\,\Omega$ working) is so low compared with the resistance of the part of the potentiometer across which it is connected that it effectively short-circuits the output, producing very nearly zero p.d. across it.

21 **B**

22 $1\,\Omega$. Students should find, and perhaps be able to prove, that the power delivered has a maximum value when R is $1\,\Omega$, that is, the resistance of the external circuit is equal to the internal resistance of the cell.

23a **B**
b **B**

24 **E**

25 **C**

26 Briefly:
a X and Y are at the same potential.
b $8 \times 10^{-6}\,\Omega\,\mathrm{m}$
c From Y to X, since the potential of X is now more negative.
Note: Fuller answers would of course be expected in an examination.

27a From the sliding contact into the meter.
b $I_1 - I_2$
c Three; only two are needed.
d $83\,\mu\mathrm{A}$

28a The current is zero at first; rises sharply to 2 A; remains steady for ten hours; drops sharply to zero at 6 p.m.
b 2 C
c 7.2×10^4 C
d The charge passed is represented by the shaded area in figure Q14.
e The charge passed is still shown by the area between the graph and the time axis.
f About 5×10^4 C.

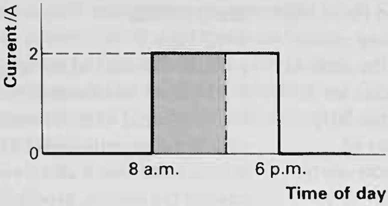


Figure Q14

29a The same as meter 1.
b They move 5 divisions to the left, and then return to zero.
c None!
d Charging, anti-clockwise; discharging, clockwise.
e When the capacitor is being charged the equality of meter readings 1 and 2 shows that as much charge flows onto one plate as flows off the other.

30a As in figure Q15.

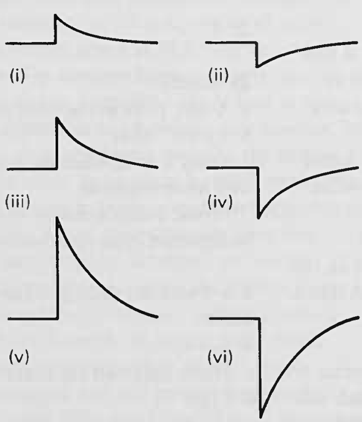


Figure Q15

b*i–iv* As for **a***i*. *v* As for **a***vi*.
c Equal changes in p.d. produce equal flows of charge (identical traces).

31a 1 mA
b 0.1 C
c 0.01 C
d 0.01 F
e Zero (and indeed at every other instant during the process the total net charge on the plates is zero).

32a 10^{-3} C
b 6.25×10^{15}
c The resistance would have to be decreased at a steady rate.
d 10^{-4} A
e See figure Q16.

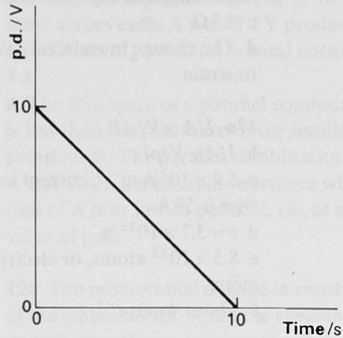


Figure Q16

33a 1 mA
b About 1.5×10^{-4} C.
c 15 μF
d As in figure Q17.

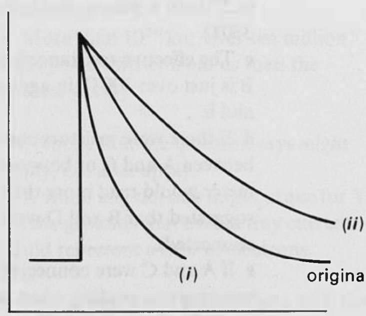


Figure Q17

34a 100 μC
b 200 μC
c 10 V
d 10 V
e 300 μC
f 30 μF
g Yes!

35a*i* No *ii* No
b No, from **a**. (Or consider the isolated part of the circuit consisting of the

righthand plate of C_1 and the lefthand plate of C_2.)
c Q/C_1
d Q/C_2
f $C = Q/V$

36a 3 μF
b 6 μF

37a 0.01 F
b 0.02 C
c 0.38 J. The taller shaded strip in figure Q18.

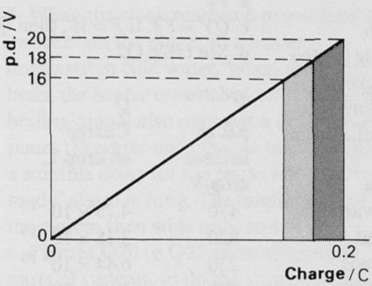


Figure Q18

d 0.34 J
e The two shaded strips in figure Q18.
f 0.38 J + 0.34 J + ... + 0.20 J = 2 J
g The total area of the triangle under the graph ($\frac{1}{2}$ × base × height) represents the energy transformed.

38a 4.5 J
b 9 ms

39a 2.1 kV
b Some points to consider:
The physical size of the bank of capacitors (the plates have to be far apart to withstand the high p.d.).
The energy will be delivered as a pulse.
Recharging arrangements.
Safety.

40a When the diaphragm bulges, there is as much more water on one side as there is less on the other. When a capacitor is charged, there is as much extra positive charge on one plate as there is missing on the other. If the pump were removed and a pipe substituted, the extra water would flow round the circuit. If a capacitor is connected to a wire, the extra charge on one plate flows round to the other. No water enters or leaves the system as a whole. No charge is created or destroyed in a capacitor circuit. The analogy is rather good.
b Yes.
c Make the diaphragm easier to stretch.
d Pressure differences across tank, corresponding to p.d., and amount of extra water in one side (and equal amount less in the other side) corresponding to charge.
e The capacitor conducts slightly through the insulating layer between the plates.

41a Identical to figure B89(b) in the *Students' guide*, but negative current.
b The currents will have initial values half that of **a** (50 μA) but will last for twice the time (time constant doubled).

42a 100 kΩ (about 40 seconds for the current to fall to $1/e$ of its starting value).
b 5 V
c Over the 80 seconds the area represents about 1700 μC. Using $Q = CV$ gives 2000 μC.
d The current is three times larger at any instant of time, so the time taken to decay to any given proportion of the initial value is unchanged.

43a See figure Q19.

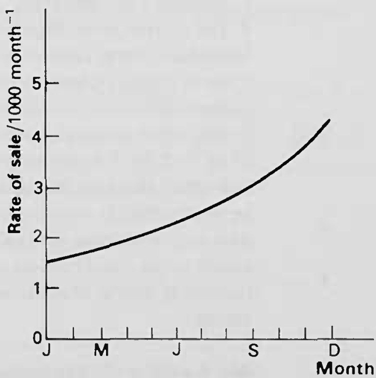


Figure Q19

b 31 310 (This can also be represented as the area under the graph.)
c The ratio is constant at about 1.08 except for the last two months.
d Christmas?
e The constant factor of 1.08 for most of the year indicates a constant *proportional* growth rate of 8 per cent per month. The growth is therefore exponential, *i.e.* the rate of increase is proportional to the total number sold. There is unlikely to be one single reason why this should be so; students should list some factors which affect demand.
f Approximately 47 months from January, assuming the growth rate for the first 10 months. Presumably the growth rate would have flattened out before this time as the market approached saturation.

44 One might use a circuit with a capacitor discharging through a variable resistor, with the discharge current holding the relay on while it exceeds 0.1 mA. The time constant of this circuit determines how long the relay is closed and hence for how long the lamp lights. The variable resistor could be calibrated for the required lighting time. Students are invited to sketch the curve of discharge current against time for these approximate values: resistance 10 kΩ, capacitance 700 μF, charging p.d. 10 V.

45a About 6×10^{15} electrons.
b About 6×10^{9} electrons.
c About 6×10^{3} electrons.
d The current is too high for the effect of individual charge carriers to show up. Even in as short a time as 10^{-12} s several thousand electrons hit the screen. Perhaps at very much lower currents (of the order of 10^{-6} or 10^{-9} A) the arrival of individual electrons on the screen might be observable. At very low currents, or over very short time intervals one might expect to see some random variation in the rate of arrival of electrons at the screen.

46a $6 \times 10^{5}\ \mathrm{m\,s^{-1}}$ approximately.
b $60 \times 10^{5}\ \mathrm{m\,s^{-1}}$ approximately.

47 The current is the ratio of power to potential difference. An estimate of 10 W power gives a current of 5×10^{-4} A. This is the same as 3×10^{15} electrons per second which, on an area of $10^{5}\ \mathrm{mm^2}$, gives a density of 3×10^{10} electrons per $\mathrm{mm^2}$. Notice that the number of electrons per square millimetre of screen must be large otherwise it might be possible to see random fluctuations in the brightness as the electrons arrived randomly.

48a QV
b Fd
c $QV = Fd$. If $F = W$, then $Q = Wd/V$.
d See table Q2.

V **p.d. to balance drop/V**	Q **Charge on drop/C**	n **Multiple**
470	4.75×10^{-19}	3
820	3.18×10^{-19}	2
230	6.44×10^{-19}	4
770	1.64×10^{-19}	1
1030	1.57×10^{-19}	1
395	7.83×10^{-19}	5

Table Q2

e Basic charge e is about 1.6×10^{-19} C.
f The values of Q and n in the table lead to the following values for e:
1.58×10^{-19} C, 1.59×10^{-19} C, 1.61×10^{-19} C, 1.64×10^{-19} C, 1.57×10^{-19} C, 1.56×10^{-19} C. Given that the true value of e is 1.60×10^{-19} C, the uncertainty in these calculated values is about 3 per cent, suggesting that the *sum* of the percentage uncertainties in V, W, and d is also about 3 per cent. From the results, V, which is usually given to two significant figures only, seems to have the largest uncertainty, and d the least.

UNIT C
Digital electronic systems

1a Suppose the numbers are 1342 and 5768. Select the end digits of each number (2 and 8). Add them. The sum is 10. Store the digit 0 as the first digit of the answer, and carry the digit 1 to the next step. Select the next two digits (4 and 6). Add them, and the 1 carried over, getting 11. Store 1 and carry 1. Then continue. A better system would have a stopping device that turned the adding process off when all of the incoming digits (carry as well) became zero. Without it a mindless computer would go on for ever, producing the answer

......0000000000000000000000000007110

b When the clock reaches a preset time it switches on the power to a heater immersed in cold water. When the water boils, the heater is switched off. The 'water boiling' signal also operates a device that pours the water onto the tea leaves. After a suitable delay for the tea to infuse, a 'tea ready' alarm is rung. The human part of the system then adds milk and sugar.

c Figures Q20 to Q22 show systems, or parts of systems, to do the suggested jobs.

2 Figure Q23 shows the series of events involved when listening to a song on a record player.

4 The power supply provides the energy required. For example, a logic gate may be arranged to light a light-emitting diode (L.E.D.). In a sense the gate directs the energy to the L.E.D. under certain conditions. The energy comes from the power supply. All the systems in question **1c** have a power source. The system controls the use of that energy.

5 There are only two digits in the binary system: 0 and 1. It is easy to define two voltage levels – high and low; supply voltage and zero volts – representing the digits 0 and 1. The binary system readily lends itself to electronic systems. Defining intermediate levels to represent more digits is much more complicated.

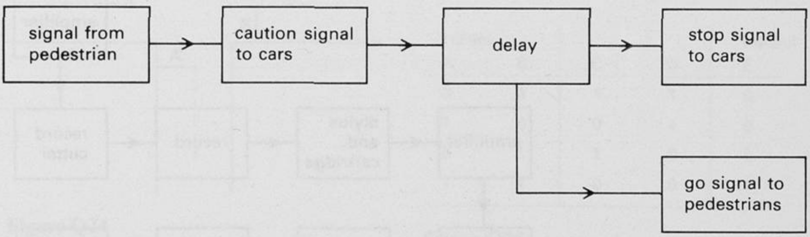


Figure Q20
Part of a system for controlling cars. Other parts should be added to operate go and stop signals for cars and pedestrians.

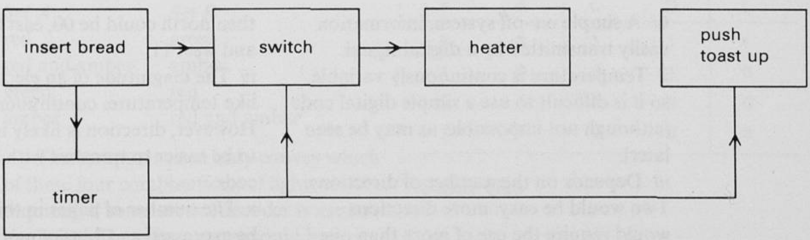


Figure Q21
A system for a 'pop-up' toaster.

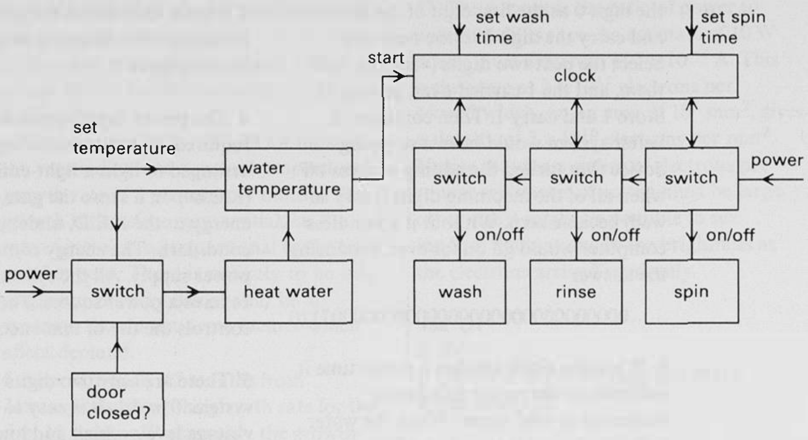


Figure Q22
A system for an automatic washing machine.

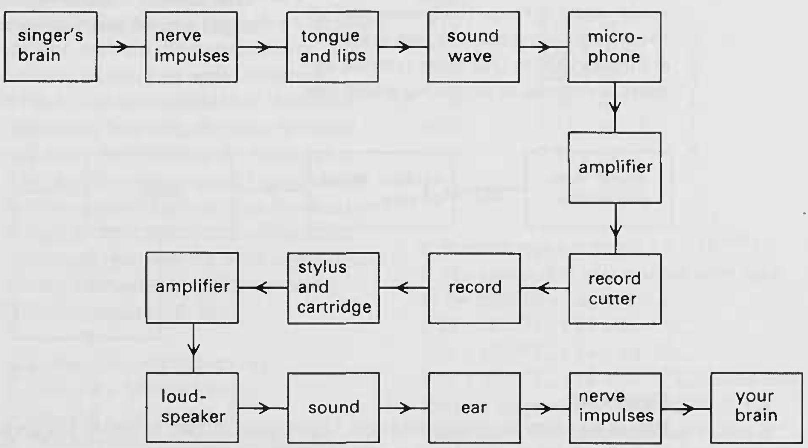


Figure Q23

6*i* A simple on–off system, information easily transmitted by a digital signal.
ii Temperature is continuously variable, so it is difficult to use a simple digital code (although not impossible, as may be seen later).
iii Depends on the number of directions. Two would be easy: more directions would require the use of more than one binary digit. For example, four directions could be coded using two 'bits'. If the directions are north, east, south, and west, then north could be 00, east 01, south 10, and west 11.
iv The magnitude of an electric current is like temperature: continuously variable. However, direction is likely in most cases to be easier to transmit with a binary code.
v The number of pages in this book can be expressed as a binary number which can be transmitted as a sequence of 1s and 0s. For example, if there are 76 pages: decimal 76 is binary 1001100.

vi The area of the page may also be expressed as a number and so can be expressed in binary form.
vii Again, this is a number and can be transmitted in digital form.

7a Crossroads, such as the one in figure Q24, require two sets of lights to control the traffic. Set A controls vehicles approaching from north and south; set B controls vehicles from east and west. Each set has 3 different lights (red, amber, and green) making 6 different lights to control altogether. Since each light is either on or off the information could be conveyed with six bits.

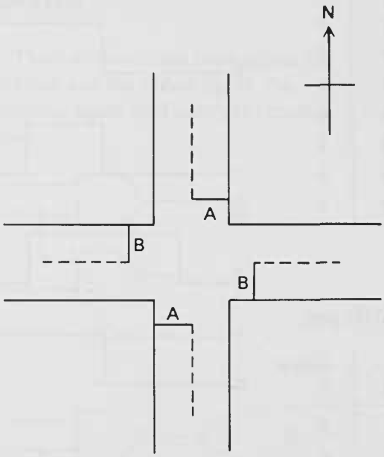


Figure Q24

However, not all possible combinations of lights are used. In fact, only the four shown below are used:

Set A	*Set B*
red	green
red and amber	amber
green	red
amber	red and amber

The computer need only convey which of these four combinations of lights is required, if some sort of decoder is used at the lights. These four states could be communicated from the computer to the lights with only 2 bits, representing the binary numbers from 00 to 11.

b $1024 = 2^{10}$. The term 'kilo-' is used because 1024 is roughly 1000.
c One byte is 8 bits. Each bit can be 1 or 0. This gives 2^8 ($=256$) possible combinations. So 256 different characters can be coded and communicated.
d It is usual to use one byte for each character. Fifty-two different characters are needed to represent the alphabet in upper and lower case form. There are also numbers and other characters: full stops, commas, brackets, dashes, etc. A code is also required for spaces and 'carriage returns' at the end of lines. (For more information about the ASCII code, see, for example, Horowitz and Hill *The art of electronic* section 10.17)

8 Circuits (a) and (b) are both inverters.
(c) This circuit functions as an inverter.
(d) In this circuit the output is always low.
(e) In this circuit the output is always high.
(f) This circuit functions as an inverter.

9 The circuit is an AND gate.

inputs A	B	C	D	output E
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

Figure Q25

10a This circuit is a NOR gate.

inputs A	B	C	D	output E
0	0	1	1	1
1	0	0	1	0
0	1	1	0	0
1	1	0	0	0

Figure Q26

b This circuit is an Exclusive OR gate.

inputs A	B	C	D	E	output F
0	0	1	1	1	0
1	0	1	0	1	1
0	1	1	1	0	1
1	1	0	1	1	0

Figure Q27

c This circuit is a three-input NOR gate.

inputs A	B	C	D	E	output F
0	0	0	1	1	1
1	0	0	0	1	0
0	1	0	0	0	0
0	0	1	1	0	0
1	1	0	0	0	0
1	0	1	0	0	0
0	1	1	0	0	0
1	1	1	0	0	0

Figure Q28

d This circuit is a three-input AND gate.

inputs A	B	C	D	E	output F
0	0	0	1	1	0
1	0	0	1	1	0
0	1	0	1	1	0
0	0	1	1	1	0
1	1	0	0	1	0
1	0	1	1	1	0
0	1	1	1	0	0
1	1	1	0	0	1

Figure Q29

11 The solution to this problem is called a Parity gate (figure Q30). It is the inverse of an Exclusive OR gate (see question **10b**).

inputs A	B	output C
0	0	1
1	0	0
0	1	0
1	1	1

Figure Q30
A Parity gate.

12 The signals are shown in figure Q31.

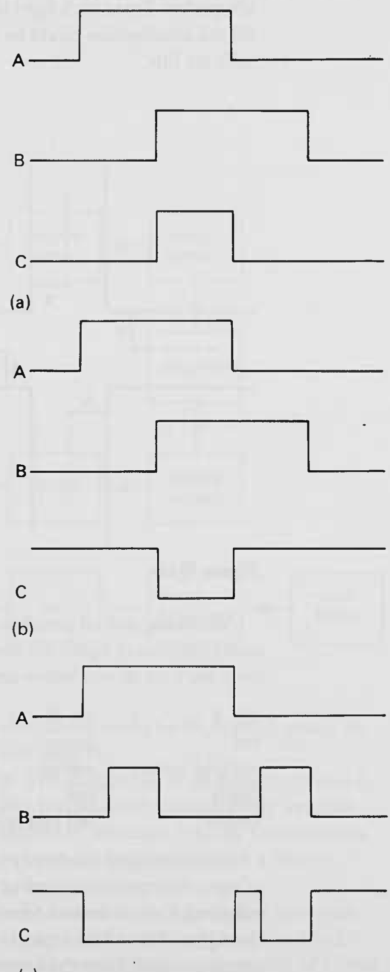


Figure Q31

13 If the system is thought of as a box with four inputs and three outputs, then truth tables can be constructed to satisfy the conditions specified in the question. For example, the red L.E.D. is controlled by sensors A and B in the way that the truth table in figure Q32 shows.

A	B	output red
1	1	1
1	0	0
0	1	0
0	0	0

Figure Q32

There are two other truth tables for the green and the yellow lights. The circuits in figure Q33 satisfy the truth tables.

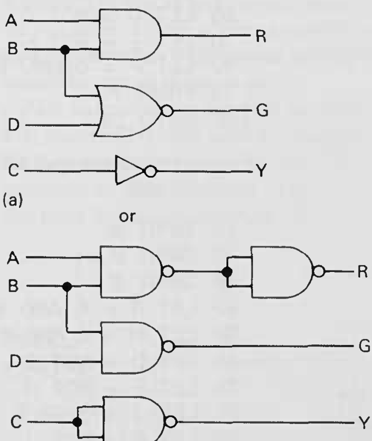


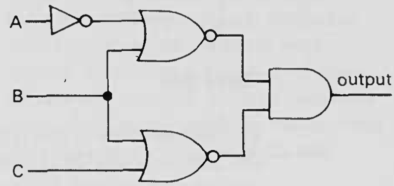
Figure Q33

14 This circuit will have three inputs and one output. Without showing the whole truth table, the necessary combination of inputs to make the output high is shown in figure Q34(a).

The circuit in figure Q34(b) satisfies this condition. Any other combination of inputs gives a low output.

A	B	C	output
1	0	0	1

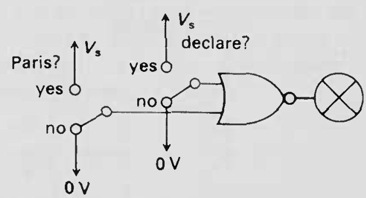
(a)



(b)

Figure Q34

15 The circuit in figure Q35(a) is suitable. Both switches must be marked 'Yes' in the V_s position. The truth table is shown in figure Q35(b).



(a)

inputs		output
I have something to declare	I have come from Paris	you may proceed
no	no	yes
no	yes	no
yes	no	no
yes	yes	no

(b)

Figure Q35

If passengers have nothing to declare and have not come from Paris they may proceed without customs examination.

16 The circuit in figure Q36 shows how a third switch can be added so the 'proceed' lamp only lights when passengers have passed it.

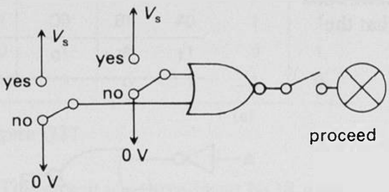
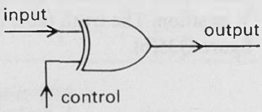


Figure Q36

17 The truth table is that of the Exclusive OR gate, although the second input is now labelled 'control'. Examination of this truth table shows that the 'input' is passed unchanged to the 'output' if the 'control' is 0. If 'control' is 1, then the 'output' is the inverse of the 'input'.



input	control	output
0	0	0
0	1	1
1	0	1
1	1	0

Figure Q37

18a Note that the actual value of the variable used depends on the computer used. Some machines (for example, Apple and Sinclair) should be used with input values of 0 or 1. Other machines (for example, Research Machines, B.B.C.) require values of 0 or −1.

b Program for Exclusive OR gate (question **10b**):

```
 10 INPUT A
 20 INPUT B
 30 LET G = A AND B
 40 LET C = NOT G
 50 LET H = A AND C
 60 LET D = NOT H
 70 LET J = C AND B
 80 LET E = NOT J
 90 LET K = D AND E
100 LET F = NOT K
110 PRINT F
120 GOTO 10
```

Program for three-input NOR gate (question **10c**):

```
 10 INPUT A
 20 INPUT B
 30 INPUT C
 40 LET G = A OR B
 50 LET H = B OR C
 60 LET D = NOT G
 70 LET E = NOT H
 80 LET F = D AND E
 90 PRINT F
100 GOTO 10
```

Program for three-input AND gate (question **10d**):

```
 10 INPUT A
 20 INPUT B
 30 INPUT C
 40 LET G = A AND B
 50 LET H = B AND C
 60 LET D = NOT G
 70 LET E = NOT H
 80 LET J = D OR E
 90 LET F = NOT J
100 PRINT F
110 GOTO 10
```

19a 5 volts. All the kits will work from a 5 V supply.
b When the output of one gate is connected to the input of another, the 'output low' of the first gate must be less than or equal to the 'input low' of the second gate, if the logic is to be preserved.

For the same reason, 'output voltage "high"' is higher than 'input voltage "high"'.

c Basic unit and TTL gates have operating limits that are defined by fixed power supply limits. Examine the characteristic curves in figure C13 (*Students' guide* page 167).

CMOS gates have a very sharp transition from high to low, but the level of this depends on the supply voltage. Since the supply voltage (V_s) can vary over a wide range, it is best to express the limits in terms of V_s.

20a The two bare wires in figure C70 in the *Students' guide* would be arranged so that rain water would short them. This would cause the input of the gate to go high and the indicator to light.

b The circuit in figure C71 in the *Students' guide* would not light the indicator brightly as soon as rain shorts the two wires. A large current is needed to light the lamp but only a small current is needed to control the logic gate.

c Pure water does not conduct electricity.

d A pump will require more power than the logic gates can control directly. The pump can be controlled by a relay energized by the gate (figure Q38).

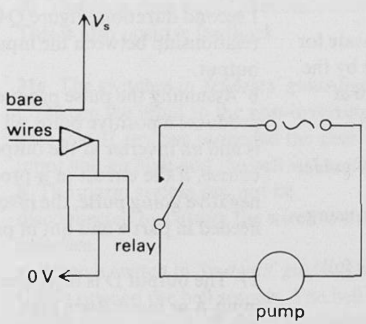


Figure Q38

21a Figure C72(b) in the *Students' guide.* As the resistance of the thermistor drops so does the potential difference across it and therefore the input to the inverter goes low. So the output of the inverter becomes high, lighting the warning lamp.

The circuit shown in figure C72(a) in the *Students' guide* will warn against low temperatures.

b The circuit in Figure C73 in the *Students' guide* would be used to warn against decreases or increases in temperature. The indicator, which in this circuit is on when conditions are safe, will go out if either A or B goes high. The resistors are chosen so that the inputs are low when the temperature is within the safe range. If an OR gate were used instead, the indicator would be off when both inputs were low. It would then light when either input went high, that is, when the temperature went out of the safe range.

22a 5 V

b 800 Ω

c 1250 Ω

d Preferred values are
1, 1.2, 1.5 ... kΩ (5 and 10 %);
1, 1.1, 1.2, 1.3 ... kΩ (2 % and 1 %).

e The gate switches when the p.d. between 0 V and the input to the gate is 5 V. So the p.d. across R_2 is 5 V and the p.d. across the L.D.R. is 4 V. I_1, the current through the L.D.R.

$$= \frac{4\,\text{V}}{1\,\text{k}\Omega} = 4\,\text{mA}$$

So $I_2 = I_1 - 1\,\text{mA} = 3\,\text{mA}$

and $R_2 = \dfrac{5\,\text{V}}{3\,\text{mA}} = 1\frac{2}{3}\,\text{k}\Omega = 1.7\,\text{k}\Omega$, to 2 s.f.

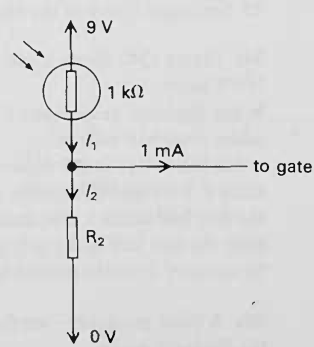


Figure Q39

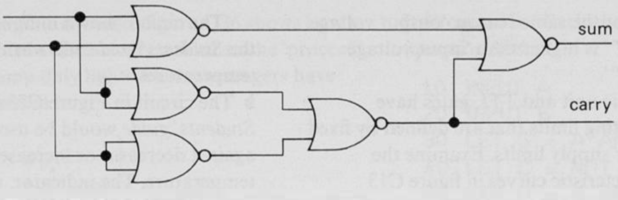


Figure Q40
A half adder circuit using NOR gates.

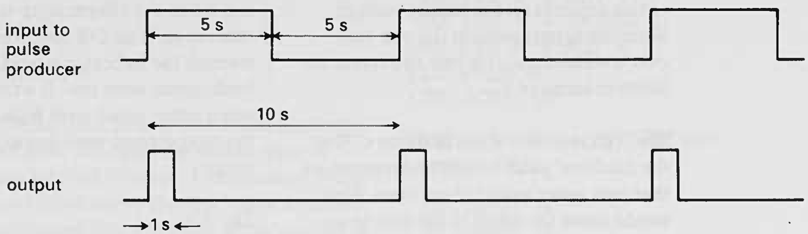


Figure Q41

So to allow for current drawn by the gate itself R_2 must be larger. (It's like connecting a low resistance voltmeter in a circuit. It actually reduces the p.d. measured.) If the circuit uses a variable resistor that can be increased to 2 or 3 times the calculated value, then problems like this can be overcome. (In fact 1 mA is huge for a CMOS gate – almost unheard of; it is even large for the base current of a transistor.)

f It is easy to adjust R_2 to compensate for the loading of the potential divider by the gate; or to make the indicator come at different levels of illumination.

23 See pages 168–9 of the *Students' guide*.

24a Figure Q40 shows a half adder using NOR gates.

b See *Students' guide* figure C17 for full adder from half adders.

c See figure C17 in the *Students' guide*. Only if A = 1 and B = 1 is the carry from the first half adder 1. But then the sum from the first half adder is 0, so there can be no carry from the second half adder.

25a A pulse producer – see figure C19 in the *Students' guide*.

b Input 1 goes from low to high; output 1 goes low, stays there for a while, and then returns to high. When output 1 goes high this triggers circuit 2, whose output then goes low, stays there for a while, and then goes high again.

c The process would continue indefinitely – just like an astable circuit.

26a Use a pulse producer which is triggered by a voltage rising from zero to V_s. Adjust the RC value of the pulse producer so that it produces a pulse of 1 second duration. Figure Q41 shows the relationship between the input and output.

b Assuming the pulse producer used in **a** produces a positive pulse, all we need do is add an inverter at the output. (Of course, if the circuit in **a** produces a negative going pulse, the inverter will be needed in part **a** and not in part **b**.)

27 The output D is high when either input A **or** input B are high.

A	B	C	D
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

Figure Q42

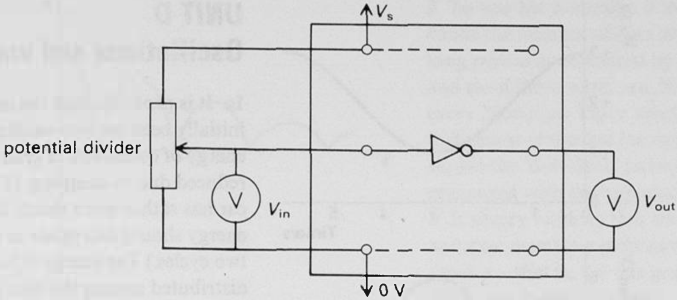


Figure Q43

28a C is high.
b D is low; D keeps A low.
c D is high; feedback keeps A high.
d D goes high and feedback makes A go high as well. The circuit has changed its state.
e A can only stay low when both A and B are low. Both must be made low.
f It has two stable states; with D (and A) low or high.

29a The output of the second also goes from low to high.
b Nothing happens to it next. It stays high. The output from the second keeps it high.

30 The shop runs out of chocolate sooner. This is also positive feedback.

31a The switches in *Students' guide* figure C79 keep the input to the non-inverting gate low. If any one is opened the gate input will go high and the bell will sound.
b The alarm system can not be disconnected by cutting the wires to the switches.
c When a switch in *Students' guide* figure C79 is opened the bell sounds. The bell can be easily turned off again by closing the switch. In *Students' guide* figure C80 the output of the OR gate is fed back to one input. When a switch is opened the output of the gate goes high and stays high even if the switch is closed again.

32 B

33 B

34a The additional circuit components are shown in figure Q43.
b The graphs are shown in figure Q44 (below and on the next page).

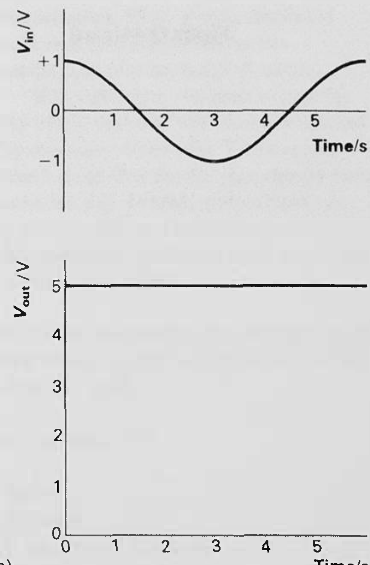


Figure Q44 (part)

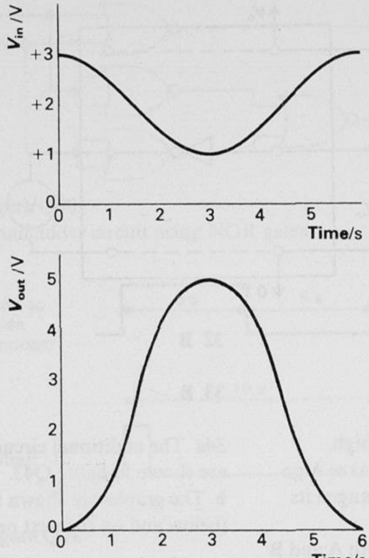


Figure Q 44 (part)

UNIT D
Oscillations and waves

1a It is probable that the car body has initially been set into oscillation, and the energy of oscillation is gradually being reduced due to damping. (This particular car has rather worn shock absorbers – the energy should disappear in only one or two cycles.) The energy is being distributed among the many atoms making up the surroundings of the car body. The reverse process – concentration of energy from many individual atoms into oscillations of one object – never happens.

But the motion shown could be reversed if the car body were being given a properly timed series of pushes by an external agent – that is, being set into forced oscillation.

b Something like the graphs in figure Q45 should appear.

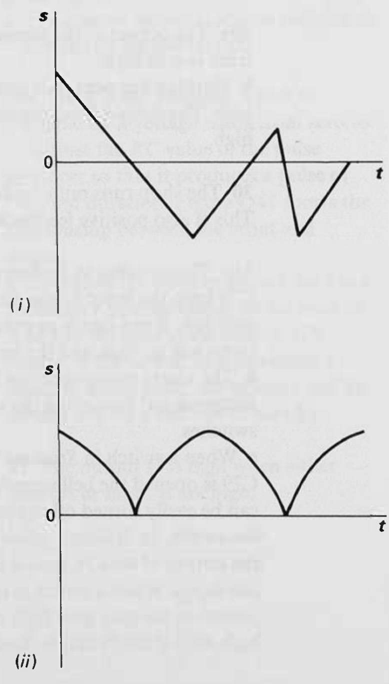


Figure Q 45 (part)

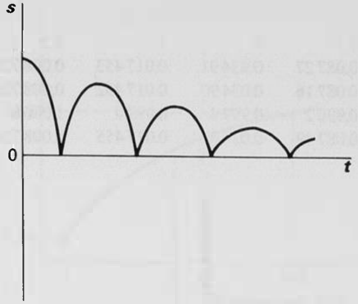


Figure Q 45 (part)

2a 1.0 m
b*i* $\frac{1}{4}$ cycle
ii 0; −1.0 m
c*i* −0.5 m
ii 1.5 s, 4.5 s, 7.5 s, etc.

3 The story of Harrison and his chronometers is told in a booklet obtainable from the National Maritime Museum.

The point of carrying a clock for navigation is to compare the time of noon (say) at the ship's unknown longitude with that, indicated by the clock, at some other definite longitude, which has gradually come to be that of Greenwich. If, for instance, local noon is six hours earlier or later than at Greenwich, the ship is one-quarter of the way around the Earth from Greenwich. At the Equator, a time difference of one minute corresponds to a distance in longitude of nearly thirty kilometres, so an accurate clock is needed.

The way to test the clock is to determine, by making astronomical observations, the longitude of the place reached (Jamaica), and to calculate the expected time of noon compared with that at Greenwich. The clock, if it is accurate, will still show Greenwich time, and its reading can be compared with the calculated time.

4a *1* To test for regularity, it would compare the number of dots punched out in fixed intervals of time as indicated by its own clock, and do so at several times of day.

2 To test for accuracy, it would need to count the number of dots produced over a long period as measured by its own clock, and see if there were, say, 5000 dots for every 2000 pips. There would be no point in doing so if the test for regularity had shown the 'dot-clock' to be irregular, compared with the 'pip-clock'.
b It is very unlikely that the clock can be accurate over long periods if it is not regular – that is, if it disagrees even with itself. But it can be regular, yet not punch dots at exactly $\frac{1}{5}$ second intervals.
c Only by *deciding* that some clock or other is going to be treated as a regular time marker. This standard time marker cannot be tested, though the choice would fall on one of a number of such clocks that show each other to be regular, and for which, if possible, there are theoretical reasons for thinking that the rate will not be affected by most kinds of laboratory disturbances. Thus, atomic clocks are preferred to clocks based on the oscillations of some lump of matter.

If the laboratory decides to trust its pip-clock, the issue will have been settled by decision, not by trial. The inventor could claim that the decision should have gone his way. Indeed, atomic clocks at present in different countries do show tiny disagreements, and there is no way to tell which one is 'right'.

5 This is discussed in the reading 'Quartz and atomic clocks' on pages 237–9 of the *Students' guide*.

6 2 radians; 115°

7a*i* $r\theta$
ii $r \sin\theta$
b $\sin\theta \approx \theta$ if θ is small.

8

θ/degrees	20	10	5	2	1	0.5	0.1
θ/radians	0.349 1	0.174 5	0.087 27	0.034 91	0.017 453	0.008 726 6	0.001 745 33
$\sin\theta$	0.342 0	0.173 6	0.087 16	0.034 90	0.017 452	0.008 726 5	0.001 745 33
$\cos\theta$	0.939 7	0.984 8	0.996 2	0.999 4	0.999 8	1.000 0	1.000 0
$\tan\theta$	0.364 0	0.176 3	0.087 49	0.034 92	0.017 455	0.008 726 9	0.001 745 33

Table Q3

ai 0.5 %
ii 0.006 %
iii 0
bi $\cos\theta \approx 1$
ii $\tan\theta \approx \theta$

9 6.7 m s^{-1}

10a

	Velocity	Acceleration
Q	zero (but about to move rapidly down)	large, downward
R	large, upward	zero
S	zero	large, downward

Table Q4

b*i* Tension in the Slinky (T) – a large tension will mean that for a given displacement there will be a large restoring force, accelerating displaced coils back towards their mean position more quickly.
ii Mass per unit length of the Slinky (μ) – a given restoring force acting on a large mass, will accelerate displaced coils more slowly towards their mean position.
In fact

$$c = \sqrt{\frac{T}{\mu}}$$

11 Both. Any parts of the spring in motion possess K.E.; and any parts in tension or compression compared with their rest state possess P.E. The superposition to give no net compression or expansion is instantaneous: at that moment all the energy is K.E.

12 See figure Q46.

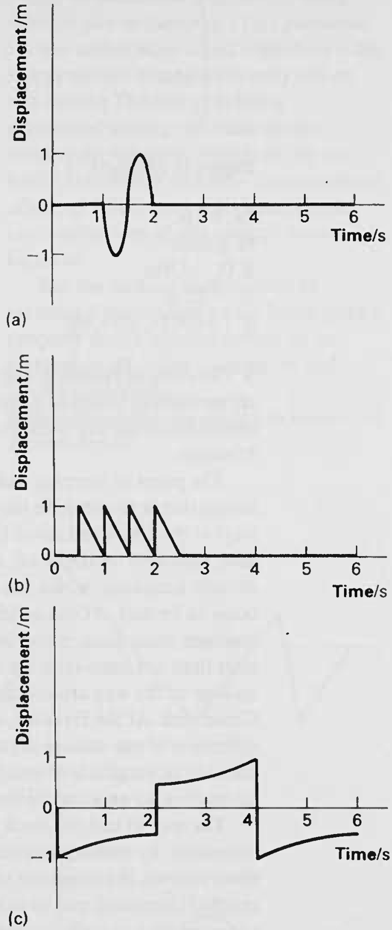
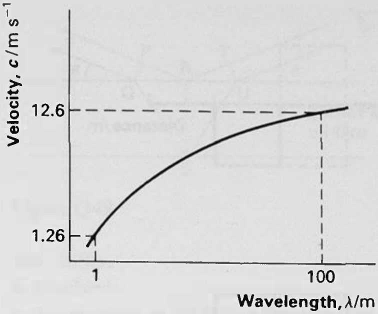
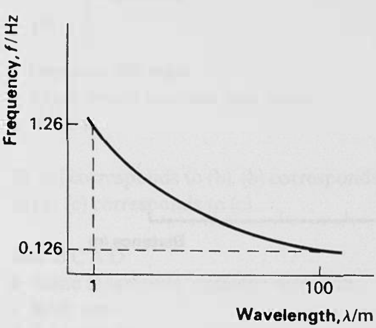


Figure Q46

13a*i* Measure the frequency directly (measure the time for, say, 10 waves to pass a point using a stopwatch).
ii Find the magnitude using $a = \omega^2 A = 4\pi^2 f^2 A$.
At P the acceleration is downwards; at Q it is to the left.
b See figure Q47.



(i) For a straight line, plot c^2 against λ



(ii) For a straight line, plot f^2 against $1/\lambda$

Figure Q47

c*i* All waves in this range of wavelengths will be felt. (Minimum speed needed by waves to reach ship $= 1.16\,\mathrm{m\,s^{-1}}$.)
ii $7.9\,\mathrm{m\,s^{-1}}$. The boat will move in a looped path forwards, with $v_1 = 9.9\,\mathrm{m\,s^{-1}}$, and $v_2 = 5.9\,\mathrm{m\,s^{-1}}$. The frequency of the loops is 0.106 Hz.
iii $6.3\,\mathrm{m\,s^{-2}}$; maximum force $\approx \frac{2}{3}$ weight.
d $1.26\,\mathrm{m\,s^{-1}}$; $31.6\,\mathrm{m\,s^{-1}}$.

14a $\lambda = S_2C - S_1C$.
b*i* The pattern spreads out.
ii and *iii* The pattern closes up (distances like AB become less).
iv Minima at A, C; maxima at B, D.
v The pattern changes with time. At any instant, an interference pattern would be visible over the whole surface, but it isn't stationary. At any one point, there are beats: each point is alternately at a maximum then at a minimum.

c The amplitude would only be zero at a minimum if the amplitudes of the disturbances arriving from S_1 and S_2 were equal. The wave energy is spreading out from the sources, so the amplitude of an individual wave decreases with distance travelled. Since in general waves from S_1 and S_2 will have travelled different distances to reach a minimum point, they will not have equal amplitudes.

15 The path difference has increased by an odd number of half-wavelengths. So λ could be 2 m, or $\frac{2}{3}$ m, or $\frac{2}{5}$ m, or ...

16 See figure Q48.

17 There are many answers. Wavelength measurements usually involve using the fact that waves superpose to give a large or small resultant effect, depending on whether the path difference is an even or an odd number of half-wavelengths.

Two-source experiments are possible in practice with waves of reasonable wavelength, for the sources need not then be very close together. But the sources must emit in phase. For light, two-source experiments are still possible, but the two sources must be imitated by splitting the light from one narrow source.

Diffraction gratings can be used, and are especially suitable for visible light and for X-rays. The grating spacing must not be very much larger than one wavelength if the diffraction angle is to be reasonably large. For X-rays, the grating is tilted so that the X-rays graze its surface, and the spacing looks small to the X-rays.

In the microwave and v.h.f. region (wavelength from 0.01 m to 1 m roughly), simple arrangements of reflectors which introduce a path difference can be used. A problem here is to have big enough reflectors, for a reflector must be bigger than one wavelength in linear dimensions to reflect an appreciable amount of wave energy.

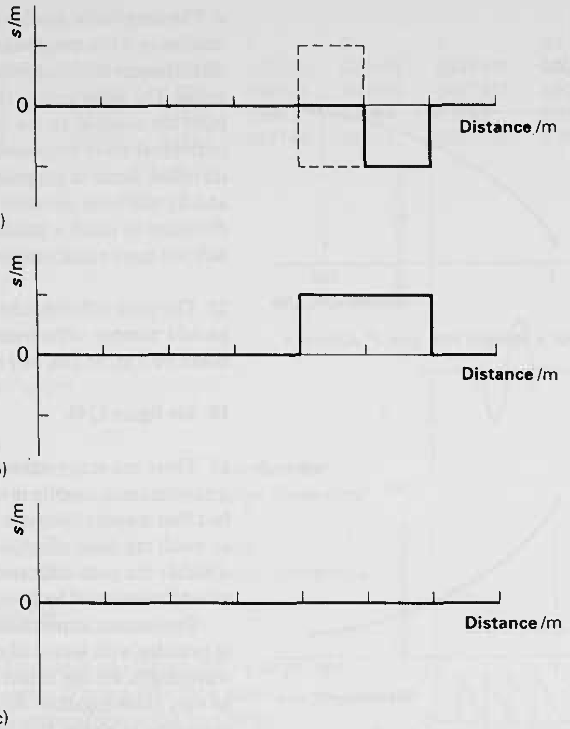


Figure Q48

18a 120 mm
b As S_2 is moving further from T, it is reflecting a decreasing amount of wave energy; and as R is also now further away, a smaller proportion of this amount reaches R. Hence the two superposing signals will no longer have equal amplitudes, which they must have to cancel out exactly.

19 **D**

20a Out of phase.
b In phase.
c The light following paths A and C superposes destructively – thus reducing the intensity reflected. B and D, however, give constructive superposition, increasing the intensity transmitted.
d There will be relatively little light intensity reflected in the middle of the visible band, and relatively more at the ends – that is, blue and red. These combine to make the reflected light appear purple.

21 156 m

22 In general, superposition occurs between light reflected at the top and the bottom of the oil film. If there are no changes of phase, the path difference in figure Q49 is (QS + SU) – (PR + RT). If the film is very thin, this may equal one wavelength for one particular colour – in which case the observer will see that colour at that angle θ. At a slightly different angle, the path difference will alter, and he will see a different colour.

23a Doubled.
b Doubled.
c Doubled.
d Same time.

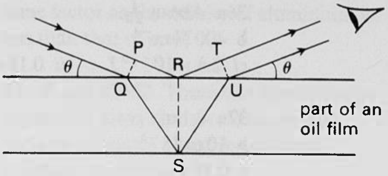


Figure Q49

24a Stiffer.
b Doubled.
c Acceleration in (b) is four times that in (a).
d k in (b) is four times k in (a).
e $T^2 \propto \frac{1}{k}$.
f Decrease the mass.
g Mass would increase four times.
h $T^2 \propto m$

25 (a) corresponds to (b). (b) corresponds to (a). (c) corresponds to (c).

26a B C A D
b Same magnitude, opposite direction.
c Both zero.
d 0, D and F.
e 0, D and F.
f $0B = T/4$, $0D = T/2$, $0F = T$, $BE = T/2$, $DF = T/2$.

27 (b) will oscillate. (Do not confuse the 'weightlessness' of the mass with lack of inertia. Its mass is the same as on the Earth, and it behaves as an oscillator with the same period, neglecting any relativistic effects.) Students can discuss the others.

t/s	s/m	a/m s^{-2}	v/m s^{-1}	Δs/m
0	0.1	−1	0	
0.05			−0.05	−0.005
0.10	0.095	−0.95		
0.15			−0.145	−0.0145
0.20	0.0805	−0.805		
0.25			−0.226	−0.0226
0.30	0.0580	−0.580		
0.35			−0.283	−0.0283
0.40	0.0296	−0.296		
0.45			−0.313	−0.0313
0.50	−0.0017			

Table Q5

28a 0.32 s
b 20 rad s^{-1}
c From diagram, *i* $\omega = 1100\,°\,s^{-1}$
ii $\omega = 19$ rad s^{-1}.

29a -1 m s^{-2}
b -0.05 m s^{-1}
c -0.05 m s^{-1}
d -0.005 m
e 0.095 m
f -0.95 m s^{-2}
g -0.095 m s^{-1}
h -0.145 m s^{-1}
Table Q5 shows the other values.
i The graphs are shown in figure Q50.
j 2.0 s. It compares well (1.99 s).
k These answers would all be doubled; the period would be shorter.
l These answers would all be halved; the period would be longer.

31 The description is plausible; but the lack of definition in the photograph, and the associated uncertainty in plotting points on a graph, make it difficult to tell for certain.

32a $2x\rho gA$
b The restoring force is proportional to the displacement x.
ci Acceleration $= -\frac{2xg}{l}$
ii Angular velocity $= \sqrt{\frac{2g}{l}}$
iii Periodic time $= \pi\sqrt{\frac{2l}{g}}$

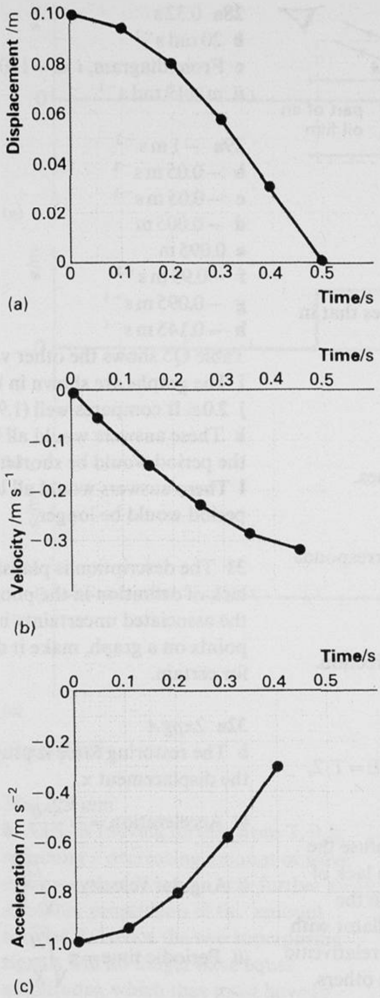


Figure Q50

33 $2\pi\sqrt{\dfrac{hd}{\rho g}} = 2\pi\sqrt{\dfrac{L}{g}}$

34 It must be doubled.

35a 0.52 rad h^{-1}
b $d = 5\cos(0.52 \times t)$ metres. (t measured in hours from high tide.)
c 1.14 p.m.

36a About $\frac{1}{20}$.
b 400 N m^{-1}
c*i* 2.8×10^{-20} J   *ii* 0.18 eV

37a 0.1 m
b 10 rad s^{-1}
c 0.16 s
d 1 m s^{-1}

38 A

39 B

40a See figure Q51.

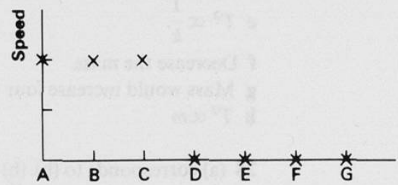


Figure Q51

b Spring constant k, mass of each trolley m, length of each section.
c One possible point is that we believe metal atoms are arranged in regular lines (Unit A); another point from Unit A is the macroscopic elasticity of metals, suggesting the existence of spring-like bonds between atoms.
d The model has only a single line of trolleys and springs, whereas a bar must have a three-dimensional array of atoms and bonds; real metals have dislocations and grain boundaries in their structures; and there is no feature of the metal directly analogous to the frictional damping of the trolleys.

41a 8×10^{12} m s^{-1}
b 5600 m s^{-1}
c Compares quite well (within about 10 %).

42a ρx^3
b $\sqrt{E/\rho}$
c 5100 m s^{-1}
d The Young modulus for aluminium must be less than that for steel by the

same factor as the density of aluminium is less than that of steel.

43–45 and **47–52** These questions involve explaining ideas about resonance, and making estimates based on students' practical experience.

46a Both sets of ions would experience forces, Na^+ to the right and Cl^- to the left.
b 3.8×10^{-26} kg
c 1.2×10^{13} Hz (effective $k = 200\ \mathrm{N\,m^{-1}}$).
d*i* 2.6×10^{-5} m
ii Infra-red.

53a The mass has constant velocity; therefore no unbalanced forces act upon it. The springs will therefore be stretched symmetrically on both sides of it.
b The mass is accelerating to the left; therefore there must be an unbalanced force acting upon it to the left. This must be caused by the lefthand spring being more stretched, and the righthand spring being less stretched. Thus the mass is displaced to the right.
c $\dfrac{ma}{k}$
d See figure Q52
e Zero friction would be a bad idea, because the mass would always oscillate, and never settle to a steady reading.
f If Q is the quality factor of the oscillator, then the oscillations will die away in time $\approx QT$. For a usable instrument, $QT \ll t$. Since $Q \geqslant 1$, we want $T \ll t$. (If $Q < 1$, damping is heavy and the instrument may not give a reading at all.)
g 0.02 m
h k/m would be $1\ \mathrm{s^{-2}}$. The maximum displacement would need to be 2 m, requiring an instrument 4 m long – impractical. The springs would be long, but would require $k = 0.1\ \mathrm{N\,m^{-1}}$ if $m = 0.1$ kg, say; these springs are very soft, and would sag.

54a 10^7 cycles
b 6×10^{14} Hz
c 1.7×10^{-8} s
d 5 m

55a,b,c In each case a low Q is desirable; the more important factor, though, in **a** and **b** is to ensure that the natural oscillation frequency is well away from any frequency at which the system may receive a periodic driving force.
d,e High.
f Low. In this case the natural frequency of the arm should be near to 50 Hz.

56a See figure Q53.
b*i* 4 m *ii* 2 m *iii* 4/3 m
c $60\ \mathrm{m\,s^{-1}}$
d The system responds with large amplitude only at special resonant frequencies. The mass-on-spring has only one resonant frequency, however, whereas the string has a series of resonant frequencies: its fundamental frequency,

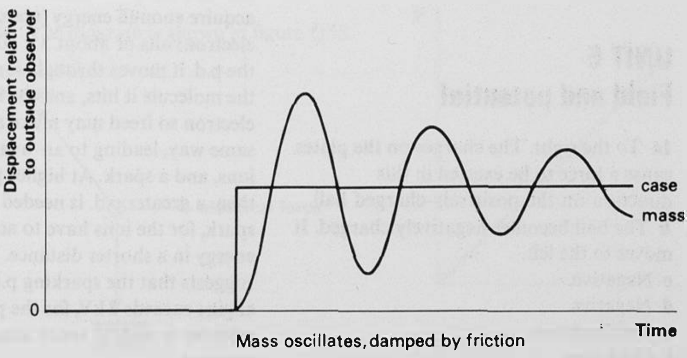


Figure Q52

and multiples of this. The resonance of the string depends on waves arriving back from the far end in phase with the new waves being generated: and this can happen when the path of the waves (4 m) is any whole number of wavelengths.

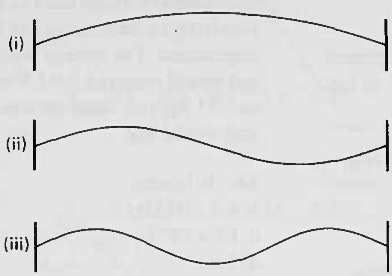


Figure Q53

57 192 Hz, 320 Hz.

58 C

59a $f=\dfrac{1}{2L}\sqrt{\dfrac{T}{A\rho}}$
b $f=650$ Hz
$L=0.5$ m
$A=0.1\times10^{-6}\,\mathrm{m}^2$
$\rho=6000\,\mathrm{kg\,m^{-3}}$
c $T=4l^2A\rho f^2\approx300$ N
d A and ρ, in **b**, are estimated to 1 significant figure: so the answer can only be quoted to 1 significant figure. Even this one figure (3) depends heavily on the estimated quantities: really only the order of magnitude is reliable.

UNIT E
Field and potential

1a To the right. The charges on the plates cause a force to be exerted in this direction on the positively-charged ball.
b The ball becomes negatively charged. It moves to the left.
c Negative.
d Negative.
e Anticlockwise.
f Clockwise.
g So that charge can flow on and off.
h The p.d., V, of the supply, and the separation, d, of the plates. The frequency rises with increasing V or decreasing d.
i Charge is displaced over the conducting surface of the ball; the larger force on the near side of the ball causes it to be attracted to the plate, whereupon it acquires the same charge as the plate and the process continues as before.

2 At the smaller spacing with the same p.d. the field strength (volts per metre) is larger, and the foil hangs at a larger angle.
The field strengths are 62.5 kV m^{-1} and 125 kV m^{-1}.
To get the same field strength at 40 mm separation, reduce the p.d. to 2500 V.

3a Fd
b QV
c $F/Q=V/d$
d V m^{-1}
e*i* joules/coulomb
ii newtons × metres
iii $\dfrac{\mathrm{V}}{\mathrm{m}}=\dfrac{\mathrm{J}}{\mathrm{C\,m}}=\dfrac{\mathrm{N\,m}}{\mathrm{C\,m}}=\dfrac{\mathrm{N}}{\mathrm{C}}$
f*i* 10^4 V m^{-1}
ii 10^{-9} C
iii 10^4 N

4a Use $E=V/d$. $V\approx2000$ V.
b Briefly, the higher the pressure the shorter the distance a particle moves before a collision with another. If in that distance the occasional ion or electron can acquire enough energy (about 30 electronvolts or about 5×10^{-18} J) from the p.d. it moves through, it may ionize the molecule it hits, and the ion or electron so freed may make another in the same way, leading to an 'avalanche' of ions, and a spark. At higher pressures, then, a greater p.d. is needed to start a spark, for the ions have to acquire a fixed energy in a shorter distance. The question suggests that the sparking p.d. in a car engine exceeds 2 kV, for the pressure in the cylinder is several times atmospheric pressure.

5 The completed diagrams are shown in figure Q54.

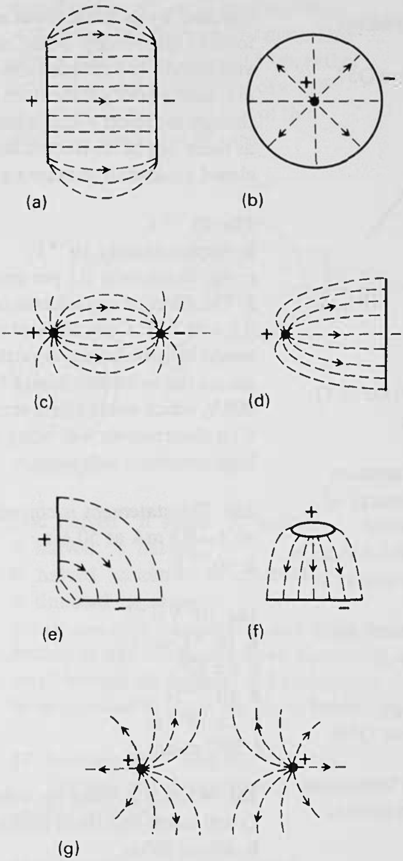


Figure Q54

6a The diagram is shown in figure Q55.

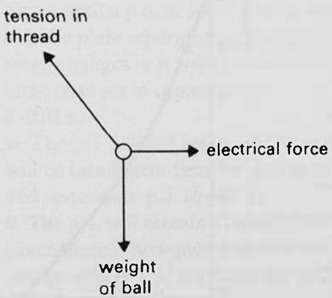


Figure Q55

b 10^{-3} N (equal to the weight of the ball).
c 10^{-8} C. Negative.

7a A large charge would affect the charges on the plates, completely altering the field between them. A small charge, however, would give such a small force that it could not be measured.
b With no flame, the probe may acquire an induced charge and thus affect the field around it, altering the potential at the tip. The flame contains ions which discharge the tip of the probe so that there is no potential difference between it and its surroundings. The electroscope gives little or no indication (unless a charged plate is touched) without the flame.
c Any source of ions would do, for example, a radioactive source giving out alpha particles. These are in fact used in probes in balloons in the upper atmosphere.
d Set up as described, it will acquire positive charge.
e Towards the electroscope.
f If the charge does not leak off the electroscope, then this current will drop to zero. In practice, it settles at a small steady value, enough to keep the electroscope 'topped up'.

8a Positive.
b To the left.
c Negative.
d Increase.
e See figure Q56.

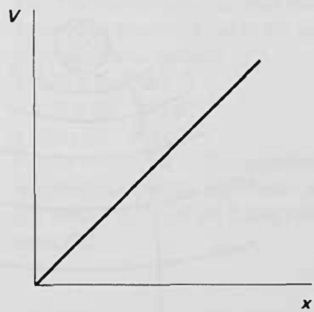


Figure Q56

f Positive.

g $E = -\dfrac{\Delta V}{\Delta x}$ or

'field strength = − potential gradient'.

9a The graph is shown in figure Q57.

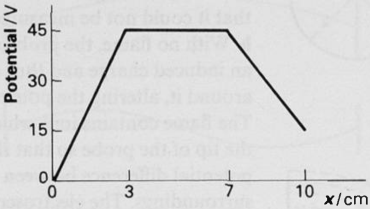


Figure Q57

b For CG_1 it is $-1500\ \mathrm{V\,m^{-1}}$; for G_1G_2 it is $0\ \mathrm{V\,m^{-1}}$; and for G_2A it is $+1000\ \mathrm{V\,m^{-1}}$.

c Between C and G_1 there is constant acceleration up to maximum energy of 45 eV. Between G_1 and G_2 there is constant speed. Between G_2 and A there is constant deceleration down to energy of 15 eV.

d Since it has only 10 eV it cannot reach A, so will 'fall back' towards G_2.

10 The gradient gives the magnitude of the electric field strength (figure Q58).

11 Discharge is most likely to take place around sharp points where the electric field strength is greatest.

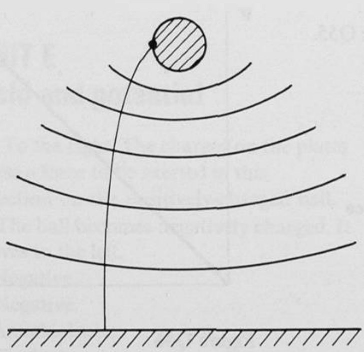


Figure Q58

a will be most dangerous, then **c** (the tree conducts, especially if wet, and charge will flow if it is struck). The horse is 'earthed' by its metal shoes and the rider forms a fairly sharp 'point', so would be vulnerable. The rubber tyres of a bicycle, *if dry*, might offer a little more insulation, *though the safest places would be* **d** *or* **b**, as there can be no electric field *inside* a closed conductor containing no charge.

12a 10^{-11} F.

b Approximately 10^{-8} F.

c Approximately 0.1 per cent.

d The effective capacitance is now about 0.5×10^{-11} F. Only 50 % of the charge would be transferred; nevertheless, the p.d. across the voltmeter would be about 500 V, which could cause serious damage if an electrometer was being used as the high resistance voltmeter.

13a The statement is correct: $Q = 10\ \mu\mathrm{C}$, so $I = 0.5$ mA at 50 Hz.

b 10^{-9} F.

14a $10^3\ \mathrm{V\,m^{-1}}$.

b $10^{-8}\ \mathrm{C\,m^{-2}}$

c 6×10^8

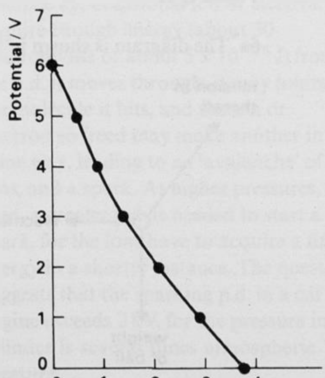
d 10^{-16} N

e 2×10^{-7} m

f 10^{10} atoms

15a When it is rolled up, otherwise B and D will touch and short circuit.

b About 200 m.



c 2
d It would be halved, *i.e.*, about 100 m.
e The cross-sectional area can be considered as πr^2, or as length (100 m) × thickness of sandwich (0.2 mm). This gives a diameter of about 0.15 m, rather larger than one might want of a 1 μF capacitor. In practice paper of this thickness would not be used.

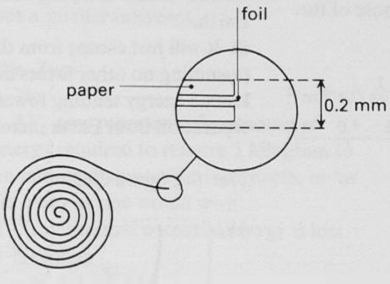


Figure Q59

16a*i* halved *ii* same *iii* halved
iv halved *v* halved.
b*i* halved *ii* same *iii* doubled
iv doubled *v* same.
c In **a** energy is transformed and heats the battery as half the charge flows 'the wrong way' through the battery. In **b** energy has to be supplied to move the plates apart.

17 Between 10^{-14} and $10^{-15}\,\Omega^{-1}\,m^{-1}$.
The area and distance from the bench are not required.

18a*i* Increases, since $C = \varepsilon_0\, A/d$.
ii Decreases, since $V = Q/C$: Q is the same, C is bigger.
b Charge would flow onto the capacitor plates until a p.d. of 300 V was restored.
c If the plate separation is small, then small changes in it will produce relatively large changes in capacitance.
d 0.01 s
e*i* The p.d. will fall below 300 V (as there will be insufficient time for charge to flow and restore the p.d. to 300 V).
ii The p.d. will remain at about 300 V (since there is now plenty of time for charge to flow and maintain the p.d. at 300 V).
f*i* For slow vibrations, there is always time for charge to flow and the p.d. of 300 V to be maintained; therefore the p.d. across BC varies little for 50 Hz vibrations but may vary much more for 10 kHz vibrations.
ii The time constant could be increased (for example, by decreasing d and thereby increasing C) so that even 50 Hz appeared to be a fairly 'high' frequency.

19a $4\,m\,s^{-1}$
b $4\,m\,s^{-1}$
c*i* 8 J *ii* 8 J *iii* 640 J
d Approximately 1 m apart for 10 J intervals of energy; 100 m apart for 1000 J intervals.
e Contour lines, as on a map, joining points at the same potential energy.

20a*i* 50 000 J *ii* 200 000 J
iii −50 000 J (*i.e.*, it *loses* P.E.)
b*i* 50 J *ii* 200 J *iii* −50 J
c*i* $50\,J\,kg^{-1}$ *ii* $200\,J\,kg^{-1}$
iii $-50\,J\,kg^{-1}$
d*i* $150\,J\,kg^{-1}$ *ii* $-100\,J\,kg^{-1}$
e 120 000 J

21a 10 m
b*i* 50 N *ii* 50 N
c*i* 1500 J *ii* 500 J
d 1000 J
e*i* $28\,m\,s^{-1}$ *ii* $32\,m\,s^{-1}$

22a 101.5 mm
b $6.66 \times 10^{-11}\,N\,m^2\,kg^{-2}$

23a The Earth's gravitational pull slows it down since it acts almost in the opposite direction to the motion.
b $7.65 \times 10^{-3}\,m\,s^{-2}$
c $7.65 \times 10^{-3}\,N\,kg^{-1}$
d $7.9 \times 10^{-3}\,N\,kg^{-1}$
e $2.2 \times 10^{-4}\,N\,kg^{-1}$. Its contribution is beginning to become significant, although still small (≈3 % of the Earth's field strength).

24a

Mean distance $r/10^6$ m	Mean acceleration $g/\mathrm{m\,s^{-2}}$
27.7	−0.453
55.4	−0.122
96.5	−0.042
170.4	−0.013

Table Q6

b Numerical values of the gravitational field strength are identical to those of the acceleration; units are N kg^{-1}.
c See figure Q60.

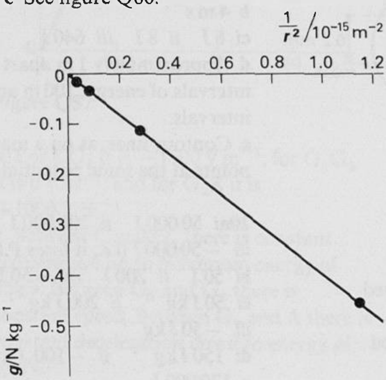


Figure Q60

25a One step. Assuming that g is constant over such a large interval leads to a poor estimate of the area under the graph.
b 1000 steps. This yields the best estimate of ΔV_g.
c One could use an even smaller step size.
d The program would take a very long time to run.
e 2 %
f Nearest the Earth, since this is where g varies most.

26a See figure Q61.

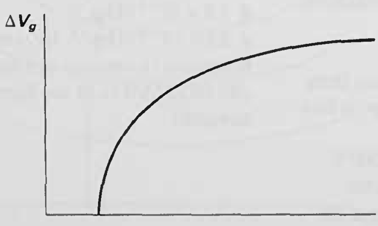


Figure Q61

b*i* 42.5×10^9 J
ii 10.6×10^9 J
iii 2×10^6 J
c 62.5 MJ
d*i* It will reach a distance of about 32×10^6 m from the centre of the Earth before falling back towards it.
ii It will escape completely from the Earth, its kinetic energy tending towards 7.5 MJ when it is a long distance from the Earth.
iii It will just escape from the Earth (assuming no other forces act on it), its kinetic energy tending towards zero as its separation from Earth increases.

27a*i* See figure Q62.

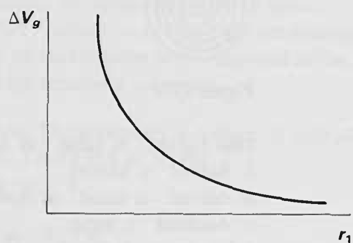


Figure Q62

ii The field strength is decreasing as r_1 increases, so the energy required to move one kilogram a given distance also decreases.
iii Students' suggestions might include $\Delta V_g \propto 1/r_1$; $\Delta V_g \propto 1/r_1{}^2$; $\Delta V_g \propto e^{-r_1}$
But note that whereas $\Delta V_g = 0$ when $r_1 = 50 \times 10^6$ m, ΔV_g will never become zero in any of the above relationships, however large r_1 becomes.
b*i* See figure Q63.

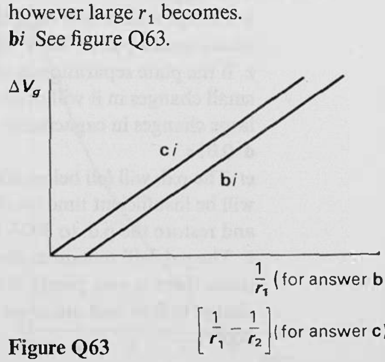


Figure Q63

ii Gradient $\approx 4.0 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$.

iii Intercept on $\frac{1}{r_1}$ axis is very close to the value of $\frac{1}{r_2}$, *i.e.*, $2 \times 10^{-8}\,\mathrm{m^{-1}}$.

iv $\Delta V_g = 4 \times 10^{14} \times \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$.

c*i* See figure Q63.

d It would have had the same gradient but a smaller intercept.

28a Zero

b No

c ΔV_g now represents the amount of energy required to remove 1 kilogram to an infinite distance from the Earth, or 'as far away as one would wish'.

d Gravitational potential energy is lost.

e Negative

f $-62.5 \times 10^6\,\mathrm{J\,kg^{-1}}$

g See figure Q64.

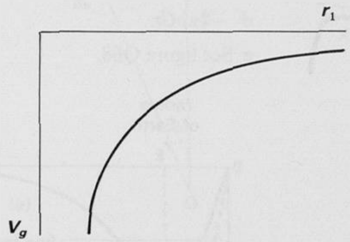


Figure Q64

29a $\Delta V_g \approx \frac{4 \times 10^{14}}{r_1}$

b $V_g \approx -\frac{4 \times 10^{14}}{r_1}$

c $GM = 3.98 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$. The two expressions are equal.

d $63 \times 10^6\,\mathrm{J\,kg^{-1}}$, using the approximate value for GM.
$62.5 \times 10^6\,\mathrm{J\,kg^{-1}}$, using the more accurate value.

30a They are equal in magnitude and opposite in sign, assuming that no other forces act.

b $-34.41 \times 10^6\,\mathrm{J\,kg^{-1}}$

c $34.41 \times 10^6\,\mathrm{J\,kg^{-1}}$

d Zero.

e $35.33 \times 10^6\,\mathrm{J\,kg^{-1}}$

f See figure Q65.

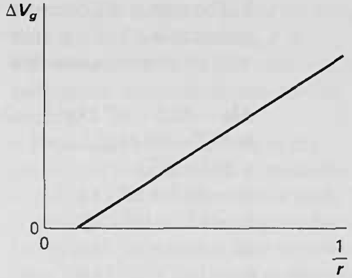


Figure Q65

g The gradient is very close to $3.98 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$, the value of GM_E.

h The intercept is approximately 2×10^{-9}, giving a value of r_0 of 5×10^8 m.

i Zero.

j $\Delta V_g = \frac{GM_E}{r}$

31a 10^5 J

b 10 N. This is approximately the force at the Earth's surface, though it decreases with height.

c 61.5×10^6 J

d Because the force is much less than 10 N for most of the distance.

e 62.5×10^6 J

f $11\,200\,\mathrm{m\,s^{-1}}$

g The energy needed is $\frac{GMm}{r} = \frac{1}{2}mv^2$.

m cancels so a larger mass will require the same *velocity*, though of course it has greater *energy*.

h See figure Q66.

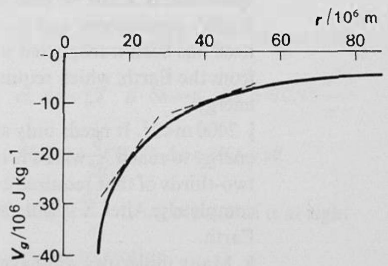


Figure Q66

i 1.0 N kg^{-1}; 0.25 N kg^{-1}.
j They represent the magnitudes of the field strengths at those points.
k The ratio is 4:1. Since the ratio of the distances is 1:2, this is quite consistent with an inverse-square field.

32a -62.5×10^6 J kg^{-1}; -2.8×10^6 J kg^{-1}
b 59.7×10^6 J kg^{-1}
c 597×10^9 J
d*i* -62.5×10^6 J kg^{-1}
ii -4.02×10^6 J kg^{-1}
iii -1.36×10^6 J kg^{-1}
iv -1.32×10^6 J kg^{-1}
v -1.35×10^6 J kg^{-1}
vi -3.94×10^6 J kg^{-1}
e–g See figure Q67.
h 612×10^9 J

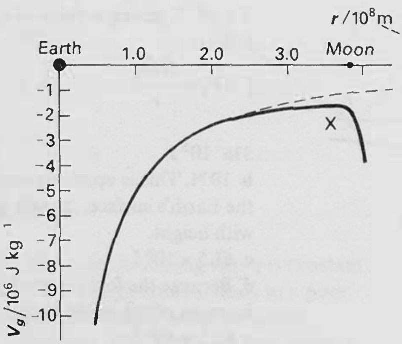


Figure Q67

i Extra energy will be gained in 'falling' from X towards the Moon. This must be transformed using reverse thrust. Also energy will have been 'lost' as the spacecraft is heated in passing through the Earth's atmosphere; and much of the fuel itself has been transported some distance from the Earth, which requires further energy.
j 2400 m s^{-1}. It needs only sufficient energy to reach X, which is only about two-thirds of that required to escape completely. After X it will 'fall' towards Earth.
k Many molecules will have speeds in excess of the escape velocity (which is the same for all masses) and so will escape. However many molecules might have been present, a good proportion will always escape, so any atmosphere will soon dwindle to zero.

33a $r_1\theta$
b $\pi r_1^2\theta^2$
c $\sigma\pi r_1^2\theta^2$
d,e $G\dfrac{(\sigma\pi r_1^2\theta^2)}{r_1^2} = G\sigma\pi\theta^2$
f The result is of the same magnitude.
g Zero. The contributions are equal in size but opposite in direction.
h Zero. There is no field anywhere inside the hollow sphere.
i No.

34a Zero
b $-\dfrac{GM}{r^2}$
c $\frac{4}{3}\pi r^3\rho$
d $-\frac{4}{3}\pi\rho Gr$
e See figure Q68.

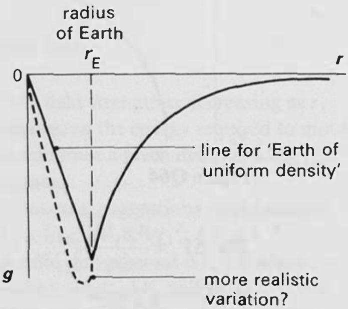


Figure Q68

35 $a = -\frac{4}{3}\pi\rho Gr$
b The equation for acceleration is of the form $a = -\omega^2 s$, where s represents the displacement from the centre and $\omega^2 = \frac{4}{3}\pi\rho G$, which is constant. Therefore the motion is simple harmonic.
c None.
d More.
e It is lower (*i.e.* more negative). The potential difference between the centre and the surface of the Earth can be

calculated by finding the area under the g against r graph (*e.g.*, the answer to question **34e**) between these two points. The potential at the centre is lower than that at the surface by this amount.

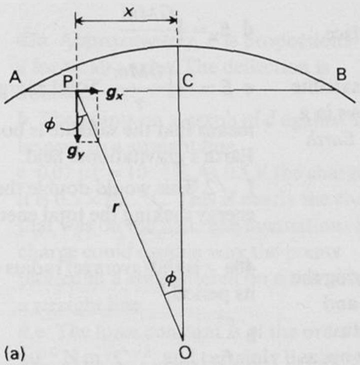
f $\sin\phi = \frac{\mathrm{PC}}{\mathrm{OP}} = \frac{x}{r}$

Component of g towards C is $g \sin\phi = g\frac{x}{r}$

$= -\frac{4}{3}\pi\rho G r \frac{x}{r}$

$= -\frac{4}{3}\pi\rho G x.$

So acceleration $\propto$ $-$displacement from centre $\Rightarrow$ the motion is S.H.M.



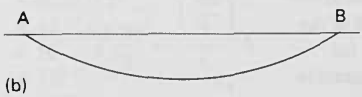


Figure Q69

g None.
h See figure Q69(b). This would be the kind of slope in which one would expect an object to perform S.H.M. It traces out the route between A and B which requires the minimum energy. In designing an underground railway, engineers might take this into consideration, as a route like that above requires the least acceleration and braking between stations and therefore uses least fuel.

36a The ball continues in a straight line (towards Q) at constant velocity because no overall force acts on it.
b An overall force (the pull of the string) now acts on the ball causing it to accelerate towards O. This acceleration is sufficient to maintain circular motion about O.
c Explanations should refer to the centripetal force required to make the contents of the dryer move in a circle. This is provided for the clothes by the drum but not for the water (at least not where there are holes in the drum) so that water which has already started moving (being carried along with the clothes by friction) continues in a straight line making its way out of the drum at a tangent. (Viewed from the inside the effect is of water being 'flung outwards'.)

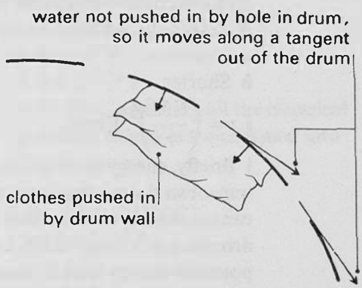


Figure Q70

37a*i* $\Delta t = \frac{1}{2}T$ *ii* $\Delta v = 2v$

iii $a = \frac{2v}{\frac{1}{2}T} = \frac{4v}{T}$ *iv* $T = \frac{2\pi r}{v}$

v $a = \frac{4}{2\pi} \times \frac{v^2}{r} \approx 0.64\frac{v^2}{r}$

b*i* $\Delta t = \frac{1}{4}T$ *ii* $\Delta v = \sqrt{2}v$ *iii* $a \approx 0.90\frac{v^2}{r}$

c*i* $\Delta t = \frac{1}{6}T$ *ii* $\Delta v = v$ *iii* $a \approx 0.95\frac{v^2}{r}$

d*i* $\Delta t = \frac{\theta}{2\pi}T$ *ii* $\Delta v \approx \mathrm{PQ} = v\theta$

iii $a = \frac{v^2}{r}$ *iv* Acceleration is at right angles to motion.

e Centripetal acceleration $= \frac{v^2}{r}$

f Centripetal force, $F = \dfrac{mv^2}{r}$

g A negative sign, $F = -\dfrac{mv^2}{r}$

38a $\dfrac{2\pi r}{T}$

b $-\dfrac{v^2}{r}$

c $-\dfrac{4\pi^2}{T^2}r$

d $-\dfrac{GM}{r^2}$

e $\left(\dfrac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$. Only one particular value of r gives the correct period.

f 36×10^6 m above the Earth's surface. $53 \times 10^6\,\mathrm{J\,kg^{-1}}$.

g Because nowhere else could the satellite stay directly above a point and move in a circular orbit *with the centre of the Earth as its centre.*

h Shorter.

i $v = \left(\dfrac{GM}{r}\right)^{\frac{1}{2}}$. Faster.

j Briefly, energy is transformed during the impact so the satellite loses energy and cannot remain in its present orbit. In dropping to a lower orbit, it speeds up, as potential energy is now transformed into kinetic.

k A satellite in 'geostationary' orbit above the Equator can 'see' nearly a third of the Earth's surface and so can be used for telecommunications over a very extensive area. However, with nearly 200 communications satellites in this orbit, space is becoming a little crowded, especially in the popular region over the Atlantic. (Signals from nearby satellites must be modified to prevent 'interference'.)

Satellites in low orbits lose energy quickly as they encounter the upper levels of the Earth's atmosphere. Sometimes their short life is an acceptable price to pay for the better 'view' they afford, especially if they are designed for monitoring or military purposes. The more stable orbits further out tend to accommodate satellites serving more civilian purposes such as weather forecasting. Many satellites engaged in scientific research have extremely eccentric elliptical orbits which enable them to penetrate wide regions of the magnetosphere. All in all the first 25 years of space exploration have seen over 3000 satellites go into orbit.

39a $E_p = -\dfrac{GMm}{r}$

b $-\dfrac{mv^2}{r}$

c $-\dfrac{GMm}{r^2}$

d $E_k = \frac{1}{2}\dfrac{GMm}{r}$

e $E = -\frac{1}{2}\dfrac{GMm}{r}$. Total energy negative means that the satellite is bound in the Earth's gravitational field.

f $\sqrt{2}$. This would double the kinetic energy making the total energy zero.

40a r is the (average) radius of the orbit, T its period.

b $\dfrac{v^2}{r}$

c $\dfrac{2\pi r}{T}$

d $\dfrac{\left(\dfrac{2\pi r}{T}\right)^2}{r} = \dfrac{4\pi^2 r}{T^2}$

e $\dfrac{r^3}{T^2}$ is constant so the acceleration is proportional to $\dfrac{1}{r^2}$.

f The force is also inverse-square.

41a Twice the distance.

b Twice the height.

c The same.

d The same.

e Four times the area.

f One-quarter.

g One-ninth.

h It is a constant (called the 'flux' of the light).

i In a parallel beam of light both intensity and area remain constant with distance and so does the flux.

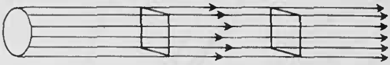


Figure Q71

But wherever light converges to or diverges from a focal point, the change in intensity with distance must be inverse-square if flux is to remain constant.

The 'conservation of flux' is a key concept in more advanced physics and the idea also applies to another type of field – magnetism.

42a Approximately, F is proportional to d for small angles. The deflection is doubled.
b The points on a graph of d against $1/r^2$ lie near to a straight line.
c $0.01\ \mu F = 10^{-8}$ F. At 0.5 V the charge on it is 0.5×10^{-8} C. This is nearly the charge that was on the ball. The fluctuations in charge could explain why the points plotted in **b** are scattered on either side of a straight line.
d, e The force constant is of the order of $10^{10}\ \mathrm{N\,m^2\,C^{-2}}$, and certainly lies between 5×10^9 and $5 \times 10^{10}\ \mathrm{N\,m^2\,C^{-2}}$.

43a 10^{-4} N
b 10^{-9} C
c 10^{-11} A
d 10^3 V
e $10^{14}\ \Omega$

44a See figure Q72.

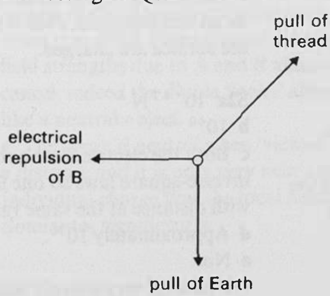


Figure Q72

b 6×10^{-5} N
c 8×10^{-9} C

45a $\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$
b $2r$
c $4Q$
d $\dfrac{1}{4\pi\varepsilon_0}\dfrac{4Q}{(2r)^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$
e*i* Upwards.
ii Small.
iii Approximately uniform.
f Approximately inverse-square.

46a 1.5×10^5 V
b 2.5×10^{-6} C
c $1.0 \times 10^6\ \mathrm{V\,m^{-1}}$
d 0.3 m, assuming a roughly uniform field between the sphere and the hand.

47a $3 \times 10^{20}\ \mathrm{N\,C^{-1}}$
b 1×10^2 N
c 7×10^6 V
d 2×10^{-12} J
e 12 MeV, assuming all the electrical potential energy is transformed into kinetic energy.

48a $5.8 \times 10^{11}\ \mathrm{N\,C^{-1}}$
b 9.2×10^{-8} N
c 28.8 V
d -28.8 eV
e Zero.
f Zero.
g 57.6 V
h -57.6 eV
i 14.4 eV
j -43.2 eV
k The ion has negative potential energy and so is a bound system; the two protons without the electron would fly apart. This very simple view of a 'covalent' bond has the electron 'shared' by both protons. It is not a complete view, as the electron has kinetic energy as well, for instance, but it gives results of the right order of magnitude.

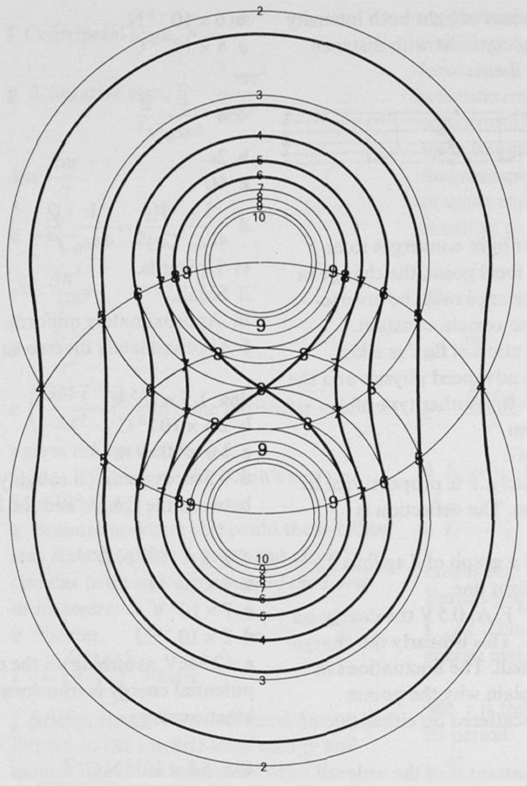


Figure Q73

49a Students may produce a pattern something like that shown in figure Q73.
b See figure Q74.
c The field patterns should be similar to those obtained between two 'point' electrodes in experiment E3 with semolina particles.

50a*i* 270 V *ii* 150 mm
iii 1.5×10^{-8} C
b Less than 900 V. Mutual repulsion between the charges causes them to be displaced somewhat toward the outer sides of the spheres, thereby increasing the effective separation of the charges and thus reducing the potential.

51a 900 V
b 1200 V at X, 1080 V at Y; by simple addition of the potentials due to A and B.
c Yes, because there is a potential gradient across it.
d Charge would flow towards Y until the potential gradient fell to zero.
e No.
f The surface of a conductor must always be an equipotential, even if some parts of the surface are charged.

52a 10^{-47} N
b 10^{40}
c Both the electric force and gravity obey inverse-square laws so one force falls off with distance at the same rate as the other.
d Approximately 10^{36}.
e No.
f It falls off very sharply indeed and is quite insignificant outside the nucleus.

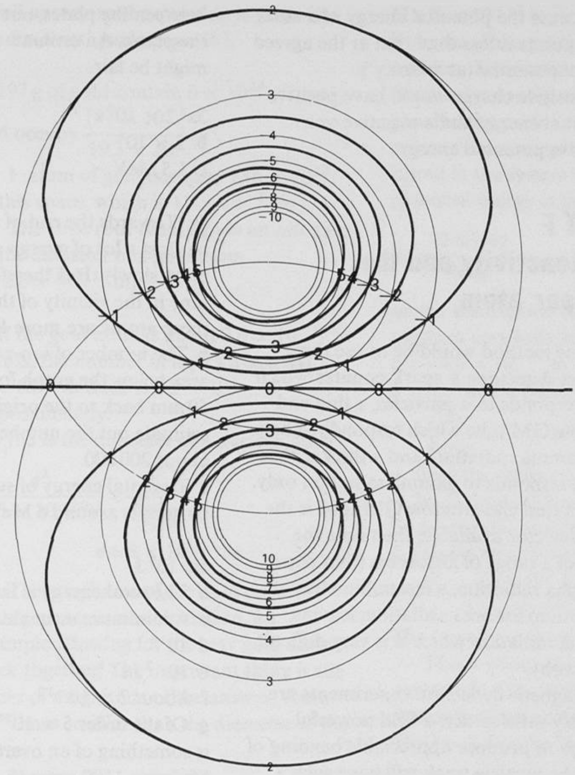


Figure Q74

53a $1.4 \times 10^{11}\,\mathrm{N\,C^{-1}}$ (=1 unit)
b $\frac{1}{100}$ unit
c $-\frac{1}{9}$ unit at C; $-\frac{1}{144}$ unit at D
d $\frac{8}{9}$ unit at C; $\frac{44}{14\,400}$ unit at D
e $\frac{11}{3200} \approx \frac{1}{290}$
f More rapidly (particularly if one considers that the distance of D from the middle of the dipole is only just over five times that of C). From a good distance the field strengths due to A and B almost cancel; indeed the dipole 'looks' almost like a neutral object.
g The array is neutral when 'viewed' from a distance and it is only very near any individual charge that the local field dominates significantly.

54 Lack of space does not permit a full answer here. (Examination question.)

55a Electrical/gravitational field strength is the force acting on unit charge/mass.
b By taking the negative of the gradient of the graph.
c By computing the area under the graph between the two points.
d By multiplying by the mass or charge.
e The negative of the force.
f The force is always attractive, so the force vector is directed *inwards*, in the opposite direction to the separation or displacement vector.
g This is taken care of by the charges which may be positive or negative, giving both attractive (negative) and repulsive (positive) forces.

h Because the potential energy of a mass at all points is less than that at the agreed zero of potential (at 'infinity').
i A positive charge would have positive potential energy and a negative one negative potential energy.

UNIT F
Radioactivity and the nuclear atom

1a One method would be to use three distinct detectors: a spark counter which only responds to α-particles, a thin end-window GM tube which responds to beta and gamma radiation, and a thicker tube which responds to gamma radiation only.

If a thin end-window GM tube is the only detector available, then note the effect of a range of absorbers (thin paper for alpha radiation, a few millimetres of aluminium for beta radiation, leaving gamma radiation which it is very difficult to absorb).

Magnetic deflection experiments are not very satisfactory; a field powerful enough to produce appreciable bending of an alpha particle track will have such a major effect on the track of the beta particles that they will be sent back to the source.
b Some energy is transferred to atoms during elastic collisions and some in exciting and in ionizing atoms.
c*i* The magnetic field is too weak to produce measurable bending of the path of the alpha particles.
ii Use a very strong magnetic field and work *in vacuo*.

2a $10^6\,\mathrm{V\,m^{-1}}$
b $1.6 \times 10^{-13}\,\mathrm{N}$
c $1.6 \times 10^{16}\,\mathrm{m\,s^{-2}}$
d $3.3 \times 10^{-10}\,\mathrm{s}$
e $9 \times 10^{-4}\,\mathrm{m} \approx 1\,\mathrm{mm}$
f If $L = l$, the deflection y will be more than twice as big as s, because the straight path from the edge of the plates to the screen cuts the undeflected path through the plates (if projected back) somewhere between the plates, not at the far end of the plates. An estimate of 3 or 4 mm for y *might be fair.*

3a $2 \times 10^8\,\mathrm{s^{-1}}$
b 5×10^4
c 1.5 MeV

4a Towards the end of a track the particle has lost a lot of energy and is travelling more slowly. It is therefore spending more time in the vicinity of the atoms of the air; these atoms are more likely to be ionized.
b The number of ion-pairs is given by the area below the graph from the distance 50 mm back to the origin. One rough estimate put the number of ion pairs at about 200 000.
c The total energy of such an alpha particle is around 6 MeV.

5a $10^9\,\mathrm{J}$
b The total energy is larger, $5 \times 10^9\,\mathrm{J}$.
c A minimum estimate.
d 10^{16}
e $10^{-2}\,\mathrm{J\,s^{-1}}$
f About 2×10^{-12}.
g Of the order $5 \times 10^{11}\,\mathrm{s}$. Note that this is something of an overestimate, the half-life being 1600 years ($5 \times 10^{10}\,\mathrm{s}$). But the errors involved in the approximations made could easily amount to a factor of ten.

6a $8.7 \times 10^{-19}\,\mathrm{J}$
b $8 \times 10^{-13}\,\mathrm{J}$
c $2.4 \times 10^{12}\,\mathrm{J}$
d The energy released in the radioactive decay of radium is greater than that released on oxidation by a factor of over a million.
e Even with 1 mole of radium (226 g – a very large sample), the energy is dissipated over thousands of years. In the first 1600 years, 0.5 mole will have decayed.

7 Students' laboratory notes for this Unit are prefaced by a note about the precautions which should be observed when handling radioactive substances. Make sure that students read this.

A full answer to the question is given in the *Students' guide* page 485.

8 197 g of gold contain 6×10^{23} atoms and occupy $\dfrac{197 \times 10^{-3}}{19.3 \times 10^{3}}\,\mathrm{m}^3$.

1 atom of gold occupies $1/(6 \times 10^{23})$ of this space, which is $17 \times 10^{-30}\,\mathrm{m}^3$.

The cube root of this gives an estimate of the diameter of a gold atom ($\approx 2.6 \times 10^{-10}$ m).

9 If the gold atom is about 2.6×10^{-10} m across, the number of layers n in a foil 6×10^{-7} m thick is 2.3×10^3 approximately.

If d is the diameter of a nucleus, then

$$\frac{d^2}{(2.6 \times 10^{-10})^2} \approx \frac{1}{8000n}$$

$$d \approx 6 \times 10^{-14}\,\mathrm{m}$$

The roughness of the data does not justify an accurate calculation, for example allowing for the way gold atoms pack together. The important thing is the order of magnitude of the answer, some 10^4 times smaller than the diameter of the atom.

10a The incident particle will stop. The other particle moves on in the same direction at $10^6\,\mathrm{m\,s^{-1}}$.

b Momentum is conserved in any collision. After the collision the velocity of the alpha particle is $0.6 \times 10^6\,\mathrm{m\,s^{-1}}$ in the original direction; the velocity of the proton is $1.6 \times 10^6\,\mathrm{m\,s^{-1}}$, also in the original direction.

The mathematics produces another solution, in which the alpha particle goes straight on without change of speed and the proton stays at rest. That is, the alpha particle misses the proton! The algebra wasn't told that they must collide!

c The gold nucleus, mass number 197, is much more massive than the alpha particle and its velocity after impact can only be very small. Even if the momentum change of the alpha particle and the nucleus is as large as possible ($2mv$), the *velocity* of the nucleus is still small.

11a 8×10^{-13} J

b As the alpha particle approaches the nucleus, this kinetic energy is converted to electrical potential energy. When the speed of the particle is zero, this conversion is complete and the energy stored in the system is 8×10^{-13} J. The potential energy is given by

$$E_\mathrm{p} = \frac{(2e)(79e)}{4\pi\varepsilon_0 r}$$

on the assumption that the gold nucleus acquires very little kinetic energy.

c $r = 4.6 \times 10^{-14}$ m

d The electric potential at this distance is

$$V = \frac{79e}{4\pi\varepsilon_0 r} = 2.5 \times 10^6\,\mathrm{V}.$$

e Since the alpha particle carries charge $+2e$, its original kinetic energy in eV will be double this number, *i.e.*, 5.0×10^6 MeV.

f The electric field at this distance

$$E = \frac{V}{r} = \frac{2.5 \times 10^6\,\mathrm{V}}{4.6 \times 10^{-14}\,\mathrm{m}} = 5.4 \times 10^{19}\,\mathrm{V\,m^{-1}}.$$

The force on a charge of $2e$ is about 17 N.

This is about the same as the weight of a mass of 1.5 kg – an enormous force to find exerted on a single atomic particle. The book was the best guess.

12a An attractive force which decreases with distance from A.

b A repulsive force which decreases with distance from A.

c No significant force at the distances shown.

d A repulsive force which suddenly affects P at a small distance from A.

13a The alpha particles are being repelled by the nucleus N.

b The force on B will be one-quarter of that on A.

c The forces are along the lines NA and NB.

d See figure Q75.

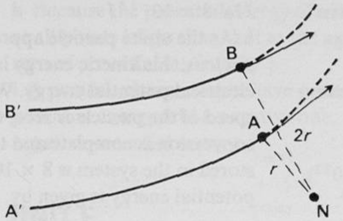


Figure Q75

e The velocity at A will be less than that at A′ as some of the kinetic energy of the alpha particle has been converted to potential energy.
f See figure Q76. At X the potential energy will be a maximum and the speed of the alpha particle a minimum.

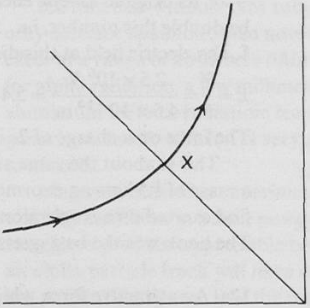


Figure Q76

g The potential energy at B is half that at A, so that the speed at B will be greater than that at A. Less kinetic energy has been transformed to potential energy.
h No. The force between the particles is always along the line joining them and so there is no force deflecting the alpha particles out of the original plane surface.

14a*i* Not very good. It describes roughly what happens, but may give the wrong impression about why. The particles come in by chance, and if the chance of arrival stays the same, over a long time there will be a steady average rate.
ii We tried to make this a correct description.
iii Wrong. The time between arrivals may have any value, but not all intervals are equally likely. The average interval is most likely; much longer or shorter intervals are less likely.
b To say that in the kinetic model the molecules of a gas have random motion implies that a molecule may be moving in any direction at any instant (not one of which is preferred). Molecular speeds are distributed randomly about a mean value; this mean value is the most likely to be found whilst far higher or far smaller speeds are unlikely.

15a In the same short time, say one second, more sodium atoms than strontium atoms will disintegrate, because the sodium has a shorter half-life.
b See figure Q77.

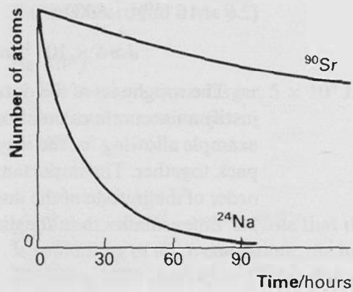


Figure Q77

c 27 years is about 1.6×10^4 times longer than 15 hours, so the sodium will initially be about this many times more active than the strontium. In 15 hours the activity of the sodium will fall to one-half; in twice as long to one-quarter; and so on. It will have fallen by a factor 1.6×10^4 after n periods of 15 hours, where

$2^n = 1.6 \times 10^4$, and therefore

$n \approx 14$

Fourteen times fifteen hours is about 9 days; much less than the 27 year half-life of the strontium, so the strontium activity will not have diminished very much at all. So 9 days may be a fair estimate of the time taken for the two activities to become equal.

16 A common answer is that a radioactive sample has an infinite lifetime 'in theory but not in practice'. Although not absolutely wrong, this seems to us to be a bit feeble.

The smooth mathematical model of exponential decay does not exactly fit the behaviour of radioactive atoms. As one finds by experiment, the rate of decay of a sample fluctuates considerably. The average rate of decay is close to the value to be expected from the mathematical model, but need not be equal to it.

When the sample is reduced to only a few atoms, the smooth exponential model is a poor fit. Yet it is not enough to say that 'the theory breaks down'. The smooth exponential decay is a consequence of supposing that there is a constant chance of decay for each atom in each time interval, and also that there are very many atoms. When there are not many atoms left, the smooth decay is not to be expected. But so far as is known, the chance of decay remains constant. When one atom remains, that atom may decay in the next second, or in the next hundred years. It is possible to say how long it will last on average: that is, how long one atom will last in many trials. But it is not possible to say how long one particular atom will last.

17 The background count is likely to be 119/4 counts per minute $\approx$ 30 counts per minute. After a further 30 s the total count is likely to be 135.

18a The background (30 counts per minute) should be deducted from each of the counts per minute tabulated.
b The half-life is about 80 minutes.
c The count rate would not change significantly if the counting were continued for a longer period; very little of the radioactive substance remains after 8 hours, anyway.
d No; random fluctuations within the background count are probably responsible for the increase.
e The experiment could be repeated and readings could be taken at intervals closer than one hour during the first few hours.

19a Both are exponential. The lengths of the straws decrease in a fixed proportion.
b Plot the natural logarithm of y against x; or check whether y changes by a constant factor as x increases in equal steps.

20a The data do fit the model; the constant ratio over the first six hours averages 0.61, and the graph of ln (activity) against time is a straight line.
b Since $N = N_0 e^{-\lambda t}$, $\ln N = \ln N_0 - \lambda t$, where λ is the gradient of the graph of ln (activity) against time and λ is the decay constant. The slope and therefore the decay constant is $-0.0084\ \text{min}^{-1}$.
Since $t_{\frac{1}{2}} = \dfrac{0.693}{\lambda}$, $t_{\frac{1}{2}}$ is 83 minutes.

21a Although the graph of ln (capacity in GW) against time does not follow a straight line, there are periods when it does so to a good accuracy. From 1951 to 1958 it was very nearly straight (and the growth exponential), and also from 1964 to 1970.
b The growth constant diminished in about 1962. During the 1970s growth ceased and the capacity remained approximately constant.
c The 'doubling times' were
i about 10 years and
ii about 11 years.

22 A logarithmic plot shows that from 1956 to 1962 the rise is closely exponential.

23 The slope of the plot of ln (population) against time has increased since the period 1650–1750, when the growth was closely exponential. (See figure Q78.)

24a The change in the number of families with video recorders.
b The rate at which the number of such families changes.

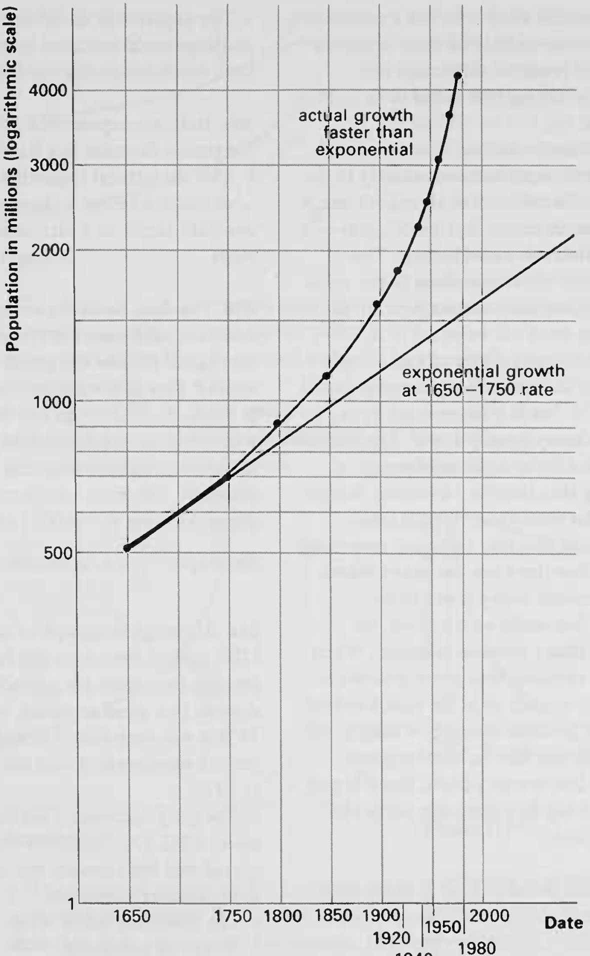


Figure Q78

c The model $\Delta N/\Delta t = kN$ suggests that the rate at which the number of families with video recorders grows is directly proportional to the number of families already having video recorders. This would give an exponential growth. In real life this could be the result of 'keeping up with the Joneses'.
d There could be a limit on the supply of money available, costs might rise unexpectedly, or the supply of the equipment might become limited. Such changes would reduce the value of k after a time.
e The existence of television could increase the value of k.

f See figure Q79.

25a Press, in order, the keys 2, x^y, 7, = (or the keys 2, logarithm, ×, 7, =, 10^x; or multiply the logarithm of 2 by 7 and find the number whose logarithm is this product).
b $N = 1.414$
c, d The graphs start at $N = 1$ because, at $t = 0$, $N = a^0 = 1$. (See figure Q80).
e At each step of 0.2 in t, the value of N is multiplied by the same factor, which is $2^{0.2}$, or 1.15.
f $2^{0.2}$ is 1.15; $2^{3.2}$ is 9.20.

g In the series a^t, a^{2t}, a^{3t}, etc., each exponent is equal to the previous one with the addition of t. Adding t to the exponent of a means multiplying by a^t; similarly, adding x to the exponent of a means multiplying by a^x.

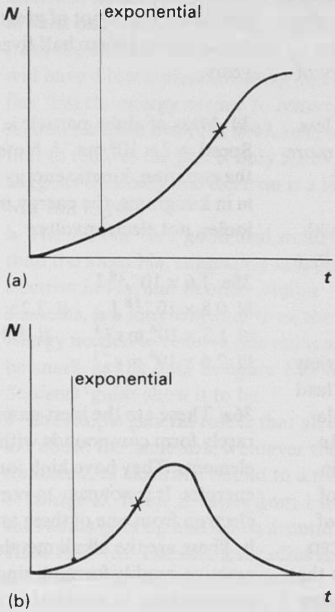


Figure Q79

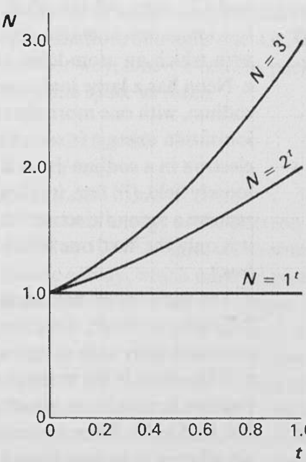


Figure Q80
Plots of $N = 1^t$, $N = 2^t$, and $N = 3^t$.

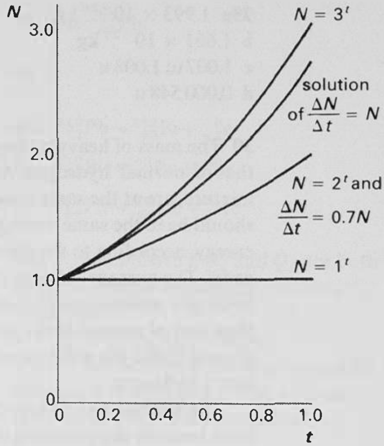


Figure Q81

26a, b See figure Q81.
a ΔN is a little larger in the second 0.1 s interval because the starting value, N, is now larger (1.1 instead of 1.0).
b Growth is slower because the constant ratio is now 0.7, not 1.0.
c If $N = 2^t$, then $N = 2$ at $t = 1$.
d $N = a$ at $t = 1$.
e For $\Delta N/\Delta t = (1.0)N$, $N \approx 2.7$ at $t = 1$, so $a \approx 2.7$. For $\Delta N/\Delta t = (0.7)N$, $N \approx 2.0$ at $t = 1$, so $a \approx 2.0$.

27a $e^{0.7} = 2.01 \approx 2.0$.
b Since $(e^{0.7})^t = e^{0.7t}$, the best speculation is $a = e^k$.

28a The steps are:
i Calculate N, the number of ^{40}K atoms in 0.6×10^{-6} g; use $dN/dt = -\lambda N$ to calculate λ.
ii Use $t_{\frac{1}{2}} = 0.693/\lambda$ to calculate $t_{\frac{1}{2}}$ (in *seconds*).
b Total mass of ^{40}K originally present was 4.8×10^{-6} g. The amount remaining, 0.6×10^{-6} g, is 1/8 of this. $8 = 2^3$, so it takes three half-lives for the amount of ^{40}K to decrease to 1/8 of its initial value.
c Background count is likely to be very much greater than $0.16\,s^{-1}$.

29a 1.993×10^{-26} kg
b 1.661×10^{-27} kg
c 1.007 u; 1.008 u
d 0.000 548 u

30 The mass of heavy hydrogen is twice that of 'normal' hydrogen. As the two, in a mixture, are at the same temperature, they should have the same average kinetic energy, according to the kinetic theory of gases. The average velocity of heavy hydrogen molecules will therefore be less than that of normal hydrogen. Thus, more normal hydrogen will evaporate than heavy hydrogen.

The method will be less effective with neon because the velocities will differ by a smaller factor.

31 The data suggest that, while the average molar mass of lead from a variety of sources is around 207 g mol^{-1}, the lead in particular minerals may have a molar mass which is more or less than this. In particular, it is possible that the lead in thorite comes in part from the decay of thorium. A glance at the decay series of ^{232}Th (see REVISED NUFFIELD ADVANCED SCIENCE *Book of data* page 29) shows that the series ends with ^{208}Pb. On the other hand, lead from the decay of ^{238}U is the isotope ^{206}Pb. It seems probable that the lead in the first four minerals mentioned comes at least in part from the decay of uranium and not from the decay of thorium to any appreciable extent. Pitchblende was the uranium mineral studied by Becquerel and by the Curies, who extracted radium from it.

32a One gram of carbon contains 0.5×10^{23} atoms, since 12 g contain 6×10^{23} atoms. Of these, about 0.5×10^{13} atoms are the isotope ^{14}C. In one second, 4×10^{-12} of the ^{14}C atoms will decay, so one may expect around 20 decays per second, an activity of 20 Bq.
b 11 000 years is close to two half-lives, so the activity might be around 5 Bq from one gram.
c The rate of decay is small, and the smaller it is, the harder it will be to measure the rate accurately unless one is prepared to measure it over a very long period. Even with counting times of about a day, the ^{14}C method is subject to pretty large errors for times above about two half-lives, and is not of great assistance much beyond three half-lives, say 20 000 years.

34 Mass of alpha particle $\approx 7 \times 10^{-27}$ kg. Speed $\approx 7 \times 10^{6}$ m s^{-1}. Note that to use the equation, kinetic energy $= \frac{1}{2}mv^2$, with m in kilograms, the energy must be in joules, not electronvolts.

35a 1.6×10^{-18} J
b*i* 0.8×10^{-18} J *ii* 3.2×10^{-18} J
c*i* 1.3×10^{6} m s^{-1} *ii* 1.9×10^{6} m s^{-1}
iii 2.6×10^{6} m s^{-1}

36a These are the inert gases, which rarely form compounds with other elements. They have high ionization energies. It is not easy to remove an electron from one of these atoms.
b These are the alkali metals, all very reactive, readily forming singly charged positive ions. In a crystal of sodium chloride, the sodium atoms are all ionized, and the crystal is a vast assembly of Na^+ and Cl^- ions. All the alkali metals have low ionization energies, indicating the ease with which an atom loses an electron.
c Neon has a large ionization energy; sodium, with one more electron, has a low ionization energy; it seems that the 'last' electron in a sodium atom is rather loosely held. (In fact, it takes 47 eV to remove a second electron from sodium, so it is only the 'last' one which is nearly free.)
d Yes, each comes first on the rise following a trough. They are members of another family with chemical similarities.
e If francium is the element one before radium, it must be an alkali metal, like Li, Na, K, Rb, Cs. Their ionization energies are all seen to be low, from 3 to 5 eV. The value drops slowly as one goes along the

list, so francium will probably have an ionization energy nearer the lower end of this range.

37a An electron a long way from a nucleus will be weakly bound to it, as electrical forces diminish with distance. If at least one electron is a long way out from the nucleus, the atom will be big, and will have a low ionization energy. (The fact that the energy needed to remove a second electron from Na is 47.3 eV, while that to remove the first is only 5.1 eV, suggests that only one electron is a long way out in Na.)
b The ion Na^+ is a good deal smaller than the atom Na, suggesting that just one electron in Na, and in other similar elements, is a long way out. If so, the energy needed to remove this electron will be small, as the data in figure F31 of the *Students' guide* show it to be.
c The simple general rule is that all atoms are about the same size, whatever the number Z of electrons bound to a nucleus of charge Z. While Z varies from 1 to 90, the radius goes up and down around 1.5×10^{-10} m. The smallest radius is 0.3×10^{-10} m; the largest shown is 2.7×10^{-10} m, a factor of only 9. This is surprising. Despite the extra attraction of a strongly charged nucleus, the electrons of a massive atom occupy much the same space as those of a light one. Or, to put it another way, ninety electrons pack into the same space as three.

38a -2.18×10^{-18} J
b $-e^2/4\pi\varepsilon_0 r$
c 1.0×10^{-10} m
d The *total* energy of the electron is -2.18×10^{-18} J. Since it has some kinetic energy, which must be positive, the potential energy is more negative than -2.18×10^{-18}, *i.e.*, the electron is closer to the proton than this calculation suggests. (In fact, as will be seen in Unit L, 'Waves, particles, and atoms', the electron is not precisely located at one distance from the proton. But this calculation gives the correct order of magnitude for the proton–electron distance.)

39 X is $^{207}_{82}Pb$.

40 A

41a $^{212}_{82}Pb \rightarrow {}^{212}_{83}Bi + {}^{0}_{-1}e$
b $^{212}_{83}Bi \rightarrow {}^{212}_{84}Po + {}^{0}_{-1}e$
c $^{212}_{84}Po \rightarrow {}^{208}_{82}Pb + {}^{4}_{2}He$

42a F_e is repulsive if Q_1 and Q_2 are both positive or both negative; F_g is always attractive.
b $F_e/F_g = Q_1Q_2/4\pi\varepsilon_0\, Gm_1m_2$
c Both forces vary in the same way with distance (inverse-square law), so their ratio is the same at all distances.
d 1.2×10^{36}
e No; it is much smaller than the repulsive electrical force.
f Nuclei of neighbouring atoms would be attracted to each other: they would not stay about 10^{-10} m apart.

43a ^{12}C is the more stable.
b ^{2}H, ^{6}Li, ^{4}He, ^{235}U, ^{12}C, ^{208}Pb, ^{56}Fe.
c Helium is at a minimum in the curve. The average binding energy per nucleon is lower for some heavier nuclei (*e.g.*, ^{12}C, ^{16}O), but to form nuclei like ^{6}Li which are likely to be involved in intermediate steps in the build-up of heavier nuclei requires an input of energy.
d C, O, Fe.

Students can read more about the evolution of the elements in 'Our nuclear history' in the Reader *Particles, imaging, and nuclei.*

44a 1050 kJ or 1.05 MJ.
b 1.2×10^{-11} kg
c About one part in 4×10^{11}.

45a 0.001 kg (1 gram).
b 0.5 J (taking $k = 100\ \mathrm{N\,m^{-1}}$, and extension = 0.1 m).
c 6×10^{-18} kg
d 6×10^{-13} per cent.

46a $2m_p + 2m_n + 2m_e = 4.0330$ u.

The mass of an atom of $^{4}_{2}He$ is 0.0304 u *less* than the mass of its parts.

b*i* -4.54×10^{-12} J *ii* -28.4 MeV
c -7.1 MeV
(Note that because the information refers to a neutral helium atom – nucleus plus electrons – the binding energy calculated includes the binding energy of the electrons as well as that of the nucleons. However, since electrons are so much more weakly bound than nucleons, the value obtained is still a good estimate for the average binding energy per nucleon.)

UNIT G
Energy sources

1a*i* The potential energy of the mass is the work which would be done by the gravitational field if the mass were allowed to fall to some reference point (often the surface of the Earth). The mass needs the presence of the Earth in order to have potential energy, which is therefore a property of the Earth–mass system.
ii The gravitational field acting on the mass transfers energy to the generator by working. The mass loses energy and the generator gains energy (to an equal extent in an ideal system).
iii This means that a quantity of energy (15 J in this example) would be transformed if the generator were brought to rest.
b Boxes should be inserted at every stage of conversion. They all represent internal energy of the surroundings.
c The boxes should have solid boundaries since they represent continuing 'losses' to the system.
d It depends what you mean by the system. If this is taken to be the 'Universe' then the energy is constant, though clearly it is not so if the mass–generator–lamp arrangement is regarded as the system. In this latter case energy is lost from the system. Because the 'losses' are only losses in this limited sense, quotation marks are used.

2 The Sankey diagram is shown in figure Q82.

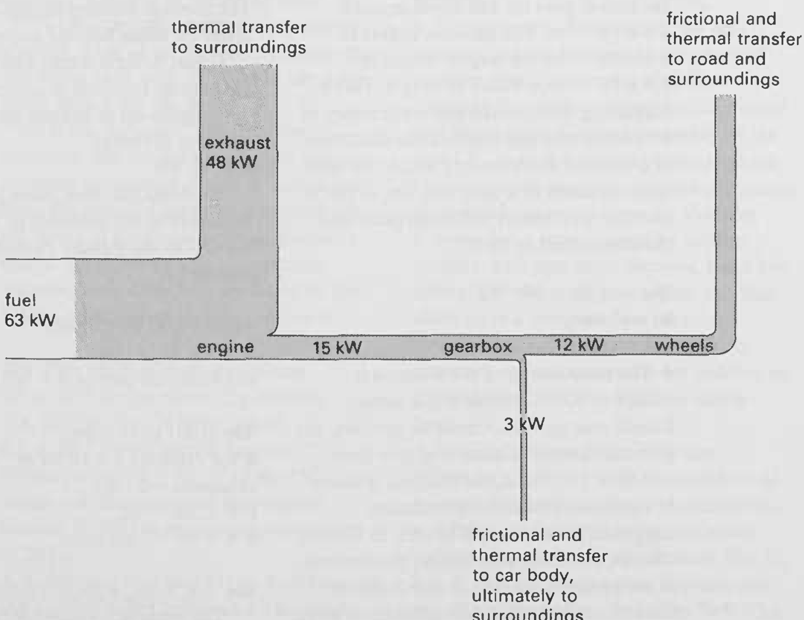


Figure Q82
Sankey diagram for a car.

3a 10–100 W depending on the activity and the length of time for which it is sustained.

b*i* 1–10 MJ approximately depending on the answer to **a**.

ii The cost of approximately one-third to three units. The minimal diet would be about $\frac{2}{3}$ kg of rice whose cost would be considerably more than that of the electricity. A more balanced diet would be even more expensive.

iii About 25 per cent (range 10–100 %).

iv About 115 W. Power from 500 people $\approx$ 60 kW. This seems a lot – but it is difficult to say whether it is enough to heat a theatre without other information. Heating would be required for raising the temperature to comfortable levels but thereafter the 'body heat' of the audience might be enough. The latter part of Section G3 deals with heating buildings.

c*i* The central parts of figure Q83 cannot easily be quantified. For example, the proportion of the energy 'contained' in food which the body can actually convert depends on the composition of the food.

ii Reducing food intake will apparently reduce all the subsequent energy flows, including that for building body parts, *i.e.*, slimming occurs. In fact this may not happen at all, or only in the short term, since the metabolic rate may have reduced. Similarly, increasing the quantity of work which is done without changing the food input should lead to slimming since less energy is available to contribute to building body material.

Again, reducing the need for energy to heat the body means that more may be available to contribute to building body material. Thus battery farms are maintained at high enough temperatures for the animals not to need to use as much energy as normal to maintain body temperatures, in order to maximize their rate of growth.

4a 3 6 MJ
b About 35 MJ.
c About 29 × 10^9 J.
d 3.6 × 10^{12} J
e 10^3 J
f About 250 kJ.
g 2.9 × 10^{20} J
h 1.6 × 10^{-11} J
i 1060 J

5a This use of the term ignores the fact that the appliances will have different power ratings and many will also be used infrequently.

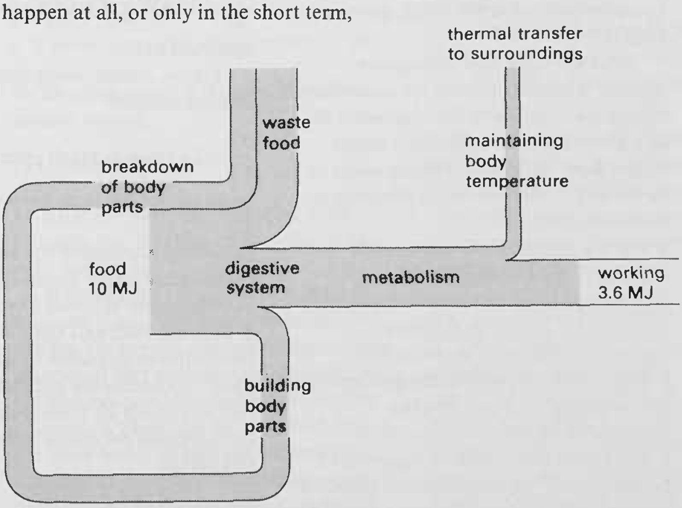


Figure Q83
Sankey diagram for a mammal.

b This version deals with part of the problem but does not take account of the time for which each appliance may be used.
c According to the Law of Conservation of Energy, energy cannot be 'consumed', if this is taken to mean 'used up'. But energy can be transferred from one part of the system (for example, the generating station) to another part (such as appliances) either by working or by thermal transfer.
d The value will usually be much bigger. This relates to the data in table G5, page 419 on the proportion of fuel used for space heating.

6a The fuel input (1983 figures in 10^9 therms) is 18.8 from coal, 1.9 from oil, 0.1 from gas, and 4.7 from nuclear and hydro. This last figure is conventionally taken to be the quantity of coal or oil which would produce the same quantity of electrical energy as is actually produced by nuclear or hydroelectric power stations. Thus the total input is 25.5.

The electrical output seems to be labelled as 7.5, but it is not clear whether the small unlabelled loss is included in this value or not. Adding the final energy use figures provides a cross-check, giving a total of 7.4.

Since the main loss from power stations is labelled as 17.4, the unlabelled loss seems to be about 0.6, and seems to be a distribution loss. Thus the actual output from the power stations seems to be about 8.1. The operating efficiency is therefore about 32 %.
b For the reason given in **a** it is not possible to say exactly, but the distribution losses are small – assumed to be 0.67×10^9 therms in **a**. Hence distribution efficiency is about 93 %.
c The overall efficiency is the product of the operating efficiency and the distribution efficiency – about 29 %.
d The total of the crude oil figures is 61 (in units of 10^9 therms). Some of this (29.6) is exported or bunkered, so 31.4 actually goes to refineries.

The output of refined fuel is (clockwise from the top) $(1.9+22.4+2.3+3.4+7.2) = 37.2$, of which 6.1 was imported already refined, and 2.3 represents losses. Hence the output from the refineries is 28.8 and the efficiency is $\frac{28.8}{31.4} \times 100 = 92\ \%$.

Note that the total input and output figures do not balance but the difference is slight (0.3).
e About 91 %.
f Roughly 3.5 times as much as the useful output.

7a Each kW h of energy delivered electrically to the final user can produce 1 kW h of heating. Table G1 in the *Students' guide* shows that gas boilers are about 75 % efficient. Hence $1\frac{1}{3}$ kW h of end-use energy are needed for each 1 kW h of heating. For equal heating costs electricity can be about 1.3 times as expensive as gas.
b 1 kW h of heating by gas requires $1\frac{1}{3}$ kW h of end-use energy and hence a little more than this of primary fuel (question **6e**). 1 kW h of heating by electricity requires about

$\frac{100}{32} = 3.1$ kW h of primary fuel

(question **6a**).

Hence, overall, using electricity for heating requires

$\frac{3.1}{1.3} = 2.4$ times as much primary fuel as using gas for the same purpose.

8a World population has been growing exponentially since about 1950 with a doubling-time of about 37 years.
b Between 1925 and 1981 World fuel consumption increased by a factor of about 7, whereas the population grew by a factor of about $2\frac{1}{2}$. Clearly the fuel consumption *per capita* increased during this period.

9a The range of values is such that the graphs can only be plotted over about 12 hours, from 0–12 or from 12–24 hours. Breeding rate is taken to be the number of new bacteria at the end of each hour, rather than the ratio of new bacteria to old ones (see figure Q84).

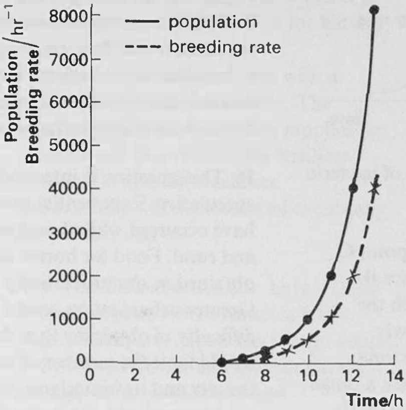


Figure Q84
Population and breeding rate of bacteria.

b One hour before it is full, *i.e.*, after 23 hours.
c Probably not until $\frac{1}{8}$ or $\frac{1}{4}$ of the nutrient is used up. This means at 21 or 22 hours, *i.e.*, with only two or three hours to go before all the resource is used.
d 1 jar; 2 jars.
e It would only give one more hour's life.

10 Doubling-time is 35 months, to the nearest month.

11 $t_D = 0.693/g$

12 Coal 226 years; oil, including shale, tarsands, etc., 57 years or 29 years for oil only; natural gas 53 years.

It is not clear in table G10 in the *Students' guide* whether the reserves of oil and gas are proved reserves or recoverable reserves. If they represent proved reserves, the lifetimes would be less for the smaller proportion which can be recovered. On the other hand, oil and gas discoveries continue to be made which will increase the lifetimes. The main point is, however, that the lifetimes are very short, even assuming zero growth.

13a Oil consumption increased by 8.7 per cent over ten years from 1972 to 1982. Assuming a growth rate of 0.87 % *per annum* gives a doubling-time of $0.693/0.0087 \approx 80$ years.

Similarly, the doubling-time for coal consumption is 27 years and for gas also 27 years.
b Taking the growth rate as an average of ten years' growth smooths out the variations which occur over a shorter period. If, however, the growth is very rapid (as in the case of nuclear power from 1972 to 1982) the average value over ten years is not the same as the year-on-year value which applies throughout the period.

Thus an annual growth rate of 0.87 % leads to growth in ten years by a factor of $e^{(0.0087 \times 10)} = 1.091$, rather than 1.087. Thus, using the 'average' growth rate overestimates the growth. The difference here is quite small – the annual growth rate required to give the correct 10-year growth is 0.83 %. However, for nuclear power the 10-year growth was 464 %. The annual growth rate required to produce this is about 15.4 % rather than the average of 46.4 %.
c Using 1982 figures the lifetimes of the fuels are: oil – 26 years; coal – 76 years; natural gas – 34 years.
d The additional resources increase the lifetimes as follows: oil – 46 years (assuming the oil shale and tarsands reserves are all recoverable); coal – 152 years; natural gas – no change since no additional resources are listed.
e If the price of fuel rises, the figures for recoverable reserves also rise since reserves which could not previously be recovered economically now become viable.

14a The broken line shown in figure Q85.
b About 23 hours. If the impending lack of nutrient had had no effect on breeding rate at all then point B (half maximum population) would be reached one doubling-period before 24 hours, *i.e.*, at 23 hours. If the impending shortage had begun to have an effect then half maximum population would be reached more slowly, *i.e.*, in more than 23 hours.

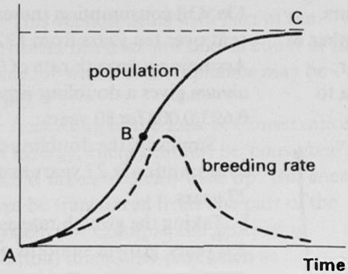


Figure Q85
Population and breeding rate of bacteria with limited resources.

c The time corresponding to point C might be said to be infinite since the population value will approach the maximum more and more slowly.

For practical purposes the time corresponding to C can be given a range of values depending on the form of the growth curve. If the curve is symmetrical, the time corresponding to C is twice that corresponding to B, *i.e.*, about 48 hours or so. Asymmetry in the growth curve could reduce or increase this value.
d The main effect in this analogy is that the organism's breeding rate would be reduced (or its death rate increased). The effect of decreasing fuel supplies would be to reduce the standard of living in various ways – some of them quite drastic – depending on the size of the reduction and what steps were taken to deal with the problem.

15a, b See figure Q86.

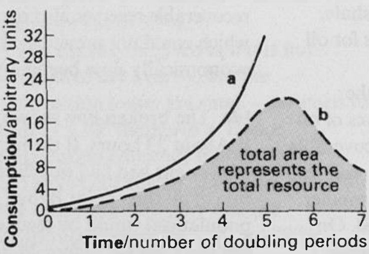


Figure Q86
Consumption of fuel.
a Constant growth rate.
b Effect of increasing difficulty of extraction.

c The area under the graph up to a particular time represents the resource used up to that time. The total area under the graph represents the total resource, which must be the same for all graphs, regardless of initial growth rate.
d If the maximum consumption is high the lifetime will be correspondingly low.
e In order to prolong the lifetime of fossil fuel resources we should start reducing consumption as soon as possible.

16 This question is intended to be speculative. Exponential growth could have occurred while London was small and rural. Food for horses could be obtained *in situ* or relatively easily. Greater urbanization would increase the difficulty of obtaining food for horses and would limit the number of horses which the city and its hinterland could support.

Alternatively, the limitation in numbers of horses could have been due to the development of new technology, for example, vehicles powered by steam.

Implications for the use of fuels could be either the need to match demand to resources or the possibility of developing a new technology such as fusion.

17a Area of Earth's surface $= 5.1 \times 10^8\,\mathrm{km}^2$.
b Land area $= 1.5 \times 10^8\,\mathrm{km}^2$.
c Area per person $= 0.03\,\mathrm{km}^2$.
d Cultivated land per person $= 2.1 \times 10^{-3}\,\mathrm{km}^2$. Grassland per person $= 8 \times 10^{-3}\,\mathrm{km}^2$.

Apart from forests the rest is rocks, desert, tundra, snowcap, *etc.*

Some nations produce too much food and store or even destroy it for economic reasons.
e Doubling-time for population = 37 years

Hence 2018 is 1 doubling period after 1981. In 2018 the cultivated land area available per person would be $1.05 \times 10^{-3}\,\mathrm{km}^2$, and in 2055 (after two doubling periods) $0.53 \times 10^{-3}\,\mathrm{km}^2$ would be available.

18a Columns **1** and **3**.

The reasons for high productivity are complicated. They might include the

ability to make use of powered machinery and fertilizers, but factors like the development of high-yielding strains of crops could also be important.
b Using columns **1** and **2**: input per hectare year for rice growing is 48 kW h; for grain growing, 8200 kW h; for battery farming, 4900 kW h.

All of these are consistent with **a**.
c The statement is reasonable. The implications of depleting fuel supplies are that less will be available for fertilizer production and for machines. Productivity will decrease and eventually people may starve.

19a The U.K. and U.S.S.R. have a slight overall surplus, North America has a slight deficit, and Japan has a large deficit.
b For water power and nuclear energy the figures for production and consumption are identical. Only one set of figures is published, presumably because storage of electrical energy on any reasonable scale is not possible. The figures for coal, natural gas, and oil can be processed as in part **a**. The calculations are simple but the real point of the question is dealt with in parts **c** and **d**.
c Short-term shortfalls can be dealt with by importing from nations which have surplus resources. The importing nations must have some means of earning the currency to buy the fuel and the fuel-exporting nations may have some degree of control over the economies of the importers (for example, the Organization of Petroleum Exporting Countries (OPEC) in the late 1970s).
d Nations like Japan, with continuing shortfalls, are likely to wish to reduce their dependence on imported fuels. They may have to develop alternatives much more purposefully than do nations with fossil fuel resources. To some extent, what happens to Japan today happens to the World tomorrow.

20a The energy equivalent of food for the average West European is 10 MJ per day (question **3**). This is equivalent to about 0.09 toe *per annum* The value for the less developed countries will be rather less than this.
b On the basis of burning 3 kg of wood per head per day the equivalent in terms of oil is about 0.2 toe per head per annum. The value will depend on the estimates made but precision is not required here.

The total of **a** and **b** is about 0.3 toe per head per year.
c If the diet is assumed to be rice, $\frac{2}{3}$ kg of rice would provide the daily requirement of 10 MJ (assuming, as usual, complete conversion). The annual requirement of 250 kg would have cost, say $100 in 1972 when the data for figure G12 were produced.

The effect of incorporating these figures into G12 would be to raise the points and move them to the right, but both changes are quite small.

21a*i* The area needed is 0.13 km^2.

The total area needed for the U.K. population is $1.8 \times 10^6\,km^2$. This is greater than the area of the U.K. (approximately $0.25 \times 10^6\,km^2$).
ii 0.007 miner for the needs of a family or about 100 000 miners for the whole population. About 200 000 miners were employed by the N.C.B. in 1983. Much of their production goes, of course, to power stations (see part **b** of this question) and either directly or indirectly to providing for the needs of industry – not the focus for this question.
iii Area of panel needed = 70 m^2 for a family, or about 1000 km^2 for the whole population. The feasibility depends on the mismatch of times of maximum supply and demand, and the consequent storage problem. The material resources needed for constructing solar panels on this scale are also a problem.
b Roughly three times as much resource would be needed.

22 Conservation measures could reduce space heating demands. (The asterisked categories in table G5 in the *Students'* *guide* are essentially space heating.) If these were to be halved, demands for transport reduced by 20 % and for fuel used in industrial production by 10 %, the total saving could be about 30 %.

A saving of 20 % in North America and Europe would be 10 % of the World's total. While this is not a vast saving it is worth while and might be influential in reducing expectations.

23a 0.04 %
b 4 %
c 5.5 %
d 7.5 %
e 0.25 %

24a A base load capacity of about 39 GW is required. This is the level for which area A = area B (figure Q87). In practice, taking an average of maximum and minimum power levels is reasonably satisfactory.

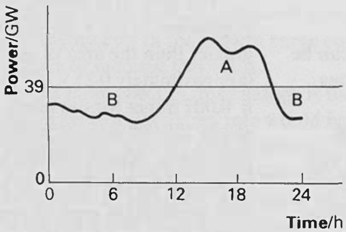


Figure Q87
Variation in power demand during 24 hours.

About 9 GW of hydroelectric power will also be needed.
b 9000 m³ of water would be needed per second to provide the peak power output. The total energy required is about 98 GW h. This would be provided by the contents of a lake of area about 12 km² and 30 m depth falling through 100 m. This arrangement would be quite feasible.
c The simplest annual demand curve might be as shown in figure Q88.

The overall base load can be assumed to be 27 GW. The daily energy requirement is 12 × 24 GW h during the winter months, *i.e.*, a total of 12 × 24 × 180 GW h. This would require a total area of pumped storage reservoir of about 6000 km², *i.e.*, about 75 km square or 2000 km of flooded valley of average width 3 km. This does not seem to be feasible. A more realistic demand curve would make no essential difference to this estimation exercise.

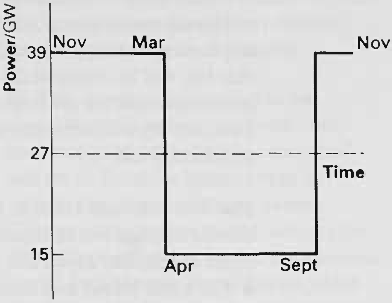


Figure Q88
Simplified annual demand curve.

25a Volume of the ring main = 56 000 m³.
b The pressure falls.
c About 1.4×10^6 m³.
d About 3.9×10^6 m³.
e The increase in pressure which would result from storing all this gas in the main would be about 70×10^5 Pa (*i.e.* from 25×10^5 to 95×10^5 Pa) which takes the main pressure above its limit. Over half ($\frac{40}{70}$) would have to be stored elsewhere.

26 Some means must be found of estimating the fuel consumption of London. If fuel consumption is assumed to be proportional to population, then London consumes $\frac{1}{6}$ of the fuel used in the U.K. (see, for example, table G11 in the *Students' guide*), hence the rate of use is about 4×10^{10} W. The area of London is about 1800 km² hence fuel is consumed at a rate of about 25 W m^{-2}.

Insolation is about 175 W m^{-2}. Hence, overall fuel consumption per unit area is in excess of 10 % of the insolation, and this is likely to raise the temperature. In fact, temperatures in Greater London are generally greater than those of its surroundings.

27 For population, $t_D = 37$ years and $g = 1.9\,\%$.

For fuels, $t_D = 17$ years and $g = 4\,\%$ over much of this time.

Hence, growth in demand for fuels is not merely due to increasing population.

28 Raising global *per capita* fuel consumption to Western European levels would double World fuel consumption. If this occurred over 30 years the annual rate of growth for this reason alone would be 2.3 %.

29 From question **27** increasing expectations seem to account for a rate of growth of 2.1 %. Table G11 in the *Students' guide* shows much larger increases for less developed countries than for developed ones.

30 For the purpose of this question the Earth can be regarded as a flat disc which is always illuminated by the Sun. This takes care of the day–night and inclination variables and gives a value for the energy incident at the bottom of the atmosphere of 2.8×10^{24} J per year. The World's annual fuel consumption is about 2.8×10^{20} J. Thus the average energy delivered by the Sun in an hour is about the same as the Earth's fuel consumption in a year.

31a*i* No. The number of electrons is the same on each side.

ii Change in mass is 0.192 u or 3.18×10^{-28} kg.

iii 2.86×10^{-11} J

b*i* $\dfrac{Q_1 Q_2}{4\pi\varepsilon_0 d}$

ii 4.61×10^{-11} J

iii The fission fragments are very close together at first, perhaps close enough for the nuclear force to have some effect. The fission fragments may be attracted initially as well as being subject to electrostatic repulsion. The kinetic energy of the fragments would then be less than that calculated on the basis of electrical potential energy.

c*i* 9×10^{13} J

ii About the same.

iii 143 kg

iv 143 tonnes

v About 30 times as much material needs to be used to provide the same amount of energy from coal as from uranium.

32a a/A

b $\dfrac{nAva}{A} = nav$

c NAd

d $nav \times NAd$

e $R = \dfrac{nav \times NAd}{A \times d} = nNav$

f $a = R/nNv$

g Measure the rate of interaction per second per unit volume, R; measure the neutron density (number of neutrons per unit volume), n; the neutron velocity, v; and the number of target nuclei per unit volume, N.

33 0.1 eV. This is a factor of about 10^7 less than the energy of a neutron released in fission of ^{238}U.

34a*i* and *ii* See table Q7.

iii See table Q8.

Chain reaction is most likely for energies < 1 eV.

Isotope, interaction	**^{238}U fission**	**^{238}U scattering**	**^{235}U fission**
i $\sigma/10^{-28}\,m^2$	0.63	8	1.3
ii Probability of interaction for 2 MeV neutrons	63	800	1.3

Table Q7
Cross-sections and interaction probabilities for 2 MeV neutrons.

Energy	Cross-section/$10^{-28}\,m^2$		Probability/arbitrary units		Ratio $\frac{^{235}U\text{ fission}}{^{238}U\text{ capture}}$
	^{235}U fission	^{238}U capture	^{235}U fission	^{238}U capture	
10 keV	8	0.63	8	63	0.13
100 eV	40	16	40	160	0.25
1 eV	100	0.63	100	63	1.6
0.01 eV	950	4	950	400	2.4

Table Q8
Cross-sections and interaction probabilities for neutrons of various energies.

iv For 3 % enriched uranium the ratio of ^{238}U to ^{235}U is 33:1. For natural uranium the ratio is 143:1. Taking 100 instead of the more exact values makes no significant difference. It can increase the numbers in the last column in table Q8 by up to three times, or could decrease them at worst by 33 %.
b*i* Arrange the fuel in thin rods or pellets surrounded by a large quantity of moderator – enough to thermalize (slow down) the neutrons before they reach another fuel element, as in figure Q89.

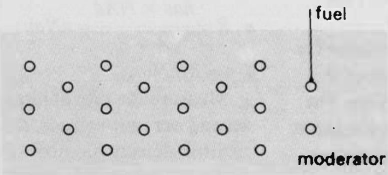


Figure Q89

ii There may be energy levels for neutrons in the ^{238}U nucleus. Each spike in the cross-section graph corresponds exactly with the difference in energy between two particular energy levels.

35a Kinetic energy of the fission fragments.
b The fission fragments cause a very large number of ionizing events and slow down rapidly. The ionized atoms eventually revert to the ground state. The energy released increases the internal energy of the material, heating it up.
c By ionizing atoms of air in the chamber. This mechanism cannot be used for neutrons since they are not charged and do not produce ionization.
d Collision with atoms (or nuclei).

36 Momentum is conserved:

$$mu = mv + MV \qquad [1]$$

Kinetic energy is conserved in an elastic collision:

$$\tfrac{1}{2}mu^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}MV^2 \qquad [2]$$

Initial kinetic energy, $E_k = \frac{1}{2}mu^2$

Change in kinetic energy of A,
$$\Delta E_k = \tfrac{1}{2}mu^2 - \tfrac{1}{2}mv^2 = \tfrac{1}{2}MV^2 \qquad [3]$$

The rest is algebra; the key step is to eliminate v.

From [3]:

$$m(u^2 - v^2) = MV^2 \Rightarrow m(u+v)(u-v) = MV^2 \qquad [4]$$

From [1]:

$$m(u-v) = MV$$

Substitute MV for $m(u-v)$ in equation [4]:

$$MV(u+v) = MV^2$$
$$\Rightarrow u + v = V$$
$$\Rightarrow v = V - u$$

Now substitute $(V-u)$ for v in equation [1]:

$$mu = m(V-u) + MV$$
$$\Rightarrow 2mu = (m+M)V$$
$$\Rightarrow V = \frac{2mu}{m+M}$$

So

$$\Delta E_k = \tfrac{1}{2}M\left(\frac{2mu}{m+M}\right)^2$$
$$= \frac{4mM}{(m+M)^2} \times \tfrac{1}{2}mu^2$$
$$= \frac{4mM}{(m+M)^2}E_k$$

37a $\Delta E_k/E_k = \dfrac{4mM}{(M+m)^2}$.
b For maximum value of $\Delta E_k/E_k$, M must be 1 u.
c See table Q9.

M/u	1	2	10	12	16	112	238
$\dfrac{\Delta E_k}{E_k}$	100	89	33	28	22	3.5	1.7
Relative value	$4\frac{1}{2}$	4	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{1}{6}$	$\frac{1}{13}$
Element	H	D	B	C	O	Cd	U

Table Q9
Percentage energy loss for neutrons in head-on collision with certain nuclei.

d Hydrogen.

38 54. This is less than the actual number since the collisions will generally not be head-on in practice.

39a Low absorption (capture) cross-section. High scattering cross-section.
b See table Q10.
c On the basis of these figures the choice would be oxygen or deuterium. When account is also taken of relative energy loss the choice is confirmed and the choice of hydrogen on the basis of energy loss only is overidden.
d Boron, cadmium.

40a 3.1×10^{19}
b 1.2×10^{-5} kg
c Mass of ^{235}U used per year = 383 kg. Total mass is three times this, 1150 kg.
d 383 kg represents 2.2 % of the uranium present. The mass of the core is therefore 52×10^3 kg.
e The main reason is the low efficiency at which internal energy is converted to electrical energy.

41a Thermal flow of energy.
b Temperature difference.
ci $C\,s^{-1}$, A.
ii Temperature difference $\propto$ rate of thermal flow of energy; joule per second, watt; power.
d Measure the power transmitted through the conductor for different temperature differences. Problems are loss of energy from sides of conductor, and difficulties in measuring the power, etc.
e $K\,W^{-1}$
f Ω m
g $W\,m^{-1}\,K^{-1}$

42a $0.63\,K\,W^{-1}$
b 1.3 kW

43a $2.7 \times 10^{-3}\,K\,W^{-1}$
b $16 \times 10^{-3}\,K\,W^{-1}$
ci 1.3 kW
ii 7.5 kW
iii 8.8 kW
d*i* $I_T = I_1 + I_2$
ii The two calculations are analogous.

44a $19 \times 10^{-3}\,K\,W^{-1}$
b 570 W

45a 10.6 MW
b 170 W

Element	H	D	B	C	O	Cd	U
$\dfrac{\sigma_s}{\sigma_c}$	61.5	6396	4.7×10^{-3}	1397	13925	2×10^{-3}	1.2
$\dfrac{\Delta E_k}{E_k} \times \dfrac{\sigma_s}{\sigma_c}$	277	25600	7.2×10^{-3}	1850	13925	0.3×10^{-3}	0.1

Table Q10
Cross-sections and relative energy loss for neutrons in collision with various nuclei.

46a $X/2$, $X/3$, X/A.
b $\mathcal{R} = X/A$
c $m^2\,K\,W^{-1}$
d $X = l/k$

47a $0.17\,m^2\,K\,W^{-1}$
b $0.13\,m^2\,K\,W^{-1}$
c The one which has the bigger resistance coefficient, *i.e.*, brick.

48a $2 \times 10^{-3}\,K\,W^{-1}$
b 10 kW
c Much too high.

49a Convection currents in the air and how easily these can flow; forced convection (winds) reduces the resistance of external surfaces.

50a Series
b $0.194\,m^2\,K\,W^{-1}$. The power loss is reduced to about one-fiftieth of its previous value.
c Outer surface of glass is at 6.2 °C. Inner surface of glass is at 6.6 °C.

51a, b The graph is shown in figure Q90.
c $0.26\,m^2\,K\,W^{-1}$. If the surface layers are assumed to be of zero thickness then the resistance coefficient of the cavity would be independent of thickness. In practice, the surface layers are of finite thickness so the resistance coefficient would be likely to increase with thickness until the cavity is two surface layers thick. Thereafter the resistance would be constant.
e A thin cavity behaves more like a slab of conductor than a thicker one. A faced horizontal cavity with downward flow of energy continues to behave like a slab of conductor as its thickness increases above 20 mm, whereas the behaviour of the other types of cavity is increasingly unlike that of a slab of conductor.
f Facing the cavity increases the resistance coefficient. If the effect were simply due to conduction, the metal foil would marginally decrease the resistance coefficient. The data suggest that radiation is also a mechanism in thermal transfer of energy. The effect is important – facing the surface increases the resistance coefficient by a factor of about three.
g Convection occurs when there is a possibility of upward movement of heated fluid, for example with hor↑ cavity. Compared with the hor↓ cavity the thermal resistance is approximately halved – hence convection is an important effect.

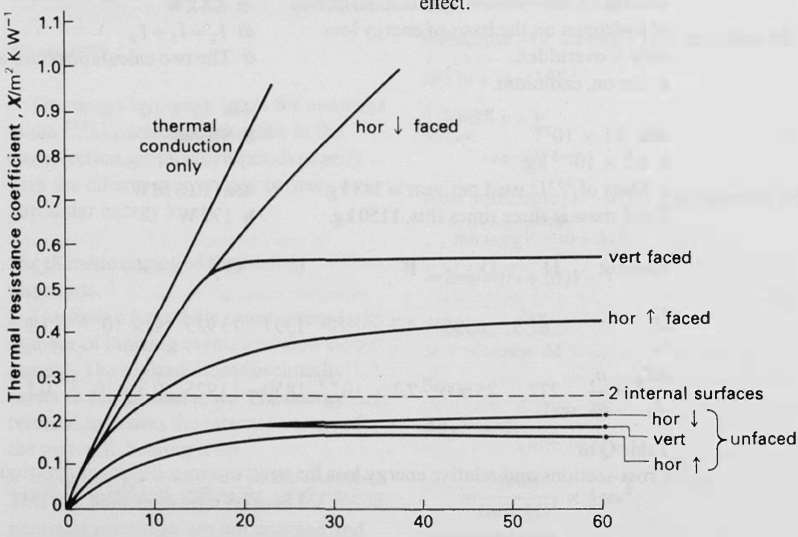


Figure Q90
Thermal resistance coefficients for various cavities.

h No, since none of the measured graphs coincides with the graph for two surfaces, except for a thickness up to about 8 mm. Perhaps this indicates that each surface layer has an effective thickness of about 4 mm for the conditions under which the measurements were taken.
i Measured values are always less than values calculated assuming that conduction is the only mechanism, often much less.
j Resistances need to be connected in parallel.

52b Power loss through the double-glazed window is about 120 W, *i.e.*, reduced to 0.6 of the single-glazed value.
c, d See figure Q91.
e No. The dependence of resistance on thickness is likely to be non-linear and hence the calculation is invalid. However, the non-linearity of resistance and, consequently, of temperature, has no effect on calculations which are solely concerned with the components in their entirety. We are using what amounts to a 'systems' approach to the problem.

53 A drawn curtain creates a cavity near the window.

54 The filling reduces the possibility of convection and hence increases the thermal resistance of the cavity.

55b A single-glazed window has $X = 0.194\ \mathrm{m^2\,K\,W^{-1}}$, while a double-glazed window has $X = 0.338\ \mathrm{m^2\,K\,W^{-1}}$.
c Wood is a poorer conductor than glass since a wood-framed window has a higher overall value of thermal resistance coefficient than that calculated for glass alone. The converse holds for a metal frame.
d Choose a single-glazed window with a relatively large area of metal frame, provided that when double-glazed the frame can be insulated to give a higher resistance coefficient than the double glazing.

56a See figure Q92.
b $0.114\ \mathrm{K\,W^{-1}}$
c 158 W
d 15.6 °C
e $0.074\ \mathrm{K\,W^{-1}}$
f 244 W
g 11.7 °C
h On the glass; it is colder.

57 Area of window $= 1.25\ \mathrm{m^2}$.

58a Reasonable values only need to be given, for example, temperature at B could be about 20 °C, and A could be between 5 and 10 °C.

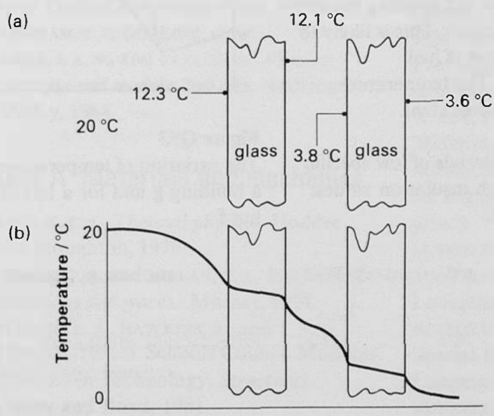


Figure Q91
(a) Temperature at surfaces in a double-glazed window.
(b) Temperature variation with distance near a double-glazed window.

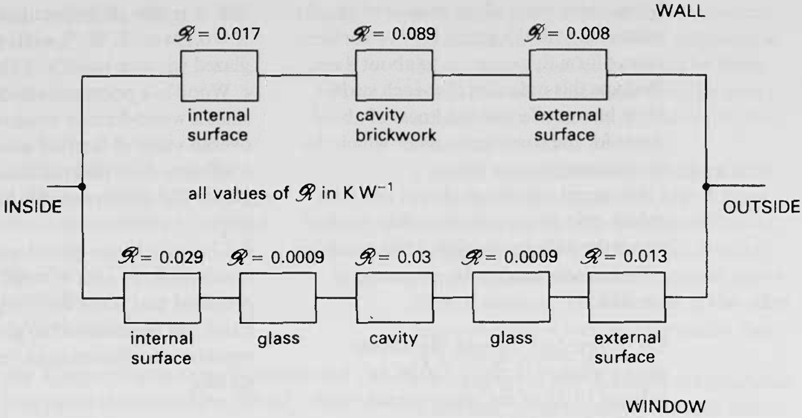


Figure Q92
Thermal resistances for wall with window.

b The rate of loss of energy depends on both the external temperature and the internal temperature. If the external temperature is 0 °C then the rate of loss of energy for an internal temperature of 20 °C is twice that for an internal temperature of 10° C or four times that for a temperature of 5 °C.

c Mass of building material, specific heating capacity of material, insulation value of the material.

d The heating system has to be operating for a long time before the building reaches its operating temperature. This is likely to require a large amount of fuel.

e Use a thermostat. The temperature would oscillate to some extent.

f, g See figure Q93.

h Light building materials of low specific heating capacity; high insulation values; minimum ventilation.

59 Power needed to heat the air = 1.3 kW. This is substantially greater than the loss through either single- or double-glazed windows in questions **50** and **52**.

60a For a group of 15 the power needed to heat the air is about 5.5 kW, assuming that the air has to be heated through 20 K.
b The power provided by the occupants is 1.5 kW, about 30 % of the power needed.

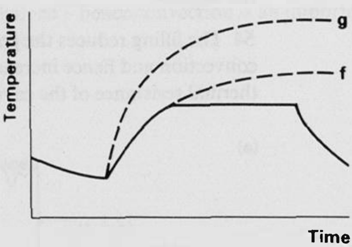


Figure Q93
The variation of temperature with time for a building **g** and for a lightly constructed hut **f**.

REFERENCE MATERIAL

TEXTBOOKS AND FURTHER READING

The students have a list of textbooks and other references on pages 491–2 of *Students' guide 1.* Here we give a list of books and references for teachers and the library.

Books useful throughout the course

AKRILL, T. B., BENNET, G. A. G., and MILLAR, C. J. *Physics.* Arnold, 1979.

BOLTON, W. *Patterns in physics.* McGraw-Hill, 1974.

DUNCAN, T. *Physics: a textbook for advanced level students.* Murray, 1982.

OR

DUNCAN, T. *Advanced physics: materials and mechanics.* 2nd edn. Murray, 1981.

DUNCAN, T. *Advanced physics: fields, waves, and atoms.* 2nd edn. Murray, 1981.

FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. The Feynman lectures on physics *Volume 1: Mainly mechanics, radiation, and heat.* Addison-Wesley, 1963.

NUFFIELD REVISED PHYSICS *Years 1 and 2, Year 3, Year 4,* and *Year 5.* Longman, 1978–80.

ROGERS, E. M. *Physics for the inquiring mind.* Oxford University Press, 1960.

WENHAM, E. J., DORLING, G. W., SNELL, J. A. N., and TAYLOR, B. *Physics: concepts and models.* 2nd edn. Addison-Wesley, 1984.

Unit A Materials and mechanics

ADKINS, C. J. *Thermal physics.* Hodder and Stoughton, 1976.

AKRILL, T. B. and MILLAR, C. J. *Mechanics, vibrations and waves.* Murray, 1974.

BLUNDELL, A., HAWKINS, R., and LUDDINGTON, D. Schools Council Modular Courses in Technology, *Structures.* Oliver and Boyd, 1981.

BOLTON, W. Study Topics in Physics, *Volume 1: Motion and force. Volume 2: Materials.* Butterworths, 1980.

BRONOWSKI, J. *The ascent of Man.* Futura, 1981. (First published by B.B.C., 1973.)

COLLIEU, A. McB. and POWNEY, D. J. *The mechanical and thermal properties of materials.* Arnold, 1973.

CRAC Hobsons Science Support Series, *Gases and gas laws.* Hobsons Press, 1983.

CRAC Hobsons Science Support Series, *Stress, strain and strength.* Hobsons Press, 1982.

FARRAR, R. A. Methuen Studies in Science, *The mechanical properties of materials.* Methuen, 1971. (Out of print.)

GORDON, J. E. *The new science of strong materials.* Pitman, 1979. (First published by Penguin, 1968.)

GORDON, J. E. *Structures.* Pitman, 1979. (First published by Penguin, 1978.)

HOCKEY, S. W. and MILLS, J. R. *Physics by experiment.* Wheaton, 1973.

LOFTAS, A. A. and GWYNNE, P. *Advances in materials science.* London University Press, 1967.

McSHEA, J. Schools Council Modular Courses in Technology, *Materials technology.* Oliver and Boyd, 1981.

MARTIN, J. W. and HULL, R. A. *Elementary science of metals.* Wykeham, 1975.

MARTIN, J. W. 'Strength of materials'. *Contemporary physics reprint.* Taylor and Francis, 1968.

SCIENTIFIC AMERICAN *Materials.* W. H. Freeman, 1967.

MOFFAT, W. G., PEARSALL, G. W., and WULFF, J. *The Structure and Properties of Materials.* Wiley, 1964.

NUFFIELD ADVANCED PHYSICS *Physics and the engineer.* Longman, 1973. (Out of print.)

NUFFIELD REVISED ADVANCED CHEMISTRY *Students book I* and *Teachers' guide I.* Longman, 1984.

NUFFIELD REVISED ADVANCED CHEMISTRY Special Studies *Metals as materials.* Longman, 1985.

NUFFIELD REVISED CHEMISTRY *Teachers' guide II.* Longman, 1978.

NUFFIELD REVISED CHEMISTRY *Handbook for pupils.* Longman, 1978.

OGBORN, J. *Molecules and motion.* Longman, 1973. (Out of print.)
OVERMAN, M. *Roads, bridges, and tunnels: modern approaches to road engineering.* Aldus, 1968.
PSSC *College Physics.* 5th edn. Heath, 1981.
ROSENBERG, H. M. *The solid state: an introduction to the physics of crystals.* Oxford University Press, 1979.
SCHOOLS COUNCIL Engineering Science Project, *The use of materials.* Macmillan, 1975. (Out of print.)
TABOR, D. *Gases, liquids, and solids.* Cambridge University Press, 1980.
WARREN, J. W. *Understanding force.* Murray, 1979.
WILLIAMS, D. J. *Force, matter, and energy.* Hodder and Stoughton, 1974.

The Institution of Metallurgists publishes several leaflets on metals and materials; particularly relevant to this Unit are:
Which materials?
Materials in Action Series: *Materials and the human body, Concorde,* and *Superlastic alloys.*
These can be obtained free from:
The Education Officer
The Institution of Metallurgists
PO Box 471
1 Carlton House Terrace
London SW1Y 5BE
Tel: 01-839 1963.

Books on theories and models

BONDI, H. Science Study Series No 12, *The Universe at large.* Heinemann, 1962. (Out of print.)
BRONOWSKI, J. *The common sense of science.* Heinemann, 1979.
POPPER, K. R. *Conjectures and refutations: growth of scientific knowledge.* 3rd edn. Routledge, 1969.
WATSON, J. D. *The double helix.* Penguin, 1970. (First published by Weidenfeld and Nicholson, 1968.)

Unit B Current, circuits, and charge

CRAC Hobsons Science Support Series, *Instrumentation systems* and *Stress, strain, and strength.* Hobsons Press, 1982.

Unit C Digital electronic circuits

A large and rapidly growing number of books is available covering the digital electronics in this Unit. We have found these listed below useful. Teachers may also find the books suggested for students useful (*Students' guide 1* page 491). Teachers and students may find other books and magazines equally useful and should be encouraged to use them.

Suggestions for teaching

BEVIS, G. and TROTTER, M. (Eds) *Microelectronics: practical approaches for schools and colleges.* BP Educational Service.
GOUGH, C. E. *et al. Notes for guidance for the electronics option in Physics (Advanced).* Joint Matriculation Board. 1981.

Textbooks on electronics

GIBSON, J. R. *Electronic logic circuits.* 2nd edn. Arnold, 1983.
HOROWITZ, P. and HILL, W. *The art of electronics.* Cambridge University Press, 1980.
JONES, M. H. *A practical introduction to electronic circuits.* Cambridge University Press, 1977.
TOCCI, R. J. *Digital systems: principles and applications.* Prentice-Hall, 1980.

Computers

THOMPSON, D. L. *Inside the micro.* Unilab, 1982.

Designing circuits

LANCASTER, D. E. *CMOS cookbook.* SAMS, 1978.
LANCASTER, D. E. *TTL cookbook.* SAMS, 1978.
McWHORTER, G. *Understanding digital electronics.* Texas Instruments Inc., 1978.
MULLARD *CMOS digital integrated circuits,* Book 4, part 4. 1983.
TEXAS INSTRUMENTS *Designing with TTL integrated circuits.* McGraw-Hill, 1971.

Nerve impulses

ADRIAN, R. H. Carolina Biology Readers No. 67, *The nerve impulse.* 2nd edn.

Carolina Biological Supply Company, distributed by Packard Publishing, 1980.
HODGKIN, A. L. *The conduction of the nervous impulse.* Liverpool University Press, 1964.

Unit D Oscillations and waves

As well as the general list of textbooks on page 501, these are particularly useful for Unit D.

Reference to school experiments

ERICSON, T. J. 'Velocity of sound in a steel bar'. *School Science Review* **57**, 1976, page 741.
FORD, D. and SOPER, P. 'Velocity of sound in metallic rods'. *School Science Review*, **60**, 1978, page 114.
MACE, W. K. 'Speed of sound in a steel rod'. *School Science Review*, **61**, 1980, page 527.
MACE, W. K. 'Resonance experiments for pupils'. *School Science Review*, **61**, 1980, page 537.
SHIPSTONE, D. M. 'Experiments with travelling waves on water'. *School Science Review*, **50**, 1969, page 816.
CONTEMPORARY PHYSICS 'Sources of physics teaching, part 1'. *Contemporary Physics.* Taylor and Francis, 1968.

Books

AKRILL, T. B. and MILLAR, C. J. *Mechanics, vibrations and waves.* Murray, 1974.
BISHOP, R. E. D. *Vibration.* 2nd edn. Cambridge University Press, 1979.
CARNELL, R. C. *Physics principles at work; a resource book for teachers.* BP Educational, 1983.
CHAUNDY, D. C. F. Longman Physics Topics, *Waves.* Longman, 1972.
CRAC Hobsons Science Support Series, *Vibrations.* Hobsons Press, 1983.
CRAC Hobsons Science Support Series, *Waves and sound.* Hobsons Press, 1982.
CRAWFORD, F. S. *Berkeley Physics Course* Volume 3, *Waves.* McGraw-Hill, 1968.
FEATHER, N. Introduction to Physics, *Volume 1 Mass, length and time.* Penguin, 1961. (First published by Edinburgh University Press, 1959.)
GOULD, R. T. *John Harrison and his timekeepers.* 4th edn. National Maritime Museum, 1978.
GRIFFIN, D. R. Science Study Series No 4, *Echoes of bats and men.* Heinemann, 1962. (Out of print.)
HOWSE, D. *Greenwich time and the discovery of the longitude.* Oxford University Press, 1980.
LAITHWAITE, E. R. *Propulsion without wheels.* Hodder and Stoughton, 1965.
OPEN UNIVERSITY *Science foundation course S101.* Unit 4, 'Earthquake waves and the Earth's interior'. Open University Press, 1979.
PSSC *Physics.* 5th edn. Heath, 1981.
SCSST Physics Plus, *Sonar.* Hobsons Press, 1985.
SHIVE, J. N. and WEBER, R. L. *Similarities in physics.* Hilger, 1982.
TABOR, D. *Gases, liquids and solids.* Cambridge University Press, 1980.
TRICKER, R. A. R. *Bores, breakers, waves, and wakes.* Mills and Boon, 1965. (Out of print.)
WALKER, J. R. *The flying circus of physics.* Wiley, 1978.
WARREN, J. W. *Understanding force.* Murray, 1979.

Scientific American references

Note: Although *Scientific American* Offprints can no longer be purchased in Europe, the titles listed here may still be available in many schools and colleges.

BASCOM, W. 'Ocean waves'. *Scientific American.* Volume **201**(2), Aug. 1959. (Offprint No. 828.)
BERNSTEIN, J. 'Tsunamis'. *Scientific American.* Volume **191**(2), Aug. 1954. (Offprint No 829.)
BULLEN, K. E. 'The interior of the Earth'. *Scientific American.* Volume **193**(3), Sept. 1955. (Offprint No 804.)
GRIFFIN, D. R. 'More about bat radar'. *Scientific American.* Volume **199**(1), July, 1958. (Offprint No 1121.)
LYONS, H. 'Atomic clocks'. *Scientific American.* Volume **196**(2), Feb. 1967. (Offprint No 225.)
OLIVER, J. 'Long earthquake waves'. *Scientific American.* Volume **200**(3), Mar. 1959. (Offprint No 227.)

Unit E Field and potential

ARONS, A. B. *Development of concepts of physics.* Addison-Wesley, 1965. (Out of print.)

ASIMOV, I. *The collapsing Universe: the story of black holes.* Corgi, 1978.

BRONOWSKI, J. *The ascent of Man.* Futura, 1981. (First published by B.B.C., 1973.)

CALDER, N. *The key to the Universe.* B.B.C., 1977. (Out of print.)

COHEN, I. B. 'Newton's discovery of gravity'. *Scientific American.* Volume **244**(3), March 1981, p. 122.

DAVIES, P. C. W. *The forces of nature.* Cambridge University Press, 1979.

HALLIDAY, D. and RESNICK, R. *Physics Part 1.* 3rd edn. Wiley, 1977.

HALLIDAY, D. and RESNICK, R. *Physics Part 2.* 3rd edn. Wiley, 1978.

KOESTLER, A. *The sleepwalkers.* Penguin, 1970. (First published by Hutchinson, 1968.)

NEWTON, I. (MOTTE, A. trans.) *Principia.* University of California Press, 1962.

PSSC *College physics.* 5th edn. Heath, 1981.

Unit F Radioactivity and the nuclear atom

Books for teachers

CONN, G. K. T. and TURNER, H. D. *The evolution of the nuclear atom.* Illiffe, 1965.

GRIFFITH, J. A. R. and TEBBUTT, M. J. *Notes for guidance for the nuclear physics option.* Joint Matriculation Board, 1981.

LITTLEFIELD, T. A. and THORLEY, N. *Atomic and nuclear physics.* 3rd edn. van Nostrand Reinhold Co., 1979.

SEMAT, H. and ALBRIGHT, J. R. *Introduction to atomic and nuclear physics.* 5th edn. Chapman and Hall, 1973.

WEHR, M. R., RICHARDS, J. A., and ADAIR, T. W. *Physics of the atom.* 4th edn. Addison-Wesley, 1984.

WEIDNER, R. T. and SELLS, R. L. *Elementary modern physics.* 3rd edn. Allyn and Bacon, 1980.

Data books

NUFFIELD REVISED ADVANCED SCIENCE *Book of data.* 2nd edn. Longman, 1984.

TENNANT, R. M. (Ed.) *Science data book.* Oliver and Boyd, 1971.

Other books

BENNET, G. A. G. *Electricity and modern physics.* 2nd edn. Arnold, 1974.

CARO, D. E., McDONELL, J. A., and SPICER, B. M. *Modern physics.* 3rd edn. Arnold, 1978.

DAVIES, P. C. W. *The forces of nature.* Cambridge University Press, 1979.

LEWIS, J. L. Longman Physics Topics, *Electrons and atoms.* Longman, 1972.

LEWIS, J. L. and WENHAM, E. J. Longman Physics Topics, *Radioactivity.* Longman, 1970.

NUFFIELD REVISED ADVANCED CHEMISTRY *Students' book I* and *Teachers' guide I.* Longman, 1984.

PROJECT PHYSICS Text Unit 6, *The nucleus.* Holt, Rinehart, and Winston, 1981.

TABOR, D. *Gases, liquids and solids.* Cambridge University Press, 1980.

WARREN, J. W. *Understanding force.* Murray, 1979.

WRIGHT, S. (Ed.) *Classical scientific papers – physics.* Mills and Boon, 1964. (Out of print.)

Journal reference

PEACOCK, T. A. H. 'Nuclear chemistry: some applications of radioisotope techniques in chemistry and biology'. *School Science Review* **45**, 157, 1964.

Rutherford Appleton Laboratory Monographs

CLARK, D. H. *The Universe and Man.* 1981.

CLOSE, F. E. *Atoms, particles, leptons and quarks.* 1980.

DAMERELL, C. J. S. *Experimental particle physics.* 1981.

These can be obtained free from:
The Library
Rutherford and Appleton Laboratories
Chilton
Didcot
Oxfordshire OX11 0QX

Unit G Energy sources

BASSETT, C. R. and PRITCHARD, M. D. W. *Heating.* Longman, 1969.
CHAPMAN, P. *Fuel's paradise.* Penguin, 1980.
CRAWLEY, G. M. *Energy.* Collier Macmillan, 1975.
ELKINGTON, J. *Sun traps.* Penguin, 1984.
FOLEY, G. *The energy question.* 2nd edn. Penguin, 1981.
GRIFFITH, J. A. R. and TEBBUTT, M. J. *Notes for guidance for the nuclear physics option.* Joint Matriculation Board, 1981.
*LEWIS, J. L. (Ed.) *Science in society.* Heinemann, 1981.
McMULLEN, J. T., MORGAN, R. and MURRAY, R. B. *Energy resources.* 2nd edn. Arnold, 1983.
RAMAGE, J. *Energy: a guide book.* Oxford University Press, 1983.
*SOLOMON, J. Science In A Social Context Series. Blackwell/ASE, 1983. (Several of the titles in this series will be relevant to this Unit.)
WARREN, J. W. *The teaching of physics* Butterworth, 1967.

*Though neither the *SISCON in schools* nor the *Science in Society* materials (published by Heinemann) are specially intended for A-level physics students, several of the publications produced by these two projects are relevant to this Unit.

Journals references

OGBORN, J. M. 'Dialogue concerning two old sciences'. *Phys. Educ.* **11**, 272–6, 1976.
SCHMID, G. B. 'Energy and its carriers'. *Phys. Educ.* **17**, 212–218, 1982.
SUMMERS, M. K. 'Teaching heat: an analysis of misconceptions'. *School Science Review.* **64**, 670–6, 1983.
WARREN, J. W. 'The teaching of the concept of heat'. *Phys. Educ.* **7**, 41–44, 1972.
WARREN, J. W. 'The nature of energy'. *Eur. J. Sci. Educ.,* **4**(3), 295–297, 1982.
WARREN, J. W. 'Energy and its carriers: a critical analysis'. *Phys. Educ.* **18**, 209–212, 1983.

COMPUTER PROGRAMS AND VISUAL AIDS

Unit A Materials and mechanics

16 mm films

Films available from CFL Vision, Distribution Centre, Chalfont Grove, Gerrards Cross, Buckinghamshire, SL9 8TN. Tel: (02407) 4433. All except 'River to Cross' can be hired free of charge.

'Materials for the engineer', 15 minutes, 1972 (UK 2665).
'Let's make a model', 22 minutes, 1979 (UK 2874).
'The slender span', 12 minutes, 1966 (UK 2787).
'River to cross', 20 minutes, 1950 (UK 1253).
'Dynamic behaviour of tall buildings', 10 minutes, 1976 (UK 2794).
'We build for the world', 11 minutes, 1967 (UK 1878).

Television or video

'Electron diffraction', in the Granada TV series *Experiment: Physics.* Notes available from Granada Television Ltd, Manchester, M60 90EA, or Education Officer (Schools Information) of regional ITV companies.
'Understanding materials' (1982): a series of five video cassettes available on free loan from the Institution of Metallurgists, Northway House, High Road, Whetstone, London, N20 9LW, or from Department of Ceramics, Glasses and Polymers, Sheffield University, Elmfield, Northumberland Road, Sheffield, S10 2TZ.

Unit B Currents, circuits and charge

16 mm film

'Are there electrons? (The Millikan experiment)'. Colour, sound, 13 minutes. Reference no. 21.7772. Rank Film Library, Rank Audio Visual Ltd, P.O. Box 70, Great West Road, Brentford, Middlesex.

Unit D Oscillations and waves

Film loops

Highly recommended:
'Tacoma Narrows Bridge collapse'. Ealing Scientific No. 80-2181/1 (super 8).
Also useful:
'Measurement of "G"'. Ealing Scientific No. 80-2124/1 (super 8).
'Soap film oscillations'. Ealing Scientific No. 80-2660/1 (super 8).
'Vibrations of a drum'. Ealing Scientific No. 80-3924/1 (super 8).
These are all available from: BFA Educational Films, 56 Copthorne Road, Leatherhead, Surrey. In due course they may also be available on videotape.
'Wind-induced oscillations'. Penguin. No. XX1671 (standard 8).
This is no longer available, but some schools may still have it.

Television or video

'Oscillations' – Physical Science Series. BBC TV.
This is no longer broadcast, but some schools may still have access to a recording.
'Vibrations of music' – Open University, Programme 9 for course S271 *Discovering physics.*
Open University programmes may be recorded by licence-holders. A school licence costs £40 and a college licence costs £80. Both may be obtained from: Guild Organization Ltd, Guild House, Oundle Road, Peterborough, PE2 9PZ.

Computer programs

'Longitudinal waves'. Heinemann Computers in Education.
'Transverse waves II'. Heinemann Computers in Education
'Waves' by R. G. Roscoe (obtainable through MUSE).
'SHM'. Longman Micro Software.
'Dynamic modelling system'. Longman Micro Software.

Unit E Field and potential

Computer programs

'Software for Nuffield Advanced Physics' and 'Dynamic modelling system'. Longman Micro Software.

Television or video

'The determination of the Newtonian constant of gravitation'. Programme 7 in the Granada TV series *Experiment: Physics.* Notes available from Granada Television Ltd, Manchester, M60 9EA, or Education Officer (Schools Information) of regional ITV companies.

Unit F Radioactivity and the nuclear atom

16 mm films

'The Rutherford model of the atom'. Colour, sound, 16 minutes. Reference no. 21.7852. Rank Film Library, Rank Audio Visual Ltd., P.O. Box 70, Great West Road, Brentford, Middlesex.

Television or video

'The determination of a radioactive half-life', and 'Measurement of ionization potential' in the Granada TV series *Experiment: physics.* Notes available from Granada Televison Ltd, Manchester, M60 9EA, or Education Officer (Schools Information) of regional ITV companies.

Computer programs

'Dynamic modelling system'. Longman Micro Software.
BEARE, R. 'Alphafoil', Warwick Science Simulations Physics Pack 2, Longman Micro Software.
CAMPBELL, J. 'Simulation of alpha scattering' in *Physics software Pack 1*, Hutchinson Software, 1983.
HARRIS, J. 'Particle scattering' (Chelsea Science Simulations). Arnold, 1982.
SOFTWARE PRODUCTION ASSOCIATES
'ALPHA' is one of six programs in their *A-level Physics Suite.* From Software Production Associates, P.O. Box 59, Royal Leamington Spa, Warwickshire, CV31 3QA.

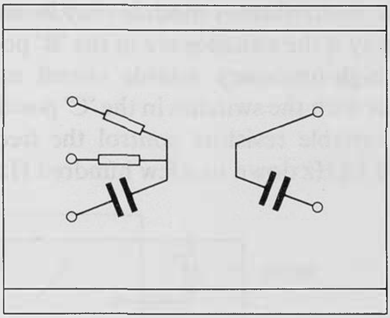
APPENDICES

APPENDIX I

Using the 'basic unit' electronics kit

A single 'basic unit' kit comprises the following:
3 basic units
2 indicator units
1 AND gate
1 switch module
1 bistable module
1 multivibrator module
1 beam splitter module

The three basic units may be used as NOR gates by considering only the two resistive inputs and the direct output. It might be helpful to affix some sort of label to make this clear to the students (see figure 1).



becomes

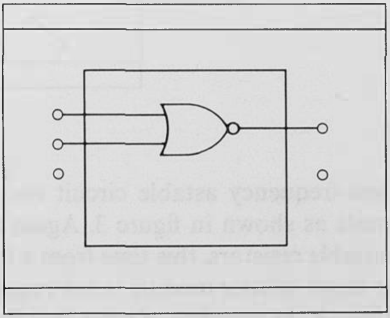


Figure 1

The indicator units and the AND gate may be used unmodified, as may the switch module, provided it has a bounce-free electronic switch circuit.

The bistable module may be used unmodified, but it differs in two ways from the specification of the new digital electronics kit. Firstly, the inputs and outputs are marked differently:

Old name		*New name*
input 1	becomes	C (clear)
input 2	becomes	S (set)
trigger	becomes	clock
output 1	becomes	Q
output 2	becomes	$\overline{\mathrm{Q}}$

Again it may be helpful to affix some sort of label.

Secondly, the bistable module in the basic unit kit changes state when the trigger or clock input goes from a high state to a low state. Where this difference is important, appropriate instructions are given in *Teachers'* and *Students' guides.*

The multivibrator module may be used as a bistable module in the same way if the switches are in the 'R' position.

A high-frequency astable circuit may be made with the multivibrator with the switches in the 'C' position using the circuit in figure 2. Both variable resistors control the frequency which may vary from around 1 kHz down to a few hundred Hz.

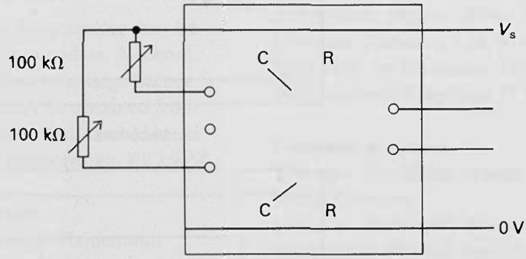


Figure 2

A low-frequency astable circuit may be made by connecting two basic units as shown in figure 3. Again the frequency is controlled by both variable resistors, this time from a few Hz down to perhaps 0.1 Hz.

The beam splitter module is not required for the revised course.

Although the quantity of gates that this provides is enough for many basic experiments to be performed by the class (with some sharing of modules for the more extensive experiments), it provides little introduction to NAND logic, except that which may be demonstrated by inverting the output from an AND gate with a NOR gate (basic unit). We would therefore recommend the addition of two further NAND gates for each student, or perhaps the addition of the quad NAND board, as described in the *Apparatus guide*, which may be used with the basic unit kit.

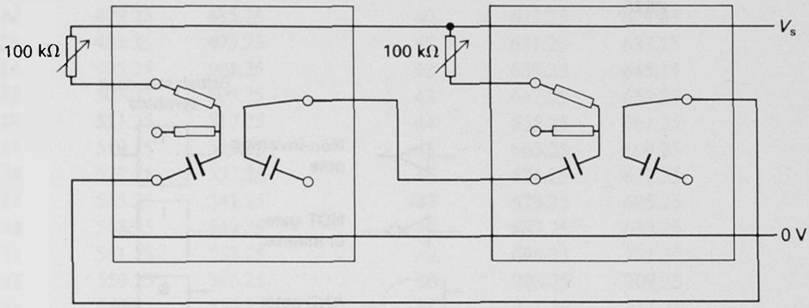


Figure 3
Low-frequency astable circuit from two basic units.

APPENDIX II

British Standard logic symbols

Teachers and students may come across the following British Standard symbols in addition to those used in the course. Note that for both systems, the presence of a circle at the output indicates an inverting gate.

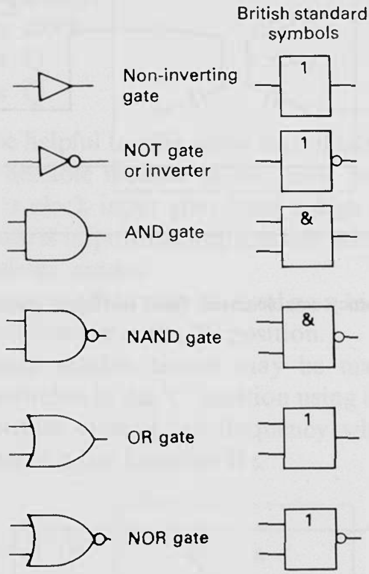


Figure 4

APPENDIX III

Television channels and nominal carrier frequencies (U.K. allocations)

Band IV

Channel	Carrier frequencies/MHz Vision	Sound
21	471.25	477.25
22	479.25	485.25
23	487.25	493.25
24	495.25	501.25
25	503.25	509.25
26	511.25	517.25
27	519.25	525.25
28	527.25	533.25
29	535.25	541.25
30	543.25	549.25
31	551.25	557.25
32	559.25	565.25
33	567.25	573.25
34	575.25	581.25

Band V

Channel	Carrier frequencies/MHz Vision	Sound
39	615.25	621.25
40	623.25	629.25
41	631.25	637.25
42	639.25	645.25
43	647.25	653.25
44	655.25	661.25
45	663.25	669.25
46	671.25	677.25
47	679.25	685.25
48	687.25	693.25
49	695.25	701.25
50	703.25	709.25
51	711.25	717.25
52	719.25	725.25
53	727.25	733.25
54	735.25	741.25
55	743.25	749.25
56	751.25	757.25
57	759.25	765.25
58	767.25	773.25
59	775.25	781.25
60	783.25	789.25
61	791.25	797.25
62	799.25	805.25
63	807.25	813.25
64	815.25	821.25
65	823.25	829.25
66	831.25	837.25
67	839.25	845.25
68	847.25	853.25

APPENDIX IV

Variable phase oscillator

This inexpensive and easily constructed module has been found invaluable in demonstrating the addition of waves of the same frequency and amplitude which differ in phase by any angle in the range 0–π radians.

It has been used in the lower school as a demonstration to explain superposition effects (especially the formation of Young's Fringes) and as a student experiment in the sixth form.

The phase shifter module is built around a 741 op-amp as shown in figure 5.

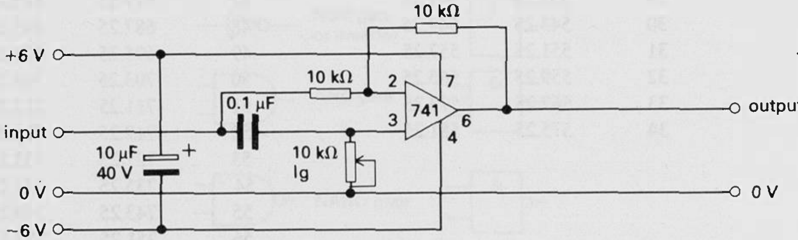


Figure 5

The cost is very low; most of it is the cost of the box and 4 mm sockets.

The module is connected in the circuit as shown in figure 6.

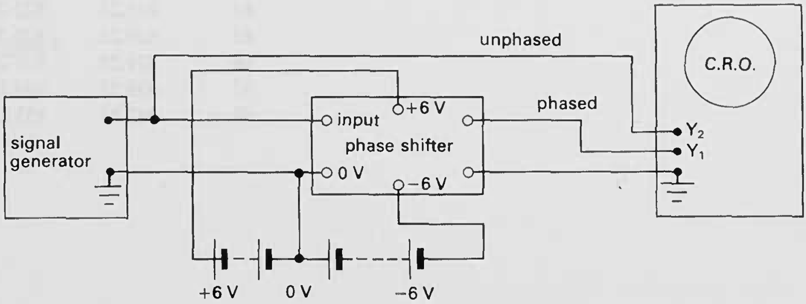


Figure 6

The output is shown on a dual beam oscilloscope; 'add' and 'X–Y' facilities are an advantage. Possible outputs are:

1 The two traces shown together, the shift in phase being immediately apparent.

2 The two traces added to show 'constructive interference' – two traces in phase; and 'destructive interference' as the two signals are brought into antiphase.

3 The Lissajou figures as the phase difference moves from 0 to $\pi/2$ to π radians.

With the component values given, the full phase shift is obtained in the range 1–10 kHz; best input frequency is obtained by tuning the signal generator input to obtain the full range in the Lissajou figure. A logarithmic potentiometer facilitates adjustment at the end of the range.

References

This module was developed by R. P. Ballingall.

The circuit shown was originally adapted from an article by Haglberg (*Phys. Teach.* **16** 58–59, 1978) and substantially the same circuit has since been published by Lopes, Melo, and Oliviera (*Phys. Educ.* **17** 238–240, 1982).

APPENDIX V

Lines of force

In earlier courses students may have used lines to sketch the shapes of magnetic fields. We do not stress the concept of lines of force in the course as it can lead to confusion and error. In figure 7(a), for instance, the field strength can be estimated at any point by reference to the spacing of equipotentials. If field lines are to be drawn so that the line density indicates the strength of the field then one must decide how many lines to draw and where, on the conductors, they should terminate. This is easier if the total charge and its distribution are known, but this is rarely the case.

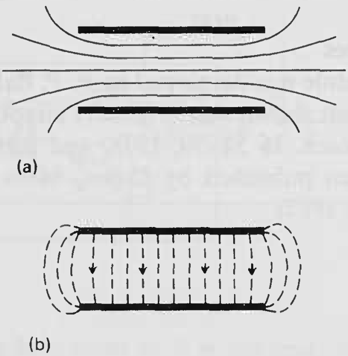


Figure 7
Two ways of representing a field.
(a) Equipotentials.
(b) Field lines.

Figure 7(b), for instance, suggests that charges only exist on the conductors where the field lines terminate. What happens between the lines – is there no field here? Of course there *is* a field here but students may easily fall into this confusion.

Also, in some circumstances, field lines cannot be correctly drawn. For example, beneath the surface of the Earth (figure 8) the gravitational field strength decreases with depth as shown by the increasing spacing of the equipotentials, yet the (erroneous) figure shows field lines getting closer together. The reason is that field lines will not be continuous through space which contains mass (remember Gauss's theorem) so it becomes very difficult to use them to represent the field.

However, the equipotentials still give a clear and true representation of the field at all points (even though the intervals between them must essentially be arbitrary).

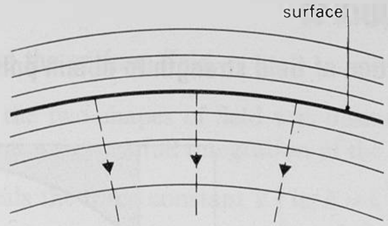


Figure 8
Equipotentials true, field lines false.

APPENDIX VI

Integration of field strength to obtain potential

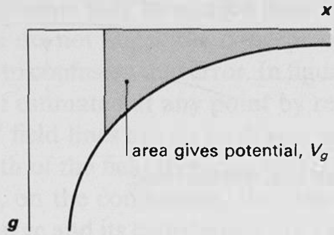


Figure 9
Area under a g against x graph.

The gravitational potential at a point is defined as the amount of energy required to bring unit mass from infinity (where its potential is zero) to that point. This may be found directly from a g against x graph by integration. Note, however, that the expression for g is negative ($g = -GM/x^2$), that the integration proceeds *from* infinity *to* the point, and that there is a negative sign in the relationship between field and potential gradient. This leads eventually to an expression for V_g which is also negative.

Since $g = -\dfrac{dV_g}{dx}$

then $$V_g = -\int_{\infty}^{r} g\,dx$$

$$= -\int_{\infty}^{r} -\frac{GM}{x^2}\,dx$$

$$= +GM\int_{\infty}^{r} \frac{dx}{x^2}$$

$$= GM\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= GM\left[-\frac{1}{r} - \left(-\frac{1}{\infty}\right)\right]$$

$$= -\frac{GM}{r}$$

APPENDIX VII

The radial field and the uniform field

The link between the two shapes of field was discussed in the text in physical terms. Here we give a full integration of the effect of a sheet of charge which reveals the force constant k $\left(\text{in } F = k\dfrac{Q_1 Q_2}{r^2}\right)$ as $\dfrac{1}{4\pi\varepsilon_0}$.

Integration of the field produced by a flat sheet of charge

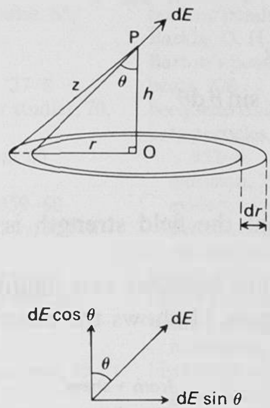


Figure 10
Field of one element of a ring of charge.

Consider a flat sheet with charge density σ. To calculate the total field at P, at height h above the sheet, consider a ring of charge centred on O of radius r and width dr.

The total area of this ring is $2\pi r\mathrm{d}r$

so it carries charge $2\pi\sigma r\mathrm{d}r$

A point charge of this value would make a contribution dE to the field at P of

$2\pi k\sigma r\mathrm{d}r/z^2$

By symmetry, the horizontal components of contributions from the ring cancel out leaving only the vertical component ($\mathrm{d}E\cos\theta$). To calculate the total field strength at P we must add up contributions from all rings from $\theta = 0$ to $\pi/2$ (*i.e.*, covering an infinite sheet).

$$z = \frac{h}{\cos\theta} = h\sec\theta$$

$$r = h\tan\theta \Rightarrow \mathrm{d}r = h\sec^2\theta\,\mathrm{d}\theta$$

$$E = 2\pi k\sigma \int_0^{\pi/2} \frac{r\mathrm{d}r\cos\theta}{z^2}$$

$$= 2\pi k\sigma \int_0^{\pi/2} \frac{h\tan\theta\, h\sec^2\theta\,\mathrm{d}\theta\cos\theta}{h^2\sec^2\theta}$$

$$= 2\pi k\sigma \int_0^{\pi/2} \sin\theta\,\mathrm{d}\theta$$

$$= 2\pi k\sigma$$

Note that the field strength is independent of the height h: it is uniform.

Let us now consider two infinite parallel plates carrying opposite charges. Figure 11 shows the contributions to the field strength from each plate.

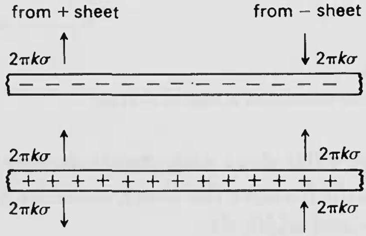


Figure 11
Adding up fields from 2 parallel sheets.

Adding the contributions from the two plates shows that there is no field outside the plates. The field between them is given by

$$E = 4\pi k\sigma$$

Comparing this with the expression we already have for parallel plates (page 312), that

$$E = \sigma/\varepsilon_0$$

we deduce that

$$k = \frac{1}{4\pi\varepsilon_0}$$

INDEX

General editor, Revised Nuffield Advanced Physics
John Harris

Consultant editor
E.J. Wenham

Editors of Units in this *Guide*
Mark Ellse
David Grace
Roger Hackett
Peter Harvey
Paul Jordan
Charles Milward
Trevor Sandford
Maurice Tebbutt
Nigel Wallis

Contributors to this *Guide*
Michael Carrick
Bev Cox
Mark Ellse
John Gardner
David Grace
Tom Gregory
Roger Hackett
Peter Harvey
Paul Jordan
April Bueno de Mesquita
Charles Milward
John Minister
Susan Ross
Trevor Sandford
Richard Spiby
Maurice Tebbutt
Mark Tweedle
Nigel Wallis

This Teachers' guide **supports the first seven of the twelve Units in the Revised Nuffield Advanced Physics course.**

The Units are: Unit A, 'Materials and mechanics'; Unit B, 'Currents, circuits, and charge'; Unit C, 'Digital electronic systems'; Unit D, 'Oscillations and waves'; Unit E, 'Field and potential'; Unit F, 'Radioactivity and the nuclear atom'; and Unit G, 'Energy sources'.

For each Unit there is a plan summarizing the relationship of the work to topics covered earlier. There are very many teaching suggestions, including details of the suggested experiments and demonstrations; and there are references to other resources, for example, books, videos, and computer programs. Full answers to the questions in the Students' guide **are provided.**

ISBN 0 582 35417 X

Longman