

# Unit K

## ENERGY AND ENTROPY

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**K**

# SUMMARY OF THE UNIT

## INTRODUCTION

### Change at the molecular level

I am the undertow  
Washing tides of power  
Battering the pillars  
Under your things of high law.

I am a sleepless  
Slowfaring eater,  
Maker of rust and rot  
In your bastioned fastenings,  
Caissons deep.

I am the Law  
Older than you  
And your builders proud.  
I am deaf  
In all days  
Whether you  
Say 'Yes' or 'No'.

I am the crumbler: tomorrow.

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Much of physics deals with single lumps of matter, whether on a large or small scale, and tells us why things happen: how the pull of the Earth's gravity makes the Moon orbit the Earth, or how the charge on a nucleus deflects an alpha-particle passing close by. But physics has also to deal with changes involving large numbers of particles, such as diffusion or Brownian motion, thermal conduction, or electric current. For these, regular and predictable behaviour on the laboratory scale has lying behind it the random, chaotic, unpredictable motions of a multitude of particles. The problem is to see how the chaotic does in fact lead to the predictable: how, for example, the random motion of molecules results in scent diffusing across a room, in spite of the fact that it is absurd to suppose that the individual molecules could care which way they go. This Unit looks at how molecular chaos leads to large-scale regularity, even certainty.

Section K1 introduces the idea of *entropy*, and gives one example of how to calculate an entropy change. The entropy change is seen to be connected with energy changes and with temperature.

Section K2 shows how to understand the entropy change when energy is given to matter at a certain temperature. This involves rethinking what temperature itself means.

Section K3 is more practical, showing how the Boltzmann factor helps us to understand such things as rate of evaporation, rate of flow, and rate of reaction.

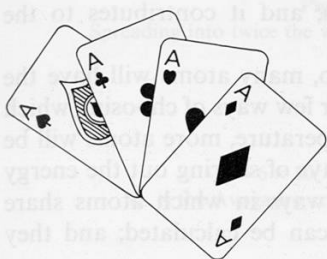
Section K4 is about how entropy changes predict the direction of processes, and about how to exploit entropy changes to control events.

The whole Unit is just an introduction to these ideas. One important reason for studying them is that they are very different from ideas met elsewhere in physics, so that they help to complete a picture of what science is like. A second reason is their fundamental importance and very wide application, the latter illustrated by the variety of examples in Sections K3 and K4. These ideas apply, not just to one kind of process, but to any kind of process. They are thus important to physicists, chemists, biologists, and to engineers of all kinds.

## Section K1 WHAT IS ENTROPY?

When ice melts or water boils, there is no sense in talking of the individual molecules 'melting' or 'boiling'. They just move about somewhat differently. No hidden guiding hand directs those motions to ensure that the result is boiling or melting.

Now physicists, chemists, and engineers *do* want to direct what happens. A physicist may want many atoms in a gas to be ionized. A



1 in 6.5 million

**Figure K1**  
Four aces.

$k = R/L = 1.38 \times 10^{-23} \text{ J K}^{-1}$   
Unit A, 'Materials and mechanics'

chemist may want many hydrogen and nitrogen molecules to combine to make ammonia for fertilizers. An engineer may want water to boil under pressure to provide steam for a turbine. All need to be able to predict when such things will happen, so as to be able to choose the conditions under which the random behaviour of many molecules will have the desired effect.

When the underlying behaviour is random, what happens most frequently is simply that which happens in most ways. If you deal a hand of cards, you hardly ever get all cards of one suit, simply because a mixture of suits can be dealt in more ways. So we have to count the numbers of ways in which events can happen: to know, for example, when one gas diffuses into another, whether the number of ways of arranging the molecules changes or not.

Suppose  $W$  is the number of ways of arranging the molecules of some piece of matter, at a certain pressure, temperature, and so on. Then we define a quantity called its *entropy*,  $S$ , related to  $W$ , by

$$S = k \ln W$$

The entropy is just the logarithm of the number of ways of arranging the molecules, multiplied by the Boltzmann constant  $k$ . The units of entropy are the same as those of  $k$ : joules per kelvin.

It is often enough just to calculate by how much  $W$  changes, and so to consider only entropy *changes*  $\Delta S$ ,

$$\Delta S = k \Delta \ln W$$

$$\Delta S = k \Delta \ln W$$

#### QUESTIONS 1 to 3

In some simple cases we can say directly in what direction events will go. We know that gas molecules will mix and diffuse, and once mixed will not of their own accord separate again. We know that the energy of a bullet hitting a target will spread out amongst the molecules of the target, warming it, and that this energy will never 'come together' and send the bullet flying back again. When the gas molecules mix, the number of ways of arranging molecules amongst molecules increases; when the target warms up the number of ways of arranging energy amongst molecules increases. The reverse processes, which do *not* occur, would decrease the number of ways.

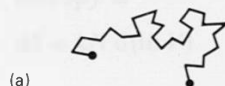
#### QUESTIONS 4 to 7

Unit A, 'Materials and mechanics'  
random walk

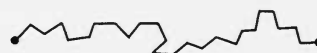
#### Some examples of numbers of ways

Rubber consists of chains of isoprene molecules, with C—C bonds around which rotation can occur. So a chain follows a zig-zag path. In unstretched rubber, the chains zig-zag a lot, so that the places at which they cross-link to other chains are close together (figure K2). In stretched rubber, the chains are more straightened out. There are fewer ways a chain can be arranged between two points far apart than between two points close together. The number of chain arrangements for a given stretch can be calculated; and it contributes to the entropy of the rubber.

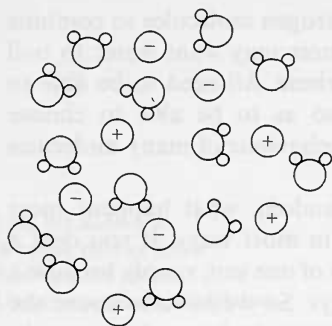
**Figure K2**  
Paths between cross-links.  
(a) Many paths.  
(b) Fewer paths.



(a)



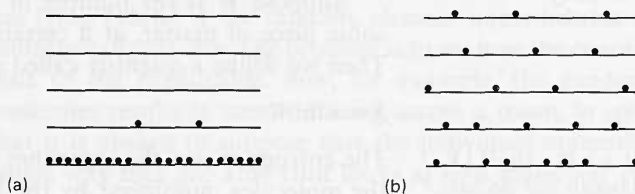
(b)



**Figure K3**  
Water molecules and ions in solution.

In salt water, there are sodium and chloride ions (figure K3). For a given saltiness, it doesn't matter where exactly the ions are. They can change places with water molecules without our tongues tasting the difference. The number of ways of interchanging ions and molecules can be calculated, for a given concentration; and it contributes to the entropy of the salty water.

In a piece of metal near absolute zero, many atoms will have the least energy possible. There are then rather few ways of choosing which atoms have what energy. At a higher temperature, more atoms will be at various energies and there are more ways of sharing out the energy (figure K4). Changes in the numbers of ways in which atoms share energy, for a given rise in temperature, can be calculated; and they contribute to the change in entropy.

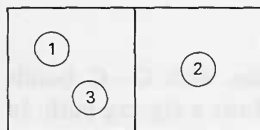


**Figure K4**  
Collections of particles with different total energies.  
(a) Most particles at lowest energy.  
(b) Particles with many different energies.

In each case we are talking about some system which has a certain large-scale state (stretch, concentration, temperature). Consistent with that state, the particles of which it is composed can vary in their detailed state (position, energy, orientation, etc.).  $W$  is the number of such different detailed states which are possible given the large-scale state.  $S = k \ln W$  is a convenient quantity to describe this degree of variability.

### Calculation of an entropy change: particles spreading out

#### GAME K1 Particle shuffling



**Figure K5**  
One of  $2^3$  arrangements.

A game in which counters are shuffled between two halves of a box gives an example of an entropy change which can be calculated. The result can be applied to the expansion of a gas or to the dilution of solutions.

If  $N$  counters move between two halves of a box, there are  $2^N$  ways of arranging them between the two halves (figure K5). Only one of these ways has all  $N$  counters in one half (say the left). If we start with all counters in one half, and move them purely at random, there is a tendency for the counters to become spread between the two halves. If  $N$  is large (100 is enough), there is essentially no chance that all the particles will concentrate again in one half.

Doubling the space open to  $N$  particles multiplies the number of ways  $W$  of arranging them by  $2^N$ , if all the space is equally likely to be occupied and the position of a particle is not affected by the positions of others (figure K6).

QUESTIONS 8 to 11



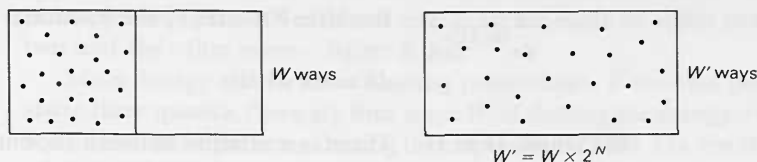


Figure K6

Spreading into twice the volume.

$$L = 6 \times 10^{23} \text{ mol}^{-1}$$

( $L$  is the Avogadro constant)

$$W'/W = 2^{6 \times 10^{23}}$$

(where  $W'$  is the new, larger number of ways).

The entropy change is

$$\Delta \ln W = \ln(W'/W)$$

$$\ln 2^{6 \times 10^{23}} \approx 4 \times 10^{23}$$

$$k \approx 1.38 \times 10^{-23} \text{ J K}^{-1}$$

$$\Delta S = k \Delta \ln W$$

$$= k \ln 2^{6 \times 10^{23}}$$

$$\approx 6 \text{ J K}^{-1}$$

#### QUESTION 12

which is a much more 'thinkable' number. A modest entropy change corresponds to a large addition to  $\ln W$ , and to an enormous factor multiplying  $W$ . (If written out, the number representing the factor would stretch beyond the nearer stars.)

When there is no change in the number of ways (for example particles moving about at random in a constant volume)  $W' = W$ , and  $W'/W = 1$ . Since  $\ln 1 = 0$ , it follows that  $\Delta S = k \ln(W'/W) = 0$  when there is no change in the number of ways, *i.e.* at equilibrium.

#### Expansion of a gas

#### QUESTION 13

The entropy change  $\Delta S \approx 6 \text{ J K}^{-1}$  is in fact rather a good approximation to the entropy change when a gas doubles its volume. (This is so, even though gas molecules, unlike counters, cannot be distinguished from one another.) The number 2 in  $2^N$  was just the ratio of volumes before and after. In general, if the volume changes from  $V_1$  to  $V_2$ , we can write  $V_2/V_1$  in place of the number 2 in  $2^N$ , since we believe that a molecule is just as likely to be in one part of the total volume as in any other. This gives:

$$S_2 = S_1 + Nk \ln(V_2/V_1)$$

$$S_2 = S_1 - Nk \ln(p_2/p_1)$$

$$\Delta S = kN \ln(V_2/V_1) = kN \Delta \ln V$$

for any volume change, as long as the temperature is constant.

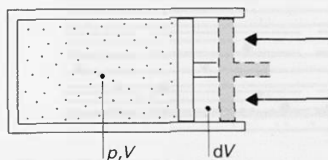


Figure K7

Expansion of a gas.

If the gas expands by an infinitesimal amount  $dV$ , then the change in entropy is

$$dS = kN d(\ln V)$$

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<b>[Q] Quit</b>	<b>[P] Probability</b> OFF ON

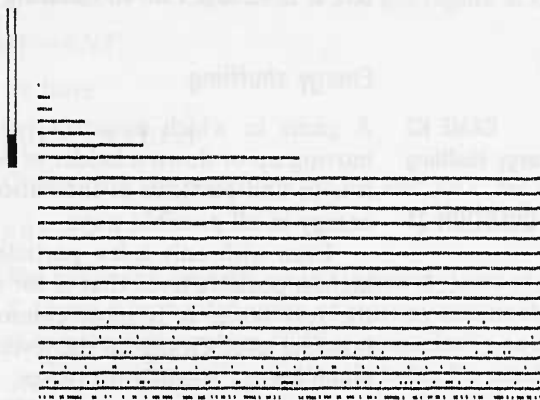


Figure K9 (part)

Computer simulation of energy shuffling between 200 particles.

(b) After a considerable number of moves.

For equal (and large) numbers of particles and quanta, it happens that the ratio of numbers in adjacent levels is just  $1/2$ . Thus if a particle has a certain chance of being at any one level, it has twice the chance of being one level lower and half the chance of being one level higher.

If the same number of particles share more energy, more of them climb higher on the levels. The pattern is still exponential, but steeper. If the particles share less energy, fewer reach the higher levels. The pattern is again exponential, but less steeply graded (figure K10).

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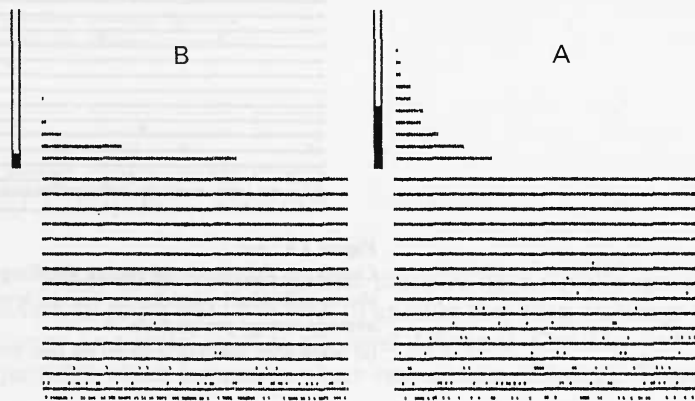
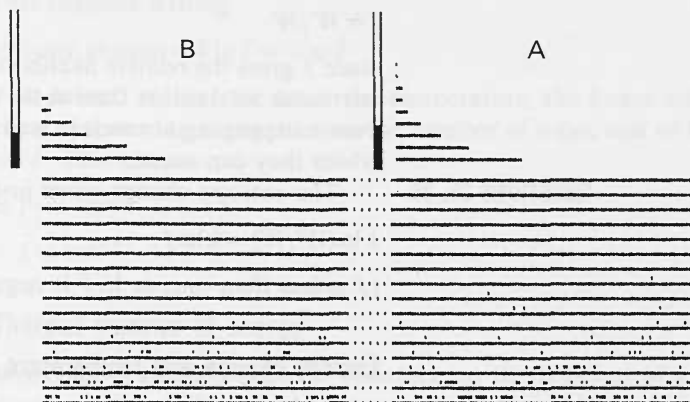
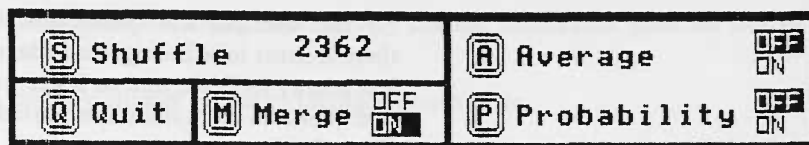


Figure K10

Two isolated systems, with different energies per particle.



**Figure K11**

The two systems of figure K10 in contact and nearing equilibrium.

### Energy shuffling and ways of sharing energy

Let  $f$  be the fraction

$$f = \frac{\text{chance of finding a particle at a given level}}{\text{chance of finding a particle at next lower level}}$$

$f$  is the Boltzmann factor, Section K3

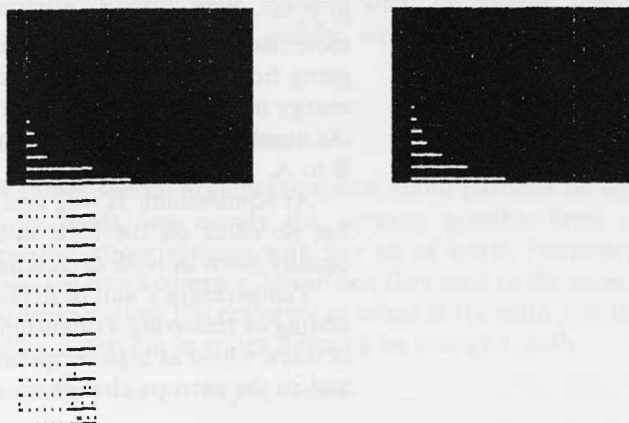
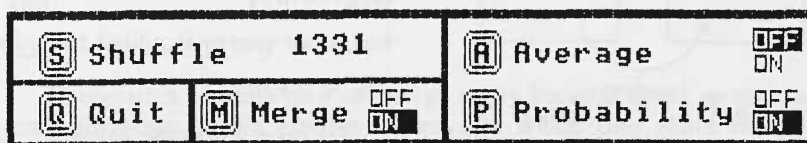
for your information:

$$f = n/(N + n)$$

for  $N$  particles sharing  $n$  quanta  
(if  $N$  and  $n$  are large)

in the equilibrium distribution. For a given distribution,  $f$  is a constant, less than unity.

Figure K12 shows one particle, exchanging energy with a number of others. As expected, the probability of finding the lone particle at each level matches the probability of finding particles at each level in the larger set.



**Figure K12**

One particle sharing energy with ten others. Display shows that probability of finding a particle at a particular level is the same for the single particle (on the left) as for the set of ten particles (right).

The lone particle spends less time at higher levels because to get there it must take energy from the others. If it does so, the others have less energy to share, and so fewer ways  $W$  of sharing energy. Let taking one quantum from the set of particles reduce the number of ways from  $W$  to  $W'$ . Then

$$f = W'/W$$

since  $f$  gives the relative likelihood that the lone particle will be at any particular level rather than at the next lower level, and the frequency of events happening at random is in proportion to the number of ways in which they can occur.

#### QUESTIONS 25, 26

The entropy change when just one quantum is removed is

$$k \ln(W'/W) = k \ln f.$$

( $f$  is less than one, so  $\ln f$  is negative and the entropy falls.)

### Entropy change and temperature

#### COMPUTER SIMULATION K4 Thermal equilibrium

Temperature differences are just what decide the direction of thermal flow of energy: from hot to cold and not from cold to hot. Since entropy changes decide the direction of change, we expect a connection between entropy change, energy change, and temperature.

The energy-shuffling simulation can be used to look at energy flow between two sets of particles (figures K10 and K11). System A (figure K10) must have been hotter than system B, because energy went spontaneously from A to B when they were in contact (figure K11). When they reach equilibrium, they must be at the same temperature, and energy then passes equally in both directions.

Suppose one quantum goes from A to B, when they are at temperatures  $T_A$  and  $T_B$  (figure K13). The entropy of A is reduced:

$$\Delta S_A = k \ln f_A$$

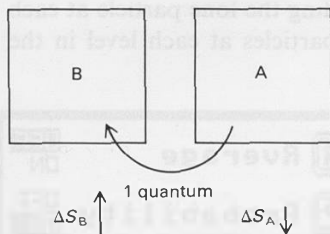
but, since energy is added to B, the entropy of B increases:

$$\Delta S_B = -k \ln f_B$$

To take a specific example, suppose  $f_A = 0.8$  and  $f_B = 0.5$ . Then  $\ln f_A = -0.22$  and  $-\ln f_B = +0.69$ . The increase in the entropy of B is more than the decrease in the entropy of A. The net effect of one quantum going from A to B is to increase the total number of ways of sharing energy in the two systems. But a quantum going from B to A reduces the number of ways. Thus quanta more often go from A to B than from B to A.

At equilibrium,  $f_A = f_B$  and one quantum going in either direction has no effect on the total number of ways. Energy therefore passes equally often in both directions.

Temperature  $T$  can be defined in terms of  $\ln f$ . At low temperatures, adding or removing a quantum must have a large effect on the number of ways, whilst at high temperatures the effect must be small. Thus  $\ln f$ , and so the entropy change, should be inversely related to temperature.



**Figure K13**  
One quantum passing from one system to another.

In the case of a gas, the entropy change associated with an energy change has the form

$$\text{entropy change} = \text{energy change} / \text{temperature}$$

In the present case, the energy change is  $-\varepsilon$  (remove one quantum). This suggests writing

$$\text{entropy change} = k \ln f = -\varepsilon / T$$

$T$  is defined so that the *lower* the temperature, the *larger* will be the effect of removing a quantum on the number of ways, and so the effect on  $f$ . This gives

$$\ln f = -\varepsilon / kT$$

$$f = e^{-\varepsilon / kT}$$

## Thermal transfer of energy

### QUESTIONS 27 to 30

Working – raising a mass or speeding up the rotation of a wheel – does not necessarily alter the entropy. But a thermal transfer of energy from hot to cold does always increase the total entropy. When one more quantum of energy  $\varepsilon$ , enters a system at temperature  $T$ , the entropy increases by  $\varepsilon / T$ . If many quanta enter, with total energy  $Q$  transferred thermally, the increase in entropy is

$$\Delta S = Q / T$$

so long as the energy  $Q$  is small enough not to raise  $T$  appreciably. The entropy increases simply because more energy means more ways to share energy. In moving towards thermal equilibrium, if a hot body A gives energy  $Q$  to a cold body B, their entropy changes are

$$\Delta S_A = -Q / T_A \quad \Delta S_B = +Q / T_B$$

and

$$\Delta S_{\text{total}} = \Delta S_A + \Delta S_B > 0$$

Energy-rich systems give up energy easily because doing so makes little difference to the number of ways in which they share energy internally. Energy-poor systems suck up energy greedily because adding a little energy greatly increases the number of sharing possibilities.

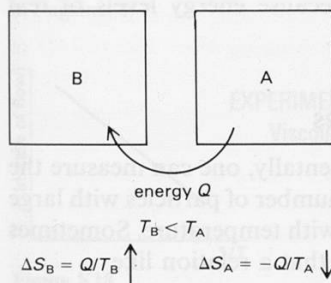


Figure K14

Thermal transfer from hot to cold.

## Thermodynamic temperature

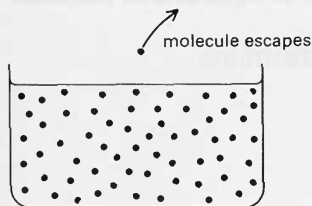
The rather special arguments above, about particles on equally spaced energy levels, are merely the simplest possible form of a general argument about systems with any set of levels. Naturally such arguments are more complex, but in fact they lead to the same basic result, that temperature  $T$  is definable in terms of the ratio  $f$  of the chances of finding a particle in states differing by energy  $\varepsilon$ , with

$$f = e^{-\varepsilon / kT} \quad \text{or} \quad \ln f = -\varepsilon / kT$$

For these reasons, the definition of temperature and its relationship to entropy change are general, not special. Temperature defined in this way is called *thermodynamic temperature*. It is also chosen so as to agree with temperature defined on the ideal gas scale, by making the constant  $k$  the Boltzmann constant.

## Section K3 THE BOLTZMANN FACTOR

### Activation processes



**Figure K15**  
Evaporation as an activation process.

Puddles of water evaporate because a few molecules acquire enough energy to break free from the surface (figure K15). Silicon conducts electricity, and so transistors and microchips can work, because a few electrons get enough energy to be mobile.

Processes like these, which depend on a few particles acquiring enough energy to do something, are called *activation processes*. If the necessary energy  $E$  is to be acquired by chance from the particles of the material all sharing energy randomly at temperature  $T$ , then the Boltzmann factor

$$f = e^{-E/kT}$$

is often a good guide to the probability that a particle will gain the excess energy  $E$ , especially if  $E$  is large compared with the energy  $kT$ . The Boltzmann factor can be interpreted as the fraction by which the number of ways of sharing energy in the material is multiplied, when energy  $E$  is removed from the material and taken by a single particle.

The Boltzmann factor is not necessarily a precise guide to the chance of a particle acquiring energy, because energy levels of real systems are not in general equally spaced.

### Experimental study of activation processes

In studying an activation process experimentally, one can measure the rate of the process, which depends on the number of particles with large enough energy  $E$ , and how the rate varies with temperature. Sometimes the energy  $E$  can be obtained directly, so that a relation like

$$\text{rate} \propto e^{-E/kT}$$

can be checked empirically. Sometimes a value of  $E$  has to be found from the variation of rate with temperature. It is common to plot  $\ln(\text{rate})$  against  $1/T$ :

$$\ln(\text{rate}) = \text{constant} - E/kT$$

A number of processes to study are suggested below. You are not expected to consider more than one in detail.

#### Vapour pressure of water

##### EXPERIMENT K5

Vapour pressure of water

The energy  $E$  is the energy needed to get a molecule out of the liquid:

$$E = \Delta H_{\text{evap}}/L$$

### QUESTIONS 31 to 33

$\Delta H_{\text{evap}}$ , in joules per mole, can be measured directly with an electrical heater. The fraction of molecules in the vapour is indicated by the pressure of the vapour, so we expect

$$p \propto e^{-\Delta H_{\text{evap}}/LkT}$$

(see figure K16).

#### QUESTIONS 34, 35

The Boltzmann factor is the reason why only a modest temperature rise produces a large increase in pressure in a pressure cooker.

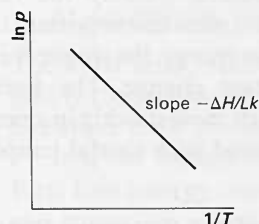


Figure K16  
ln  $p$  against  $1/T$ .

#### EXPERIMENT K6 Current in a thermistor

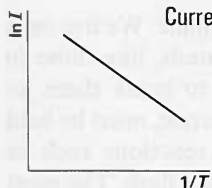


Figure K17  
ln  $I$  against  $1/T$ .

#### Current in a semiconductor

The energy  $E$  is that needed to free an electron which is bound to an atom (leaving also a mobile 'hole') in the semiconductor. The current  $I$  at a given potential difference is approximately proportional to the number of free charge carriers. Thus we expect

$$I \propto e^{-E/kT}$$

The Boltzmann factor is the reason why (pure) semiconductors conduct at all, and why they are rather temperature sensitive, so that devices like a thermistor can be used to monitor temperatures.

#### EXPERIMENT K7 Viscous flow

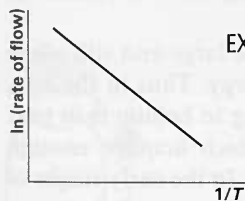


Figure K18  
ln (rate of flow) against  $1/T$ .

#### Viscosity

Liquids flow because molecules acquire enough energy to break out of the 'cage' temporarily formed by their nearest neighbours, moving to a nearby 'cage'. The energy  $E$  is the average energy needed to push aside neighbouring molecules.

If we measure the rate of flow of a liquid under a fixed pressure gradient, we might expect approximately

$$\text{flow rate} \propto e^{-E/kT}$$

#### EXPERIMENT K8 Creep

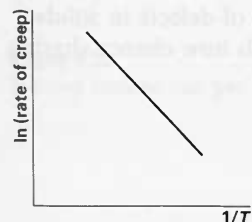


Figure K19  
ln (rate of creep) against  $1/T$ .

#### Creep

Solids can flow under stress, usually less rapidly than liquids. For an atom or molecule to move past its neighbours, an extra energy  $E$  is needed, so we might expect approximately, for a given stress,

$$\text{rate of creep} \propto e^{-E/kT}$$

For both liquids and solids, the energy  $E$  depends on the external stress, which supplies molecules with energy when they are displaced.

EXPERIMENT K9  
Rate of reaction

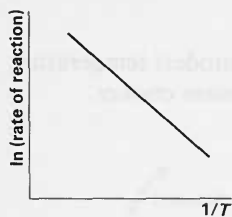


Figure K20  
ln (rate of reaction) against  $1/T$ .

### Rate of reaction

Some chemical reactions require an energy barrier to be overcome before two or more molecules can combine, with the particles passing through a short-lived, high-energy activated state. The chance of getting the activation energy  $E_A$  from random energy exchanges may be given by the Boltzmann factor, and we may expect

$$\text{rate of reaction} \propto e^{-E_A/kT}$$

Other factors, notably the rate of collision of molecules, also matter, but are often less important than the Boltzmann factor. The larger the activation energy, the slower the reaction, and the more sensitive it is to temperature change. The Boltzmann factor explains why food cooks so much more quickly in a pressure cooker. It is also a reason why our bodies need such careful temperature control.

### Importance of the Boltzmann factor

QUESTIONS 36 to 39

The Boltzmann factor is very sensitive to the ratio  $E/kT$ . For an energy twice  $kT$  it is around  $1/10$ , but for an energy twenty times  $kT$  it falls to about  $10^{-9}$ .

This sensitivity is fundamental to the possibility of life. We live on a planet at a temperature of some 300 K. Covalent bonds, like those in rocks, need an energy of several hundred times  $kT$  to break them, so rocks last a long time and do nothing. People, by contrast, must be held together by bonds which break often enough for reactions such as digestion to occur, but not so easily that life is over in a flash. The most important bond in biological material is the hydrogen bond, with breaking energy around ten times  $kT$ . Amongst other essential things, hydrogen bonds play an important role in holding the two strands of DNA together.

READING

'Life in the Universe' (page 267)

'Our nuclear history' in the Reader  
*Particles, imaging, and nuclei*

At very high temperatures, the energy  $E$  can be large and still allow an appreciable fraction of particles with high energy. Thus in the Sun, the rate of the nuclear reaction of hydrogen fusing to helium is in part controlled by the small fraction of protons which acquire enough energy to 'touch' despite their electrical repulsion. In the early stages of the birth of the Universe (the 'Big Bang'), it was hot enough for some protons to gain enough mass-energy to exist as neutrons, and the temperature at that time decided the present ratio of protons to neutrons in the Universe.

Less dramatic examples of applications of the Boltzmann factor include lasers, ionization in flames, the emission of electrons from a hot cathode in a television tube, and the occurrence of defects in solids.

In all such examples the Boltzmann factor tells how chance sharing of energy works out.



## Section K4 EXPLOITING ENTROPY CHANGES

### The Second Law of Thermodynamics

When energy is transferred thermally from hot to cold, the entropy increases. The idea is quite general:

Second Law of Thermodynamics

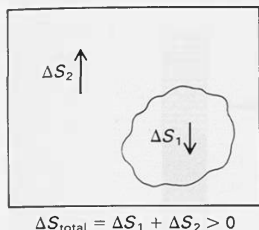


Figure K21

Total entropy increases in a closed system.

All changes occur in the direction for which the total entropy change is an increase.

The total entropy never decreases.

At equilibrium, the total entropy change is zero.

The word 'total' refers to all entropy changes produced. The entropy of part of the system can decrease, but if so there must be a bigger increase elsewhere if the process is to happen (figure K21).

It is the Second Law which explains our feeling that, despite the First Law (energy conservation), energy is in fact easily 'lost'. A moving car comes to a halt, because its kinetic energy is shared out in many more ways amongst warmed-up road and brakes. A warm room cools off as energy spreads through the walls and windows, again being shared in more ways by more particles. What is lost is not energy, but the potential for further change.

#### QUESTIONS 40 to 42

Any spontaneous change in an isolated system (such as a mixture of liquids in a stoppered vacuum flask) must increase the number of arrangements of molecules and their energy. The number of ways could only decrease if the molecules were to avoid getting into certain of the arrangements which they could in fact get into. But this would mean that their behaviour was not random.

More interesting are cases where entropy increases in one part of the system are exploited to achieve entropy decreases in another part. Thus letting carbon dioxide out of a cylinder increases the entropy by expanding the gas, and this permits the smaller entropy decrease involved in cooling and even solidifying some of the carbon dioxide (figure K22). An increase in one kind of numbers of ways can pay for a decrease in another.

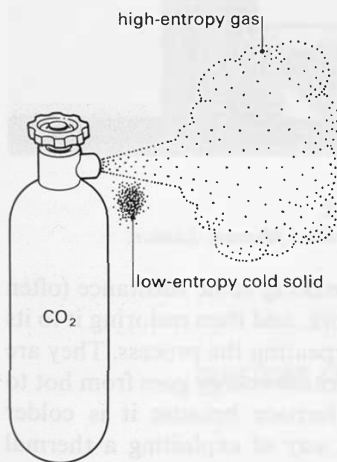


Figure K22

Entropy increase can 'pay' for a decrease.

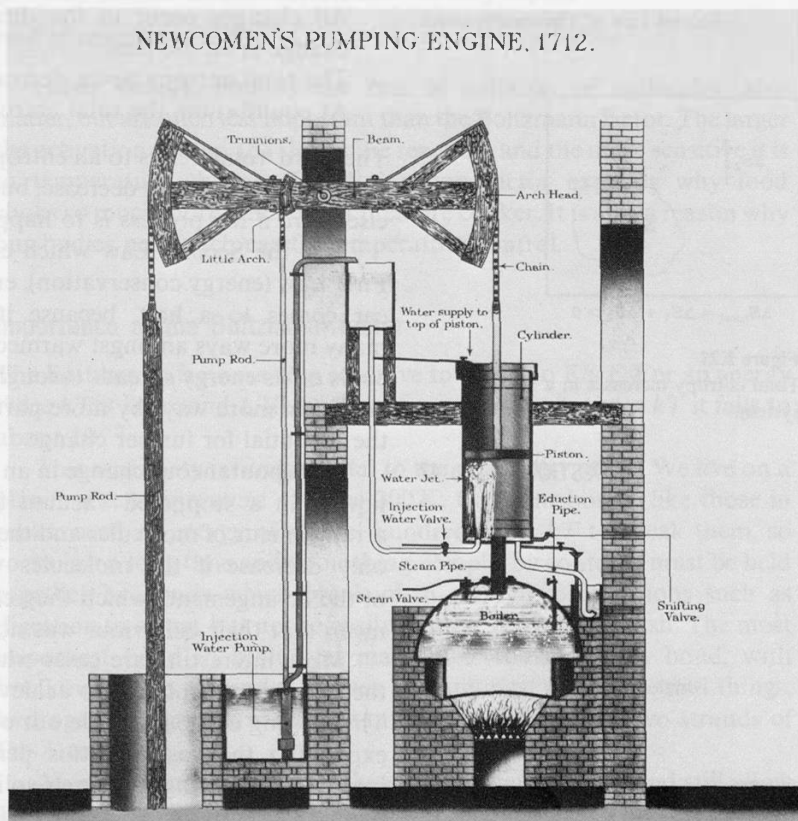
This is how entropy changes can be exploited, in nature or by human intervention. Any process which is going to happen depends on the existence of some disequilibrium, and will involve an entropy increase as it moves towards equilibrium. By some device, it may be possible to use the process to run another which produces a desirable entropy decrease, as long as the *total* entropy still increases. Thus entropy increase both permits and sets limits to exploiting natural processes.

In this way we get combustion to run engines, extract pure metals from their ores, desalinate sea-water, freeze food for storage, or make chemical reactions in batteries run electric motors.

## The inefficiency of engines

Unit G, 'Energy sources',  
efficiency of power stations

It was in studying steam engines, so important for the Industrial Revolution, that the science of thermodynamics began. The French engineer Carnot (1796–1832) argued that all such engines, no matter what their design, must be inefficient to some extent.



**Figure K23**

Drawing of Newcomen's pumping engine circa 1712.

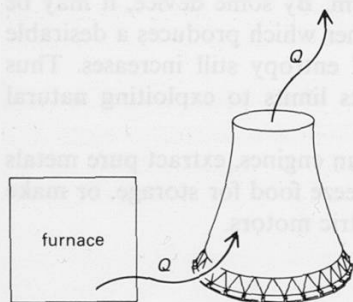
Reproduced by permission of The Trustees of the Science Museum, London.

A whole variety of engines work by making some substance (often steam) hot, getting it to expand and do work, and then restoring it to its original state, making it hot again, and repeating the process. They are all called *heat engines*. They work only because energy goes from hot to cold: the steam gets energy from the furnace because it is colder than the furnace. So a heat engine is a way of exploiting a thermal disequilibrium.

A thermal transfer of energy across a temperature difference increases entropy. If energy  $Q$  is transferred thermally from a furnace at temperature  $T_{\text{hot}}$  to the atmosphere at  $T_{\text{cold}}$ , the net entropy change is

$$\Delta S = Q/T_{\text{cold}} - Q/T_{\text{hot}}$$

Only because  $T_{\text{hot}} > T_{\text{cold}}$  does the energy flow occur, and the entropy increase.

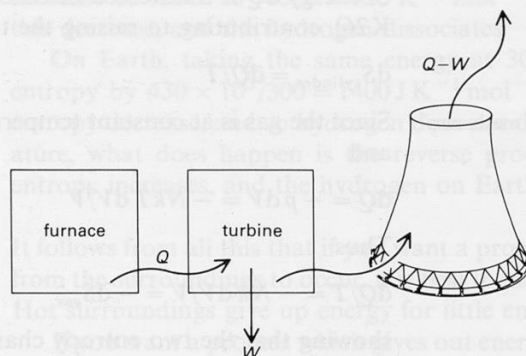


**Figure K24**

A power station merely heating the countryside.

A heat engine is a way of diverting some of the energy flow, so that it does useful work. In the long run, this energy still goes to the surroundings, but it is channelled through whatever dissipates energy in the job the engine does – whether driving a car, pumping water as Newcomen’s engine did, or driving machinery. In the long run the entropy increases as much as if the engine wasn’t there, but in the short run there is a smaller increase.

The best we could possibly do would be to have the entropy stay constant in the short run. If the engine was well designed we could



**Figure K25**

A power station not increasing entropy more than is necessary.

imagine diverting work  $W$  usefully, so that only energy  $(Q - W)$  was delivered immediately to the cool surroundings (figure K25). The entropy change would then be

$$\Delta S = (Q - W)/T_{\text{cold}} - Q/T_{\text{hot}}$$

$\Delta S$  cannot be less than zero, so the best that can be done is to have

$$(Q - W)/Q \geq T_{\text{cold}}/T_{\text{hot}}$$

or

$$W/Q \leq 1 - T_{\text{cold}}/T_{\text{hot}}$$

But  $W/Q$ , the ratio of useful energy output to total energy input, is the efficiency of the engine. If  $T_{\text{hot}}$  is 1000 K and  $T_{\text{cold}}$  is 300 K, at least 3/10 of the energy from the furnace must be delivered to the surroundings and the efficiency cannot exceed 70 %.

The argument is very general, and it may seem as if there ought to be some ingenious way round it. But there isn't, *because* it is so general. It depends on nothing more than the fact that (as Carnot put it) in getting motive power from fire, the substance which gets energy from the fire must be cooler than the fire.

### Further applications

The further applications which follow all use the idea of entropy. You should study at least one.

QUESTIONS 43 to 45

## EXPERIMENT K10

### Entropy changes in a gas

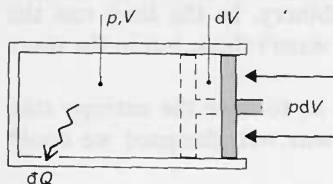


Figure K26  
Compressing a gas.

Unit A, 'Materials and mechanics'

### Energy and entropy changes in a gas

If a gas is gently compressed or expanded, the smallest change in the force on the piston can bring it to rest. So the system is always very close to equilibrium, and the entropy change should approach zero.

But if the gas is compressed, its entropy falls:

$$dS_{\text{gas}} = Nk dV/V$$

A conducting cylinder can keep the gas at a constant temperature. As soon as the gas is warmed slightly by the compression, a small amount of energy  $\delta Q$  is transferred thermally to the walls of the cylinder (figure K26), contributing to raising the total entropy by

$$dS_{\text{cylinder}} = \delta Q/T$$

Since the gas is at constant temperature, its internal energy is constant, and

$$\delta Q = -p dV = -NkT dV/V$$

Thus

$$\delta Q/T = -Nk dV/V = -dS_{\text{gas}}$$

showing that the two entropy changes are then equal and opposite.

Of course, if there is friction on the piston, or turbulence in the gas,  $\delta Q$  is numerically larger than  $p dV$  and there is a net entropy increase.

### QUESTION 46

### QUESTIONS 47, 48

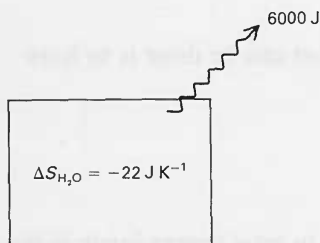


Figure K27  
Freezing a mole of water.

### Equilibrium

At equilibrium, the total entropy change is zero. But we do have to take into account all entropy changes. Consider the freezing of water. When water changes to ice, its entropy decreases by  $22 \text{ J K}^{-1} \text{ mol}^{-1}$ , as a result of the formation of the ice crystal structure. But at the same time,  $6000 \text{ J mol}^{-1}$  are liberated to the surroundings, increasing the total entropy on that account, by an amount depending on the temperature of the surroundings (figure K27).

	$\Delta S/\text{J K}^{-1} \text{ mol}^{-1}$
water to ice	-22
surroundings at 250 K	+24
surroundings at 300 K	+20
surroundings at 273 K	+22

Table K1

$$-22 + \frac{6000}{250} = +2$$

$$-22 + \frac{6000}{300} = -2$$

$$-22 + \frac{6000}{273} = 0$$

At 250 K the entropy increase  $Q/T$  of the surroundings is larger than the decrease due to the change in structure. There is a net entropy *increase* of  $2 \text{ J K}^{-1} \text{ mol}^{-1}$ . Such an event can happen, and it does: water at 250 K freezes.

At 300 K the entropy of the surroundings increases by less than that of the water decreases. There is a net entropy *decrease*. Such a thing cannot happen, and indeed water does not freeze at 300 K.

At 273 K the two changes are equal, and freezing and melting are in equilibrium. The net entropy change is *zero*.

## EXPERIMENT K11

### 'Hot ice'

$$+100 - \frac{430 \times 10^3}{6000} > 0$$

$$+100 - \frac{430 \times 10^3}{300} < 0$$

$$-100 + \frac{430 \times 10^3}{300} > 0$$

$$T = d(\text{energy})/d(\text{entropy})$$

The example seems trivial, but just this reasoning is used to determine whether any change can happen, and where its equilibrium will lie.

Another example: hydrogen is dissociated in the Sun, but not on Earth. If hydrogen dissociates, there are two particles where before there was one, and so an increase in entropy, in fact of  $100 \text{ J K}^{-1} \text{ mol}^{-1}$ . But to dissociate  $\text{H}_2$  means breaking bonds, taking energy from the surroundings, amounting to  $430 \text{ kJ mol}^{-1}$ .

Taking this energy from the Sun at  $6000 \text{ K}$  gives an entropy decrease of  $430 \times 10^3/6000 = 72 \text{ J K}^{-1} \text{ mol}^{-1}$ . The increase outweighs this decrease, and the hydrogen dissociates.

On Earth, taking the same energy at  $300 \text{ K}$  would decrease the entropy by  $430 \times 10^3/300 = 1400 \text{ J K}^{-1} \text{ mol}^{-1}$ . There would be a net entropy decrease, and so hydrogen does not dissociate. At this temperature, what does happen is the reverse process, for which the total entropy increases, and the hydrogen on Earth is strongly associated.

It follows from all this that if you want a process which absorbs energy from the surroundings to occur, it is best to make the surroundings hot. Hot surroundings give up energy for little entropy decrease.

If you want a process which gives out energy to the surroundings to occur, it may be best to make the surroundings cool. Cool surroundings have a large entropy increase for the receipt of given energy. On the other hand, making things cool also slows down reactions, so although the reaction may be possible it may not happen fast enough to be useful. Then you may need a catalyst, to reduce any energy barrier.

### A cell which runs on entropy

## EXPERIMENT K12

### Concentration cell

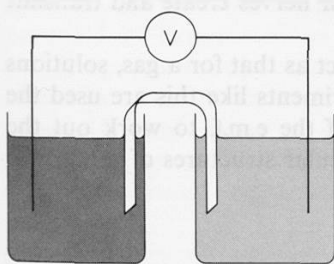


Figure K28  
e.m.f. from a concentration difference.

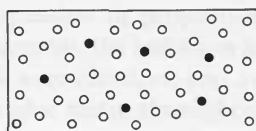
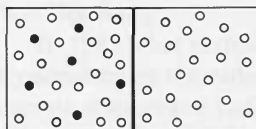


Figure K29  
Doubling the dilution.

A difference in concentration between two beakers of copper sulphate solution, is enough to produce an e.m.f. (figure K28).

The electrode in the more concentrated solution is positive.  $\text{Cu}^{2+}$  ions have arrived from the solution, so making the solution more dilute. At the negative electrode,  $\text{Cu}$  atoms go into solution, making it more concentrated. This is of course the direction in which the concentrations would have changed if the solutions just diffused.

At the molecular level, the particles in the solution are continually moving about, changing places with one another. In how many more ways can they be arranged if we dilute a solution? Suppose we dilute the solution by doubling the number of solvent molecules (figure K29).

Each solute molecule (or ion) can now change places with a solvent molecule in twice as many ways as before. Doubling the dilution multiplies the number of ways of arranging  $N$  solute particles amongst solvent particles by the factor  $2^N$ . Just as for expanding a gas,

$$W' = W \times 2^N$$

$$\Delta S = kN \ln 2$$

The number 2 is now the ratio of two dilutions. The general form of the relation is

$$\Delta S = -kN \ln(\text{concentration ratio})$$

with entropy increasing as concentration falls. In real solutions, the entropy change may be given approximately by such an expression.

#### QUESTIONS 49, 50

The action of the cell is equivalent to taking some of the more concentrated solution, diluting it, and putting it in the dilute solution. Thus the equation also gives the net entropy increase if  $N$  ions of  $\text{Cu}^{2+}$  are removed from the concentrated side, and produced at the dilute side. The concentration ratio is now the ratio of concentrations in the two parts of the cell. But the entropy need not increase: it must simply never decrease. The cell can take energy from the surroundings and do electrical work.

If the cell e.m.f. is  $\mathcal{E}$ , then the maximum energy it can deliver when  $N$  ions pass is  $2eN\mathcal{E}$  (two electrons pass for each  $\text{Cu}^{2+}$  ion). If the entropy decrease of the surroundings balances the entropy increase in the cell, then

$$k/e \approx 8.6 \times 10^{-5} \text{ JK}^{-1} \text{ C}^{-1}$$

$$\ln(1/10) = -2.3$$

$$\mathcal{E} \approx 30 \text{ mV}$$

$$2eN\mathcal{E}/T = -kN \ln(\text{concentration ratio})$$

$$\mathcal{E} = -(kT/2e) \ln(\text{concentration ratio})$$

This gives a few tens of millivolts for a concentration ratio of 10 to 1 at  $T = 300 \text{ K}$ . The e.m.f.s from such concentration differences are an important part of the means by which our nerves create and transmit electrical signals.

The calculation is nothing like so exact as that for a gas, solutions being much more complex. In fact, experiments like this are used the other way round – from observations of the e.m.f. to work out the entropy change, to learn about the molecular structures of solutions.

# READING

## LIFE IN THE UNIVERSE

The Universe is a place of dramatic extremes: dense, white-hot stars set in the cold, black emptiness of space. It is clearly very far from equilibrium. We, the human race, live on a planet populated with an immense variety of kinds of biological pattern, from viruses through algae and plants to the animals. Yet all this must have come about through the universal tendency of things to decay; to become more, not less even.

The story begins with the birth of the Universe. Three simple facts lead us to suppose that the Universe is not infinitely old nor infinitely big, so that there must have been such a birth. These facts are:

the sky is dark at night  
there are stars in the sky  
there is matter in the Universe.

The German astronomer Olbers (1758–1840) pointed out that if the Universe were infinite, we should see a star in every direction we looked, so that the sky would be as white hot as the surface of a star. But, as Newton realized, if matter occupies only part of space, its gravitational attraction will pull it all together. The Universe is either infinite, but would then be white hot, or not infinite and not stable. The Universe cannot be infinitely old, because it only takes a finite time for a system to reach equilibrium, so the bright stars we see would not be there.

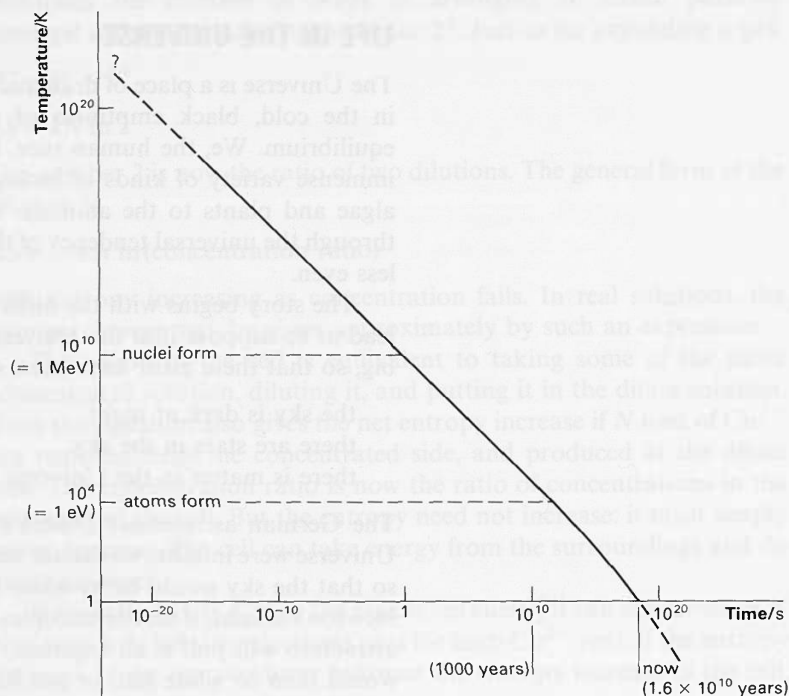
We now think that the Universe is ‘young’, and expanding. Friedmann showed in 1922 that Einstein’s equations of General Relativity, replacing Newton’s Laws of Gravitation, share the problem of stability, so that a Universe containing matter must either expand or contract. Recently, Hawking and Penrose have shown that space–time itself is gravitationally unstable, and that the Universe must contain at least one ‘singularity’ (point of collapse).

In the 1930s Hubble showed that the distant galaxies were running away from us, at a rate indicating a compressed origin about 16 billion years ago. Gamow in the 1940s used nuclear physics to work out how the elements might have been built up in that compressed state. He found that to get the observed preponderance of hydrogen with some helium, he had to suppose that the early Universe was very hot – the ‘hot Big Bang’.

In 1965 what is thought to be the faint glow of that bang was found by accident by the radio engineers Penzias and Wilson, who discovered cosmic microwave radiation filling space and travelling equally in all directions. As the Universe expands, the wavelength of radiation stretches in proportion and its temperature falls ( $\lambda T = \text{constant}$ ). At present, the radiation has a temperature of about 3 K, a wavelength of a few centimetres, and an intensity equivalent to about  $10^9$  photons per cubic metre throughout space. This compares with about one proton

K

mass per cubic metre. Tracing back the size and temperature, we can reconstruct the thermal history of the Universe (figure K30).



**Figure K30**  
Cooling of the Universe.

At about time 1 second after the birth of the Universe, the temperature was high enough for protons to be converted into the more massive neutrons, by randomly acquiring the extra energy needed. But at about this time, the expansion meant that the particles got too far apart to collide often enough, so the reaction effectively stopped and the present proportion of neutrons to protons (around 1 to 10), and so of hydrogen to helium, was 'frozen' into the composition of the Universe.

After about 100 000 years, the temperature was low enough (about 1000 K) for electrons to combine with protons to make hydrogen atoms. Before this, the intense radiation kept matter from collapsing, but now with matter in the form of neutral atoms, gravity could begin to magnify local irregularities and clump the gas into clouds which became the galaxies.

But gravitational attraction delivers energy, so the collapsing matter grew hot, nuclear reactions began, and there were stars. The first stars were made of hydrogen and helium, but as they grew old, there was time (as there had not been in the 'Big Bang') to synthesize heavier nuclides, such as carbon and oxygen. Still heavier nuclides, like iron and up to uranium, were made in the explosive death throes of such stars (novae and supernovae). The dust and gas from such dead stars could again be collapsed by gravity, and new stars like our Sun formed, probably with planets like Earth made of the heavy-element ashes of ancestor stars.



The thermal disequilibrium of our present Universe thus came from gravitational instability as a source of increasing entropy. Then the thermal disequilibrium acted as a further source of increasing entropy, driving further change. So far as we understand it, this is how life began.

The Sun's photons arrive at the Earth at a radiation temperature of about 6000 K, and are re-radiated at about 300 K. With photons delivering 1 kW per square metre, the entropy increase per square metre per second is about  $3 \text{ J K}^{-1}$ . This is enough to drive reactions such as the photosynthesis of sugars or cellulose from carbon dioxide and water by plants. We and other animals profit by eating these low-entropy forms so as to drive other reactions in our bodies. We live by the Second Law of Thermodynamics, not in spite of it.

Life has another special property: self-replication. All organisms on Earth use the same method, a genetic code-carrying molecule (DNA) which replicates itself. The ingredients of DNA have already been made in the laboratory just by exposing more primitive chemicals known to exist in the Universe to heat and light. Nobody knows how the first self-replicating molecule arose, though von Neumann has proved that sufficiently complicated structures are capable of it. Once self-replication happens, the inevitability of mistakes, due again to the Second Law, gives it enormous power. Poor copies which replicate a little more slowly are rapidly suppressed, but if conditions change and what was a poor copy has an advantage, it soon takes over. In this way, given a Sun delivering energy and a continual entropy increase for a few billion years, structures could grow and evolve, which may be the reason that we are now here to wonder at it all.

### Questions

- a Prove that Olbers was right. Assume  $n$  stars per unit volume, and write down the number of stars  $dn$  in a spherical shell of radius  $r$ , thickness  $dr$ . Use the inverse-square law to write down the intensity  $dI$  contributed by this shell. Why do all such shells, no matter what their radius, contribute the same intensity? Why does this mean that Olbers was right?
- b What is the energy per cubic metre in the cosmic background radiation, if its wavelength is about 5 cm? How does this compare with the energy  $mc^2$  of one proton mass per cubic metre?
- c If the temperature of the cosmic radiation is now 3 K, how many times smaller was the Universe than it is now, in the year 100 000 when the temperature was about 1000 K ( $\lambda T = \text{constant}$ )? How many proton masses were there per cubic metre at that time? How many per cubic metre at time 1 s, when the temperature was about  $10^{10}$  K?
- d Use  $\Delta E = c^2 \Delta m$  to find the energy equivalent of the mass difference between neutrons and protons. At what temperature, according to the Boltzmann factor  $e^{-\Delta E/kT}$ , would their numbers have been in the ratio 1 to 10?

$$(m_p = 1.67265 \times 10^{-27} \text{ kg}; m_n = 1.67495 \times 10^{-27} \text{ kg})$$

- e It requires about  $20 \times 10^{-19} \text{ J}$  to ionize a hydrogen atom. What, from the Boltzmann factor, is the probability of an atom acquiring this energy by random collisions if the temperature is *i* 10 000 K  
ii 1000 K?

- f Study of the dynamics of the expansion of the Universe suggests that as the early Universe expanded and cooled, its density  $\rho$  and temperature  $T$  at time  $t$  were given by

$$\rho t^2 = 3/(32\pi G)$$

$$T^4 = 2 \times 10^{31} \rho$$

Estimate the density and temperature at  $t = 1 \text{ s}$ .

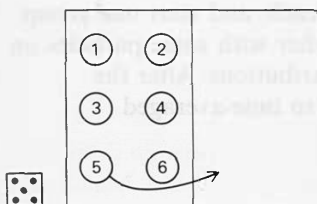
- g Show, using  $Q(1/T_{\text{Earth}} - 1/T_{\text{Sun}})$  and data in the Reading, that 1 kW of sunlight per square metre corresponds to an entropy increase of about  $3 \text{ J K}^{-1} \text{ m}^{-2} \text{ s}^{-1}$ .
- h Suppose cells of an early life-form (say blue-green algae) can reproduce once by cell division in one hour. How many copies are made in 24 hours, if none die? Suppose a genetic 'mistake' increases the time to reproduce by 10 per cent, to 66 minutes. How many generations are produced in 24 hours? How many fewer descendants than the original does the mutation have in 24 hours? What happens if changed conditions mean that the mutation has an advantage and reproduces in 10 per cent less time, 54 minutes?

# LABORATORY NOTES

## GAME

### K1 Particle shuffling

6 counters  
die  
graph paper



**Figure K31**  
Counters in a box.

Draw a box with two equal halves on squared paper. Place six numbered counters in one half. Throw the die, and move the counter whose number comes up to the half it is not presently occupying. Continue moving, recording how many counters are in each half.

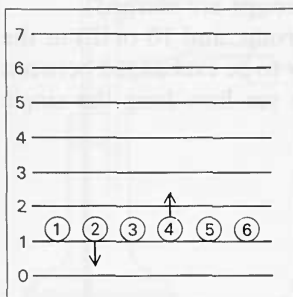
What *must* happen on the first throw? What is likely to happen on the second throw? What distribution of counters between the halves happens most often?

Is it possible that all the counters will go back into the half they started in? Is it likely?

## GAME

### K2 Energy shuffling

20 counters (or more)  
*either*  
2 dice  
*or*  
other means of picking counters at random  
graph paper



**Figure K32**  
Counters on 'levels'.

Rule lines for 'energy levels' on a sheet of paper. Put numbered counters down in some initial pattern (for example, all on level 1). Start with just six counters; then try a larger number – 20 or more.

The rules of the game are:

- 1 Pick a counter at random and move it down one level. (If the counter you pick is on the lowest level, pick again.)
- 2 Pick a counter at random and move it up one level.
- 3 Repeat.

Record how many counters are on each level, after making several moves. A class average should show a graded distribution.

Given your initial pattern, what is the highest level any counter can reach? Why is it rare for a counter to get high up on the levels?

How does the rule of the game conserve 'energy'?

## COMPUTER SIMULATION

### K3 Equilibrium distribution of energy

microcomputer  
program 'Quantum shuffling'

K

## EXPERIMENT

### K6 Current in a thermistor

thermistor (e.g. RS Components TH3)  
 cell holder with two cells  
 milliammeter, 10 mA d.c.  
 milliammeter, 100 mA d.c.  
 2 crocodile clips  
 thermometer,  $-10$  to  $110^{\circ}\text{C}$   
 tripod  
 gauze  
 Bunsen burner  
 beaker,  $250\text{ cm}^3$   
 retort stand base, rod, boss, and clamp  
 leads

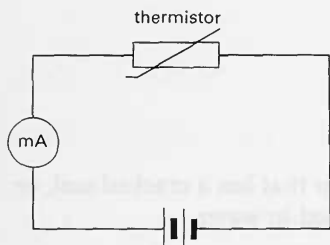


Figure K35

Measure the current in the thermistor for a fixed potential difference. Heat the thermistor by suspending it in water heated in a beaker.

Is the plot of  $\ln I$  against  $1/T$  linear? From its slope, what is the energy  $E$  in  $e^{-E/kT}$ ? Why would differences in how easily electrons move at different temperatures also affect the results?

## EXPERIMENT

### K7 Viscous flow

Ostwald viscometer  
 stopwatch  
 tall beaker  
 thermometer,  $-10$  to  $110^{\circ}\text{C}$   
 tripod  
 gauze  
 Bunsen burner

Any method suited to measuring the rate of flow of a liquid at different temperatures will serve. The Ostwald viscometer is convenient. Other possibilities include measuring the torque on a drum suspended in a rotating can of liquid, the torque needed to stir a liquid, or the time taken for a vessel to empty through a small outlet.

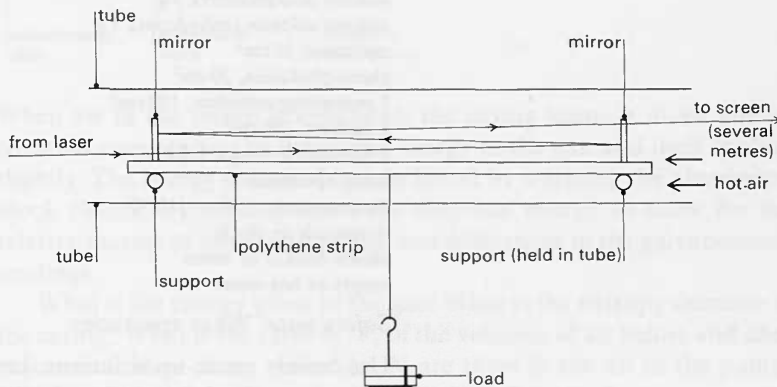
For many liquids, including water, the dependence of rate of flow on temperature is governed by the Boltzmann factor. Certain oils, with complex molecules, may not show this behaviour.

If the method used measures the time of flow,  $t$ , for a given quantity, the rate is proportional to  $1/t$ . Is a plot of  $\ln(1/t)$  against  $1/T$  linear? If you have measured viscosity  $\eta$ , plot  $\ln(1/\eta)$  against  $1/T$ .

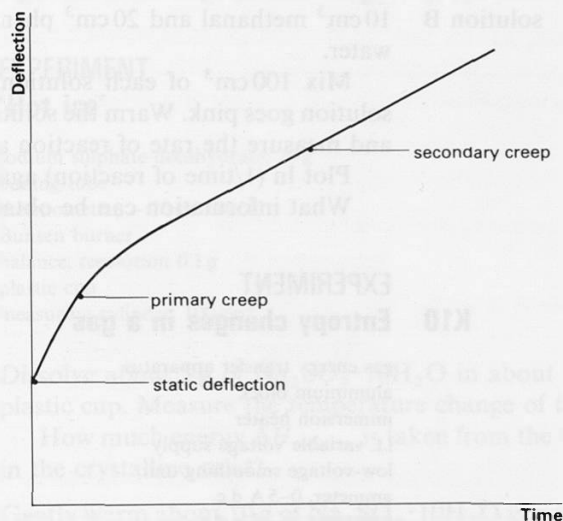
## EXPERIMENT

### K8 Creep

polythene strip  
hanger and slotted masses, 10 g  
2 plane mirrors  
laser (or compact light source and lens)  
screen (e.g. piece of white paper on wall)  
75 mm diameter tube (e.g. drain pipe)  
supports (e.g. retort stand rods, short)  
hair dryer  
thermometer,  $-10$  to  $110^{\circ}\text{C}$   
Plasticine



**Figure K36**  
Apparatus to study creep.



**Figure K37**  
Primary and secondary creep.

**Safety note:** If you use a laser make sure that the beam cannot enter your eye, either directly or after reflection from a mirror.

You can study creep if you bend a polythene strip by hanging a load at its centre. Mirrors on each end of the strip tilt as it bends, and deflect a beam of light cast on a screen several metres away (figure K36).

The beam first deflects as soon as the load is applied (static deflection) and then creeps rather rapidly for a time (primary creep).

Then it settles down to a linear rate of creep (secondary creep). It is this secondary rate of creep which you should investigate (see figure K37).

The temperature can be controlled by enclosing the strip in a tube (which can carry supports for the strip), and blowing hot air from a hair dryer down the tube.

The experiment is lengthy, and you may only be able to study the creep at two or three temperatures.

## EXPERIMENT

### K9 Rate of reaction

sodium disulphate(iv), 5 g  
sodium sulphite (anhydrous), 1 g  
methanal, 10 cm<sup>3</sup>  
phenolphthalein, 20 cm<sup>3</sup>  
2 measuring cylinders, 100 cm<sup>3</sup>  
2 beakers, 250 cm<sup>3</sup>  
2 volumetric flasks, 1 dm<sup>3</sup>  
safety spectacles  
thermometer, -10 to 110 °C  
stopwatch or clock  
plastic bucket or basin  
supply of hot water

*Safety note:* Wear spectacles.

Use freshly made-up solutions, kept in stoppered flasks:

solution A 5 g sodium disulphate(iv) and 1 g sodium sulphite in 1 dm<sup>3</sup> water  
solution B 10 cm<sup>3</sup> methanal and 20 cm<sup>3</sup> phenolphthalein made up to 1 dm<sup>3</sup> with water.

Mix 100 cm<sup>3</sup> of each solution, and measure the time before the solution goes pink. Warm the solutions in hot water (and/or cool in ice), and measure the rate of reaction at different temperatures.

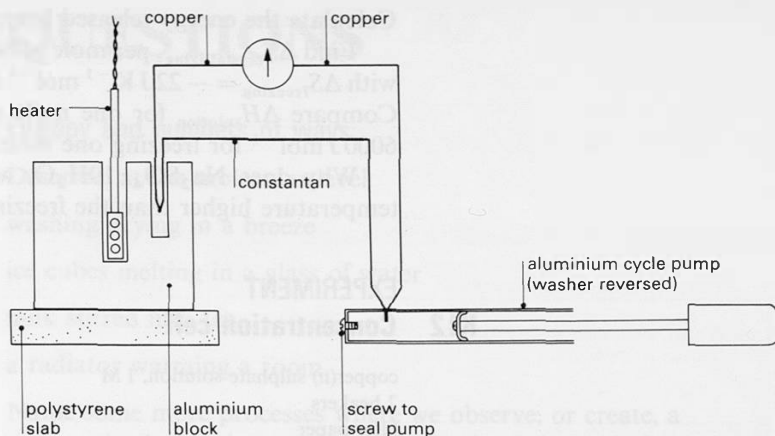
Plot  $\ln(1/\text{time of reaction})$  against  $1/T$ .

What information can be obtained from the graph?

## EXPERIMENT

### K10 Entropy changes in a gas

gas energy transfer apparatus  
aluminium block  
immersion heater  
l.t. variable voltage supply  
low-voltage smoothing unit  
ammeter, 0–5 A d.c.  
voltmeter, 0–15 V d.c.  
sensitive galvanometer  
copper wire, 0.28 mm diameter (bare)  
constantan wire, 0.28 mm diameter (covered)  
G-clamp  
slab of expanded polystyrene  
transparent adhesive tape  
measuring cylinder, 100 cm<sup>3</sup>  
balance, resolution 0.1 g  
stopwatch or clock  
leads



**Figure K38**  
Gas energy transfer.

When air in the pump is expanded, the casing keeps it at an almost constant temperature by delivering energy to the gas, and itself cooling slightly. The energy delivered can be found by warming the aluminium block electrically, scaling down the electrical energy to allow for the relative masses of block and pump, and differences in the galvanometer readings.

What is the energy given to the gas? What is the entropy decrease of the casing? What is the ratio  $V_2/V_1$  of the volumes of air before and after expansion? How many molecules  $N$  are there in the air in the pump? What is the entropy increase  $Nk \ln(V_2/V_1)$  of the gas?

## EXPERIMENT

### K11 'Hot ice'

sodium sulphate decahydrate, 20 g  
boiling-tube  
thermometer,  $-10$  to  $110^\circ\text{C}$   
Bunsen burner  
balance, resolution 0.1 g  
plastic cup  
measuring cylinder,  $100\text{ cm}^3$

- a Dissolve about 5 g of  $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$  in about  $50\text{ cm}^3$  of water in a plastic cup. Measure the temperature change of the water.

How much energy  $\Delta H_{\text{solution}}$  is taken from the water to break bonds in the crystalline solid?

- b Gently warm about 10 g of  $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$  in a boiling-tube, until the crystals dissolve in their own water of crystallization. Cool slowly to about  $25^\circ\text{C}$ . Drop in a further crystal, and observe the temperature  $T_{\text{eq}}$  at which crystals and water are in equilibrium. Note the release of energy to the surroundings when the solution crystallizes.

At equilibrium,

$$\Delta S_{\text{crystallizing}} + \text{energy released}/T_{\text{eq}} = 0$$

Calculate the energy released from the results of part a.

Find  $\Delta S_{\text{crystallizing}}$ , per mole of  $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ . Compare the value with  $\Delta S_{\text{freezing}} = -22 \text{ J K}^{-1} \text{ mol}^{-1}$  for the freezing of one mole of water. Compare  $\Delta H_{\text{solution}}$  for one mole of  $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$  with the value  $6000 \text{ J mol}^{-1}$  for freezing one mole of water.

Why does  $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ , which is mostly water, 'freeze' at a temperature higher than the freezing point of water?

## EXPERIMENT

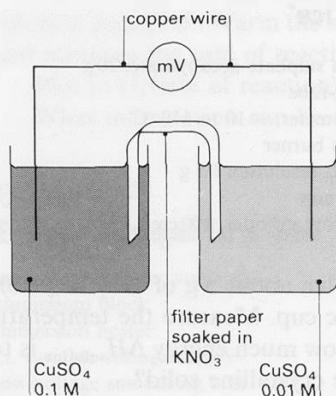
### K12 Concentration cell

copper(II) sulphate solution, 1 M  
2 beakers  
filter paper  
potassium nitrate solution  
high-impedance voltmeter, 100 mV d.c.  
distilled water

Which side of the cell is electrically positive? Which side would be positive if  $\text{Cu}^{2+}$  ions were removed from solution on that side and produced at the other? For one mole of  $\text{Cu}^{2+}$  ions taken from the concentrated side and produced at the dilute side, what is the entropy increase  $-kL \ln(\text{lower concentration/higher concentration})$ ?

What is the work delivered electrically when two moles of electrons pass at the observed cell e.m.f.  $\mathcal{E}$ ? If all this work were taken from the surroundings, what would be the entropy decrease of the surroundings?

How would you expect altering the temperature of the cell to change its e.m.f.?



**Figure K39**  
Concentration cell.



# QUESTIONS

## Entropy and numbers of ways

- 1(I)** Describe, at the molecular level:
- a** washing drying in a breeze
  - b** ice cubes melting in a glass of water
  - c** milk stirred into tea
  - d** a radiator warming a room

Name some more processes where we observe, or create, a systematic change, but at the molecular level the underlying behaviour is purely chaotic.

- 2(L)** New packs of cards are sold arranged in one particular order. If they are shuffled, they can be arranged in many ways. The first card can be one of 52, the second one of 51, the third one of 50, and so on.

$$W = 52 \times 51 \times 50 \times \dots \times 2 \times 1 = 52!$$

Use a calculator with the factorial function (!) to find  $W$ ,  $\ln W$ , and  $k \ln W$ . What is the entropy increase when a pack of cards is shuffled?

- 3(L)** When the Mint makes new coins, they are all produced the same way up (say 'heads'). They are arranged in one way. But if everybody in the country put their change down, each coin could show 'heads' or 'tails'.

- a** Estimate the number  $N$  of coins in circulation in Britain.
- b** Argue that  $W = 2^N$  is the number of ways in which the coins could be put down.
- c** Use a calculator to find  $\ln W = \ln 2^N$ . Why is it not possible to get  $2^N$  itself on a calculator?
- d** Calculate the entropy change  $\Delta S = k \ln 2^N - k \ln 1$ .

- 4(P)** The genetic code has four 'letters', each consisting of a particular organic base. The base molecules are about  $10^{-9}$  m in size.

- a** The DNA chain molecule carrying the genetic 'message' for a fruit fly is about  $10^{-2}$  m long. What is the number  $N$  of 'letters' in the genetic 'message'?
- b** How many different 'messages' (*i.e.* distinct sequences of 'letters') could be carried by such a chain?
- c** If a fruit fly DNA molecule is decomposed into its component bases, what is the entropy increase associated with losing its 'message'?

K

- 5(P)** Figure K40 shows marbles being shaken in a container. The marbles are identical except for their colour. Why can you be sure that the sequence of pictures in (a) must be read from right to left? Why can you not be sure of the order in (b)?

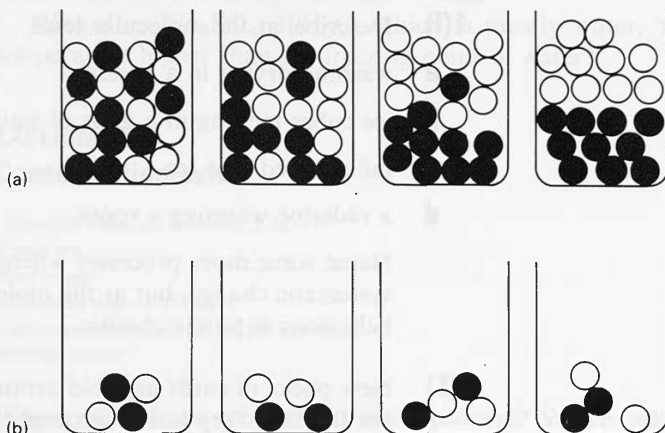


Figure K40

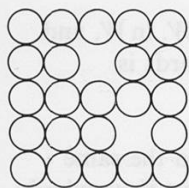


Figure K41

- 6(L)** Crystalline solids can have defects in the crystal lattice. Figure K41 shows a number of atoms, with two empty lattice sites.

- Draw some other pictures with the same number of atoms and vacancies.
- How many different pictures are there with  $N$  atoms and *one* vacancy?

- 7(P)** Suppose that in a strongly magnetized material all the molecular magnets point the same way, as in figure K42(a). Suppose that when the material is demagnetized, the molecular magnets can point 'up' or 'down' equally, as in figure K42(b).

- Argue that there are  $2^N$  possible arrangements of figure K42(b), if the direction of one magnet does not affect the others.
- Calculate the entropy change

$$\Delta S = k \Delta \ln W$$

for going from (a) with  $W=1$  to (b) with  $W=2^N$ , if  $N=L=6 \times 10^{23}$ .

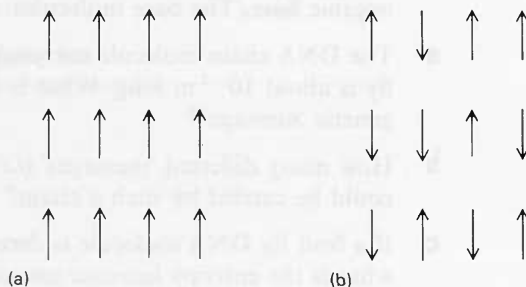


Figure K42

### Number of ways to multiply

- 8(I)** If a menu has four starters, six main dishes, and five sweets, followed by tea or coffee, how many meals can you order making one selection of starter, main dish, sweet, and drink? Why do the possibilities *multiply*?
- 9(I)** 100 people on holiday are allowed to choose freely between a coach trip or going to the beach. In how many different ways can the people arrange themselves given just two choices? If the organizer loses the list of choices, and makes one at random, how likely is he to pick the original set of choices?
- 10(P)** Plot a graph of  $Y = 2^N$  for  $N = 1, 2, \dots, 8$ . What would the graph of  $\lg Y$  against  $N$  be like? ( $\lg Y = \log_{10} Y$ .)

### Particles spreading out

- 11(L)** Calculate the chance of getting all of 100 particles, moving at random between the two halves of a box, back in one half, as follows:
- a** Use a calculator to find  $2^{100}$ .
  - b** Check the value, given that  $\lg 2 \approx 0.3$  and that  $\lg 2^{100} = 100 \lg 2$ , and writing  $2^{100}$  as a power of ten.
  - c** Suppose you look at the box every microsecond. For how many seconds would you have to look on average, before by chance all the particles were in one half?
  - d** The Universe is about  $10^{10}$  years old, and one year is about  $3 \times 10^7$  seconds. How many Universe lifetimes is your answer to part c?
- 12(L)a** About how big is a jar of air (atmospheric pressure and room temperature) which contains  $10^{22}$  molecules?
- b** Write down  $\lg 2^{10^{22}}$  as a power of 10.
- c** From the logarithm, to base 10, of a number  $n$  we can tell the number of digits in  $n$  when  $n$  is written out in full up to the decimal point. How many digits are there in  $2^{10^{22}}$ ? If you wrote out  $2^{10^{22}}$  with numerals only 1 mm across, how many metres long would it be? How many light years is that? ( $c = 3 \times 10^8 \text{ m s}^{-1}$ , 1 year  $\approx 3 \times 10^7 \text{ s}$ .) (Can you name a star at that distance?)
- 13(L)** Suppose that, God-like, you are deciding into which of many small parts  $dV$  of the volume of a gas every molecule will go. There are  $N$  molecules. (See figure K43, page 282.)
- a**
    - i* If you do it by throwing a die, why does the die need  $V/dV$  sides?
    - ii* What does the number  $W = (V/dV)^N$  represent?

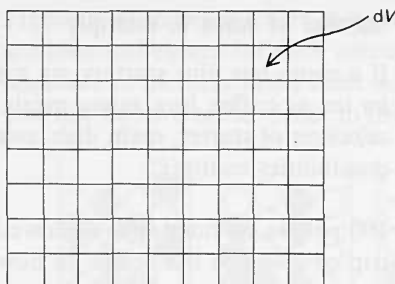


Figure K43

- b** If the volume is made bigger by  $dV$ , the die now needs  $(V + dV)/dV$  sides.

*i* What does the number  $W' = [(V + dV)/dV]^N$  represent?

*ii* What does the ratio  $W'/W = [(V + dV)/V]^N$  represent?

*iii* Express  $dS = k \ln(W'/W)$  in terms of  $V$ ,  $dV$ , and  $N$ .

- 14(L)** Use a calculator or computer with the natural logarithm function to find  $\ln 10$ ;  $\ln 10.5$ ;  $\ln 10.5 - \ln 10$ ;  $0.5/10$ .

Compare the last two answers. Try, with any number  $x$ ,  $\ln x$ ;  $\ln(x + dx)$ ;  $\ln(x + dx) - \ln x$ ;  $dx/x$ , where  $dx$  is small compared with  $x$ .

- 15(P)** What is the entropy change of the gas when, at constant temperature:

- 1 mole of neon expands from a pressure of 1 atmosphere (atm) to  $\frac{1}{2}$  atm?
- 1 mole of neon is compressed from  $\frac{1}{4}$  atm to  $\frac{1}{2}$  atm?
- 1 mole of neon expands from 1 atm to  $\frac{1}{4}$  atm?
- 2 moles of neon expands from 4 atm to 1 atm?
- 1 mole of helium contracts from 1 atm to 4 atm?
- 2 moles of helium expands from 16 atm to 1 atm?

- 16(E)** Estimate the entropy increase when a car tyre is punctured.

- 17(L)a** Estimate the volume of a cycle pump, or of a cylinder of a typical car engine.

- Pick any such volume. About how many molecules of ideal gas would there be in it at standard temperature and pressure?
- Use  $pV = NkT$  to plot a  $p$ - $V$  curve for the gas at 300 K.
- Take any two volumes  $V_1$  and  $V_2$ , and find the work involved in going from one to the other, from the area below the curve (figure K44).
- Find  $\ln V_1$  and  $\ln V_2$ , and calculate  $\Delta S = Nk(\ln V_2 - \ln V_1) = Nk \ln(V_2/V_1)$
- Check that the work from **d** is equal to  $T\Delta S$ .

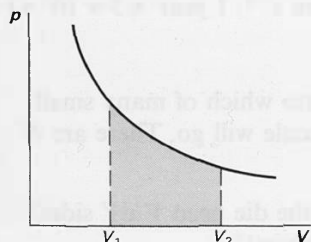


Figure K44

- 9 Suppose the gas were thermally insulated from its surroundings while the volume changed. What would happen to its temperature, and to its internal energy? Why must there be a thermal exchange of energy with the surroundings if the temperature is to remain constant?

### Particles and energy levels

- 18(I) There are five ways for two particles to share four quanta (assume equally spaced levels). Draw them. Guess how many ways for two particles sharing six quanta.

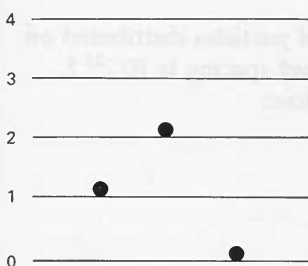


Figure K45

- 19(I) There are ten ways for three particles to share three quanta. Figure K45 shows one. Draw some more. Why must there be more than ten ways for three particles to share *four* quanta?
- 20(I) How many ways are there for  $N$  particles to share just one quantum?

- 21(E) Show that the following segment of computer program follows the rule of the energy shuffling game.

```
800 LET N = 100
900 DIM L(100)
1000 LET N1 = RND(N)
1010 LET N2 = RND(N)
1020 IF L(N1) = 0 THEN LET C = 0 ELSE LET C = 1
1030 LET L(N1) = L(N1) - C
1040 LET L(N2) = L(N2) + C
1050 GO TO 1000
```

( $RND(N)$  is a function choosing an integer from 1 to  $N$  randomly,  $L(N)$  is an array storing the level of each of  $N$  particles.) You could extend the program to give initial levels of particles and keep records of and display the number of particles on each level.

- 22(P)a Sketch the typical equilibrium pattern of numbers of particles on equally spaced energy levels.
- b Show how the pattern would change if the same number of particles shared more energy, and explain why there will now be fewer particles on the very lowest level.
- 23(P) If there is a fraction  $f = 0.5$  as many particles on each level as on the next lower level, and there are 256 particles on the lowest level, calculate the numbers expected on each level up to the eighth. Above what level will it be common to find no particles? How do the answers change if  $f = 0.25$ , and  $f = 0.75$ ?
- 24(P) Why does the fact that the number of particles on a given level is a constant fraction of the number on the level below mean that the

distribution is correctly called *exponential*? What function of the number of particles on each level would you plot to obtain a *linear* slope of the distribution?

**25(R)a** Suppose one set X of particles on equally spaced energy levels has  $f$  (the Boltzmann factor) equal to  $2/3$ , and another set Y has  $f = 1/3$ . Which is the hotter?

**b** Removing a quantum multiplies  $W$  by  $f$ . Adding a quantum multiplies  $W$  by  $1/f$ . What is the effect on  $W$  for the set X of losing one quantum? What is the effect on  $W$  for the set Y of gaining one quantum? What is the effect on the product ( $W$  for X)  $\times$  ( $W$  for Y)?

**26(P)** Suppose the Boltzmann factor  $f$  for a set of particles distributed on equally spaced levels is 0.5. If the energy level spacing is  $10^{-21}$  J, what is the entropy change if the particles lose:

**a** 1 quantum

**b** 100 quanta

**c**  $10^{-18}$  J

**d** 1 J

**e** 1000 J

assuming that there are enough particles for  $f$  to remain nearly constant?

### Thermal transfer of energy

**27(P)a** If there are as many quanta as particles, the Boltzmann factor  $f = 0.5$  and  $\ln f = -0.693$ . If the energy level spacing is  $10^{-21}$  J, what is the temperature of such a system?

**b** What is the factor  $f$  for the same level spacing if  $T = 300$  K?

**c** If there are  $N$  particles sharing  $n$  quanta,

$$f = \frac{n}{n + N}$$

*i* At 300 K, with level spacing  $10^{-21}$  J, how many quanta are there per particle?

*ii* About what fraction of those on the lowest level would be on the tenth level?

**28(L)** When one quantum is added to a system,  $W$  is multiplied by  $1/f$ , and the entropy increases. In the questions below assume that  $f$  remains constant.

**a**  $f$  is less than unity. Show that  $\Delta S = -k \ln f$  is positive.

**b** What is the entropy change if *two* quanta are added?

- c** If energy  $Q$  is added, with level spacing  $\varepsilon$ , how many quanta are added?
- d** Show that if  $\Delta S = -nk \ln f$  for adding  $n$  quanta, and  $\ln f = -\varepsilon/kT$ , adding energy  $Q$  increases the entropy by  $\Delta S = Q/T$ .

**29(L)a** Complete table K2:

$f$	$-\ln f$
1	0
0.5	
0.1	
0.01	

Table K2

- b** When  $f$  is *small*, why is the entropy change  $\Delta S = -k \ln f$  for adding one quantum *large*?
- c** As the temperature is made lower, how does the entropy change for adding fixed energy vary?
- d** If  $f$  were 1, the entropy change for adding further energy would be zero. What is the temperature?
- e** If  $f$  were infinitely small,  $-\ln f$  would be infinitely big. What would be the temperature?

**30(E)** Estimate the entropy changes for

- a** Warming a cupful of water:  
*i* by one degree at room temperature;  
*ii* from room temperature to the normal body temperature.
- b** Cooling an ice-cube's worth of water from room temperature to  $0^\circ\text{C}$ .

### The Boltzmann factor

**31(L)a** Use a calculator to complete table K3.

$E/kT$	$e^{-E/kT}$
1	0.37
2	
5	
10	
20	
50	
100	

Table K3

- b** What value of  $E/kT$  corresponds to a Boltzmann factor of  $10^{-6}$ ?

**32(P)** Obtain the temperatures at which the Boltzmann factor is  $1/10$  for

- a**  $E = 1.6 \times 10^{-19} \text{ J}$  (1 eV)

- b**  $E = 1.6 \times 10^{-13} \text{ J}$  (1 MeV)  
**c**  $E = 4 \times 10^{-21} \text{ J}$  ( $kT$  at room temperature)

**33(P)** For energy  $E = 1.6 \times 10^{-19} \text{ J}$  (1 eV), plot a graph of the Boltzmann factor against temperature, over the range 300 K to 350 K. Describe the form of the graph.

### Activation processes

**34(P)** The energy needed to evaporate water is about  $2.26 \text{ MJ kg}^{-1}$ .

- a** How many joules per mole?  
**b** How many joules per molecule evaporated?  
**c** When a water molecule evaporates, on average two hydrogen bonds are broken. What is the energy  $E$  to break a hydrogen bond?  
**d** At  $T = 300 \text{ K}$ , what is  $kT$ ? What is  $E/kT$ ? What is  $e^{-E/kT}$ ?

**35(P)**  $\Delta H/LkT$  for the evaporation of water is about 16 at 300 K, and about 13 at 373 K. Calculate  $e^{-16}$  and  $e^{-13}$ . Why is the second about 20 times larger than the first? Why does water evaporate more readily at 373 K than at 300 K?

**36(P)a** If the rate  $r$  of an activation process is given by

$$r = Ce^{-E/kT}$$

where  $C$  is a constant, show that for temperatures  $T_1$  and  $T_2$

$$\ln\left(\frac{r_2}{r_1}\right) = \frac{E}{k}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

- b** *i* What is the activation energy  $E$  of a process whose rate is multiplied by 20 when the temperature goes from 300 K to 330 K?  
*ii* By what factor would the rate be multiplied if the activation energy were half as big?

**37(R)** (*Hard*) The graphs (figure K46) show the collector current  $I$  (logarithmic scale) for an RCA 40389 power transistor, for a range of values of  $V_{BE}$ , the base-emitter voltage, at two temperatures. It is expected that

$$I = I_0 e^{eV_{BE}/kT}$$

- a** Why do the graphs *i* differ in slope, *ii* but not very much?  
**b** Estimate the slopes of the two graphs, and hence obtain estimates for  $\Delta \ln I / \Delta V_{BE}$  and for  $e/k$ .  
**c** Discuss the experimental difficulties of measuring currents over a range  $10^{-10} \text{ A}$  to  $10^{-3} \text{ A}$ .  
**d** One way to measure such currents is to use an operational amplifier, with resistive feedback, as shown in figure K47. Explain how this system measures small currents.



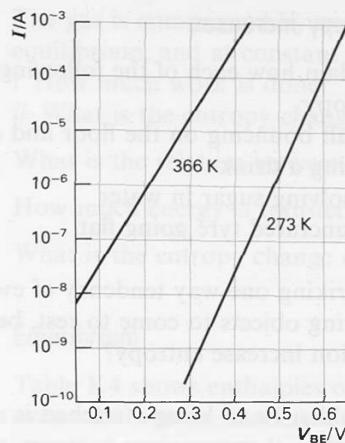


Figure K46

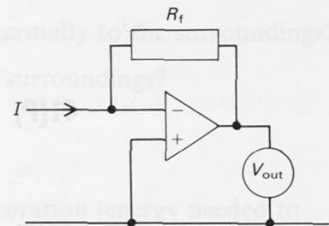


Figure K47

**38(R)** Power is generated in the Sun by the fusion of protons to form helium nuclei (the process is complex).

- For the process to occur, protons must collide with energy of the order of that needed to bring them to a separation of about  $1.5 \times 10^{-15}$  m, against their electrical repulsion. What is this energy?
- The temperature in the centre of the Sun is about  $10^7$  K. Is the rate of proton fusion likely to be fast or slow?
- If a star contracts, its central temperature rises. What will happen to the rate of proton fusion?

**39(P)** The escape of electrons from a hot filament can be thought of as like evaporation, an electron needing energy equal to the work function  $\phi$  to escape, with the electron current being given approximately by

$$I = I_0 e^{-\phi/kT}$$

The graph, figure K48, shows data for platinum. Estimate the work function  $\phi$  for platinum.

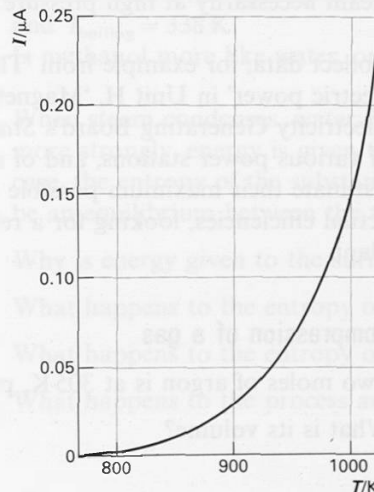


Figure K48

From: CROWTHER, J. A. Ions, electrons, and ionizing radiations. 8th edn. Edward Arnold, 1949.

### Entropy increases

- 40(P)** Explain how each of the following changes involves an increase in entropy:

A ball bouncing on the floor and coming to rest

Mixing a drink

Dissolving sugar in water

A punctured tyre going flat.

- 41(P)** A striking one-way tendency of everyday life is the tendency of moving objects to come to rest, because of friction. How does friction increase entropy?

- 42(E)** A rubber band being stretched is rather like a gas being compressed. Its internal energy does not vary very much when it is stretched, the chains of molecules simply becoming less kinked. When a band is stretched, it becomes warm (feel the temperature change on your lip). The work of stretching is 'dumped' into thermal motion of the molecules (which tends to kink the chains).

Without any calculation, explain why stretching the band must *decrease* the entropy of arrangement of chains, but *increase* the entropy associated with the energy shared amongst the molecules.

### Efficiency of heat engines

- 43(P)** What is the maximum efficiency of a heat engine which

- a** has a furnace at 2000 K and surroundings at 300 K?
- b** runs on the 15 K temperature difference between surface ocean water at about 288 K and the deep water at nearly 273 K? Suggest why such an engine might still be a good idea.

- 44(P)** Why, from the point of view of efficiency, is it desirable to run steam turbines with super-heated high pressure steam? Why is super-heated steam necessarily at high pressure?

- 45(E)** Collect data, for example from 'The generation and transmission of electric power' in Unit H, 'Magnetic fields and a.c.' or the Central Electricity Generating Board's *Statistical yearbook*, on the efficiency of various power stations, and of the temperature of the steam used. Estimate their maximum possible efficiencies, and compare with actual efficiencies, looking for a relationship with the age of the plant.

### Compression of a gas

- 46(P)** Two moles of argon is at 305 K, pressure 10 atmospheres (1.0 MPa).

- a** What is its volume?

- b** The gas is compressed by one per cent, keeping it close to equilibrium, and at constant temperature.
  - i* How much work is done?
  - ii* What is the entropy change of the gas?
- c** What is the relation between **bi** and **bii**?
- d** How much energy is transferred thermally to the surroundings?
- e** What is the entropy change of the surroundings?

### Equilibrium

- 47(R)** Table K4 shows enthalpies of evaporation (energy needed to evaporate one mole at constant pressure), and the boiling-points at standard pressure, of some liquids.

Liquid	$\Delta H_{\text{evap}}/\text{kJ mol}^{-1}$	$T/\text{K}$
carbon disulphide	27.2	319
trichloromethane	29.3	335
hexane	28.8	342
octane	34.9	399
sulphuric acid	50.2	617
water	40.6	373

Table K4

- a** Calculate the entropy changes  $\Delta S_{\text{evap}} = \Delta H/T$ .
  - b** Guessing roughly that the volume changes by a factor of 1000 on evaporation, estimate  $\Delta S_{\text{volume change}} \approx Nk \ln 1000$  for 1 mole.
  - c** How much of the entropy changes in **a** are explained by **b** (excepting water)? Would the existence of a certain amount of structure in the liquid help explain the difference?
  - d** Water has hydrogen bonds, which give the liquid considerable structure. How does this help explain the value of  $\Delta S_{\text{evap}}$  for water? Methanol also has hydrogen bonds, and has  $\Delta H_{\text{evap}} = 35.2 \text{ kJ mol}^{-1}$  and  $T_{\text{boiling}} = 338 \text{ K}$ .  
Is methanol more like water, or like hexane?
- 48(R)** When steam condenses, water freezes, or a material is magnetized more strongly, energy is given to the surroundings, and also, in each case, the entropy of the substance decreases. In each case, there can be an equilibrium between the two states at a certain temperature.
- a** Why is energy given to the surroundings, in each case?
  - b** What happens to the entropy of the surroundings?
  - c** What happens to the entropy of the substance?
  - d** What happens to the process as the temperature is raised?

## Concentration cells

**49(P)** Volumes of 1 M sodium chloride containing 1/10 mole NaCl are diluted

i to 0.5 M

ii to 0.1 M

iii to 0.01 M

- a** How many particles are there in solution?
- b** Estimate roughly the entropy changes in each case.

**50(R)a** Why is raising the temperature likely to increase the e.m.f. of a concentration cell?

- b** Why, on a simple picture of dilution, would concentration cells made from pairs 0.1 M and 0.01 M, or 1 M and 0.1 M, have the same e.m.f.?
- c** Why is the e.m.f. of a concentration cell higher for monovalent ions than for divalent ions, for the same concentration and temperature?

# Unit L WAVES, PARTICLES, AND ATOMS

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L

# SUMMARY OF THE UNIT

## INTRODUCTION: THE QUANTUM REVOLUTION

This Unit is about some of the strange and powerful ideas invented by physicists in the first third of the twentieth century. The ideas seem strange because they make us think about apparently familiar and well understood things like light and electrons in a new way. We need these strange ideas because matter on a very small scale – the scale of atoms and electrons – just does not behave in the same way as everyday-sized objects do. The power of these ideas lies in the fact that they help us to explain, for example, the stability and the size of atoms, and the existence of energy levels in atoms. But quantum theory has much wider scope than this: it is essential for our modern understanding of chemical bonds, electrical conductivity, magnetism, the nature of solids, and much more besides. For example, without quantum physics we would have no satisfactory theory of semiconductors and so no transistors, integrated circuits, or microelectronics.

The new quantum ideas truly revolutionized physics, and what may seem at first highly theoretical and abstract ideas affect the lives of people who may know nothing of the ideas that make possible many of the devices they use.

## Section L1 PHOTONS

### What is electromagnetic radiation?

Unit J, 'Electromagnetic waves'

So far in this course electromagnetic radiation has been seen to have two characteristics. It has wave properties, and all wavelengths travel (in a vacuum) at the same speed – very nearly  $3 \times 10^8 \text{ ms}^{-1}$ . But different wavelengths have different effects: our eyes are sensitive to visible light but not to radio waves; infra-red doesn't affect normal photographic film, but visible and ultra-violet light do; photosynthesis needs light having wavelengths of about 600 nm and 450 nm; and so on.

DEMONSTRATION L1  
Detection of electromagnetic radiation

Long wavelength radiation (radio, television) is detected by aerials which respond to the continuously varying electric and magnetic fields; short wavelength radiation (for example gamma rays) is detected as a series of discrete events in a GM tube. The gamma ray seems more like a stream of particles than a wave.

### Waves or particles: the photoelectric effect

Whether light is a wave or a particle is an old question. Isaac Newton, in his *Opticks* (1704) wrote of 'very small bodies emitted from shining substances'. In 1803 Thomas Young explained his famous two-source interference experiment, in which light plus light can produce darkness, by comparing light with water waves.

DEMONSTRATION L2  
Photoelectric effect

Crucial evidence comes from the photoelectric effect, discovered more or less by chance, when Heinrich Hertz was experimenting with

radio waves in 1888. When ultra-violet light falls on a metal surface it can cause electrons to be emitted from the surface. The key facts of the photoelectric effect are:

- a** If the wavelength is too long, no electrons are emitted however intense the light may be.
- b** Light of a certain colour produces electrons with a certain maximum energy. Brighter light gives *more* electrons, it does *not* increase the maximum energy.
- c** If the wavelength is reduced, the maximum energy increases.

#### READING

The photoelectric effect (page 308)

In 1905 Albert Einstein produced a theory which accounted for these facts. His explanation is summarized in

$$K.E._{\max} = hf - \phi$$

#### DEMONSTRATION L3

Colour of light and energy of photoelectrons

The theory was given support by experiments done by Millikan (1916) and can be tested in the school laboratory.

The theory assumes that light energy comes in packets or quanta (photons); if the frequency of the light is  $f$ , the energy,  $E$ , of each photon is  $hf$ . The constant  $h$  has the value  $6.6 \times 10^{-34} \text{ J s}$ . (It is known as Planck's constant, after the German physicist Max Planck who first introduced the idea that energy is quantized in 1900.)

#### QUESTIONS 1 to 9

Check that J s is the appropriate unit for  $h$

### Light quanta, energy levels, and spectra

The sharp line spectra of gases are evidence that atoms have discrete energy levels. Measuring the frequency of spectral lines, and using  $E = hf$  to find the energy of the photon emitted or absorbed and hence the energy change in the atom, is a powerful method of mapping out energy levels. Thus the line at 253 nm in the absorption spectrum of mercury vapour corresponds to a transition from the lowest or ground state of the atom to a level  $7.84 \times 10^{-19} \text{ J}$  above the ground state. And when an excited mercury atom returns from this level to the ground state it emits radiation of wavelength 253 nm. The other lines in the mercury spectrum correspond to transitions between other pairs of energy levels.

#### DEMONSTRATION L4

The spectrum of mercury vapour

#### QUESTIONS 10 to 13

### The spectrum and energy levels of the hydrogen atom

EXPERIMENT L5  
The hydrogen spectrum

Hydrogen is the simplest element, and its spectrum is easy to interpret. The complete spectrum is conveniently grouped into several families among which the most important are the Lyman series, which is in the ultra-violet; the Balmer series, mostly in the visible (figure L1); and the Paschen series in the infra-red.

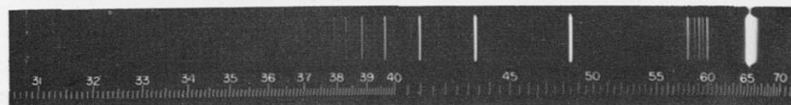


Figure L1

Part of the hydrogen spectrum. The scale shows wavelengths in units of  $10^{-8}$  metre.

Table L1 gives the wavelengths and frequencies for many of the lines in the Lyman, Balmer, and Paschen series.

Lyman series		Balmer series		Paschen series	
wavelength/nm	frequency/ $10^{14}$ Hz	wavelength/nm	frequency/ $10^{14}$ Hz	wavelength/nm	frequency/ $10^{14}$ Hz
121.57	24.659	656.47	4.5665	1875.6	1.5983
102.57	29.226	486.26	6.1649	1282.2	2.3380
97.25	30.824	434.16	6.9044	1094.1	2.7399
94.97	31.564	410.29	7.3064	1005.2	2.9822
93.78	31.966	397.12	7.5487	954.84	3.1395
93.07	32.208				
92.62	32.365	389.01	7.7060		
.....	.....				
.....	.....				
Limit	$32.881 \times 10^{14}$ Hz				

**Table L1**  
The hydrogen spectrum.

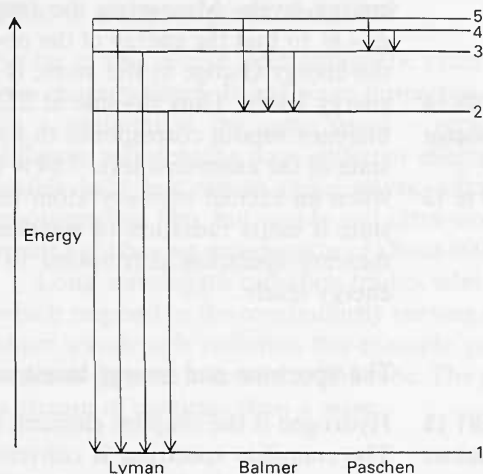
Each series follows the pattern of figure L1: the lines converge at higher frequencies. Each series has a limit: for the Lyman series it is  $32.881 \times 10^{14}$  Hz.

#### QUESTION 14

The frequency of a line in the Balmer series turns out to be equal to the *difference* in frequency between two lines in the Lyman series.

The three series are accounted for by a scheme of energy levels like that shown in figure L2.

The Lyman series arises from transitions to the lowest level; the Balmer from transitions to level 2; the Paschen from transitions to level 3.



#### QUESTIONS 15 to 17

**Figure L2**

Some energy levels of a hydrogen atom. The higher energy levels get more and more closely spaced.

### Why energy levels are negative

$$E = hf$$

$$h = 6.63 \times 10^{-34} \text{ J s}$$

The limit of the Lyman series ( $32.88 \times 10^{14}$  Hz) corresponds to an energy of  $2.18 \times 10^{-18}$  J. This is equivalent to 13.6 electronvolts, the ionization energy of hydrogen, the energy needed to remove the



## QUESTION 18

## QUESTION 19

electron completely from the hydrogen atom. By convention we call the energy zero when two charges are very far apart. Since there is a *decrease* in potential energy when an electron and a proton approach, the energy of the hydrogen atom, or any system where unlike charges are bound together, is *negative*. In particular, the energy of the lowest state of the hydrogen atom (level 1 in figure L2) is  $-2.18 \times 10^{-18} \text{ J}$ . The next level (level 2) is  $1.63 \times 10^{-18} \text{ J}$  above this, that is, at  $(-2.18 + 1.63) \times 10^{-18} \text{ J} = -0.55 \times 10^{-18} \text{ J}$ . The next one is at  $(-2.18 + 1.935) \times 10^{-18} \text{ J} = -0.242 \times 10^{-18} \text{ J}$ , and so on. If the other levels are worked out in this way they form a regular sequence, see table L2.

Energy level/ $10^{-18} \text{ J}$	Number, $n$
-2.18	1
-0.55	2
-0.242	3
-0.136	4

Table L2

## QUESTION 20

There is a simple rule here: the energy of a level  $n$  is

$$-\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$

Explaining how a hydrogen atom has stable states with these energies – and no others – is one of the success stories of twentieth century physics. And it depends on a new way of thinking about the electron – as a wave.

## Light: waves or particles – or both?

## EXPERIMENT L6

## Interference of single photons

The particle model of light is needed to explain the photoelectric effect, and to make sense of spectra and the energy levels of atoms. But one of the instruments used to analyse spectra – the diffraction grating – depends on the wave behaviour of light.

## QUESTION 21

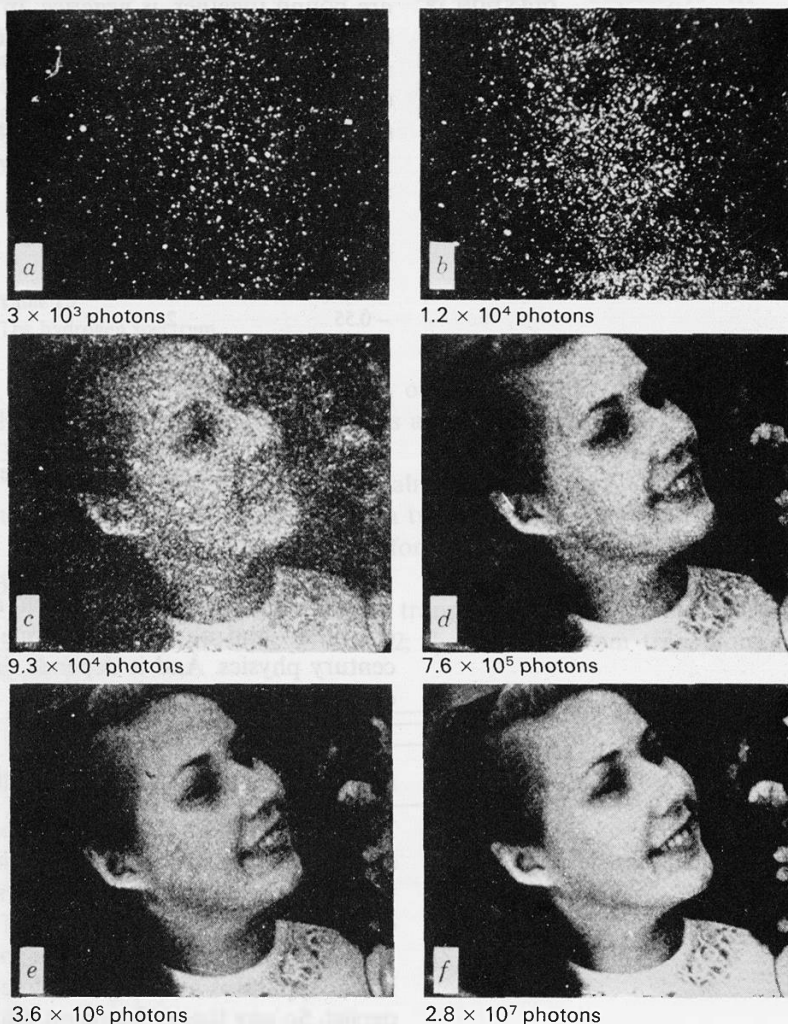
Even when the light level is so low that there can only be one photon in the apparatus at any given time, diffraction and interference effects persist. So any theory based on photons interacting with each other as they go through a grating fails in this case.

Both models are needed and are inextricably bound together. To find the energy of a photon, which we think of as a *particle*, we need to know the frequency, which we probably get from measuring the length of a *wave* in a superposition experiment. And we can only explain these experiments in terms of light's wave-like behaviour.

## Relating the particle and wave model

On the particle model one photon carries energy  $hf$ . So the intensity of light at any point depends on the number of photons arriving there each second. But photons do not arrive at a steady rate: at very low

intensities the random rate of arrival becomes quite apparent (figure L3). There is a higher chance of a photon arriving (in any short time interval) at a place where the intensity is high, than where the intensity is low.



**Figure L3**

Series of photographs showing the quality of picture obtainable from various numbers of photons. In very dim light the picture breaks up into a number of randomly spaced dots, just as if a number of lumps of light energy had been delivered to particular places.

ROSE, A. *Advances in biological and medical physics*, 5, 211, 1957.

For an oscillator energy  $\propto$  (amplitude)<sup>2</sup>

On the wave model intensity depends on (wave amplitude)<sup>2</sup>. If the two models are to agree it must be the case that

chance of photon arriving  $\propto$  (wave amplitude)<sup>2</sup>

The relationship of probability to (amplitude)<sup>2</sup> is an important part of quantum theory's picture of electrons in atoms.

### So what *is* light?

**READING**  
'What *is* light?' (page 308)

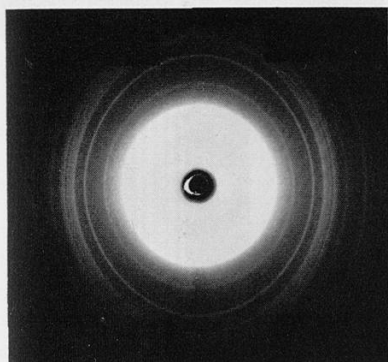
### QUESTION 22

In some experiments (for instance diffraction grating) light behaves like a wave; in others (for example photoelectric effect) it behaves like particles. Neither the wave, nor the particle model on its own gives a complete and satisfactory explanation of light.

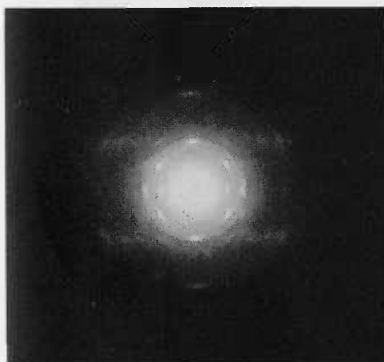
## Section L2 ELECTRONS

### Electrons: particles or waves?

An electron is a small particle with mass  $9.1 \times 10^{-31}$  kg and charge  $-1.6 \times 10^{-19}$  C. But figures L4, L5, and L6 show that electrons, like X-rays, can produce diffraction effects. And diffraction is a characteristic of a wave.



(a)



(b)

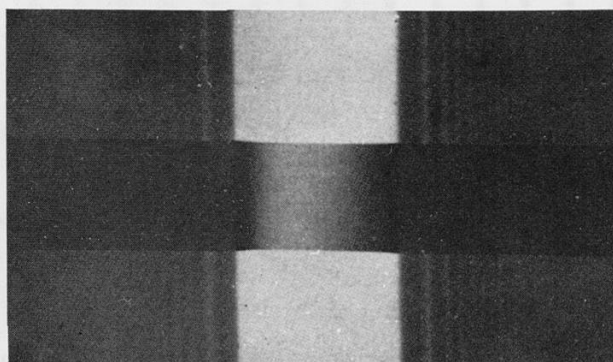
**Figure L4**

(a) X-ray diffraction – polythene.

*ICI P.L.C., Plastics Division.*

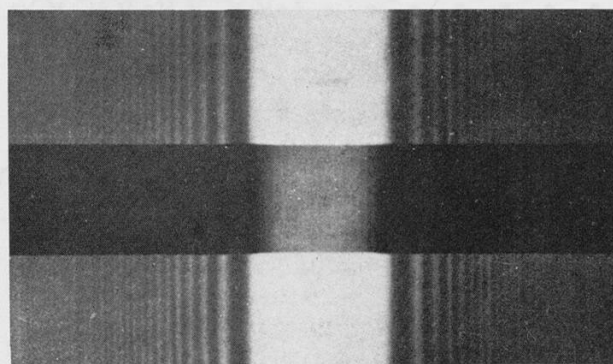
(b) Electron diffraction – stretched rubber.

*Malaysian Rubber Producers' Research Association.*

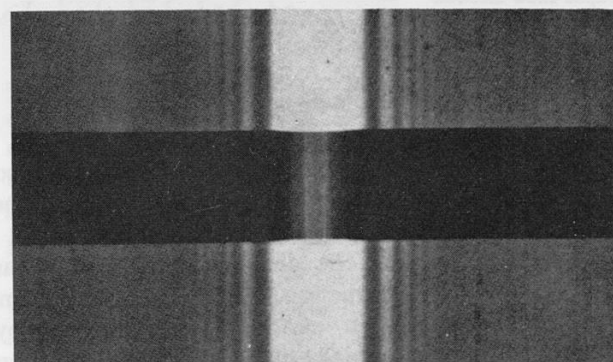


width of wire

0.044 mm



0.038 mm



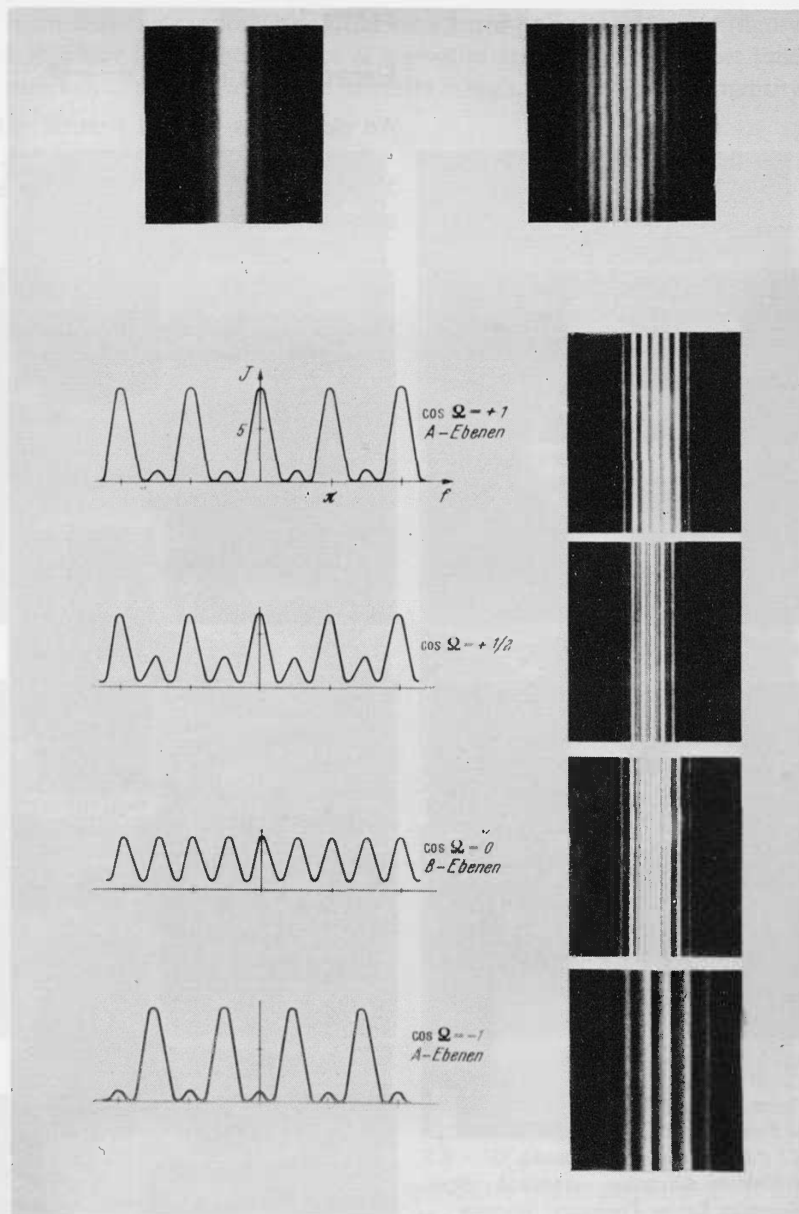
0.019 mm

**Figure L5**

Diffraction of X-rays produced by narrow wires.

*KELLSTROM, G. Nova Acta Regiae Societatis Scientiarum Upsaliensis. Series IV, 8, 5, 1932.*

**Figure L6**  
 Diffraction of electrons at narrow slits.  
 The top two photographs show electron  
 diffraction at single and double slits. The  
 pictures below show diffraction at three  
 slits and the theoretical intensity curves.  
 JONSSON, C. Zeitschrift für Physik, **161**,  
 1961.



### DEMONSTRATION L7

#### Electron diffraction by graphite

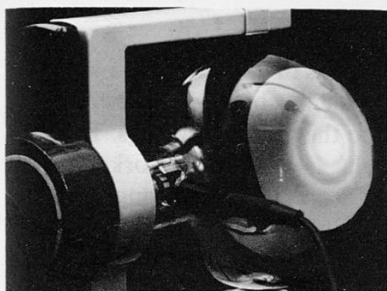
QUESTIONS 23, 24

When a beam of electrons, accelerated to a few kilovolts, passes through a thin film of graphite, the electrons are diffracted to form a pattern of rings on the fluorescent screen (figure L7).

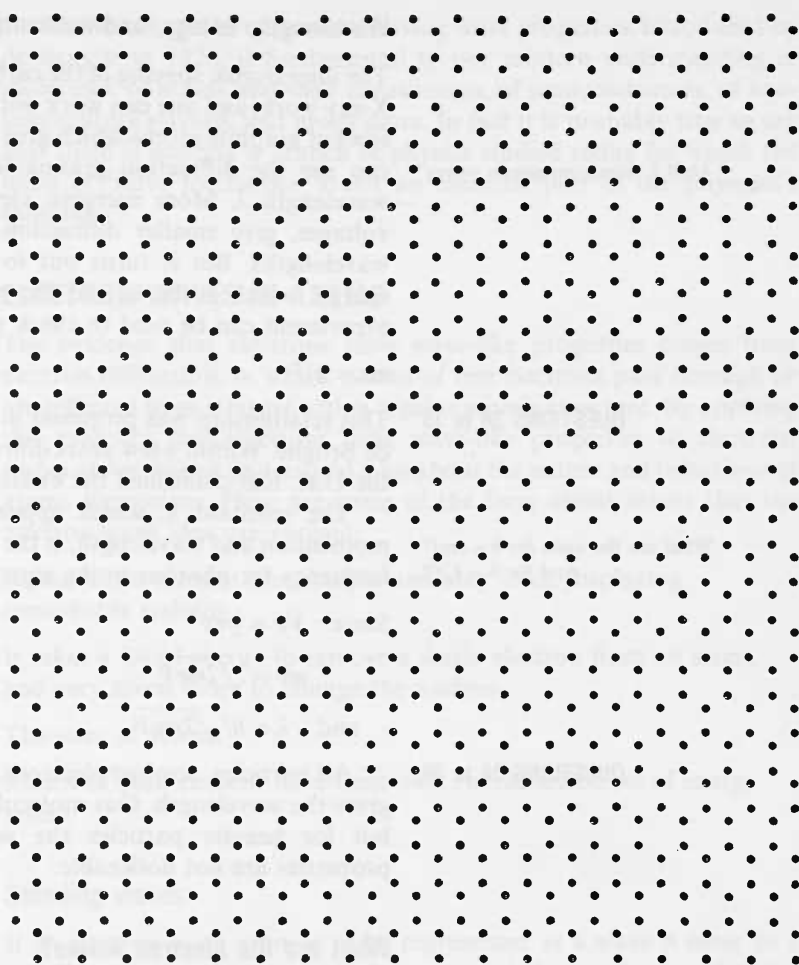
The diameter of the rings can be measured and the angle of diffraction can be calculated from the geometry of the tube. If the electrons are given more energy – by being accelerated through a higher voltage – the diffraction angle decreases.

The rings are formed because the layers of graphite in the film all act as diffraction gratings.

Figure L8 shows an idealized diagram of the hexagonal arrangement of carbon atoms in a graphite layer.



**Figure L7**  
Electron diffraction tube.  
*Teltron Ltd.*



**Figure L8**  
Arrangement of carbon atoms in a graphite layer. (Atoms in the target of the electron diffraction tube are arranged rather less perfectly.)

QUESTIONS 25, 26, 27

If you hold figure L8 up to eye level and look along the page you will see that the atoms are arranged in rows. It is these rows of atoms, acting rather like the lines of a diffraction grating, that give rise to the rings. The spacing between lines of carbon atoms is fixed by the hexagonal structure, but the orientation of these lines of atoms varies from one part of the target to the next, and between the atomic layers of graphite.

#### EXPERIMENT L8

Optical analogue of electron diffraction

QUESTIONS 28, 29

A similar effect can be seen if you rotate a fine diffraction grating held in front of your eye while you look through it at a point source of light with a colour filter in front of it.

The diffraction of an electron beam by graphite gives *two* rings because there are two characteristic spacings between rows of atoms in the graphite layers.

## Wavelength, energy, and momentum of electrons

The interatomic spacing of the carbon atoms in graphite is known from X-ray work, and one can work out the spacings between the two sets of rows of graphite atoms which give rise to the diffraction rings. Then we can use the diffraction grating formula  $\lambda = s \sin \theta$  to work out the wavelength  $\lambda$ . More energetic electrons, accelerated through greater voltages, give smaller diffraction rings, so they must have smaller wavelengths. But it turns out to be the *momentum*, not the kinetic energy, which is the key to the wavelength. The electron diffraction experiment can be used to check the prediction

$$mv = h/\lambda$$

### QUESTIONS 30 to 33

What are the units for  $h = mv\lambda$ ?  
And for  $h = E/f$ ?

This relationship was proposed in 1923 by the French physicist Louis de Broglie. Within a few years diffraction experiments in the U.S.A. and the U.K. had confirmed the existence of electron waves.

The constant  $h$ , which appears here in a relationship between momentum and wavelength, is the same constant that relates energy to frequency for photons in the equation  $E = hf$ .

Since  $Ve = \frac{1}{2}mv^2$

$$mv = \sqrt{2meV}$$

and  $\lambda = h/\sqrt{2meV}$

### QUESTIONS 34 to 36

All particles, not just electrons, have wave properties and  $\lambda = h/mv$  gives the wavelength. Gas molecules have been successfully diffracted, but for heavier particles the wavelength is so small that wave properties are not noticeable.

## What *are* the electron waves?

Electrons are not waves – but they do have wave-like properties. The wave associated with an electron tells us where we are likely to find it. The chance of finding an electron at a particular point depends on the amplitude of the wave at that point. If superposition effects are involved – as in a diffraction experiment – the amplitudes of the waves must be added, paying attention to the phase of each. Then the chance of finding an electron at a particular point is proportional to the square of the resultant amplitude at that point:

$$\text{chance of finding electron} \propto (\text{amplitude})^2$$

This of course parallels the interpretation of amplitude for light waves:

$$\text{chance of finding photon} \propto (\text{amplitude})^2$$

QUESTIONS 37 to 40

The whole notion of particles having wave properties, introduced by de Broglie in 1923, is fundamental to our modern understanding of molecules, of atoms and their constituents, of semiconductors, of low-temperature physics, and much more. In fact it is probably true to say that there is scarcely a branch of physics studied today for which the ideas of 'wave mechanics' aren't an essential part of the physicist's thinking.

## Section L3 ELECTRON WAVES IN ATOMS

DEMONSTRATION L7  
Electron diffraction by graphite

The evidence that electrons have wave-like properties comes from electron diffraction, in which beams of free electrons pass through or are reflected from a target with a regular atomic structure. By applying this evidence that electrons have wave-like properties to electrons *within atoms* we can understand a lot about the nature and behaviour of atoms themselves. Here are some of the facts about atoms that the electron-wave idea can explain:

QUESTION 41

- a** Atoms of an element are all alike and stay alike, displaying remarkable stability.
- b** It takes a lot of energy to remove a single electron from an atom, and very much more to change the nucleus.
- c** The sizes of atoms.
- d** Atoms of each element have their own characteristic set of energy levels.

### Standing waves

If an electron in an atom is to be represented as a wave it must be a standing wave, otherwise the electron won't be confined within the atom. Experiments with standing waves on cords show that when a standing wave is formed it always has 1, 2, 3, ..., or any whole number of loops. Each loop is half a wavelength long. If there is one loop on a cord of length  $l$ , then the wavelength  $\lambda = 2l$ , and this is the longest wavelength standing wave that can be set up.

DEMONSTRATION L9  
Standing waves

QUESTION 42

Unit D, 'Oscillations and waves'

Standing waves in two dimensions can be demonstrated on a membrane; three-dimensional standing waves also exist, though they are more difficult to show.

### Facts about the hydrogen atom

Size: about  $10^{-10}$  m

Ionization energy: 13.6 eV or  $2.18 \times 10^{-18}$  J

Energy levels: 
$$-\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$



## Fact about electron waves

$$\text{momentum} = h/\lambda$$

From these facts it follows that if we imagine a hydrogen atom to be a spherical box the longest wavelength standing wave (one loop) that would fit inside a hydrogen atom has a wavelength of about  $4 \times 10^{-10} \text{ m}$ , as shown in figure L9.

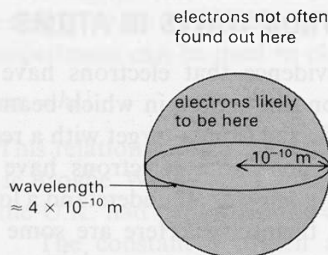


Figure L9

An electron wave in a hydrogen atom 'box'.

### QUESTION 43

The momentum can be calculated ( $h/\lambda$ ), and from it the electron's kinetic energy. The kinetic energy turns out to be about  $1.4 \times 10^{-18} \text{ J}$ .

So we imagine an electron with this kinetic energy rattling around in an atom-sized box. Clearly some force is needed to keep the electron confined within this space. This is the electrical force of attraction between it and the proton. The energy needed to separate the electron from the proton when they are  $10^{-10} \text{ m}$  apart can be calculated from  $e^2/4\pi\epsilon_0 r$ . It turns out to be  $2.3 \times 10^{-18} \text{ J}$ . So the electron, with kinetic energy  $1.4 \times 10^{-18} \text{ J}$  does not have enough energy to get away from the proton. It would need at least  $2.3 \times 10^{-18} \text{ J}$  to escape.

Similar calculations for different sizes of box show that the *total* energy (K.E. + P.E.) is a minimum for a particular size of box. If the box were smaller, the electron would have a smaller wavelength and more momentum, and the K.E. would rise more than the P.E. falls. If the box were made bigger, the P.E. would rise more than the K.E. falls.

A more systematic attack on the problem is to write an expression for the total energy of the atom,  $E$ , in terms of its radius,  $r$ , and find the value of  $r$  for which  $E$  is a minimum by differentiation.

## Some successes for the theory

The crude wave-in-a-box theory explains more than why atoms are the size they are.

It tells why solids are very hard to compress: making each atom smaller means increasing the kinetic energy of the electrons. Doing this to every atom for even a small compression takes a lot of energy.

### QUESTION 47

It also tells us why there can not be electrons in the nucleus. Nuclei are about  $10^{-14} \text{ m}$  across; the kinetic energy of an electron confined in such a small space is of the order of  $10^{-9} \text{ J}$  or  $10^{10} \text{ eV}$ . There is no known force that could hold down electrons with such big energies.

Unit E, 'Field and potential'

### QUESTIONS 44, 45

### QUESTION 46



# QUESTION 48

Unit F, 'Radioactivity and the nuclear atom'

We can use the same wave-in-a-box idea to calculate what momentum and hence kinetic energy a nuclear particle (proton or neutron) must have in its nuclear-sized box. The answer turns out to be of the order of 10 MeV. So the potential energy with which nuclear forces bind the particles together must be enough to overcome this. In fact, nuclear binding energies do turn out to be of about this magnitude.

## A better model for the hydrogen atom

The wave-in-a-box model is a very crude one. The 'box' which holds the electron near the nucleus is really the potential energy, and the box model takes no account of how the potential energy of the atom varies with the electron's distance from the proton.

Figure L10 shows how the potential energy of the atom varies with distance between proton and electron ( $E_p = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$ ).

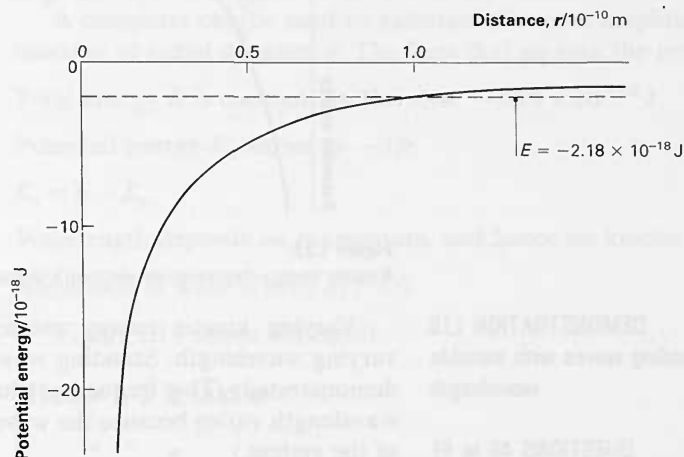


Figure L10

Variation of potential energy with distance for an electron near a proton.

Figure L11 is a representation of two 'potential wells'. One corresponds to the box model; the other suggests, in three dimensions, the way in which the potential energy of an electron actually varies with distance from the nucleus.

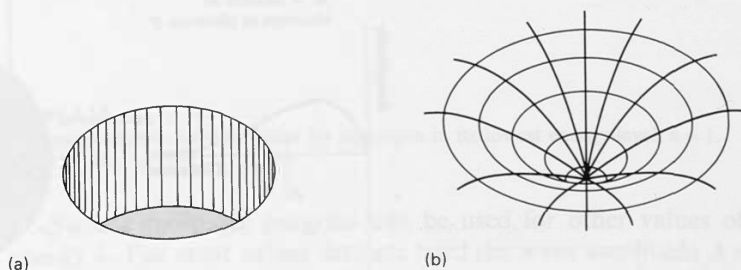
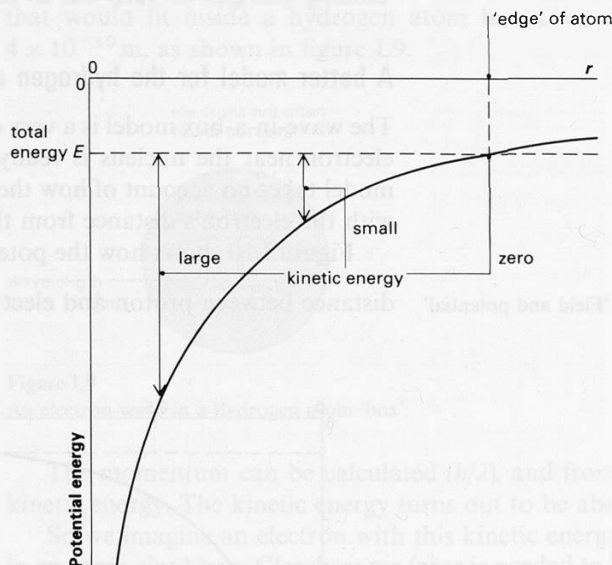


Figure L11

## SECTION L1

The lowest energy level, the ground state, is at  $E = -2.18 \times 10^{-18} \text{ J}$ . This is the *total energy*  $E$ . Figure L12 shows how the total energy, the potential energy,  $E_p$ , and the kinetic energy,  $E_k$ , of the electron are related:

$$E_k = E - E_p$$



**Figure L12**

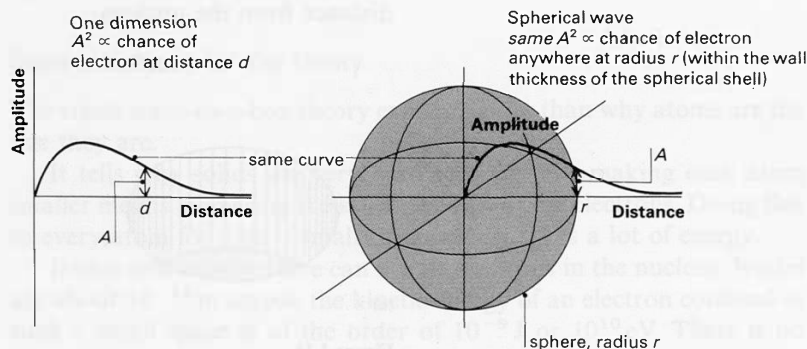
Kinetic energy decreases as electron's distance from proton increases.

### DEMONSTRATION L10 Standing waves with variable wavelength

#### QUESTIONS 49 to 51

Varying kinetic energy means varying momentum, and hence varying wavelength. Standing waves with varying wavelength can be demonstrated. (The frequency must be the same everywhere – the wavelength varies because the wave speed is different in different parts of the system.)

But an atom is a three-dimensional object, so we have to imagine standing waves of varying wavelength in three dimensions. For simplicity we consider the case where the waves have spherical symmetry: there is a mathematical theorem which allows us to relate spherically symmetrical standing waves to the much simpler one-dimensional standing waves along a line (figure L13).



**Figure L13**

Volume of shell =  $4\pi r^2 dr$

$A \rightarrow 0$  as  $r \rightarrow 0$

If a one-dimensional standing wave has amplitude  $A$  at some point the chance of finding an electron *near that point* depends on  $A^2$ . For the spherically symmetrical case we are considering,  $A^2$  at any radius gives the chance of finding the electron *near that radius, in any direction*.  $A^2$  is the chance of finding the electron in a spherical shell of constant thickness at radius  $r$ . Since the volume of such a shell approaches zero as  $r$  becomes very small, so does the wave amplitude  $A$ . The value of  $A$  at any point is given by Schrödinger's equation, which in simple form for a spherical atom is:

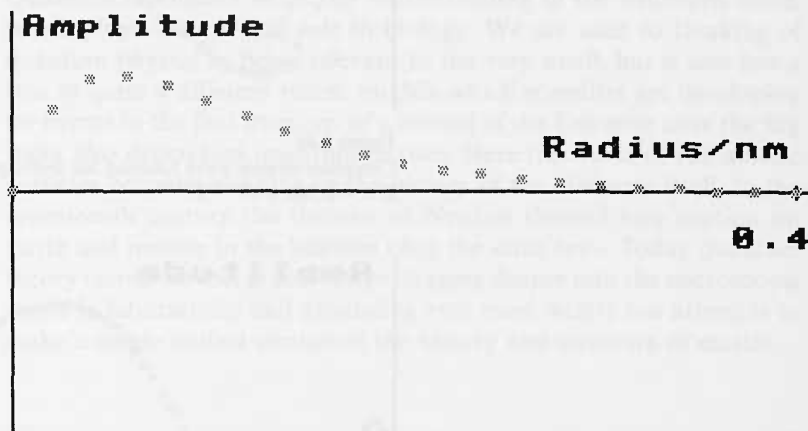
$$\frac{d^2 A}{dr^2} = -\left(\frac{2\pi}{\lambda}\right)^2 A$$

(Many treatments of three-dimensional standing waves plot a function  $\psi$  – Greek 'psi' – rather than  $A$ .  $\psi^2 dV$  represents the chance of finding the electron in a small volume  $dV$ .  $A$  and  $\psi$  are related by  $A = \psi r$ , so although  $A$  is zero when  $r = 0$ ,  $\psi$  is not. In general the shapes of graphs  $A$  and  $\psi$  against  $r$  are different.)

A computer can be used to calculate the wave amplitude,  $A$ , as a function of radial distance,  $r$ . The facts that go into the program are:

- a** Total energy  $E$  is constant, in this case  $-2.18 \times 10^{-18}$  J.
- b** Potential energy  $E_p$  varies as  $-1/r$ .
- c**  $E_k = E - E_p$ .
- d** Wavelength depends on momentum, and hence on kinetic energy.
- e** Amplitude of wave is zero at  $r = 0$ .

Figure L14 shows the result.



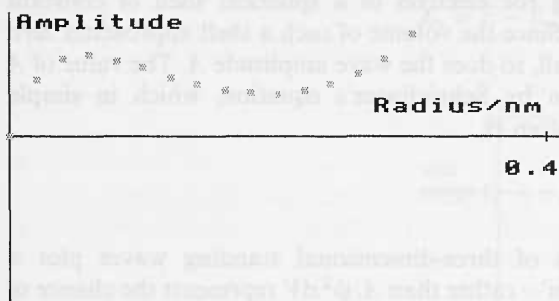
**Figure L14**

Computer-drawn wave function for hydrogen in its lowest energy level,  $n = 1$ .  
 $E = -2.18 \times 10^{-18}$  J.

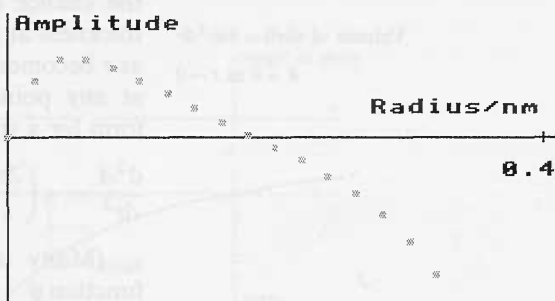
Such a computer program can be used for other values of total energy  $E$ . For most values that are tried the wave amplitude  $A$  shoots off towards infinitely high or low values: there are no closed loops, no standing waves and so an electron cannot be bound to an atom for these

QUESTIONS 52, 53

values of  $E$  (figure L15). But when  $E = -2.18 \times 10^{-18} \text{ J}$ , closed loops are formed: the wave mechanics interpretation of the electron in the atom successfully predicts Balmer's  $1/n^2$  rule for the hydrogen atom (figures L16, L17).



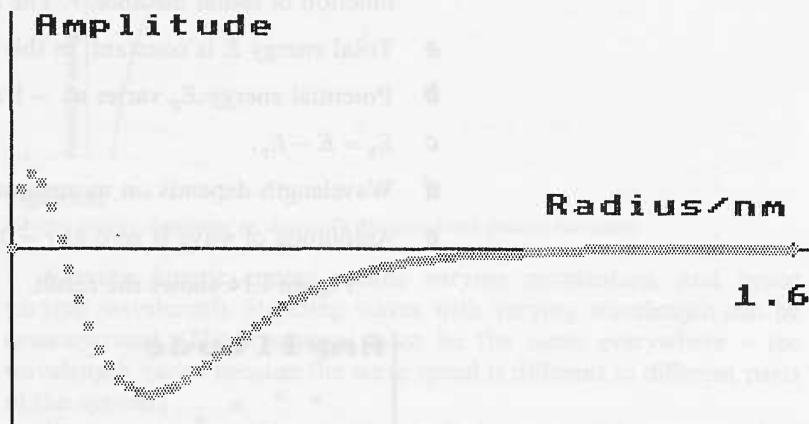
(a)



(b)

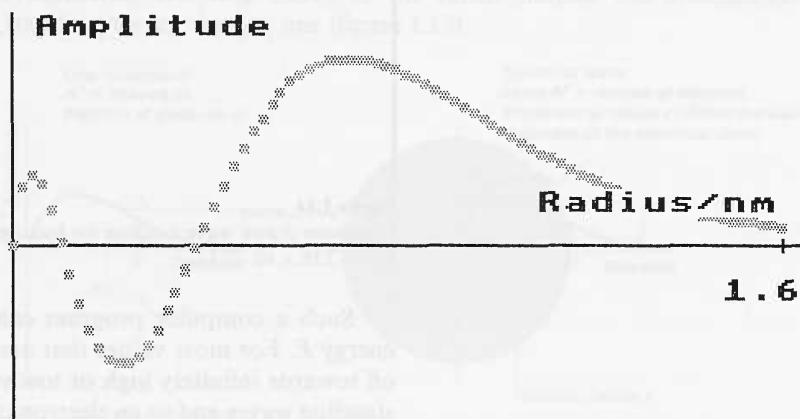
**Figure L15**

Computer-drawn wave functions for hydrogen. (a)  $E = -2.3 \times 10^{-18} \text{ J}$ . (b)  $E = -2.0 \times 10^{-18} \text{ J}$ .



**Figure L16**

Computer-drawn wave function for hydrogen, at the energy of the level  $n = 2$ .  
 $E = -0.546 \times 10^{-18} \text{ J}$ .



**Figure L17**

Computer-drawn wave function for hydrogen, at the energy of the level  $n = 3$ .  
 $E = -0.243 \times 10^{-18} \text{ J}$ .

The first closed solution ( $E = -2.18 \times 10^{-18} \text{ J}$ ) has one loop, the second ( $E = -2.18 \times 10^{-18}/2^2 \text{ J}$ ) has two loops, and so on. As the energy level increases so does the size of the atom.

#### QUESTION 54

A crude, but fairly simple algebraic argument can be used to predict that the energy levels are given by  $E \propto -1/n^2$ .

### An end and a beginning

The explanation of the size, stability, and energy levels of the hydrogen atom brings together ‘classical’ ideas including standing waves, field and potential, kinetic and potential energy, and momentum, and also the ‘new’ idea that associated with a particle there is a wave with wavelength  $\lambda = h/mv$ . While this is the end of this Course, it is, in a very real sense, the beginning of today’s physics, for the ideas of wave mechanics now pervade nearly every part of the subject. The hydrogen atom – one proton and one electron – is the simplest example. To predict the size and energy levels of the helium ion  $\text{He}^+$  – a single electron in the potential well of a nucleus containing two protons – uses exactly the same ideas. Other examples that require only a little extension of the ideas used in this Unit include the 2, 8, 18, ... pattern in the Periodic Table of the elements, and the escape of an alpha particle from a radioactive nucleus.

The ideas of wave mechanics were first developed in the 1920s. Today they find application at all levels from the sub-atomic through chemical bonding and the electrical, thermal, optical, and mechanical properties of matter in bulk, to cosmology. For example, fundamental research in semiconductors, essential for today’s and tomorrow’s microelectronics, would be impossible without quantum mechanics. Quantum mechanics helps our understanding of the hydrogen bond, which plays such a vital role in biology. We are used to thinking of quantum physics as being relevant to the very small, but it also has a role in quite a different realm: models which scientists are developing for events in the first fractions of a second of the Universe after the Big Bang also depend on quantum physics. Here the world of sub-atomic particles becomes the clue to the history of the Universe itself. In the seventeenth century the theories of Newton showed how motion on Earth and motion in the heavens obey the same laws. Today quantum theory continues this grand design, digging deeper into the microscopic world in laboratories and expanding ever more widely our attempts to make a single unified picture of the history and structure of matter.



**Figure L18**  
Erwin Schrödinger (1887–1961), one of the founders of wave mechanics.  
*Mary Evans Picture Library.*

‘The particles and forces of nature’ and  
‘Our nuclear history’ in the Reader  
*Particles, imaging, and nuclei*

# READINGS

## THE PHOTOELECTRIC EFFECT

Einstein, who first proposed using the quantum picture to explain the photoelectric effect, explains it as clearly as anybody:

‘According to the concept that the incident light consists of energy quanta of magnitude  $hf$ , however, one can conceive of the ejection of electrons by light in the following way. Energy quanta (photons) penetrate into the surface layer of the body, and their energy is transformed, at least in part, into kinetic energy of electrons. The simplest way to imagine this is that a light quantum delivers its entire energy to a single electron; we shall assume that this is what happens ... An electron to which kinetic energy has been imparted within the body will have lost some of this energy by the time it reaches the surface. Furthermore, we shall assume that in leaving the surface of the body each electron must perform an amount of work,  $\phi$ , characteristic of the substance of which the body is composed. The ejected electrons leaving the body with the largest normal velocity will be those that were directly at the surface. The kinetic energy of such electrons is given by  $K.E._{\max} = hf - \phi$ .

‘If the emitting body is charged to a positive potential difference relative to a neighbouring conductor, and if  $V$  represents the potential difference which just stops the photoelectric current ... [then]

$$eV = hf - \phi$$

where  $e$  denotes the electronic charge.

‘If the deduced formula is correct, a graph of  $V$  versus the frequency of the incident light must be a straight line with a slope that is independent of the nature of the emitting substance ...’

From EINSTEIN, A. *Annalen der Physik*, **17**, 132, 1905, as reproduced in translation, in ARONS, A. B. *Development of concepts of physics*, Addison-Wesley, 1965. (Einstein’s symbols have been modified.)

### Questions

You should be able to answer questions 2, 3, and 4, on page 324, after reading this passage.

## WHAT IS LIGHT?

Richard Feynman is one of the present-day research physicists who have learned to live with the problem. He works in theoretical quantum mechanics, and accepting the conflict is the only way he and others have found to make progress.

In the Feynman lectures on physics *Volume 1*, Chapter 37, he says:

‘Things on a very small scale behave like nothing that you have any direct experience about. They do not behave like waves, they do not

behave like particles, they do not behave like clouds, or billiard balls, or weights on springs, or like anything that you have ever seen.

'Newton thought that light was made up of particles, but then it was discovered ... that it behaves like a wave. Later, however, .... Now we have given up. We say, "It is like *neither*".

'Because atomic behaviour is so unlike ordinary experience, it is very difficult to get used to and it appears peculiar and mysterious to everyone, both to the novice and the experienced physicist. Even the experts do not understand it the way they would like to, and it is perfectly reasonable that they should not, because all of direct human experience and of human intuition applies to large objects. We know how large objects will act, but things on a small scale just do not act that way.'

About the idea that photons arrive in large numbers where a wave would have a large amplitude, Feynman says:

'One might still like to ask: "How does it work? What is the machinery behind the law?". No one has found any machinery behind the law. No one can "explain" any more than we have just "explained". No one will give you any deeper representation [model] of the situation.'

### Question

This extract may help you answer question 22, on page 328.

## WAVES AND PARTICLES

The dual nature of matter – and of radiation – is one of the keys to understanding modern physics. It is also one of the most puzzling aspects of the subject. Much has been written about it. Some recommended passages are listed below, followed by a list of questions. You should read at least one of the passages and while you do so bear in mind the questions.

FEYNMAN, LEIGHTON, and SANDS, *The Feynman lectures on physics Volume 1*, Chapter 37. This is difficult, but worth it. Concentrate on 37–1 to 37–5, without worrying about the algebra in 37–3. Feynman describes beautifully how the quantum world behaves, and you should read the words rather than the mathematics.

PSSC *Physics* 3rd edition, Chapter 33.

PSSC *College physics* 5th edition, Chapters 31, 33, 34.

The chapter from the PSSC course gives information at an appropriate level. It is worth reading carefully. One interesting section compares the photon and the electromagnetic wave models of light. The three *College physics* chapters expand the same material.

ROGERS, *Physics for the inquiring mind*, Chapter 44. This is a long chapter, and although you may like to read it all, the especially relevant parts are pages 723 (about photons) to 727, and pages 737 (particles and waves) to 742. The diagrams on pages 723, 725, 740, and 741 are very useful, as is the table at the top of page 738.

BORN, *The restless Universe*, Chapter III. Max Born was one of the physicists who first explored the quantum world. He first suggested that chance or probability had to be used to describe the behaviour of particles. Pages 106–117 will be revision for most students. But read pages 117–121 about photons. You could skip pages 122–133 on spectra and Bohr's model of the atom; also skip pages 133–139 on the Compton effect and pages 139–151 which introduce electron waves. You should read pages 151–158 carefully. See especially plate III(b). The final pages, 159–165, are interesting, but could be missed.

TOULMIN and GOODFIELD, *The architecture of matter*, Chapter 12. This is a fairly condensed historical account of the development of quantum ideas. Concentrate most on the section, 'Radiation is atomized'. You could stop at the point where Rutherford's nuclear model of the atom is discussed.

ROTHMAN, *The laws of physics*, Chapter 10. This is also historical, but less condensed than Toulmin and Goodfield. It is worth following up to page 180, but the final part ('The uncertainty principle') could be omitted. (Energy is measured on page 169 in ergs. 1 erg is equal to  $10^{-7}$  joule.) If you have little time, omit pages 172–178 ('The size of a photon').

PROJECT PHYSICS Reader *Unit 5*. This contains a good chapter by Banesh Hoffman.

CARO, MCDONELL, and SPICER, *Modern physics*, Chapter 3. This chapter concerns the photoelectric effect, and the behaviour of photons. It is useful as a guide to which ideas come from experiment and which from theory. Part of Chapter 9 deals with matter waves.

TOLANSKY, *Revolution in optics*, Chapter 2. Do not bother with pages 33–37 on 'black body' radiation. Concentrate on pages 37 ('The photon') to 43. The remaining pages, 43–50, link up usefully with other parts of our work, but are less essential.

PROJECT PHYSICS Text *Unit 5*, Chapter 18. Section 18.4 discusses the photoelectric effect, and its interpretation is considered in 18.5. Section 18.6, on X-rays, is useful but less necessary. Chapter 20, sections 20.2 and 20.3, gives a good outline of the wave and particle behaviour of photons and electrons. (You will have to accept one formula from relativity.) Section 20.4 outlines the developments in quantum theory that will form most of the remaining work in our course.

HOFFMANN, *The strange story of the quantum*. Chapter 8 discusses the idea of de Broglie and the Davisson–Germer experiment. Electrons as waves and particles are discussed on pages 166–173. Chapter 3 discusses the photoelectric effect.

OPEN UNIVERSITY, Science Foundation Course S101, Unit 9 *Light: waves or particles?* Section 6 (The photoelectric effect) and Section 7 (Waves and particles) are particularly relevant. Units 10 and 11 are called *Atomic structure*: section 5 deals with spectra and energy levels. (Other parts of Units 10 and 11 such as Section 3 'Atomic structure – the



nuclear model', and section 4 'Radioactivity and nuclear reactions' are relevant to other parts of this course.)

BOLTON, *Patterns in physics*. Chapter 7 deals briefly with spectra, the photoelectric effect, energy levels, and the wave properties of electrons. You may find the section on 'The energy necessary for sight', and the part dealing with lasers interesting though they are not necessary for this course.

DUNCAN, *Advanced physics: fields, waves, and atoms*, or *Physics: a textbook for advanced level students*. The sections relevant to this work are 'Photoelectric emission', 'Quantum theory', 'Energy levels in atoms', 'Evidence of energy levels', 'Wave particle duality of matter'.

AKRILL, BENNET, and MILLAR, *Physics*. Parts of Chapter 18 (especially 18.3 The photoelectric effect, 18.4 Energy levels) are useful.

BENNET, *Electricity and modern physics*. Section 13.4 on the photoelectric effect. (Note that the selenium photovoltaic cell is quite different from the photocells used to test Einstein's relation.) Section 13.5 briefly deals with quantum theory, wave – particle duality, spectra and wave mechanics.

WENHAM, DORLING, SNELL, and TAYLOR, *Physics: concepts and models*. 'Electricity, matter and light' deals with photoelectricity, energy levels, and spectra; 'The atom and its electrons' deals with the wave nature of electrons. (In the second edition of this book this work appears in the Section 'The atom, electrons and radiation'.)

### Questions

Now here are some questions to ask yourself as you read:

- a Does the author say that  $E = hf$  comes from experiment, from theory, from both, or from elsewhere?
- h Does the author say that  $mv = h/\lambda$  comes from experiment, theory, both, or neither? Are the answers for photons the same as for electrons, or not?
- c Does the author say that the behaviour of electrons and photons can be deduced from deeper ideas?
- d What does the author say about why the description of the quantum world is widely accepted?
- f Does the author mention 'probability'? What is said to be probable or not probable?
- g What is said to be random, unpredictable?
- h What is said to be predictable?
- i Does the author explain why random effects are not seen on the large scale, for photons, or for electrons?

- j** Does the author say whether photons can vanish, unlike electrons? (Watch out for words like ‘absorbed’.)
- k** Are photons said to have mass as well as momentum?
- l** Do you have the impression that in the development of these ideas, experiment led theory, theory led experiment, or a bit of both happened?
- m** Does the author talk about ‘models’ or ‘analogues’ or ‘pictures’ (or some similar word) of photons or electrons?
- n** Does the author say we have no adequate model of photons and electrons, or that we have two good models, or that we have a complicated model, or that we shouldn’t expect to have a model? What do you think about these questions?

Questions 38, 39, and 40 on pages 333 to 334 give an opportunity to write about these ideas yourself.

# POSTSCRIPT

Sir Nevill Mott began research in physics soon after Schrödinger and Heisenberg laid the foundations of modern quantum theory in 1925. He shared the 1977 Nobel prize for physics for work which involved the applications of quantum ideas to disordered structures.

A personal note on the origin and future of quantum mechanics by Professor Sir Nevill Mott.

The organizers of the Revised Advanced Physics Project have asked me to write a few words about quantum mechanics, both as it looked in the mid 1920s when I started research and as it looks now. Then Bohr's theory of the atom held the field. We were brought up on it, our text books and lectures contained beautiful pictures of elliptical orbits, and I think we students really believed they existed. They explained so much: the Balmer series in the spectrum of hydrogen, X-ray spectra, and a lot more besides.

I believe that the leaders of physics at that time were divided into those who were trying to make the Bohr theory better and better and those – many fewer – who felt it would in the end have to be replaced by something quite different. Among the latter was the German physicist Heisenberg, and he and others produced the first breakthrough, the so-called 'matrix mechanics' which did promise to be quite general. It promised to be a form of mechanics replacing Newtonian mechanics, capable of answering any question that Newtonian mechanics could answer, but of giving different answers at any rate for problems about atoms. The mathematics promised to be difficult; physicists who were good at mathematics thought they would have a lot of fun.

Schrödinger's equation was a bit of a bombshell. Schrödinger used 'easy' mathematics, differential equations, which are part of any university course. I remember a talented mathematical contemporary saying to me, 'All the fun has gone out of quantum theory, I'm going to study Law' – and he did. He was wrong; the later developments of quantum mechanics are difficult enough in all conscience. But what I remember most vividly about this equation was that Schrödinger himself did not know what it meant! He thought his 'wave intensity' must be interpreted as density of charge. We know now that this is only true in a statistical sense, and that the amplitude gives the *probability* that a particle will be found somewhere. This was first clearly stated by Max Born, a German physicist who moved to England in 1933.

To start research just after Schrödinger's equation was – perhaps – like being an explorer just after Columbus. The facts of physics and chemistry were wide open. In a very few years Schrödinger's equation was used to explain why atoms form molecules, why some solids conduct electricity and others do not, how radioactive decay occurs, and the details of the spectra of helium and most other atoms. It did everything that Bohr's theory could do and a lot more. Most convincing of all, it made predictions. For instance, it showed that the Rutherford scattering formula – really the basis of all of nuclear physics – did not work if an alpha particle hit a nucleus, namely of helium, of just the same kind as itself. When this prediction was made, the Cavendish Laboratory quickly mounted the experiments which showed that it was so. And even more important, quantum mechanics predicted

that moving protons, quite slow by the standards of nuclear physics, could penetrate into an atomic nucleus of a light element, and so encouraged Cockcroft and Walton to make their famous first disintegration of the nucleus with artificially accelerated particles. What quantum mechanics did *not* do was to predict the neutron! And if one looks at quantum mechanics as it is today, it is in nuclear or particle physics that it is still in a state of flux. At the time of writing there is considerable disagreement between experts about the nature of the solutions of Schrödinger's equation for this case. Here it is just like Bohr's theory was, brilliantly successful sometimes, failing at others, constantly being modified, and calling on all the techniques of advanced mathematics. No-one would pretend that here it was the last word.

But outside the nucleus, the least one can say is that no one has proved it wrong. In the theory of molecules the theoretical chemists use all the resources of modern computers to calculate the properties of molecules and achieve greater and greater success. Schrödinger's equation gets more and more complicated as the number of electrons in a molecule increases, so the computer comes into its own here. In solids, particularly metals, there are problems of real mathematical difficulty. Superconductivity is one, understood in principle but not in detail. Our whole thinking about semiconductors and transistors is based on quantum mechanics and has become a very exact branch of science. At the time of writing there is a great deal of interest in non-crystalline semiconductors, which present a more difficult problem – how does the electron find its way among a jumble of molecules put together without pattern? But no-one doubts that quantum mechanics is competent to give the answer, and will in the not too distant future.

What of the more distant future? I see quantum mechanics applied to more and more complicated systems – the molecules of biology, technically useful alloys, conducting glasses, and polymers. Quantum mechanics, I dare guess, will not change, but the methods of using it will change, as they have already in the last few decades. And in the nucleus and in stellar interiors? It would be arrogant to guess. Will a billion dollar accelerator give us the final understanding of the nuclear particles, or will some brilliant insight in theory? Only the future can show.

# LABORATORY NOTES

## DEMONSTRATION

### L1 Detection of electromagnetic radiation

#### L1a Radio waves

portable radio

Tune the radio to a broadcast. You should be able to show that the radio is responding to electrical or magnetic waves. (Try pointing the aerial or ferrite rod in different directions; try screening the radio with a metal sheet (e.g. aluminium foil) or grid.)

#### L1b Gamma rays

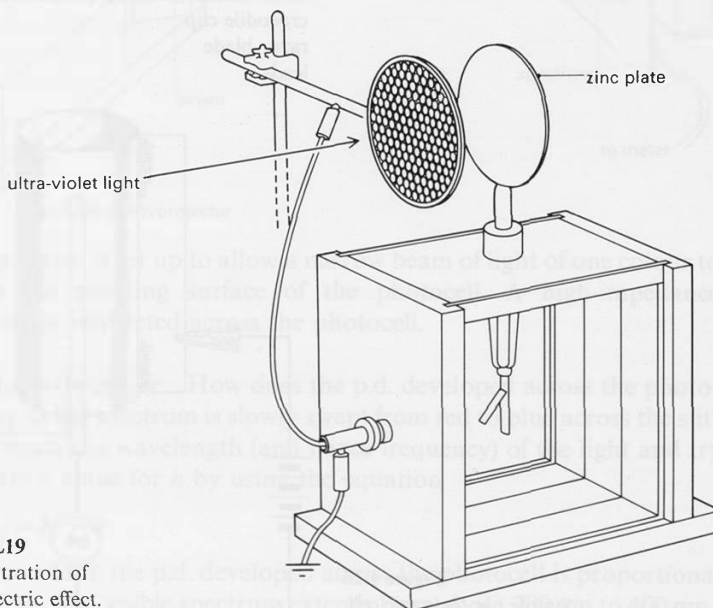
gamma GM tube  
GM tube holder  
scaler  
pure gamma source  
source holder

Show that this detector responds as if electromagnetic radiation consists of discrete quanta.

## DEMONSTRATION

### L2 Photoelectric effect

#### L2a Demonstration of photoelectric effect with zinc plate and gold-leaf electroscope



**Figure L19**  
Demonstration of  
photoelectric effect.

gold-leaf electroscope  
 zinc plate attachment  
 wire mesh about 5 cm diameter  
 e.h.t. power supply  
 piece of fine emery cloth  
 glass plate about 25 cm square  
 ultra-violet lamp  
 retort stand base, rod, boss, and clamp  
 leads

*Safety note:* Take care not to look directly at the ultra-violet lamp.

The surface of the zinc plate must be cleaned with the emery cloth. The electroscope case and the wire mesh are earthed.

Give the zinc plate a negative charge (from the e.h.t. supply), and then shine ultra-violet light on the zinc plate. There should be evidence of some movement of charge from the electroscope. What sign has that charge?

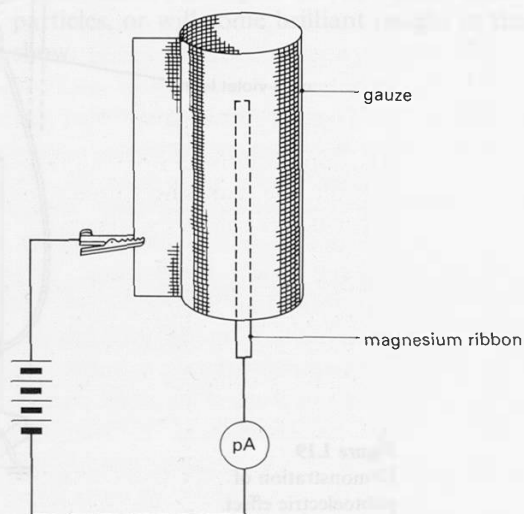
Repeat the experiment with a sheet of glass between the ultra-violet lamp and the zinc plate.

Repeat the experiment starting with positive charge on the zinc plate.

Explain the effects.

## L2b Simple photoelectric cell using magnesium ribbon

picoammeter  
 cell holder with four cells  
 ultra-violet lamp  
 magnesium ribbon 100 mm long  
 glass plate about 25 cm square  
 wire gauze, 70 mm  $\times$  60 mm, e.g. 20 mesh copper  
 retort stand base, rod, boss, and clamp  
 crocodile clip  
 razor blade  
 leads



**Figure L20**  
 Simple photoelectric cell.

*Safety note:* Take care not to look directly at the ultra-violet lamp.

The magnesium ribbon must be scraped clean with the razor blade before one end of it is put into the meter's input socket. The gauze is used to make a cylinder around, *but not touching* the ribbon.

You should be able to show that a current flows between the ribbon and the gauze when ultra-violet light falls on the clean magnesium surface. If the current is due to movement of negatively charged electrons which way are they travelling?

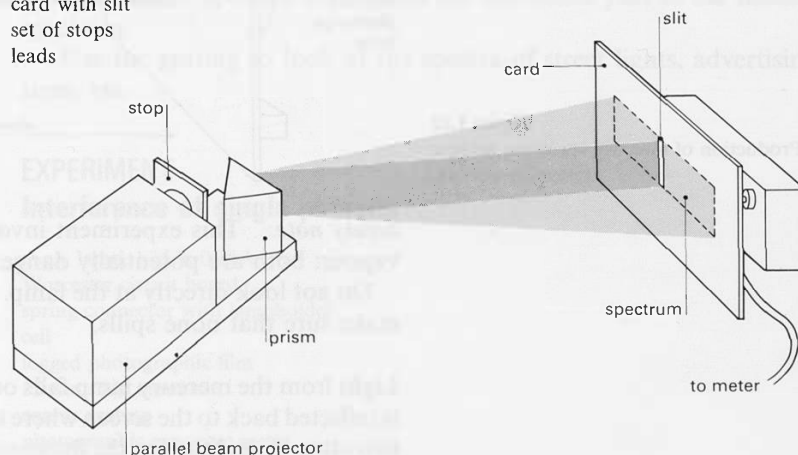
What happens if you put a sheet of glass between the lamp and the 'photocell'? Explain the effect.

You may be able to experiment with other metals in place of magnesium.

## DEMONSTRATION

### L3 Colour of light and energy of photoelectrons

parallel beam projector  
l.t. variable voltage supply  
high-dispersion prism  
photoelectric cell  
high-impedance voltmeter  
cell holder with one cell  
card with slit  
set of stops  
leads



**Figure L21**

Projection of a spectrum on a photocell.

The apparatus is set up to allow a narrow beam of light of one colour to fall on the emitting surface of the photocell. A high-impedance voltmeter is connected across the photocell.

*Variation with colour* How does the p.d. developed across the photocell vary as the spectrum is slowly swept from red to blue across the slit?

Estimate the wavelength (and hence frequency) of the light and try to obtain a value for  $h$  by using the equation

$$K.E._{\max} = hf - \phi$$

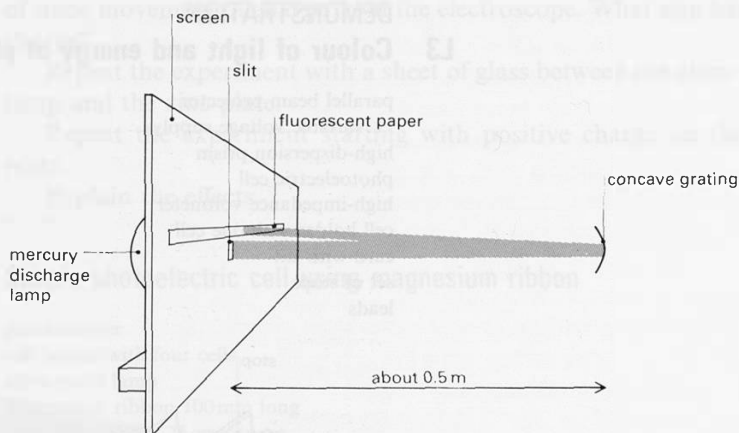
( $\phi$  is a constant; the p.d. developed across the photocell is proportional to  $K.E._{\max}$ . The visible spectrum extends from about 700 nm to 400 nm.)

**Variation with intensity** When blue light is falling on the slit try reducing the intensity by placing stops over the lens of the projector. Does the p.d. change as much as the light intensity?

## DEMONSTRATION

### L4 The spectrum of mercury vapour

mercury discharge lamp  
concave reflection grating  
screen with slit (see below)  
strip of green fluorescent paper, 20 mm wide, 0.5 m long  
retort stand base, rod, boss, and clamp  
a little mercury in a polythene bottle  
microscope slide  
fine diffraction grating



**Figure L22**

Production of mercury spectrum using a reflection grating.

**Safety note:** This experiment involves ultra-violet light and mercury vapour: both are potentially dangerous.

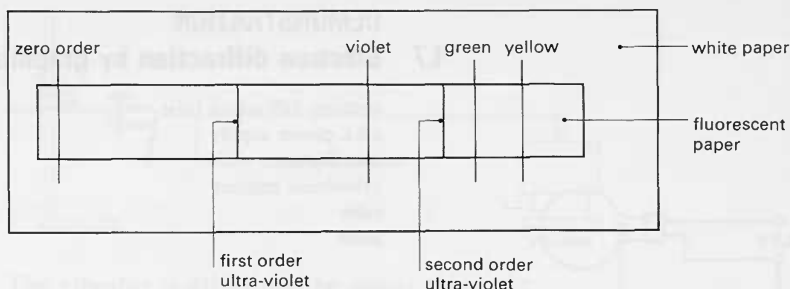
Do not look directly at the lamp. Take care when using mercury and make sure that none spills.

Light from the mercury lamp falls on the concave reflection grating and is reflected back to the screen where a spectrum is formed. You should see two ultra-violet lines (on the fluorescent paper only, figure L23). They are in fact the first and second order lines due to light of the same wavelength. What happens to the spectrum when light from the mercury lamp passes through glass, *e.g.* a microscope slide?

The green line in the mercury spectrum is at 546 nm ( $546 \times 10^{-9}$  m). Use this to estimate the wavelength of the ultra-violet line.

If a polythene bottle containing a few drops of mercury is held just below the grating and squeezed gently, light from the mercury lamp passes through vapour containing cold mercury atoms. What effect does this have on the spectrum?





**Figure L23**

Spectrum of mercury using fluorescent paper to detect ultra-violet lines.

## OPTIONAL EXPERIMENT

### L5 The hydrogen spectrum

fine diffraction grating (or direct vision spectroscope)  
hydrogen spectrum tube  
holder for spectrum tube  
e.h.t. power supply  
leads

Run the hydrogen spectrum tube from the e.h.t. power supply. Look at it through the grating held close to your eye, or through the spectroscope.

If the tube contains atomic hydrogen you should see the regular arrangement of spectral lines which are the visible part of the Balmer spectrum.

Use the grating to look at the spectra of street lights, advertising signs, etc.

## EXPERIMENT

### L6 Interference of single photons

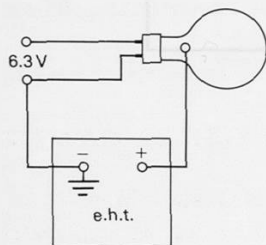
m.e.s. lamp, 1.25 V, 0.25 A  
Worcester circuit board  
spring connector with lampholder  
cell  
fogged photographic film  
35 mm slide mounts  
coarse grating  
photographic exposure meter  
slide projector

Look at the lamp through the diffraction grating and observe the spectrum. Put filters in front of the lamp – in a darkened room the spectrum should still be visible if you stand about 0.5 m from the lamp.

The amount of light transmitted by the filters can be estimated by using the exposure meter and slide projector.

Question 21 (page 328) shows how you can estimate the number of photons entering your eye each second. From this, and the fact that they travel at  $3 \times 10^8 \text{ m s}^{-1}$  you can calculate how 'far apart' they are.

Do you still see the diffraction spectrum with the filters in place, when comparatively few, well spaced photons enter your eye each second?



**Figure L24**

Circuit for electron diffraction tube.

## DEMONSTRATION

### L7 Electron diffraction by graphite

electron diffraction tube  
e.h.t. power supply  
transformer  
cylindrical magnet  
ruler  
leads

Earth the negative terminal of the e.h.t. supply, and connect it to one of the terminals in the tube base. Connect the positive e.h.t. terminal to the anode, not using the 50 M $\Omega$  resistor.

You should see a pattern of rings on the fluorescent screen.

What happens to the pattern as you vary the anode voltage?

Measure the diameter of the rings at several voltages.

What happens if you put a magnet near the neck of the tube?

## OPTIONAL EXPERIMENT

### L8 Optical analogue of electron diffraction

*either*  
compact light source  
transformer  
*or*  
m.e.s. lamp, 2.5 V, 0.3 A, in holder  
cell holder with 2 cells  
fine diffraction gratings  
colour filter (*e.g.* green)  
leads

Put the filter in front of the light source. Stand about 3 m away, hold the diffraction grating in front of your eye, and look at the lamp.

How does the position of the diffracted spots of light depend on the orientation of the grating?

Use this to suggest why the electron diffraction experiment shows rings, not spots.

## DEMONSTRATION

### L9 Standing waves

#### L9a Standing waves on a rubber cord

signal generator  
vibrator  
xenon flasher  
rubber cord, (0.5 m long, 3 mm square cross-section)  
retort stand base, rod, boss, and clamp  
2 small wooden blocks (to clamp rubber)  
2 small G-clamps  
leads

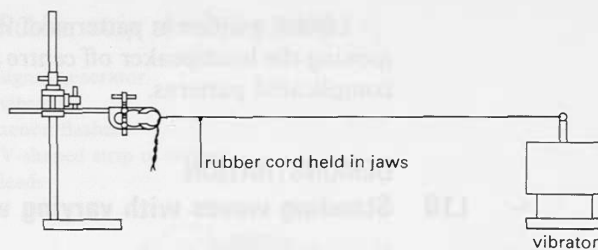


Figure L25

The vibrator is driven by the signal generator.

Does the cord vibrate when driven at any frequency, or only at certain frequencies?

What is the lowest frequency at which the cord will vibrate?

How many 'loops' are seen when it is vibrating at its lowest frequency?

What is the relationship between the wavelength of this one-loop standing wave and the length of the cord?

## L9b Vibrations in a rubber sheet

signal generator  
large loudspeaker  
xenon flasher  
sheet of rubber  
3 retort stand bases and rods  
boss and clamp  
large aluminium ring  
rubber band  
leads

Start with the loudspeaker under the centre of the rubber sheet, and slowly increase the frequency from about 10 Hz.

What is the lowest frequency that will excite standing waves? At this frequency does the wavelength have its lowest or highest value?

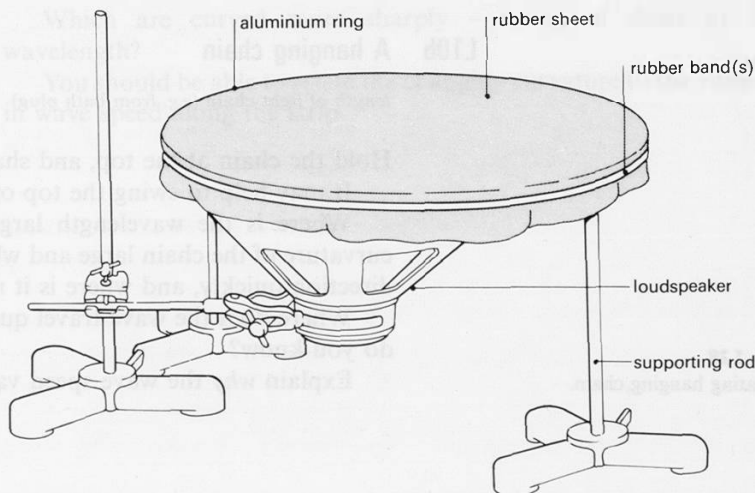


Figure L26  
Standing wave experiments.

Look for different patterns of vibration at higher frequency, and try moving the loudspeaker off centre (as shown in figure L26) to see more complicated patterns.

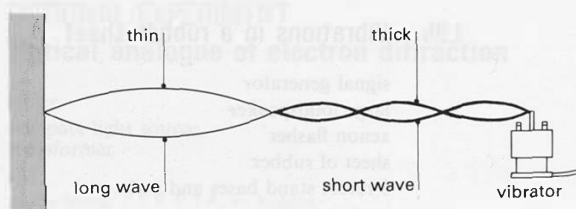
## DEMONSTRATION

### L10 Standing waves with varying wavelength

You can set up standing waves with varying wavelength in different systems.

#### L10a Rubber cords of different thickness

signal generator  
vibrator  
rubber cord, 0.5 m long, 3 mm square cross-section  
light rubber cord, 0.5 m long (e.g. dress-making elastic)  
leads



**Figure L27**  
Oscillating rubber cords.

Tie the two cords together.

Adjust the frequency of the signal generator until you get standing waves of large amplitude.

Do both cords vibrate with the same frequency?

In which cord is the wavelength shorter?

In which cord does the wave travel more slowly? How do you know?

Explain why the wave travels more slowly in that cord.

#### L10b A hanging chain

length of light chain (e.g. from bath plug)

Hold the chain at the top, and shake it to set up standing waves.

It may help to swing the top of the chain around in a small circle.

Where is the wavelength large, and where small? Where is the curvature of the chain large and where small, *i.e.*, where does it change direction quickly, and where is it more nearly straight?

Where does the wave travel quickly, and where more slowly? How do you know?

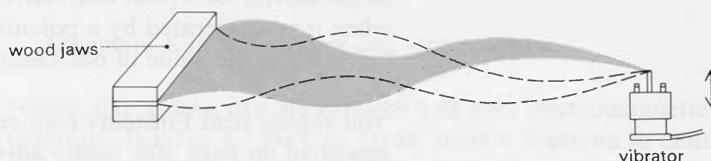
Explain *why* the wave speed varies.



**Figure L28**  
Oscillating hanging chain.

## L10c Rubber strip of varying width

signal generator  
vibrator  
xenon flasher  
V-shaped strip of rubber  
leads



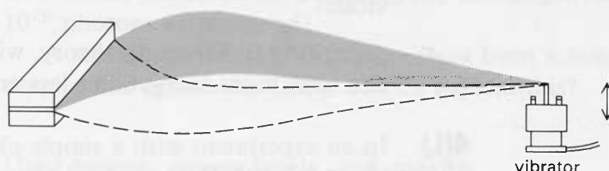
**Figure L29**

An oscillating V-shaped rubber strip.

Adjust the frequency of the signal generator until large-amplitude standing waves are formed.

Is the wavelength of the standing waves constant? If not where is it larger and where smaller?

Adjust the frequency to obtain a single loop (figure L30).



**Figure L30**

Lowest mode of V-shaped rubber strip.

Is the peak in the middle?

Where is the rubber curved more sharply, and where less so?

The mass per unit length of the rubber strip varies along its length.

At which end will the wave speed be least?

Which are curved more sharply – waves of short or long wavelength?

You should be able to relate the changing curvature to the variation in wave speed along the strip.

# QUESTIONS

The photoelectric effect;  $E = hf$

- 1(I)** The energy of a light photon, or of an electron, may be expressed in joules or in electronvolts. (One electronvolt is the energy gained by an electron when it is accelerated by a potential difference of one volt.)

What is the value of one electronvolt (1 eV) in joules?

You should read Einstein's own explanation of the photoelectric effect reprinted on page 308, before answering questions 2 to 4.

- 2(P)** What is the minimum frequency of light needed to ionize a helium atom, whose ionization energy is about 24 eV?
- 3(P)** Light of frequency  $8 \times 10^{14}$  Hz releases electrons from sodium metal.
- What is the wavelength of this light?
  - What region of the spectrum is it in – infra-red, visible, or ultra-violet?
  - According to Einstein's theory, will light of *shorter* wavelength than this eject more energetic or less energetic electrons?
- 4(L)** In an experiment with a simple photocell it was found that red light ( $f = 4.5 \times 10^{14}$  Hz) ejected electrons with a maximum energy of about 0.2 eV, while violet light ( $f = 7.5 \times 10^{14}$  Hz) gave electrons of 1.4 eV. Use Einstein's relationship to obtain a value for Planck's constant,  $h$ .

- 5(R)** Light falls on a metal surface. Electrons ejected from the metal surface are collected by a collector, which is connected via a battery and a meter to the metal, as shown in figure L31.

Which one of the following statements about the photoelectric effect in such an experiment is true?

- The number of electrons ejected per second from the metal is proportional to the potential difference between the metal and the collector.
- The number of photoelectrons produced per second is proportional to the frequency of the light.
- The maximum energy of the photoelectrons is proportional to the intensity of the light.
- The maximum energy of the photoelectrons is dependent on the frequency of the light.
- The maximum energy of the photoelectrons is independent of the particular metal used.

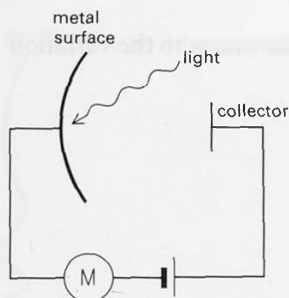


Figure L31

(Coded answer paper, 1979)

**6(P)** Use  $E = hf$  and  $c = f\lambda$  to complete the blanks in table L3.

	BBC Radio 4	Microwaves	Visible light	Gamma rays
$\lambda$	1500 m	$3 \times 10^{-2}$ m		
$f$			$5 \times 10^{14}$ Hz	
$E$				$10^6$ eV

Table L3

Calculate  $E$  in both joules and electronvolts.

**7(P)** Nuclei, like atoms, can also absorb and emit electromagnetic radiation. It takes 2.2 MeV to break apart a deuteron (a neutron plus proton). What wavelength of radiation is needed to do this? Is your answer the smallest or the largest wavelength that will do?

**8(P)** The school microwave apparatus ( $\lambda \approx 3$  cm) has a power of about  $10^{-3}$  watt. About how many photons does it emit each second? In each cycle of oscillation?

**9(P)** Your answer to question 8 may be about  $10^{20}$  photons per second. How much power would a gamma ray source deliver if it emitted  $10^{20}$  photons each second?

A school gamma ray source does not burn a hole in its box, despite this calculation. How do you explain this?

### Light quanta, energy levels, and spectra

**10(I)** What is the energy, in J and in eV, of a photon of light of wavelength 253 nm?

**11(L)** Table L4 shows the energy levels of a mercury atom, measured from the lowest level, A.

Energy/J	Energy/eV		
$1.41 \times 10^{-18}$	8.84	F	<b>a</b> Identify the origin of the line at wavelength 253 nm in the mercury spectrum, <i>i.e.</i> , say which are the energy levels involved.
$1.24 \times 10^{-18}$	7.73	E	<b>b</b> What is the wavelength associated with a jump between the levels marked D and F?
$1.07 \times 10^{-18}$	6.70	D	<b>c</b> Will light be emitted or absorbed if the transition is <i>from</i> F to D?
$0.875 \times 10^{-18}$	5.46	C	
$0.784 \times 10^{-18}$	4.90	B	

0 — 0 A

Table L4

- 12(R)** Suppose that a certain atom had only four possible energy levels. What would be the maximum number of different frequencies there could be in the spectrum of the radiation it could emit?

A 3      B 4      C 5      D 6      E 8

(Coded answer paper, 1978)

- 13(R)** When electrons of energy 4.9 eV bombard mercury atoms in mercury vapour, some atoms in the lowest energy level (ground state) are excited to an energy 4.9 eV above the ground state, after which they return to the ground state, emitting a photon of ultraviolet radiation.

If the number of bombarding electrons is reduced, but their energy is still 4.9 eV, which of the following correctly describe(s) changes to the radiation emitted?

- 1 The intensity (brightness) of the radiation is reduced.
- 2 The wavelength of the radiation is reduced.
- 3 The energy of each photon of the radiation is reduced.

A 1 only      B 2 only      C 1 and 3 only  
D 2 and 3 only      E 1, 2, and 3

(Coded answer paper, 1979)

### The spectrum and energy levels of hydrogen

- 14(l)a** Calculate the energy of a photon whose frequency is  $32.881 \times 10^{14}$  Hz. (Use  $h = 6.63 \times 10^{-34}$  J s.)
- b** Look up the ionization energy for hydrogen. Compare with your answer to a.

- 15(l)** See figure L32.

The transitions marked A, B, C, and D give rise to four members of the Lyman series. Will the transition A be the longest or the shortest wavelength line in the series?

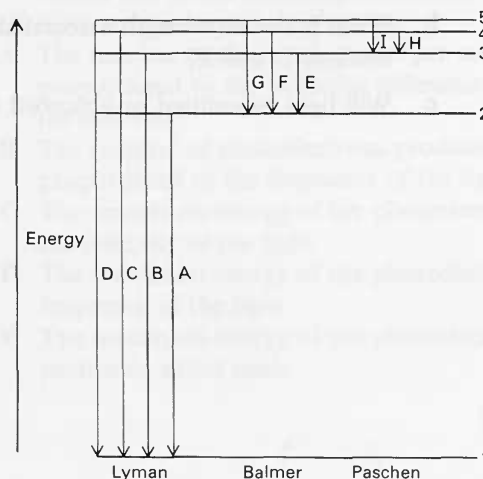


Figure L32

Some energy levels of a hydrogen atom.



**16(L)** See figure L32.

The frequencies corresponding to the transitions marked A and B are  $24.659 \times 10^{14}$  Hz and  $29.226 \times 10^{14}$  Hz respectively.

- a** What frequency will be emitted by the transition marked E?
- b** The third line in the Balmer series (corresponding to the transition marked G) has a frequency of  $6.9044 \times 10^{14}$  Hz. Calculate the energy difference between levels 2 and 5.

**17(L)** The frequencies of the first four lines in the Lyman series, (corresponding to the transitions marked A, B, C, and D in figure L32) are given in table L5.

Frequency/ $10^{14}$ Hz	Transition
24.659	A
29.226	B
30.824	C
31.564	D

Table L5

Calculate the corresponding photon energies in joules. (Use  $h = 6.63 \times 10^{-34}$  J s.)

**18(L)** This question asks you to calculate the total energy per kilogram of the Earth in its orbit around the Sun. You should answer in terms of the gravitational constant,  $G$ , the mass of the Sun,  $m$ , and the mean radius of the Earth's orbit,  $r$ .

- a** What is the potential (gravitational potential energy per kilogram) of the Earth in its orbit round the Sun?
- b** Use the gravitational force on the Earth and the centripetal force needed to keep it in orbit at distance  $r$  from the Sun to obtain an expression for  $v$ , the speed of the Earth in its orbit, assumed circular.
- c** What is the kinetic energy per kilogram of the Earth?
- d** What is the total energy per kilogram of the Earth?

Your answer to **d** should be a negative quantity. If the Earth had so much kinetic energy that the total energy were positive, then the gravitational attraction of the Sun would not be enough to hold it in orbit. In any bound system, including the hydrogen atom (an electron bound to a proton), the total energy is negative.

**19(L)** To answer this question you need to know the photon energies (in J) of the first four lines in the Lyman series (see question 17). Level 1 in figure L32 is the ground state of the hydrogen atom. The ionization energy of hydrogen is 13.6 eV ( $2.18 \times 10^{-18}$  J).

- a** At what energy is level 1?
- b** Work out the energies (in J) of levels 2, 3, 4, and 5.

**20(L)a** Check that the expression  $-2.18 \times 10^{-18}/n^2$  J correctly predicts the energy levels calculated in question 19.

- b** Calculate the 8th energy level.

Light: waves or particles – or both?

**21(L)** A small lamp passes 0.2 A when the p.d. across it is 1.5 V.

- a** What is the total power transformed?
- b** Perhaps 3% of this is visible light. About how many visible photons are emitted each second?  
(Take  $f = 6 \times 10^{14}$  Hz.)
- c** Suppose your eye is about 0.3 m from the lamp. About what fraction of these photons will enter your eye?
- d** From the number of photons entering your eye each second work out the average time interval between the arrival of photons at the eye.
- e** If these photons are travelling at  $3 \times 10^8$  m s<sup>-1</sup> about how far behind is the next photon when one enters your eye?
- f** Suppose the intensity of light is reduced by a factor of  $10^4$  (e.g. by grey filters). How 'far apart' are the photons that now enter your eye?

**22(E)** Summarize what is known about the *behaviour* of light, and the *models* of light that are used to explain the phenomena. Say which model(s) is(are) useful in explaining different phenomena. *Why* do we seem to need two quite different models?

Electron diffraction

**23(I)** In optical diffraction by a grating, is red light diffracted more or less – i.e., through larger or smaller angles – than blue light?

**24(I)** In the electron diffraction experiment the diffraction angle decreases if the electrons have higher energy. Thinking about the wave-like behaviour of electrons, does higher energy mean longer or shorter wavelength?

- 25(L)** Suppose that the two rings formed at angles  $\theta_1$  and  $\theta_2$  in the electron diffraction experiment with graphite come from rows of atoms with spacings  $s_1$  and  $s_2$ . Use the grating formula  $\lambda = s \sin \theta$  to get an expression comparing the sines of the angles  $\theta_1$  and  $\theta_2$ . Does the expression depend on wavelength,  $\lambda$ ?

**26(I)** Figure L33 shows the geometry of the electron diffraction tube.

- What is the value of  $\theta$ , in radians?
- What is  $\sin \theta$ , approximately?

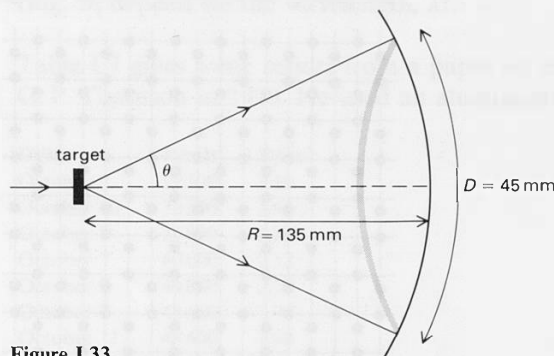


Figure L33

- 27(L)** Table L6 shows the diameter of the two rings formed at different accelerating voltages in an electron diffraction experiment with graphite.

Electron gun p.d./V	$D_1$ /mm	$D_2$ /mm
5000	26	45
4000	28.5	49.5
3000	34	56

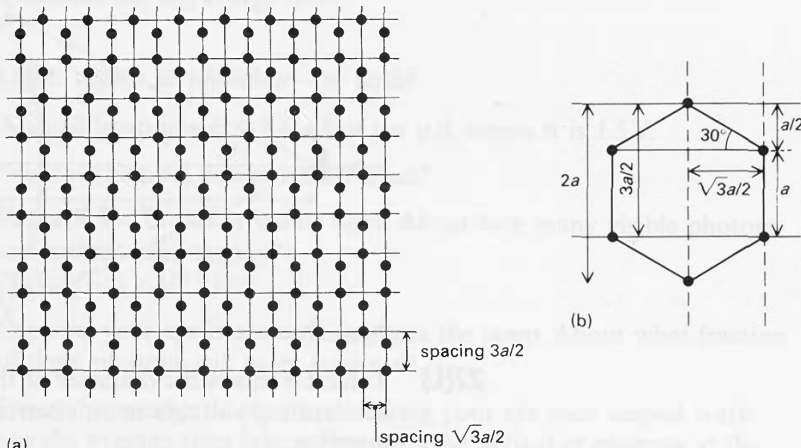
Table L6

Diffraction ring diameters

- Calculate  $D_2/D_1$  for each voltage, and the average value of  $D_2/D_1$ .  
Because the diffraction angle  $\theta$  is small (see question 26) the ring diameter is approximately proportional to  $\sin \theta$ .
- Use your value of  $D_2/D_1$  ( $\approx \sin \theta_2/\sin \theta_1$ ) to calculate the relative spacings of the rows of atoms responsible for these rings.
- Does the wider ring (diameter  $D_2$ ) come from rows of atoms with larger or smaller spacing than the rows that give the ring with diameter  $D_1$ ?
- Does the angle of diffraction increase or decrease as the accelerating voltage is increased? What does this suggest happens to the wavelength associated with the electrons as the energy of the electrons is increased?

**28(L)** This question is about the spacings between different rows of carbon atoms in the hexagonal arrangement of graphite.

In figure L34 two sets of rows are marked which could be responsible for the two diffraction rings. The hexagons of carbon atoms have sides  $1.42 \times 10^{-10} \text{ m}$  long – distance marked  $a$  in figure L34(b). The vertical lines show rows of atoms  $\frac{\sqrt{3}}{2} \times (1.42 \times 10^{-10} \text{ m})$  apart, while the horizontal rows are  $\frac{3}{2} \times (1.42 \times 10^{-10} \text{ m})$  apart. The geometry involved is shown in figure L34(b).



**Figure L34**

- (a) Two row spacings in a graphite layer.  
(b) Geometry of row spacings.

- a** What is the ratio of the spacing of these two sets of rows?
- b** If these rows were the ones responsible for the electron diffraction rings, what would be the ratio of the diameters of the rings? Compare with your answer to question 27a.
- c** Which of the rows marked in figure L34(a) – horizontal or vertical – correspond to the rings labelled  $D_1$  in question 27, and which to  $D_2$ ?

**29(R)** Which of the following facts directly support(s) the view that electrons have wave properties?

- 1 When light of short wavelength falls on a suitable metal, electrons are emitted from the metal.
- 2 A beam of electrons can be diffracted by a suitable crystalline material.
- 3 There is an oscillatory force on an electron placed in a beam of radio waves.

- A 1 only    B 2 only    C 1 and 3 only  
D 2 and 3 only    E 1, 2, and 3.

(Coded answer paper, 1979)

## Wavelength, energy, and momentum of electrons

- 30(L)** In 1923 the Frenchman Louis de Broglie suggested that waves might be associated with particles. He predicted that momentum multiplied by wavelength would be constant

$$mv \propto 1/\lambda$$

- If the accelerating p.d. is  $V$ , will the velocity  $v$  of the electrons be proportional to  $V$ ,  $1/V$ ,  $1/\sqrt{V}$ , or  $\sqrt{V}$ ?
- In an electron diffraction experiment how does the diameter of the ring,  $D$ , depend on the wavelength,  $\lambda$ ?

- 31(L)** Table L7 gives some results from a paper on electron diffraction by G. P. Thomson in 1928. He used an aluminium target.

Date	$V/\text{volt}$	$D/\text{cm}$
October 7	17 500	3.10
October 10	30 500	2.45
October 7	31 800	2.32
October 7	40 000	2.12
October 7	44 000	2.08
October 7	48 600	1.90
October 11	48 600	1.98
October 12	56 500	1.83
October 12	56 500	1.80

**Table L7**

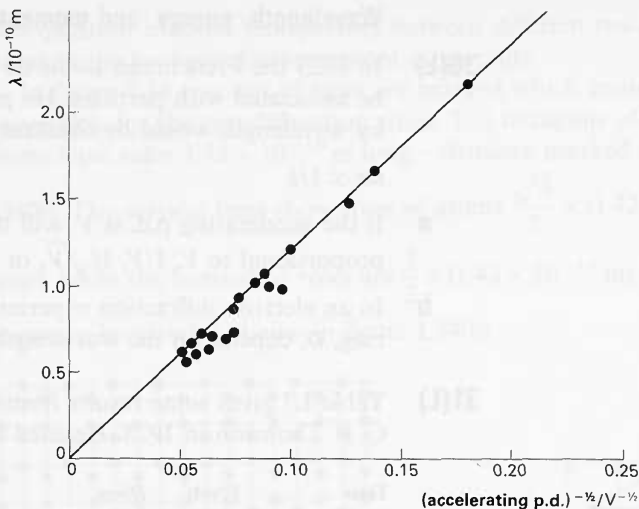
From THOMSON, G. P. 'Experiments on the diffraction of cathode rays'. Proceedings of the Royal Society, A. **117**, 1928.

Use your answers to question **30** to predict some function of  $D$  and  $V$  (such as  $D \times V$ ,  $D\sqrt{V}$ ,  $V/D$ , ...) which you would expect to be constant. Then check whether this function is constant for these results.

- 32(L)** In an electron diffraction experiment using graphite the larger ring, formed by rows of carbon atoms  $1.23 \times 10^{-10}$  m apart, was formed at an angle of 0.167 radian.

- What is the wavelength?
- Write an expression for the kinetic energy of an electron ( $\frac{1}{2}mv^2$ ) in terms of its charge,  $e$ , and the accelerating voltage,  $V$ .
- Obtain an expression for momentum,  $mv$ , in terms of  $e$ ,  $V$ , and  $m$ .  
The accelerating voltage was 5000 V.
- Work out the constant  $h$  in  $mv = h/\lambda$ .
- Are the units of this constant (momentum  $\times$  wavelength) the same as those of the constant in  $E = hf$ ?

- 33(L)** Figure L35 is a graph prepared by Davisson in 1928 collecting several people's results together. Wavelength  $\lambda$  is plotted against  $1/\sqrt{V}$  ( $V$  is accelerating p.d.).



**Figure L35**

Wavelength of electrons plotted against reciprocal of the square root of the accelerating voltage.

Davisson, Clinton J. 'Are electrons waves?'  
Journal of the Franklin Institute, **205**,  
1928.

In your answer to question 32c you should have found

$$mv = \sqrt{2meV}$$

- a** Show that the slope of the graph is  $h/\sqrt{2me}$ .
- b** Find  $h$  from the slope. ( $V$  was measured in volts,  $\lambda$  in units of  $10^{-10}$  m.)

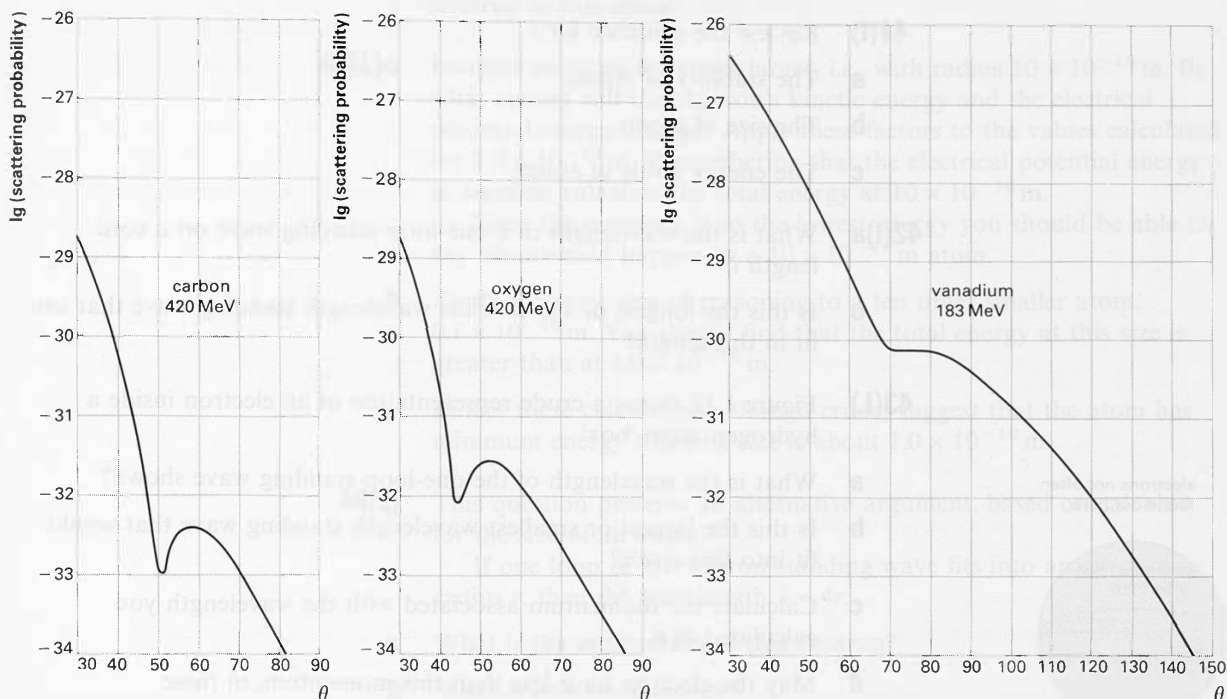
### Matter waves

- 34(P)** How much more slowly will a neutron have to travel than an electron if they are to have the same wavelength?
- 35(E)** An electron has a wavelength of about  $10^{-10}$  m when its velocity is of the order of  $10^7$  m s<sup>-1</sup>. Estimate the wavelength associated with a ball in a game of tennis.
- 36(R)** What happens to a diffraction pattern as the wavelength gets smaller and smaller? What happens as the wavelength is made larger? Why are X-rays and not visible light used to study crystal structures by diffraction methods? Why are diffraction effects with tennis balls never seen?
- 37(L)** When light waves are diffracted by a spherical object of diameter  $b$  the first minimum of intensity occurs at an angle  $\theta$ , given by  $\sin \theta = 1.22\lambda/b$ . If the wavelength is known, the size of the object can be estimated from the diffraction pattern.

When high-energy electrons are scattered by nuclei, the nuclear size can be estimated from the scattering pattern. Figure L35 shows how the scattering probability for electrons varies with angle for several nuclei, and different electron energies. The scattering probability determines the light intensity in a diffraction pattern.

For these very energetic electrons the momentum,  $p$ , and energy,  $E$ , are connected by the relativistic formula  $p = E/c$ .

- a** Calculate the momentum and hence (from  $\lambda = h/p$ ) the wavelength of 420 MeV and 183 MeV electrons.
- b** Use the information in figure L36, and  $\sin \theta = 1.22\lambda/b$  to estimate the size of the nuclei of carbon, oxygen, and vanadium.



**Figure L36**

From: Physics (Advanced) Notes for the guidance of teachers on the nuclear physics option. JMB, 1981.

- 38(R)** Write an account of the evidence for the existence of  
*either a* discrete energy levels within the atom  
*or b* wave properties associated with the electron.

(Special paper, 1979)

- 39(R)** The passage below was written by someone who knows something of the ideas involved, but is confused and puzzled about them.

‘The idea of matter as like waves is really a very silly invention. It’s fairly easy to think of electrons as very tiny billiard balls – and that used to be all right. But to say that when you go to very tiny things they start behaving like waves instead, then it’s almost an insult to one’s intelligence. Let’s hope they can think of a better idea some day.’

Write a short explanation of the passage. Your explanation should first point out any mistakes or ambiguities and then make a more accurate and complete statement to help the author to understand.

(Part question, Long answer paper, 1978)

- 40(E)** Write an explanation of ‘wave – particle duality’ for an intelligent non-scientist of your own age.

### Electron waves in a hydrogen atom box

- 41(I)** Review the evidence for :

- a** The stability of atoms.
- b** The size of atoms.
- c** The energy levels of atoms.

- 42(I)a** What is the wavelength of a one-loop standing wave on a cord length  $l$ ?

- b** Is this the longest or the shortest wavelength standing wave that can fit in this length?

- 43(L)** Figure L37 shows a crude representation of an electron inside a hydrogen atom ‘box’.

- a** What is the wavelength of the one-loop standing wave shown?
- b** Is this the largest or smallest wavelength standing wave that would fit into this space?
- c** Calculate the momentum associated with the wavelength you calculated in **a**.
- d** May the electron have less than this momentum, in these circumstances?
- e** Calculate the kinetic energy of the electron.
- f** Is this the maximum or minimum possible kinetic energy for the electron?
- g** What prevents the electron, with this kinetic energy, flying away from its proton?
- h** Calculate the energy that must be transferred to electrical potential energy to move an electron from  $1.0 \times 10^{-10}$  m to a long distance from a proton, that is, to free the electron.
- i** Has an electron with the kinetic energy you calculated in **e** enough energy to escape from a distance of  $1.0 \times 10^{-10}$  m from a proton?

- 44(L)** Suppose an electron were confined to a space ten times smaller,  $0.1 \times 10^{-10}$  m.

- a** By what factors will the wavelength, momentum, and kinetic energy differ from those calculated in question **43**? For each quantity say whether it is larger or smaller.

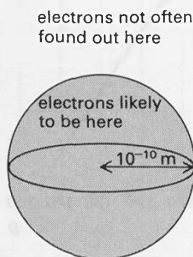


Figure L37



- b** How much energy must be transferred to electrical potential energy to remove an electron from a distance  $0.1 \times 10^{-10} \text{ m}$  from a proton?
- c** Compare your answer to **b** with the kinetic energy of an electron in this space. Would the electrical attraction be enough to keep an electron in this space?

**45(L)a** Imagine an atom ten times larger, *i.e.*, with radius  $10 \times 10^{-10} \text{ m}$ . By what factors will the electron's kinetic energy and the electrical potential energy change? Apply these factors to the values calculated for  $1.0 \times 10^{-10} \text{ m}$ . Remembering that the electrical potential energy is *negative*, calculate the total energy at  $10 \times 10^{-10} \text{ m}$ .

Since the tendency is to the lowest energy you should be able to say what would happen to a  $10 \times 10^{-10} \text{ m}$  atom.

- b** Apply the same line of reasoning to a ten times smaller atom,  $0.1 \times 10^{-10} \text{ m}$ . You should find that the total energy at this size is greater than at  $1.0 \times 10^{-10} \text{ m}$ .

In fact these calculations, though crude, suggest that the atom has minimum energy when its size is about  $1.0 \times 10^{-10} \text{ m}$ .

**46(L)** This question presents an alternative argument, based on calculus, for the size of an atom.

If one loop of an electron standing wave fits into an atom of radius  $r$ , then the wavelength  $\lambda = 4r$ .

- a** What is the momentum of this electron?
- b** What is its kinetic energy?
- c** What is the potential energy when the electron is at distance  $r$  from a proton?
- d** What is the total energy,  $E$ , of the atom?
- e** The total energy will be a minimum when  $dE/dr = 0$ . Work out  $dE/dr$ .
- f** Calculate the value of  $r$  for which  $dE/dr = 0$ .

### Particles in the nucleus?

**47(L)** An atomic nucleus is about  $10^{-14} \text{ m}$  in size.

- a** Calculate the kinetic energy of an electron with  $\lambda \approx 10^{-14} \text{ m}$ .
- b** Calculate the electrical potential energy for a proton and electron  $10^{-14} \text{ m}$  apart.
- c** Is the electrical attraction sufficient to keep the electron within the nucleus?

- 48(L)a** Find the least kinetic energy that a neutron or proton must have if it is to be contained in a space such that  $\lambda \approx 10^{-14}$  m.
- b** Protons and neutrons are bound together within the nucleus by the 'strong' nuclear force. Because this force only operates over very short distances, it is not so well understood as the electrical force. But what can you say about the potential energy due to this force, from your calculation of the nuclear particle's kinetic energy?

### A better model for the hydrogen atom

- 49(P)** Figure L38 shows two cords, one heavy and one light tied end to end. One end is fixed, the other is vibrated.

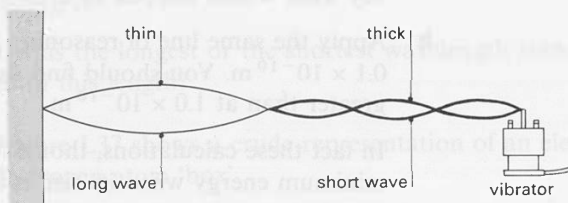


Figure L38

- a** Do both cords vibrate with the same frequency?
- b** On which cord does the wave travel more slowly?
- c** Why does it travel more slowly on this part of the cord?

- 50(R)a** Figure L39 shows a standing wave in a long narrow spring which is vibrating from side to side on a smooth horizontal surface.

What factors together determine the frequency of the vibrations?



Figure L39

- b** Figure L40 shows a standing wave in the same spring when it is arranged vertically between two supports; the speed of propagation of the wave, and hence the wavelength, is now decreasing towards the lower end. Why does the speed decrease towards the lower end?
- c** Suggest one similarity and one difference between the standing waves shown in figure L40 and electron standing waves in an atom.



Figure L40

(Short answer paper, 1979)

- 51(L)** Parts (a)–(d) in figure L41 show some possible waves of varying wavelength 'fitted into' the space between  $r = 0$  and the 'edge' of the atom. Which of the waves shown in (a)–(d) have the right kind of variation of wavelength with radius?

(Remember how kinetic energy varies with distance from the centre of the atom, and how wavelength depends on kinetic energy.)

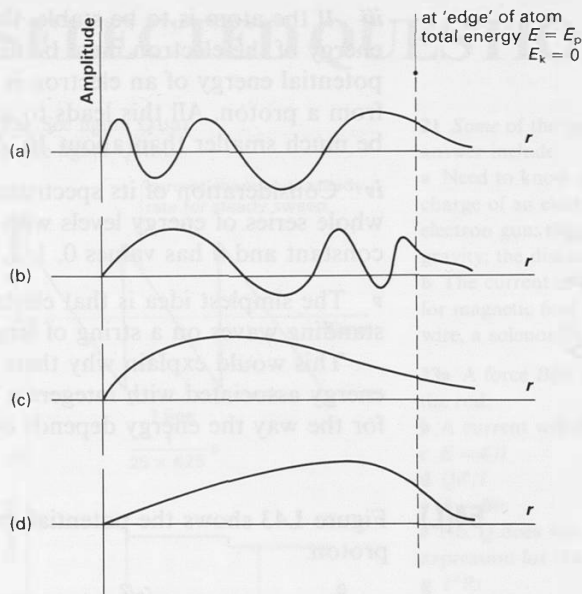


Figure L41

**52(R)** Which of the following sets of ideas are needed to provide a theory which can account for the size and energy of a hydrogen atom?

- 1 The electrical energy due to forces between charged particles.
- 2 The wavelength associated with moving particles (de Broglie waves).
- 3 The mechanics of the motion of massive particles (*e.g.* momentum, kinetic energy).

A 1 only    B 1 and 2 only    C 2 and 3 only  
D 1 and 3 only    E 1, 2, and 3.

(Coded answer paper, 1978)

**53(R)** This question is about the ideas and evidence in the wave theory of atoms.

The passage below consists of numbered statements *i* to *v*. For each of these you are asked to give arguments which *explain* and *support* them.

The arguments that you use may be of many kinds and you should indicate what kind they are – for example theories, models, calculations, evidence, etc.

#### Passage

*i* It can be shown that electrons have wave properties, having a wavelength  $\lambda$  related to their momentum by  $mv = h/\lambda$ .

*ii* If a wave-like electron is confined in a 'box' of size  $10^{-10}$  m, its momentum can't be less than a certain size, and so its kinetic energy has a lower limit too. The smaller the box, the bigger the kinetic energy of the electrons.

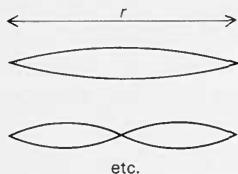


Figure L42

*iii* If the atom is to be stable, the sum of potential and kinetic energy of the electron must be negative. We know that the electrical potential energy of an electron is  $-10\text{eV}$  when it is about  $10^{-10}\text{m}$  from a proton. All this leads to an explanation of why atoms cannot be much smaller than about  $10^{-10}\text{m}$ .

*iv* Consideration of its spectrum shows that a hydrogen atom has a whole series of energy levels with energies given by  $c/n^2$ , where  $c$  is a constant and  $n$  has values  $0, 1, 2, 3, \dots$ , etc.

*v* The simplest idea is that electrons in hydrogen behave like standing waves on a string of length  $r$  (figure L42).

This would explain why there are discrete values of kinetic energy associated with integers  $n$  but it would give the wrong rule for the way the energy depends on  $n$ .

(Long answer paper, 1980)

**54(L)** Figure L43 shows the potential energy curve for an electron near a proton.

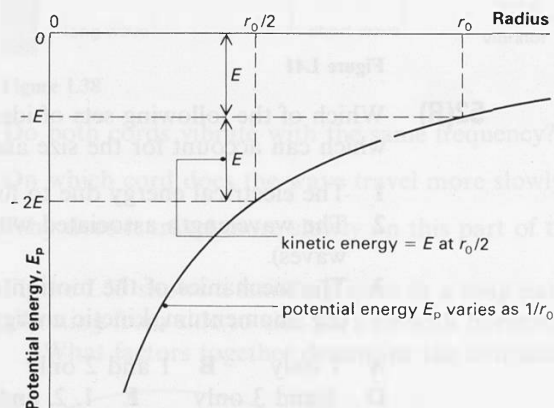


Figure L43

The total energy,  $E$ , of the atom is constant and is a negative quantity. It is the sum of potential and kinetic energies. At the edge of the atom,  $r_0$ , the kinetic energy is zero, and the potential energy  $E_p = -E$ . At  $r_0/2$   $E_p$  is  $-2E$  (from the  $1/r$  rule). For the total energy to be constant kinetic energy  $= E - E_p = -E + 2E = E$ . We will use this value of the kinetic energy at  $r_0/2$  as the average value of kinetic energy between  $0$  and  $r_0$ .

- Average kinetic energy  $\propto E$ . How does average momentum depend on  $E$ ?
- How does average wavelength depend on  $E$ ?
- How does total energy  $E$  depend on the size of the atom,  $r_0$ ? (Remember that at the 'edge' of the atom  $E$  is the potential energy  $E_p$ , at that radius, and  $E_p \propto -1/r$ .)
- Suppose  $n$  loops of mean wavelength  $\lambda$  fit into an atom of radius  $r_0$ . How does  $n$  depend on  $\lambda$  and  $r$ ?
- Use your answer to **b** and **c** in **d** to find a relationship between  $E$  and  $n$ .

# ANSWERS TO SELECTED QUESTIONS

## UNIT H

### Magnetic fields and a.c.

- 1a Perpendicular to both  $x$  and  $y$  axes.  
b A brick, a box, a die, a lump of sugar.

- 3a  $0.04\text{ T}$   
b  $0.002\text{ N}$   
c The force is  $T$  newtons per metre.

- 4a  $0.004\text{ N m}$   
b  $0.004\text{ N m}$   
c 0 (the forces are in the plane of the coil and tend to push the sides in or out, not to turn the coil).

- 5a  $1300\text{ A}$   
b This is an enormous current. It would melt the wire. A thicker wire would need a greater current and would get hotter. An aluminium wire (lower density) would need less current. Note that the current would have to flow at all times, and because the Earth's magnetic field has a vertical component there would also be a horizontal force on the wire.

- 7a  $Ne$   
b  $Ne/t$   
c  $vt$   
d  $F = BNe v$   
e  $Be v$   
f  $BQv$

- 9a  $BQv$   
b Towards the front edge.  
c The moving charge carriers are pushed towards the front edge, so the density there will build up. It will continue to increase until there is an equally strong electrical force in the opposite direction, i.e., from front to back.  
d The electric force ( $EQ$ ) must be equal in magnitude to the magnetic force,  $BQv$ . So the electric field  $E = Bv$ .  
e Front edge negative, rear edge positive.  
f  $V = Ed = Bvd$   
g  $bd$   
h  $n = BI/VbQ$

- 11a  $eV$   
b  $v = \sqrt{2eV/m}$   
c  $F = Be v$ ; at right angles to  $v$  and  $B$ .  
d  $a = Be v/m$   
e No  
f  $e/m = 2V/B^2 r^2$

13a See figure Q1(a).

b See figure Q1(b).

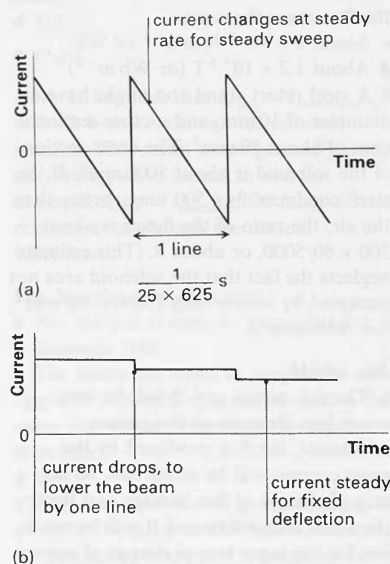


Figure Q1

- (a) Current in coils for horizontal deflection.  
(b) Current in coils for vertical deflection.

The field giving horizontal deflection must vary at a steady rate as the spot sweeps across the screen at a steady speed. If the middle of the screen corresponds to zero deflection and zero field, the current must vary in sign. The steps shown in the vertical deflection current are there to keep the spot at a fixed height for the whole of each line, dropping this height by one line spacing every time a line is completed. In practice, a very slowly changing current is used. Each line on the screen is tilted by an angle of about  $1/625$  radian. Such a steadily changing current would be much easier to produce than a series of small sharp steps.

- 17a The field is  $k\text{ N A}^{-1}\text{ m}^{-1}$   
b  $z$   
c The force is  $k$  newtons  
d  $x$   
e  $2 \times 10^{-7}\text{ N A}^{-2}$   
f  $4\pi \times 10^{-7}\text{ N A}^{-2}$

- 19a  $1.5 \times 10^{-3}\text{ T}$   
b  $0.75 \times 10^{-3}\text{ T}$

21 Some of the points to be made in an answer include:

- a Need to know or estimate the mass and charge of an electron; the p.d. of the electron gun; the acceleration due to gravity; the distance from gun to screen.  
b The current in the wire; the formula for magnetic field strength (near a straight wire, a solenoid?); the value of  $\mu_0$ .

23a A force  $BQv$  parallel to the length of the rod.

- b A current will flow.  
c  $E = \mathcal{E}/l$   
d  $Q\mathcal{E}/l$   
e  $\mathcal{E} = Blv$   
f No;  $Q$  does not appear in the expression for the e.m.f.  
g  $I^2 R t$   
h No. The force  $BIl$  must act against the motion of the rod. If it helped the motion one would only have to start the rod moving for it to go faster and faster, gaining energy without drawing on any supply.

- i  $vt$   
j  $BIlt$   
k  $IR = Blv$ . Since  $IR$  is the p.d. across the resistor, which must be equal to the induced e.m.f.  $\mathcal{E} = Blv$ . Notice that the argument of steps g to k, which was about energy, has given the same result as the argument about force in steps a to e.  
l No. Charges will be pushed to one end of both conductors as they move together through the  $B$ -field, but after this initial movement of charge, the rod and wire travel along with one end of the pair positively charged with respect to the other end, but with no current circulating.

26a  $I = V/R$

b See figure Q2.

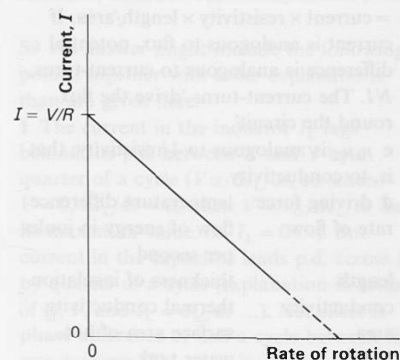


Figure Q2

**c** The torque will fall as the rate of rotation increases.

**d** The rate of rotation must fall. If the motor is not producing enough torque to turn the larger load, it will slow down. As it does so, the induced e.m.f. in its rotor falls. The constant supply p.d. can now drive a larger current through the rotor, so the current rises. It rises until the motor has slowed down enough to deliver sufficient torque to turn the load. In this way, an electric motor is a self-adjusting device.

**27a** As long as the motor is turning, an e.m.f. is induced in the rotor and the current in it is given by  $V - \mathcal{E} = IR$ . The applied p.d.,  $V$ , is constant at 5 V, so  $I$  is always less than  $V/R = 5 \text{ A}$ , the value it would have if the rotor were stationary and  $\mathcal{E} = 0$ . The faster the rotor turns, the larger  $\mathcal{E}$  and the smaller  $I$  become.

**b** The field,  $B$ , in which the rotor turns is constant, so the torque is proportional to the rotor current. The larger the load, the larger the current must be. The motor slows down as much as is necessary for this larger current to flow, if that is possible without it stalling.

**c** Raise the p.d. applied to the rotor, so that it exceeds the induced e.m.f. at 27 revolutions per second (about 4 V) by as much as is needed to drive 3.1 A through  $1 \Omega$  (3.1 V). This assumes that the torque required remains the same, and so, since the torque is proportional to the current, that the current will remain 3.1 A.

**28a**  $B = 0.75 \times 10^{-3} \text{ T}$ ;  $\Phi = 3.7 \times 10^{-6} \text{ Wb}$

**b**  $B = 0.75 \times 10^{-3} \text{ T}$ ;  $\Phi = 1.9 \times 10^{-6} \text{ Wb}$

**c**  $B = 0.75 \times 10^{-3} \text{ T}$ ;  $\Phi = 6.3 \times 10^{-6} \text{ Wb}$

**31a** Resistance = resistivity  $\times$  length/area.

**b** Potential difference = current  $\times$  resistance = current  $\times$  resistivity  $\times$  length/area. If current is analogous to flux, potential difference is analogous to current-turns,  $NI$ . The current-turns 'drive the flux round the circuit'.

**c**  $\mu_r \mu_0$  is analogous to  $1/\text{resistivity}$ ; that is, to conductivity.

**d** driving force rate of flow temperature difference flow of energy in joules per second

length thickness of insulation  
conductivity thermal conductivity  
area surface area of hot water tank

**e** Reluctance corresponds to resistance.

**32a** About  $5 \times 10^7 \text{ A}^2 \text{ m}^{-1} \text{ N}^{-1}$

**b** If  $l$  is doubled (keeping the same number of turns per metre), then so is  $N$ , the total number of turns. But doubling  $l$  also doubles the reluctance. Since both current-turns and reluctance are doubled, the flux stays the same.

**c** About  $6 \times 10^{-6} \text{ N mA}^{-1}$  or  $\text{Wb}$

**d** About  $1.2 \times 10^{-3} \text{ T}$  (or  $\text{Wb m}^{-2}$ )

**e** A steel retort stand rod might have a diameter of 10 mm, and a cross-sectional area of about  $80 \text{ mm}^2$ . The cross-section of the solenoid is about  $5000 \text{ mm}^2$ . If the steel 'conducts' flux 500 times better than the air, the ratio of the fluxes is about  $500 \times 80/5000$ , or about 8. (This estimate neglects the fact that the solenoid area not occupied by iron is only  $(5000 - 80) \text{ mm}^2$ , not  $5000 \text{ mm}^2$ .)

**36a** 0.05 H

**b** The p.d. across coil B will be very much less. Because of the greater reluctance, the flux produced by the same current will be much less. So the rate of change of flux linkage and hence the e.m.f. induced in coil B will be much less for the same rate of change of current.

**c** The p.d. across coil B will be doubled: having twice as many turns in A will double the flux for the same current. The rate of change of flux is doubled and so therefore is the induced e.m.f.

**d** Doubling the number of turns in coil B doubles the flux linkage. The rate of change of flux linkage doubles, and so therefore does the induced e.m.f.

**e** Because the flux is decreasing the p.d. across B will have the opposite sign. Because the rate of change of current, and therefore of flux and of flux linkage, has half its previous value the magnitude of the p.d. across coil B will be 0.75 V.

**37a** The p.d. across R is only 0.05 V when the current is 0.5 A, so the p.d. across L is very nearly 2 V. The rate of rise of current is  $50 \text{ A s}^{-1}$ , so the inductance is  $2/50 \text{ H}$ , or  $0.04 \text{ H}$ , approximately.

**b** By connecting an oscilloscope across R.

**c** The p.d. across R when the current is 0.5 A is still only 0.1 V, so that the p.d. across L is still not much under 2 V. The rate of rise of current will be much the same.

**d** The initial rate of rise of current will be the same,  $50 \text{ A s}^{-1}$ . When the current is 0.5 A, the p.d. across R is 1 V, leaving only 1 V across L. The rate of rise of current must now be about half what it

was when there was nearly 2 V across L, being now about  $25 \text{ A s}^{-1}$ .

**43**

Number of turns on primary, P	Number of turns on secondary, S	Brightness of lamp, L	Alternating p.d. across S/volts
20	50	normal	2.5
50	20	dim	0.4
20	30	dim	1.5
40	100	normal	2.5
20	80	bright	4.0

**Table Q1**

Note that, if the coils are wound from fine wire, it may well be that the resistance of a coil with many turns is important, and the lamp may then not light as it would if the coil's resistance were negligible.

**44a** The flux through the secondary coil must be pretty much the same wherever the coil is on the core. The greater proportion of the flux must circle round within the core, with little flux passing through the air.

**b** The flux has to cross the air gaps. The system behaves for flux as an electric circuit behaves for current. The air gaps correspond to lengths of high-resistivity material inserted into a circuit of thick copper wire. The current then depends more on the high-resistance part of the circuit than on the low-resistance part. Similarly, a small air gap in an iron-filled 'magnetic circuit' reduces the flux in the 'circuit' by a large factor.

**c** The result **b** suggests that the flux is determined by the whole 'magnetic circuit'. If there are air gaps, the flux has a low value right round the circuit. (Similarly, one would have a low current in a thick copper bar if somewhere else in the electric circuit there were a high resistance.)

**d** The flux links each coil, and each has an e.m.f. induced in it. Current can be drawn from each, a corresponding current flowing in the single primary. That each coil can light a lamp is no stranger than that three times as much current could be drawn from one coil.

**e** The primary current should be about three times larger.

**47** Suggested demonstrations using small laminated C-cores and 120 + 120 turn coils might include:

Connect primary to a d.c. source (e.g. 1.5 V cell), then to an a.c. source (2 V, 50 Hz) to

show that transformers work only for a.c., not for d.c. (Take care that current in primary is not large enough to cause overheating on d.c.) Use 1.5 V lamp on secondary.

Show effect of turns ratio: for given a.c. input (say 2 V) and about 10 primary turns wound from insulated wire, increase secondary turns until lamp lights.

Show effect of using 2 V, 4 V, 6 V, ..., a.c. input on output voltage with, say, 120-turn coils as primary and secondary, using a.c. voltmeters to measure  $V_p$ ,  $V_s$ ; hence establish that  $V_s/V_p \approx N_s/N_p$ ; show use as step-up and step-down transformer.

Measure current in primary and secondary coils (a.c. meter 1 or 5 A) to show that although transformer can be used to step up voltage there is no gain in power, and in fact power out is slightly less than power in.

Increase load on secondary (perhaps by starting with no load on secondary, then connecting one lamp, then two or more lamps in parallel) to show that current in primary rises as current in secondary rises.

Show importance of magnetic circuit linking the two coils by making a small, then a large, air gap between two C-cores, and/or show effect of no iron core linking the two coils.

Point out that iron core is laminated – to reduce losses due to eddy currents.

Show that these losses become more important at high frequencies (signal generator, oscilloscope).

**48a**  $P/V$

**b**  $I^2 R = (P/V)^2 R$

**c** Reduce  $I$  (i.e. increase  $V$ ); reduce  $R$ .

**d** a.c. can be stepped up and down using transformers; d.c. cannot. Although power should be transmitted at high voltage to reduce losses, generators do not produce such high-voltage a.c., nor do consumers use it.

**e** 3:1

**f** 1:9

**g** Nine times more for the 132 kV system.

**h** Insulators, e.g. between the conductor and the supporting tower, must be larger. Also clearances between conductors and surroundings must be greater.

**49ai**  $I^2 R$

**ii**  $I^2 R/2$

**iii**  $I^2 R$

**b**  $I/\sqrt{2}$

**c** Its square is the average, or mean, of the square of the varying current. So it is the root of the mean of the square of the varying current. The usual abbreviation is r.m.s.

**d**  $1/2$

**e**  $I/\sqrt{2}$

*Note:* It happens that  $I_{r.m.s.} = I_{\text{maximum}}/\sqrt{2}$  in both **b** and **e**, though for different reasons. This result is *not* generally true for all varying wave forms. In **aiii**, for example,  $I_{r.m.s.} = I_{\text{maximum}}$  for fairly obvious reasons, even though the current 'alternates'.

**51a** No; the current is zero.

**b** No; the p.d. is zero, so the coil is not in a magnetic field.

**c** The forces are equal in magnitude and opposite in direction as the current is the same size at both times but reverses its direction.

**d** Zero

**e** The wattmeter would give a reading. It must, as the lamp would light and would be dissipating energy. If it does, the current and p.d. across the two together can no longer have the phase relationship shown in figure H103, for that phase relationship gives zero average force on the wattmeter coil. In fact, if the lamp is the only component in the circuit, the current and p.d. will be in phase.

**53a** The same maximum current would have to be attained from zero in half the time, so the rate of change of current would have doubled.

**b** The time for the change of current has halved; to keep the rate of change constant, the maximum current must have halved, so that half the change happens in half the time, at the same rate.

**c** The maximum output p.d. is the maximum p.d. across  $R$ , which is  $IR$  if  $I$  is the maximum current. As the latter has halved, so has the output p.d. Of course this only works out so simply if  $R$  is very small, so that  $IR$  is not an appreciable part of the input p.d. Nevertheless, the output as a fraction of the input always falls as the frequency is raised in this circuit.

**d** Less of a high frequency input appears across the output, than of a low frequency. The circuit acts as a 'filter' which only passes lower frequency signals to the next stage. The circuit is used to discourage unwanted mains frequency voltages from mixing with steady (zero frequency)

voltages in devices which draw their power from the a.c. mains.

**55a** The p.d. across the coil is proportional to the rate of change of current,  $dI/dt$ . When  $I$  has its maximum value  $dI/dt$  is zero.

**bi** OA, BC

**ii** AB, CD

When the product  $VI$  is positive the source is supplying energy; when this product is negative it is receiving energy.

The energy comes from the magnetic field set up by the current in the coil.

**56a** Stored in the springs.

**b** Stored in the capacitor.

**c** The force exerted by the springs and the mass of the trolley.

**d** The p.d. across the capacitor and the inductance of the coil.

**e** The velocity is momentarily constant, because there is no net force on the trolley.

**f** The current is momentarily constant, because there is no p.d. across the coil.

**g** The springs begin to exert a force, slowing down the trolley.

**h** A p.d. builds up across the capacitor, and the current decreases.

**i** The system has as much energy as before, stored in the springs again. If energy is conserved, one could infer that the trolley had energy equal to the total energy, when it was in the centre position, and the undisplaced springs had no energy. This is, of course, the trolley's kinetic energy,  $\frac{1}{2}mv^2$ .

**j** The system has as much energy as before, stored in the capacitor again. If energy is conserved, one could infer that the inductor had energy equal to the total energy, when the capacitor was uncharged and had no energy. The energy of an inductor is the electrical analogue of kinetic energy, and is given by  $\frac{1}{2}LI^2$ .

**59** An answer might include the following points, together with fuller explanations than are given here.

**1** The current in the inductor  $I_L$  lags behind the p.d. between X and Y by a quarter of a cycle ( $V \propto dI_L/dt$ , so when  $V = 0$ ,  $dI_L/dt = 0$ ; when  $V = V_0$ ,  $dI_L/dt$  has its maximum value, i.e.  $I_L = 0 \dots$ ). But current in the capacitor leads p.d. across it by quarter of a cycle (explanation in terms of  $Q$ ,  $V$ , and  $I_C = dQ/dt \dots$ ). Net effect is phase difference of half a cycle between the two currents. Thus one is positive when the other is negative, and vice versa:



charge circulates around the  $LC$  circuit first in one direction then in the other, then ...

**2** Values of  $I_L$  and  $I_C$  depend on values of  $L$ ,  $C$ , and frequency  $f$ .  $I_C \propto f$ ;  $I_L \propto 1/f$ . At resonant frequency  $I_L = I_C$  in magnitude. Because these currents are in opposite directions charge circulates around the  $LC$  circuit first in one direction, then in the other, without much charge entering or leaving the circuit at  $X$  or  $Y$ . If frequency is increased  $I_L$  decreases,  $I_C$  increases: current is then drawn from the supply to 'make up the difference'.

**3** When  $C$  is fully charged the current in the  $LC$  circuit is momentarily zero (at resonant frequency). Energy ( $= \frac{1}{2} CV^2$ ) is stored in the electric field between capacitor plates (*cf.* trolley at rest with spring fully compressed). The capacitor discharges through the inductor, current grows (*cf.* spring pushes trolley which accelerates). Current in the inductor causes a magnetic field around it which can store energy (*cf.* kinetic energy of moving trolley). The capacitor charges up in the opposite sense ...

## UNIT 1

### Linear electronics, feedback and control

**1a**  $V \times \frac{R_2}{R_1 + R_2}$

- b**  $\frac{2}{3} V$   
**c**  $\frac{2}{3} mA$   
**d**  $\frac{2}{3} V$ ,  $0.067 mA$

**2**  $1.5 V$

**3a**  $V_1 + V_2$

**b**  $\frac{R_1}{R_2}$

**c** **a** Doubled, **b** the same.

- 4a**  $0.2 mA$   
**b**  $0.2 mA$   
**c**  $20 V$   
**d**  $X$   
**e** Negative.  
**f** Equal resistances.

- 5a**  $20 \mu C$   
**b**  $100 \mu C$

- 6a**  $0.5 mC$   
**b**  $0.67 \times 10^{-3} F$   
**c** Very high resistance voltmeter (*e.g.*

electrometer), or oscilloscope. (Charge would leak away through a normal voltmeter.)

**7a** Exponential decay curve.

- b** Current  $\propto$  slope of first graph.  
**c** Linear rise.

**13a** 20, 26, 32, 38.

- b**  $10^5$   
**c**  $2 \times 10^5$

- 15a**  $0 V$   
**b**  $0.2 V$   
**c**  $2 \times 10^{-5} A$   
**d**  $2 \times 10^{-5} A$   
**e**  $-2 V$   
**f** Negative  
**g**  $2 \times 10^{-6} A$   
**h**  $2 V$

**16e**  $V_{out}/V_{in} = -R_f/R_{in}$

- 18a**  $-1 mV$   
**b**  $1001 mV$   
**c**  $1001 mV$   
**d**  $-1002 mV$   
**e**  $+2000 mV$   
**f**  $500 mV$ ,  $+499 mV$ .  
**g** Opposite direction.  
**h** Much too large.  
**i**  $0.5 mV$  rise.  
**j**  $-0.5 mV$   
**k**  $+1000 mV$ ; nearly.

**19a**  $0 V$

- b**  $3 V$   
**c**  $3 \times 10^{-5} A$   
**d**  $5 \times 10^{-5} A$   
**e**  $8 \times 10^{-5} A$   
**f**  $8 V$   
**g**  $-8 V$   
**h**  $V_{out} = -(V_A + V_B)$   
**i**  $V_{out} = -(2V_A + V_B)$   
**j**  $-7 V$   
**k**  $V_{out} = -2(V_A + V_B)$ ;  $V_{out} = -1.6 V$ .  
**l**  $V_{out} = -10(V_A + V_B)$ ;  $V_{out} = -8 V$ .

**21a**  $R_C = 5R_f$

- b**  $R_B = \frac{5}{2} R_f$   
**c**  $R_A = \frac{5}{4} R_f$   
**d** *e.g.*  $R_C = 40 k\Omega$ ,  $R_B = 20 k\Omega$ ,  $R_A = 10 k\Omega$ ,  $R_f = 8 k\Omega$   
**e** Assume 63 will be represented by  $12.6 V$ , then resistors  $R_A \dots R_f$  should be in the ratios  $1: \frac{1}{2}: \frac{1}{4}: \frac{1}{8}: \frac{1}{16}: \frac{1}{32}: 2$ .

**22a** Ramp and analogue negative terminal to comparator 1; ramp and analogue positive terminal to comparator 2;

comparators to bistable inputs  $A$  and  $B$  respectively; bistable and clock into  $AND$ ;  $AND$  to counter  $1 Mz$  input; comparator 1 to counter input  $P$ .

**b** Display goes to zero at start of each count and begins to count up. If voltage was near level of top of ramp, the reading would be displayed for negligible time. Either add (unspecified) circuitry to 'hold' (latch) the reading until next reading is ready, or make the ramp generator have long rest spaces between ramps so that display is steady for a large proportion of the time.

**24a**  $1 V$

- b**  $1 \mu A$   
**c** The capacitor.  
**d** Yes  
**e**  $10 \mu C$ ,  $20 \mu C$   
**f**  $0.1 V$ ,  $0.2 V$   
**g** Left hand positive.  
**h** Falling steadily  
**i**  $-0.01 V s^{-1}$   
**j** Fall at  $0.02 V s^{-1}$ .  
 $V_{out}$  moves with uniform gradients from (0 s, 0 V) to (2 s,  $-0.02 V$ ) to (7 s,  $-0.17 V$ ) to (8 s,  $-0.15 V$ )  
**l** All rates of change multiplied by 10,  $V$  values  $0 V$ ,  $-0.2 V$ ,  $-1.7 V$ ,  $-1.5 V$ .

**28a**  $0.2 mA$

- b**  $2 V$   
**c**  $+2 V$   
**d**  $+2 V$   
**e**  $V_{out} = 3V_{in}$   
**f** In place of the 1:2 resistance ratio of the diagram, it would need a 1:9 ratio.  
**g** Straight line runs from (0  $\Omega$ ,  $+6 V$ ) to (30 k $\Omega$ , 0 V).  $X$  is at (20 k $\Omega$ , 2 V).  
**h** All  $V$  values half those in g.

**35ai**  $V_X = (V_{out} + 3)/2$ ,

- ii**  $V_Y = 5/2 V$   
**b**  $V_{out} = 2V$   
**c**  $V_X = (V_{out} + V_A)/2$ ,  $V_Y = V_B/2$ , whence  $V_{out} = V_B - V_A$   
**d** Two intersecting straight lines  $AC$  and  $BD$ .  $A$  is (0  $\Omega$ , 3 V),  $B$  is (0  $\Omega$ , 5 V),  $C$  is (2 M $\Omega$ , 2 V),  $D$  is (2 M $\Omega$ , 0 V), lines intersect at  $X \equiv Y$  (1 M $\Omega$ , 2.5 V)  
**e**  $A$  is (0, 1),  $B$  is (0, 3),  $C$  is (1.5, 4),  $D$  is (1.5, 0), intersecting at  $X \equiv Y$  (0.5, 2);  $V_{out} = 4 V$ .

**39a** They are equal.

- b**  $dV/dt = -(1/CR)V$ ,  $V = V'e^{-t/CR}$  ( $V = V'$  when  $t = 0$ ).  
**c**  $1.84 V$   
**d**  $0.68 V$   
**e**  $2.27 \times 10^{-4} V$



40g  $V = 0.1e^{10t}$   
 i 0.107 s

41b 1600 Hz (2 significant figures).

44ai  $Z/1000$   
 ii  $S - Z/10$   
 b  $Z/S = 9.9(0)$   
 c  $Z/S = 9.99$

d  $G = \frac{1}{\beta + 1/A}$  or  $\frac{A}{A\beta + 1}$

or  $\frac{A/\beta}{A + 1/\beta}$

e  $G = 1/\beta$ : depends only on  $\beta$ .

45 The answers to this question depend upon interpretation of what is being controlled: level, or inflow.  
 Feedback: position of ball controls inflow.  
 Reference signal: setting of ball relative to valve position, or intended water level.  
 Error signal: discrepancy between ball level required to stop flow, and present level.  
 Input: could mean reference signal, or water input.  
 Output: could mean rate of flow into tank, or water outflow.  
 Disturbance: change of water pressure and therefore flow rate, or drop in level when water is being used.

49a

No.	3	4	5	6	7	8	9	10	11
m/g	90	100	110	110	100	90	90	100	110

Table Q2

b Reducing the correction by 50 % will give damped oscillation, but not immediate correction. Moving the sensor one place to the left will give stable correction.

## UNIT J Electromagnetic waves

7 Minima of intensity are detected at 0.30, 0.64 rad, etc.

8b 0.23 m

9 Solving these should not be taken too seriously; it can be quite fun and quite a valuable exercise in detective work. See figure Q3 for some clues.

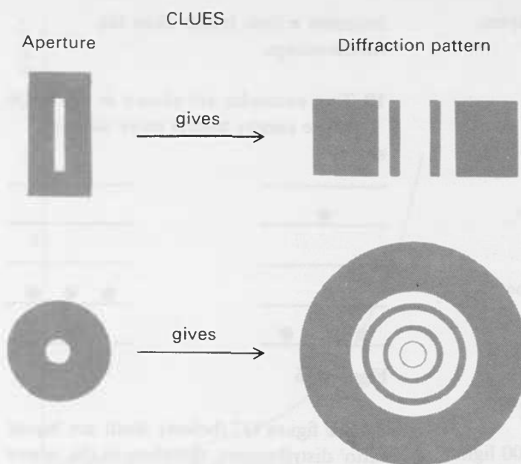


Figure Q3

14a  $\theta = 0.008$  rad  
 b About  $1.6 \times 10^{18}$  m (160 light years).

17a  $0.6 \times 10^{-3}$  rad  
 b  $8 \times 10^{17}$  km  
 c 1.5 km

18b Maxima occur at angles  $\theta$  where  $\sin \theta = \left[ \frac{(2n+1)}{4} \right] 0.015$ , and where  $n = 0, 1, 2, 3$ , etc.

22 About 3400 Hz.

24a  $1.6 \times 10^{-6}$  m  
 b 420 nm

25a  $30^\circ$

26c  $n\lambda = sx_n/L$  or  $\lambda = sx_1/L$ .

27a Nearly  $5 \times 10^{-7}$  m.

28c  $5 \times 10^{-6}$  m

31a  $10^{-4}$  m

32a  $\theta_1 = 24^\circ$   
 b  $\theta_2 = 53^\circ$

35  $c = 2.8(5) \times 10^8$  m s $^{-1}$

37b 0.3 m

38b  $2.5 \times 10^8$  m s $^{-1}$

40a  $5 \times 10^{-15}$  W

43b  $3 \times 10^8$  m s $^{-1}$

## UNIT K Energy and entropy

1a By random collisions, a water molecule gets enough energy to break the bonds holding it to other water molecules or to the fabric, and moves into the air nearby.

If there is a breeze, water molecules are swept away, reducing the number near the wet clothes and so reducing the chance that any will go back into the wet clothes.  
 b Water molecules in ice are bonded in a regular lattice, with permanent neighbours. As the ice melts, a molecule is still closely surrounded by others (the density actually increases) but has shifting neighbours, and itself moves about.

c The water already has tannin and other molecules from the tea dispersed rather uniformly through it. When milk is stirred in fat, casein, and other molecules, previously close together, become dispersed so that there is a roughly equal average distance between molecules of the same kind, and the chance of having a neighbour of a given kind is proportional to the total number of that kind of molecule in the mixture.

d Energy leaves the radiator in two ways, by emission of photons of infra-red radiation, and by giving extra energy to nearby air molecules.

2  $W \approx 8 \times 10^{67}$ ;  $\ln W \approx 156$ ;  
 $k \ln W \approx 2 \times 10^{-21}$  J K $^{-1}$ .  
 $\Delta S = k \ln W - k \ln 1 = 2 \times 10^{-21}$  J K $^{-1}$ .

4a  $10^7$   
 b  $4^{10^7}$   
 c  $1.9 \times 10^{-16}$  J K $^{-1}$

6a Figure Q4 shows two examples.

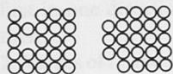


Figure Q4

b  $N$

9  $2^{100} \approx 10^{30}$ ; 1 chance in  $10^{30}$ .

11a  $2^{100} \approx 10^{30}$

b  $\lg 2^{100} = 100 \lg 2 \approx 30$ ; so  $2^{100} \approx 10^{30}$ .

c  $10^{24}$  s

d  $3 \times 10^6$

12a About  $4 \times 10^{-4} \text{ m}^3$  (0.4 litres).

b  $3 \times 10^{21}$

c  $3 \times 10^{21}$  digits;  $3 \times 10^{18}$  m; 300 light years. Betelgeuse (alpha Orionis).

14  $\ln 10 = 2.302$

$\ln 10.5 = 2.351$

$\ln 10.5 - \ln 10 = 0.049$ ; compared with  $0.05 = 0.5/10$

$\ln(x + dx) - \ln x \approx dx/x$  if  $dx \ll x$ .

17a Cycle pump: about 0.2 m long; cross-section about  $2 \times 10^{-4} \text{ m}^2$ ; volume  $\approx 4 \times 10^{-5} \text{ m}^3$ .

b About  $10^{21}$  molecules in  $4 \times 10^{-5} \text{ m}^3$ .

c See figure Q5.

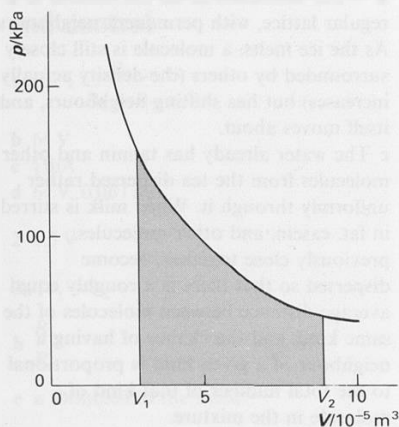


Figure Q5

d Area from  $V_1 = 3 \times 10^{-5} \text{ m}^3$  to  $V_2 = 9 \times 10^{-5} \text{ m}^3 \approx 4.5 \text{ J}$ .

e  $\ln V_1 = -10.41$ ;  $\ln V_2 = -9.32$ ;  $\ln(V_2/V_1) = 1.09$

$\Delta S \approx 1.5 \times 10^{-2} \text{ J K}^{-1}$

f  $T\Delta S \approx 4.5 \text{ J}$

g The gas would become hotter when compressed; its internal energy would increase. To keep the temperature constant, the gas must give energy to the surroundings, which it can do if it briefly

becomes a little hotter than the surroundings.

19 Two examples are shown in figure Q6.

More energy means more ways to share.



Figure Q6

22 See figure Q7 (below). Both are 'equal ratio' distributions, therefore in (b), where more energy is being shared by the same number of particles, more are on higher levels, fewer are on the very lowest level.

23 See table Q3

Level	$f=0.5$ Number	$f=0.25$ Number	$f=0.75$ Number
0	256	256	256
1	128	64	192
2	64	16	144
3	32	4	108
4	16	1	81
5	8		$\approx 61$
6	4	less	$\approx 45$
7	2	than 1	$\approx 34$
8	1		$\approx 26$

Table Q3

25a X is hotter.

b  $W'_X = \frac{2}{3} W_X$

$W'_Y = \frac{3}{1} W_Y$

$W'_X W'_Y = 2 W_X W_Y$

26a  $-9.6 \times 10^{-24} \text{ J K}^{-1}$

b  $-9.6 \times 10^{-22} \text{ J K}^{-1}$

c  $-9.6 \times 10^{-21} \text{ J K}^{-1}$

d  $-9.6 \times 10^{-3} \text{ J K}^{-1}$

e  $-9.6 \text{ J K}^{-1}$

28a  $-\ln x = \ln 1/x$ , so if  $x < 1$ ,  $\ln x$  is negative and  $-\ln x$  is positive.

b  $-2k \ln f$

c  $Q/\varepsilon$

d  $n = Q/\varepsilon$ , so  $\Delta S = -(Q/\varepsilon)k \ln f$ .

If  $\ln f = -\varepsilon/kT$ ,  $\Delta S = Q/T$ .

29a  $f$   $-\ln f$

1	0
0.5	+0.69
0.1	+2.3
0.01	+4.6

Table Q4

b If  $f$  is small,  $\ln f$  is large and negative,  $-\ln f$  is large and positive.

c Increases.

d Infinite.

e Zero.

31a	$E/kT$	$e^{-E/kT}$
1	0.37	
2	0.14	
5	0.0067	
10	$4.5 \times 10^{-5}$	
20	$2.1 \times 10^{-9}$	
50	$1.9 \times 10^{-22}$	
100	$3.7 \times 10^{-44}$	

Table Q5

b 14

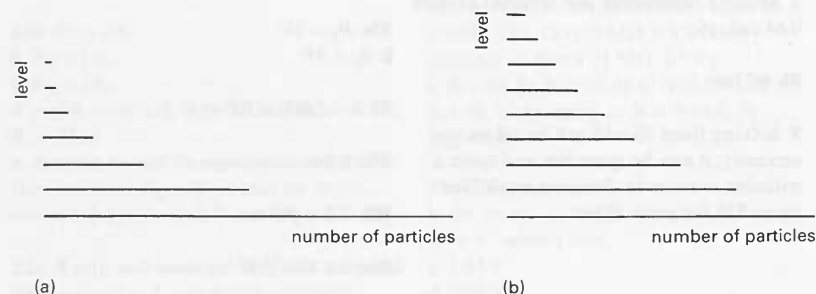


Figure Q7

33 See figure Q8. The Boltzmann factor increases very rapidly with temperature.

34a  $41 \text{ kJ mol}^{-1}$

b  $6.8 \times 10^{-20} \text{ J molecule}^{-1}$

c  $3.4 \times 10^{-20} \text{ J bond}^{-1}$

d  $4.1 \times 10^{-21} \text{ J}; 8.3; 2.5 \times 10^{-4}$ .

36a  $\ln r_1 = \ln C - E/kT_1$

$\ln r_2 = \ln C - E/kT_2$

$$\ln\left(\frac{r_2}{r_1}\right) = \ln r_2 - \ln r_1 = \frac{E}{k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

bi  $1.4 \times 10^{-19} \text{ J}$

ii 4.5

39 The current increases by a factor of about 5 between  $T = 900 \text{ K}$  and  $T = 1000 \text{ K}$ .  $\Phi \approx 2 \times 10^{-19} \text{ J}$ .

40 The bouncing ball passes kinetic energy to the molecules of the ball and floor, making ball and floor warmer, and sharing the energy out amongst many more particles than before.

Mixing a drink means that particles can exchange places with unlike particles in many more ways than before. Dissolving sugar in water is in essence the same.

Air escaping from a tyre occupies a larger volume, so that the molecules can be arranged in many more ways than before.

43a 85 %

b 5.2 %. The temperature difference is maintained by the world climate, and affects large masses of water, so that a substantial amount of renewable energy might be extracted, provided that doing so did not materially affect the temperature gradient.

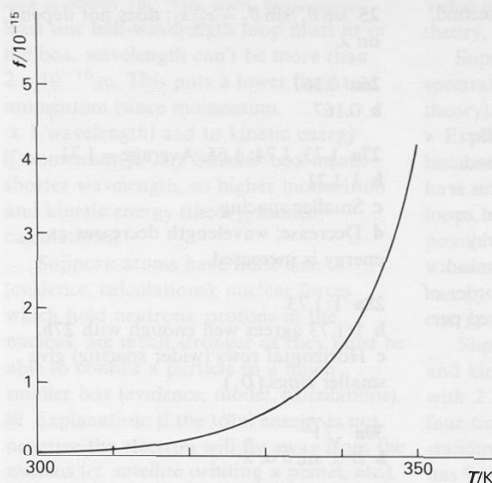


Figure Q8

47b About  $60 \text{ J K}^{-1}$ .

c About 70 %. If there is structure in the liquid and not in the vapour, the entropy increase on evaporation must be larger than that due to the volume change above.

d The entropy change for water is larger (by some  $20 \text{ J K}^{-1}$ ) than for the other liquids. If there is more structure in the water, this might explain some of the excess. Methanol,  $\Delta S = 104 \text{ J K}^{-1}$ , is more like water than the other liquids.

48

a energy of surroundings increase because:

b entropy of surroundings:

c entropy of substance:

d effect of raising temperature:

Steam condensing

bonds form

increases

decreases

more steam

Water freezing

bonds form

increases

decreases

more liquid water

Material more magnetized

molecular magnet aligned along field has less energy  
increases  
decreases  
less magnetized

Table Q7

## UNIT L

### Waves, particles, and atoms

1  $1.6 \times 10^{-19} \text{ J}$

2  $5.8 \times 10^{15} \text{ Hz}$

4  $6.4 \times 10^{-34} \text{ J s}$

6 See table Q8.

	BBC Radio 4	Microwaves	Visible light	Gamma rays
$\lambda$	1500 m	$3 \times 10^{-2} \text{ m}$	$6 \times 10^{-7} \text{ m}$	$1 \times 10^{-12} \text{ m}$
$f$	$2.0 \times 10^5 \text{ Hz}$	$1 \times 10^{10} \text{ Hz}$	$5 \times 10^{14} \text{ Hz}$	$3 \times 10^{20} \text{ Hz}$
$E$	$\begin{cases} 1.3 \times 10^{-28} \text{ J} \\ 8.2 \times 10^{-10} \text{ eV} \end{cases}$	$\begin{cases} 7 \times 10^{-24} \text{ J} \\ 4 \times 10^{-5} \text{ eV} \end{cases}$	$\begin{cases} 3 \times 10^{-19} \text{ J} \\ 2 \text{ eV} \end{cases}$	$\begin{cases} 2 \times 10^{-13} \text{ J} \\ 1 \times 10^6 \text{ eV} \end{cases}$

Table Q8

47a

Liquid	$\Delta H_{\text{evap}} / \text{kJ mol}^{-1}$	$T/\text{K}$	$\Delta S_{\text{evap}} / \text{J K}^{-1} \text{ mol}^{-1}$
carbon disulphide	27.2	319	85.3
trichloro-methane	29.3	335	87.5
hexane	28.8	342	84.2
octane	34.9	399	87.5
sulphuric acid	50.2	617	81.4
water	40.6	373	108.8

Table Q6

8 About  $1.5 \times 10^{20}$  photons per second;  
 $1.5 \times 10^{10}$  photons per cycle of  
oscillation ( $f \approx 10^{10}$  Hz).

9 About  $2 \times 10^7$  W.

The gamma ray source emits far  
fewer than  $10^{20}$  photons in a second.  
(The strength of radioactive sources is  
given in disintegrations per second or  
becquerel (Bq). A school gamma ray  
source may have an activity of around  
0.2 MBq. This means that of the order of  
 $10^5$  gamma ray photons are emitted per  
second.)

11a Energy levels A and B.

b 582 nm

c Emitted.

13 A

16a  $4.567 \times 10^{14}$  Hz

b  $0.458 \times 10^{-18}$  J

17  $1.63 \times 10^{-18}$  J;  $1.94 \times 10^{-18}$  J;  
 $2.04 \times 10^{-18}$  J;  $2.09 \times 10^{-18}$  J. (Note that  
since  $h$  is given to 3 significant figures,  
the energies cannot be given more  
precisely, even though frequencies are  
given to 5 significant figures.)

18a  $-Gm/r$

b  $\sqrt{Gm/r}$

c  $Gm/2r$

d  $-Gm/2r$

19a  $-2.18 \times 10^{-18}$  J

b  $-0.55 \times 10^{-18}$  J;  $-0.24 \times 10^{-18}$  J;  
 $-0.14 \times 10^{-18}$  J;  $-0.09 \times 10^{-18}$  J.

21a 0.3 W

b About  $2.3 \times 10^{16}$  photons per second.

c About one in 50 000 (the area of your  
eye pupil divided by total area of sphere  
0.3 m radius).

d About  $5 \times 10^{11}$  photons enter your eye  
per second; time between photons is  
about  $2 \times 10^{-12}$  seconds.

e About  $6 \times 10^{-4}$  m.

f About 6 m.

22 Behaviour: interference, diffraction,  
photoelectric effect, reflection, refraction,  
polarization.

Models: wave model (superposition  
effects: diffraction, interference,  
polarization); particle model  
(photoelectric effect). Both models can  
explain reflection and refraction.  
Models which refer to objects and  
phenomena we can see aren't adequate  
to explain matter on a very small scale.

25  $\sin \theta_1 / \sin \theta_2 = s_2 / s_1$ ; does not depend  
on  $\lambda$ .

26a 0.167

b 0.167

27a 1.73; 1.74; 1.65. Average = 1.71.

b 1:1.71

c Smaller spacing.

d Decrease; wavelength decreases as  
energy is increased.

28a 1:1.73

b 1:1.73 agrees well enough with 27b.

c Horizontal rows (wider spacing) give  
smaller rings ( $D_1$ ).

30a  $\sqrt{V}$

b  $D \propto \sin \theta \propto \lambda$

31  $D\sqrt{V}$  should be constant. Values are  
410; 427; 413; 424; 436; 419; 435; 429.  
(Since values of  $V$  were estimated from  
the size of a spark gap and so are not  
accurately known, the agreement is quite  
good.)

32a  $0.205 \times 10^{-10}$  m

b  $\frac{1}{2}mv^2 = V_e$

c  $mv = \sqrt{2mV_e}$

d  $h = 7.78 \times 10^{-34}$  J s (the accepted value  
is  $6.6 \times 10^{-34}$  J s).

e Yes; both are J s.

33a Slope is  $\frac{\lambda}{1/\sqrt{V}} = \lambda\sqrt{V} = \frac{h}{mv} \times \sqrt{V}$   
 $= \frac{h}{\sqrt{2mV_e}} \times \sqrt{V} = \frac{h}{\sqrt{2me}}$

b From the graph the slope is approxi-  
mately  $2 \times 10^{-10}/0.15 = 13 \times 10^{-10}$ .

So  $h \approx 13 \times 10^{-10} \times \sqrt{2me}$   
 $\approx 7.1 \times 10^{-34}$  J s.

35  $\lambda \approx 10^{-34}$  m

37a Wavelengths of 420 MeV and  
183 MeV electrons are  $2.96 \times 10^{-15}$  m  
and  $6.79 \times 10^{-15}$  m respectively.

b Estimates of nuclear diameters:

C  $4.7 \times 10^{-15}$  m

O  $5.2 \times 10^{-15}$  m

V  $9.1 \times 10^{-15}$  m

39 Points to make include:

Mistake or ambiguity: they still behave  
as 'very tiny billiard balls', and as waves  
as well.

Explanation: neither model complete;  
need both ideas to understand behaviour  
of matter on a very small scale. Most  
scientists accept the need for two models,

but some are unhappy with this and  
search for 'a better idea'.

40 Points to make include:

Light: some behaviour (superposition)  
explained on wave model; other  
behaviour (photoelectric effect) needs  
particle model.

Particles: small particles (e.g.  
electrons, but also neutrons, protons,  
etc.) show wave-like behaviour (electron,  
neutron diffraction).  
Wavelength  $\ll$  wavelength of light.

Duality: neither model complete –  
need both, depending on behaviour to be  
'explained'.

43a  $4 \times 10^{-10}$  m

b Largest

c  $1.65 \times 10^{-24}$  kg m s $^{-1}$

d No

e  $1.5 \times 10^{-18}$  J

f Minimum.

g Electrical attraction between proton  
and electron.

h  $2.3 \times 10^{-18}$  J

i No

44a Wavelength: ten times smaller;  
momentum: ten times larger; kinetic  
energy: hundred times larger.

b  $23 \times 10^{-18}$  J

c No, because kinetic energy would be  
 $150 \times 10^{-18}$  J.

45a Electron's kinetic energy will decrease  
by factor of 100; electrical potential energy  
will decrease in magnitude by factor of 10.  
Total energy  
 $= 0.015 \times 10^{-18}$  J  $- 0.23 \times 10^{-18}$  J  
 $= -0.215 \times 10^{-18}$  J.

Total energy is more (i.e. a smaller  
negative quantity) than for the atoms with  
radius  $1 \times 10^{-10}$  m.

b Total energy (for atom with radius  
 $0.1 \times 10^{-10}$  m) is  $(150 \times 10^{-18})$  J  $-$   
 $(23 \times 10^{-18})$  J =  $127 \times 10^{-18}$  J.

46a  $\frac{h}{4r}$

b  $\frac{h^2}{32mr^2}$

c  $-\frac{e^2}{4\pi\epsilon_0 r}$

d  $E = \frac{h^2}{32mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$

e  $\frac{dE}{dr} = -\frac{h^2}{16mr^3} + \frac{e^2}{4\pi\epsilon_0 r^2}$

$$f \quad r = \frac{4\pi\epsilon_0 h^2}{16me^2}$$

$$= 1.3 \times 10^{-10} \text{ m}$$

**50a** Tension in the spring; mass per unit length of the spring.

**b** Speed decreases because tension is less near the lower end (less weight to support).

**c** Similarity: electron standing waves in atoms have variable wavelength.

Difference: electron standing wave in atom shows chance of finding electron, it does not represent displacement of a material object like a spring.

**51 (c)**

**52 E**

**53i** Explanation: some properties of electrons are only understood by thinking of them as waves, but this does not mean they *are* waves.

Support: electron diffraction (experimental evidence); success of wave mechanics in explaining, for example, energy levels of hydrogen atom (theory, evidence).

**ii** Explanation: electron confined to a box must be a standing wave (model). If size of

box is about  $10^{-10}$  m, then, because at least one half-wavelength loop must fit in the box, wavelength can't be more than  $2 \times 10^{-10}$  m. This puts a lower limit to momentum (since momentum  $\propto 1/\text{wavelength}$ ) and to kinetic energy ( $[\text{momentum}]^2/2m$ ). Smaller box means shorter wavelength, so higher momentum and kinetic energy (theory, model, calculations).

Support: atoms have finite size (evidence, calculations); nuclear forces which hold neutrons, protons in the nucleus, are much stronger as they must be able to confine a particle to a much smaller box (evidence, model, calculations).

**iii** Explanation: if the total energy is not negative the electron will fly away from the nucleus (*cf.* satellite orbiting a planet, etc.). Wave mechanics shows that kinetic energy of electron exceeds 10 eV if it is confined within a space much smaller than  $10^{-10}$  m (theory, model, calculation).

Support: calculations of this kind correctly predict the size of atoms (calculations, evidence).

**iv** Explanation: atoms cannot have any arbitrary amount of energy: only specific, well-defined amounts called energy levels are allowed. The values of energy which the atom can have are  $c/1, c/4, c/9, \dots, c/n^2$ .

Value of  $c$ :  $-2.18 \times 10^{-18}$  J (evidence, theory, calculations).

Support: measurement of frequencies of spectral lines; (evidence, calculations, theory).

**v** Explanation: electron is standing wave because it is confined. Standing wave must have a whole number of half-wavelength loops, so only certain wavelengths are possible. Since kinetic energy depends on wavelength, only discrete values of kinetic energy are possible. This model predicts energy  $\propto n^2$ ; in fact energy  $\propto 1/n^2$ .

Support: momentum  $\propto 1/\text{wavelength}$ , and kinetic energy  $\propto (\text{momentum})^2$ . Wave with 2 loops has half the wavelength, so four times the kinetic energy of one-loop standing wave. Three-loop standing wave has 9 times, etc. The  $1/n^2$  rule arises because potential as well as kinetic energies contribute to total energy of atom; also because the standing waves with more loops occupy more space (model, theory, and evidence.)

**54a** Average momentum  $= \sqrt{2mE}$

**b** Average wavelength  $= h/\sqrt{2mE}$

**c**  $E \propto -1/r_0$

**d**  $n \propto r_0/\lambda$

**e**  $E \propto -1/n^2$

# REFERENCE MATERIAL

## Textbooks and further reading

Textbooks that are useful throughout the course are listed here. Other books and other references that are particularly relevant to specific Units are listed individually.

- AKRILL, T. B., BENNET, G. A. G., and MILLAR, C. J. *Physics*. Edward Arnold, 1979.
- BOLTON, W. *Patterns in physics*. McGraw Hill, 1974.
- DUNCAN, T. *Physics: a textbook for advanced level students*. Murray, 1982.
- OR
- DUNCAN, T. *Advanced physics: fields, waves, and atoms*. 2nd edn. Murray, 1981.
- DUNCAN, T. *Advanced physics: materials and mechanics*. 2nd edn. Murray, 1981.
- WENHAM, E. J., DORLING, G. W., SNELL, J. A. N., and TAYLOR, B. *Physics: concepts and models*. 2nd edn. Addison-Wesley, 1984.

## Unit H Magnetic fields and a.c.

### Textbooks for reference

- BENNET, G. A. G. *Electricity and modern physics*. 2nd edn. Arnold, 1974.
- NUFFIELD REVISED PHYSICS *Pupils' text year 5*. Longman, 1980.

### Further reading

- CARO, D. E., MCDONNELL, J. A., and SPICER, B. M. *Modern physics*. 3rd edn. Edward Arnold, 1978.
- MORGAN, D. V. and HOWES, M. J. Wykeham Science Series No. 20. *Solid state electronic devices*. Wykeham, 1972.
- STAFFORD, G. H. A Rutherford Appleton Laboratory Monograph. *The use of high energy machines in particle physics*. Rutherford Appleton Laboratories, 1980. (Out of print.)
- WILSON, R. R. and LITTAUER, R. Science Study Series No. 15. *Accelerators: machines of nuclear physics*. Heinemann, 1962.
- WRIGHT, J. P. *The vital spark*. Heinemann, 1974. (Out of print.)

## Unit I Linear electronics, feedback and control

### Further reading

- BOLTON, W. *Engineering instrumentation and control*. Butterworth, 1980.

- BRAUN, E. and MACDONALD, S. *Revolution in miniature: history and impact of semiconductor electronics*. 2nd edn. Cambridge University Press, 1982.
- CLOSE, K. J. and YARWOOD, J. *Electronics*. University Tutorial Press, 1976.
- ELECTRONIC SYSTEMS TEACHING PROGRAMME ESP700 Book 4 *Feedback systems*. Feedback Instruments Ltd.
- ENGINEERING CONCEPTS CURRICULUM PROJECT *The man-made world*. McGraw-Hill, 1971.
- ENGINEERING SCIENCE PROJECT *Electronics, systems, and analogues*. Macmillan, 1975. (Out of print.)
- HARDY, R. N. Studies in Biology No. 63. *Homeostasis*. 2nd edn. Edward Arnold, 1983.
- MARSTON, R. M. *110 operational amplifier projects*. Hayden, 1975.
- NUFFIELD ADVANCED PHYSICS Students' book Unit 6 *Electronics and reactive circuits*. Penguin, 1971.
- NUFFIELD REVISED ADVANCED BIOLOGY Study guide I. Longman, 1985.
- PLANT, M. *Operational amplifier applications*. NCST Trent Polytechnic, 1974.
- PROJECT TECHNOLOGY Handbook 14 *Simple computer and control logic*. Heinemann/Schools Council, 1972.
- RAMSEY, D. C. *Engineering instrumentation and control*. Stanley Thornes, 1981.
- ROBERTS, M. B. V. *Biology, a functional approach*. 2nd edn. Nelson, 1976.
- Science in Society Book M *Engineering 1*. Heinemann/ASE, 1983.

## Unit J Electromagnetic waves

### On optics

- FRENCH, A. P. MIT Introductory Physics Series. *Electromagnetic waves and optics*. Nelson, 1968.
- FRENCH, A. P. MIT Introductory Science Series. *Vibrations and waves*. Van Nostrand Reinhold (U.K.) Co. Ltd., 1982.

### On images

- CANNON, T. M. and HUNT, B. R. 'Image processing by computer'. *Scientific American* Volume 245(4), Oct. 1981.
- TAYLOR, C. A. Wykeham Science Series, *Images*. Wykeham, 1978.

### On historical background

- MASON, P. *The light fantastic*. Pelican Books, 1981

### On radioastronomy

- HENBEST, N. 'Jodrell under Merlin's spell'. *New Scientist* Volume 96, 1332, Nov. 1982.
- KELLERMANN, K. I. 'Intercontinental radio astronomy'. *Scientific American* Volume 226(2), Feb. 1972.
- READHEAD, A. C. S. 'Radioastronomy by very-long-baseline interferometry'. *Scientific American* Volume 246(6), June 1982.

### On electromagnetic waves

- BENNET, G. A. G. *Electricity and modern physics*. 2nd edn. Edward Arnold, 1975.
- WHELAN, P. M. and HODGSON, M. J. *Essential principles of physics*. Murray, 1978.

### On relativity

- BONDI, H. *Relativity and common sense: a new approach to Einstein*. Heinemann, 1964.
- EPSTEIN, L. C. *Relativity visualized*. Insight Press, San Francisco, 1985. (Available through Adam Hilger Ltd., Bristol.)
- HOFFMANN, B. *Relativity and its roots*. A Scientific American Book. W. H. Freeman and Co., 1983.
- LANDAU, L. D. and RUMER, G. B. *What is relativity?* Oliver and Boyd, 1960.
- SANDAGE, A. R. 'The red shift'. *Scientific American* Volume 195(3), Sept. 1956. (Offprint No. 240.)
- SHANKLAND, R. S. 'The Michelson-Morley experiment'. *Scientific American* Volume 211(5), Nov. 1964. (Offprint No. 327.)

## Unit K Energy and entropy

- ANGRIST, S. W. and HEPLER, L. G. *Order and chaos*. Basic Books, 1967.
- ATKINS, P. W. *The creation*. W. H. Freeman, 1981.
- CRAWLEY, G. M. *Energy*. Collier-Macmillan, 1975.
- DAVIES, P. C. W. *The accidental Universe*. Cambridge University Press, 1982.
- DAVIES, P. C. W. *The runaway Universe*. Cambridge University Press, 1982.
- NUFFIELD REVISED ADVANCED CHEMISTRY Students' books 1 and 2. Longman, 1984.

PSSC *College Physics*. 5th edn. Raytheon, 1981.  
*Energy and power*. (A *Scientific American* book.) W. H. Freeman, 1971.  
 WEINBERG, S. *The first three minutes*. Fontana, 1983. (First published by André Deutsch, 1977.)

## Unit L Waves, particles, and atoms

In addition to the general list of text books on page 348, the following are particularly useful for this Unit. Particular sections or chapters in several of them are referred to in the Reading section 'Waves and particles' on page 309.

BENNET, G. A. G. *Electricity and modern*

*physics*. 2nd edn. Edward Arnold, 1974.  
 BORN, M. *The restless Universe*. Dover, 1951.  
 CARO, D. E., MCDONNEL, J. A., and SPICER, B. M. *Modern physics: an introduction to atomic and nuclear physics*. 3rd edn. Arnold, 1978.

CONN, G. K. T. and TURNER, H. D. *The evolution of the nuclear atom*. Iliffe, 1965.  
 FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. *The Feynman lectures on physics Volume 1: Mainly mechanics, radiation, and heat*. Addison-Wesley, 1963.  
 HOFFMANN, B. *The strange story of the quantum*. Penguin, 1970. (Also available in Dover Press edition.)

MILLIKAN, R. A. Phoenix Science Series. *The electron*. University of Chicago Press, 1963.

OPEN UNIVERSITY Science Foundation Course S101 Unit 9 *Light: waves or*

*particles?* Open University Press, 1979.  
 PROJECT PHYSICS Text and Reader Unit 5, *Models of the atom*. Holt, Rinehart, and Winston, 1971.

PSSC *College physics*. 5th edn. Raytheon, 1981.

PSSC *Physics*. 3rd edn. Heath, 1971.  
 ROGERS, E. M. *Physics for the inquiring mind*. Oxford University Press, 1960.

ROTHMAN, M. A. *The laws of physics*. Penguin, 1966.

TAYLOR, R. J. Unilever Educational Booklet Advanced Series No. 5. *Water*. 2nd edn. Unilever, 1969.

TOLANSKY, S. *Revolution in optics*. Penguin, 1968.

TOULMIN, S. E. and GOODFIELD, J. *The architecture of matter*. Hutchinson, 1962.  
 WRIGHT, S. (ed.) *Classical scientific papers – physics*. Mills and Boon, 1964. (Out of print.)

# DATA; FORMULAE AND RELATIONSHIPS; SYMBOLS

## Data

(Values are given to three significant figures, except where more – or less – are useful)

### Physical constants

speed of light	$c$	$3.00 \times 10^8 \text{ m s}^{-1}$
permittivity of free space	$\epsilon_0$	$8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ (or $\text{F m}^{-1}$ )
electric force constant	$\frac{1}{4\pi\epsilon_0}$	$8.98 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ ( $\approx 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ )
permeability of free space	$\mu_0$	$4\pi \times 10^{-7} \text{ N A}^{-2}$ (or $\text{H m}^{-1}$ )
charge on electron	$e$	$-1.60 \times 10^{-19} \text{ C}$
mass of electron	$m_e$	$9.11 \times 10^{-31} \text{ kg} = 0.00055 \text{ u}$
mass of proton	$m_p$	$1.673 \times 10^{-27} \text{ kg} = 1.0073 \text{ u}$
mass of neutron	$m_n$	$1.675 \times 10^{-27} \text{ kg} = 1.0087 \text{ u}$
mass of alpha particle	$m_\alpha$	$6.646 \times 10^{-27} \text{ kg} = 4.0015 \text{ u}$
Avogadro constant	$L, N_A$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Planck constant	$h$	$6.63 \times 10^{-34} \text{ J s}$
Boltzmann constant	$k$	$1.38 \times 10^{-23} \text{ J K}^{-1}$
molar gas constant	$R$	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
gravitational force constant	$G$	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

### Other data

standard temperature and pressure (s.t.p.)		$273 \text{ K}$ ( $0^\circ \text{C}$ ), $1.01 \times 10^5 \text{ Pa}$ (1 atmosphere)
molar volume of a gas at s.t.p.	$V_m$	$2.24 \times 10^{-2} \text{ m}^3$
gravitational field strength at Earth's surface (in the U.K.)	$g$	$9.81 \text{ N kg}^{-1}$
mass of Earth		$5.98 \times 10^{24} \text{ kg}$
$GM$ for Earth		$\approx 4 \times 10^{14} \text{ N m}^2 \text{ kg}^{-1}$
mass of Moon		$7.35 \times 10^{22} \text{ kg}$
average separation of Earth and Moon		$3.82 \times 10^8 \text{ m}$
mean radius of Earth		$6.37 \times 10^6 \text{ m}$
mean radius of Moon		$1.74 \times 10^6 \text{ m}$



## Conversion factor

unified atomic mass unit                      1 u                      =  $1.661 \times 10^{-27}$  kg

## Numerical constants

the number e, the base of natural logarithms    e                      2.718 ...

ratio of circumference to diameter of circle     $\pi$                       3.14 ... ( $\approx \sqrt{10}$ )

## Formulae and relationships

### Motion and forces

linear momentum =  $mv$                       (mass  $m$ , velocity  $v$ )

force = rate of change of momentum

$F = ma$                       if mass is constant (force  $F$ , acceleration  $a$ )

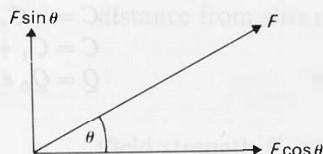
impulse =  $F\Delta t$

translational kinetic energy =  $\frac{1}{2}mv^2$

gravitational potential energy difference =  $mg\Delta h$                       (uniform field strength  $g$ , height  $\Delta h$ )

energy transformed (work) = component of force  $\times$  displacement

components of force in two perpendicular directions:



moment of force about a point = force  $\times$  perpendicular distance from point to line of action of force

static equilibrium conditions:

$$\Sigma F = 0$$

$$\Sigma \text{ moments} = 0$$

limiting friction

$F = \mu N$                       (coefficient of friction  $\mu$ , normal force  $N$ )

circular motion

$a = v^2/r$                       (acceleration  $a$ , speed  $v$ , radius  $r$ )

$F = mv^2/r$                       (centripetal force  $F$ , mass  $m$ )

### Solids

For a material in tension

Hooke's Law:

$F = kx$                       (tension  $F$ , spring constant  $k$ , extension  $x$ )

stress = tension/cross-sectional area

strain = extension/original length

Young modulus = stress/strain

elastic strain energy =  $\frac{1}{2}kx^2$

elastic strain energy per unit volume =  $\frac{1}{2}$  stress  $\times$  strain

## Gases

### Ideal gas equation

for  $n$  moles

$$pV = nRT$$

(pressure  $p$ , volume  $V$ , molar gas constant  $R$ , temperature  $T$ )  
(molar volume  $V_m$ )

for one mole

$$pV_m = RT$$

### Kinetic theory of gases

$$pV = \frac{1}{3}Nmc^2$$

(number of molecules  $N$ , mass of molecule  $m$ , mean square speed  $c^2$ )  
(density  $\rho$ )

$$p = \frac{1}{3}\rho c^2$$

mean kinetic energy of translation of

$$\text{one mole of an ideal gas} = \frac{3}{2}RT$$

## Electricity

flow

$$I = AvnQ$$

(current  $I$ , area  $A$ , velocity of carriers  $v$ , carrier density  $n$ , charge  $Q$ )

resistance

$$R = V/I$$

(resistance  $R$ , potential difference  $V$ )

$$R = \rho l/A$$

(resistivity  $\rho$ , length  $l$ , area  $A$ )

$$R = R_1 + R_2 + \dots$$

(resistors in series)

$$1/R = 1/R_1 + 1/R_2 + \dots$$

(resistors in parallel)

charge

$$\Delta Q = I\Delta t$$

(charge  $Q$ , time  $t$ )

capacitance

$$C = Q/V$$

(capacitance  $C$ )

$$\text{energy stored} = \frac{1}{2}QV$$

$$1/C = 1/C_1 + 1/C_2 + \dots$$

(capacitors in series)

$$C = C_1 + C_2 + \dots$$

(capacitors in parallel)

discharge of capacitor

$$Q = Q_0 e^{-t/RC}$$

(initial charge  $Q_0$ , time  $t$ , time constant  $RC$ )

## Oscillations

### Simple harmonic motion

equation of motion

$$a = -(k/m)s$$

(acceleration  $a$ , force per unit displacement  $k$ , mass  $m$ , displacement  $s$ )

displacement-time relation

$$s = A \cos \omega t$$

(amplitude  $A$ , angular frequency  $\omega$ , time  $t$ )

$$\omega^2 = k/m$$

$$T = 2\pi/\omega$$

(periodic time  $T$ )

$$= 2\pi \sqrt{\frac{m}{k}}$$

$$f = 1/T = \omega/2\pi$$

(frequency  $f$ )

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$v_{\max} = \omega A$$

(maximum velocity  $v_{\max}$ )

$$a_{\max} = \omega v_{\max} = \omega^2 A$$

(maximum acceleration  $a_{\max}$ )

$$\text{kinetic energy} = \frac{1}{2}mv^2$$

$$\text{potential energy} = \frac{1}{2}ks^2$$

$$\text{total energy} = \frac{1}{2}kA^2$$

Quality factor

$$Q = 2\pi \frac{\text{energy stored in oscillator}}{\text{energy lost per cycle}}$$

## Waves

Wave speeds

for all waves

$$c = f\lambda \quad (\text{wave speed } c, \text{ frequency } f, \text{ wavelength } \lambda)$$

compression wave in mass-spring system

$$c = x \sqrt{\frac{k}{m}} \quad (\text{spacing } x, \text{ force per unit displacement } k, \text{ mass } m)$$

sound in a solid

$$c = \sqrt{\frac{E}{\rho}} \quad (\text{Young modulus } E, \text{ density } \rho)$$

transverse wave on string

$$c = \sqrt{\frac{T}{\mu}} \quad (\text{tension } T, \text{ mass per unit length } \mu)$$

electromagnetic waves in free space

$$c = 1/\sqrt{\epsilon_0 \mu_0} \quad (\text{permittivity of free space } \epsilon_0, \text{ permeability of free space } \mu_0)$$

Diffraction

narrow slit

$$n\lambda = b \sin \theta \quad (\text{order } n, \text{ slit width } b, \text{ angles of minima } \theta)$$

Rayleigh criterion

$$\theta \geq \lambda/b$$

diffraction grating

$$n\lambda = s \sin \theta \quad (\text{grating spacing } s, \text{ angles of maxima } \theta)$$

Young's double slits

$$\lambda/x \approx s/L \quad (\text{fringe separation } x, \text{ slit spacing } s, \text{ distance from slits to fringes } L)$$

## Field and potential

All fields

$$E = -dV/dr \quad (\text{field strength } E, \text{ potential gradient } dV/dr)$$

electric field

$$E = F/Q$$

uniform field between parallel plates

$$E = V/d \quad (\text{electric field strength } E, \text{ force } F, \text{ charge } Q)$$
$$= \sigma/\epsilon_0 \quad (\text{potential difference } V, \text{ separation } d)$$

parallel plate capacitor

$$C = \epsilon_0 \epsilon_r A/d \quad (\text{charge density } \sigma, \text{ permittivity of free space } \epsilon_0)$$
$$C = \epsilon_0 \epsilon_r A/d \quad (\text{capacitance } C, \text{ relative permittivity } \epsilon_r, \text{ area } A, \text{ separation } d)$$

point charges

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \quad (\text{charges } Q_1, Q_2, \text{ separation } r)$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \quad (\text{electric field strength } E)$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad (\text{electric potential } V)$$

gravitational field

$$g = F/m \quad (\text{gravitational field strength } g, \text{ force } F, \text{ mass } m)$$

$$F = -Gm_1 m_2 / r^2 \quad (\text{gravitational constant } G, \text{ masses } m_1, m_2, \text{ separation of centres } r)$$

$$g = -GM/r^2 \quad (\text{mass of Earth, or other body, } M)$$

uniform gravitational field

$$V_g = -GM/r$$

$$\Delta V_g = GM(1/r_1 - 1/r_2)$$

$$\Delta V_g = g\Delta h$$

(gravitational potential  $V_g$ )  
(gravitational potential difference  $\Delta V_g$ )  
(height  $h$ )

## Atomic and nuclear physics

Radioactive decay

$$dN/dt = -\lambda N$$

$$N = N_0 e^{-\lambda t}$$

(number  $N$ , decay constant  $\lambda$ )  
(initial number  $N_0$ )

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$= \frac{0.693}{\lambda}$$

(half-life  $T_{1/2}$ )

mass-energy relationship

$$\Delta E = c^2 \Delta m$$

(energy  $E$ , mass  $m$ , speed of light  $c$ )

energy-frequency relationship  
for photons

$$E = hf$$

(photon energy  $E$ , Planck constant  $h$ ,  
frequency  $f$ )

wavelength-momentum relationship  
for particles

$$\lambda = h/p$$

(wavelength  $\lambda$ , momentum  $p$ )

## Electromagnetism

Magnetic fields

force on a current carrying  
conductor

$$F = BIl$$

(flux density  $B$ , current  $I$ ,  
length  $l$ )

force on a moving charge

$$F = BQv$$

(charge  $Q$ , velocity perpendicular  
to field  $v$ )

flux densities

inside a long solenoid

$$B = \mu_0 NI/l$$

(permeability of free space  $\mu_0$ ,  
turns  $N$ , length  $l$ )

near a long straight wire  
at centre of circular coil

$$B = \mu_0 I/2\pi r$$

(radial distance  $r$ )

$$B = \mu_0 NI/2r$$

(radius of coil  $r$ )

magnetic circuit

$$\text{reluctance} = l/\mu_0 \mu_r A$$

(length  $l$ , relative permeability  $\mu_r$ ,  
area  $A$ )

$$\text{flux} = \text{current turns/reluctance}$$

Induction

induced e.m.f. = rate of change of flux linked

$$\mathcal{E} = N d\Phi/dt$$

(induced e.m.f.  $\mathcal{E}$ , rate of change of  
flux  $d\Phi/dt$ , number of turns linked  $N$ )

$$\mathcal{E} = M dI/dt$$

(e.m.f. in secondary  $\mathcal{E}$ ,  
mutual inductance  $M$ )

$$V = L dI/dt$$

(p.d. across coil  $V$ , self inductance  $L$ )

Transformers

$$\mathcal{E}_s = V_p \times N_s/N_p$$

(e.m.f. in secondary  $\mathcal{E}_s$ , p.d.  
across primary  $V_p$ , turns  $N_p$ ,  $N_s$ )

$$I_p V_p > I_s \mathcal{E}_s$$

(currents  $I_p$ ,  $I_s$ )

Alternating current

$$I_{\text{r.m.s.}} = I_0 / \sqrt{2}$$

(root mean square current  $I_{\text{r.m.s.}}$ , peak current  $I_0$ )

Electrical oscillation

$$2\pi f = \omega = 1/\sqrt{LC}$$

(resonant frequency  $f$ , angular frequency  $\omega$ , inductance  $L$ , capacitance  $C$ )

Energy and entropy

Energy transfer

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

$$\text{efficiency of heat engine} = 1 - \frac{T_2}{T_1}$$

(temperature of source  $T_1$ , temperature of sink  $T_2$ )

$$\Delta T = \phi \mathcal{R}$$

(temperature difference  $\Delta T$ , rate of thermal transfer of energy  $\phi$ , thermal resistance  $\mathcal{R}$ )

$$\mathcal{R} = l/kA$$

(area  $A$ , length  $l$ , thermal conductivity  $k$ )

$$X = \mathcal{R}A$$

(thermal resistance coefficient  $X$ )

Entropy

$$\Delta S = (\Delta Q/T)_{\text{reversible}}$$

(entropy change  $\Delta S$ , energy transferred thermally and reversibly  $\Delta Q$ )

$$\Delta S = k \Delta \ln W$$

(number of ways  $W$ , Boltzmann constant  $k$ )

$$\Delta S = Nk \ln (V_2/V_1)$$

(number of particles  $N$ , volume change from  $V_1$  to  $V_2$ ; for ideal gas)

$$pdV = TdS$$

(pressure  $p$ , volume  $V$ ; for ideal gas, reversible isothermal change)

Boltzmann factor

$$f = e^{-\varepsilon/kT}$$

(Boltzmann factor  $f$ , difference between energies of particle  $\varepsilon$ )

$$\text{rate} \propto e^{-E_A/kT}$$

(activation energy  $E_A$ )

## Electrical circuit symbols

Some of the symbols used in circuit diagrams are shown below:

*Wires, junctions, terminals*

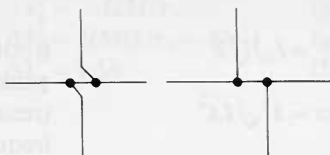
crossing of wires,  
no electrical contact



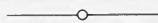
junction



double junction



terminal



aerial



earth



frame or chassis connection



*Lamps*

signal lamp



lamp for illumination



*Transducers*

microphone



earphone



loudspeaker



motor



*Amplifier*

(or non-inverting gate)



*Logic gates*

Invert or NOT gate



OR gate



NOR gate



AND gate



NAND gate



Exclusive OR  
(XOR) gate



## Capacitors

general symbol



polarized (electrolytic) capacitor



## Inductors

general symbol



inductor with core

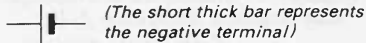


transformer with  
ferromagnetic core



## Batteries

primary or secondary cell



battery with tapings



## Diodes

diode/rectifier



light sensitive diode



light emitting diode  
(L.E.D.)



## Measuring instruments

voltmeter



ammeter

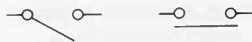


galvanometer



## Switches, relays

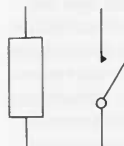
normally open switch



normally closed switch



relay coil and contact

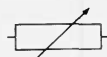


## Resistors

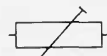
general symbol



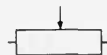
variable resistor



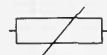
variable resistor with preset adjustment



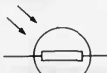
potentiometer



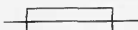
resistor with inherent variability (e.g. thermistor)



light-dependent resistor



fuse





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**This Students' guide contains the last five of the twelve Units in the A-level physics course.**

**The Units are: Unit H, 'Magnetic fields and a.c.'; Unit I, 'Linear electronics, feedback and control'; Unit J, 'Electromagnetic waves'; Unit K, 'Energy and entropy'; and Unit L, 'Waves, particles, and atoms'.**

**Each of the Units follows the same pattern: a brief introduction is followed by *Summaries of the most important ideas in the Unit*; there are then *Readings*, possibly extracts from specialist journals or other publications, followed by questions, which should help you to develop the skill of reading with a purpose; the *Laboratory notes* cover all the experiments and demonstrations suggested for the Unit; there are some suggestions for *Home experiments*, practical activities you can do at home; finally, there is a large number of *Questions*. This Guide ends with *Answers to selected questions* and some useful *Data*, *Formulae and relationships*; *Symbols*.**

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$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

## QUESTIONS 14 to 17

entropy change =  
energy transfer/temperature

units of entropy  $\text{JK}^{-1}$

But  $d(\ln V) = dV/V$ , the fractional change in volume, so

$$dS = kN dV/V$$

There is a relation between the entropy change  $dS$  and the work  $p dV$  involved in the expansion of the gas (figure K7, page 251). Since

$$pV = kNT$$

we have

$$p dV = kNT dV/V$$

Comparing the equations for  $dS$  and  $p dV$ , we obtain the result

$$p dV = T dS$$

The value of  $k$  depends on the (arbitrary) size of a degree on the scale of  $T$ . Using  $k$  in the definition of entropy means that for a gas expanding at a given temperature,  $T dS$  is numerically equal to the work  $p dV$ .

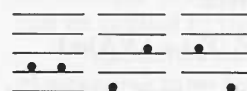
The relation  $p dV = T dS$  only holds for an ideal gas, for which there are no changes in internal energy. But it shows why we give entropy, via the Boltzmann constant, the units of energy divided by temperature.

## Section K2 ENTROPY, ENERGY, AND TEMPERATURE

The connection of entropy with energy arises because, just as molecules can be arranged in space in many ways, so they can share energy in many ways.

### Particles and energy levels

The energy of any bound particle is *quantized*: its energy can have only discrete values. Thus we can think about counting the number of particles at each energy level, and so think about counting the number of different arrangements of particles on the various levels, sharing out the total energy in different ways.



(a)



(b)

**Figure K8**

Ways of sharing energy.

- (a) All the ways for 2 particles sharing 2 quanta.  
(b) All the ways for 2 particles sharing 3 quanta.

Suppose, for simplicity, that particles have equally spaced energy levels. In going from one level to another they then exchange energy in lumps or quanta equal to the level difference. Consider just two such particles which happen to share two quanta of energy. There are just

three possibilities: both can have one quantum each, or either can have two and the other none – figure K8(a).

More energy means more sharing possibilities. If the two particles share three quanta, there are four ways  $W$  of sharing the energy – figure K8(b). For three particles sharing three quanta,  $W = 10$ . For ten sharing ten, it is 92 378. For 100 sharing 100, it is  $8 \times 10^{59}$ .

## Energy shuffling

GAME K2  
Energy shuffling

QUESTION 21

A game in which counters representing particles exchange energy, moving up or down a ladder of equally spaced levels, shows how likely it is to find particles at the various levels, if they share the fixed total energy in all possible ways.

Even with only a few particles, it is rare for a particle to be at the highest possible level: that is, for one particle to have all the energy and the rest none. It is most common to find a particle on the lowest possible level. Going up the levels, the chance of a particle getting that much energy steadily decreases.

COMPUTER SIMULATION K3  
Equilibrium distribution of energy

QUESTIONS 22 to 24

A computer program can play the game with many particles. With (say) 200 particles sharing 200 quanta, it is rare for a particle to get as many as 10 quanta, common for it to have one or two, and most frequent for it to have none. The distribution fluctuates around an *exponential* form: one in which the number on any one level is a constant fraction of the number on the next lower level (figure K9).

<input checked="" type="checkbox"/> Shuffle	55	<input checked="" type="checkbox"/> Average	OFF ON
<input checked="" type="checkbox"/> Quit		<input checked="" type="checkbox"/> Probability	OFF ON

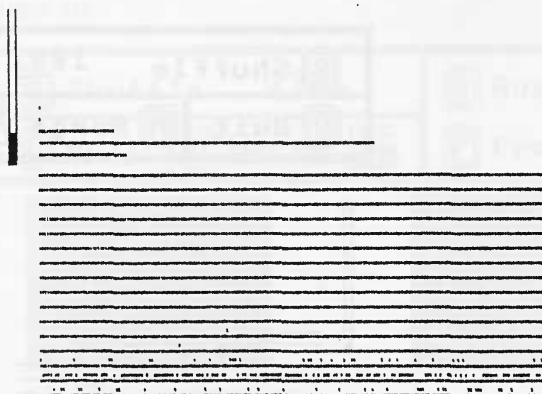


Figure K9 (part)

Computer simulation of energy shuffling between 200 particles. Lower part of display shows individual particles on energy levels; above that total number of each level and average energy per particle.

(a) Soon after starting with all on one level.

The program should let you:

split the particles into two groups;  
merge the two groups into one;  
show instantaneous, or time-averaged distributions, or probabilities;  
place particles initially anywhere on the lower levels.

- a Firstly, have one group of particles. Try first placing all (say 200) on the lowest level but one (level 1). Observe the form around which the instantaneous distribution fluctuates. Try with fewer particles.
- b Make two groups of particles, say 100 in each, and start one group off with particles on low levels, and the other with some particles on rather higher levels. Compare the two distributions. After the distributions become fairly steady, switch to time-averaged distributions for a clearer comparison.

## COMPUTER SIMULATION

### K4 Thermal equilibrium

microcomputer  
program 'Quantum shuffling'

See experiment K3 for notes on the program.

Make two groups of particles, about 125 in each, starting one group with a larger mean energy per particle. Observe them reach equilibrium independently, with the two groups isolated.

Now merge the two groups, to allow energy to pass from one to the other. What happens to the two distributions?

It is instructive to start one group with all its particles on the lowest level (level 0). Why do these particles *not* alter levels when the groups are isolated? What *must* happen when the groups are merged?

Then have just *one* particle in the first group, and 10 or 20 in the second. Merge the two groups to allow energy to be exchanged between them. Watch the display of probabilities to see how long the single particle spends on each level.

## EXPERIMENT

### K5 Vapour pressure of water

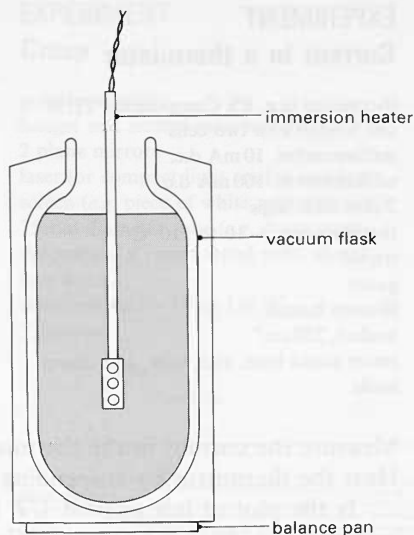
#### K5a Energy needed to evaporate water

vacuum flask  
balance, resolution 0.1 g  
immersion heater  
l.t. variable voltage supply

*either*  
joulemeter  
*or*

ammeter, 10 A a.c., and voltmeter, 25 V a.c.

leads

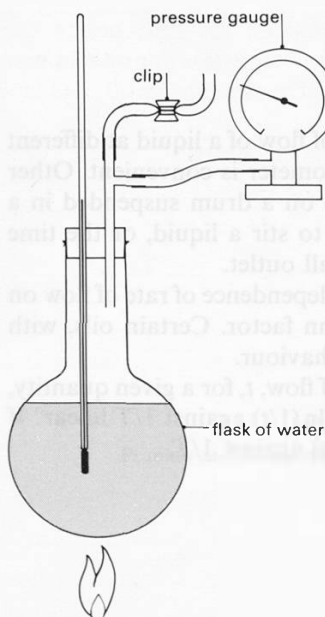


**Figure K33**

Boiling water in a vacuum flask.

*Safety note:* Do not use an immersion heater that has a cracked seal, or allow the heater to cool while it is immersed in water.

Use the immersion heater to boil away water in a vacuum flask, resting on a top-pan balance. Record the mass of water boiled away in (say) 20 minutes, and the energy supplied electrically. Calculate the energy needed to evaporate one mole (18 g) of water. How much energy is that per molecule?



**Figure K34**

Measuring vapour pressure with a pressure gauge.

## K5b Variation of vapour pressure with temperature

round-bottomed flask  
thermometer,  $-10$  to  $110\text{ }^{\circ}\text{C}$   
T-tube  
Bourdon gauge  
pressure tubing  
Hoffmann clip  
tripod  
gauze  
Bunsen burner  
safety spectacles  
retort stand base, rod, boss, and clamp

*Safety note:* You must wear safety spectacles for this experiment.

In this experiment you measure the vapour pressure as water in the flask and vapour above it cool, starting with the water boiling at atmospheric pressure and the flask full of vapour.

Half fill the flask with water and heat it to boiling *with the clip open*. Continue boiling with the clip open for some minutes to displace air. Remove the flame, close the clip, and allow the flask to cool. At intervals, warm the water *gently* with a *small flame* until it just boils: record the pressure and temperature.

Is the plot of  $\ln p$  against  $1/T$  linear? How does its slope compare with  $-\Delta H_{\text{evap}}/Lk$ ?



- 55(R)** Figure H107 shows how the alternating potential difference  $V$  applied to the ends of a coil made from thick copper wire, and the current  $I$  through the coil, vary with time  $t$ .

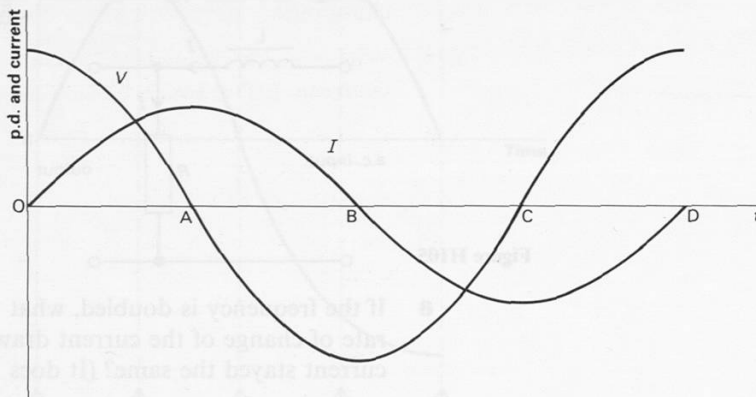


Figure H107

- a** Why is the applied potential difference zero when the current  $I$  has a maximum value?
- b** During which of the periods of time OA, AB, BC, and CD would the source of power be *i* supplying energy, *ii* receiving energy?  
How did you decide on these answers?

When the source of power *receives* energy, where does the energy come from?

(Short answer paper, 1978)

### Electrical oscillations

- 56(L)a** Figure H108(a) shows a trolley tethered between springs fixed to rigid walls. The trolley is pulled to one side, stretching and compressing the springs. The system has gained energy. Where is this energy?

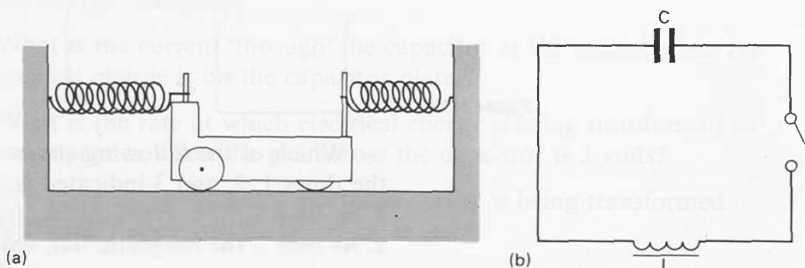


Figure H108

- b** Figure H108(b) shows a capacitor about to be connected across an inductor. The capacitor has been connected to a battery, giving it a charge. The system has gained energy. Where is this energy?
- c** The trolley is released. What determines its initial acceleration?

- d The switch is closed. What determines the initial rate of rise of current?
- e In the middle position, is the velocity of the trolley constant, zero, or changing?
- f When the capacitor has no charge is there a steady, a zero, or a changing current in the circuit?
- g Because the trolley is still moving when the springs are not displaced, it soon does displace them. What effect does this have on the trolley's velocity?
- h Because there is a current when the capacitor is uncharged, the capacitor soon becomes charged again. What effect does this have on the current?
- i There comes a time when the trolley is at rest again, displaced from the centre by as far as it was to begin with, if there was negligible friction. How much energy has the system and where is this energy?
- j There comes a time when the current is zero again, with the capacitor charged by as much as it was to begin with, if there was negligible resistance in the circuit. How much energy has the system, and where is it?

57(R)

When the switch in the circuit of figure H109 is closed, the current in the circuit oscillates for a while. After some time, the oscillations stop. When the oscillations have stopped, which of the following correctly describe(s) the potential differences across the resistor  $R$ , the inductor  $L$ , and the capacitor  $C$ ?

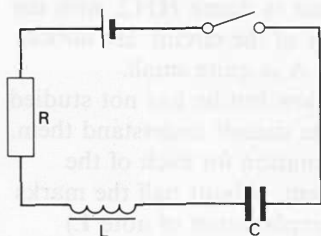


Figure H109

- 1 There is then *no* potential difference across  $R$ .
  - 2 There is then *no* potential difference across  $L$ .
  - 3 There *is* then a potential difference across  $C$ , equal to the voltage of the battery.
- A 1 only    B 2 only    C 1 and 3 only  
D 2 and 3 only    E 1, 2, and 3

(Coded answer paper, 1979)