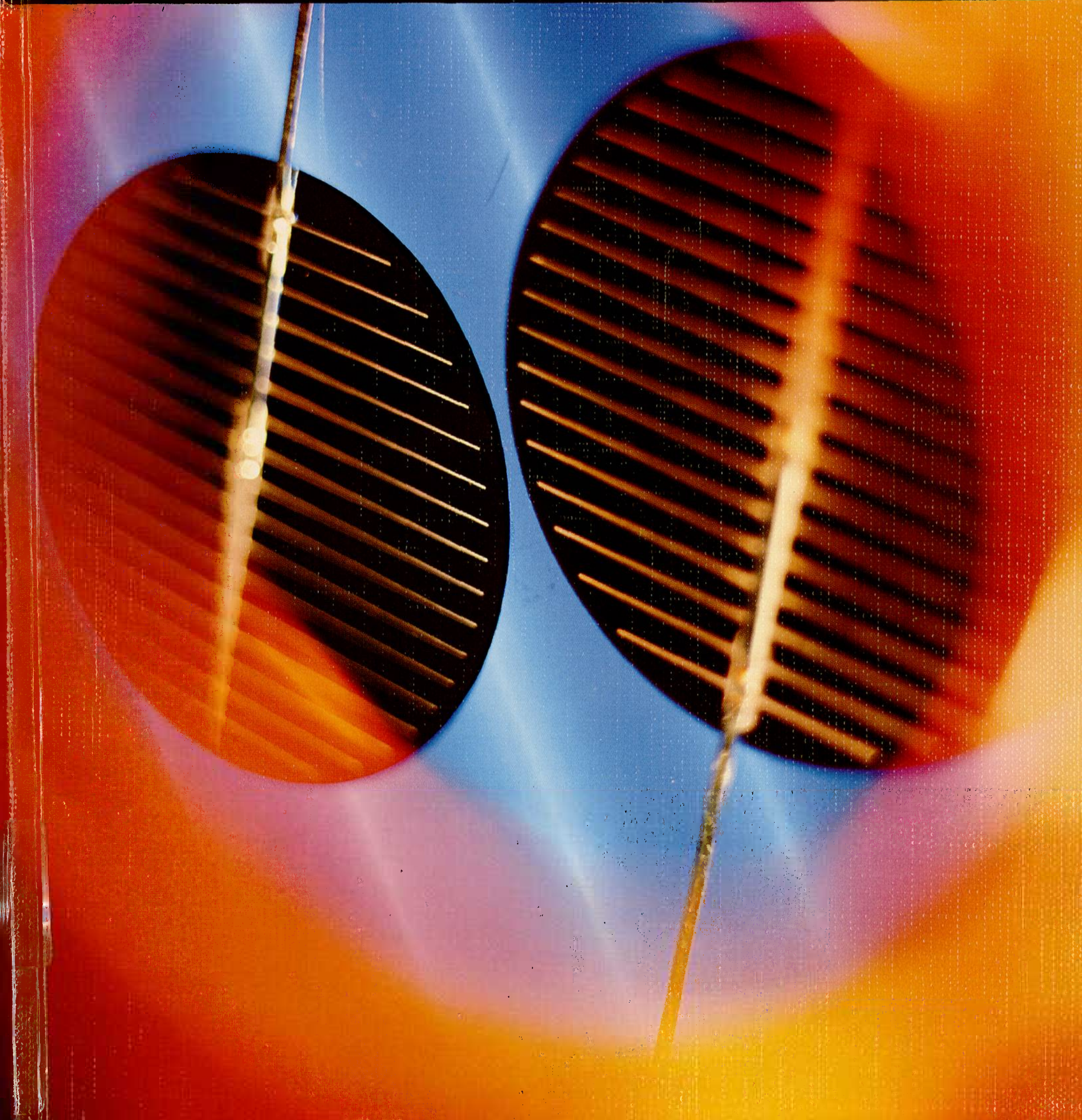
REVISED

NUFFIELD ADVANCED SCIENCE

PHYSICS

STUDENTS' GUIDE 2 UNITS H to L



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PHYSICS

STUDENTS' GUIDE 2

UNITS H to L

Revised Nuffield Advanced Science

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The Nuffield–Chelsea Curriculum Trust is grateful to the authors and editors of the first edition:

Organizers: P. J. Black, Jon Ogborn; **Contributors:** W. Bolton, R. W. Fairbrother, G. E. Foxcroft, Martin Harrap, John Harris, A. L. Mansell, A. W. Trotter.

PHYSICS STUDENTS' GUIDE 2

UNITS H to L

REVISED NUFFIELD ADVANCED SCIENCE

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Cover

Silicon solar cells. Many of these cells go to make up a solar panel for the direct generation of electricity from solar energy.

Paul Brierley

CONTENTS

H

I

J

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FOREWORD

When the Nuffield Advanced Science series first appeared on the market in 1970, they were rapidly accepted as a notable contribution to the choices for the sixth-form science curriculum. These courses were devised by experienced teachers working in consultation with the universities and examination boards, and subjected to extensive trials in schools before publication and they introduced a new element of intellectual excitement into the work of A-level students. Though the period since publication has seen many debates on the sixth-form curriculum, it is now clear that the Advanced Level framework of education will be with us for some years in its established form. That period saw various proposals for change in structure which were not accepted but the debate to which we contributed encouraged us to start looking at the scope and aims of our A-level courses and at the ways they were being used in schools. Much of value was learned during those investigations and has been extremely useful in the planning of the present revision.

The revision of the physics course under the general editorship of John Harris has been conducted with the help of a committee under the chairmanship of K. F. Smith, Professor of Physics, University of Sussex. We are grateful to him and to the committee, whose other members were W. F. Archenhold, J. Bausor, Professor P. J. Black, Professor R. Chambers, A. E. De Barr, Roger Hackett, John Harris, Wilf Mace, Robert Northage, Professor Jon Ogborn, A. J. Parker, and Maurice Tebbutt. We also owe a considerable debt to the Oxford and Cambridge Schools Examination Board which for many years has been responsible for the special Nuffield examinations in physics and to the Assistant Secretary of the Board, Mrs B. G. Fraser, who has been an invaluable adviser.

The Nuffield–Chelsea Curriculum Trust is also grateful for the advice and recommendations received from its Advisory Committee, a body containing representatives from the teaching profession, the Association for Science Education, Her Majesty's Inspectorate, universities, and local authority advisers; the committee is under the chairmanship of Professor P. J. Black, educational consultant to the Trust.

Our appreciation also goes to the editors and authors of the first edition of Nuffield Advanced Physics, who worked with Jon Ogborn and P. J. Black, the project organizers. Their team of editors and writers included: W. Bolton, R. W. Fairbrother, G. E. Foxcroft, Martin Harrap, John Harris, A. L. Mansell, and A. W. Trotter. Much of their original work has been preserved in the new edition.

I particularly wish to record our gratitude to the General Editor of the revision, John Harris, Lecturer at the Centre for Science and Mathematics Education, Chelsea College, and a member of the team responsible for the first edition. To him, to E. J. Wenham, Consultant Editor of the revision, and to the editors of the Units in the revised course – all teachers with a wide experience of the needs of students and of the current state of physics education – Roger Hackett, Nigel Wallis, David Grace, Mark Ellse, Charles Milward, Trevor Sandford, Paul Jordan, Peter Harvey, Maurice Tebbutt, David Chaundy, Wilf Mace, Stephen Borthwick, Peter Bullett, and Jon Ogborn, we offer our most sincere thanks.

I would also like to acknowledge the work of William Anderson, publications manager to the Trust, his colleagues, and our publishers, the Longman Group, for their assistance in the publication of these books. The editorial and publishing skills they contribute are essential to effective curriculum development.

K. W. Keohane,
Chairman, Nuffield–Chelsea Curriculum Trust

INTRODUCTION

This is the *Students' guide* for the second year of the Revised Nuffield A-level physics course. It covers Units H, I, J, K, and L. An introduction to the whole course, and an explanation of the way in which the *Students' guides* are organized is given under the heading 'About the course and about this book', in *Students' guide 1*.

ACKNOWLEDGEMENTS

One of the pleasantest aspects of the development of *Revised Nuffield Advanced Physics* has been the willing way in which so many people have contributed and become involved in the work. Above all, teachers have helped in many ways, and the very number who have done so makes it impossible to acknowledge the contribution of each individual. Many have offered suggestions at meetings or have written in with ideas for questions, demonstrations, and so on. We have tried to consider carefully all the suggestions put forward and, inevitably, it is impossible to give proper credit to the source or origin of every idea we have used. One who has made a particularly valuable contribution in this way is Colin Price. To him and the many others whose contributions go unacknowledged, we offer our sincere thanks.

Other teachers have helped by conducting trials of some of the more radically changed parts of the course, and of a major innovation – the 'Dynamic modelling system'. The trial schools are: Aylesbury Grammar School; Beechen Cliff School, Bath; Bexley–Erith Technical High School, Bexley; Bishop Hedley High School, Merthyr Tydfil; Cheltenham College; Esher College; Forest Hill School, London; Godolphin and Latymer School, London; The Grammar School, Batley; The Greenhill School, Tenby; Haverstock School, London; Heathland School, Hounslow; Henbury School, Bristol; Highfield School, Wolverhampton; Howell's School, Llandaff; King Edward VI College, Nuneaton; Kingsbridge School; Lady Margaret High School, Cardiff; Malvern College; Marlborough College; Netherhall School, Cambridge; North London Collegiate School; Northgate High School, Ipswich; Oulder Hill Community School, Rochdale; Richmond-upon-Thames College; Royal Grammar School, High Wycombe; Rugby School; Samuel Ward Upper School, Haverhill; and Sutton Manor High School.

We are grateful to the Inner London Education Authority for trying some of our material on electronics in their 1983 Summer School for sixth-form students at the North London Science Centre.

Mark Ellse has read and commented on much of the draft material, and has made particularly useful suggestions about the up-dating of some experiments and pieces of equipment.

Thanks are due to a group of teachers, convened by Bob Fairbrother, who met several times to discuss assessment. Their suggestions led to some changes in the structure of the examination.

Others, as well as teachers, have helped, of course. While he was working as a technician at the Centre for Science and Mathematics Education, Chelsea College, Phil Webb found time in a busy schedule to try out ideas for demonstrations and experiments, and to suggest ideas for new apparatus.

CLEAPSE School Science Service reviewed all the suggested experiments and demonstrations and made useful suggestions on the safety aspects of some of them.

Industry has helped too, and, among others, we are indebted to Rank Xerox, Amersham International P.L.C., and the CEGB for technical help and information.

Examination questions in the *Students' guide* are reprinted by permission of the Oxford and Cambridge Schools Examination Board. All are taken from Oxford and Cambridge Nuffield A-level Physics papers. Where guide lines for answers to examination questions are provided it must be understood that these are not the Examination Board's responsibility.

The Consultative Committee have, I believe, been asked to work harder and contribute more than is usually expected of such a group. As well as attending many meetings they have read and commented in detail on draft manuscripts – sometimes in a far from ideal state – and they have done all this most willingly.

It is a pleasure to acknowledge E. J. Wenham's help and sound advice. Much of what is written in these books has benefited from his knowledge and experience as teacher and author.

All of us who have contributed to these books owe a great debt of gratitude to Nina Konrad and her colleagues in the Publications office of the Nuffield–Chelsea Curriculum Trust for their thorough and painstaking work in preparing our manuscripts for the printers and our sometimes quite inadequate drawings for the artists.

Finally, I would like to express my sincere thanks to Paul Black and Jon Ogborn. Their help and support has been invaluable. During a period when both have been particularly busy, they have still found time to give advice both on general matters and on points of detail. They were, of course, the chief architects of the original Nuffield Advanced Physics course. Their willingness to be involved with what must at times have seemed like a severe distortion of their original plans, says much about their generosity of spirit.

John Harris

Unit H
MAGNETIC FIELDS AND A.C.

David Chaundy
Malvern College

H

SUMMARY OF THE UNIT

SUMMARY OF THE UNIT

INTRODUCTION

Magnetic fields are more complex than either electric or gravitational fields. The strength of a magnetic field when iron is present is difficult to calculate. This is why the treatment here is largely practical rather than theoretical, and why it is aimed at showing how to make magnetic fields and how to apply them effectively.

Perhaps the most important applications are to be found in the generation of alternating current supplies for the mains distribution system and, at much higher frequencies, for use in radio communication and television. The Unit is also concerned with some applications of a.c. which are not possible with d.c.

Section H1 MAGNETIC FIELDS

Dry cells, solar cells, and fuel cells do not depend on magnetic fields. However, they are incapable of providing the amounts of energy upon which our industry, commerce, agriculture, transport, and our homes depend. For this we need electrical generators capable of transforming vast amounts of energy derived from the combustion of fossil or nuclear fuels. All such generators depend on electromagnetic induction in strong magnetic fields. This Section will show how electric currents produce magnetic fields, how these fields are measured, and how they are used to handle atomic particles.

Forces on currents

DEMONSTRATION H1
Forces on currents;
forces between currents

Two long flexible conductors attract one another if they carry currents in the same direction (as if they were trying to become one wire). They repel each other if the currents are in opposite directions (figure H1). These forces are very small compared, for example, with those needed to drive an electric train.

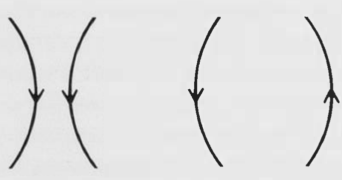


Figure H1
Forces on currents.

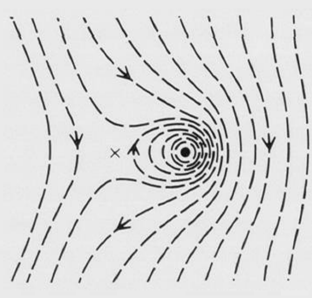


Figure H2
The catapult field.

OPTIONAL DEMONSTRATION H2
The catapult field

A wire carrying a current is pushed aside when there is a magnetic field at right angles to the wire. The 'catapult field' demonstration shows how the magnetic field due to the current in the wire and the external field combine (figure H2).

DEMONSTRATION H3
Forces on induced currents

An explanation of the 'jumping ring' demonstration (figure H3) also involves the magnetic forces between currents.

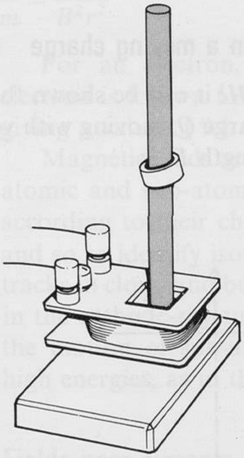


Figure H3
The jumping ring.

Gravitational field strength is given by

$g = F/m$

and the force acts in the direction of the field.

Electric field strength is given by

$E = F/Q$

and the force acts in the direction of the field.

DEMONSTRATION H4
Directions of forces in magnetic fields; measuring magnetic fields

The strength of a magnetic field is given by

$B = F/Il$

and the force acts in a direction *perpendicular* to both the current and the field, given by the left hand rule. See figure H4.

g is measured in $\mathrm{N\,kg^{-1}}$; E is measured in $\mathrm{N\,C^{-1}}$; B is measured in $\mathrm{N\,A^{-1}\,m^{-1}}$ or tesla (T).

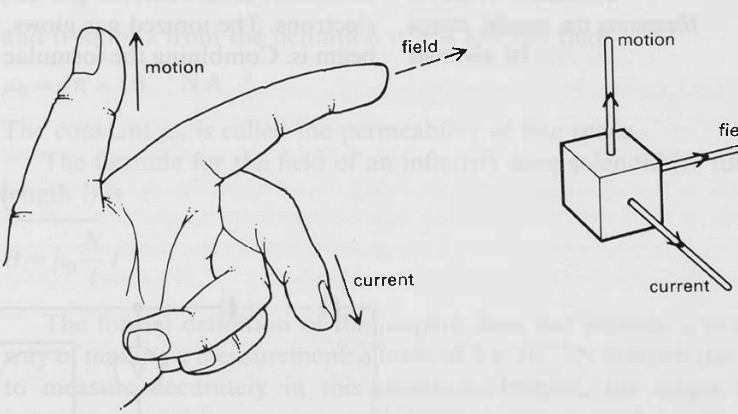


Figure H4
Force–direction rule.

The strength of a magnetic field may be measured by putting a wire of length l in the field, sending a current I through it, and measuring the force, F, on the wire. B is also called the flux density of a magnetic field.

QUESTIONS 1 to 5

Like g and E, B is a vector quantity.

The force on a moving charge

QUESTION 7

From $F = BIl$ it can be shown that the force on an individual particle carrying charge Q, moving with velocity v at right angles to a magnetic field of strength B, is

$F = BQv$

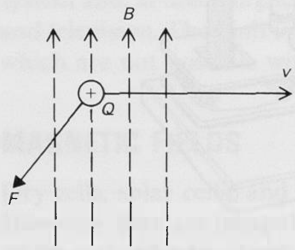


Figure H5
$F = BQv$.

QUESTIONS 8 to 10

READING
Electromagnetic flowmeters
The 'Hall-effect' flowmeter (page 18)

DEMONSTRATION H5
Hall effect; Hall probe

In a demonstration of the Hall effect in a semiconductor the magnetic force on a moving charge is balanced by the electric force on it. The electric field is V/d and so the electric force is QV/d.

Thus $BQv = QV/d$, giving the Hall voltage $V = Bvd$.

A Hall probe can therefore be used to compare magnetic fields by comparing voltages provided that the speed of the charge carriers, v, is kept constant. This means that the current in the Hall probe must be kept constant.

Measurement of e/m

QUESTIONS 6, 11

DEMONSTRATION H6
Measuring the specific charge for electrons

A beam of electrons can be bent into a circular path by a uniform magnetic field at right angles to the electrons' direction of motion. A fine-beam tube contains gas at low pressure which is ionized by the electrons. The ionized gas glows, enabling us to see where the electron beam is. Combining the formulae for centripetal force (mv^2/r), magnetic

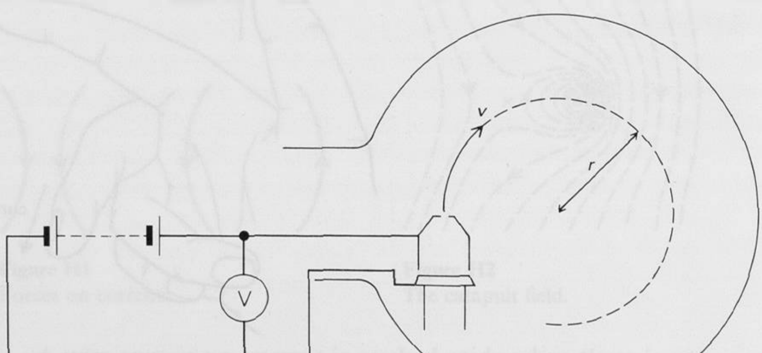


Figure H6
Measuring the specific charge of an electron.

force on an electron (Bev), and for energy of an electron accelerated through a p.d. V ($\frac{1}{2}mv^2 = eV$) leads to an expression for the specific charge (that is, charge per unit mass) for the electron:

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

For an electron, $e/m = 1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$. The charge on the electron is known to be $1.60 \times 10^{-19}\,\mathrm{C}$ (from Millikan's experiment), giving a value of $9.1 \times 10^{-31}\,\mathrm{kg}$ for the electron's mass.

Magnetic fields are used in many devices and experiments involving atomic and sub-atomic particles. They are used to separate particles according to their charge-to-mass ratio, as in the mass spectrometer, and so to identify isotopes, and to identify particles from their curved tracks in cloud and bubble chambers. They are used to deflect electrons in the cathode-ray tube of a monitor or television set, and to control the motion of sub-atomic particles as they are accelerated to very high energies, as in the cyclotron and synchrotron.

QUESTIONS 12 to 16

Fields near currents

The field strength near a long straight wire can be shown to be inversely proportional to the distance from the wire and proportional to the current flowing in it.

EXPERIMENT H7
Fields near electric currents

Thus

$$B \propto I/r$$

The ampere is defined as that current which, if maintained in two infinitely long conductors of negligible diameter, placed 1 metre apart in a vacuum, would give a force between the conductors of $2 \times 10^{-7}\,\mathrm{N}$ per metre length. The ampere is the basic unit from which all other electrical and magnetic units (coulomb, volt, tesla, etc.) are derived.

The constant of proportionality is written as $\mu_0/2\pi$, so

QUESTIONS 17 to 22

$$B = \frac{\mu_0 I}{2\pi r}$$

and it follows from the definition of the ampere that

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$$

The constant μ_0 is called the permeability of free space.

The formula for the field of an infinitely long solenoid (N turns in length l) is

QUESTION 19

$$B = \mu_0 \frac{N}{l} I$$

The formal definition of the ampere does not provide a practical way of making a measurement: a force of $2 \times 10^{-7}\,\mathrm{N}$ is much too small to measure accurately in this situation. Instead, the larger forces between solenoids are measured using instruments for calibrating ammeters, like that shown in figure H7.

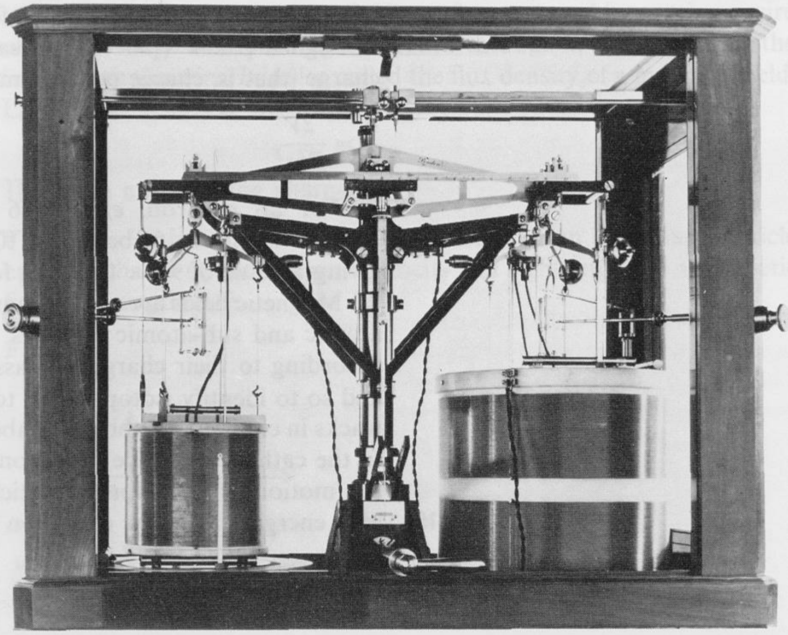


Figure H7
National Physical Laboratory current balance.
National Physical Laboratory, Crown Copyright.

Section H2 ELECTROMAGNETIC INDUCTION

About 150 years ago Michael Faraday found that there were many ways in which an e.m.f. could be *induced* in a circuit without any electrical contact with the circuit. All that was needed was some movement or some change of a magnetic field.

One very important use of electromagnetic induction is in generators, which move coils or magnets to produce electricity. It is also used in transformers, where nothing can be seen moving, and in induction motors, which have no electrical connection to the rotor. Another example is an inductor, where the e.m.f. is induced in the circuit which is itself producing the magnetic field.

DEMONSTRATION H8
The e.m.f. induced in a moving wire

The force which acts on a charged particle moving in a magnetic field leads to an induced e.m.f. in a conductor moving through a magnetic field. If a length of wire l is perpendicular to a magnetic field B and moves at right angles to both its length and the field at a speed v, the magnetic force on each electron is Bev (figure H8). Electrons begin to move along the wire and there is a separation of charge leading to a potential difference between the ends of the wire. This p.d. is called the *induced e.m.f.*, $\mathscr{E}$. If the length of the wire is l, the electric field in it is $\mathscr{E}/l$ and the electric force on each electron is $e\mathscr{E}/l$. When the electrons are in equilibrium the electric and magnetic forces balance:

$$e\mathscr{E}/l = Bev$$

QUESTIONS 23 to 25

$$\mathscr{E} = Bvl$$

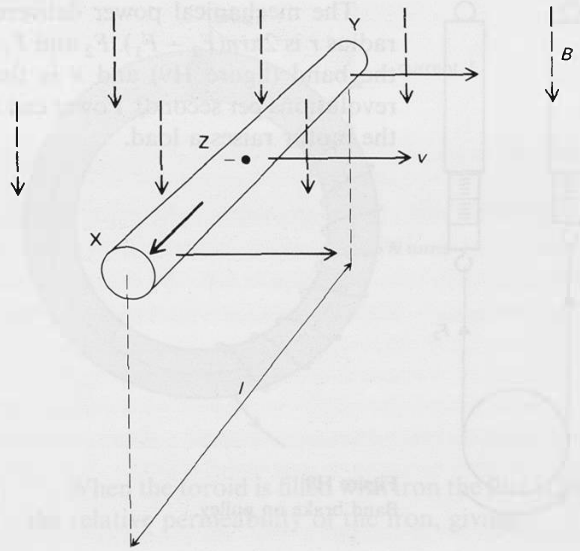


Figure H8
Conductor moving across a magnetic field.

Direct current generators and motors

DEMONSTRATION H9
Inducing e.m.f.s in a motor/dynamo

When the rotor of a generator is turned, the rotor coils move through a magnetic field. An e.m.f. proportional to the rate of rotation is induced. If an external circuit is connected a current will flow.

The torque of a motor is the couple produced by the rotor. From $F = BIl$ it follows that, for a given motor, the torque is proportional to I, the rotor current. Small motors use permanent magnets so the field, B, is fixed. Larger motors use iron-cored coils to produce the field, so B and hence the torque increase with the current in these field coils.

In a motor, the turning rotor is a conductor moving in a magnetic field. So when it rotates an e.m.f. is induced in it which is proportional to the field, B, and to the rate of rotation. The current in the circuit is driven by the applied p.d., V, and opposed by the e.m.f., $\mathscr{E}$, induced in the rotor. The rotor current is therefore given by

$$V - \mathscr{E} = IR$$

where R is the resistance of the rotor coils.

QUESTIONS 26, 27

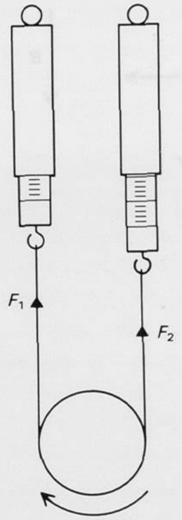
If R is small, $\mathscr{E}$ is almost as big as V. If the load on the motor is increased more torque is needed and the rotor current I must increase. The rotor slows down, $\mathscr{E}$ is reduced, and I grows.

The equation $V - \mathscr{E} = IR$ leads to

$$VI = I^2R + \mathscr{E}I$$

DEMONSTRATION H10;
OPTIONAL EXPERIMENT H11
Behaviour and efficiency of motors

VI is the total power taken from the supply. I^2R is the power which heats the rotor and it is wasted. $\mathscr{E}I$ is the power transferred to the load, used to overcome mechanical losses (for example in the bearings), etc.



The mechanical power delivered by a motor to a band brake of radius r is $2\pi rn(F_2 - F_1)$. F_2 and F_1 are the tensions on the two sides of the band (figure H9) and n is the rate of rotation of the rotor (in revolutions per second). Power can also be found from the rate at which the motor raises a load.

Figure H9
Band brake on pulley.

Flux

QUESTIONS 28, 29

The flux, Φ, through any circuit is the product of B and the area A of the circuit. If the flux links with N turns of wire the total flux linkages are NBA. The unit of flux is the weber (Wb). Since $B = \Phi/A$ the magnetic field strength is also the flux per unit area or *flux density.*

Faraday's Law of electromagnetic induction

DEMONSTRATION H12
Moving wires and changing flux
DEMONSTRATION H13
A continually changing field
STUDENT DEMONSTRATION H14
Induction using a.c.

It is useful to know Faraday's Law in two slightly different forms:

When a conductor cuts magnetic flux, the e.m.f. induced is equal to the rate at which flux is cut.

When the flux linked with a circuit changes the e.m.f. induced is equal to the rate of change of flux linkages.

Lenz's rule

The magnitude of the induced e.m.f., $\mathscr{E}$, is given by $\mathscr{E} = N\mathrm{d}\Phi/\mathrm{d}t$. The direction of the induced e.m.f. is such that it tends to oppose the motion or change causing it. This must follow from the conservation of energy.

READING
Electromagnetic flowmeters
The turbine flowmeter (page 16)

The effect of iron

Inside a toroid (figure H10) the flux density is the same as in an infinitely long cylinder with the same number of turns per unit length

$$B = \mu_0 \frac{N}{l} I$$

If the toroid has a cross-sectional area A, the flux inside it is

$$\Phi = BA = \mu_0 \frac{N}{l} IA$$

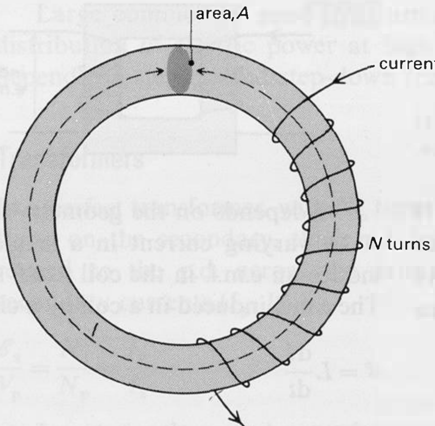


Figure H10
A toroid.

DEMONSTRATION H15
The effect of iron in a solenoid

When the toroid is filled with iron the flux is increased by a factor μ_r, the relative permeability of the iron, giving

$$\Phi = \mu_0 \mu_r \frac{N}{l} IA$$

μ_r may be as high as 1000 for soft iron; it varies greatly as the flux in the iron changes. The equation can be rearranged to give

$$\Phi = \frac{NI}{(l/\mu_0 \mu_r A)}$$

The product NI is often referred to as 'current turns', and the quantity $l/\mu_0 \mu_r A$ is known as the *reluctance.* The equation

QUESTIONS 31 to 35

flux = current turns/reluctance

can be compared with

current = potential difference/resistance.

EXPERIMENT H16
Increasing the iron in a magnetic circuit

When a solenoid has an iron core with an air gap, the two reluctances (iron and air) are first added like resistances in series, and then the flux can be calculated using the formula above. High flux densities ($\Phi/A = B$) are needed if an electromagnetic machine (motor, generator, or transformer) is to be efficient, which is why they contain a lot of iron with only small air gaps.

Inductance

DEMONSTRATION H17
Mutual inductance of two coils

A varying current in one coil sets up a varying magnetic field which can induce an e.m.f. in a second coil nearby (figure H11). This induced e.m.f. depends on the rate of change of current in the first coil.

$$\mathscr{E} \propto \mathrm{d}I/\mathrm{d}t$$

The constant of proportionality is called the *mutual inductance*, M,

QUESTION 36

$$\mathscr{E} = M \frac{\mathrm{d}I}{\mathrm{d}t}$$

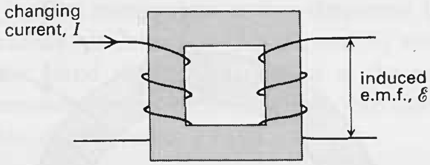


Figure H11
Mutual inductance.

DEMONSTRATION H18
Self induction
DEMONSTRATION H19
Measurement of self inductance

M depends on the geometry of the coils and any iron present.

A varying current in a single coil causes changes of flux which induce an e.m.f. in the coil itself, so every coil has a *self inductance*, L. The e.m.f. induced in a coil by a changing current in the coil is given by

$$\mathscr{E} = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

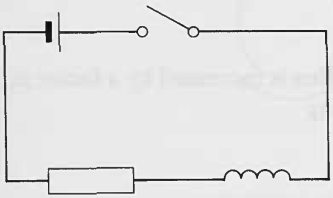


Figure H12
LR circuit.

When a steady p.d. is connected to a circuit with resistance and inductance (figure H12),

$$V = IR + L\frac{\mathrm{d}I}{\mathrm{d}t} \qquad \text{(compare the equation for a motor)}$$

QUESTIONS 37 to 41

When I is very small, $\mathrm{d}I/\mathrm{d}t \approx V/L$, and the initial rate of rise of current depends only on V and L. A steady applied p.d. produces a current which initially grows at a uniform rate.

When the current has stopped rising, $\mathrm{d}I/\mathrm{d}t$ is zero and so $I = V/R$: the final current depends only on V and R. See figure H13.

If the current is switched off suddenly, $\mathrm{d}I/\mathrm{d}t$ is very large and a very big e.m.f. is induced.

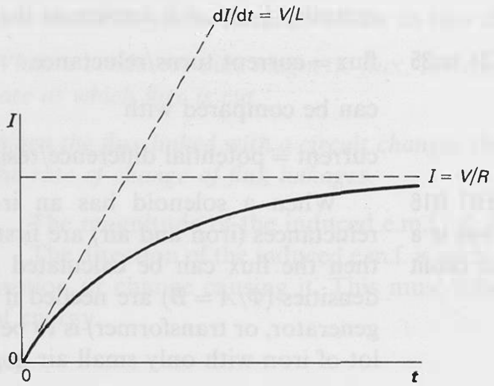


Figure H13
Growth of current in *LR* circuit.

Section H3 ALTERNATING CURRENT

An alternating current will do things which a direct current will not do. If a steady p.d. is applied to a circuit containing a capacitor there may be a short-lived pulse of current – but no steady current is possible. However, if an alternating p.d. is applied to such a circuit, an alternating current will be maintained as long as the p.d. is applied. And since the flux due to an alternating current is constantly changing, an a.c. will induce e.m.f.s continually in an inductor, whereas with steady d.c. the induced e.m.f.s occur only when the current is switched on or off.

READING
The generation and transmission of electric power (page 20)

Large commercial generators are a.c. machines. The economical distribution of electric power at high voltage on the National Grid depends on step-up and step-down transformers.

QUESTION 48

Transformers

EXPERIMENT H20
Investigation of transformer action

In a perfect transformer, with N_p turns on the primary winding and N_s turns on the secondary, the e.m.f. induced in the secondary ($\mathscr{E}_s$) is related to the p.d. across the primary (V_p) and the primary and secondary currents (I_p, I_s) by

QUESTIONS 42 to 47

$$\frac{\mathscr{E}_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

In practice there are losses of energy both in the windings (due to the resistance of the wire) and in the iron core, so that these equations do not hold exactly. In a real transformer the primary current is a little greater, and the number of turns on the secondary needs to be higher.

Eddy currents

DEMONSTRATION H21
Eddy currents

When the flux in a solid piece of iron changes, an e.m.f. is induced in the iron, causing large eddy currents and loss of energy. To reduce such eddy currents the iron core of a transformer is made of laminations which are insulated from each other (figure H14). Eddy currents can also occur in direct current machines, such as motors, if the flux in the iron is changing, and so their iron cores are laminated too.

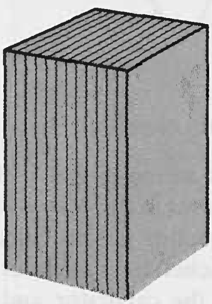


Figure H14
Laminated transformer core.

Alternating current in a resistor

DEMONSTRATION H22
Power in a resistive circuit

When a direct current is adjusted to make a lamp glow at the same brightness as with a sinusoidal a.c., the steady p.d. across the lamp is rather more than half the peak value of the alternating p.d.

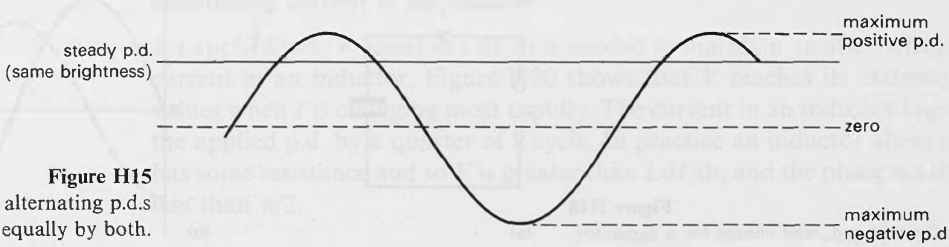


Figure H15
Steady and alternating p.d.s across a lamp lit equally by both.

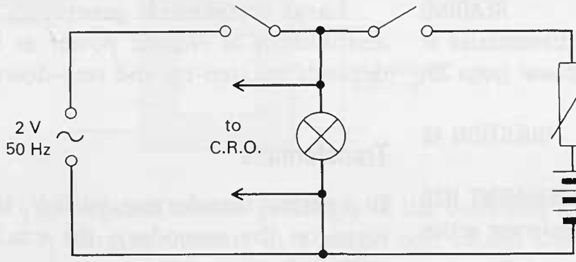


Figure H16
Comparing the brightness of a lamp lit from a.c. and d.c.

QUESTION 49

When current flows through a resistance R and the p.d. across the resistor is V, the power dissipated at any moment is V^2/R. For an a.c. with a sinusoidal waveform, V^2 will vary with time as shown in figure H17: V^2 varies with twice the frequency of V. Because power $\propto V^2$ the total energy is proportional to the area under the graph of V^2 against time. Since this graph is symmetrical about $\frac{1}{2}V_0{}^2$, the mean power is $\frac{1}{2}V_0{}^2/R$. Hence the effective voltage is $\dfrac{1}{\sqrt{2}}V_0$, which is called the root mean square (r.m.s.) voltage. This is the value shown by the line labelled 'steady p.d.' in figure H15.

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}} \approx 0.707\ V_0$$

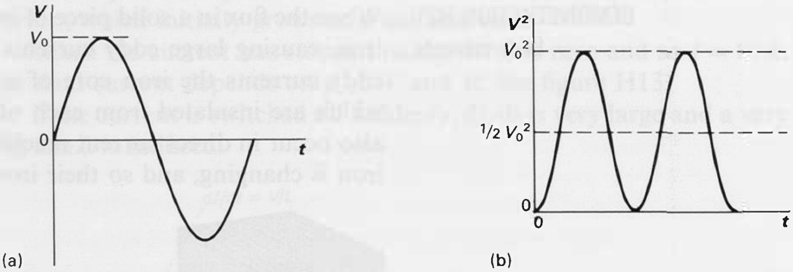


Figure H17
Variation of (a) V and (b) V^2 with time.

Capacitors in a.c. circuits

DEMONSTRATION H23
Slow a.c. in a circuit containing a capacitor

Positive current in the circuit of figure H18(a) charges the capacitor positively, as shown in the graph – figure H18(b). When the current falls to zero the capacitor has its maximum positive charge and the p.d. across it has reached its maximum positive value. The negative current first discharges the capacitor and then charges it negatively and so, when the current has risen to zero again, the p.d. across the capacitor has its maximum negative value. The current through the capacitor thus leads the voltage by one-quarter of a cycle (90° or $\pi/2$). See figure H18(b).

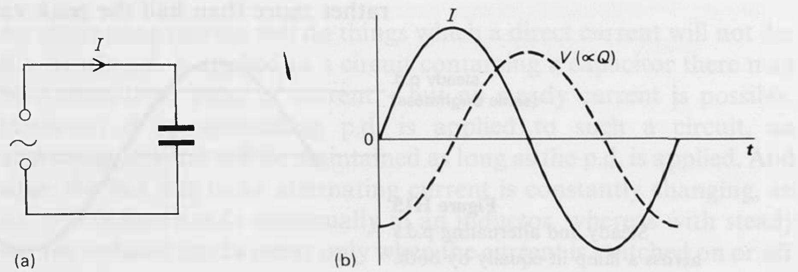


Figure H18
Current, p.d., and charge for a capacitor.

QUESTION 50 To reach the same maximum p.d. at a higher frequency, the same charge must flow in a shorter time, and so the current is greater.

With a larger capacitance at a given frequency, more charge is needed in the same time and so more current flows, for the same p.d.

This means that $I \propto fC$.

Power in a capacitor

DEMONSTRATION H24
Power in a capacitor

QUESTIONS 51, 52

The power at any instant is given by VI and is alternately positive and negative (see figure H19). When a circuit containing capacitance only is connected to an a.c. supply, no net energy is taken from the supply: energy which is taken from the supply while the capacitor is charging up (either positively or negatively) is returned when the charge drops to zero.

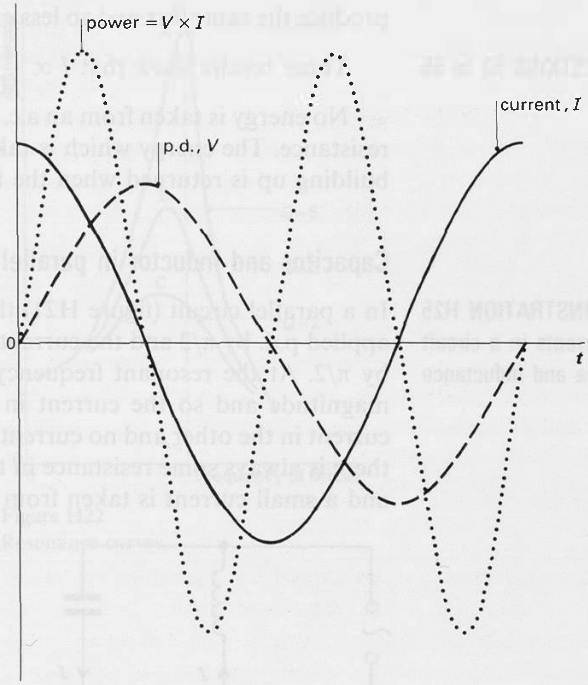
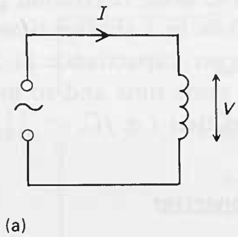


Figure H19
Variation with time of current, p.d., and power in a capacitor circuit.

Alternating current in an inductor

An applied p.d., V, equal to $L\mathrm{d}I/\mathrm{d}t$ is needed to maintain an alternating current in an inductor. Figure H20 shows that V reaches its extreme values when I is changing most rapidly. The current in an inductor lags the applied p.d. by a quarter of a cycle. In practice an inductor always has some resistance and so V is greater than $L\mathrm{d}I/\mathrm{d}t$, and the phase lag is less than $\pi/2$.



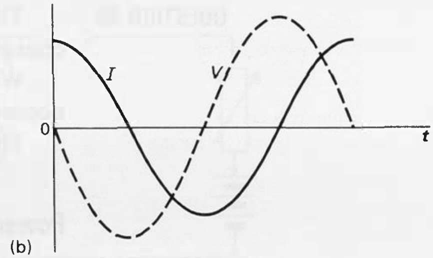


Figure H20
Current and p.d. for an inductor.

At a higher frequency a smaller current can give the same rate of change of current and so, for the same p.d., less current flows in an inductor as the frequency is raised.

With more inductance at a fixed frequency a smaller current can produce the same flux and so less current flows for the same applied p.d.

QUESTIONS 53 to 55

These results show that $I \propto \dfrac{1}{fL}$.

No energy is taken from an a.c. supply by a pure inductance with no resistance. The energy which is taken from the supply while the flux is building up is returned when the flux returns to zero.

Capacitor and inductor in parallel

DEMONSTRATION H25
Alternating currents in a circuit containing capacitance and inductance

In a parallel circuit (figure H21) the current in the capacitor leads the applied p.d. by $\pi/2$ and the current in the inductor lags the applied p.d. by $\pi/2$. At the resonant frequency these two currents have the same magnitude and so the current in one is just right for supplying the current in the other and no current is taken from the supply. In practice there is always some resistance in the circuit, especially in the inductor, and a small current is taken from the supply.

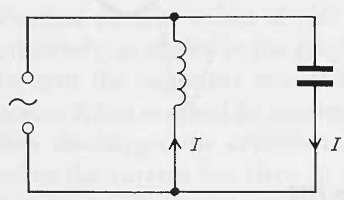


Figure H21
Alternating current in a parallel *LC* circuit.

EXPERIMENT H26
Oscillations in a parallel *LC* circuit

DEMONSTRATION H27
A simple radio

The resonant frequency of such circuits depends on the values of L and C. They can be used as oscillators and as filters to select one particular frequency, as in the tuning circuit of a radio. The lower the resistance of the circuit, the sharper its resonance.

Mechanical analogue of an LC circuit

QUESTIONS 56 to 59

'Electromechanical similarities' in the Reader *Physics in engineering and technology*

Electrical oscillations in an LC circuit can be compared with the mechanical oscillations of a mass-and-spring system. Inductance, L, is analogous to mass, m; and $1/C$ (C is capacitance) to the spring constant, k. The formulae for frequency of natural oscillations of the two systems are:

mechanical	**electrical**
$f_{\text{nat}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$	$f_{\text{nat}} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$

Section D4, Forced vibrations and resonance

Resistance in an electrical oscillator corresponds to friction in a mechanical oscillator: both cause the oscillations to die away; both cause broadening of the resonance curve. The Q factor is a measure of the width of the resonance peak (figure H22).

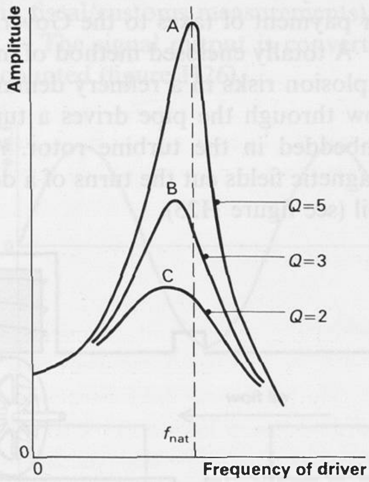


Figure H22
Resonance curves.

READINGS

ELECTROMAGNETIC FLOWMETERS

This reading passage, which is based on articles in *Physics at work* and *Physics principles at work* (both published by BP Educational Service, copyright BP International Limited, and reproduced with their kind permission), describes how the principles of electromagnetism are used in two types of flowmeter which are in day-to-day use at oil refineries and chemical manufacturing plant.

The turbine flowmeter

Twenty-five million tonnes of crude oil entered BP's Grangemouth Refinery in 1979 and accurate measurement is needed for costing and for payment of taxes to the Government (£478 000 000 in 1979).

A totally enclosed method of measurement is needed since fire and explosion risks in a refinery demand strict safety precautions. The oil flow through the pipe drives a turbine fan which has small magnets embedded in the turbine rotor. As the turbine rotates the moving magnetic fields cut the turns of a detector coil inducing an e.m.f. in the coil (see figure H23).

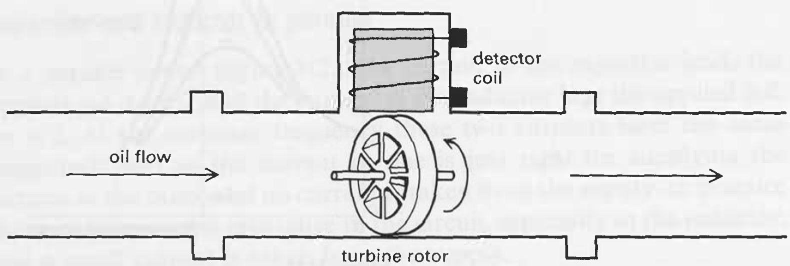


Figure H23
Principle of turbine flowmeter.

One type of flowmeter has a rotor with 8 blades and has 27 magnets embedded in the stainless steel rim (figure H24). The diameter is 203 mm (8 inches).

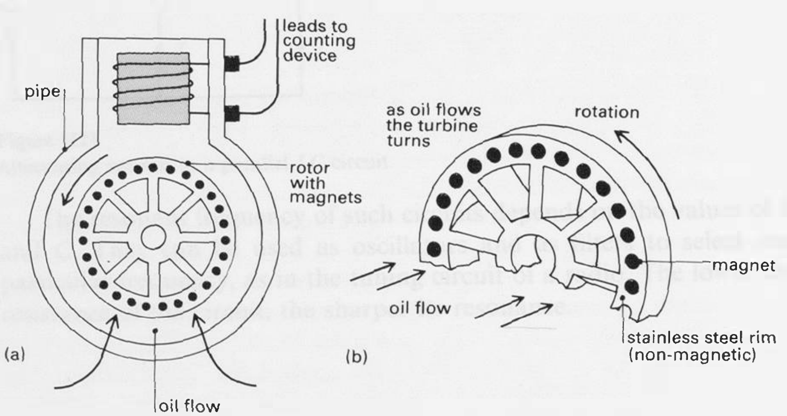


Figure H24
Detail of turbine.

As the rotor turns the magnets are swept past the detector coil (resistance 1850 Ω). The induced e.m.f. is shown in figure H25.

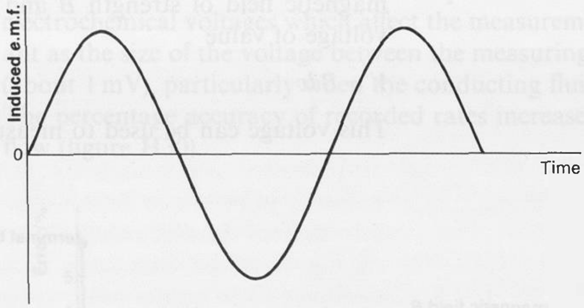


Figure H25
Variation of induced e.m.f. with time.

More magnets produce more pulses per revolution and help to improve the accuracy of the measurements (an important consideration in fiscal/customs measurements).

The signal output is converted into square pulses which are then counted (figure H26).

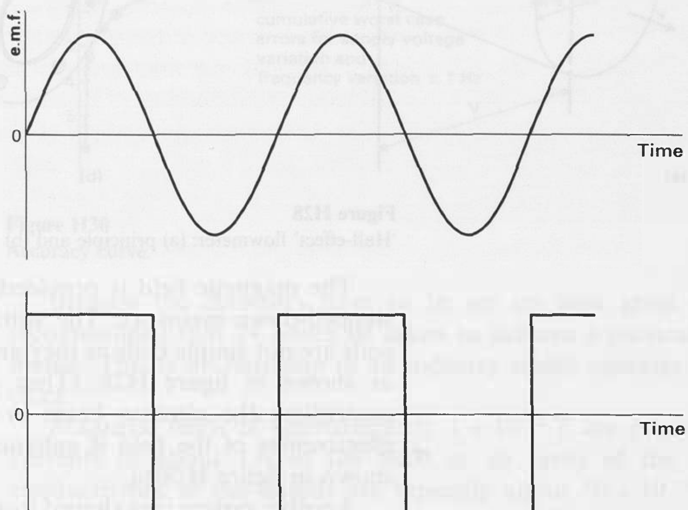


Figure H26

From the number of pulses counted in a given time, the volume flow rate can be obtained (figure H27). Rates of flow between 120 and 1130 cubic metres per hour (about 0.03 and 0.3 $m^3 s^{-1}$) can be measured in this way.

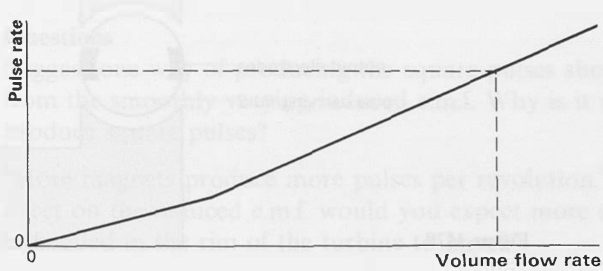


Figure H27
Calibration curve for turbine flowmeter.

The 'Hall-effect' flowmeter

A conducting fluid flowing along a pipe with velocity v, through a magnetic field of strength B and pipe of diameter d, will produce a voltage of value

$$V = Bdv$$

This voltage can be used to measure the volume flow rate.

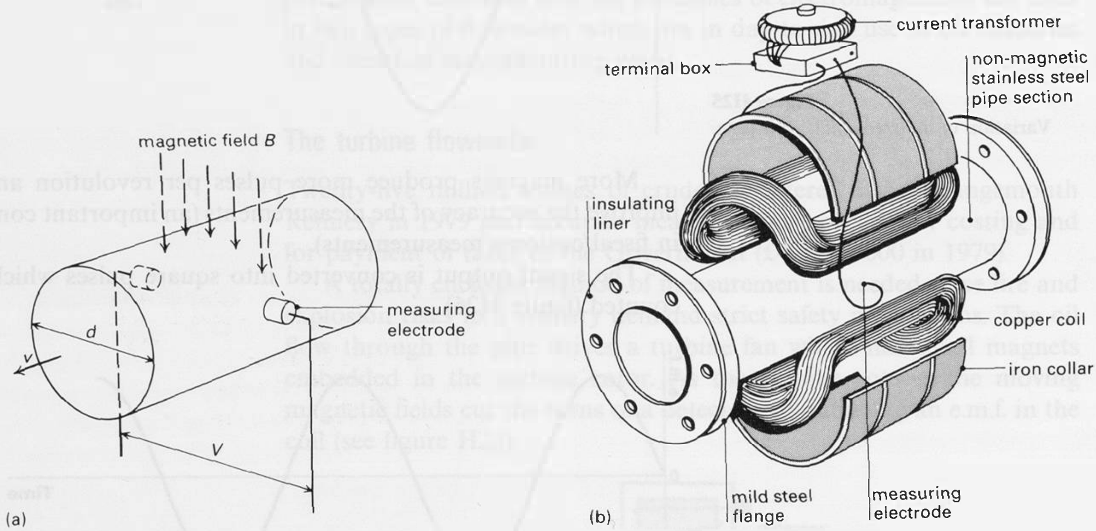


Figure H28
'Hall-effect' flowmeter: (a) principle and (b) design.

The magnetic field is provided by field coils which operate using stepped-down mains a.c. The voltage V is thus alternating. The field coils are not simple coils as they are designed to fit the cylindrical pipe as shown in figure H28. (They are similar in shape to the coils controlling the electron beam in a household television tube.) The effectiveness of the field is enhanced by the use of an iron collar as shown in figure H28(b).

Another system uses shaped iron pole-pieces which are magnetically energized using a field coil (figure H29).

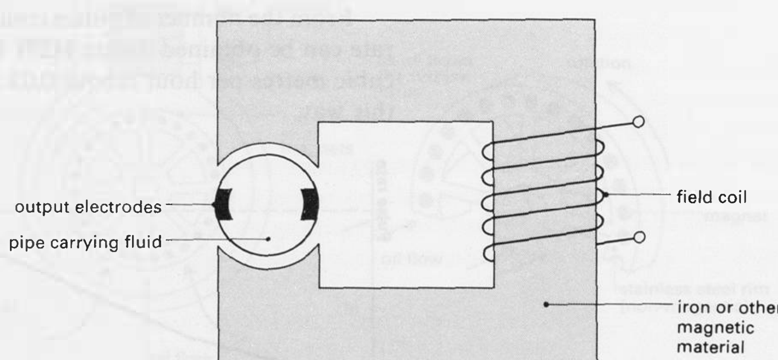


Figure H29

In either system, the pipe close to the measuring device must be non-magnetic and the shape of the electrodes and coil design are very important. The use of a.c. avoids the presence of thermoelectric and electrochemical voltages which affect the measurement. This is important as the size of the voltage between the measuring electrodes is small (about 1 mV), particularly when the conducting fluid is flowing slowly. The percentage accuracy of recorded rates increases with the speed of flow (figure H30).

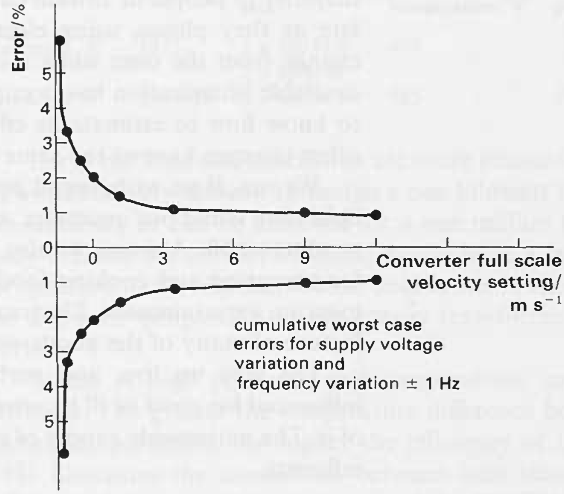


Figure H30
Accuracy curve.

Because the detectors have to be set up with great care, it is recommended that 24 hours be taken to achieve a particular performance. This is no hardship in an industry which operates round the clock.

Magnetic fields of approximately 1×10^{-2} T are produced using currents of about 1 A in the 1000 or so turns of the coils. The conductivities of the liquids are typically about $50 \times 10^{-4}\ \Omega^{-1}\ \mathrm{m}^{-1}$ (engineers sometimes refer to this unit of conductivity as mhos per metre). The liquid has to be sufficiently conducting to allow enough current to operate the voltage detectors.

The above method of measurement has been successfully applied to the measurement of flow in the human body where blood acts as the conducting fluid.

Questions

a Suggest one way of producing the square pulses shown in figure H26 from the smoothly varying induced e.m.f. Why is it necessary to produce square pulses?

b ‘More magnets produce more pulses per revolution.’ What other effect on the induced e.m.f. would you expect more magnets imbedded in the rim of the turbine to have?

c Explain why the field coils are wound on iron (figures H28(b) and H29).

d Suggest advantages and disadvantages of the two types of flowmeter described.

THE GENERATION AND TRANSMISSION OF ELECTRIC POWER

In industrial communities, it is no longer dark at night. The great majority of people in Britain can sit and read, work, or play games as late as they please, using electricity costing only a few pence. The change from the time when a dim candle or rush light was the only available illumination has occupied several generations, and it is hard to know how to estimate its effect on the quality of life, especially as other changes have at the same time made at least as large an impact.

We can, if we wish, spend perhaps 10 to 20 per cent longer in good light than could our ancestors, and so have an 'extra' ten or so years of productive life. At home we also use electricity in all kinds of appliances for preparing and cooking food, washing clothes, and so on – not to mention entertainment. Electrical machinery is essential to the manufacture of many of the goods we buy.

The way we live, and perhaps the kind of people we are, are influenced for good or ill by science and the technologies that grow out of it. The nationwide supply of electricity is a good example of such an influence.

Demand versus amenities

Most of us depend on electricity. The Central Electricity Generating Board (CEGB) now sells about 200 GW h every year in England and Wales, where the population is about 50 million. But few want a power station in the view from their windows, and many are concerned at the effect on the countryside of pylons carrying the overhead lines that bring electricity to their houses. Are underground cables the answer? Could electrical power not be produced by many small, independent generators, with a dynamo in every house? Why do we have large power stations, some producing over 1000 MW each, linked by miles of transmission lines using voltages up to 400 kV?

Why have large power stations?

At first sight, a million one-kilowatt generators would seem to be as good as a single one thousand million watt station. But the effect of making things smaller or larger is not negligible. A large turbine (660 MW is now quite common) turns out to have a smaller capital cost per unit of power produced than the equivalent number of small ones.

Table H1 illustrates the trend towards larger generating sets in Britain. Table H2 gives comparative data for an old and a relatively new power station. Notice that as well as larger generating sets, the newer station operates at higher steam temperatures and pressures.

Table H1

Age and power of steam-driven generating sets.

Data from Central Electricity Generating Board Statistical yearbook 1982–83.

Age (years)	Number of sets: Below 100 MW	100 MW up to 500 MW	500 MW and over
0–4	–	–	9
5–9	–	1	11
10–14	1	8	33
15–19	9	25	8
20–24	31	54	–

Table H2

Comparison of two coal-fired power stations. (Note that Battersea power station is no longer in use.)

Data from Central Electricity Generating Board Statistical yearbook 1982–83.

Station	Date	Generator sets	Steam temperature/°C	Steam pressure/Pa	Efficiency
Battersea 'A'	1933	1 × 100 MW 2 × 69 MW	427	42×10^5	16 %
Drax	1974–76	3 × 660 MW	565	160×10^5	37 %

Large turbines and generators are more efficient for several reasons. It would be very difficult to design a one kilowatt turbine with as high an efficiency as can be achieved for a one million kilowatt machine, if only because the smaller machine would have a larger surface area in proportion to its volume, so that heat losses (although smaller) would be a greater proportion of the energy transformed in the boiler and turbine.

Steam at high pressures and temperatures can be used in large turbines. The greater the temperature difference between the inlet and outlet temperatures, the higher the efficiency of the machine. Figure H31 illustrates the association between inlet steam temperature and efficiency for power stations in one of the CEGB's regions.

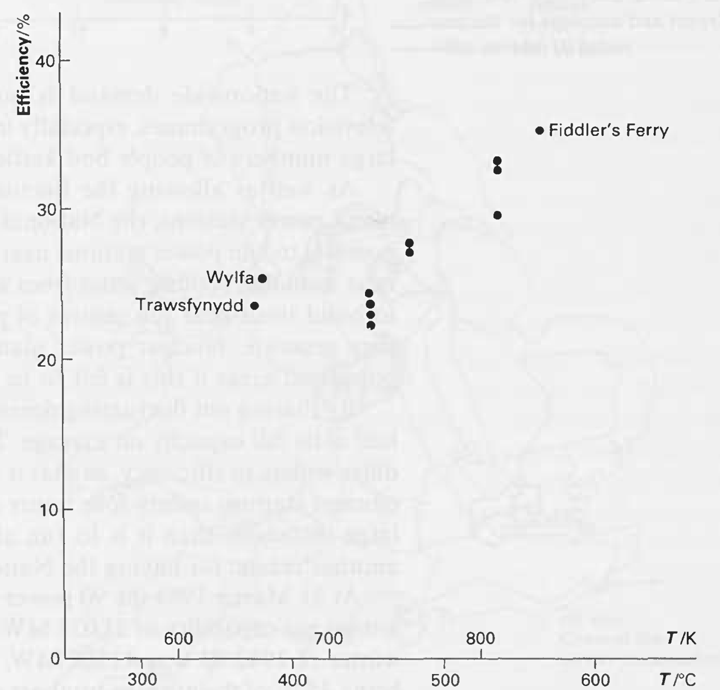


Figure H31

Efficiency and steam temperature for power stations in the North-West Region.

Data from Central Electricity Generating Board Statistical yearbook 1983–84.

Electricity cannot be stored up

Unlike fuels, or water stored behind a dam, electrical energy cannot be stored in quantity. It must be produced on demand. This is another reason why a small domestic plant is not viable: the demand fluctuates widely, and to cope with the occasional use of a cooker, a capacity of ten or a hundred times the capacity needed at other times would have to be installed.

When aggregated, many small fluctuating demands become a total demand of relatively smooth daily profile which can be connected to geographically distributed power stations by a grid of transmission lines. Even so the task is not easy, as the daily demand curve (figure H32) shows.

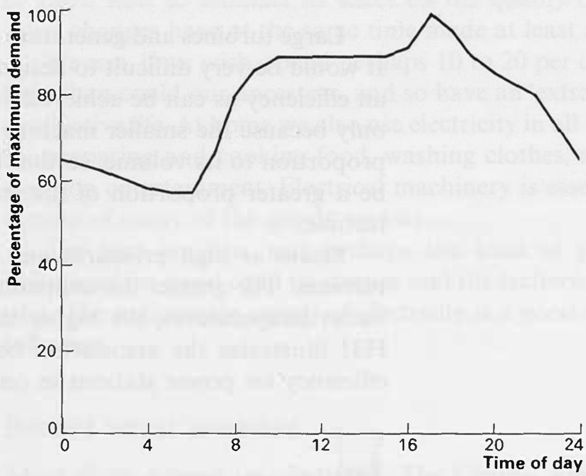


Figure H32
Typical daily demand curve at time of winter maximum demand.
Data from Central Electricity Generating Board Report and accounts for the year ended 31 March 1984.

The nationwide demand is quite noticeably affected by popular television programmes, especially in breaks between programmes when large numbers of people boil kettles for tea.

As well as allowing the fluctuating demand to be shared among many power stations, the National Grid of transmission lines makes it possible to site power stations near the sources of the fuels they use, or near available cooling water from a river or the sea, rather than having to build them near the centres of population which use the electricity they generate. Nuclear power plants can be built away from densely populated areas if this is felt to be important.

By sharing out fluctuating demand the system can be run using only half of its full capacity on average. The power stations we actually have differ widely in efficiency, so that it is more economical to run the most efficient stations twenty-four hours a day, transmitting their power over large distances, than it is to run all stations part of the time. This is another reason for having the National Grid linking stations together.

At 31 March 1984 the 90 power stations in England and Wales had a total net capability of 51 028 MW. The maximum demand met in the winter of 1982/83 was 43 802 MW, the average load through the year being 45 % of the average total net capability. As table H3 shows, most of the stations were steam operated. The majority of these, 52 out of 63, used coal as their primary fuel; there were 11 oil-burning stations. Nuclear stations, although shown separately, are also in a sense steam operated since the energy from the nuclear disintegrations in the reactors is used to generate steam to drive turbines, just as the energy from burning a conventional fuel is.

Table H3
Types of power station and energy supplied.
Data from Central Electricity Generating Board Statistical yearbook 1983–84.

Type	Number of stations	Electricity supplied/GW h
Steam	63	182 000
Nuclear	9	31 000
Gas turbine	9	52
Hydro	7	202

One interesting recent development is pumped storage. The same piece of electrical machinery can be used either as a generator or as a motor. When demand is low and there is power to spare it can be used to pump water up to a high-level reservoir. When demand is high the water is allowed to fall down again, driving the machine as a generator and recovering some of the energy used to pump it up.

The National Grid

A generator usually operates at 10 to 20 kV, while users are supplied at 240 V. Power is transmitted at much higher voltages, of 275 kV or 400 kV on the Supergrid, or 132 kV on the old grid system.

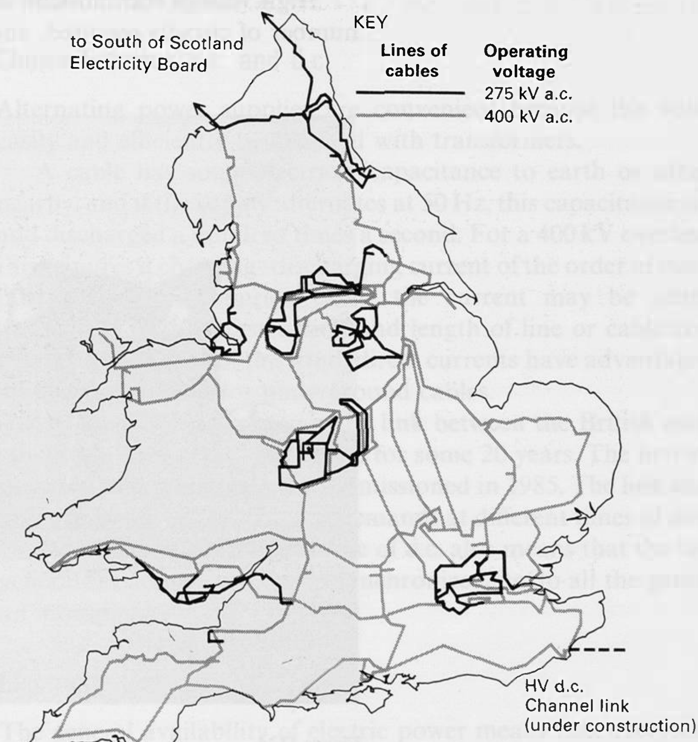


Figure H33
Supergrid system as at 31 March 1984.
From Central Electricity Generating Board Statistical yearbook 1983–84.

The reason for using high voltages is simple: the power losses are smaller, as is shown below.

Two of the 400 kV Supergrid conductors, capable of handling 1000 MW, have a combined resistance of 0.034 ohm per kilometre. (They are made of aluminium on a steel core, cross-sectional area 2.6 cm^2.)

To make a simple calculation, we treat them as d.c. lines carrying 1000 MW at 400 kV. The current is 2500 A, and a power of 210 kW is dissipated in each kilometre of cable, a loss of 2.1% in a hundred kilometres.

Had the voltage been ten times lower (40 kV), the current would be ten times higher (25 000 A) and the power loss (proportional to the square of the current) a hundred times greater, amounting to 21 MW in each kilometre.

In this much simplified calculation, the lower-voltage lines would be dissipating 21 kW per *metre*, and would be as hot as electric fire elements. (The resistance of the hot line would have risen, invalidating the calculation as it stands.)

In general, for a given power, energy losses for a given cable cross-section, or cross-section for a given loss, vary inversely as the square of the transmission voltage.

It is cheaper to pay for the cost of nearly a thousand transformer substations, connected to 16 000 kilometres of high-voltage cables, than to transmit the power through thicker cables at lower voltages.

High-voltage transmission also allows a significant reduction in the number of circuits required, and therefore of supporting towers.

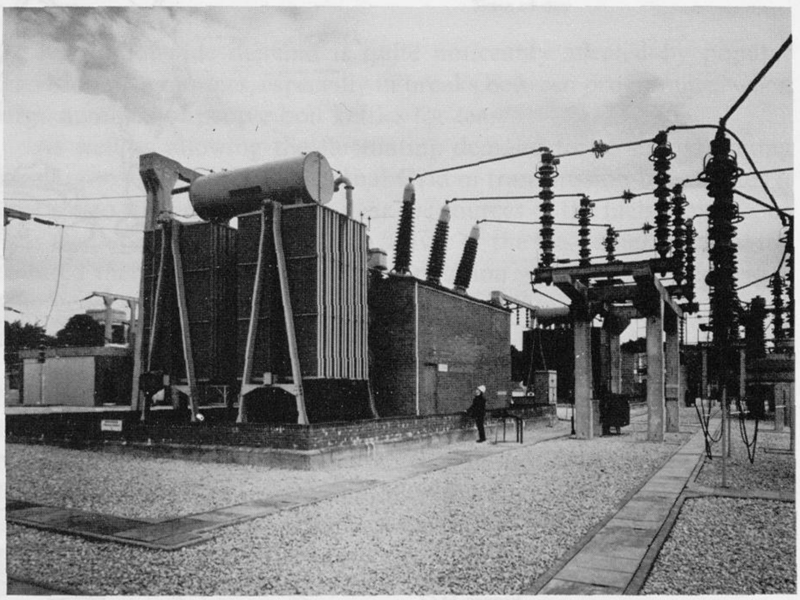


Figure H34
Bishops Wood 132/275 kV substation near Stourport-on-Severn, Worcestershire.
Central Electricity Generating Board.

Overhead lines versus underground cables

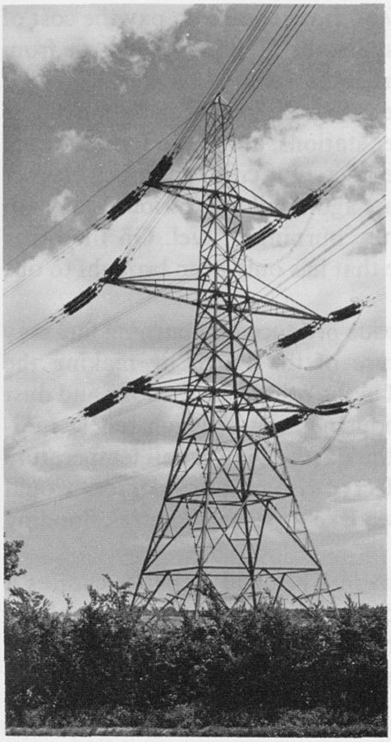


Figure H35
A 400 kV tower, part of the CEGB's 400 kV Sizewell–Sundon transmission line.
Central Electricity Generating Board.

The National Supergrid 400 and 275 kV transmission system has more than 13 000 circuit km of overhead lines. About 70 % of the overhead lines operate at 400 kV. There are also about 900 km of underground cables, mostly in the cities.

Many people would prefer to see fewer pylons straddled across open country. Ultimately, the choice depends on how much one is prepared to pay not to see the overhead lines, assuming that one wants the electrical power delivered.

The CEGB estimates that at 132 kV, underground cables are more than ten times as expensive to install as an equivalent overhead line. They put the capital cost of a double circuit 400 kV underground cable at between £6m and £7m per kilometre, compared with the cost of the equivalent overhead line at some £500 000, a factor of fourteen against the use of cable.

The high cost of underground cables is mainly a consequence of the fact that soil is not a good thermal conductor. Overhead cables are cooled by the air, but buried cables can easily overheat. To avoid this they need to be made thicker and to be cooled, so therefore they are more expensive. A 400 kV cable of 2250 mm^2 cross-section can, when water-cooled, provide a continuous rating approximately equal to that of two 400 mm^2 overhead lines operating at 65 °C. The electrical insulation round the cable, which also costs money, adds to the cooling problem.

Choice between a.c. and d.c.

Alternating power supplies are convenient because the voltage can easily and efficiently be changed with transformers.

A cable has some electrical capacitance to earth or other cables nearby, and if the supply alternates at 50 Hz, this capacitance is charged and discharged a hundred times a second. For a 400 kV overhead cable, this requires a charging–discharging current of the order of one ampere, but for a similar buried cable, the current may be nearer 40 A, depending on the construction and length of line or cable concerned. For considerations of this kind, direct currents have advantages at high voltages, especially for underground cables.

An interesting example is the link between the British and French systems. The first link was in use for some 20 years. The first stage of a new 2000 MW d.c. link was commissioned in 1985. The link enables the two systems, which have peak demands at different times of day, to feed each other spare power. This use of d.c. also means that the two sets of generators do not have to be synchronized, as do all the generators in an a.c. linked system.

Electricity and choice

The general availability of electric power means that everyone can do more and for longer than would be possible without it. Choices have to be made, however, about the cost of supplying this power in terms of

H

money and damage to the environment. If we choose to pay the cost of putting cables underground, we must also be prepared to choose from what other desirable things we divert resources.

There has long been concern over the pollution of the atmosphere by flue gases from fuel-burning power stations, although domestic coal fires can be worse offenders in this respect. Our cities are much cleaner now than they were 50 or even 25 years ago thanks to various 'clean air' measures which restrict or control the burning of fuel. On the other hand the problem of 'acid rain' is one that has only been brought to our attention in the last few years.

The large amounts of carbon dioxide produced by burning fuel may slightly raise the average temperature of the Earth by making the atmosphere more like a greenhouse. On the other hand, smoke and dust may reduce the temperature by producing clouds which reflect radiation from the Sun before it reaches the Earth. Quite small temperature changes would have a significant effect: a rise of a few degrees would melt the polar icecaps and raise the sea level by tens of metres, flooding most of the World's large cities.

Clearly, choices will have to be made. They will be better choices if they are made in the light of an understanding of the issues involved. We may have to balance the advantages of having larger-scale energy resources for each person against the consequences of providing them.

Questions

a The 50 million people in England and Wales 'use' about 2200 GW h of electrical energy every year.
i How much is this, *per capita*?
ii Estimate how much electrical energy you use in an average day, and hence your yearly consumption.
iii A healthy person can work steadily at a rate of about 50 W. For how many hours would you have to work to generate as much electrical energy as you use in a day?
iv Account for any difference between your answers to *i* and *ii*.

b *i* Show that for two objects of the same shape (*e.g.* spheres, or cubes) the ratio of surface area to volume is greater for a small object than for a large one.
ii What shape should an object have to minimize this ratio?
iii Give another example of the importance of size in reducing the (relative) rate at which energy is transferred thermally.
iv Suggest another reason – not concerned with energy transfer – why a large electromagnetic machine may be more efficient than a small one.

c 'Electrical energy cannot be stored up in quantity.'
i Name two systems which *can* be used to store electrical energy, and, for one of them, make an estimate of the energy that could be stored in an installation of reasonable size. Apart from limited capacity, what other disadvantages do such systems have? Where, on the other hand, do they find application?

ii List the factors which determine the capacity of a pumped storage system. Estimate values for them and hence estimate the capacity of such a system.

d Show, algebraically, that 'for a given power, energy loss for a given cable cross-section, or cross-section for a given loss, vary inversely as the square of the transmission voltage.'

e *i* Why should the charging and discharging current for an underground cable be so much greater than that for the equivalent overhead line? (Think about the factors that determine capacitance.)
ii Estimate the capacitance of the cable for which the charging–discharging current at 400 kV, 50 Hz is 40 A.

f Why do all generators in an a.c. linked system have to be synchronized?

H

LABORATORY NOTES

DEMONSTRATION
H1 Forces on currents; forces between currents

l.t. variable voltage supply
2 Magnadur magnets
mild steel yoke
aluminium cooking foil, 1 cm wide and 1 m long
2 clip component holders
retort stand base and rod
2 bosses
demonstration meter, 10 A d.c.
scissors
leads

H1a Forces on currents

Pass a current of about 2 A through the strip of foil. Where must the magnet be placed to lift the strip of foil off the bench?

Reverse the direction of the current. What happens to the force on it? What other change can you make to lift the foil off the bench?

Draw a diagram showing the relative directions of the current, *I*, the force, *F*, and the magnetic field, *B* (*i.e.* the direction of the force on the North pole of a plotting compass).

H1b Forces between currents

Hang two strips of foil vertically between a pair of clip component holders. The strips should be about 50 cm long and 1 cm apart.

Pass a high current (5–8 A) through the strips. What is the direction of the force between the strips when the currents are in opposite directions? When the currents are in the same direction?

What is the direction of the magnetic field near a long, straight, current-carrying conductor? How is the direction of the force on one conductor related to the direction of the current in that conductor and the magnetic field due to the current in the other conductor?

OPTIONAL DEMONSTRATION
H2 The catapult field

l.t. variable voltage supply
retort stand base
3 retort stand rods
7 bosses
4 nails, 15 cm long
piece of card with hole (see figure H36)
2 Magnadur magnets
iron filings
0.45 mm PVC-covered copper wire
demonstration meter, 5 A d.c.
leads

The large coil of wire in figure H36 is about 20 cm square. It should have at least ten turns of PVC-covered wire. The magnets are set up with unlike poles facing to produce a uniform field between them.

Use the iron filings to show the field due to the magnets alone. Remove the magnets and the iron filings. Pass a current of about 3 A through the coil, and sprinkle iron filings again to show the field due to the current alone.

If both current and magnets are present, what shape would you expect the field to have? Where will the two fields combine to produce a stronger field? Where they are in opposition the combined effect will be a weaker field.

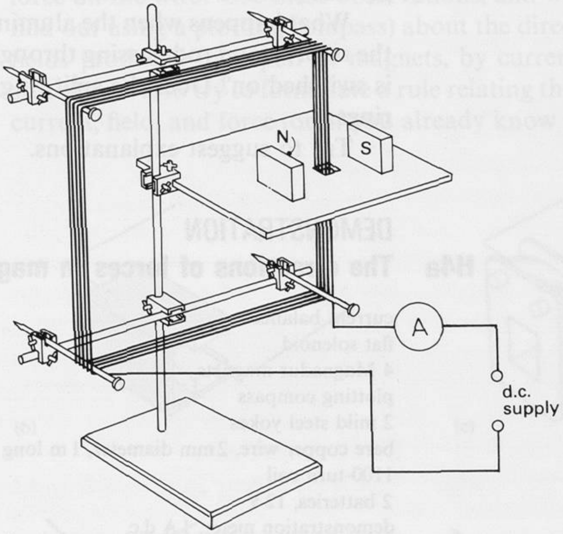


Figure H36
Demonstration of the 'catapult field'.

Now use iron filings to test your prediction for the shape of the field due to both current and magnets together.

Of course there are no invisible stretched elastic bands in the magnetic field, but its 'catapult' shape may help you remember that there is a sideways force on the wire. (You can work out the direction of the force by sketching the two independent fields, and then combining them.) The magnets give a nearly uniform field from N to S. Use the right hand fingers and thumb rule* to find the direction of the circular field around the wire (figure H37). On one side of the wire the two fields are in opposite directions. At one point the combined field will be zero. The magnetic force on the wire pushes it towards this point.

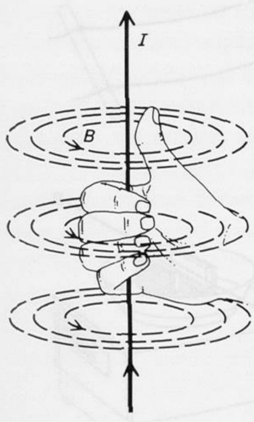


Figure H37

* If the thumb points in the direction of the current, then the curled fingers of the right hand show the direction of the magnetic field.

DEMONSTRATION

H3 Forces on induced currents

l.t. variable voltage supply
coil with 120 + 120 turns
retort stand base and rod (mild steel)
aluminium ring
split aluminium ring
leads

The coil is placed over the retort stand rod. An alternating current of up to 5 A from a 12 V a.c. supply can be passed through the coil for *short periods* without overheating it.

What happens when the aluminium ring is placed over the coil (with the retort stand rod passing through both coil and ring) and the current is switched on? Does the split ring behave in the same way? Feel both rings.

Try to suggest explanations.

DEMONSTRATION

H4a The directions of forces in magnetic fields

current balance
flat solenoid
4 Magnadur magnets
plotting compass
2 mild steel yokes
bare copper wire, 2 mm diameter, 1 m long
1100-turn coil
2 batteries, 12 V
demonstration meter, 1 A d.c.
demonstration meter, 5 A d.c.
rheostat, 10–15 Ω, 5 A
roll of tickertape
scissors
leads

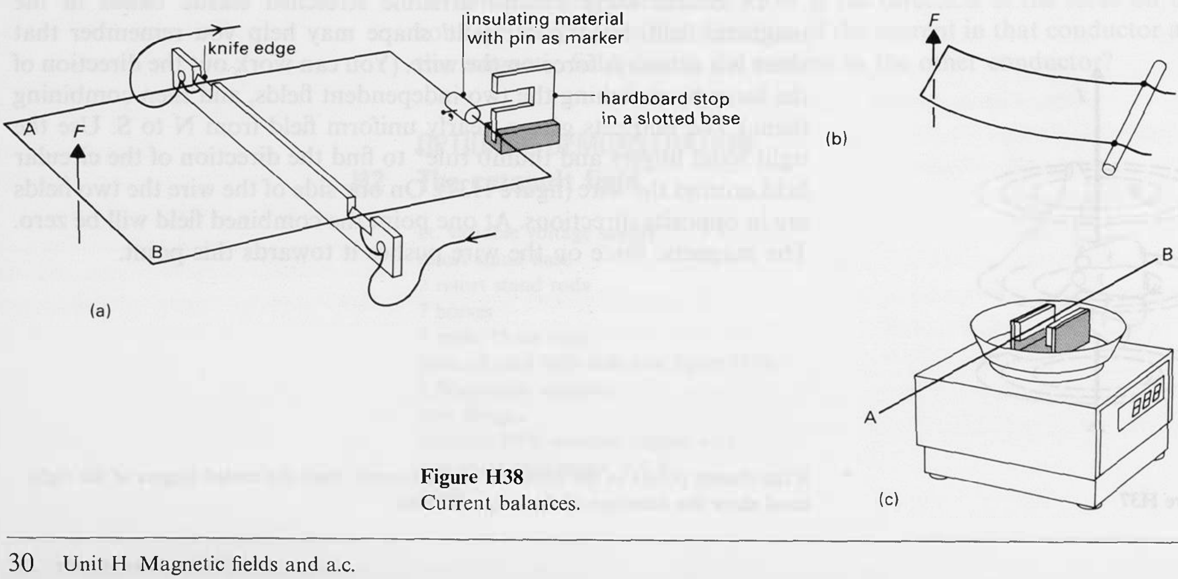


Figure H38
Current balances.

A current balance can be used to detect magnetic fields, and to measure them.

Various designs of current balance are available. Figure H38(a) shows a wire rectangle pivoted on razor blades or brass strips. A current of about 3 A is fed to the wire through the razor blades. Current only passes between the pivots along one side of the rectangle – there is a small gap in the wire on the other side. In figure H38(b) the wire is not pivoted, but any vertical force on it causes it to bend. In figure H38(c) any force on the magnets due to a current in the wire can be calculated from the change in reading of the balance.

In which of the situations depicted in figure H39 is there a vertical force on the wire? Use these observations, and what you know (or can find out using a plotting compass) about the directions of the magnetic fields produced by a pair of magnets, by current in a coil, and by a straight wire, to try to formulate a rule relating the relative directions of current, field, and force (or, if you already know such a rule, to test it).

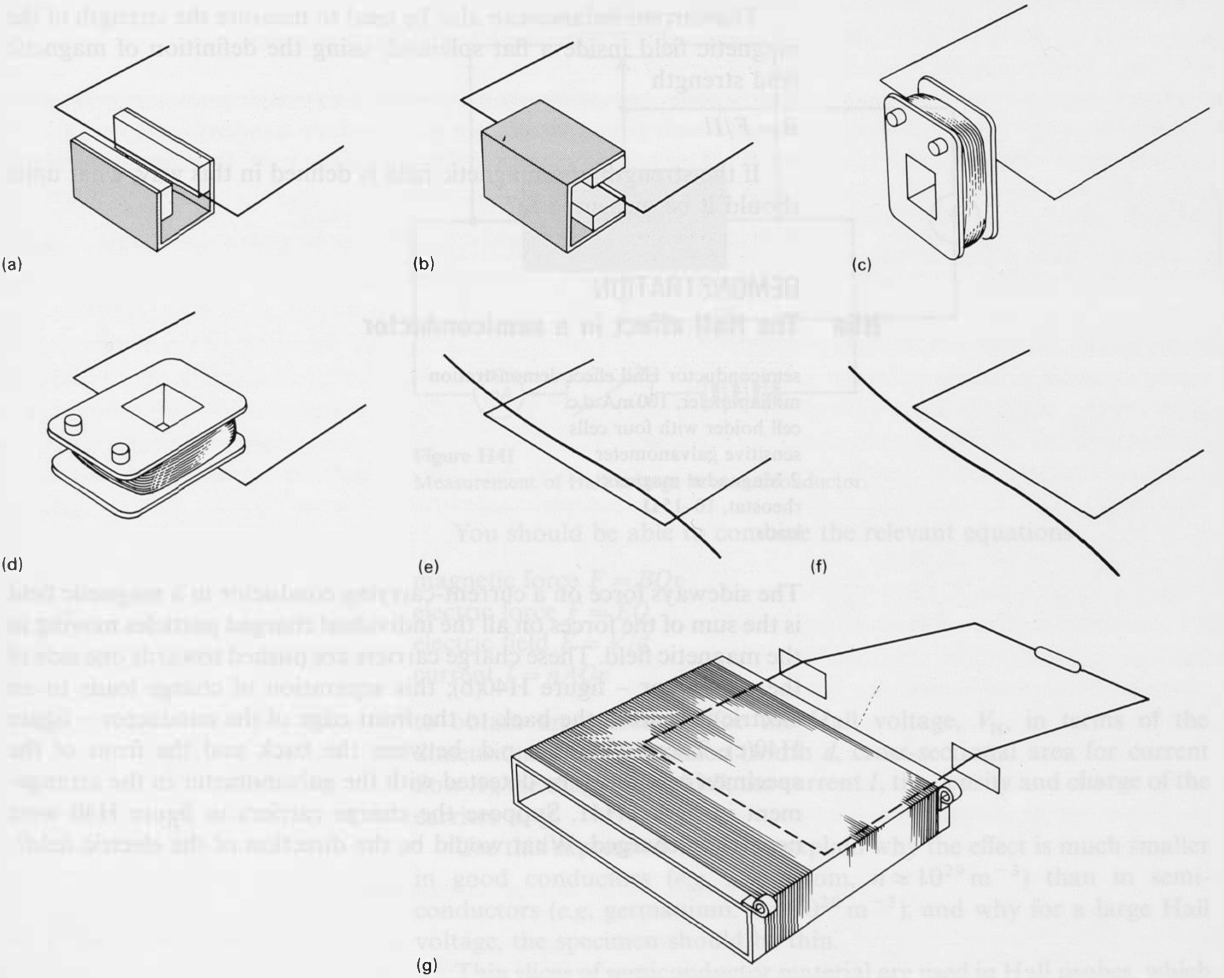


Figure H39
Forces on currents.

DEMONSTRATION

H4b Measuring a magnetic field

The force on the wire in the current balances shown in figures H38(a) and (b) is measured by adding small pieces of paper or wire to the deflected current balance and adjusting the current until the balance returns to its original position. The magnetic force on the current-carrying wire is then equal to the weight of the paper added to it.

In figure H38(c) the force on the magnets, which (by Newton's Third Law) is equal in magnitude to the force on the wire, is measured directly from the change in reading of the balance. (But remember that balances are used to measure *mass*: the *force* needed in this experiment is the weight of that mass.)

With the current balance and the pairs of magnets it should be possible to test how, for a given magnetic field, the force on a current-carrying wire depends on I, the current in the wire, and l, the length of the field.

The current balance can also be used to measure the strength of the magnetic field inside a flat solenoid, using the definition of magnetic field strength

$$B = F/Il$$

If the strength of a magnetic field is defined in this way, what units should it be measured in?

DEMONSTRATION

H5a The Hall effect in a semiconductor

semiconductor Hall effect demonstration
milliammeter, 100 mA d.c.
cell holder with four cells
sensitive galvanometer
2 Magnadur magnets
rheostat, 10–15 Ω
leads

The sideways force on a current-carrying conductor in a magnetic field is the sum of the forces on all the individual charged particles moving in the magnetic field. These charge carriers are pushed towards one side of the conductor – figure H40(b); this separation of charge leads to an electric field from the back to the front edge of the conductor – figure H40(c). There will be a p.d. between the back and the front of the specimen which can be detected with the galvanometer in the arrangement in figure H41. Suppose the charge carriers in figure H40 were negatively charged. What would be the direction of the electric field?

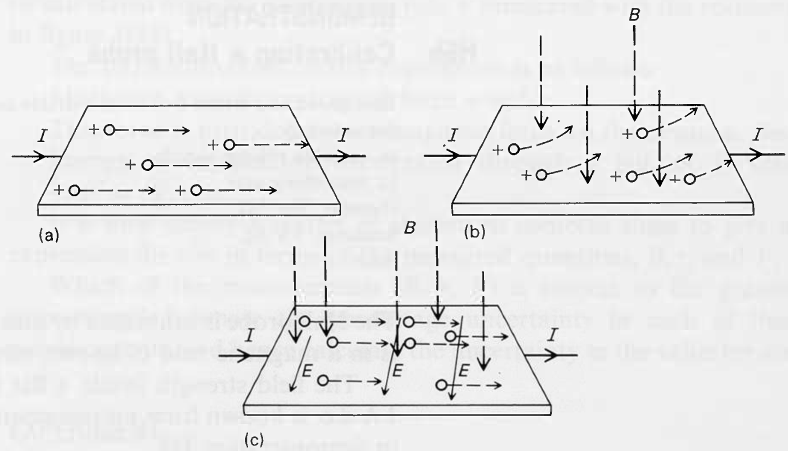


Figure H40
Forces on positive charges moving in a conductor with a transverse magnetic field.

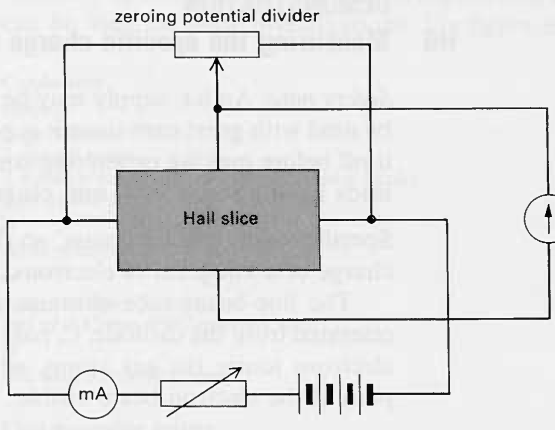


Figure H41
Measurement of Hall voltage in a semiconductor.

You should be able to combine the relevant equations

magnetic force $F = BQv$
electric force $F = EQ$
electric field $E = V/d$
current $I = nAQv$

to obtain an expression for the Hall voltage, V_H, in terms of the dimensions of the specimen (width d, cross-sectional area for current flow A), the magnetic field B, the current I, the density and charge of the carriers n, Q.

Use this expression for V_H to explain why the effect is much smaller in good conductors (*e.g.* aluminium, $n \approx 10^{29}\ \text{m}^{-3}$) than in semiconductors (*e.g.* germanium, $n \approx 10^{20}\ \text{m}^{-3}$); and why for a large Hall voltage, the specimen should be thin.

Thin slices of semiconductor material are used in Hall probes, which are much more convenient devices than the current balances for comparing magnetic field strengths.

DEMONSTRATION
H5b Calibrating a Hall probe

Hall probe and circuit box (with suitable meter)
flat solenoid
l.t. variable voltage supply
l.t. smoothing unit
rheostat, 10–15 Ω
ammeter, 5 A d.c.
leads

The Hall probe is calibrated by measuring the voltage produced when it is in a magnetic field of known strength.

The field strength inside a flat solenoid carrying a current of, say, 1 A d.c. is known from measurements of the force on a current balance in demonstration H4.

DEMONSTRATION
H6 Measuring the specific charge for electrons

Safety note: An h.t. supply may be used in this demonstration. It must be used with great care since it is potentially dangerous. Always switch it off before making or altering connections to it. It is advisable to use leads having shrouded 4 mm plugs.

Specific means 'per unit mass'; so the specific charge for electrons is the charge of a kilogram of electrons.

The fine-beam tube contains gas at low pressure. Electrons, accelerated from the cathode, C, pass through a hole in the anode, A. The electrons ionize the gas atoms, which then emit light, so making the path of the electron beam visible.

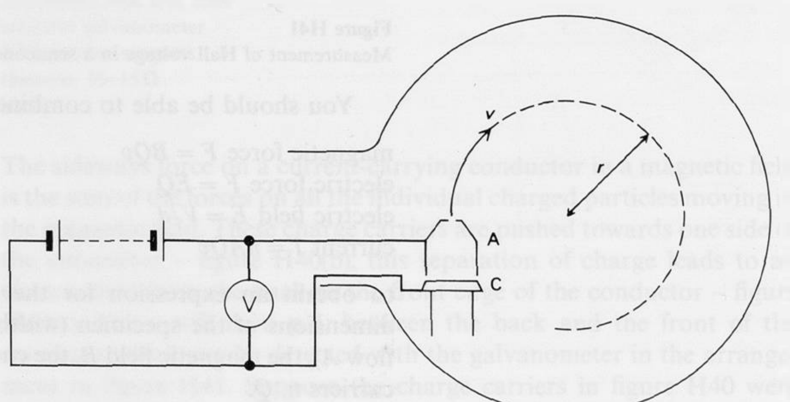


Figure H42
Electron beam bent into circular path in fine-beam tube.

In a uniform magnetic field, perpendicular to their direction of motion, the electrons move in a circle. The experiment consists of measuring the diameter of this circle, and the magnetic field strength, B. The size of the circle also depends on the electrons' speed, v, which can be calculated from the accelerating p.d., V (measured with the voltmeter in figure H42).

The physics involved in this experiment is as follows:

Motion in a circle: centripetal force $= mv^2/r$.

This force is provided by the magnetic force on the electron, Bev.

Energy of an electron accelerated through a p.d. of V volts: $\frac{1}{2}mv^2 = eV$.

It is now simply a matter of algebra to combine these to give an expression for e/m in terms of the measured quantities, B, r, and V.

Which of the measurements (B, r, V) is subject to the greatest uncertainty? Estimate the percentage uncertainty in each of these measurements, and hence calculate the uncertainty in the value for e/m.

EXPERIMENT
H7 Fields near electric currents

A variety of conductors, field-measuring devices, and sources of current can be used for these investigations. See figure H43.

Conductors

either

large Slinky
2 slotted bases
2 wooden strips (*e.g.* rulers) to support Slinky
2 crocodile clips

or

set of solenoids

or

magnetic field board
reel of 0.45 mm PVC-covered wire

or

coil with 120 + 120 turns

Field-measuring devices

either

Hall probe and circuit box
sensitive galvanometer

or

axial search coil
lateral search coil
oscilloscope

Sources of current

either

12 V battery
rheostat, 10–15 Ω, 5 A
ammeter, 10 A d.c.

or

transformer
rheostat, 10–15 Ω, 5 A
ammeter, 10 A a.c.

or

signal generator
ammeter, 1 A a.c.

leads

The Hall probe or a search coil can be used to explore magnetic fields due to currents in straight wires, in coils, in solenoids, and so on. If you use the search coil which responds to a changing magnetic field, then the current in the wire, coil, etc., must be a.c.

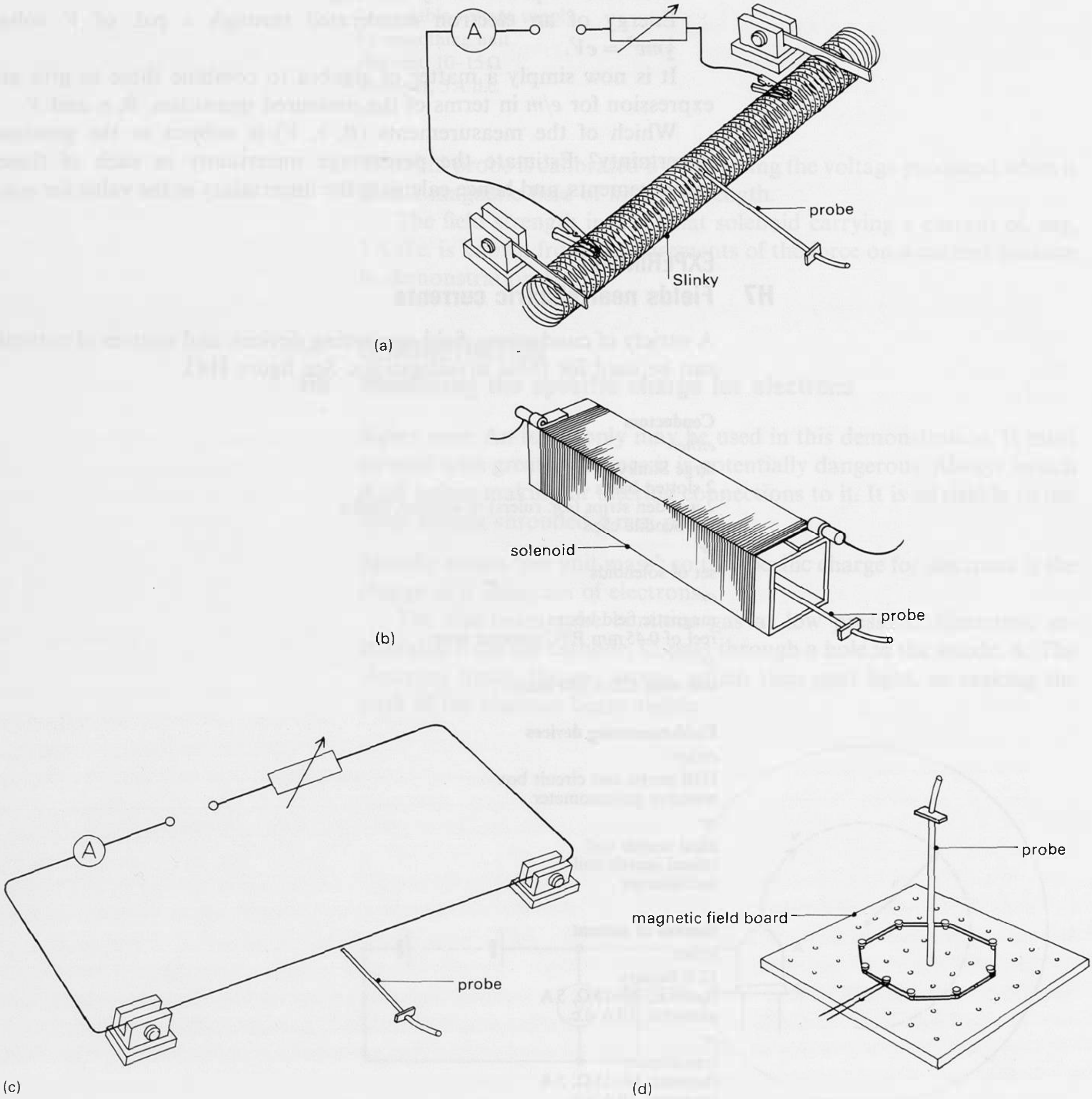


Figure H43
Measuring magnetic fields near currents. (a) Field in a Slinky solenoid. (b) Field in a solenoid. (c) Field near a long, straight wire. (d) Field of a coil.

Straight wire

The magnetic field near a single straight wire is rather weak, so it is necessary to use as large a current as possible, without overheating (turn off the current whenever you can). Several parallel wires, close together and each carrying a large current, will also increase the field strength. Use the rheostat to control current in the wire.

What orientation of the probe gives a maximum reading for a fixed distance from the wire?

Theory says that $B \propto I/r$. Do tests of this relationship, and suggest reasons for any departure from it.

Solenoids

A current of 1 to 2 A will be needed.

What orientation of the probe gives the maximum reading? Always use the probe in this orientation.

You should be able to answer the following questions:

Is the field strength constant across the width of the solenoid?

How does it vary along the length of the solenoid?

How does the field strength inside the small solenoid compare with that inside the large one?

How does the field strength depend on current?

How does it depend on the spacing of the turns? (Use the solenoid with closely wound turns and the one with spaced turns, or use the Slinky stretched to different extents.)

Coils

You can make flat coils of various shapes and sizes on the magnetic field board. To get sizeable fields, each coil should have about 10 turns. Use a current of about 5 A.

Some things you can test:

How does the field at the centre of the coil depend on the radius? On the number of turns?

What is the direction of the field at points on the axis of the coil?

For which position on the axis of the coil is the field strength a maximum? You may be able to look up and test a formula for the variation of field strength with distance from the coil, for points on the axis.

How does the field strength vary with position for points in the plane of the coil? How does its value at the centre compare with its value nearby?

DEMONSTRATION

H8 The e.m.f. induced in a moving wire

sensitive galvanometer
2 dynamics trolleys
10 Magnadur magnets
5 mild steel yokes
bare copper wire, 2 mm diameter
reel of 0.45 mm PVC-covered copper wire
clip component holder
adhesive tape
leads

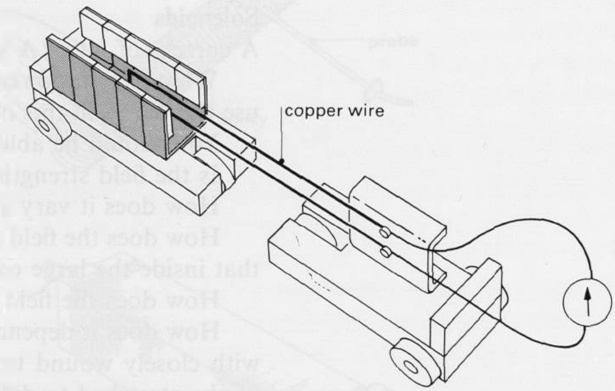


Figure H44
Moving wire or moving magnet.

Magnadur magnets with opposite poles facing (figure H44) are used to produce a region of about 15 cm length in which there is a nearly uniform magnetic field.

It is possible to move the magnet past the copper wire, or the copper wire past the magnet. Explore whether either movement produces an e.m.f.

How does the direction of any e.m.f. that is induced depend on the motion of magnet or wire? What factors determine its magnitude?

Is it a magnetic or an electric field that causes a force on an electron in the wire as it moves through the magnetic field? When the magnet moves past the wire we would say that any force on the electrons in the stationary wire must be due to an electric field acting on them. But the demonstration shows that the result is the same whether magnet or wire moves. It is only the *relative* movement of the magnet and the wire that matters.

Einstein was able to link magnetic and electric effects. Coulomb's Law (which applies to charges at rest) and the ideas of relativity theory show that what we call magnetic effects are to be expected from charges in motion, that is, from currents.

DEMONSTRATION
H9 Inducing e.m.f.s in a motor/dynamo

fractional horsepower motor
battery, 12 V
demonstration meter, 1 A d.c.
demonstration meter, 5 V d.c.
2 rheostats, 10–15 Ω, 5 A
geared hand drill
rubber pressure tubing to fit 5 mm shaft, about 10 cm long
steel rod, 5 mm diameter, about 10 cm long
lamp, 12 V, 6 W
lampholder, s.b.c., on base
leads

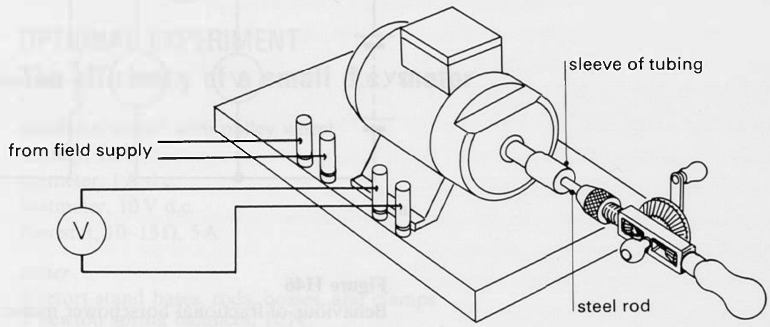


Figure H45
Fractional horsepower motor as a dynamo.

The magnetic field of the motor (figure H45) is provided by a current of about 0.5 A to the field coils. The hand drill is connected to the shaft of the motor by a short piece of rubber tubing. Turning the hand drill makes the rotor rotate.

As the rotor turns, its coils sweep through a strong magnetic field and an e.m.f. is induced.

It should be possible to demonstrate how the induced e.m.f. depends on the rate of rotation, and by varying the current in the field coils, that the e.m.f. depends on the strength of the magnetic field.

When the output of the rotor is connected to a (high-resistance) voltmeter, very little current flows, and so very little energy is transformed. But if a lamp is connected across the rotor terminals it will light, and the rotor is now harder to turn. You should be able to explain this effect firstly in terms of energy (why do you have to work harder to turn the rotor now?); and secondly by considering the fact that there is now a current in the rotor. (Which way will the force on this current in a magnetic field tend to turn the rotor? Why *must* it be in this rather than the opposite sense?)

DEMONSTRATION
H10 Behaviour of a fractional horsepower motor

Not all the electrical energy supplied to a motor is transferred to the load. There are losses due to heating the rotor windings, magnetic losses in the iron core of the motor, and mechanical losses, for example due to friction in the bearings.

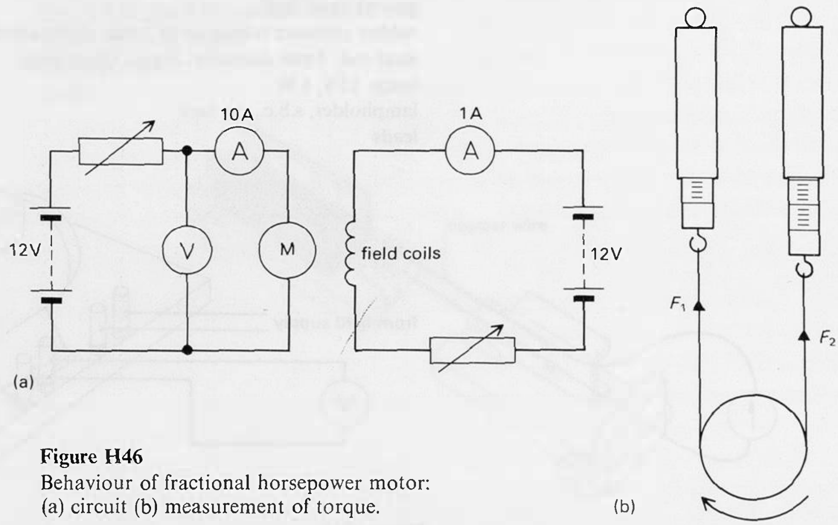


Figure H46
Behaviour of fractional horsepower motor:
(a) circuit (b) measurement of torque.

It is easy to measure the resistance of the rotor coils when they are stationary. (Use $R = V/I$.) The resistance will probably be quite low: thick copper wires are used.

With a p.d. of about 2.5 V across the rotor terminals the field current (about 1 A) is turned on. What happens to the current in the rotor as it begins to turn and speeds up? What might cause the rotor current to vary in this way? (Remember that we now have a conductor moving in a magnetic field, and think about the effects observed when the rotor was turned by the hand drill in demonstration H9.)

By Lenz's rule (*i.e.* conservation of energy), the induced e.m.f., $\mathscr{E}$, must act in the opposite direction to the p.d. applied to the rotor terminals, V. We can use Kirchhoff's Second Law to describe the circuit: $V - \mathscr{E} = IR$, and from measurement of I, $\mathscr{E}$ can be calculated.

Torque

The torque produced by the motor is proportional to BIl. B is constant, as long as the current to the field coils is held steady. If the load on the motor is increased (the p.d. applied to the rotor being kept the same), the motor slows down. How will $\mathscr{E}$ change as the motor slows down? Use $V - \mathscr{E} = IR$ to predict how I will change, and hence how the torque will change.

Power

The total power transformed by the motor is of course VI. The power which heats the rotor wires is I^2R. The remainder provides the useful mechanical output power and unavoidable magnetic and mechanical losses.

The mechanical output power can be measured using a band brake – figure H46(b). It is

$$2\pi rn(F_2 - F_1)$$

F_2 and F_1 are the tensions on either side of the band, r the radius of the wheel which the brake rubs on, and n the number of rotations per second.

OPTIONAL EXPERIMENT

H11 The efficiency of a small d.c. motor

small d.c. motor with pulley wheel
battery, 12 V
ammeter, 1 A d.c.
voltmeter, 10 V d.c.
rheostat, 10–15 Ω, 5 A

either
2 retort stand bases, rods, bosses, and clamps
2 newton spring balances, 10 N
string
means of measuring rotational frequency, *e.g.*, photodiode assembly with light source and oscilloscope (hand stroboscope and stopwatch would do)
or
hanger and slotted masses, 10 g
string
stopwatch
metre rule

leads

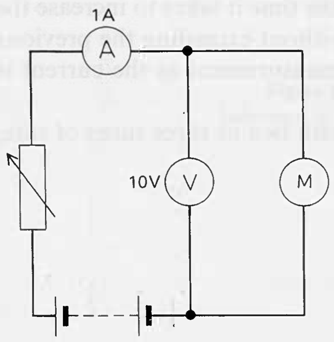


Figure H47
Energy input to a d.c. motor.

In a small d.c. motor the magnetic field is supplied by permanent magnets; the only current supplied to the motor is to the rotor.

To calculate the useful power output of the motor you can measure the steady rate at which the motor raises a load; or you can use a band brake, as in demonstration H10.

Calculate the electrical power supplied to the motor; and compare this with the useful power output. Calculate the efficiency of the motor. You can also calculate the power which goes to heating up the rotor coil (I^2R).

Do not expect to account for all losses. Such small motors are usually quite inefficient. Suggest where some of the other losses might be.

If you have time it is instructive to plot graphs of rate of rotation against load, and of current against load.

DEMONSTRATION

H12 Moving wires and changing flux

H12a Moving wires

demountable transformer kit (use 300-turn coil)
l.t. variable voltage supply
low-voltage smoothing unit
rheostat, 10–15 Ω, 5 A
demonstration meter, 5 A d.c.
sensitive galvanometer
stopclock
leads

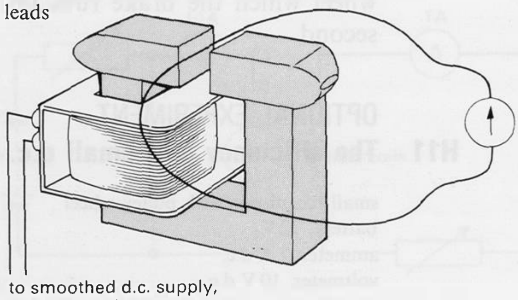


Figure H48
Moving a wire and changing a field.

The 300-turn coil supplied with 3 A produces a magnetic field between the pole pieces.

Record the time taken to move the wire down between the pole pieces of the magnet and into the 'U' of the electromagnet without exceeding a certain reading on the galvanometer. Can the wire be brought back up out of the 'U' in less time, without exceeding the same reading (in the opposite sense)?

Now, with the wire in the 'U', record the time it takes to increase the current in the coil from 0 to 3 A, again without exceeding the previous galvanometer reading. Repeat the time measurement as the current is decreased from 3 A to 0.

It is instructive to repeat these tests with two or three turns of wire, and with a folded piece of wire.

H12b Changing flux

Apparatus as for demonstration H12a plus:
set of solenoids
(demountable transformer kit not needed)

This version of the demonstration uses two close-wound solenoids. The outer one carries 3 A d.c.; the inner one is connected to a sensitive galvanometer. The minimum time in which the current in the outer coil can be reduced from 3 A to zero without exceeding a certain galvanometer reading is recorded. It is compared with the least time taken to withdraw the inner solenoid completely while a current of 3 A flows in the outer one, again without exceeding the same reading.

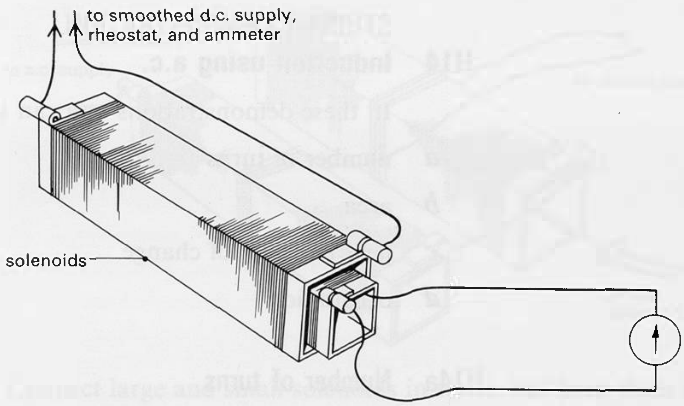


Figure H49
Changing the flux in a coil in two ways.

DEMONSTRATION
H13 A continually changing field

demountable transformer kit (use 300-turn coil)
signal generator
0.45 mm PVC-covered copper wire
rheostat, 10–15 Ω, 5 A
double-beam oscilloscope
demonstration meter, 1 A d.c.
wire strippers
leads

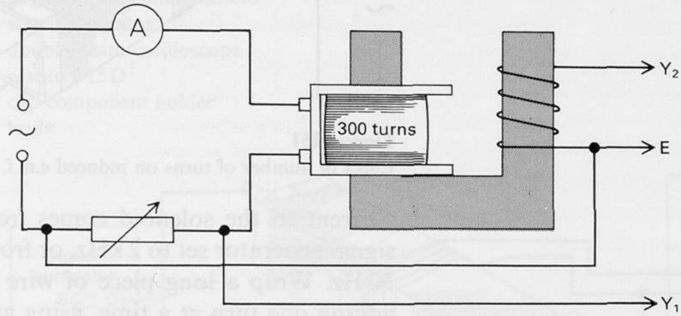


Figure H50
Induction of a.c.

Alternating current in the 300-turn coil sets up a continually changing field and so the flux through the wire wound round the other limb of the iron core is continually changing.

This demonstration allows an investigation of how various factors affect the alternating e.m.f. induced in the wire:

a the number of times this wire is wound around the iron;

b the size of the current in the 300-turn coil;

c the frequency of this alternating current;

d whether or not the iron transformer core has an iron 'yoke' across the top, making a complete ring of iron.

When, in relation to the alternating current in the 300-turn coil, does the alternating e.m.f. induced in the wire reach its maximum value?

STUDENT DEMONSTRATION
H14 Induction using a.c.

In these demonstrations you can show the effect on induced e.m.f. of:

a number of turns

b area

c field and rate of change

d orientation

H14a Number of turns

large, close-wound solenoid
signal generator (or transformer)
oscilloscope
0.45 mm diameter PVC-covered copper wire
leads

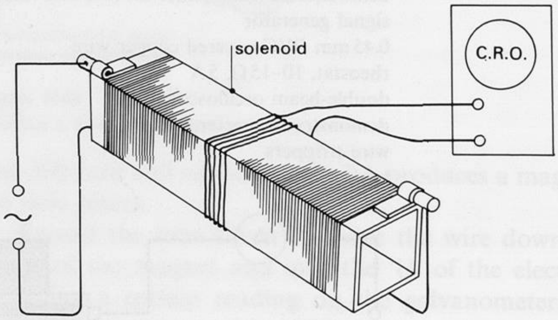


Figure H51
Effect of number of turns on induced e.m.f.

Current to the solenoid comes from the low-impedance output of a signal generator set to 2 kHz, or from a transformer giving about 3 A at 50 Hz. Wrap a long piece of wire around the centre of the solenoid, adding one turn at a time, using an oscilloscope to monitor the e.m.f. induced in the wire.

H14b Area

large and small close-wound solenoids
transformer
either
Hall probe with circuit box and meter
or
search coil

double-beam oscilloscope
0.45 mm PVC-covered copper wire
leads

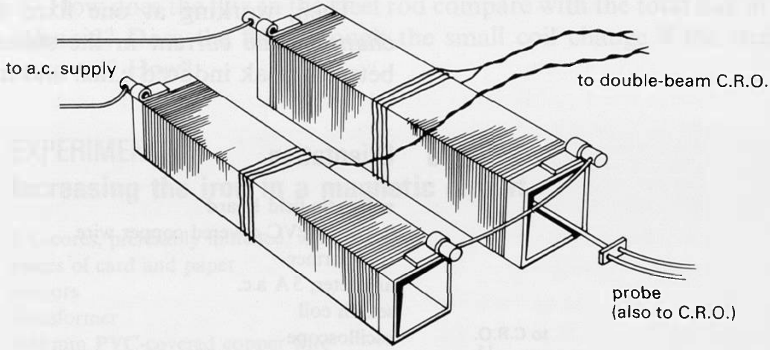


Figure H52
Effect of area on induced e.m.f.

Connect large and small solenoids in series, but keep them well apart. The current is the same in each (*e.g.* from a 12 V, 50 Hz supply).

Use a search coil or Hall probe to compare the magnetic field strengths inside the two solenoids. Use wires wrapped tightly around the outside of the two solenoids and connected to the two inputs of a double-beam oscilloscope to compare the induced e.m.f.s. Since induced e.m.f. depends on rate of change of flux, this will enable you to compare the total fluxes in the two solenoids. What is the ratio of the cross-sectional areas of the two solenoids?

H14c Field and rate of change

large, close-wound solenoid
signal generator
double-beam oscilloscope
resistor, 15 Ω
clip component holder
leads

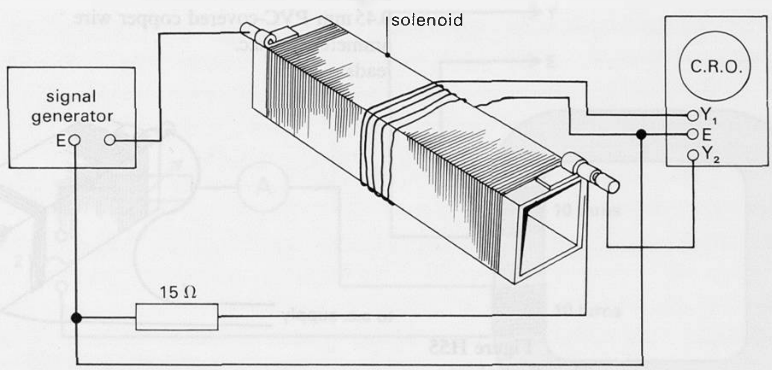


Figure H53
Effect of rate of change of flux on induced e.m.f.

Use the double-beam oscilloscope to monitor both the current in the solenoid and the e.m.f. induced in ten turns of wire wrapped around it. Note what happens to the peak value of the induced e.m.f., and also to the maximum slope of the current trace on the oscilloscope, as you vary the frequency in the region of 1 kHz. Use the low-impedance output of the signal generator, and adjust its output, if necessary, to make sure that the peak current in the solenoid is the same at each frequency.

Then, working at one fixed frequency, investigate the effect of changing the current in the solenoid. Again look for a relationship between peak induced e.m.f. and maximum rate of change of current.

H14d Orientation

magnetic field board
0.45 mm PVC-covered copper wire
transformer
ammeter, 5 A a.c.
search coil
oscilloscope
leads

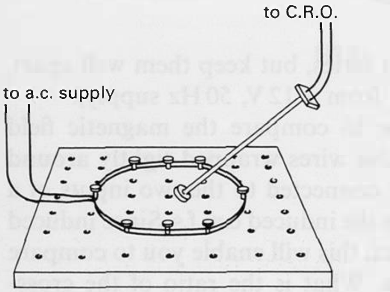


Figure H54
Effect of orientation on induced e.m.f.

Use an oscilloscope to find out how the e.m.f. induced in the search coil depends on its orientation with respect to the coil on the magnetic circuit board, which should have about ten turns. Pass an alternating current of several amperes and use an oscilloscope to find ~~out~~ how the e.m.f. induced in the search coil depends on its orientation with respect to the coil on the board. Keep the search coil in the same position (*e.g.* at the centre of the coil), and tilt it as shown in figure H54. Try to formulate a rule which relates the e.m.f. to the angle between the plane of the board and the plane of the search coil.

DEMONSTRATION
H15 The effect of iron in a solenoid

retort stand rod of mild steel (not stainless)
transformer
double-beam oscilloscope
large close-wound solenoid
0.45 mm PVC-covered copper wire
ammeter, 1 A a.c.
leads

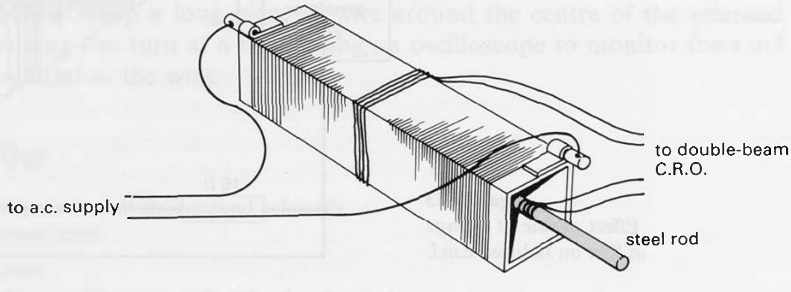


Figure H55
The effect of iron.

A coil of wire is wrapped around the solenoid, and another one with the same number of turns is wrapped around the steel rod (figure H55). A double-beam oscilloscope is used to compare the e.m.f.s induced in these two coils when there is an alternating current in the solenoid. Since e.m.f. depends on rate of change of flux, and since both fluxes change at the same frequency, comparing the e.m.f.s allows us to compare the fluxes in the two coils.

How does the flux in the steel rod compare with the total flux in the solenoid? Does the flux through the small coil change if the steel is removed? How?

EXPERIMENT
H16 Increasing the iron in a magnetic circuit

2 C-cores, preferably matched, with clip
pieces of card and paper
scissors
transformer
0.45 mm PVC-covered copper wire
oscilloscope
ammeter, 1 A a.c.
voltmeter, 5 V a.c.
rheostat, 10–15 Ω, 5 A
micrometer screw gauge
leads

Wind two ten-turn coils round one C-core. Feed one coil with 1 A a.c. from a transformer. Connect the other to an oscilloscope which displays the induced e.m.f., and hence gives a measure of the flux linked with the second coil.

How does the flux linked with this 'pick-up' coil vary with its position on the C-core?

What happens if it is slid off the end of the C-core?

Now try using two C-cores as shown in figure H56. Does the flux linked with the pick-up coil change when the magnetic circuit is completed, *i.e.* when the second C-core is added? Does the flux linked with the coil depend on its position on the C-cores? What is the effect of putting a piece of card between the two C-cores?

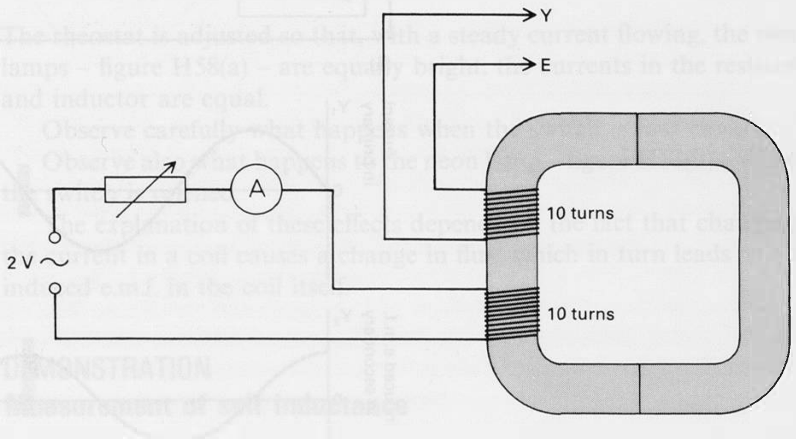


Figure H56
Increasing the iron in a magnetic circuit.

Compare the flux linked with the pick-up coil when the two coils are close together in air (no iron), with the flux when both coils are on the continuous magnetic circuit with no gaps between the two C-cores. From this measurement use $\Phi = \mu_0 \mu_r NIA/l$ to estimate μ_r, the relative permeability of the iron. (Φ is the flux produced by a current I in a coil of area A having N turns in length l. The constant μ_0 is the permeability of free space.)

DEMONSTRATION
H17 Mutual inductance of two coils

The e.m.f. induced in a secondary coil depends on the rate of change of flux, and therefore on the rate of change of current in the primary coil. If the e.m.f. is directly proportional to the rate of change of current ($\mathscr{E} \propto dI/dt$) then a constant dI/dt, *i.e.* a steadily changing current in the primary, should give a steady output at the secondary. In this case the constant of proportionality, M, the mutual inductance, can be calculated from $M = \mathscr{E}/(dI/dt)$.

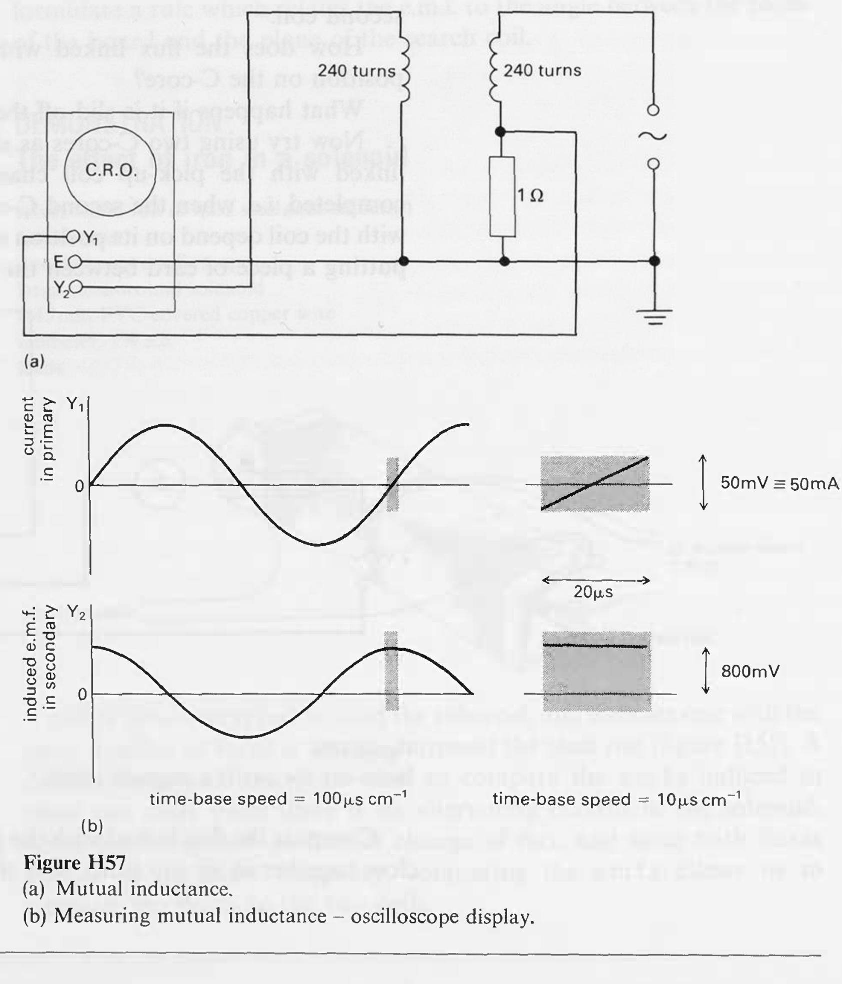


Figure H57
(a) Mutual inductance.
(b) Measuring mutual inductance – oscilloscope display.

It is possible to investigate how M depends on the numbers of turns in the two coils. M also depends on the size of the core, and the properties of any iron. Since the magnetic properties of iron vary with the flux density, M may be found to depend on the current in the primary coil.

DEMONSTRATION
H18 Self induction

high-inductance coil
double C-core with clip
2 m.e.s. lamps, 2.5 V, 0.3 A
m.e.s. neon lamp
3 m.e.s. lampholders
cell holder with two cells
mounted bell push
rheostat, 10–15 Ω, 5 A
leads

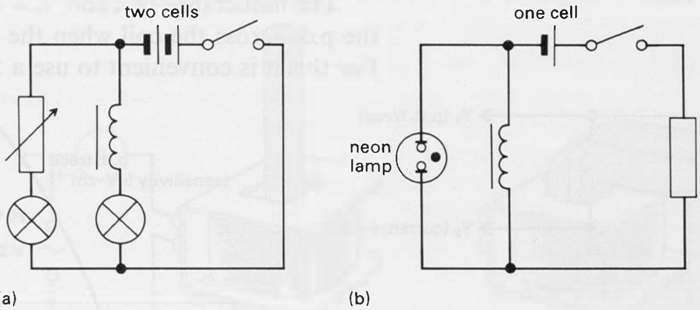


Figure H58
Simple inductor experiments.

The rheostat is adjusted so that, with a steady current flowing, the two lamps – figure H58(a) – are equally bright: the currents in the resistor and inductor are equal.

Observe carefully what happens when the switch is first closed.

Observe also what happens to the neon lamp – figure H58(b) – when the switch is opened.

The explanation of these effects depends on the fact that changing the current in a coil causes a change in flux, which in turn leads to an induced e.m.f. in the coil itself.

DEMONSTRATION
H19 Measurement of self inductance

In this demonstration the oscilloscope is used to monitor both the p.d. across an inductor and the current through it (Y_1 and Y_2, respectively, in figure H59).

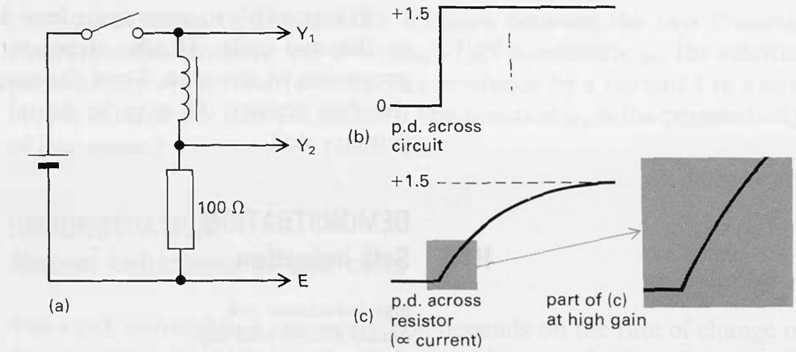


Figure H59
Measurement of self inductance.

Observe carefully what happens when the switch in the circuit of figure H59 is closed. What effect does the inductor have on the current in the circuit? Is it a permanent or a transient effect?

In a d.c. circuit an inductor does not affect the final steady current; it only affects how quickly the current reaches this value.

The inductance of a coil, $L = V/(\mathrm{d}I/\mathrm{d}t)$, can be found by measuring the p.d. across the coil when the current is changing at a known rate. For this it is convenient to use a 50 Hz supply as shown in figure H60.

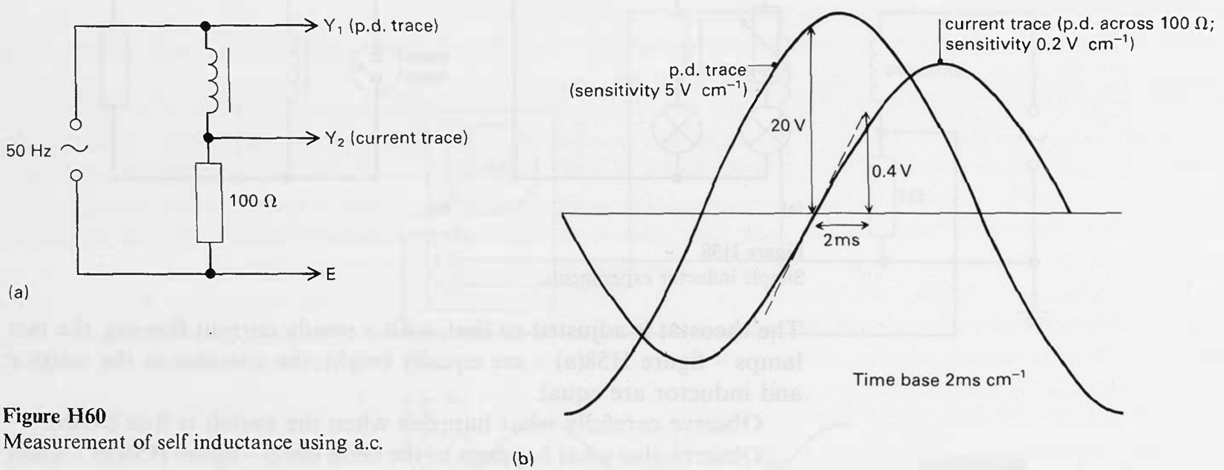


Figure H60
Measurement of self inductance using a.c.

The oscilloscope is used to make measurements of V and $\mathrm{d}I/\mathrm{d}t$. Of course the voltage sensitivity of the two traces must be known, and also the time-base speed.

Notice that in figure H60 the Y_1 input of the oscilloscope is connected across the resistor as well as the inductor. How big an error does this introduce into the measurement of V, the p.d. across the inductor? How can this error be kept small?

EXPERIMENT

H20 Investigation of transformer action

transformer to provide 6 and 12 V a.c.
2 coils with 120 + 120 turns
double C-core and clip
2 ammeters, 5 A a.c.
2 s.b.c. lamps, 12 V, 6 W
2 s.b.c. lampholders on bases
retort stand rod, steel
rheostat, 10–15 Ω, 5 A
oscilloscope
leads

H20a What is the effect of iron in the circuit?

In figure H61(a) the coil and lamp are connected in series to a 12 V a.c. supply. Observe and explain what happens:
i when an iron rod is inserted into the coil, and
ii when there is a complete C-core in the coil – figure H61(b).

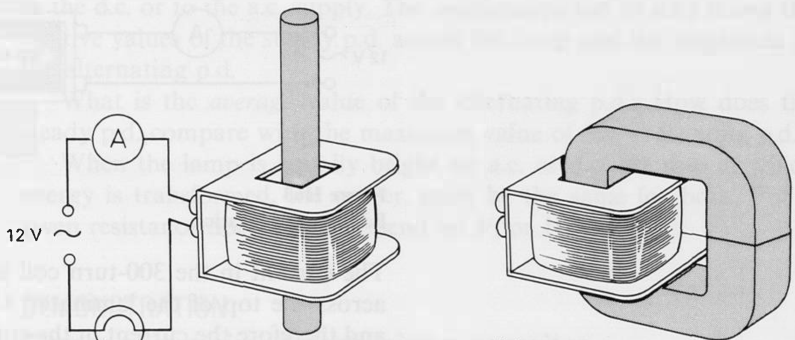


Figure H61 (a) (b)

H20b What is the effect in a second coil on the same core?

Now add a second coil, with the complete C-core linking both coils. For a 6 volt input to one coil investigate how the e.m.f. induced in the secondary coil depends on the ratio of the number of turns in the two coils. Can you use a 6 volt input to light a 12 V lamp?

H20c How does the current in one coil depend on the current in the other?

Use the rheostat to control the resistance of the secondary circuit (figure H62). Measure pairs of values of current in the two coils. How does the current ratio depend on the turns ratio?

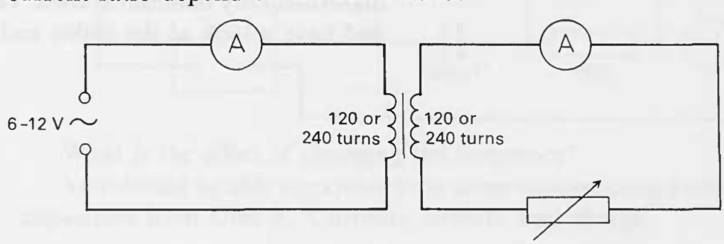


Figure H62

H20d How do the input and output powers compare?

Use the circuit shown in figure H62 to compare the input and output powers of the transformer. Loss of power is due to heating of the wires in the primary and secondary coils (I^2R), and also to heating of the core by eddy currents (and magnetic hysteresis).

DEMONSTRATION

H21 Eddy currents

transformer to provide 12 V a.c.
demountable transformer kit
demonstration meter, 5 A a.c.
mild steel retort stand rod (or even a flat file)
leads

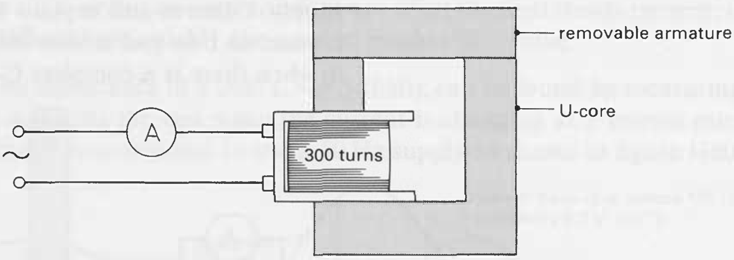


Figure H63
Eddy current heating.

The current in the 300-turn coil is measured with various iron pieces across the top of the laminated U-core (figure H63). The inductance, and therefore the current in the coil, depend on the amount and type of iron in the magnetic circuit.

The changing flux in the iron can set up induced currents in the iron itself. One can't put an ammeter inside the iron, so how might such currents be detected? (Think back to demonstration H3, Forces on induced currents.)

These internal currents in the core, called eddy currents, can be reduced by using laminated iron. Look for laminations in this core, in the C-cores used in experiment H20, and also in the iron of any other electrical machinery available – motors, generators, and so on.

Eddy currents become more important at high frequency (why?). High-frequency currents circulate in the coil of a radio tuning circuit (about 1 MHz for the medium wave band). The coil needs a core to increase its inductance. To cut down on losses due to eddy currents, a high-resistivity material is used. Take off the back of a radio if you can and have a look at the coil(s) and the core.

DEMONSTRATION

H22 Power in a resistive circuit

cell holder with three cells
transformer to provide 2 V a.c.
rheostat, 10–15 Ω, 5 A
oscilloscope
2 mounted bell pushes (or s.p.d.t. switch)
m.e.s. lamp, 2.5 V, 0.3 A, in holder
leads

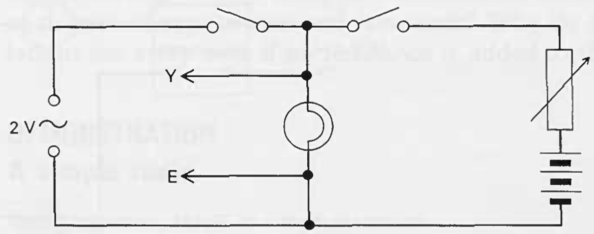


Figure H64
Comparing the brightness of a lamp lit from a.c. and d.c.

The rheostat is adjusted until the lamp is equally bright when connected to the d.c. or to the a.c. supply. The oscilloscope (set to d.c.) shows the relative values of the steady p.d. across the lamp and the amplitude of the alternating p.d.

What is the *average* value of the alternating p.d.? How does the steady p.d. compare with the maximum value of the alternating p.d.?

When the lamp is equally bright on a.c. or d.c. the rate at which energy is transformed, the power, must be the same for both. For a given resistance does power depend on V, or V^2, or ...?

DEMONSTRATION

H23 Slow a.c. in a circuit containing a capacitor

When a.c. passes through a resistor the p.d. across the resistor always 'keeps up' with the current through it: the two are in phase.

Two meters, or a double-beam oscilloscope, can be used to investigate the phase relationship between p.d. across a capacitor and current in the circuit, using a slowly alternating current (figure H65).

Is there a phase difference in a circuit containing capacitance?

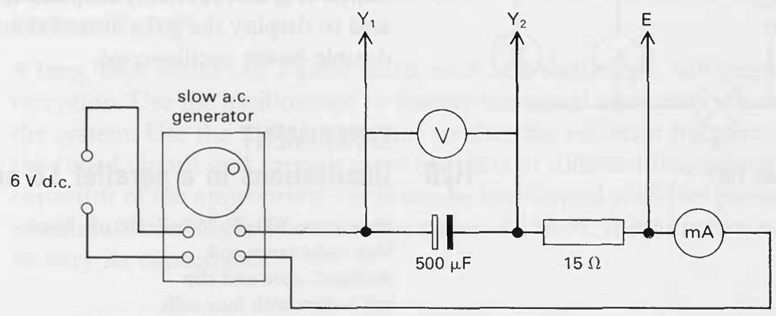


Figure H65

What is the effect of changing the frequency?

You should be able to explain your observations using knowledge of capacitors from Unit B, 'Currents, circuits, and charge'.

DEMONSTRATION
H24 Power in a capacitor

When a lamp or resistor is connected across a joulemeter the joulemeter reading is equal to IV. But when a capacitor is connected across the meter the reading is quite different from IV. What is it?

Use the phase relationship between I and V for a capacitor and power $= IV$ to explain what's going on here.

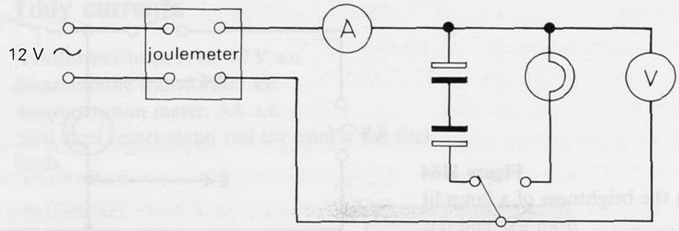


Figure H66

DEMONSTRATION
H25 Alternating currents in a circuit containing capacitance and inductance

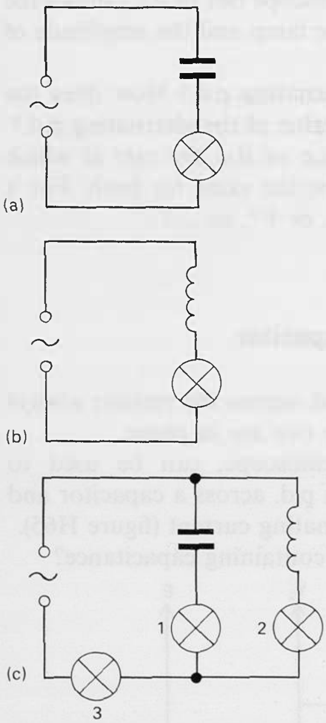


Figure H67

For a given p.d. the alternating current through a resistor does not depend on the frequency of the supply: resistance does not depend on frequency.

When the frequency is increased in figure H67(a) the lamp glows more brightly – more current passes for the same p.d. Why is this? (Look back at demonstration H23, Slow a.c. in a circuit containing a capacitor, if necessary.)

Before seeing the effect in the circuit shown in figure H67(b), try to predict what will happen. Would you expect the current in the inductor to depend on the frequency? If so, how? (Remember that $V = L\,\mathrm{d}I/\mathrm{d}t$.)

Another demonstration uses both capacitor and inductor in a parallel circuit – figure H67(c). After you have seen this demonstration, try to explain the result, *i.e.* the relationships between the currents in lamps 1, 2, and 3. It may help to replace the lamps by low-value resistors and to display the p.d.s across them (proportional to the currents) on a double-beam oscilloscope.

EXPERIMENT
H26 Oscillations in a parallel *LC* circuit

capacitors, 500 µF, 250 µF, 100 µF, 50 µF
high-inductance coil
double C-core and clip
cell holder with four cells
potentiometer, 1 kΩ (100 Ω if available)
oscilloscope
s.p.d.t. switch (if available)
leads

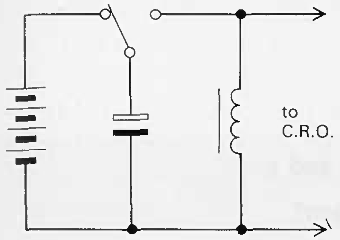


Figure H68
Oscillations in an LC circuit.

Use the oscilloscope, with its time-base running slowly, to show the p.d. when the capacitor is discharged through the inductor.

What effect does changing the capacitance, C, for example from 500 μF to 100 μF, have on the frequency of oscillation? You can reduce the inductance L by slightly separating the C-cores in the inductor. What effect does this have?

Now add extra resistance in series with the inductor. What is the effect on the oscillations you observe? If inductance in an electrical circuit corresponds to mass in a mechanical oscillator, what are the analogues of capacitance and resistance? Why do the electrical oscillations die away even if no resistance is added to the LC circuit?

H

DEMONSTRATION

H27 A simple radio

tuning capacitor, 365 pF to 500 pF maximum
set of solenoids (or other coil)
diode
general-purpose amplifier
loudspeaker (if not in amplifier)
oscilloscope
reel of 0.45 mm PVC-covered wire (for aerial)
leads

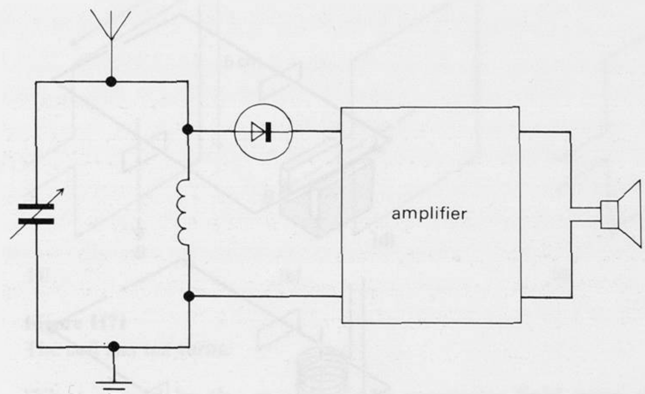


Figure H69
Simple radio circuit.

A long, high aerial and a good earth, such as a water pipe, will improve reception. Use the oscilloscope to display the signal at various stages of the system. Use the tuning capacitor to alter the resonant frequency of the tuned circuit and to pick up broadcasts at different frequencies. A capacitor of the appropriate value can be improvised with two pieces of aluminium cooking foil between the pages of a book: there are two ways to vary its capacitance.

QUESTIONS

Forces on currents, $F = BIl$

1(I) On a graph the axes are often labelled x and y.

a If there is also a z-axis, what is its direction?

b List some common objects which have three lines on them, all at right angles to each other.

2(L) Figure H70(a) shows a form of 'current balance' which consists of a pivoted wire frame resting on knife edges through which current enters the frame. The balance is tilted by a vertical force on the wire AB.

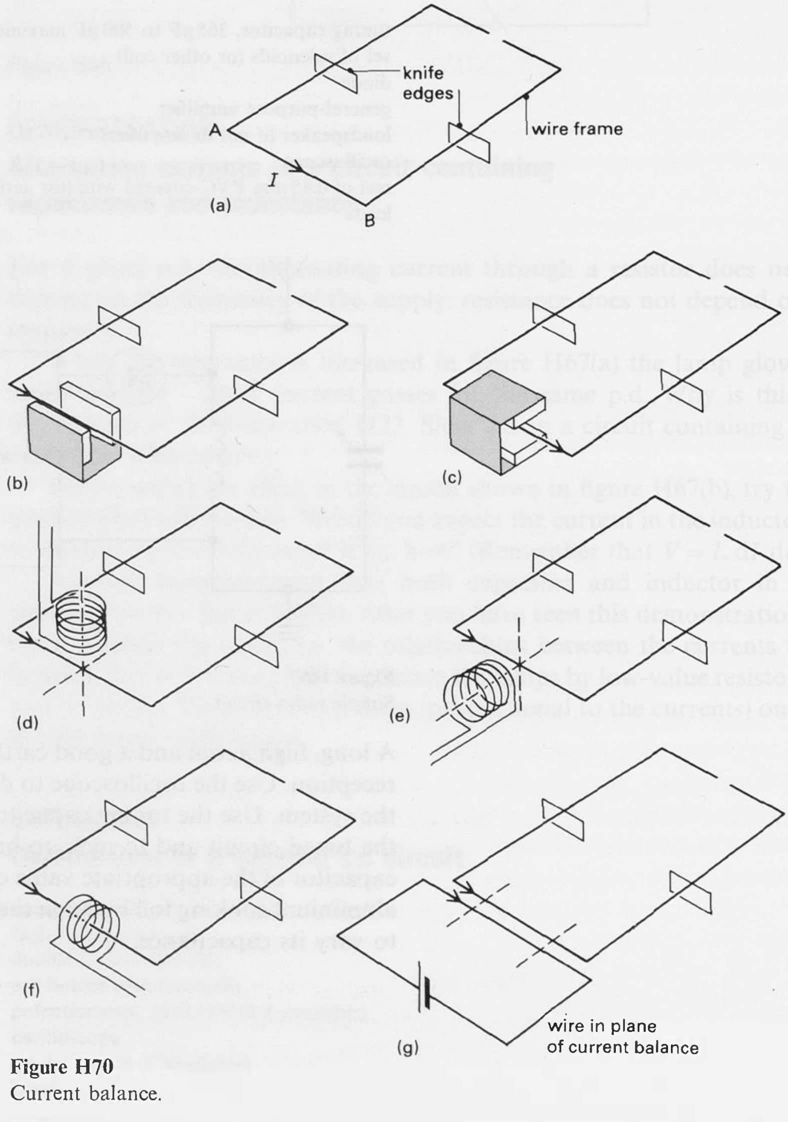


Figure H70
Current balance.

The force on a current is at right angles to the current and to the direction of the B-field. Figures H70(b) to (g) show a series of attempts to tilt a current balance using a magnetic force. Which of them will produce a force on the wire which will tilt the balance?

3(P) In these questions you should assume that the magnetic field is perpendicular to the current.

a What magnetic field strength is needed to give a force of 0.01 N (about the weight of 1 gram) on a wire 5 cm long carrying a current of 5 A?

b Calculate the force per centimetre length of wire on a straight wire carrying a current of 2 A in a magnetic field of 0.1 tesla.

c Write down the force per metre on a straight wire carrying a current of one ampere in a magnetic field of T tesla.

4(P)a A 4 cm by 5 cm rectangular coil having 10 turns is situated in a magnetic field of 0.2 tesla in such a way that the 4 cm sides are perpendicular to the field – figure H71(a). What would be the couple, or torque, on the coil if it carried a current of 1 A?

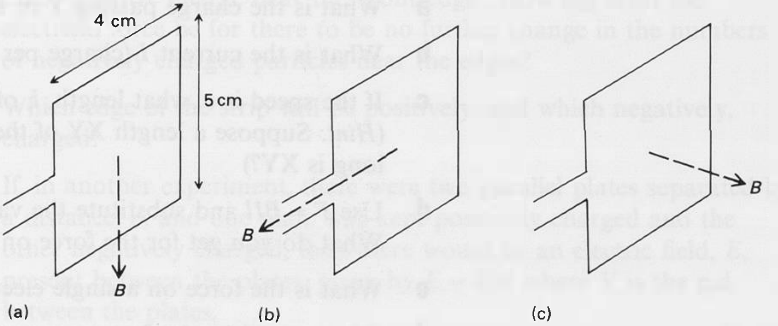


Figure H71
The coil has ten turns.

b What would be the couple if the magnetic field were perpendicular to the 5 cm sides and parallel to the 4 cm sides – figure H71(b)?

c What would be the couple on the coil if the magnetic field were perpendicular to both the 4 cm and the 5 cm sides – figure H71(c)?

5(E)a What current would need to flow in a horizontal copper wire of cross-sectional area 1 mm^2 to make it self-supporting in the Earth's magnetic field. In Great Britain the Earth's magnetic field has a strength of 1.7×10^{-4} T in a direction about 66° below the horizontal. The density of copper is 8900 kg m^{-3}.

b Discuss whether this might be a practical way of supporting power cables.

Force on a moving charge, $F = BQv$

6(I) A satellite is moving in a circular orbit around the Earth.

a Why does the satellite
i not move in a straight line?
ii not speed up or slow down?

b If the net force on the satellite is always perpendicular to its motion, can it ever speed up or slow down?

7(L) This question goes from the equation $F = BIl$ to $F = BQv$. It refers to electrons but the same argument can be applied to any charge carriers, for example ions in a mass spectrometer.

If a current I runs at right angles to a B-field, the force F on length l of the current is given by

$$F = BIl$$

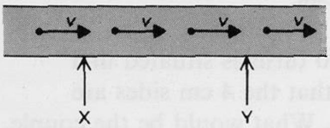


Figure H72

The question is about finding a similar expression for the force on an electron having a charge e and a velocity v. Suppose that, in a time t, N electrons pass any place in the beam, such as Y in figure H72.

a What is the charge passing Y in time t?

b What is the current I (charge per second) at Y?

c If the speed is v, what length, l, of beam passes Y in time t? (*Hint:* Suppose a length XY of the beam will pass Y in time t. How long is XY?)

d Use $F = BIl$ and substitute the value of I from **b** and of l from **c**. What do you get for the force on N electrons?

e What is the force on a single electron?

f What would be the force on a particle with charge Q?

Hall effect

8(I)a A mass of 1 kg which is attached to a slack spring is allowed to fall. When the mass has stopped oscillating, what forces are acting on the mass?

b Why does the mass not fall or rise?

9(L) A result from Unit B, 'Currents, circuits, and charge' is useful. If I is the current when there are n charge carriers per unit volume, each with negative charge Q passing through an area A with a velocity v, then:

$$I = nAQv$$

a When the magnetic field B is switched on, what force will be exerted on one moving charge carrier?

b What is the direction of the force? Mark it on a copy of figure H73(a).

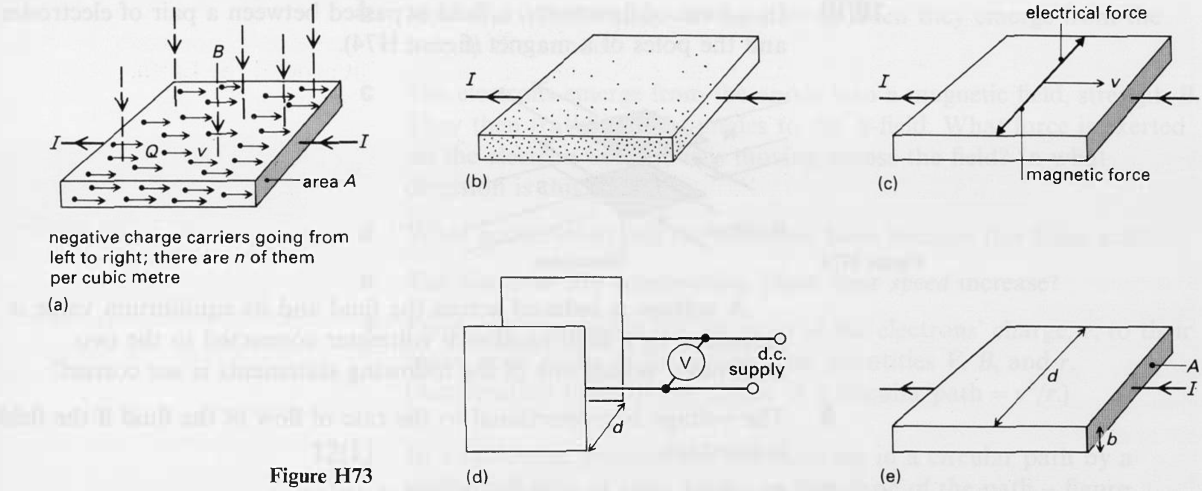


Figure H73

c After a time there will be higher density of negative charge near the front edge than near the rear edge, as shown in figure H73(b). Why?

d A charge carrier in the middle, figure H73(c), will still experience a force due to the magnetic field, but it will also be repelled by the extra negative charge near the 'front edge'. How big must the electrical force be for there to be no further change in the numbers of negatively charged particles near the edges?

e Which edge of the strip will be positively, and which negatively, charged?

f If, in another experiment, there were two parallel plates separated by a distance, d, and one plate was kept positively charged and the other negatively charged, then there would be an electric field, E, present between the plates, given by $E = V/d$ where V is the p.d. between the plates.

A charge, Q, placed between the plates would experience a force due to this electric field:

force $= EQ$

The electric force acting on a charge moving in the metal strip has been given by your answer to **d**. What is the potential difference which has been produced between the front and rear edges of the strip?

g But $I = nAQv$. What is A if the strip has thickness b?

An equation for the number of charge carriers is wanted which does not contain the velocity of the charge carriers.

h Use $I = nAQv$ to eliminate v from your equation for the potential difference and obtain an equation relating the number of charge carriers per unit volume to the potential difference. (Check that your expression for n has the units metre^{-3}.)

10(R) In a form of flowmeter, a fluid is passed between a pair of electrodes and the poles of a magnet (figure H74).

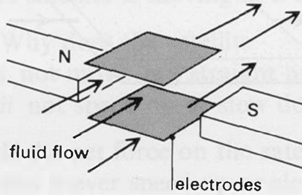


Figure H74

A voltage is induced across the fluid and its equilibrium value is measured by a high resistance voltmeter connected to the two electrodes. Which one of the following statements is *not* correct?

A The voltage is proportional to the rate of flow of the fluid if the field is constant.

B The fluid must be a conductor of electricity.

C The voltage is proportional to the size of the magnetic field if the rate of flow is constant.

D The voltage would be increased by making the fluid flow parallel to the magnetic field.

E There is no resultant force on particles in the fluid in a direction perpendicular to the magnetic field and to the direction of flow.

(Coded answer paper, 1974)

Forces on charged particles

11(L) This question is about the bending of beams of charged particles into curved paths.

a Electrons are emitted from the cathode, in a fine-beam tube. They are accelerated towards an anode by a potential difference between the anode and the cathode. If the charge on an electron is e and the potential difference between cathode and anode is V, how much energy does each electron acquire by the time it reaches the anode?

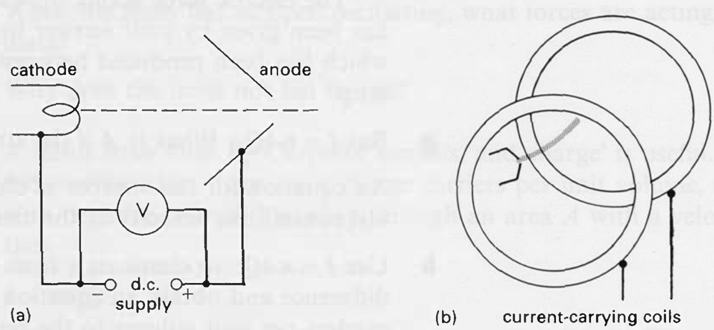


Figure H75

b How fast will the electrons be moving when they emerge from the hole in the anode?

c The electrons emerge from the anode into a magnetic field, strength B. They then travel at right angles to the B-field. What force is exerted on the electron because it is moving across the field? In what direction is this force?

d What acceleration will the electrons have because this force acts?

e The electrons are accelerating. Does their *speed* increase?

f Derive an expression for the ratio of the electrons' charge, e, to their mass, m, in terms of the measurable quantities V, B, and r. (Acceleration towards the centre of a circular path $= v^2/r$.)

12(L) In a cyclotron, protons are kept moving in a circular path by a uniform B-field at right angles to the plane of the path – figure H76(a). In the first cyclotron, made by E. O. Lawrence, protons were accelerated to an energy of 13 keV.

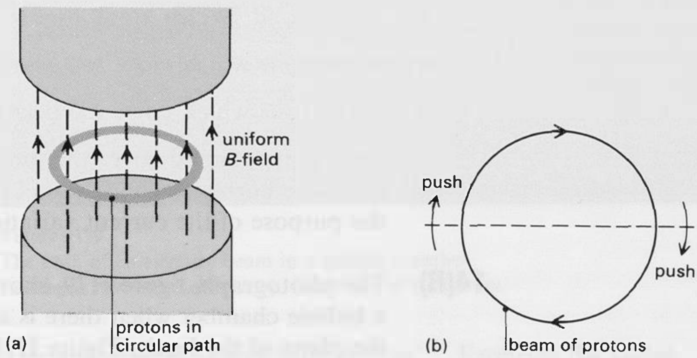


Figure H76

a What velocity has a proton with this kinetic energy?

b The largest possible path had a radius of about 50 mm. What strength of B-field must have been used?

c What radius path would a proton with *half* this maximum energy follow in this field?

d How long must it have taken the 13 keV protons to travel once round their path? How long for those with half this energy?

The cyclotron worked by giving particles of any energy, on their respective paths, a push every time they had completed half an orbit – figure H76(b).

e How was it possible for particles of differing energy all to be accelerated together?

mass of proton $= 1.7 \times 10^{-27}$ kg

charge on proton $= 1.6 \times 10^{-19}$ C

13(P) Figure H77 shows a cathode-ray tube of the sort used in a television set. The coils around its neck are magnetic deflection coils, used to sweep the electron beam to and fro.

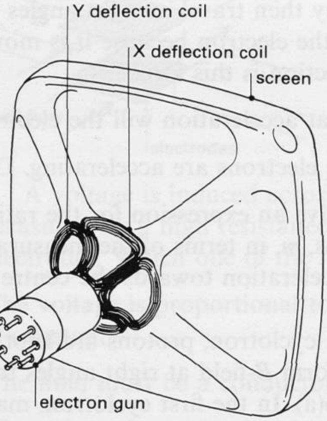


Figure H77
The X and Y deflection coils in a television cathode-ray tube.

The beam has to paint out 625 lines to make a full picture every 1/25 second. Sketch a graph of the variation with time of the current:

a in the coils which move the beam in a horizontal direction;

b in the coils which move the beam in a vertical direction.

Your graph should cover the time needed to draw out, say, three complete horizontal lines. Label the graph as necessary to indicate the purpose of the current variations.

14(R) The photograph, figure H79, shows the path of an electron beam in a bubble chamber when there is a magnetic field at right angles to the plane of the beam. Figure H78 shows the apparatus.

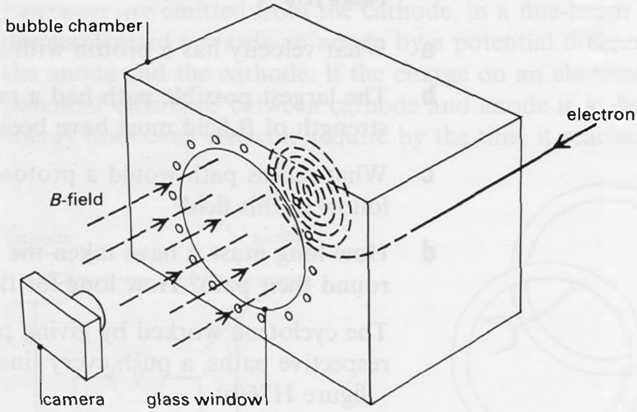


Figure H78

a The path is a spiral. Consider what could be changing as the electron moves in the bubble chamber, and suggest why the path is not a circle.

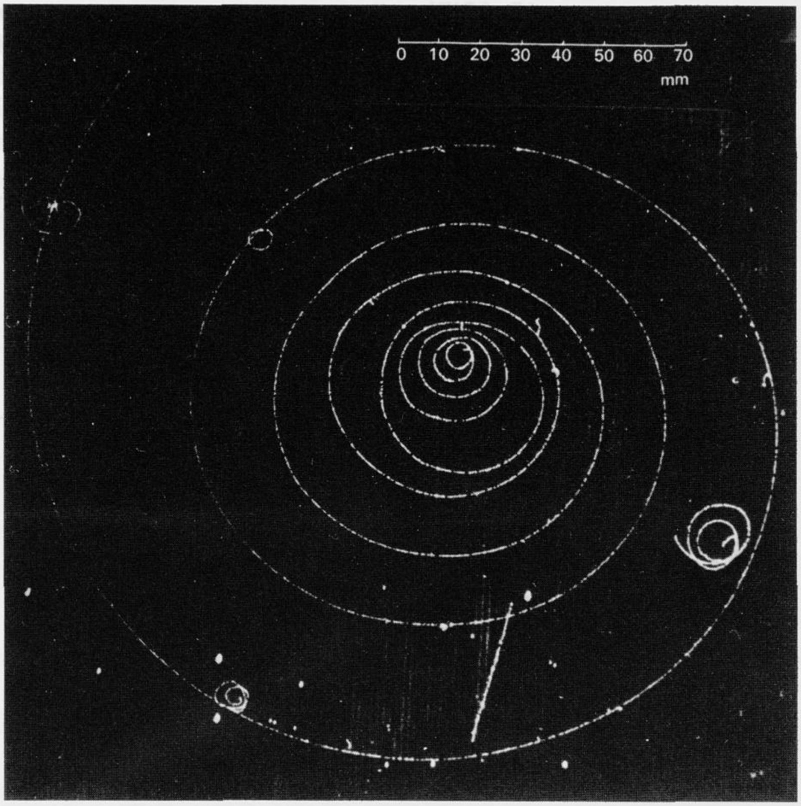


Figure H79
The path of an electron beam in a bubble chamber.
Lawrence Berkeley Laboratory, University of California.

b The magnetic field was $1.2\,\mathrm{N\,A^{-1}\,m^{-1}}$. Estimate the initial momentum of the electron using the scale shown.

c *Optional extra.* Plot a graph showing how the momentum of the electron beam changes as the distance covered in the chamber changes.

Note: In fact, the speed of the electron hardly changes, even though its momentum decreases. This is because its speed, v, is near to the speed of light, c. The theory of relativity says that the momentum is not mv but $m_0 v/\sqrt{1 - v^2/c^2}$, where m_0 is the rest mass (the mass measured at low velocities).

15, 16(R) A particle P with charge Q, mass m, and constant speed v crosses line XY into a region where a uniform magnetic field B causes it to move in a circular path of radius r, determined by the equation $BQv = mv^2/r$, so that $r/v = m/BQ$. The particle P takes a time $t = \pi r/v$ to travel in a semicircle from X to Y (figure H80).

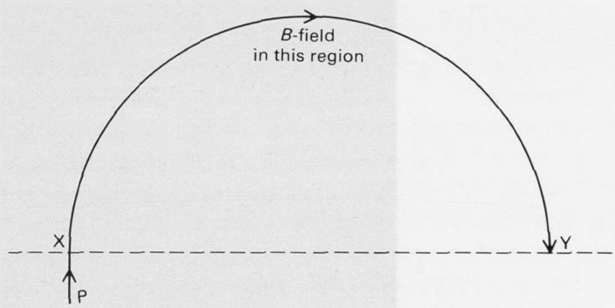


Figure H80

What time will a particle take to go around from X so as to recross the straight line XY if it is identical with P, except that it has:

15 speed $2v$?

16 charge $2Q$?

A $t/4$ **B** $t/2$ **C** t **D** $2t$ **E** $4t$

(Coded answer paper, 1977)

Fields near currents

17(L) It can be shown experimentally that near a long straight wire $B \propto I$ and $B \propto 1/r$. This means that the magnetic field of an infinite straight wire can be expressed as $B = kI/r$ where k is some constant. The ampere is defined as that current which, flowing in two infinitely long wires of negligible diameter, one metre apart, gives a force of 2×10^{-7} N per metre length of the wires.

Figure H81 shows two long straight wires, A and B, one metre apart, each carrying a current of one ampere.

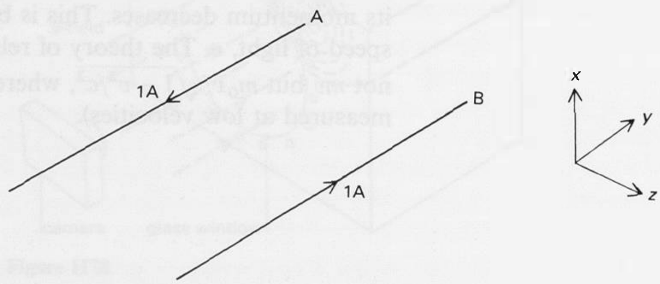


Figure H81

a What is the field due to the current in wire A, 1 metre away, in terms of k?

b Is the field in the direction x, y, or z?

c How big is the force on 1 metre of wire B, in terms of k?

d Is the force in the direction x, y, or z?

e The force is 2×10^{-7} N. What value does this give the constant k?

The field of a straight wire is normally given as $B = \mu_0 I/2\pi r$. (The extra 2π is added so that the denominator is actually the circumference of the circle, distance r from the wire, at which B is calculated. It also ensures that the formula for a uniform field, such as that inside a solenoid, does not include a π.)

f What is the value of μ_0, and what are its units?

In theory the two wires should be in a vacuum. In practice the presence of air makes very little difference. It changes the force between the wires by a very small factor (less than 1.000 001).

18(P) A current of 10 A flows in a long straight wire. What is the B-field:

a *i* 1 cm from the wire?
ii 10 cm from the wire?

b What would be the force on another wire of length 10 cm carrying a current of 5 A if it were placed parallel to the first at distances of 1 cm and 10 cm from the first wire?

c Would these forces be enough to support 1 cm of tickertape with a mass per metre length of 0.62 g?

19(P)a Use the formula $B = \mu_0 NI/l$ for an infinite solenoid to calculate the B-field at the centre of a solenoid of length 30 cm with 360 turns carrying a current of 1 A.

b What is the field at one end of the solenoid? (Remember that $B = \mu_0 NI/l$ assumes that the solenoid extends 'to infinity' in both directions from the point at which B is calculated.)

20(R) A space of about 10 cm × 10 cm × 10 cm with no magnetic field is needed for experiments on small living organisms. To cancel the Earth's magnetic field it is proposed to pass a current through a 20-turn square coil of side 1 m, with the experiment placed at the centre of the coil.

a Using the formula for a straight wire, $B = \mu_0 I/2\pi r$, find an approximate value for the current which would be needed to cancel the Earth's field of 1.7×10^{-4} T.

b The formula does not apply to short wires. Estimate the actual current which would be needed.

c The direction of the Earth's B-field in the UK is about 7° West of North and about 66° below the horizontal. In what plane would the coil have to be placed?

H

21(R) Explain how you would estimate the quantities **a** and **b** below.

Say what would be done at each step of the calculation, giving reasons for each step. Say what numerical information you would need. Indicate where you would make simplifications or approximations.

Your explanation should be such as to enable someone, provided with the numerical information, to perform the calculation.

a The distance an electron falls under gravity as it travels from the electron gun to the screen of a television set.

b The magnetic field (B) that you could obtain close to the wire by connecting a thick piece of copper wire direct from terminal to terminal of a car battery.

(Special paper, 1977)

22(R) The graph in figure H82(a) shows the variation with distance of the magnetic field strength (B) near a long straight wire carrying a current of 10 A. The small circle with a cross in it – figure H82(b) – to the right of the graph, represents the wire carrying the current perpendicularly into the paper.

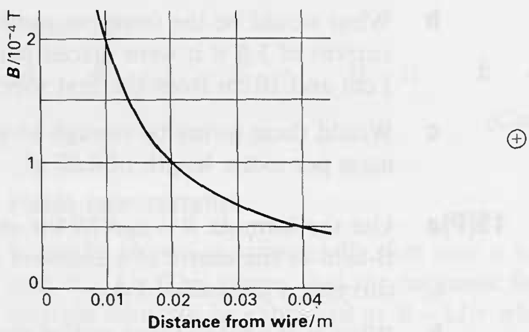


Figure H82 (a) (b)

a Show, by drawing on a copy of figure H82(b), the shape and direction of the magnetic field near the wire.

Does your drawing also represent the variation of field strength shown by the graph? Say either how your drawing does this, or how it could be modified to do so.

b Using information from the graph, determine:
i the strength of the magnetic field at a distance of 0.005 m from the wire when the current is 10 A;
ii the current which, at a distance of 0.02 m from the wire, would give a value of B equal to 2×10^{-5} T. In each case show how you obtain your answer.

c A second wire of length 0.3 m, which also carries a current of 10 A, is placed parallel to the original wire and at a distance of 0.01 m from it. What is the magnitude of the force on this second wire due to the current in the original wire? Show the steps in your calculation.

(Short answer paper, 1980)

Induction

23(L) This question is about the e.m.f. induced in a conductor moved in a magnetic field.

Figure H83 shows a conducting rod moving at right angles to its length, l, across a B-field. The rod, and all the charge carriers in it (each with charge Q) are carried along at velocity v.

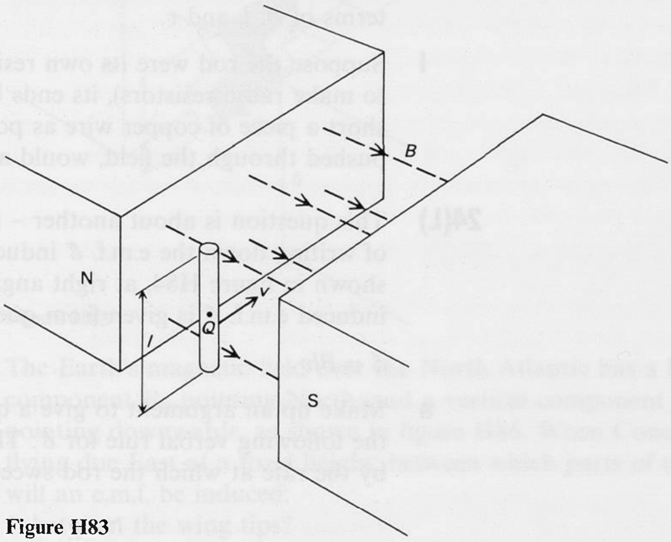


Figure H83

a What magnetic force will be exerted on one carrier? Will the force be parallel to the rod, the B-field, or the velocity?

b What will happen if a galvanometer is connected to the ends of the rod? (The galvanometer is stationary.)

c The carriers tend to pile up at one end of the rod, if there is now no external circuit (such as the galvanometer) for them to get through. An induced e.m.f., $\mathscr{E}$, develops in the rod. If $\mathscr{E}$ is steady, no further piling up of charge carriers can be going on. What electric field E then exists across the rod, length l?

d What force EQ acts on each charge carrier in the rod?

e When the carriers are no longer piling up more and more, the electric force (from **d**) must counterbalance the magnetic force (from **a**). Write an expression for $\mathscr{E}$ involving only B, l, and v.

f Will the magnitude and sign of the induced e.m.f. depend upon the magnitude and sign of the charge Q on each carrier?

g Suppose now that the moving rod is connected to an external resistor (at rest) so that current I flows through resistance R. How much energy will be dissipated in R, in time t, in terms of I, R, and t?

h When the rod moves there is a current I, and so there is a force $F = BIl$ on the rod. Could this force be in such a direction as to increase the speed of the rod? Explain.

i What distance does the rod move in time t?

j Energy has to be delivered to the rod to keep it moving, since the moving rod is delivering energy to the resistor. Write down the work involved in pushing the rod for a time t, from previous answers for force and distance.

k Compare the answers to **j** and **g**. Express the induced e.m.f. $\mathscr{E}$ in terms of B, l, and v.

l Suppose the rod were its own resistor (say a rod of the material used to make radio resistors), its ends being connected together by as short a piece of copper wire as possible. If this combination were pushed through the field, would any current flow?

24(L) This question is about another – at first sight not very helpful – way of writing down the e.m.f. $\mathscr{E}$ induced in a rod of length l, moving as shown in figure H84, at right angles to the field B, at velocity v. The induced e.m.f. $\mathscr{E}$ is given from question **23** by

$\mathscr{E} = Blv$

a Make up an argument to give a quantitative algebraic expression for the following verbal rule for $\mathscr{E}$: The e.m.f. $\mathscr{E}$ is equal to B multiplied by the rate at which the rod sweeps out area A perpendicular to B.

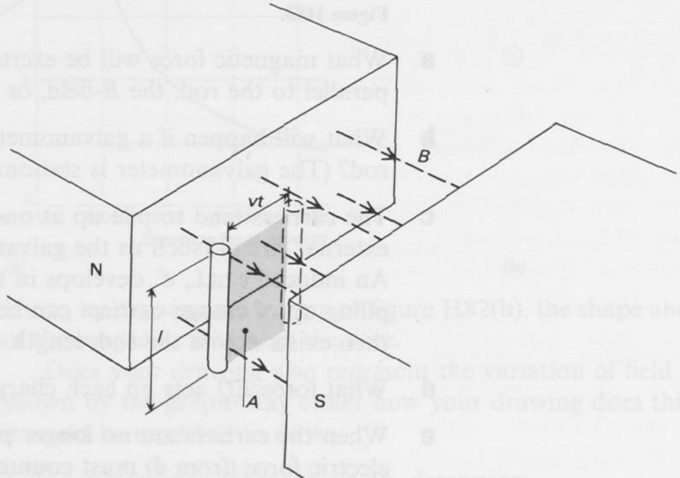


Figure H84

b (*Harder*) Suppose that the rod were a zig-zag shape, and its length were larger than the distance l between its ends, as shown in figure H85. Would the induced e.m.f. be larger than $\mathscr{E} = Blv$?

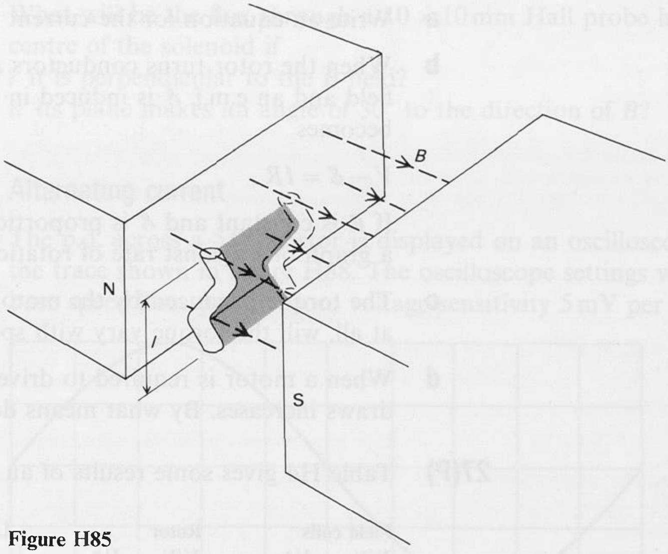


Figure H85

25(R) The Earth's magnetic field over the North Atlantic has a horizontal component B_H pointing North, and a vertical component B_V pointing downwards, as shown in figure H86. When Concorde is flying due East at a fixed height, between which parts of the aircraft will an e.m.f. be induced:
1 between the wing tips?
2 between the nose and the tail?
3 between the top of the tail and the bottom of the fuselage?
A 1 only **B** 2 only **C** 1 and 3 only
D 2 and 3 only **E** 1, 2, and 3.

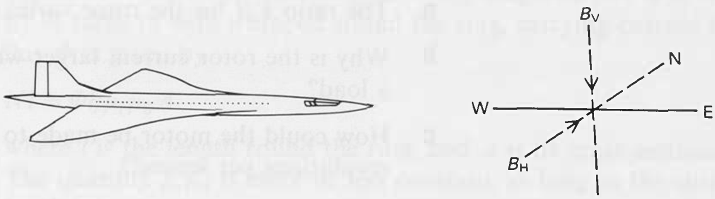


Figure H86

(Coded answer paper, 1979)

Direct current motors

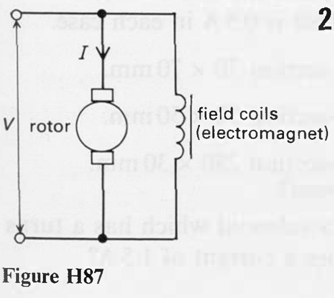


Figure H87

26(L) In many electric motors the magnetic field in which the rotor rotates is provided by an electromagnet. One way of connecting the rotor and field coils (electromagnet) is in parallel, as shown in figure H87. Since the field coils are in parallel with the rotor, the current through them, and hence the magnetic field, stay pretty constant irrespective of what happens to the rotor.

The rotor has resistance R, carries current I, and has a p.d. V across it. In this question, the steady current drawn by the field coils is ignored.

a Write an equation for the current I if the rotor is held still.

b When the rotor turns conductors are being moved in a magnetic field and an e.m.f. $\mathscr{E}$ is induced in the coil. The equation from **a** becomes

$V - \mathscr{E} = IR$

If R is constant and $\mathscr{E}$ is proportional to the rate of rotation, sketch a graph of I against rate of rotation.

c The torque produced by the motor depends on the current I. How, if at all, will the torque vary with speed of rotation?

d When a motor is required to drive a larger load, the current its rotor draws increases. By what means does this happen?

27(P) Table H4 gives some results of an experiment using a d.c. motor.

Field coils V/V	I/A	**Rotor** V/V	I/A	**Load**	**Rate of rotation** /s^{-1}
5	0.35	5	1.0	zero	27
5	0.35	5	1.9	light	21
5	0.35	5	3.1	heavy	13

Table H4

Resistance of rotor = 1 Ω.

The field and rotor windings were supplied independently, from two different sources. Readings of applied p.d. V and current I for both field coils and rotor were taken.

a The ratio V/I for the rotor varies, and is never equal to 1 Ω. Why?

b Why is the rotor current larger when the motor is working against a load?

c How could the motor be made to turn the 'heavy' load at 27 revolutions per second?

Flux

28(P) The magnetic field strength B (flux density) in a long solenoid carrying a current I is given by $B = \mu_0 NI/l$, where there are N turns in length l. Calculate the field strength B, and the flux Φ in the following rectangular solenoids. The current is 0.5 A in each case.

a A 360-turn solenoid, 300 mm long, cross-section 70 × 70 mm.

b A 360-turn solenoid, 300 mm long, cross-section 50 × 50 mm.

c A 240-turn solenoid, 200 mm long, cross-section 280 × 30 mm.

29(P)a What is the flux density at the centre of a solenoid which has a turns density of 1200 turns per metre and carries a current of 1.5 A?

b What will be the flux through a 10 × 10 mm Hall probe held at the centre of the solenoid if
i it is perpendicular to the *B*-field?
ii its plane makes an angle of 30° to the direction of *B*?

Alternating current

30(I) The p.d. across a 5 Ω resistor is displayed on an oscilloscope, giving the trace shown in figure H88. The oscilloscope settings were: time-base speed 2 ms per division; voltage sensitivity 5 mV per division.

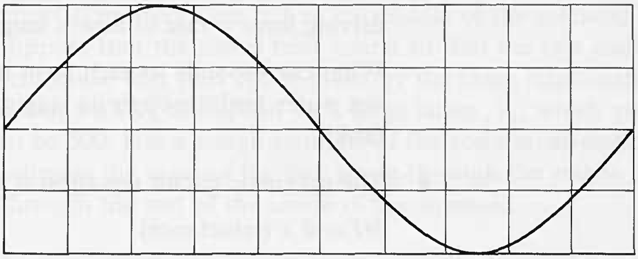


Figure H88

a What is the frequency of the alternating current in the resistor?

b What is the *peak* value of this current?

c What is the *average* value of the p.d. across the resistor, and of the current in it?

Flux and reluctance

31(L) Figure H89 shows an iron ring, in which magnetic flux Φ is set up by N turns of wire wrapped round the ring, carrying current I. The flux Φ is given by

$$NI = \Phi l/\mu_r \mu_0 A$$

where l is the length round the ring, and A is its cross-sectional area. The quantity $\mu_r \mu_0$ is more or less constant, as long as the current is not large.

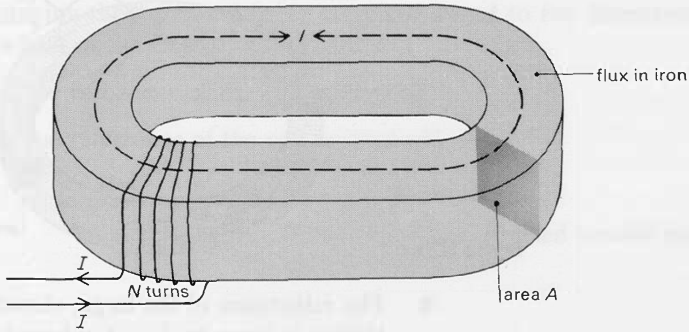


Figure H89

In a copper wire, the steady current flowing depends on the applied p.d. and the resistance. The resistance depends on the length of wire and the cross-sectional area of the wire.

a Write an expression for the resistance of a wire in terms of its length and area.

b Write an equation analogous to that for current-turns, flux, length, and area, but for the p.d. driving current in a wire. Current is to be analogous to flux. What is the p.d. analogous to in the 'magnetic circuit'?

c To what quantity does the combination $\mu_r \mu_0$ compare in the analogy?

d The general form of the relationship is

driving force = rate of flow × length/(conductivity × area)

What corresponds to each term in the case of energy leaking from a hot water tank through its lagging (the fibre-filled jacket over the tank)?

e The magnetic circuit equation is sometimes written

$NI = \Phi \times$ (reluctance)

To what electrical quantity does reluctance correspond, in the analogy with current?

32(L) A solenoid is rather like a pipe which carries magnetic flux, and the flux in a solenoid can be calculated just as if one were calculating the flow of electric current along a copper bar, the thermal flow of energy ('heat flow') along a conducting bar, or (neglecting many of the real-life complications) the flow of fluid down a tube.

A calculation of the 'flow' of flux in a straight solenoid, such as that shown in figure H90(b), is complicated by the difficulty of working out what happens at the ends. One way of avoiding the difficulty is to regard the solenoid as part of a very long one whose ends are brought round to meet each other, as in figure H90(a).

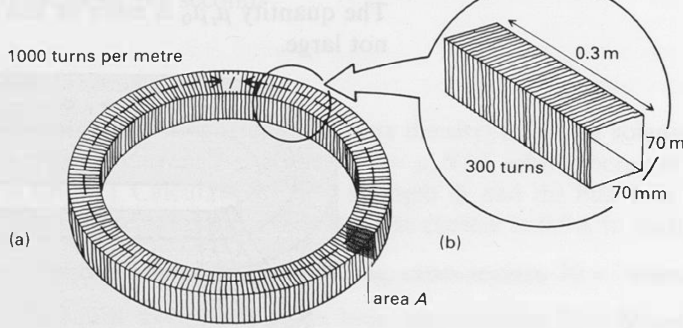


Figure H90

a The reluctance of the large, closed solenoid pipe shown in figure H90(a) is given by $l/\mu_0 A$, where l is the length all around the pipe, and A is its cross-sectional area. Calculate the contribution made to this reluctance by the shorter, almost straight section in figure H90(b).

b The flux 'going round' the large closed solenoid is given by

current-turns = flux × reluctance.

Explain why, if the length l is altered, keeping the number of turns per metre and the current the same, the flux is unaltered.

c Calculate the flux 'going through' any section of the solenoid, using the answer to **a**, if the solenoid carries one ampere.

d What is the value of B within the solenoid?

e Suppose that a steel rod having roughly the cross-sectional area of a retort stand rod were put in the middle of the solenoid. You may suppose that the rod is bent round so that the two ends meet. The reluctance of a steel rod is given by the same relationship as that in **a**, but with μ_0 multiplied by a large factor, μ_r, which you may take to be 500. Use a rough estimate of the rod's cross-sectional area to estimate the ratio of the flux going through the rod to the flux going through the rest of the inside of the solenoid.

33, 34(R) In figure H91(a) a potential difference V across a long iron rod produces a current I in the rod, the resistance of the rod being V/I.

In figure H91(b) current-turns NI around the rod produce a flux Φ in the rod, the reluctance of the rod being NI/Φ.

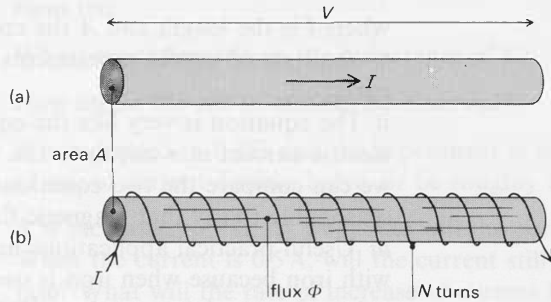


Figure H91

If the area A of cross-section of the rod were doubled (the length staying the same and being large compared to the diameter), by which of the factors A to E below

33 is the *resistance* of the rod multiplied?

34 is the *reluctance* of the rod multiplied?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 1 **D** 2 **E** 4

(Coded answer paper 1981)

35(R) This question asks you to say more about four brief statements about magnetic flux.

a The passage below consists of four main statements, numbered *i*, *ii*, *iii*, and *iv*. For each of these statements you are asked to give arguments which explain and support the statement; these might refer to theoretical principles, or to experimental evidence, or both. Your arguments should include explanation of the principles and description of the evidence.

b Statement *iii* could be described in general terms as a statement about practical applications based on an experimental result (the value of μ_r). For each of statements *i*, *ii*, and *iv* give a similar description to say what sort of ideas are in the statement, for example, it is a definition based on previous ideas; or it is a summary of experimental results; or some other description you think appropriate; give a *brief* justification for each description.

Passage

i We can show from experiments with solenoids in air that magnetic flux Φ is related to the current I and number of turns N by an equation of the form

$$NI = \frac{\Phi l}{\mu_0 A}$$

where l is the length and A the cross-sectional area; however, it takes a whole set of careful experiments to sort out the various factors correctly.

ii The equation is very like the equation which relates the flow of electric current in a circuit to the various factors which determine it; we can compare the two equations term by term, so that it seems reasonable to say that magnetic flux is like the flow of something.

iii Useful practical applications nearly always use solenoids filled with iron because when iron is used μ_0 is replaced by $\mu_0\mu_r$, and the value of μ_r can be as much as 1000.

iv The idea of flux as a flow around a magnetic circuit is often helpful. For example, it can easily be shown that if an iron ring electromagnet has a small air gap cut in it there is a large reduction in Φ – just like the effects on the current when a large resistance is inserted in series in an electrical circuit.

(Long answer paper, 1979)

36(L) Mutual inductance; $\mathscr{E} = M\mathrm{d}I/\mathrm{d}t$.

Two coils A and B are wound on an iron core. The current in A increases steadily from 0 to 30 mA in 1 ms. The p.d. across coil B is found to be 1.5 V.

a What is the mutual inductance, M?

Explain, using such terms as flux, flux linkage, and rate of change, the effect of each of the following independent changes.

b The iron core is removed, and the current is increased from 0 to 30 mA in 1 ms as before.

c The number of turns of wire in coil A is doubled.

d The number of turns in coil B is doubled.

e The current is decreased from 30 mA to 0 in 2 ms.

Inductance

37(P) When the switch in figure H92 is closed, the current in the circuit rises to 0.5 A in the first 0.01 s. For this period, the rate of rise of current is very nearly steady. R is 0.1 Ω, and L has negligible resistance.

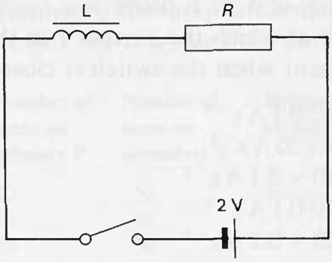


Figure H92

a What, approximately, is the inductance of L?

b How might the rise of current be observed?

c R is increased to 0.2 Ω and the experiment is repeated. Will the rate of rise of current double, halve, or be roughly the same as before?

d R is increased to 2 Ω. At what rate will the current rise initially? When the current is 0.5 A, will the current still be rising at a uniform rate? What will the rate of increase of current be at this stage?

38(R) A coil of inductance L (and zero resistance), and a resistance R are connected via a switch to a battery of voltage V (and no internal resistance). See figure H93.

Which one of the following statements about the circuit is correct?

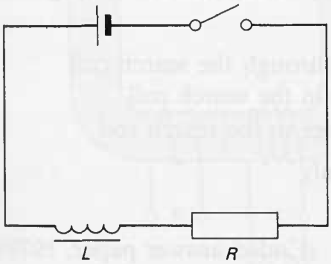


Figure H93

A Just as the switch is closed, the rate of change of the current is V/R.
B Just as the switch is closed, the current is V/L.
C After the switch is closed, the final steady current does not depend on the value of L.
D After the switch is closed, the final steady current does not depend on the voltage V.
E When the current is finally steady, there is a constant large induced voltage across the inductor.

(Coded answer paper, 1979)

39, 40(R) A coil of inductance 0.1 H is connected, as shown in figure H94, to a resistor of resistance 20 Ω, and to a 10 V battery (of negligible internal resistance). The switch is initially open.

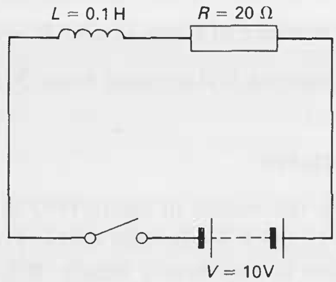


Figure H94

39 Which of **A** to **E** below is a correct calculation of the initial rate dI/dt at which the current I in the circuit changes with time, at the moment when the switch is closed?

A $10/0.1\ \mathrm{A\,s^{-1}}$
B $10/20\ \mathrm{A\,s^{-1}}$
C $10 \times 0.1\ \mathrm{A\,s^{-1}}$
D $20/0.1\ \mathrm{A\,s^{-1}}$
E $20 \times 0.1\ \mathrm{A\,s^{-1}}$

40 Which of **A** to **E** below is a correct calculation of the *final* value of the current in the circuit, a long time after the switch has been closed?

A $10/0.1$ A
B $10/20$ A
C 10×0.1 A
D $20/0.1$ A
E 20×0.1 A

(Coded answer paper, 1978)

41(R) A solenoid carries a sinusoidal alternating current. A search coil is placed at its centre, with the coil's axis parallel to that of the solenoid.

If the frequency of the current in the solenoid is doubled, but the amplitude of the current is kept the same, which of the following quantities will also be doubled?

1 the maximum rate of change of flux through the search coil
2 the amplitude of the induced voltage in the search coil
3 the frequency of the alternating voltage in the search coil

A 1 only **B** 2 only **C** 1 and 3 only
D 2 and 3 only **E** 1, 2, and 3

(Coded answer paper, 1979)

Transformers

42(P) A transformer for a toy railway set converts the 240 V alternating mains supply to a 12 V alternating voltage.

a What is the turns ratio?

b What do you think determines the actual number of turns?

c Could the transformer be used with a steady d.c. input?

43(P) In figure H95, P is a primary coil of 20 turns, joined to a 1 V alternating supply. S is a secondary of 50 turns joined to a small 2.5 V lamp, L, which lights with what we may call 'normal brightness'. These facts are expressed in the top line of table H5.

Copy and complete the table. For the third column, write 'normal' if the lamp is normally bright, 'dim' for dim or not alight, 'bright' for brighter than normal (including 'burnt-out').

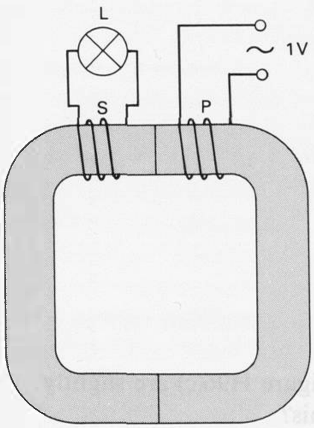


Figure H95

Number of turns on primary P	Number of turns on secondary S	Brightness of lamp L	Alternating p.d. across S/volts
20	50	normal	2.5
50	20		
20	30		
40	100		
20	80		

Table H5

44(P) Figure H96 shows three different arrangements of the same transformer. P is a primary coil of 20 turns, always connected to a one-volt alternating supply. S is a secondary coil of 50 turns, joined to a 2.5 V lamp. In figure H96(c) the secondary is wound on top of the primary. The coils are wound on a double C-core.

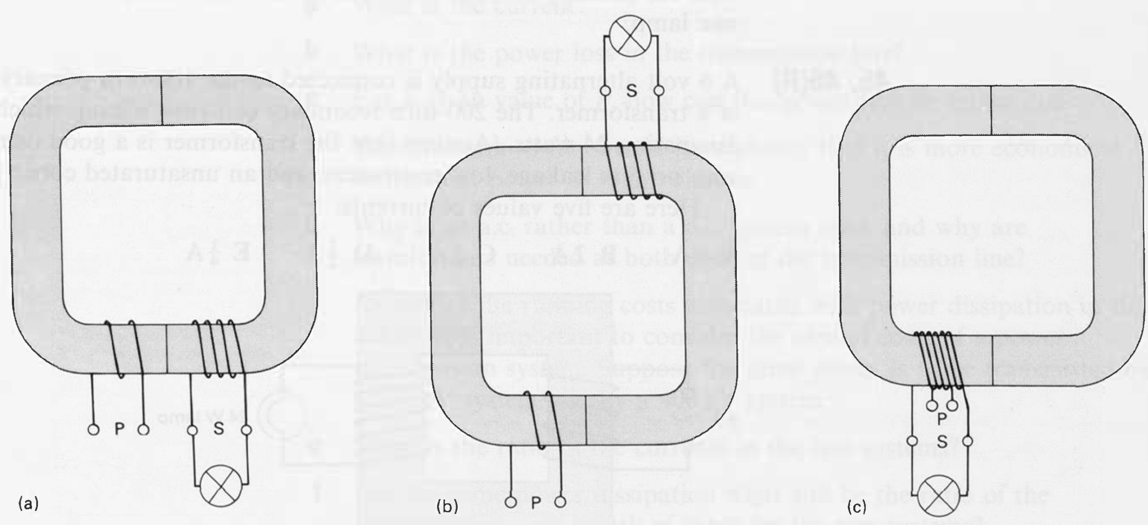


Figure H96

a There seems to be little difference between the three arrangements: in every case the lamp lights normally although the secondary coil is in different places. How do you explain this?

b Figure H97 shows the arrangement of figure H96(a) with the two C-cores slightly separated. This gives a different result: the lamp lights only very dimly. Why does separating the two iron cores of a transformer cause it to work much less efficiently?

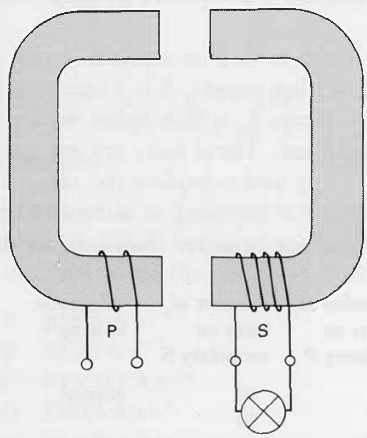


Figure H97

c If the iron C-cores in the arrangement of figure H96(c) are slightly separated, the lamp lights dimly. Why is this?

d Look again at figure H96. Suppose that a transformer is made by winding all three secondary coils and the single primary coil on the same pair of C-cores. Each secondary is joined to a lamp, and all three lamps are found to light normally. How do you explain this result?

e How would you expect the primary current in this case to compare with the primary current when there is one secondary coil with one lamp?

45, 46(R) A 6 volt alternating supply is connected to the 100-turn primary coil of a transformer. The 200-turn secondary coil runs a lamp which is dissipating 24 watts. (Assume that the transformer is a good one, with no flux leakage, low resistances, and an unsaturated core.)

Here are five values of currents:

A 4 A **B** 2 A **C** 1 A **D** $\frac{1}{2}$ A **E** $\frac{1}{4}$ A

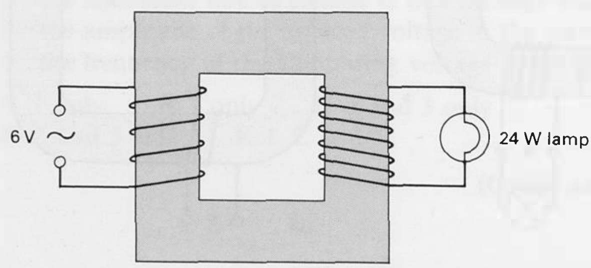


Figure H98

45 Which is the best estimate of the current in the secondary circuit?

46 Which is the best estimate of the current in the primary circuit?

(Coded answer paper, 1979)

47(R) Describe a series of demonstrations you could do to show an A-level Physics class the principles and behaviour of alternating current transformers. Include losses and saturation besides fundamental principles. For each demonstration:

a say clearly what idea(s) you are showing, and how the demonstration illustrates them.

b discuss the selection of coils, cores, voltages, currents, and frequency, and the choice of measuring instruments, such that the effect to be shown can in fact be detected easily.

(Special paper, 1983)

Transmission of electric power

48(L) Figure H99 shows a very simplified representation of a transmission line delivering power P from a generator to a load. The potential difference across the load is V. The total resistance of the cables between generator and load is R.

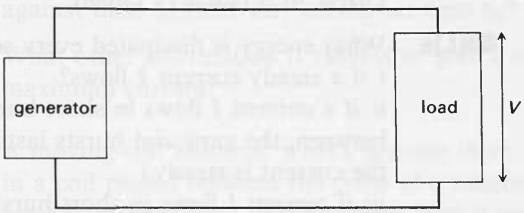


Figure H99

a What is the current?

b What is the power loss in the transmission line?

c For a given value of P, how can this power loss be minimized?

Your answers so far should have shown that it is more economical to transmit power at high voltages.

d Why is an a.c. rather than a d.c. system used, and why are transformers needed at both ends of the transmission line?

As well as the running costs associated with power dissipation in the cables, it is important to consider the capital costs of a power transmission system. Suppose the same power is to be transmitted by a 132 kV system and by a 400 kV system.

e What is the ratio of the currents in the two systems?

f For the same power dissipation what will be the ratio of the resistance per unit length of cable for the two systems?

g How therefore would you expect the amount of metal needed and hence the cost per kilometre length of the two transmission lines to compare?

Figure H100 shows that the cost of the cables is not the only consideration.

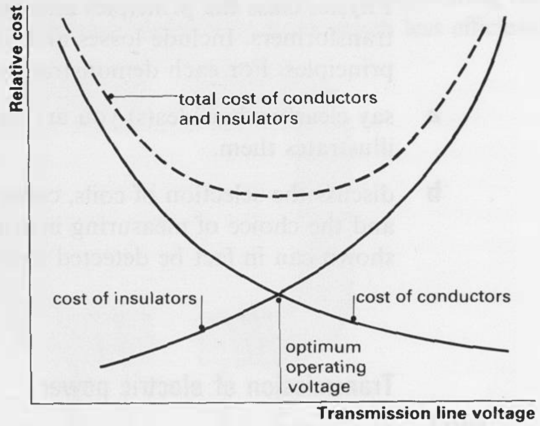


Figure H100
Transmission line capital costs. There exists an optimum operating p.d. for which the installation cost is a minimum. *Based on* WRIGHT, J. P. The vital spark. *Heinemann, 1974.*

h Why does the cost of insulators rise as the transmission voltage is increased?

Power in a resistive circuit

49(L)a What energy is dissipated every second in a resistance R, on average,
i if a steady current I flows?
ii if a current I flows in short bursts, with gaps of no current in between, the gaps and bursts lasting equally long? (During a burst the current is steady.)
iii if current I flows in short bursts, first in one direction and then in the other, taking no time to change direction? (Again, the current is steady during a burst.)

b What steady current would dissipate energy at the same rate as the current I in **a***ii*, which comes in bursts?

c Suggest why the current in **b** is called the root mean square current.

d The alternating current from the a.c. mains varies as $\sin\theta$. Figure H101 is a sketch graph of $\sin^2\theta$, over one cycle. The two shaded areas are equal. What is the average value of $\sin^2\theta$ over one cycle?

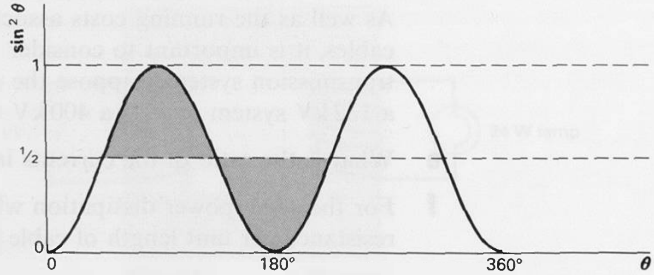


Figure H101

e What steady current would dissipate as much energy in a resistor as an alternating current of maximum value I?

Alternating current in circuits containing capacitors

50(L) The curve in figure H102 shows one cycle of a sinusoidal variation of the p.d. V across a 100 μF capacitor.

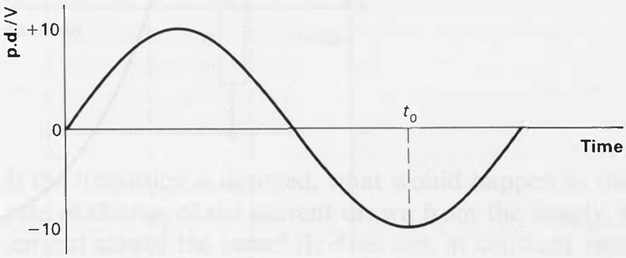


Figure H102

a What is the charge, Q, on the capacitor when $V = +10$ V, 0 V, -10 V? Sketch a graph of Q against t. Is there a charge on the capacitor at time t_0?

b When is the current flowing into or out of the capacitor at a maximum, and when at a minimum? Sketch a graph of current against time. Is there any current at time t_0?

c What other information is needed to give a rough estimate of the maximum current?

51(L) A moving-coil ammeter works because there is a force on a current in a coil placed between the poles of a magnet. The magnitude of the force is proportional to the current, and it reverses direction if the current reverses.

The magnitude of the force is also proportional to the strength of the magnetic field. A wattmeter can be made by replacing the magnet of an ammeter with a fixed coil which carries a small current proportional to the p.d. across the circuit, because this makes the strength of the magnetic field proportional to the p.d. The force on the coil is now proportional to the product of the current and p.d., that is, to the power.

Figure H103 shows just half of a cycle of the variation of current and p.d. for a capacitor in an a.c. circuit.

a Will there be a force on the wattmeter coil at time 2? Why?

b Will there be a force on the wattmeter coil at times 0 and 4? Why?

c At times 1 and 3 the magnetic field in the wattmeter will be exactly the same, for V has the same value at these times. How will any forces on the coil compare at these times?

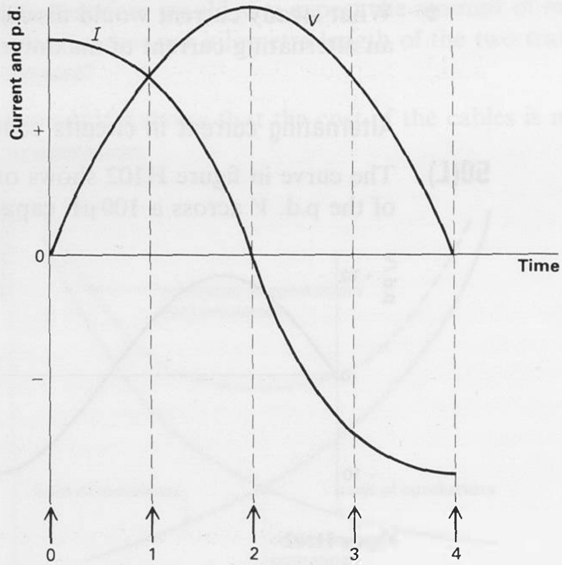


Figure H103

d What does the average wattmeter reading seem likely to be, if it is set to measure the average power delivered to a capacitor by an alternating supply?

e What would be different if a lamp were now placed in series with the capacitor?

52(L) In figure H104 the oscilloscope's Y-sensitivity is set at $1\,\mathrm{V\,cm^{-1}}$.

a What is the amplitude of the alternating p.d. (the biggest p.d. at any time during the cycle) across the capacitor?

b What is the p.d. across the capacitor when the greatest charge at any time during the cycle is on the capacitor plates?

c When the greatest charge is on the capacitor plates, at what rate is the charge changing?

d What is the current 'through' the capacitor at the instant when the greatest charge is on the capacitor plates?

e What is the rate at which electrical energy is being transformed in the capacitor when the p.d. across the capacitor is 3 volts?

f What is the rate at which electrical energy is being transformed when the p.d. across the capacitor is zero?

g Is the energy stored in the capacitor increasing or decreasing while the p.d. across it is increasing?

h Is the energy stored in the capacitor increasing or decreasing while the p.d. across it is decreasing?

i What is the average rate at which electrical energy is being converted to other forms, taken over many cycles?

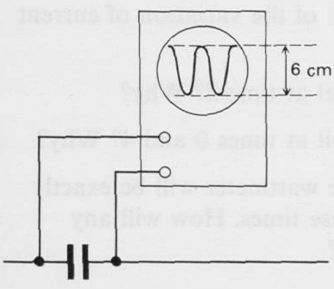


Figure H104

Alternating current in circuits containing inductance

53(L) Suppose that the input to the circuit of figure H105 is an alternating voltage of constant amplitude, but variable frequency.

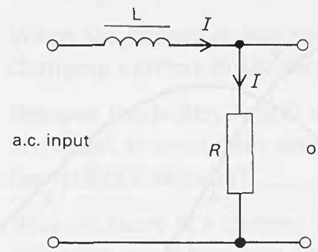


Figure H105

a If the frequency is doubled, what would happen to the maximum rate of change of the current drawn from the supply, if the maximum current stayed the same? (It does not, at constant voltage.)

b If R is small, nearly all the p.d. is across L, equal to the supply p.d. If this is unaltered, the rate of change of current must be the same as before, since V, the p.d. across L, is given by $L\,\mathrm{d}I/\mathrm{d}t$. How can the rate of change of current be unaltered, though the frequency has doubled?

c If the maximum current has roughly halved, how has the maximum output p.d. changed?

d Why do you think such a circuit is often called a 'low-pass filter'?

54(R) An alternating voltage, V, varying with time sinusoidally as shown in figure H106, is applied across an inductor, which has negligible resistance.

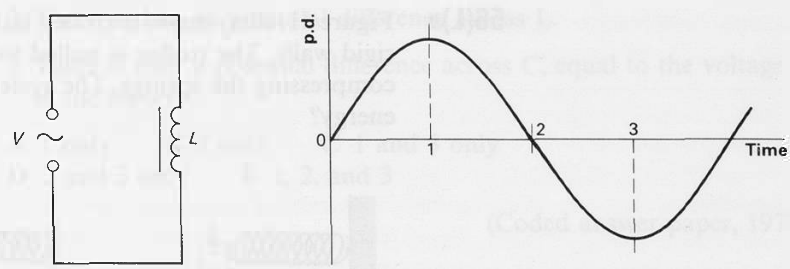


Figure H106

Which of the following statements about what is happening at the times 1, 2, and 3 indicated is/are correct?

1 At time 1 the current in the circuit is a maximum.
2 At time 2 the magnetic flux linking the turns of the inductor is a maximum.
3 At time 3 the rate of change of current in the inductor is a maximum.

A 1 only **B** 2 only **C** 1 and 3 only
D 2 and 3 only **E** 1, 2, and 3

(Coded answer paper, 1983)

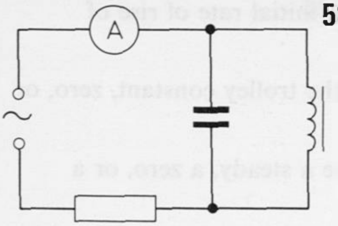


Figure H110

58(R) The ammeter records the alternating current I drawn by the inductor and capacitor in parallel, as shown in figure H110, from an alternating source of constant voltage, with a resistor in series with it. Which of the graphs in figure H111 correctly represents the variation of the current I as the frequency f is varied?

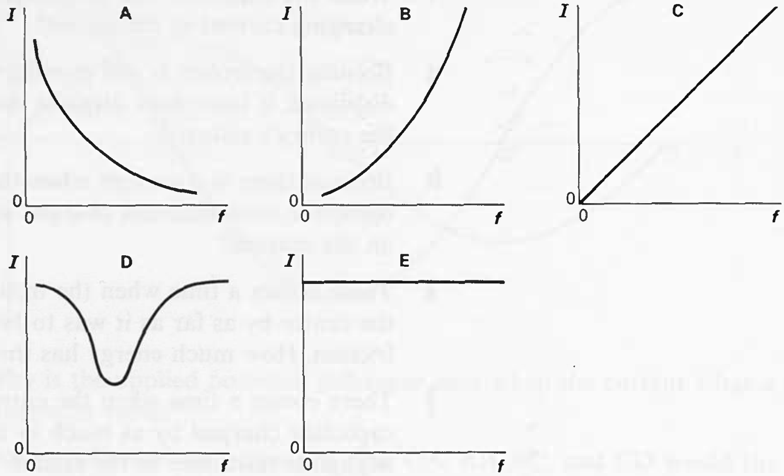


Figure H111

(Coded answer paper, 1980)

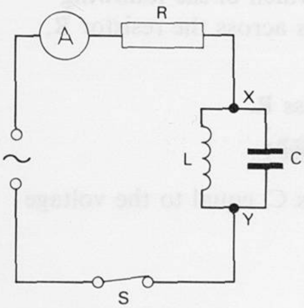


Figure H112

59(R) This question is about the explanation of resonance in an LC circuit.

A student is shown the circuit illustrated in figure H112, with the a.c. generator set at the resonant frequency of the circuit. He notices that the current indicated on the ammeter A is quite small.

He is told to study the notes **1**, **2**, **3** below but he has not studied much a.c. theory before and he finds that he cannot understand them.

You are asked to write out a full explanation for each of the notes **1**, **2**, and **3** in order to help the student. (About half the marks for this question will be available for the explanation of note **1**.)

Notes

1 The currents between X and Y, through L and through C, are in opposite directions at all times even though they have the same potential difference across them.

2 Because the frequency is just right, these currents will be about equal so that there will be little or no current in R even though there is a large current flowing in both the L and the C arms. However, if the frequency were altered (increased or decreased), there would be a current in R.

3 Just as with a trolley oscillating on the end of a spring, the energy is continually changing between potential (in the spring) and kinetic (in the trolley), so in this circuit energy changes continually between being stored in the field associated with L and being stored in the field associated with C.

(Long answer paper, 1978)

Unit I
LINEAR ELECTRONICS, FEEDBACK AND CONTROL

Wilf Mace
King Edward VII School, Sheffield

SUMMARY OF THE UNIT

I

SUMMARY OF THE UNIT

INTRODUCTION

Unit C of this course was about digital electronics – devices like gates and multivibrators whose output is either high or low, 0 or 1. This Unit is concerned with linear electronics – devices and circuits whose output can vary continuously between minimum and maximum levels. The output of course depends on the input, and may be proportional to it.

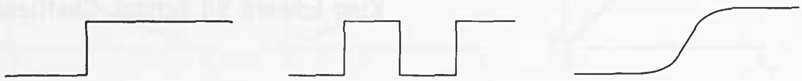


Figure I1
Digital and continuously variable signals.

DEMONSTRATION I1
Introduction to linear electronics and control

As well as the details of some particularly useful circuits, this Unit is concerned with some more general and powerful ideas related to feedback and control. Feedback, usually negative feedback, is a feature of many electronic circuits, and many control systems use electronics. But the ideas of feedback and control can be applied in many other fields as well, including mechanics, biology, economics, and perhaps even psychology.

Figure I2

As in Unit C, 'Digital electronic systems', we are not concerned with the details of how a particular electronic component works, or what there is inside it. What we *are* interested in is what the output is for various inputs, and how the component can be used to do a wide range of jobs. The work of the Unit involves much experimental work, and the solving of practical problems. It also makes use of many basic ideas about electric circuits: current, resistance, p.d., capacitance, and so on.

QUESTIONS 1 to 7

Section I1 BASIC OPERATIONAL AMPLIFIER CIRCUITS

The operational amplifier

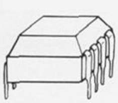


Figure I3
A 'chip'.

One of the most useful devices available to electronic engineers today is the operational amplifier. The operational amplifier is an integrated circuit, or 'chip', a product of the microelectronics revolution. That revolution, which started in the 1960s when lighter and more reliable electronic components were demanded by the space race, shows no sign of slowing down. It has been called a second Industrial Revolution, and it may well affect our lives, and those of coming generations, in just as profound a way as the mechanical inventions of the eighteenth and nineteenth centuries changed the lives of our ancestors.

An operational amplifier can be used to amplify and to do mathematical operations (add, integrate, etc.)

Figure I4 shows the internal circuit of one common operational amplifier: a single integrated circuit on a silicon chip, less than 2 mm square and about 0.2 mm thick, may contain the equivalent of tens of individual transistors, resistors, and capacitors.

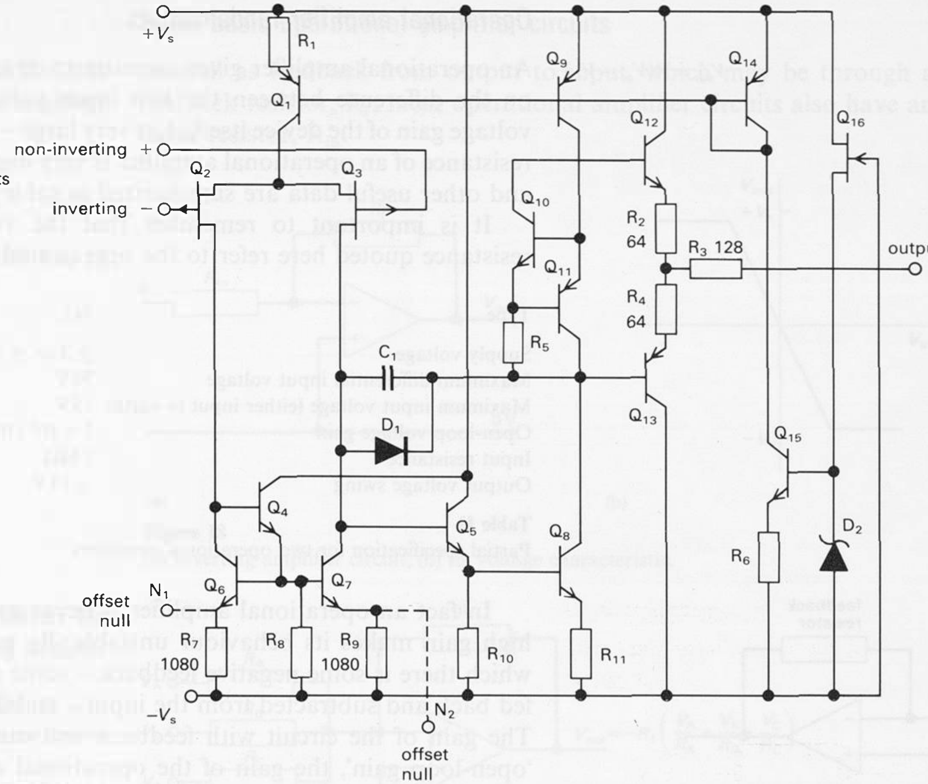


Figure I4
Schematic diagram of the TL081 operational amplifier.
From The BIFET Design manual, *Texas Instruments Ltd.*

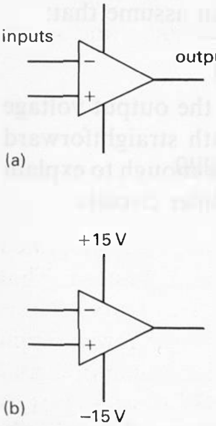


Figure I5

The operational amplifier is conventionally represented by a triangle. Its two inputs, inverting and non-inverting, are shown by a − and + respectively. A useful circuit will consist of one or more operational amplifiers with various resistors and capacitors (and perhaps transducers such as thermistors, light-dependent resistors, and so on) connected around them. For greater clarity the power supply is often omitted from simplified circuit diagrams – but the operational amplifier won't work without it. A typical operational amplifier is designed to operate from a power supply that gives + 15, 0, and − 15 V, but many will operate from as little as + 3, 0, and − 3 V.

The operational amplifier has one output terminal – shown at the right of the conventional symbol. The output voltage can be positive or negative with respect to the 0 V line of the power supply. But it cannot be more than the positive or less than the negative supply voltage, in fact if the supply voltage is, say, ± 15 V, the output voltage range will be a little less, perhaps ± 13 V. Within this range the output voltage can have any value; in other words, it can vary continuously.

Operational amplifier fundamentals

$V_{out} = A(V_+ - V_-)$

An operational amplifier gives an output voltage (V_{out}) which depends on the difference between the two input voltages, V_+ and V_-. The voltage gain of the device itself, A, is very large – typically 10^5. The input resistance of an operational amplifier is very high – perhaps 2 MΩ. This and other useful data are summarized in table I1.

It is important to remember that the voltage gain and input resistance quoted here refer to the operational amplifier on its own.

Type	741	081
Supply voltage	± 3 to ± 18 V	± 3 to ± 18 V
Maximum differential input voltage	30 V	± 30 V
Maximum input voltage (either input to earth)	15 V	$\pm V_s$ (supply voltage)
Open-loop voltage gain	2×10^5 (106 dB)	2×10^5 (106 dB)
Input resistance	2 MΩ	10^{12} Ω
Output voltage swing	± 13 V	± 13.5 V

Table I1
Partial specification for two operational amplifiers.

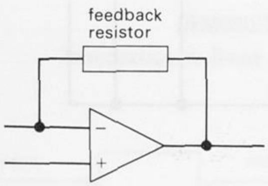


Figure I6

In fact an operational amplifier is never used on its own. Its very high gain makes its behaviour unstable. By putting it in a circuit in which there is some negative feedback – some of the output voltage is fed back and subtracted from the input – stability is greatly increased. The gain of the circuit with feedback will certainly be less than the 'open-loop gain', the gain of the operational amplifier alone; and its high input resistance may be reduced too. In spite of these apparent drawbacks, almost all useful operational amplifier circuits have some negative feedback.

OPTIONAL DEMONSTRATION I5
Current and voltage in an operational amplifier

Two important simplifications follow from the high gain and high input resistance of the operational amplifier. We can assume that:
a the two inputs are at the same potential;
b there is no current into either input.
These two assumptions (which are valid as long as the output voltage doesn't reach its upper or lower limit), together with straightforward application of basic ideas about electrical circuits, are enough to explain or predict the behaviour of many operational amplifier circuits.

QUESTIONS 12, 13, 14

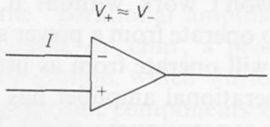


Figure I7

QUESTION 44

It turns out, surprisingly enough, that the behaviour of the circuit does not depend on the gain or input resistance of the operational amplifier itself. As long as they are high enough for us to make the above simplifying assumptions, the properties of the circuit depend only on the values of the resistors, capacitors, etc., around the operational amplifier. So any changes in the operational amplifier's behaviour, such as variation of A with temperature, or at different frequencies, are unimportant.

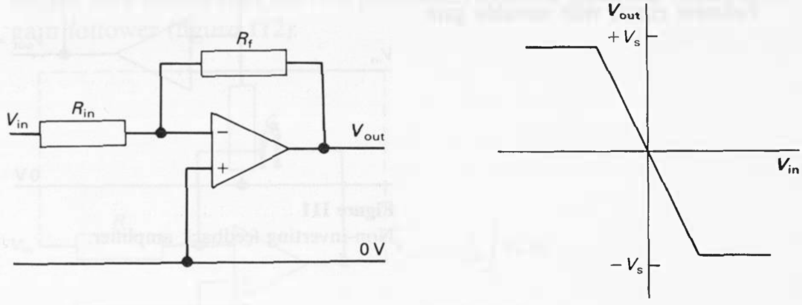
Some basic operational amplifier circuits

EXPERIMENTS I2, I3, I4
Operational amplifiers

As well as feedback from output to input, which may be through a feedback resistor, R_f, most operational amplifier circuits also have an input resistor, R_{in}.

$V_{out}/V_{in} = -R_f/R_{in}$

QUESTIONS 8 to 18



(a) (b)

Figure I8
(a) Inverting amplifier circuit; (b) its voltage characteristic.

EXPERIMENT I6a
Summing amplifier

An important application is in digital-to-analogue conversion

QUESTIONS 19 to 23

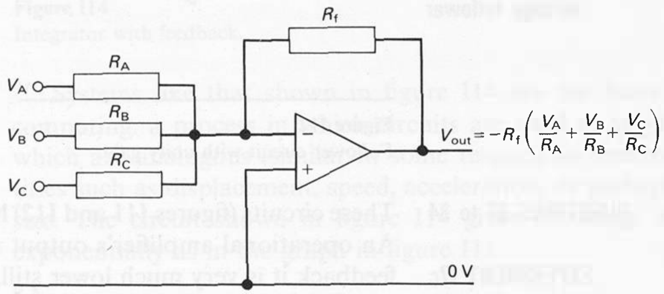
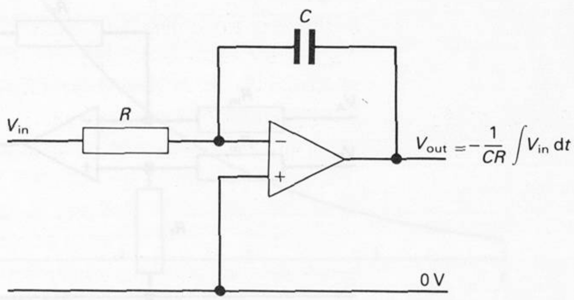


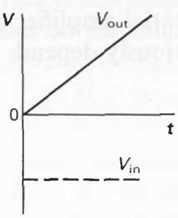
Figure I9
Summing amplifier.

EXPERIMENT I6c
Integration

QUESTIONS 24 to 27



(a)



(b)

Figure I10
(a) Integrator circuit.
(b) Behaviour of integrator circuit.

Circuits using the non-inverting input

QUESTIONS 28 to 31

EXPERIMENT I7a
Follower circuit with variable gain

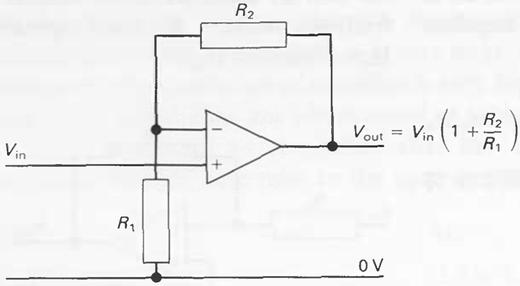


Figure I11
Non-inverting feedback amplifier.

unity gain

EXPERIMENT I7b
Input and output currents of voltage follower

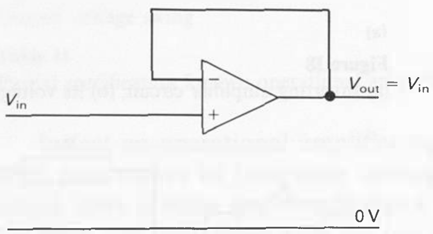


Figure I12
Follower circuit with unity gain.

QUESTIONS 32 to 34

EXPERIMENT I7c
Use of voltage follower circuit

These circuits (figures I11 and I12) have extremely high input resistance. An operational amplifier's output resistance is low, and with negative feedback it is very much lower still, so circuits like those in figures I11 and I12 can supply much bigger currents – perhaps to a moving-coil meter or chart recorder – than they draw from the input source.

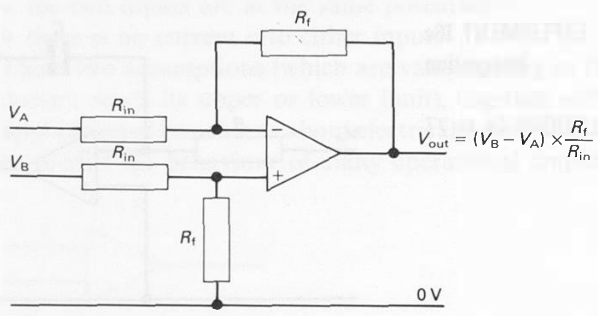


Figure I13
Differential amplifier.

QUESTIONS 35 to 38

The differential amplifier is useful in many control applications. Its sensitivity obviously depends on the ratio R_f/R_{in}.

Section I2 MORE FEEDBACK; CONTROL

EXPERIMENT I8
Integrator with feedback

In most of the circuits shown so far, the feedback from output to input has been via a resistor or capacitor. Such feedback tends to reduce the difference in potential between the input and output. Feedback by a simple wire means that the two potentials must be equal, as in the unity-gain follower (figure I12).

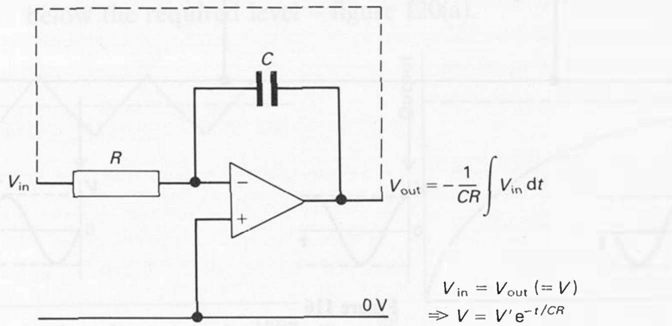


Figure I14
Integrator with feedback.

'Electromechanical similarities' in the Reader *Physics in engineering and technology*

QUESTIONS 39, 40

Systems like that shown in figure I14 are the basis of analogue computing: a process in which circuits are used to produce voltages which are analogous (similar in some respect) to time-varying quantities such as displacement, speed, acceleration, or perhaps population size. The circuit shown in figure I14 gives a voltage which decays exponentially as in the graph in figure I15.

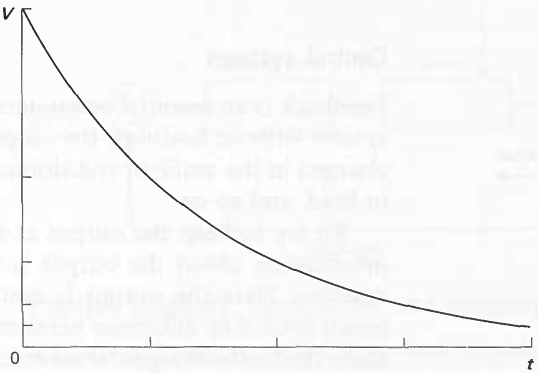


Figure I15

DEMONSTRATION I9
A circuit to produce oscillation

Other circuits can be built to model exponential growth, or to produce oscillations. The time constants or frequencies of these circuits depend on the values of the resistors and capacitors in them.

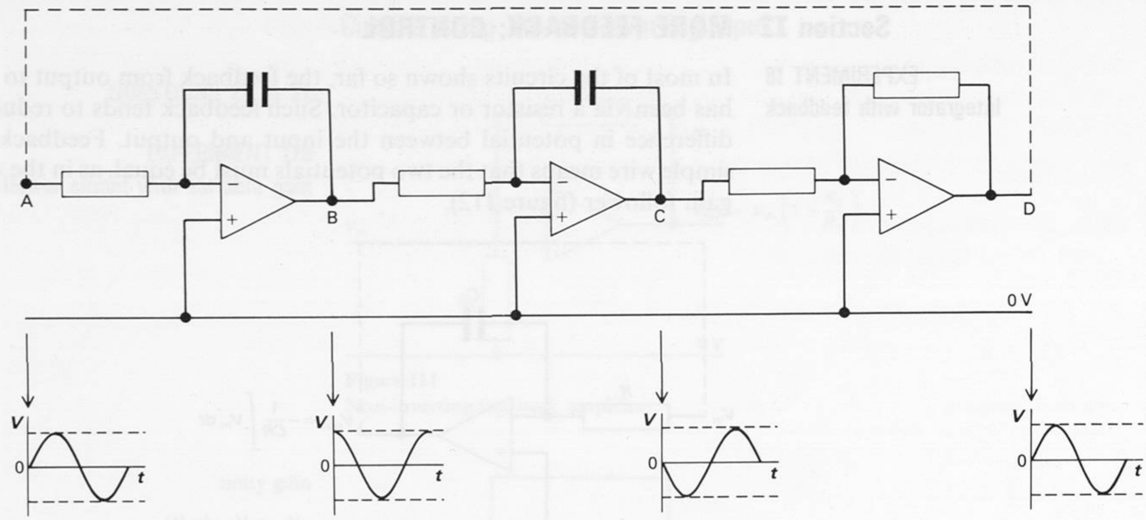


Figure I16
An oscillator circuit, showing the voltages at various points.

QUESTIONS 41 to 43

DEMONSTRATION I10
Feedback in public address systems

OPTIONAL DEMONSTRATION I11
Oscillation in a feedback system

In the circuit of figure I16 each of the integrators introduces a phase lag of 90° ($\pi/2$); the inverter introduces another 180° phase lag. The total delay around the circuit is thus $90° + 90° + 180° = 360°$. So the signal feedback to point A is in phase with the signal there.

This positive feedback causes oscillations in other systems as well. The annoying howl sometimes produced by a public address system is an example. Here the time taken for the sound to travel from loudspeaker to microphone introduces the extra phase change.

Control systems

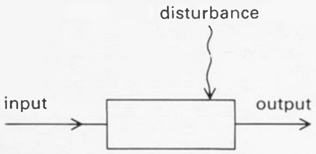


Figure I17
Open-loop system.

Feedback is an essential component of any efficient control system. In a system without feedback the output is subject to variations caused by changes in the ambient conditions, disturbance to the system, increase in load, and so on.

To try to keep the output at a constant level it is monitored and information about the output is fed back to a comparator or error detector. Here the output is compared with the reference signal or preset level. Any difference between the two gives rise to an error signal. Since the feedback signal is subtracted from the reference signal, this is a case of negative feedback. The error signal is used to control power to the 'plant', that is the lamp, motor, or whatever is producing the output. If there is no error signal then the output is already at the required level, and there is no change to power.

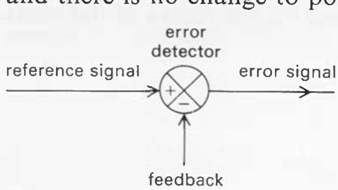


Figure I18

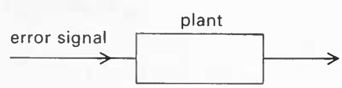


Figure I19

DEMONSTRATIONS I12, I13
On–off and continuous control of illumination

An *on–off* system is one in which power to the plant is switched on if the error signal rises above a certain level, and goes off if the error signal is below another, slightly lower level. In a *continuous* system the power varies steadily with the size of the error signal. In principle, at least, a continuous system can settle down to produce an output which is exactly the preset value – figure I20(b). But the output of an on–off system will be continually rising and falling a little above and a little below the required level – figure I20(a).

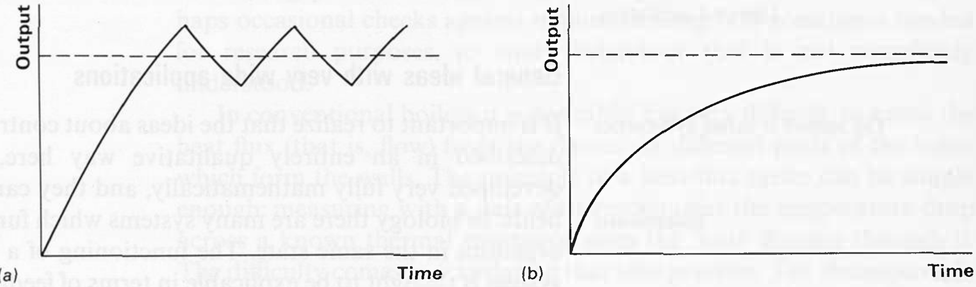


Figure I20
(a) On–off, and (b) continuous control.

QUESTIONS 44 to 48

Most practical systems require some kind of amplification between the error detector and the power unit, or 'plant'. For example, the error detector might be a bridge circuit or an operational amplifier used as a comparator: neither of these could supply enough power to light a lamp or drive a motor, so a power amplifier or relay of some sort is necessary (control unit in figure I21).

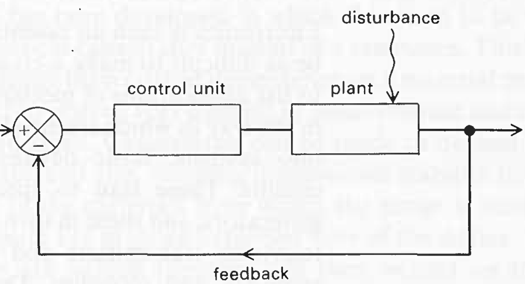


Figure I21

Positive feedback, oscillations, and damping

'The scope and relevance of control engineering' in the Reader *Physics in engineering and technology*

A control system is designed to have negative feedback. But if there is any delay in the system – perhaps due to inertia – the feedback may become positive. Then the feedback signal will be added to the reference signal. If a disturbance causes the output to increase, the error signal will now cause it to increase further. Under these conditions the system is likely to go into oscillation. If it is an electrical system the period of oscillation will depend on the resistance, capacitance, and inductance of the circuit; in a mechanical system the masses of moving parts and the forces between them are the important factors.

DEMONSTRATION I14
Temperature control with thermal inertia

DEMONSTRATION I15
Light follower

In a simple mechanical oscillator, like a pendulum, more friction reduces the amplitude and – unless the oscillator is driven – causes the oscillations to die away more quickly. Similarly, oscillations in a

QUESTION 49

feedback control system are reduced by damping, that is dissipation of energy in some part of the system.

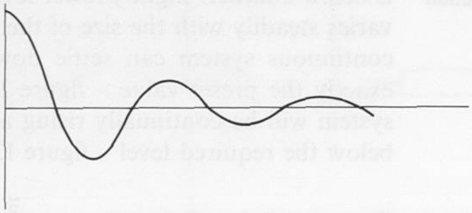


Figure I22
Damped oscillation.

General ideas with very wide applications

The subject is called cybernetics

homeostasis

J. M. Keynes, English economist (1872–1945)

It is important to realize that the ideas about control systems, which are described in an entirely qualitative way here, have in fact been developed very fully mathematically, and they can be applied in many fields. In biology there are many systems which function to maintain an organism in the same state. The functioning of a (capitalist) economic system is thought to be explicable in terms of feedback – the interaction of supply and demand, and so on. And, at least according to one school of thought, the economic system can be controlled – that is, its output (goods, services) brought to a desired level by adjustments to the feedback and input (investment, taxation, and so on).

QUESTION 50

The same basic ideas have been applied to all these, and many more apparently diverse systems.

Section I3 PUTTING ELECTRONICS TO USE

Electronics is such an essential part of technology today, that it would be as difficult to make a classified list of applications as it would to try to list applications of mechanical devices. The power of electronics lies in the way in which essentially simple building blocks can be assembled into systems. Basic devices such as transistors lead to integrated circuits. These lead to specific units such as amplifiers and signal generators, and these in turn are assembled, for example, into radio and television transmitters and receivers, radar systems, radio telescope receivers, and recorders. Devices can be constructed for monitoring electrically a wide variety of variables (position, velocity, acceleration, flow rate, strain, pressure, temperature, humidity, illumination, noise level, vibration), and these can be linked at a simple level to warning systems for human operators, or to automatic control systems which need no human intervention.

QUESTIONS 47, 49

The field of control engineering embraces manufacturing systems, navigation (radar, automatic pilot), electrical power generation and distribution, air traffic control, medicine (pacemakers, life support systems), transport (road traffic control, railway signalling, automated railway sidings), high-energy physics (particle accelerators and detectors), and a host of other essential activities and services. The 'mini-projects' suggested in the Laboratory notes are a selection of real-world problems from the fields of instrumentation, communications, computing, and automatic control.

EXPERIMENT I16
Putting electronics to use

READING

MAKING MEASUREMENTS IN HOSTILE ENVIRONMENTS

(This passage is adapted from NOLTINGK, B. E., *Physics in technology*, Vol. **10**, 1979.)

Instrumentation is sometimes needed operationally, to provide data for controlling plant: this may be minute-by-minute adjustments or perhaps occasional checks against malfunctioning. It is sometimes needed for research purposes, to study behaviour that is not completely understood.

In conventional boilers it is desirable, but very difficult, to know the heat flux (that is, flow) from the flames on different parts of the tubes which form the walls. The principle of a heat-flux meter can be simple enough: measuring with a pair of thermocouples the temperature drop across a known thermal resistance gives the 'heat' flowing through it. The difficulty comes in translating that into practice. The thermocouple leads must follow a route where they will not be too vulnerable. Implicit assumptions about the direction of heat flow must not cause invalidities in the calculations or in the deductions from any calibration.

Some materials in generating plant are used near their limits of strength. It is not surprising, therefore, that there is a call to study behaviour in service. Structural engineers are accustomed to using bonded resistance strain gauges in surveys, but conventional gauges exhibit too much drift to be effective for measuring static strains at temperatures above 300 °C or 400 °C. Accordingly, a new type of strain gauge has been developed in which the strain to be measured changes an electrical capacitance instead of a resistance. This has the advantage of escaping from critical dependence on a material property – resistivity – that has proved very difficult to hold constant and consistent in severe environments. Capacitance can be made to depend almost exclusively on shape and size, allowing dimensional stability to be the one quality asked of the materials from which the gauge is made.

Figure I23 gives an exploded view of the device. The feet of the two arches are welded together and then welded on the structure to be examined. Strain in that structure differentially alters the height of the arches, thus changing the separation of the electrodes mounted between them and so changing their capacitance.

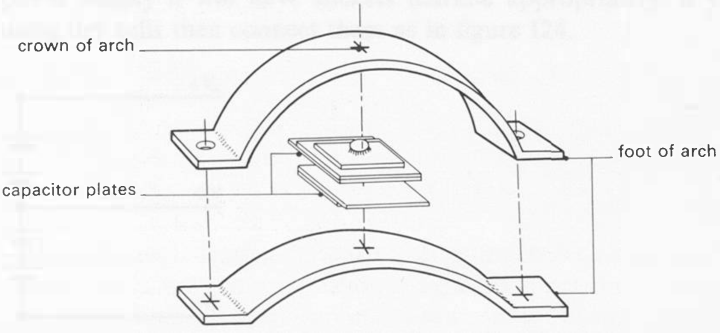


Figure I23
Exploded view of a capacitor strain gauge for use at high temperatures.

By taking great care over materials and construction techniques it has been found possible for the gauge to retain its dimensions and hence the strain it indicates, even over long periods at temperatures up to 600 °C. It has thus opened up new possibilities for studying creep and thermal strains and the cracking they may cause.

Measurements of small movements are often desirable at the phase of investigating questionable performance. One method uses a novel configuration that allows a gap to be measured using the fringing capacitance between two electrodes that are both placed on the same side of the gap.

Questions

a Give an example of data being needed
i 'to control plant',
ii to provide 'occasional checks against malfunctioning'.

b *i* Explain in simple descriptive terms why 'heat flux' can be deduced from two thermocouple readings.
ii In what units is thermal resistance measured?

c One place where assumptions have to be made about the 'direction of heat flow' through a wall would be near to a corner. Draw a sketch to illustrate this, and explain how different assumptions would lead to different conclusions about the flux.

d What is meant by 'drift'?

e Explain in a simple case how capacitance is related to 'shape and size'.

f 'Dimensional stability' is 'the one quality asked of the materials'. Explain this statement in simple language.

g Draw 'before' and 'after' diagrams showing the changes that occur in this type of strain gauge when the structure it is welded to is strained.

h How can 'thermal strains' (or 'creep and thermal strains') result in cracking?

i Explain in principle how you think a gap width could be monitored using 'fringing capacitance'.

LABORATORY NOTES

NOTE ON APPARATUS LISTS IN UNIT I

For each experiment you are told in the list what values of *potentiometers*, *resistors*, or *capacitors* are needed. You may find that these are already fitted where you want them on the operational amplifier units. If they are not, then use separate components and clip component holders.

DEMONSTRATION

I1 Introduction to linear electronics and control

I1a Manual control of illumination

I1b Automatic control of illumination

I1c Light follower

Essential parts of any control system include a sensor or transducer, a feedback path, and some means of comparing two signals. Try to identify these components in each of the demonstrations you see.

Why is a power amplifier necessary in many control systems? If the system includes an operational amplifier, what is its function?

A control system may start to oscillate in certain circumstances. What factor or factors may cause this?

The demonstrations will be repeated later, and details are given under demonstrations I13 and I15.

EXPERIMENT

I2 Introduction to operational amplifiers

operational amplifier unit with power supply and resistors, 10 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
leads

Figure I25 (page 100) shows the basic circuit for many operational amplifier experiments. The diagram does not show the power supply, but you must make sure that the power supply is correctly connected to the three terminals marked +, 0 (or earth), and −. If you have a special power supply it will have sockets marked appropriately. If you are using dry cells then connect them as in figure I24.

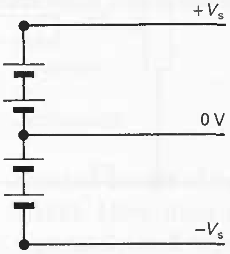


Figure I24

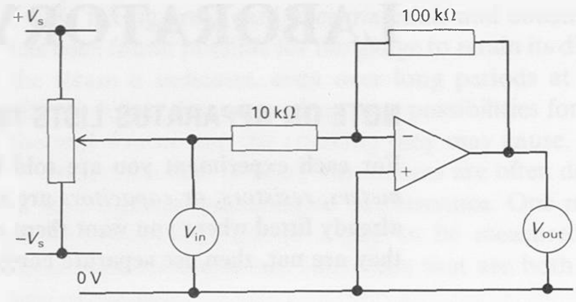


Figure I25
Circuit for experiment I2a (power supply connections not shown).

In each experiment (I2a to I2e) answer the questions, concerning the output voltage V_{out} and the input voltage V_{in} (these voltages are always measured with respect to the 0 V power line).

I2a Input and output voltages

Apparatus as for experiment I2 with:
either
2 voltmeters, 1 V, 10 V d.c.
or
oscilloscope
voltmeter

Use the potential divider to vary the input voltage (figure I25). Try positive and negative values, up to the + and − power supply voltage. Use a voltmeter or oscilloscope to measure V_{in}, V_{out}. What is the maximum value of V_{out}? What is the minimum value? When is V_{out} positive? When is it negative?

I2b Response to a.c.

Apparatus as for experiment I2 with:
either
signal generator
or
transformer
oscilloscope

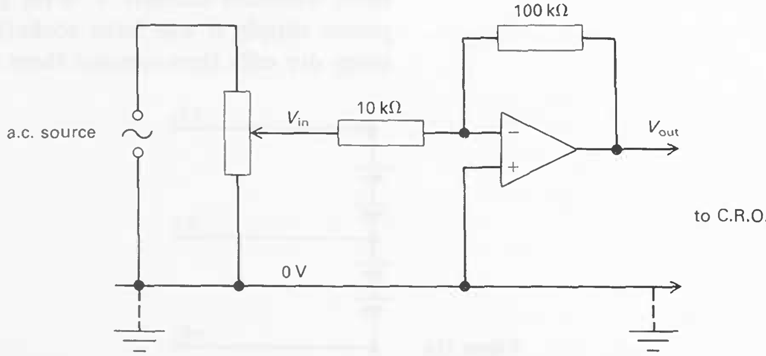


Figure I26
Operational amplifier with a.c. input.

Use the circuit of figure I26. Note that the a.c. source and potential divider are now connected to the 0 V line, not to $+V_s$ and $-V_s$. If your source is a signal generator with one side earthed, this side must be connected to the 0 V line, and so must the earthed terminal of the oscilloscope (see dashed lines in figure I26). If your source is a transformer, use its 2 V output.

Use the potential divider to increase the input steadily from zero, and observe what happens to the output signal on the oscilloscope. Does its behaviour confirm or add anything to your observations in experiment I2a?

I2c Effect of changing input resistance

Apparatus as for experiment I2 with:
either
resistance substitution box
or
additional resistors

Use the circuit shown in figure I25. What difference does it make to the behaviour of the circuit when the value of the input resistance (the 10 kΩ resistor in figure I25) is: *i* halved; *ii* doubled; *iii* multiplied by 10?

I2d Effect of changing feedback resistance

Apparatus as for experiment I2 with:
either
resistance substitution box
or
additional resistors

Use the circuit of figure I25. What difference does it make to the behaviour of the circuit when the value of the feedback resistance (the 100 kΩ resistor in figure I25) is: *i* doubled; *ii* halved; *iii* divided by 10?

I2e Amplifier circuit with two inputs (optional)

Apparatus as for experiment I2 with:
potentiometer, *e.g.* 1 kΩ linear

either
resistor, 10 kΩ
or
resistance substitution box

either
voltmeter
or
oscilloscope

Use the circuit of figure I27 (page 102), in which each input resistance is 10 kΩ. How does the output voltage depend on the input voltages, separately and together?

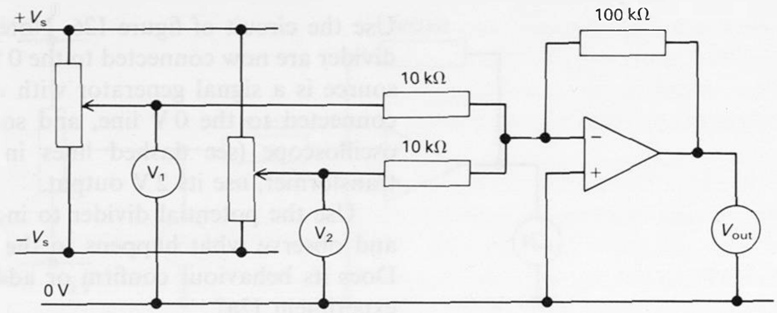


Figure I27
Operational amplifier circuit with two inputs.

DEMONSTRATION

I3 Behaviour of a feedback amplifier circuit

operational amplifier unit with power supply and resistors, 10 kΩ, 100 kΩ
signal generator
double-beam oscilloscope
leads

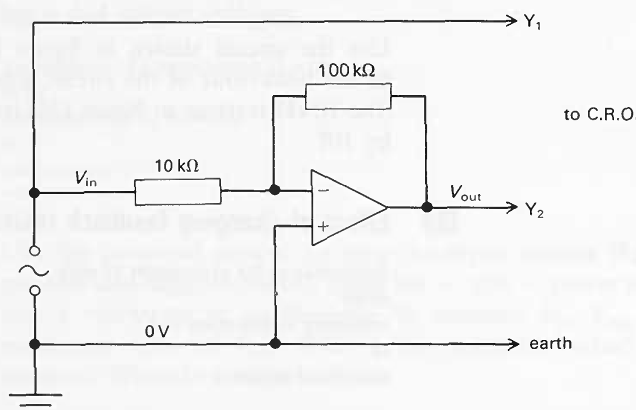


Figure I28
Measurement of input and output voltages on double-beam oscilloscope.

Note that the earthed side of the signal generator and also that of the oscilloscope must be connected to 0 V.

It is helpful to have the oscilloscope's time-base switched off for parts of this demonstration.

Set up a demonstration to show (visually, not by detailed graph plotting) that $V_{out} \propto -V_{in}$; that there is an upper (and lower) limit to the value of V_{out}; that a small sinusoidal signal is faithfully amplified, but a bigger one is not.

If there is time, show the effect of changing the value of either of the resistors. What is the amplification factor when the resistors are equal?

EXPERIMENT

I4 Input–output characteristic of a feedback amplifier circuit

operational amplifier unit with power supply and resistors, 10 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
voltmeter, 1 V d.c.
voltmeter, 10 V d.c.
leads

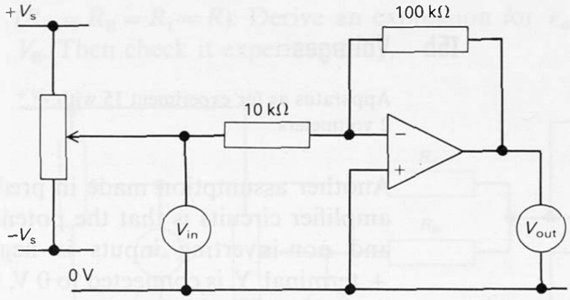


Figure I29
Measurement of input–output characteristics.

Use the potential divider connected across the operational amplifier power supply to provide a variable input voltage, V_{in}. Plot a graph of V_{out} against V_{in} for the whole range of possible input voltages.

What is the slope of the graph (where it is a straight line)?

If you have time, repeat the experiment with other resistors.

OPTIONAL DEMONSTRATION

I5 Investigation of currents and voltages in an operational amplifier

operational amplifier unit with power supply and resistors, 10 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
leads

I5a Currents

Apparatus as for demonstration I5 with:
2 microammeters, 100 μA

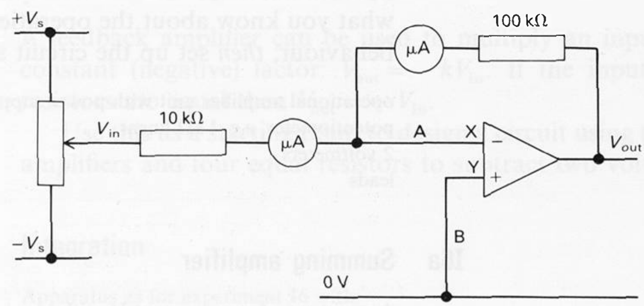


Figure I30
Measurement of currents.

Connect the circuit of figure I30.

According to an operational amplifier's specification the input resistance is very high, and in analysing operational amplifier circuits we usually assume that no current enters or leaves the device itself.

Use the microammeter readings to show that the current through the 100 kΩ resistor is the same as that through the 10 kΩ resistor. (How small a difference between these currents could you detect?)

If the connections to the unit allow, move one of your meters to the point A and verify that no detectable current is present.

You may also want to check for any current in the lead to the non-inverting input, by inserting a meter at B.

I5b Voltages

Apparatus as for experiment I5 with:
2 voltmeters

Another assumption made in predicting the behaviour of operational amplifier circuits is that the potential difference between the inverting and non-inverting inputs is negligibly small. In your circuit the + terminal, Y, is connected to 0 V, so the inverting input, X, should also be virtually at 0 V.

Connect your voltmeters so as to measure V_{in} and V_{out} simultaneously. Use the voltmeter readings and the values of the input and feedback resistances to calculate the potential at A.

Why would it not be sensible simply to connect a voltmeter between A and the 0 V line? (Think about the input resistance of the operational amplifier.)

Assumptions and limits
Parts **a** and **b** of this demonstration were to check some assumptions which are *usually* true of operational amplifiers.

Try to find out (by varying the input voltage) when these approximations are valid and when they are not.

EXPERIMENT
I6 More uses of the feedback amplifier

This group of experiments deals with slightly more complicated operational amplifier circuits. Whichever experiment you do, *first* use what you know about the operational amplifier to predict the circuit's behaviour; *then* set up the circuit and check your predictions.

operational amplifier unit with power supply and resistors, 10 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
2 voltmeters
leads

I6a Summing amplifier

Apparatus as for experiment I6 with:
potentiometer, *e.g.* 1 kΩ linear

either
2 resistance substitution boxes
or
additional resistors

either
voltmeter
or
oscilloscope

The two input voltages (V_A and V_B) can be varied independently.

Suppose all the resistors in the circuit (figure I31) have the same value

$(R_A = R_B = R_f = R)$. Derive an expression for V_{out} in terms of V_A and V_B. Then check it experimentally.

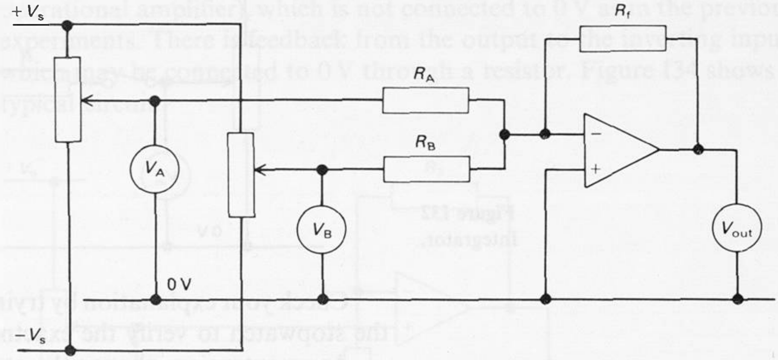


Figure I31
Summing amplifier.

Suppose the values of the resistors are not equal. What is V_{out} in terms of V_A, V_B, R_A, R_B, and R_f? Use your result to set up a circuit for which $V_{out} = -10(V_A + V_B)$, and another one for which $V_{out} = -(10V_A + V_B)$.

I6b Subtractor

Apparatus as for experiment I6 with:
operational amplifier unit with resistors, 100 kΩ, 100 kΩ, 100 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear

either
voltmeter
or
oscilloscope

A feedback amplifier can be used to multiply an input voltage by a constant (negative) factor: $V_{out} = -kV_{in}$. If the input and feedback resistors are equal then $V_{out} = -V_{in}$.

Use this as a starting point to design a circuit using two operational amplifiers and four equal resistors to subtract two voltages.

I6c Integration

Apparatus as for experiment I6 with:
capacitor, 1 μF

either
switch
or
mounted bell push

stopwatch

With the switch open make sure that the capacitor is discharged. (See figure I32, page 106.) Then make the input voltage a few volts negative and observe what happens to V_{out} when the switch is closed.

You should be able to explain what you have observed using what you know about the operational amplifier and capacitors.

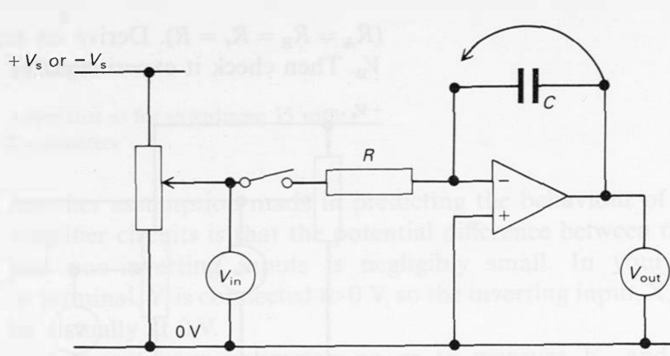


Figure I32
Integrator.

Check your explanation by trying other values of V_{in}, R, or C, and use the stopwatch to verify the exactness of the circuit's behaviour.

As an extension, change the component values to give a very rapid rate of change of output voltage (say 100 volts per second) and for input V_{in} use a signal generator set to square-wave output. Think out how you expect V_{out} to behave. Check your conclusions by using an oscilloscope.

I6d Differentiation

Apparatus as for experiment I6 with:
capacitance substitution box
oscilloscope
signal generator

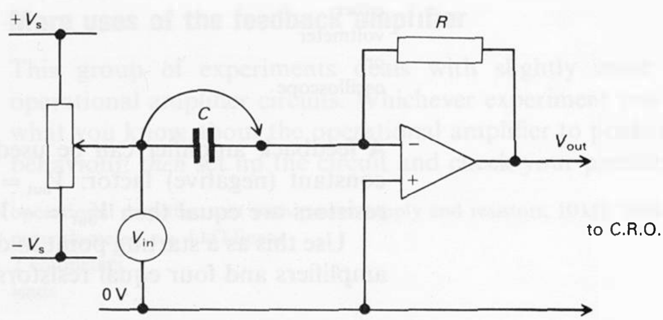


Figure I33
Operational amplifier circuit for differentiation.

Use the oscilloscope to observe the output voltage. Start with $C = 1\ \mu F$ and $R = 1\ M\Omega$. Initially the capacitor should be discharged. What happens to V_{out} if V_{in} is changed to a new value quickly, or slowly? What effect do the values of R and C have on the behaviour of this circuit?

Use a signal generator to provide 'square-wave' or 'saw-tooth' input signals, and observe the output on the oscilloscope. Explain how the circuit acts to differentiate the input signal.

EXPERIMENT
I7 Using the non-inverting input

operational amplifier unit with power supply and resistors, 100 kΩ, 100 kΩ
voltmeter, 10 V d.c.
leads

In this series of experiments the input signal goes to the non-inverting input of the operational amplifier (marked + in the symbol for an operational amplifier), which is not connected to 0 V as in the previous experiments. There is feedback from the output to the inverting input, which may be connected to 0 V through a resistor. Figure I34 shows a typical circuit.

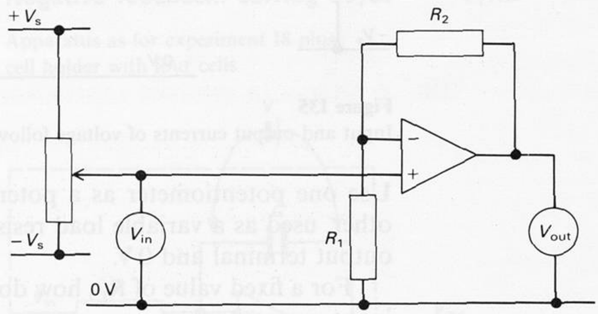


Figure I34
Using the non-inverting input.

I7a Follower circuit with variable gain

Apparatus as for experiment I7 with:
potentiometer, *e.g.* 1 kΩ linear
voltmeter
either
2 resistance substitution boxes
or
additional resistors

Start with $R_1 = R_2 = 100\,\text{k}\Omega$.

Use a potentiometer to vary the input voltage V_{in}; try positive and negative values.

What is the relationship between V_{out} and V_{in} for this circuit?

Is the term 'non-inverting' appropriate for this input?

You should be able to explain why the circuit behaves as it does, and predict the ratio V_{out}/V_{in} for other values of R_1 and R_2. (Questions **28** and **29** might help.)

Test your predictions.

What would happen if $R_2 = 0$ or R_1 is infinite?

Set up this circuit and test your prediction.

Why is 'voltage follower' an appropriate name for this circuit?

I7b Input and output currents of voltage follower

Apparatus as for experiment I7 plus:
potentiometer, 5 kΩ or 10 kΩ linear
milliammeter, 10 mA d.c.
microammeter, 100 μA d.c.

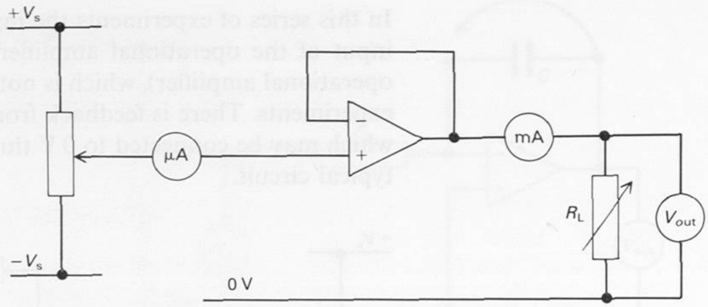


Figure I35
Input and output currents of voltage follower.

Use one potentiometer as a potential divider to vary the input. The other, used as a variable load resistance R_L, is connected between the output terminal and 0 V.

For a fixed value of R_L, how does the output current depend on the input current?

For a fixed value of input voltage, how does the output current depend on R_L?

How large an output current can we take, if the output voltage is not to drop by more than 1 %? 20 %?

If output current is much greater than input current, where does the extra current come from?

Suggest a use for this circuit.

I7c Use of voltage follower circuit

Apparatus as for experiment I7 plus:
capacitor, 50 μF
cell holder with one cell

Charge the capacitor to 1.5 V and then connect the voltmeter across its terminals. What happens to the voltmeter reading? Why?

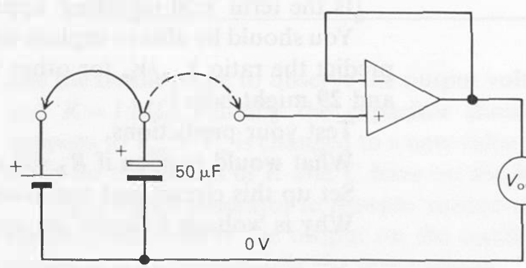


Figure I36
Unity-gain voltage follower.

Now try again using the unity-gain voltage follower circuit as a buffer between the capacitor and voltmeter (figure I36). Charge the capacitor, then connect it to the input of the circuit.

What happens to the voltmeter reading this time? You should be able to explain the effect of the buffer circuit, and suggest other uses for it. At least one piece of equipment you are familiar with incorporates a buffer circuit.

EXPERIMENT

I8 Integrator with feedback

operational amplifier unit with power supply and resistor, 1 MΩ, capacitor, 1 μF
oscilloscope
leads

I8a Negative feedback: solving $dV/dt = -V/RC$

Apparatus as for experiment I8 plus:
cell holder with four cells

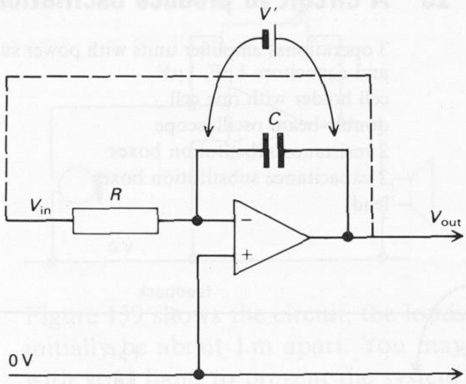


Figure I37
Integrator with feedback.

Without the feedback wire (shown dashed in figure I37) the output of the integrator circuit is

$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt + \text{constant}$$

(see experiment I6c). Or we can write

$$dV_{out}/dt = -V_{in}/RC.$$

The rate of change of the output depends on the size of the input.

If the output is connected to the input by a feedback wire then $V_{out} = V_{in}$ ($= V$, say). So $dV/dt = -V/RC$. This should be a familiar equation and you should know its solution (*i.e.*, how V varies with t).

Set up the circuit and display V against t on the oscilloscope. You will need to have a starting value of V: the capacitor must be charged at the start of the experiment using the cell (V').

What would be the effect of increasing R and/or C? Or of decreasing either?

If possible, check your prediction.

I8b Positive feedback: solving $dV/dt = V/RC$

Apparatus as for experiment I8 plus:
operational amplifier with resistors, 100 kΩ, 100 kΩ

The circuit shown in figure I37 solves $dV/dt = -V/RC$. A feedback amplifier circuit can be used to multiply an input by -1. Use these two ideas to build a circuit using two operational amplifiers with feedback to solve $\mathrm{d}V/\mathrm{d}t = +V/RC$. You will need a lead to discharge the capacitor first.

This is the rate-of-change equation for exponential growth. A quantity which is growing exponentially continues to grow indefinitely, and at an ever-increasing rate. Does your circuit model this behaviour accurately? Can you explain any differences?

DEMONSTRATION

I9 A circuit to produce oscillation

3 operational amplifier units with power supply and resistors, 1 MΩ, 1 MΩ, 1 MΩ, 1 MΩ, and capacitors 1 μF, 1 μF
cell holder with one cell
double-beam oscilloscope
2 resistance substitution boxes
2 capacitance substitution boxes
leads

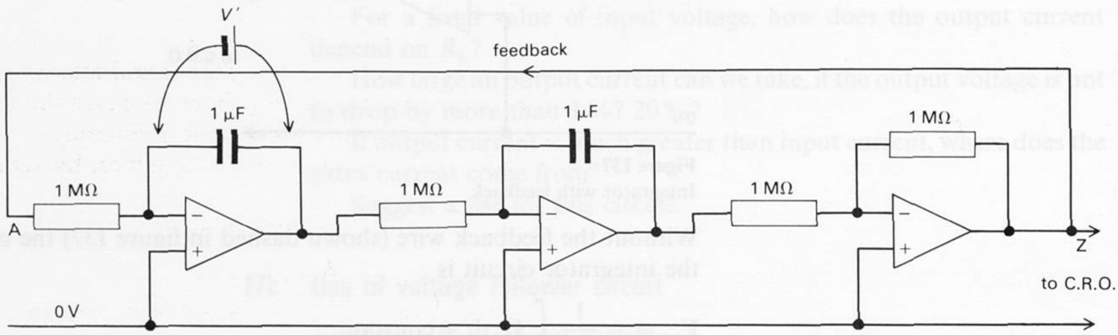


Figure I38
Circuit to produce oscillation.

This circuit should produce oscillation when a feedback connection is made by a piece of wire between Z and A. (If it does not start to oscillate try applying 1.5 V momentarily across one of the capacitors.)

Measure the frequency of the oscillation. The circuit consists of three sub-circuits you are familiar with. What does each unit do? Imagine that a sinusoidal signal is fed into the circuit at A. How will it be changed by each of the units?

How will the signal at Z compare with the signal at A? What is the effect of feedback from Z to A?

Use the double-beam oscilloscope to compare the output of each unit with its input to check your answers to the questions above.

How could you change the frequency of the oscillation? Try it! What change(s) or addition(s) would you need to make to produce a 'square-wave' output?

DEMONSTRATION

I10 Feedback in public address systems

operational amplifier unit with power supply

either
potentiometer, 1 MΩ
or
resistance substitution box

microphone
small loudspeaker (not earpiece)
leads

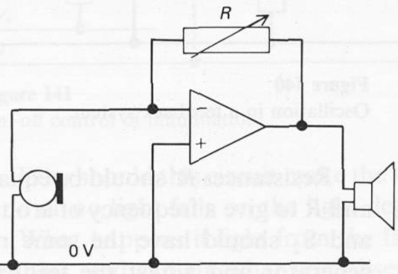


Figure I39
Howl produced by positive feedback.

Figure I39 shows the circuit; the loudspeaker and microphone should initially be about 1 m apart. You may have to cover the microphone with your hand to prevent the system from oscillating.

Show that the circuit is acting as an amplifier, by scratching the microphone.

Investigate the oscillation behaviour by:

a changing the gain by adjusting *R*, and

b putting loudspeaker and microphone close together and then moving them slowly apart.

OPTIONAL DEMONSTRATION

I11 Oscillation in a feedback system

3 operational amplifier units with power supply and resistors, 100 kΩ, 100 kΩ or 1 MΩ, 1 MΩ
potentiometer
4 resistance substitution boxes
2 capacitance substitution boxes
signal generator
oscilloscope
cell holder with one cell
leads

To the circuit of demonstration I9 two things are added: an extra input from the signal generator, and an extra feedback loop from the output of the first operational amplifier to the input resistor S_2 (see figure I40, page 112). In terms of a mechanical system, this is equivalent to adding an extra force to a moving object, proportional to its velocity and in the opposite direction, that is, frictional damping.

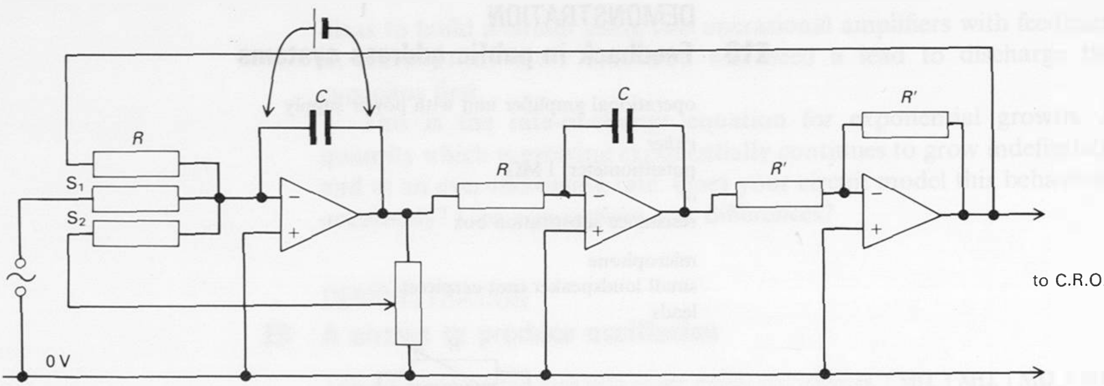


Figure I40
Oscillation in a feedback system.

Resistances R' should be equal: 1 MΩ or 100 kΩ. Choose values of C and R to give a frequency of around 150 Hz (*e.g.* 0.001 μF and 1 MΩ). S_1 and S_2 should have the same resistance as R. Switch off the signal generator and adjust the feedback potentiometer for zero feedback. Touch the leads from the 1.5 V cell briefly across the capacitor as indicated in figure I40: this should start the oscillation. This needs to be made to die away within about 10 seconds, and to achieve this you may have to adjust the feedback potentiometer or change the value of S_2.

Now switch on the signal generator and set it to about 150 Hz. Vary the frequency around this value, and observe on the oscilloscope how the circuit behaves. Try increasing or decreasing the damping. Try disconnecting the signal generator and connecting 1.5 V d.c. in its place.

DEMONSTRATION

I12 On–off control of illumination

operational amplifier unit with power supply not more than + 6, 0, − 6 V (see below)
light-dependent resistor
potentiometer, 1 kΩ linear
2 general-purpose diodes
relay
lamp (12 V, 24 W) and holder

either
l.t. variable voltage supply
or
transformer

leads

Note that the power supply to the operational amplifier needs here to be + 6, 0, − 6 V, in order to operate the relay.

In this demonstration the operational amplifier is functioning simply as a (digital) logic gate. What kind of gate is it?

When the input to the relay is low the contacts are open; when it goes high they close and the lamp goes on. (Why is a relay needed – why not use the gate to control the lamp directly?)

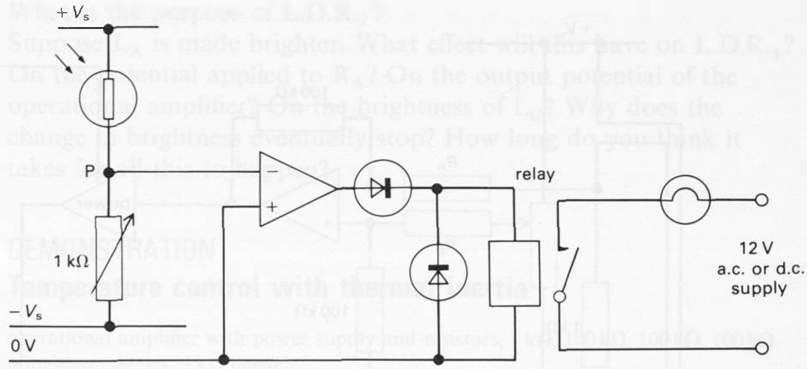


Figure I41
On–off control of illumination.

Why should this circuit make the lamp come on in the dark, that is if little or no light falls on the light-dependent resistor? Try it.

What happens if light from the lamp itself falls on the transducer? Try this, and explain what you observe.

What controls the level of darkness at which the light comes on?

Why can't this system be used to provide a controlled level of illumination?

DEMONSTRATION
I13 Continuous control of illumination

operational amplifier with power supply and resistors, 1 kΩ, 1 kΩ, 10 kΩ, 10 kΩ, 100 kΩ, 100 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
2 lamps, 12 V, 24 W
2 light-dependent resistors (L.D.R.s)
either
ohmmeter
or
cell holder with one cell and milliammeter

power amplifier
2 variable power supplies, 0–25 V d.c.
leads
retort stand base, rod, 2 bosses, 2 clamps

A fairly dark room is needed to make the best of this demonstration. The circuit is shown in figure I42 (page 114). The operational amplifier supply voltage could be any value down to $+3, 0, -3$ V. R_A will need to be 1 kΩ, 10 kΩ, and 100 kΩ in turn (see below). R_B should be 100 kΩ throughout, to avoid making the 'set level' control too sensitive. (In the unlikely event of the set level range being insufficient, R_B can be changed to 10 kΩ.) Long leads will be needed to L.D.R.$_1$ and to the controlled lamp L_C.

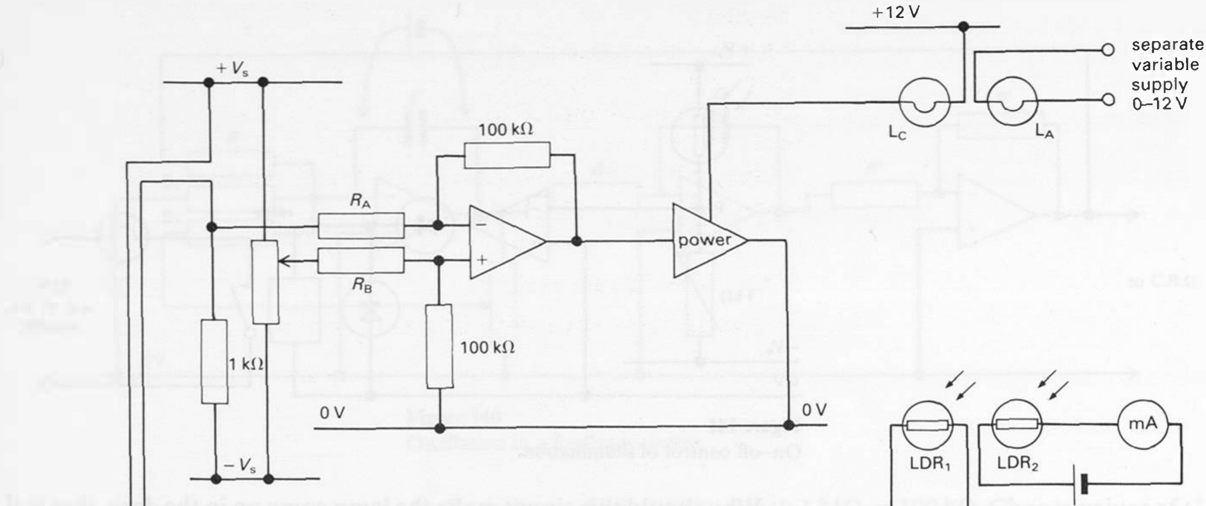


Figure I42
Automatic control of illumination.

Arrangement

The two lamps should be fixed as close together as possible about 50 cm above bench level, with the L.D.R.s close together on the bench below them, facing upwards. The meter for L.D.R.$_2$ should also be where light will fall on it.

Adjustment

Set R_A at 1 kΩ. Turn up the lamp L_A to full brightness. Turn up the voltage of the power amplifier supply to 12 V. Turn the potentiometer to maximum positive potential, so that L_C is bright. Further raise the power amplifier supply voltage as necessary until L_C is as bright as L_A. Finally, adjust the potentiometer again until L_C is only just visibly glowing dull red.

Demonstration

Gradually turn down the brightness of L_A. L_C should brighten, and the meter reading should change very little, if at all. Show by covering L.D.R.$_1$ that it is controlling the brightness of L_C. Show by making rapid changes in the light from L_A (including interposing the hand or even switching off and on) that the system corrects instantly and precisely – much better than a human operator.

It might be worth increasing R_A to 10 kΩ and then 100 kΩ, to see how a reduction in gain produces less than total compensation for changes in ambient illumination.

Questions
What is the purpose of L.D.R.$_2$?
Suppose L_A is made brighter. What effect will this have on L.D.R.$_1$? On the potential applied to R_A? On the output potential of the operational amplifier? On the brightness of L_C? Why does the change in brightness eventually stop? How long do you think it takes for all this to happen?

DEMONSTRATION
I14 Temperature control with thermal inertia

operational amplifier with power supply and resistors, 1 kΩ, 100 kΩ, 100 kΩ, 100 kΩ
potentiometer, *e.g.* 1 kΩ linear
either
capacitor, *e.g.* 0.001 μF
or
capacitance substitution box

resistance substitution box
bead thermistor, *e.g.* GL23, 2 kΩ–115 Ω
aluminium foil
dull black paint (Aquadag)
lamp, holder, and stand
voltmeter, 12 V d.c.
power amplifier
l.t. variable voltage supply
leads

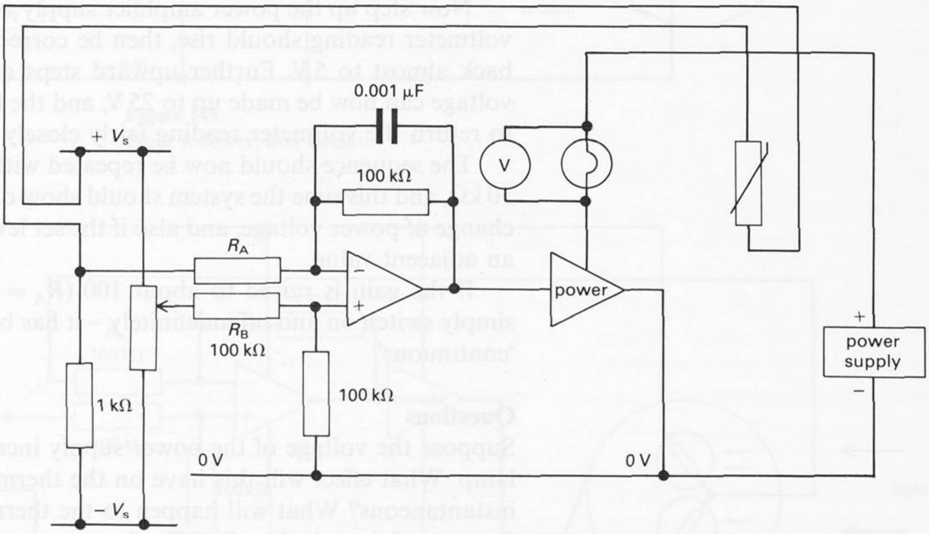


Figure I43
Temperature control with thermal inertia.

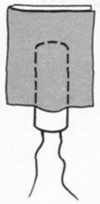


Figure I44
Thermistor mounting.

Preparation
Cut a piece of cooking foil about 2 cm × 1 cm, fold it in half and press it tightly round the thermistor as shown in figure I44. Paint one side dull black. Mount the thermistor level with the 24 W lamp, with the black surface facing the lamp and about 1 cm away from it.

Notes

i The behaviour of this circuit will inevitably depend on such local factors as the degree of thermal contact between thermistor and foil, and on the precise characteristics of the power amplifier, so some modifications may have to be imposed on the notes given below.

ii A possible variation is to cover the thermistor and lamp with an inverted beaker, representing a miniature 'room' being heated. The transfer of energy from lamp to thermistor will then probably depend more on convection than on radiation.

The circuit is shown in figure I43 (page 115). The operational amplifier supply voltage can be any value down to $+3, 0, -3$ V. R_A should be the resistance substitution box, set initially at 100 kΩ. R_B remains at 100 kΩ. The 'set level' adjustment of the potentiometer is very sensitive and can be disturbed by capacitive effects due to a hand nearby: the feedback capacitor serves to eliminate this disturbance, and its value is not critical. Long leads will be needed to the thermistor and to the lamp.

Adjustment

Set the power amplifier supply to 0 V. Turn the potentiometer to maximum positive potential, then raise the power amplifier supply voltage until the voltmeter shows 8 V. Carefully adjust the potentiometer until the voltmeter reading falls to 5 V, and continue adjusting as necessary until the radiation feedback to the thermistor holds the reading at this value (response will be sluggish).

Now step up the power amplifier supply voltage by a few volts. The voltmeter reading should rise, then be corrected within a few seconds back almost to 5 V. Further upward steps of power amplifier supply voltage can now be made up to 25 V, and the feedback should continue to return the voltmeter reading fairly closely to 5 V.

The sequence should now be repeated with R_A at 47 kΩ, 22 kΩ, and 10 kΩ, and this time the system should show damped oscillation at each change of power voltage, and also if the set level is changed from 5 V to an adjacent value.

If the gain is raised to about 100 ($R_A = 1$ kΩ), the system should simply switch on and off indefinitely – it has become 'on–off' instead of 'continuous'.

Questions

Suppose the voltage of the power supply increases, brightening the lamp. What effect will this have on the thermistor? Will it be instantaneous? What will happen to the thermistor's resistance? To the potential applied to R_A? To the output of the operational amplifier? To the brightness of the lamp?
Why does increased gain lead to oscillation?

DEMONSTRATION

I15 Light follower

For first circuit (figure I45)
power amplifier
2 variable power supplies, 0–25 V d.c.
potentiometer, 1 kΩ linear
2 light-dependent resistors (L.D.R.s)

either
lamp 12 V, 24 W
or
other source of illumination, *e.g.* reading lamp

turntable and motor
insulating tape
leads

Additional for second circuit (figure I46)
operational amplifier with power supply and resistors, 10 kΩ, 10 kΩ, 100 kΩ, 100 kΩ, 100 kΩ

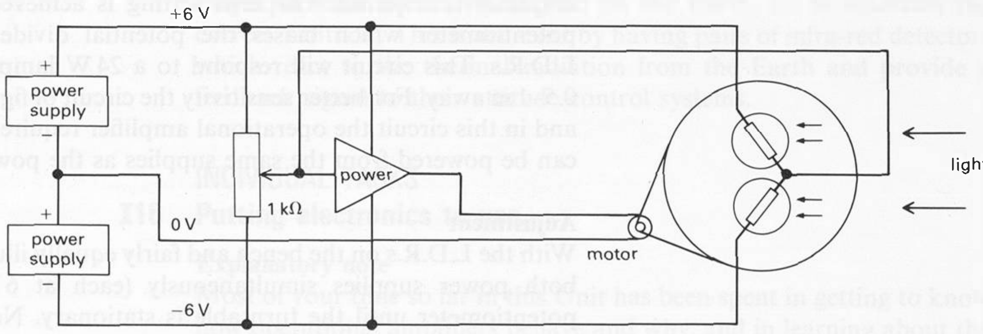


Figure I45
Light follower: first circuit.

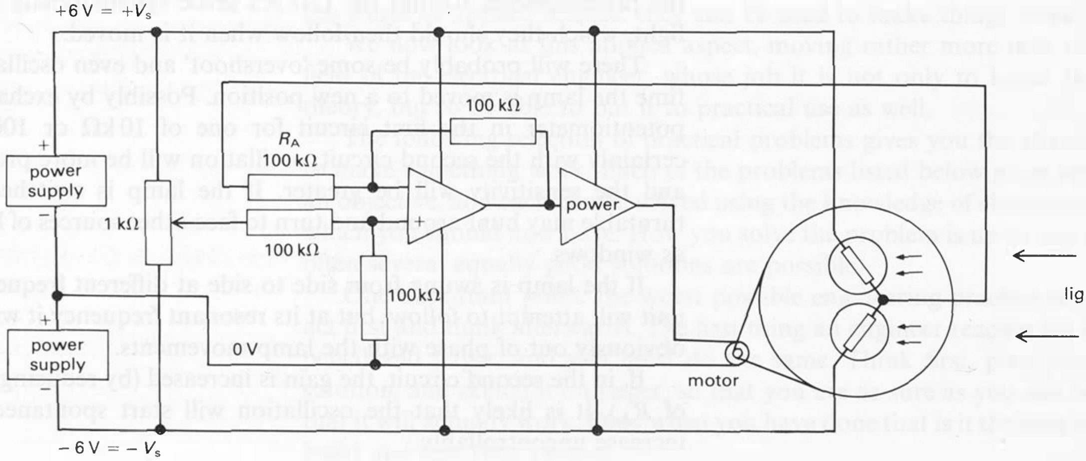


Figure I46
Light follower: second circuit.

Preparation
The two L.D.R.s should be taped firmly to a block of wood so that they face horizontally and are at 90° to one another. The three leads from them should be as light and as flexible as possible, and about 2 m long: 0.45 mm insulated copper wire is suitable. The block should not at first be put on the turntable.

Important note
The motor for the turntable can ordinarily be driven using dry cells, since it needs little current. However, some simple power amplifier units provided for schools may draw very much more current than this (typically 3 A between + and − leads when supplying zero output current). Consequently, the 6 V supplies must be provided from two mains-operated power units, not from dry cells.

The circuits
A simple version is shown in figure I45, in which no operational amplifier is required. The zero setting is achieved by means of the potentiometer which biases the potential divider provided by the L.D.R.s. This circuit will respond to a 24 W lamp at distances up to 0.5–1 m away. For better sensitivity the circuit of figure I46 can be used, and in this circuit the operational amplifier requires + 6, 0, − 6 V, and can be powered from the same supplies as the power amplifier.

Adjustment
With the L.D.R.s on the bench and fairly equally illuminated, switch on both power supplies simultaneously (each at 6 V) and adjust the potentiometer until the turntable is stationary. Next place the block with the L.D.R.s on the centre of the turntable, and place a 24 W lamp or reading lamp facing it and about 0.5 m away. If the assembly rotates away from the light, reverse the motor leads. Make a fine adjustment of the potentiometer so that the L.D.R.s settle symmetrically facing the light, which they should then follow when it is moved.

There will probably be some 'overshoot' and even oscillation each time the lamp is moved to a new position. Possibly by exchanging the potentiometer in the first circuit for one of 10 kΩ or 100 kΩ, and certainly with the second circuit, oscillation will be more pronounced, and the sensitivity will be greater. If the lamp is switched off, the turntable may hunt around and turn to face other sources of light, such as windows.

If the lamp is swung from side to side at different frequencies, the unit will attempt to follow, but at its resonant frequency it will be very obviously out of phase with the lamp movements.

If, in the second circuit, the gain is increased (by reducing the value of R_A), it is likely that the oscillation will start spontaneously and increase uncontrollably.

Questions

Suppose a light-dependent resistor is set facing a lamp. If it is now turned partly away from the lamp, what change will there be in the amount of light which it intercepts? What effect will this have on its resistance?

If the two L.D.R.s in this apparatus are equally inclined to the incoming light, their resistances will be equal. If the direction of the light changes, what will happen to the resistance of each? Explain why the potential at their common junction will change. Assuming the motor is connected the right way round, what effect will this have on the motor output, the orientation of the L.D.R.s, the light intercepted by each, and the resistance of each?

The turntable tends to overshoot when it moves. Why? Why does high gain increase this?

Information

Communications satellites need to keep their receiving and transmitting equipment accurately aligned on the Earth, *i.e.* to maintain the correct 'attitude'. This is achieved by having pairs of infra-red detectors which react to the thermal radiation from the Earth and provide a feedback signal to their attitude control systems.

INDIVIDUAL TASKS

I16 Putting electronics to use

Explanatory note

Most of your time so far in this Unit has been spent in getting to know how operational amplifiers behave and why, and in learning about the range of operations they can be made to perform: amplification, addition, integration, etc. – that is, pure electronics. At the same time you have seen that these operations have obvious potential for use in the world of applied science: they can be used to make things work.

We now look at this applied aspect, moving rather more into the field of the electrical engineer, whose job it is not only to know the theory, but to be able to put it to practical use as well.

The following selection of practical problems gives you the chance to make something work. Each of the problems listed below gives you an objective, and each can be solved using the knowledge of electronics which you should now have. How you solve the problem is up to you – often several equally good solutions are possible.

One important point: the worst possible engineering practice is to act first and think afterwards. The first thing an engineer reaches for is pencil and paper, and you must do the same. Think first, plan your solution, and sketch it on paper, so that you are as sure as you can be that it will actually work. Only when you have done that is it the time to build and test your system.

List A: easier

A1 Make a 4-digit digital-to-analogue converter with four input switches to represent the binary numbers and the analogue output registering on a meter (see question **21**, page 133).

A2 Make a light meter, assuming that the resistance of a light-dependent resistor is inversely proportional to light intensity (which it very nearly is). The meter should have two ranges, one for dim lighting and one for brighter conditions.

A3 Make an ohmmeter which gives directly the value of a resistance as a linear reading on a meter scale. Provide two ranges.

A4 Make a high-impedance voltmeter in one or more of the ways listed:

a a simple follower;
b an inverting amplifier (provide ranges 0.1 V, 1 V, and 10 V);
c use a non-inverting circuit and use an ammeter in the feedback loop as your measure of voltage.

You could use your meter to measure the e.m.f. generated by a photovoltaic cell, and you could verify the circuit's high input impedance by trying to measure the p.d. across a charged capacitor.

A5 Set up a comparator (see question **37**, page 141) as a switch for one of the purposes below. For a warning signal use a lamp, a light-emitting diode, a bell, a buzzer, or an 'audible warning device'. Suggestions for applications:

a to switch on traffic bollard lights as daylight fades;
b to warn if temperature gets too high, or too low (and possibly to switch on a heater);
c to warn if water level becomes too high;
d to warn when a kettle emits steam.

A6 Make an intercom between two stations, in which each has an earpiece or small loudspeaker which also acts as a microphone, and can be switched from 'speak' to 'receive'. The amplifier is situated at one of the two stations. Special points:

a When an amplifier is used for a.c. it may be desirable to feed into it and out of it via capacitors; try 0.1 μF.
b You may need to provide a second stage of amplification, since neither earpiece nor loudspeaker are likely to be very efficient as microphones.

A7 Make an a.c. signal mixer with fade-in facility, *i.e.* two inputs with variable gains down to zero, giving mixed output in any proportion. *Note:* when an amplifier is used for a.c. it may be desirable to feed into it and out of it via capacitors; try 0.1 μF.

A8 Make a short-interval timer to measure times of a few thousandths of a second. Use it to measure the contact time in some bouncing process. An integrator is recommended, and a reset button will be required. Adapt your circuit to measure human reaction time.

List B: harder

B1 Make a direct reading fluxmeter. When a coil of N turns is moved into a magnetic field so that a flux change of Φ becomes linked with it, the e.m.f. induced in the coil is $\mathscr{E} = N\mathrm{d}\Phi/\mathrm{d}t$. Try a 5000-turn lateral search coil and a strong magnet at first.

Extension

When a piece of iron is placed in an increasingly strong magnetic field, B_0, the flux density in the iron, B, increases until magnetic saturation is reached, and will then increase no further. If the applied field is reduced to zero, B will decrease but not to zero – there will be 'residual magnetization' which can only be removed using a reverse field. The total behaviour of a magnetic material can be studied by putting it in an alternating field and monitoring the changes in B.

Figure I47 shows a typical result, which is called a magnetization cycle. Try various combinations of 240- and 1100-turn coils side by side (co-axially) with a short iron rod (say 5 cm long) inside them. Feed a.c. (maximum 4 V, from a mains transformer) into one coil and use the other as a search coil. Use Ext X on an oscilloscope to plot B_0, with B as Y deflection.

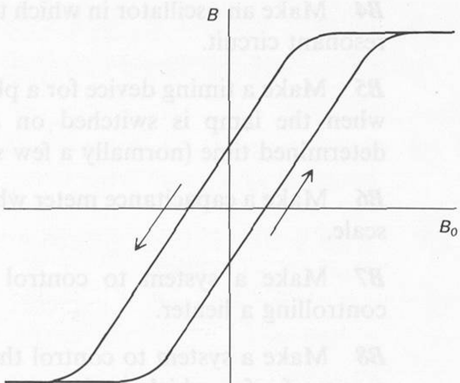


Figure I47
Magnetization cycle.

B2 Use a comparator (see question **37**, page 141) as a bridge balance detector in a Wheatstone bridge circuit. The power supply for the bridge could be the + and − supply to the operational amplifier, *but* the resistances must be large enough to prevent current drain on the supply being more than 100 mA. If this is not possible, extra resistors must be included between the bridge and the supply.

The output of the comparator will flick from one saturated state to the other on either side of the balance point (and you might consider how the meter could be replaced by a pair of light-emitting diodes, one for each direction of the output, thus saving weight, space, and cost). In order to *measure* changes in resistance you will need to have a standard resistance box as one of the arms of the bridge, and adjust it for balance each time the resistance changes. Alternatively, you could use a differential amplifier (see question **35**, page 140) in place of the comparator, so that small changes in resistance show as changes in the output meter reading, which can then be calibrated to read ohms or fractions of an ohm.

Suggested uses are:

a Measure the change in resistance as a wire is strained.
b Set up a resistance thermometer. (Note that resistance wire such as constantan is no use, being designed to have negligible temperature coefficient: a pure metal is needed. As an alternative there is always the thermistor, though obviously this would be unsuitable for high temperatures.)
c Make a hot-wire anemometer: one of the arms of the bridge is a thin wire made warm by the current through it. It cools in a current of air, changing its resistance.

B3 Make up an analogue computing circuit for one of the following:

a vertical motion under gravity, with means of starting the motion with any chosen values of height and of initial velocity;
b radioactive decay: A decays into B which decays into stable C;
c radioactive decay: A is being produced at a virtually steady rate from a substance with enormously long half-life. A decays into B which is also radioactive. Start from the point where all of A and B have been extracted from the mixture, and will begin to appear again.

B4 Make an oscillator in which the frequency is determined by an *LC* resonant circuit.

B5 Make a timing device for a photographic enlarger which will start when the lamp is switched on and will switch it off after a predetermined time (normally a few seconds).

B6 Make a capacitance meter which gives a direct reading on a meter scale.

B7 Make a system to control the temperature of something by controlling a heater.

B8 Make a system to control the temperature of a heated object by means of a fan which cools it.

B9 Make a linear variable transformer to display the position of its core as a reading on a d.c. meter, and possibly to control or measure something.

Two coils are set side by side, with the movable iron core (any suitable short rod) inside them, as shown in figure I48. Alternating current is supplied to one coil, and the position of the core determines the amplitude of the alternating e.m.f. induced in the other coil.

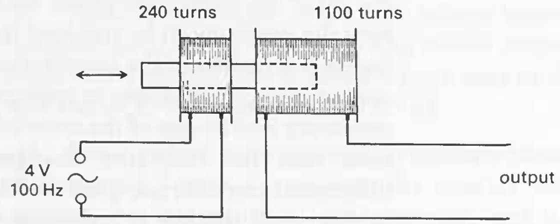


Figure I48
Linear variable transformer.

B10 Make a variable-reluctance transducer, in which very small movements of a piece of iron change an a.c. voltage. This in turn is made to change a d.c. current which is used to measure or control something.

B11 Use an operational amplifier to make an astable multivibrator (see question **38**, page 142) whose frequency depends upon one of the following:

a temperature
b illumination
c water level.

B12 Make a system to maintain a constant current in spite of variations in supply voltage.

QUESTIONS

Potential difference, current, and resistance

1(I)a Two resistors (R_1, R_2) are connected across a power supply giving a p.d. of V (figure I49). What is the potential at A (*i.e.* the potential difference between point A and 0 V)? (Assume no current through the voltmeter.)

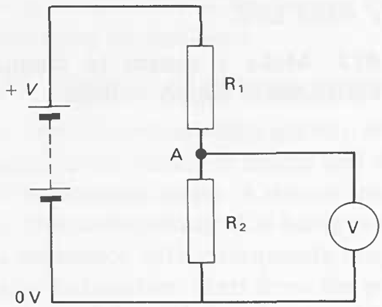


Figure I49

b Suppose $V = 10\,\text{V}$, $R_1 = 5\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$. What is the potential at A?

c Using the same values as in part **b**, calculate the current in the resistors.

d Suppose that R_1 is changed to $50\,\text{k}\Omega$ and R_2 to $100\,\text{k}\Omega$. What is the potential at A now? What is the current in the resistors?

2(I) What is the potential difference between the two ends of the resistor shown in figure I50?

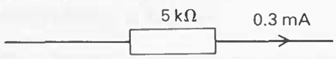


Figure I50

3(I) In figure I51 the current in each resistor is the same.

a What is the total p.d. across both resistors?

b What is the ratio of the p.d.s across the two (V_1/V_2)?

c Suppose the current is doubled. Will the values of **a** or **b** change?

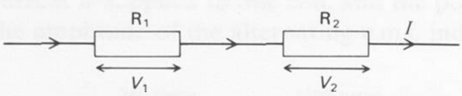


Figure I51

4(I) An operational amplifier with negative feedback has the remarkable property of holding the potential at a point in a circuit fixed at very nearly 0 V (earth), but *without any current flowing to or from earth* at that point. So in figure I52 point X is held at 0 V, although there is no current to earth at this point, and hence the current in the two resistors is the same.

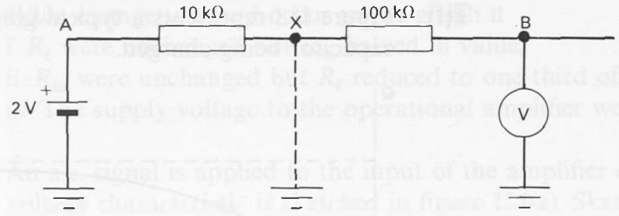


Figure I52

The potential at A is $+2$ V, *i.e.* there is a p.d. of 2 V between A and earth.

a What is the current in the 10 kΩ resistor?

b What is the current in the 100 kΩ resistor?

c What is the p.d. across the 100 kΩ resistor?

d Which end of the 100 kΩ resistor is at the higher potential, X or B?

e Is the potential at B positive or negative, *i.e.* is it above or below 0 V?

This circuit is sometimes compared to a see-saw. Point X is fixed, and if the potential at one end goes up the potential at the other goes down, and vice versa.

f Suggest resistor values that would make the changes at the two ends of the see-saw equal in magnitude, *i.e.* if A goes down by 2 V, B goes up by 2 V.

Capacitors

5(I) How much charge is stored on a 10 μF capacitor when the p.d. across it is

a 2 V?

b 10 V?

6(I) A steady current of 0.1 mA is used to charge a capacitor.

a How much charge goes on to the capacitor in 5 seconds?

b The p.d. across the capacitor is now found to be 0.75 V. What is the capacitance of the capacitor?

c Suggest how this p.d. might be measured. (Why would it not be a good idea to use a voltmeter?)

7(I) Figure I53 represents a typical graph of charge against time for a capacitor being charged.

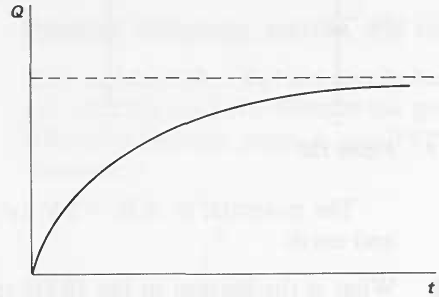


Figure I53

a Sketch a graph showing how the current varies with time as the capacitor charges up.

b What is the relationship between the two graphs?

Suppose the charging current is instead somehow maintained at a constant level.

c How will the charge on the capacitor and the p.d. across it vary with time?

Operational amplifier circuit characteristic

8(P) Say what you understand by the terms *inverting* and *limiting value* applied to an operational amplifier circuit.

9(P) Figure I54(a) shows a graph of V_{out} against V_{in} for the operational amplifier circuit shown in figure I54(b).

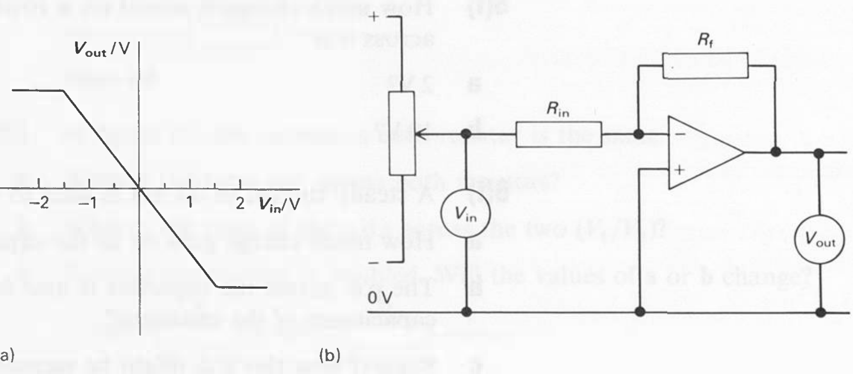


Figure I54

a The sloping part of the graph is straight, downwards to the right, and goes through the origin. What important facts does this indicate about the amplifier?

b The graph flattens at the ends. Describe accurately in words what this means about the way the output changes as the input is changed. (One sentence should suffice; two at the most.)

c What change(s) would occur in the graph if
i R_f were unchanged but R_{in} halved in value;
ii R_{in} were unchanged but R_f reduced to one third of its value;
iii The supply voltage to the operational amplifier were increased?

10(P) An a.c. signal is applied to the input of the amplifier circuit whose voltage characteristic is sketched in figure I54(a). Sketch the output signal you would expect if the input had a peak-to-peak voltage of

a 1 V

b 4 V

11(E) A student is confused about *amplifiers* and *transformers*. Both seem to do the same thing – turn small voltages into big ones. Explain what the differences are.

The operational amplifier

12(P) The very high gain of an operational amplifier and its very high input resistance allow two simplifications to be made about its behaviour. What are they?

13(L) *Optional* The gain of an operational amplifier is often quoted in dB (decibels). The arithmetical gain A (ratio of output to input voltage) is related to the gain in dB by

$$\text{gain (dB)} = 20 \log_{10} A$$

a Calculate the gain in dB for $A = 10, 20, 40, 80$. You should see that the decibel gain increases by the same amount every time A is multiplied by a constant factor.

b What is A if the gain is 100 dB?

c What is A if the gain is 106 dB? (Use your answer to part **a** – there is no need to use logarithms.)

14(P) An operational amplifier has a voltage gain of 10^5, and an input resistance of 1 MΩ. When the output voltage is +5 V

a What is the differential input voltage, the p.d. between non-inverting and inverting terminals $V_+ - V_-$?

b What will the input current be?

15(L) Question **14** showed that for output voltages V_{out} of a few volts, the p.d. between X and Y (figure I55, page 128) is a small number of microvolts, and the current taken by the operational amplifier is a small number of picoamperes. For a large range of uses to which the operational amplifier is put, these are regarded as negligible. In this question you are to assume that both are *zero*.

a In figure I55, Y is connected to 0 V. What is the p.d. between X and 0 V?

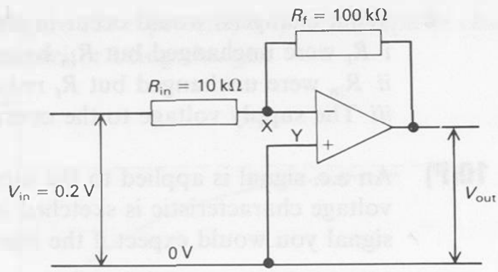


Figure 155

b What then is the p.d. across the $10\,k\Omega$ input resistor?

c What will be the current through this resistor?

d Since no current enters or leaves the operational amplifier at X, what is the current through the $100\,k\Omega$ feedback resistor?

e What then will be the p.d. across this resistor?

f V_{in} is $+0.2$ V, that is, positive with respect to 0 V. What is the sign of the actual potential V_{out}?

g If instead of the values given, R_{in} and R_f were $100\,k\Omega$ and $1\,M\Omega$ respectively, and the input voltage V_{in} were still 0.2 V, what would be the current through these resistors?

h What would the output voltage V_{out} be?

16(L) In this question you are to make the assumptions explained in the paragraph at the beginning of question **15**. The circuit concerned here is that of figure I56 in which the potentials marked are all with respect to 0 V.

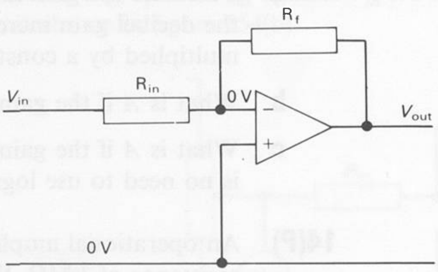


Figure I56

a In terms of the quantities given on the diagram, what is the *fall* in potential (left to right) across R_{in}?

b What then is the current (from left to right) in R_{in}?

c In terms of the quantities given on the diagram, what is the *fall* in potential (from left to right) across R_f?

d What therefore (in terms of these same quantities) is the current (from left to right) in R_f?

e Since no current enters or leaves the input of the operational amplifier, the two currents you have calculated are equal. Write this fact as an equation, using your answers to **b** and **d**, and rearrange the equation to give: $V_{out}/V_{in} = \ldots$

This question and the next two may help you to understand how certain operational amplifier circuits work. The key is to realize that the resistors connected around the operational amplifier can be seen as potential dividers. Question **17** is a preliminary one about potential dividers alone.

17(P) Resistances of 10 kΩ and 5 kΩ are connected to form a potential divider as shown in figure I57. A current of 0.2 mA flows through the resistors to earth.

a What are the potentials at A, B, and C?

A helpful semi-graphical way of portraying the above potential-dividing process is illustrated in figure I58, which might be called a 'potential ladder diagram'. Horizontal distances represent the size of the resistances, and potential is plotted vertically.

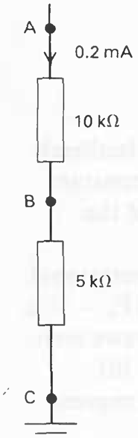


Figure I57

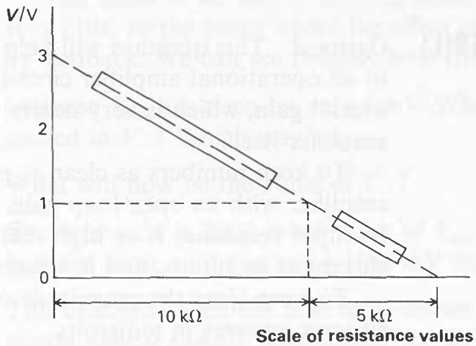


Figure I58

b Why must both 'resistances' be drawn with the same gradient on this diagram?

c *i* Draw a diagram of this sort to represent the potential divider of figure I59(a), and show on your diagram the values of the potentials at A and B.
ii Draw a diagram to represent the potential divider of figure I59(b).

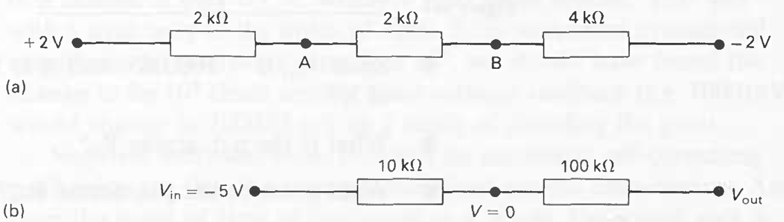


Figure I59

d On the same axes, illustrate what happens to the potentials in *ii* above, if the value of V_{in} is changed from −5 V to +3 V.

e Draw potential ladder diagrams to represent (using two diagrams on the same axes) the changes in potentials in the operational amplifier circuit of figure I60, under the following conditions:

i $V_{in} = +2\,V$, changing to $V_{in} = -1\,V$
ii $V_{in} = +2\,V$, and R_f is changed from $20\,k\Omega$ to $15\,k\Omega$
iii $V_{in} = +2\,V$, and both resistances are doubled.

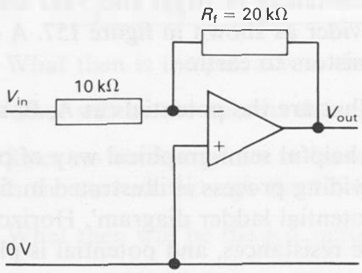


Figure I60

18(L) *Optional* This question will help you understand how the feedback in an operational amplifier circuit functions to maintain a constant overall gain, which is very nearly independent of the gain of the amplifier itself.

To keep numbers as clear as possible, we imagine an operational amplifier with an open-loop gain, A, of only 1000. $V_{out} = A(V_+ - V_-)$. Its input resistance is so high that we can assume that it draws zero current at its inputs, and it is connected as shown in figure I61.

To keep clear the magnitude of the quantities involved, express all your answers in millivolts.

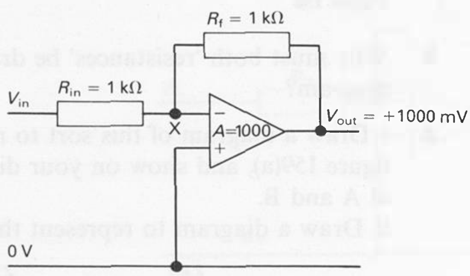


Figure I61

a Since V_{out} is $+1000\,mV$ and A is 1000, what is the potential (V_-) of X?

b What is the p.d. across R_f?

c What must be the p.d. across R_{in}?

d What then is the input potential V_{in}?

Now suppose some disturbance (perhaps a change in temperature) causes A to increase to 2000.

e If no other changes occurred in the circuit, so that V_- remained fixed at $-1\,mV$ by the feedback current, what would the value of V_{out} now be?

f If V_{out} *did* rise from $+1000\,mV$ to $+2000\,mV$, this in turn would raise the potential at X because of the feedback (V_{in} being fixed). By how much would this make V_- change? What would the new value of V_- be?

g Would this change in V_- be increasing the effect of the disturbance (*i.e.* raising the value of V_{out} even further), or would it be in the opposite direction?

h Would this change due to feedback be sufficiently large to compensate for the disturbance (*i.e.* bring the output back to $+1000\,mV$), or much too small, or much too large?

At this point you should have found that a 1000 mV rise in V_{out} would cause a rise in V_- of 500 mV, which is in the right direction to compensate for the disturbance (*i.e.* to bring V_{out} down again), but is much too big!

The result of all this is that V_{out} *starts* to rise, but can only rise very little, to the point where the effect of the disturbance is nullified by feedback. We can see roughly how this works as follows.

i Suppose V_{out} has risen by only 1 mV. What change will this have caused in V_-? Which way?

j What will now be the value of V_-?

k Since the gain is 2000, what value of V_{out} ought this to produce? Is it the same as the value $+1001\,mV$ we assumed it to have?

This calculation should now have shown that a rise of 1 mV in V_{out} would almost exactly compensate for the disturbance by raising V_-, the input, by 0.5 mV. (This gives the correct value for an output of 1000 mV, though it is not quite negative enough for an output of 1001 mV.) In fact, careful calculation shows that input and output will match the gain *and* compensate for the disturbance if V_- changes to $-0.500\,499\,5\,mV$ and V_{out} changes to $+1000.999\,mV$ (to 7 significant figures). The precise values are not important. The important thing is that a disturbance which without feedback would cause a 100 % change in output can, with negative feedback, result in a change of only 0.1 %, which is 1000 times smaller. This was with a gain only of the order of 1000. If, as with most operational amplifiers, the gain were more like 10^5, we should have found the change to be 10^5 times smaller than without feedback (*e.g.* 1000 mV would change to 1000.01 mV as a result of doubling the gain).

Negative feedback, then, provides an automatic self-correcting mechanism, so that the circuit is *stabilized* against disturbances. And from the point of view of the circuit as a whole, the *overall gain* is stabilized at a value depending virtually entirely on the values of R_f and R_{in}. If the input and feedback resistors are equal, then $V_{out} = -V_{in}$.

More uses of the operational amplifier

19(L) In the circuit shown in figure I62 (page 132) there are two input points, which are held at the potentials shown.

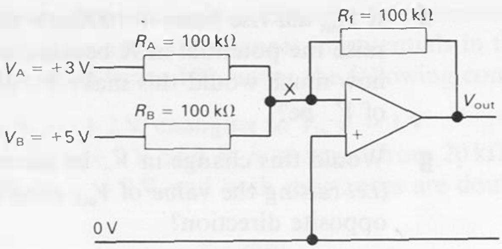


Figure 162

a What is the potential at X?

b What is the drop in potential (left to right) across R_A?

c What is the current (left to right) in R_A?

d What is the current in R_B?

e What is the value of the current leaving X?

f What must be the p.d. across R_f?

g What is the potential V_{out}?

h How is the output potential, therefore, related to the input potentials?

Suppose now that R_A is changed to $50\,k\Omega$, all the other values shown in the diagram remaining the same.

i Repeat the calculations above, and state how the output is now related to the inputs.

j What is now the output, if $V_A = 2\,V$, $V_B = 3\,V$?

Write down the relationship between output and input voltages in the following cases, and in each find the value of V_{out}, if $V_A = 0.3\,V$, $V_B = 0.5\,V$.

k $R_A = 50\,k\Omega$, $R_B = 50\,k\Omega$, $R_f = 100\,k\Omega$.

l $R_A = 100\,k\Omega$, $R_B = 100\,k\Omega$, $R_f = 1\,M\Omega$.

20(L) Consider the circuit in figure 163.

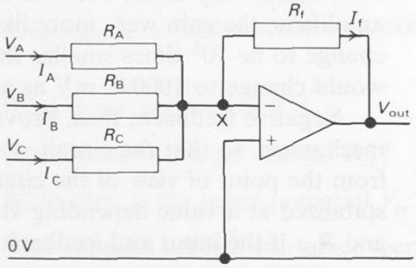


Figure 163
Summing amplifier.

a What is the potential at X?

b What is I_A (in terms of V_A, R_A)? Obtain similar expressions for I_B and I_C.

c Write down an expression relating I_f to I_A, I_B, and I_C.

d Express I_f in terms of R_f and V_{out}, and use this expression together with your answer to **c** to express the output potential in terms of the input potentials and the resistances in the circuit (be careful to get the sign right).

e What can you say about the values of R_A, R_B, R_C, and R_f if the circuit is to be used to add three potentials?

21(L) *Optional* This question is about one form of digital-to-analogue converter, that is, a circuit which will convert information in binary code (*e.g.* five is represented by 101, six by 110, etc.) into a voltage proportional to the number. Modern computers handle data in digital form, so if the computer output is to be displayed on a meter it will need to be converted to an analogue signal. For the reverse process, analogue-to-digital conversion, see question **22**.

Digital-to-analogue conversion can be accomplished using the summing amplifier as discussed in questions **19** and **20**. In the circuit shown in figure I64, each input can only be either 'high' or 'low'; typically, 'high' means $+5$ V, 'low' means 0 V. Binary numbers are represented by the state (high or low) of the inputs, for example as follows: for binary 101, V_A is high, V_B is low, and V_C is high.

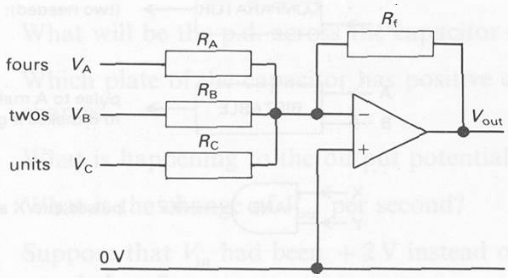


Figure I64

If only V_C is high ($+5$ V), this is required to give an output of -1 V.

a Express R_C in terms of R_f.

If only V_B is high, this is required to give an output of -2 V.

b Express R_B in terms of R_f.

If only V_A is high, this is required to give an output of -4 V.

c Express R_A in terms of R_f.

d Suggest suitable numerical values for the four resistances if this system is to work.

e You need to extend the circuit to cope with *six* binary digits. However, 111111 is denary 63, and your operational amplifier saturates at about -13 V. You therefore need to arrange that one unit is represented in the output as -0.1 V. Suggest suitable values for all the resistances.

22(R) *Optional* Digital-to-analogue conversion is easily done using an operational amplifier, as in question **21**. The opposite, analogue-to-digital conversion, is needed for example in digital display meters, and in interface units for feeding the values of real voltages into a computer. There are several systems, and one, voltage-to-time conversion, works as follows. A ramp generator provides a voltage which rises steadily from zero, repeatedly. This is compared with the potential of the negative terminal of the analogue input, and then with the potential of its positive terminal.

In each case a pulse is sent out when the compared voltages are equal. One pulse starts a binary counter driven from a 1 MHz clock, the other stops it. The resulting reading is held until the next cycle of the ramp generator.

a Use the units described in figure I65 to make a block diagram of this system.

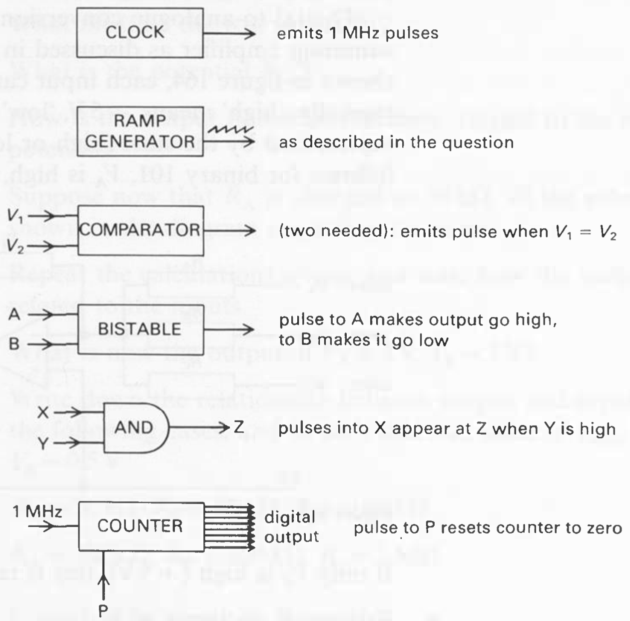


Figure I65

b If this system were used for a digital display meter, how would the display actually behave? What modification or specification would be needed to ensure that it was easily readable?

23(R)a Design a circuit using operational amplifiers to give an output voltage $V_{\text{out}} = +(V_A + 10V_B)$, where V_A and V_B are two input voltages.

b Design a circuit using operational amplifiers whose output $V_{\text{out}} = 10V_A - 3V_B$.

24(L) Consider the circuit shown in figure I66.

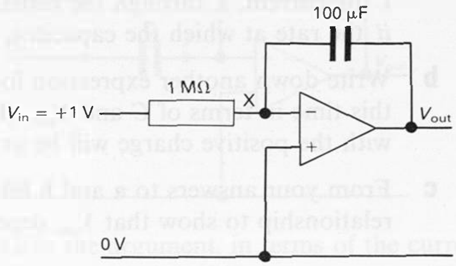


Figure I66

a What is the p.d. across the 1 MΩ resistor?

b What current will flow in it?

c The + 1 V input is kept constant by the input source, and the potential at X is kept at zero by the properties of the operational amplifier. The current must therefore remain constant. Where must it flow to?

d Will there be any current on the righthand side of the capacitor?

e How much charge will there be on the capacitor plates after 10 s? After 20 s?

f What will be the p.d. across the capacitor after 10 s? After 20 s?

g Which plate of the capacitor has positive charge, and which negative charge?

h What is happening to the output potential V_{out}?

i What is the change of V_{out} per second?

j Suppose that V_{in} had been + 2 V instead of + 1 V. How would V_{out} now behave?

k Sketch a graph showing how V_{out} would change if V_{in} were + 1 V for 2 s, then + 3 V for 0.5 s, then − 2 V for 1 s.

l If in the circuit the capacitor had been 10 μF instead of 100 μF, what difference would this have made to the behaviour of V_{out}?

25(L) This question is about the circuit shown in figure I67 whose output voltage is the integral of the input voltage multiplied by a constant.

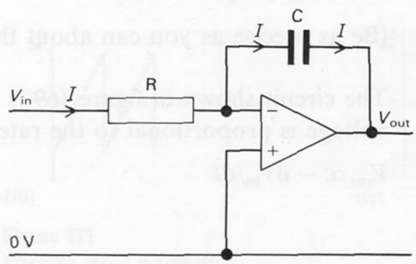


Figure I67
Integrating circuit.

a Write down expressions in terms of V_{in} and R for:
i the current, I, through the resistor, R;
ii the rate at which the capacitor, C, is charged.

b Write down another expression for the rate at which C is charged, this time in terms of C and V_{out}. (Remember that the capacitor plate with the positive charge will be at the higher potential.)

c From your answers to **a** and **b** relate V_{out} to V_{in}, C, and R. Use this relationship to show that V_{out} depends on the time integral of V_{in}.

26(R) The circuit of figure I67 (page 135) performs the operation:

$$V_{out} = -\frac{1}{CR}\int V_{in}\,dt + \text{constant}$$

You may sometimes find it easier to use the alternative statement:

$$\frac{dV_{out}}{dt} = -\frac{1}{CR}V_{in}$$

If $R = 1\,\text{M}\Omega$ and $C = 1\,\mu\text{F}$, sketch how the output varies with time for each of the following inputs. Assume V_{out} at the start is 0 V in each case.

a V_{in} is constant $= 0.5$ V.

b V_{in} is a 'square wave' between $+2$ V and -2 V, of frequency 20 Hz. See figure I68(a).

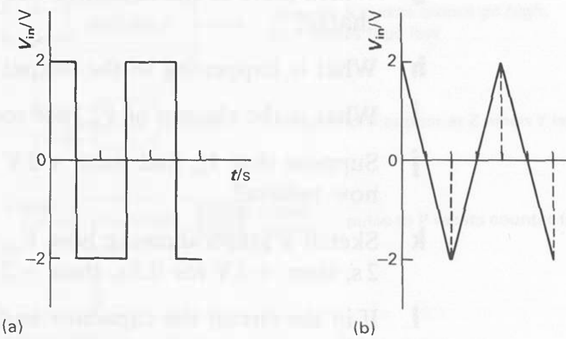


Figure I68

c V_{in} is a 'triangular wave' between $+2$ V and -2 V, of frequency 0.5 Hz. See figure I68(b).

(Be as precise as you can about the shape of the graph of V_{out}.)

27(L) The circuit shown in figure I69 is a differentiating circuit; the output voltage is proportional to the rate of change of the input voltage:

$V_{out} \propto -dV_{in}/dt$

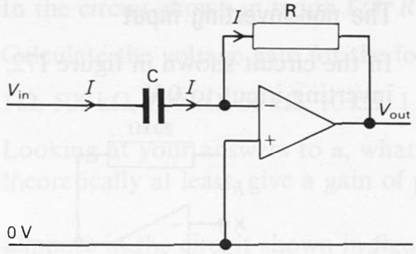


Figure I69
Differentiating circuit.

a Give the argument, in terms of the current, I, showing why V_{out} and V_{in} are related in this way.

b Various *output* waveforms of a differentiating circuit are shown in figure I70. For each of them pick the appropriate 'ideal' input from figure I71.

c Why is the word 'ideal' used in **b**?

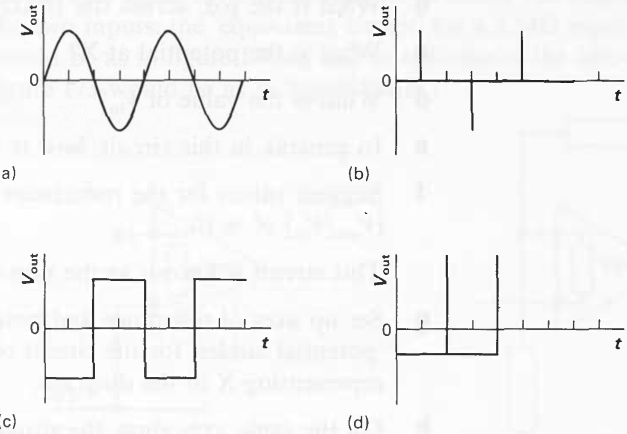


Figure I70
Output waveforms from differentiating circuit.

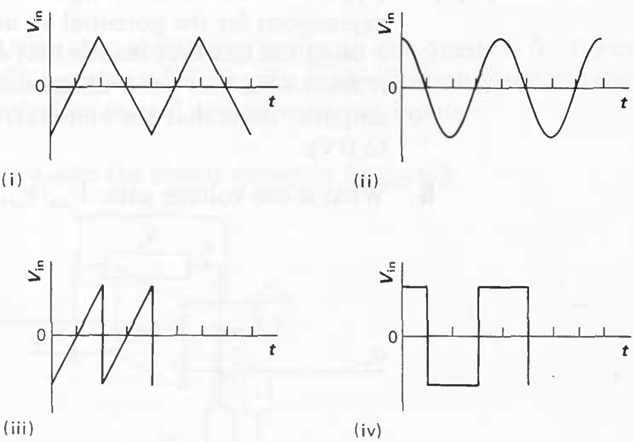


Figure I71
Possible input waveforms.

The non-inverting input

28(L) In the circuit shown in figure I72, the 10 kΩ resistor connects the inverting input to 0 V.

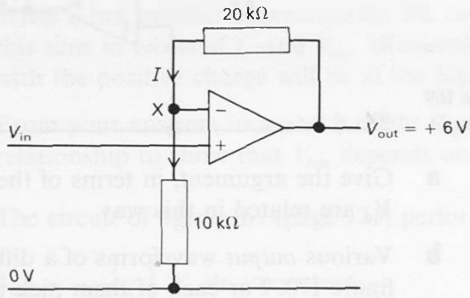


Figure I72

a What is the value of the current I?

b What is the p.d. across the 10 kΩ resistor?

c What is the potential at X?

d What is the value of V_{in}?

e In general, in this circuit, how is V_{out} related to V_{in}?

f Suggest values for the resistances which would give an overall gain (V_{out}/V_{in}) of + 10.

This circuit is known as the non-inverting amplifier.

g Set up axes of resistance and potential as in question **17**, and draw a 'potential ladder' for the circuit of figure I72. Mark on it the point representing X in the diagram.

h On the same axes show the situation if V_{in} is reduced to half its original value.

29(L)a For the circuit shown in figure I73 obtain two independent expressions for the potential V_X at X:
i using the fact that resistors R_1 and R_2 form a potential divider;
ii from what you know about the input potentials of an operational amplifier (note that the non-inverting input is not connected directly to 0 V).

b What is the voltage gain, V_{out}/V_{in}, of this circuit?

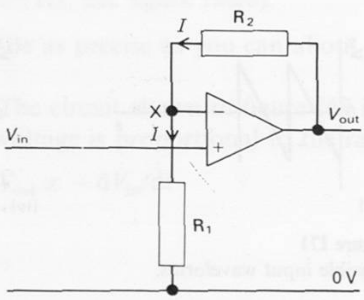


Figure I73

30(L) In the circuit shown in figure I73, R_1 is 100 kΩ.

a Calculate the voltage gain for the following values of R_2:

1 Ω, 500 kΩ, 100 kΩ, 50 kΩ, 10 kΩ, 1 kΩ.

b Looking at your answers to **a**, what value of R_2 (or of R_1) would, theoretically at least, give a gain of precisely 1?

31(P) Suppose in the circuit shown in figure I73 that $V_{in} = 1.5$ V and $R_1 = R_2$.

a Calculate V_{out}.

b Calculate I, if $R_1 = R_2 = 100$ kΩ.

(*Note:* in the circuits you studied before this one, the non-inverting input was connected to 0 V. You learned that the input resistance is very high, so the input current to the operational amplifier is very small. This input current actually flows inside the amplifier between the two inputs: the 'equivalent circuit' for a 2 MΩ input resistance would be as in figure I74(a), and in the case of the above circuit figure I73 would be as in figure I74(b).)

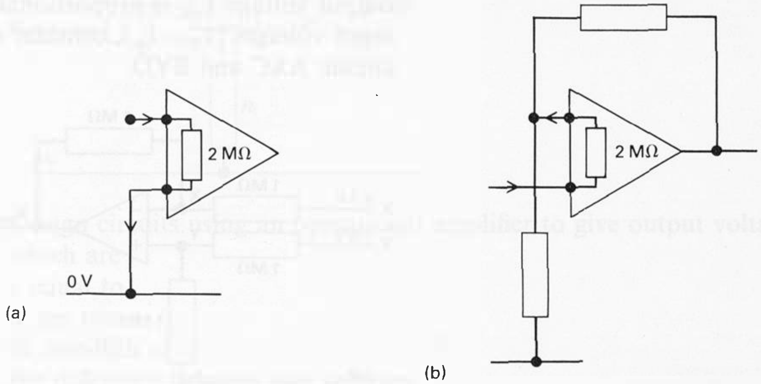


Figure I74

c Using the values given earlier in this question for the circuit of figure I73 together with the typical specification given in table I1 (page 90), calculate the p.d. between the two inputs.

32(P) Consider the circuit shown in figure I75.

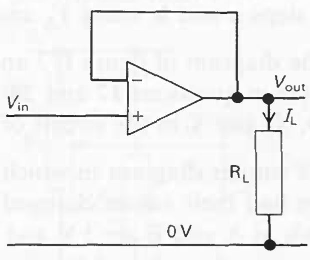


Figure I75
Unity-gain voltage follower.

a How are V_{out} and V_{in} related in this circuit?

b Obtain a relationship between the input voltage, V_{in}, and the current I_L delivered to the load resistor R_L.

c Explain, without calculation, why the input current to the amplifier is very much less than this.

33(P) Draw an operational amplifier circuit whose output is a (measured) current which is proportional to an input voltage. The input *voltage* is in the range -1 to $+1$ V, but the source cannot supply more than 1 μA. The output current should be in the range -1 to $+1$ mA.

34(P) Design an operational amplifier circuit which gives a measured voltage output which is proportional to the current drawn from a source. The input current can vary between -10 and $+10$ mA; the output voltage should be in the range -1 to $+1$ V. Suggest suitable values for components in the circuit.

35(L) Figure I76 shows how an operational amplifier can be used in a differential amplifier circuit. To see how it comes about that the output voltage V_{out} is proportional to the difference between the two input voltages ($V_B - V_A$), consider the following two paths in the circuit: AXC and BYD.

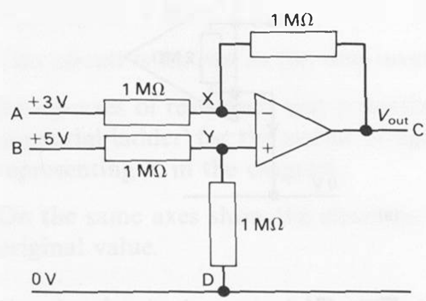


Figure I76

a *i* Using the path AXC, express the potential at X (V_X) in terms of V_{out}.
ii Using the path BYD, write down the value of the potential at Y (V_Y).

b Write an expression stating that V_X and V_Y must be the same, and rearrange it in the form $V_{out} = \ldots$

c Repeat steps **a** and **b**, using V_A and V_B in place of $+3$ V and $+5$ V.

d Copy the diagram of figure I77 and on it sketch the 'potential ladder' (as in questions **17** and **28**) to represent the potentials at A, B, C, D, X, and Y in the circuit of figure I76.

e Sketch a similar diagram in which the two input resistors AX and BY have had their values changed to 0.5 MΩ each, and the potentials at A and B are 1 V and 3 V respectively. Deduce from your diagram the value of V_{out}.

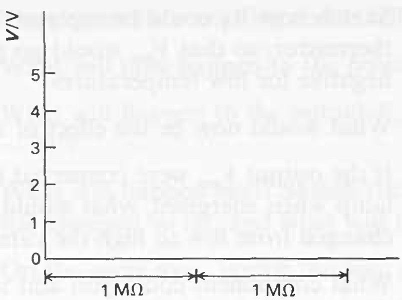


Figure I77

f *Optional* Use the generalized values shown in figure I78 (in which two resistors have the value R_{in} and two have the value R_f) to show algebraically that

$$V_{out} = -\frac{R_f}{R_{in}}(V_A - V_B).$$

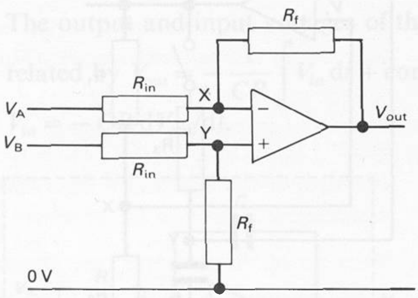


Figure I78

36(R)a Design circuits using an operational amplifier to give output voltages which are
i equal to
ii ten times
iii one-fifth of
the difference between two voltages.

b Suggest a situation in which *ii* would be more useful than *i*.

37(L) The circuit of figure I79 is known as a comparator. What is the output V_{out}:

a if V_+ is greater than V_-?

b if V_+ is less than V_-?

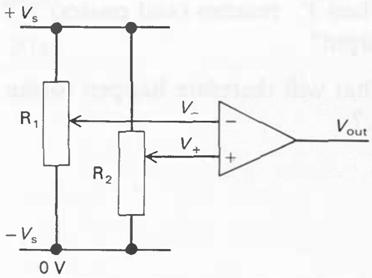


Figure I79
The comparator.

c Sketch how R_1 could be replaced by a fixed resistor and a thermistor, so that V_{out} would go positive for high temperatures and negative for low temperatures.

d What would now be the effect of adjusting the setting of R_2?

e If the output V_{out} were connected to a relay which would switch on a lamp when energized, what would happen as the temperature changed from low to high (be careful!)?

f What component could you add to the circuit (and where) so that the lamp was on for high temperatures and off for low temperatures?

38(L) *Optional* In the circuit of figure I80, R_1 and R_2 are equal resistors. Suppose the output of the operational amplifier is at +6 V, the limiting value due to the power supply.

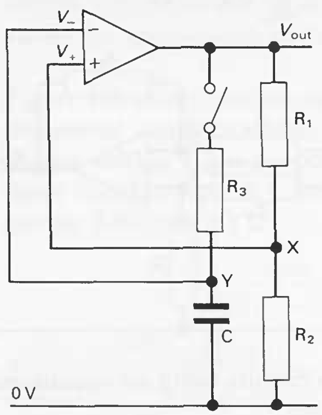


Figure I80

a What is the potential at X (and therefore V_+)?

Suppose C is not charged.

b What is the potential at Y (and therefore V_-)?

c Are these potentials V_+ and V_- such as to keep the output at the limiting value?

Now suppose the switch is closed.

d What will happen to the potential at Y? What will happen to V_-?

e When V_- reaches (and passes) +3 V, what will happen to the output?

f What will therefore happen to the potential at X and therefore to V_+?

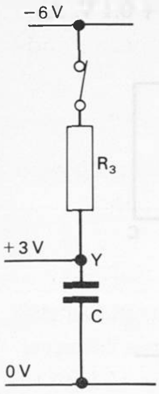


Figure I81

The potentials across R_3 and C will now be as shown in figure I81.

g What will now happen to the potential at Y?

h What will happen to the output V_{out} when this potential reaches -3 V?

i What will happen next? Sketch the way in which the potential of Y has changed so far, and what will happen from this point onwards.

j On the same axes, sketch the way in which V_{out} varies.

k What difference would it make if R_2 were decreased, so that X were set at ± 1 V instead of ± 3 V? Sketch another graph to illustrate this.

This circuit is an *astable multivibrator*, and its frequency can be changed by changing the value of any of the resistors.

Integrators with feedback

39(L) The output and input voltages of the circuit (figure I82) are related by $V_{out} = -\dfrac{1}{CR}\int V_{in}\,dt + \text{constant}$, or, alternatively $V_{in} = -CR\,dV_{out}/dt$.

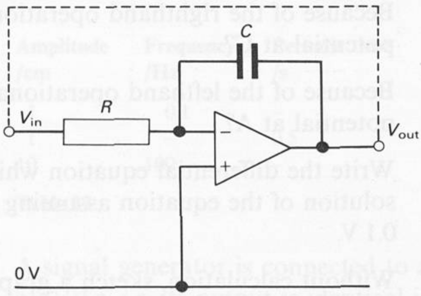


Figure I82

Suppose the output and input are connected by a piece of wire.

a What is the relationship between V_{out} and V_{in} now?

b What differential equation will the circuit now solve, and what is the solution?

Suppose $R = 1\ \mathrm{M\Omega}$, $C = 10\ \mu\mathrm{F}$, and the capacitor is initially charged to 5 V. What is the output voltage after

c 10 s?

d 20 s?

e 100 s?

40(P) In the circuit of figure I83 the potential of B is initially $+0.1$ V. At this instant,

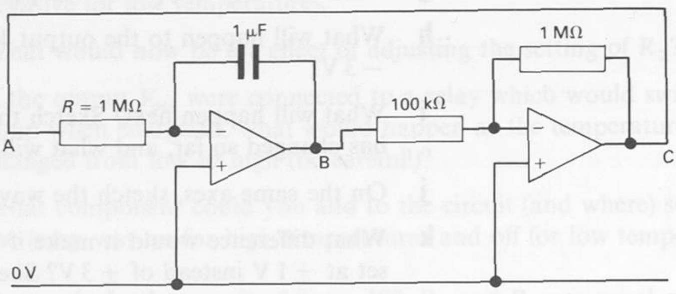


Figure I83

a what is the potential at C?

b what is the potential at A?

c what current will flow in R, and which way?

d what effect will this have on the capacitor, and what will therefore happen to the potential at B? (Numerical answers required.)

Assuming the same values as in the diagram, suppose that (at some time t) the potential at B is V.

e Because of the righthand operational amplifier, what must be the potential at C?

f Because of the lefthand operational amplifier, what must be the potential at A?

g Write the differential equation which this system solves, and the solution of the equation assuming that the value of V at $t = 0$ is 0.1 V.

h Without calculation, sketch a graph showing in general terms how you would expect V to change with time.

i How long will it take for V to reach 3 V?

41(R) In a single operational amplifier integrating circuit with resistance R and capacitance C, the output potential V_{out} is related to the input potential V_{in} by: $dV_{out}/dt = -(1/RC)V_{in}$.

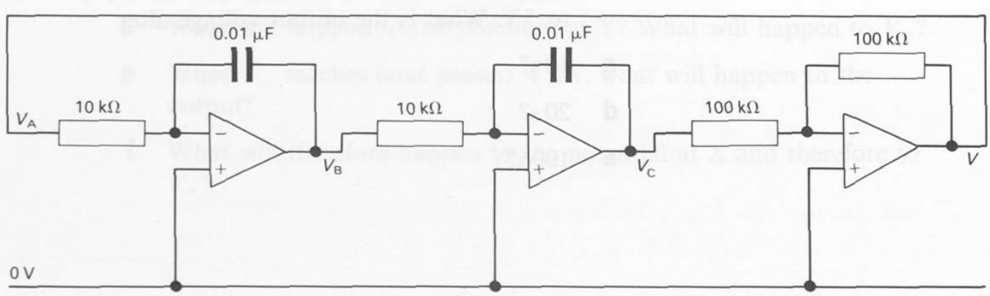


Figure I84

a What differential equation is represented by the circuit of figure I84?

b At what frequency will it oscillate?

42(I) The graph of figure I85 illustrates two oscillations, each with amplitude 2 cm and frequency 10 Hz, with a time lag between one and the other of 0.01 s, representing a phase difference of 1/10 cycle, or 0.2π.

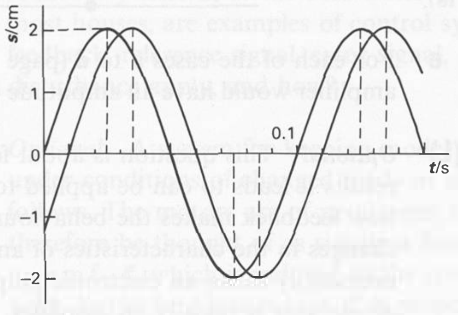


Figure I85

Sketch similar pairs of oscillations as follows, and in each case state the values not given in table I2.

Amplitude /cm	Frequency /Hz	Periodic time /s	Time lag /s	Phase difference
1	0.1	–	2.5	–
1	–	0.5	–	$\pi/2$ ($\frac{1}{4}$ cycle)
10	100	–	0.005	–

Table I2

43(R) A signal generator is connected to a circuit which produces a delay of 0.01 s, *i.e.* its output is identical in amplitude to its input but occurs 0.01 s later (figure I86).

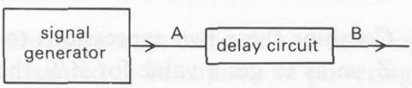


Figure I86

What will be the phase difference between the input and output of the delay circuit if the frequency provided by the signal generator is

a 100 Hz?

b 50 Hz?

c 25 Hz?

d 200 Hz?

Suppose the input signal (at A in figure I86) and the delayed signal (at B) both have an amplitude (peak value) of 1 V. The points A and B are then connected to a summing amplifier as shown in figure I87.

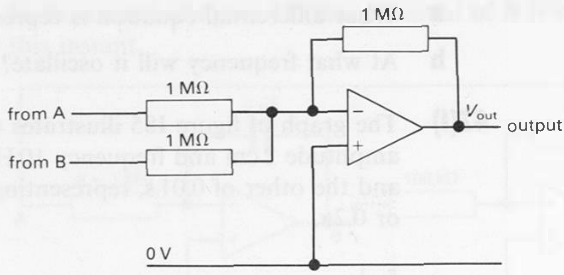


Figure I87

e For each of the cases **a** to **d** (page 145) say whether the output of the amplifier would have an amplitude of 0 V, 2 V, or something in between.

44(L) *Optional* This question is about feedback systems in general. The results it leads to can be applied to many kinds of system. It shows how feedback makes the behaviour of the system insensitive to changes in the characteristics of an amplifier. ('Amplifier' need not necessarily mean an electronic amplifier: a car engine with an accelerator is equally an amplifier, since the small human input force controls the large output force of the engine.)

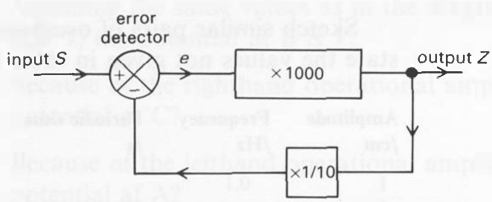


Figure I88

In the closed loop system of figure I88, one-tenth of the output of the amplifier is fed back to the error detector. This is subtracted from the input, S, giving the error, e, as input to the amplifier.

a Write down two expressions for the value of e
i in terms of Z and the gain of the amplifier, and
ii in terms of S and the feedback from Z.

b Combine these two expressions to obtain a relation between S and Z, so as to get a value for Z/S, the overall gain of the system.

c Repeat the calculation, changing the amplifier gain to 10 000.

Figure I89 shows the same system in general terms, with amplification, A, and feedback fraction, β. (β is a number *less* than 1, and the output is *multiplied* by β to give the feedback.)

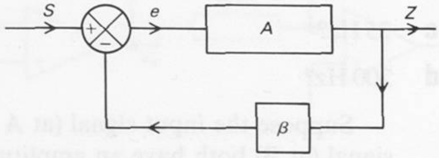


Figure I89

d Using the same method as above, obtain an expression for the overall gain, G, of the system ($G = Z/S$).

e Use the assumption that A is very much bigger than $1/\beta$ (or $1/A$ is very much less than β, or $A\beta$ is very much bigger than 1) to show that your expression can be written to a close approximation without A in it. What does it say about the overall gain?

Control systems

45(P) A lavatory cistern, or the cold water tank to be found in the loft of most houses, are examples of control systems. Which of the terms feedback, reference signal, error signal, input, output, and disturbance apply, and how?

46(R) *Optional* A system for keeping model trains to a constant speed under conditions of changed loads or changing gradients works as follows. The motors are of permanent magnet type, and can therefore be thought of as simply a fixed resistance, R, in series with an e.m.f., $\mathscr{E}$, which is induced in the rotor as it turns in the magnetic field. As the field is constant, $\mathscr{E}$ is proportional to the rate of rotation.

a Show that in the circuit of figure I90 the p.d. between A and B is $\mathscr{E}/2$.

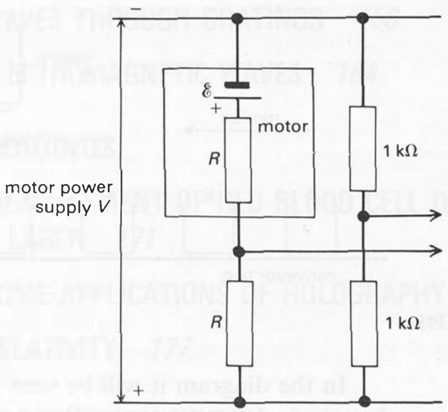


Figure I90

b Devise a way, using one or more operational amplifiers (one can be assumed to be capable of powering the motor), of using the result of **a** to maintain a steady speed.

47(P)a A 'robot' traffic light system controlled by sensors in the road which are activated by approaching cars is an example of a system in which the feedback needs some 'processing' before it is used. Explain, with some detail.

b Describe (or invent) some other system involving the processing of feedback.

48(P) Here are two problems about possible feedback in driving a car.

a If after travelling straight, a car veers sideways due to road irregularities or a gust of wind, the driver's body tends to move sideways relative to the car. Why?

b This movement may be transmitted to the steering wheel through the driver's arms. Consider two possibilities:
i the driver is holding the wheel at points above the axis of the wheel ('2 o'clock and 10 o'clock'), and
ii he is holding it below the axis.
For each, consider what might be the effect on the path of the car.

c Will the effects be the same or opposite, if the car is reversing?

d If a car accelerates or decelerates, the driver's body, including his accelerator foot, tends to move forwards or backwards relative to the car. Explain how this might account for the fact that a learner driver, when reversing, may cause the car to move in jerks ('kangaroo-hopping'). What might determine the frequency of the jerks?

49(P) In a factory, jars on a conveyor belt are each supposed to be filled with 100 g of powder. Each jar passes over a sensor (figure I91). If the mass is, for example, 1 g low, a correction is immediately made to the valve of the hopper causing it to deliver 1 g more; other corrections are proportional.

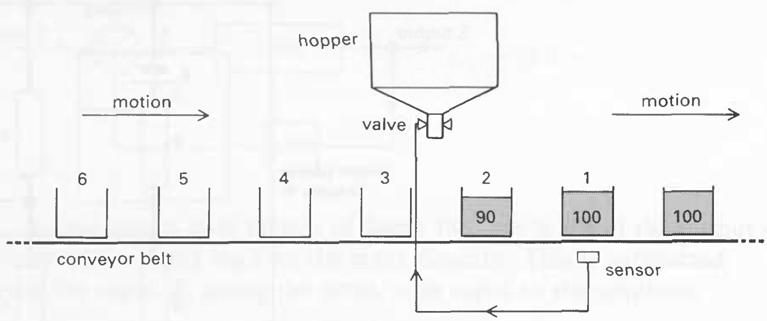


Figure I91

In the diagram it will be seen that starting with jar number 2, the hopper is delivering only 90 g.

a What amounts will be delivered to jars 3 to 11?

b How might the system be modified to improve its performance?

50(E) The term 'self-fulfilling prophecy' is used to describe the idea that the very fact that people believe a certain event is likely and act accordingly makes the event itself more likely to happen. For example, a completely unfounded rumour that sugar is in short supply can cause a shortage of sugar in the shops; if nervous investors think that a particular bank is likely to crash, their actions may help this happen. Describe how these effects happen, in these and other examples you can think of. Is the concept of feedback relevant? Of course there are many situations in which human belief and behaviour will not affect the course of events. Give examples. What distinguishes these from the examples of 'self-fulfilling prophecy'?

Unit J
ELECTROMAGNETIC WAVES

Stephen Borthwick
Marlborough College

Peter Bullett
Rugby School

J

SUMMARY OF THE UNIT

SUMMARY OF THE UNIT

INTRODUCTION

Much of our information about the outside world comes to us in the form of waves: sound, light, and radio waves are particularly important. However we detect them, the waves are inevitably restricted by some sort of aperture, such as the ear, the pupil of the eye, or the dish aerial of a radio telescope. Whenever waves pass through an aperture, they are diffracted and spread out. The effect depends on the size of the aperture in relation to the wavelength. This is why radio telescopes, which use radiation with wavelengths about a million times greater than the wavelength of visible light, have to be much bigger than optical telescopes. Diffraction patterns can be predicted using the principle of superposition (see Unit D, 'Oscillations and waves').

Diffraction limits the amount of information we can obtain, and leads to the limited resolving power of the eye and instruments such as the telescope. But diffraction effects are useful too. For example, we use diffraction gratings to gain knowledge about the waves themselves, and hence about the sources emitting them, as in the study of spectra (taken up again in Unit L, 'Waves, particles, and atoms'). Or, if we know the wavelength of the radiation, we can learn about the structure of the object causing the diffraction, as in X-ray crystallography.

Holography is an application of the principle of superposition which has increasing importance as a means of information storage.

This Unit is specifically concerned with electromagnetic waves. All parts of the electromagnetic spectrum, from the longest radio waves, through visible light, to gamma rays, have properties in common. In the last section of this Unit ideas about electric and magnetic fields developed earlier in the course are brought together to suggest an explanation of the nature of electromagnetic radiation itself.

Section J1 WAVES THROUGH AN APERTURE

Single aperture experiments

QUESTIONS 1, 2

EXPERIMENT J1
Looking through a slit and through a pin-hole

The two most obvious features of diffraction at a single aperture (figure J1) are that the waves spread into the region normally expected to be in shadow, and that within this region there are maxima and minima of intensity.

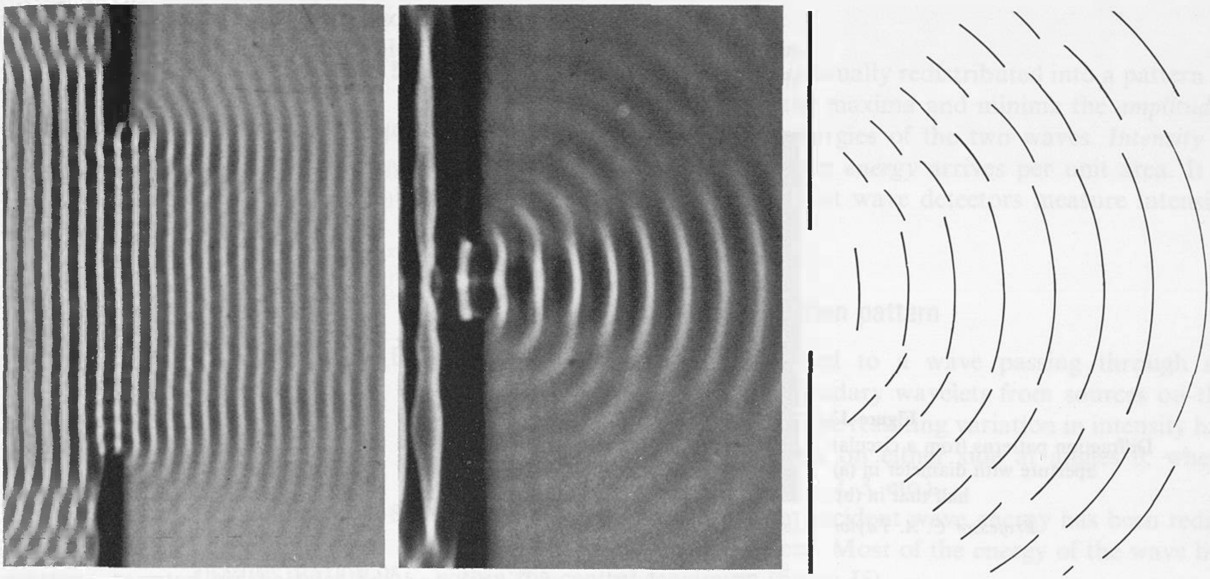


Figure J1
Photographs showing the diffraction of plane waves at a wide and a narrow gap. The sketch on the right is a diagrammatic representation of the photograph next to it. Note the phase differences between the fan-like portions.
PSSC Physics. *2nd edition, 1965; D.C. Heath and Company with Educational Development Centre Inc., Newton, Mass.*

CIRCUS OF EXPERIMENTS J2
Diffraction

HOME EXPERIMENT JH1
A homemade slit

The amount of spreading depends on the ratio of the slit width, b, to the wavelength, λ. If b is large compared with λ then there is little spreading; but if b is small compared with λ then there is much more spreading.

In a single-slit diffraction pattern, the first minimum of intensity is at an angle θ from the central maximum where $\sin\theta = \lambda/b$; so $\theta \approx \lambda/b$ is a good approximation for small values of θ.

There are further minima at angles 2θ, 3θ, etc. on both sides of the central maximum, thus

$$\sin\theta = n\frac{\lambda}{b}$$

where n is 1, 2, 3, ... The central fringe of the diffraction pattern is twice as wide as the equally-spaced fringes on either side. The fringes become less intense with increasing θ (see figure J2).

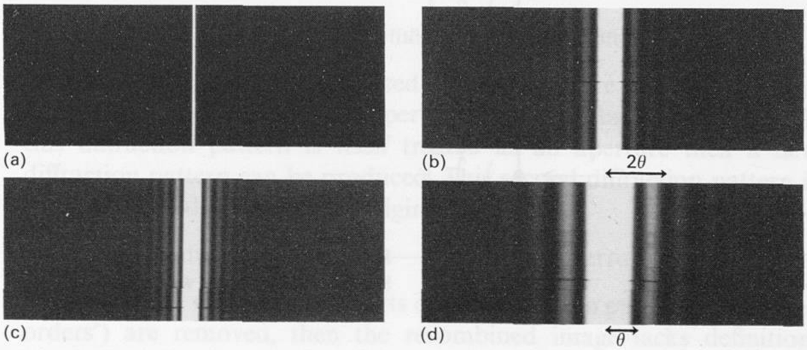


Figure J2
Single-slit diffraction patterns for light of one wavelength using slits of width $b \gg \lambda$ in (a); width $b \approx 5\lambda$ in (b); width $b \approx 3\lambda$ in (c); and width $b \approx \lambda$ in (d).
G. R. Graham, Cambridgeshire College of Arts and Technology

If the aperture is circular, the pattern is a set of concentric rings whose spacing is about 20 per cent bigger than the single-slit pattern for the same b and λ (figure J3).

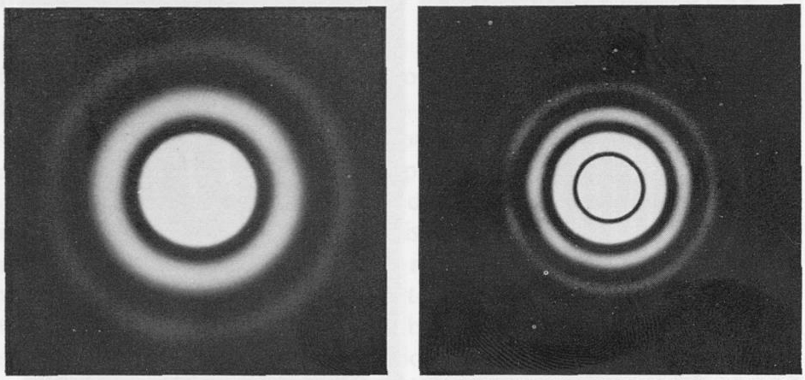

(a) (b)

Figure J3
Diffraction patterns from a circular aperture with diameter in (a) half that in (b).
Professor C. A. Taylor

The production of fringes and the relationship between aperture size, wavelength of radiation, and the intensity pattern of the spread wave can be explained in terms of the superposition of waves. First, however, it is necessary to show that a single aperture acts as a multiple source of waves.

Huygens's construction

EXPERIMENT J3
Huygens's construction

QUESTIONS 3, 4

The Dutch scientist Christiaan Huygens (1629–1695) suggested that every point on a wavefront behaves as a point source of waves sending out energy in the direction of the wave's propagation. To find the new wavefront he used the principle of superposition to add the contributions from these sources on the original wavefront (figure J4). This hypothesis can be used to predict the spreading of waves in a ripple tank. Huygens's construction helps us to understand many aspects of wave behaviour, including the laws of reflection and refraction for waves in any medium, the single-slit diffraction pattern, the diffraction grating, X-ray diffraction, and holography. It is an excellent example of a simple model which can increase our understanding of a wide range of related phenomena.

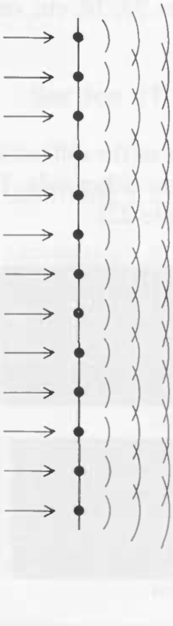


Figure J4
Huygens's secondary wavelets.

Wave amplitude and energy

QUESTION 5

DEMONSTRATION J4
Wave amplitude and energy
Intensity $\propto$ (amplitude)^2

Energy of harmonic oscillator $\propto$ (amplitude)^2
(Unit D, 'Oscillations and waves')

When waves superpose energy is usually redistributed into a pattern of maxima and minima. To find the maxima and minima the *amplitudes* are added together – *not* the energies of the two waves. *Intensity* is defined as the rate at which wave *energy* arrives per unit area. It is proportional to (amplitude)^2. Most wave detectors measure intensity rather than amplitude.

Explaining the single-slit diffraction pattern

QUESTION 6

QUESTIONS 7, 8

DEMONSTRATION J5
Measuring a diffraction pattern

Huygens's construction is applied to a wave passing through an aperture by superposing the secondary wavelets from sources on the plane wavefront at the aperture. The resulting variation in intensity has a central maximum with minima on either side at angles θ, where $\sin\theta = n\lambda/b$ and n is an integer (1, 2, 3, etc.).

The construction shows that incident wave energy has been redistributed into a diffraction pattern. Most of the energy of the wave lies within the central maximum (figure J5).

It is not possible to predict the position or intensity of maxima (which do *not* occur midway between minima) using the simple theory. However, a phasor treatment, in which the amplitude and relative phase of the secondary wavelets are represented by rotating vectors, can be used to predict the intensity pattern shown quantitatively in figure J5.

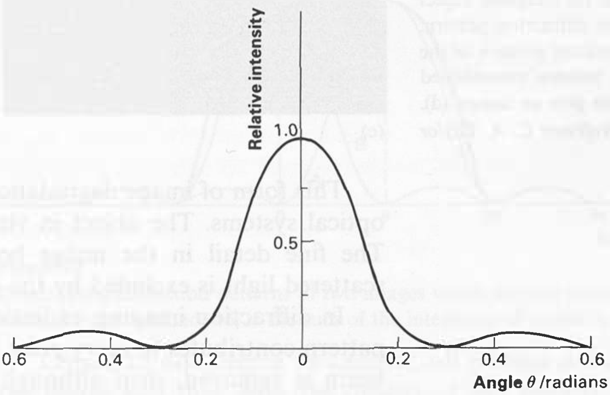


Figure J5
Single-slit diffraction pattern drawn for the case where $b \approx 3\lambda$.

Diffraction at an aperture and image recombination

DEMONSTRATION J6
Diffraction and image recombination

QUESTIONS 9, 10

The diffraction pattern produced by an aperture contains all the information necessary for the aperture's shape to be reconstructed. If this diffraction pattern is itself treated as an aperture then a new diffraction pattern can be produced. This second diffraction pattern is the recombined image of the original aperture.

aperture $\longrightarrow$ diffraction pattern $\longrightarrow$ image of aperture

If the more widely spread parts of the diffraction pattern (the 'higher orders') are removed, then the recombined image lacks definition.

Sharp corners and other fine details vanish progressively as less of the pattern is allowed to contribute to the image (figure J6).

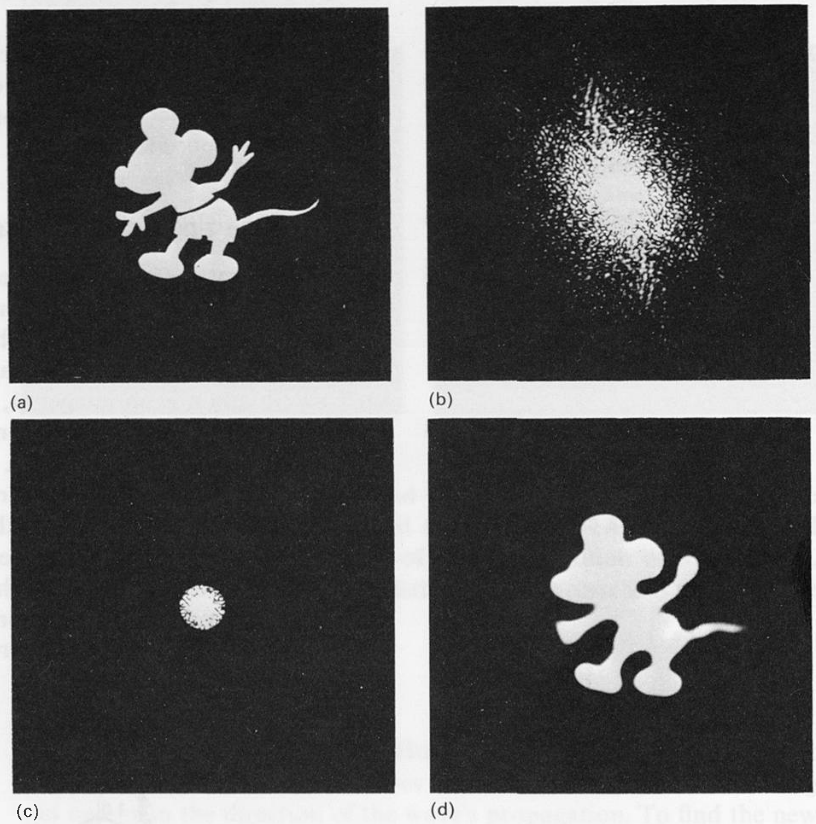


Figure J6
Image recombination. (a) Irregular object aperture. (b) Its diffraction pattern. (c) A restricted portion of the diffraction pattern, recombined to give an image, (d).
Professor C. A. Taylor

This form of image degradation is very important in the design of optical systems. The object in view scatters light in many directions. The fine detail in the image becomes poorer as the more widely scattered light is excluded by the restricted aperture.

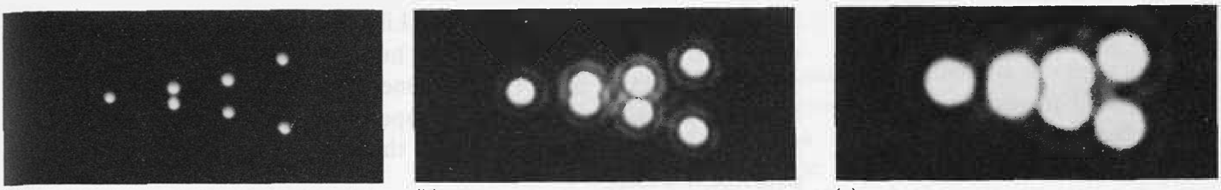
In diffraction imaging, radiation from *each point* of the diffraction pattern contributes to *every point* on the eventual image. If part of the beam is removed, then although the image loses detail, its general outline is preserved. This is in contrast to a conventionally produced image as, for example, that produced by a slide projector, where removing part of the beam removes an entire section of the image.

Resolution

QUESTION 11

EXPERIMENT J7
Resolution

It is not possible to resolve (detect as separate) two small objects if their angular separation is too small. Diffraction at the entrance aperture to the system (for example by a telescope or microscope) is one important factor which limits resolution (figure J7).



(a) (b) (c)

Figure J7
A constellation of seven close point sources photographed using a telescope with a progressively smaller aperture.
G. R. Graham, Cambridgeshire College of Arts and Technology

Rayleigh (1842–1919) suggested that if the centre of the diffraction pattern due to one object coincides with the first minimum of that due to the other, then the objects can just be distinguished as separate (figure J8). So, adopting Rayleigh's criterion, λ/b is a good guide to the limit of angular resolution for a system with slit width b. For a circular aperture of diameter b, $\theta \approx 1.2\lambda/b$.

J

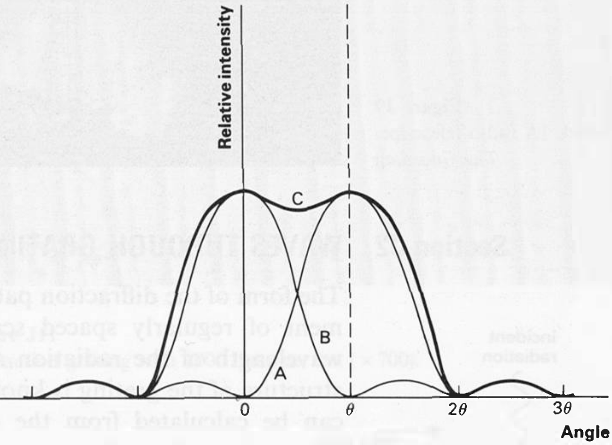


Figure J8
Overlapping diffraction patterns of two images which are just resolved. Curve C, which the eye responds to, is the sum of the intensities of curves A and B.

Lenses, to one degree or another, all possess defects which distort the image and thus limit the quality of any optical instrument. Our ability to see fine detail is also determined in part by the closeness of the retinal receptors, rods, and cones, whose density varies across the retina. Since the sensation of light is achieved by the eye operating in tandem with the brain, there are clearly psychological and neurological factors which affect visual perception as well as the physiological and physical limitations.

QUESTIONS 12 to 16

Radio astronomy

Radio telescopes have become increasingly important instruments for astronomical study, in particular for observing radio galaxies and in leading to the discovery of quasars and pulsars. Since such sources typically emit wavelengths about 10^6 times longer than light, a radio

QUESTIONS 17 to 19

DEMONSTRATION J8
Model of radio interferometer

telescope needs to be hundreds of kilometres in diameter to achieve a resolution equivalent to that of the human eye. The interferometer type of radio telescope achieves the benefits of a large aperture by using aerials some distance apart and superposing the signals. The received intensity is, of course, much less than for a continuous detector of equivalent width.

Figure J9
Jodrell Bank, the Mark 1A radio telescope.
The Guardian

Section J2 WAVES THROUGH GRATINGS

The form of the diffraction pattern produced by a grating (an arrangement of regularly spaced scattering centres) is determined by the wavelength of the radiation and the structure of the grating. If the structure of the grating is known then the wavelength of the radiation can be calculated from the diffraction pattern. Since most of our information about the structure of atoms and molecules comes from spectral analysis, the accurate measurement of wavelength over a wide range of the electromagnetic spectrum is of great importance.

If, on the other hand, we know the wavelength, the diffraction pattern can yield information about the structure of the grating. Reconstruction of this information can take place either directly, by diffraction imaging (as in holography) or indirectly, as in X-ray diffraction.

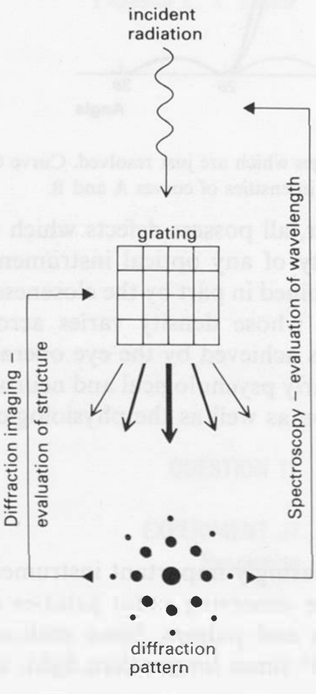


Figure J10
Waves through gratings.

Grating experiments

CIRCUS OF EXPERIMENTS J9
Looking through gratings

HOME EXPERIMENT JH2
Simple spectroscopy

DEMONSTRATION J10
The diffraction grating

HOME EXPERIMENT JH3
Ear and eye

An optical grating has many slits, usually parallel grooves on a piece of plastic or glass (figure J11). Looking at a line filament light source through such a grating reveals a diffraction pattern with lines of maximum intensity in each of the colours or wavelengths present. Longer wavelengths are diffracted through larger angles than shorter ones.

If the light source emits only one wavelength, then the diffraction pattern consists of a series of equally spaced bright lines (figure J12).

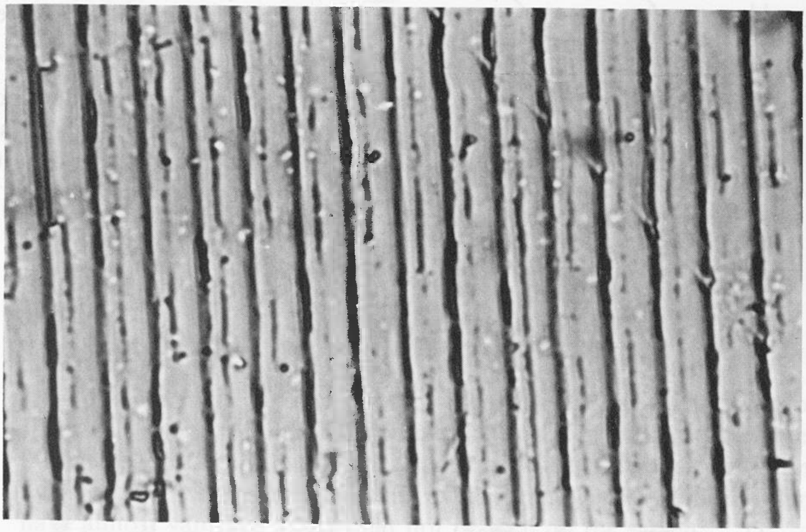


Figure J11
Diffraction grating with 1000 lines per cm ($\times$ 700).

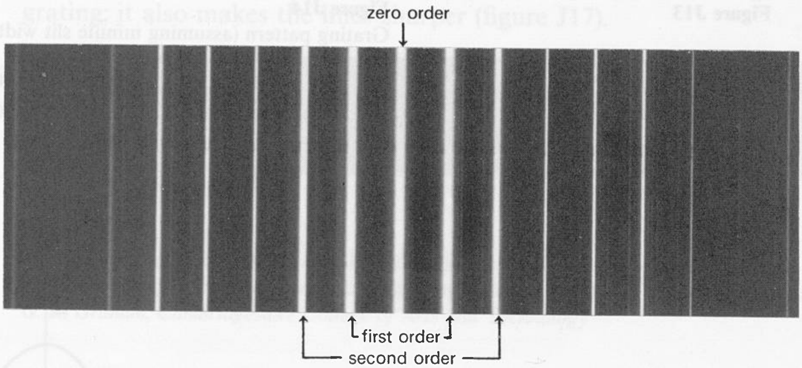


Figure J12
Diffraction pattern with monochromatic light.
G. R. Graham, Cambridgeshire College of Arts and Technology

J

Grating formula

DEMONSTRATION J11
Ripple tank demonstration of grating pattern

Huygens's construction can be used to model the scattering action of a grating. Figure J13 shows the path differences for consecutive waves which produce the various maxima or orders.

QUESTION 20

Zero Order ($n = 0$)
Path difference between waves from adjacent slits $= 0$
1st Order ($n = 1$)
Path difference between waves from adjacent slits $= s \sin \theta_1 = \lambda$
2nd Order ($n = 2$)
Path difference between waves from adjacent slits $= s \sin \theta_2 = 2\lambda$

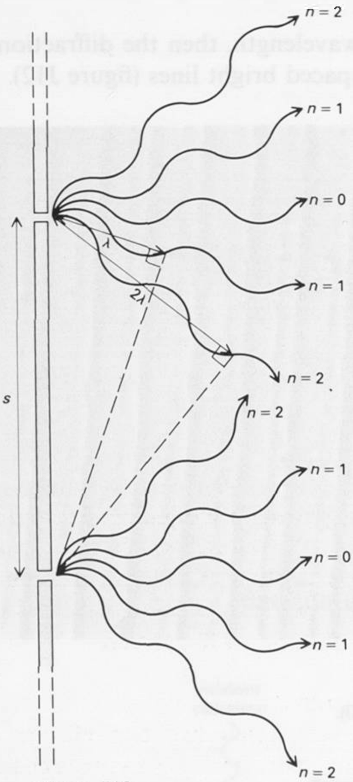


Figure J13

The general formula for a superposition maximum is:

$$n\lambda = s \sin \theta_n$$

where n is the diffraction order and s is the grating spacing. (There is a limit to the value n can take since $\sin \theta$ must be $\leqslant 1$, thus the maximum number of orders will depend on the particular values of s and λ.)

A simple analysis predicts that the intensity of all maxima would be the same (figure J14).

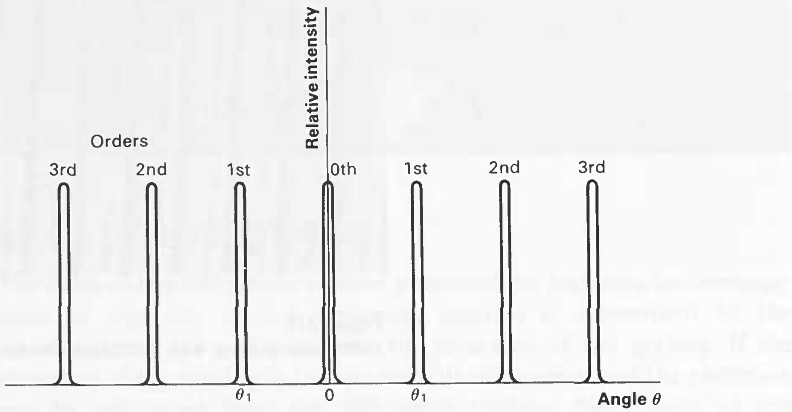


Figure J14
Grating pattern (assuming minute slit width), $\sin \theta_1 = \lambda/s$.

But light does not leave a single slit of width b equally in all directions – there are directions of zero intensity at angles given by $\sin \theta = n\lambda/b$ (figure J15).

QUESTION 6

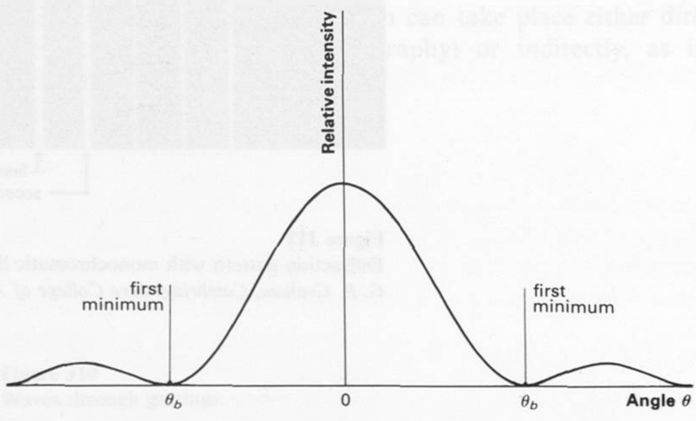


Figure J15
Single-slit pattern, $\sin \theta_b = \lambda/b$.

If no light leaves any slit at one of these angles, for example θ_b in figure J15, then even if the grating condition $n\lambda = s \sin\theta_n$ is obeyed, there will be *no* maximum at this angle. The effect is to combine the two patterns; the single-slit pattern being the 'envelope' which contains the grating pattern (figure J16).

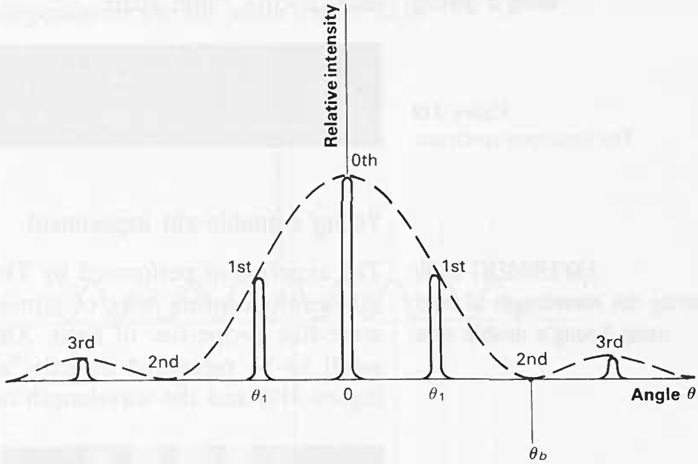


Figure J16
Overall pattern.

QUESTIONS 21 to 24, 32, 34

Notice that the maxima due to the grating are equally spaced (for small values of θ) and that the single-slit 'envelope' which determines the intensity of each maximum, has, by contrast, a central maximum twice as wide as the others. In the example shown the second orders have almost vanished and would not be seen on the screen.

Effect of number of slits on grating pattern

DEMONSTRATION J12
Sharpness of maxima and number of slits

Increasing the number of equally spaced slits has two effects. It increases the brightness of each line, since more light passes through the grating; it also makes the lines sharper (figure J17).

Figure J17
Grating pattern produced by (a) 2 slits, (b) 4 slits, (c) 50 slits. (The exposure times of the photographs vary to accommodate the very different intensities in the three cases.)
G. R. Graham, Cambridgeshire College of Arts and Technology

QUESTION 25

EXPERIMENT J13a
Measuring the wavelength of light using a grating

These two effects are crucial in the study of spectra (figure J18). The individual maxima (corresponding to the various wavelengths) are bright, and also very sharp, thus making it easier to distinguish two close wavelengths. Such a grating is said 'to have good resolution'. A typical grating used in optical spectroscopy might have 15 000 slits, each 2×10^{-3} mm apart.

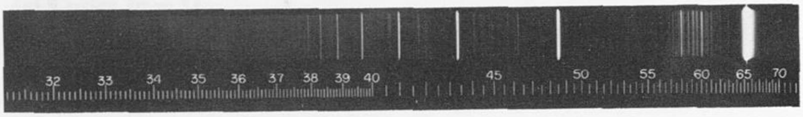


Figure J18
The hydrogen spectrum.

Young's double-slit experiment

EXPERIMENT J13b
Measuring the wavelength of light using Young's double slits

The experiment performed by Thomas Young at the beginning of the nineteenth century was of considerable importance in showing the wave-like properties of light. Although the path differences are too small to be measured directly, a superposition pattern is produced (figure J19) and the wavelength can be calculated.

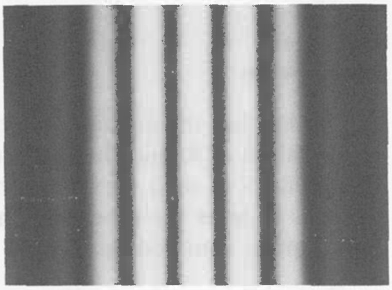


Figure J19
Young's double-slit pattern.
G. R. Graham, Cambridgeshire College of Arts and Technology

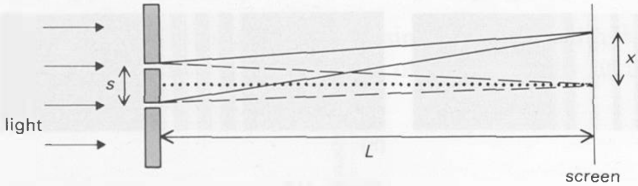


Figure J20
Young's double-slit experiment.

QUESTION 26

The relationship between the wavelength, λ, the slit spacing, s, the fringe separation, x, and the distance between the slits and the screen, L, (see figure J20) is known as the Young's fringes or the double-slit equation:

$\lambda/x \approx s/L$

QUESTIONS 27, 28, 31, 33

This relationship is just a special case of the general diffraction formula. It provides quite an easy method for measuring or comparing different wavelengths of light, though it is less precise than using a grating with many slits. If the source used is not monochromatic then a number of overlapping, coloured fringes is produced.

As with a grating, the *positions* of the maxima in the pattern depend on the slit *separation*. The *intensity* of the various maxima in the pattern depends on the slit *width* (figure J21).

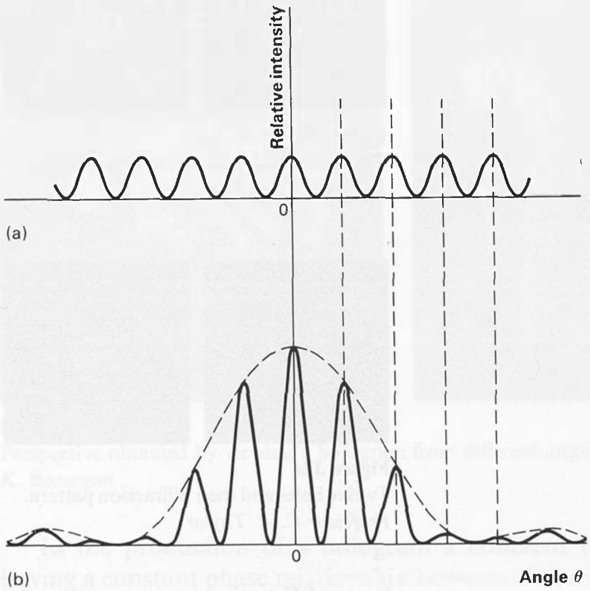


Figure J21
Relative intensity of Young's fringes for (a) infinitely narrow slits (b) slits of finite width.

Reflection gratings

DEMONSTRATION J14
A spectrum using a concave reflection grating

DEMONSTRATION J15
Reflection grating at glancing incidence

Reflection gratings can be made by scratching lines on a suitably reflective surface. The areas between the lines act like the slits in a transmission grating, that is, as Huygens's sources of secondary wavelets. If the line spacing is fairly large compared with the wavelength of the radiation, then the grating must be used at a glancing incidence, which greatly increases the path difference between waves leaving consecutive lines. It is just possible to measure the wavelength of X-rays by this method using the finest optical gratings available.

Complex gratings

DEMONSTRATION J16
Diffraction at complex gratings

The diffraction pattern of a simple multiple-slit grating contains information both about the slit spacing and the slit width (figure J16). Similarly, the diffraction pattern of a complex grating (one which consists of a regularly repeated array of apertures) contains information about the way the apertures are arranged in the grating, and also about the shape and size of the apertures themselves (figures J22, J23, and J24). If the grating has no regularity of structure then the resulting diffraction pattern is as shown in figure J25 – it has no regular fine structure.

READING
Measurement of red blood cell diameters using a laser (page 171)

Huygens's construction suggests that both a point and a pin-hole scatter waves in much the same way: so an array of scattering obstacles behaves just like an array of scattering apertures.

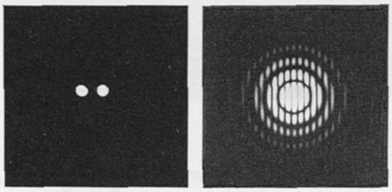


Figure J22
Two holes and their diffraction pattern.
Professor C. A. Taylor

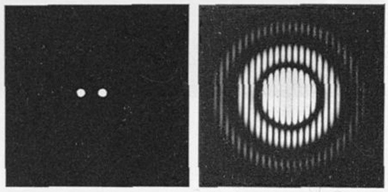


Figure J23
Two smaller holes but with same spacing as figure J22 and their diffraction pattern.
Professor C. A. Taylor

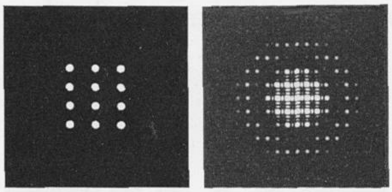


Figure J24
Twelve holes and their diffraction pattern.
Professor C. A. Taylor

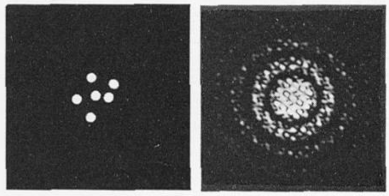


Figure J25
Random array of holes and their diffraction pattern.
Professor C. A. Taylor

X-ray diffraction

QUESTION 29

An array of atoms in a crystal can act as a complex grating for radiation of a small enough wavelength. X-rays have a wavelength roughly equal to the size of an atom. Since X-rays cannot be focused, the diffraction pattern cannot be recombined, so decoding the information in the diffraction pattern is not straightforward. Furthermore, the scattering array is three-dimensional, and the diffraction pattern is highly dependent on the orientation of the X-ray beam relative to the crystal. One way to decode the information in the diffraction pattern is to start from a 'ball-and-stick' model of the structure previously constructed from other physical and chemical data and to make an optical grating corresponding to the material in the X-ray beam (see figure J120, page 232). If the optical diffraction pattern of this grating compares favourably with that from the X-ray experiment, then the model is clearly valuable. An additional benefit of this method is that it is possible to find out about the shape and size of atoms, or groups of atoms, from the intensity distribution within the pattern.

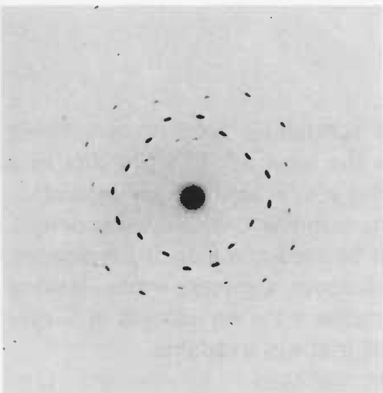


Figure J26
Early X-ray diffraction photograph of zinc sulphide.
FRIEDRICH, KNIPPING, and *LAUE.*
Sitzungsberichte der Königlich Bayerischen Academie der Wissenschaften,
Munich, 1912.
By courtesy of The British Library

High-speed computers are often used by crystallographers to predict the diffraction pattern produced by a suggested structure. Again, if this pattern compares well with that obtained in the X-ray diffraction experiment then there is a close fit between the suggested and the actual structures.

Holography

QUESTION 30

In holography light illuminates an object and produces a particular set of superposition patterns called a hologram, which can be recorded on a photographic plate. This plate carries all the amplitude and phase information needed to create a true, three-dimensional image of the original object when correctly illuminated.

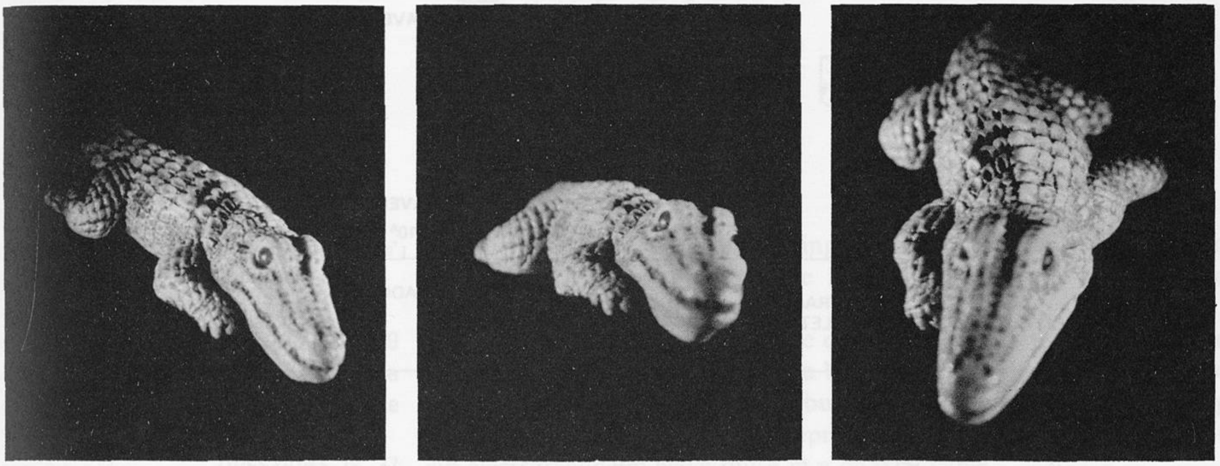


Figure J27
Perspective obtained by viewing a hologram from different angles.
K. Bazargan

In the production of a hologram a coherent beam of light (one having a constant phase relationship between all the waves, for example a laser beam) is split into two. One beam strikes the object and the other, called the reference beam, travels directly to the plate. The light which is scattered from each point on the object sets up a superposition pattern with the reference beam at the plate. The plate thus records a pattern from every illuminated part of the object – the result is an immensely complex set of diffraction patterns. Since each point on the object scatters waves in very many directions, each part of the hologram contains some contribution from *every* part of the object in the form of the superposition pattern. Thus each piece of the hologram (providing it is not too small) is able to reconstruct a complete image of the object.

DEMONSTRATION J17
Reconstructing the image from a hologram

In the reconstruction stage, the developed plate is illuminated by the reference beam from its original direction. The image reconstructed from each of the patterns on the plate is actually the image of a point on the original object and the observer thus sees a complete set of images corresponding to one particular view of the object.

'Imaging' in the Reader *Particles, imaging, and nuclei*

From a different viewpoint, the observer sees a new set of recombined images corresponding to a different view of the original object. The overall image is thus three-dimensional.

READING
Some applications of holography
(page 174)

Holography is becoming increasingly important as a technique for detecting and measuring minute movements, and in the storage and retrieval of three-dimensional images.

Section J3 ELECTROMAGNETIC WAVES

Radio waves, microwaves, light, X-rays, and gamma rays all belong to the family of electromagnetic waves called the *electromagnetic spectrum* (figure J28). The discovery, generation, and application of these waves have formed an important thread in the development of science and engineering.

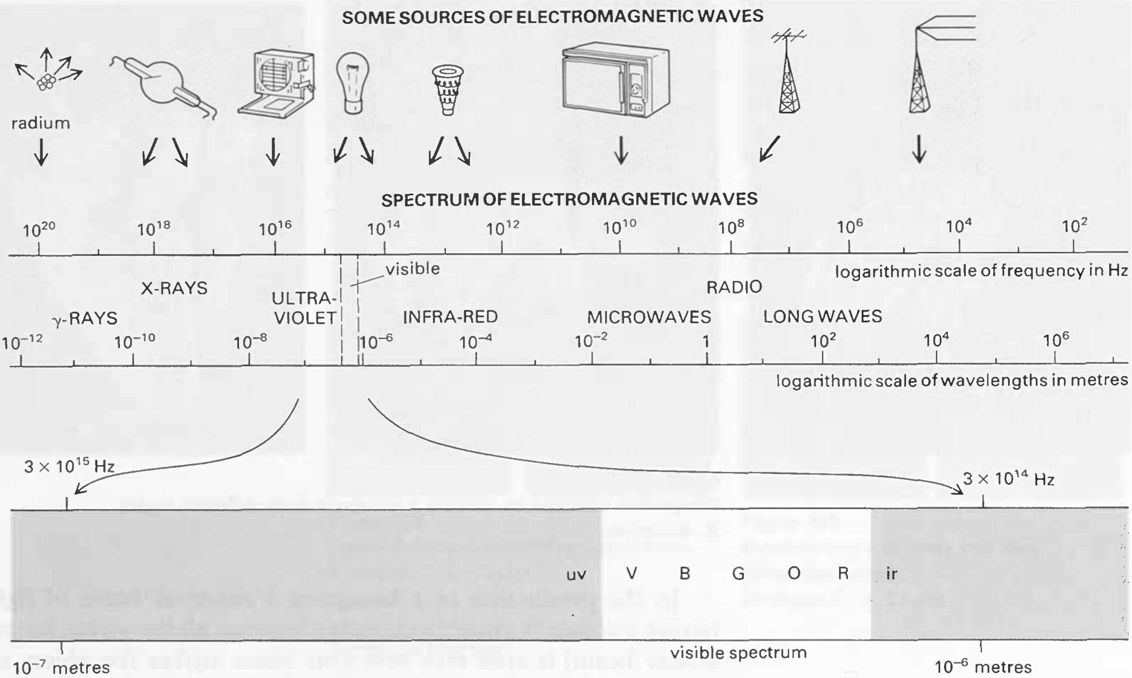


Figure J28
Schematic representation of the electromagnetic spectrum.

Waves that are 'tied' to a 'guide'

An electromagnetic wave which moves along a pair of long parallel plates or rods is directly associated with the charges in the plates. It is said to be *tied* to the plates, which form a *waveguide*. An oscillator connected across the ends of the plates produces an alternating p.d. at a very high frequency, and alternating regions of excess positive and negative charge are set up on the plates. These charged regions have an electric or E-field associated with them.

Because they are moving they constitute a current, so there is also a magnetic or B-field. The patches of moving charge and the associated pattern of E- and B-fields (which we call the electromagnetic wave) move along the guide at a very high, though finite speed. (The individual charges, for instance electrons, move at a very much lower speed.) The plates do not continue indefinitely and the wave is reflected at the far end. As the reflected and incident waves have the same frequency and speed but move in opposite directions, a standing wave is produced.

DEMONSTRATION J18a
Guided or 'tied' waves

QUESTION 35

The antinodes of the standing wave can be located using a pair of conducting rods forming a dipole aerial (figure J29). The changing concentrations of positive and negative charge on the plates set up an alternating e.m.f. between the ends of the dipole. This is rectified using a diode.

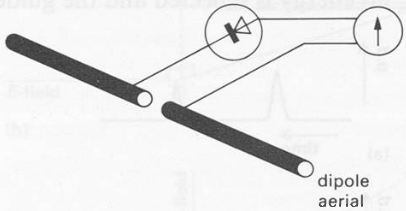


Figure J29
Dipole aerial.

Unit D 'Oscillations and waves'

Since the antinodes in a standing wave pattern are $\lambda/2$ apart, we can calculate the speed of the wave if we know, or can estimate, the frequency of the oscillator.

DEMONSTRATION J19
The speed of a pulse along a coaxial cable

The speed of the wave can be calculated more directly if the time taken for a pulse to travel down a long coaxial cable is measured. The value is close to 3×10^8 m s^{-1}, but less, since the wave is travelling partly in plastic. Figure J30 is a representation of the *E*- and *B*-fields for an electromagnetic wave pulse in a coaxial cable.

QUESTIONS 36, 37

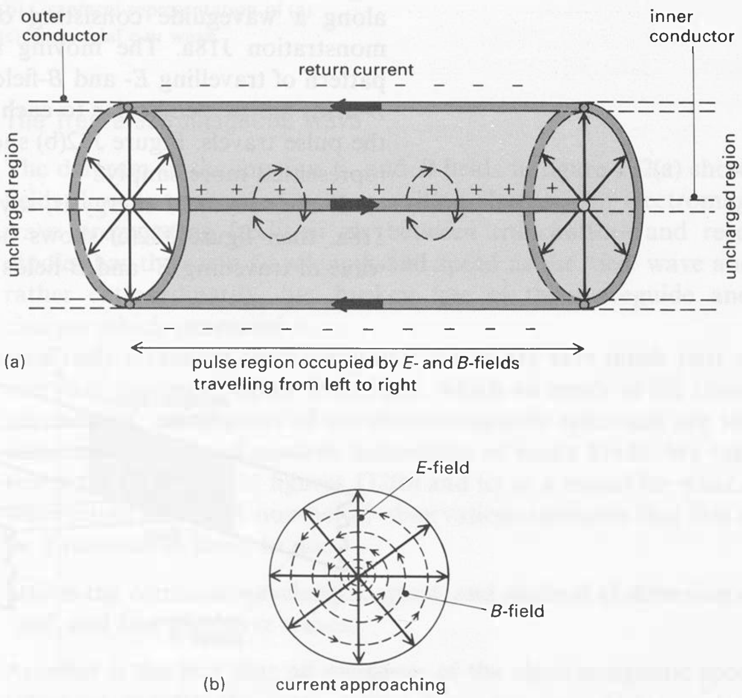


Figure J30
E- and *B*-fields in a coaxial cable. (a) Side view; (b) end view.

When the pulse reaches an open end it is reflected (with some loss of energy). The direction of the *E*-field in the reflected pulse is the same as that of the incoming pulse, while the direction of the B-field is reversed. If the conductors are short-circuited at the end, the B-field is unchanged but the E-field is reversed. This results in a reflected pulse of opposite sign when viewed on an oscilloscope (figure J31). With a suitable resistance joining the two conductors it is possible to absorb the pulse: no energy is reflected and the guide behaves as if it were infinitely long.

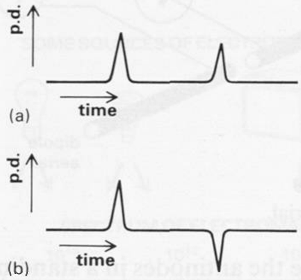


Figure J31
Oscilloscope traces of incident and reflected pulses. (a) Open circuit; (b) short circuit.

QUESTIONS 38, 39

'Systems' in the Reader *Physics in engineering and technology*

Avoiding such reflections, which generate spurious signals, is very important in all systems where waveguides or cables are used to join aerials, receivers, and transmitters. This is an example of the important process called *impedance matching*.

Some suggestions for the geometry of the travelling wave

Figure J32(a) shows how an electromagnetic wave pulse might travel along a waveguide consisting of two long, flat plates, as in demonstration J18a. The moving regions of opposite charge have a pattern of travelling E- and B-fields associated with them. The E- and B-fields are perpendicular to each other and to the direction in which the pulse travels. Figure J32(b) shows how the two fields alone can be represented more simply.

If the source of energy is, say, an oscillator, as in demonstration J18a, then figure J32(c) shows how the continuous electromagnetic wave of travelling E- and B-fields might propagate.

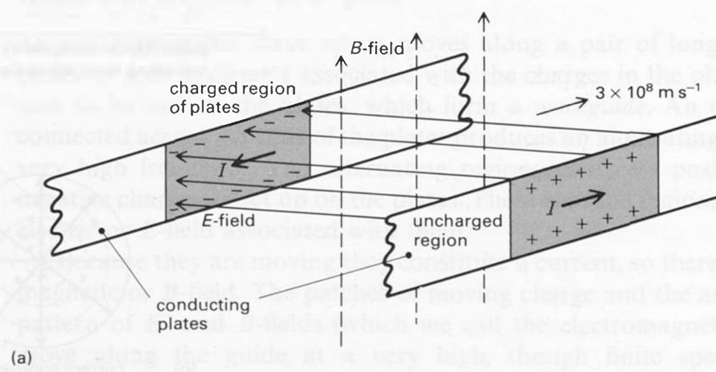
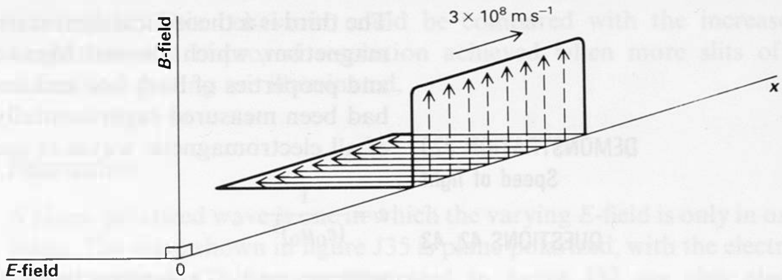
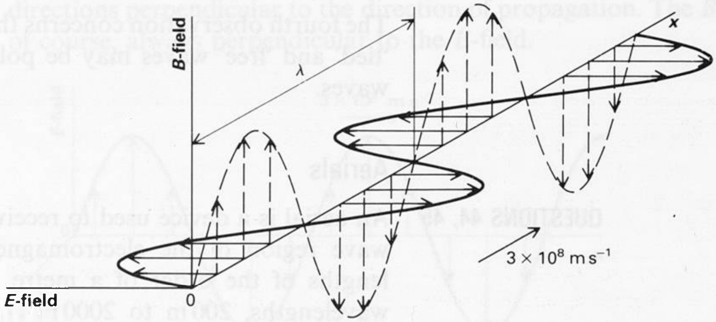


Figure J32 *(part)*
(a) Electromagnetic wave pulse.



(b)



(c)

Figure J32 *(part)*
(b) Graphical representation of (a).
(c) Sinusoidal e-m wave.

The free electromagnetic wave

DEMONSTRATION J18b
Free waves

The diagram of the moving *E*- and *B*-fields in figure J32(a) shows the fields directly associated with travelling charges. An electromagnetic wave propagating freely in air between transmitting and receiving dipoles has the same wavelength and speed as the 'tied' wave and yet, rather extraordinarily, has broken free of the waveguide and the charges which generated it.

QUESTIONS 40, 41

Freely travelling electromagnetic waves are very much part of our everyday existence. Apart from light, which so much of life obviously depends on, many parts of the electromagnetic spectrum are vital to communications and modern technology of many kinds. We take the tied wave illustrated in figures J32(b) and (c) as a model for what a free wave might be like. A number of observations indicates that this might be a reasonable thing to do:

One is the common wavelength, speed, and method of detection of the 'tied' and free gigahertz waves.

Another is the fact that *all* members of the electromagnetic spectrum travel at exactly the same speed in a vacuum. The speed of an electromagnetic wave in a vacuum is one of the most important physical constants, and has been measured with great precision.

Wavelength/m	Speed/10^8 m s^{-1}
6.4	2.9978 ± 0.0003
1.8	2.99795 ± 0.00003
1.0	2.99792 ± 0.00002
1.0×10^{-1}	2.99792 ± 0.00009
1.2×10^{-2}	2.997928 ± 0.000003
4.2×10^{-3}	2.997925 ± 0.000001
5.6×10^{-7}	2.997931 ± 0.000003
2.5×10^{-12}	2.983 ± 0.015
7.3×10^{-15}	2.97 ± 0.03

Table J1
The speed of electromagnetic waves.
Adapted from FRENCH, A. P. Special relativity. *Nelson, 1968.*

The third is a theoretical derivation, based on the laws of electricity and magnetism, which allowed Maxwell (1831–1879) to predict the speed and properties of both free and tied electromagnetic waves before they had been measured experimentally. The analysis shows that the speed of all electromagnetic waves *in vacuo* is:

DEMONSTRATION J20
Speed of light

$$c = \frac{1}{(\varepsilon_0 \mu_0)^{\frac{1}{2}}}$$

QUESTIONS 42, 43

where ε_0 and μ_0 are, respectively, the permittivity and the permeability of free space (that is, a vacuum).

The fourth observation concerns the phenomenon of polarization. Both 'tied' and 'free' waves may be polarized and thus must be transverse waves.

Aerials

QUESTIONS 44, 45

An aerial is a device used to receive or transmit radiation in the radio wave region of the electromagnetic spectrum. For receiving wavelengths of the order of a metre a dipole is convenient. For longer wavelengths, 200 m to 2000 m (1.5 MHz 'medium wave' to 150 kHz 'long wave'), a coil is used. The coil is wrapped round a ferrite rod to increase the B-field and hence the alternating e.m.f. induced in the coil (figure J33). The rod should be oriented parallel to the B-field to maximize the flux through the coils.

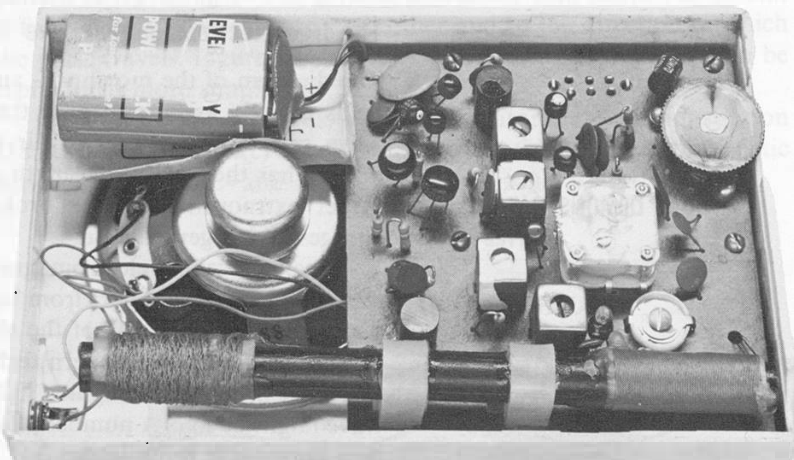


Figure J33
Portable radio with ferrite rod aerial.
Michael Plomer

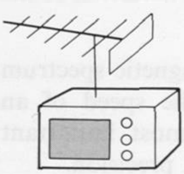


Figure J34
Television multi-element aerial array.

A dipole aerial can be improved by adding a metal reflector behind the dipole and/or one or more director rods in front of it. The principle of superposition is the key to designing such an aerial array: the waves reflected by the reflector and those scattered by the director rods must arrive at the dipole in phase with the original wave if the signal is to be increased. The more director rods an aerial array has the stronger the signal, but such an aerial must be accurately directed towards the transmitter. This behaviour could be compared with the increased brightness and improved resolution achieved when more slits of a diffraction grating are illuminated.

Polarization

A plane-polarized wave is one in which the varying E-field is only in one plane. The wave shown in figure J35 is plane polarized, with the electric vector vertical. (The waves illustrated in figure J32 are also plane polarized.) In an unpolarized wave the E-field has components in all directions perpendicular to the direction of propagation. The B-field is, of course, always perpendicular to the E-field.

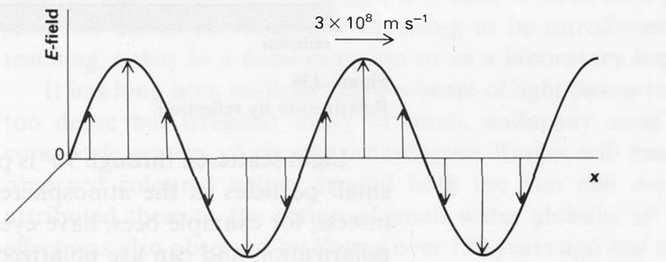


Figure J35
Polarized electromagnetic wave.

EXPERIMENTS J21a, b
Polarization

The aerial array illustrated in figure J34 is oriented to detect a polarized wave with the E-field horizontal. A dipole aerial receives no signal when it is at 90° to the transmitting dipole; the signal is maximum when the two dipoles are parallel. If a grid of wires is placed between the transmitter and receiver, no signal is received when the grid is parallel to the dipole rods. When the dipole transmits an electromagnetic wave with the electric field vector parallel to the rods, the grid does not transmit energy because the incident E-field sets the free electrons in the rods oscillating along the length of the rods, thus removing energy from the wave. The same phenomenon is displayed by the microwave equipment.

Polarization of light

QUESTIONS 46, 47

Light waves are usually generated by a large number of random, independent atomic processes, so they have no single plane of polarization and are said to be unpolarized. However, a material such as Polaroid transmits light with only one plane of polarization. Polaroid consists of long-chain molecules oriented parallel to each other; these act for light in much the same way as the parallel grid of rods does for microwaves.

EXPERIMENT J21c
Polaroid

The transmitted wave has no component of oscillating E-field in a direction parallel to the long-chain molecules in the material.

Polarization by reflection and scattering

EXPERIMENT J21d
Polarization by reflection and scattering

When unpolarized light is reflected by materials such as glass and water the light becomes partially polarized. This is because the component of the E-field parallel to the surface is reflected more strongly than the component perpendicular to the surface, the degree of polarization

HOME EXPERIMENT JH4
Polarized light

depending on the angle of incidence. The reflected beam is partly absorbed by Polaroid oriented to transmit light whose E-field is perpendicular to the surface. This is used to good effect in certain sunglasses, which reduce glare from such horizontal surfaces as the sea or a shiny road.

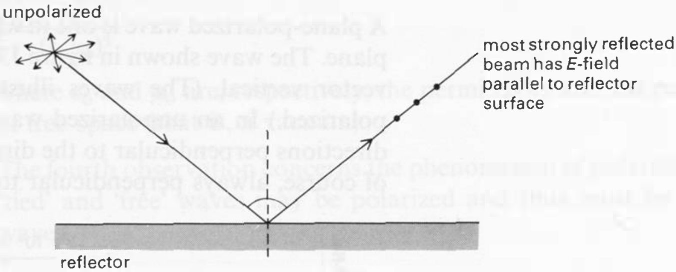


Figure J36
Polarization by reflection.

Light scattered through 90° is polarized. Sunlight scattered from the small particles in the atmosphere is partially plane polarized. Some insects, for example bees, have eyes which are sensitive to the plane of polarization and can use polarized light from the sky to navigate.

READINGS

MEASUREMENT OF RED BLOOD CELL DIAMETERS USING A LASER

(This article, by C. Bowlt, first appeared in *Physics education*, **6**, 1971, pages 13–15.)

Introduction

The experiment described here is not new, but the application of laser light to it, although obvious, does not seem to have been described. It seems to me to be sufficiently arresting to be introduced into optics teaching, either as a demonstration or as a laboratory experiment.

It has long been realized that if a beam of light passes through a not too dense but irregular array of small, uniformly sized particles, a concentric system of circular interference fringes will result. Newton observed coloured haloes around both the Sun and the Moon and attributed them to the action of small water globules in the air. The effect was also observed by Young over 150 years ago and he attempted to use it as a method for measuring the diameters of very small particles, including red blood cells. However, as a measuring technique the effect was not seriously investigated until 100 years later. The method requires a strong source of monochromatic light. Prior to the advent of the laser around 1960, such sources were not available and a number of the early investigators using non-monochromatic light concluded that the method was unreliable. In 1928 Allen and Ponder, using weak monochromatic light, showed that experimental results did accord with theory. The technique continued to be used for blood cell measurements for a number of years, but was eventually abandoned for clinical diagnostic purposes; principally, it would seem, because of the lack of an intense monochromatic light source.

The laser is a device producing a very intense beam of highly monochromatic, coherent light and is thus the ideal source for use with the method, providing a simple and elegant way for measuring the average diameter of a sample of several thousand blood cells or any other circular or spherical particles. At about the same time as the invention of the laser, the Coulter counter became available for the counting and measurement of small particles and the volumes of individual blood cells can now be found with ease using it. Had the laser been invented earlier, the diffraction method of measuring blood cells might not have been abandoned.

Theory

Diffraction of a parallel beam of light passing through a narrow slit produces on a screen (placed effectively at infinity) a series of line fringes centred on the straight-through position. The condition for a dark fringe to occur is $n\lambda/b = \sin\theta$, where n is the fringe order 1, 2, 3, …, λ the

J

wavelength of light, b the slit width, and θ the angular deviation of the diffracted beam.

For a circular hole, interference occurs between all rays from the whole area producing a series of concentric fringes. The exact solution in this case is more complicated, but turns out to be of the same form as for a slit, except that n is non-integer (that is, $n = 1.22, 2.23, 3.24, 4.24, 5.25, \ldots$).

Babinet's principle, which applies to any point outside the area illuminated by the undiffracted beam, states that the illumination is unaltered if the transparent parts of an aperture become opaque and the opaque parts transparent. Thus the same pattern of fringes will be produced by a circular hole in an opaque screen and by a circular disc of the same diameter as the hole, so that a single blood cell of diameter b will also produce a series of concentric fringes. In a blood smear there are many cells distributed randomly. The only effect of this is to enhance the intensity of the diffraction pattern due to a single cell.

If the angular deviation is small, $\sin\theta \approx \theta \approx S_n/L$, where S_n is the radius of the nth dark fringe and L is the distance between the diffracting particles and the screen, then $b = n\lambda L/S_n$ ($n = 1.22, 2.23$, etc.).

Experiment

The fringe pattern shown in figure J37 was produced on a screen by a parallel beam of light from a helium–neon gas laser passing through a

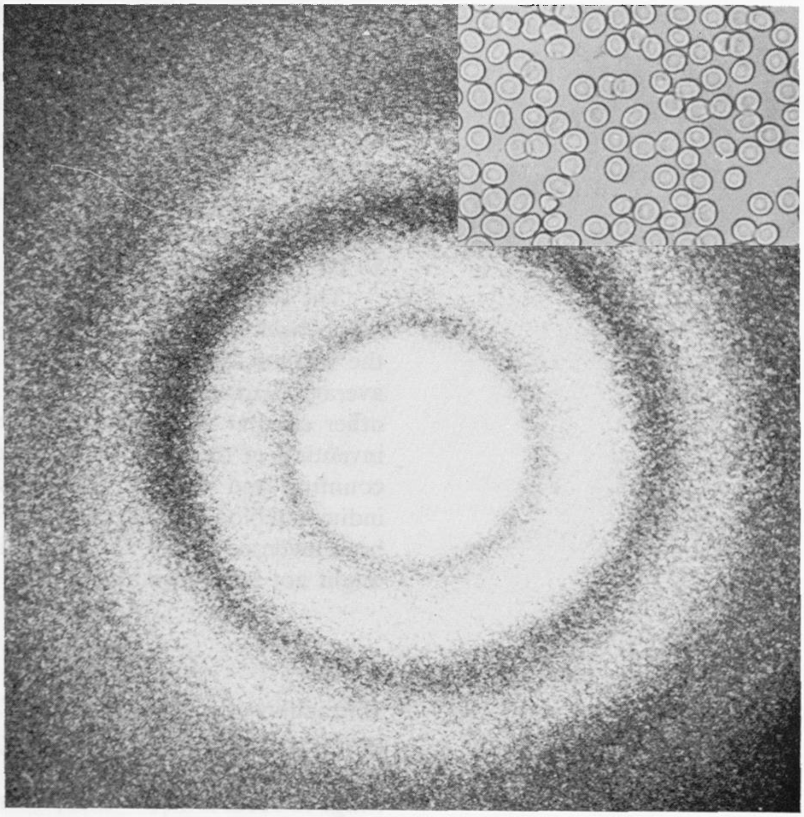


Figure J37
Interference fringes produced by diffraction of laser light by red cells.
Inset: A photomicrograph of a dried smear of red blood cells.

blood smear on a microscope slide. The great range in light intensity makes it difficult to show on a photograph all the orders observable by eye. The granular appearance of the fringe pattern is characteristic of laser light reflected from a surface. A very small quantity of blood was required and the only difficulty in producing a suitable smear was to spread the blood thinly enough to prevent the red cells overlapping (see inset, figure J37). Some adjustment of the position of the smear in the beam was required to find the area of the smear giving the most distinct fringes.

The thin film of blood will quickly dry unless covered with a cover slip. Fringes are produced by both wet and dry films, but it was found easier to produce distinct fringes with dry films. It should be borne in mind that the diameter of red blood cells shrinks about 10 per cent on drying. No focusing of the diffracted light was required. With the particular laser used, and with the distance between the blood smear and screen of the order of 20 cm so that the percentage error in its measurement was small, five fringes were visible. Best results were obviously obtained in a darkened room, but it was possible to see three fringes in ordinary lighting. Table J2 shows the diameters of the dark fringes at a distance (L) of 15.5 ± 0.15 cm from a dried blood smear obtained using laser light with wavelength (λ) of 6.328×10^{-7} m.

Dark fringe	n	Fringe diameter, $2S_n$/cm
1st	1.22	3.2
2nd	2.23	5.9
3rd	3.24	8.5
4th	4.24	11.4
5th	5.25	14.3

Table J2

Figure J38 shows the fringe *diameters* plotted against n. Since theory requires a straight line passing through the origin, results such as these are a nice illustration that for diffraction by circular objects the orders (n) are non-integer, and that the values 1.22 and so on which are difficult to derive theoretically certainly produce a better fit. The slope of this line gives $n/2S_n$, and since

$$b = \left(\frac{n}{S_n}\right)\lambda L$$

$$b = (2 \times \text{slope})\lambda L$$

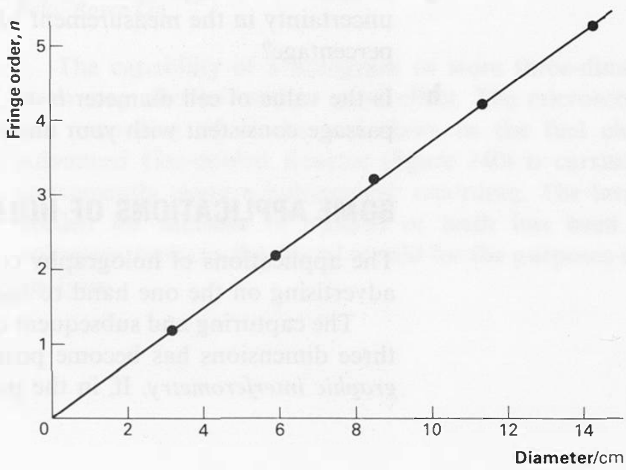


Figure J38
Plot of fringe diameter against fringe order, n.

This experiment gives the value $b = 7.3 \pm 0.2 \times 10^{-6}$ m. This is in agreement with Ponder's (1947) findings of 7.3 to 7.6×10^{-6} m, according to whom the actual value depends on the conditions of drying. It must be remembered that this is a mean diameter of a large number of cells. To produce such a result by direct measurement using a microscope would be very tedious, but a calibrated microscope can be used to show quickly that the diffraction result is certainly of the right order. An interesting point arises in contrasting these two methods of measurement. In using the microscope much care is needed to eliminate the diffraction rings from the field of view and obtain a sharp focus, whereas in the method described here it is these diffraction rings which provide the measurement.

Questions

a Explain what is meant by 'monochromatic' and 'coherent' (paragraph 3).

b *i* What fringe pattern would you get in the experiment if you did not use monochromatic light?
ii Explain why the calculation of blood cell diameter would thus be very difficult.

c How can 'the action of small water globules in the air' (paragraph 2) produce coloured haloes around the Sun and the Moon?

d With the help of a diagram, explain what is meant by 'fringe order' and 'angular deviation of the diffracted beam' (paragraph 4).

e According to Babinet's principle (paragraph 6), what would the fringe pattern caused by a straight human hair look like? To what condition must you adhere for this principle to apply?

f Taking values of n, λ, L, and S_n from the text, show that the calculated value of b is correct.

g If figure J37 is approximately life size, what is the minimum uncertainty in the measurement of fringe diameter expressed as a percentage?

h Is the value of cell diameter $b = 7.3 \pm 0.2 \times 10^{-6}$ m given in the passage consistent with your answer to question **g**? Explain.

SOME APPLICATIONS OF HOLOGRAPHY

The applications of holography cover an extremely wide range, from advertising on the one hand to scientific research on the other.

The capturing and subsequent comparison of minute movements in three dimensions has become possible using a technique called *holographic interferometry*. If, in the production of a normal transmission hologram the subject moves very slightly, then the superposition patterns on the hologram change. The reconstructed image then contains fringes which yield information about the object's movement. Such techniques have been used in, for example, a study of the vibrational characteristics of the fan blades in an aero engine; the investigation of distortion in the cone of a moving-coil high fidelity loudspeaker; the design of artificial hip joints; and in aerodynamics (see figure J39).

Figure J39
Double exposure holographic interferogram showing the flow field around an aerofoil, mounted in a wind tunnel.
Rolls–Royce Ltd

The capability of a hologram to store three-dimensional information may also be used to good effect. The microscopic examination and recording of cracks and flaws in the fuel elements from an Advanced Gas-cooled Reactor (figure J40) is carried out safely and conveniently using a holographic recording. The large storage space needed for millions of moulds of teeth has been reduced as the hologram replaces the dental mould for the purposes of inspection and analysis.

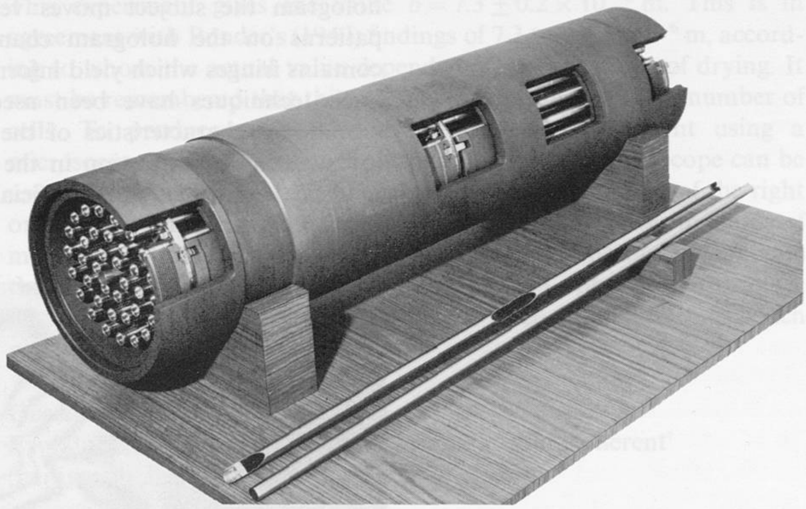


Figure J40
AGR fuel element.
Central Electricity Generating Board

Precious art objects reproduced as high quality images from a hologram make it possible for people in remote regions to experience cultural treasures in a most life-like form (see figure J41). Indeed, artists themselves have been quick to develop holography into an exciting new artform in its own right.

Figure J41
Holograms of icons and jewellery.
Hologram by NIKFI (USSR)
French Museum of Holography, Paris/Eve Ritscher Associates Ltd

If a hologram is made of a scene incorporating a magnifying lens, then the viewer of the hologram can look through the lens and see a magnified image of the subject that lies behind it, as if he were viewing the scene itself through the lens. This use of optical components in holography is finding increasing commercial application.

Devices to read bar codes on supermarket items are not new, but the laser scanner has made a considerable improvement. The beam detector can read the code within an angle of 180° of the scanning window, so the package need not be so carefully slid past the small window (figure J42). The scanner uses a spinning holographic optical element to split a laser beam into a large number of intersecting beams in space. It replaces the previous, cumbersome method which used rotating mirrors.

Figure J42
Scanned laser beam 'reading' the bar code on a product.
Richard Turpin. Courtesy of IBM United Kingdom Ltd

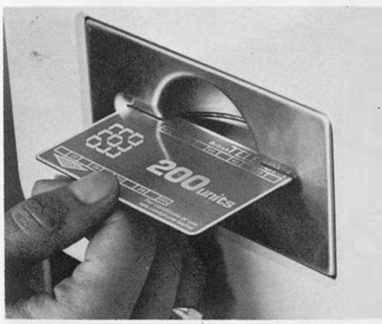


Figure J43
British Telecom Phonecard.
British Telecom/Landis & Gyr Ltd

In order to increase both the data storage capacity and the protection from forgery, credit card companies have begun to replace the magnetic strip on their credit cards with one which is holographically encoded (figure J43).

RELATIVITY

This article is concerned with some important, and perhaps surprising consequences of two pieces of experimental evidence about the speed of electromagnetic waves. The experimental evidence was graphically outlined by Einstein in 1920.

'There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with a velocity $c = 300\,000\,\mathrm{km\,s^{-1}}$. At all events we know with great exactness that this velocity is the same for all colours, because if this were not the case, the minimum of emission would not be observed simultaneously for different colours during the eclipse of a fixed star by its dark neighbour. By means of similar considerations based on observations of double stars, the Dutch astronomer Willem de Sitter was able to show that the velocity of propagation of light cannot depend on the velocity of motion of the body emitting the light....

'... In short, let us assume that the simple law of the constancy of the velocity of light c (in vacuum) is justifiably believed by the child at school. Who would imagine that this simple law has plunged the conscientiously thoughtful physicist into the greatest intellectual difficulties?'

From EINSTEIN, A. *Relativity: the special and the general theory.* Methuen, 1920.

Figure J44
Albert Einstein in 1905, the year in which his paper which provided the Special Theory of Relativity was published.
The Hebrew University of Jerusalem

The high but *finite* speed of $3 \times 10^8 \, m \, s^{-1}$ shared by all freely propagating electromagnetic waves can have some surprising consequences as illustrated by this account of a dream Einstein is reputed to have had. It is about one of the ideas that led him to his theory of special relativity, though the theory itself is not involved here.

Einstein dreamed that he was in a field with seven cows of an unusually agile breed. Their grazing was controlled by an electric fence to which a high-voltage pulse was applied every few seconds. But at the time of Einstein's dream the battery had run down, and the cows had become accustomed to touching the fence gently, and were grazing right up to it. The farmer was replacing the battery at one end of the fence and Einstein was standing near the other end of the fence watching. When the farmer switched on Einstein saw the cows jump back.

The farmer came across, and Einstein said to him, 'Your cows react very quickly. They all jumped back from the fence the instant you switched on, and they kept in a line perfectly parallel with the fence.' The farmer replied, 'They are better than that. The first had jumped back before the electricity had got to your end of the fence. Although they kept in a straight line, the line was at an angle to the fence after they had started to move.' He took some paper out of his pocket and sketched what he had seen – figure J45(a). Einstein drew a different sketch, shown in figure J45(b), and the farmer looked at it. 'Is this how you saw them at the same moment my sketch describes?' he asked.

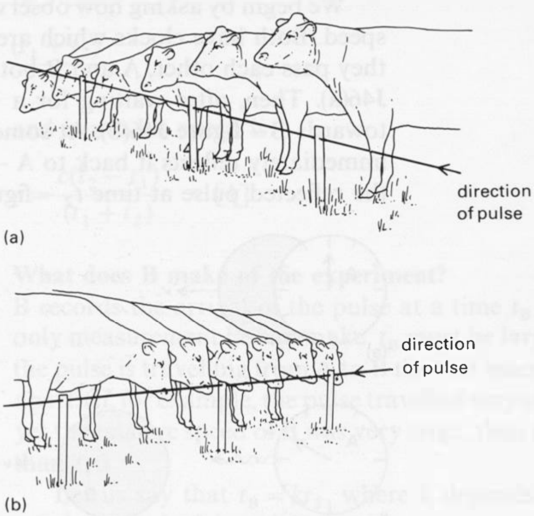


Figure J45
Einstein's dream.
(a) The farmer's sketch.
(b) Einstein's sketch.

How would you expect Einstein to have answered and explained it all?

Because the speed of light is finite, Einstein and the farmer gave two quite different descriptions of the same event.

The second piece of experimental evidence, is the perhaps surprising fact that the speed of light is constant and independent of the velocity of the emitter. Imagine that one is heading towards the Sun in a spacecraft, travelling very fast, at, say, $10^8\,\mathrm{m\,s^{-1}}$. Will sunlight go past at $4 \times 10^8\,\mathrm{m\,s^{-1}}$ (as it would if light were like any other object coming the other way)? No, all measurements show that light goes past everyone at the same speed, regardless of any relative motion between the source emitting the light and the observer.

As well as the experimental evidence of de Sitter, and also of Michelson and Morley, there are cogent theoretical arguments for the constancy of the speed of light. If it changed with the velocity of the emitter, it would be possible to determine the velocity of the emitter absolutely. But this would be at variance both with the laws of electromagnetism which had allowed Maxwell to predict *c*, but which dealt only in *relative* velocities, and also with the laws of mechanics, which are unchanged when a body moves with constant velocity and thus cannot be used to determine its absolute speed.

Einstein assumed that it can never be possible to measure the absolute motion of a body. A direct consequence is that the velocity of light must be the same regardless of the speed of the source and the measurer. The Special Theory of Relativity is the mathematical working out of Einstein's fundamental assumption. It concerns the descriptions which two observers (moving at constant velocity relative to one another) give of the same event. We can see some of the surprising predictions made by the theory by considering an imagined experiment with two observers, A and B, on spaceships moving with a relative velocity, v. They communicate by sending radio signals.

We begin by asking how observer A might set about determining B's speed. Both have clocks which are initially set to zero. At the moment they pass each other, A and B both start their onboard clocks – figure J46(a). Then, after waiting for a time t_1, A sends out a radio pulse towards B – figure J46(b). At some later time, B detects this pulse and immediately reflects it back to A – figure J46(c). Eventually A receives the reflected pulse at time t_2 – figure J46(d).

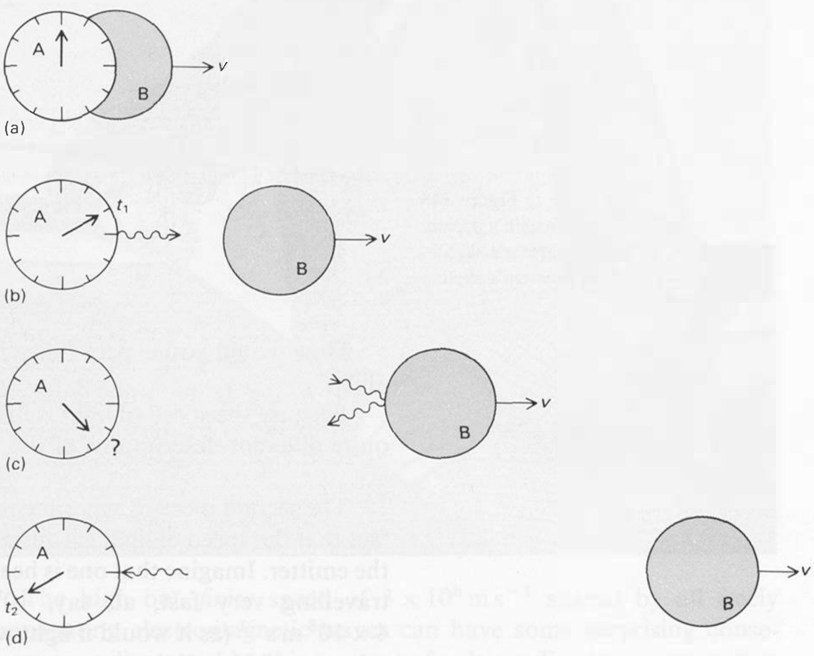


Figure J46

A makes the following calculation

If the pulse travels at the same speed in both directions (as indeed it must) then A reasons that the pulse must have been reflected at a time midway between when she sent it out (t_1), and the time that she received the reflection (t_2). Hence she calculates that the reflection took place $\dfrac{(t_2 - t_1)}{2}$ after she sent the pulse, that is, at a time $\dfrac{(t_2 - t_1)}{2} + t_1 = \dfrac{t_1 + t_2}{2}$ after the spacecraft passed and A and B started their clocks.

The separation between the spaceships when the reflection occurs

= speed of the pulse × time the pulse takes to reach B

$$= c \times \frac{(t_2 - t_1)}{2}$$

Thus A calculates that B travels a distance

$$\frac{c(t_2 - t_1)}{2}$$

in a time

$$\frac{(t_1 + t_2)}{2}$$

and hence his speed is

$$v = \frac{c(t_2 - t_1)}{(t_1 + t_2)} \qquad [1]$$

What does B make of the experiment?

B records the arrival of the pulse at a time t_B on his clock. This is the only measurement he can make. t_B must be larger than t_1 because while the pulse is travelling from A to B the two spacecraft are moving further apart. (If, for example, the pulse travelled very slowly – c was very small – yet the relative speed of B was very large, then t_B would be much greater than t_1.)

Let us say that $t_B = kt_1$, where k depends on the pulse speed and relative speed of A and B.

What happens on reflection?

The speed of the pulse remains c regardless of relative motion. For both A and B the pulse moves away at the same speed, c; and both observe the other spacecraft receding at speed v. If this were NOT true then it would be possible for A and B to determine their absolute motions.

Thus when B measures the time of departure of the reflected pulse as t_B and A measures its time of arrival as t_2, then t_2 is larger than t_B by the same factor k as before.

Thus $t_2 = kt_B$

so $\quad t_2 = k^2 t_1 \qquad [2]$

The solution to equations [1] and [2] is:

$$k = \sqrt{\frac{(1 + v/c)}{(1 - v/c)}} \quad \Rightarrow \quad k = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}$$

Time dilation

From our imagined spacecraft experiment we can see an important consequence of the principle of relativity. *According to A* the pulse arrives at spaceship B at time $\frac{1}{2}(t_1 + t_2)$. Let us call this time t_A.

According to B the pulse arrives at time t_B:

Now $\quad t_B = kt_1 \quad \Rightarrow \quad t_1 = t_B/k$

and $\quad t_2 = kt_B \quad \Rightarrow \quad t_2 = t_B k$

Hence $t_A = \frac{1}{2}(t_1 + t_2) = \frac{1}{2}t_B(k + 1/k)$

But $\quad k = \sqrt{\dfrac{(1+v/c)}{(1-v/c)}}$

$$\text{So} \quad t_A = \tfrac{1}{2}t_B\left(\sqrt{\frac{(1+v/c)}{(1-v/c)}} + \sqrt{\frac{(1-v/c)}{(1+v/c)}}\right)$$

$$= \tfrac{1}{2}t_B\left(\frac{(1+v/c)+(1-v/c)}{\sqrt{1-v/c}\sqrt{1+v/c}}\right)$$

$$= t_B\frac{1}{\sqrt{(1-v^2/c^2)}}$$

$$\Rightarrow \quad t_B = t_A\sqrt{(1-v^2/c^2)}$$

This is the time dilation equation. It says that t_B, the time interval *measured* by B with his clock, is always smaller by a factor $\sqrt{1-v^2/c^2}$ than the time t_A of the equivalent time interval as *calculated* by A from measurements made with her own clock.

Figure J47 shows how the ratio t_A/t_B varies with the relative velocity v. Only when v is an appreciable fraction of the speed of light is the ratio t_A/t_B significantly different from unity. Such high velocities are not part of our everyday experience, and so time dilation and other relativistic effects seem surprising and perhaps hard to accept. But time dilation is tested every day in high energy physics laboratories where experiments using particles moving at speeds very close to c are routinely performed.

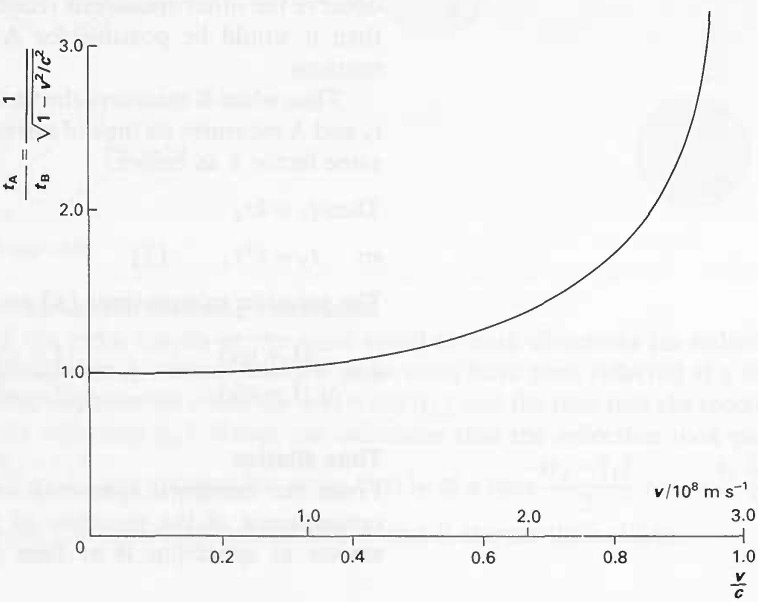


Figure J47

One particular example of time dilation is the unexpectedly long lives that rapidly moving π-mesons appear to have. π-mesons are unstable particles with a mass in between those of an electron and a proton. They are produced when rapidly moving protons bombard atomic nuclei in a target such as aluminium. Figure J48 is a plan of the proton accelerator at CERN (the European nuclear research laboratory) which was used in some experiments performed in the 1960s. This accelerator is quite small by today's standards, being about 200 m in diameter. A beam of particles, including π-mesons, was produced from targets placed in the proton beam in the accelerator ring. The π-mesons were detected in a bubble chamber (not shown in the diagram) beyond the experimental hall, more than 200 m from the point where they were produced.

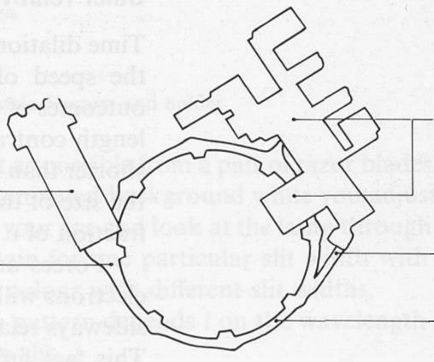


Figure J48
Plan of the proton accelerator used in the experiments with π-mesons.

π-mesons are unstable, and on average live for only 2.5×10^{-8} s (the lifetime being measured when the particles are at rest, or nearly so) before they decay into other particles. Moving at very nearly $3 \times 10^8 \text{ m s}^{-1}$ a π-meson will travel only 7.5 m in this time. This is an average value, but the fraction living for much longer is small. Yet the experimental hall is at least 100 m long, and the path laid out for the mesons is even longer. Were the designers very foolish, expecting the mesons to travel between ten and twenty times as far as an average meson could go before decaying? How many actually reach the bubble chamber?

In fact, the set-up does not waste a good beam of mesons. Because they are travelling so fast, they seem to us to live longer, and the effect is big enough for most of them to get to the far end of the hall. This is the time dilation effect predicted by relativity theory. In this case, the effect is by no means a small one: the mesons live much longer than they 'ought' to. We can use the time dilation formula to show how fast the π-mesons must be travelling to produce such a phenomenon.

t_B is the average lifetime of a π-meson measured by the π-meson itself and is thus equal to the lifetime measured when the particles are at rest. t_A is the time calculated by an observer in the laboratory from measurements made on the high speed π-mesons. If t_A is about $20 \times t_B$, then the speed of the π-mesons is given by v where:

$$t_B = t_A\sqrt{1 - v^2/c^2}$$

$$1 = 20\sqrt{1 - v^2/c^2}$$

$$1/400 = 1 - v^2/c^2$$

$$v = 0.999c$$

(From the meson's point of view, however, the length of the experimental hall seems to be the few metres it could expect to travel in a normal lifetime.)

Other relativistic effects

Time dilation is just one of the surprising consequences of the fact that the speed of light is finite and the same for all observers. Other outcomes of the Special Theory of Relativity include mass increase and length contraction. A moving object appears to be more massive and shorter than the same object at rest in the laboratory. Like time dilation the size of these effects depends on v^2/c^2, so unless v is an appreciable fraction of c the effects are negligible.

Forces are altered too. For example, the force between a pair of electrons will seem to be reduced by about v^2/c^2 if they move together sideways relative to the instruments or observer investigating the force. This fact links electricity and magnetism. Although the velocities of charge carriers in electric currents are very small (see Unit B, 'Currents, circuits, and charge') the fact that very large numbers of them are involved gives a sizeable effect. Magnetism is an everyday example of a relativistic effect.

The fact that the speed of light does not depend on the velocity of the source relative to the observer has to be taken into account in an analysis of the Doppler effect. This is the name given to the effect (well known before the theory of relativity was developed) whereby the frequency of light emitted by a moving source appears to depend on the relative motion of the source and the observer. If the source and observer are moving apart the frequency observed decreases. This is the well known red shift, widely used by astronomers to find the velocities of distant stars and galaxies. It has led to our present picture of an expanding Universe in which distant galaxies are receding at speeds proportional to their distances from us.

LABORATORY NOTES

EXPERIMENT

J1 Looking at a lamp through a slit and through a pin-hole

holder with two halves of a razor blade, to be used as a single slit
set of colour filters (red, blue, green)
aluminium foil
35 mm slide mounts
copper wire, 0.2 mm diameter, bare
steel or nichrome wire, 0.2 mm diameter, bare
lamp, holder, and stand
transformer

either
matt white reflecting screen
or
white card
mains lamp with 30 cm single filament and holder

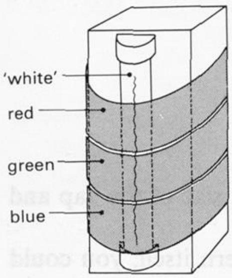


Figure J49
Single-filament lamp with colour filters.

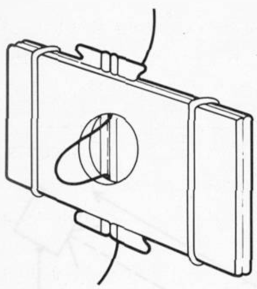


Figure J50
Adjustment of slit width to diameter of steel or nichrome wire.

Make as narrow a slit as possible from a pair of razor blades, by viewing the slit against an illuminated background while you adjust the blades. Hold the slit close to your eye and look at the lamp through it. Describe the patterns you obtain for one particular slit width with each of the colours, and for one colour with different slit widths.

Describe how the pattern depends *i* on the wavelength of light and *ii* on the width of the slit.

Make a circular aperture in some aluminium foil by pricking it carefully with the copper wire which has been stretched and broken. Make sure the wire is pushed through to its maximum diameter. Look at the lamp through the hole and note what you see.

As a final element in your experiment you can compare the patterns produced by a hole and slit of equal 'widths' by using the steel or nichrome wire to set the razor blade spacing equal to the hole diameter.

How are the patterns similar? How do they differ?

CIRCUS OF EXPERIMENTS

J2 Diffraction

J2a Water waves going through a gap

ripple tank kit
illuminant
transformer
cell holder with two cells
rheostat, 10 to 15 Ω
hand-held stroboscope

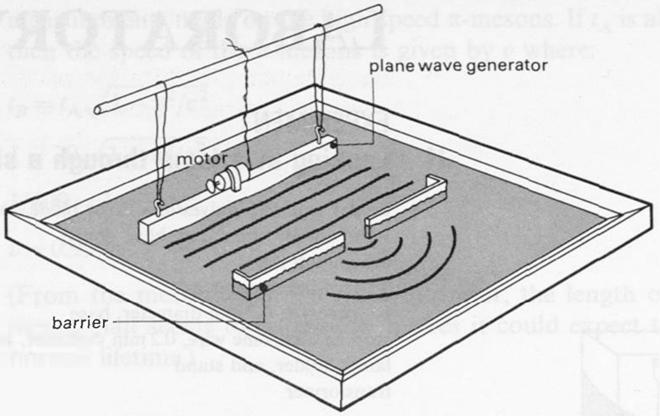


Figure J51
Diffraction of water waves at a gap.

How is the spreading of the wave affected by *i* the size of the gap and *ii* the wavelength of the waves?

You should be able to see the diffraction pattern itself; you could even make some measurements to show how the pattern depends on the wavelength and the gap size.

J2b Microwaves going through a slit

microwave transmitter
microwave receiver
general purpose amplifier
loudspeaker (if not part of above)
wax lens or plastic lens filled with paraffin (if available)
2 metal screens about 0.3 m by 0.3 m
metre rule
leads

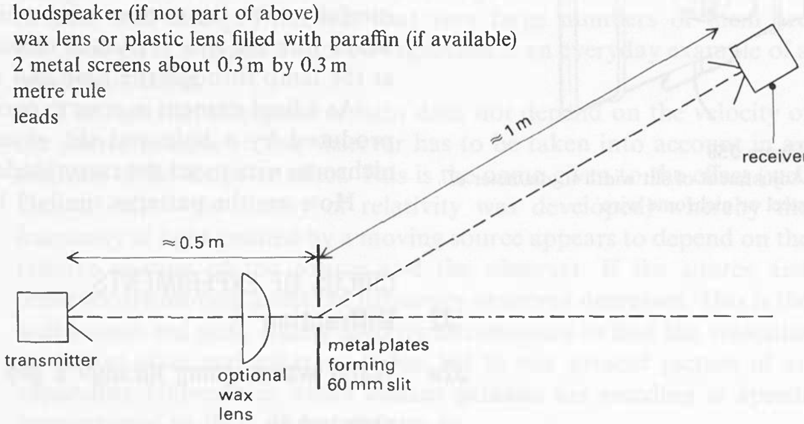


Figure J52
Diffraction of microwaves.

It is important to have either the transmitter a long way from the slit or a correctly adjusted lens. Why?

Set the metal plates to form a slit 60 mm wide. By moving the receiver in an arc in front of the slit (figure J52), observe the diffraction pattern and measure the angle between the straight-through direction and that at which the first 'drop in intensity' occurs.

Try changing the slit size. How does this affect the pattern?

Why is this experiment likely to be of poor accuracy?

To see how 'less can mean more', place the transmitter and receiver as in figure J53. Slide the plate A into the beam until it cuts off the radiation to the receiver. Now slowly move plate B into the beam to vary the slit width from about 60 to 30 mm, until the receiver output increases. Less wave energy now passes through the slit, yet more energy is detected by the receiver. Why is this?

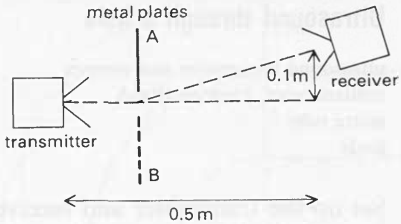


Figure J53
Superposition experiment.

J2c Light through an adjustable slit

big stop to stand on bench
small translucent screen
eyepiece
adjustable slit
planoconvex lens, + 2D, diameter 37 mm
holder for lens of diameter 37 mm
2 holders (for eyepiece and adjustable slit)
set of colour filters (red, blue, green)
lamp, holder, and stand
transformer
holder and two halves of a razor blade
transparent ruler or 0.5 mm graticule

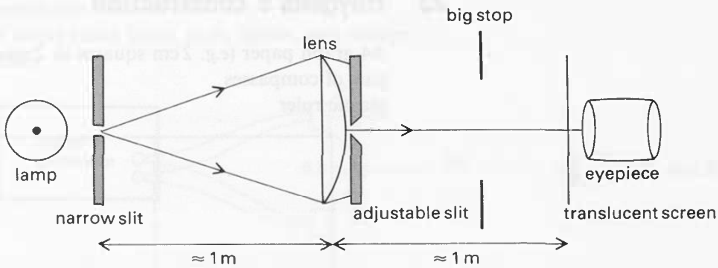


Figure J54
Diffraction of light by a single slit.

This experiment is really a rather more careful version of experiment J1.

Arrange the lamp, lens, and screen as shown in figure J54, to form a sharp image of the vertical filament on the screen. Use the big stop to prevent any light which has not passed through the lens reaching the screen. Adjust the razor blades to form a parallel-sided slit between 1 and 2 mm wide. Put this against the lamp. Making sure that the narrow beam of light passes through the centre of the lens, move the lens a small distance further from the lamp to focus a sharp image of the slit on the screen. Put the adjustable slit just beyond the lens and gradually reduce its width until you see faint fringes on the screen.

To see the pattern more clearly, focus the eyepiece on the fringes, remove the screen, and position the transparent ruler so that its scale may be read easily. Measure the separation of a fixed number of minima or maxima for red, green, and blue light. The ratio of pairs of distances gives an approximate value for the ratio of pairs of wavelengths.

How does the diffraction pattern vary with wavelength?

J2d Ultrasound through a hole

ultrasound transmitter and receiver
milliammeter, 1 mA or 10 mA
metre rule
leads

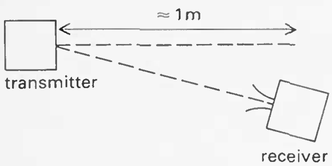


Figure J55
The diffraction of ultrasound.

Set up the transmitter and receiver about a metre apart (as shown in figure J55). Put them on the edges of two facing stools or tables to minimize troublesome reflections. Trial and error is the way to find the best arrangement.

The effective width of the aperture is the diameter of the piezo-electric transducer in the transmitter.

What is the function of the horn on the receiver?

It may be helpful to use the modulation facility for the transmitter – you will need to ask about this.

If the speed of sound in air is about $330\,\mathrm{m\,s^{-1}}$, what is the wavelength of the waves?

You should measure the diffraction pattern carefully, in particular the angle between the straight-through direction and the first minimum of intensity on either side.

EXPERIMENT

J3 Huygens's construction

A4 graph paper (*e.g.* 2 cm squares in 2 mm graduations)
pair of compasses
plastic ruler

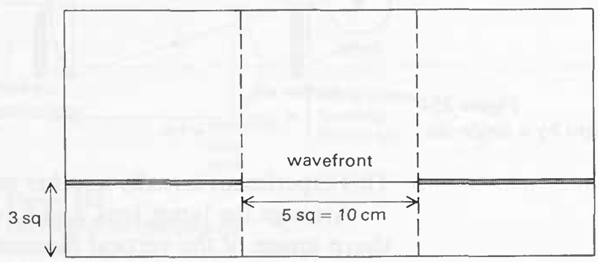


Figure J56

Mark out the barrier on graph paper and draw a wavefront which has just reached it, as in figure J56. Starting at one end of the wavefront, draw arcs from centres 1 cm apart with a radius of 3 cm into the region beyond the barrier.

Draw in the envelope of these wavefronts in ink, drawing extra arcs in any region where it is not clear.

Next, starting from this new wavefront, repeat the construction to produce a further wavefront 3 cm ahead. This process should be repeated until the 'frozen' picture of the diffracting waves becomes clear (figure J57).

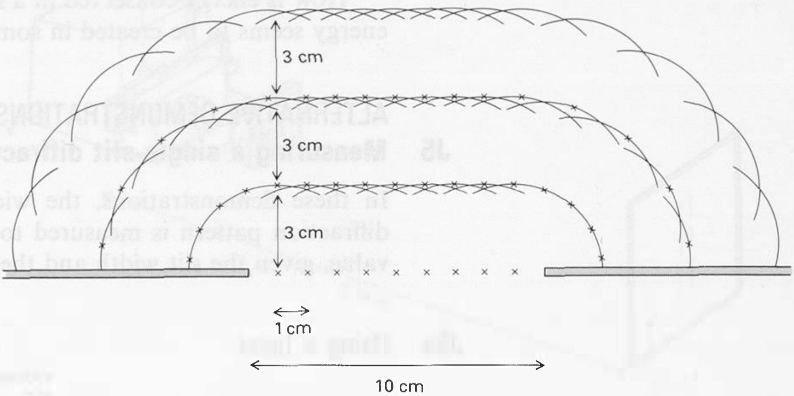


Figure J57
Huygens's construction.

DEMONSTRATION
J4 Wave amplitude and energy when waves superpose

signal generator
pre-amplifier
oscilloscope
2 loudspeakers
microphone
rheostat, 10 to 15 Ω
metre rule
3 retort stand bases, rods, bosses, and clamps
leads

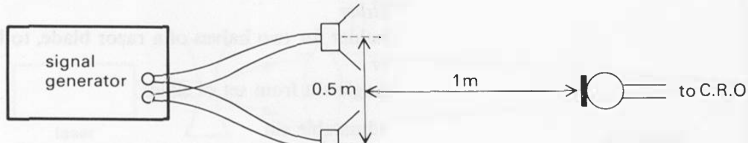


Figure J58
Amplitude and energy of a signal.

The frequency of the signal generator is set to 4 kHz and the two loudspeakers produce a superposition pattern. The loudspeakers and the microphone should be put on the edges of benches about a metre apart. The microphone is connected to an oscilloscope (via a pre-amplifier if necessary) and moved to the central maximum.

By how much is the height of the oscilloscope trace reduced if one loudspeaker is covered up? Does this show that the microphone is an 'amplitude detector'? You should be able to explain your answer.

If the oscilloscope were replaced by a resistor, how much less energy would be dissipated when one loudspeaker was covered up?

If the amplitude of the wave detected by the microphone doubles, by how much does the energy of the wave increase?

The eye and photographic film respond to intensity – the rate of arrival of energy per unit area. Intensity $\propto$ (amplitude)^2.

How is energy conserved in a superposition experiment like this, if energy seems to be created in some places and destroyed in others?

ALTERNATIVE DEMONSTRATIONS

J5 Measuring a single-slit diffraction pattern

In these demonstrations, the width of the central maximum of a diffraction pattern is measured to see if it conforms to the predicted value, given the slit width and the wavelength of the light.

J5a Using a laser

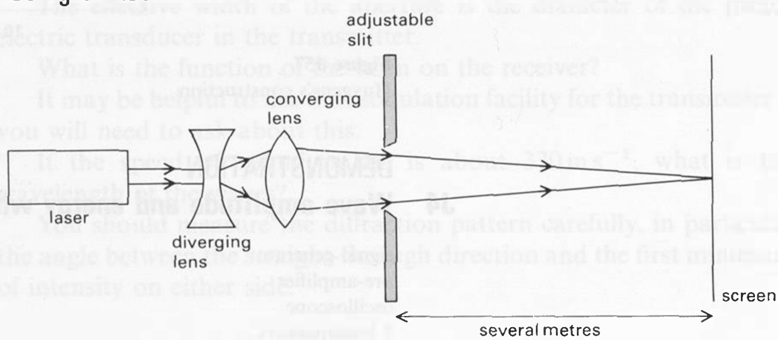


Figure J59
Measurement of single-slit diffraction pattern using a laser.

J5b Using a slide projector

either
holder for two halves of a razor blade, to be used as a single slit
or
single slit from set of slides

adjustable slit
small translucent screen
transparent ruler with mm graduations
metre rule
powerful slide projector

Whichever method is used to produce the diffraction pattern on the screen, the angle of the first order minimum, θ, is calculated from measurements of the separation of the two first order minima and the distance from the slit to the screen.

The slit width is measured by holding the slit in the plane of the carriage of a slide projector, and focusing and marking the position of the edges of its image on a distant screen. A transparent plastic ruler then replaces the slit in the projector and is moved until its image is in focus on the screen – the slit width is then read off. An alternative, for use with the laser, is to use steel or nichrome wire to set the slit width as in experiment J1.

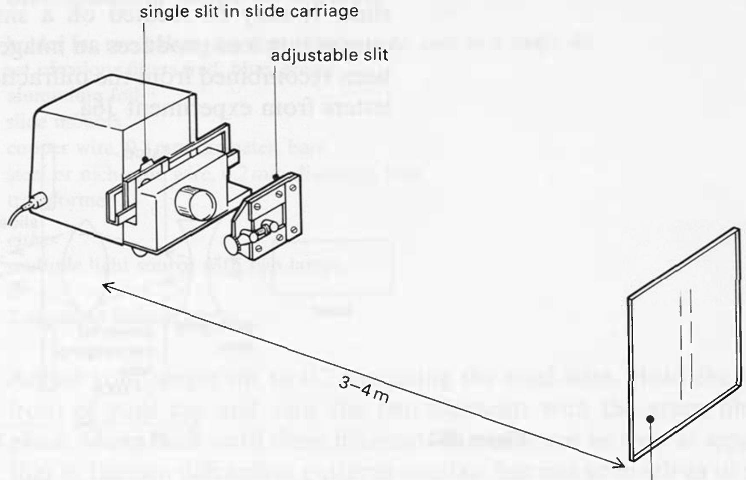


Figure J60
Measurement of single-slit diffraction pattern using a projector.

Use your values of θ and b, plus a value for the wavelength (taking either the value in the laser manufacturer's notes or an average value for white light) to check that the formula $n\lambda = b \sin\theta$ holds for the first order minimum.

DEMONSTRATION

J6 Diffraction and image recombination

J6a Diffraction at an aperture

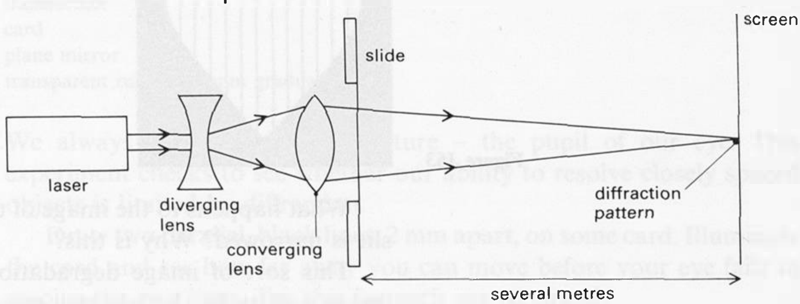


Figure J61
Diffraction at an aperture.

Diffraction patterns of slides with various apertures are obtained on the screen (figure J61). The diffraction patterns for a single slit and a small hole should be familiar to you from earlier experiments.

Try some slides of letters of the alphabet. The diffraction patterns become quite complex, but you should be able to deduce from them which letter produces which diffraction pattern.

J6b Recombination of a diffraction image

In figure J62 a more powerful converging lens is used (+ 10D), and the diffraction pattern of the slide is now very small and much closer to the slide. It may be located on a small translucent screen. The second converging lens produces an image of the slide on the screen which has been recombined from the diffraction pattern. Try some of the slides of letters from experiment J6a.

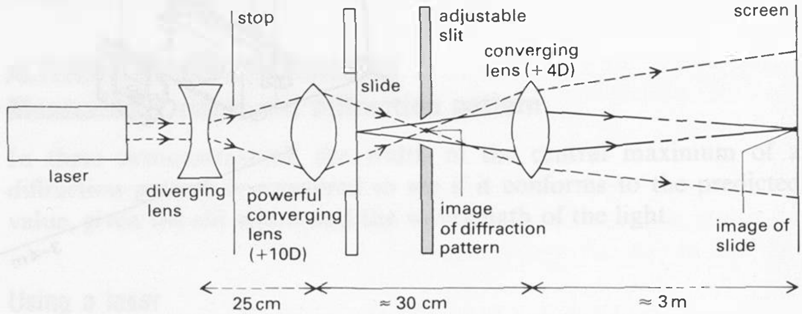


Figure J62

A phenomenon unique to diffraction imaging can be observed if an adjustable vertical slit is placed so as to cut off the higher orders of the diffraction pattern. What happens to the recombined image of the aperture on the slide as the diffraction pattern is increasingly restricted? Why is the finer detail removed before the coarse detail? This sort of image degradation causes problems in optical systems with limited apertures.

The previous slide is removed and one consisting of an aperture with fine vertical lines is substituted (figure J63).

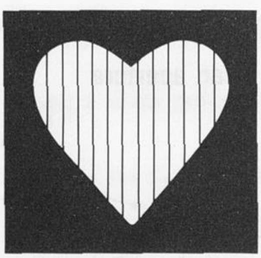


Figure J63

What happens to the image of the slide when the adjustable vertical slit is narrowed? Why is this?

This sort of image degradation can actually be used to *improve* image quality by the selective removal of certain unwanted features.

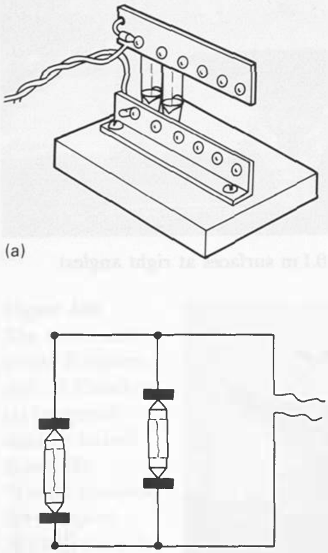
The apparatus is returned to the arrangement of figure J61. Now use a slide of the diffraction pattern of a pin-hole (see figure J3). What do you see on the screen?

Try the slide of the diffraction pattern of the letter 'X'. An image of the pinhole or letter has been reconstructed by forming a diffraction pattern of its own diffraction pattern. This method of image reconstruction has close links with the process of holography.

EXPERIMENTS

J7 Resolution

J7a Distinguishing lamps as separate



(a)

(b)

Figure J64
Resolution of two sources. (a) Multiple light source; (b) 2 filaments, vertically displaced.

holder for two halves of a razor blade, to be used as a single slit
set of colour filters (red, blue, green)
aluminium foil
slide mounts
copper wire, 0.2 mm diameter, bare
steel or nichrome wire, 0.2 mm diameter, bare
transformer

either
multiple light source with two lamps
or
2 mounted festoon lamps

Adjust your single slit to 0.2 mm using the steel wire. Hold the slit in front of your eye and view the two filaments with the green filter in place. Move back until these filaments can *only just* be seen as separate, that is, the two diffraction patterns overlap, but not so much as to make the source seem to have only one filament. Replace the green filter with firstly a blue and secondly a red filter, keeping the slit width and your distance from the lamps the same.

What do you notice? What does this tell you about the ability to resolve objects clearly and the wavelength of light used?

Try changing the slit width. What effect does this have?

Try changing the slit for a hole of equal size in aluminium foil. How does the resolution of the two lamps compare?

J7b Resolving detail with the eye

lamp, holder, and stand
transformer
card
plane mirror
transparent ruler with mm graduations

We always see through an aperture – the pupil of our eye. This experiment checks to see whether our ability to resolve closely spaced objects is limited by diffraction.

Draw two parallel, black lines, 2 mm apart, on some card. Illuminate the card and see how far away you can move before your eye fails to resolve the two lines. Try this for each eye separately.

The angular separation of the lines at your eye in radians is 2 mm/(distance between your eye and the card in mm). Calculate this value.

To calculate the limit of resolution predicted by the Rayleigh criterion, you will have to take an 'average' value for the wavelength of visible light ($\lambda \approx 5 \times 10^{-7}$ m) and measure the diameter of your pupil. This can be done by holding a ruler to your eye and looking at its reflection in a mirror. What is the predicted limit of resolution, λ/b?

How does this compare with your measured value? Suggest what factors other than diffraction might limit the ability of your eye to resolve detail.

DEMONSTRATION
J8 Model of a radio interferometer

microwave transmitter
microwave receiver
2 wax lenses (if available)
2 metal plates about 0.3 m square
general purpose amplifier
loudspeaker (if not part of above item)
metal plate (0.2 m square, bent to form two 0.2 m × 0.1 m surfaces at right angles)
table on wheels, or trolley, or sheet of hardboard

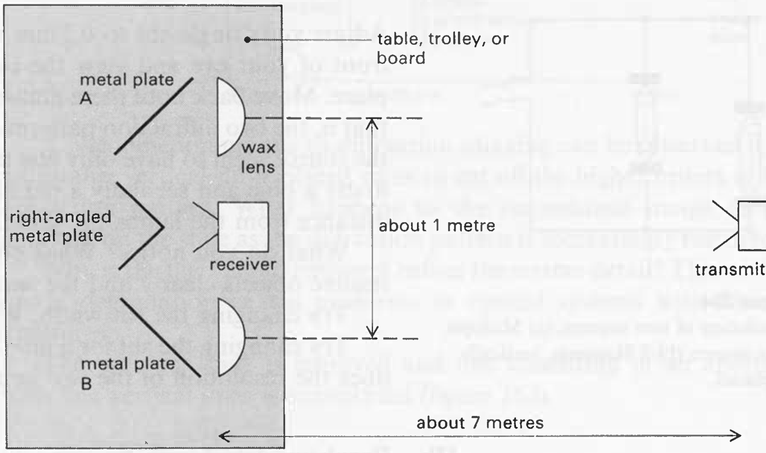


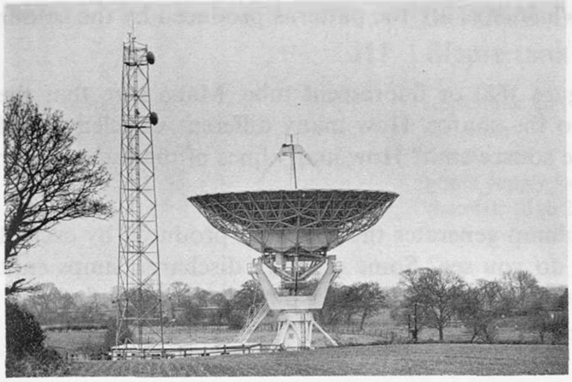
Figure J65
Microwave analogue of radio interferometer.

The receiver is pointed *directly* at the transmitter and the angle over which an appreciable signal is received is noted.

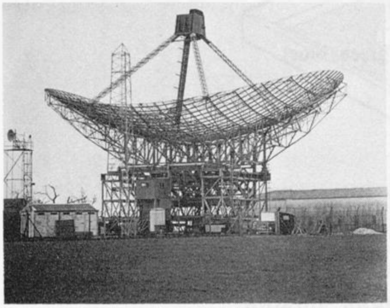
Then the receiver is positioned as in figure J65 and it is rotated together with the various reflectors (and lenses). What happens to the response of the system?

The two signals from the metal plates A and B are being superposed by the receiver. Suggest why the new arrangement gives improved resolution.

A radio interferometer at Cambridge uses several aerials spaced along a railway line. This idea has been taken further with MERLIN – Multi-Element Radio-Linked Interferometer Network – which consists of six individual radio telescopes with 25 m dishes linked electronically to a computer. It stretches across much of the West Midlands and East Wales, and gives the resolution equivalent to a 113 km dish. Having fifteen interferometer pairs of different orientations and baselines, it enables astronomers to make a detailed map of radio sources. (See figures J66 and J67.)



(a)



(b)

Figure J66
The tower next to the E-system dish at Knockin (a) transmits data to Jodrell Bank; the Wardle telescope (b) is one of MERLIN's older dishes.
Jerry Mason

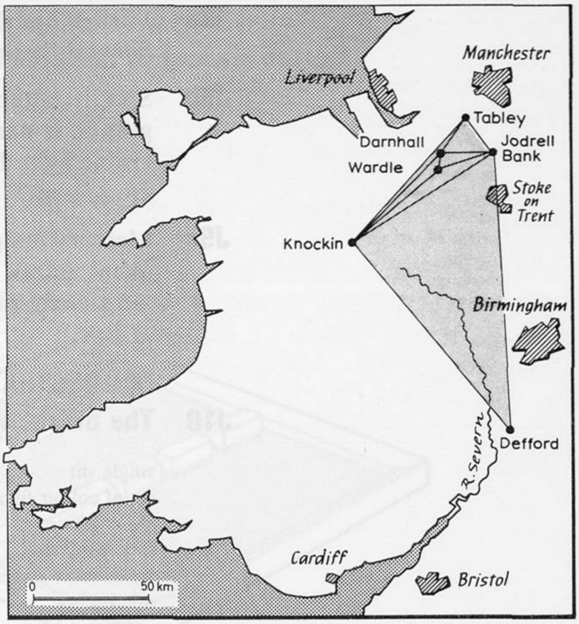


Figure J67
The network already covers much of the West Midlands, and it may be extended to include Cambridge.
New Scientist, *96, 1332, Nov. 1982, page 446*

J

CIRCUS OF EXPERIMENTS

J9 Looking through gratings

coarse grating, 100 lines mm^{-1}
fine grating, 300 lines mm^{-1}
neon (and/or hydrogen) spectrum tube (fluorescent tube will do)
e.h.t. supply
mercury discharge lamp (optional)
mains lamp with 30 cm straight filament and holder
set of colour filters (red, green, blue)
fine black chiffon (or umbrella material)

Hold each of the gratings and the chiffon in turn in front of your eye and look at the various light sources.

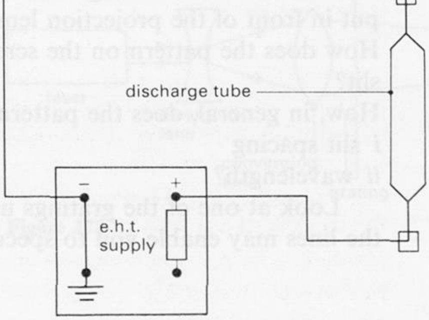


Figure J68

J9a Filament lamp. In what ways are the patterns produced by the colour filters different?

J9b Spectrum tubes (figure J68) or fluorescent tube. Make sure that the grating is parallel to the source. How many different wavelengths of visible light does the source emit? How many lines of the same colour are present?

J9c Mercury lamp. The lamp generates the spectrum produced by excited metal atoms. What do you see? Some mercury discharge lamps emit ultra-violet radiation: you should not look directly at such a lamp.

DEMONSTRATION

J10 The diffraction grating

single slit
set of colour filters (red, green, blue)
set of gratings
slide projector
translucent screen
microscope

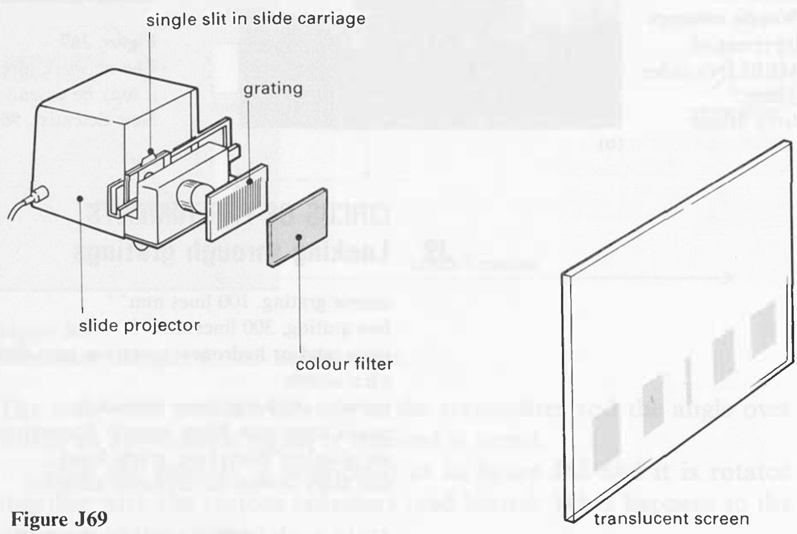


Figure J69

The projector must be focused so that (with no grating) a sharp image of the slit in the slide carriage is formed on the screen. The grating is then put in front of the projection lens.
How does the pattern on the screen differ from that given by a single slit?
How, in general, does the pattern depend on
i slit spacing
ii wavelength?

Look at one of the gratings under a microscope – the poorness of the lines may enable you to speculate how they were made.

OPTIONAL DEMONSTRATION

J11 Ripple tank demonstration of grating pattern

ripple tank kit
illuminant
transformer
power supply for motor
rheostat, 10 to 15 Ω
hand stroboscopes

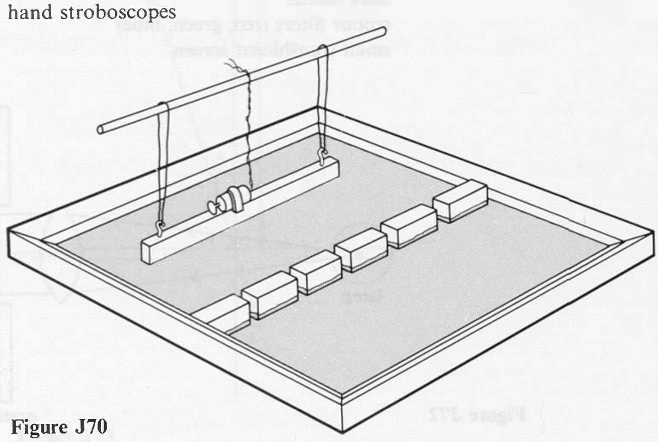


Figure J70

Look for semi-circular ripples emerging from the slits. If you try looking along the water surface you should be able to see a wave moving straight ahead, and waves moving at various angles either side of this one. This is sometimes difficult to see, so do take trouble over it.

What happens to the spacing of the strong reinforcement lines when the wavelength is changed?

It may help to use a stroboscope to freeze the pattern. Make sure you have tackled question **20**.

DEMONSTRATION/EXPERIMENT

J12 Sharpness of maxima depends on number of slits

Demonstration

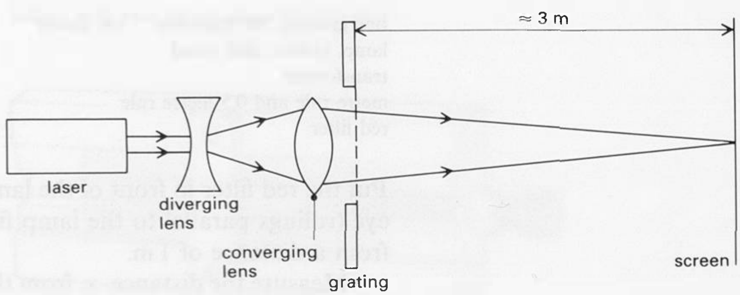


Figure J71

Experiment

lamp, holder, and stand
transformer
2 lenses, $+1D$ or $+0.5D$
eyepiece
set of coarse gratings
set of parallel slits
slide holder
colour filters (red, green, blue)
small translucent screen

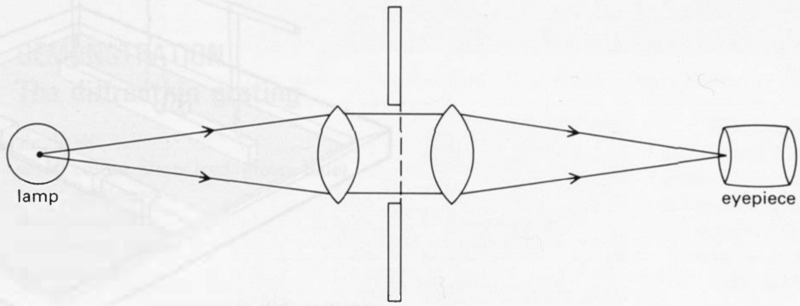


Figure J72

The set of parallel slits contains slides with 1, 2, 3, 4, 5, and 6 slits. All have the same slit width and spacing as each other and as the coarsest of the gratings.

Starting with this grating, describe what you see when you observe the diffraction pattern for red light through the grating, then in turn through 6, 5, 4, 3, and 2 slits.

Return to the coarsest grating in the set. Some of the higher orders are missing. Suggest a reason for their absence. (*Clue:* look carefully at the very coarse grating pattern, then at the pattern given by the single slit of the *same slit width.*)

EXPERIMENT

J13 Measuring the wavelength of light

J13a Using a grating

fine grating, 300 lines mm^{-1} (or more)
lamp, holder, and stand
transformer
metre rule and 0.5 metre rule
red filter

Put the red filter in front of the lamp. Hold the grating in front of your eye (rulings parallel to the lamp filament) and look towards the lamp from a distance of 1 m.

Measure the distance, x, from the middle of the central maximum to the middle of the first order maximum. You will need to use a pointer and it may help to have the services of a friend. From x, L (the grating to screen distance), and s (the grating spacing), calculate λ_{red}. What is the uncertainty in your value?

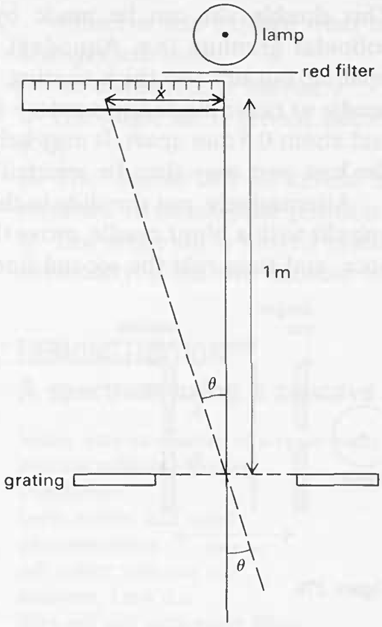


Figure J73

J13b Using Young's double slits

lamp, holder, and stand
transformer

either
microscope slide
colloidal graphite
slide holder for ruling slits
needle
or
slide with two slits

translucent screen
hand lens
set of colour filters (red, green, blue)
transparent millimetre scale
0.5 metre rule
holder with two halves of a razor blade

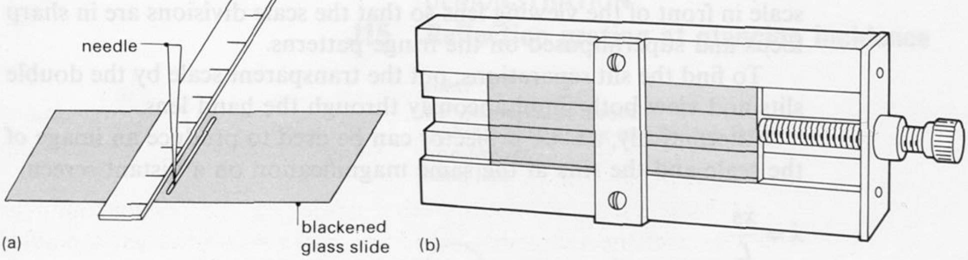


Figure J74

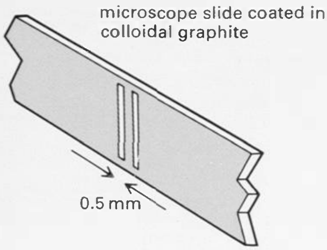


Figure J75

This double slit can be made by coating a microscope slide with colloidal graphite (*e.g.* Aquadag), then ruling a pair of lines on the opaque, but not too thick coating. The lines can be made with a blunt needle or razor blade and a ruler – figure J74(a). They should be parallel and about 0.5 mm apart. It may help to make several such pairs of lines, the best pair may then be selected by trial and error.

Alternatively, put the slide in the special holder – figure J74(b). Rule one slit with a blunt needle, move the slide 0.5 mm by turning the screw once, and then rule the second line.

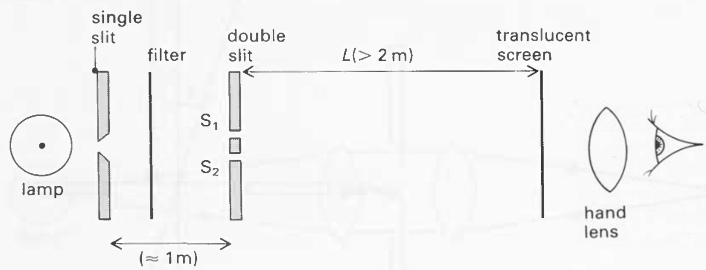


Figure J76

Make sure that the vertical filament, slits, and screen are in line and that the adjustable (razor blade) slit provides enough light to cover the double slit.

The fringes may be viewed closer than 2 m from the slits if a magnifying lens is used, and there may then be no need for the screen.

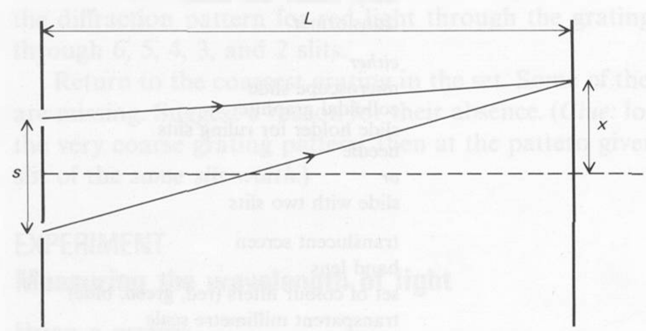


Figure J77

The fringe separation, x, can be found by putting the transparent scale in front of the viewing lens so that the scale divisions are in sharp focus and superimposed on the fringe patterns.

To find the slit separations, put the transparent scale by the double slits and view both simultaneously through the hand lens.

Alternatively, a slide projector can be used to produce an image of the scale and the slits at the same magnification on a distant screen.

$$\lambda = \frac{xs}{L}$$

Obtain a value for the average wavelength transmitted by the filter, and estimate the uncertainty in your result.

Observe and explain what happens to the pattern if the following changes are made.
i The distance L is halved.
ii The single slit is moved sideways in the plane of the diagram (figure J76).
iii The double slit is moved sideways, the single slit having been returned to its original position.
iv The single slit is moved towards the double slit. What may happen, eventually, if the two become too close? Why?

DEMONSTRATION
J14 A spectrum using a concave reflection grating

holder with two halves of a razor blade
concave reflection grating
transformer
lamp, holder, and stand
phototransistor
cell holder with one cell
ammeter, 1 mA d.c.
infra-red and ultra-violet filters
white screen (non-fluorescent)
fluorescent paper (green)
leads

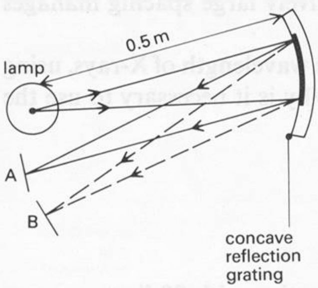


Figure J78
Concave reflection grating.

The grating is stuck to a concave spherical surface so that it both produces a conventional grating diffraction pattern by reflection and focuses it at the same time. In figure J78, the image of the filament is formed at A on the screen. If the screen is moved to B, first and higher order spectra should be visible. An additional benefit is that the reflection grating does not absorb any ultra-violet (u.v.) or infra-red (i.r.) radiation, whereas most other gratings do. Why is this?

Why are special methods needed to detect u.v. and i.r. radiation?

Over what part of the spectrum is the phototransistor sensitive to radiation? To which wavelength of radiation is it most sensitive?

Over what part of the spectrum is the fluorescent paper sensitive?

What sort of detectors might be used for radiation beyond i.r. and u.v.?

DEMONSTRATION
J15 Reflection grating at glancing incidence

either
LP gramophone record
multiple light source and festoon lamp
transformer
hand lens

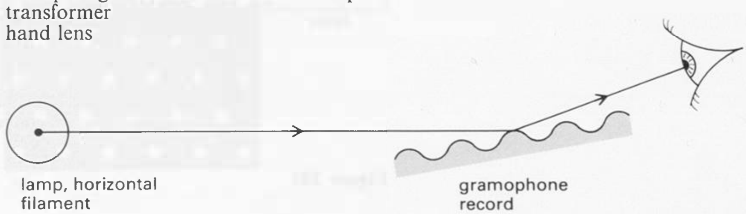


Figure J79
Reflection grating at glancing incidence using a gramophone record.

or
metal rule with 0.5 mm divisions
laser
adjustable height stand (Labjack)
white screen
Plasticine

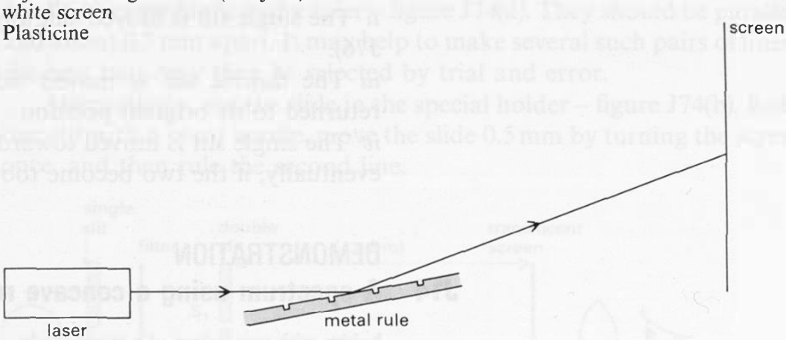


Figure J80
Reflection grating at glancing incidence using a laser and metal rule.

Both the record and the rule act as diffraction gratings for light even though the grooves are very many wavelengths apart. The pattern can be seen if the light strikes the 'grating' at an angle of incidence of very nearly 90°. (*N.B. DO NOT* look down the laser beam but look at the pattern on the screen.) Describe what you see.

i Look at a record with the hand lens. Estimate how many wavelengths of visible light would fit into one groove spacing.

ii Explain how a grating with such a relatively large spacing manages to form a diffraction pattern.

iii The same method is used to measure the wavelength of X-rays, using a grating ruled for use with visible light. Why is it necessary to use the grazing angle method?

DEMONSTRATION

J16 Diffraction at complex gratings

i What does the diffraction pattern of a grating with 80 lines per mm look like?

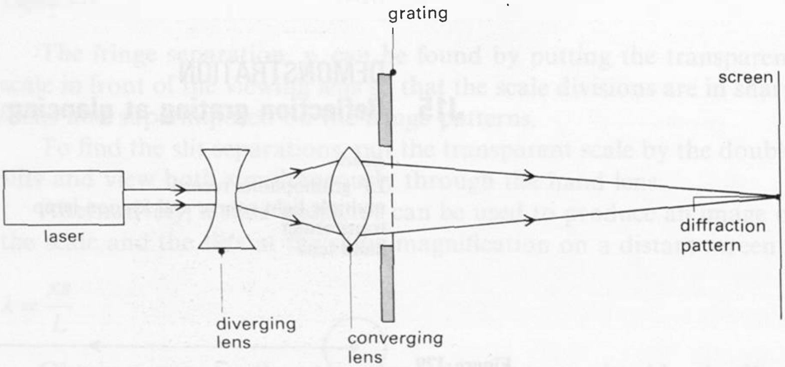


Figure J81

ii Sandwich two such gratings together, with their rulings perpendicular. Sketch the diffraction pattern. It is *not* merely a single row of vertical and horizontal dots as you might expect. Suggest a reason for the increased complexity.

iii Try the piece of black chiffon that you used in experiment J9. It has a square weave but is much coarser than the gratings used above. How could you tell this from its diffraction pattern? Another feature of its diffraction pattern is the absence of spots in certain regions of the pattern. What might cause this?

iv The two crossed gratings form an array of regularly spaced square holes – figure J82(a). The diffraction pattern produced by such an array is simply a scaled version of the pattern produced by the chiffon. By looking at the diffraction patterns produced by the arrays shown in figures J82(a), (b), (c), (d), and (e) try to identify which characteristics of an array determine

a the pattern and spacing of the dots

b the relative intensities of the dots in its diffraction pattern.

The same factors determine the arrangement and intensities of the dots in the X-ray diffraction pattern produced by an array of atoms, as in a solid.

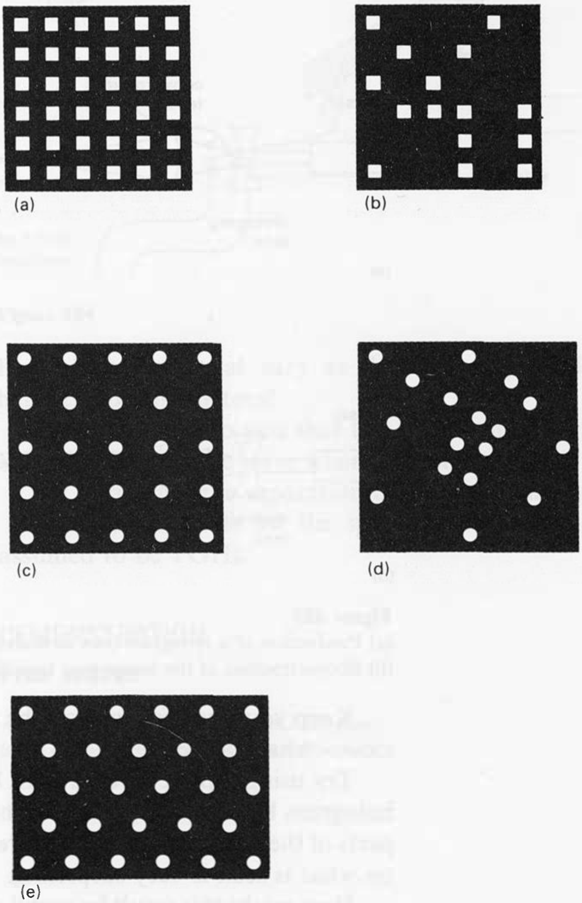


Figure J82
Aperture arrays.

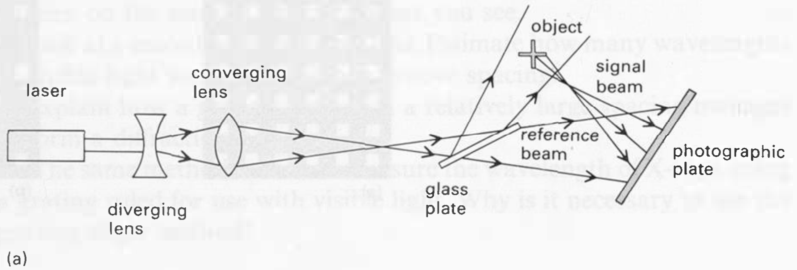
DEMONSTRATION

J17 Reconstructing the image from a hologram

The production of a hologram is extremely difficult in school using a low power laser. A schematic layout of the apparatus is shown in figure J83(a).

A glass plate acts as a beam-splitter and the object is illuminated by the laser light reflected by the plate. The reference beam produces a uniform patch of light on the photographic plate. Light scattered from the object, the signal beam, strikes the plate and forms a complex superposition pattern with the reference beam which is recorded on the photographic plate.

The reconstruction process shown in figure J83(b) is quite straightforward to carry out in schools and consists, essentially, of illuminating the hologram with a laser from the same direction as the reference beam in the production stage. To view the hologram you should look for the reconstructed virtual image *behind* the plate and *not on* the plate. It helps to locate the image if you move your head from side to side looking through the plate all the time.



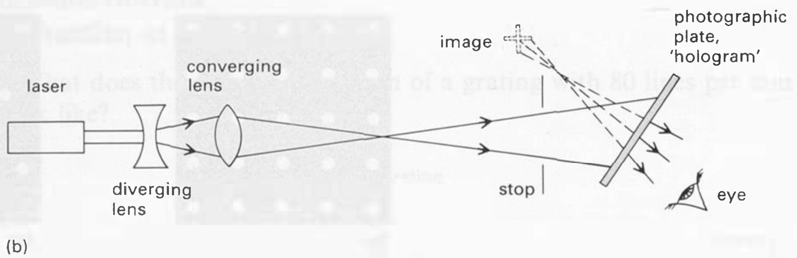


Figure J83
(a) Production of a hologram (not to scale).
(b) Reconstruction of the image (not to scale).

Keep your eyes focused on the image but change your direction of view – what happens?

Try using a piece of card with holes in as stops placed near to the hologram both to reduce the illuminated area and to select particular parts of the hologram. Clearly there is altogether less light but the effect on what is seen is very surprising.

How might this result be useful in high-density information storage?

DEMONSTRATION

J18a Guided or 'tied' waves

15 cm dipoles and oscillator
sensitive galvanometer

either
2 long retort stand rods
or
2 metre rules covered with aluminium foil

2 crocodile clips
leads

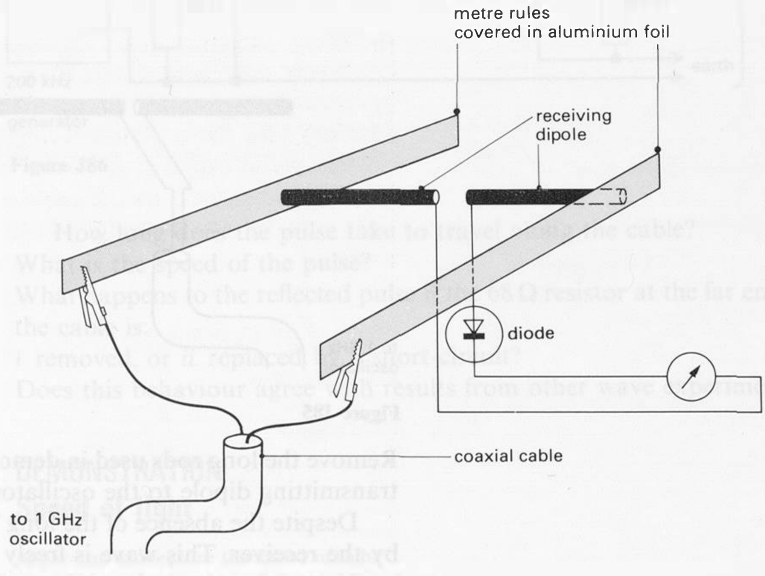


Figure J84

How does the signal vary as the receiving dipole is moved along between the conductors?

What suggests to you that this is a 'standing wave' phenomenon? What happens to the wave when it reaches the ends of the conductors?

By measuring the separation of maxima, calculate the wavelength.

Calculate a value for the speed of the waves if the frequency is assumed to be 1 GHz.

DEMONSTRATION

J18b Free waves

Apparatus as for demonstration J18a plus
large metal sheet, about 30 cm × 30 cm

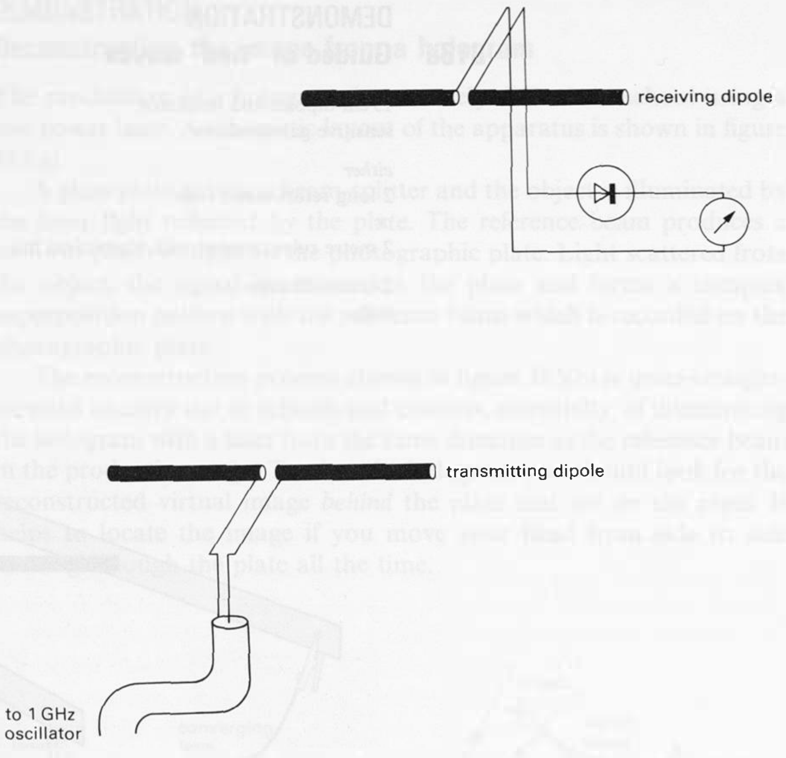


Figure J85

Remove the long rods used in demonstration J18a and attach the usual transmitting dipole to the oscillator as in figure J85.

Despite the absence of the long rods a strong signal is still detected by the receiver. This wave is freely propagating in the laboratory.

Use a large sheet of metal to reflect the wave and set up a standing wave pattern. Measure the wavelength and so calculate the speed of the wave.

What is the difficulty in measuring the speed of this wave *directly* (that is, timing its travel over a known distance)?

DEMONSTRATION
J19 The speed of a pulse along a coaxial cable

200 kHz pulse generator
200 m drum of coaxial cable
double-beam oscilloscope
2 resistors, 68 Ω
2 clip component holders
1 kΩ potentiometer
leads

The pulse generator switches a p.d. on and off 200 000 times a second across the end of a long length of coaxial cable. This is rather like demonstration J18a, except here the pair of conducting rods is very long and the signal is a pulse instead of a sinusoidal wave.

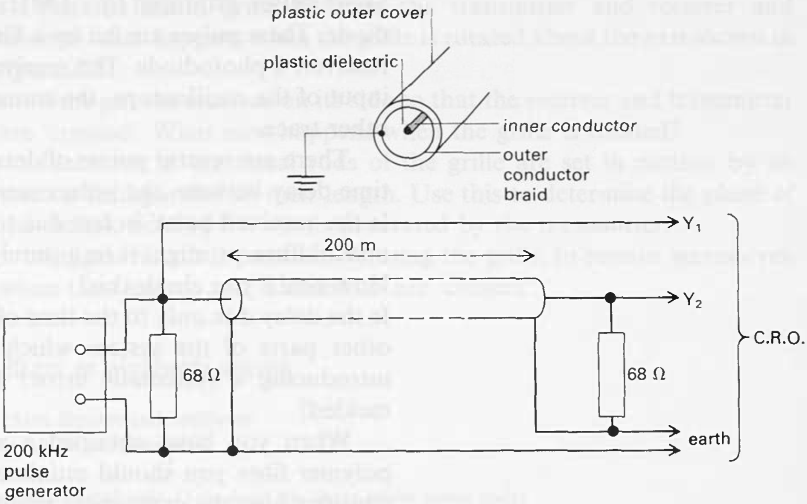


Figure J86

How long does the pulse take to travel along the cable?
What is the speed of the pulse?
What happens to the reflected pulse if the 68 Ω resistor at the far end of the cable is:
i removed, or *ii* replaced by a short-circuit?
Does this behaviour agree with results from other wave experiments?

DEMONSTRATION
J20 Speed of light

apparatus to measure the speed of light
double-beam oscilloscope, 0.1 μs div^{-1} or better
2 cell holders with 6 cells (or other 9 V d.c. supply)
leads

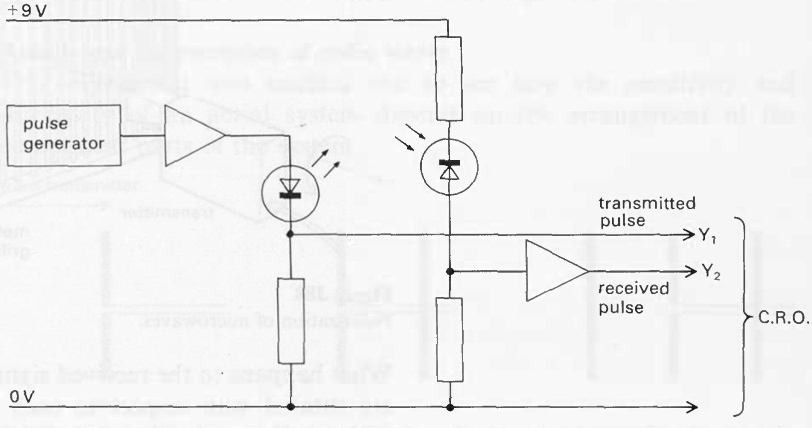


Figure J87
Measurement of the speed of light.

Short pulses, produced by a 1 MHz oscillator, are fed to a light-emitting diode. These pulses are fed by a fibre optics cable of known length to a receiver, a photodiode. The received signal is amplified and fed to one input of the oscilloscope, the transmitted pulse being displayed on the other trace.

There are several points of detail to think about in interpreting the time delay between the pulses seen on the screen.
Is the 'received pulse' in fact due to light which has travelled along the optical fibre, or might it be a purely electrical signal within the system? How could you check this?
Is the delay due only to the time of flight of the light pulse, or are there other parts of the system which might be causing a delay and so introducing a systematic error? If so, how might this difficulty be tackled?

When you have obtained a value for the speed of light in the polymer fibre you should estimate the uncertainty in your value and consider how the experiment might be improved.

By careful use of lenses and a mirror, it might be possible to adapt this method to measure the speed of light in air. What distance would a light pulse have to travel for you to have a chance of measuring the time of flight?

CIRCUS OF EXPERIMENTS

J21 Polarization

J21a Microwaves

microwave transmitter
microwave receiver
general purpose amplifier
loudspeaker (if not included in amplifier)
metal grille

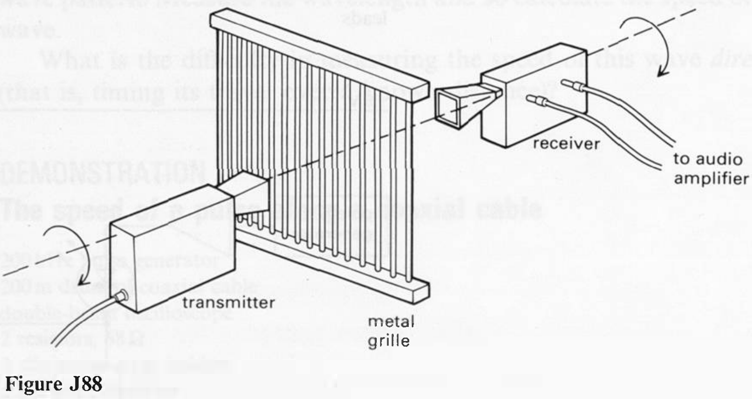


Figure J88
Polarization of microwaves.

What happens to the received signal when the transmitter and receiver are rotated with respect to each other as shown in figure J88 (but without the metal grille in place)? Does this result suggest that the wave from the transmitter is plane polarized or unpolarized?

Put the metal grille between the transmitter and receiver and observe what happens when the grille is rotated about the axis shown in figure J88.

Now put the receiver on its side so that the receiver and transmitter are 'crossed'. What now happens when the grille is rotated?

Electrons in the metal rods of the grille are set in motion by an electric field parallel to their length. Use this to determine the plane of the electric field of the waves generated by the transmitter.

Suggest how it is possible, by using the grille, to receive waves even when the receiver and transmitter are 'crossed'.

J21b 30 cm or gigahertz waves

15 cm dipoles and oscillator
sensitive galvanometer
leads
several metal rods 15 cm long (or short retort stand rods)
40 cm square rotatable platform (hardboard or similar)

J

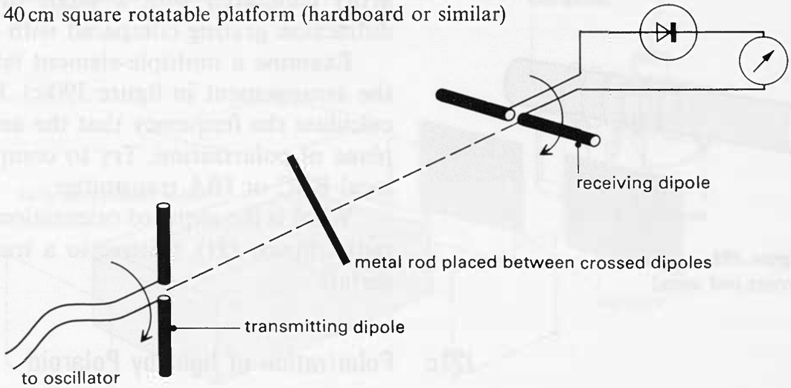


Figure J89
Polarization of 30 cm waves.

With this equipment you can do the same experiments and answer the same questions as with the microwave equipment – see experiment J21a above. Use a 15 cm metal rod in place of the grille of rods.

Aerials and the reception of radio waves
This experiment also enables you to see how the sensitivity and directivity of an aerial system depend on the arrangement of the component parts of the system.

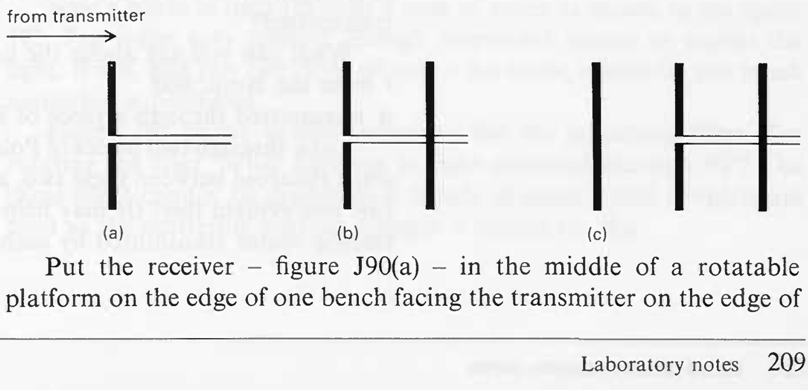


Figure J90
Aerial systems.

Put the receiver – figure J90(a) – in the middle of a rotatable platform on the edge of one bench facing the transmitter on the edge of another about 1 m away. Reflections are unavoidable, but their effects can be minimized by reducing unnecessary movement and, if possible, making the final adjustments from a distance using a metre rule.

What happens to the received signal as a rod is brought up behind the receiver dipole as in figure J90(b)?

Bring up a rod in front of the dipole as in figure J90(c). Suggest a reason for the changes in signal.

Adjust the multi-element array of figure J90(c) until it produces the biggest signal. Rotate the whole array until the signal has fallen by a factor of between 5 and 10. Now remove the front and rear rods. What happens to the signal? Return the receiver to its original position facing the transmitter and note the size of the signal. How does the directivity of the simple dipole compare with that of the three-element array?

By considering the rods as sources of secondary wavelets, use the principle of superposition to suggest how the array has increased sensitivity and directivity. In what way could the behaviour of an aerial array compared with a single dipole be said to be like that of a diffraction grating compared with a single slit?

Examine a multiple-element television aerial and compare it with the arrangement in figure J90(c). Estimate the wavelength and hence calculate the frequency that the aerial is designed for and identify the plane of polarization. Try to compare your result with data from the local BBC or IBA transmitter.

What is the required orientation of a ferrite rod and coil, such as in a radio (figure J91), relative to a transmitter for it to be effective as an aerial?

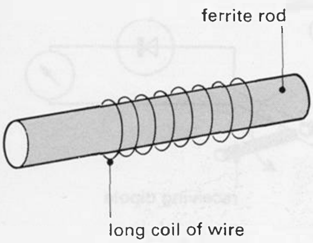


Figure J91
Ferrite rod aerial.

J21c Polarization of light by Polaroid

3 polarizing filters (Polaroid)
lamp, holder, and stand
transformer
transparent adhesive tape
cellophane paper
microscope slides

Look at the lamp, on its own, through a polarizing filter (Polaroid). Try rotating the filter about an axis parallel to the light beam. What effect does it have?

Try looking at the lamp through two pieces of Polaroid and rotating one relative to the other. What effect does this have on the light transmitted?

What can you say about the light
i from the lamp, and
ii transmitted through a piece of Polaroid?

Look through two pieces of Polaroid in the 'crossed' position. Add a third Polaroid between these two, and rotate it. What do you see? How can you explain this? (It may help to think about the direction of the electric vector transmitted by each of the successive Polaroids.)

You may already have seen how polarized light can be used to detect strain, for example when polythene is stretched between crossed polarizers. Transparent adhesive tape or cellophane wrapping material also give beautiful colour effects when viewed between crossed polarizers. Try it.

J21d Polarization of light by reflection and by scattering

2 polarizing filters (Polaroid)
rectangular plastic tank
lamp, holder, and stand
housing shield
2 barriers
plano-cylindrical lens, + 7D
transformer
small sheets of glass, polythene, and metal (*e.g.* aluminium foil)
milk or powdered milk

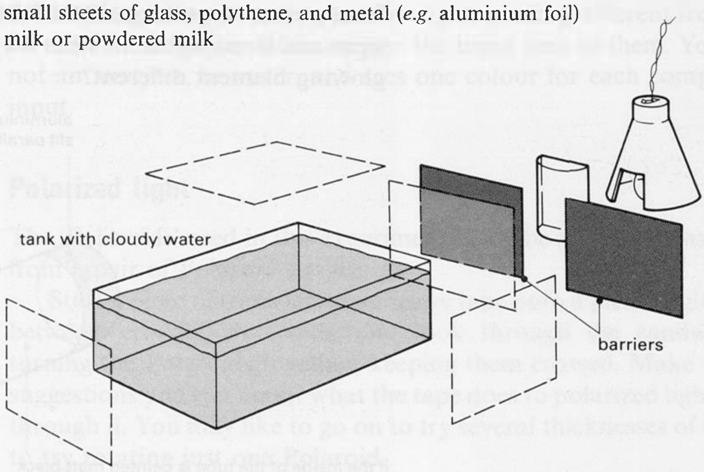


Figure J92
Scattering by cloudy water. (Broken rectangles show positions for a Polaroid filter.)

Using just one Polaroid sheet look at light reflected from
i the bench,
ii a sheet of glass,
iii a shiny metal surface, and
iv a polythene sheet.

Try different angles of reflection. In each of the cases, is the reflected beam polarized?

Send a beam of light through a tank of water as shown in the figure J92. Tap water may contain enough suspended matter to scatter the light. If not, add just one drop of milk – no more, otherwise too much scattering is produced.

Figure J92 shows various positions for the polarizing filter. Try rotating the filter. What happens to light scattered through 90°? The same effect cannot be produced if clouds of small water droplets are used as the scattering centres. Suggest a reason for this.

HOME EXPERIMENTS

JH1 A homemade slit

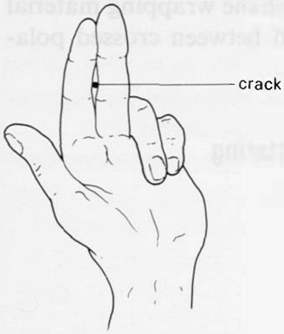


Figure J93

Hold your first and second fingers together so that there is a narrow crack between the middle parts of the two fingers, although they touch at the top, as in figure J93. Look at a distant small lamp through the crack, and press the two fingers gently together. As the crack closes, the lamp appears dimmer (obviously), but when the crack is very narrow, the lamp also seems to get wider. Why?

JH2 Simple spectroscopy

You can make a simple spectroscope and look at a reflection of sunlight from a piece of white card (*NEVER* look directly at the Sun), or at shop signs and street lights. In what ways are the spectra of the Sun and of a glowing filament different?

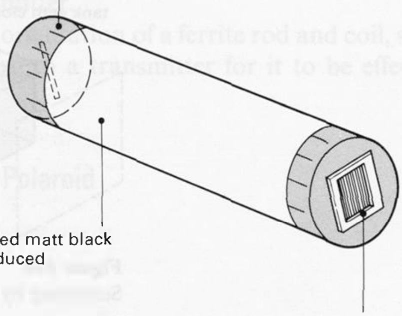


Figure J94

Light from the hot outer surface of the Sun has to pass through the Sun's cooler (though still hot) atmosphere. This atmosphere contains atoms and ions of many different elements, which absorb radiation at the same frequencies as those at which they emit. Since the emitted radiation is in all directions, the Sun's 'white light' spectrum has light missing from it at a number of frequencies. If the spectrum is formed from a slit, the spectrum seems to be crossed by narrow dark lines. These lines in the absorption spectrum were found by Fraunhofer, and are often named after him.

JH3 Ear and eye

This experiment is about an interesting difference between what your ears and eyes do to signals containing more than one frequency. A physicist would say it shows that your ear is a 'Fourier analyser', but that your eye is not.

Get someone to play you three or four notes on a piano, one at a time. Give them arbitrary names, say P, Q, R, etc. Then get your helper to play two at once, and see if you can say which two notes are being played.

Now supply your eyes with two colours of light simultaneously, as follows. Stand in front of a pair of electric lamps with a diffraction grating held over your eye, and move about until part of the lefthand first-order spectrum from one lamp seems to fall over part of the righthand first-order spectrum from the other lamp. Suppose red overlaps blue: do you see 'red and blue', or some other, single colour?

Both eye and ear were supplied with a complex oscillation made up of at least two frequencies. Your ear and brain, at least partially, separate the complex oscillation into its constituent frequencies. A diffraction grating does the same for light, sending different frequencies off at different angles. Both analyse the input sent to them. Your eye is not an analyser, however, and sees one colour for each complex light input.

J

JH4 Polarized light

The 'Polaroids' used in this experiment could be a pair of 'lenses' taken from a pair of Polaroid sunglasses.

Stick a piece of transparent adhesive tape onto a piece of glass, put it between 'crossed' Polaroids, and look through the sandwich. Try turning the Polaroids together, keeping them crossed. Make whatever suggestions you can about what the tape does to polarized light passing through it. You may like to go on to try several thicknesses of tape, and to try rotating just one Polaroid.

You may recall, from Unit A, 'Materials and mechanics', that materials like polythene sheet which do not behave like the tape can be made to do so if they are strained. Try cutting a V-shaped notch in the side of a long strip of polythene, and pull it longways between crossed Polaroids.

On a sunny day, stand so that the Sun is on your right or on your left, and look at a patch of blue sky through a piece of Polaroid. Choose a patch of sky from which the light to your eye is at right angles to the direction from it to the Sun.

Rotate the Polaroid, and try to explain what you see. (*Harder.*) Refer to question **46**, and try to decide along which direction in your Polaroid sheet the iodine-loaded, long-chain polymer molecules generally lie.

QUESTIONS

Superposition

1(I) Figure J95 shows circular ripples on water, produced by a disturbance of the water surface.

In what sense do the ripples not affect one another? In what sense do the ripples affect one another? Explain what physicists mean when they say that waves 'superpose' on one another. You may be able to think of an example where water waves do not superpose, at least not in the simple sense intended by physicists when they use the term.

Figure J95
Ripples on water.
Barnaby's Picture Library

Figure J96
Morro Bay, California.
University of California

Diffraction

2(I) Figure J96 is an aerial photograph of a narrow harbour mouth formed by two angled jetties, open to the sea.

How can there be waves inside the harbour, well within the 'shadow' of the jetty which runs parallel to the waves in the open sea? Make a sketch showing what you expect to see in a ripple tank if straight waves reach a barrier which blocks off half the tank.

Huygens's construction

3(L)a Figure J97 is based on one in Huygens's *Treatise on light* (1690), and is part of his explanation of how a wave theory of light could explain the fact that light is reflected from a flat, mirror-like surface at an angle equal to that at which the light strikes the surface.

AB is the flat mirror, seen edge on. Identify the lines AC and NB. The distance CB is drawn equal to the distance AN. Suggest how Huygens used this diagram to advance his explanation.

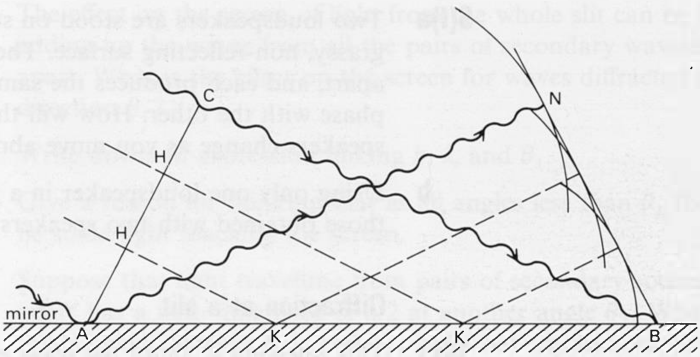


Figure J97

b Figure J98 is part of Huygens's explanation of the refraction of light as it passes from, say, air into water or glass. AB is the flat surface of, say, water. Identify the lines AC, HK, and NB.

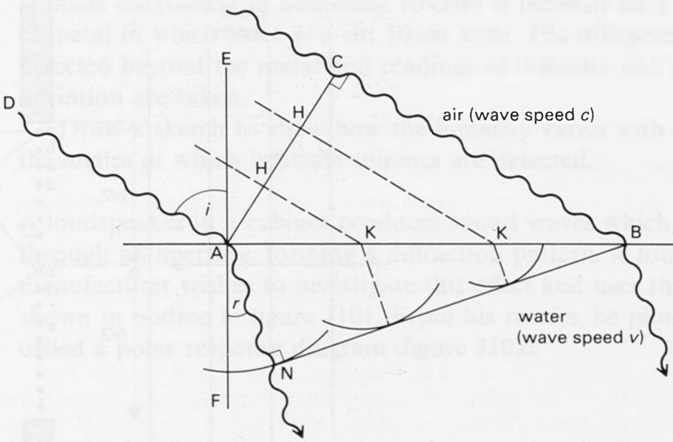


Figure J98

Suggest how Huygens explained refraction, particularly the fact that the ratio of the sines of angles DAE and FAN is constant for any one pair of materials. You will need to know that light travels more slowly in water or glass than it does in air.

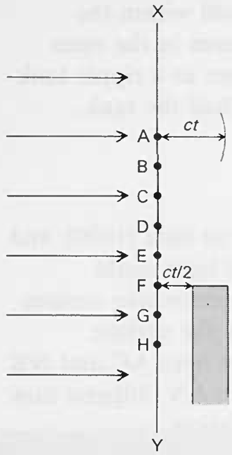


Figure J99

c Show that $\sin i/\sin r = c/v$, where i and r are the angles of incidence and refraction (angles DAE and FAN respectively in figure J98). This ratio, the ratio of the speed of light in air (strictly in a vacuum) to its speed in the transparent material is called the refractive index of the material.

4(L) A single wavefront is represented by a line XY in figure J99. The circular arc is a secondary wavelet drawn from A with radius ct. The obstacle is distance $ct/2$ from the wavefront. Copy and complete the diagram by drawing a series of secondary wavelets from B, C, D, etc., and sketch the shape of the wavefront after time t. Also sketch the shape of the wavefront after $2t$.

Superposition

5(I)a Two loudspeakers are stood on stools out-of-doors on a rough, grassy, non-reflecting surface. They are mounted about a metre apart, and each produces the same musical note, and oscillates in phase with the other. How will the sound heard from the two speakers change as you move about in front of them?

b Using only one loudspeaker in a room, you can get effects similar to those obtained with two speakers outside. How do you explain this?

Diffraction at a slit

6(L) This question is about the wave energy diffracted in a particular direction at an angle θ to the straight-through direction when a plane wavefront, wavelength λ, is incident at a slit, width b (figure J100).

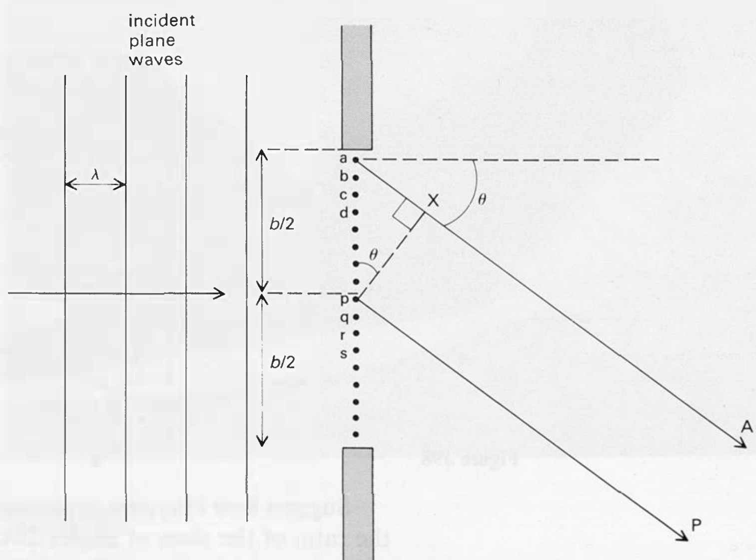


Figure J100

a Consider the plane wavefront reaching the slit to consist of a row of secondary wavelet sources a, b, c, etc. (as in Huygens's construction). All the sources are equally spaced, and p is just over half way across the slit. What is the distance between a and p, b and q, and c and r in terms of the slit width b?

b What is the path difference (aX) between waves from a and p travelling in the directions A and P at an angle θ to the original direction? Express this in terms of b and θ.

c If the path difference between a and p at a particular angle θ_1, is $\lambda/2$, what will be the effect of superposing waves from a and p?

d What will be the effect on a distant screen of light coming from the pairs of sources b and q; c and r; d and s; etc. all travelling in this same direction θ_1?

e The effect on the screen of light from the whole slit can be found by adding up the waves from all the pairs of secondary wavelets $b/2$ apart. What is the effect on the screen for waves diffracted in the direction θ_1?

f Write down an expression linking b, λ, and θ_1.

g Give a reason for thinking that at all angles less than θ_1 there will be some light reaching the screen.

h Suppose that light travelling from pairs of secondary sources $b/4$ apart has a path difference of $\lambda/2$ at another angle θ_2. What is the relationship between b, λ, and θ_2, and what will be seen on the screen at this angle?

i Write down a general equation predicting the directions of minima in a single-slit diffraction pattern.

7(P) A plane microwave of frequency 10 GHz is incident on a large piece of metal in which there is a slit 10 cm wide. The diffracted wave is detected beyond the metal and readings of intensity and angular deviation are taken.

Draw a sketch to show how the intensity varies with angle. Show the angles at which intensity minima are detected.

8(P) A loudspeaker in a cabinet produces sound waves which pass through an aperture, forming a diffraction pattern. A loudspeaker manufacturer wishes to investigate this effect and uses the equipment shown in outline in figure J101. From his results, he plots what is called a polar response diagram (figure J102).

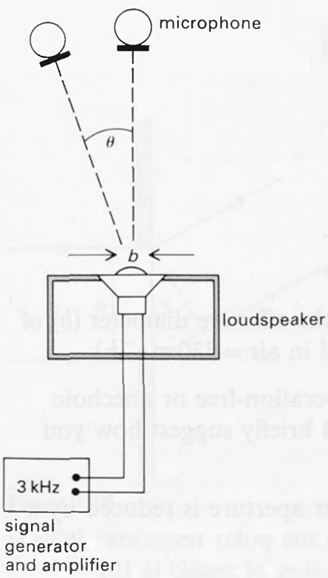


Figure J101

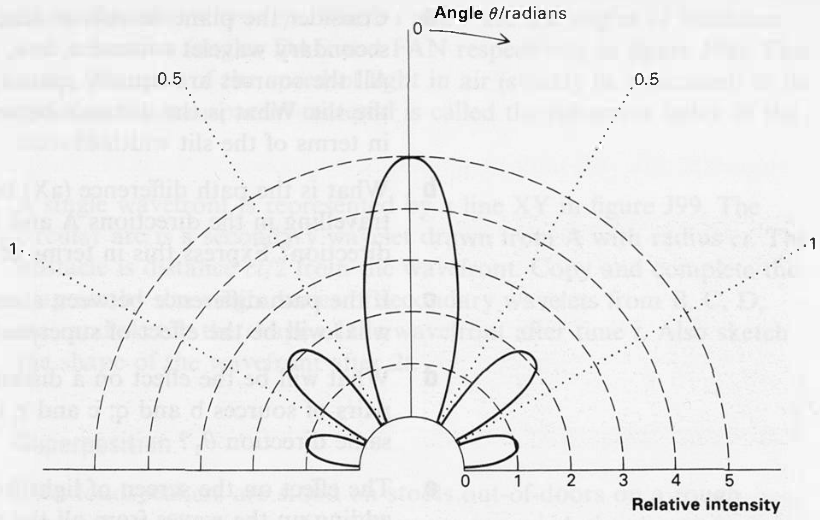


Figure J102

a Figure J103 shows the same data, plotted in a form familiar from earlier work. Explain how it is related to figure J102. Why may figure J102 be more useful?

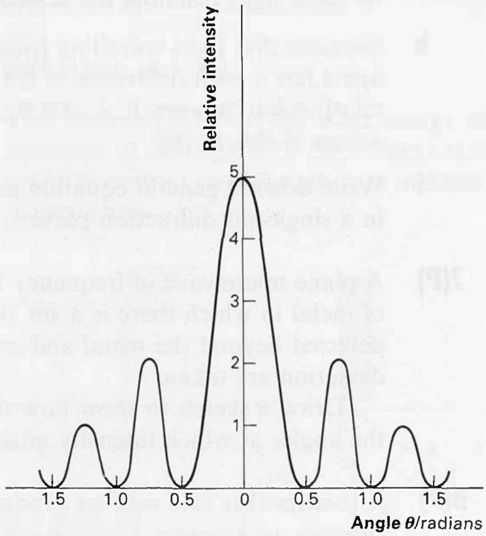


Figure J103

b From the results shown above, calculate the effective diameter (b) of the loudspeaker aperture. (Speed of sound in air $= 330\,\mathrm{m\,s^{-1}}$.)

c Such an experiment really requires reverberation-free or anechoic conditions. Say why this is important and briefly suggest how you would produce a suitable environment.

d If the effective diameter of the loudspeaker aperture is reduced by a factor of ten, what effect will this have on the polar response? Why is this advantageous to the faithful reproduction of music in the domestic setting?

The polar response is a very important property of many wave systems: microwave transmitters and receivers, radio and television aerials, as well as loudspeakers and microphones. (See 'Aerials', page 168.)

9(P) Diffraction patterns of six letters of the alphabet are shown in figure J104.

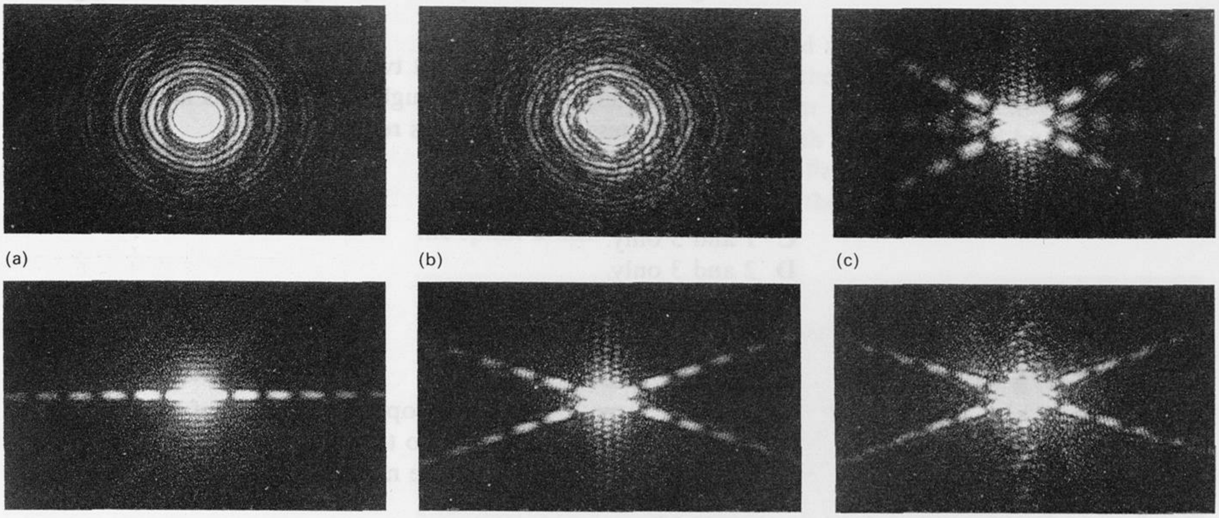


Figure J104
Diffraction patterns of some letters of the alphabet.
Dr A. Winter

Though not in order, the letters are: A, I, O, Q, V, Y. By considering the single-slit and circular hole diffraction patterns, try to decide which pattern belongs to which letter.

10(R) Figure J105 is used to illustrate an argument which shows that for diffraction from a single slit, the intensity first falls to zero when $b \sin\theta = \lambda$.

Here are three deductions made from this result.

1 If a longer wavelength is used the main diffraction peak will be broader.

2 If white light is used the edges of the main diffraction peak will be coloured.

3 If a slit with smaller b is used the main diffraction peak will be broader.

Which of these deductions is/are true?

A 1 only.
B 2 only.
C 1 and 2 only.
D 1 and 3 only.
E 1, 2, and 3.

(Coded answer paper, 1970)

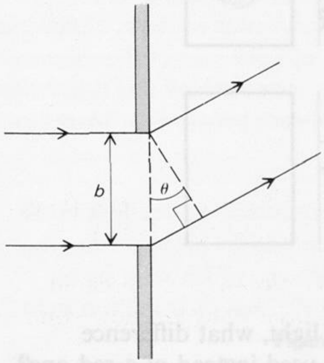


Figure J105

Resolution

11(R) The image of a star so distant as to be a point source appears in a telescope as a fuzzy disc, because of diffraction at the objective aperture of the telescope.

If the diameter of the objective aperture is halved, which of the following statements correctly describes aspects of the resulting effect?

1 The fuzzy disc becomes about twice as wide.
2 The light energy coming through the aperture is halved.
3 The brightness of the image is reduced.

A 1 only.
B 1 and 2 only.
C 1 and 3 only.
D 2 and 3 only.
E 1, 2, and 3.

(Coded answer paper, 1977)

12(P) The World's largest optical telescope has a mirror of 5 m diameter whilst the 'mirror' of a large radio telescope may be as large as 80 m across. Why is the radio telescope mirror so much bigger?

13(L) A telescope is set to view a star. The image at the focus of its main lens, when viewed through an eyepiece or when photographed, is not a sharp point of light, but is blurred out into a disc surrounded by some fainter rings. Figure J106(a) suggests the way the intensity varies across the pattern. A line from point X on the pattern to the middle of the lens makes an angle θ with a line from point Y at the middle of the pattern to the lens.

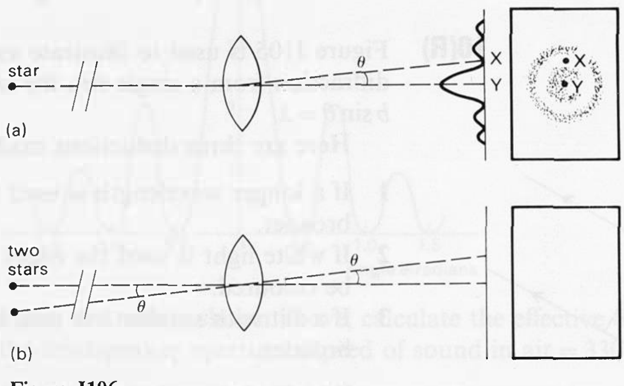


Figure J106

a If the pattern shown is obtained with red light, what difference would it make, if any, if a blue filter were used instead of a red one?

b The same telescope is pointed at a pair of stars which happen to subtend at the lens the angle θ mentioned above, as indicated in figure J106(b).

Make a rough sketch of what the pattern of light at the focus of the telescope would look like now, supposing that the stars are equally bright.

c The telescope is pointed at a planet whose diameter subtends this same angle θ at the telescope. Would the pattern shown in figure J106(a) be altered much?

14(P) Figure J107 shows the dish-shaped reflector and aerial of the radio telescope at Dwingeloo in the Netherlands. It was used to plot the distribution of hydrogen gas in our galaxy by detecting the characteristic radiation (wavelength 0.21 m) emitted by hydrogen atoms in space. The galaxy has a flattish, multiple spiral structure with the Sun about two-thirds of the way from the centre in one of the spiral arms.

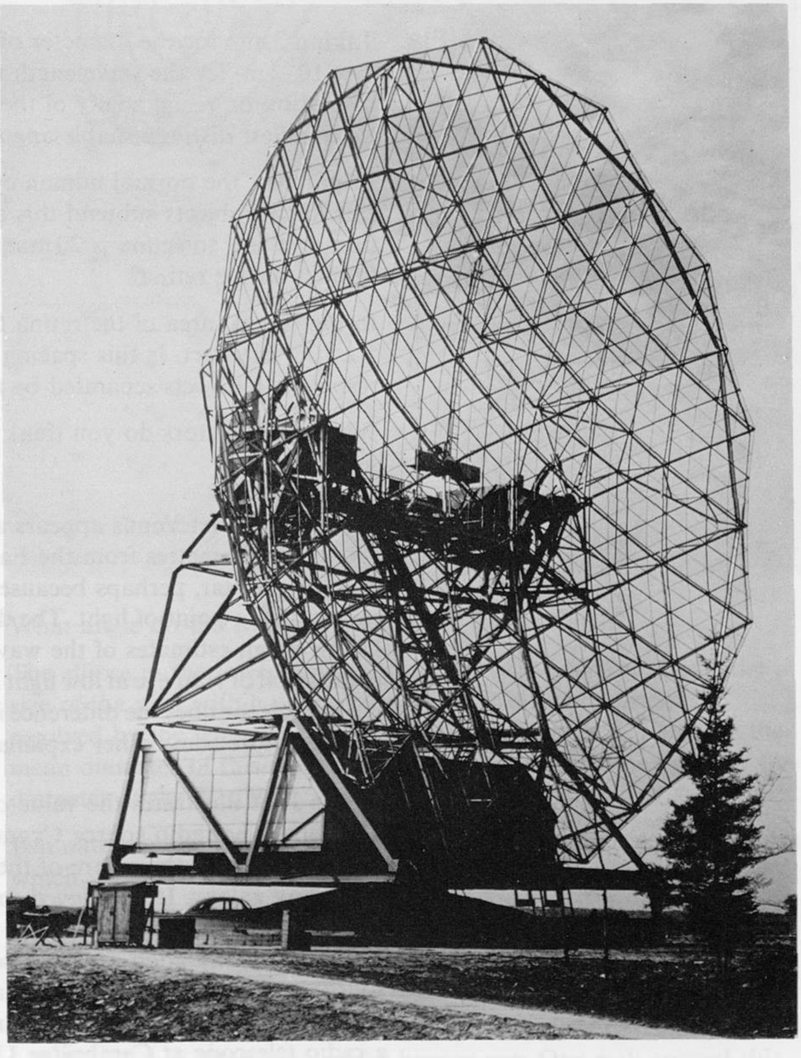


Figure J107
The radio telescope at Dwingeloo in the Netherlands.
By courtesy of the Royal Netherlands Embassy

a Suppose that the reflecting dish of diameter 25 m is equivalent to a slit of this width (actually, it is more nearly equivalent to a slit 25/1.22 m wide). Over what angle either side of the central position will the telescope detect an appreciable intensity when directed towards a 'point source' of 0.21 m radiation?

b A region of our galaxy under examination might be about 2×10^{20} m ($\approx$ 20 000 light years) from the telescope. What is the minimum separation of two hydrogen 'clouds' if the telescope is to be able to resolve them as separate sources?

c It is proposed that a similar telescope be built with a dish diameter ten or a hundred times bigger than that at Dwingeloo. You will see from figure J107 that this telescope can be steered to point at different places in the sky. What arguments might there be for and against this proposal?

15(P)a Taking 2 mm for the diameter of the eye pupil and 500 nm (5×10^{-7} m) for the wavelength of light, estimate the limit of resolution or visual acuity of the eye due to diffraction (that is, the smallest distinguishable angular separation).

b In practice, the normal human eye has a limit which is about twice this. If two objects subtend this angle at the eye and the distance from eye lens to retina is 20 mm, how far apart are the two images formed on the retina?

c In the central area of the retina the light receptors are about 5×10^{-6} m apart. Is this spacing adequate to enable the eye to resolve two objects separated by the angle which you calculated in **b**?

d What other factors do you think may limit the ability of the eye to see fine detail?

16(E) When the planet Venus appears as the Morning Star it is about 150×10^6 kilometres from the Earth. It looks perceptibly different from a true star, perhaps because the unaided eye sees it as a disc rather than a point of light. The diameter of Venus is about 12 000 km. Using rough estimates of the wavelength of light and of the diameter of the pupil of your eye at low light intensity, do you think this suggestion is plausible, or does the difference in appearance between the planet and a star require some other explanation?

17(P) Figure J108 illustrates the value of good resolving power in a radio telescope. The radio source Cygnus A seems to be located near the blurred object at the centre of the photograph, which may be an exploding galaxy. It is believed to be 5×10^{21} km (500 million light years) from the Earth.

A sufficiently good radio telescope can show that the radio source corresponding to this visual object is actually a pair of sources. The figure shows contours of radio intensity, obtained with a radio telescope at Cambridge University. The two sources are some 3×10^{18} km apart.

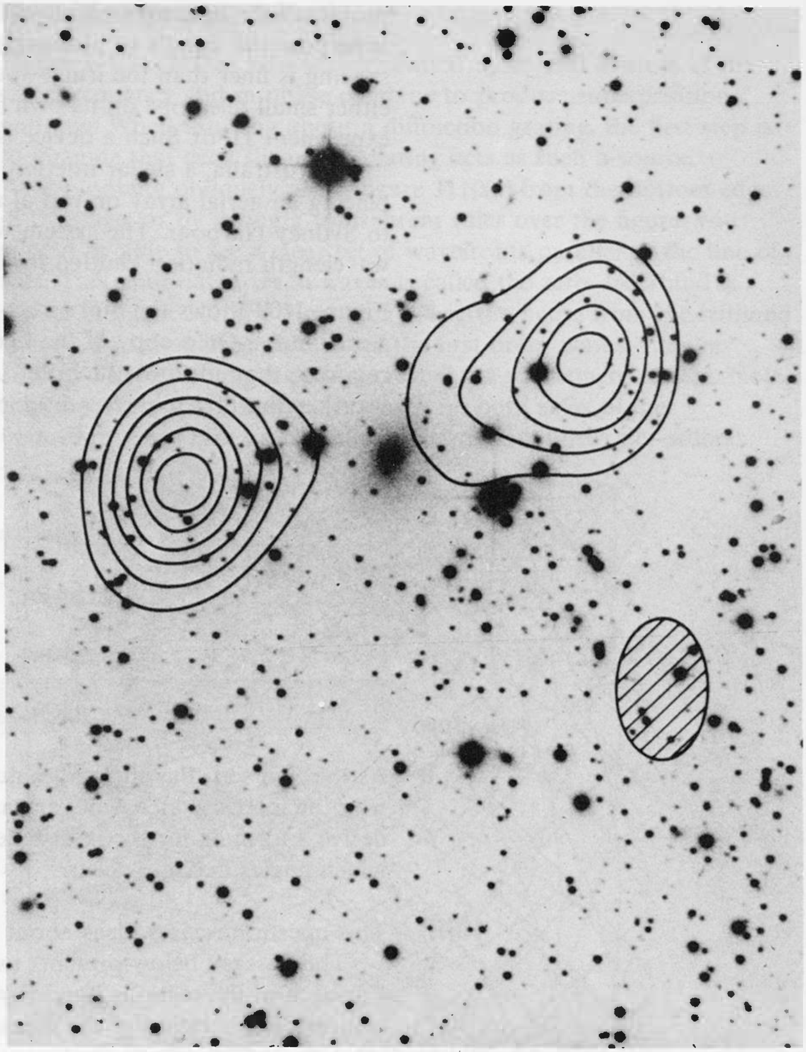


Figure J108
The radio source Cygnus A, showing radio intensity contours superimposed on a photograph of the same part of the sky, taken with the 5-metre Palomar telescope in California.
MOFFET, A. T. Annual review of astronomy and astrophysics, 4, *149*, *1966*.

a What angle do the two sources subtend at the Earth?

b The ellipse in the lower righthand corner of the picture shows the size of the area within which pairs of radio sources cannot be resolved by the telescope at the distance of Cygnus A. Estimate the mean diameter of this region in km, and the angle subtended by this diameter at the Earth.

c Estimate the diameter of the dish aerial (such as that in question **14**) which would be needed to resolve sources as well as the telescope used to obtain the result in figure J108, at a wavelength of 0.15 m. Explain why a dish aerial was not used.

18(L) The ability of a radio telescope to resolve two sources is limited by its aperture, see question **14**. Even one with a dish 100 m across has much poorer resolution than the human eye. One way around this problem is to use two small telescopes a long way apart and to superpose the signals to produce a superposition pattern whose spacing is finer than the fringe spacing of the diffraction pattern from either small telescope on its own (this is like Young's double slits, experiment J13b). Such a device is called a *stellar interferometer*.

In Australia, a stellar interferometer has been constructed by placing an aerial array on top of a 100 m high cliff near the entrance to Sydney Harbour. The system was designed for the study of 1.5 m wavelength radiation emitted from the Sun.

a Figure J109 shows the Sun's rays when the received intensity at A is a maximum. On a copy of the figure, mark a point Y on the direct ray from the Sun to A, such that AX–AY is an odd number of half wavelengths. (The phase is reversed on reflection by the sea.)

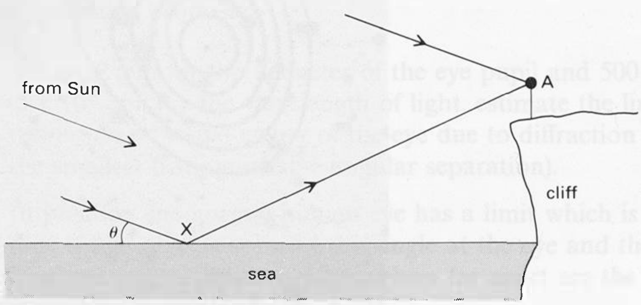


Figure J109

b As the Sun sets, the angle θ gradually changes. Explain how and why the intensity at A will vary as the Sun sets. You may be able to derive a formula for the path difference in terms of θ to predict at which angles maxima occur.

19(R) This question revises ideas about diffraction by an aperture.

The passage below presents two sets of ideas about diffraction. For each of the sections **i** and **ii** you are asked to write a more complete explanation of the ideas. Your explanation may include:

fuller explanations of the theory;
quantitative calculations to illustrate the ideas;
discussion of possible experiments.

i Light from a point source appears to cast sharp shadows and this leads to the familiar idea that it travels in straight lines. However, this is not exactly true: the shadows are not perfectly sharp, although special experiments are needed to show the effect because it is so small. This unfamiliar property is called diffraction and is explained by a wave model of light.

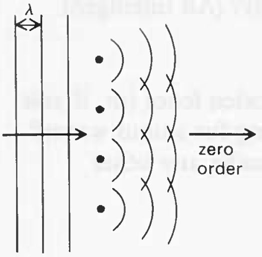
ii The consequence is that the eye, or a camera, or even the best possible telescope, doesn't produce a perfect image. Instead it gives an image which is slightly blurred. When we try to make a telescope magnify more to show finer details of the stars this blurring effect can become an obstacle.

(Long answer paper, part question, 1980)

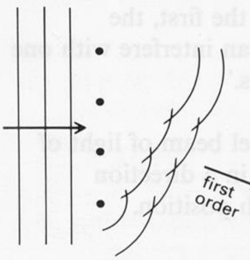
Waves through gratings

20(L) Figure J110(a) shows how waves emitted by several sources of the same frequency and in phase combine to produce superposition patterns. When thinking about a diffraction grating, the first step is to imagine that each slit in the grating acts as such a source.

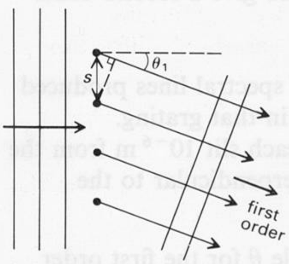
By looking obliquely along figure J110(a) from the bottom edge of the page, or by laying a transparent ruler over the figure, you should be able to see a clear set of wavefronts parallel to the line of slits. This undeviated set of waves is called the zero order and is shown in figure J110(b). Now look along the figure from the lefthand corner. You should be able to see the first order waves – figure J110(c). Further changes of angle reveal the subsequent orders. Note that there is a set of first order waves on both sides of the undeviated zero order set, similarly for second, third, etc. orders.



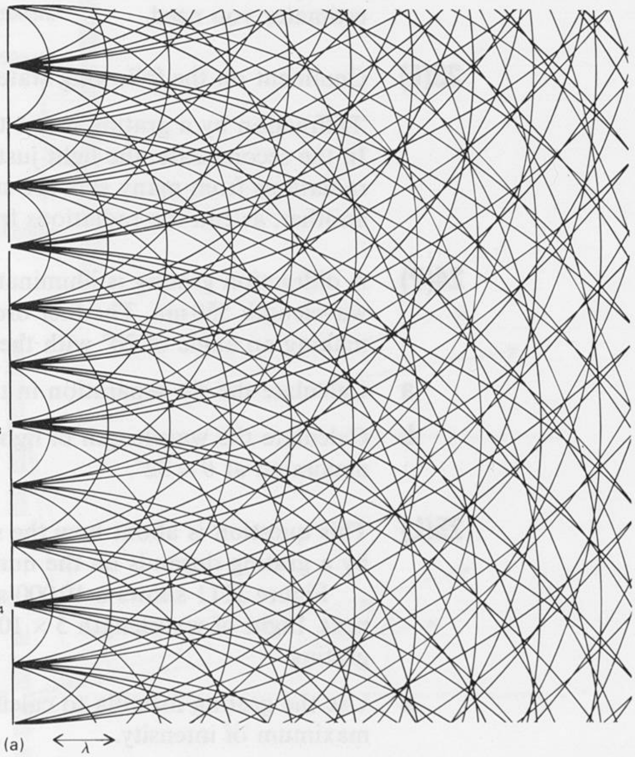
(b)



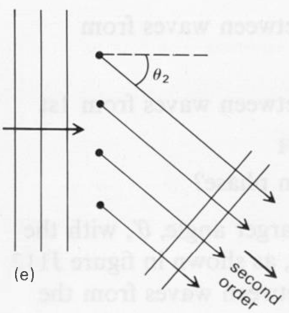
(c)



(d)



(a)



(e)

Figure J110

a Lay a rule along this first order wavefront. What is the path difference between waves from consecutive sources? (This is, of course, why they are 'in phase'.)

b Figure J110(d) is a modified figure, drawn to make clear the path differences between waves. Use this figure to explain why the constructive superposition (maximum) occurs when $\lambda = s \sin \theta_1$. (It may help to refer back to question **6**.)

c Use figure J110(e) to explain in a similar way why there is another maximum for $2\lambda = s \sin \theta_2$.

d Write down a general equation for the angles θ at which maxima will occur for waves of wavelength λ incident on a grating spacing s. (This assumes that the angle of incidence is zero.)

21(P) A grating composed of many narrow slits in an otherwise opaque sheet gives, with a monochromatic source of light, a set of bright lines. Then every other slit is blocked out. What happens to the positions and intensities of the bright lines?

What do you think might happen if half the slits were blocked out but selected at random rather than regularly? (An intelligent guess – don't spend too long on it.)

22(E) At roughly what frequency would a slatted wooden fence (or, if you prefer, iron railings) be a good diffraction grating for sound waves? The speed of sound in air is about $330\,\mathrm{m\,s^{-1}}$; make any other estimates you need.

23(E) Comment on the following statement.

'Diffraction by a grating is not the same as diffraction at a single slit. In the second case, the light just spreads out; in the first, the radiations from many slits are superposed and can interfere with one another, as can the radiations from a pair of slits.'

24(P) A diffraction grating is illuminated with a parallel beam of light of wavelength 550 nm. The first order maximum is in a direction making an angle of 20° with the straight-through position.

a Calculate the slit separation in the grating.

b Calculate the wavelength of light which would give a second order maximum at $\theta = 32°$.

25(L) This question is about how the sharpness of spectral lines produced by a grating depends on the number of slits in that grating.

Figure J111 shows a 10 000-slit grating, each slit 10^{-6} m from the next. Light of wavelength 5×10^{-7} m falls perpendicular to the grating.

a Use the grating formula to calculate the angle θ for the first order maximum of intensity.

b What is the path difference (in terms of λ) between waves from adjacent slits?

c What is the path difference (in terms of λ) between waves from 1st and 5001st slit? This is equal to BC.

d Why are all waves emerging along line AC in phase?

e Now consider waves emerging at a slightly larger angle, θ', with the condition that path difference $BC' = BC + \lambda/2$, as shown in figure J112. What is the path difference (in terms of λ) between waves from the 1st and 5001st slit now?

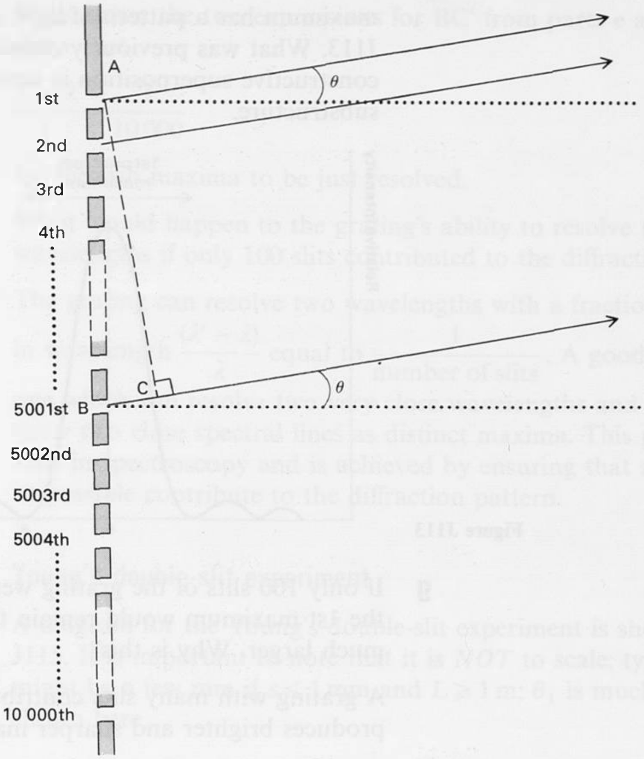


Figure J111

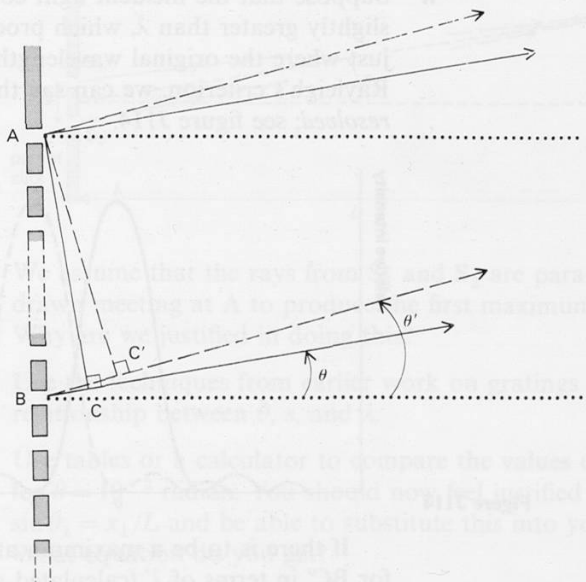


Figure J112

f What is the resultant of the waves from all the slits in this new direction?

The determination of the positions of no light is similar to the analysis of single-slit diffraction (question 6). The first principal maximum has a pattern of light and dark fringes as drawn in figure J113. What was previously considered to be a single position of constructive superposition is seen to be spread out and to have substructure.

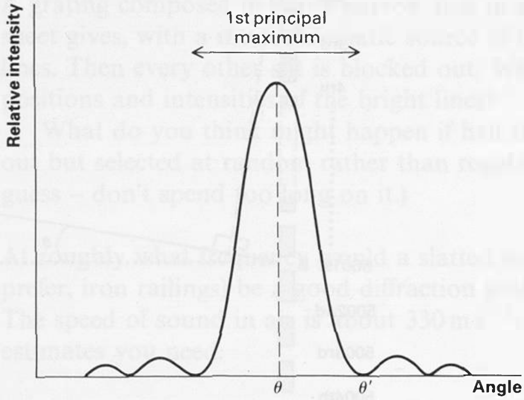


Figure J113

g If only 100 slits of the grating were illuminated, then the angle θ for the 1st maximum would remain the same but θ' would now be very much larger. Why is this?

A grating with many slits contributing to the diffraction pattern produces brighter and sharper maxima than one with fewer slits.

h Suppose that the incident light contains another wavelength, λ', slightly greater than λ, which produces a 1st order maximum at θ' just where the original wavelength gives no resultant. According to Rayleigh's criterion, we can say that the two maxima are *just resolved*; see figure J114.

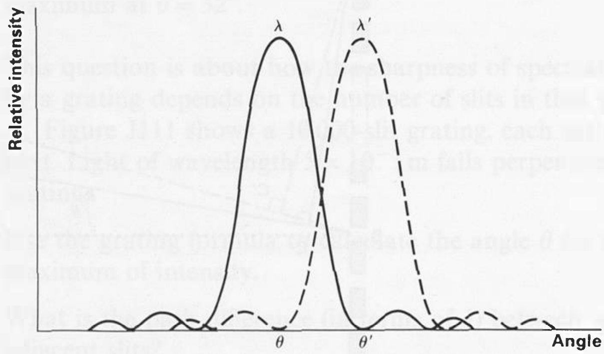


Figure J114

If there is to be a maximum at θ' for λ', write down an expression for BC′ in terms of λ' (calculated as in part **c**).

i Write down the two expressions for BC′ from parts **e** and **h** and show that

$$\frac{\lambda' - \lambda}{\lambda} = \frac{1}{10\,000}$$

for the two maxima to be just resolved.

j What would happen to the grating's ability to resolve the two wavelengths if only 100 slits contributed to the diffraction pattern?

The grating can resolve two wavelengths with a fractional difference in wavelength $\frac{(\lambda' - \lambda)}{\lambda}$ equal to $\frac{1}{\text{number of slits}}$. A good grating is one which can resolve two very close wavelengths and is thus able to show two close spectral lines as distinct maxima. This property is vital in spectroscopy and is achieved by ensuring that as many slits as possible contribute to the diffraction pattern.

Young's double-slit experiment

26(L) A diagram for the Young's double-slit experiment is shown in figure J115. It is important to note that it is *NOT* to scale; typically x_1 might be a few mm if $s \leqslant 1$ mm and $L \geqslant 1$ m; θ_1 is much smaller than shown here.

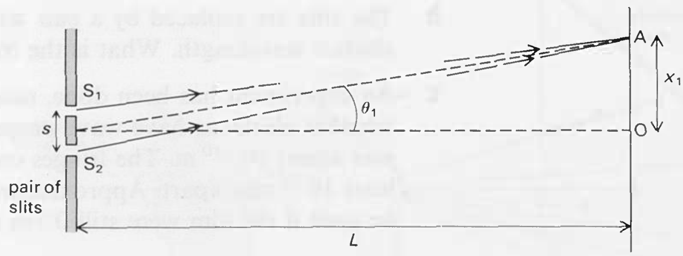


Figure J115

a We assume that the rays from S_1 and S_2 are parallel, yet they are drawn meeting at A to produce the first maximum or bright fringe. Why are we justified in doing this?

b Use the techniques from earlier work on gratings to find the relationship between θ, s, and λ.

c Use tables or a calculator to compare the values of $\sin\theta$ and $\tan\theta$ for $\theta = 10^{-2}$ radian. You should now feel justified in writing $\sin\theta_1 = x_1/L$ and be able to substitute this into your answer to **b**. What equation do you get?

d Suppose that the $(n+1)$th fringe is measured, giving a new value of $OA = x_{n+1}$. What is the grating formula now?

e Show that if x is the fringe separation, $\lambda/x = s/L$.

27(P) Light from a colour filter is used to produce Young's double-slit fringes. The slit separation is 0.4 mm. The distance between the slits and the screen on which the fringes are formed is 1.4 m, and the distance between successive dark spaces (or bright fringes) is 1.7 mm.

a Find the average wavelength of the light used.

b Why 'average'?

28(P) Light of wavelength 500 nm from a very small source falls on a pair of slits 0.1 mm apart, and forms fringes 2.5 mm apart on a photographic film, 0.5 m from the slits.

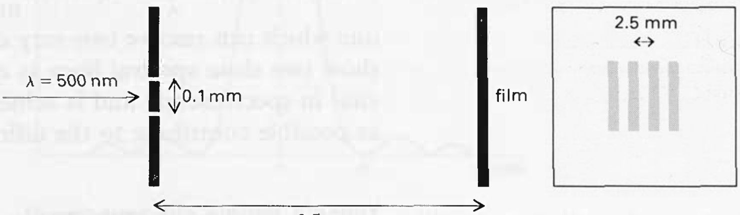


Figure J116

a What would the fringe spacing become if light of wavelength 250 nm were used? Would the fringes be visible if you looked at a screen placed where the film was held?

b The slits are replaced by a pair with half the spacing, still using the shorter wavelength. What is the fringe spacing now?

c An experiment has been done, using this arrangement, to test whether electrons have wave properties. The expected wavelength was about 10^{-10} m. The fringes could be detected if they were at least 10^{-2} mm apart. Approximately what slit spacing would have to be used if the film were still 0.5 m from the slits?

X-ray diffraction

29(L) This question is about the diffraction of X-rays by a crystal and its optical analogue.

In 1912 Max von Laue suggested that, just as a grating with a periodic array of apertures produces a diffraction pattern when illuminated by radiation of comparable wavelength, so the regular arrangement of atoms in a crystal should also give a diffraction pattern when illuminated appropriately – in this case with X-rays. It was hoped that the analysis of such diffraction patterns would yield much useful information about the structure of crystals.

Figure J117 shows a photograph of an X-ray diffraction pattern for a crystal of ZnS taken with the X-ray camera shown in figure J118. The essential geometry of the rather complex apparatus is shown in figure J119.

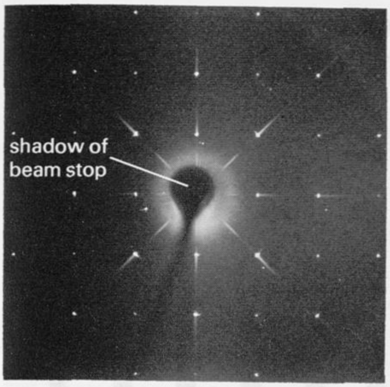


Figure J117
X-ray diffraction pattern for ZnS.
Professor C. A. Taylor

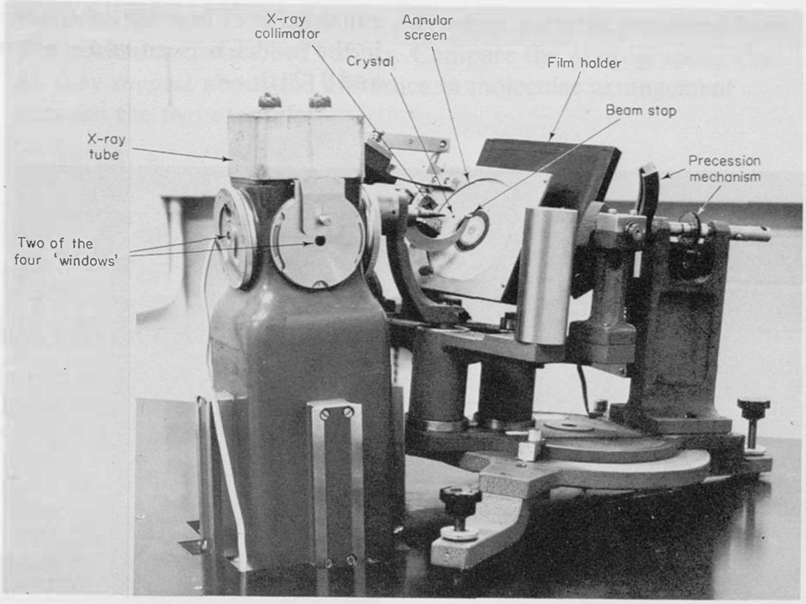


Figure J118
X-ray camera used to achieve figure J117.
Professor C. A. Taylor

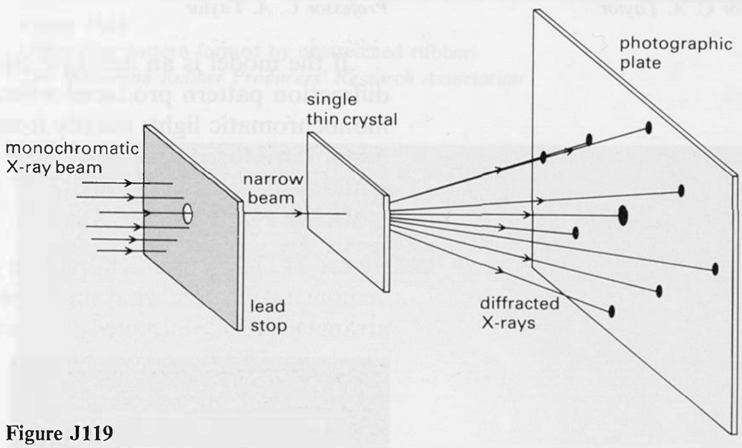


Figure J119
Simple geometry for X-ray diffraction.

a As we have seen in demonstration J6, the diffraction pattern contains all the information necessary to enable an enlarged image of the crystal to be produced by recombination. Why cannot this be done using X-rays?

b Not only can an image of the crystal not be formed, but it proves very difficult to determine its structure by analysing the diffraction pattern directly. A way forward is to make an optical analogue of the experiment. It is possible to make an informed guess at the crystal structure from its chemical and physical properties. A 'ball-and-stick' model is constructed and a mask made of dimensions suitable for diffracting light, where each aperture represents an atom in the model, as seen in a shadow to simplify the detail (figures J120 and J121).

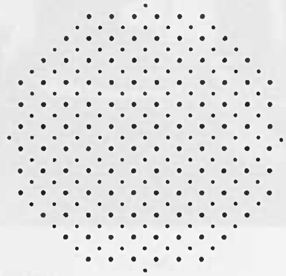


Figure J121
Optical diffraction grating of holes from ZnS model.
Professor C. A. Taylor

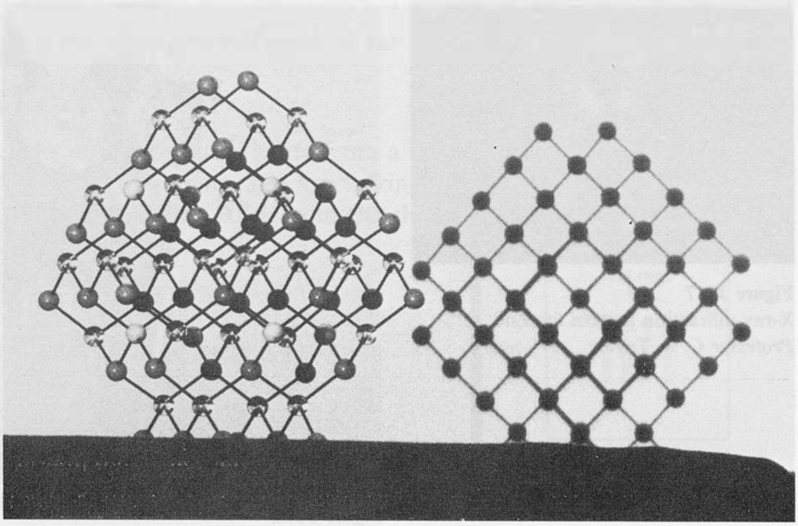


Figure J120
Ball-and-stick model of ZnS, with the 'shadow' corresponding to the position which produces the diffraction pattern shown in figure J122.
Professor C. A. Taylor

If the model is an accurate representation of the crystal, the diffraction pattern produced when the negative is illuminated with monochromatic light, usually from a laser, should be similar to the pattern produced with the crystal using X-rays. Compare the two patterns in figures J122 and J117 and say why you think the model of ZnS is a good one or not.

c Figure J123 shows the diffraction pattern for distilled water. From demonstration J16, what might be a reasonable interpretation of the arrangement of molecules in water?

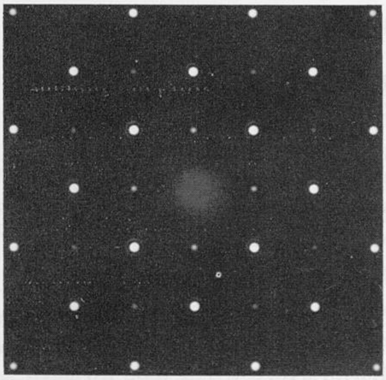


Figure J122
The optical diffraction pattern, with the direct beam removed.
Professor C. A. Taylor

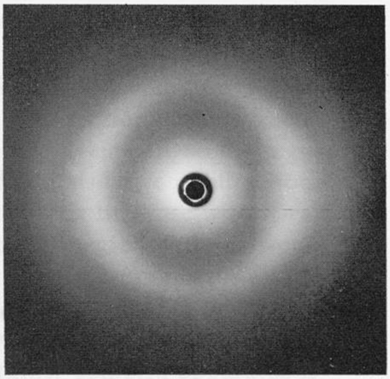


Figure J123
Distilled water (X-ray diffraction).
Pilkington Brothers P.L.C.

d Figures J124 and J125 show the diffraction patterns produced from unstretched and stretched rubber. Compare the photographs; what do they suggest about the difference in molecular arrangement between the two examples?

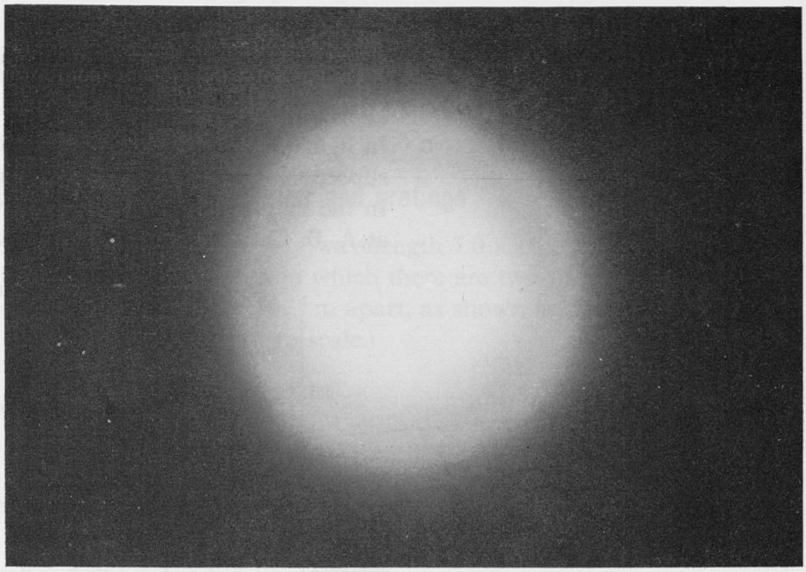


Figure J124
Diffraction pattern formed by unstretched rubber.
The Malaysian Rubber Producers' Research Association

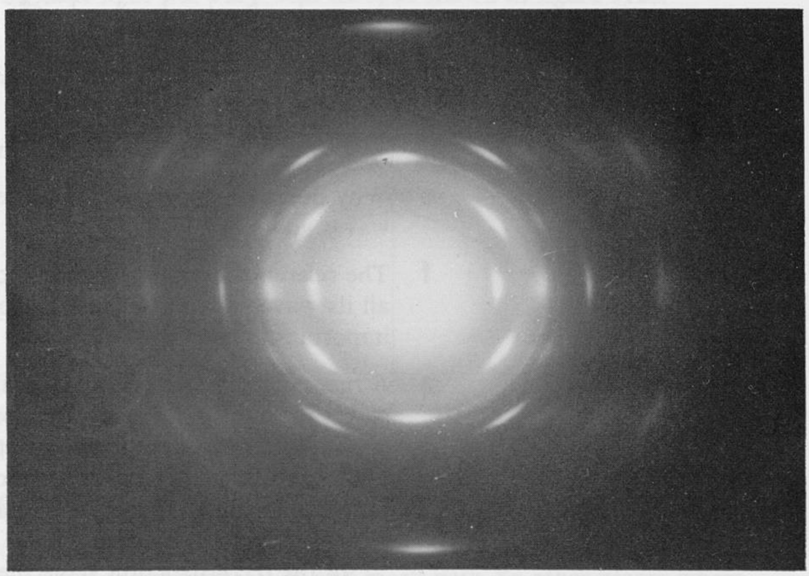


Figure J125
Diffraction pattern formed by stretched rubber.
Professor E. H. Andrews, Department of Materials, Queen Mary College, University of London

Holography

30(L) Imagine that in the production of a hologram the object consists of just a single small point.

a What shape would the scattered wavefronts which constitute the signal beam have?

b If the reference beam is virtually parallel, what shape do the reference wavefronts have?

c In figure J126 the scattered and reference waves are shown superposing along a line where the photographic plate has been put. In the diagram the waves are shown frozen in time. What happens at A, B, C, D, and E? Explain your answer.

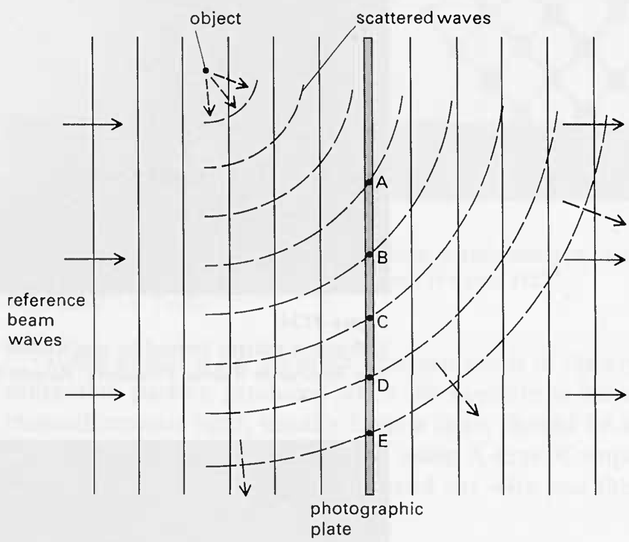


Figure J126

d What happens between these points on the plate?

e Why does this superposition pattern remain the same even as the waves advance?

f The reference beam must have a constant phase relationship between all its waves, that is, it must be coherent. Explain why this is important.

g A real object would consist of many such points, each scattering waves in slightly different directions, and each scattered wave would produce its own superposition pattern with the reference wave. The photographic plate records an enormous number of such diffraction patterns, for all the illuminated parts of the object. The developed plate is called a hologram.

In the reconstruction stage the developed plate is illuminated by the reference beam from its original direction. Each of the diffraction patterns on the plate produces a diffraction image of the original scattering point and in this way a complete image of the original object is recombined.

If the viewpoint is changed, then light is received from a changed direction and consequently from a different set of diffraction patterns on the hologram. Thus a different set of images is recombined, with the new overall image corresponding exactly to the viewer's changed perspective of the object.

Each part of the hologram contains some contribution from *every* part of the object. Explain why this is so, and why it is possible to recombine a complete image from a small (though not too small) piece of the hologram.

Revision questions on slits and gratings

31(R) A parallel beam of light of wavelength 7.0×10^{-7} m falls normally on to an opaque screen in which there are two parallel and identical slits, S_1 and S_2, 3.5×10^{-4} m apart, as shown in figure J127. (Note that the drawing is not to scale.)

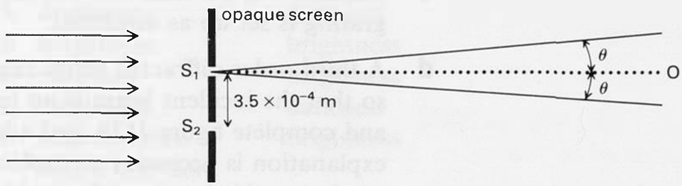


Figure J127

If slit S_2 is covered up, the variation of the intensity of the light transmitted by slit S_1 on both sides of the line S_1O parallel to the incident light will vary with $\sin\theta$, as shown in figure J128.

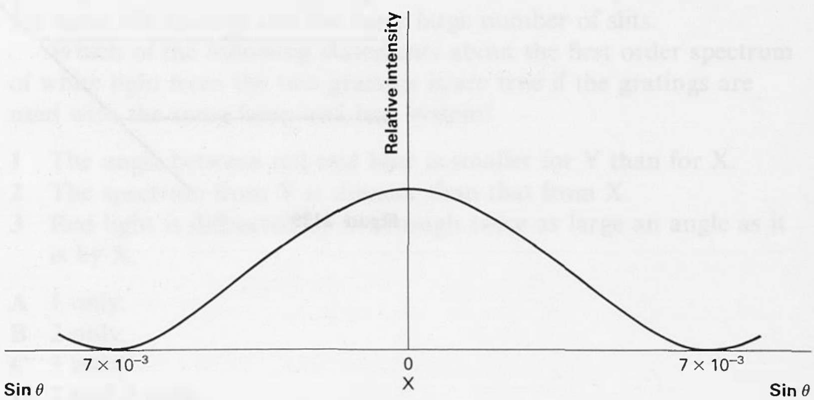


Figure J128

a Calculate the slit width, showing your working.

b *i* If slit S_2 is now uncovered, there is still a maximum at X, where $\sin\theta = 0$. Copy figure J128 and mark with Xs other places where there are now maxima. Show how you calculate these positions.
ii Now show how the intensity of the light transmitted by both slits varies with $\sin\theta$, indicating also how the intensity scale may need to be changed.

c State two ways in which the graph you have drawn in **b***ii* would differ if the width of each slit were doubled, their distance apart remaining unchanged.

(Short answer paper, 1978)

32(R) A narrow parallel beam of light of wavelength 5.0×10^{-7} m falls normally on to a diffraction grating with a spacing of 12.5×10^{-7} m. Diffracted beams are observed on the other side of the grating from the light source.

a Calculate the angle between the first order diffracted beam and a line perpendicular to the grating. Show how you arrive at your answer.

b Calculate the angle between the second order diffracted beam and a line perpendicular to the grating. Show how you arrive at your answer.

c Explain why no third and subsequent orders are produced when this grating is set up as described.

d A third order diffracted beam can be produced by tilting the grating so that the incident beam is no longer normal to the grating. Copy and complete figure J129, and add whatever further written explanation is necessary to explain how the tilted grating can produce a third order diffracted beam.

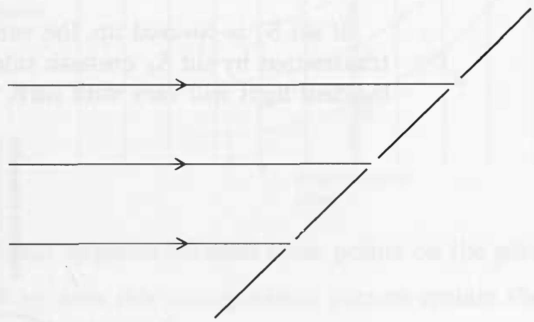


Figure J129

(Short answer paper, 1976)

33(R) Light from a distant point source falls on a single slit, and also on a double slit, as shown in figure J130 parts (a) and (b).

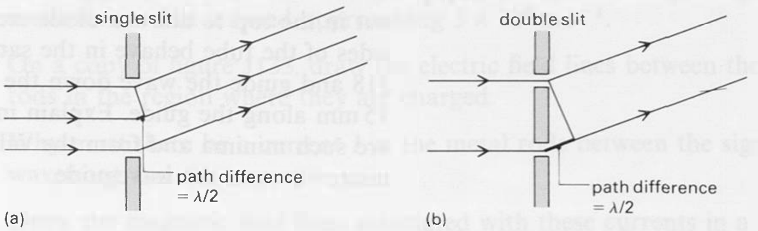


Figure J130

If the path difference indicated in each diagram is half a wavelength, which one of the following correctly describes for both what will be observed at the angle shown?

	single slit	*double slit*
A	darkness	darkness
B	brightness	brightness
C	darkness	brightness
D	brightness	darkness
E	edge between bright and dark	brightness

(Coded answer paper, 1979)

34(R) Figure J131 shows three slits from each of two diffraction gratings X and Y. The open slits of grating X are twice as wide as those of grating Y, otherwise the gratings are identical. Both gratings have the same slit spacing and the same large number of slits.

Which of the following statements about the first order spectrum of white light from the two gratings is/are true if the gratings are used with the same lamp and lens system?

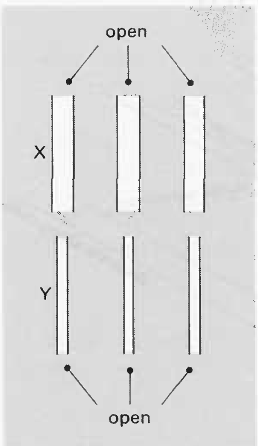


Figure J131

1 The angle between red and blue is smaller for Y than for X.
2 The spectrum from Y is dimmer than that from X.
3 Red light is diffracted by Y through twice as large an angle as it is by X.

A 1 only.
B 2 only.
C 3 only.
D 1 and 2 only.
E 1, 2, and 3.

(Coded answer paper, 1976)

'Tied' and travelling waves

35(P) The hollow tube of rectangular cross-section in figure J132 has a slot cut in the top to allow a diode receiver probe to be inserted. The sides of the tube behave in the same way as the rods in demonstration J18 and guide the wave down the tube. Minima are detected every 15 mm along the guide. Explain in terms of superposition why there are such minima and from the values given calculate the speed of the microwaves in the waveguide.

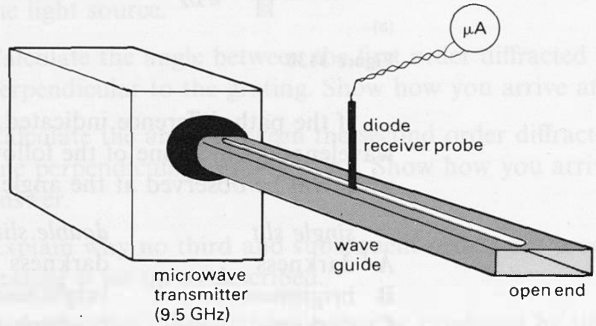


Figure J132

36(L) This question suggests some links between the 'tied' electromagnetic wave, moving charges, and travelling *E*- and *B*-fields. It uses a very simple geometrical arrangement similar to that in demonstration J18a, but with the essential difference that the signal applied to the guide is not oscillating; it is a step pulse produced when a battery is connected across the nearer ends of a pair of long metal rods, by closing the switch in the circuit shown in figure J133.

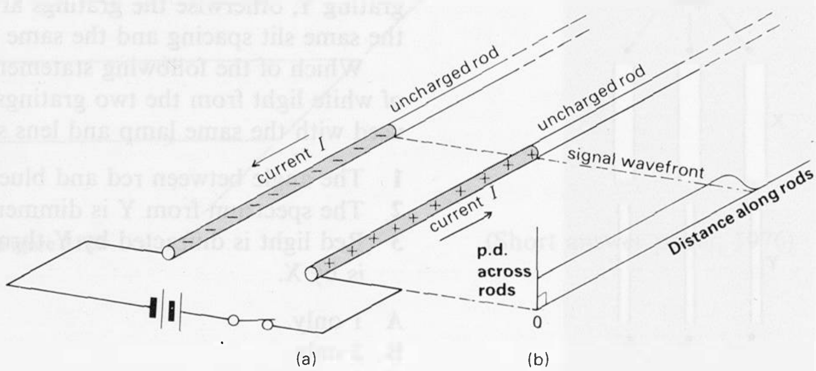


Figure J133
Signal travelling along a pair of metal rods.

a When the switch is closed, positive charges move off the lefthand rod and flow onto the righthand rod, leaving them charged negatively and positively respectively. Figure J133 shows the charged regions a short time after the battery had been switched on. The speed of charge carriers in metal was discussed in Unit B, 'Currents, circuits, and charge'. What is a reasonable estimate for the drift speed of individual charges in the rods? This diminutive value is *not* the speed of the signal wavefront, however, as the edges of the charged regions move forward at a speed approaching $3 \times 10^8\ \mathrm{m\,s^{-1}}$.

b On a copy of figure J133, draw the electric field lines between the rods in the region where they are charged.

c Why must there be a current I in the metal rods between the signal wavefront and the battery?

d Draw the magnetic field lines associated with these currents in a contrasting colour on your answer to **b**.

e At what speed are these E- and B-fields spreading into the region which was previously uncharged?

The signal moving along the guide is an electromagnetic pulse. The pattern of E- and B-fields which constitutes the pulse is associated with moving regions of charge and propagates at a speed of $3 \times 10^8\ \mathrm{m\,s^{-1}}$ in a vacuum. The individual charges drift along very much more slowly.

37(R) Figure J134 shows a length of coaxial cable, made with an outer sheath of conductor wrapped round a long roll of insulator, inside which is a conducting wire running along the axis of the whole cable. The capacitance of each metre of cable is 200×10^{-12} F. Some time before the instant shown, the switch S was switched to the 1.5 V battery for 10^{-9} s and then back to the position shown. At the time shown, the inner wire carries positive charge in the region BC and the outer conductor carries negative charge. The regions AB, CE are uncharged.

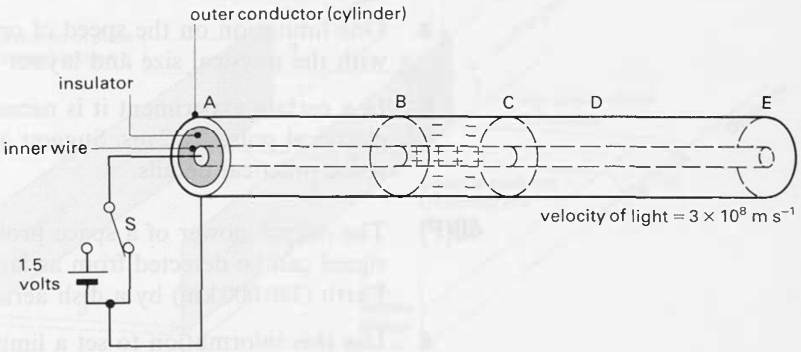


Figure J134

a Give a reason why no electricity has yet reached the distant place E.

b Estimate roughly the length of the charged region BC (the insulator actually reduces the velocity, but you may ignore this).

c Make a sketch showing the directions of the electric field around the central wire within the region BC.

d Explain why the charge on each conductor in the region BC is about 9×10^{-11} C (again neglect any effect of the insulator).

e What do you think will happen when the charged region reaches the end of the cable beyond E if the conductors end abruptly and are not connected to anything else or to each other?

(Paper I, 1970)

38(R) The signal from a receiving aerial is brought by coaxial cable to the input of a television set. The spot where the electron beam meets the television screen sweeps across the screen at $10^4\,\mathrm{m\,s^{-1}}$. Another length of coaxial cable is also connected to the aerial, in parallel with the first, but the far end of this second cable is left free and unconnected. When this is done, a second, fainter picture appears on the screen, 10^{-2} m to the right of the first.

a Suggest a reason for the existence of this second picture.

b If the second cable is 125 m long, how fast does the signal travel along the cable?

c It is suggested that the lengths of otherwise identical coaxial cables remaining on a number of big, partly-used reels could be estimated by measuring the displacement of the second picture. Explain, using appropriate numerical estimates, how accurate this method is likely to be.

(Short answer paper, 1972)

39(E) Advances in solid state electronics, in particular the production of integrated circuits, have made it possible to design computers which can process data in times of the order of a nanosecond.

a One limitation on the speed of operation of a computer is connected with the physical size and layout of the circuits. Explain this.

b In a certain experiment it is necessary to delay the passage of an electrical pulse by 2 ms. Suggest a simple way of doing this and give some practical details.

40(P) The output power of a space probe's radio might be about 100 W. Its signal can be detected from as far away as the Moon's distance from Earth (380 000 km) by a dish aerial about 10 m in diameter.

a Use this information to set a limit on the power that modern receivers can detect.

b If you assume that the power follows an inverse-square law, what are you supposing about the space probe's aerial?

c The receiving dish aerial is strongly directional. Does this fact make the use of an inverse-square law in a invalid?

41(E) *For discussion*

A television programme may be received in one of two ways:

1 The signals are transmitted from an aerial, and picked up in a receiving aerial connected to a television set. (In the Direct Broadcast Satellite television system, developed in the early 1980s, the transmitter aerial is an orbiting satellite.)

2 The signals are transmitted along a coaxial cable which is connected directly to the television set (there may be an amplifier somewhere in the transmission line). This is commonly called the Cable Television system.

Describe how each system conveys the programme information and explain how it is that the final results are very similar.

The speed of electromagnetic waves

42(L) This question is about the speed of an electrical signal along a pair of long, flat conducting plates (see figure J135). It is very similar to the geometry of the 'guided' or 'tied' wave demonstration J18a and question **36**, but the gap between the plates is very much smaller than their width, so that any length of the plates behaves like a parallel plate capacitor. The electrical signal applied to the guide is not oscillatory but merely a step pulse produced by switching on a battery across one end. This electrical signal travels at speed v leaving behind a charged region – the left plate is negatively charged and the right positively charged.

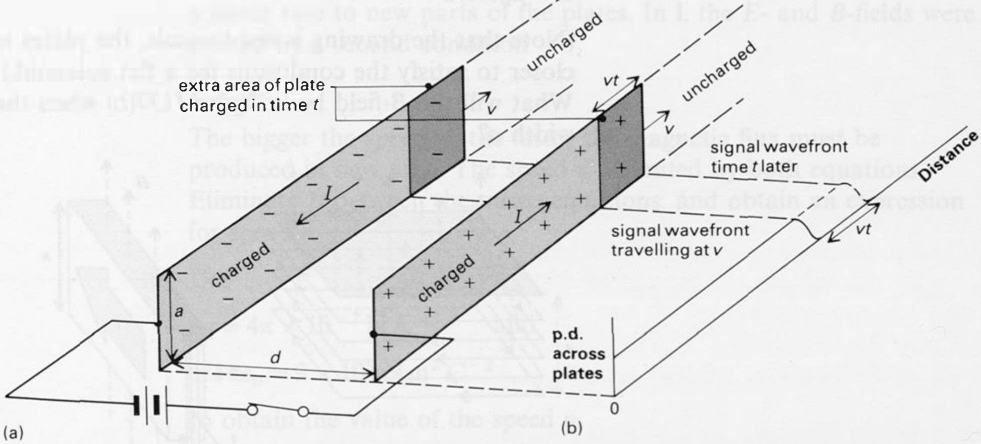


Figure J135

a The wavefront travels a distance vt along the plates in time t, thus an extra area of plates becomes charged in time t. Where does this extra charge come from?

b Why must there be electric current I flowing in the left and right plates between the battery and the signal wavefront?

c If the current is I, what charge flows onto the extra area of plate in time t?

d Over what area will the charge in **c** be spread, if the width of each plate is a?

e Treating the two newly charged regions as a parallel plate capacitor $\left(\text{capacitance } C = \dfrac{\varepsilon_0 A}{d}\right)$, write down an expression for the electric field E between the plates in terms of the current I using the answers from parts **c** and **d** (figure J136).

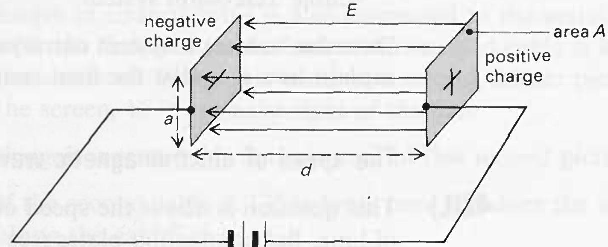


Figure J136

f If the plates carry current there must be a B-field between them. To help visualize how the plates produce a B-field, and to link up with work on solenoids, it is useful to think of the plates as rather like a flat solenoid in which the many wires wound around a solenoid have become one wire – the plates themselves. In figure J137(a), if the solenoid has N turns in width a, the B-field inside it and well away from its ends is given by:

$B = \mu_0 NI/a$

(Note that the drawing is *not* to scale, the plates have to be much closer to satisfy the conditions for a flat solenoid.)
What will the B-field be in figure J137(b) when there is *one* turn in width a?

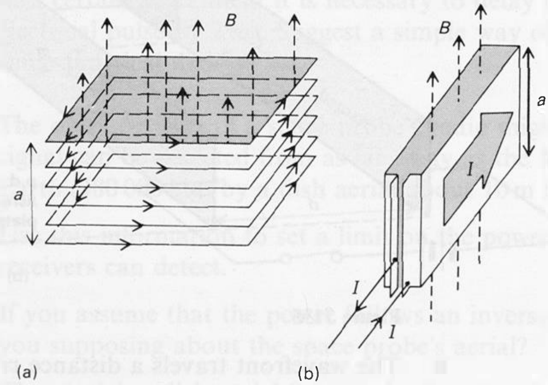


Figure J137

g Use the answers to **e** and **f** to write as simple an equation as you can, giving B in terms of E.

h Your answer to g should show that the larger the speed, v, the larger the B-field for a given E-field. The E-field is fixed by the battery p.d. and is just that p.d. divided by the plate spacing. Why is B larger if the speed, v, is larger? (Think about how much current must flow.)

i Wherever currents flow in the plates there will be a B-field near the plates. Currents flow up to the present position of the travelling leading edge of the pulse. Thus the B-field extends further and further along the plates, just as the E-field does, while the pulse is propagating along the plates. Into what new length between the plates does the B-field extend in time t?

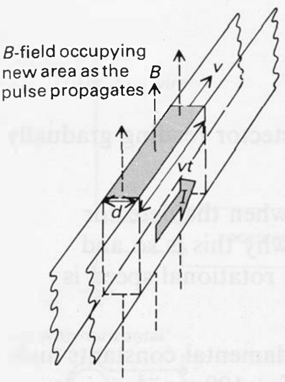


Figure J138

j Figure J138 illustrates the B-field occupying new area as the pulse propagates. An area dvt has new B-field at right angles to it in time t. What is the new flux through this area? What is the rate of change of flux?

k A p.d., equal to the rate of change of flux, is needed to increase the magnetic flux in a region of space. What p.d. across the plates produces the rate of change of flux found in j?

l If there is a p.d. V across the plates, there is an electric field E between them, equal to V/d. Write an expression for this electric field, E, in terms of B, using the answer to **k**.

m In g, the electric field between the plates was found to be related to the B-field between the plates by:

$$B = \varepsilon_0 \mu_0 E v$$

The larger the speed, v, the bigger the B-field, because a greater current would be needed to deliver the same charge per unit area at a faster rate to new parts of the plates. In **l**, the E- and B-fields were related by a second condition

$$E = Bv$$

The bigger the speed, v, the faster the magnetic flux must be produced in new area. The speed v is limited by both equations. Eliminate B between these two equations, and obtain an expression for speed v.

n Use

$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$ and

$1/4\pi\varepsilon_0 = 9 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$

to obtain the value of the speed v.

o Show that the units of $1/\sqrt{\varepsilon_0 \mu_0}$ are $\mathrm{m\,s^{-1}}$, using the units given in **n**.

p Why does the speed, v, remain the same even if the size of the plates, their spacing, or the battery p.d. are changed?

43(P) In an experiment to measure the speed of light in air, light from a laser is shone through one of the holes of a stationary disc and is reflected back from a mirror 5 km away through the adjacent hole. It is received by a photosensitive detector behind this hole.

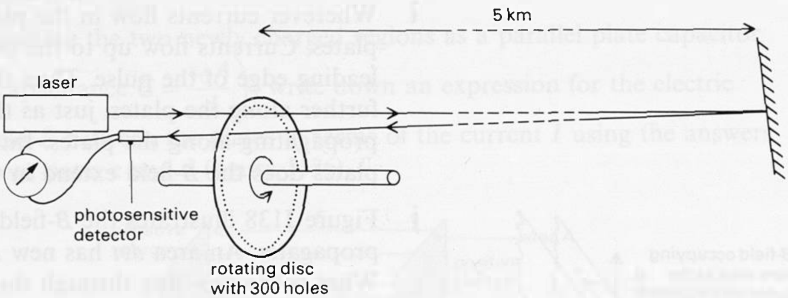


Figure J139

a The wheel is then made to rotate and the detector reading gradually falls. Why is this?

b As the speed increases, there comes a point when the detector reading again reaches a maximum. Explain why this is so and calculate a value for the speed of light if this rotational speed is $100\,\text{rev}\,\text{s}^{-1}$.

The speed of light *in vacuo* is one of the fundamental constants and can now be measured with an uncertainty of $\pm 100\,\text{m}\,\text{s}^{-1}$, *i.e.*, to better than 1 part in 10^6. Early attempts at its determination were astronomically based, relying upon a deduction from the time light took to travel a relatively large distance, for example Rϕmer's method of 1676. Later attempts, similar to the one in the question above, utilized either a toothed wheel (Fizeau, 1849) or a rotating mirror (Foucault, 1850, and Michelson, 1926) to chop up or modulate a continuous beam into pulses. By measuring the time interval separating pulses which have travelled a known distance, the speed can readily be calculated.

Aerials

44(R) A television programme is broadcast at a frequency of 600 MHz with the electric vector vertical. Figure J140 illustrates the waves at a considerable distance from the transmitter.

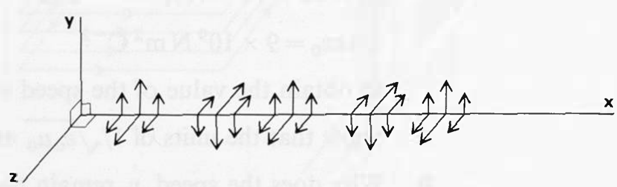


Figure J140

a Make a rough copy of the diagram, label the axes, and explain how it is meant to represent the waves.

b Draw another diagram to show what this wave would look like 5×10^{-10} s later. ($c = 3 \times 10^8\,\text{m}\,\text{s}^{-1}$.)

c Show on another diagram
i how you would place a short straight wire so as to get the maximum e.m.f. induced in it,
ii how you would place a small loop of wire so as to get the maximum e.m.f. induced in it,
iii where you would place a large metal sheet so as to increase the e.m.f. in *i*.

d What features of the diagram of the wave indicate that the wave is polarized? How would you decide whether the waves carrying a particular television broadcast were polarized?

(Paper I, 1970)

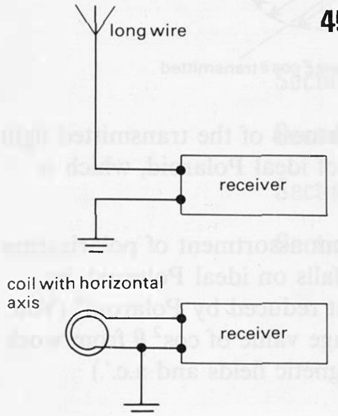


Figure J141

45(R) A medium wave radio receiver may have two kinds of aerial, as shown in figure J141: either a long straight wire or a coil. (The coil is actually wound on a ferrite rod and is usually enclosed within the receiver.)

a Reception of a particular station using the long straight wire aerial is best with the wire vertical. Explain how the radio waves cause alternating currents in the aerial, and why it matters that the wire is vertical.

b Reception of the same station using the coil aerial is good when the axis of the coil is at right angles to the direction of the transmitter and poor when the axis of the coil points at the transmitter. Why?

c The aerial currents produce alternating currents in the tuning circuit of the receiver (figure J142). Why can turning the knob which varies C tune in a particular signal?

(Short answer paper, 1979)

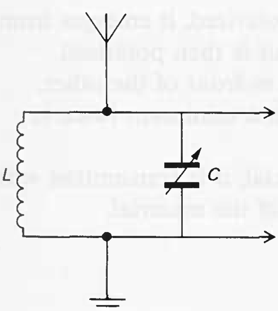


Figure J142

Polarization

46(L) Figure J143 (page 246) shows how much light 'ideal' Polaroid transmits. In figure J143(a), the electric field of the incoming light is at right angles to long polymer chains, to which iodine atoms are attached. The presence of the iodine allows electrons to migrate along the chains, so that the chains behave like a very fine grid of conducting wires. With the electric field perpendicular to the grid, all the incident light is transmitted.

In figure J143(b), the electric field is parallel to the grid, and no light is transmitted.

In figure J143(c), the electric field makes an angle θ with the direction it had in figure J143(a). A component $E \cos\theta$ lies along that original direction, and is transmitted. (A component $E \sin\theta$ lies along the direction in figure J143(b), and is not transmitted.)

The amplitude of the transmitted electric field oscillation in figure J143(c) is $E \cos\theta$, if the amplitude of the incoming wave is E. The brightness, or intensity, of the transmitted light is proportional to the square of the amplitude, and so to $\cos^2\theta$.

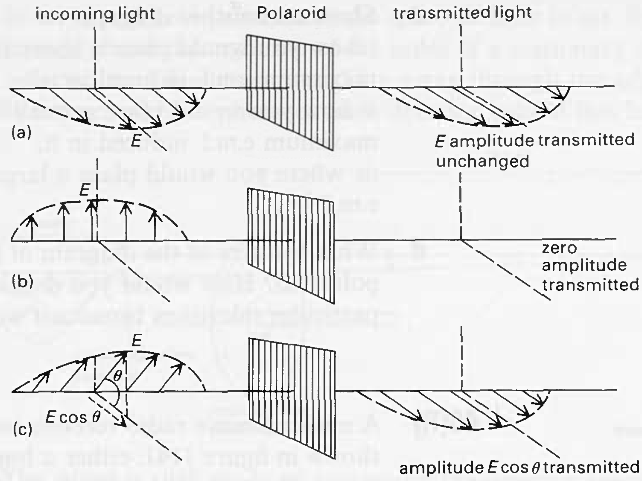


Figure J143

a Sketch a graph of the variation in brightness of the transmitted light if plane-polarized light falls on a sheet of ideal Polaroid, which is rotated through 360°.

b If unpolarized light, containing a random assortment of polarization directions, with no direction favoured, falls on ideal Polaroid, by what factor is the brightness of the light reduced by Polaroid? (You should recall something about an average value of $\cos^2 \theta$ from work on alternating currents in Unit H, 'Magnetic fields and a.c.'.)

47(R) Which of these remarks about the use of plane-polarizing material (such as Polaroid) with light is/are correct?

1 If the light falling on the material is unpolarized, it emerges from the material with unchanged intensity, but is then polarized.

2 If two sheets of the material are put one in front of the other, and one is turned, the light transmitted is a minimum twice in each complete revolution.

3 If plane-polarized light falls on the material, it is transmitted with equal intensity whatever the orientation of the material.

A 1 only.
B 2 only.
C 1 and 3 only.
D 2 and 3 only.
E 1, 2, and 3.

(Coded answer paper, 1982)