Unit D
OSCILLATIONS AND WAVES

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SUMMARY OF THE UNIT

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SUMMARY OF THE UNIT

INTRODUCTION

This Unit deals with oscillations in mechanical systems and the waves they set up. Engineers are especially concerned with such mechanical vibrations, which have important implications for the stability and even the safety of the structures they build. For example, the designer of a new car must consider how the parts of the vehicle might oscillate and the effect such oscillations would have on passenger comfort, road-holding, and so on.

The study of mechanical oscillations and waves is important too as a preparation for understanding electrical oscillations (Unit H, 'Magnetic fields and a.c.') and electromagnetic waves (Unit J, 'Electromagnetic waves'). Analogies between mechanical and electrical oscillations and waves are easy to draw, and the mathematical models developed in this Unit to describe mechanical situations transfer directly to equivalent electrical situations.

You will use several of the ideas developed in Unit A, 'Materials and mechanics', including those of interatomic forces and spacings, the Young modulus, and the spring constant. And it will be helpful if you can recall some of the important features of wave behaviour from earlier science courses.

Section D1 INTRODUCTION TO OSCILLATIONS

What are oscillators and why study them?

Anything that exhibits a rhythmic, repetitive, or to-and-fro motion may be considered as an oscillator. Although the cause of the oscillations and the nature of the oscillator differ from example to example, we can describe common properties.

Unit H, 'Magnetic fields and a.c.'

Unit I, 'Linear electronics, feedback and control'

Unit J, 'Electromagnetic waves'

READING
Quartz and atomic clocks (page 237)

'Buildings, bridges, and wind' in the Reader *Physics in engineering and technology*

Almost every object in the universe, large or small, can oscillate in some way or another; and if oscillating electric and magnetic fields, currents, and potential differences are considered, then the study of oscillations is a major theme in physics, as the list below indicates.

Quartz crystals as used in clocks and watches.
Atoms oscillating in solids.
Metal structures oscillating (leading to fatigue) as, for example, bridges, aircraft wings.
A car bouncing on its suspension.
An oil platform oscillating in rough seas.
A boat pitching and rolling.
The Earth's atmosphere after an explosion such as that of Krakatoa.
The larynx in the human voice-box.

Any real system, electrical or mechanical, subject to a sudden change will begin to oscillate, unless damping (for example, by friction) is very large.

As well as the fact that oscillations can occur in so many different systems, as the list above suggests, there are four particular reasons why they are worth studying:

i some oscillators have a constant period and so can be used for timekeeping;

ii some oscillations can be destructive if uncontrolled;

iii mechanical waves may originate from an oscillating body if the body can cause particles or other objects within the surrounding medium to oscillate and so transmit the wave;

iv electromagnetic waves are radiated into free space by oscillating charged particles; the same oscillators are also able to absorb electromagnetic radiation.

Time traces of oscillators

EXPERIMENT D1
How do oscillators move?

You should be familiar with the words:
displacement
amplitude
period
frequency

QUESTION 1

Some oscillators are isochronous; that is, the period of the oscillation is constant (from the Greek *iso* = equal, *khronos* = time). Some are not. Some have smooth graphs of displacement against time and some have not.

A typical near-isochronous time trace is given by the loaded lath oscillator (see figure D1). Note that:

i one cycle of the graph resembles a cosine graph;

ii the amplitude dies away due to damping – the oscillator loses energy to the surroundings;

iii the period is independent of the amplitude.

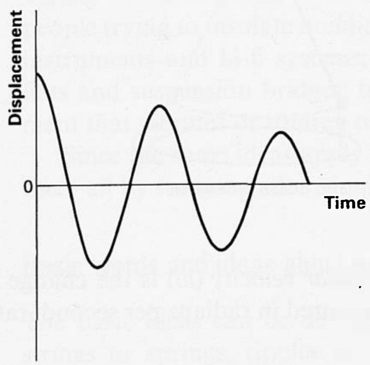

Figure D1

Oscillations and clocks

HOME EXPERIMENT DH1
Making a chronometer

READING
Quartz and atomic clocks (page 237)

Isochronous oscillators from the pendulum to the precise oscillations of caesium atoms in the atomic clock obviously have a use in measuring time. But is our desire to measure time more and more accurately the end of the story? Such questions arise as: 'What is time?', 'Does time run steadily?', 'Could time run backwards?', and 'Would we know if it was doing so?'. These questions are not just amusing, they have great importance to physicists studying fundamental particles, and influenced both Newton and Einstein. So it is well worth reading about time and its measurement, the history of timekeeping, the use of timekeeping in navigation, and the problems that have arisen with our ideas of time.

Oscillations and circular motion

QUESTIONS 3 to 5

DEMONSTRATION D2
Oscillators and circular motion

If a swinging pendulum and an object rotating on a turntable are viewed from the side, the two motions can appear identical. They seem to have a great deal in common, and the relationship between them is a very useful one. This demonstration leads to two important quantities associated with oscillations: phase, ϕ, and angular velocity, ω.

Phase (ϕ) If a pendulum is the right length for its natural frequency to equal the frequency of rotation of the turntable, then the shadows cast on a screen by the pendulum and the rotating object will move together and be 'in step' (providing of course that the pendulum is released at the right moment). We say that they are *in phase* – figure D2(a). If the pendulum is released at some other instant, there will be a constant time interval between one shadow reaching the outermost limit of its swing and the other reaching that position. The fraction of a complete oscillation by which one is ahead of the other is known as the *phase difference*. It can be expressed as a fraction of a revolution or oscillation, or, more usually, as an angle. See figure D2(b). Such an angle is usually measured in radians (see below). If the pendulum is too long or too short, the two movements will not stay in step, and the phase difference will alter continuously.

QUESTION 2

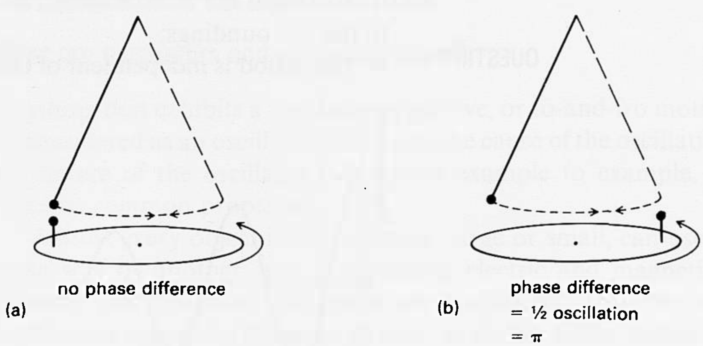

Figure D2

Angular velocity (ω) is the change in angle per unit time. It is usually measured in radians per second, rather than degrees per second.

Definition of the radian

QUESTION 6

One radian is the angle at the centre of a circle of radius r subtended by an arc of length r.

If an arc of length r subtends an angle of 1 radian, then the whole circumference (length $2\pi r$) will subtend an angle of 2π radians. That is,

2π radians are equivalent to $360°$

$$\text{therefore 1 radian} = \frac{360°}{2\pi} = 57.3°$$

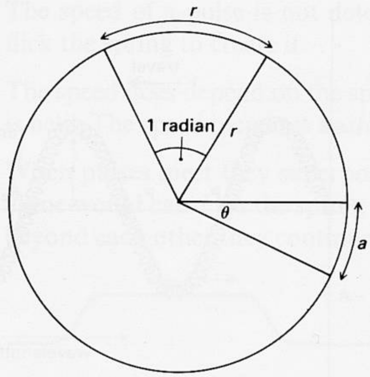

Figure D3
Radian measure of angles.

In general for any angle θ, if θ is measured in radians, then $\theta = \text{arc/radius} = \dfrac{a}{r}$

QUESTIONS 7, 8

For small enough angles $\tan\theta \approx \sin\theta \approx a/r$. So $\tan\theta = \sin\theta = \theta$ (in radians) is quite a good approximation when θ is small.

Section D2 MECHANICAL WAVES AND SUPERPOSITION

D

READING
Applications of ultrasonics (page 232)

All waves are produced by some sort of oscillator; the wave transfers energy from the oscillator to other points. Sometimes this is useful, for example the oscillations of a loudspeaker producing musical sound waves; sometimes a nuisance, as when a loose oscillating panel in a bus produces a non-musical rattle; and sometimes it can be dangerous – an earthquake wave causing buildings to crack or collapse. The enormous variety of waves means that they are of great practical importance: to people trying to insulate buildings against noise; to designers of musical instruments and hi-fi systems; to architects and builders of high-rise flats and suspension bridges; to installers and designers of any equipment that vibrates or rotates; to geophysicists and many others.

Since the same ideas apply to all types of wave, we can learn about them all by studying a few simple systems.

Basic words and ideas about waves

DEMONSTRATION D3
Basic ideas about waves

HOME EXPERIMENT DH2
Make your own wave machine

The basic ideas can be demonstrated using mechanical waves along strings or springs, ripples on water, or special wave machines. The system carrying the waves is called the *medium*. A *displacement against distance graph*, or *wave profile*, shows the displacement of points along the medium at one instant of time.

In figure D4, waves on the spring are being started by oscillations of the end P_1. At the instant shown, P_1 has completed two oscillations. All other points along the medium perform the same oscillations as P_1, only later in time. P_5 is just starting to oscillate; P_3 has performed one complete oscillation; P_2 and P_4 are also oscillating, half a cycle (π) out of phase with P_1. The distance between any two adjacent points which are in phase is one *wavelength* (λ).

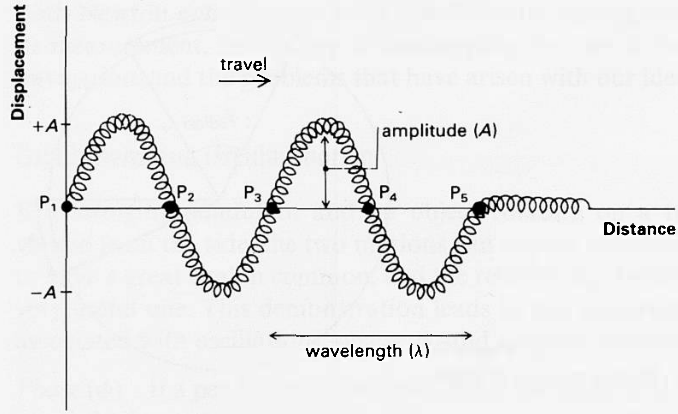

Figure D4
Wave profile: displacement–distance.

Another graph, of *displacement against time*, could be plotted for any particular point along the medium. From this graph the *period*, T, and frequency, f, could be obtained. Figure D5 shows a displacement against time graph for the point P_5, assuming that figure D4 shows the profile at $t = 0$.

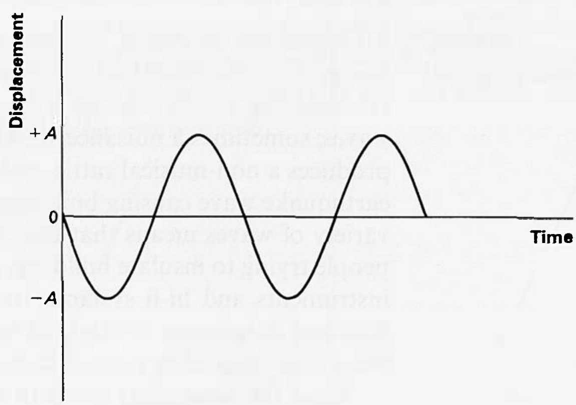

Figure D5
Displacement–time for P_5.

QUESTIONS 9, 12

The speed of travel of the wave, c, is given by $c = f \times \lambda$. If P_1 oscillates continuously, a *continuous wave* travels along the spring. If, however, P_1 is just displaced once and then remains at its equilibrium position, a *single pulse* travels along the spring.

Pulses on springs: experimental results

Experiments show that:

EXPERIMENT D4
Properties of mechanical waves

The shape of a pulse on a spring is determined by the nature of the flick creating it: a quick flick gives a short pulse, whereas a slow flick gives a long pulse.

Friction makes a pulse grow smaller in amplitude as it travels – its energy spreads out to its surroundings.

The speed of a pulse is not determined by its shape, nor on how you flick the spring to create it.

The speed does depend on the spring – and on the tension with which it is held. The speed increases as the tension is increased.

When pulses meet they superpose – the displacements that each pulse alone would cause on the spring add together; but when the pulses pass beyond each other they continue with their original shape (figure D6).

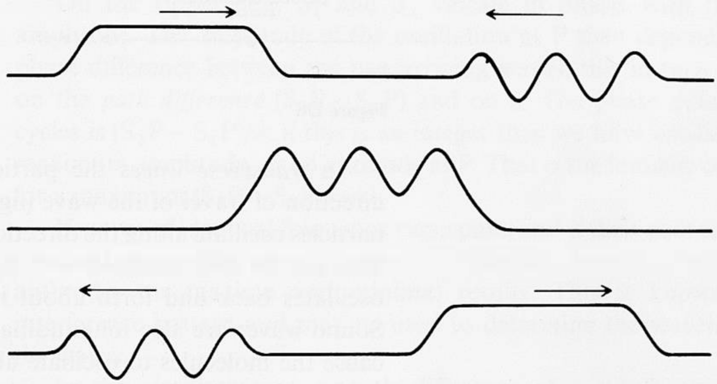

Figure D6
Superposition of pulses.

QUESTIONS 10, 11

When a series of pulses is reflected, the returning pulses form a stationary pattern as they superpose with those pulses still moving outward (figure D7).

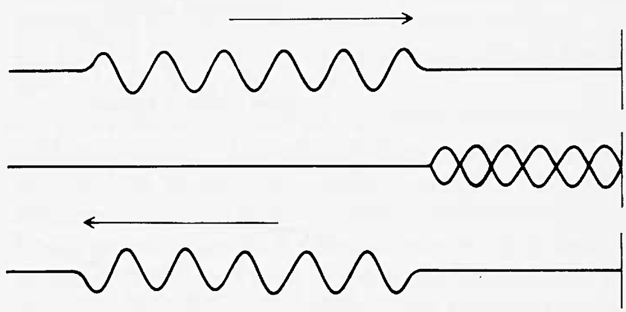

Figure D7
Stationary pattern produced when waves superpose.

EXPERIMENT D4c

A pulse reflected at a fixed end suffers a phase change of π – it turns upside down. If reflected at an open end, it suffers no phase change.

How does a mechanical wave travel?

Section D3 considers the speeds of waves like this in more detail

When a wave travels along a trolley-and-spring model each trolley moves in turn, because of a force on it from the preceding spring. The speed of the wave depends on how long it takes the trolley to acquire enough displacement to exert force on the next spring. This depends on the mass, m, of the trolley and the stiffness, k, of the springs, and it can be shown that the speed, c, is proportional to $\sqrt{k/m}$ for this wave.

Longitudinal waves

DEMONSTRATION D5
Longitudinal waves on a Slinky spring

As a wave travels through a medium the individual particles of the medium oscillate about their rest positions.

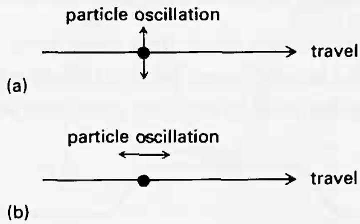

Figure D8

In *transverse waves* the particles oscillate at right angles to the direction of travel of the wave (figure D8(a)). In *longitudinal waves* the particles oscillate along the direction of travel of the wave (figure D8(b)). This can be demonstrated on a Slinky spring; each part of the spring oscillates back and forth about its rest position as the wave passes. Sound waves are also longitudinal: here pressure variations in the gas cause the molecules to oscillate about their mean positions.

Longitudinal waves behave in much the same way as transverse waves.

One difficulty arises in drawing the longitudinal wave: often it is represented in the same way as a transverse wave, and this can be misleading. Remember that the displacement is really in the direction of travel of the wave (see figure D9).

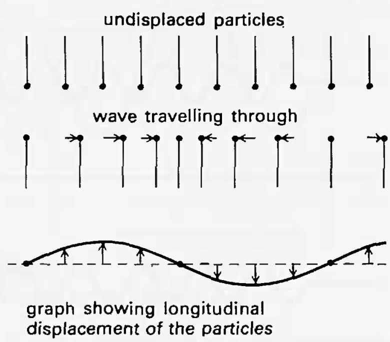

Figure D9

Superposition of waves

DEMONSTRATION D6
What happens when waves meet?

QUESTION 16

In figure D10, waves from both S_1 and S_2 are arriving at P. The principle of superposition is that at any moment the displacement at P is the sum of the separate displacements that the waves from S_1 and S_2 would cause individually.

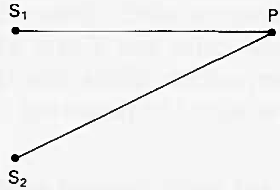

Figure D10

There will be a maximum disturbance at P if $S_2P - S_1P = n\lambda$ (providing S_1 and S_2 are emitting in phase)

On the ripple tank S_1 and S_2 vibrate in phase, with the same amplitude. The amplitude of the oscillation at P then depends on the phase difference between the two arriving waves; this in turn depends on the *path difference* ($S_2P - S_1P$) and on λ. The phase difference in cycles is $(S_2P - S_1P)/\lambda$; if this is an integer then we have oscillations of maximum amplitude, or an antinode at P. That is the familiar condition for a maximum $(S_2P - S_1P) = n\lambda$.

If waves of identical frequency superpose, and if their sources have a constant phase difference (or none), a stationary pattern of nodes and antinodes (or maxima and minima) results. This is known as an interference pattern and may be used to determine the wavelength of the waves.

DEMONSTRATION D7
Path differences and phase difference

In the simplest case, a path difference of a whole number of wavelengths means that the two waves arrive in phase, giving a maximum, or antinode; but this only applies if the waves are emitted in phase with each other and if no other factors, such as reflection, affect the phase of either wave. Reflections in some cases result in the wave changing phase by half a cycle, or π. Such phase changes must be taken into account in the calculation of wavelength. If the two waves are exactly out of phase, a minimum, or node, results.

If either the frequency or the speed is already known, the other may now be calculated using $c = f\lambda$.

Superposition at a point can occur with waves from two (or more) different sources – though a stationary pattern will only result if they have a fixed phase relationship; it can also occur with waves from a single source which have travelled by different paths to the point.

EXPERIMENTS D8
Superposition of waves

QUESTIONS 14, 15, 17 to 19

As an example, the 3 cm radio receiver, R, in figure D11 will receive *minimum* radiation if both T and R are very close to M. This is because the difference in path between TMR (reflected path) and TR (direct path) is almost zero; but the reflection at M changes the phase of the reflected wave by π: hence the waves arrive out of phase by π. To obtain a phase difference of one cycle, M must be moved away from T and R until the path difference is 1.5 cm (half a wavelength). See figure D12.

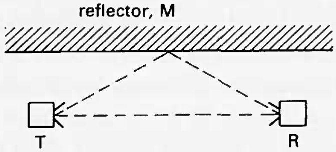

Figure D11

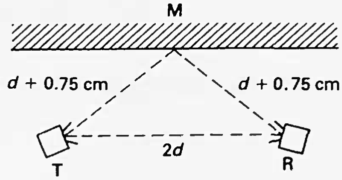

Figure D12

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Unit L, 'Waves, particles, and atoms'

Superposition effects demonstrate the wave nature of radiations such as light, radio, and X-rays where the waves themselves cannot be seen. Superposition effects also show that electrons (and other 'particles') have wave-like properties.

Examples and applications of superposition

QUESTIONS 20 to 22

Familiar examples include the colours seen in 'rainbow bubbles' and on oily puddles, and the colours seen on the surface of a long-playing record when it is tilted around in sunlight. The abrupt occurrence and subsequent disappearance of gigantic ocean waves up to 30 metres high are also due to superposition.

Section D3 MECHANICAL OSCILLATIONS

EXPERIMENT D1
How do oscillators move?

DEMONSTRATION D2
Oscillators and circular motion

This Section develops the link between oscillatory and circular motion, and uses it to derive a mathematical description which can be applied, more or less exactly, to many different oscillators. The reason why many oscillators are isochronous is also dealt with.

Features common to all mechanical oscillators

Every mechanical oscillator, isochronous or not, has these features:
i it is displaced successively to one side then the other of an equilibrium position;
ii it is accelerated towards the equilibrium position by a force; the force is related to its displacement in some way;
iii it has inertia, which means that it continues through the equilibrium position, rather than coming to rest there;
iv it possesses kinetic energy as it passes through the equilibrium position, potential energy at the extreme ends of its motion, and usually a combination of both at points in between;
v there are resistive forces against which it must do work; as a result the oscillator loses energy.

QUESTIONS 26, 27

The mass and spring oscillator

k is the force which would displace the mass m a distance of 1 m

The rest of this Section deals solely with one particular type of oscillator; a mass, m, which oscillates horizontally or vertically, and is attached to springs which provide the restoring force. By Hooke's Law, $F = -ks$, where F and s are force and displacement (positive when measured to the right or downward), and k is the spring constant of the whole assembly of springs. This is illustrated in figure D13.

This system is chosen for detailed study because many other systems are analogous to it: if quantities corresponding to m and k can be identified in another system, then the results obtained for the mass and spring system can be used to describe this other system.

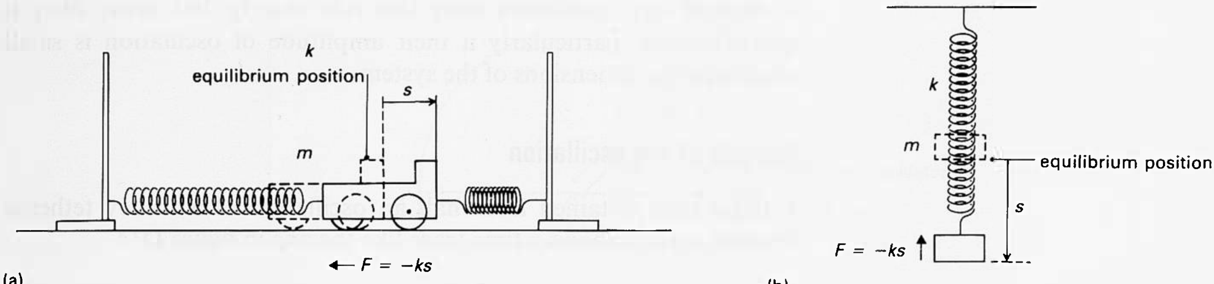


Figure D13
Mass and spring oscillators.

Damping is ignored, partly to simplify the mathematics, and partly because it does not affect perhaps the most important property of the oscillator, its period.

Periodic time, T

EXPERIMENT D9
Factors affecting the period of an oscillator

Experiments show that for this system
i T does not depend on A, the amplitude;
ii T is proportional to $\sqrt{m}$;
iii T is proportional to $\sqrt{1/k}$.

A qualitative argument explaining why T does not depend on A runs as follows:

HOME EXPERIMENT DH3
A mechanical oscillator

Consider one quarter of an oscillation, first with one amplitude, then with double that amplitude.

In the second case, the object starts with double the displacement;
⇒ twice the force acts on it
⇒ it has twice as much acceleration
⇒ velocity it gains in a given short time doubles
⇒ it covers twice the distance in a given short time
⇒ new amplitude is covered in the same time as the old amplitude.

QUESTIONS 23, 24

Similar qualitative arguments can be used to explain the dependence of T on m and k.

Simple harmonic motion

Simple harmonic motion (S.H.M.) is the name given to the motion of objects moving in such a way that:

restoring acceleration

$a \propto$ displacement s or, with our sign convention,

QUESTION 25

$a \propto -s$.

The mass in the mass-and-spring system obeys this rule, since

$a \propto F$ (Newton's Second Law) and

$F \propto -s$ (Hooke's Law)

EXPERIMENT D10
Oscillation of a tethered trolley

As shown above, the resulting oscillation has a period independent of its amplitude.

Few, if any, oscillators obey this rule exactly; but many obey it approximately, particularly if their amplitude of oscillation is small relative to the dimensions of the system.

Analysis of the oscillation

A ticker-tape obtained from half an oscillation of a trolley tethered between springs shows a time trace like the one in figure D14.

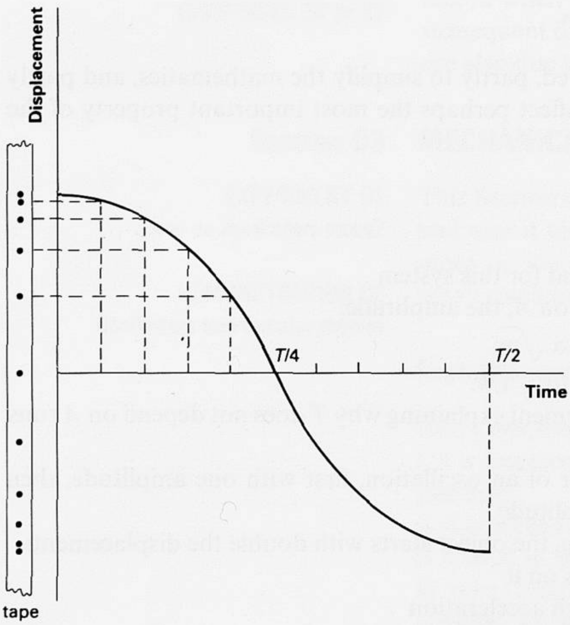


Figure D14
Time trace for a tethered trolley.

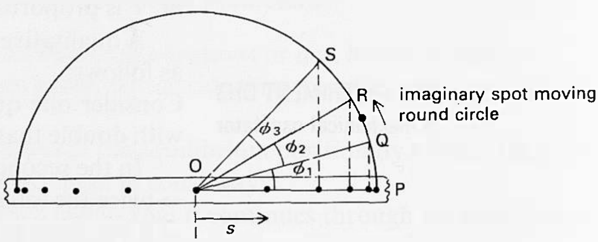


Figure D15
Projection of time trace onto a semicircle.

$\phi = \omega t$
if $t = 0$ when oscillator is at P

The procedure illustrated in figure D15 shows that the motion can be generated by the shadow (projection) of a spot moving round a circle at constant speed. In the time interval between the ticker-tape dots shown, the spot always moves through the same angle round the circle, since measurements show that

$$\phi_1 = \phi_2 = \phi_3\ (= \Delta\phi)$$

By definition, the angular velocity of the spot $\omega = \dfrac{\Delta\phi}{\Delta t}$.

Also,

$$\omega = \frac{2\pi}{T} \qquad \text{(if } \phi \text{ is in radians; } T = \text{period)}$$

ω can be measured from figure D15; if ω is also calculated from the directly measured T, agreement should be good.

QUESTION 28

It can be seen from figure D16 that the displacement of the oscillator $s = A\cos\phi = A\cos\omega t$.

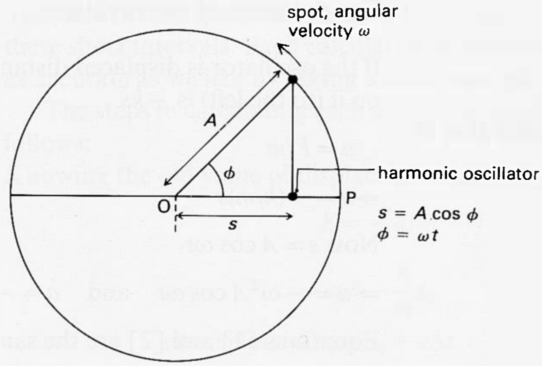


Figure D16
Projection of circular motion onto diameter.

Thus $A \cos \omega t$ gives the complete detail of where the oscillator will be at any given time t after it starts from P.

An alternative definition of S.H.M. is to say that it is a motion which is described by $s = A \cos \omega t$.

Velocity and acceleration of the oscillator

Since velocity is the rate of change of displacement, the velocity at any time is the gradient of the displacement against time curve. Similarly, the acceleration can be obtained from the graph of velocity against time (see figure D17).

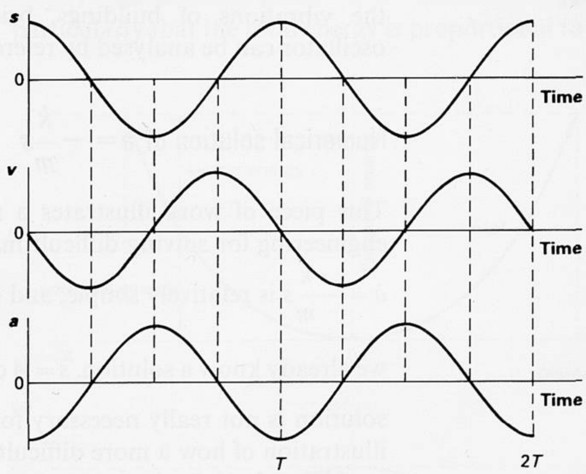


Figure D17
Displacement, velocity, and acceleration of an oscillator.

This result can be obtained more formally by differentiation:

QUESTION 31

$$s = A \cos \omega t$$

$$\Rightarrow v = \mathrm{d}s/\mathrm{d}t = -A\omega \sin \omega t$$

$$\Rightarrow a = \mathrm{d}v/\mathrm{d}t = -A\omega^2 \cos \omega t = -\omega^2 s$$

D

The dynamics of the oscillator

If the oscillator is displaced distance s to the right, the unbalanced force on it (to the left) is $-ks$.

QUESTIONS 32 to 39

$a = F/m$

$\Rightarrow a = -(k/m)s$ equation [1]

Now $s = A\cos\omega t$

$\Rightarrow a = -\omega^2 A\cos\omega t$ and $a = -\omega^2 s$ (see above) equation [2]

Equations [1] and [2] are the same, provided

$$\omega^2 = \frac{k}{m}$$

ω, previously calculated, should compare (within experimental error) with the value of k/m measured during the experiment.

Note also that

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

This ties up with earlier experiments and qualitative reasoning.

The formula $T = 2\pi\sqrt{\frac{m}{k}}$ has many applications. For example, atoms in solids vibrate as though they were masses held by springs (later in this Section this idea is used to derive the speed of sound in a solid); and the vibrations of buildings, bridges, and almost any mechanical oscillator can be analysed by reference to this mass-and-spring model.

Numerical solution of $a = -\frac{k}{m}s$

This piece of work illustrates a method widely used in science and engineering for solving difficult mathematical problems. The equation $a = -\frac{k}{m}s$ is relatively simple, and can be solved exactly by integration: we already know a solution, $s = A\cos\sqrt{\frac{k}{m}}t$. So the numerical method of solution is not really necessary for this problem, but provides a good illustration of how a more difficult problem, impossible by integration, could be solved. Such problems are common in more complex fields like engineering. There is an example from physics in Unit L, 'Waves, particles, and atoms'.

The problem: To obtain a detailed graph of how s varies with t, knowing the constants of the system (k and m), the initial values of s and v, and that $a = -\frac{k}{m}s$.

The principle: The position of the oscillator is calculated at successive moments, which are separated by short time intervals Δt.

The oscillator's speed is assumed to remain constant during each of these short intervals. Each calculation is approximate, but can be made as accurate as we like by taking a small enough value for Δt.

The steps in calculating each successive displacement value, s, are as follows:

Knowing the old value of displacement, s_0

$$F = -ks_0$$

$$\Rightarrow a = -\frac{k}{m}s_0$$

$$\text{new velocity, } v_1 = v_0 + a\Delta t$$

$$\text{new displacement } s_1 = s_0 + \Delta s = s_0 + v_1\Delta t$$

QUESTION 29 The new value of s is now used to work out a at the new position, hence the new v, etc.

This so-called iterative method, in which the same steps are repeated (re-iterated) many times, is of course ideally suited for programming on a digital computer. Furthermore, in a computer program one could easily include other factors, such as damping, or a regularly applied driving force.

QUESTION 30

D

Energy of an oscillator

The potential energy stored in the springs at any position is $\frac{1}{2}ks^2$. When the oscillator is at maximum displacement (and stationary), this is equal to $\frac{1}{2}kA^2$, which must, therefore, be the total energy of the system. Note particularly that the total energy is proportional to A^2.

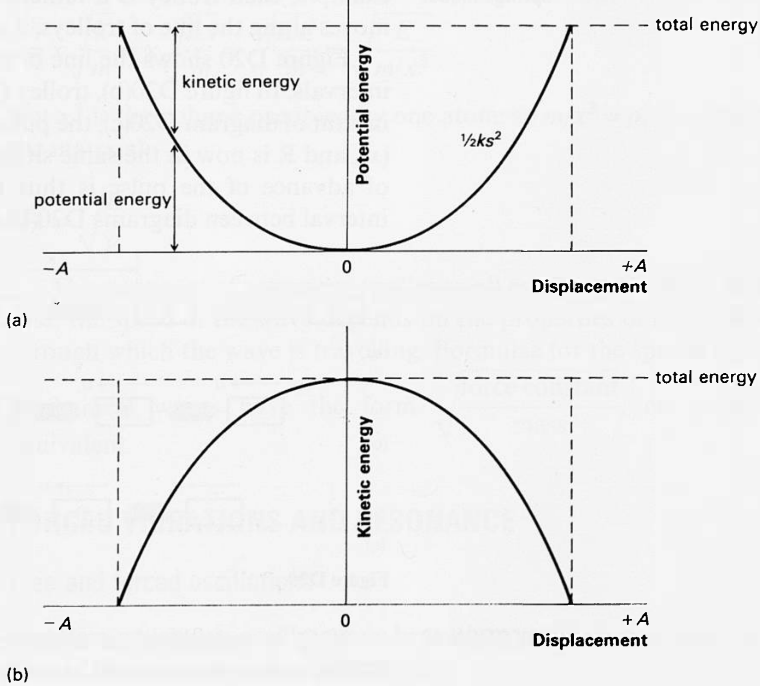


Figure D18
Energy of an oscillator.

If no energy spreads from the oscillator to the surroundings,

total energy = P.E. + K.E.

$\Rightarrow$K.E. $= \frac{1}{2}kA^2 - \frac{1}{2}ks^2$

Thus P.E. and K.E. vary with s, as in figures D18(a) and D18(b). P.E. and K.E. vary with time like this:

P.E. $= \frac{1}{2}ks^2 = \frac{1}{2}kA^2 \cos^2 \omega t$

K.E. $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$

(See figure D19.)

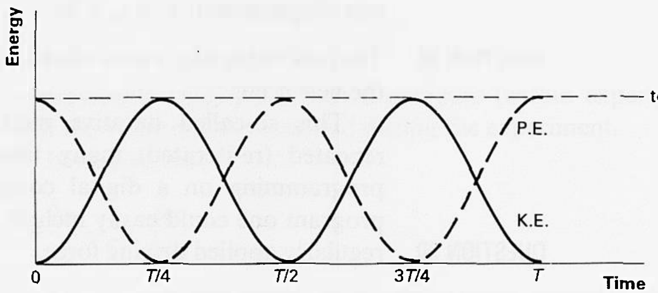


Figure D19
K.E., P.E., and total E of a harmonic oscillator.

Oscillations and the speeds of waves

DEMONSTRATION D11
Longitudinal wave on a trolleys-and-springs model

The passage of a mechanical wave involves energy passing through a medium from one oscillator to the next. In demonstration D11, for example, each trolley is a tethered-trolley oscillator; as a single pulse moves along the line of trolleys, it sets each one into motion in turn.

Figure D20 shows the line of trolleys at three successive short time intervals. In figure D20(b), trolley Q is just being set into motion. By the instant of diagram D20(c), the pulse has advanced by one section-length (x), and R is now in the same situation as Q was previously. The speed of advance of the pulse is thus the distance x divided by the time interval between diagrams D20(b) and D20(c).

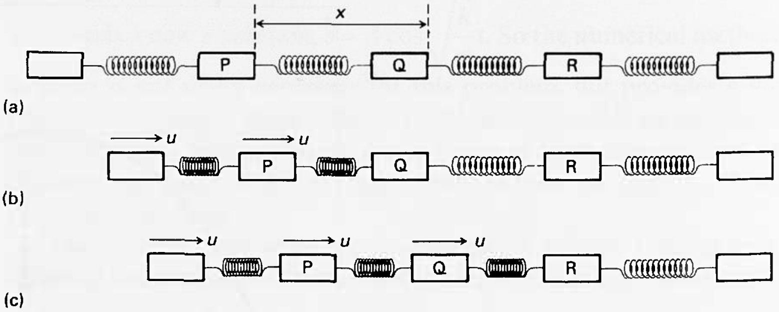


Figure D20

QUESTION 40

If Q is considered as a tethered-trolley oscillator, then in figure D20(b) it is at the lefthand extreme of an oscillation; by figure D20(c) it has reached its equilibrium position between trolleys P and R. This is one-quarter of an oscillation; the time interval for the pulse to be handed on to the next oscillator thus appears to be one-quarter of the period of the oscillation. In fact, a more thorough analysis shows that the fraction is not $\frac{1}{4}$ but $\frac{1}{2\pi}$ of an oscillation.

c is the speed of the pulse

$$c = \frac{x}{\text{time interval}} = \frac{x}{\frac{1}{2\pi} \times \text{period}}$$

$$\text{period} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow c = \frac{x}{\frac{1}{2\pi} \times 2\pi\sqrt{\frac{m}{k}}}$$

$$= x\sqrt{\frac{k}{m}}$$

The speed of sound in a solid

In Unit A, 'Materials and mechanics', a solid was pictured as consisting of atoms connected by springy bonds. This model was used to calculate the Young modulus for steel. Now it can be used to calculate the speed of a wave in a solid. The atoms and bonds are modelled by trolleys connected by springs. From Unit A, the Young modulus, $E = k/x$, where k is the spring constant of the interatomic bonds.

So

$$c = x\sqrt{\frac{k}{m}} = x\sqrt{\frac{Ex}{m}} = \sqrt{\frac{Ex^3}{m}} = \sqrt{\frac{E}{m/x^3}}$$

But x^3 is the volume occupied by one atom; so $m/x^3 = \rho$, the density of the material

$$\Rightarrow c = \sqrt{\frac{E}{\rho}}$$

QUESTIONS 41, 42

DEMONSTRATION D12
Speed of sound in a metal rod

The same sort of argument applies to all mechanical waves. In each case, the speed of the wave depends on the properties of the substance through which the wave is travelling. Formulae for the speeds of many mechanical waves have the form $\sqrt{\frac{\text{force constant}}{\text{mass}}}$, or something equivalent.

Section D4 FORCED VIBRATIONS AND RESONANCE

Free and forced oscillations

A system which can oscillate may be set into oscillation in many ways. Two of these are particularly important.

Section D3 dealt with 'free' or natural oscillations. In this case the oscillator is given an initial displacement or velocity, and then released.

The second important situation is when a periodic repetitive driving force is applied to the oscillator in some way. This in general causes 'forced' oscillations. When the frequency of the driving force equals the oscillator's natural frequency, then the amplitude of the oscillations may build up to a large value. This special situation is called *resonance*.

Resonance

DEMONSTRATION D13
Forced vibrations of a mass on a spring

'Buildings, bridges, and wind' in the Reader *Physics in engineering and technology*

Resonance has wide ranging practical applications. Any machine or structure is likely to be subjected to periodic forces, either as a result of its own operation (*e.g.*, the motor in any vehicle imposes an oscillation or vibration on every part of the vehicle) or through the action of some external agent (*e.g.*, wind exerts a periodic force on buildings and structures through vortex shedding). If you keep your eyes and ears open you will notice countless examples of forced oscillations.

Forced oscillations can prevent machines operating efficiently, as when an unevenly loaded spin drier cannot achieve its normal working speed because much of its energy is being diverted into a violent wobbling. More seriously, forced oscillations can result in fatigue failure of metal components at stresses well below the tensile strength of the metal, simply as a result of repeated flexing (like breaking a piece of wire by bending it to and fro). If resonance occurs, forced oscillations can be violent and may have catastrophic results (as in the Tacoma Narrows bridge collapse). An understanding of forced oscillations is clearly essential to engineering.

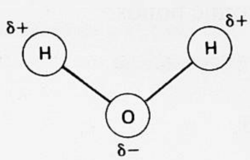


Figure D21

Forced oscillations are not always destructive; sometimes engineers and scientists can make positive use of them. Nor is the phenomenon confined to mechanical oscillations. Microwave ovens heat food as a result of a forced oscillation of the molecules within the food, particularly water molecules, which are polar (they are permanently charged positive at one end and negative at the other, see figure D21). Infra-red absorption spectroscopy, which is an important technique for chemists, involves the forced oscillation of atoms or groups of atoms within a molecule. The conversion of radio waves to electric currents in an aerial is an example of a forced oscillation, and the operation of a tuning circuit in a radio relies on resonance.

READING
Spectroscopy (page 236)

Many more examples are given in the books recommended for this Unit.

EXPERIMENT D14
Investigations of resonance

DEMONSTRATION D15
Barton's pendulums

An investigation of a resonant system reveals the following points.

i When a driving force (driver) acts on something which can vibrate, the initial *transient oscillations* are irregular, with varying amplitude.

ii These transient oscillations give way, in a time which depends on the degree of damping, to a *steady state*, in which the driven oscillator oscillates at the forcing frequency, regardless of its own natural frequency. (Damping is the result of friction-type forces which always act against the motion of an oscillator.)

iii The amplitude of the driven oscillation depends on the forcing frequency and rises to a maximum if the forcing frequency is equal to the natural frequency of the driven oscillator. These large amplitude vibrations are called resonant oscillations. See figure D22.

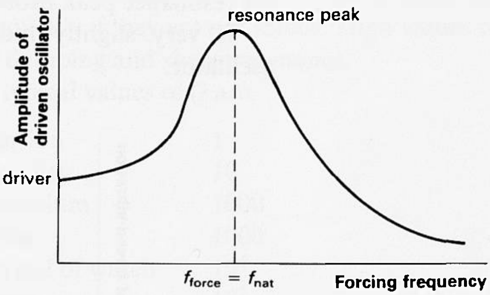


Figure D22
Resonance.

iv At resonance the driver and the driven oscillator are not in phase. The driver leads by one quarter of a cycle.

The photograph of Barton's pendulums in figure D23(c) was taken when the driver was at its maximum displacement to the left. The resonating pendulum is just passing through the centre of its oscillation, and moving to the left. It is one quarter of a cycle, or $\pi/2$, behind the driver. The shorter pendulums at the top of the picture, with higher natural frequency, are moving approximately in phase with the driver. The long pendulums with lower natural frequencies, are approximately in antiphase with the driver (phase difference of π). Notice how the pendulums which have natural frequencies close to the forcing frequency (that is, pendulums of similar length to the driver), oscillate with larger amplitude than the others.

The amplitude of the forced vibrations also depends on the degree of damping. The photographs in figures D23(a) and (b) illustrate how the amplitude of resonant vibrations is reduced by damping.

D

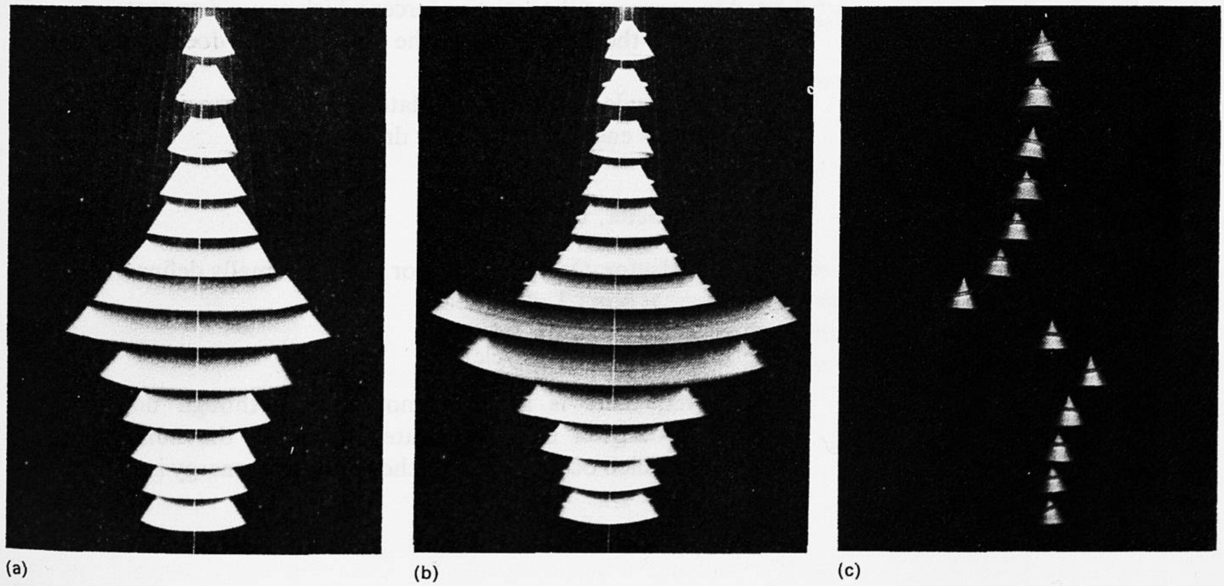


Figure D23
Photographs of Barton's pendulums.
(a) Time exposure (damped).
(b) Time exposure (less damped).
(c) Instantaneous.
A. W. Trotter.

Resonance curves (figure D24) reveal the effects of damping in more detail. Damping reduces the amplitude at all frequencies. It also makes the resonance peak broader (reduces the sharpness of resonance).

It very slightly reduces the resonant frequency of the driven oscillator.

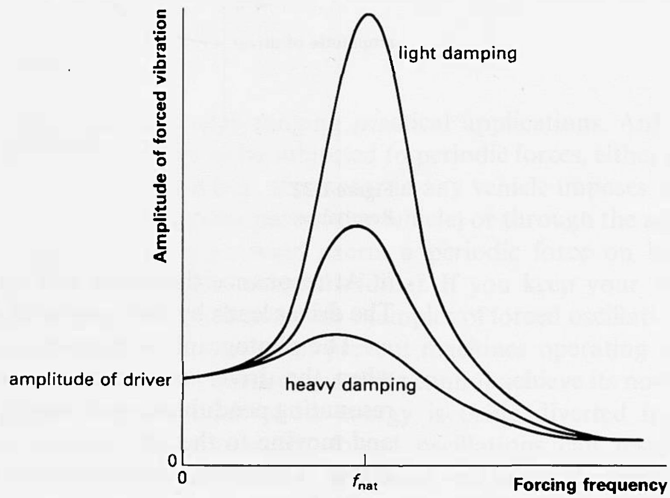


Figure D24
Resonance curves.

Energy in forced oscillation

QUESTIONS 43 to 53

The driver delivers energy to the forced oscillator during each cycle of oscillation. This energy may be:
i stored in the oscillator, increasing the amplitude (energy stored $\propto$ amplitude^2);
ii used to overcome the resistive forces which cause damping;
iii returned to the driver, later in the cycle (but this does not happen at resonance).

The amplitude of a forced oscillator goes on increasing until energy loss per cycle = energy provided by driver per cycle.

The quality factor, Q

The quality factor, Q, of an oscillator can be formally defined like this:

$$Q = 2\pi \times \frac{\text{energy stored}}{\text{energy lost per cycle}}$$

However, there is a much more useful, though non-rigorous, description of Q: it is approximately equal to the number of free oscillations which occur before all the oscillator's energy is gone.

Q is related to the degree of damping of the oscillator, and to the sharpness of its resonance peak. Low values of Q are associated with heavily damped oscillations which do not resonate violently and which die away quickly if they are not forced. High values of Q are associated with light damping and sharp resonance.

Some typical values of Q are:

Car suspension	1
Tethered trolley	10
Simple pendulum	1000
Guitar string	1000
Quartz crystal of watch	10^5
Excited atom	10^7
Excited nucleus	10^{12}

QUESTIONS 54, 55

Consider the guitar string, for example. The energy is emitted as sound waves, with a fundamental frequency of, say, 512 Hz (the C above middle C). If $Q = 1000$, then roughly 1000 oscillations occur before all the energy is gone. Thus the plucked string will cease to oscillate after $1000/512 \approx 2$ seconds: which agrees roughly with experience.

Standing waves and resonance

A standing wave is formed when identical waves travelling in opposite directions superpose.

In figure D25, P and Q represent points along a rope. At the instant shown, two wave trains travelling in opposite directions are just about to overlap at point P. Points L_1 to L_4, R_1 to R_4 represent peaks or troughs along the wave train.

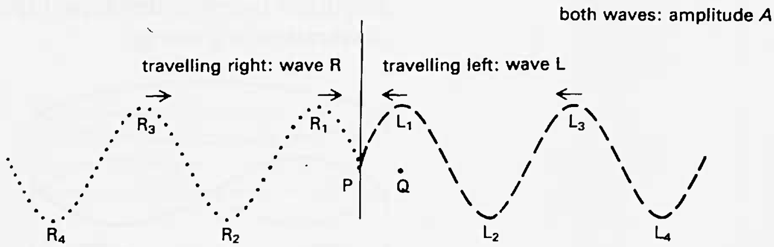


Figure D25
Formation of a standing wave.

Superposition at point P causes P to oscillate with amplitude $2A$, since peaks L_1 and R_1 arrive there simultaneously, followed half a cycle later by troughs L_2 and R_2, etc. Careful inspection shows that superposition at Q will result in Q remaining stationary in space all the time.

Points such as P are called *antinodes* (A); points such as Q are called *nodes* (N). Adjacent nodes are distance $\lambda/2$ apart. The motion of a section of rope on which a standing wave is occurring can be represented as in figure D26.

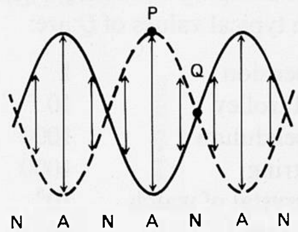


Figure D26
Representation of a standing wave.

All points between adjacent nodes oscillate in phase with each other; they are in antiphase with all points in the next half-wavelength section.

Resonance of a string with both ends fixed

DEMONSTRATION D16
Standing waves on a rubber cord

A wave in a stretched string cannot escape beyond either end; it must be reflected.

QUESTIONS 56, 59

If a string is made to vibrate near one end, waves travel to and fro along the string, being reflected each time they reach an end. If the length of the return trip for these waves is a whole number of wavelengths, that is, $2L = n\lambda$, where L is the length of the string, they will always pass the vibrator in phase with the wave which it is producing, even after several return trips. A standing wave of large amplitude therefore develops. Figure D27 illustrates some of the modes of vibration of a string.

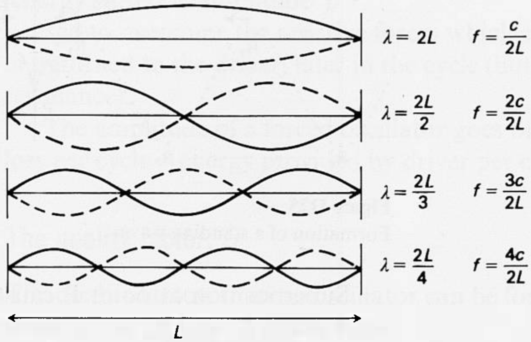


Figure D27

Large amplitude standing waves only occur for these well-defined wavelengths.

Standing waves in bounded systems

DEMONSTRATIONS D17
More complicated standing waves

HOME EXPERIMENTS DH4 to DH6

QUESTIONS 57, 58

Unit L, 'Waves, particles, and atoms'

The edges of any solid object act as boundaries to waves. Superposition of waves travelling towards the boundary with those reflected from it can lead to standing waves, if the object is vibrated at an appropriate frequency (unless the vibrations are damped). In a similar way, standing waves can be set up in fluids if they are contained (air in a trumpet, water in the bath). The same ideas are used to explain the energy levels of atoms.

These more complex standing waves have the following features in common with waves on a string:

i There is a series of definite modes of oscillation, corresponding to different frequencies, at each of which the response is large (resonance).

ii The patterns developed depend on the frequency, there being more nodes at higher frequencies.

iii The standing waves must 'fit' into the system.

D

READINGS

APPLICATIONS OF ULTRASONICS

Ultrasonic waves are compression waves travelling through a medium with frequencies higher than those of audible sound waves. Their existence may be demonstrated using an ordinary signal generator and loudspeaker to create them, and a microphone and C.R.O. to detect them. However, most practical ultrasonic systems use transducers which depend on either the piezoelectric effect or the magnetostrictive effect. (A 'transducer' converts energy, in this case from electrical energy to ultrasonic wave energy, or vice versa.)

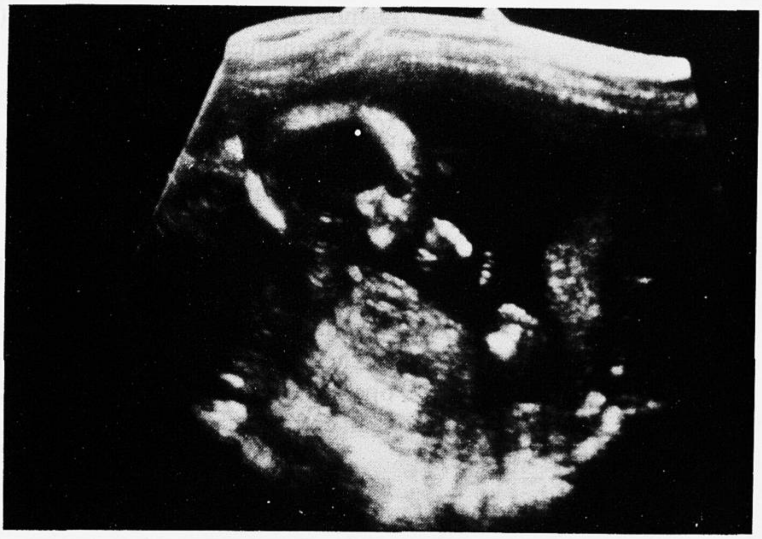
The piezoelectric effect occurs in certain crystals, such as natural quartz. When a p.d. from an external supply is applied across it, the crystal will alter its shape slightly; an alternating p.d. at high frequency will thus cause it to act like a miniature tuning fork, generating ultrasonic waves. Conversely, if the crystal is compressed or stretched by a mechanical force, such as occurs repeatedly when an ultrasonic compression wave arrives, a p.d. appears across it; hence it can also be used to detect the waves.

Magnetostriction involves the change of shape of certain metal alloys when they are magnetized, or the complementary effect of a change in their magnetization when they are stressed.

A number of applications involve an echo-sounding technique. A transducer sends out a pulse of ultrasonic wave energy; the same transducer is then used to detect any returning energy reflected from discontinuities in the medium. The time between transmission of the pulse and its returning echo, together with a knowledge of the wave speed in the medium, allows the distance to the discontinuity to be computed.

Ultrasonic flaw detection This is an example of a non-destructive testing method. Such methods are used where the material to be tested must not be cut up, broken down chemically, or even removed from its working position. A transducer using a frequency in the MHz range is placed in contact with the material to be inspected. The pulses are reflected from the rear surface of the material; they are also reflected from any cracks or flaws within it, including any which may be invisible from the outside. If the returning signals are displayed on a C.R.O., the position of a flaw and its approximate size can be judged from the trace. This technique is particularly valuable in the inspection of railway tracks and welded pipes.

Ultrasonic foetal scanning Many expectant mothers now receive an ultrasonic scan as part of their routine check-up on the progress of their as-yet-unborn baby. Ultrasonic waves penetrate the mother's skin, but are reflected back selectively by discontinuities in the tissues beneath. By examining the whole area of the foetus bit by bit, a complete 'picture' of it can be built up. (See figures D28(a) and (b).) This is another example of non-destructive testing: ultrasonic waves apparently cause no harm to mother or baby, whereas X-rays, which could be used to obtain a similar picture, would harm them.



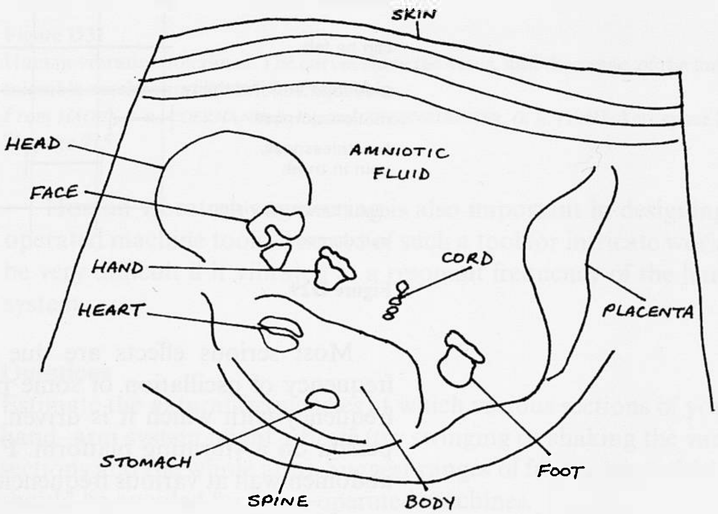


Figure D28
Ultrasound scan of 18-week foetus and explanatory diagram.
Department of Medical Illustration, St. Bartholomew's Hospital, London.

Ultrasonic flow measurement If the pulse is projected into a stream of liquid flowing in a pipe, then energy reflected back from minute discontinuities in the liquid will show a Doppler frequency shift depending on the flow rate. This is used as the basis of a 'non-invasive' flowmeter, so-called because a transducer can send and receive sound through the wall of the pipe, and there is no need to obstruct the flow as in many meters which, for instance, measure the rotation of a paddle-wheel inserted in the pipe.

The last example, ultrasonic flow measurement, is quoted from B. Jolly (Ed.) Hobsons Science Support Series, *Waves and sound.* CRAC Publications, Hobsons Press (Cambridge) Ltd, 1982.

Questions

a An ultrasonic transducer converts alternating p.d.s into ultrasonic wave energy (or vice versa). How might this be done using the phenomenon of magnetostriction? (What would be the essential parts of the transducer?)

b The speed of compression waves in a metal is of the order of $5000\,\mathrm{m\,s^{-1}}$. If your best laboratory C.R.O. is to be used for flaw-detection in a metal sample, estimate the length of the smallest sample that could satisfactorily be tested.

THE EFFECTS OF VIBRATION ON PEOPLE

We know that the slow oscillations of a rolling ship can produce sea-sickness, although the origin of car-sickness is less clear. Machine operators are subject to more rapid vibrations; the pneumatic road drill is an extreme example.

Some effects of oscillations of various frequencies are shown in figure D29.

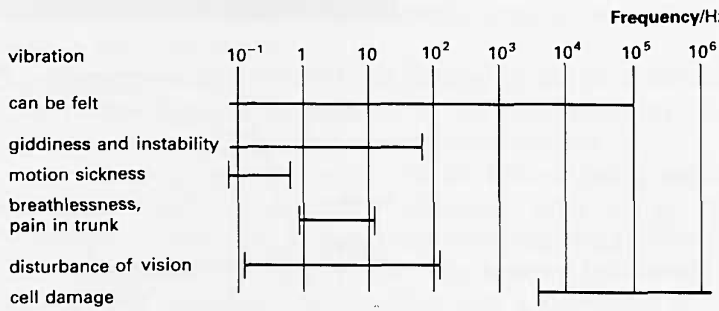


Figure D29

Most serious effects are due to *resonance* – when the natural frequency of oscillation of some part of the body is the same as the frequency with which it is driven. This has been studied by sitting a person on a vibrating platform. Figure D30 shows the motion of the abdomen wall at various frequencies.

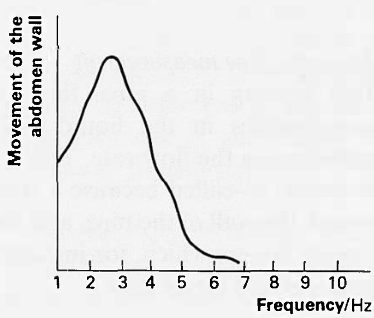


Figure D30

The tolerance of human beings to vibration varies with frequency. You can see this in figure D31, which is a graph showing the results of a study of human vibration tolerance. Such studies are of especial importance in designing aircraft and space probes.

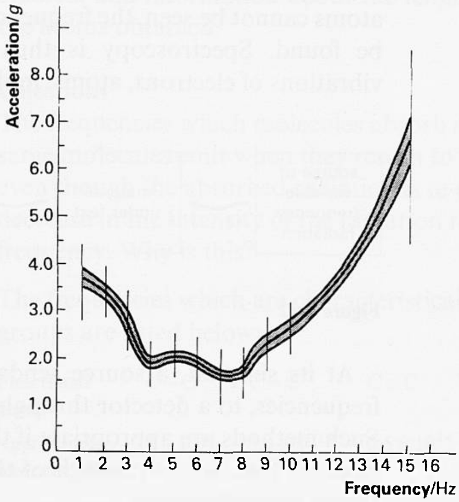


Figure D31
Human vibration tolerance. The curves show the value, and the range, of the limit of tolerable acceleration at various frequencies.
From MAGID, E. B., COERMANN, R. R., *and* ZIEGENRUECKER, G. H. (1960) Aerospace Medicine, **31**, *page 915*.

Human vibration engineering is also important in designing hand-operated machine tools. The use of such a tool for intricate work would be very difficult if it vibrated at a resonant frequency of the hand–arm system.

Questions

a Estimate the natural frequencies at which various sections of your hand–arm system might vibrate (try swinging or shaking the various sections and the whole arm). Suggest ranges of frequencies which should be avoided for hand-operated machines.

b Figure D31 shows that humans are very intolerant of acceleration when they are subjected to vibrations between 3 to 9 Hz. Use information from figure D29 to suggest what discomforts might be experienced under these conditions.

c Figure D31 shows the maximum tolerable acceleration, during one cycle of an oscillation, plotted against oscillation frequency. Using the formula $a_{max} = -\omega^2 A$, compute the maximum tolerable *amplitudes* of oscillations at 1 Hz, 4 Hz, 8 Hz, and 15 Hz. Hence sketch a graph of maximum tolerable amplitude of oscillation against frequency (use logarithmic scales).

d Comment on figure D30 (showing the movement of the abdomen wall against frequency) in the light of your knowledge of resonance.

SPECTROSCOPY

If one could see the atoms in a molecule vibrating, and time their oscillations, one could obtain useful information about the stiffness, k, of the bonds between them, using $2\pi f = \sqrt{k/m}$. Although the vibrating atoms cannot be seen, the frequency at which they absorb radiation can be found. Spectroscopy is thus a valuable tool for studying the vibrations of electrons, atoms, molecules, or ions.

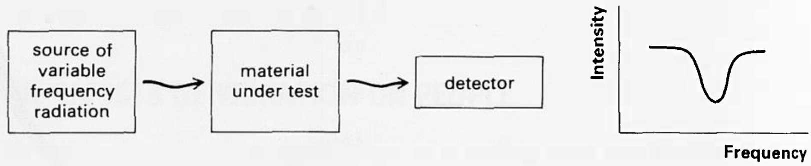


Figure D32

At its simplest, a source sends radiation, at a range of controlled frequencies, to a detector through the material under test (figure D32). Such methods are appropriate if the frequency of vibration is relatively slow, so that the wavelength of the electromagnetic radiation is more than a few millimetres.

Many interesting vibrations are faster, but we cannot vary continuously the frequency of sources of infra-red or visible light.

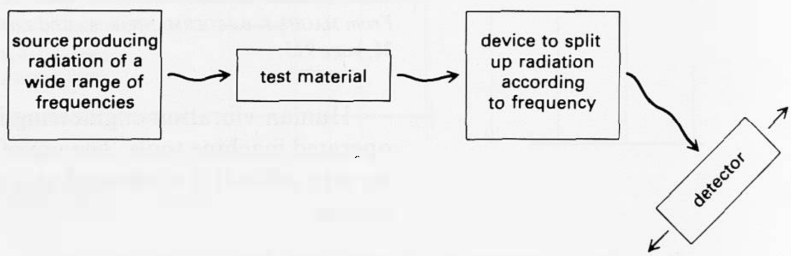


Figure D33

In figure D33, radiation of a wide range of frequencies is shone on the test material, and a device (a grating or prism, for example) sends the radiation of each frequency off in a different direction. Alternatively, the detector could in principle be tuned to each frequency in succession, although this is less useful in practice.

Uses of spectroscopic information

The stiffness of bonds in molecules, or in solids, may be found. For example, the bond stiffness of H_2 is $5.2 \times 10^2\ \mathrm{N\,m^{-1}}$.

The analysis of complex organic compounds is assisted by studying their infra-red absorption spectra, for many types of bond tend to absorb at much the same frequency even though the atoms form part of different molecules. The spectrum can then be used as a means of indicating which bonds are present. For instance, aliphatic C—C bonds oscillate in an in-and-out (stretching) manner at a little below 10^{14} Hz.

Dyes, whose function is to colour, must absorb visible radiation strongly at selected frequencies. It is possible to design molecules which will absorb at a desired frequency.

At microwave frequencies, the spinning motion of molecules can be studied, and information about the length of bonds and the masses of the atoms obtained.

Questions

a The frequencies which molecules absorb are the frequencies which the same molecules emit when they return to their unexcited state. But even though the absorbed radiation is re-emitted, there is a detectable decrease in the intensity of the radiation reaching the detector at this frequency. Why is this?

b The frequencies which are characteristically absorbed by certain groups are listed below:

Functional group	C–C	C=C	C≡C	C–H
Approximate frequency/Hz	4×10^{14}	5.3×10^{14}	6.6×10^{14}	9.0×10^{14}

i Compare the stiffnesses of the single, double, and triple bonds between carbon atoms (*i.e.*, calculate the ratios of the stiffnesses).
ii Why does the C—H group have the highest frequency of oscillation among those tested?

QUARTZ AND ATOMIC CLOCKS

(from G. W. Dorling, Longman Physics Topics, *Time*. Longman, 1973.

The quartz clock

In the 1930s a new type of clock started to replace the most accurate pendulum clocks as a standard for measuring time. This was the quartz crystal clock. The time keeper in this case is a quartz crystal instead of a pendulum. A quartz crystal will vibrate elastically with a natural period of its own, just like a tuning fork. In this case, however, electrical charges constantly build up and die away on its surface in time with the vibrations. It is this effect, the piezoelectric effect, which makes it so easy both to keep the crystal vibrating and to use the vibrations to control the frequency of electrical oscillations in other circuits.

It is these electrical oscillations, accurately controlled by the vibrations of the quartz crystal, that drive the hands of the clock, or control its display.

The frequency of the quartz crystal vibrations is sharply defined by the dimensions of the crystal. It is much less affected by variations in external conditions than the pendulum.

Why do we believe that these clocks are so much more reliable than pendulum clocks? There is an important test we can do. We can ask how well these clocks keep time with each other. Suppose two quartz clocks are adjusted to read exactly the same time and then left to run without adjustment. Comparisons of their time readings at various times later have indicated a difference of no more than 0.0005 second per day over a period of a week or so. This suggests that a quartz clock will measure a time interval of 1 day, or 86 400 seconds, to within 0.0005 seconds; an accuracy of better than 1 in 10^8. This is ten times better than that which could be obtained with the best pendulum clock.

Such comparisons are made continuously as quartz clocks are usually run in groups of three. This is because the likeliest disturbance to a quartz clock's time keeping is a failure of one of the electronic components. Simultaneous failure of all three clocks is most unlikely.

Quartz clocks were initially developed in response to the demand from scientists and engineers for more and more precise time standards, for the purposes of radio communication, navigation, and pure research. It was also in response to this demand that the atomic clock was developed in 1954.

The atomic clock

Atoms can emit and absorb energy only at very sharply defined frequencies. Provided a suitable atom is chosen, they can be used to control the frequency of radio waves from an electronic oscillator.

In 1958 a clock, based on a beam of caesium atoms, was successfully constructed on this principle. The electronic oscillator is controlled by a quartz crystal whose vibrations are in turn controlled by the effect on the beam of caesium atoms of radio waves produced by the oscillator. As in the case of the quartz clock, it is these accurately maintained electrical oscillations which ultimately drive the clock.

Comparison of the timekeeping of two of these clocks showed that they could be relied upon to an accuracy of 1 part in 10^{11} over an apparently indefinite period of time. To put this in a slightly different way, this meant that they could be relied upon to within 1 second in 3 000 years!

A new time-scale

The development of a clock of such high reliability highlighted the strain that this increasing demand for precision had thrown on the astronomical unit of time. Both quartz and now atomic clocks showed greater consistency amongst themselves than they did with the sidereal day. Again, appeal to the laws of physics showed good reason why the Earth's period of rotation should vary from day to day and year to year. At first variations which could be calculated were incorporated into adjustments of the astronomical time-scale to make it more uniform. Atomic time showed, however, that there were some important irregular variations in the Earth's rotation as well.

In 1964 general agreement was reached for a new time-scale based on the atomic clock. The frequency of the energy level transition of the caesium atom involved in the atomic beam clock was determined as precisely as possible in terms of the then accepted value of the second and was found to be

9 192 631 770 ± 20 Hz

The ± 20 Hz represented the uncertainty in the value of the astronomical second rather than the uncertainty in the value of the frequency.

The atomic second was then defined as exactly 9 192 631 770 periods of the oscillation associated with the caesium atom for this particular transition.

In this way the atomic second is the same time interval as the previously defined second based on astronomical time. This is essential, for we must still be able to use the new scale to tell the time of day.

Because the length of the mean solar day and the year vary by small amounts when viewed against the atomic time-scale, occasional adjustments are made to the atomic scale to keep it in step with the year, in much the same way that the number of days in the year is occasionally adjusted to keep *them* in step with the year. Astronomical observations must remain the basis for determining the time of day, and they are constantly compared with the atomic time-scale. In this way the demand for a time-scale of precisely repeated equal intervals is brought into line with the need for a time-scale to tell the time of day and season of the year.

D

Questions

a Show that the frequency of the energy level transition of caesium used to define the second is consistent with a radio wavelength. What is the wavelength?

b A similar redefinition to that which befell the second has occurred with another fundamental unit, the metre. What is now the definition of one metre?

c *i* Explain the meaning of this sentence more fully: 'The ± 20 Hz represented the uncertainty in the value of the astronomical second rather than the uncertainty in the value of the frequency.'
ii Roughly what uncertainty, in Hz, would be attributable to uncertainty in the value of the frequency? (Information which enables you to answer this question is given in the text.)

LABORATORY NOTES

GROUP OF EXPERIMENTS
D1 How do oscillators move?

You will be asked to find some method of showing how the displacement of the oscillators listed below varies with time. A graph of displacement against time is called a time trace. The basic equipment for the oscillator will be available, but you may need to ask for other equipment when you have decided how you are going to produce its time trace.

Whichever oscillator you work with, consider these questions:
i Does the method of obtaining the time trace affect the period?
ii Does the method of obtaining the time trace affect the damping (resistance to motion)?
iii If you are relying on several different 'runs' to take, for example, a series of times, can you be sure that all runs are carried out under the same conditions?
iv What assumptions are you making if you interpolate between the points you have obtained (for example, by joining them to form a continuous graph)?
v Is the oscillation isochronous?
vi Whether or not it is isochronous, on what factors does the period of the oscillation depend? Try some experiments to confirm your ideas. Can you find any quantitative rules?

Here are some suggestions for setting up the oscillators and measuring the time traces.

D1a Pendulum

There are many ways of producing its time trace, from a simple sand pendulum to the electrical methods shown in figures D34(a) and (b). But remember these are not the only possibilities.

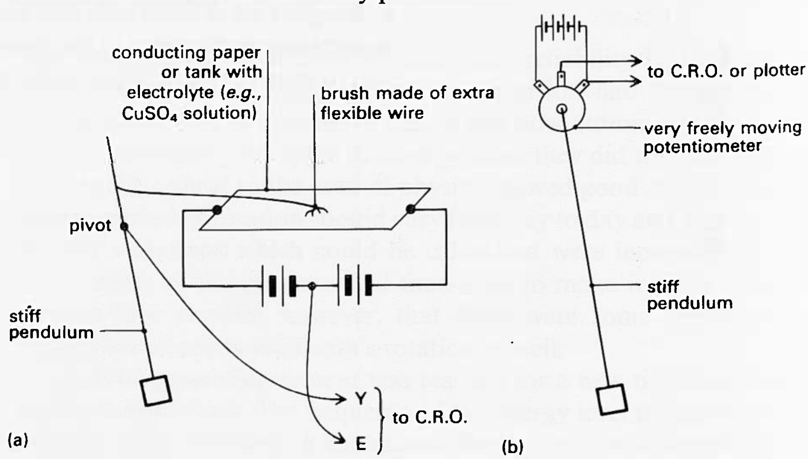


Figure D34
Obtaining the time trace of a pendulum.

D1b Torsion pendulum

string
2 retort stand bases
3 retort stand rods and bosses
G-clamp, large

Hang a retort stand rod horizontally on two parallel lengths of string. The ends of the rod may be loaded with bosses. It might be possible to adapt the potentiometer method illustrated in figure D34(b) to obtain this time trace.

D1c Lath with load

either
metre rule
or
long lath

2 G-clamps, large
clean smooth paper (*e.g.*, computer print-out paper)
felt-tip pen or brush with ink
2 rubber bands
2 masses, 1 kg

Clamp the metre rule to a stool or table so that it oscillates in a horizontal plane.

Figure D35 shows one possible way of obtaining the time trace.

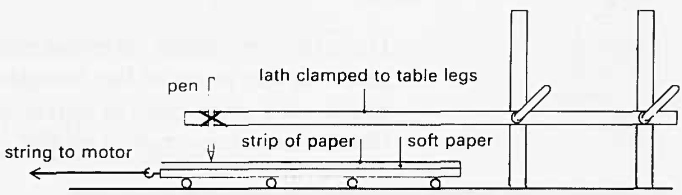


Figure D35
Lath with load.

D1d Inertia balance (wig-wag)

inertia balance
2 G-clamps, small

D1e Ball rolling on curved tracks

3 lengths of curtain rail
large ball-bearing, 1 or 2 cm diameter

Three curvatures to investigate are shown in figure D36.

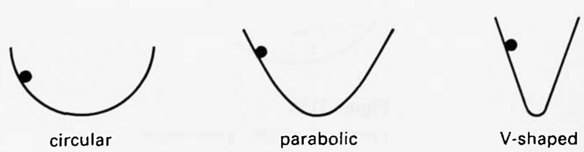


Figure D36
Ball rolling on curved tracks.

D1f Mass oscillating vertically on spring

expendable spring
retort stand base, rod, boss, and clamp
G-clamp, large
hanger with masses totalling 400 g

D1g Undamped light beam galvanometer

light beam galvanometer
cell holder with one cell
switch
resistance substitution box
leads

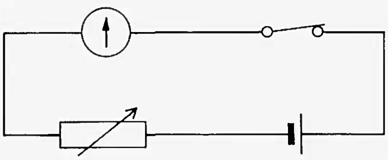


Figure D37

Set up the circuit with the galvanometer on its least sensitive scale; then increase the sensitivity until, with a resistance of over 500 kΩ, the spot reaches almost a full-scale deflection with the switch closed. Then, with the galvanometer on its 'direct' setting, open the switch: the spot will oscillate about its central zero position.

D1h Bar magnet suspended over another magnet

cylindrical magnet
horseshoe magnet
retort stand base, rod, boss, and clamp
nylon fishing line

Hang the bar magnet on nylon or cotton so that it is horizontal and lies just over the poles of the horseshoe magnet resting on its back. You might use a small piece of mirror attached to the suspension to observe the oscillations by optical means.

D1i Large-amplitude pendulum

turntable clamped vertically (or large gyroscope)
boss (or small G-clamp)
retort stand base, rod, and boss

With the boss or G-clamp on the edge of the turntable or gyroscope, the system can be made to execute large-amplitude oscillations (see figure D38).

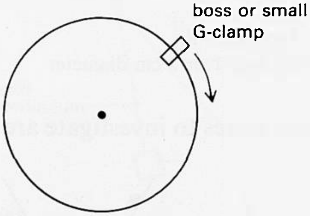


Figure D38
Large-amplitude pendulum.

D1j Air track vehicle running between elastic barriers

air track with rubber bands at both ends, and blower
air track vehicle

D1k U-tube containing liquid

large U-tube filled with water or potassium manganate(VII) solution

DEMONSTRATION
D2 Oscillators and circular motion

either
record-player turntable
or
fractional horsepower motor, with gearbox, turntable, and band
l.t. variable voltage supply

2 pendulum bobs
retort stand base, rod, boss, and clamp
compact light source
screen

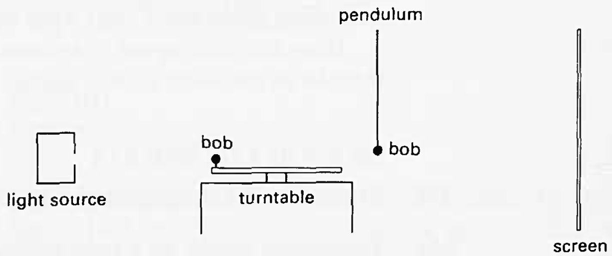


Figure D39
Oscillator and circular motion.

Observe the shadows of the two pendulum bobs as they move across the screen (figure D39). What do you see on the screen if the pendulum is exactly the right length to synchronize with the rotating bob, and the pendulum is released just as the rotating bob passes it? Why do you think this happens?

What happens if the pendulum is not released at the correct moment? What happens if the pendulum is not of exactly the right length? Use the words 'phase difference' in describing your observations.

DEMONSTRATION

D3 Basic ideas about waves

either
ripple tank kit
or
long spring
or
large Slinky spring
or
any other wave machine

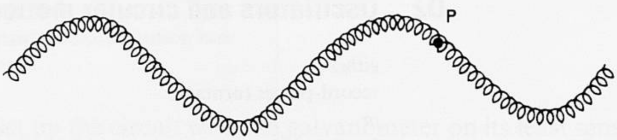


Figure D40
Wave profile.

Define the terms *wavelength*, *frequency*, and *amplitude* for a wave like that shown in figure D40. What is meant by the *undisturbed position* and the *displacement* of a point such as P? What determines the frequency of the wave, and what unit is frequency measured in?

How are wave speed, wavelength, and frequency related? How does P move as the wave travels along?

GROUP OF EXPERIMENTS

D4 Properties of mechanical waves

D4a Transverse waves on a long narrow spring

D4b Transverse waves on a Slinky spring

long spring
Slinky spring
metre rule
optional
string
large curtain ring
retort stand base, rod

It is easier to see what is happening if you make single hump-like pulses by giving the end of the spring or Slinky a single sideways flick.

Observe the pulse as it travels, with a view to answering questions such as:
Does the speed depend on the shape of the pulse – its height or length?
Does the speed depend on how rapidly you flick the end of the spring?
Does the speed depend on the spring – how could the speed be made larger or smaller?
Does friction make any difference to the speed of the pulse? to its shape?
What decides the shape of a pulse?
What happens when pulses, starting from opposite ends of the spring, meet?

What happens when the pulse reaches the far end of the spring, and that end is not free to move?

If you have time, try the following:

i Attach a large nylon curtain ring to the end of the spring and slide the ring onto a retort stand rod. When the pulse reaches this end, the end is free to move. What happens to the pulse?

ii Tie a piece of thick string or cord onto the end of the spring. What happens to the pulse as it moves from the spring to the string or vice versa? Can you explain this?

D4c Transverse waves on a trolleys-and-springs model

12 dynamics trolleys
44 expendable steel springs
12 masses, 1 kg (or 12 more trolleys)
stopwatch or stopclock

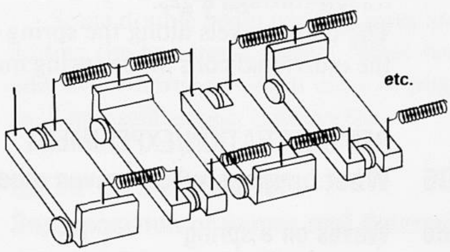


Figure D41
Transverse wave model made of trolleys and springs.

The model is made of a row of trolleys linked by springs, as shown in figure D41, with the trolleys spaced out so that the springs are in tension. It is best to set the model up on the floor, or on a surface with raised barriers along the edges to prevent the trolleys running off.

Make a transverse pulse travel along the model by moving the end trolley sharply from side to side. You should be able to answer these questions:

Does the speed of the pulse depend on its shape – its height or length?

Does the speed of the pulse depend on how quickly you move the end trolley?

Does the speed depend on the spacing of the trolleys?

What happens to the speed if the mass of each trolley is doubled? (Do this by adding loads, or stacking a second trolley on each one.)

What happens to the speed if the tension between the trolleys is doubled? (Do this by putting an extra spring in parallel with each existing spring.)

What happens to the speed if both of these changes are made at the same time?

By what factor does the speed change in each of these cases?

Explain why these changes affect the speed.

B

DEMONSTRATION

D5 Longitudinal waves on a Slinky spring

large Slinky spring

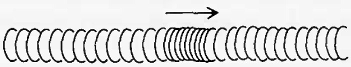


Figure D42
Longitudinal pulse on a Slinky.

The Slinky should be on a smooth surface.

Make a longitudinal pulse by moving the end of the spring sharply to and fro once only in the direction of the spring. (See figure D42.)

Look carefully at a pulse as it passes and see if you can answer the questions:

What makes the pulse move along the spring?

Do compression and expansion pulses travel at the same speed?

How does the speed at which the individual coils of the spring move compare with the speed at which the pulse moves along the spring?

Use this model to explain how sound, which is a longitudinal wave, travels through a gas.

The pulse travels along the spring from one end to the other. How do the individual coils of the spring move?

DEMONSTRATION/EXPERIMENT

D6 What happens when waves meet?

D6a Waves on a spring

long spring

What happens where transverse pulses meet? Does this meeting have any permanent effect on the pulses?

What happens when two wave trains of equal frequency and similar amplitude meet? (You can create this situation by sending a single wave train from one end of the spring with the other end fixed: the second wave train is caused by reflection of the first at the fixed end.)

D6b Ripples on water

ripple tank kit

Waves of the same frequency spread from each dipper. Observe the effects on the resulting pattern of adjusting the frequency of the waves and of altering the separation of the two dippers. Explain these effects.

DEMONSTRATION

D7 Path differences and phase difference

signal generator
loudspeaker
2 microphones
double beam oscilloscope
metre rule
leads

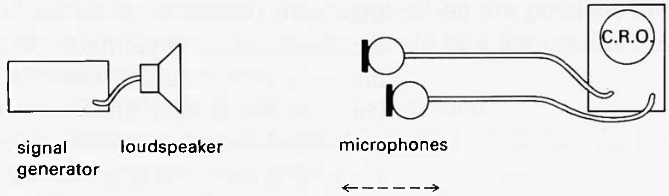


Figure D43
Path differences and phase difference.

The oscilloscope traces show the electrical oscillations produced in the two microphones by the sound waves (see figure D43). The sound from the single loudspeaker has to travel different distances to the two microphones. Because of this, the two traces will probably not be in phase. For what path differences will the oscillations be *i* in phase, *ii* in antiphase (phase difference of half a cycle)? Use this to determine the wavelength of the sound waves.

Some double beam oscilloscopes are able to display the resultant of adding the two input signals. What resultant would you expect if you add two vibrations which are *i* in phase, *ii* in antiphase? If you have such an oscilloscope, then try this.

D

GROUP OF EXPERIMENTS

D8 Superposition of waves and determination of wavelength

In each of these experiments you should be able to demonstrate that the radiation you are using has wave properties; you should also try to measure the wavelength. Then if you know the frequency you can calculate the wave speed.

D8a 1 GHz radio waves

15 cm dipoles and oscillator

either
microammeter
or
galvanometer (*e.g.*, internal light beam)
or
general purpose amplifier and loudspeaker

2 metal screens
leads
metre rule

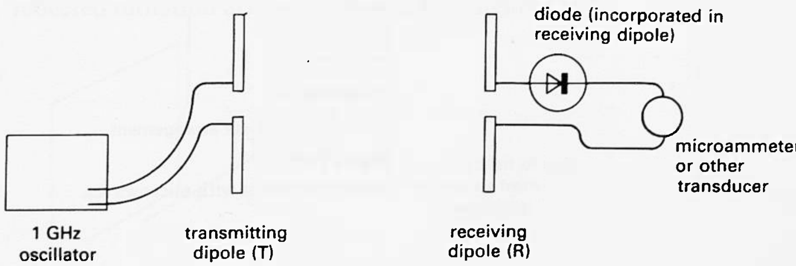


Figure D44
Transmitting and receiving 1 GHz radio waves.

Start with a simple investigation of the waves (figure D44). For example: are they blocked by your arm, or by metal screens? Does their strength diminish with distance? Does the orientation of the receiving dipole (vertical, horizontal) affect the magnitude of the signal? Does the position of the receiver (different positions around the transmitter; above the level of the table) affect the signal received?

Now look for superposition. Use the metal screen to reflect radiation to the receiving dipole so that waves can follow two different paths to reach it: directly from the transmitter and indirectly via the screen.

Figures D45 and D46 show two possible arrangements.

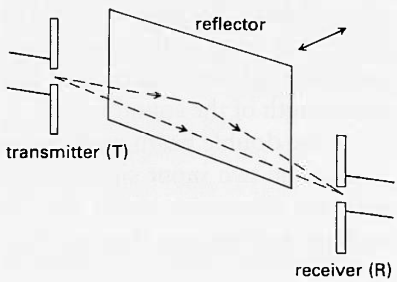


Figure D45

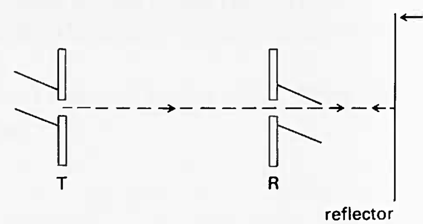


Figure D46

Look for maxima and minima. Hence measure the wavelength of the 1 GHz waves, and estimate the percentage uncertainty in your result.

From your measurement calculate the speed of 1 GHz waves.

D8b Microwaves

microwave transmitter
microwave receiver
2 metal reflectors
narrow metal plate
general purpose amplifier
loudspeaker
microammeter (may be incorporated in receiver)
diode probe receiver
metre rule
leads

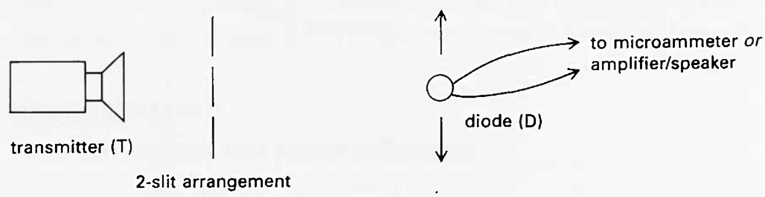


Figure D47
2-slit experiment with microwaves.

Various arrangements for superposition are possible, one of which is shown in figure D47. The slits should be a few centimetres wide, and a similar distance apart.

Will the waves be in phase at the two slits?

At positions where the signal falls to a minimum the intensity is not necessarily zero. Why not?

Microwaves from the transmitter must travel in different directions to reach the two slits. If they continued in these directions they would not overlap and no interference would result. Why do they interfere? What must have happened? Can you use the equipment to check your answer?

Other arrangements to try are suggested in figures D48(a) and (b).

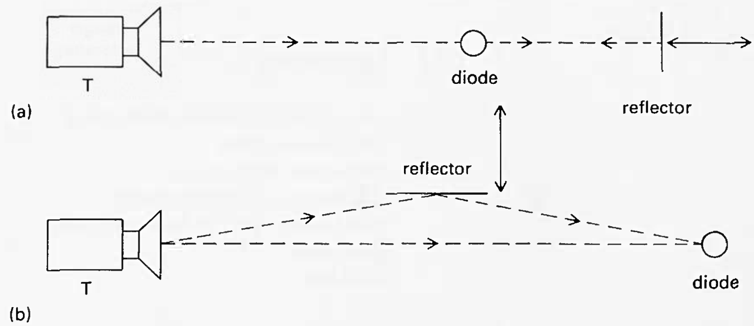


Figure D48
Simple superposition experiments with microwaves.

The apparatus you use may be labelled '3 cm wave equipment', but the actual wavelength is unlikely to be exactly 3 cm. Make the best measurement of the wavelength that you can (estimate the percentage uncertainty in your result).

D8c v.h.f. radio waves or u.h.f. television waves

portable radio capable of receiving v.h.f.
television set
television aerial
coaxial cable
metal reflector
metre rule

You will need to find out the direction to the radio or television transmitter and then place a reflecting screen so that both direct and reflected radiation arrives at the receiver (figure D49).

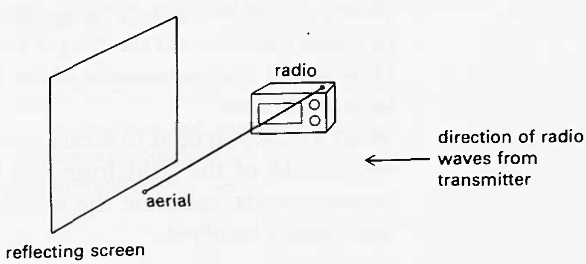


Figure D49

A similar arrangement may be used for u.h.f. television broadcasts, but if the aerial is not rigidly connected to the television set then the set itself need not be moved.

For the v.h.f. radio the reflecting screen should be as large as possible: say 1 m × 1 m. For the television waves, the same screens as were used in experiments D8a and D8b will do.

With the aerial very close to the reflector, is the signal received a maximum or a minimum? Explain why.

Try to find evidence that radio/television radiations have wave properties. Measure the wavelength(s), and estimate the percentage uncertainty in the result. If you know or can find out the frequency, calculate the wave speed.

D8d Light waves

sodium lamp or sodium flame pencil
2 microscope slides
micrometer screw gauge
thin paper, *e.g.*, cigarette paper
retort stand base, rod, boss, and clamp
glass plate
hand lens

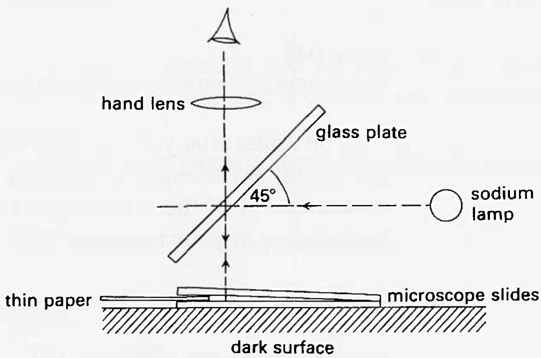


Figure D50
Superposition of light waves by reflection at a wedge.

Take care that your head, hair, or clothes do not get too close to the Bunsen flame.

The microscope slides must be clean.

The light and dark fringes are caused by the superposition of light waves and are clearer when viewed through a microscope.
Where do the two sets of waves come from?
In which direction do the fringes run? Why?
How would the appearance of the fringes differ if a different colour of light were used?
What would you need to measure, or find out, in order to determine the wavelength of the light from this arrangement? Make the necessary measurements, calculate the wavelength, and estimate the percentage uncertainty involved.

D8e Sound waves

signal generator
2 loudspeakers
microphone
pre-amplifier
oscilloscope
metal reflector
metre rule

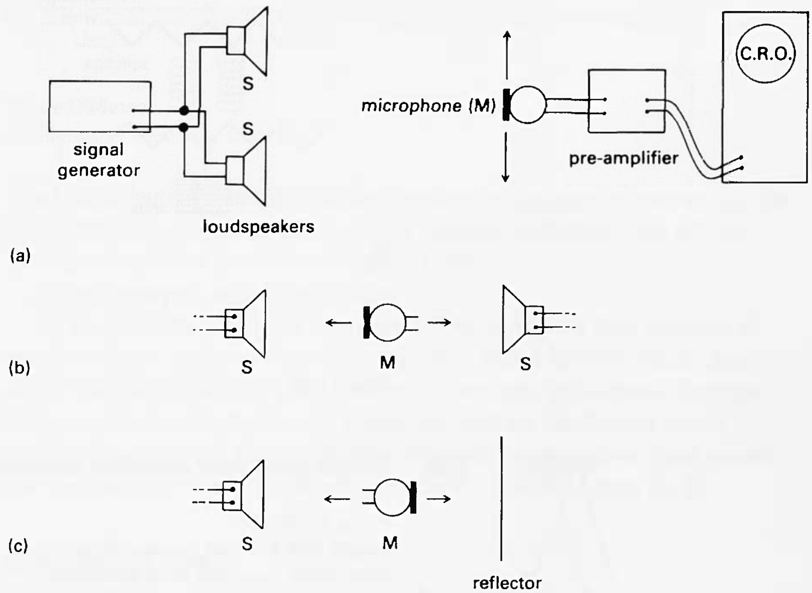


Figure D51
Superposition of sound waves.

Two sets of overlapping waves may be produced either by using two loudspeakers, figure D51(a) and (b), or by using one loudspeaker and a reflector, figure D51(c). Set up a demonstration of superposition and use it to measure the wavelength of the sound. Try with a frequency of about 3000 Hz; measure λ, calculate c, and estimate the percentage uncertainty. Then halve the frequency, and repeat the experiment. Do you arrive at the same value for c? Is this what you expect?

EXPERIMENT
D9 Factors affecting the period of an oscillator

D9a Mass on spring

4 expendable steel springs
hanger with 8 slotted masses, 100 g
retort stand base, rod, boss, and clamp
stopwatch or stopclock
stiff wire and pliers

Try the following investigations:
i How (for a given arrangement of masses and springs) does T, the time for one oscillation, depend on the amplitude, A?
ii How does T depend on m (for a given arrangement of springs)?
iii How (for a given mass, m) does T depend on k, the spring constant of the arrangement? (What is the effective k of several springs arranged in series and in parallel as in figure D52?)

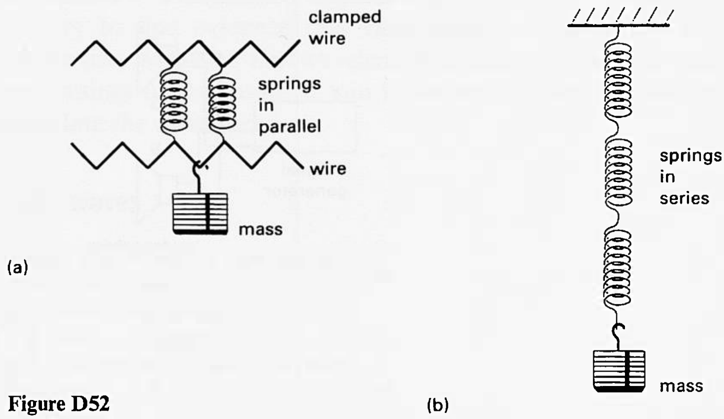


Figure D52

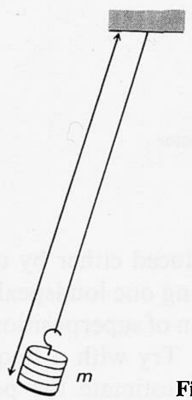


Figure D53

D9b Simple pendulum *(optional alternative)*

string, 2 m length
hanger with 8 slotted masses, 100 g
retort stand base, rod, boss, and clamp
2 metal strips (as jaws)
stopwatch or stopclock
metre rule

Try the following investigations:
i How (for a given mass m and length l) does T, the time for one oscillation, depend on the amplitude, A?
ii How does T depend on m (for a given length l)?
iii How does T depend on l (for a given mass m)?

EXPERIMENT
D10 Oscillation of a tethered trolley

dynamics trolley
2 retort stand bases and rods
2 G-clamps, large
runway for trolley
6 expendable steel springs
newton spring balance, 10 N
ticker-tape vibrator, carbon paper disc, gummed ticker-tape
transformer
leads
metre rule
masses, 100 g and 10 g
Plasticine

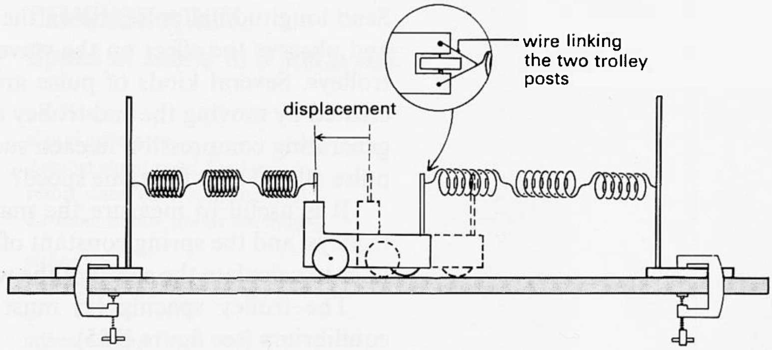


Figure D54
Oscillation of a tethered trolley.

Find the effective k of the complete system of springs by measuring the force needed to displace the trolley a suitable distance, with all the springs connected as shown in figure D54.

Find the mass, m, of the trolley.

It is desirable to adjust the mass of the trolley so that k/m has a simple value – your teacher may suggest a value for the whole class. Since the trolley is only going to travel one way, you should friction-compensate the slope. Obtain a tape for half an oscillation of the trolley. You can use this to obtain a displacement against time graph for the motion; or your teacher may have a different plan for it.

DEMONSTRATION

D11a Longitudinal wave on trolleys-and-springs model

11 dynamics trolleys
11 masses, 1 kg (or 11 extra trolleys)
20 compression springs
20 spring holders
metre rule
newton spring balance, 10 N

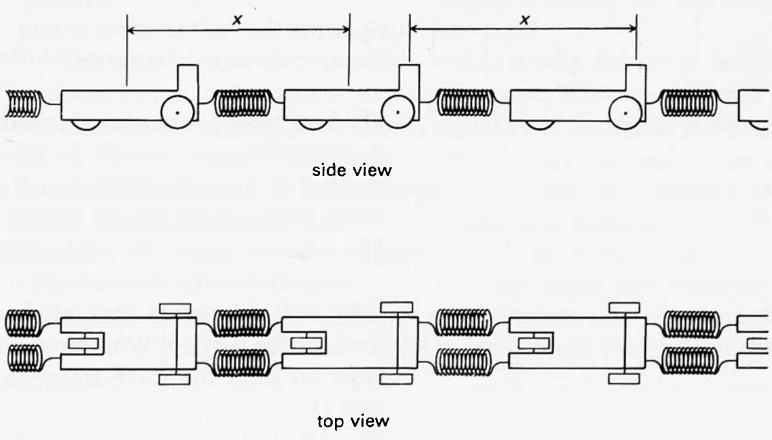


Figure D55
Trolleys linked by compression springs.

D

Send longitudinal pulses down the line of trolleys shown in figure D55, and observe the effect on the wave speed of increasing the mass of the trolleys. Several kinds of pulse are possible; the one analysed later is created by moving the end trolley at a steady speed towards the others, generating compression in each successive spring. Do different kinds of pulse all travel at the same speed?

It is useful to measure the mass of a trolley, the spacing between trolleys, and the spring constant of the springs now, as they will be used later to calculate the speed of the wave theoretically.

The trolley spacing (x) must be measured with the trolleys in equilibrium (see figure D55).

$$\text{The spring constant } (k) = \frac{\text{force needed to extend one pair of springs}}{\text{extension produced by force}}$$

DEMONSTRATION

D11b Measuring the speed of the wave

Apparatus as for experiment D11a with:
aluminium block
timer, resolution 10 ms
insulated copper wire
2 crocodile clips
leads

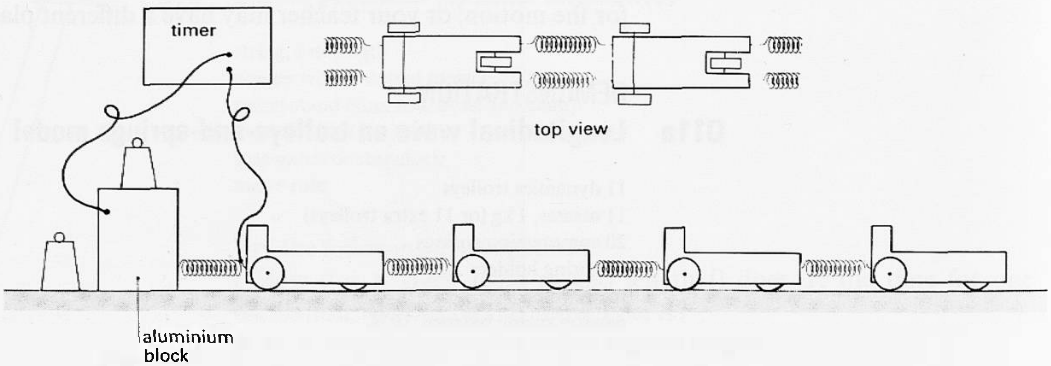


Figure D56
Timing a pulse up and down a row of trolleys.

The four trolleys are moved together towards the block. When the first spring makes contact with the block, a compression pulse sets out down the line of trolleys; it reflects off the open end as a rarefaction pulse, which travels back, finally pulling the end spring away from the block. The timer measures the total contact time.

Some points to think about:
Why is it important that all four trolleys move at the same speed towards the barrier? What time is the timer measuring?
How far does the wave travel in this time? Hence what is the wave speed?
What will be the major source of uncertainty in this experiment?
How would you expect the time recorded to change if the trolleys were more massive? If the springs were stiffer?

DEMONSTRATION

D12 Speed of sound in a metal rod

oscilloscope
signal generator
2 retort stand rods, 1 m long
retort stand base, rod, and boss
2 rubber bands, about 10 cm long

either
crocodile clip
or
adhesive tape

hammer, club or claw head, at least 0.5 kg
leads

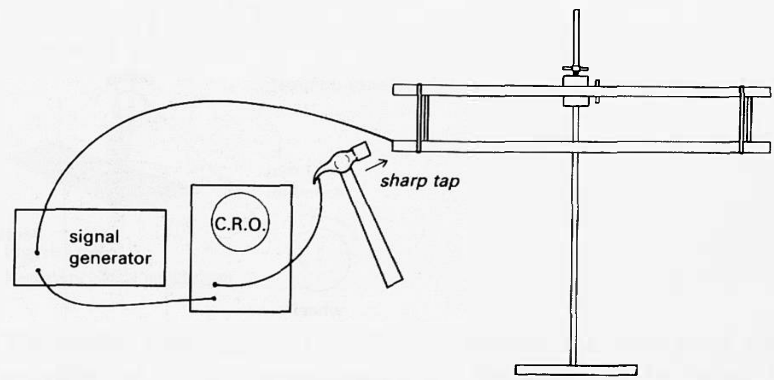


Figure D57
Speed of sound in a metal rod.

Compare the situation shown in figure D57 with the one in demonstration D11b. The steel rod replaces the row of trolleys, and the hammer acts as the aluminium block. What determines how long the rod remains in contact with the hammer? (Remember what happened when the row of trolleys hit the block.) Does it matter that the hammer moves towards the rod, rather than vice versa?

The contact time is very small, which is why the timer has been replaced by a signal generator and oscilloscope. When the hammer and rod are in contact the high frequency signal from the signal generator is fed to the oscilloscope. A frequency of about 25 kHz is used, so the time for one oscillation is 1/25 ms. So if four cycles of the signal appear on the screen, then the hammer and rod have been in contact for $4 \times 1/25$ ms. Measure the contact time, and the distance the wave has travelled in this time. Hence calculate the speed of the wave, and estimate the percentage uncertainty involved. Suggest reasons why the percentage uncertainty in this experiment is high. How could the experiment be improved?

DEMONSTRATION

D13 Forced vibration of a mass on a spring

2 expendable springs
slotted masses and hanger
Perspex tube or wide glass tube
light string

either
vibrator and signal generator
or
wheel with pin offset about 1 cm
fractional horse power motor
variable voltage supply
single pulley

leads

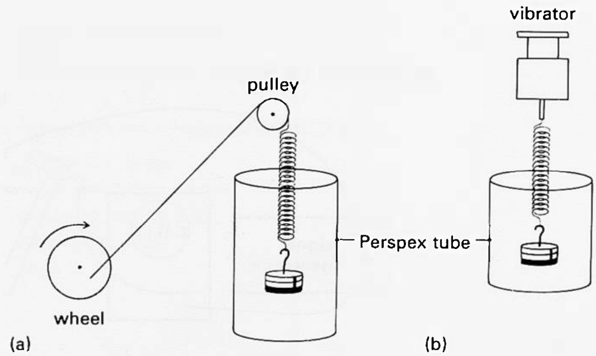


Figure D58
Forced vibration.

How does the mass on a spring behave when it is subjected to a periodic force? How does its behaviour depend on the frequency of the driving force? Why is the tube necessary?

EXPERIMENTS

D14 Investigations of resonance

Find out what you can about forced oscillations. Devise your own experiment(s) using one of the two sets of equipment shown below. Make observations and measurements which reveal in detail some aspect(s) of the relationship between the motion of the forced oscillator and the driver. Investigate the effect of damping (energy loss).

D14a Resonance of a pendulum

resonance kit
metre rule
retort stand base, rod, boss, and clamp
stopclock
card

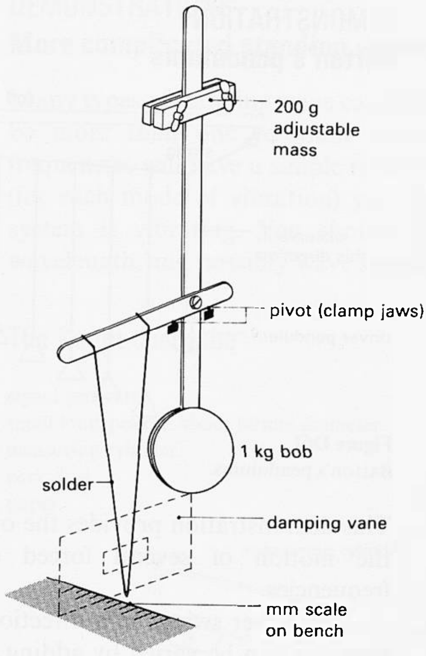


Figure D59
Resonance of a pendulum.

The solder pendulum is the driven oscillator; the horizontal strip provides the driving force. The driving frequency can be varied by moving the adjustable mass.

D14b Resonance of a mass on a spring

2 expendable springs
mass, 50 g
thread
single pulley with suitable support
large beaker or measuring cylinder
metre rule

either
signal generator
vibrator
or
wheel with pin offset about 1 cm
motor
power supply for motor

drinking straws

The mass on the spring is the driven oscillator. The thread supplies the driving force. Try the experiment with and without damping.

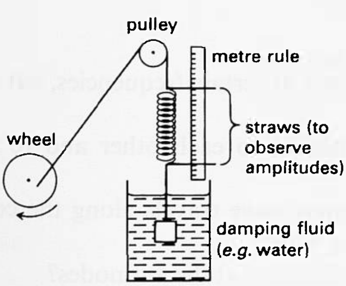


Figure D60
Resonance of a mass on a spring.

D

DEMONSTRATION
D15 Barton's pendulums

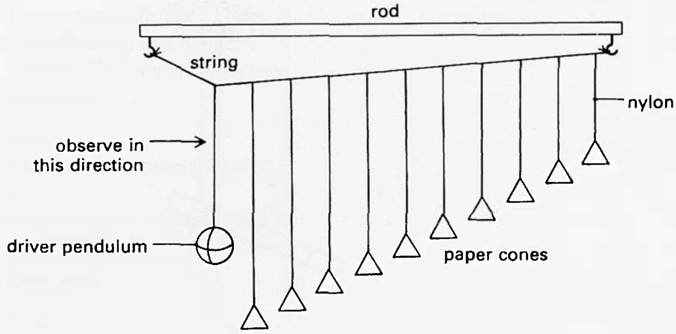


Figure D61
Barton's pendulums.

This demonstration provides the opportunity to observe and compare the motion of several forced oscillators with differing natural frequencies.

The driver swings in a direction in and out of the paper. Relative damping can be varied by adding extra mass to the paper cones, *e.g.*, using plastic curtain rings. What factors control the amplitude, phase, and frequency of the forced vibrations of the paper cones?

EXPERIMENT
D16 Standing waves on a rubber cord

signal generator
vibrator
xenon flasher
rubber cord (0.5 m long, 3 mm square cross-section)
2 retort stand bases, rods, bosses, and clamps
4 metal strips (as jaws)
2 G-clamps, large
leads

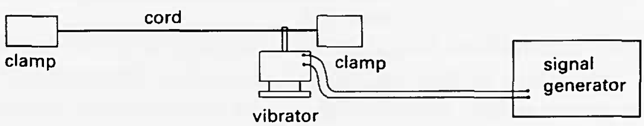


Figure D62
Standing waves on a rubber cord.

Why does the cord show a large response at certain frequencies, but not at other frequencies?
How are the resonant frequencies related to each other and to the length of the cord?
When the vibrator is first switched on a wave travels along the cord. How does this develop into a standing wave?
What factors affect the amplitude of vibration at the antinodes?
Is there an optimum position for the vibrator? Is the vibrator always at a node or an antinode?

DEMONSTRATIONS

D17 More complicated standing waves

Many types of standing wave can be demonstrated. There will generally be more than one resonant frequency, and in some cases these frequencies will have a simple relationship. At each resonant frequency (for each mode of vibration) you should try to understand how the system is vibrating. You should be able to make an estimate of wavelength, and possibly wave speed.

D17a The Kundt dust tube

signal generator
small loudspeaker, about 60 mm diameter
measuring cylinder
cork dust
paper

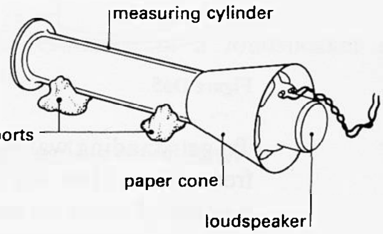


Figure D63

A thin layer of cork dust on the bottom of the glass tube shows the position of nodes and antinodes.

D17b Longitudinal standing waves in rods

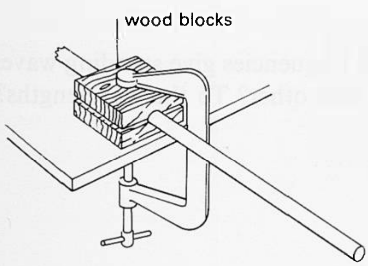


Figure D64

glass, steel, or brass rod, about 10 mm diameter and about 1.5 m long
G-clamp
wooden blocks
cloth
rosin (for metals)
alcohol (for glass)

Rub the rod with the rosined or dampened cloth. If you can find a way of estimating the frequency of the sound emitted, then you can go on to estimate the speed of sound in the rod. What is the wavelength of the standing wave being produced?

D17c Vibrations of circular wire rings

signal generator
vibrator
xenon flasher
copper wire, 0.9 mm diameter, or thinner steel wire
leads

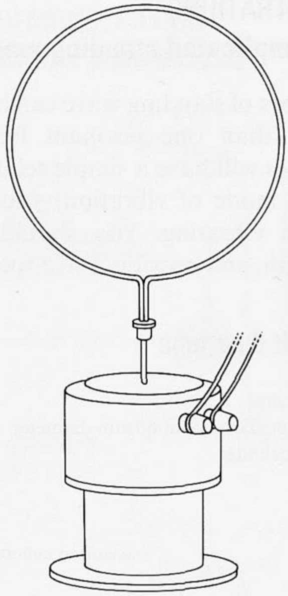


Figure D65

To get standing waves on the wire loop, it must be vibrated at specific frequencies. How are these frequencies related to each other? To the number of nodes on the loop?

D17d Longitudinal standing waves

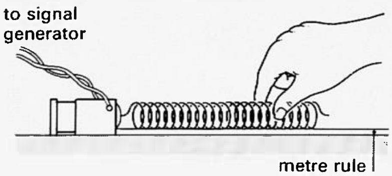


Figure D66

signal generator
vibrator
xenon flasher
long spring
metre rule
leads

The spring should be stretched. What frequencies give standing waves? How are these frequencies related to each other? To the wavelengths?

D17e Vibrations in a rubber sheet

signal generator
large loudspeaker
xenon flasher
sheet of rubber
2 retort stand bases and rods
big metal ring (*e.g.*, embroidery ring)
leads

A rubber diaphragm stretched over a ring can be excited by placing it over a loudspeaker. Lines drawn on the rubber help to show up the vibration patterns. You should be able to see several modes of vibration. Is there a simple relationship between the resonant frequencies? Between the patterns obtained?

D17f Chladni figures

signal generator
vibrator
square or round metal plate
sand
leads

The metal plate is attached centrally to the vibrator. Sand is used to reveal the vibration pattern. Many modes of vibration exist. Measure the resonant frequencies. Are they simply related?

D17g Vibrations of a loudspeaker cone

signal generator
large loudspeaker
xenon flasher
leads

Watch the resonance of a loudspeaker cone under stroboscopic illumination.

D17h Standing waves in a round bowl

signal generator
vibrator with dipper attached
Petri dish
large plastic bowl
wooden block

Arrange a dipper to generate circular ripples at the centre of the bowl or dish. You may be able to see stationary ring patterns on the water surface at various frequencies.

D17i Standing waves – musical instruments

oscilloscope
microphone
assorted musical instruments

Standing waves are set up when musical instruments are either plucked, blown, struck, or stroked. Usually the standing wave pattern is a complex one, and waves of several different frequencies are present. You can see the different waveforms of the sounds produced by different instruments on the oscilloscope.

HOME EXPERIMENTS

DH1 Making a chronometer

Throughout the history of science and technology one of the most difficult problems has been the development of an accurate, robust chronometer. The task is to make a device which can measure, accurate to 1 second, any time interval between 0 and 3 minutes. Compare your design with others in the class and see who can produce the most accurate device.

DH2 Make your own wave machine

Fix drinking straws at intervals along a piece of sewing tape, as shown in figure D67.

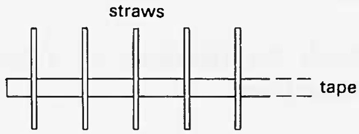


Figure D67
Simple wave machine.

You can add mass to each straw by putting small screws into the ends. Suspend the system vertically from one end of the tape; or clamp each end of the tape so that it is horizontal. You are now ready to do experiments with the wave machine; try, for example, sharply twisting one of the straws around the tape at an angle of about 45°.

DH3 A mechanical oscillator

Using either a home-made spring or combinations of springs that you can find in the laboratory, make a mechanical oscillator that vibrates, say, at 5 Hz. You should not approach this problem on a 'Trial and error' basis, but rather from detailed knowledge of the spring constant of your device. You must also try to think of accurate ways to measure the frequency of your oscillator. Maybe you might use a strobe, or perhaps some voltage-inducing device coupled with an oscilloscope.

DH4 Standing waves in a rectangular tank

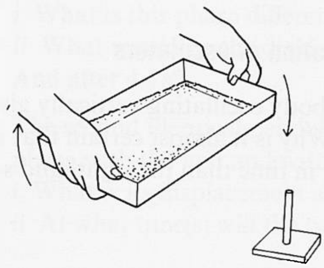


Figure D68

It is easy to excite water into a 'slopping' mode, which is why it is hard to carry pans of water. What can you say about the Q (quality factor) of this system? You may be able to excite other modes of oscillation using the plunger, for example, by moving it up and down in the middle of a large tank of water, and in other positions.

DH5 Standing waves under a running tap

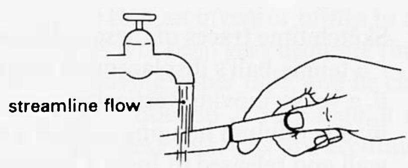


Figure D69

Hold a knife-blade about 2 cm below a smoothly running tap. Look for standing waves in the water flow. How does the pattern depend on the speed of water flow? On the separation of knife and tap?

DH6 Step waves under a running tap

Water flowing down from a tap onto a flat surface, like the under-surface of a baking tray, shows a strange discontinuity: as the water flows away from the impact point, there is a step-like increase in its depth, at a distance r all round the impact point. (This is nothing to do with standing waves, but is a nice piece of physics relevant to this Unit!) Find out what factors affect r and try to explain this phenomenon.

D

QUESTIONS

The motion of oscillators

1(I)a A car body oscillating vertically gives a time trace as shown in figure D70. Why is it almost certain that the lefthand side of the graph is earlier in time than the righthand side?

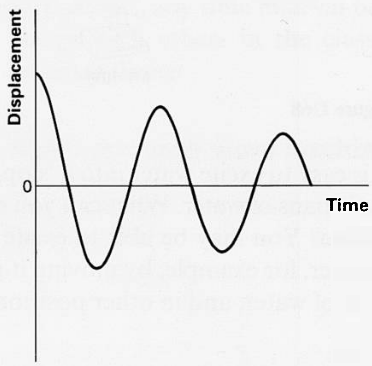


Figure D70

b Sketch time traces of these motions:
i a tennis-ball's displacement measured from the net during a rally;
ii a yo-yo moving vertically;
iii a pendulum hanging against a wall, after it is pulled away from the wall and released (it loses a fraction of its energy on each collision with the wall).

2(P) The graph in figure D71 shows the time trace of a pendulum (about 10 m long) during one complete oscillation.

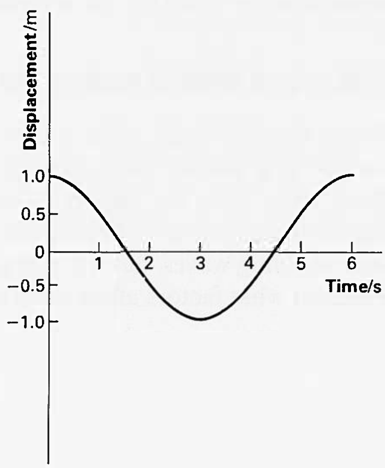


Figure D71

a What is the amplitude of this oscillation?

b Consider another identical pendulum oscillating with the same amplitude as the first one, but with a phase lag of $\pi/2$ radians.
i What is this phase difference as a fraction of one cycle?
ii What would be the displacement of this second pendulum after 3 s? And after 4.5 s?

c The second identical pendulum is now stopped, and released again in phase with the first and with an amplitude of 0.5 m.
i What is its displacement at $t = 3$ s?
ii At what time(s) will the two pendulums have the same displacement?

Time and its measurement

3(E) In 1761, the Board of Longitude, which had been set up to consider claims for a government prize of £20 000 for a clock which could keep accurate time at sea, arranged a test of a clock made by John Harrison. This clock was more accurate and reliable than any other mechanical clock that had ever been made (except pendulum clocks, which will not work well on a rolling ship). Harrison and his clock were sent on a sea voyage to Jamaica, to test the clock. How could a test possibly be made, if there were no better clocks to compare it with?

D

4(E) Suppose that an inventor brings to a standards laboratory a clock that he claims will keep very accurate time. His clock punches dots onto a roll of moving paper tape, and he claims:
1 That it does so at very regular intervals.
2 That these intervals are accurately $\frac{1}{5}$ second.

The laboratory has a standard clock of its own which produces an audible 'pip' once every half second.

a How would the laboratory set about testing the two claims made by the inventor?

b Is it possible that claim *1* can be true, but claim *2* not true? Is the reverse possible?

c The laboratory finds that claim *1* is not true; it judges that the time-dots come irregularly. The inventor replies that it is the laboratory clock that is irregular, not his. Can the conflict of opinion be resolved?

5(E) Until 1964, a time interval of one second was defined as $\dfrac{1}{24 \times 60 \times 60}$ of a 'mean solar day'. To-day one second is the time for 9 192 631 770 oscillations of a particular radiation from a caesium atom. Discuss:

a whether one of these definitions is more 'correct' than the other;

b whether one definition is more practical and useful than the other;

c why the change was made.

Angles

6(P) A circle has a radius of 2 m. What angle is subtended at the centre of the circle by an arc of length 4 m? Give your answer in radians and also to the nearest degree.

Angle approximations

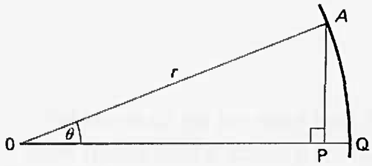


Figure D72

7(L)a In terms of θ (in radians) and r, what is the length (in figure D72) of:
i the arc AQ
ii the straight line AP?

b If θ is very small, it can be seen from the diagram that arc length AQ and straight line AP are approximately equal. From your answer to **a**, what does this suggest about $\sin\theta$ and θ, if θ is small?

8(L) Use a calculator to make a table of θ (in radians) and $\sin\theta$, $\cos\theta$, and $\tan\theta$ for $\theta = 20°, 10°, 5°, 2°, 1°, 0.5°, 0.1°$.

a What is the percentage error made in assuming $\sin\theta = \theta$ (radians) for:
i $\theta = 10°$
ii $\theta = 1°$
iii $\theta = 0.1°$?

b As θ becomes small, what are suitable approximations for
i $\cos\theta$, *ii* $\tan\theta$?

The behaviour of waves

9(P) Two fishing floats on a lake are 20 m apart. Waves travel along the water surface from a point in line with the floats, so that each float bobs up and down 30 times per minute. Someone notices that when one of the floats is on a wave crest, the other is in a trough, and there is one crest between them. Calculate the speed of the wave.

10(P) A wave pulse travels along a Slinky. An instant in the process is sketched in figure D73. Q, R, and S are points on the Slinky.

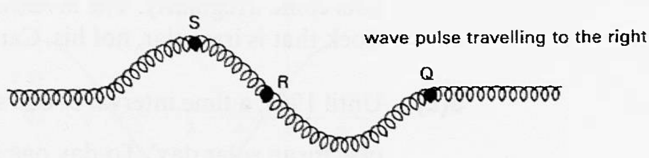


Figure D73

a Describe the states of motion (velocity and acceleration) of the points Q, R, and S at this instant.

b What two physical factors affect the speed of the wave along the Slinky? Explain qualitatively why each factor affects the speed.

11(E) A compression pulse travelling along a Slinky spring carries energy. Is it kinetic energy (because the coils of the spring are moving) or potential energy (because the spring is squashed by the pulse) or both?

What happens to the energy when a compression pulse going one way coincides with an expansion pulse going the opposite way, and the two superpose to give no net compression or expansion?

12(P) Figure D74 shows three hypothetical graphs of displacement, s, against distance for three different travelling waves at time $t = 0$. In each case, draw a graph of displacement, s, against time, t, for the point marked P, for the range $t = 0$ to $t = 6$ seconds. Show as much numerical information on the graphs as you can.

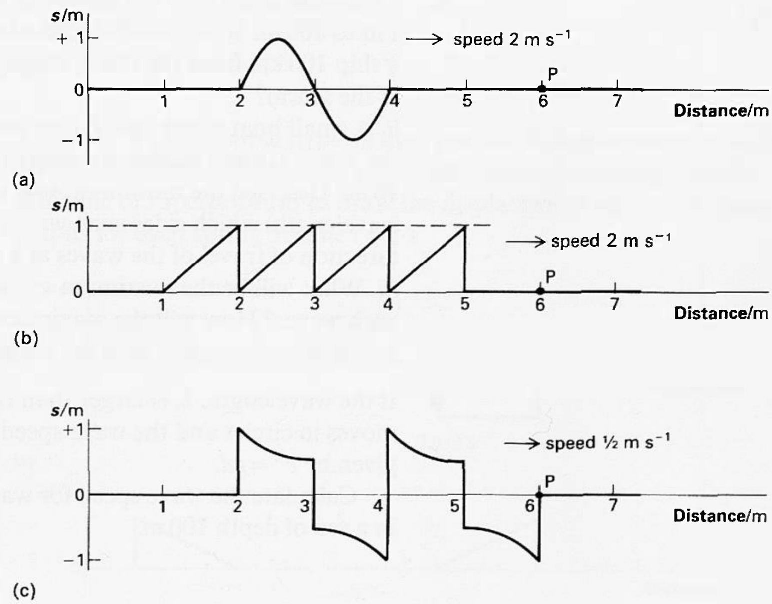


Figure D74

13(R) This question is about the behaviour of water waves.

Figure D75 shows a wave in water deep enough for the wavelength to be much less than the depth. As the wave moves to the right, the water at P acquires in succession the velocities v_1, v_2, v_3, v_4, and v_5 ($v_5 = v_1$) and it can be shown that it moves in a circle with constant speed, where the radius of the circle is equal to the wave amplitude, A.

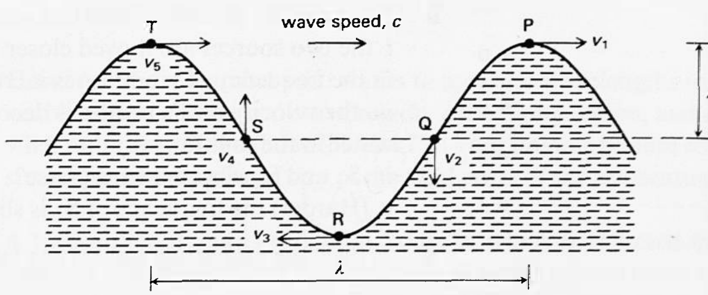


Figure D75

a *i* How would you find the time it takes for a water particle to go once round in a circle?
ii If the water particles are moving in circles, how would you find the magnitudes, and what are the directions, of their accelerations at P, and at Q?

b The speed, c, of deep water waves is given by $c^2 = \lambda g/2\pi$.
Sketch graphs of *i* speed, c, and *ii* frequency, against wavelength, λ, for deep water waves in the range $\lambda = 1$ m to $\lambda = 100$ m. Indicate on your sketch the orders of magnitude of speed and frequency for wavelengths 1 m and 100 m. What would you plot in order to obtain straight line graphs relating wavelength, λ, to *i* speed, *ii* frequency?

c *i* Suppose a storm at sea generates waves of wavelengths in the range 1 m to 100 m. What wavelengths of waves from the storm will be felt by a ship 100 km from the storm during the 24 hours following the onset of the storm?
ii A small boat at sea has to ride such waves. What will be the speed of the water around circles in waves of wavelengths 100 m and amplitude 10 m? Describe the motion of such a boat (short compared to the wavelength) which rides such waves and also travels forward in the direction of travel of the waves at a mean speed of about $2\,\mathrm{m\,s^{-1}}$.
iii What will be the maximum vertical acceleration of the water in such waves? How will the maximum force on the yacht causing this acceleration compare with the weight of the yacht?

d If the wavelength, λ, is larger than the depth, d, the water no longer moves in circles and the wave speed for such shallow water waves is given by $c^2 = gd$.
Calculate the wave speed for waves of wavelengths 1 m and 1000 m in a sea of depth 100 m.

(Long answer paper, 1981)

Superposition of waves

14(P) In figure D76, S_1 and S_2 are two water wave sources in a ripple tank. They are vibrating at the same frequency and amplitude. There is a maximum disturbance at A, a minimum at B, another maximum at C, and so on.

a Write an expression for the wavelength of the ripples.

b How will the pattern of maxima and minima change if:
i the two sources are moved closer to each other?
ii the frequency of vibration is increased?
iii the velocity of the ripples is decreased (by reducing the depth of the water in the tank)?
iv S_1 and S_2 vibrate in antiphase?
v (Harder) the frequency of S_1 is slightly greater than that of S_2?

c Why is the amplitude unlikely to fall to zero at the minima?

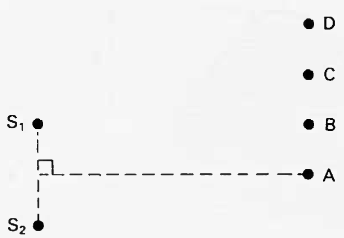


Figure D76

15(P) With the arrangement of wave transmitter, T, receiver, R, and reflector shown in figure D77(a), the signal strength received at R is a maximum. When the reflector is removed to the position shown in figure D77(b), the signal reaches a minimum. Why? Suggest a value for the wavelength of the radiation. Why can you not be sure of the value? What are some other possible values?

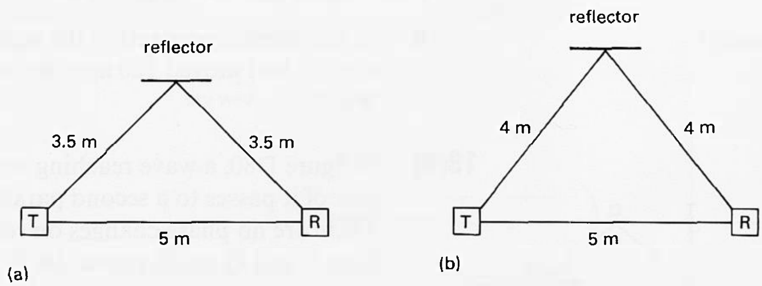


Figure D77

16(P) Figure D78 shows three hypothetical graphs of displacement, s, against distance at time $t = 0$ for wave pulses on a stretched spring. Use the principle of superposition to draw the displacement against distance graphs for each spring at time $t = 1$ s.

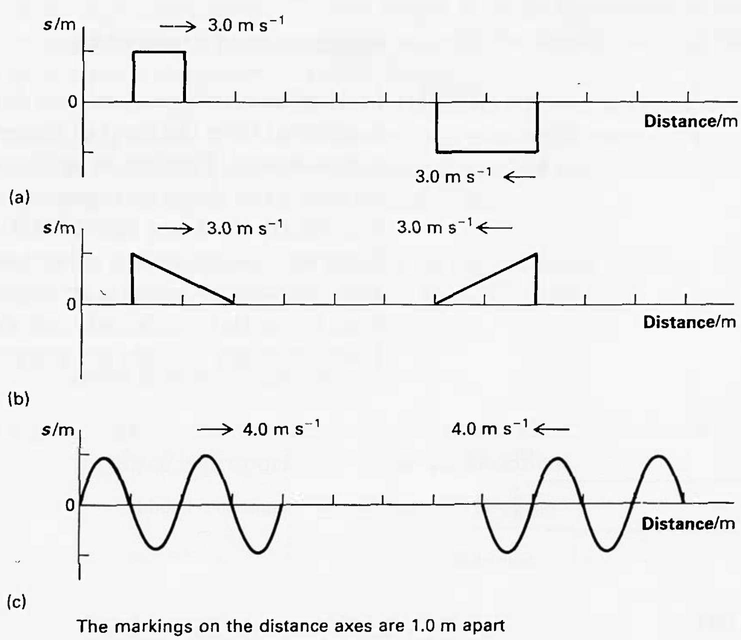


Figure D78

17(E) How would a researcher who wants to measure wavelengths in *a* the visible region, *b* the microwave region, *c* the X-ray region, and *d* the v.h.f. radio region set about the task? In your answer be sure to give some idea of the order of size of the vital parts of the apparatus.

18(P) A transmitter, T, emits radiation, some of which is reflected from a partially reflecting screen, S_1, and some of which carries on to be reflected from a second screen, S_2 (figure D79). The radiation reflected

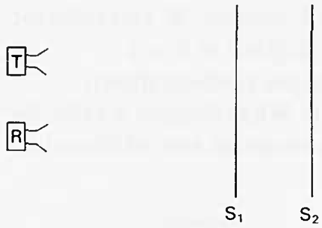


Figure D79

back from S_1 and S_2 is detected by a receiver, R, placed alongside T. At a certain separation of S_1 and S_2 the receiver records zero signal. S_2 is then moved away from S_1. As S_2 is being moved, the detector records a signal minimum and S_2 is moved on until the detector again records a minimum signal, a total movement of 120 mm.

a What is the wavelength of the radiation?

b At the original separation the signal detected was very nearly zero; but after S_2 had moved 120 mm the minimum signal was quite perceptible. Why?

19(R) In figure D80, a wave reaching surface 1 is partly reflected at P, but part of it passes to a second parallel surface where it is reflected at Q. There are no phase changes on reflection or transmission. The waves from P and Q are in step at JK if

A $\lambda = 2PT$ **B** $\lambda = PQ$ **C** $\lambda = PS$
D $\lambda = PQ + QR - PS$ **E** $\lambda = PS - QR$

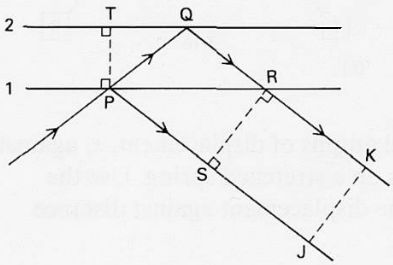


Figure D80

(Coded answer paper, 1979)

Applications of superposition

20(P) In designing a camera lens, it is desirable that as little light as possible is reflected from the front of the lens, so that as much light as possible is transmitted. This can be achieved by 'blooming', coating the outer surface of the lens with transparent material one-quarter of a wavelength thick (see figure D81) — as the following questions will illustrate. (If light strikes either boundary from *above* and is reflected back *upwards*, its phase is changed by π; if it strikes either boundary from *below* and is reflected back *downwards*, its phase is unchanged.) Possible routes for light are shown in figure D82.

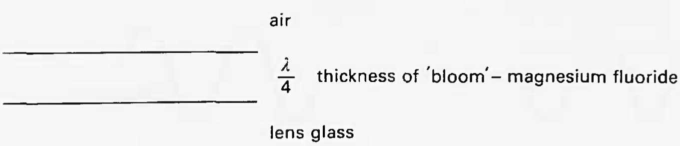


Figure D81

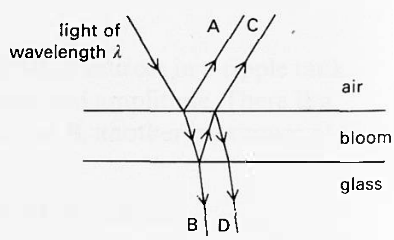


Figure D82

a Is the light following path A in phase or out of phase with the light following path C?

b Compare similarly light following paths B and D.

c Explain why the $\lambda/4$ bloom reduces the light reflected by the lens surface, but increases the light transmitted.

d The thickness of the bloom can only be exactly $\lambda/4$ for one particular value of λ. The chosen value of λ for cameras is usually in the middle of the visible band (4×10^{-7} m to 7×10^{-7} m) – that is about 5.5×10^{-7} m. Explain why the lenses of good cameras usually look purple.

21(P) In the medium wave radio band, waves may reach a receiver by two routes: the ground wave travels direct, while the sky wave is reflected off the ionosphere – see figure D83.

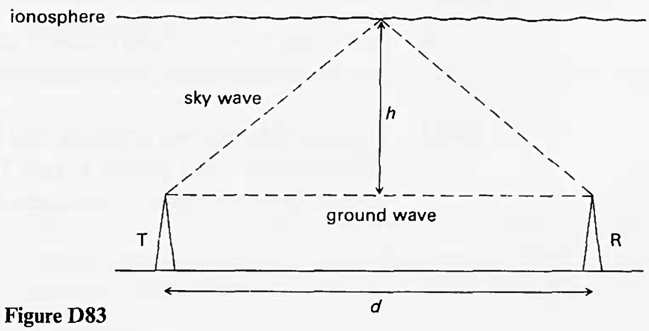


Figure D83

Thus superposition may occur. At night the sky wave is particularly strong; its amplitude is comparable with that of the ground wave. The receiver may receive a strong signal, or almost none at all, depending on the effective height, h, of the ionosphere at that moment. Since h varies over a few seconds, the signal rises and falls – an irritating phenomenon called 'fading'.

Suppose that $\lambda = 250$ m, $d = 120$ km, and h at one moment is effectively 80 km. If, at that moment, the receiver is receiving a maximum signal, then by what distance would h have to change in order for this signal to become a minimum?

22(E) Why do soap films and oily patches on roads appear brightly coloured? (Look this up if you don't know the answer.)

Qualitative motion of oscillators

23(L) Figure D84 illustrates a simple kind of oscillator, for which the restoring force is proportional to the displacement.

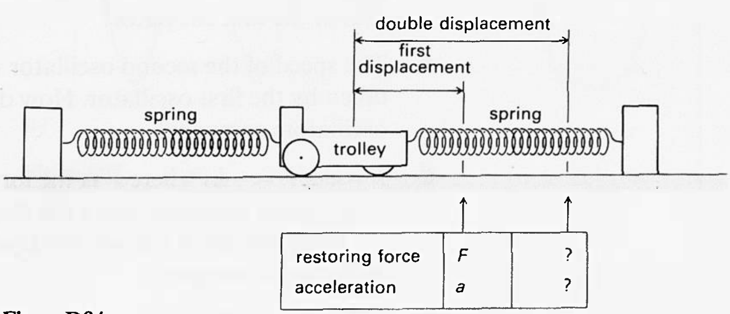


Figure D84

Suppose the trolley is given a small displacement and released. It oscillates, taking a certain time to go from the extreme position to the centre of the motion (one-quarter of an oscillation).

Now suppose it is given twice that initial displacement.

a How has the average restoring force on it been changed?

b How must its average acceleration have changed?

c In the same time, how will the speed acquired compare with the first trial?

d How long will the trolley take to cover the double distance to the centre of the motion, by comparison with the first trial?

24(L) Figure D85(a) shows a trolley tied by two springs, and its displacement–time graph. Figure D85(b) shows the same trolley, with different springs. It oscillates twice as rapidly as before.

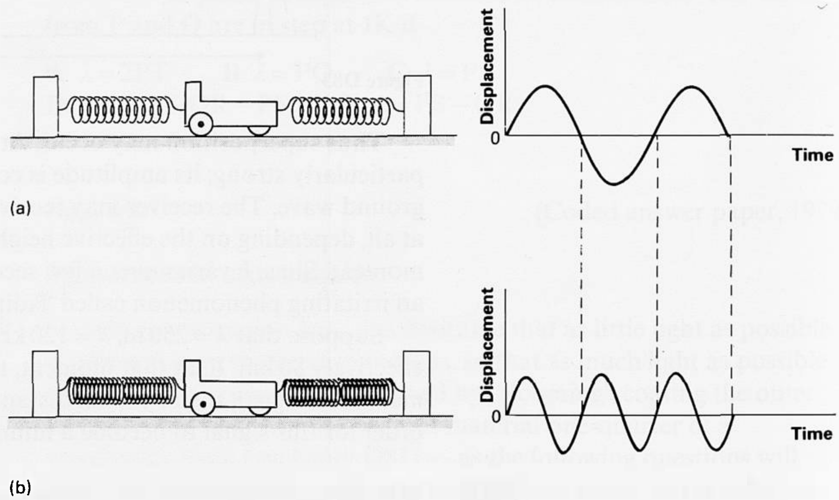


Figure D85

a Are the springs in figure D85(b) stiffer or weaker than those in figure D85(a)?

b In figure D85(b), the oscillation time is halved. For comparable positions, how does the speed of this second oscillator compare with that of the first oscillator?

c The speed of the second oscillator must be attained in half the time taken by the first oscillator. How do the accelerations of the two oscillators compare?

d $F = ma$, $F = -ks$ where F is the force, m the mass, a the acceleration, k the spring constant, and s the displacement.
If the masses of the two trolleys are the same, how do the spring stiffnesses k compare?

e If T is the oscillation time, which relationship below agrees with the answer to **d**?

$T \propto k^2$; $T^2 \propto k$; $T \propto \frac{1}{k^2}$; $T^2 \propto \frac{1}{k}$.

f To *decrease* the oscillation time, by changing the mass of the trolley, would one *increase* or *decrease* the mass?

g The answers to **a** to **c** above show that halving the oscillation time means quadrupling the acceleration, or that doubling the time means having one-quarter the acceleration. How would the mass have to be changed to achieve one quarter the acceleration (and so twice the oscillation time)?

h Which of the following relationships agrees with the answer to **g**?

$$T^2 \propto m;\ T \propto m^2;\ T^2 \propto \frac{1}{m};\ T \propto \frac{1}{m^2}.$$

25(P) The graphs in figure D86 indicate how the force, F, necessary to displace a mass varied with displacement, s, from the rest position for different cases. Each mass oscillates when it is released.

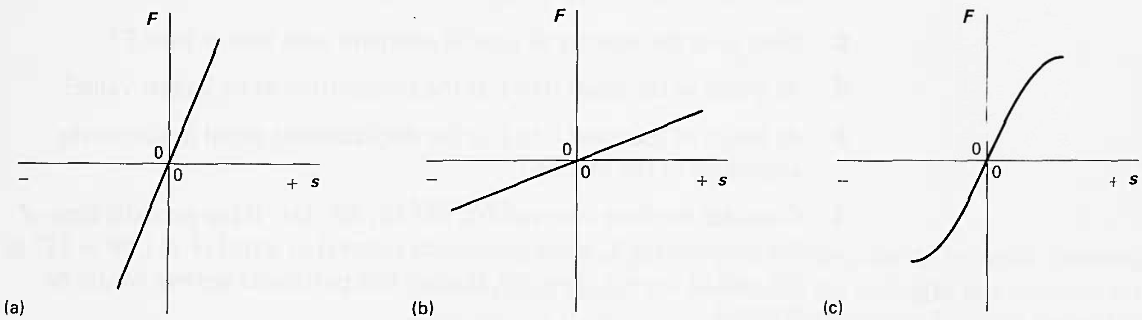


Figure D86

The second set of graphs (figure D87) shows the displacement–time traces for the above cases. Which trace corresponds with which force–displacement graph?

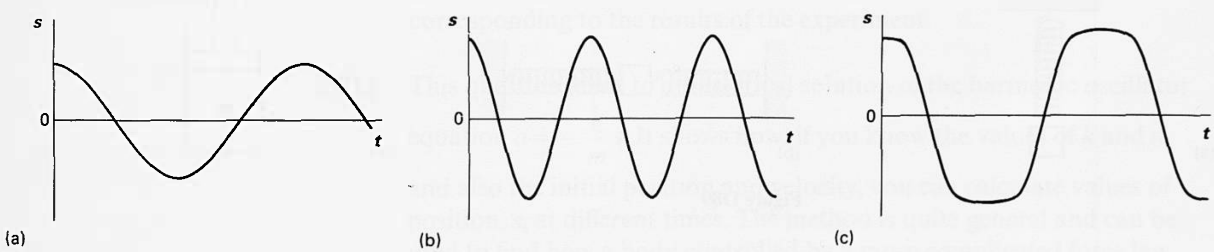


Figure D87

26(P) Figure D88 shows the displacement–time graph of an oscillator.

a Consider the speed of the oscillator at the four times labelled A, B, C, D. Arrange the times A, B, C, D in order of *decreasing speed.*

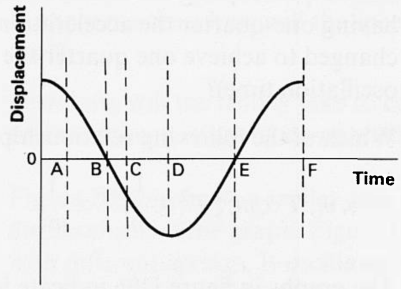


Figure D88

b How does the velocity at time B compare with that at time E?

c How does the velocity at time D compare with that at time F?

d At which of the times 0 to F is the acceleration at its largest value?

e At which of the times 0 to F is the displacement equal in size to the amplitude of the motion?

f Consider the time intervals 0B, 0D, 0F, BE, DF. If the periodic time of the oscillator is T, write down each interval in terms of T. ('$0F = 3T$' is the sort of answer expected, though this particular answer would be wrong.)

27(E) Figure D89 shows three things which would oscillate in a laboratory on Earth. Which, if any, would oscillate in a spacecraft going at steady speed a long way from the Earth and from any planet or star?

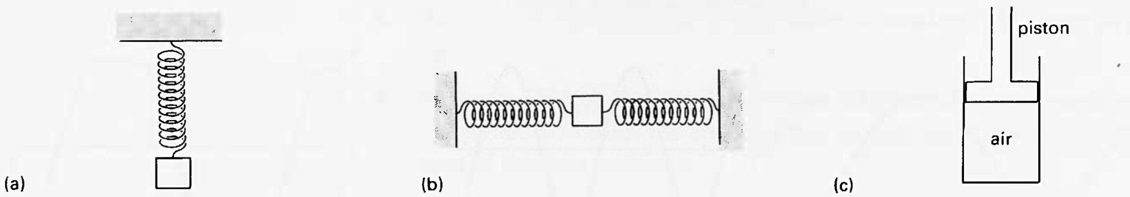


Figure D89

Analysis of a simple harmonic oscillation

28(L) Figure D90 represents a multiflash photograph of the motion of a trolley held between two springs. The distances from the mean position are given in table D1.

Figure D90

Point	Time/s	Displacement/cm
0	0	5.0
1	0.01	4.9
2	0.02	4.6
3	0.03	4.1
4	0.04	3.5
5	0.05	2.7
6	0.06	1.8
7	0.07	0.9
8	0.08	0

Table D1

a What is the value of T for the motion?

b What is the value of ω for the motion?

c Draw a semicircle of radius 5 cm. Mark the points 0 to 8 on to the diameter (8 is the centre of the circle), then project them vertically upwards onto the semicircle, as illustrated in figure D91.

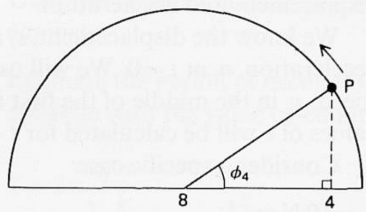


Figure D91

Measure the angles ϕ_1, ϕ_2, etc. They should be equal, showing that ω is constant. Hence the position of the trolley at any moment is the projection of P onto the diameter of the circle. P moves round the imaginary circle at constant angular rate ω. Calculate ω
i in degrees s^{-1}
ii in rad s^{-1}.

d In the experiment, the mass of the trolley was 0.1 kg, and the spring constant was 40 N m^{-1}. Show that this gives a value of ω corresponding to the results of the experiment.

29(L) This question leads to a numerical solution of the harmonic oscillator equation $a = -\frac{k}{m}s$. It shows how, if you know the values of k and m and also the initial position and velocity, you can calculate values of position, s, at different times. The method is quite general and can be used to find how a body controlled by a more complicated force law moves. The essence of the numerical method is to calculate what happens during each of a series of short time intervals. (The shorter the intervals the more accurate the result.) The body is assumed to move with constant velocity during each of these short time intervals.

The acceleration of the harmonic oscillator, a, depends on its position s ($a = -\frac{k}{m}s$). From the acceleration we can find the change in velocity during a short time interval Δt:

$\Delta v = a\Delta t$

If we know the velocity at the beginning of this time interval, we can calculate the velocity at the end:

$$v_{\text{new}} = v_{\text{old}} + \Delta v$$

We can use the velocity to find how far the object moves in one time interval:

$$\Delta s = v \Delta t$$

And hence the new position from:

$$s_{\text{new}} = s_{\text{old}} + \Delta s$$

One step is now complete. We have a new s from which we can get a new a, and we can repeat the whole process for the next time interval. The accuracy of the calculation is improved if we work out the velocity at times midway between the times at which we want to calculate displacement and acceleration.

We know the displacement, s, at $t = 0$, and hence can work out the acceleration, a, at $t = 0$. We will use this value of a to work out the speed, v, in the middle of the first interval Δt, that is, at $t = \frac{1}{2}\Delta t$. Later values of v will be calculated for $t = 1\frac{1}{2}\Delta t$, $2\frac{1}{2}\Delta t$, etc.

Consider a specific case:

$k = 10\,\mathrm{N\,m^{-1}}$
$m = 1\,\mathrm{kg}$

initial values $s = 0.1\,\mathrm{m}$
$v = 0\,\mathrm{m\,s^{-1}}$
at $t = 0\,\mathrm{s}$

a What is the acceleration at $t = 0\,\mathrm{s}$?

Take time increments $\Delta t = 0.1$ s.

b What is the change in velocity in the first *half* interval, that is, between $t = 0\,\mathrm{s}$ and $t = 0.05\,\mathrm{s}$?

The oscillator is at rest at $t = 0\,\mathrm{s}$.

c What therefore is the velocity at $t = 0.05$ s?

d Use this value to work out the distance travelled, Δs, between $t = 0\,\mathrm{s}$ and $t = 0.1\,\mathrm{s}$.

The displacement at $t = 0\,\mathrm{s}$ was 0.1 m.

e What is the displacement at $t = 0.1\,\mathrm{s}$?

f What is the acceleration at $t = 0.1\,\mathrm{s}$?

g From the acceleration at $t = 0.1\,\mathrm{s}$, work out the change in velocity between $t = 0.05$ and $t = 0.15\,\mathrm{s}$.

h What therefore is the velocity at $t = 0.15\,\mathrm{s}$?

The calculations now become routine, repeating steps **c** to **h** for each successive time step.

It is convenient to record your values in a table. The starting values and the velocity at 0.05 s have been entered. Other values you need to calculate are shown ▒

t/s	s/m	a/m s^{-2}	v/m s^{-1}	Δs/m
0	0.1	−1	0	
0.05			−0.05	−0.005
0.1	▒	▒		
0.15			▒	▒
0.20	▒	▒		
0.25			▒	▒
0.30	▒	▒		

Continue in this way at least until s becomes zero, preferably until it reaches its greatest negative value.

i Use your results to sketch graphs of s against t, v against t, and a against t.

j Estimate the period of oscillation from your results, and compare this estimate with the value calculated from

$$T = 2\pi\sqrt{\frac{m}{k}}$$

k Suppose k were twice as great, 20 N m^{-1}. How would the answers to **a**, **b**, **c**, and **d** change? Would the period of this oscillator be greater or smaller?

l If m were 2 kg (but k still 10 N m^{-1}) how would the answers to **a**, **b**, **c**, and **d** change? What would be the effect on the period of the oscillator?

30(E) The routine for solving the problem of the harmonic oscillator in question **29** can easily be made into a computer program.

Write a program that will print out values of v at $t = \Delta t/2$, $3\Delta t/2$, $5\Delta t/2$ ..., and values of s and a at $t = 0$, Δt, $2\Delta t$ ....

Some investigations you can make with the program:

a Does it show that the oscillator is isochronous, that is that the period does not depend on the initial displacement?

b Explore the effect on period T of changing k and m.

c How sensitive is the program to the value of Δt?

d Add a line to your program so that it represents a damped oscillator. Explore the effect of a constant frictional force, friction proportional to velocity, or friction proportional to (velocity)^2.

31(R) 'Figure D92 is a stroboscopic photograph of the motion of a harmonic oscillator, released from rest at the extreme left of the photograph. It is, however, detached from its restoring force when it reaches its central position, and as a result continues on at constant velocity.'

Check, by drawing a graph of velocity against time, whether the above description is plausible.

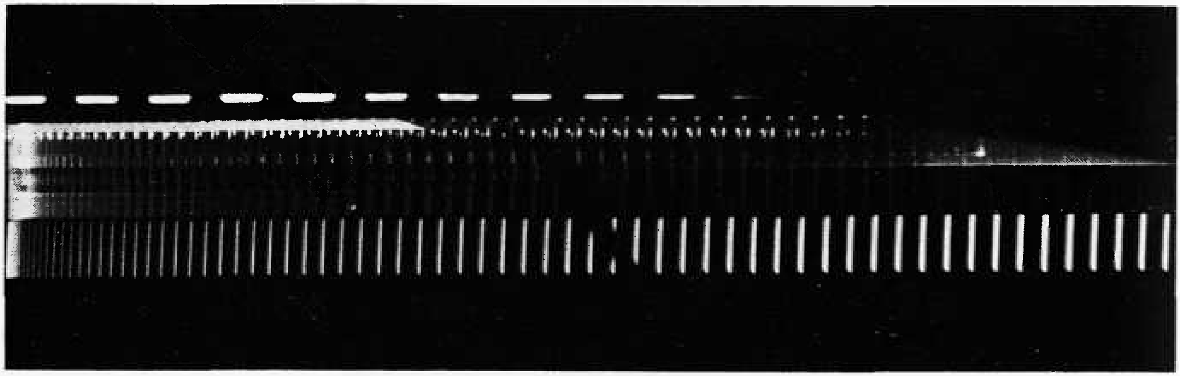


Figure D92

Further examples of oscillations and S.H.M.

32(R) Figure D93 shows a liquid in a U-tube of uniform cross-section. In figure D93(b) the liquid is displaced as shown.

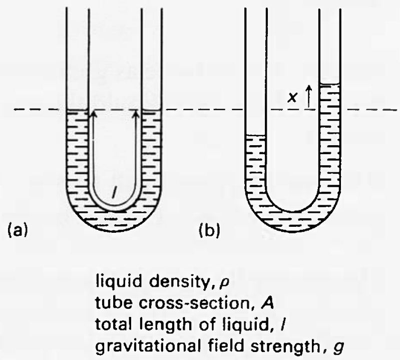


Figure D93

a In figure D93(b), what is the value of the force tending to return the liquid to its equilibrium position?

b Explain how this situation fulfils the necessary condition for simple harmonic motion.

c Find expressions, in terms of the symbols given, for
i the acceleration, a, of the liquid;
ii the angular velocity, ω, of the S.H.M.;
iii the periodic time, T, of the S.H.M.

33(R) Figure D94 shows a cylindrical buoy floating in water. In equilibrium, figure D94(a), the buoy floats so that a length L, of its total height, h, is below the water surface. The buoy is displaced a further depth, x, then released, figure D94(b). Find an expression for the periodic time of the oscillations, in terms of the symbols given, and show that it has the dimensions of time.

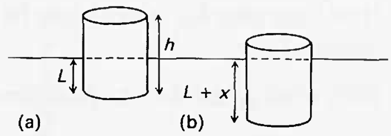


Figure D94

34(P) An object oscillates on the end of a spring. If the mass of the object is multiplied by four, what change must be made in the amplitude of the oscillations for the maximum speed of the object to be unaltered?

35(P) In a small harbour, the depth of water at high tide is 10 m, and at low tide is exactly zero. The depth of water follows approximate S.H.M., with amplitude 5 m and periodic time 12 hours. (Really about $12\frac{1}{2}$ hours, but use 12 hours for this question.) Fishing boats can only leave the harbour when the depth of water is 9 m or more. On one day, high tide is at noon.

a What is the value of ω (in radians per hour)?

b If d is the depth of water *above* the mean depth (5 m), write an equation of the form $d = A\cos\omega t$, substituting the appropriate values for A and ω.

c Calculate the latest time in the afternoon at which boats can leave the harbour.

36(L) At room temperature the atoms of a particular solid vibrate with S.H.M. of frequency 10^{13} Hz and amplitude 12×10^{-12} m. (This is a much simplified model of a solid, but it is a useful starting point.) The mass of each atom is 10^{-25} kg.

a About what fraction of a typical atomic separation is this amplitude?

b Calculate the approximate value of the force constant, k, between two atoms.

c What is the total energy of vibration of one atom
i in joules,
ii in eV?

37(P) A baby in a 'baby-bouncer' is a real-life example of a mass-on-spring oscillator. The baby sits in a sling suspended from a stout rubber cord, and can bounce himself up and down if his feet are just in contact with the ground (an example of resonance). If suspended out of contact with the ground, he oscillates if displaced and released. Suppose a baby of mass 5 kg is suspended from a cord with spring constant 500 N m^{-1}.

a What is the initial (equilibrium) extension of the cord?

b The baby is pulled down a further distance, 0.1 m and released. What is the value of ω?

D

c How long after his release does he pass through his equilibrium position?

d With what speed does he pass through his equilibrium position?

38,39(R) Figure D95 shows the tip of a stereo record-player stylus resting in the right-angled groove of a record. It rests on the lefthand and righthand walls of the groove, labelled L and R.

As the record turns, the stylus is moved by either wall moving at right angles to itself, as indicated by the broken lines. Suppose that both walls move with simple harmonic motion. Here are five possible combinations of wall motions, of the same frequency, and except where specified, the same amplitude.

A L and R move up and down in phase.
B L and R move up and down out of phase (180° difference).
C L oscillates, but R has zero amplitude of motion.
D The upward displacement of L is a maximum when that of R passes through zero going downwards.
E The upward displacement of R is a maximum when that of L passes through zero going downwards.

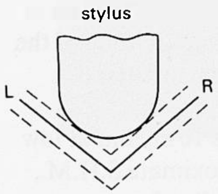


Figure D95

38 In which one of the above does the stylus move only in the vertical direction?

39 In which one of the above does the stylus move only in the horizontal direction?

(Coded answer paper, 1979)

The speed of compression waves

40(R) Figure D96(a) shows a snapshot of a row of identical trolleys joined to each other by identical springs. The trolleys here are at rest and their positions are shown by the scale above the trolleys. Figure D96(b) shows the same trolleys at a later instant. Trolley A has been pushed in such a way that, at this instant, each of the trolleys A, B, and C is moving at a steady speed to the right. Figure D97(a) shows the displacement of each trolley from its original position, at this instant.

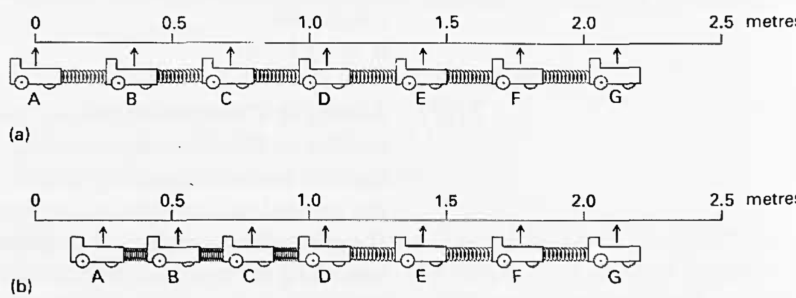
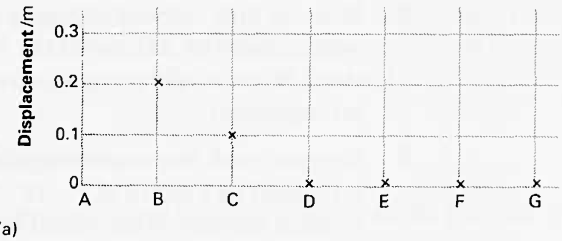


Figure D96

Figure D97(b) will show the speed (in arbitary units) of each trolley at this same instant. The speed of trolley A has been plotted as a cross (X) on this graph.



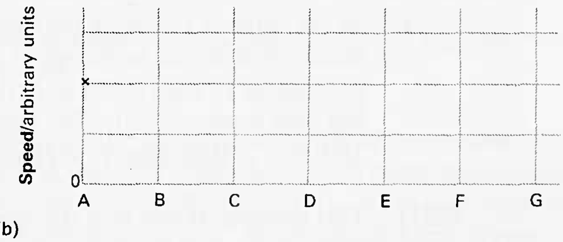


Figure D97

a Mark crosses on a copy of figure D97(b) to indicate approximately the speeds of all the other trolleys at this instant.

b State what factors affect the speed with which the initial displacement of the first trolley passes along the line of trolleys.

It is suggested that a row of spring-connected trolleys might be a model which would help us to understand the way in which a compression pulse travels along a metal bar.

c Give one reason why you consider that such a row of trolleys would be a good model for this purpose.

d Suggest a way in which you think the model is unlike a metal bar.

(Short answer paper, 1974)

41(L) This question shows how the wave speed for a row of trolleys, discussed in question **40**, can be applied to discussing the speed of sound in a solid.

The speed, c in a compression wave travelling along a row of trolleys linked by springs is given by:

$$c = x\sqrt{k/m}$$

where x is the distance between the centres of successive undisturbed trolleys, m is the mass of a trolley, and k the constant relating the force causing the compression of a spring and the amount of compression. Trolleys linked by springs might be compared to the atoms in a solid linked by interatomic forces. Table D2 gives some comparative values of x and m.

	Trolleys and springs	Atoms in steel
x	0.35 m	2.5×10^{-10} m
m	0.95 kg	9.3×10^{-26} kg
c	$2.5\,\mathrm{m\,s^{-1}}$	?

Table D2

k is roughly $50\,\mathrm{N\,m^{-1}}$ both for the spring linking a pair of trolleys and for the bond between a pair of atoms in steel.

a Suppose that, without changing anything else, the mass of a trolley were reduced by a factor of 10^{-25} (to about the mass of an atom in steel). What would be the speed of a compression wave in such an arrangement?

b Suppose, with this new arrangement, that the distance between trolleys is reduced by a factor of 7×10^{-10} (or 10^{-9} if you don't mind a rougher answer). What would be the speed of a compression pulse in such an arrangement?

c As the value of k is much the same for both steel and spring, the answer to **b** should be the speed of a compression wave in steel if it is permissible to scale down from the trolleys to atoms, and if the scaling has been done correctly. The measured speed of sound in steel is about $5100\,\mathrm{m\,s^{-1}}$. How does your estimate compare?

42(L) This question shows how the speed of sound in a metal can be written down in terms of the Young modulus, E, and the density, ρ.

The speed, c, of a compression pulse travelling along a row of trolleys linked by springs is given by:

$$c = x\sqrt{k/m}$$

where x is the distance between the centres of successive undisturbed trolleys, m is the mass of a trolley, and k is the spring constant in the equation

force $= k \times$ change in length

Unit A, 'Materials and mechanics', question **30** showed, for a specially simplified case, that if E is the Young modulus and x the spacing between atom centres, the atomic bond spring constant, k, is given by $k = Ex$ if the atoms are in a simple cubic array. Consult that question again to see why.

a Suppose the density of the material is ρ, and that each atom occupies a volume x^3. What is the mass, m, of each atom?

b Find a new expression for the speed, c, of compression waves (or sound waves) in a solid in terms of E and ρ, by substituting for k and for m.

c What is the speed of sound in an aluminium rod?

Young modulus $= 7.0 \times 10^{10}\,\mathrm{N\,m^{-2}}$

density $= 2700\,\mathrm{kg\,m^{-3}}$

Note: The atoms in aluminium are not arranged in a simple cubic array, but the answer to **b** does give a correct expression for the speed of sound in an aluminium rod. For arrays more complicated than cubical arrays, the relations between k and E and between m and ρ are both more complicated than above, but the spacing, x, still cancels out as it did in **b**, together with all the extra geometrical factors which allow for the more complex atomic arrangement.

d The speed of sound in aluminium is very nearly the same as its speed in steel, but steel is several times more dense than aluminium. What must be true of their Young moduli?

Resonance

These questions have to do with various practical problems involving vibration and resonance. You will need to use $T = 2\pi\sqrt{\frac{m}{k}}$ or $2\pi f = \sqrt{k/m}$, and to know that the total energy of a harmonic oscillator is equal to $\frac{1}{2}kA^2$.

43(E) Estimate the spring constant of the suspension of a car. Imagine a man sitting over one wheel: how much might the suspension deflect? What frequency of oscillation might a wheel have, considered as a mass on the end of this spring? What sort of repeated ruts on a road would give trouble at, say, 50 kilometres an hour? Why does the car body move with smaller amplitude than the wheels?

44(R) This question is about explaining to non-scientists some applications of scientific ideas which they find confusing. Give an explanation, suitable for a non-scientist, which explains the ideas involved and clears up any errors or confusions in the statement below. Assume that your explanation will include experimental demonstrations and describe these in detail.

'People take it for granted that a car needs to be "well sprung" so that the ride is comfortable. It is not so obvious how to get the suspension right: it can be too hard or stiff, so that the car jolts on every bump; it can be too soft so that the suspension sags when the car is loaded and the car sways about over the wheels. It must certainly not make the car body oscillate up and down after the wheels hit a bump, and there could be a danger that the suspension will resonate if the car travels over a regular series of bumps.'

What does need to be taken into account in choosing a suspension, and how are these difficulties avoided?

(Long answer paper, 1981, part question)

45(P) Diatomic molecules such as HF or HCl can vibrate by extension and compression of the bond between the atoms. They behave like a pair of masses held by a spring. For these two molecules, the H atom is much less massive than the F or the Cl atom to which it is bonded, and so, roughly, it will be good enough to imagine the H atom vibrating at the end of its bond. (Compare a small mass linked with a spring to a large mass – will the large mass move much?)

Now HF absorbs infra-red radiation very strongly at a wavelength of 2.4×10^{-6} m. The corresponding wavelength for HCl is 3.3×10^{-6} m.

D

Why can you say straight away that the HF bond is probably rather stiffer (larger force for the same extension) than the HCl bond?

Show that the bond stiffnesses are roughly:

$1000\ \mathrm{N\,m^{-1}}$ for HF

$500\ \mathrm{N\,m^{-1}}$ for HCl

The mass of a hydrogen atom is nearly 1.7×10^{-27} kg. The velocity of light is $3 \times 10^{8}\ \mathrm{m\,s^{-1}}$.

46(L) The model of sodium chloride illustrated in figure D98 shows just one line of ions in a crystal.

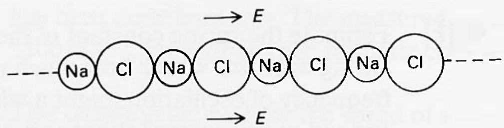


Figure D98

a If an electric field, E, is switched on in the direction shown, what would be its effect on
i the Na^{+} ions,
ii the Cl^{-} ions?

b Calculate an approximate value for the mass of each Na ion. (Atomic mass of Na = 23 u; the Avogadro constant = $6 \times 10^{23}\ \mathrm{mol^{-1}}$.)

c The 'spring constant' between *each* pair of ions is roughly $100\ \mathrm{N\,m^{-1}}$. Calculate a rough value for the frequency (in Hz) at which a Na^{+} ion would oscillate. (Treat each ion as a mass tethered between two springs.)

d If the electric field oscillates at this frequency, resonance can occur. This can be achieved by directing electromagnetic radiation of the right frequency at the crystal.
i What would be the wavelength of the radiation corresponding to the required frequency?
ii To what part of the electromagnetic spectrum do these waves belong?

47(P) The ammonia molecule, NH_3, can vibrate with the nitrogen atom passing to and fro 'between' the three hydrogen atoms (figure D99). The frequency happens to be 23 870 MHz. This vibration has been used as the basis of the 'atomic clock'.

Would the rate of vibration be affected by using molecules of ND_3, with deuterium (heavy hydrogen, ^{2}H) atoms in place of the hydrogen atoms? (A deuterium atom has a nucleus with one neutron and one proton, whereas the hydrogen nucleus is just one proton.)

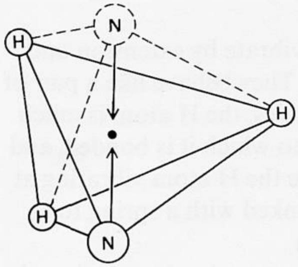


Figure D99

48(E) Stand on one foot and allow your other leg to swing freely and easily like a pendulum. What connection do you think there is between the time of these swings and the speed at which you usually walk?

49(E) A car accelerates away from traffic lights, and the driver notices that as the car accelerates past a certain speed, his view in the driving mirror goes 'blurred' and then becomes sharp again. Suggest a reason for this, and a practical way of reducing or eliminating the effect.

50(E) Ultrasonic vibrations may be used to kill bacteria in liquids. How big are bacteria? Discuss the choice of a suitable frequency.

51(E) A record-player pick-up has a stylus or needle that runs in the record groove and is oscillated sideways by the wavy walls of the groove (see figure D100). The performance of a pick-up is often specified by giving the effective tip mass of the stylus and the compliance or flexibility of the stylus when it is pushed sideways. The compliance is the reciprocal of the stiffness, k, which is measured in newtons per metre. The compliance is thus measured in metres per newton (deflection for a given force).

A suggested standard for high fidelity equipment is:

effective tip mass $< 2\,\text{mg}$
compliance $> 4 \times 10^{-3}\,\text{m N}^{-1}$

If the mass and compliance have these values, at what frequency will the stylus resonate? Would you regard this as satisfactory? You might go on to consider whether it would be desirable to have low or high values of tip mass and compliance.

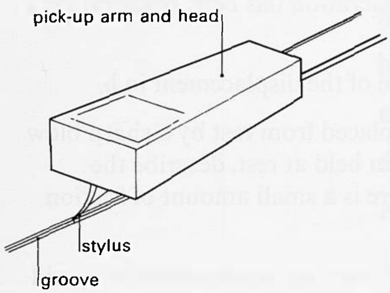


Figure D100

52(E)a Some high fidelity sound systems have the loudspeaker mounted in a 'bass reflex cabinet'. The cabinet in figure D101 is sealed except for a port, and the air in the port (shaded) behaves like a mass acted on by the springiness of the air in the cabinet. Suppose you made such a cabinet and found that it 'boomed' whenever notes of about 300 Hz were reproduced. How might it be improved? Can you suggest any reason why the speaker is not just used on its own, without a cabinet?

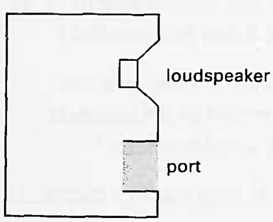


Figure D101

b A violin, or a guitar, has its strings held taut over a hollow wooden box which contains air and has holes in it (or a hole). Air in such a box can vibrate, the air near the hole acting as a mass driven in or out by the springiness of the air in the box.

What will be the effect on the sound produced when a string is sounded at a frequency near to the resonant frequency of the air in the body of the instrument? What other parts of a violin or guitar can resonate to notes from the strings?

53(R) This question is about measuring the acceleration, and so the velocity and displacement, of a moving vehicle, by making observations on masses carried within the vehicle.

Figure D102 shows the principle of one sort of accelerometer (a device for measuring acceleration). A mass, m, is free to move horizontally within a case, but is restrained by springs fixed to the case. A pointer on the mass can move over a scale fixed to the case. When the case and mass are at rest, the pointer is opposite the zero mark on

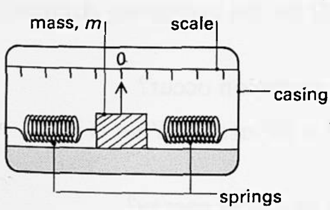


Figure D102

the scale. When the pointer shows a displacement, x, from zero, the net force exerted by the springs is kx.

a Explain why, when the mass and case are moving at constant velocity in the horizontal direction, the pointer still reads zero. Assume that the velocity has been constant for a long time.

b If the casing is in a state of steady acceleration a to the *left*, explain carefully in words why the pointer now has a fixed displacement, saying whether the displacement is to the left or to the right and explaining why. Assume that the acceleration has been constant for a long time.

c Give an expression for the magnitude of the displacement in **b**.

d If the casing were to be suddenly displaced from rest by a sharp blow from a hammer, for example, and then held at rest, describe the subsequent motion of the mass if there is a small amount of friction between it and the casing.

e It is suggested that, in use to measure varying accelerations, it would be good to have zero friction between the mass and the casing. Argue briefly for or against this idea.

f Suppose that, in use, appreciable changes of acceleration are expected to occur over times not exceeding time t. Give an argument to help decide whether the period, T, of natural oscillation of the mass and springs should be large, or should be small, compared with t.

g In designing an accelerometer for use in a car, a period, T, of $\pi/5$ seconds was chosen and it was assumed that accelerations up to $2\,\mathrm{m\,s^{-2}}$ should be measured. What would be the displacement at an acceleration of $2\,\mathrm{m\,s^{-2}}$? (The values of m and k are not needed.)

h Suppose that it is decided that an accelerometer for use in a car accelerating at up to $2\,\mathrm{m\,s^{-2}}$ should have a period of 2π seconds. What problems would arise in designing this accelerometer?

(Long answer paper, 1979)

Quality factor, Q

54(L) When an excited atom emits its excess energy, this is equivalent to an oscillating electron losing energy by damping. The energy is emitted as light or other electromagnetic wave radiation, each cycle of the wave corresponding to one oscillation of the electron. Suppose that one particular atom returns to its unexcited (ground) state, emitting a burst of radiation with $\lambda = 5 \times 10^{-7}$ m; and that Q for the oscillating electron is 10^7.

a Roughly how many complete cycles of the radiation occur?

b The speed of the electromagnetic wave is $3 \times 10^8\,\mathrm{m\,s^{-1}}$. What is its frequency?

c How long does it take for the atom to lose its excess energy?

d What length in space does the burst of radiation occupy?

Notice that this burst of radiation takes a finite time to be emitted (part **c**). During this time the atom itself (if gaseous) will have moved a significant distance. This causes a change in λ for the radiation, due to the Doppler effect. This is one reason why spectral lines, even using the best possible optical equipment, are not perfectly sharp.

55(E) Each of the following objects oscillates, possibly in resonance, during its normal operation. Discuss whether each should be designed with a value of Q which is high (>100), intermediate, or low (<2).

a The platform of a suspension bridge.

b The mountings for a machine like a lathe.

c A car-body on its suspension.

d The balance wheel of a mechanical watch.

e The tuning circuit of a radio.

f The moving arm of a ticker-timer.

Standing waves

56(P) An elastic string is clamped at both ends as shown in figure D103. Near one end a vibrator is loosely attached to it. The vibrator oscillates with fixed amplitude and variable frequency. A graph of maximum oscillatory amplitude along the string against frequency of the vibrator looks like figure D104.

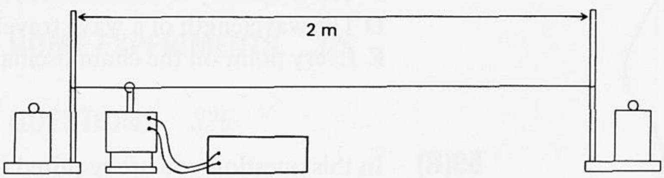


Figure D103

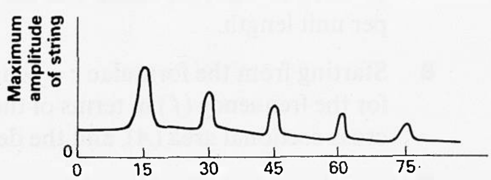


Figure D104

a Sketch the instantaneous appearance of the string at
i 15 Hz,
ii 30 Hz, and
iii 45 Hz

b What is the wavelength of the waves on the string at
i 15 Hz,
ii 30 Hz, and
iii 45 Hz?

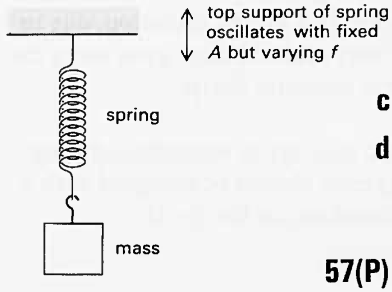


Figure D105

c What is the speed of travel of the waves along the string?

d How is the above situation
i similar to,
ii different from the case in figure D105.

57(P) An organ pipe (or any wind instrument) closed at one end can allow standing waves which have a node (N) at that end and an antinode (A) at the other (neglecting a small 'end-correction'); see figure D106. One such pipe has a fundamental note of 64 Hz. Consider what other wavelengths are possible for standing waves in this pipe, and calculate the frequencies of the pipe's second and third harmonics (the next two higher frequencies at which the pipe resonates).

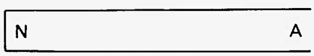


Figure D106

58(R) Figure D107 represents a hanging chain of constant mass per unit length. The support at S is gently oscillated from side to side sending transverse waves down the chain. These are reflected at the free bottom end and a stationary mode of oscillation (a standing wave) is set up.

Which one of the following statements is correct?

A The tension at all points of the chain is the same.
B The speed of a wave at all points along the chain is constant.
C The frequency of oscillation of each part of the chain is constant.
D The wavelength of a wave travelling along the chain is constant.
E Every point on the chain oscillates with the same amplitude.

(Coded answer paper, 1974)

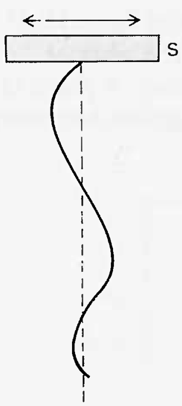


Figure D107

59(R) In this question you are required to estimate the tension in a violin string which vibrates at a natural frequency of 650 Hz. The string is made of steel. The speed of transverse waves along a stretched wire is given by $c = \sqrt{T/\mu}$, where T is the tension in the wire, and μ is its mass per unit length.

a Starting from the formulae $c = f\lambda$ and $c = \sqrt{T/\mu}$, obtain an expression for the frequency (f) in terms of the tension (T), the length (L), the cross-sectional area (A), and the density (ρ) of the string.

b Estimate values for the quantities you will need to know in order to calculate the tension using the above expression.

c Combine your estimates in order to obtain a value for the tension in the string.

d Say in a few words how you decide on the appropriate number of significant figures to give in the answer.

(Short answer paper, 1981)

Unit E
FIELD AND POTENTIAL

Trevor Sandford
Henbury School, Bristol

SUMMARY OF THE UNIT

E

SUMMARY OF THE UNIT

INTRODUCTION

This Unit is about electric and gravitational fields and potential. You have probably used the word 'field' before to describe effects due to magnetism, electricity, or gravity, but here we develop further what is meant by the term and explore the relationships between fields, forces, energy, and potential for both charges and masses. Electricity and gravity have much in common – their inverse-square force laws, for example, and the mathematics that describe them. They also have enormous differences: gravity only becomes noticeable when huge masses are involved but its effect is felt over astronomical distances. By contrast, electrical forces dominate the behaviour of atoms and molecules on the microscopic scale.

Although this area of physics can be treated in a rather abstract mathematical way, the emphasis in this course is on understanding the physics and being able to apply fundamental ideas to a variety of situations. These range from space travel and satellites to the structure of atoms, molecules, and crystals; from printing, painting, and photocopying to sparks, 'static', and thunderstorms.

The Unit uses ideas about capacitors, charge, and potential difference from Unit B, 'Currents, circuits, and charge'; vectors and dimensions, introduced in Unit A, 'Materials and mechanics'; and Newton's Laws of Motion. Ideas from this Unit are used later in Unit F, 'Radioactivity and the nuclear atom', Unit J, 'Electromagnetic waves', and Unit L, 'Waves, particles, and atoms'.

Section E1 THE UNIFORM ELECTRIC FIELD

Fundamental ideas

New ideas: E, V, $\Delta V/\Delta x$

Parallel plates: V, d, A, Q, ε_0, ε_r

The fundamental ideas developed here are the concept of field strength, electric potential, equipotentials, and potential gradient. There is a detailed study of the field between charged parallel plates and how it relates to the p.d. between the plates, their separation and their area, the charge stored, and the medium which separates them. Applications of these ideas cover a range of situations from industrial processes to domestic problems.

Observing a uniform electric field

DEMONSTRATION E1
The 'shuttling ball'

DEMONSTRATION E2
Charged foil strip as a field detector

Experiments on a charged ball between charged parallel plates show that the force, F, on the ball depends on the p.d. between the plates, V, and on their separation, d (figure E1). A charged foil 'detector' shows that the force is uniform over most of the region between the plates, and that its size and direction depend on the size and sign of the charge, Q, on the foil as well as on V and d (figure E2).

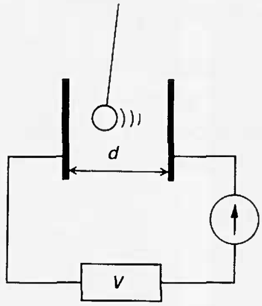


Figure E1

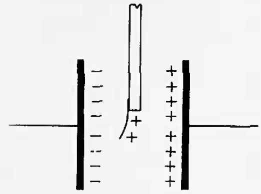


Figure E2

Definition of electric field strength, E

QUESTIONS 1 and 2

The strength of a gravitational field is the force per unit mass; by analogy electric field strength is defined as the force per unit charge, $E = \frac{F}{Q}$.

units of E are NC^{-1}

If, then, we could measure the charge on the foil and the electric force exerted on it, we could calculate the field strength. But both of these measurements prove impracticable: a small charge has too small a force on it, but a larger charge disturbs the field and gives a misleading result. This problem is solved by considering the links between F, Q, V, and d.

QUESTION 3

Millikan's experiment, Unit B

energy = force × distance

A charged drop is moved across the space between two charged plates. If a constant force, F, is exerted over distance d, then the energy transformed $= Fd$. But charge Q is then moved through a p.d. V, so the electrical energy transformed is QV.

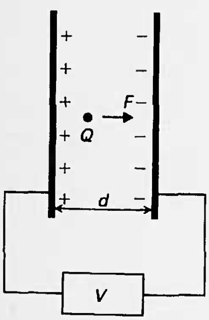


Figure E3

p.d. $= \frac{\text{energy}}{\text{charge}}$

Hence

$$Fd = QV.$$

and

$$\frac{F}{Q} = \frac{V}{d}$$

But $\frac{F}{Q}$ is equal to the electric field strength E.

$E = \frac{V}{d}$ for a *uniform* field *only*

Note that this is only the case when the force, F, is uniform over all the region – in other words, for a *uniform field*. However, it is extremely useful since the field strength can be measured simply using a voltmeter and a ruler, and expressed in $V\,m^{-1}$.

Alternative units of $E = V\,m^{-1}$ (equivalent to $N\,C^{-1}$)

$E = \frac{V}{d}$ is true *only* for a uniform field, but $E = \frac{F}{Q}$ is true in every situation: it is quite general, though in practice less useful.

QUESTIONS 4 and 6

Patterns of electric field

DEMONSTRATION E3
Electric field patterns

QUESTION 5

Semolina particles floating on an organic liquid orientate themselves in the presence of an electric field to reveal its shape, rather as iron filings do in a magnetic field. Parallel, point, and circular electrodes may be used to show different shapes of field (figures E4 and E5).

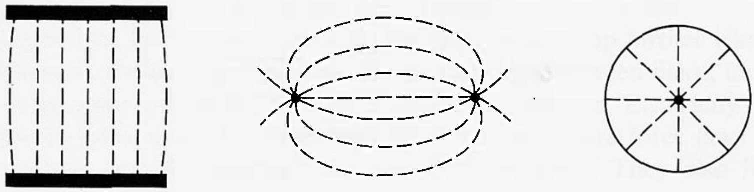


Figure E4

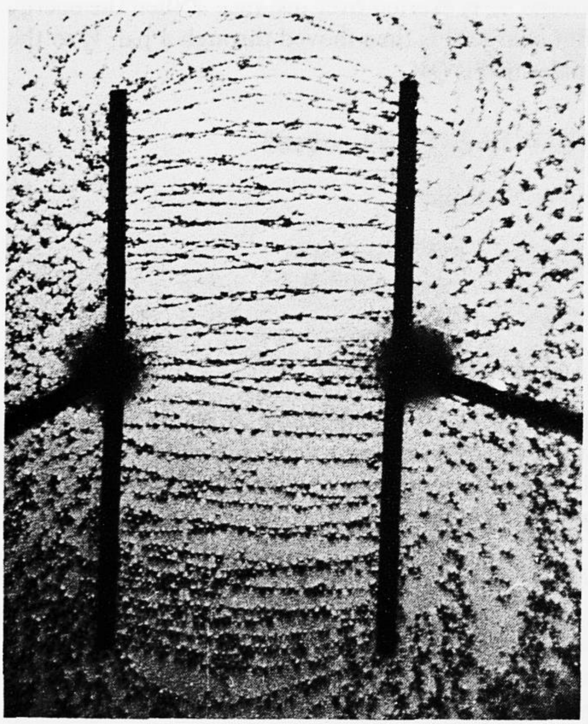
Using a flame probe

DEMONSTRATION E4
Measuring potentials in a uniform field using a flame probe

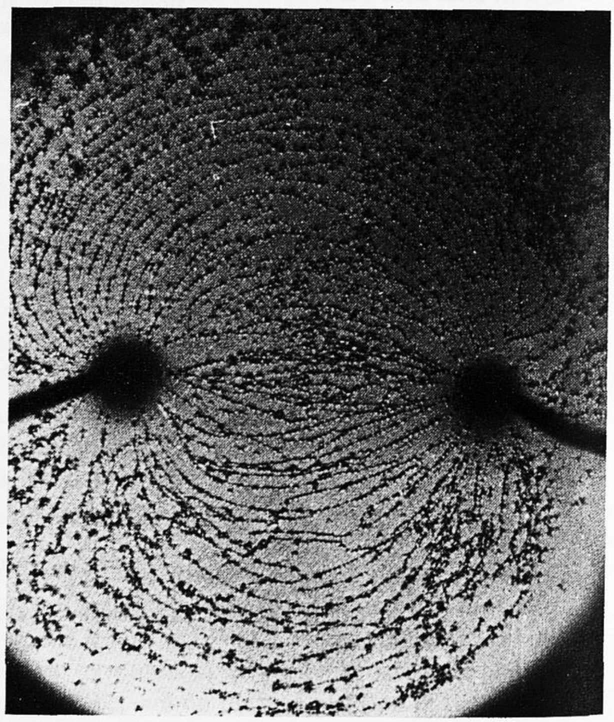
The field strength between charged parallel plates, then, is uniform over the region between them and at all points is equal to the potential difference per metre in that region. The p.d. between the *plates* can be measured with a voltmeter or oscilloscope; the p.d. between one of the plates and a point *in space* between them can be measured with the flame probe.

QUESTION 7

The flame prevents any charge building up on the probe and disturbing the field. If one of the plates (usually the earthed one) is

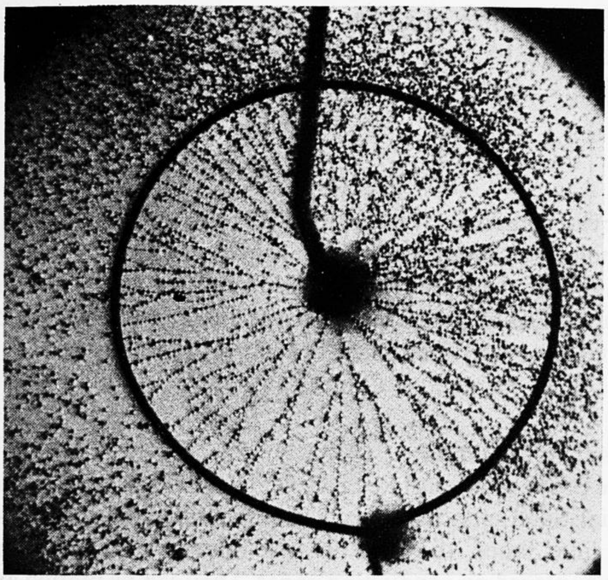


(a)

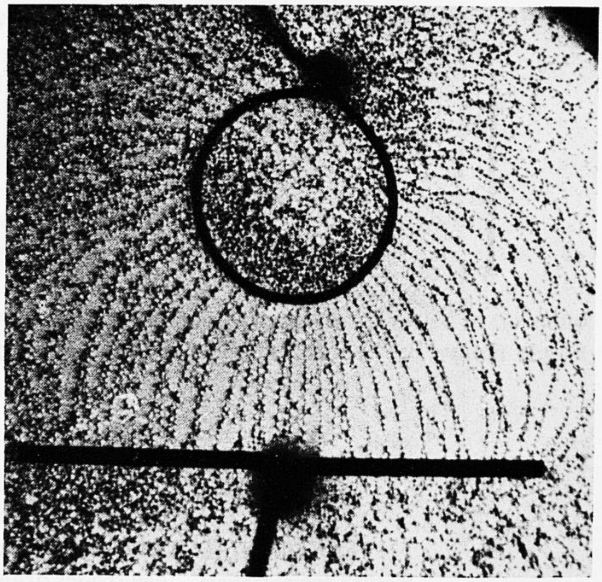


(b)

Figure E5 (part)



(c)



(d)

Figure E5 (part)
Electric field patterns. (Note that there is no field inside the ring in (d).)
Colin Price

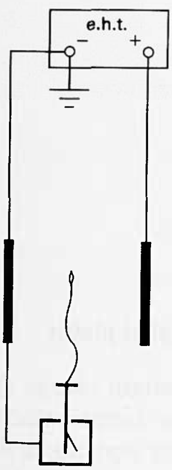


Figure E6
Flame probe.

referred to as our zero of potential, then we can define the *potential* at any point in the space as being equal to the potential difference between that point and the reference plate (figure E6).

Field and potential gradient

Moving the probe steadily from one plate to the other reveals a steady change in potential. A graph of potential, V, against distance, x, from the reference plate is a straight line; its gradient $\frac{\Delta V}{\Delta x}$ is equal in magnitude to $\frac{V}{d}$, the field strength. So the *potential gradient* at any point gives the strength of the field: the uniform field has uniform potential gradient (figure E7). Since V increases towards the positive plate, and E is

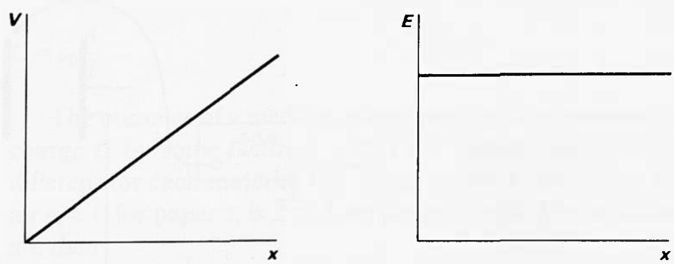
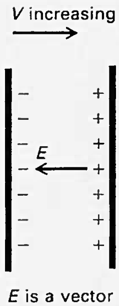


Figure E7

Figure E8



directed toward the negative plate (the direction of the force on a *positive* charge, see figure E8), we must insert a negative sign to show the direction in which E, a vector, acts:

$$E = -\frac{\Delta V}{\Delta x}$$

QUESTION 8

QUESTION 9

Now since we can consider any bit of any field to be uniform over an infinitesimally small distance Δx, this relationship can be applied to any shape of field, not just the uniform field, provided we speak of the potential gradient at a *point* only. This is very useful later in Sections 2 and 3 of this Unit when we deal with fields which vary with distance.

Equipotentials

EXPERIMENT E5
Plotting equipotentials

QUESTIONS 10 and 11

Moving the flame probe along surfaces parallel to the plates reveals no change in potential. Such surfaces are called equipotentials. Equipotentials appear as lines when the field is two dimensional, between electrodes drawn or placed on conducting paper. They can be plotted at given intervals to reveal the variation in potential between the electrodes and are analogous to contours drawn on a map. The direction of the electric field is always at right angles to the equipotentials; the field has no component along the equipotential (figure E9).

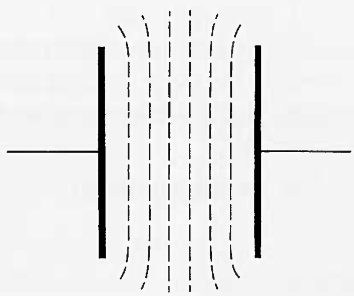


Figure E9

Factors affecting the strength of the field between parallel plates

EXPERIMENTS E6, E7
Charge on parallel plates
(coulombmeter, reed switch)

Charged parallel plates form a capacitor, storing a certain charge Q when a p.d. V is applied. For a capacitor, $Q \propto V$. Other factors which affect the charge stored are the area of the plates, A, their separation, d,

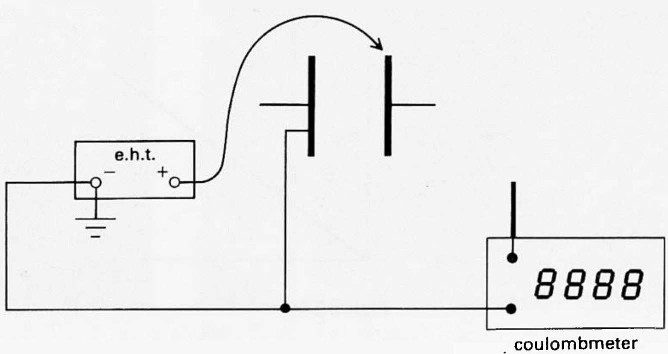


Figure E10

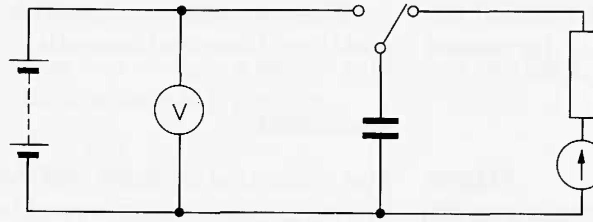


Figure E11

and the medium which separates them. These can be varied in turn and the charge Q measured using a coulombmeter or reed switch circuit (figures E10, E11).

QUESTION 13

Results show that Q is proportional to V, to A, and to $\frac{1}{d}$:

$$Q \propto \frac{VA}{d}$$

or $Q = \varepsilon_0 \frac{VA}{d}$

where ε_0 is a constant.

This relationship can be arranged in two useful ways:

i $\frac{Q}{A} = \varepsilon_0 \frac{V}{d}$

$\frac{Q}{A}$ is the charge per unit area, or surface charge density usually denoted by σ. The quantity $\frac{V}{d}$ is of course the field strength E.

QUESTION 12

$\varepsilon_0 \approx 8.85 \times 10^{-12}$ F m^{-1}

So $\sigma = \varepsilon_0 E$

ε_0 is called the 'permittivity of free space' and is a fundamental constant linking field strength and charge density.

ii $\frac{Q}{V} = \varepsilon_0 \frac{A}{d}$

$\frac{Q}{V}$ is the capacitance C so

$$C = \varepsilon_0 \frac{A}{d}$$

The presence of a medium other than air (or a vacuum) increases the charge Q by some factor ε_r called the 'relative permittivity', which is different for each material but, being purely a factor, has no units. For air $\varepsilon_r \approx 1$, for paper ε_r is 2 to 3, for water $\varepsilon_r \approx 80$. The modified equations are then

QUESTIONS 14 to 18

$$\sigma = \varepsilon_r \varepsilon_0 E \qquad \text{and} \qquad C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

HOME EXPERIMENT EH1
The capacitor

The second equation is used in the design of capacitors: one can be made from household materials.

Applications

READING
Applications of electrostatics (page 305)

Many industrial processes make use of electric fields, while 'static' can be a big problem in industry and in the home.

Section E2

GRAVITATIONAL FIELD AND POTENTIAL

This Section is concerned with gravitational fields – in particular that of the Earth. The fundamental ideas and the way they are linked are very similar to those in the last Section – except that, whereas electric fields act on charges, gravity acts on masses.

A uniform gravitational field

$g = \frac{F}{m}$

$g \approx 9.81\ \mathrm{N\,kg^{-1}}$ on Earth's surface

The gravitational field strength, g, is defined as the force on unit mass. Measurements show this to be virtually uniform near the surface of the Earth, and certainly so in the laboratory.

Gravitational potential difference

$\Delta(\text{g.p.e.}) = mg\Delta h$

$\Delta V_g = \frac{\Delta(\text{g.p.e.})}{\text{mass}} = g\Delta h$ in uniform field

$\Delta(\text{g.p.e.}) = \Delta V_g \times \text{mass}$

QUESTIONS 19 to 21

In the uniform field, changes in gravitational potential energy (g.p.e.) are given by $mg\Delta h$, where Δh indicates a change in height. It is useful to know the change in potential energy per kilogram; this is called *gravitational potential difference* (ΔV_g). In this context it is simply $g\Delta h$.

Contours on a map join points at the same height above sea level. The energy involved in moving from one contour to another can be calculated from the gravitational potential difference (ΔV_g) between the two contours. Movement along a contour involves no change in g.p.e.: contours can be called gravitational *equipotentials*.

The inverse-square law for gravitational fields

Newton first deduced that the gravitational force obeys an inverse-square law. The force, F, between two masses m_1 and m_2 separated by distance r is

$G \approx 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

$$F = -G\frac{m_1 m_2}{r^2}$$

G is a universal constant, which applies to all masses everywhere.

The '–' sign is used to indicate that the force, F, is inwards, that is, attractive.

QUESTIONS 22 to 24

Because gravitational forces between laboratory sized objects are so small, very sensitive equipment is needed to measure G.

Testing the inverse-square law

Since field strength is defined as force per unit mass, the field strength, g, due to a mass M is given by

$$g = -\frac{GM}{r^2}$$

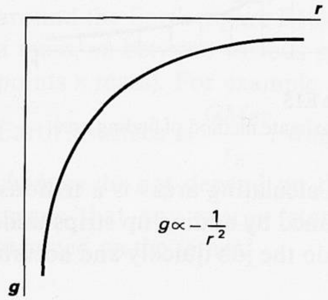


Figure E12

The inverse-square law can be tested over large distances by analysing spaceflight data. Indeed it is used to plan the trajectories of spacecraft with great accuracy.

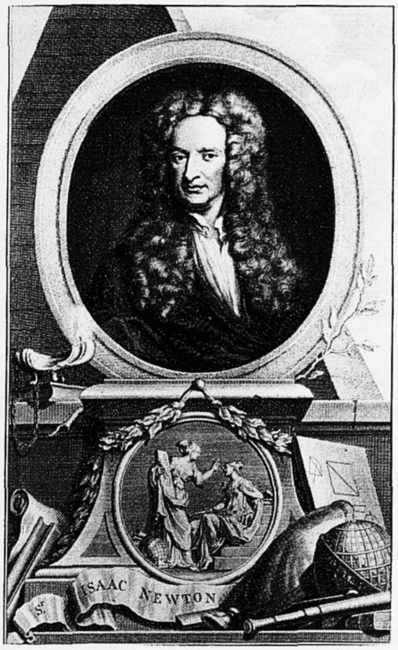


Figure E13
Sir Isaac Newton (1642–1727).
The Mansell Collection

Calculating energy changes from force–distance graphs

Energy changes can be calculated by computing areas under a force–distance graph as shown in figure E14(a). Since field strength is force per unit mass, the area under a graph of field strength against distance indicates the energy change per unit mass, or the *gravitational potential difference*, ΔV_g, shown in figure E14(b).

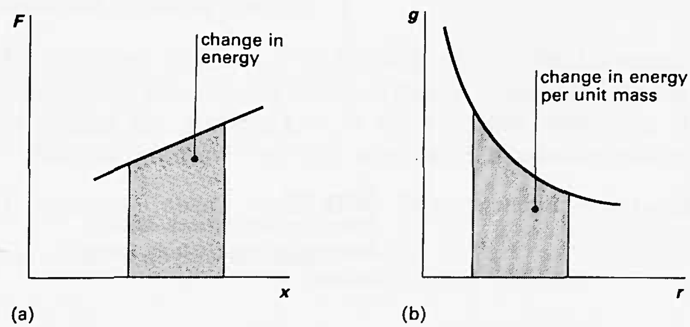


Figure E14

E

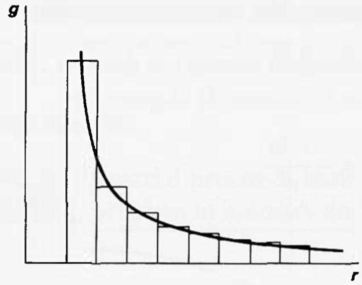


Figure E15
Approximate method of finding area.

QUESTION 25

Computer program 'GFIELD'

Calculating areas is a tedious process: approximate results can be obtained by adding up strips under the graph (figure E15). A computer can do the job quickly and accurately.

Calculating ΔV_g from field–distance graphs

QUESTION 26

QUESTION 27

The amount of energy required to move unit mass from the Earth's surface to various distances r_2 from the centre of the Earth increases with r_2, but approaches a finite limit, even if r_2 is very large (figure E16). Conversely, the energy required to move unit mass from a point distance r_1 from the Earth's centre, to some larger distance r_2 decreases as r_1 is increased (figure E17). Measurements from graphs show that this value of ΔV_g is given by

$$\Delta V_g \approx 4 \times 10^{14}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

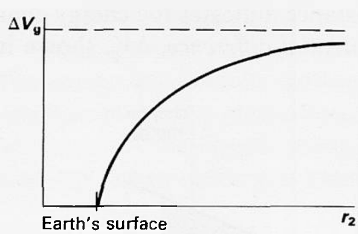


Figure E16
Energy needed to reach different distances r_2, starting from Earth's surface.

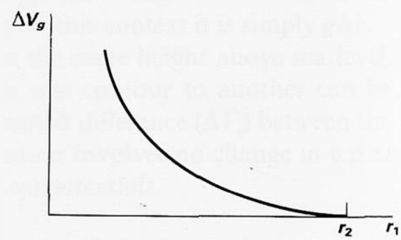


Figure E17
Energy needed to reach distance r_2 depends on starting position r_1.

Gravitational potential V_g

QUESTION 28

Potential is negative for all attractive fields

If we make r_2 '*infinity*' or 'as far away as we like' we can easily calculate the energy required to pull one kilogram completely free of the Earth's gravity. It seems sensible to agree on some reference point to use as our zero of gravitational potential energy. This 'point' is agreed to be 'at infinity', or 'as far away from any mass as you like'. There the gravitational potential energy of any object is zero. But since it loses potential energy and gains kinetic energy by 'falling' towards the Earth (or any other planet), then we must consider the potential energy at points nearer the Earth to be negative. (This is so for all attractive fields.) The formula for gravitational potential at distance r from mass M is

QUESTIONS 29 and 30

$$V_g = -\frac{GM}{r}$$

Equipotentials, lines joining points at equal potential, can be drawn around the Earth (figure E18). From these the energy required to move a mass, m, between various points can be calculated (ΔV_g between the points × mass). For example, the energy for complete escape from the Earth's surface is $\frac{GMm}{r_E}$, where r_E = radius of the Earth. Such energy changes do not depend on the route taken between the points. This means that any energy 'stored up' on an outward journey can be regained on the return.

QUESTION 31

g field is 'conservative'

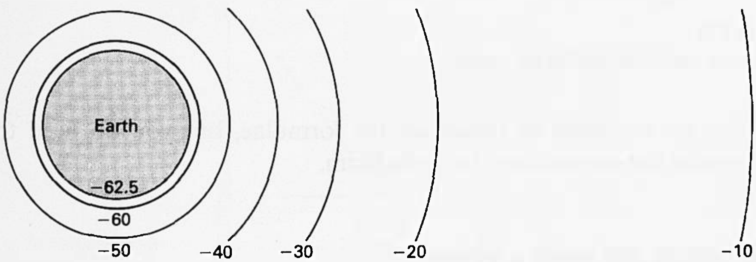


Figure E18
Equipotentials around the Earth (at intervals of 10×10^6 J kg^{-1}).

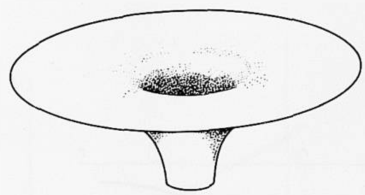


Figure E19
$\frac{1}{r}$ profile potential well or rubber sheet model.

Field and potential gradient

Potential can be found from the area under a field strength–distance graph. Field strength can be found from a potential–distance graph by measuring the gradient. As in the electrical case 'field strength = − potential gradient'. It is easy to show by measuring gradients that if V_g varies as $\frac{1}{r}$, then g varies as $\frac{1}{r^2}$. This connection will be used in the next Section, for electric fields.

for gravity: $g = -\frac{dV_g}{dr}$

for electricity: $E = -\frac{dV}{dx}$

for both: $\frac{1}{r}$ potential ⇔ $\frac{1}{r^2}$ field

QUESTION 32

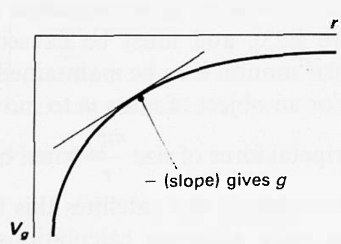


Figure E20

Formulae and relationships for gravity

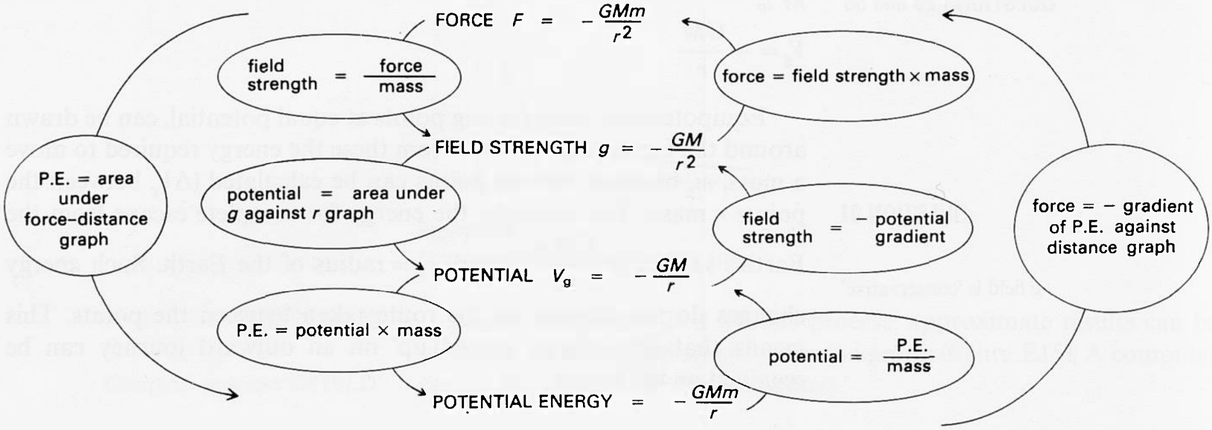


Figure E21
Formulae and relationships for gravity.

You do not need to *remember* the formulae, but you do need to *understand* the connections between them.

Field outside and inside a sphere

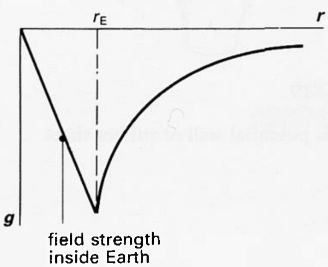


Figure E22
Field strength outside and inside a uniform sphere.

QUESTIONS 33 to 35

The inverse-square law, defined for 'point' masses, works outside a *solid* sphere of uniform density (which the Earth is, very approximately), or a hollow sphere, as if all the mass were concentrated at the centre. This is an invaluable simplification, enabling field and potential to be calculated even quite close to the surface of a large sphere. It will be of even greater use later as it applies also to electric fields around a charged sphere.

The value of g falls to zero at the centre of a sphere of uniform density (figure E22) with some interesting, if theoretical, consequences.

Inside a *hollow* sphere there is no field.

Circular motion

QUESTIONS 36 and 37

An object moving in a circle at constant speed changes direction and therefore *velocity*. Its acceleration is towards the centre or 'centripetal' (figure E23), and must be caused by an overall *centripetal force* if circular motion is to be maintained.

For an object of mass m to move at speed v in a circle of radius r, a centripetal force of size $\frac{mv^2}{r}$ must be provided towards the centre.

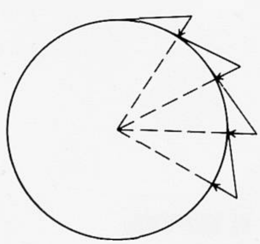


Figure E23
Centripetal force acts inwards.

QUESTIONS 38 to 40

For planets and satellites this force is provided by gravity. We can make quite accurate calculations of the positions and energies of satellites, assuming circular orbits, though we know that satellite and planetary orbits are in fact elliptical, with varying degrees of eccentricity.

Section E3 THE ELECTRICAL INVERSE-SQUARE LAW

Unit F, 'Radioactivity and the nuclear atom'

Unit L, 'Waves, particles, and atoms'

Many applications of electrostatics can be understood using the uniform field alone. However, if we want to delve into the structure of atoms, which are in essence made up of charged particles, we must know how spherical and point charges behave.

Potential near a charged sphere

DEMONSTRATION E8a
Investigating the variation of potential around a charged sphere

The field of a spherical mass varies as $\frac{1}{r^2}$ and the associated potential as $\frac{1}{r}$. The electric potential around a charged sphere is found to vary as $\frac{1}{r}$ (figures E24 and E25). By analogy with gravity we deduce that the electric field varies as $\frac{1}{r^2}$.

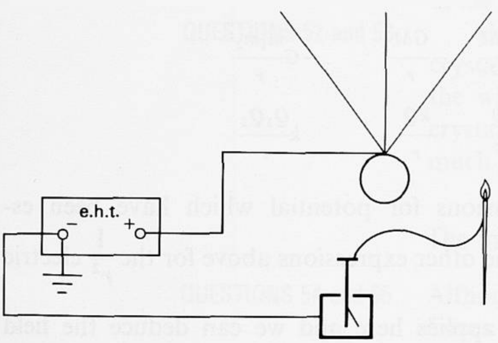


Figure E24

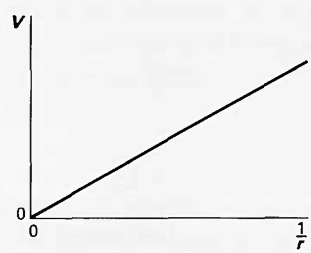


Figure E25

DEMONSTRATION E8b
Measuring the value of k in $V = \frac{kQ}{r}$

The potential, V, depends also on the charge, Q, on the sphere (figure E26).

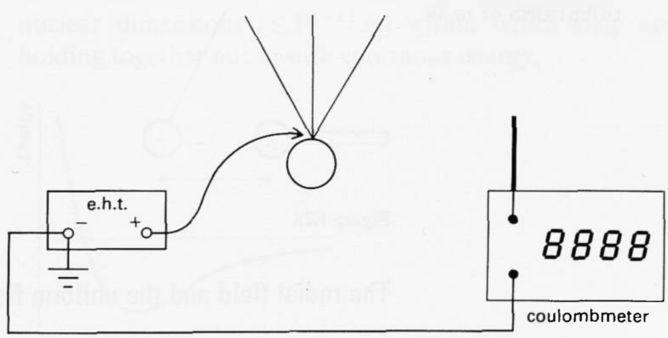


Figure E26

If Q is positive, the potential is positive: a positive charge nearby has positive potential energy and will be repelled. Conversely a negatively charged sphere has negative potential and would attract a nearby positive charge (figure E27).

By contrast all gravitational potentials are negative because all gravitational forces are attractive.

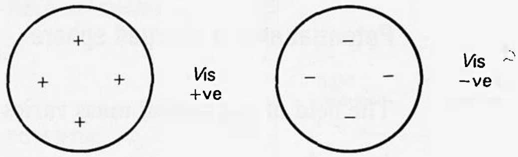


Figure E27

Analogy between electricity and gravity

	Force	Field	Potential	Potential Energy
Gravity	$-G\dfrac{m_1 m_2}{r^2}$	$-\dfrac{GM}{r^2}$	$-\dfrac{GM}{r}$	$-G\dfrac{m_1 m_2}{r}$
Electricity	$k\dfrac{Q_1 Q_2}{r^2}$	$\dfrac{kQ}{r^2}$	$\dfrac{kQ}{r}$	$k\dfrac{Q_1 Q_2}{r}$

Comparing the expressions for potential which have been established we can deduce the other expressions above for the $\dfrac{1}{r^2}$ electric field. $E = -\dfrac{dV}{dr}$ of course applies here and we can deduce the field strength from the gradient of a graph of potential against distance. The force expression may be tested experimentally with some care (figure E28). Coulomb first established it as a fundamental law of nature in 1785.

EXPERIMENT E9
Experiments to test the inverse-square law for electric forces

Compare Newton's Law for Gravity

QUESTIONS 41 to 44

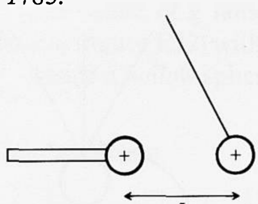


Figure E28

The radial field and the uniform field

QUESTION 45

Single charges have a $\dfrac{1}{r^2}$ field: yet many charges together on a large flat plate give a uniform field (Section E1). This is because of the way in which contributions from each bit of charge add up to give an overall effect (figure E29). The mathematics of this adding up process show that the constant, k, is in fact $\dfrac{1}{4\pi\varepsilon_0}$.

QUESTIONS 46 to 51

Computer program 'EFIELD'

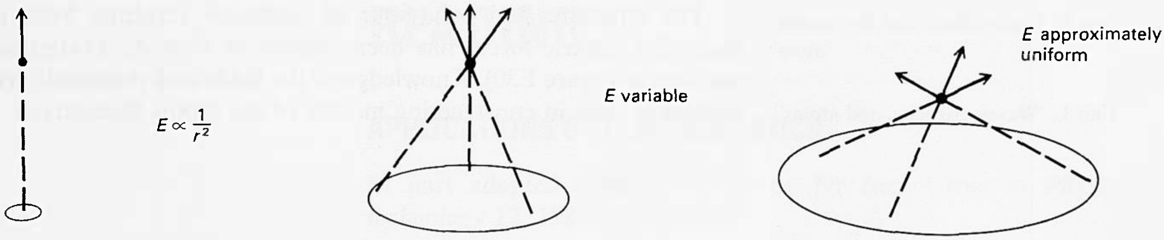


Figure E29

Sizes of electrical and gravitational forces

$G \approx 7 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$

The electrical and gravitational force constants are very different in size. Comparison of the two forces between an electron and a proton within an atom shows that the electrical forces are bigger by a factor of nearly 10^{40}. For the charged spheres of the Coulomb's Law experiment the ratio is less, but still about 10^{10}. This means that very few of the atoms of the ball carry charge.

QUESTIONS 52 and 53

Although very large forces exist between neighbouring ions within a crystal, say, the almost perfect balance of + and − charges ensures that the whole crystal is neutral and exerts no electrical force on other crystals. More massive objects experience gravitational forces which are much greater than the electrical forces between them.

The four known interactions

QUESTIONS 54 and 55

'Forces and particles' in the Reader *Particles, imaging, and nuclei*

Although they both obey inverse-square laws, electric and gravitational forces are quite distinct, acting as they do on charges and masses respectively. All other 'everyday' forces, (friction, surface tension, 'contact' forces between objects and even magnetism) are due to the interactions of charged particles surrounding atoms at rest or in motion. However, two other apparently distinct kinds of force do exist – the 'strong' and the 'weak' nuclear forces. Their range is limited to nuclear dimensions ($\lesssim 10^{-15}\,\mathrm{m}$) within which they are capable of holding together nuclei with enormous energy.

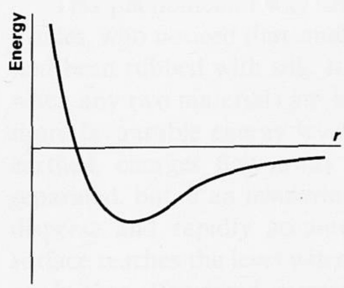


Figure E30
Intermolecular energy.

Unit F, 'Radioactivity and the nuclear atom'

Unit L, 'Waves, particles, and atoms'.

The structure and behaviour of materials resulting from intermolecular electric forces has been studied in Unit A, 'Materials and mechanics' (figure E30). Knowledge of the fields and potentials around charges is used in constructing models of the atoms themselves.

READING

APPLICATIONS OF ELECTROSTATICS

In part adapted from an article by Dr Jean Cross in *Physics in technology* **12,** 1981, pages 54–59.

Introduction

Electrostatic forces are generally weak compared with gravitational effects and have little influence on macroscopic bodies. However, for small particles (1–100 μm), electrostatic forces can exceed gravitational and aerodynamic effects and this has led to the development of a wide range of industrial processes which rely upon electrostatic phenomena.

Charging small particles

In the macroscopic world electrostatic forces have little impact. Sparks may be observed which demonstrate the high potential which can be achieved when insulating materials are rubbed, but the energy stored is low and although the sparks may be a hazard in a flammable atmosphere, their energy cannot be usefully harnessed to control large bodies. However, the electrostatic charge stored (and hence the force experienced in an electric field) depends on surface area. The gravitational force, on the other hand, depends on mass or volume, so, as size decreases, electrostatic forces become gradually more important and can dominate the motion of particles less than a few hundred micrometres in diameter.

This principle has been used in a wide range of applications in which the motion of small charged particles is controlled with considerable accuracy by an electric field. Particles and droplets are nearly always naturally charged but for most applications charge is added artificially both to maximize the magnitude and to achieve uniformity. Particles can be given a charge by a number of different techniques of which friction is the simplest and best known.

This phenomenon was first observed more than 2500 years ago by Thales, who noticed that small particles were attracted to amber which had been rubbed with silk. It is now known that charge is transferred when any two materials are brought into contact as electrons move to more favourable energy levels. If both materials are conducting and earthed, charges flow away instantaneously when the surfaces are separated, but, if an insulating material is involved, the charge cannot disperse and rapidly accumulates until the electric field above the surface reaches the level where the air ionizes and charge can build up no further. Powdered materials easily acquire charge by friction in passing through a tube and even most metals are sufficiently insulating in powdered form for charge to accumulate if the particles move rapidly.

Frictional charging is not easy to control and may vary considerably from day to day, therefore many industrial applications of

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electrostatics rely on corona charging. In this process a high voltage is applied to a sharp point, causing the air to ionize in the high field region around the point. Ions of the opposite polarity are attracted by the point but those of the same polarity are repelled, forming a stream of unipolar ions. These ions will be attracted to any powder or dust particles present because the difference in relative permittivity between the powder and air causes a distortion in the electric field (figure E31) and thus ions are pulled in to the particle. Charging stops when the repulsion due to the charges on the particle balances the attraction due to the field distortion.

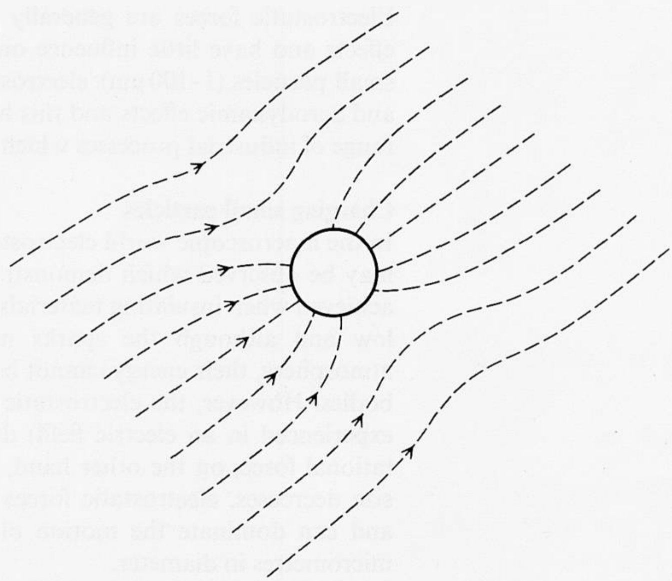

Figure E31
Distortion of field by a dust particle.

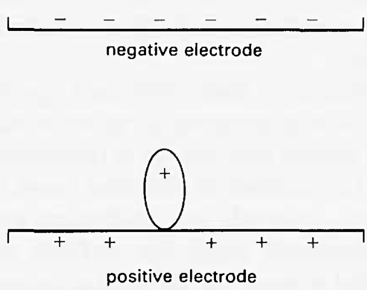

Figure E32
Induction charging.

A third method of applying charge – induction – can be used only for conducting particles and is most often used in practice to charge liquid droplets. The principle is illustrated in figure E32. A particle on a positive electrode in an electric field will acquire a positive charge, simply because the charge is attracted towards the negative electrode. The force of attraction may be sufficient to detach the particles from the electrode, creating a free charged particle. A liquid surface is distorted in an electric field and is pulled up into a cone. The induced charge on this cone can cause droplets to break off – the process of electrostatic atomization. The resistivity of the liquid is critical for the formation of uniformly charged droplets. If it is too conducting, a corona discharge may form or the cone break up erratically. If it is too insulating, charge cannot flow to help form the cone.

Electrostatic generators

One of the earliest applications was the design of electrostatic generators which use mechanical energy to build up a high voltage. Most are high voltage, low current devices. The Van de Graaff machine is probably the best known and is still used today for the production of voltages exceeding 1 MV for the acceleration of elementary particles in nuclear research.

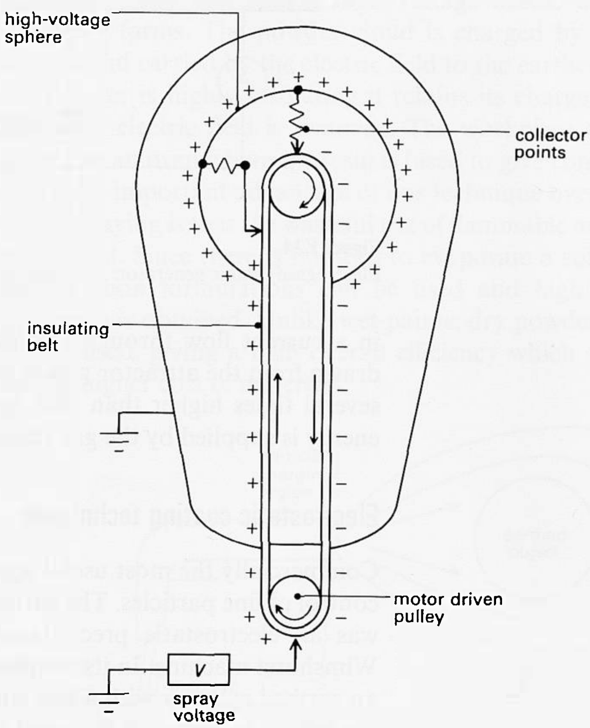

Figure E33
Van de Graaff generator.

A Van de Graaff generator is shown schematically in figure E33. Charge is sprayed onto the bottom of a conveyor belt by corona from a sharp point. (This charge may also be produced by friction.) As the belt moves round, charge is transferred to the large sphere by ionization from collector points. Resistors maintain a voltage between the sphere and the upper pulley so as to allow a negative discharge from the collector points, leaving a positive charge on the sphere. The larger the sphere, the higher the voltage which can build up without discharge. The leakage from the sphere limits the voltage which is produced. Mechanical belt generators have been produced for high voltage laboratory supplies of 50–200 kV, although at these levels conventional multipliers are more commonly used.

Electrogasdynamic generators also produce voltages of this level and have been used in electrostatic paint and powder spray guns. In this device ions are created, usually by a corona discharge between a point and an attractor, and then forced down a tube by a high velocity air stream. Usually a vapour or a powder is introduced to collect ions and reduce their mobility so they are not collected at the attractor but are swept on to the collector. It can be seen from figure E34 that this results

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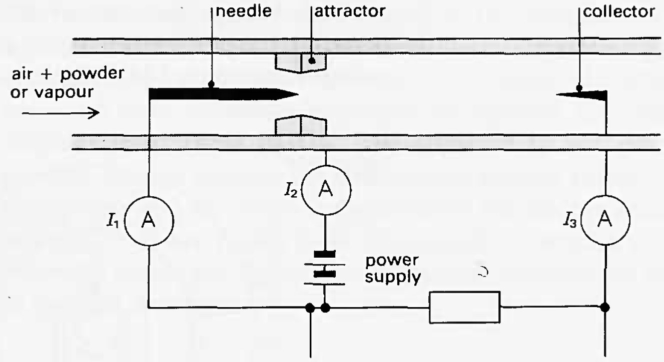


Figure E34
Electrogasdynamic generator: I_1 needle current; I_2 attractor current; I_3 collector current.

in a current flow through the load which is higher than the current drawn from the attractor power supply. The voltage on the collector is several times higher than that applied to the attractor and the extra energy is supplied by the gas stream motion.

Electrostatic coating techniques

Commercially the most useful applications of electrostatics involve the control of fine particles. The earliest device to be put into industrial use was an electrostatic precipitator built in 1890 and powered by a Wimshurst machine. In its simplest form an electrostatic precipitator is an earthed cylinder with a fine wire along its axis which gives a corona discharge. As dusty air is passed through the cylinder, the particles are charged by the corona ions and repelled by the wire to be deposited on the cylinder walls. Commercial precipitators are now generally two-stage devices with separate charging and collecting zones. The discharge electrodes are wires suspended between plates (figure E35) and the planar collecting region can be cleaned without disturbing the electrodes. Efficiencies of over 99 per cent can be achieved down to particle sizes of a few micrometres. Precipitators were first developed for large scale industrial processes such as steel works, but small devices

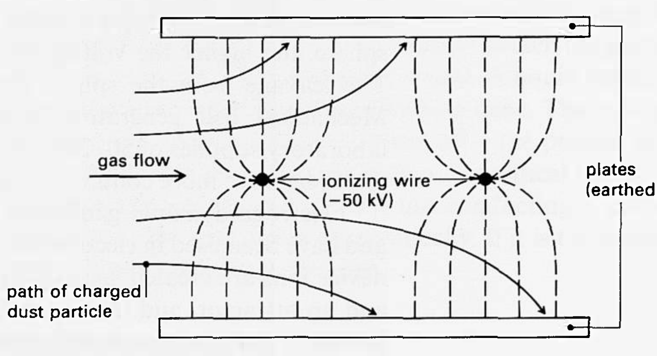


Figure E35
Electrostatic precipitation.

have also been developed for collecting cigarette smoke from public rooms.

In the electrostatic powder coating process shown diagrammatically in figure E36, a thermoplastic or thermosetting resin powder is blown out of a spray gun past a high voltage needle at which a corona discharge forms. The powder cloud is charged by the ions from the corona and carried by the electric field to the earthed workpiece. Since the powder is highly insulating it retains its charge and adheres even when the electric field is removed. The workpiece can then be transported to an oven where the resin is fused to give continuous paint film. The most important advantage of this technique over conventional wet paint spraying is that the wasteful use of flammable and toxic solvents is eliminated. Since there is no need to evaporate a solvent, better cross-linking resin formulations can be used and high scratch and chip resistance is obtained. Unlike wet paints, dry powders can be collected and re-used, giving a high overall efficiency which often compensates for the higher cost of materials.

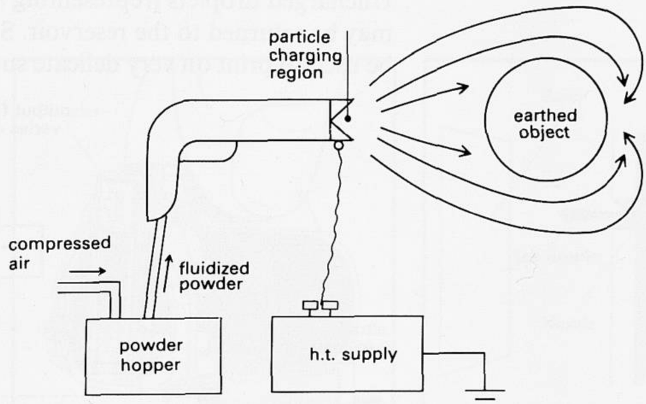


Figure E36
Electrostatic powder coating.

Electrostatic deposition may also be used with wet paints. The paint is atomized by an air jet, by a rapidly spinning disc or bell, or occasionally simply by electrostatic forces which tend to break up large drops until the electrostatic repulsion is counteracted by the surface tension of the drop. As with any electrostatic process, the paint travels along the electric field lines to areas not directly within the field of view of the gun – a phenomenon known as wrap round. Recently the techniques developed in the 1960s for electrostatic paint spraying have also been applied to crop spraying. Efficiencies have been increased considerably and there is a great improvement in deposition on the underside of leaves, which is particularly important for fungicides.

The ink-jet printer – printing without pressing

One of the problems with modern computers and data processing is that whilst a computer can happily churn out 10^6 numbers per second, a printer which can produce only 100–200 characters per second is obviously slowing down production somewhat. The development of a printer capable of dealing with over 1000 characters per second with a resolution of over thirty points per centimetre is clearly an 'order of magnitude' improvement.

There are several types of ink-jet printers being developed or in use. In the 'deflect-to-print' type, which is shown in figure E37, ink from a reservoir is pushed through a narrow jet (about 35µm in diameter) which is modulated ultrasonically at a frequency of about 500 kHz and breaks up into a fine stream of droplets. A charging cylinder through which they pass induces on each drop a charge which varies according to the p.d. applied to the cylinder. This p.d. is determined directly by the computer so that each drop can be given a unique charge. The drops now pass between deflector plates, rather as in an oscilloscope, across which there is a steady p.d. so that the deflection depends on the charge of the drop. As the jet moves over the paper (or vice versa), characters can be built up at extremely high speed and with great precision. Uncharged droplets (representing spaces) are collected in a gutter and may be returned to the reservoir. Since no contact is made, the jet may be used to print on very delicate surfaces – even butterfly wings!

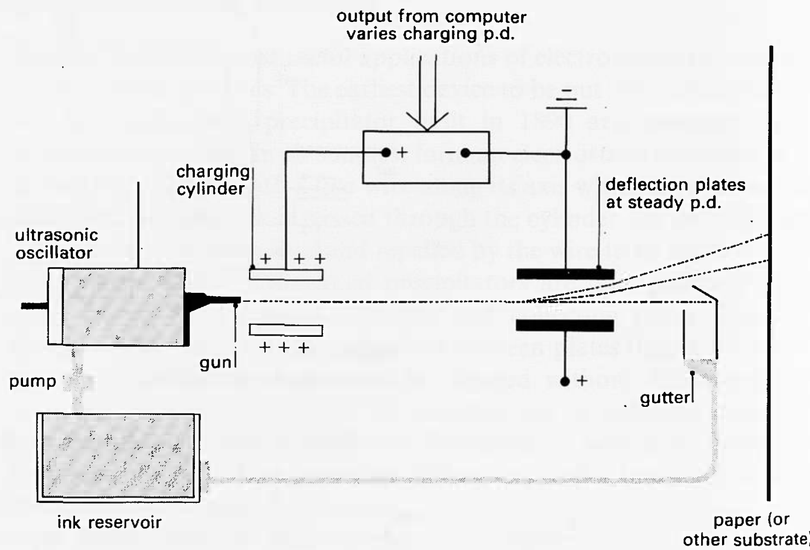


Figure E37
Ink-jet printer. Deflect-to-print type.

This 'continuous flow' method has some advantages over the simpler and cheaper 'drop-on-demand' printer where the ink jet is held back by surface tension and switched on at low pressure only when required. The drop-on-demand printer shown in figure E38 has been designed for use with personal computers. Considerable research and development has gone into producing an ink which does not dry in the jet, blocking it up, but which *does* dry quickly on paper. A third method charges the drops *not* required for printing and deflects them away to an earthed plate. With the use of three coloured inks a large full-colour page can be printed to a high degree of resolution in a few minutes. This

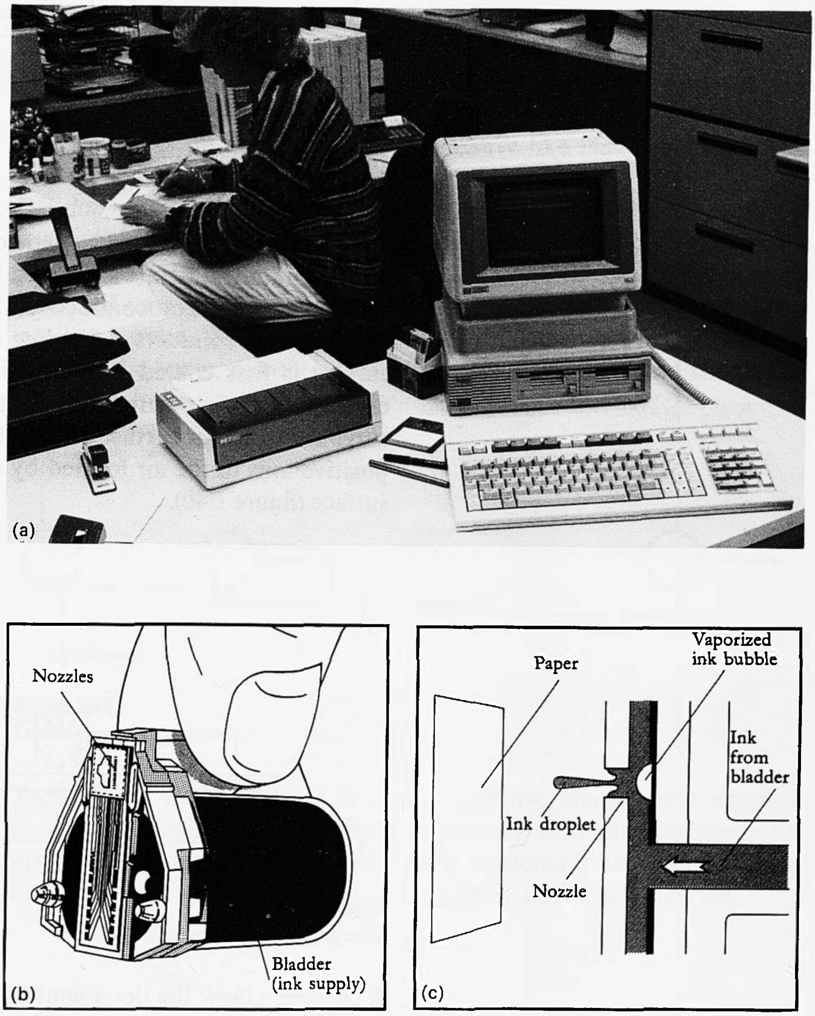


Figure E38
'ThinkJet', a drop-on-demand type ink-jet printer. The printer is small and portable and is used here with an office microcomputer. It also has the advantage of operating below 50 dB. Diagrams (b) and (c) show how the printer works.
Hewlett-Packard Ltd.

opens up potential applications in cartography and the production of hard copies of aerial and satellite images.

Xerography

This is the process used in almost all copying machines. In the U.K. one leading firm's machines alone produce 12 billion copies a year; typical office machines handle up to 120 copies per minute.

The name derives from two Greek words meaning 'dry writing', and it was indeed a great step forward when copies of documents could be made almost at once on ordinary dry paper without the messiness of earlier methods of printing. The basic process has about six main stages: charging, exposure, development, transfer, fusion and cleaning.

Charging The plate or rotating drum on which the image of the

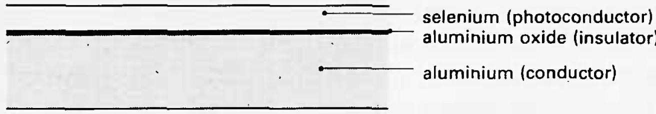


Figure E39
The photoreceptor plate.

document is to be formed is made of a thin (60 μm) layer of selenium on an aluminium substrate separated by a thin layer of oxide (figure E39).

Selenium is photoconductive, that is, it is effectively an insulator in darkness but conducts well when exposed to light. The plate or drum surface is first coated with positive charge by being traversed by a corotron, or fine wire at high positive potential (typically 850 V), surrounded by an earthed metal shield. Since the plate is also earthed, positive ions in the air formed by corona discharge are forced onto its surface (figure E40).

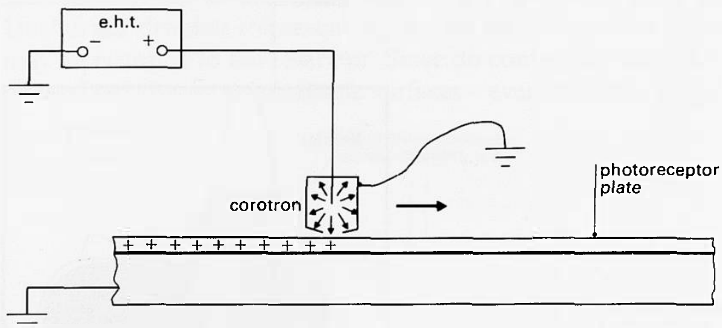


Figure E40
Charging the photoreceptor plate.

Exposure Now the document to be copied is illuminated. A lens, and usually also mirrors, to reduce the physical size of the device, focus a real image (laterally inverted) onto the photoreceptor plate. Where light falls on the selenium it conducts and the charge drains away through the earthed aluminium substrate (the oxide layer is thin enough to allow this). However, dark areas (for example writing) on the document cause

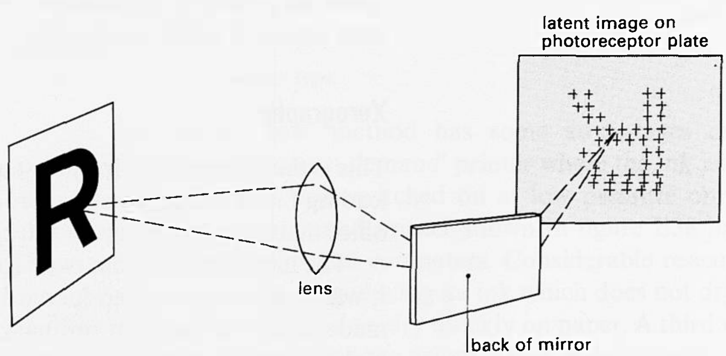


Figure E41
Exposure.

dark areas on the image so the charge remains at these points. So a 'latent image' is formed on the plate (figure E41).

The selenium is most sensitive to blue light, whose photon energy is greatest (see Unit L, 'Waves, particles, and atoms') and least to red, so that red areas on a photograph will behave like black, but light blue areas, like white, will be difficult to reproduce.

Development A dry black thermoplastic powder called toner is used to develop this latent image. The toner powder is mixed with carrier beads (metal shot or glass); the carrier and toner obtain opposite charge by frictional contact and the now negative powder adheres lightly to the positive carrier. When this mixture is sprinkled over the latent image, most of the toner powder sticks to the positive image and the carrier falls off to be re-used (figure E42).

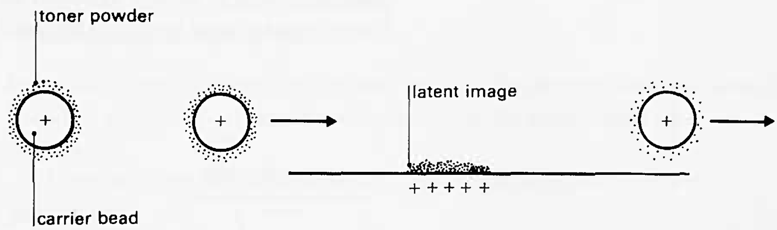


Figure E42
Development using toner.

Transfer A sheet of paper is given a positive charge by a separate corona wire and placed carefully in contact with the photoreceptor plate. A good deal of the toner powder is attracted to the paper forming on it an image which is now like the original, that is, not inverted.

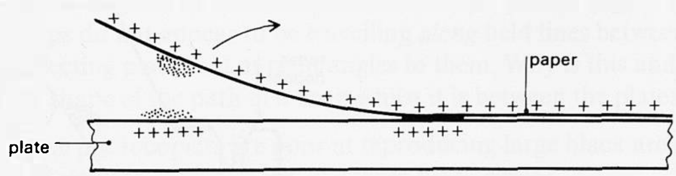


Figure E43
Transfer onto paper.

Fusion The paper is transported underneath a radiant heater or between hot rollers to melt the plastic toner (at about 140 °C) giving a firm, permanent coating to the paper (figure E44). Alternatively, the fusing may be done using a chemical vapour which breaks down the plastic in a similar way to melting.

Cleaning Some toner remains sticking to the plate or drum. This 'residual image' must be cleaned off before the next copy is taken. First the plate or drum is neutralized, generally by a corona discharge from an alternating current source, to loosen the powder. Next the powder is

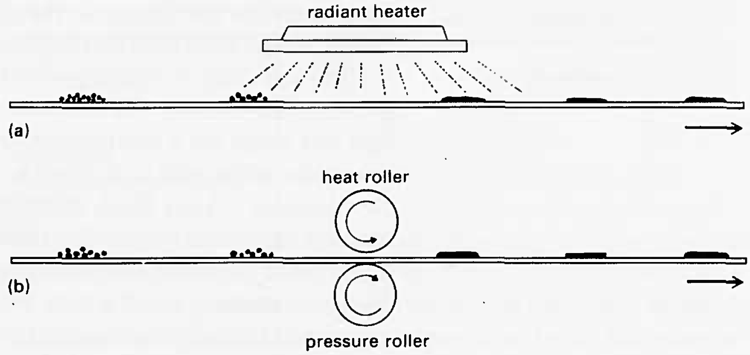


Figure E44
Fusing to make a permanent copy.

scraped, wiped, or brushed off. Since this process may charge up the plate again, it is finally exposed to light to render the selenium surface free of charge and ready to be used again (figure E45).

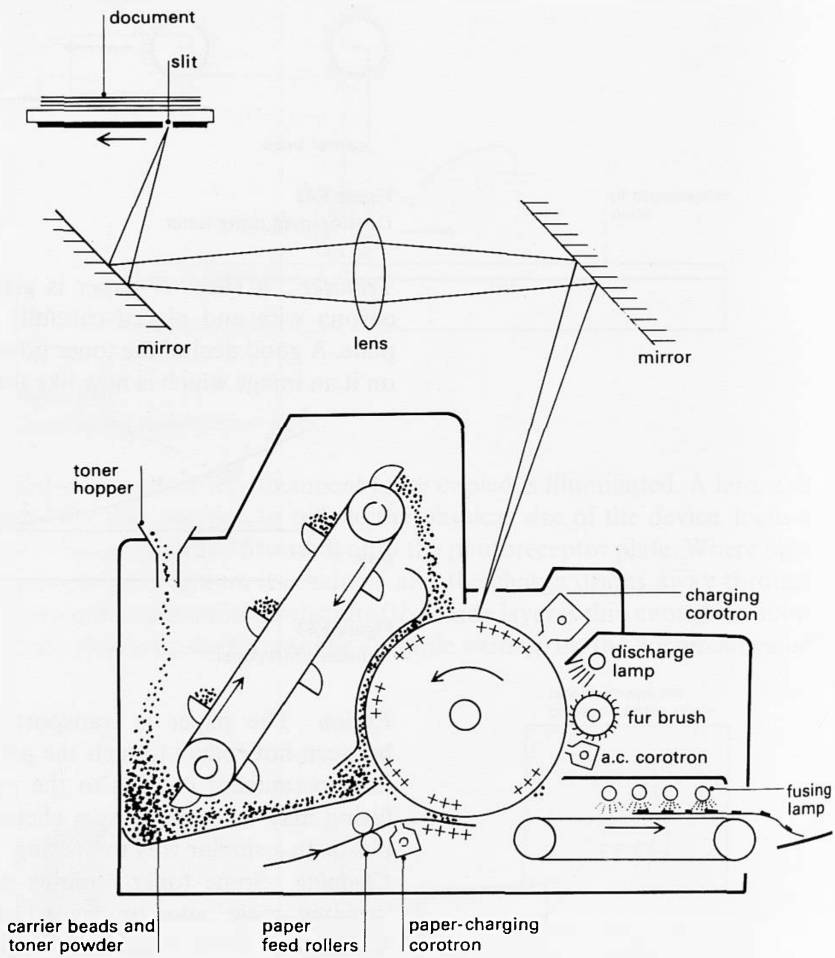


Figure E45
The complete process for a rotating drum copier. In this case a slit traverses the document forming an image on the drum as it rotates.
(*Information courtesy of Rank Xerox (U.K.) Limited, who may supply further details on request.)*

Questions

a *i* The article states that 'charge stored depends on surface area'. This implies a limit on the charge density on a surface. Why is this limited? (Think about the field strength at the surface and support your argument with relevant formulae.)
ii Why do electrostatic forces have more effect on small particles?

b Describe the three most common methods by which particles are given an electric charge, explaining the limitations of each method.

c Discuss the effect on the potential attained by a Van de Graaff generator of each of the following (separately):
i increasing the radius of the sphere;
ii increasing the resistance between the sphere and the upper pulley;
iii increasing the speed of the belt;
iv increasing the spray voltage.

d In an electrogasdynamic generator, why is the current flowing through the load greater than that flowing between the needle and the attractor?
How will the gas stream be affected by giving energy to the generator?

e What are the advantages of electrostatic dry powder paint spraying over conventional liquid paint spraying? Are there any disadvantages? Explain how the technique could be applied to crop spraying.

f The article states that 'paint travels along the electric field lines'. Sketch some equipotentials and lines of electric field between a paint nozzle and a spherical earthed object.

g In the diagram of the deflect-to-print ink-jet printer (figure E37) the drops do not appear to be travelling *along* field lines between the deflecting plates but at right angles to them. Why is this and what is the shape of the path of a drop whilst it is between the plates?

h Some photocopiers are poor at reproducing large black areas.
i Sketch the 'latent image' of a large black circle.
ii Considering the mutual repulsion of the charge on this image, suggest why the central area may not come out fully black.

i Describe what happens during one complete rotation of the drum in figure E45.

j 'Static' can be a hindrance, indeed a hazard, as well as being of use in commerce and industry. Find out about some problems of static, for example, on plastic-hulled ships, in pumping liquids in oil tankers, in the manufacture of semiconductor devices.

E

LABORATORY NOTES

DEMONSTRATION

E1 Forces on a charged ball between charged plates – the 'shuttling ball'

e.h.t. power supply
2 metal plates with insulating handles
table tennis ball coated with colloidal graphite
nylon sewing thread
galvanometer (*e.g.*, internal light beam)
polythene strip
2 retort stand bases, rods, bosses, and clamps
leads

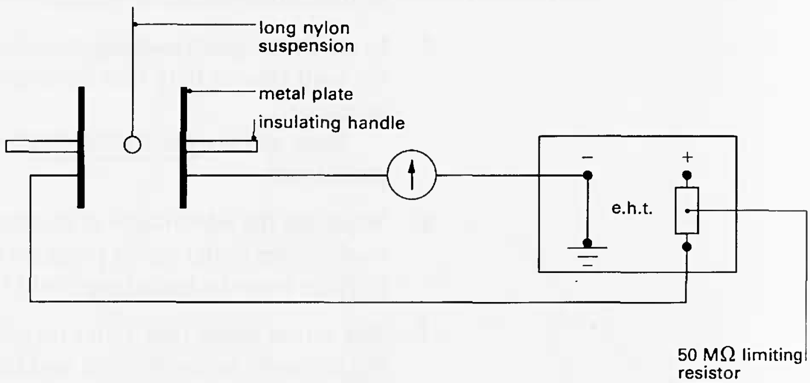


Figure E46
The 'shuttling' ball.

What causes the ball to move from one plate to the other?
Why does it keep changing direction?
What sign of charge is being 'ferried' across the gap by the ball? Why does the galvanometer not show pulses of current in *both* directions?
What factors affect the force on the charged ball? How do they affect it?

DEMONSTRATION

E2 Using a charged foil strip as a field detector

pair of capacitor plates
2 slotted bases
2 square polythene tiles
polythene strip
foil
adhesive tape
razor blade or scissors
e.h.t. power supply
leads
means of projection (optional)

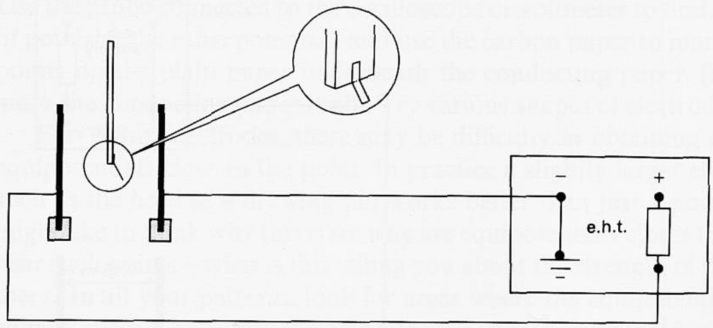


Figure E47
Charged strip field detector.

How can you judge the size of the electric force acting on the foil? How does this force vary from place to place
a between the plates,
b outside the plates?
How are the direction and size of the force on the foil affected by
a the p.d. between the plates,
b the distance between them, and
c the charge on the foil itself?
How might you set about measuring both the charge and the force on the foil? Explain whether or not these would be easy tasks.

DEMONSTRATION

E3 Patterns of electric field for different geometries (using semolina)

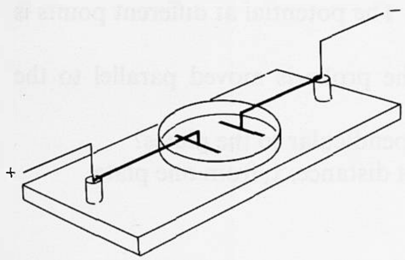


Figure E48
Apparatus for demonstrating electric field patterns.

either
e.h.t. power supply
or
Van de Graaff generator

electric field apparatus
1,1,1-trichloroethane
castor oil
semolina
bare copper wire, about 2 mm diameter, for electrodes
leads

You should be familiar with the pattern produced between two parallel electrodes, two point electrodes, one point and one straight electrode, and a point in the centre of a circular electrode.
What might happen if the liquid conducted electricity? Would you still obtain a field pattern? Explain your answer.

DEMONSTRATION

E4 Measuring potentials in a uniform field using a flame probe

The electroscope, calibrated as a voltmeter, measures the potential difference between the tip of the probe, connected to the cap, and the

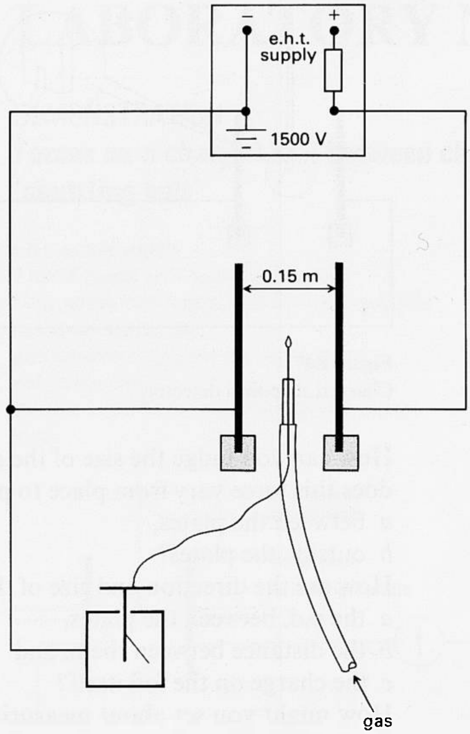


Figure E49
Using a flame probe to measure the potential between parallel plates.

case, which is earthed. The flame ionizes the air around the tip, neutralizing any charge on it, so that there is no p.d. between it and its surroundings. In effect, then, the electroscope measures the potential difference between a point in the space between the plates and earth. We call this the 'potential' at that point. The potential at different points is found by moving the probe around.

Does the potential change when the probe is moved parallel to the plates?

How does it change along a line perpendicular to the plates?

Draw a graph of potential, V, against distance, x, from one plate.

EXPERIMENT
E5 Plotting equipotentials in two dimensions for various shapes of field

4 cells in holder or l.t. variable voltage supply and smoothing unit
oscilloscope or voltmeter
pencil or ball point pen adapted as probe
copper or aluminium sheet, 0.1–0.5 mm thick
conducting paper (*e.g.* 'teledeltos' type) cut to A5 or A6 sheets
conducting putty
stapler and staples
carbon paper and white paper
drawing board or hardboard
bulldog clips
silver conducting paint (optional)

Use the probe connected to the oscilloscope or voltmeter to find a series of points at the same potential, and use the carbon paper to mark these points on the plain paper underneath the conducting paper. (Do not mark the conducting paper itself.) Try various shapes of electrodes.

For 'point' electrodes, there may be difficulty in obtaining reliable equipotentials close to the point. In practice a slightly larger electrode such as the *head* of a drawing pin works better than just a point. You might like to think why this is so: why are equipotentials closer together near such points – what is this telling you about the strength of the field there? In all your patterns, look for areas where the equipotentials are equally spaced: how does the field strength vary in these regions?

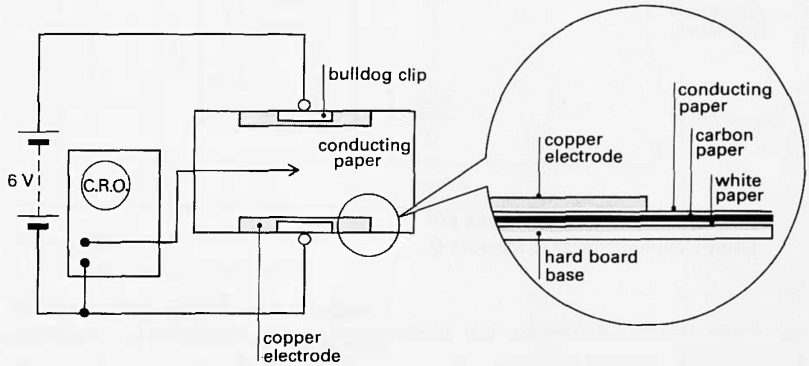


Figure E50
Plotting equipotentials on conducting paper.

Consider also the effect of the shape of the electrodes. How do sharp curves and corners affect the spacing of the equipotentials nearby?

Further investigations could include modelling some 'real' situations such as plotting equipotentials between different shapes of electrode in a cathode ray tube, say, or between the base of a thundercloud and the roofs of buildings beneath it. If silver paint is available such shapes can be easily created, though they can also be achieved using copper strip.

An alternative method for plotting equipotentials uses copper electrodes in copper sulphate solution. Teachers will be able to provide details.

EXPERIMENT
E6 Investigating factors affecting the charge on parallel plates using a coulombmeter

coulombmeter with probe rod
2 metal plates with insulating handles
e.h.t. power supply
polythene strip for use as insulating handle
2 retort stand bases, rods, and bosses
metre rule
leads

The unearthed plate is charged by a flying lead from the positive of the e.h.t. supply (with limiting resistor). The lead is held by an insulating handle. Other earthed conductors (the bench, hands, etc.) should be kept well away.

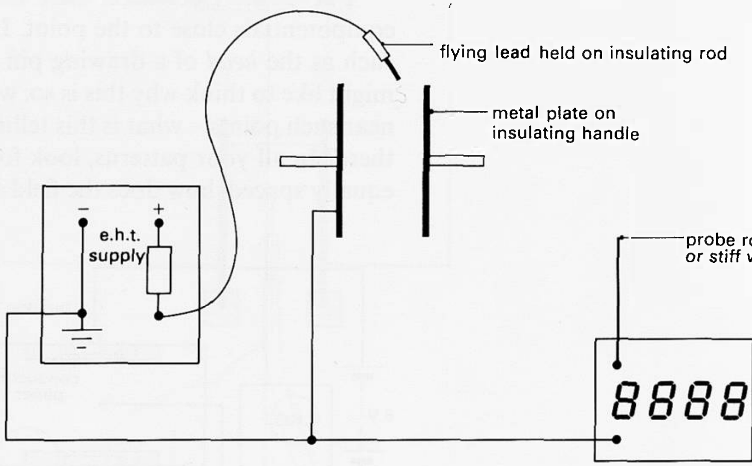


Figure E51
Using a coulombmeter to measure Q.

Caution: The flying lead *must be removed* from the plate before the charge is measured. The coulombmeter is then brought up so that the probe rod touches the plate; the charge can be deduced from the resulting reading.

Experiments to test whether $Q \propto \frac{1}{d}$ and $Q \propto V$ are possible. $Q \propto A$ would require a set of plates of various sizes. The effect of a plastic sheet or paper between the plates can also be explored.
Does the coulombmeter reading return reliably to zero when shorted, or does it suffer from zero drift?
How accurately repeatable is any given reading?
Is any charge left on the plate? How could you check this?

Plot a graph for Q against $\frac{1}{d}$. What significance has the Q intercept?

EXPERIMENT
E7 Investigating factors affecting the charge on parallel plates using a reed switch

reed switch
signal generator
1 pair capacitor plates with 16 polythene spacers, 10 × 10 × 1.5 mm
polythene sheet, 1.5 mm thickness, and paper sheets

either
l.t. variable voltage supply and smoothing unit
or
3 cell holders with four cells

voltmeter, 100 V and 10 V
galvanometer (*e.g.* internal light beam)
resistance substitution box
class oscilloscope
metre rule
mass, 1 kg
leads

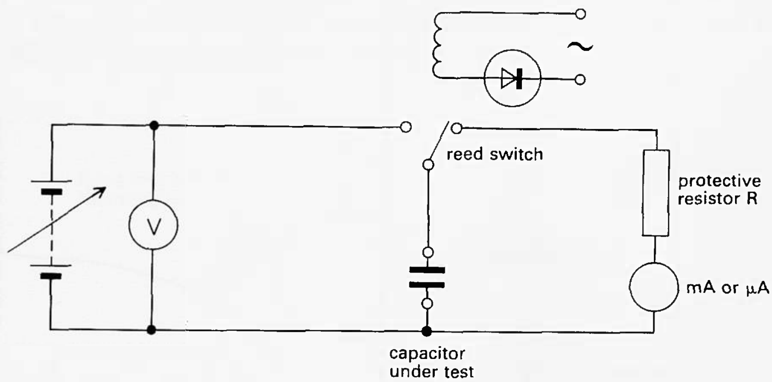


Figure E52
Using a reed switch to measure Q.

Caution: The signal generator output p.d. should be raised until the switch is heard to vibrate cleanly, and not increased further. Nor should the switch's stated operating p.d. be exceeded. The capacitor plates should never touch whilst the switch is working; turn off the signal generator if you are not taking readings, and disconnect the power supply before adjusting the plates. Keep hands and earthed objects well away from the plates when taking readings.

Use an oscilloscope across the protective resistor R to check that discharge of the capacitor is complete for each cycle. Use $R = 100\,\mathrm{k\Omega}$ and $f = 400$ Hz; try raising R or f, observing what happens.

$Q \propto V$ can be easily tested for p.d.s up to 25 V.

$Q \propto \frac{1}{d}$ can be tested by spacing the plates at different separations using spacers.

$Q \propto A$ can be tested by allowing different amounts of overlap between the plates.

Estimate the percentage uncertainties in your readings and draw error bars in your graphs. Are graphs of Q against A and Q against $\frac{1}{d}$ straight lines, within the limits of uncertainty? What significance has the Q intercept of each?

DEMONSTRATION

E8a Investigating the variation of potential around a charged sphere using a flame probe

The flame probe is constructed and the electroscope calibrated as in demonstration E4. A triple suspension of nylon thread keeps the ball

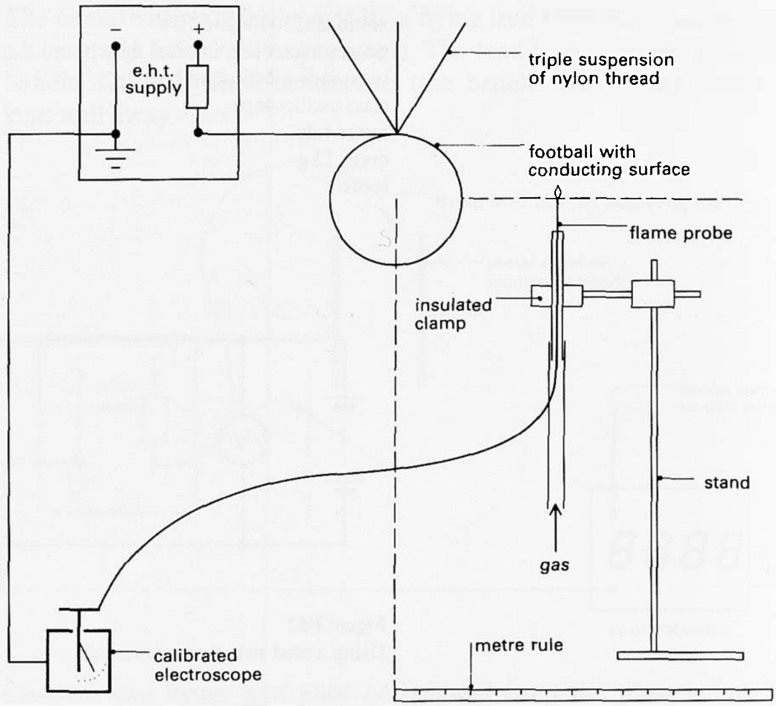


Figure E53
Using a flame probe to explore potential near a charged sphere.

stable. The ball should be as far as possible from walls, benches, and other conducting surfaces.

What changes of potential are indicated as the probe is kept a constant distance r from the centre of the sphere (*i.e.*, moved around a spherical surface concentric with the ball)? Explain this result.

How does the potential vary as the distance r is varied? Explain. In particular, note what happens when r is doubled (or halved). Do this for two values of r, one of which is equal to the radius of the sphere.

Measure the potential V at different distances r and plot a graph which you think will yield a straight line. Deduce what you can about the relationship between V and r.

DEMONSTRATION

E8b Measuring the value of k in $V = k\frac{Q}{r}$

A potential of rather less than 1000 V is suitable, say 500 V. The sphere is charged by touching with a flying lead from the e.h.t. positive terminal held, as shown, on an insulating handle. This flying lead is then removed. To measure the charge it is transferred to the coulombmeter by touching the probe rod to the sphere.

If an earthed hand were nearby whilst the sphere was being charged, how might this affect the amount of charge stored on it?

Why should the flying lead always be removed before the probe rod touches the sphere?

Measure the charge, Q, stored for one or more values of V and deduce a value for the constant k.

This experiment depends on all (or very nearly all) of the charge on the sphere being transferred to the coulombmeter. Should the coulombmeter's capacitance be large or small compared with that of the sphere?

Estimate what percentage of the original charge stays on the sphere if the coulombmeter has a capacitance of $10^{-2}\,\mu F$. (Use $C = Q/V$ to deduce the capacitance of the sphere.)

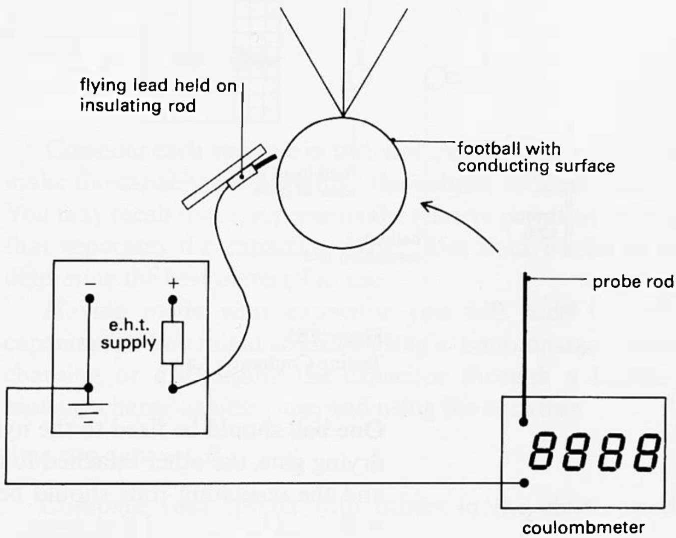


Figure E54
Measuring the value of k.

EXPERIMENT
E9 Experiments to test the inverse-square law for electric forces

small proof plane, or ball point pen barrel (to act as insulating handle)
2 metallized polystyrene balls
nylon thread for suspension
lamp, holder, and stand
transformer

either
e.h.t. power supply
or
Van de Graaff generator
or
electrophorus plate, 'rubber', and polythene tile

retort stand base, rod, boss, and clamp
graph paper
glue (Durafix or Evo-Stik 863)
adhesive tape
leads
hair dryer (if air is humid)
balance (optional)

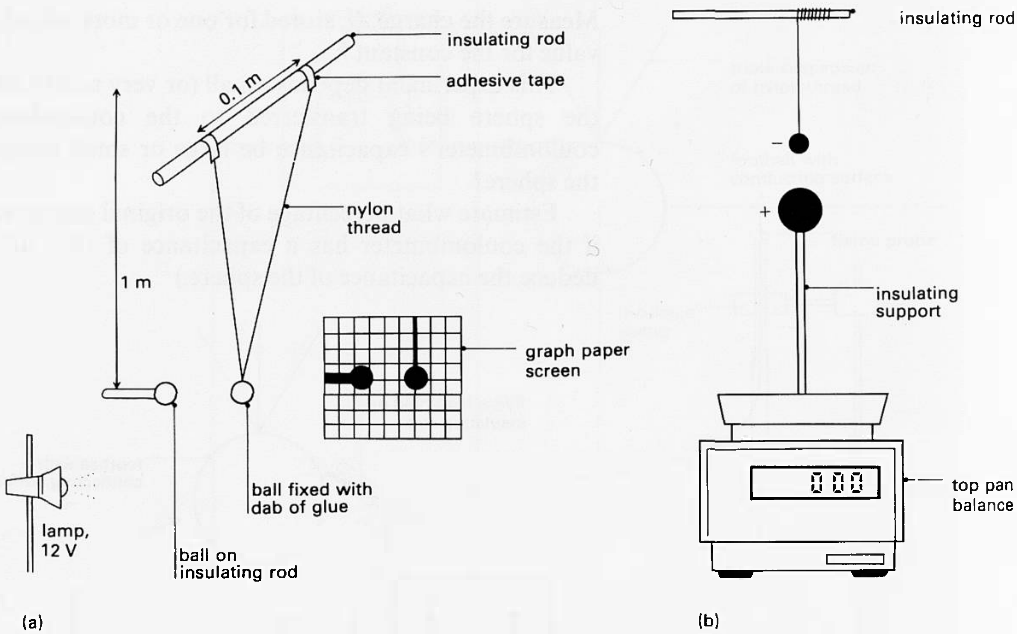


Figure E55
Testing Coulomb's Law.

One ball should be fixed to the nylon suspension using a dab of quick-drying glue, the other attached to the insulating handle. Both the nylon and the insulating rods should be clean and free of finger grease and well dried before, and if necessary during, the experiment using the hair dryer. Shadows of the balls cast onto a screen allow the magnified movements to be measured without touching the balls.

A pendulum performs simple harmonic motion when displaced. How does the sideways (*i.e.*, restoring) force vary with displacement of the ball?

Is $F \propto \frac{1}{r^2}$? Use the sideways displacement, d, of the suspended ball to investigate the force exerted at different separations, r.

Is $F \propto Q_1, Q_2$? Alter the charge on one ball (how?) to investigate this relationship. Knowing the angle at which the ball hangs and its weight, how could you find the actual force exerted on it? How could you measure the charge on each ball?

It is difficult, but not impossible, to make these measurements and thus deduce a value for k in $F = k\frac{Q_1Q_2}{r^2}$.

HOME EXPERIMENTS

EH1 The capacitor

Using only household materials, make a capacitor with the biggest capacitance to volume ratio that you can contrive. For safety reasons your capacitor volume must not be larger than that of a *small* match box.

In order to help you in your design, consider the capacitance equation

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

Consider each variable in turn and decide what must be done to it to make the capacitance big whilst the volume of the device is kept small. You may recall that ε_r represents the relative permittivity of the material that separates the capacitor plates. Use data books to help you to determine the best material to use.

Having made your capacitor you will need to find its actual capacitance. You might consider using a 'time constant' method – either charging or discharging the capacitor through a known resistance, plotting charge against time, and using the equation

$$\text{time constant} = CR$$

Compare your results with others in the class – perhaps on a competition basis.

EH2 The potential hill/well

In dealing with certain physical phenomena or events it is very often helpful to try to visualize these things in concrete terms that we can readily appreciate. The Rutherford model of the atom, for example, provides us with an 'image' of the atom which is extremely helpful though often incomplete or limiting.

In this task a three-dimensional model of a $\frac{1}{r^2}$ field may be created and you may find it very helpful.

On a large piece of graph paper draw a $\frac{1}{r}$ curve – maybe the graph of gravitational potential against r.

Put this graph on eleven other sheets of paper and cut out the graph – cutting along both axes and the line of the graph so that you have twelve graph 'envelopes'.

Stick the vertical axes of the graphs together, and arrange the graphs to radiate outwards. This is the skeleton of your potential hill or well. Put the skeleton on its wide base and clad the shape in strips of paper to make a detachable funnel shape.

When it is placed on its wide base, you see a potential or 'coulomb' hill. Holding it with its wide base up you see a potential well.

QUESTIONS

Uniform electric fields

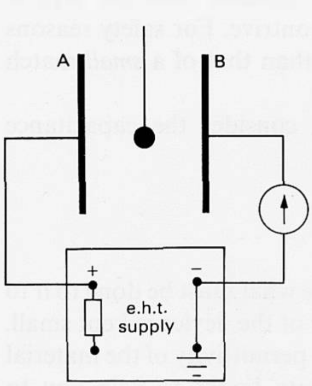


Figure E56

1(L) A table tennis ball covered in conducting paint is suspended between two metal plates, A and B, which are connected via a galvanometer to an e.h.t. power supply. The ball is initially given a positive charge.

a Which way does it move? Why?

b How is the charge on the ball affected when it touches plate B? How does it move as a result?

c As the ball continues to 'shuttle' back and forth, what sign of charge does plate B lose?

d What sign of charge does plate A gain?

e In which direction do electrons flow in the circuit (clockwise or anticlockwise)?

f In which direction does conventional current flow?

g Why does the surface of the ball have to be conducting?

h What two factors affect the frequency at which the ball shuttles backwards and forwards? How is the frequency altered by changing each factor?

i (Harder) Describe and explain what happens if the ball is initially *uncharged* but very near to one of the plates.

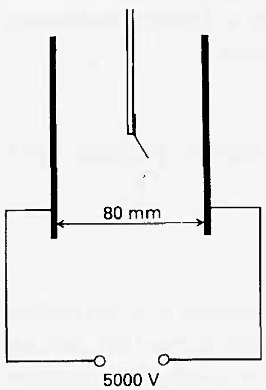


Figure E57

2(L) A small charged strip of foil on an insulating handle is held between two large charged plates which are connected to a 5000 V supply. The plates are 80 mm apart; the angle at which the strip hangs is noted.

The plates are now moved until they are only 40 mm apart, leaving them connected to the supply at the same p.d. Will the strip hang at a larger angle to the vertical (showing more force on its test charge) or a smaller angle (showing less force)?

How strong are the electric fields in the two cases?

To what value must the p.d. be changed to get the same force as at first on the charged test strip at the new, smaller spacing?

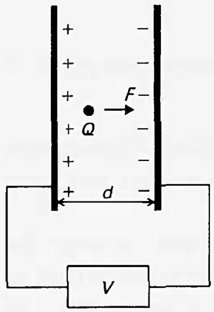


Figure E58

3(L) This question introduces the volt per metre as an alternative unit for E.

A ball carrying charge Q is moved a distance d by an electrical force, F. The force arises from a field created by a p.d. V between the plates; the force is the same everywhere between the plates as the field is uniform.

a Write down an expression for the energy transformed by the force F in moving the ball a distance d.

b Write down the energy transformed when a charge Q crosses a p.d. V.

c Use your answers to **a** and **b** to obtain an expression for the force per unit charge on the ball.

d Instead of $N\,C^{-1}$, what other units could be used for field strength?

e *i* Express volts in terms of joules and coulombs.
ii Express joules in terms of newtons and metres.
iii Hence show that $V\,m^{-1}$ can be written $N\,C^{-1}$.

f *i* What is the strength of the electric field in experiment E2, where 1500 V is applied across plates 0.15 m apart?
ii If the force on the foil is about equal to the weight of a 1 mg fly, what is its charge?
iii What would be the force on 1 coulomb?

4(P)a In the sparking plug of a motor car engine there are two electrodes separated by a spacing of about 0.67 mm (figure E59). If air begins to ionize when the electric field is about 3×10^6 volts per metre (at atmospheric pressure), roughly what p.d. must be applied across the electrodes to cause a spark in the air?

b The p.d. needed to cause a spark will depend on the gas pressure. Explain what effect you think a change of pressure will have on this p.d. Is the gas pressure in a motor car engine greater or less than atmospheric pressure when the spark is needed?

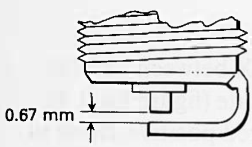


Figure E59

Some examples of non-uniform fields

5(P) Copy the diagrams of different electrodes and on them sketch lines to illustrate the shape of the electric fields between them. Draw arrows to indicate the direction of the field. Some you may already know; others you will have to guess.

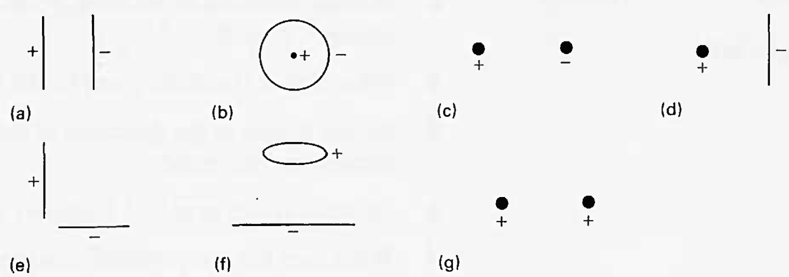


Figure E60

6(P) A small, charged ball weighing 10^{-3} N is attracted to a charged plate as shown in figure E61, and hangs at 45° to the vertical.

a Draw a diagram showing all the forces acting on the ball.

b Deduce the size of the electric force on the ball.

c If the electric field strength near the plate is $10^5\ V\,m^{-1}$, calculate the size and sign of the charge on the ball.

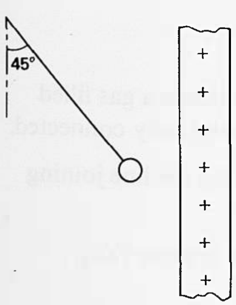


Figure E61

E

7(E) A 'flame probe' is used to measure potentials between parallel plates connected to an e.h.t. supply. The potential is indicated by an electroscope, whose case is connected to the earthed negative terminal of the e.h.t. supply.

a Why is it not practicable to measure an electric field strength by placing a charged object between the plates and measuring the force on it?

b Why is the flame necessary? What does the electroscope indicate if there is no flame?

c No gas supply is available. What could be used in place of the flame?

d What sign of charge does the electroscope acquire during the experiment?

e In which direction does current flow in the wire leading from the probe to the electroscope cap?

f Is this current steady?

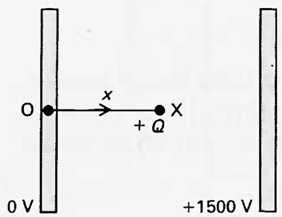


Figure E62

8(L) A small object carrying charge, $+Q$, is placed at X between parallel plates as shown, a distance x from the lefthand plate (figure E62). In this question, vectors to the right will be considered positive, those to the left, negative.

a What sign has the vector, x, indicating the displacement of the charge from O?

b In what direction is the force, F, on the charge, and the electric field strength, E, at X?

c What sign is therefore given to the field E?

d As one moves in the direction of increasing x, does the potential increase or decrease?

e Sketch a rough graph of V against x.

f What sign has the potential gradient $\frac{\Delta V}{\Delta x}$?

g Refer to your answers to **c** and **f**. Write a correct expression relating the field strength, E, to the potential gradient, $\frac{\Delta V}{\Delta x}$.

9(P) Figure E63 shows a scale drawing of the electrodes inside a gas filled tube. G_1 and G_2 are of thin wire gauze and are externally connected.

a Draw a graph showing how the potential varies along the line joining C and A.

b What is the strength of the electric field in the three regions CG_1, G_1G_2, and G_2A?

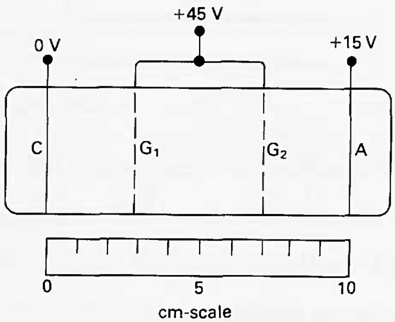


Figure E63

c Describe the motion of an electron through these three regions, stating its energy (in eV) at C, G_1, G_2, and A. Assume it starts at rest at C and does not hit either G_1 or G_2.

d (Harder) Suppose another electron ionizes a gas atom in the G_1G_2 region and loses 35 eV of energy in so doing. Describe and explain its subsequent behaviour.

10(P) The figure shows a full-scale section of equipotentials at 1 V intervals between two conductors.

On a tracing of figure E64, starting from A on the upper conductor, construct a field line until it terminates on the lower conductor. By taking measurements, plot a graph of the variation of electric potential with distance from A along this line. How can the electric field strength at a point be deduced from this graph?

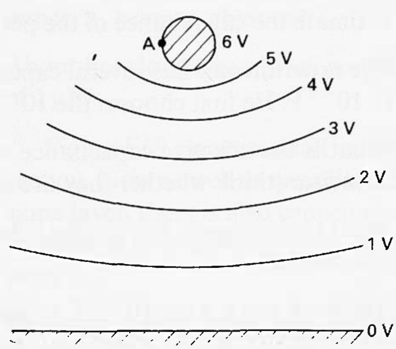


Figure E64

11(E) Figure E65 shows equipotentials drawn on a two-dimensional model representing a thundercloud over a churchyard. Assuming the charge is evenly spread over the base of the cloud, sketch lines of electric field between the cloud and earth on a copy of this diagram.

Comment on the safety of the following locations during a thunderstorm:

a On top of the church spire.

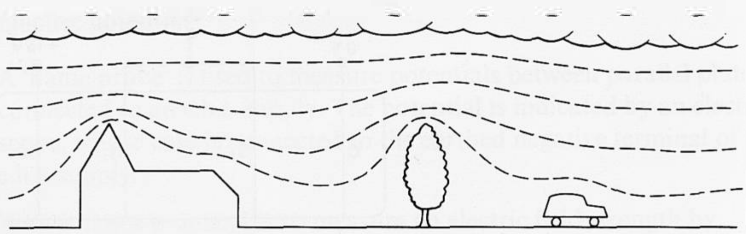


Figure E65

b Inside the church.

c Leaning against the tree.

d In a car.

e On a bicycle.

f On horseback.

Parallel plates and capacitance

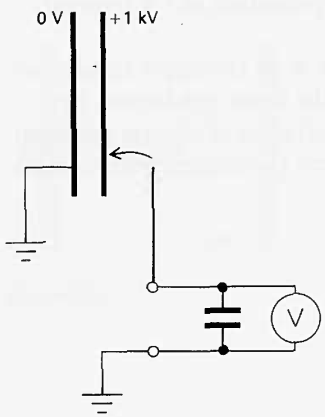


Figure E66

12(L) Two circular metal plates of radius 0.15 m are placed parallel to one another, 50 mm apart. A p.d. of 1 kV is established between them by momentary connection to an e.h.t. supply. A high resistance voltmeter is to be used to measure the charge on one of the plates, the other being earthed.

This is done by transferring its charge, via a thick wire, to a capacitor connected across the voltmeter, and measuring the resulting p.d.

a Estimate the capacitance of the parallel plates.

The experimenter has several capacitors available in the range 10^{-11} F to 10^{-8} F. He first chooses the 10^{-8} F capacitor.

b What is the effective capacitance of the parallel plates and the 10^{-8} F capacitor (think whether they are in series or parallel)?

c What percentage of the original charge on the plate is retained, not transferred?

d Answer **b** and **c** if the 10^{-11} F capacitor is used and comment on any problems or hazards which may arise as a result.

13(E)a Check the accuracy of the following statement. ‘If a reed switch is used to discharge a 1 μF capacitor through an ammeter 50 times a second, a 1 mA meter can be used safely if the capacitor is charged to 10 V.’

b What order of magnitude of capacitance can be used in a similar experiment, again using 10 V, if the reed switch discharges it 100 times a second through a meter which gives a measurable deflection for a steady current of 1 μA?

14(P) A pair of horizontal parallel plates, each measuring 0.1 m × 0.1 m, 0.01 m apart, is connected to a 10 V battery as shown in figure E67.

Figure E67

a What is the electric field strength between the plates?

b Estimate the charge density on the plates.

c How many excess electrons are there on the negative plate?

d Estimate the upward force on a water drop bearing one excess electron positioned midway between the plates.

e Treating the drop as a cube, estimate its 'diameter' if it is held stationary.

f Approximately how many atoms would it contain?

15(E) A 1 μF capacitor is to be made as follows. Long, 50 mm wide strips of thin metal foil, B and D, and of insulating paper 0.1 mm thick, A and C, are arranged in a sandwich as shown (figure E68), and then rolled up to make a cylinder. The relative permittivity of the paper is 2.

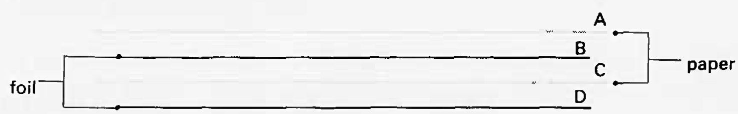


Figure E68

a At what point in the manufacture does the need for the top sheet of paper, A, become obvious?

b About how long a sandwich would be needed to get a final capacitance of 1 μF?

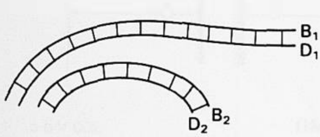


Figure E69

Figure E69 shows a small area of the rolled up cylinder in cross-section. Now as well as capacitance between 'plates' B_1 and D_1 of the same layer, there is also capacitance between 'plates' D_1 and B_2 of adjacent layers, allowing extra charge to be stored on the foils for the same p.d.

c By roughly what factor will the charge stored increase when the cylinder is completely rolled up?

d How does this affect your answer to **b** – the total length needed?

e Estimate roughly the diameter of the rolled up cylinder. You might then compare its size with that of a commercial 'paper' capacitor of about the same value.

16(P) Two conducting plates a distance d apart are connected to a battery of p.d. V. In figure E70(b) the separation is increased to $2d$ while the battery is still connected; in figure E70(c) the battery is disconnected before separation.

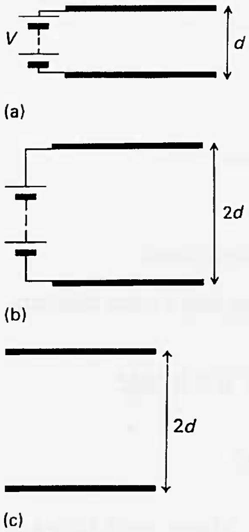


Figure E70

a In (b) what changes occur in:
i the capacitance, *ii* the p.d., *iii* the charge stored, *iv* the energy stored, *v* the electric field in the space.

b In (c) what changes occur in:
i the capacitance, *ii* the charge stored, *iii* the p.d., *iv* the energy stored, *v* the electric field.

c Account for the changes in energy in each case.

17(R) A plate of area 2×10^{-3} m^2 is held by a very well insulated support 50 mm above the bench, which conducts well (figure E71). It is charged to 4 kV, but its potential drops to 3 kV over 10 minutes. Estimate the conductivity of air, assuming that a uniform conduction across the plate area is the cause of the leakage. (You may be able to show that the question has given you two pieces of data that you need not use.)

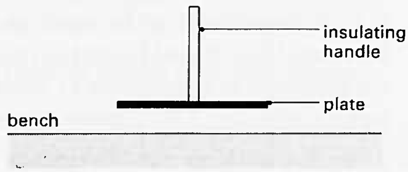


Figure E71

18(R) Figure E72 shows a section through a capacitor microphone; figure E73 shows a circuit with which the microphone is used.

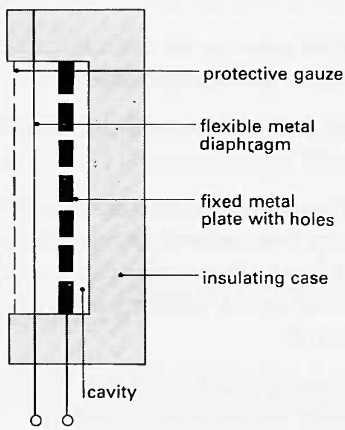


Figure E72

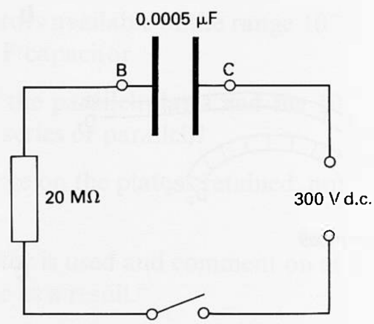


Figure E73

a The switch is initially closed. If it is opened, and the diaphragm pushed slightly inwards, explain what would happen to:
i the capacitance of the microphone,
ii the p.d. between B and C.

b Explain what would happen if, with the diaphragm still pushed in, the switch were closed.

c Why is the instrument constructed so that the diaphragm is as close to the first plate as possible?

d What is the time constant of the circuit in figure E73?

e Assuming that the switch is closed, state the changes of p.d. between B and C that you would expect to occur if a compression wave moved the diaphragm inwards in a time which was:
i short compared with the time constant of the circuit – say about 10^{-5} s.
ii long compared with the time constant of the circuit – say about 1 s.

f Two sources of sound, one of frequency 10 kHz, the other 50 Hz, are each found to produce the same amplitude of mechanical vibration in the diaphragm.
i Why is the amplitude of the resulting variations of p.d. across BC smaller for the 50 Hz vibrations than for the 10 kHz?
ii Explain what change you could make in the circuit to bring the amplitude of the electrical output from the microphone, when responding to the 50 Hz note, nearer to that produced by the 10 kHz note.

(Long answer paper, 1970)

Uniform gravitational field

19(I)a A child of mass 40 kg on a swing loses 0.8 m in height in swinging from A to B as shown. What is the child's speed at B? (Take $g = 10\,\mathrm{N\,kg^{-1}}$ in this question.)

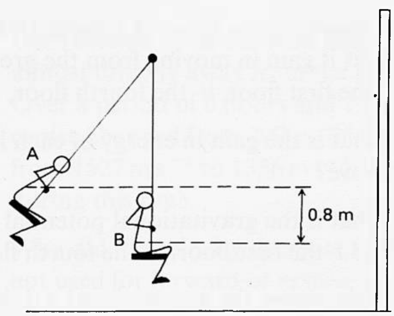


Figure E74

b Repeat **a** for an adult, in place of the child.

You may have found **b** more difficult than **a** if you tried to first work out the potential energy of the adult since you were not given the adult's mass. However, since in the equation $mgh = \frac{1}{2}mv^2$, the m cancels out, it was not required, even for **a**.

c *i* What is the extra P.E. of each kilogram of the child at A compared with B?
ii What is the extra P.E. of each kilogram of the adult at A?
iii What is the extra P.E. of the adult if his or her mass is 80 kg?

d Suppose you had to paint on the wall of the playground marks to show the P.E. gained by 1 kg at different heights. If you put a mark each time the P.E. increased by 10 J how far apart vertically would they be? How far apart for 1000 J intervals of energy? (You might need a rather high wall!)

e Suppose you continued painting lines at these vertical intervals all over the surface of the countryside. What would the overall pattern look like and what would it represent?

20(I) Figure E75 shows several cars parked in a multi-storey car park. The floors rise in 5 m steps. (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

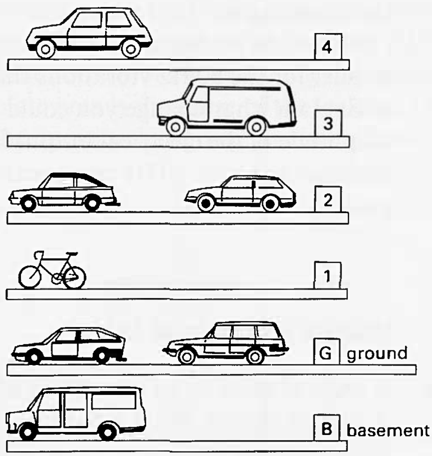


Figure E75

a A 'Sierra' has a mass of 1 tonne (1000 kg). How much potential energy does it gain in moving from the ground floor to
i the first floor, *ii* the fourth floor, *iii* the basement?

b What is the gain in energy of each kilogram of the car in *i*, *ii*, and *iii* above?

c What is the gravitational potential difference between the ground floor and *i* the first floor, *ii* the fourth floor, *iii* the basement?

d Write down the gravitational p.d. between *i* the first and the fourth floors, *ii* the first floor and the basement.

e How much potential energy does a 'Fiesta' of mass 800 kg lose in coming down from the fourth to the first floor?

21(P) Figure E76 shows part of the gravitational field near the surface of the Earth where $g = 10\,\mathrm{N\,kg^{-1}}$, and is uniform over this small region.

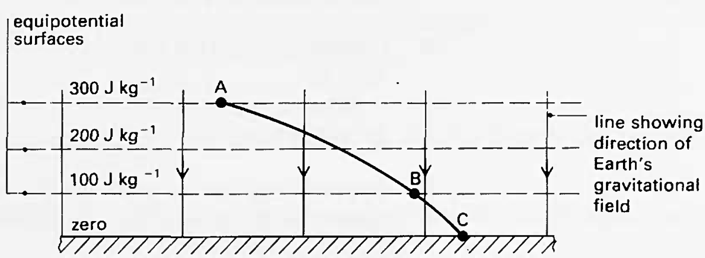


Figure E76

a How far apart are successive equipotential surfaces drawn in the figure?

b What is the gravitational force on a mass of 5 kg
i at A, *ii* at B?

c What is the gravitational P.E. of a mass of 5 kg
i at A, *ii* at B?

d How much energy must be supplied to move the mass from B to A?

e A cannonball of mass 5 kg follows the path ABC shown. At A its speed is $20\,\mathrm{m\,s^{-1}}$. What is its speed
i at B, *ii* at C?

The gravitational inverse-square law

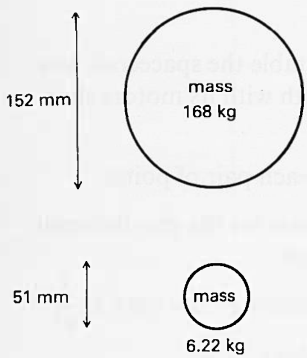


Figure E77

22(P) Figure E77 shows the two lead spheres used by Cavendish in 1798 to measure *G*.

a How far apart would the centres of the spheres be to give the maximum force of attraction between them?

b Calculate a value for *G* if this force was measured as $6.76 \times 10^{-6}\,\mathrm{N}$.

The next two questions use data from the spaceflight of Apollo 11. This made the first manned landing on the Moon in July 1969. You can use the data to test whether the Earth's gravitational field strength obeys the inverse-square law at quite large distances.

23(L) On a section of its outward flight, Apollo 11 was coasting on a path almost directly away from the Earth and well outside its atmosphere. Over a period of 6 hours and 13 minutes its distance from the Earth's centre changed from $209 \times 10^6\,\mathrm{m}$ to $241 \times 10^6\,\mathrm{m}$, and its velocity fell from $1527\,\mathrm{m\,s^{-1}}$ to $1356\,\mathrm{m\,s^{-1}}$. The thrust motors were not used during this time.

a Why did the velocity decrease, even though the rocket motors were not used for forward or reverse thrust?

b What was the average acceleration of the spacecraft over the 22 380 s period?

c Without performing any further calculation, write down the average value of *g* (the Earth's gravitational field strength) over this period.

d Newton's Law of Gravitation gives the value of *g* as $-\dfrac{GM}{r^2}$, where $GM \approx 4.0 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$ for the Earth. What value does *g* have if *r* is taken as the average distance of the spacecraft from Earth ($225 \times 10^6\,\mathrm{m}$) during the period being considered?

e The Moon also pulls on the spacecraft. *GM* for the Moon is $4.9 \times 10^{12}\,\mathrm{N\,m^2\,kg^{-1}}$. Calculate its contribution to the field strength $\left(-\dfrac{GM}{r^2}\right)$, at the same point (about $150 \times 10^6\,\mathrm{m}$ from the Moon). Does it have a significant effect?

E

24(L) Table E1 gives four other similar pairs of points between which the Earth's gravitational pull was the only significant force acting.

Time from launch h : min : s	Distance from Earth's centre $r/10^6$ m	Velocity $v/\mathrm{m\,s^{-1}}$	Mean distance $r/10^6$ m	Mean acceleration $g/\mathrm{m\,s^{-2}}$
03 : 58 : 00	26.3	5374		
04 : 08 : 00	29.0	5102		
05 : 58 : 00	54.4	3633		
06 : 08 : 00	56.4	3560		
09 : 58 : 00	95.7	2619		
10 : 08 : 00	97.2	2594		
19 : 58 : 00	169.9	1796		
20 : 08 : 00	170.9	1788		

Table E1

During the whole period covered by the table the spacecraft was travelling almost directly away from the Earth with its motors shut down.

a Calculate the average acceleration between each pair of points.

b Without further calculation deduce an estimate for the gravitational field strength at the mid-point of each interval.

c Plot a graph which enables you to check whether g does vary as $\frac{1}{r^2}$. (Think which graph will most easily reveal this.)

Gravitational potential difference, ΔV_g, and potential, V_g

Questions 25 to 29 are based on a computer program which calculates values of gravitational potential difference from a graph of field strength against distance by adding up the areas of strips under the graph. It is not absolutely necessary to have seen the computer program, for all the data you will need are given with each question.

25(L) Table E2 shows various calculations of the energy required to transport one kilogram (*i.e.*, the gravitational potential difference, ΔV_g) between the surface of the Earth and a distance of 50×10^6 m from its centre. These have been computed using the number of steps shown to calculate the area under a field–distance graph by adding the rectangular strips 'under' the graph. The time taken for each calculation to be performed by the computer is also shown.

The computer program assumes the field strength g to be constant over the full distance given by the step size.

a For which number of steps is this least true? Explain why this leads to a poor estimate of ΔV_g.

b For which number of steps is this most nearly true? Which number of steps yields the most accurate estimate of ΔV_g?

c What could be done to obtain an even better estimate of ΔV_g?

Number of steps	Step size /10^6 m	gpd(ΔV_g)MJ kg^{-1}	Time taken/s
1	43.6	218	<1
2	21.8	120	<1
5	8.73	70.3	≈1
10	4.36	59.2	≈2
20	2.18	55.8	≈3
50	0.873	54.8	5
100	0.436	54.7	14
200	0.218	54.7	26
500	0.087	54.6	74
1000	0.044	54.6	144

Table E2

d What would be the disadvantage in doing this?

e A precise calculation (or use of a *very* small step size) leads to a value for ΔV_g of 54.52 M J kg^{-1}. What is the percentage error in the estimate of ΔV_g if just 20 steps are used?

f Which part of the graph – nearest the Earth or furthest away – makes the greatest contribution to the error? Why?

26(L) Rocket physicists need to know the amount of energy required to raise one kilogram from the Earth's surface to various heights. They might use the program to calculate values of ΔV_g between R (the Earth's surface) and various distances r_2 (from the centre of the Earth). The results are shown in table E3. ($R = 6.37 \times 10^6$ m; 200 steps are used in each calculation.)

E

$r_2/10^6$ m	ΔV_g/MJ kg^{-1}
6.371	0.01
7	5.64
10	22.7
14	34.1
18	40.5
22	44.5
30	49.3
40	52.7
50	54.7

Table E3

a Plot a graph of ΔV_g against r_2.

b Use the graph to estimate the amount of energy required to lift
i a 1000 kg probe from the Earth's surface to a distance of 20×10^6 m from the Earth's centre;
ii a 200 kg satellite from the Earth's surface to a height of 36×10^6 m above the Earth's surface;
iii a 1 kg mass *from* 40×10^6 m *to* 50×10^6 m from the centre of the Earth.

c The graph seems to be approaching a limit – it does not continue to rise indefinitely. The extra energy required to transport 1 kg from 50×10^6 m to a very great distance can be worked out. It comes to about 7.8 MJ. What then is the total energy needed to lift 1 kg from the Earth's surface to as great a distance as one would wish?

d Describe what will happen to a 1 kg space probe launched with an energy of
i 50 MJ
ii 70 MJ
iii an amount equal to your answer to **c**.

27(L) An interplanetary space convention is being held on a satellite 50×10^6 m from the centre of the Earth. Delegates from Earth are at present situated at the distances given in table E4 (r_1 values). The amount of energy needed to transport 1 kg from these positions to the destination, at 50×10^6 m, has been calculated by computer. These values represent the gravitational potential difference, ΔV_g, between the two points. (200 steps are used in each calculation.)

$r_1/10^6$ m	ΔV_g/MJ kg^{-1}
6.37	54.7
7	49.0
10	31.9
14	20.5
18	14.2
22	10.2
30	5.3
40	2.0

Table E4

a *i* Plot a graph of ΔV_g against r_1 (or use a computer to do this for you).
ii Why does ΔV_g decrease as r_1 increases?
iii Can you suggest a mathematical form for the way ΔV_g varies with r_1? (You may be able to test your suggestion by having the computer plot a suitable graph.)

b *i* Now plot ΔV_g (y-axis) against $\dfrac{1}{r_1}$ (x-axis).
ii Calculate the gradient of this graph (it should be a straight line).
iii Measure the intercept on the horizontal axis and compare its value with $\dfrac{1}{r_2}$ where $r_2 = 50 \times 10^6$ m.
iv Suggest an expression for ΔV_g in terms of r_1 and r_2.

c (Optional) *i* Finally plot ΔV_g against $\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ where $r_2 = 50 \times 10^6$ m.
ii Confirm that this graph has the same gradient as that in **b**.

d Suppose r_2 had been much larger than 50×10^6 m. What effect would this have had on the graph in **b**?

28(L) This question discusses a zero for potential energy.

Suppose the venue of the interplanetary conference was moved to a very much larger distance from the Earth. From the original ΔV_g against r_2 graph (question **26a**), you can see that not much extra energy would be needed to go *beyond* 50×10^6 m to any distance one would wish. Suppose that, in question **27**, instead of making $r_2 = 50 \times 10^6$ m we made r_2 'as far away as one could wish'. The shorthand for this is ∞, infinity.

a What would be the value of $\frac{1}{r_2}$?

b Would there be any difference between the graphs of ΔV_g against $1/r_1$ and ΔV_g against $\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$?

c Express in words what is now represented by ΔV_g.

Delegates from other planets would, of course, have different graphs as they would have come different distances through different strengths of field. Therefore they would have required different amounts of energy per kilogram to reach the same point from their own planets. Yet it seems sensible to regard each of them as now having the same amount of potential energy per kilogram, or in other words *potential*. Indeed they agree to accept a convention on this: that as far away from all their planets as possible any object will be considered to have zero potential. Exchanging interplanetary pleasantries, they return to their respective homes. Ideally, each need only set off in the right direction: the gravitational pull of the home planet will do the rest. No fuel need be used; indeed, kinetic energy is gained on the homeward journey.

E

d If kinetic energy is gained then what form of energy is lost?

e If the original potential energy of 1 kg at 'infinity' is agreed to be zero, what then is the sign of the potential energy per kilogram (or 'potential') nearer a planet?

f If ΔV_g between the surface of the Earth and 'infinity' is $62.5 \times 10^6\ \mathrm{J\,kg^{-1}}$, what is the potential at the Earth's surface?

g Look at your graph for question **27a** and *sketch* a graph of gravitational potential, V_g (with reference to a zero at infinity), against r_1.

29(L) This question relates the graph of V_g against r_1 to a mathematical expression for V_g which can be found by integrating the expression for gravitational field strength:

$$V_g = -\int_{\infty}^{r_1} \left(-\frac{GM}{r^2}\right) \mathrm{d}r$$

Previous work with graphs suggests that ΔV_g varies with distance from the Earth according to:

$$\Delta V_g \approx 4 \times 10^{14} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

a If r_2 is infinity, rewrite the expression for ΔV_g.

b From question **28**, the gravitational potential, V_g, at a point is equal to ΔV_g but negative. Write down an expression for V_g in terms of r_1.

c Mathematical integration yields the expression $-\frac{GM}{r_1}$ for V_g. Calculate the value of GM for the Earth and compare the two expressions for V_g.

d Calculate the minimum energy which must be supplied per kilogram of a spacecraft on the Earth's surface to enable it just to escape from the Earth's influence.

Potential and energy changes near the Earth

30(L) (Optional) This question uses data from the Apollo 11 spaceflight to compute changes in energy and see how such changes vary with position. The rocket motors were not used over the period for which data are given.

Distance from centre of Earth $r/10^6$ m	$\frac{1}{r}/10^{-8}$ m^{-1}	Velocity v/m s^{-1}	Kinetic energy per kilogram $\frac{1}{2}v^2/10^6$ J kg^{-1}
11.0	9.09	8406	35.33
26.3	3.80	5374	14.44
54.4	1.84	3653	6.60
95.7	1.04	2619	3.43
169.9	0.59	1796	1.61
209.2	0.48	1532	1.17
240.6	0.42	1356	0.92

Table E5

a We cannot easily measure the change in potential energy per kilogram of the spacecraft, so we measure the change in kinetic energy per kilogram. How are the two related and what assumption is made in thus relating them?

b What is the change in kinetic energy per kilogram between the first and last points in table E5?

c What is the corresponding change in potential energy per kilogram? (That is, the gravitational potential difference ΔV_g between the two points.)

d On this 'outward' flight the spacecraft is clearly slowing down. It may stop at a distance r_0 from the Earth's centre. How much kinetic energy would it have then?

e What would be the change in potential energy per kilogram (ΔV_g) between the first point in the table and r_0?

f Deduce values of ΔV_g between all the points in the table and r_0, and plot a graph of ΔV_g against $\frac{1}{r}$.

g Find the gradient of the graph and compare its value with GM_E (M_E = mass of Earth).

h Find the intercept on the $\frac{1}{r}$ axis and from it deduce the distance r_0.

i If r_0 had been much larger, say 'as far away as you like', what would the intercept have been?

j Suggest an equation which describes the way ΔV_g would vary with r in that case.

31(P) This question uses values of the gravitational potential at different distances from the centre of the Earth to calculate energies required to escape from the Earth. A few values of V_g and r are already given.

$r/10^6\,\mathrm{m}$		$-\dfrac{GM}{r}/10^6\,\mathrm{J\,kg^{-1}}$
6.37	(Earth's surface)	−62.5
6.38		−62.4
10		−40
400	(distance of Moon)	−1
∞	(as far as you like)	0

Table E6

a How much energy is needed to raise 1 kg to a height of 10 km? (Use the above data.)

b What force acting on 1 kg would transform this amount of energy if it were uniform over the 10 km? Comment.

c How much energy is needed to transport 1 kg from the Earth to a distance equal to that of the Moon?

d Why is the energy not equal to the product of 10 N and this distance?

e How much energy is needed for a mass of one kilogram to escape completely from the Earth's influence?

f What velocity would the one kilogram object have to have at its launch to achieve this?

g Why would a mass of any size need this same launch velocity?

h Use the $\dfrac{1}{r}$ variation of potential to deduce values of V_g at distances of 20×10^6 m, 40×10^6 m, and 80×10^6 m. Hence plot a graph of V_g against r from $r = 10 \times 10^6$ m to 80×10^6 m.

i Find the gradients of tangents at $r = 20 \times 10^6$ m and 40×10^6 m.

j What is represented by these gradients?

k Find the ratio of the gradients and comment on the result.

32(P) This question asks you to calculate the *total* potential at various points near the Earth and Moon by summing the potentials due to each body. Take the mass of the Earth as 6.0×10^{24} kg and its radius as 6.40×10^6 m. Other data can be found on page 493.

a Calculate first the separate potentials at the surfaces of the Earth and the Moon.

b What is the gravitational potential difference between the two?

E

c Deduce the difference in gravitational potential energy of a 10 tonne (1 tonne $= 10^3$ kg) spacecraft on the Moon and on the Earth.

d Calculate the total potential due to the Earth–Moon system at the following distances from the centre of the Earth on a line towards the Moon:
i The Earth's surface (6.40×10^6 m)
ii 1.00×10^8 m
iii 3.00×10^8 m
iv 3.40×10^8 m
v 3.60×10^8 m
vi The Moon's surface (3.78×10^8 m).

e Using these draw a graph of total potential against distance from Earth from 1.0×10^8 m outwards.

f On the same diagram sketch a dotted curve which would represent the potential if the Moon were not there.

g Mark a point X where the overall field strength of the Earth and Moon is zero.

h From the graph deduce the minimum energy needed to send the 10 tonne spacecraft from the Earth to X.

i This is greater than your answer to **c**. Nevertheless a 10 tonne spacecraft must be provided with still more energy than this if it is to land on the Moon's surface (never mind returning!). Suggest reasons for this.

j Use the potential at the Moon's surface to calculate the 'escape velocity' for the 10 tonne spacecraft. Why need it be launched with a rather smaller velocity in order to return to and make a crash landing on the Earth?

k The temperature on the Moon's surface reaches up to 400 K, at which temperature oxygen molecules have an average speed of $560\,\mathrm{m\,s^{-1}}$. Use your answer to part **j** of this question to explain why the Moon has no atmosphere.

The gravitational field *inside* the earth

33(L) This question shows why there cannot be a field inside a hollow sphere. Consider a hollow shell which has a mass per unit area of surface (or surface density) of σ. The field at P can be deduced by adding up contributions from a whole series of pairs of cones (in three dimensions) like those shown in figure E78. Let us consider one such pair of cones subtending a solid angle 2θ at P, whose bases are circles on the surface of the shell (figure E79).

a What is the radius of circle A_1, in terms of r_1 and θ?

b Deduce an expression for the area of A_1.

c The mass per unit area is σ. What is the mass of the disc A_1?

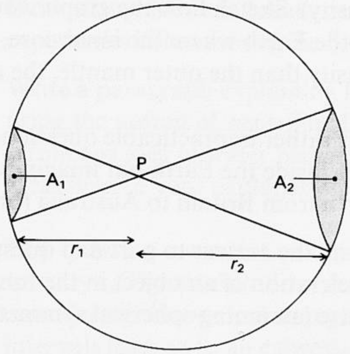


Figure E78

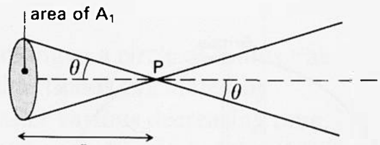


Figure E79

d What is the magnitude of the gravitational field strength at P due to this mass?

e Verify that this simplifies to $G\sigma\pi\theta^2$.

f Repeat **a** to **e** for the disc A_2 to find its contribution to the field strength at P.

g What is the total contribution of the two discs A_1 and A_2 to the field strength at P? Explain.

h What is the total contribution of all such pairs of discs to the field at P?

i Would the same be true if the field were not inverse-square? (Look at parts **d** and **e** again.)

34(P) This question explores the gravitational field *inside* the Earth, which is assumed to be uniformly dense.

Take a point, P, at distance r from the centre O. The shaded area represents parts of the Earth further from the centre than P. See figure E80.

This can be thought of as being made up of many thin shells centred on O.

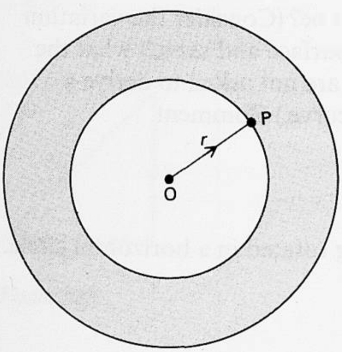


Figure E80

a What is the total field strength at P due to all these shells outside P?

b One consequence of the inverse-square law is that the field *outside* a solid or hollow sphere is the same as that produced if all the mass of the sphere were concentrated at its centre.

Write an expression for the contribution of the unshaded part of the sphere which has mass M, to the field strength g at P.

c Express M in terms of the density ρ (assumed to be uniform) and radius r of the unshaded region.

d Combine your answers to **b** and **c** to obtain an expression for g in terms of r.

e Sketch a graph showing the variation of g with distance both inside and outside the Earth (assuming, as above, that the Earth is of uniform density). Sketch how the graph would change to more truly represent g for the Earth where the inner core, mainly of molten iron, has greater density than the outer mantle, the crust being the least dense.

35(R) This rather impracticable question ignores the fact that it is rather hot deep inside the Earth but imagines a tunnel drilled right through the Earth from Britain to Australia (or thereabouts).

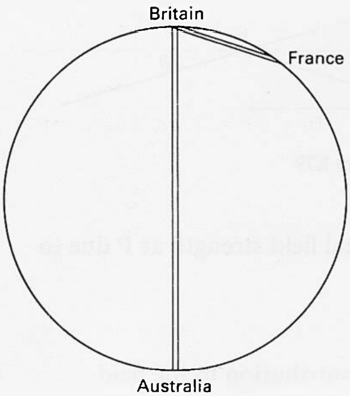


Figure E81

a Using the answer to part **d** of question **34**, write an expression for the acceleration of an object in the tunnel at different distances r from its centre (assuming spherical symmetry and uniform density).

b Deduce that an object dropped down the tunnel will perform simple harmonic motion about the centre.

c If the tunnel were pumped free of air, how much energy would be required to post a letter to Australia?

d Would more or less energy be required to send a rocket to the Moon from the centre of the Earth than from its surface?

e How does the gravitational potential at the centre compare with that on the surface? How could it be calculated (you need not do the calculation)?

f (Harder) Imagine a second tunnel cut as a chord rather than a diameter from Britain to, say, France. By considering components of gravity acting on a train in this tunnel, try to show that it should also perform simple harmonic motion.

g How much energy would be needed, in the absence of friction, for such a train to go from Britain to France?

h Imagine this (straight) track viewed from the apparently flat surface of the Earth. What shape would it appear to be? (Consider the variation of depth with distance round the curved surface and sketch what the shape of the tunnel would look like. You are not asked to derive a precise mathematical expression for the curve.) Comment.

Circular motion

36(I) Figure E82 shows a ball on a string being rotated in a horizontal circle.

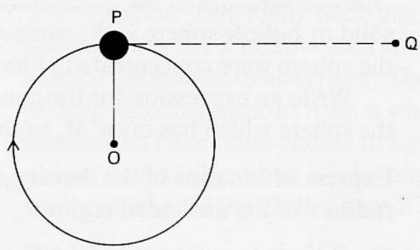


Figure E82

a If the string breaks when the ball is at P, use Newton's First Law of Motion to describe the subsequent behaviour of the ball.

b If the string does not break, use Newton's Second Law of Motion to explain the behaviour of the ball.

c Write a paragraph explaining the operation of a spin-drier without using the notion of 'centrifugal' force or without reference to the erroneous idea of water being 'flung outwards'. Diagrams will be a great help to your explanation.

37(L) This question considers an object moving in a circle of radius r at velocity v. The magnitude of its acceleration is calculated by considering the (vector) change in v after various decreasing time intervals leading to an expression for the acceleration of the object at any moment.

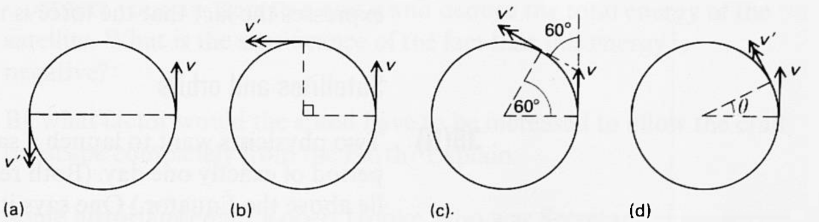


Figure E83

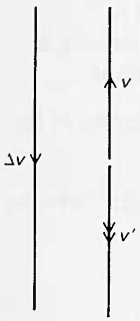


Figure E84

a First we consider half a circle, figure E83(a), where the velocity vector has simply reversed direction, as shown in figure E84.
i What is Δt, the time interval, in terms of the period T?
ii What is the size of Δv, the change in velocity in terms of v?
iii Since acceleration $a = \dfrac{\Delta v}{\Delta t}$, derive an expression for a in terms of velocity, v, and time, T.
iv Since speed is distance travelled divided by time taken, write an expression for T in terms of r and v.
v Substitute for T in your answer to *iii* and deduce that the acceleration is $\dfrac{4}{2\pi}\dfrac{v^2}{r}$ or $0.64\dfrac{v^2}{r}$

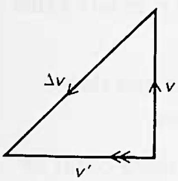


Figure E85

b Now consider a quarter of a circle, figure E83(b), and the vector diagram in figure E85.
i What is Δt now, in terms of T?
ii From the vector diagram, use Pythagoras' Theorem to deduce Δv in terms of v.
iii Hence deduce an expression for a and, by substituting for T as above express it in terms of $\dfrac{v^2}{r}$.

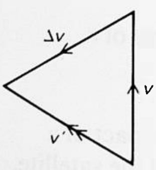


Figure E86

c Repeat **b** for figure E83(c) (a 60° or $\pi/3$ revolution). The vector diagram, figure E86, forms an equilateral triangle.

d Now consider movement through a small angle θ, figure E83(d).
i Express Δt in terms of T and θ.

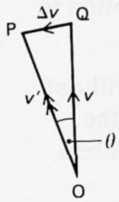


Figure E87

ii On the vector diagram (figure E87) express the *arc* length PQ in terms of v and θ. For a small angle θ this will be nearly equal to the *straight* length Δv.
iii Deduce an expression for the acceleration and simplify it as before.
iv In what direction is the acceleration, relative to the object's velocity?

e Look at the expressions for a as the time interval has decreased. They should approach a limiting value. Write down an expression for the centripetal acceleration of the object at any moment in its motion.

f If it has mass m, write down the centripetal force, F, acting on it.

g Considering the force as a vector, what addition to this formula expresses the fact that the force is inwards?

Satellites and orbits

38(R) Two physicists want to launch a satellite which orbits the Earth with a period of exactly one day. (Both realize that the most useful orbit will lie above the Equator.) One says it will have to be placed at a particular height, the other believes it can go to any height as long as its orbital speed is adjusted. They proceed to make calculations ....

a Considering the orbit to be circular, express the speed v in terms of the radius r and period T.

b What is the centripetal acceleration required to keep an object orbiting at speed v and radius r?

c Express this in terms of r and T.

d Now this acceleration will have to be provided by the Earth's gravitational field at whatever height the orbit is. Write an expression for the acceleration, g, at distance r from the centre of the Earth.

e Using your answers to **c** and **d** deduce an expression for r. Is any value of r possible, given the required period?

f Calculate the height to which the satellite must be raised and the amount of energy per kilogram required to get it up there.

g If you wanted the satellite to remain vertically over the same point on the Earth's surface, why would this point have to be on the Equator?

h Would a lower orbit mean a longer or shorter period? (Look at your answer to **e**.)

i Rearrange the equation $-\frac{v^2}{r} = -\frac{GM}{r^2}$ to express v in terms of r.
Would a lower orbit mean faster or slower motion?

j (Harder) Explain why the action of air resistance or the impact of a shower of meteorites will speed up rather than slow down the satellite.

k Discuss the pros and cons of equatorial and non-equatorial orbits, thinking of some of the uses to which the satellite might be put.

39(P) This question shows that a satellite in orbit always has half the total energy it would need to escape completely from its orbit.

a Write down an expression for the potential energy of a satellite of mass m at a distance r from the centre of the Earth, whose mass is M.

b Write down the centripetal force required to keep the satellite in orbit at this radius with velocity v.

c Write down an expression for the Earth's gravitational pull which provides this force.

d Using your answers to **b** and **c** obtain an expression for the kinetic energy ($\frac{1}{2}mv^2$) of the satellite in terms of G, M, m, and r.

e Compare your answers to **a** and **d** and deduce the total energy of the satellite. What is the significance of the fact that this energy is negative?

f By what factor would the speed have to be increased to allow the craft to escape completely from the Earth? Explain.

40(E) Some historians credit Robert Hooke, who was Secretary of the Royal Society, with the discovery of the inverse-square law of gravitation. Hooke certainly had suggested to Newton in a letter in 1679 that the centripetal force attracting a planet to the Sun varied as $\frac{1}{r^2}$, although he could not explain why. Hooke then assumed a circular orbit: Newton subsequently proved that the orbit of an object under a $\frac{1}{r^2}$ force would in fact be elliptical, although he 'sat on' the proof for twenty years until Halley (of comet fame) questioned him on the matter.

This question shows how anyone equipped with a rudimentary knowledge of algebra and who knew Kepler's Third Law, already published by that time, could have deduced the inverse-square law for a *circular* orbit.

a Kepler's Third Law states that $\frac{r^3}{T^2}$ is a constant. What do r and T represent?

b What is the size of the centripetal acceleration of a planet moving in a circle at radius r from the Sun with velocity v?

c What is the planet's velocity in terms of r and T?

d By substitution deduce that its centripetal acceleration can be expressed as $\frac{4\pi^2 r}{T^2}$.

e This can be expressed as $4\pi^2\left(\frac{r^3}{T^2}\right)\frac{1}{r^2}$.

Use Kepler's Law to show that the acceleration is proportional to $\frac{1}{r^2}$.

E

f If acceleration is inverse-square, how does the force on a planet vary with distance?

g Try to find out about the events leading up to Newton's discovery of the laws of universal gravitation as outlined in his *Principia Mathematica.*

The inverse-square law

41(L) This question seeks to throw some light on the inverse-square law.

Imagine a candle illuminating room A (figure E88). Light falling on the window leaves the candle as a pyramid (a square-based 'cone') of rays. Room B (figure E89) is identical to A, except that everything apart from the candle has been scaled up in linear dimensions by a factor of 2.

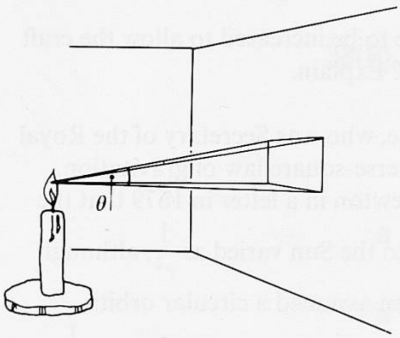


Figure E88

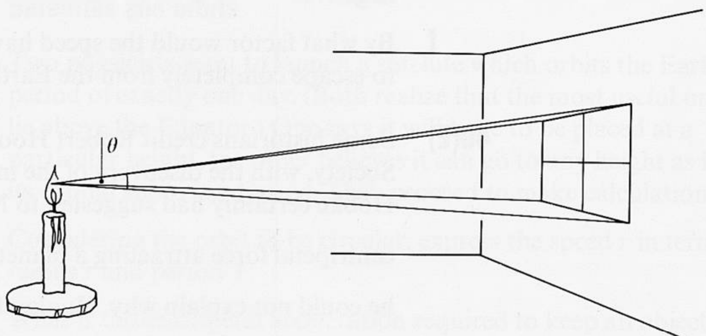


Figure E89

a How does the distance, r, of the window from the candle in room B compare with the candle-to-window distance in room A?

b How does the height of the window compare in B?

c How does the angle θ of the cone compare?

d How does the amount of light passing through this cone compare?

e How does the area of the window in B compare with the area of the window in A?

f What is the intensity of light from the candle at the window in B compared with A?

g If room C is 3 times the size of A, how will the intensity of the light hitting its window compare with that in A?

h What can you deduce about the product (intensity of light) × (area through which it passes)?

i Would the same be true if the light intensity did not obey an inverse-square law?

The electrical inverse-square law

42(L) Photographs in figure E92 show the position in which a small charged polystyrene ball, suspended like a pendulum as in figure E90, hangs when a second charged ball on an insulating rod, seen in figure E92(a)–(g), is pushed up close to it. The fine nylon thread used to suspend the first ball is not visible.

The pictures were taken with the apparatus in figure E90.

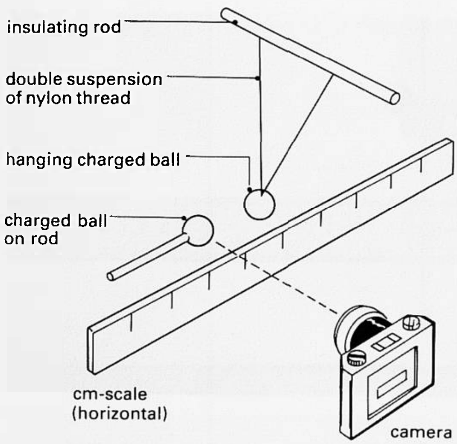


Figure E90
Taking photographs of charged balls.

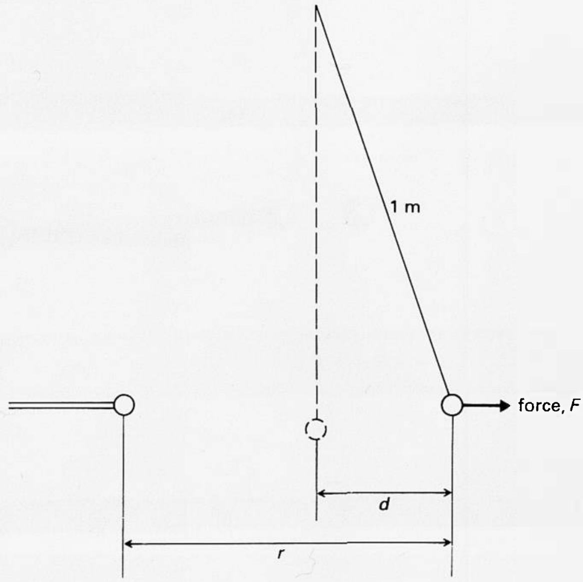


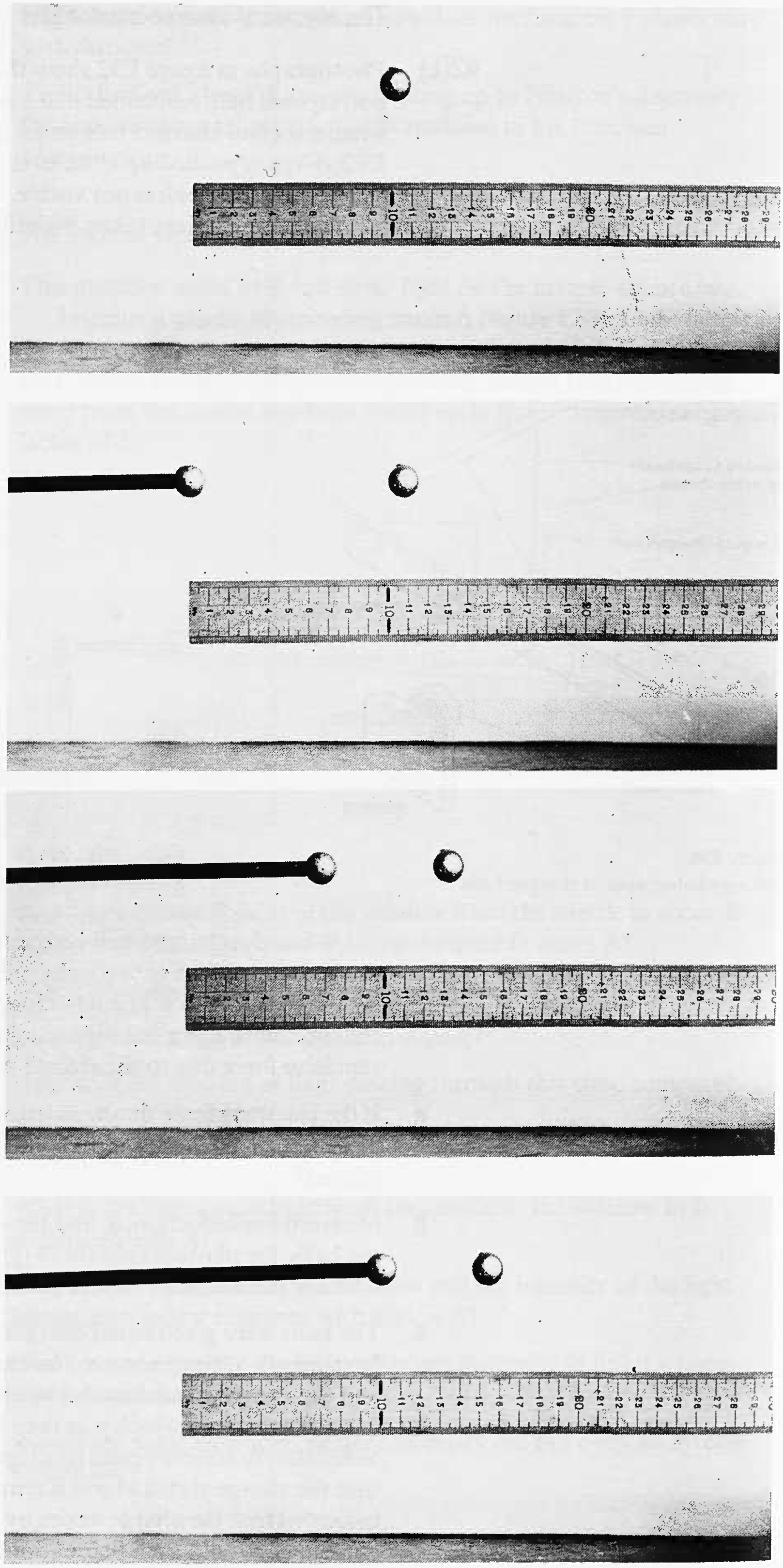
Figure E91
Sideways deflection of the ball.

When the charged ball on the rod is a long way away, the suspended ball hangs vertically, in the position shown in figure E92(a). As the second ball is brought closer, the suspended ball is pushed to the right until there is a big enough sideways force on it to balance the repulsive force due to the second ball (figure E91).

a If the sideways force on the suspended ball is doubled, how will its deflection, d, change (approximately)? How are d and the force F related?

b Measure the deflection, d, and the separation, r, between the centres of the balls, for photographs (b) to (g) in figure E92. Plot a graph to test whether the sideways force, F, on the suspended balls varies as $1/r^2$.

c The balls were given equal charges, each about 5×10^{-9} coulomb, from a high-voltage source. The charging was repeated for each picture and the charge was measured by sharing the charge on a ball with a 0.01 μF capacitor, which was then found, using a high resistance voltmeter, to have a potential difference of 0.5 volt across it. Check that the charge stated above is correct. The measurements of charge indicated that the charge varies by a few per cent on different occasions. Would such fluctuations explain any feature of your graph?

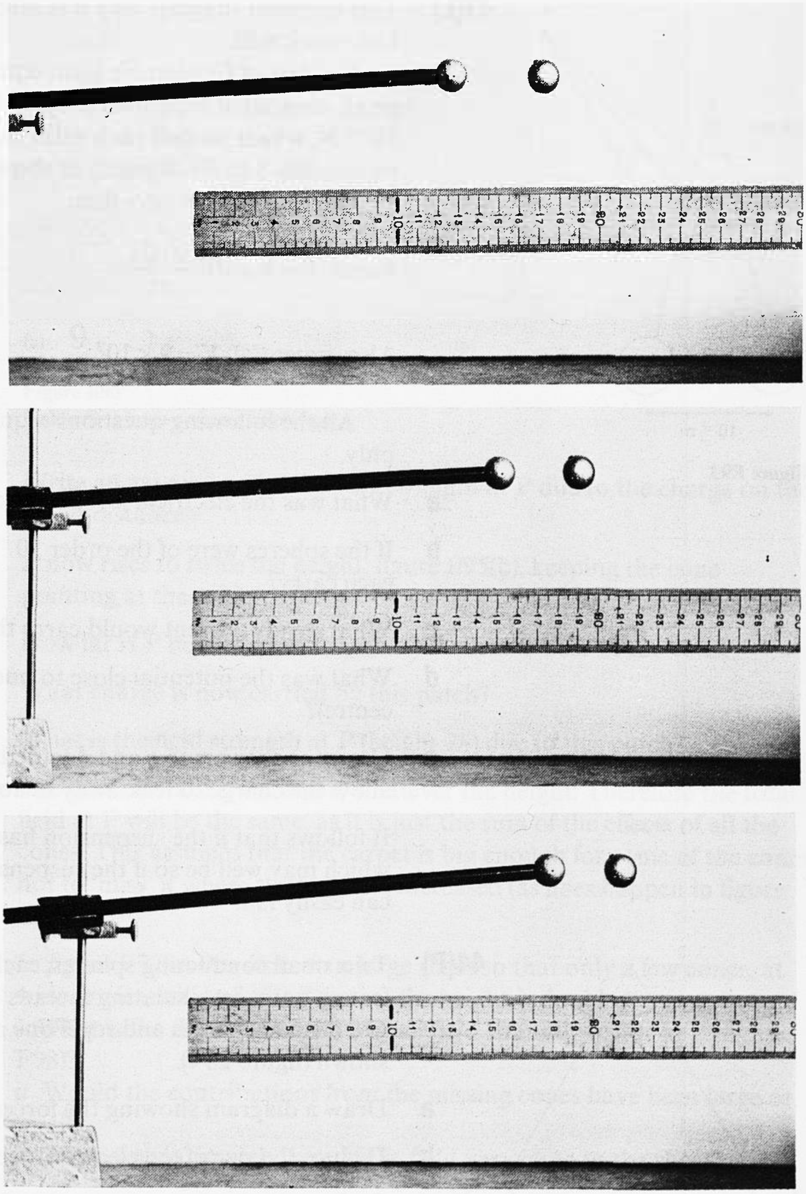


(a)

(b)

(c)

(d)



(e)

(f)

(g)

Figure E92

d Estimate the order of magnitude of the force constant in the equation:

force between balls

$$= \frac{\text{(force constant)(charge on one ball)(charge on other ball)}}{\text{(distance between balls)}^2}$$

The suspended ball weighed 1.1×10^{-3} newton.

e The charge on each ball almost certainly lies between 4 and 6×10^{-9} coulomb. Between what limits does this suggest that the force constant lies? The usual value is $9 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$.

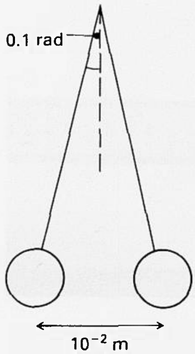


Figure E93

43(L) This question suggests why it is often hard to make tests of Coulomb's Law work well.

In a test of Coulomb's Law, equal charges were placed on two small suspended expanded polystyrene spheres each weighing about 10^{-3} N, which pushed each other aside at an angle of the order of 0.1 radian (say 5 to 10 degrees), as shown in figure E93.

Coulomb's Law says that:

$$\text{Force, } F = 9 \times 10^9 \frac{Q_1 Q_2}{r^2}$$

$$\text{Also, potential, } V = 9 \times 10^9 \frac{Q}{r}$$

All the following questions require order of magnitude answers only.

a What was the electrical force on each sphere?

b If the spheres were of the order 10^{-2} m apart, what charge did they each carry?

c What steady current would carry this charge away in 10^2 seconds?

d What was the potential close to one sphere (say 10^{-2} m from the centre)?

e Use your answers to **c** and **d** to deduce the resistance which would allow the charge to leak away in about 100 seconds.

It follows that if the suspension has a resistance of less than 10^{14} ohms, which may well be so if the suspension is at all damp, the experiment can easily fail.

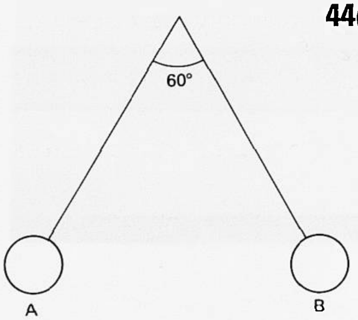


Figure E94

44(P) Two small conducting spheres, each of mass 10 mg, are suspended from a point by insulating threads 10 cm in length. The spheres are given equal charges and repel one another, settling in the position shown (figure E94).

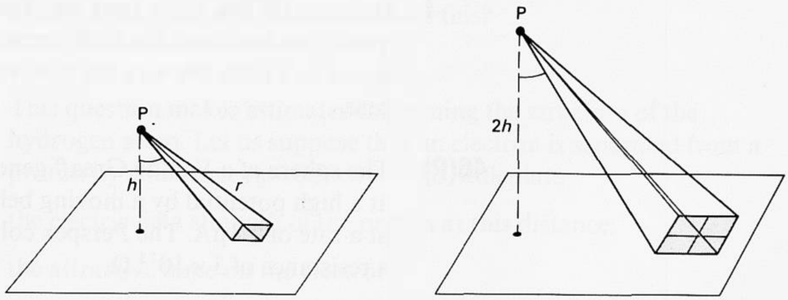
a Draw a diagram showing the forces acting on sphere A.

b Deduce the size of the electrical repulsion of A by B.

c Deduce the charge on each sphere.

Radial and uniform fields

45(L) This question helps to explain why a collection of charges each having a $\frac{1}{r^2}$ field gives a uniform field when made into a flat sheet. Imagine a carpet of charge made up of small tiles, each carrying charge Q. An observer at P, at height h above the sheet, is concerned with the field contributed by one tile at the base of a cone a distance r away, figure E95(a). The total field strength at P can be obtained by adding the contributions of all such cones.



(a) (b)

Figure E95
Field effect from a carpet of charge.

a Write an expression for the field strength at P due to the charge on the tile at distance *r*.

P now rises to twice the height, figure E95(b), keeping the cone pointing at the same angle.

b How far is P now from the patch of tiles at the base of the cone?

c What charge is now carried by this patch?

d What is the field strength at P (height $2h$) due to this patch?

The contribution is the same, whatever the height. Therefore the total field at P will be the same, as it is just the sum of the effects of all the cones. This assumes that the carpet is big enough for some of the cones not to 'miss' it where the height is increased (as *does* happen in figure E96).

e In figure E97(a) P is close to a large sheet so that only a few cones, at wide angles, will 'miss' the sheet if the height is doubled.
i What is the direction of the overall field strength vector at P (figure E98)?
ii Would the contributions from the missing cones have been large or small?
iii How, approximately, would the field vary close to the sheet?

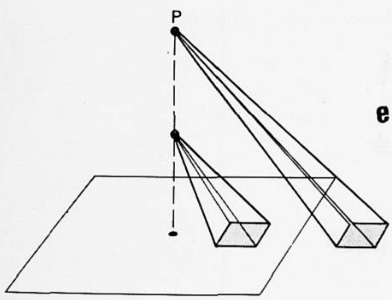
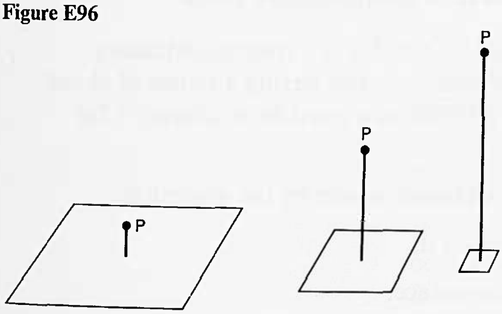


Figure E96



(a) (b) (c)

Figure E97
Variation of field with distance.

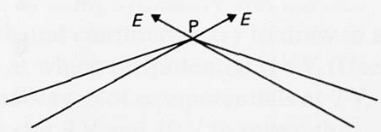


Figure E98
Contribution to the field from two cones at P.

f In figure E97(c), P is a long way from a small sheet so that many cones 'miss' the sheet, and the field strength at P is considerably less than it is closer in. Guess the way the field strength varies with distance in this case.

46(P) The sphere of a Van de Graaff generator 15 cm in radius is maintained at a high potential by a moving belt which carries charge to the sphere at a rate of 0.5 μA. The Perspex column which supports the sphere has a resistance of $3 \times 10^{11}\,\Omega$.

a What is the potential of the sphere?

b What is the charge on it?

c What is the electric field close to its surface?

d Estimate how close an earthed hand could be brought to the sphere before a spark crosses the gap. (Air conducts in a field strength of about $3 \times 10^6\,\mathrm{V\,m^{-1}}$.)

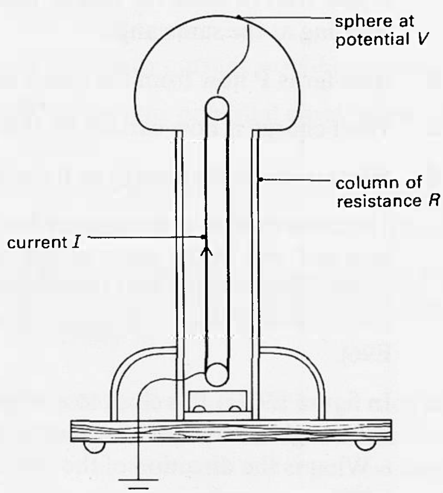


Figure E99

Electrical potential and energy in inverse-square fields

47(R) A simplified model of a uranium nucleus is a sphere containing 92 protons and rather more neutrons, and having a radius of about 2×10^{-14} m. If the nucleus releases an α-particle (of charge $+2e$) at its 'surface', estimate:

a the strength of the electric field experienced by the α-particle;

b the size of the repulsive force on it;

c the electrical potential at the surface;

d the electrical potential energy of the α-particle;

e the kinetic energy of the α-particle when it is a long way from the nucleus (express this in MeV where $1\,\mathrm{MeV} = 1.6 \times 10^{-13}$ J). What

assumption must you make to deduce this?
(Charge on proton $= 1.6 \times 10^{-19}$ C.)

48(R) This question makes estimates concerning the structure of the hydrogen atom. Let us suppose that an electron is separated from a proton by a distance $r_0 = 0.5 \times 10^{-10}$ m. Calculate

a the electric field strength of the proton at this distance;

b the attractive force on the electron;

c the potential due to the proton at this distance;

d the electrical potential energy of the system in electon volts (eV). (1 eV $= 1.6 \times 10^{-19}$ J.)

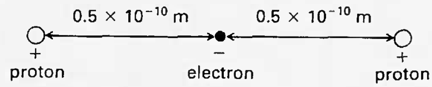


Figure E100

Now suppose another proton is brought up an equal distance from the electron, on the opposite side of it as shown in figure E100. Deduce:

e the electric field where the electron is;

f the force on it;

g the potential at this point due to both protons;

h the electrical potential energy of the electron.

i What would the electrical potential energy of the system be if the electron were removed?

j What is the total potential energy of this ion (2 protons plus 1 electron)?

k Comment on the role played by the electron in binding the ion together.

49(L) Figure E101 shows equipotentials drawn at 1 V intervals around two point charges. The object of this question is to deduce the overall equipotential pattern by adding together the potentials due to each charge.

a On a copy of Figure E101, first identify the contours by lightly labelling them 10, 9, 8, ... etc. (The smallest circle represents 10 V in each case.) Find the place where the two 4 V equipotentials meet and at this place mark a small figure 8 (representing 8 V, the total potential at this point). Mark a figure 8 also at all the positions where 5 V crosses 3 V, 6 V crosses 2 V, etc. Now, by using symmetry and the fact that the equipotentials will be smooth and continuous, try to draw in a complete curve joining all the points at which the potential is 8 V. (Use felt tip pen, or soft pencil.) In the same way, plot equipotentials at 7 V, 6 V, etc., as far as you can go, and also for 9 V and 10 V to reveal the overall equipotential pattern. What you have drawn is very similar to the potential experienced by an electron in the plane of the two protons in an H_2^+ ion, with the values scaled down.

E

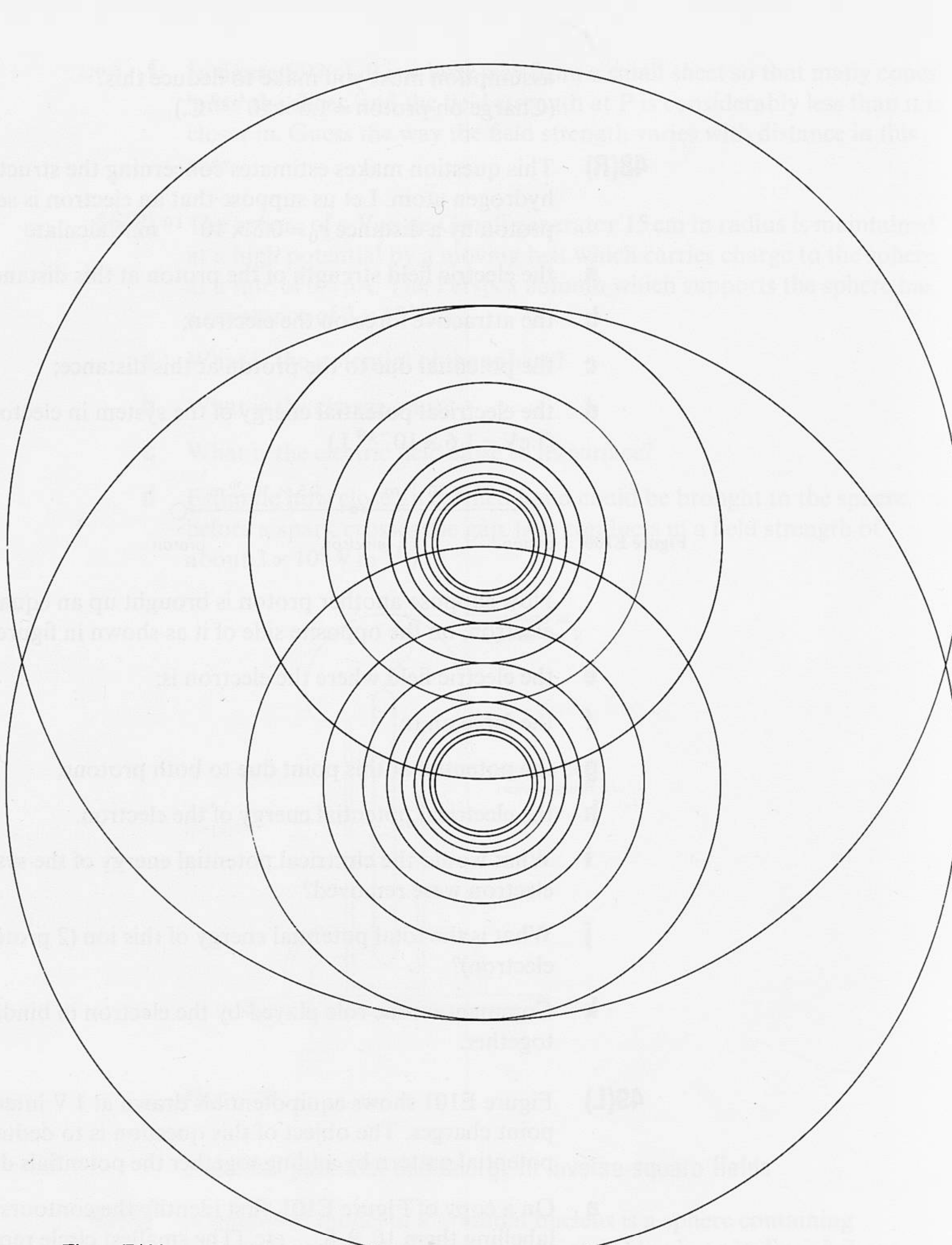


Figure E101

b Now regard one charge as positive and the other negative. On a fresh copy relabel the equipotentials +10, +9, etc. from one charge and −10, −9, etc., from the other. Using the same method as before, deduce the overall equipotential pattern. This would represent, for example, the potential between two parallel conductors carrying currents in opposite directions.

c Remembering that field is always perpendicular to the equipotentials, deduce the direction of the electric field in different places and sketch in some lines to show its overall shape. (Recall experiments E3 and E5.)

50(R) A metal sphere, A, is connected by a long fine wire to a source of potential of 900 V. An insulated stand supports the sphere sufficiently far above the bench for the effect of the latter to be negligible.

a The potential 300 mm from the centre of the sphere is 450 V. Find the following, showing in each case the steps in your calculation:
i the potential 500 mm from the centre of the sphere;
ii the radius of the sphere;
iii the charge on the sphere.

b Another identical sphere, B, connected to the same source, is similarly supported with its centre 600 mm from that of the first sphere, A, as shown in figure E102.

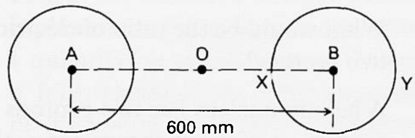


Figure E102

Would you expect the potential at O, a point midway between A and B, to be equal to, greater than, or less than 900 V? Explain why.

(Short answer paper, 1981)

E

51(R) (Hard) This question begins to explore the differences between conducting and insulating surfaces and the usefulness of adding up potentials to determine whether there is a field or not.

Suppose the metal spheres in question **50** were replaced by insulating spheres which separately had been given a uniform surface charge so that the potential on the surface of each was 900 V. These spheres are no longer connected to any source of charge.

a What would now be the potential at O in figure E102?

b What would be the potential at X and Y on the nearer and far surface of sphere B? How do you obtain these values?

c Would any electric field exist in the sphere B? Why?

d What would happen if B's surface were suddenly coated with conducting paint?

e Would there now be any differences in potential across the sphere B?

f What can you say about the potential at all points on the surface of a conductor? What if the conductor carries some charge?

Electrical and gravitational forces

52(E) In question **48b**, you calculated the electric force between a proton and an electron. It was assumed that this was responsible for holding the hydrogen atom together. Could not gravity be a factor here too?

a Estimate the gravitational force at separation, $r_0 = 0.5 \times 10^{-10}$ m, between a proton (mass $\approx 10^{-27}$ kg) and an electron (mass $\approx 10^{-30}$ kg).

b What is the ratio of the electric force to the gravitational force at this distance?

c Why is the ratio of these two forces the same even if the proton and electron were one light-year apart?

The enormous size of this ratio has seemed to be of fundamental significance to physicists although no one has yet been able to exploit it. Clearly gravity does not hold the atom together. However, could it have effect in the nucleus where electric forces between protons are repulsive?

d What would be the ratio of electric force to gravitational force between two *protons*?

e A helium nucleus has two protons and two neutrons. Assuming the neutrons are about as massive as the protons, would gravitational attraction between the nucleons overcome the electric repulsion?

f The force which *does* hold the nucleus together must be over 10^{36} times stronger than gravity at separations of about 10^{-15} m. Since no such force seems to dominate affairs at an astronomical, or even an atomic level, what can we say about the way it varies with distance?

53(P) If, as question **52** suggests, electrical forces are so much greater than gravitational ones, why do we not experience them much in everyday life? This question starts out to answer this problem.

Let us consider a dipole, that is two equal and opposite charges A and B, separated by a short distance. (The distribution of charges in many molecules leads effectively to this arrangement. An ion pair, for example Na^+ and Cl^-, will also show similar properties.) Assuming charges of $+e$ and $-e$, 2×10^{-10} m apart, we shall consider how the electric field strength varies with distance from the dipole, and compare this with the corresponding variation for a single charge.

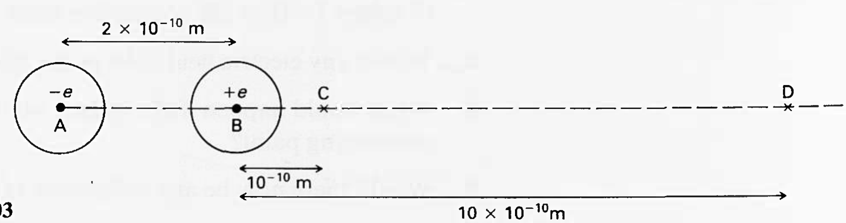


Figure E103

a First consider B alone, A being removed. Calculate the electric field strength at C due to B and call this 1 'unit'.

b D is ten times further from B than C is. Using the inverse-square law, write down the field strength in units at D, due to B alone.

c Similarly, work out the contribution to the field strength, in units, made by A (remembering the sign of its charge) at C and D.

d Work out the total field strength, in units, of the dipole at C and D.

e Calculate the ratio now of the field strength at D to that at C.

f Does the field of the dipole fall off following an inverse-square law, or more rapidly? Suggest a reason for this.

g Now imagine many dipoles forming an array of positive and negative charges in equal numbers (as for instance in an ionic crystal). Explain why the very large electric forces within the array are not experienced by charges at some distance from it.

54(R) This question is about gravity and the similarities between gravity and electric and magnetic effects.

The passage below sets out three sets of ideas about gravity. For each of the sections **a** to **c** you are asked to write a more complete explanation of the ideas: your explanations may include
i quantitative calculations to illustrate the ideas,
ii fuller explanations of the theoretical ideas,
iii discussion of possible experiments.

You should pay particular attention to the words and phrases that are in italics in each section.

a There is something peculiar about gravity: it is such a *small force* that if we didn't live on a big lump of matter called the Earth we might not notice that it affected human-size objects at all. In fact the simplest calculations can show that it is very *hard to demonstrate* that the effect exists between all pieces of matter.

b There is a close *analogy* between the theoretical ideas involved in electricity and in gravity, and this can be of great value in discussing such abstract ideas as *field* and *potential.* Thus problems such as the scattering of alpha-particles by a nucleus and the path of a spaceship round the Moon have many *similarities* though there are also *important differences.*

c However, electrical and magnetic effects are so *much bigger*, for human-size experiments, that they swamp all effects of gravity. The fact that when we come to matter on an astronomical scale, gravity is *by far the most important force* is then hard to explain – it must be due to *electrical neutrality of big objects.*

(Long answer paper, 1979)

55(R)

Gravity		**Electricity**
$F_g = -G\dfrac{m_1 m_2}{r^2}$	Force	$F_e = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{Q_1 Q_2}{r^2}$
$g = -G\dfrac{M}{r^2}$	Field strength	$E = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{Q}{r^2}$
$V_g = -G\dfrac{M}{r}$	Potential	$V_e = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{Q}{r}$
$E_p = -G\dfrac{m_1 m_2}{r}$	Potential energy	$E_p = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{Q_1 Q_2}{r}$

This question revises the links between the estimated quantities above and some of the differences between electricity and gravity.

a Using the words *mass* or *charge* where relevant write down two sentences defining electrical and gravitational field strength.

b How can one obtain the gravitational or electrical field strength from a graph of the appropriate potential against distance?

c How is the potential difference between two points obtained from a graph of field strength against distance?

d How is the potential energy of a mass or charge at a point in a field obtained from the potential at the point?

e What would the gradient of a graph of potential energy against distance indicate?

f What does it mean to say that all gravitational forces are negative?

g Why is there no negative sign in the corresponding electrical expression, though electric force vectors may sometimes be negative?

h Why is gravitational potential (and potential energy) always negative?

i In a place where the electrical potential is positive (for example, near a proton), how could one object have a positive potential energy and yet another have a negative potential energy?