Unit F
RADIOACTIVITY AND THE NUCLEAR ATOM

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SUMMARY OF THE UNIT

F

SUMMARY OF THE UNIT

Section F1 THE RUTHERFORD MODEL OF THE ATOM

The nature and properties of alpha, beta, and gamma radiation

EXPERIMENT F4
Photographic detection of radiation

The development of our present picture of the atom began in 1896 when Becquerel discovered that uranium compounds could affect photographic plates and also ionize a gas.

EXPERIMENT F3
Penetrating power of α-, β-, and γ-rays

There are three types of radiation emitted and they can be identified by their penetrating power, ionizing ability, and behaviour in a magnetic field.

EXPERIMENT F2
Number of ions produced by an alpha particle

Alpha radiation has a range of a few centimetres in air and can be stopped by a sheet of paper. It can cause intense ionization in gas and is deflected by a very strong magnetic field. The deflection by a magnetic field suggests that it consists of relatively heavy, positively charged particles. Alpha particles are in fact helium ions with a double positive charge. The energy range of alpha particles from different sources varies from about 4 to 10 MeV, and this corresponds to kinetic energies giving them speeds of about 1.5 to $2 \times 10^7\,\mathrm{m\,s^{-1}}$. The alpha particles emitted from a particular source all have the same energy.

$1\,\mathrm{MeV} = 1.6 \times 10^{-13}\,\mathrm{J}$

EXPERIMENT F1
The deflection of beta radiation in a magnetic field

Beta radiation is more penetrating than alpha radiation, having a range up to about one metre in air, and it is able to penetrate a few millimetres of aluminium. It causes less ionization and is more easily deflected by a magnetic field than is alpha radiation. The direction and amount of deflection indicates that beta radiation consists of negatively charged particles with a relatively small mass. In fact, beta particles are fast-moving electrons. The energy range of beta particles (from about 0.025 to 3.2 MeV) corresponds to kinetic energies giving them speeds very close to the speed of light ($3 \times 10^8\,\mathrm{m\,s^{-1}}$). Beta particles are emitted with an almost continuous energy spectrum.

Gamma rays have the ability to penetrate several centimetres of lead: they cause only weak ionization. They are not deflected by a magnetic field, which indicates that this radiation does not consist of charged particles. Gamma rays are in fact a form of electromagnetic radiation, travelling with the speed of light, and with wavelengths shorter than those of X-rays. Unlike beta particles, the gamma rays emitted by a particular source can have only certain sharply-defined energies. For example, cobalt-60 emits 1.2 and 1.3 MeV gamma rays.

QUESTIONS 1 to 7

Sources and activities

Bq = becquerel
1 Bq = 1 disintegration per second
1 curie (Ci) = 3.7×10^{10} Bq

Sources of radioactivity used in schools are relatively weak; they have low activities of about 18×10^4 Bq (5 μCi) or less. Typical sources are:

Radium-226 for alpha and beta particles, and gamma radiation.
Americium-241 for alpha particles (also some low-energy gamma radiation).
Strontium-90 for beta particles.
Cobalt-60 for gamma radiation.

The activity of a particular source depends on the quantity of radioactive material it contains, as well as the particular atomic species (nuclide). Thus a 2×10^5 Bq source of, say, strontium-90 contains twice as many ^{90}Sr atoms as a 1×10^5 Bq source.

Nuclear gunnery

Section F3

Many experiments in atomic and nuclear physics consist of bombarding one kind of particle with another. The ionization energy of atoms (the energy needed to remove one of the outer electrons) can be found by bombarding gas atoms with relatively slow-moving electrons 'boiled off' a hot wire. When accelerators are used to produce very fast-moving particles (speeds approaching that of light), much more damage is done: the atomic nucleus may break up and new particles can be created.

READING
'Radioactivity and the nuclear atom: a brief history' (page 373)

Some very important bombardment experiments crucial in the development of the nuclear model of the atom were carried out by Rutherford and his colleagues Geiger and Marsden in about 1913. They used alpha particles from radioactive sources to bombard metal atoms. To interpret their results it is essential to know the properties of alpha particles: their mass, electric charge, and energy.

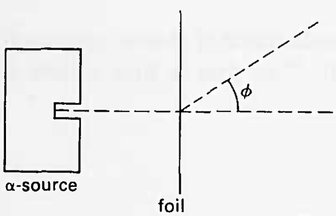

Figure F1

Angle ϕ /degrees	Number scattered at angle ϕ into fixed small area
150	33.1
135	43.0
120	51.9
105	69.5
75	211
60	477
45	1 435
37.5	3 300
30	7 800
22.5	27 300
15	132 000

Table F1

The Rutherford model of the atom

The idea that matter ultimately consists of 'uncutable' atoms (particles which cannot be subdivided any further) is a very old one. Our present ideas of how atoms are made up of smaller particles began to develop early in the twentieth century. By 1900, J. J. Thomson had found that electrons could be removed from atoms, and in 1904 he proposed a so-called 'plum pudding' model for the atom: negative electrons embedded in a sphere of uniformly distributed positive charge. Some experiments in about 1910 produced surprising results which the Thomson model could not explain. When alpha particles were fired at a metal, a few were turned back or scattered through very large angles (figure F1). This suggested to Rutherford that the atom's positive charge was concentrated in a small but massive nucleus; the scattering was due to the electrical repulsion of the alpha particle by the central small nucleus. Geiger and Marsden were then set to work to test this model experimentally.

The experiment tested whether the scattering behaved as if the particles were acted upon by an inverse-square law force. The particles could not be followed along their paths so an indirect test had to be arranged. The number of particles recorded at particular angles (table F1) was compared with the number predicted if the force acting upon them were an inverse-square one. The experimental results agreed with the theoretical predictions.

A '$1/r$ hill' is a physical model which has some of the properties of the nuclear atom. The hill is carefully constructed so that its height varies as $1/r$, r being the distance from the centre (figure F2). When a ball rolls on the hill its potential energy therefore varies as $1/r$ and the force on it as $1/r^2$.

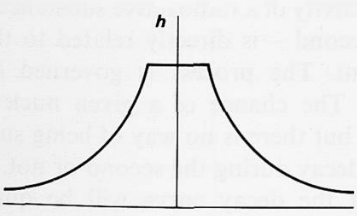

Figure F2

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DEMONSTRATION F5
Qualitative test of the gravitational hill model

Geiger and Marsden slowed down alpha particles by passing them through thin sheets of mica. They found that slower-moving alpha particles are more likely to be deflected; see table F2. Tests with the hill show that the lower the initial kinetic energy of the ball, the larger the angle it is deflected through.

Table F2

Number of sheets of mica	Range of α-particles after leaving mica	Number of α-particles detected at fixed angle
0	5.5	24.7
1	4.76	29.0
2	4.05	33.4
3	3.32	44
4	2.51	81
5	1.84	101
6	1.04	255

QUESTIONS 10 to 13

The fact that the vast majority of alpha particles either pass through the foil undisturbed or are scattered at small angles is evidence for the small size of the nucleus – the undeflected particles pass too far from any nucleus to be significantly affected.

QUESTIONS 8, 9

$r_{atom} \approx 10\,000 \times r_{nucleus}$

The size of the nucleus – defined by the distance of closest approach of an alpha particle – is of the order of 10^{-14} m; that is, four orders of magnitude smaller than the atom itself.

Section F2 EXPONENTIAL DECAY

Radioactive decay and half-life

EXPERIMENT F6
Decay and recovery of protactinium

Radioactive decay is a spontaneous process which cannot be controlled and is not affected by chemical reactions, temperature, or pressure. Radiation is emitted from the nucleus of an atom and the process does not involve the atom's outer electrons. When a nucleus emits an alpha or a beta particle the nucleus of a different atom, called the daughter atom, is formed.

EXPERIMENT F7
The decay of radon and the determination of its half-life

The half-life of a particular radioactive nuclide is defined as the time it takes for half the nuclei originally present to disintegrate. Since activity is proportional to the number of active nuclei present, the activity due to a particular nuclide also falls to half its initial value in one half-life.

$\Delta N/\Delta t \propto -N$

EXPERIMENT F8
Radioactive decay analogue

QUESTION 14

Half-lives can vary from millionths of a second to millions of years. Polonium-212 has a half-life of 3×10^{-7} s, whereas the half-life of uranium-238 is 4.5×10^9 years. The activity of a radioactive substance – the number of disintegrations per second – is directly related to the number of undecayed nuclei present. The process is governed by chance, like the throwing of a die. The chance of a given nucleus decaying in, say, one second is fixed, but there is no way of being sure whether that particular nucleus will decay during the second or not. If many undecayed nuclei are present the decay curve will be quite smooth, but when only small numbers are involved the statistical fluctuations are much more noticeable.

QUESTIONS 15 to 18

The rate of radioactive decay depends on the number of undecayed nuclei present. The decay curve has the same shape as that for the decay of charge on a capacitor, where the rate depends on the amount of charge. Other processes grow at a rate which depends on the quantity of something present – for example, the growth of a population of bacteria. Such processes are called exponential decay or exponential growth, respectively. The characteristic of an exponential change is the constant ratio property: in equal intervals of time the quantity present always grows (or decreases) by a constant factor. Exponential growth has a constant doubling time; exponential decay has a constant half-life.

QUESTIONS 19 to 23

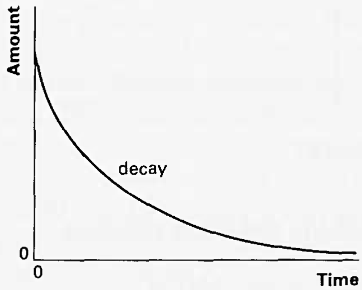

Figure F3

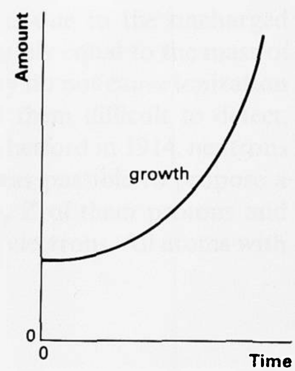

Figure F4

The exponential function

'About exponential changes' (page 394)

QUESTIONS 24 to 27

λ is the *decay constant*

In 10^2, 10^3, a^t, the quantities 2, 3, and t are called 'exponents'; they are also called 'powers' or 'indices'. This is where the word 'exponential' comes from. The growth equation $\Delta N/\Delta t = \lambda N$ can be solved numerically. The solution is of the form $N = a^t$. This gives a constant-ratio curve when values of N are plotted against t. When $\lambda = 1.0$ and $N = 1.0$ at $t = 0$, the value of a is 2.718... which is the natural number e. The general solution of the equation $\Delta N/\Delta t = \lambda N$ is $N = e^{\lambda t}$.

Similarly, the equation for decay, $\Delta N/\Delta t = -\lambda N$, has the solution $N = e^{-\lambda t}$. This assumes that $N = 1.0$ at $t = 0$, but this is not always true. The solutions become $N = N_0 e^{-\lambda t}$ for a decay and $N = N_0 e^{\lambda t}$ for a growth, where N_0 is the value of N at $t = 0$ and N is the value after time t (figures F5, F6).

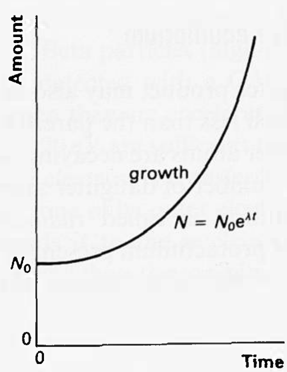

Figure F5

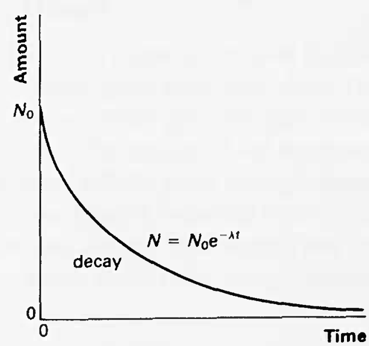

Figure F6

$N/N_0 = e^{-\lambda t}$
$\Rightarrow \ln N = \ln N_0 - \lambda t$

The constant-ratio property is a test of whether or not a curve is exponential. If the change is exponential, a graph of $\ln N$ against time will be a straight line. The slope is positive for a growth curve, and negative for a decay curve (figure F7).

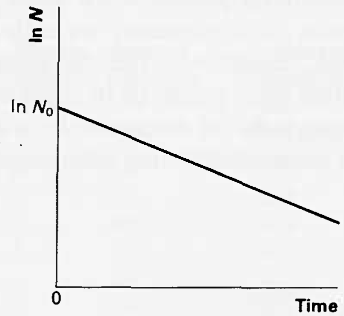

Figure F7

Half-life and decay constant

After one half-life ($t_{\frac{1}{2}}$)

$N = N_0/2 = N_0 e^{-\lambda t_{\frac{1}{2}}}$

from which it follows that

$\lambda = \ln 2/t_{\frac{1}{2}} = 0.693/t_{\frac{1}{2}}$

and

QUESTION 28

$t_{\frac{1}{2}} = 0.693/\lambda.$

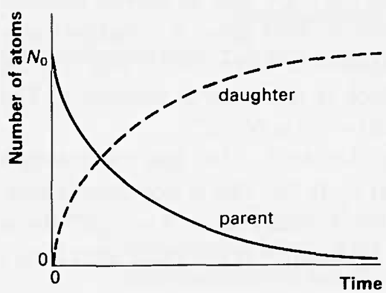

Figure F8

Radioactive decay and recovery

When alpha or beta radiation is emitted from a nucleus, a new nucleus of a different substance is formed. This new nucleus is called the daughter product. If the daughter nucleus is not radioactive, the number of daughter nuclei present at any time is just the number of parent nuclei which have decayed: $N_0 - N$. But N is given by $N = N_0 e^{-\lambda t}$.

So $N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t})$.

Radioactive equilibrium

The daughter product may also be radioactive. If the daughter half-life is very much less than the parent half-life a state is soon reached where the daughter atoms are decaying as quickly as they are being produced. Thus the number of daughter atoms present in the sample is constant. This condition is called *radioactive equilibrium.* For example, the amount of protactinium present in a sample of uranium quickly reaches an equilibrium value because the half-life of protactinium is rather short.

Section F3 THE NUCLEUS

Proton number, nucleon number, and the Periodic Table

Z = proton (or atomic) number
A (nucleon number) = number of protons + number of neutrons

$m_n = 1.675 \times 10^{-27}$ kg

$m_p = 1.673 \times 10^{-27}$ kg

QUESTION 29

Alpha-particle scattering experiments by Geiger and Marsden established Rutherford's model for the atom: a small, massive, positively-charged nucleus, surrounded by negative electrons. The amount of scattering depends, among other things, on the nuclear charge, that is, on the number of protons in the nucleus. In the Periodic Table the elements are arranged in order of increasing proton number, Z. The nucleon number of an atom, A, is usually about twice its proton number. This means that the mass of the nucleus is about twice the mass of the protons it contains. The extra mass is due to the uncharged neutrons, each of which has a mass approximately equal to the mass of a proton. Because neutrons are uncharged they do not cause ionization and can penetrate matter easily. This makes them difficult to detect. Although their existence was suggested by Rutherford in 1914, neutrons were not shown to exist until 1932. Then it was possible to propose a model for the nucleus containing A particles, Z of them protons and $(A-Z)$ neutrons. In the atom there are also Z electrons. All atoms with the same value of Z are chemically the same.

Isotopes

${}^{A}_{Z}X$

isotope means 'same place' (*i.e.*, in Periodic Table)

QUESTIONS 30 to 33

'Radioisotopes' in the Reader *Particles, imaging, and nuclei*

Atoms with the same Z but different A are *isotopes*: they are chemically the same, but have different masses. For example, seven isotopes of carbon have been identified. Most are radioactive. Some 98.9 per cent of naturally-occurring carbon is ${}^{12}_{6}C$, which is stable. On the other hand, ${}^{14}_{6}C$ is a naturally-occurring radioactive isotope with a half-life of about 5000 years, and is the basis of radiocarbon dating. Radioactive isotopes can also be manufactured, often by neutron irradiation in a nuclear reactor (for example ${}^{23}Na + {}^{1}n \rightarrow {}^{24}Na$). Radioisotopes have many uses in science, medicine, and industry.

Electrons and ionization energy

QUESTION 34

Alpha particles can be detected by a Geiger–Müller (GM) tube because they ionize the gas inside the tube. The ions created by an alpha particle also act as condensation centres in a cloud chamber. Alpha particles have energies of about 5 MeV.

QUESTION 35

DEMONSTRATION F9
Ionization by electron collision

Beta particles (high-energy electrons) also cause ionization and can be detected with a GM tube. Experiments with electrons show that when they are accelerated through a low-pressure gas, energies of only 10–20 eV are sufficient to cause ionization. The energy of the bombarding electron is transferred to the gas atom which gains enough energy for one of its outer electrons to break free from the electric force which binds it to the nucleus. Since the bound system (the atom) has *less* energy than the ion plus free electron which results, the energy binding the electron to the atom is taken to be *negative*, in the same way that the energy of a bound system such as the Earth and the Moon is taken to be negative.

Energy must be *added* to a neutral atom to free an electron, and ionization energy is conventionally taken as a *positive* quantity; it is equal in magnitude to the electron's binding energy.

Ionization energy varies from element to element, in a way which reflects the element's position in the Periodic Table (figure F9). Elements in the same column of the Periodic Table have similar ionization energies. The addition of one more proton can change the ionization energy drastically: inert gases such as helium bind their electrons firmly, whereas an alkali metal such as lithium has an outer electron which is weakly bound and has a low ionization energy. Using our knowledge of ionization energies we can confirm that an atom's outer electrons are about 10^{-10} m from the nucleus.

QUESTIONS 36 to 38

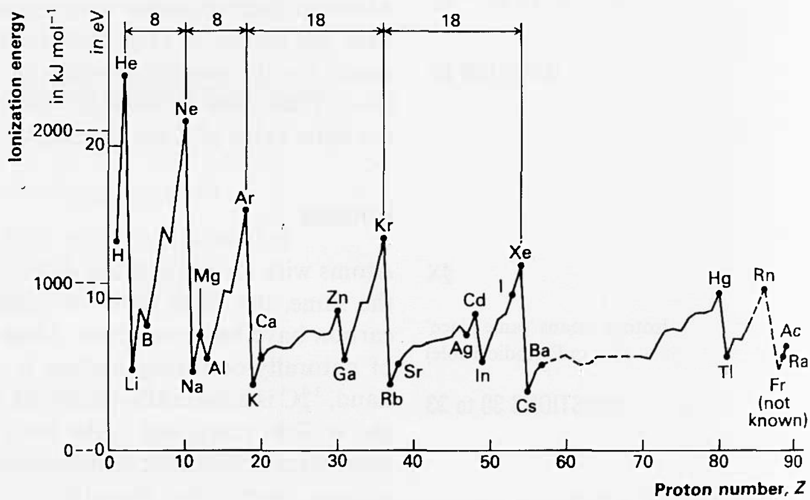

Figure F9
Ionization energies of the elements.

If the atom is bombarded by electrons with energies less than the binding energy, *inelastic collisions* may still occur. Some of the energy of the bombarding electron is absorbed by the atom but it is not enough to free an atomic electron. This process is called *excitation*. The excited atom loses this extra energy – usually very quickly – by emitting light. If the electron does not have enough energy to cause excitation then the collision will be elastic: the bombarding electron loses no energy, the atom gains none.

Transformation rules

An alpha particle is a helium nucleus emitted from the nucleus of a decaying atom. A beta particle is an electron liberated when a neutron in the nucleus decays into a proton plus an electron (plus a massless neutral particle, the neutrino). The changes in Z and A which accompany these decays are shown in table F3.

QUESTIONS 39 to 41

	change in proton number Z	change in nucleon number A
α decay	-2	-4
β decay	$+1$	0

Table F3

Unit L, 'Waves, particles, and atoms'

When a nucleus emits gamma radiation, neither Z nor A change. Gamma radiation is not a stream of particles, but is very short-wavelength electromagnetic radiation – shorter than X-rays. Atoms emit light when their electron energy levels change, and we can learn about these energy levels from the wavelength of light emitted. Gamma radiation is emitted when nuclear energy levels change, and analysis of the radiation gives information about nuclear energy levels.

What holds the nucleus together?

QUESTION 42

'The particles and forces of nature' in the Reader *Particles, imaging, and nuclei*

'Lasers probe the atomic nucleus' in the Reader *Particles, imaging, and nuclei*

A simple calculation shows that gravity is not nearly strong enough to hold the protons in a nucleus together against the electric repulsion between them. A new force, which acts on neutrons as well as protons, is at work. This force is evidently restricted to very small distances – its influence is not felt outside the nucleus.

Nuclear binding energy

This attractive force gives rise to a nuclear binding energy. Because a collection of neutrons and protons has *less* energy than the same particles have when far apart, the nuclear binding energy is *negative*. The larger the number of particles in the nucleus, the more negative the binding energy is. For each nucleon added the total binding energy decreases approximately by a further 8 MeV. Nuclear binding energies are much larger in magnitude than ionization energies.

However, the graph of nuclear binding energy (BE) against nucleon number A is not quite a straight line (figure F10), and a graph of BE/A (average binding energy per nucleon) against A shows some interesting features (figure F11).

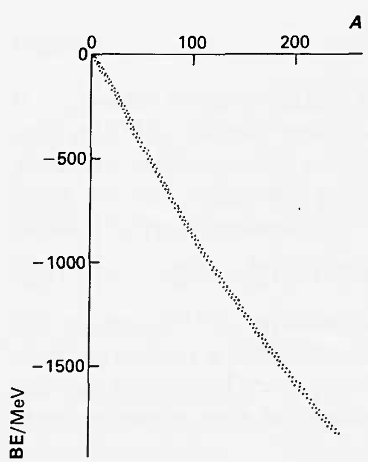

Figure F10

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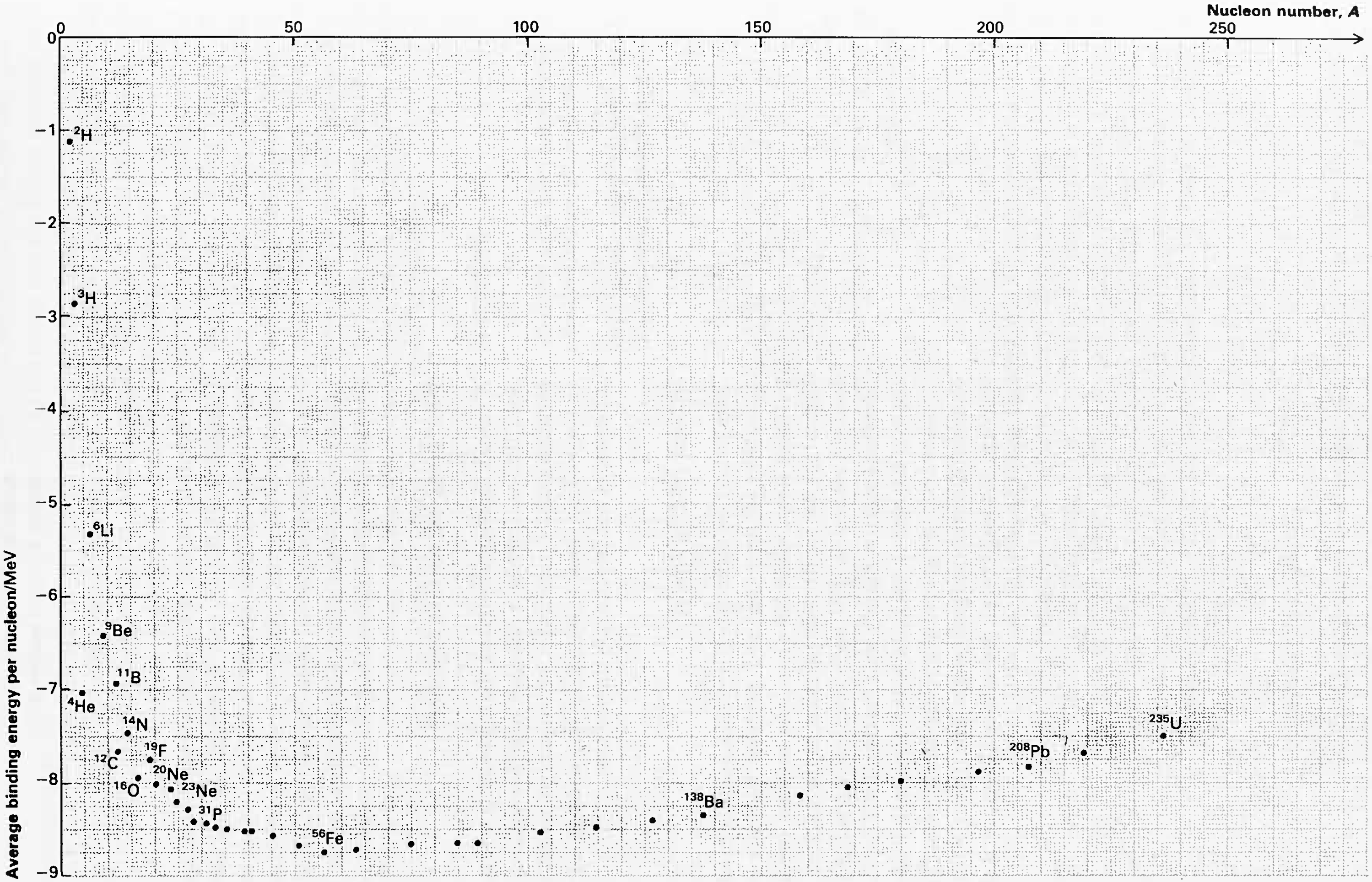

Figure F11
Average binding energy per nucleon as a function of number of nucleons.

QUESTION 43

The most stable nuclei are those with the most negative binding energies. The iron nucleus, ^{56}Fe, is the most stable. ^{4}He, ^{12}C, and ^{16}O are each more stable than their immediate neighbours. ^{235}U is considerably less stable than lighter nuclei. This suggests that if the 235 nucleons in ^{235}U could be rearranged into two or more lighter and more stable nuclei, energy would be released.

Calculation of nuclear binding energies

Unit H, 'Magnetic fields and a.c.'

$1\,u = 1.66 \times 10^{-27}\,kg$

QUESTION 29

The masses of atoms can be measured very precisely using a mass spectrometer. The masses of the elementary particles (proton, neutron, and electron) are also known with considerable precision. These very small masses are often expressed in unified atomic mass units (u). 1 u is one-twelfth of the mass of an atom of ^{12}C. It is a surprising fact that the mass of an atom – any atom – always turns out to be less than the value found by adding the masses of its constituent parts. The difference is biggest in the case of ^{56}Fe, less for ^{235}U, and considerably less for ^{6}Li. This mass loss (or *mass defect*) is a measure of the binding energy.

QUESTIONS 44 to 46

Einstein's theory of relativity linked the concepts of mass and energy. He showed that energy has mass. Experiments demonstrate that a very fast-moving electron is accelerated less by a given force than a slow-moving one: its mass is greater. The mass of a clock spring is greater when it is wound, and storing energy, than when it has run down; the mass of a beaker of hot water is greater than the mass of the same water molecules when cold. For these everyday examples the mass change is far too small to be detected. But the mass of an atom of ^{4}He is about 0.03 u (0.75 per cent) less than the mass of two protons plus two neutrons plus two electrons.

This loss of mass means that the atom has less energy than its parts. Mass change and energy change are related by

$c = 3 \times 10^{8}\,m\,s^{-1}$ (speed of light)

$$\Delta E = c^2 \Delta m$$

The energy loss corresponding to 0.03 u is 28.3 MeV, so the average binding energy *per nucleon* for ^{4}He is $-28.3/4\,MeV = -7.1\,MeV$, as shown in figure F11.

Fission

'fission' means splitting

It is possible to cause certain heavy atoms (high nucleon number) to split into two roughly equal parts. A few free neutrons will also be produced. Although this can happen spontaneously, it is a very rare event. We can trigger this process by adding an extra neutron to the nucleus. A typical example is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{90}_{36}Kr + 2^{1}_{0}n$$

The nucleus of ^{235}U 'captures' a low-energy neutron (slow neutron); the resulting nucleus is unstable and splits. (Note that the reaction shown is just one example of many possibilities.) The neutrons produced in the reaction can in turn be captured by other ^{235}U nuclei, and so the

F

chain reaction

Unit G, 'Energy sources'

reaction continues, perhaps at an increasing rate. Since the total binding energy of the products is less (more negative) than the binding energy of ^{235}U, energy is released in this reaction – about 200 MeV per nucleus. Most of this energy appears as kinetic energy of the fission fragments. 200 MeV per atom is equivalent to about 80×10^6 MJ per kilogram – very much more than the energy available in a chemical reaction, for example, when one kilogram of oil or coal is burned (about 30 MJ per kg).

Fusion

'fusion' means joining, uniting (or melting)

The 'energy well' shown in figure F11 slopes very steeply near the origin. If it were possible to move deeper into the 'well' by combining two light nuclei on this part of the curve to form a single more massive nucleus, the energy released per nucleon would be much greater than in fission.

Unit G, 'Energy sources'

Fusion is a difficult process to achieve because of the strong electrical repulsion between the nuclei when they are close to each other. At extremely high temperatures (about 100 million K) the nuclei have enough kinetic energy to overcome this repulsion. A possible fusion reaction involving deuterium (heavy hydrogen) is

$${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$$

This reaction releases 3.3 MeV, but the energy *per kilogram* is greater than in fission.

'Our nuclear history' in the Reader *Particles, imaging, and nuclei*

The Sun's energy source is a sequence of fusion reactions of this kind in which hydrogen is converted into helium.

READING

RADIOACTIVITY AND THE NUCLEAR ATOM: A BRIEF HISTORY

Our modern nuclear model of the atom derived, after a lapse of twenty years, from two remarkable discoveries. The first, by H. Becquerel in 1896, was of the effect we know as radioactivity; the second was the discovery by J. J. Thomson in 1897 that all atoms of all elements contained the same light, negatively-charged particles we now call electrons.

Becquerel's discovery

In 1896, Henri Becquerel was trying to see whether phosphorescent materials that glowed when light had shone on them emitted any penetrating rays like the X-rays that Röntgen had recently discovered. He reported in the journal of the French Academy of Sciences:*

'One of Lumière's gelatine bromide photographic plates is wrapped in two sheets of very heavy black paper so that the plate does not fog on a day's exposure to sunlight.

'A plate of the phosphorescent substance (uranium potassium sulphate) is laid above the paper on the outside and the whole exposed to the Sun for several hours. When later the photographic plate is developed, the silhouette of the phosphorescent substance is discovered, appearing in black on the negative.'

A little later, in the same volume, he wrote:

'Among the preceding experiments some were prepared on Wednesday the 26th and Thursday the 27th of February, and, as on those days the Sun only appeared intermittently, I held back the experiments that had been prepared and returned the plate holders to darkness in a drawer, leaving the thin layers of the uranium salt in place. As the Sun still did not appear during the following days, I developed the photographic plates on the first of March, expecting to find very weak images. To the contrary, the silhouettes appeared with great intensity. I thought at once that the action had been going on in darkness...'

Repeated tests soon identified the uranium as the cause of the effect.

J. J. Thomson and the electron

In 1897 Thomson published his classic paper† on his experiments with cathode rays which led him to propose that these rays consisted of

*ROMER, A. (ed. and trans.) *The discovery of radioactivity and transmutation.* Dover, 1964. From *Comptes rendus de l'Académie des sciences.* Volume **122**, 742.

†THOMSON, J. J. 'Cathode rays'. *Philosophical Magazine.* Volume **44** (5), October 1897, p. 293. (Reprinted in WRIGHT, S. (ed.) *Classical scientific papers – physics.*)

E

streams of negatively-charged particles of very small mass. These later became known as 'electrons'. The properties of these particles were just the same whatever gas was used in the tube and whatever metal was used for the cathode.

Naturally Thomson tried to relate his discovery of the electron as a universal ingredient of atoms to what was then known about atoms; and, in particular, to the perplexing problem of the periodicities recognized in the Periodic Table. Of these, the most important was the arrangement of the elements in groups containing 2–8–8– etc., elements, with properties repeating from group to group. Thomson wrote:‡

'We have seen that corpuscles§ are always of the same kind whatever may be the nature of the substance from which they originate; this, in conjunction with the fact that their mass is much smaller than that of any known atom, suggests that they are a constituent of all atoms; that, in short, corpuscles are an essential part of the structure of the atoms of the different elements. This consideration makes it important to consider the ways in which groups of corpuscles can arrange themselves so as to be in equilibrium. Since the corpuscles are all negatively-electrified, they repel each other, and thus, unless there is some force tending to hold them together, no group in which the distances between the corpuscles is finite can be in equilibrium. As the atoms of the elements in their normal states are electrically neutral, the negative electricity on the corpuscles they contain must be balanced by an equivalent amount of positive electricity; the atoms must, along with the corpuscles, contain positive electricity. The form in which this positive electricity occurs in the atom is at present a matter about which we have very little information. No positively-electrified body has been found having a mass less than that of an atom of hydrogen. All the positively-electrified systems in gases at low pressures seem to be atoms which, neutral in their normal state, have become positively-charged by losing a corpuscle. In default of exact knowledge of the nature of the way in which positive electricity occurs in the atom, we shall consider a case in which the positive electricity is distributed in the way most amenable to mathematical calculation, *i.e.*, when it occurs as a sphere of uniform density, throughout which the corpuscles are distributed. The positive electricity attracts the corpuscles to the centre of the sphere, while their mutual repulsion drives them away from it; when in equilibrium they will be distributed in such a way that the attraction of the positive electrification is balanced by the repulsion of the other corpuscles.'

Thomson went on to consider how these corpuscles (electrons) might be arranged within the sphere of positive charge and he was able to suggest a whole series of arrangements which, as one proceeded through the Periodic Table, broke up into sets of concentric rings.

This is the Thomson model of the atom, which is often referred to as a 'plum-pudding' model. It accounted for the way in which the

‡THOMSON, J. J. *The corpuscular theory of matter.* Constable, 1907.

§'corpuscle' = electron.

radiations from radioactive substances passed so readily through atoms in their path, for nowhere did they encounter anything either massive enough or with a big enough charge to deflect them much.

In spite of weaknesses, Thomson's model proved to be a very useful one in the early days, but by 1910 it was clear that something else was needed.

Rutherford and alpha particles

We now return to the other side of the story – radioactivity. Soon after Becquerel's discovery, it was found that there were three different kinds of radiation from radioactive substances – and these were named the alpha, beta, and gamma radiations.

It was the alpha particles that happened to be the tool which, almost by accident, gave Rutherford the clue which led him to propose the nuclear model for the atom. Before this could happen Rutherford had found out a great deal about alpha particles. Some of his reports are reprinted in *Classical scientific papers – physics.* In 1902 he discussed the causes of radioactivity. In 1903, he reported the deflection of alpha particles by electric and magnetic fields. In 1909, Rutherford and Royds collected the gas formed when alpha particles were trapped in a tube and showed that it was helium. This indicated that alpha particles were charged helium atoms. But Rutherford had thought that alpha particles and helium were connected long before his experiment in 1909. In a general review of the state of knowledge about radioactivity which Rutherford and Soddy published in 1903, they made a number of rough (but quite good) estimates of the charge, mass, energy, and speed of alpha particles. Without such estimates, the behaviour of alpha particles would have been much harder to understand and the alpha particle scattering experiment which gave the clue to the nucleus would have been unintelligible.

Rutherford proposed the nuclear model of the atom to account for the surprising results obtained by Geiger and Marsden in which they bombarded metal foils with alpha particles (1909). If most of the mass of the atom is concentrated in a very small positively-charged nucleus surrounded at a distance by a distribution of negative electrons, then most of the alpha particles would be expected to pass through the foil target unaffected. Those few that came close enough to the nucleus would be deflected, the angle of deflection or scattering depending on how close the alpha particle passed. He assumed that the force between nucleus and alpha particle obeyed an inverse-square law and calculated how the fraction of incident particles that are scattered through an angle ϕ should depend on ϕ. Geiger and Marsden's paper describing their tests of Rutherford's nuclear model was published in 1913. (*Philosophical Magazine.* Volume **27** (6), 1913, p. 604. (Reprinted in WRIGHT, S. (ed.) *Classical scientific papers – physics.*)

F

Questions

a In 1904, J. J. Thomson, the discoverer of the electron, suggested a model for the atom which is frequently called the 'plum-pudding model'. Describe this model and indicate how it incorporated the facts known at the time.

b How was radioactivity discovered and how were the radiations sorted out into the three different types?

c What is the evidence for the identification of
i an alpha particle with a helium ion, and
ii a beta particle with an electron?

d (Additional question) How does a Geiger–Müller (GM) tube work and what are its limitations?

LABORATORY NOTES

Handling radioactive substances

Radioactive substances should be handled with the same care and treated with the same respect as concentrated acids. You may find yourself using naturally-occurring radioactive substances, such as compounds of uranium or thorium, or specially prepared 'sealed sources' (americium-241, strontium-90, or cobalt-60). Such sources must always be handled with tongs or a special source holder and never with the fingers. They should be pointed away from the body and, indeed, held well away from it. When not in use, even temporarily, they should be returned to their lead-lined storage boxes. Under no circumstances should you probe inside such sources or allow them to come into contact with any substance that might attack or dissolve the source or its container.

When handling the salts of uranium or thorium, you must ensure that they cannot be taken into the body, nor be dispersed around the laboratory. They should be handled above a suitable spill tray, lined with absorbent paper; you should wash your hands both before and after the experiment and, if necessary, cover any cuts or scratches, and wear protective clothing. Keep any object which may have been contaminated away from the mouth; and keep your papers and books out of the way.

Used properly, these radioactive sources, which are approved for use in schools by the Department of Education and Science, will cause no harm.

F

The use of Geiger–Müller tubes

The thin end-window tube is very easily damaged, take great care not to allow any solid object to come into contact with the end-window. This is normally covered with a plastic mesh cap. Some of these tubes contain ferromagnetic material and, if you use one in conjunction with a magnet, take great care not to allow the tube to be attracted so that it moves towards the magnet.

Geiger–Müller (GM) tubes work over a limited range of voltages. You should ask what voltage to use. Too high a voltage can damage the tube; at too low a voltage it will work ineffectively or not at all.

EXPERIMENT

F1 The deflection of beta radiation in a magnetic field

scaler
GM tube holder
thin window GM tube
beta particle source
source holder
mild steel yoke
2 Magnadur magnets
2 retort stand bases, rods, bosses, and clamps
2 lead blocks
plotting compass

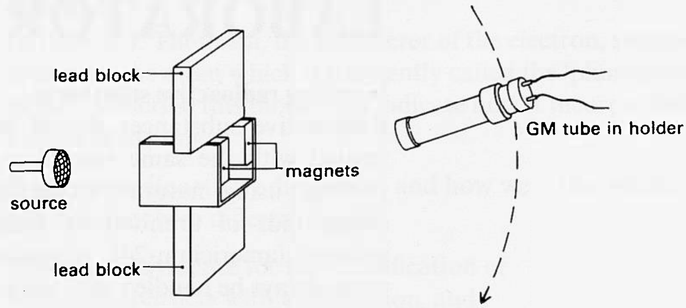

Figure F12
Arrangement of apparatus for the magnetic deflection of beta particles (seen from above).

Adjust the scaler to record beta particles arriving at the GM tube. Then arrange the apparatus so that you can examine the effect of allowing beta particles to pass through a magnetic field. Hence determine the sign of the charge on the beta particles.

EXPERIMENT

F2 Measuring the ionization current to find the number of ion pairs produced by an alpha particle and an estimation of the energy of the alpha particles emitted

picoammeter, 10^{-9} or 10^{-11} A
or
electrometer with $10^9\,\Omega$ or $10^{11}\,\Omega$ input resistance, and output meter

alpha particle source
ionization chamber
e.h.t. power supply
source holder
leads

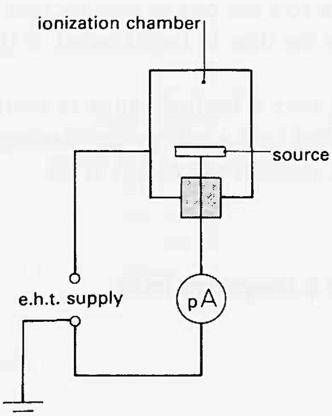

Figure F13
Measuring ionization current with a picoammeter.

DO NOT connect the e.h.t. supply directly across the meter! The range required (10^{-9} A or 10^{-11} A) depends on the source provided.

The ions resulting from the passage of alpha particles through the air in the ionization chamber move in the electric field between the source and the walls of the ionization chamber. This current can be measured and it is then possible to estimate the number of ions present.

The ionization current is very small – perhaps 10^{-9} A or 10^{-11} A. It can be found from the p.d. across a very large resistance through which it flows. An electrometer will probably be used for this measurement. Why would a standard moving-coil meter not be suitable?

Apply a small p.d. between the source and the chamber and carefully increase this until all the ions which are being produced are being collected. (How will you know when this is happening?)

Measure (or calculate) the ionization current.

Assuming the charge on a single ion to be the same as that on an electron, use your value of the ionization current to calculate the number of ion pairs produced per second.

Each ionizing event creates a pair of charged particles – say an electron and an ion with a single positive charge – which move in

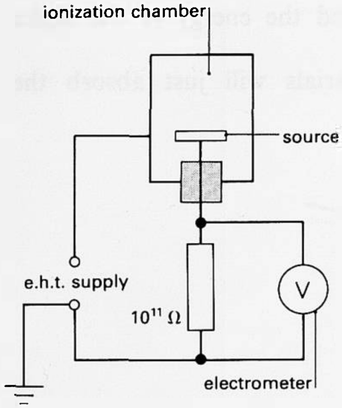

Figure F14
Measuring ionization current by measuring p.d. across a resistance.

opposite directions in the electric field. Why is the charge transferred by such a pair equivalent to *one* single charge moving right across the ionization chamber?

From the known activity of the source, find the number of disintegrations per second and hence find the number of ion pairs produced per alpha particle.

To find the energy of the alpha particle you will need to know the average energy required to produce each ion pair. The ionization energy of nitrogen (air) is 14 eV. The collision between an alpha particle and a molecule is a complex process and the average energy lost by the particle cannot be less than 14 eV per ionization. Experiments suggest an average of about 30 eV. The energy of the alpha particle is the product of the energy needed to produce one ion pair and the number of pairs produced. Calculate the energy of an alpha particle produced by the source *($1\,\text{eV} = 1.6 \times 10^{-19}$ J)*.

EXPERIMENT
F3 The penetrating power of alpha, beta, and gamma rays

alpha, beta, and gamma sources
source holder
GM tube and holder
scaler
stopwatch or clock
2 retort stand bases, rods, bosses, and clamps
metre rule
set of absorbers
Vernier callipers or micrometer screw gauge

This experiment offers a number of possibilities:

a Range of alpha particles in air.

b Range of beta particles in aluminium.

c 'Half-thickness' of lead for gamma radiation.

d The relation between the thickness of the absorber and the radiation transmitted for gamma and beta radiation.

e The relation between intensity and distance for gamma radiation.

In all cases, a count will be detected even in the absence of a source. You must make a correction for this *background count*.

F3a Range of alpha particles in air

Alpha particles are readily absorbed and it is necessary to use a very thin end-window GM tube or an ionization chamber. In the former case you will have to make an allowance for the effect of the end-window (the window of the MX 168/01 tube is equivalent to about 17 mm of air, and that of the MX 168 tube to about 30 mm).

Determine the range in air of the alpha particles from the source provided. Figure F15 shows the range in air of alpha particles with

F

different energies. Use this graph to find the energy of the alpha particles emitted by the source used.

What thicknesses of different materials will just absorb the particles?

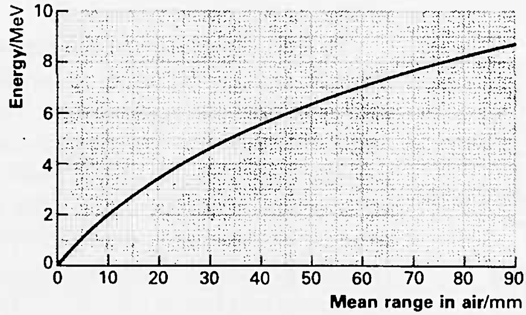

Figure F15
The mean range in air of alpha particles of different energies.

F3b Range of beta particles in aluminium

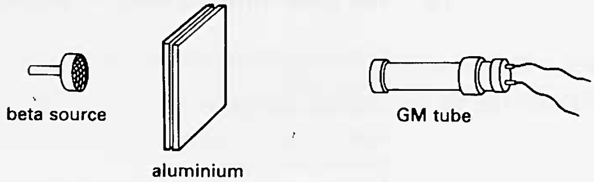

Figure F16
Range of beta particles in aluminium.

It does not follow that beta particles have more energy than alpha particles even though they may travel further in air.

Find the maximum energy of the beta particles from the source provided. Keep the distance from the source to the detector fixed (between 1 and 2 cm), and interpose varying thicknesses of aluminium to determine the range of the radiation in aluminium (figure F16). Then use figure F17 to find the maximum energy of the beta particles from the source. Remember that the beta particles have travelled through air and the end-window of the GM tube as well as through the aluminium.

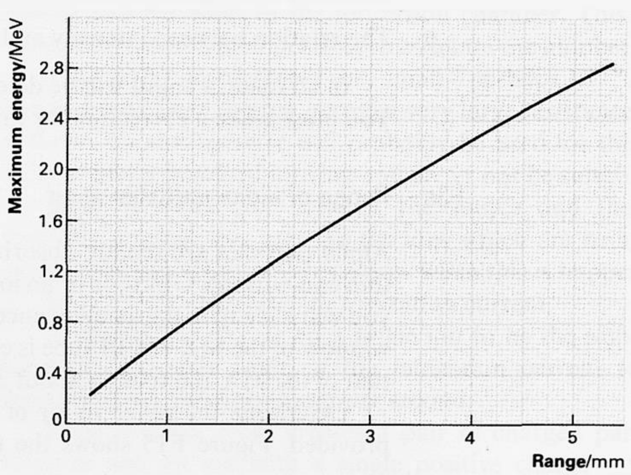

Figure F17
The range–energy curve for beta particles in aluminium.
From KATZ, *L. and* PENFOLD, *A. S. 'Range–energy relations for electrons and the determination of beta-ray end-point energies by absorption'.* Rev. Mod. Phys. **24**, *28, 1952.*

F3c 'Half-thickness' of lead for gamma radiation

Find the energy of the gamma radiation from the source provided. Keep the distance from the source to the detector fixed at about 2 cm, and by interposing varying thicknesses of lead find the thickness which will reduce the count-rate to one-half of its original value ('half-thickness'). Use figure F18 to determine the energy of the gamma radiation used.

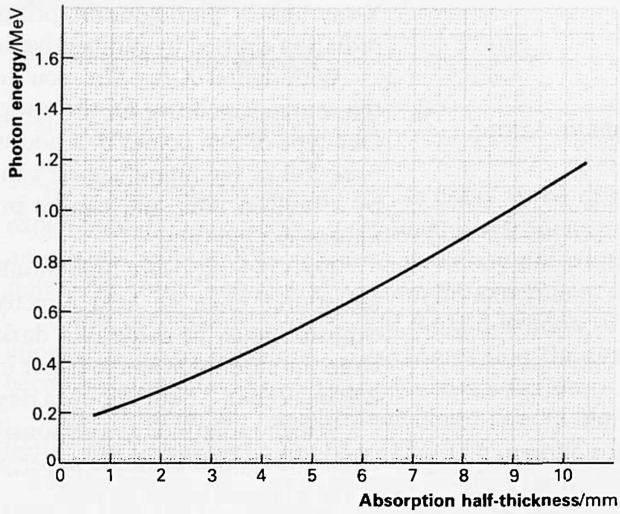

Figure F18
The half thickness–energy curve for gamma radiation in lead.
From DAVISSON, C. M. *and* EVANS, R. D. *'Gamma-ray absorption coefficients'.* Rev. Mod. Phys. **24**, *79, 1952.*

F3d The relation between the thickness of the absorber and the radiation transmitted for gamma and beta radiation

Plot a graph of count-rate against the thickness of absorber introduced between the detector and the source, keeping the distance from detector to source fixed. Try both gamma and beta radiation. Suggest a relationship between count-rate and absorber thickness in both cases.

F3e The relation between intensity and distance for gamma radiation

Try to determine how the gamma ray count-rate varies with distance from the source. Plot graphs to determine whether the relationship is inverse, or inverse-square, or exponential, or...

It is difficult to measure the distance without involving a zero error so think carefully about what might be the best graph to plot. For example, when testing for an inverse-square law, is it better to plot count-rate against $1/(\text{distance})^2$, or distance against $1/\sqrt{(\text{count-rate})}$? If you choose correctly you will be able to estimate the zero error from your graph.

EXPERIMENT

F4 Photographic detection of radiation

radium source

either
dental X-ray film
or
bromide paper with developer and fixer

X-ray film or photographic printing paper is sensitive to the ionizing radiation emitted by radioactive sources.

With dental X-ray film you do not need to use a darkroom. Place the source face down on the unopened film pack for 20 to 30 minutes. One type of film comes in a pod which contains its own developer and fixer: follow the manufacturer's instructions. Another type is protected by a built-in filter and can be processed in subdued light (but avoid fluorescent lighting).

The same exposure time should be sufficient for bromide paper; but the source must be held directly over the sensitive surface, so this exposure must be made in a darkroom. To work in the light you can wrap the sheet of bromide paper in black paper, but a much longer time will be needed – hours or even days.

Which radiations are responsible for affecting
i the unwrapped bromide paper, and
ii the wrapped paper or film?

DEMONSTRATION

F5 Qualitative test of the gravitational hill model

alpha particle scattering analogue
drawing board

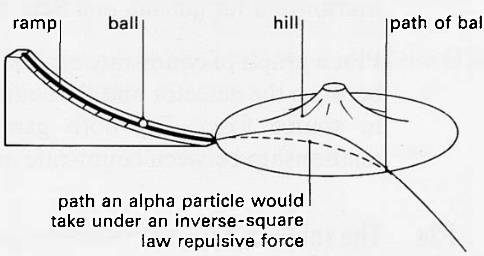

Figure F19
Gravitational analogue of alpha particle scattering.

Where must the ball be aimed for it to be turned through the largest angle?
How does the 'scattering angle' for the ball vary with the aiming error ('impact parameter')?
How does the scattering angle for a given aiming error depend on the ball's speed?
List similarities and differences between this situation and an experiment in which alpha particles are scattered by a thin metal foil.

EXPERIMENT

F6 Decay and recovery of protactinium

scaler
GM tube holder
thin end-window GM tube
small polythene bottle, 50 cm^3, containing prepared solution of uranyl(VI) nitrate
retort stand base, rod, boss, and clamp
stopwatch or clock
tray lined with absorbent paper

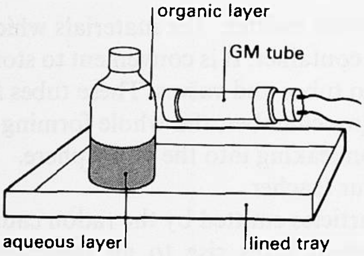

Figure F20
The decay of protactinium.

The decay chain involved is:

$$\text{uranium-238} \xrightarrow{\alpha} \text{thorium-234} \xrightarrow{\beta} \text{protactinium-234} \xrightarrow{\beta}$$

^{234}Pa decays quickly enough for its decay to be observed in a short experiment. It has, however, to be separated from its parent thorium by an organic solvent. The solutions necessary are contained in a small polythene bottle. You should perform the experiment in a spill tray.

Under no circumstances should the end-window of the GM tube be allowed to touch the bottle. This ensures that the GM tube does not become contaminated with the radioactive solutions.

First measure the background count-rate in the absence of the bottle. Shake the bottle for 10 to 15 s. The organic solvent containing the protactinium will float to the top. Position the bottle beside the GM tube so that the end-window is opposite the organic layer. Once the layers have separated, the beta particles can be counted. Take a ten-second count each half minute.

Having made allowance for the background count-rate, plot a graph of the count-rate against time and determine the half-life of protactinium from the graph. Do this by starting at several points and measure how long it takes for the count-rate to fall to one-half of the initial value chosen. The half-life is the mean of these times.

Also plot a graph of ln (activity) against time. If this should prove to be a straight line, you have added confirmation that the decay is obeying an exponential law.

Estimates of half-life based on low values of the count-rate may be unreliable. Why is this?

The thorium from which the protactinium has been removed remains in the aqueous layer in the lower half of the bottle. Protactinium will immediately start to reappear there as the thorium decays and the growth curve of protactinium can be plotted. To observe this recovery, repeat the experiment with the GM tube window opposite the lower (aqueous) layer. Plot a graph of count-rate against time.

F

EXPERIMENT

F7 The decay of radon and the determination of its half-life

picoammeter, 10^{-11} A (*e.g.*, electrometer with $10^{11}\,\Omega$ input resistance and output meter)
ionization chamber
radon generator
2 cell holders with four cells
leads

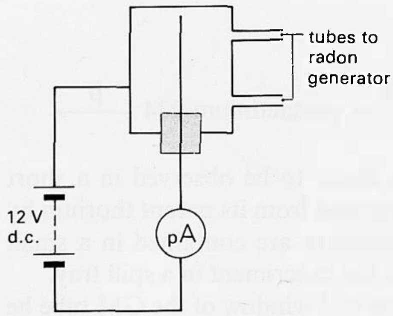

Figure F21
The decay of radon.

The decay chain involved in this experiment is

thorium-232 → radium-228 → actinium-228 → thorium-228 → radium-224 → radon-220 →

Radon-220 is a gas and an alpha particle emitter. The materials which give rise to it must be kept in a closed container. It is convenient to store them in a plastic bottle fitted with two tubes and valves. These tubes fit on to tubules attached to an ionization chamber, the whole forming a closed circuit with no risk of the radon leaking into the atmosphere.

Report any accidental spills to your teacher.

As in experiment F2, the alpha particles emitted by the radon cause some ionization in the chamber, which gives rise to an ionization current when a p.d. is applied across the chamber. The ionization current is very small: it can be calculated from the p.d. set up when it flows through a very high resistance, say $10^{11}\,\Omega$. An electrometer will probably be used for this measurement. Why could a standard moving-coil meter not be used?

Apply a p.d. of 12 V d.c. across the ionization chamber, and select an appropriate current range (that is, an appropriate input resistance for the electrometer).

Pass ^{220}Rn into the chamber by *gently* squeezing the plastic bottle until a full-scale reading is obtained. Record the ionization current every ten seconds and plot a graph of meter readings against time. Use your graph to determine the half-life of radon.

EXPERIMENT

F8 A radioactive decay analogue

100 dice
graph paper
tray

This experiment is about events which happen by chance. If each member of a collection of objects has a fixed chance of 'decay' in a given time interval, then the number remaining will decrease with time. The question concerns the pattern of this decay; does it have the same form as the decay of the radioactive substances?

In radioactive decay we do not know when any particular atom is going to decay. But, given enough atoms, we can say what proportion of them will decay in a given time interval. This fraction will vary from substance to substance.

If you throw an ordinary die, you have one chance in six of throwing, say, a five. If you threw one die many times (or many dice just once) what fraction of the throws (or of the dice) would result in, say, a five?

Given enough throws (or enough dice) you can predict that fraction with reasonable certainty. So throwing dice is analogous to radioactive decay in the sense that both processes lead to a fraction which is fixed for specific dice or for specific atoms. In neither case can we predict which of the atoms will decay or which of the dice will show a five next.

If we remove this fraction (that is, the 'fives') as if they have decayed, and then throw the remaining dice, and so on, the process imitates radioactive decay to some extent.

Throw the dice and count those that fall with a five upwards. Remove these dice. Throw the remaining dice, count and remove the fives, and so on. Continue for some ten throws. Plot graphs of the number 'decaying' and of the number remaining against the number of the throw. The shapes of the two graphs will be familiar but may look rather 'bumpier' than other examples you have met. Why is this? How could you improve the shape of the curves?

The graphs are typical of situations in which the rate at which something decays depends on the quantity or the number present. What tests can you apply to check whether or not the graph for the dice is exponential in form? Try one at least of these. Estimate the 'half-life' of a die.

F

OPTIONAL DEMONSTRATION
F9 Ionization by electron collision

Experiments in which electrons are accelerated in a gas at low pressure gave information about the structure of atoms. For each gas there is a characteristic electron energy which will ionize the gas atoms. The onset of ionization is shown by a rapid increase in current, as the number of charge carriers increases.

In an *elastic* collision between an electron ($m_e \approx 0.0005\,\mathrm{u}$) and a stationary neon atom ($m \approx 20\,\mathrm{u}$), will the energy transferred to the atom be a large or a small fraction of the electron's kinetic energy?

Must energy be added to a neutral atom to ionize it, or is energy released when the ion plus free electron is formed? (*Hint:* is the potential energy of the atom greater or smaller than that of the two charged particles, ion and electron, when they are separated?)

Careful experimentation may show evidence of inelastic collisions between electrons and gas atoms when the electron's kinetic energy is less than that required to ionize the atom. The atom may be excited – one of its electrons is promoted to a higher energy level. More precise information about ionization and excitation energies comes from careful measurement of the wavelengths of light in an element's absorption or emission spectrum.

QUESTIONS

The nature and properties of alpha, beta, and gamma radiation

1(E)a Describe how you would distinguish between alpha, beta, and gamma radiation by experiment.

b What happens to the kinetic energy of alpha and beta particles when passing through air?

c When asked to outline how he would distinguish between the signs of the charges carried by alpha and beta particles, a student drew a rough sketch (figure F22).

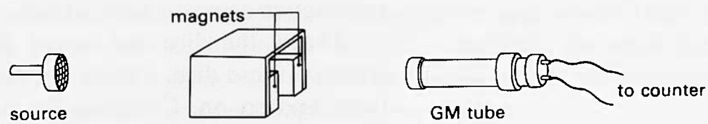

Figure F22

The Geiger tube can be moved in any direction. The student was able to detect the deflection of beta particles, but not of alpha particles.
i Why could he not detect the deflection of alpha particles?
ii What changes would you make to the apparatus to improve the chances of detecting the deflection of alpha particles?

(From Physics (Nuffield) O-level examination, Paper 2, November 1979)

2(I) Figure F23 is a sketch made by someone trying to design an arrangement that will detect the deflection of beta particles by an electric field. The idea is to have parallel plates, length l, spacing d, down the centre of which the beta particles travel. After travelling distance l they have been deflected sideways by a distance s. From there on, they travel more or less in a straight line and have been deflected by a distance y after going a further distance L to a detector.

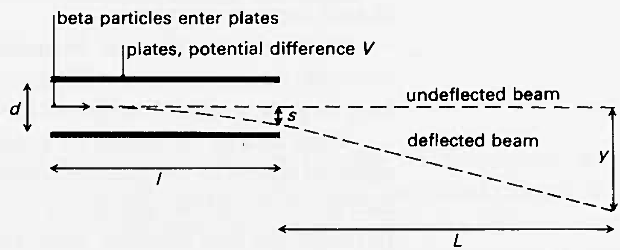

Figure F23

a What is the electric field, E, between the plates if the potential difference between them is 2 kV and d is 2 mm?

b What is the sideways force, F, on an electron, charge $e = 1.6 \times 10^{-19}$ C, between the plates?

c A fast-moving beta particle's mass, m, is about 10^{-29} kg (larger than its rest mass because of the relativistic increase of mass at high speed). What will be its acceleration, a, in the field?

d A beta particle travels at a speed approaching that of light, $3 \times 10^8 \text{ m s}^{-1}$. If l is 0.1 m, for what time does this sideways acceleration last?

e Find the sideways distance, s, through which the beta particles are deflected by the field in time t under the acceleration calculated in **c**.

f If the experiment is done in air, L cannot be much more than 0.1 m or so. Estimate the deflection y.

3(P) In an experiment to determine the energy of an alpha particle using an ionization chamber and a very sensitive ammeter, the following results were obtained:

Ionization current	$= 3 \times 10^{-11}$ A
Activity of the source	$= 3.7 \times 10^3$ Bq

Given that the average energy required to produce an ion-pair in air is 30 eV, find:

a the number of ions produced per second;

b the number of ions produced by each alpha particle (on average);

c the energy of each alpha particle.

4(P) The graph (figure F24) shows how many ion-pairs are formed per millimetre by an alpha particle at each point on its track. Note that the graph places the *end* of the track at the origin.

a Suggest why the ionization per millimetre rises as the particle approaches the end of the track.

b Estimate the total number of ion-pairs formed by an alpha particle that produces a 50 mm track.

c If 30 eV is the average energy required to produce one ion-pair, what is the total average energy of a single alpha particle?

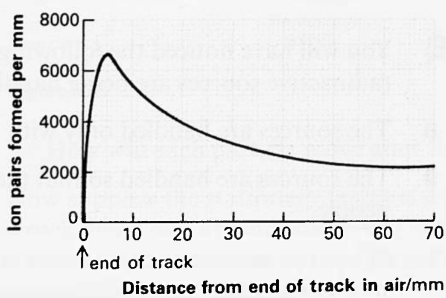

Figure F24
Number of ion pairs per mm of track versus distance for an alpha particle.
After GENTNER, W., MAIER-LEIBNITZ, H., *and* BOTHE, W. An atlas of typical expansion chamber photographs. *Pergamon, 1954.*

F

5(L) This question follows an argument set out by Rutherford and Soddy in 1903. The estimates are similar to those of Rutherford and Soddy but have been expressed in SI units.

a If each alpha particle from radium has energy 10^{-12} J, and there are 10^{21} radium atoms in a gram of radium, what total energy is emitted if each atom emits one particle?

b Each atom emits at least five particles. How does that alter the answer to **a**?

c Is the answer to **b** a minimum estimate of the energy emitted or a maximum estimate?

d The total electric current from ions produced by alpha particles emitted from one gram of pure radium in air is about 10^{-3} A. Each ion has a charge of about 10^{-19} C. How many ions are being produced each second?

e The least energy needed to produce an ion in air is about 10^{-18} J. How much energy is emitted in one second by one gram of radium?

f In one second, what fraction is emitted of the total energy which will be given out ultimately by the radium?

g Make a rough estimate of the lifetime of a sample of radium.

6(P) A lump of material containing radium is always slightly warmer than its surroundings. Burning 1 mole of radium to radium oxide releases 525×10^3 J.

a How much energy is that, *per atom of radium?*

b When radium atoms disintegrate, each emits, on average, five alpha particles each of energy about 5 MeV. How much is this in joules?

c How much energy is emitted when one mole of atoms disintegrates?

d How does the energy released in the radioactive decay of radium compare with the energy released on oxidation?

e Why are materials containing radium not very hot?

(Half-life of ^{226}Ra = 1600 years.)

7(E) You will have noticed the following precautions being observed when radioactive sources are being handled.

a The sources are handled only with tongs or special source holders.

b The sources are handled so that the radiation is not directed towards people.

c When not in use, sources are put in a box with lead shielding and kept in a locked cupboard with a warning notice on it.

Suppose a local councillor has heard that you are doing experiments with radioactive sources and is concerned lest you be exposed to radiation. Write a letter explaining the reasons for the precautions taken. (The councillor may not realize the existence of 'background' radiation.)

Alpha particle scattering and evidence for the nucleus

8(I) How big is an atom of gold?

(The density of gold is $19.3 \times 10^3\ \mathrm{kg\,m^{-3}}$ and its mass number is 197. The Avogadro constant is $6 \times 10^{23}\ \mathrm{mol^{-1}}$.)

9(L) You are now asked to estimate the size of an atomic nucleus. Geiger and Marsden's experiments suggest that about 1 alpha particle in 8000 is turned back through a large angle by a gold foil 6×10^{-7} m thick. If such a foil has n layers of atoms then $1/n$ of this number would be turned back by one layer; that is, $1/8000n$ of the particles would be turned back.

All the alpha particles would be turned back if the cross-sectional area of the layer were entirely filled with nuclei. Half would be turned back if the layer were 'half-filled' with nuclei. Since $1/8000n$ is turned back, then only $1/8000n$ of the layer is filled with nuclei. See figure F25.

If we can assume that there are only small gaps between atoms, then the nuclei represent $1/8000n$ of the total area 'seen' by approaching alpha particles. Hence

$$\frac{\text{target area of nucleus}}{\text{cross-sectional area of atoms}} \approx \frac{1}{8000n}$$

The number of layers, n, in a given thickness of the foil will depend on the diameter of the atoms:

$$n \approx \frac{\text{thickness of foil}}{\text{diameter of atom}}$$

Use your result from question **8** to make a rough estimate of the diameter of a gold nucleus.

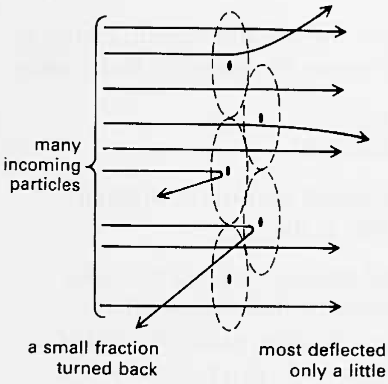

Figure F25
The 'target' area of the nuclei decides the fraction of alpha particles turned through a large angle.

10(I)a Suppose an alpha particle travelling at $10^6\ \mathrm{m\,s^{-1}}$ hits another stationary helium nucleus 'head on' (figure F26).

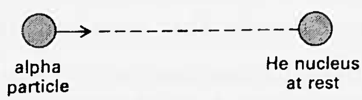

Figure F26

How will each particle move after the collision?

b Now suppose the stationary target is a proton (hydrogen nucleus) also struck 'head on'. At what speed will each particle be travelling after the collision? You may assume that the collision is elastic. What else *must* be true about the collision?

c In collisions of alpha particles with gold nuclei, it is usually assumed that the gold nucleus is not moving after the collision. Is this a fair assumption to make?

11(L) How large is a nucleus? If it is true that inverse-square law electrical forces act right down to the closest distance an alpha particle comes to a nucleus, then a nucleus must be smaller than that distance.

Consider a 5 MeV alpha particle approaching a gold nucleus 'head on'.

a What is its kinetic energy in joules? What happens to this kinetic energy as the particle approaches the nucleus?

b At some distance r from the nucleus, the speed of the alpha particle will be zero. Assume that a gold atom has a charge of $+79e$ and an alpha particle has a charge of $+2e$. How much potential energy is stored in the system at this moment?

c Assuming that the massive gold nucleus will acquire very little kinetic energy from this store, equate the two values for energy to find a value for r.

d What is the electric potential at this distance?

e What relationship does the electric potential bear to the original kinetic energy of the alpha particle? Why is this?

f What 'turn around' force does the gold nucleus exert on the alpha particle at this distance of nearest approach? Before doing the calculation, guess – is it equal to the weight of an atom, of a cell of your body, of an eyelash, of a pencil, of a book, of a person?

12(P) Describe the forces between the object A (at rest) and the bombarding particle P, that would explain the paths shown in figure F27.

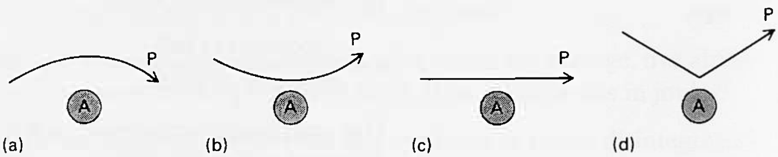

Figure F27

13(R) In figure F28, A is an alpha particle being deflected by a charged nucleus N, r metres from A. Another alpha particle B happens to be $2r$ metres from N. They come from distant points A′, B′.

a Why do the continuations (broken lines) of the paths of the alpha particles from A and B curve upwards on the sketch?

b If the forces on A and B from N are electrical, how does the force on A compare in size with that on B? (Ignore forces between A and B.)

c In what direction(s) are the forces on A and B?

d Copy figure F28 and add arrowed lines (vectors) at A and B representing the direction of the *velocity* of each particle.

e In **d** you will not have been able to know for certain how big the velocities are. But is the particle's velocity at A larger, smaller, or about the same size as its velocity at A′?

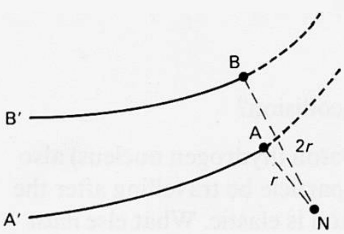

Figure F28

f On your diagram, sketch the path of an alpha particle deflected through about 90° by a nucleus. Mark in the position of the nucleus. Mark the point where the speed of the alpha particle would be least. Explain.

g If the speeds are equal at A′ and B′ in figure F28, how do the speeds at A and B compare?

h Suppose you could lay down a flat surface (a very thin sheet of card, for example) so that the incoming alpha particles travelled along the flat surface, and the scattering nucleus also lay on the surface. Would the alpha particles leave the surface as they were deflected by the nucleus? Explain.

Random events

14(E)a The arrival of particles at a Geiger tube from a radioactive substance is said to be 'random'. Do you think that the following are sensible descriptions of what 'random' means in this context?
i The particles arrive at a basically steady rate, which is disturbed by small variations.
ii In each second there is a fixed chance that a particle will arrive, but just when they do so is quite unpredictable.
iii The time between the arrival of the particles may be 0.1 s, 1 s, 5 s, or any other value: each is equally likely.

b In an earlier Unit, the word 'random' was used to describe the motion of the molecules in a gas. Give a sensible description of what 'random' means in that context.

Radioactive decay

15(P)a The half-life of ^{90}Sr is 27 years. The half-life of ^{24}Na is 15 hours. Which of two samples, one containing 10^{20} atoms of ^{90}Sr, and the other containing 10^{20} atoms of ^{24}Na, would have the highest activity?

b Sketch graphs showing how the number of atoms of ^{90}Sr and ^{24}Na changes with time.

c (*Harder*) After roughly how long would the activities become equal?

16(E) In exponential decay the number of atoms remaining after a given interval of time is always the same fraction of the number present at the beginning of the time interval. Therefore, someone might say, however long you wait there will always be some left. A sample of radioactive atoms will last for an infinite time. Is this reasonable?

17(L) No ordinary place on the Earth's surface is free from 'background radiation'. In making a measurement of this background a student set

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up a GM tube and scaler and recorded the scaler reading every 30 s. The results are shown in table F4.

Time/s	0	30	60	90	120	150	180	210	240
Count	0	13	31	48	60	75	88	102	119

Table F4

Determine the background count per minute. What would you expect the total count to be after a further 30 seconds?

18(P) A radioactive source is set up in front of the GM tube of question **17** in the same laboratory. Scaler readings of the activity of the source are taken for 1 minute at time intervals of 1 hour. These are shown in table F5.

Time/hours	0	1	2	3	4	5	6	7	8	9	10
Activity/counts minute^{-1}	828	510	320	202	135	95	70	51	41	38	37

Time/hours	11	12	13	14	15
Activity/counts minute^{-1}	31	33	34	29	35

Table F5

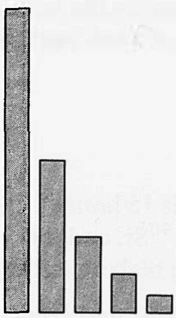
a What action should be taken about the background count (see question **17**)?

b Plot a graph to show how the activity of the radioactive substance changes with time and derive from it three different values of the half-life. Calculate the mean of these.

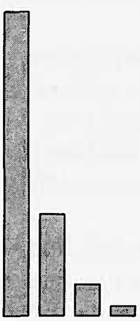
c Worried by the obvious fluctuations in the final hours of the count, one student suggests that it might be wise to continue counting for at least as long again. Is this a good idea? Explain.

d There is a slight increase in the count rate towards the end of the experiment. Is this significant? Explain.

e Suggest any steps you might take to improve the experiment.



(a)



(b)

Figure F29

Testing for exponential growth and decay

19(P) A student says she knows what an exponential curve is and offers to illustrate it by taking some drinking straws, cutting one in half, then one of the halves into half again, then one of the quarters into half again, and so on. She then places the straws side-by-side as shown in figure F29(a).

A second student says he has thought of a similar method but instead of using halves each time, he has cut thirds – figure F29(b).

a Are both models exponential?

b How could you check to see if a graph of one variable (y) against another (x) is 'exponential'?

20(P) Refer to your table of the activity of the source (less background) in question **18**.

a Test whether the data fit a mathematical model of exponential decay by
i checking the constant-ratio property and saying what that ratio is, and
ii plotting a graph of ln(activity) against time.

b Given that $N = N_0 e^{-\lambda t}$ where N_0 and N are the number of radioactive atoms at time zero and time t respectively, and λ is the decay constant, use your graph to find the decay constant and hence the half-life.

21(P) Exponential changes are not confined to physics. They occur so widely that they have great importance in other studies, for example economics, industry, population statistics.

Table F6 gives data for the capacity of the electrical generating plant installed in Great Britain for each year from 1951 to 1980.

Year	51	52	53	54	55	56	57	58	59	60
Capacity/GW	16.2	17.7	19.2	20.6	22.5	24.6	26.6	28.0	30.0	31.9
Year	61	62	63	64	65	66	67	68	69	70
Capacity/GW	33.9	37.2	39.3	40.0	43.9	46.2	50.0	53.6	55.1	60.5
Year	71	72	73	74	75	76	77	78	79	80
Capacity/GW	65.6	68.8	71.1	72.1	71.8	69.9	69.7	69.9	71.6	70.9

Table F6
Data from Central Statistical Office Annual Abstract of Statistics *(1954–82). By permission of the controller, HMSO.*

a By plotting a suitable graph, find out whether any part or parts of this development were exponential.

b There was an economic recession in the early 1960s. How was this reflected in the growth of installed generating plant about that time? What happened to this industry during the 1970s?

c What was the 'doubling time' during:
i the 1950s
ii the 1960s?

22(P) Table F7 shows the number of cars in private ownership in Britain from 1950 to 1980. Does the number rise exponentially?

Year	**Cars in millions**	**Year**	**Cars in millions**
1950	2.26	1966	9.51
1952	2.51	1968	10.8
1954	3.10	1970	11.5
1956	3.89	1972	12.7
1958	4.55	1974	13.6
1960	5.53	1976	14.0
1962	6.56	1978	14.1
1964	8.25	1980	15.1

Table F7
The number of cars in private ownership in Britain.
From Central Statistical Office Annual Abstracts of Statistics *(1954–82). By permission of the Controller, HMSO.*

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23(P) Many estimates have been made of the growth of the World's population. Show that the growth suggested by the data in table F8 indicates a rise even 'faster' than an exponential.

Year	Estimate
1650	0.51×10^9
1750	0.72×10^9
1800	0.91×10^9
1850	1.13×10^9
1900	1.59×10^9
1920	1.81×10^9
1930	2.01×10^9
1940	2.25×10^9
1950	2.51×10^9
1960	3.03×10^9
1970	3.68×10^9
1980	4.42×10^9

Table F8
Estimates of the World's population.
The figures for the years from 1650 to 1900 are means of several estimates; subsequent figures are based on UNO data.

About exponential changes

The argument that follows is made up of a number of learning questions and a commentary. Our concern is with the shape and meaning of the mathematical equations that describe exponential changes.

Changes are called exponential if the rate of change of something is in proportion to the quantity of that something already present.

But, before investigating the mathematical form of exponential change two facts have to be recalled:

Logarithms The logarithms of two numbers are added to find the logarithm of their product. So

$\lg AB = \lg A + \lg B.$

(Note that $\lg x = \log_{10} x$.)

Powers Powers are added when numbers expressed as powers are multiplied.

$10^2 \times 10^5 = 10^7;\qquad 2^3 \times 2^{0.2} = 2^{3.2}$

In 10^2, 2^3, a^t, the quantities 2, 3, and t are called 'exponents' as well as 'powers' or 'indices'. This is where the word 'exponential' originates.

24(L) Consider an equation of the form $\Delta N/\Delta t = kN$ where t is time and k is a constant. Suppose N stands for the number of families which have video recorders in their homes.

a What does ΔN stand for?

b What does $\Delta N/\Delta t$ mean?

It might be plausible to think that a family would not consider buying a video recorder unless they knew of other families like themselves who had done so, and who recommended it.

c Taking this view, suggest why a mathematical model like $\Delta N/\Delta t = kN$ might be appropriate for the way in which the number of families having video recorders changes.

d If 'keeping up with the Joneses' were the only factor, the simple mathematical model might be appropriate. What complications might make it less so?

e Think now about the spread of a fashion in, say, clothes. If such a mathematical model were appropriate for the period when a fashion was spreading, how might the existence of television in a society alter the value of k?

f Fashions do not continue to spread indefinitely. Sketch graphs of the change with time of the number of people following a fashion which starts with only a few people
i if pretty well everyone adopts the fashion in the end, and
ii if the fashion becomes unfashionable and more and more people reject it.

Mark on your graphs the points beyond which the model $\Delta N/\Delta t = kN$ is no longer appropriate, supposing that it serves quite well for the initial spreading period.

To summarize, $\Delta N/\Delta t = kN$ is a simple recipe for how N will change in the next time interval Δt. Although it may be an exact recipe for some problems, for others it might be just nearly right or perhaps only roughly right. To be useful, a simple model such as this must apply, in part at least, to a wide variety of situations.

A solution to a differential equation

The equation $s = \frac{1}{2}at^2$ tells us what the displacement s will be at some time t. The equation $\Delta N/\Delta t = kN$ does not do anything so definite as that. It says that if N has *this* value *now*, a short time, Δt, later it will differ from this value by ΔN, and it gives a recipe for working out ΔN. Such equations are called 'differential' equations.

But one still needs to know what N will be at some time t. If N_0 is the number of yeast cells in a culture of yeast now, the number to be expected after several hours may be needed. Sometimes an equation to do this can be found, sometimes not. If it can, it is called a solution of the differential equation. It is 'a' solution, not 'the' solution because it will differ according to the starting point. The recipe for change doesn't

E

say how many yeast cells there are to start with; but the number of cells at time t does depend on how many were there in the first place.

The next question shows you why $N = a^t$, where a is some number, makes a reasonable choice as a solution of $\Delta N/\Delta t = kN$.

25(L)a If $N = 2^t$ how would you use the x^y key of a calculator (or the log key, or logarithm tables) to find N when $t = 7$? (We know you could do it without, of course.)

b Use the same method to find the value of $N = 2^{0.5}$.

c Plot a graph of $N = 2^t$ from $t = 0$ to $t = 1.0$. Why does the graph start at $N = 1.0$? Use axes which allow N to go from 1.0 to at least 3.

d Sketch the graph of $N = 1^t$. Also, *either* sketch roughly the graph of $N = 3^t$ on the same axes, getting the exact values at $t = 0$ and $t = 1.0$; *or* plot a graph of $N = 3^t$.

e Consider the graph of $N = 2^t$. When t goes up in equal steps from 0 to 0.2, then to 0.4, to 0.6, and so on, the same thing happens each time to the value of N. Say carefully what that same thing is, perhaps by inspecting your graph.

f Find $N = 2^{0.2}$ from your graph. Knowing that $2^3 = 8.0$, what is $2^{3.2}$? (Remember that $2^{(a+b)}$ means $2^a \times 2^b$.)

g Look at this table:

a^t
$a^t \times a^t = a^{2t}$
$a^t \times a^t \times a^t = a^{3t}$
etc.

Use it to explain why, when t increases in equal steps, a^t is multiplied by a constant factor.

Exponential changes have, as we have seen, the 'equal ratio' property in common. The changing quantity alters by a constant *factor* when time (or another variable) increases in equal steps. The equation $N = a^t$ shares this property.

Next we must ask what value of a in $N = a^t$ goes with a particular value of k in $\Delta N/\Delta t = kN$. In particular, what value of a fits the case when $k = 1$?

26(L)a Using the same scale as you used for your graph in question **25**, plot out step by step the growth of N, starting with $N = 1.0$ when $t = 0$, if $\Delta N/\Delta t = +1.0N$. Here N might be the number of bacteria in a colony, for example. To start you off, when $N = 1.0$, $\Delta N = 1.0 \times \Delta t$. Taking time intervals Δt as 0.1 second, ΔN begins by being 0.1. See figure F30.

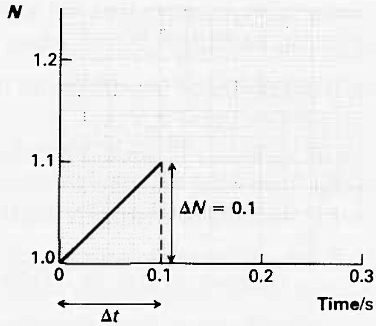

Figure F30

After 0.1 s, N has become 1.1. In the *next* 0.1 s, ΔN is a little larger than 0.1. Why?

After ten steps you reach $t=1.0$, when N will have risen to about 2.6 or 2.7. Stop at this point.

b Repeat this process of curve-drawing for the growth represented by $\Delta N/\Delta t=0.7N$, starting with $N=1.0$ at $t=0$. Why is this growth slower than the earlier one?

Now compare these curves with those from question **25**.

c What value will N reach at $t=1$ if $N=2^t$?

d What will N be at $t=1$ if $N=a^t$?

e If the curves drawn in **a** and **b** above can be represented by equations such as $N=a^t$, what values of a would be suitable for each curve (approximately)?

A value for a in $N=a^t$

There is now reason to think that the curve you have drawn for $\Delta N/\Delta t=+1.0N$ in your answer to question **26a** has the form $N=a^t$, because this equation has the proper 'constant ratio' property. You can check that your curve has this property too, by looking at it. $\Delta N/\Delta t=1.0N$ is a recipe for growth with 'constant ratio'. But what value has a?

Whatever the value of a is, it is bigger than 2, because your curve lies above the graph of $N=2^t$. However, a is less than 3 because your curve lies below the graph of $N=3^t$. Your curve, drawn from the recipe $\Delta N/\Delta t=1.0N$, could be represented by the equation $N=a^t$, if a were between 2 and 3. Your graph may have given a value around 2.6; more accurate calculations give 2.718... This number has a mathematical symbol, e. So the solution to the recipe $\Delta N/\Delta t=1.0N$ if N starts at 1.0 when $t=0$, is $N=e^t$.

A less special case

The recipe $\Delta N/\Delta t=1.0N$ is a very special case; we must move towards the much more general case $\Delta N/\Delta t=kN$ with k not equal to unity. The next question uses the graph drawn in question **26b** for which k was 0.7, to find out how to cope with the more general problem.

27 Question **26e** suggests that the solution to $\Delta N/\Delta t = 1.0N$ (the first curve) can be written $N = e^t$, where e is the number 2.718...

a Use a calculator or logarithms to find $e^{0.7}$, or read the value at $t = 0.7$ from the first curve $\Delta N/\Delta t = N$.

The value of $e^{0.7}$ should be nearly 2.0, and in question **26b** you should have seen that the solution to $\Delta N/\Delta t = 0.7N$ is approximately $N = 2.0^t$. Instead of writing $N = 2.0^t$ we can write $N = (e^{0.7})^t$ since $e^{0.7} \approx 2.0$.

So a solution to $\Delta N/\Delta t = 0.7N$ is $N = e^{0.7t}$.

b Now make a speculation. If the equation $\Delta N/\Delta t = kN$ has a solution $N = a^t$, is the value of a likely to be e^k or ke or e/k?

A solution in general

The lesson from question **27** is that if we wish to solve $\Delta N/\Delta t = kN$, the solution is like that for $\Delta N/\Delta t = 1.0N$ which is $N = e^t$, except that the constant k will appear multiplying t.

When $N = 1.0$ at $t = 0$, the equation that tells us the value of N at time t, if $\Delta N/\Delta t = kN$, is $N = e^{kt}$.

We have been thinking about growth because that is easier to think about than decay. But this result can cope with decay, too. The recipe for change, when N *decreases*, as in radioactivity decay, is just $\Delta N/\Delta t = -kN$. The negative sign arranges that the changes ΔN are all *subtracted* from the previous value of N. This is like the growth version but with a negative value of k. Following the same rule (N is e to the power t times k) gives

$$N = e^{-kt}.$$

We have seen that e^{kt} is multiplied by a constant factor when t rises in equal steps. So $1/e^{kt}$ (which is the same as e^{-kt}) is *diminished* by a constant factor when t rises in equal steps. And that is just what an exponential decay curve that obeys the recipe $\Delta N/\Delta t = -kN$ does.

Finally, what about not requiring N to be unity when $t = 0$, which is, after all, not very realistic?

Suppose there are initially $N_0 = 10\,000$ radioactive nuclei in a sample at some moment $t = 0$, and that one-fifth of them, 2000, decay in the first second, leaving 8000. In the next second we should expect one-fifth of $8000 = 1600$ to decay, leaving 6400; and so on. Clearly, the actual numbers left, N (8000, 6400, 5120,...) and the fraction of nuclei remaining, N/N_0 (0.8, 0.64, 0.512,...) diminish by the same proportion of themselves in each second: N/N_0 obeys the same recipe for change as N. But the fraction remaining, N/N_0, must be equal to unity at $t = 0$. So the solution to $\Delta(N/N_0)/\Delta t = -k(N/N_0)$ is simply $N/N_0 = e^{-kt}$. In the case of radioactive changes the letter k is usually replaced by λ, the decay constant.

In general, if $\Delta N/\Delta t = kN$, and $N = N_0$ when $t = 0$, then

$$N = N_0 e^{kt}$$

This is the exponential equation. It gives the number of active nuclei left in a sample of radioactive material after any time, or the number of bacteria in a growing colony. It has uses, often as an approximation, in studying the growth and decay of many things in the sciences, economics, resources, populations, and so on.

28(R) A scientist wished to find the age of a sample of rock in which he knew that radioactive potassium ($^{40}_{19}K$) decays to give the stable isotope argon ($^{40}_{18}A$). He started by making the following measurements:

Decay rate of the potassium in the sample	$= 0.16$ disintegrations s^{-1}
Mass of potassium in the sample	$= 0.6 \times 10^{-6}$ g
Mass of argon in the sample	$= 4.2 \times 10^{-6}$ g

a Show how he could then calculate that for the potassium:
i the decay constant (λ) was $1.8 \times 10^{-17} s^{-1}$
ii the half-life was 1.3×10^9 y.

b Calculate the age of the rock, assuming that originally there would have been no argon in the sample. Show the steps in your calculation.

c Identify and explain a difficulty involved in measuring the decay rate $0.16 s^{-1}$ given earlier in the question.

(Short answer paper, 1981)

Unified atomic mass unit u

29(L) The unified atomic mass unit (u) is defined as one-twelfth of the mass of an atom of carbon-12. The molar mass of ^{12}C atoms is $0.012 \text{ kg mol}^{-1}$. The Avogadro constant, L, is $6.022 \times 10^{23} \text{ mol}^{-1}$.

a What is the mass of one atom of ^{12}C?

b Calculate the value of 1 u, in kilograms.

The mass of a proton is 1.673×10^{-27} kg and the mass of a neutron is 1.675×10^{-27} kg.

c Express these two masses in u.

d The mass of an electron is 9.11×10^{-31} kg. What is its mass in u?

Radioactive isotopes and their applications

30(R) Heavy hydrogen, ^2H, often called deuterium, was first separated from a mixture which was mostly ^1H by evaporating several litres (dm^3) of liquid hydrogen very slowly, until only about 1 cm^3 remained.

In this residue ^2H was detected. Why should this process tend to concentrate ^2H in the residue? Would the method be as effective with liquid neon, containing mostly ^{20}Ne and a little ^{22}Ne, to separate the two?

31(P) The molar mass of lead, as tabulated in data books, is $207.21 \text{ g mol}^{-1}$. Table F9 gives the molar masses of lead taken from several different minerals.

Mineral	Origin	Molar mass/g mol^{-1}
Cleveite	Norway	206.08
Broggerite	Norway	206.01
Pitchblende	W. Africa	206.05
Kolm	Sweden	206.01
Thorite	Sri Lanka	207.8
Thorite	Norway	207.9

Table F9

What do these values suggest to you?

32(R) The radioactive isotope of carbon ^{14}C, is used for the dating of some archaeological finds. ^{14}C emits beta particles, and has a half-life of 5570 years. This isotope is present in small amounts in the carbon present as carbon dioxide in the atmosphere.

The ^{14}C is made continually by the interaction of cosmic rays with the nitrogen in the upper atmosphere. There is evidence that the rate of production of ^{14}C and its concentration in the atmosphere have remained fairly steady for at least the last 10 000 years. All living organisms have a small content of radioactive carbon when they are alive.

When an animal or plant dies, it stops exchanging carbon dioxide with the atmosphere. The ^{14}C in wood or bones then decays, reducing in amount and activity year by year.

a When an organism is alive, it contains about one atom of ^{14}C to every 10^{10} atoms of ordinary ^{12}C. In one second, a fraction 4×10^{-12} of the atoms of ^{14}C present may be expected to decay.

Estimate the activity in decays per second of 1 g of carbon from a living organism.

b What activity would you expect from 1 g of carbon taken from the bones of a bison eaten by Man some 11 000 years ago, when the great ice sheets covered much of Europe and North America?

c About how far back in time do you think the ^{14}C clock might be of use to archaeologists?

33(R) This question is about explaining ideas in physics.

Write an explanation of radioactivity suitable for a friend studying A-level physics who missed the teaching of this particular subject, giving a careful explanation of the topics: decay, radiation, isotopes.

Show also how your explanations could help your friend to understand **one** everyday application of the subject.

(Long answer paper, 1980)

Ionization

34(I) The Avogadro constant L is chosen (defined) to be the number of ^{12}C atoms in a sample of that isotope with a mass of exactly 12 g.

a What, approximately, is the mass of an alpha particle?

b What speed has an alpha particle which has a kinetic energy of one million eV?

35(I)a Calculate the energy (in joules) gained by an electron when it is accelerated through a potential difference of 10 V.

b Write down the kinetic energy (in J) of:
i a 5 eV electron
ii a 20 eV electron.

c Calculate the speeds of:
i a 5 eV electron
ii a 10 eV electron
iii a 20 eV electron.
(Use $m_e = 9.1 \times 10^{-31}$ kg.)

36(P) Figure F31 shows the energy needed to remove one electron from an atom, leaving it ionized, for atoms up to proton number about 90. The energies are given in electronvolts, and, for comparison, in kJ mol^{-1}.

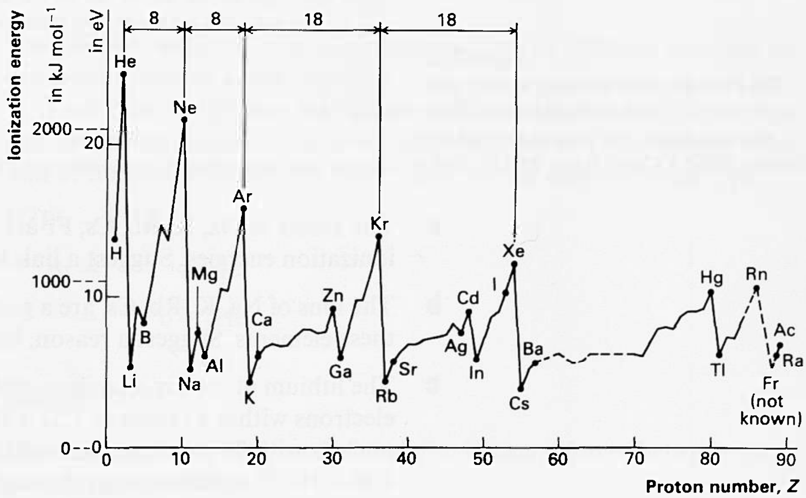

Figure F31
Ionization energies of the elements.

a The elements helium, neon, argon, krypton, and xenon come at peaks in figure F31. What chemical properties do these elements share?

b The elements lithium, sodium, potassium, rubidium, caesium, come at the lowest points of figure F31. What kind of chemical properties do these elements share?

c Look where sodium, Na, comes in figure F31. Now find the element neon, Ne, which has only one electron less than sodium. Is a sodium atom's 'last' electron tightly held to the atom?

Find a similar pair of elements about which similar remarks might be made.

d Do calcium, strontium, and barium occupy similar positions on the graph? Are they chemically similar?

e Radium belongs to the same chemical family as calcium, strontium, and barium. The ionization energy of francium, the element one before radium, is not known. Predict the value, from the overall shape of the graph.

37(L) Figure F32 shows the radii of atoms of most elements, and the radii of the ions of a few.

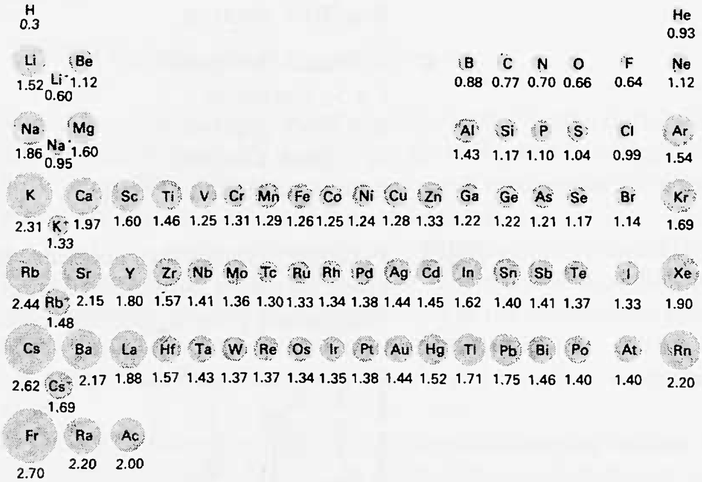

Figure F32
The Periodic Table showing atomic, and some ionic, radii/10^{-10} m.
After CAMPBELL, J. A. *'Atomic size and the Periodic Table'*. J. Chem. Educ., **23**, *525*, *1946*.

a The atoms of Na, K, Rb, Cs, Fr are large. They all have small ionization energies. Suggest a link between these two facts.

b The ions of Na, K, Rb, Cs, are a good deal smaller than the atoms of these elements. Suggest a reason, based on your answer to **a**.

c The lithium atom has a nucleus with three protons, and contains three electrons within a radius of 1.52×10^{-10} m. The gold atom has a nucleus with 79 protons, and contains 79 electrons within a radius of 1.44×10^{-10} m. What general, rough rule can you give for the order of magnitude of the radius of atoms of an element with Z protons and Z electrons? Is it a surprising rule?

38(L) The binding energy of the electron in the hydrogen atom is -13.6 eV.

a Express this in joules.

The electron and proton both carry a charge of magnitude e.

b Write down an expression for the electrical potential energy when an electron is at a distance r from a proton.

c Use your answers to **a** and **b** to estimate the proton–electron distance.

This question implies that the total energy of the hydrogen atom is the potential energy of the proton–electron pair. In fact the electron has some kinetic energy.

d How does this affect your answers?

Radioactive transformations

39(P) The isotope ^{235}U decays into another element, emitting an alpha particle. What is that element? (Use a Periodic Table.)

This element decays, and the next, and so on, until a stable element is reached. The complete list of particles emitted in this chain is:

$$^{235}_{92}\mathrm{U} \rightarrow [\alpha, \beta, \alpha, \beta, \alpha, \alpha, \alpha, \alpha, \beta, \alpha, \beta] \rightarrow \mathrm{X}$$

What is the stable element X? (You could write down each element in the series, but there is a quicker way.)

40(R) $^{226}_{88}$Ra is a naturally-occurring isotope of radium and has a half-life of 1622 years. The radioactive decay is represented by the equation

$$^{226}_{88}\mathrm{Ra} \rightarrow \mathrm{X} + \alpha + \gamma,$$

where X is a daughter nuclide, α is an alpha particle, and γ is a gamma-ray photon. The proton number of X is

A 86 **B** 88 **C** 89 **D** 224 **E** 227 (Coded answer paper, 1980)

41(P) Use tables of nucleon (mass) numbers and proton (atomic) numbers to complete reactions **a**, **b**, **c** below.

Each item should have the nucleon number at the top left (superscript) and the proton number at the lower left (subscript). For example, for uranium of nucleon number 238 write $^{238}_{92}$U; for an electron write $_{-1}^{0}$e.

a $^{212}\mathrm{Pb} \rightarrow {}^{212}\mathrm{Bi} +$

b $^{212}\mathrm{Bi} \rightarrow \qquad + {}_{-1}^{0}\mathrm{e}$

c $\qquad \rightarrow {}^{208}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$

Nuclear forces

42(L) The magnitudes of the electrical and gravitational forces between two particles are given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2} \quad \text{and} \quad F_g = G\frac{m_1m_2}{r^2} \quad \text{respectively.}$$

a For each force say whether it is attractive or repulsive.

b Obtain an expression for the ratio F_e/F_g.

c This ratio is independent of r. What is the *physical* basis for this?

d Evaluate the expression obtained in **b** for two protons.

e Can the gravitational force be responsible for holding protons together in the nucleus?

The force which does hold protons (and neutrons) together in the nucleus is called the *strong nuclear force.* It is clearly stronger (within the nucleus) than the electrical force. But it is vanishingly small at distances greater than about 3×10^{-15} m.

f Suggest some evidence for the short-range nature of the nuclear force. (Think about what would happen if the nuclear force were still greater than the electrical force at a distance of about 10^{-10} m, *i.e.*, the distance between atoms.)

Nuclear binding energy

43(L) Use the curve for average binding energy per nucleon against nucleon number A (figure F11) to answer the following questions.

a Which nucleus is more stable, ^{4}He or ^{12}C?

b Arrange the following nuclei in order of increasing stability:

^{235}U, ^{12}C, ^{6}Li, ^{4}He, ^{2}H, ^{208}Pb, ^{56}Fe

c (*Harder*) In the Sun, helium is formed by the fusion of hydrogen. Suggest why the fusion of helium to form heavier elements does not occur. More extreme conditions of temperature and pressure (for example, those found in red giant stars or supernovae explosions) are needed for the formation of heavier elements.

d From figure F11 suggest elements which are likely to be formed in large quantities in the evolution of the heavier elements.

Mass and energy

44(P) The specific heat capacity of water is $4.2\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$.

a How much energy is needed to raise the temperature of 5 kg of water from 20°C to 70°C?

b Use $\Delta E = c^2 \Delta m$ to estimate the increase in mass of the water when the temperature is raised by this amount.

c Express the mass increase as a fraction of the mass of the water.

45(P)a Estimate the mass of a rubber band.

b Estimate its force constant and obtain a value for the energy stored when the band is stretched to twice its original length.

c Use $\Delta E = c^2 \Delta m$ to estimate the increase in mass of the rubber band when it is stretched.

d By what percentage has the mass increased?

46(L) Atomic masses can be determined very precisely by mass spectrography. For example, the mass of a neutral atom of $^{4}_{2}He$ is 4.002 6 u.

a Compare the mass of this neutral atom with the total mass of its component parts ($m_p = 1.007\,3$ u, $m_n = 1.008\,7$ u, $m_e = 0.000\,55$ u). Which is greater, and how large is the difference?

b Use $\Delta E = c^2 \Delta m$ to estimate the total nuclear binding energy for $^{4}_{2}He$:
i in joules (J) *ii* in mega electronvolts (MeV).

c Calculate the average binding energy per nucleon. Your result should be comparable to the value given in figure F11.

Unit G
ENERGY SOURCES

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INTRODUCTION

This Unit is about energy sources, which means that it is also about energy itself. As you work through it you will meet familiar ideas, new ideas, and ideas which will be developed further in later Units. You will also need to try to make sense of data and graphs which are less straightforward than many others in physics. Many of the variables involved are not those normally found in physics, but this does not mean that you cannot think about them as a physicist, and find how far this gets you. Physics cannot give all the answers, but it does have its own contribution to make to the debate on energy supply. The Unit cannot cover all aspects of the topic, but will only lay the groundwork. Nor is it intended to lead you to adopt any one particular view.

Energy supply has been a topic of concern throughout the World since the early 1970s at least. This concern has been voiced in terms such as 'crisis', and expresses the fear that the World does not have enough energy for its present or future needs. But some might argue that it has too much for its own good.

It is worth looking first at some features of energy supply and demand.

SECTION G1
THE BACKGROUND TO ENERGY SUPPLY AND DEMAND

SOME BASIC IDEAS

Energy is conserved

QUESTION 1 The conservation of energy is a basic principle of physics and features explicitly or implicitly at many places in science. If energy is conserved, how could we ever be 'short of energy'? The problem is that much of the energy from fuels heats the surroundings, spreads out, and becomes less and less useful.

QUESTION 2 **Sankey diagrams**
Many of the processes which are examined in this Unit are concerned with energy transfers and are often complicated. Sankey diagrams, like the one in figure G1, are frequently used to represent these complex situations.

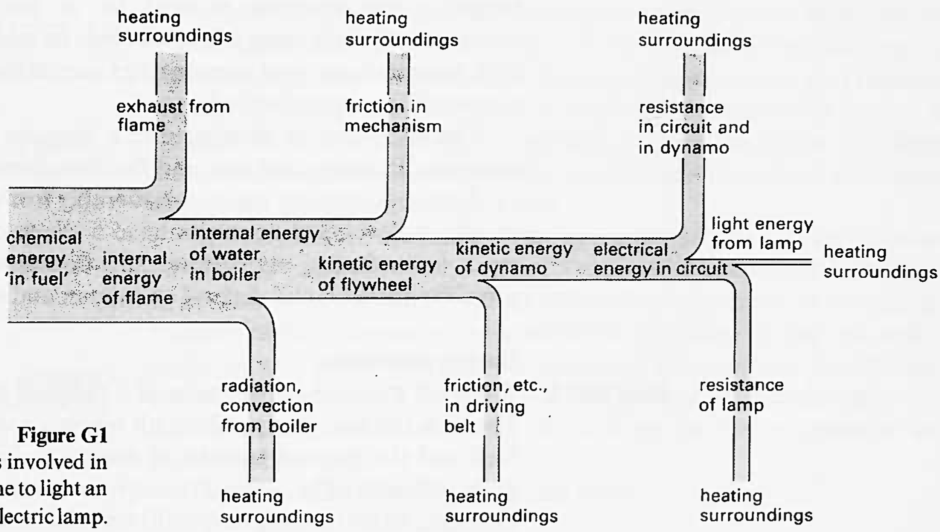

Figure G1
The energy transfer processes involved in using a steam engine to light an electric lamp.

The transfer of energy within the system is generally represented in the diagram as being like a stream flowing from left to right, with inputs and outputs usually shown vertically until they join or leave the main stream. The width of the stream represents the quantity of energy involved at that stage. (The idea of flow is used a great deal, both in this and other branches of physics. It is worth considering carefully what, if anything, flows in each case; or whether flow is just a useful way of visualizing a complex process.)

Figure G1 gives a general indication of the energies involved. Further data would be needed to make the width of the 'streams' directly proportional to the energy which each 'stream' represents.

Efficiency

Efficiency indicates what proportion of the energy input to a system which does a particular job is actually used to do that job.

$$\text{Thus efficiency is } \frac{\text{useful energy output}}{\text{total energy input}}.$$

It is expressed either as a fraction or as a percentage. What counts as 'useful' in the definition depends on the purpose of the system: whether it is being used for heating or for working, for instance. This point is taken up again in 'Energy converters', below.

'Energy slaves'

QUESTION 3

QUESTIONS 4 and 5

The human race has been very ingenious in developing machines which can do work for it, and because these machines can be called upon to help when required they can be regarded as 'energy slaves'. Although this idea usefully illustrates how our society depends on machines, 'energy slave' is not a precisely defined term and is used in various ways.

Fuels

If machines are to do work they need to be provided with energy in a form which they can use. Materials such as coal or oil, which, when burned enable machines to work for us, are called fuels. The fuel is consumed, though energy is conserved. In addition, burning occurs at high temperatures, and some energy inevitably goes to heating up the surroundings unproductively.

The term 'fuel' is often used in a broader sense than this. See, for example, 'Primary, end-use, and functional energy' on page 410.

Fuel consumption varies considerably from country to country. The people of developed countries have a range of energy slaves to draw upon for industry, agriculture, and domestic needs while the most primitive societies still depend on human and animal labour.

Energy converters

QUESTION 6

Table G1 shows the efficiencies of a range of energy transfer processes. Those in the first group, which all involve a chemical change (burning fuel) and the thermal transfer of energy to heat something up can be quite efficient. The second group involves mechanical change (for example, water running downhill) and electromagnetic processes. These too can be efficient. The members of the third group are all heat engines: all involve burning fuel and thermal transfer of energy in devices designed to do mechanical work. These processes are necessarily less efficient than the others. Efficiency depends on what you want: burning petrol heats things up well, but it is less efficient at making cars move.

Device	Efficiency/%	Processes
Space or water heaters		
Large commercial boiler	90	chemical change and thermal transfer
Domestic boiler		
gas	75	
oil	70	
coal	60	
Motors and generators		
Hydro-electric turbine	90	mechanical change and electromagnetic processes
Large electric motor	90	
Large electric generator	90	
Small electric motor	70	
Heat engines		
Steam turbine	45	chemical change, thermal transfer, mechanical work
Diesel engine	40	
Car engine	25	
Steam locomotive	10	

Table G1
Efficiencies of energy converters.
These values are indicative of efficiencies which can be obtained. The actual performance of the devices can often be worse than shown because of poor maintenance, for example.

Unit K, 'Energy and entropy'

These differences are more than chance ones and reflect a fundamental law. The treatment given here is by no means comprehensive.

A body at a high temperature (called a source) seems to be able to transfer internal energy readily to a body at a lower temperature (a sink), through the process we call thermal transfer of energy. (Thermal transfer of energy goes from hot to cold.) In practice, some energy is transferred elsewhere owing, for example, to poor insulation, but there is no fundamental reason why all the energy transferred from the source should not go to the sink (figure G2).

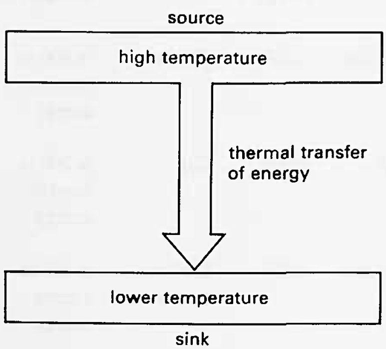

Figure G2
Thermal transfer of energy.

In figure G3 we are making the system do work. In order to cause any energy transfer from the source it must be in contact with the sink. If, in some way, the work done by the system were equal to the energy transferred from the source, none would be transferred to the sink and it might as well not be there, hence removing the condition for thermal transfer to occur at all. In fact some energy *must* be transferred to the sink and this quantity of energy depends on the temperatures of the source and the sink.

The maximum efficiency turns out to be

$$E = \frac{T_1 - T_2}{T_1} \qquad \text{(temperature measured in K)}$$

The turbines in a modern power station take in steam at, say, 530 °C (about 800 K), and reject it at 30 °C (300 K). Thus the maximum efficiency is 63 per cent, but practical efficiencies are likely to be considerably less than this.

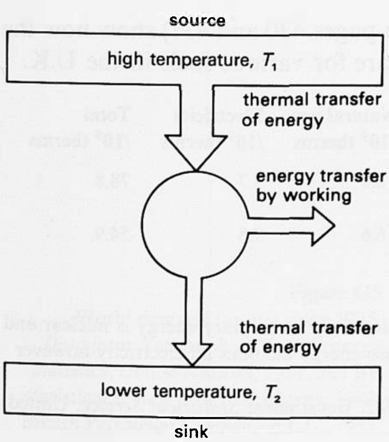

Figure G3

Primary, end-use, and functional energy

Fuels are not necessarily used in the form in which they are originally obtained. In this original form they are called primary fuels; the total energy that they have the potential to provide is called their primary energy.

QUESTION 7

Crude oil, a primary fuel, is usually converted to various secondary fuels, such as petrol, or fuel oil. In some cases the secondary fuel is in the form in which it is ultimately used to provide energy; in these cases the energy that the secondary fuel could provide (the secondary energy) may also be referred to as the end-use energy. However, the secondary fuel may undergo one or more further conversions, for example into electrical energy, which is the end-use energy in this case.

The amount of energy actually available to the final user is sometimes called functional energy and depends on the efficiency of the final energy conversion process. Functional energy is usually less than the end-use energy which is a measure of the energy as it was delivered to the final process. Many terms like these used in connection with energy are not used precisely or consistently, but table G2 may help to sort out common usage.

PRIMARY STAGE			**SECONDARY STAGE**			**END-USE STAGE**			
Primary fuel	**Primary energy/J**	**Converter**	**Secondary fuel**	**Secondary energy/J**	**Converter**	**End-use fuel**	**End-use energy/J**	**Converter**	**Functional energy/J**
Example 1									
Crude oil	1000	Refinery	Fuel oil	$\approx$950	Power station	Electricity	$\approx$300	Heater	$\approx$300 in internal energy
Example 2									
Crude oil	1000	Refinery	Petrol	$\approx$900	none	Petrol	$\approx$900	Car	$\approx$200 in kinetic energy
Example 3									
Coal	1000	Power station	Electricity	$\approx$330	none	Electricity	$\approx$330	Electric motor	$\approx$260 in kinetic energy

Table G2
Examples of energy conversion to illustrate the use of the terms primary fuel and energy; secondary fuel and energy; end-use fuel and energy; and functional energy. The values given for the energies involved are illustrative rather than exact figures.

HOME EXPERIMENT GH1
The cost of a 'cuppa'

Table G3 and figure G4 (between pages 430 and 431) show how the primary and end-use energies compare for various fuels in the U.K.

	Coal, etc. /10^9 therms	**Oil /10^9 therms**	**Natural gas /10^9 therms**	**Electricity /10^9 therms**	**Total /10^9 therms**
Primary energy	29.0	28.1	18.0	3.7	78.8
End-use energy	7.6	23.2	16.6	7.5	54.9

Table G3
U.K. energy consumption 1981. (Note that electricity as primary energy is nuclear and hydro electricity, whereas electricity as end-use energy includes all electricity however generated.)
From: United Kingdom Energy Statistics, 1982. *Government Statistical Service.* United Kingdom Energy Statistics. HMSO, 1982.

Energy units

The fuel industry uses a variety of units other than the joule for measuring energy. Some of these, like kW h (kilowatt-hour), Btu (British thermal unit), and therm are comparable to the joule, that is, they are physical units which are defined, can be measured, and have values which are constant.

QUESTION 4

Other units, such as the megatonne of coal equivalent (Mtce but often written, incorrectly, as mtce) are not defined in the same way. This unit of energy is that which would be obtained if one million tonnes of coal were completely burned. The problem is that different kinds of coal can provide different amounts of energy if burned, so that it is common to use an average or a conventional value. Not everyone agrees on the convention which should be adopted and a degree of confusion results. For this reason, the conversion factors given in table G4 (see next page) may not agree with others which may be found, and the variations are such that it does not seem sensible to quote the values in this table to more than two significant figures.

Changes in fuel supply

Figure G5 shows that World demand for fuel did not grow steadily between 1925 and 1981, but that the rate of growth increased (apart from one or two relatively short periods).

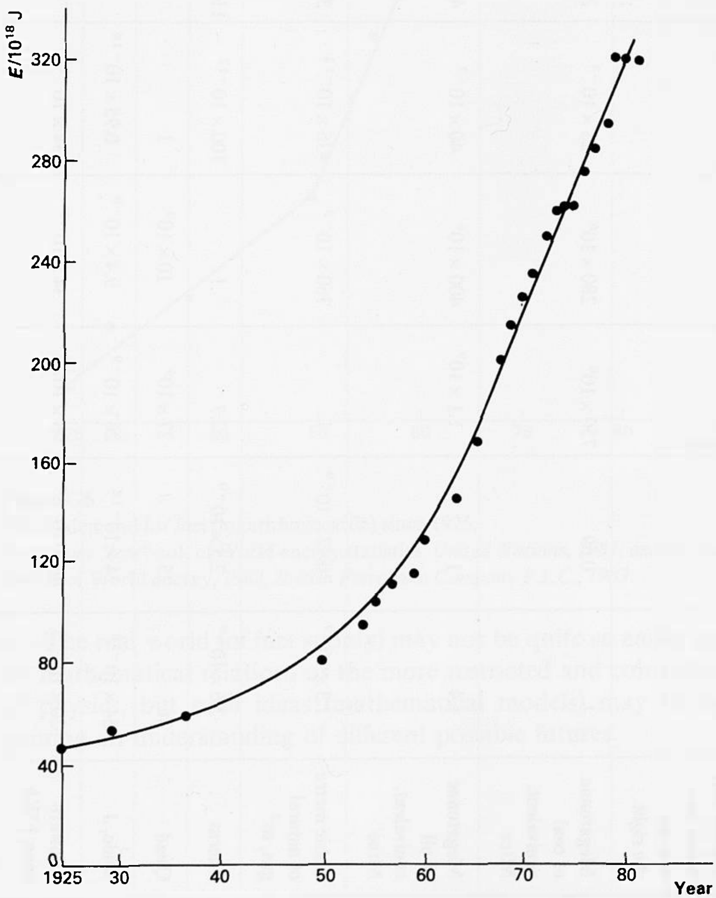

Figure G5
World demand for fuel since 1925.
Data from: Yearbook of World energy statistics, *United Nations, 1981, and* BP statistical review of World energy, 1982, *British Petroleum Company P.L.C., 1983.*

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Multiply by factor to convert unit below into unit on the right	Mtce	Mtoe	m^3	Therm	Quad	J	kW h	kcal	Btu	bbl
Megatonne of coal equivalent, Mtce	1	0.69	750×10^{6}	280×10^{6}	28×10^{-3}	29×10^{15}	8.1×10^{9}	6.9×10^{12}	28×10^{12}	4.8×10^{6}
Megatonne of oil equivalent, Mtoe	1.4	1	1.1×10^{9}	400×10^{6}	40×10^{-3}	42×10^{15}	12×10^{9}	10×10^{12}	40×10^{12}	7.0×10^{6}
Cubic metre of natural gas, m^3	1.3×10^{-9}	0.92×10^{-9}	1	360×10^{-3}	36×10^{-12}	39×10^{6}	11	9.2×10^{3}	36×10^{3}	6.4×10^{-3}
Therm	3.6×10^{-9}	2.5×10^{-9}	2.7	1	100×10^{-12}	110×10^{6}	29	25×10^{3}	100×10^{3}	18×10^{-3}
Quad	36	25	27×10^{9}	10×10^{9}	1	1.1×10^{18}	290×10^{9}	250×10^{12}	10^{15}	180×10^{6}
Joule, J	34×10^{-18}	24×10^{-18}	26×10^{-9}	9.4×10^{-9}	0.95×10^{-18}	1	0.28×10^{-6}	240×10^{-6}	950×10^{-6}	0.17×10^{-9}
Kilowatt-hour, kW h	120×10^{-12}	86×10^{-12}	93×10^{-3}	34×10^{-3}	34×10^{-12}	3.6×10^{6}	1	860	3.4×10^{3}	0.60×10^{-3}
Kilocalorie, kcal	140×10^{-15}	100×10^{-15}	110×10^{-6}	40×10^{-6}	4.0×10^{-15}	4.2×10^{3}	1.2×10^{-3}	1	4.0	0.70×10^{-6}
British thermal unit, Btu	36×10^{-15}	25×10^{-15}	27×10^{-6}	10×10^{-6}	1.0×10^{-15}	1.1×10^{3}	290×10^{-6}	0.25	1	180×10^{-9}
Barrel of oil, bbl	210×10^{-9}	140×10^{-9}	160	57	5.7×10^{-9}	6.0×10^{9}	1.7×10^{3}	1.4×10^{6}	5.7×10^{6}	1

Table G4
Conversion factors for some units commonly used in connection with energy.
For example, 10 000 m^3 of natural gas $= (10\,000) \times (39 \times 10^{6})$ J.

QUESTIONS 8 to 11 Figure G6, in which the World demand for fuel is plotted on a logarithmic scale against time, has three distinct linear sections, showing that in each section the growth in demand was exponential. A characteristic of exponential decay curves is constant half-life and the equivalent idea for exponential growth is doubling-time. Hence the three sections of figure G6 have different doubling-times.

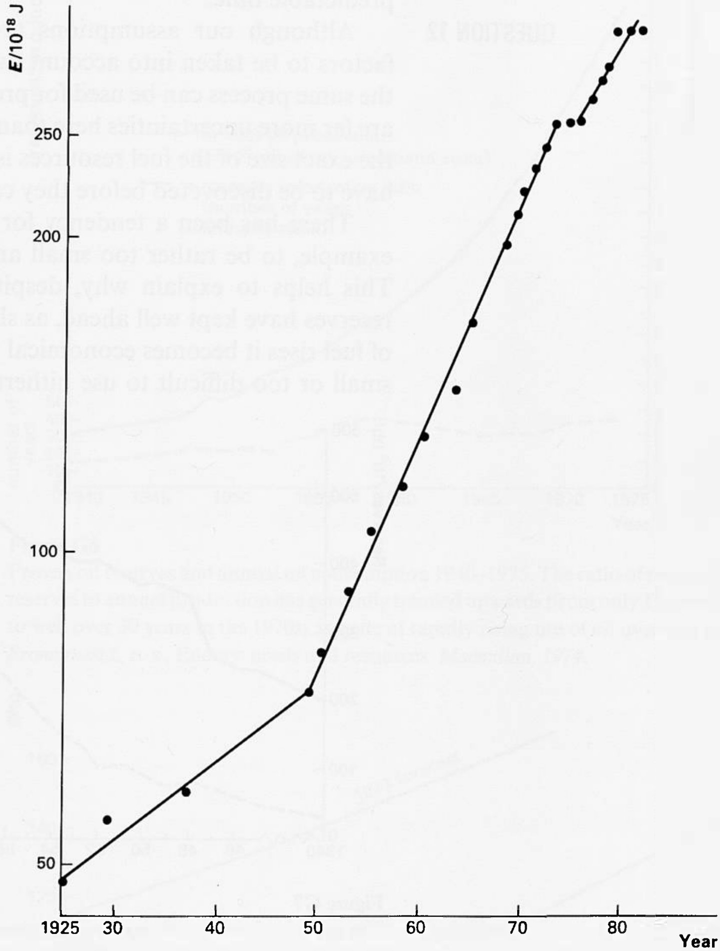

Figure G6
World demand for fuel (logarithmic scale) since 1925.
Data from: Yearbook of World energy statistics, *United Nations, 1981, and* BP statistical review of World energy, 1982, *British Petroleum Company P.L.C., 1983.*

The real world (of fuel supply) may not be quite so easily described by mathematical relations as the more restricted and controlled world of physics, but such ideas (mathematical models) may be useful in gaining an understanding of different possible futures.

Making predictions

We can predict the future of a bacterial colony in a controlled environment when we know both the stock of food available for it to live on and its rate of growth in the past. If growth in the past has been exponential, and if the conditions remain the same, we can assume that growth will continue to be exponential and the food will be used up in a predictable time.

QUESTION 12

Although our assumptions are too simple and there are other factors to be taken into account (we return to these later), in principle the same process can be used for predicting the future use of fuels. There are far more uncertainties here than in the example above. For instance, the exact size of the fuel resources is not known, since the resources first have to be discovered before they can be extracted.

There has been a tendency for initial estimates of oil reserves, for example, to be rather too small and to be increased later (figure G7). This helps to explain why, despite great increases in consumption, reserves have kept well ahead, as shown in figure G8. Also, as the price of fuel rises it becomes economical to use deposits which have been too small or too difficult to use hitherto.

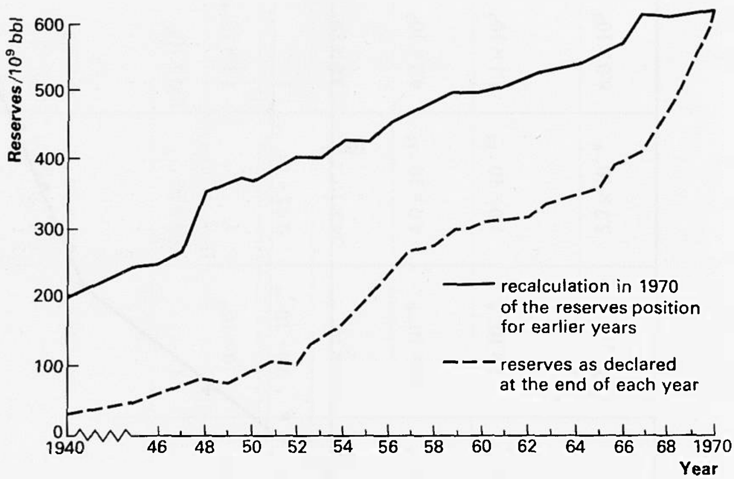

Figure G7

The growth of proven oil reserves over time. Proven reserves figures as declared at a particular time seriously understate the amount of oil which has actually been discovered

From: ODELL, P. R., Energy: needs and resources. *Macmillan, 1974.*

The pattern of demand in the future is also uncertain. The growth in demand in the past has not been uniformly exponential, so it is unlikely to be so in the future. Nevertheless, possible values for the lifetimes of resources can be calculated by making simple assumptions about the pattern of future growth. Which pattern actually comes about will depend on factors such as the growth of World population, the expectations of that population, and the way in which industry develops in both the industrialized and the less developed nations. All of these factors make forecasting difficult, and forecasts can be markedly inaccurate even in the short term, and even when made by leading experts. However, it is possible to put into perspective the estimates which are continually being made by a variety of people – expert and otherwise.

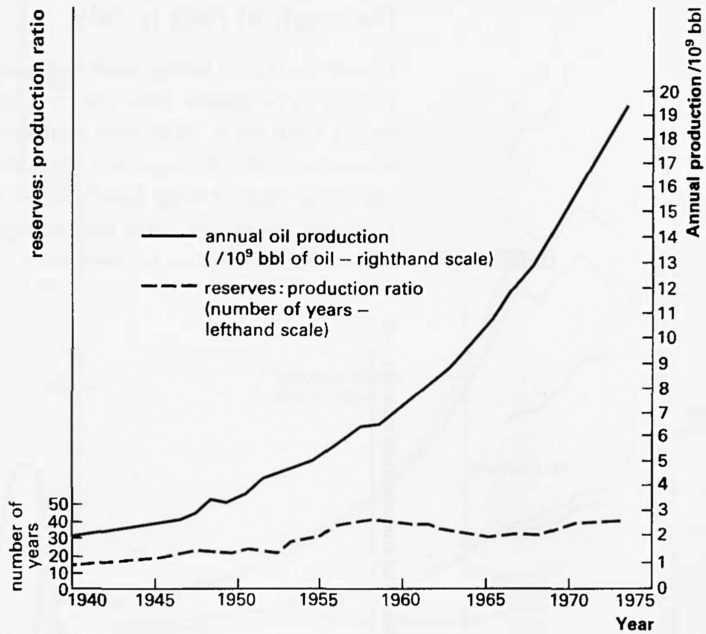

Figure G8
Proven oil reserves and annual oil consumption 1940–1975. The ratio of remaining known reserves to annual production has generally trended upwards (from only 15 years in 1940 to well over 30 years in the 1970s), in spite of rapidly rising use of oil over this period.
From: ODELL, P. R., Energy: needs and resources. *Macmillan, 1974.*

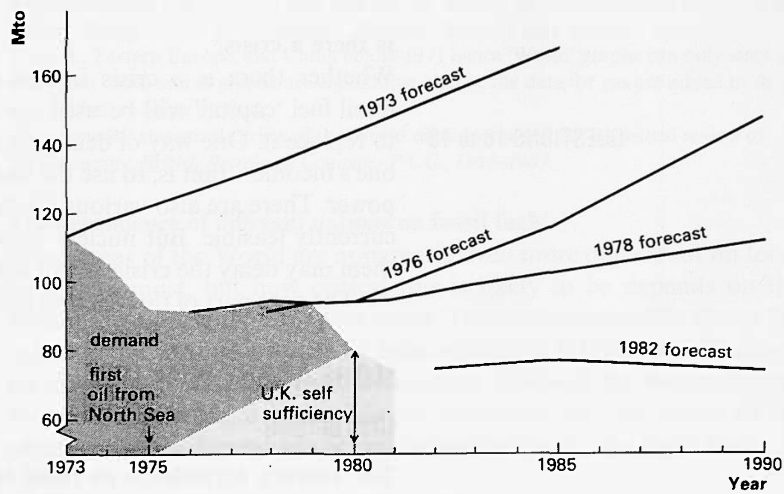

Figure G9
Variation of actual demand, and consequent variation in forecasts of demand, with time.
Source: Esso Magazine Supplement Winter 1982/83. *Public Affairs Department, Esso Petroleum Company Limited.*

QUESTIONS 13 and 14

All of the discussion of growth hitherto has assumed that if the growth is exponential it will continue unrelentingly until the resource is suddenly found to be used up. In practice, growth is unlikely to remain exponential and the growth rate will decrease as the resource becomes more difficult to obtain and more expensive.

The supply of fuels is finite

Fossil fuels are being used up and therefore will eventually run out. Figure G10 shows how the consumption of the World's stock of oil might look on a 2000 year time-scale centred on the present day. This diagram could be repeated with somewhat different time-scales for all the fossil fuels. Using fossil fuel is very like living off one's capital – it runs out sooner or later depending on the rate at which it is used and how much there was to start with.

QUESTION 15

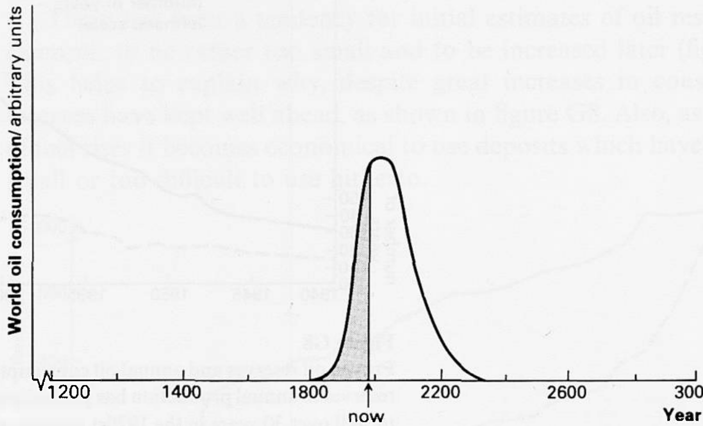

Figure G10
Variation in World oil consumption on a long time-scale.

Is there a crisis?
Whether there is a crisis in fuel supply depends on how soon the fossil fuel 'capital' will be used up and whether there will be anything to replace it. One way of dealing with the problem would be to live off one's income – that is, to use the renewable energy sources such as solar power. There are also various kinds of nuclear energy source which are currently feasible. But nuclear fuels are also finite, so, although using them may delay the crisis, it will not, in the long run, avoid it.

QUESTIONS 16 to 18

The next part of this Section deals with some of these ideas in detail.

MORE ABOUT FUEL USE

Distribution

The World's dependence on fossil fuels
Oil contributes roughly two-fifths of the World's fuel consumption, gas one-fifth, coal almost a third, and less than one-tenth is provided by hydroelectricity and nuclear power together. Given the projected lifetimes of the resources considered in the first part of this Section, the World as a whole seems to be heavily, even dangerously, dependent on those fuels which are likely to be depleted quickly. Figure G11 shows how this dependence has been growing.

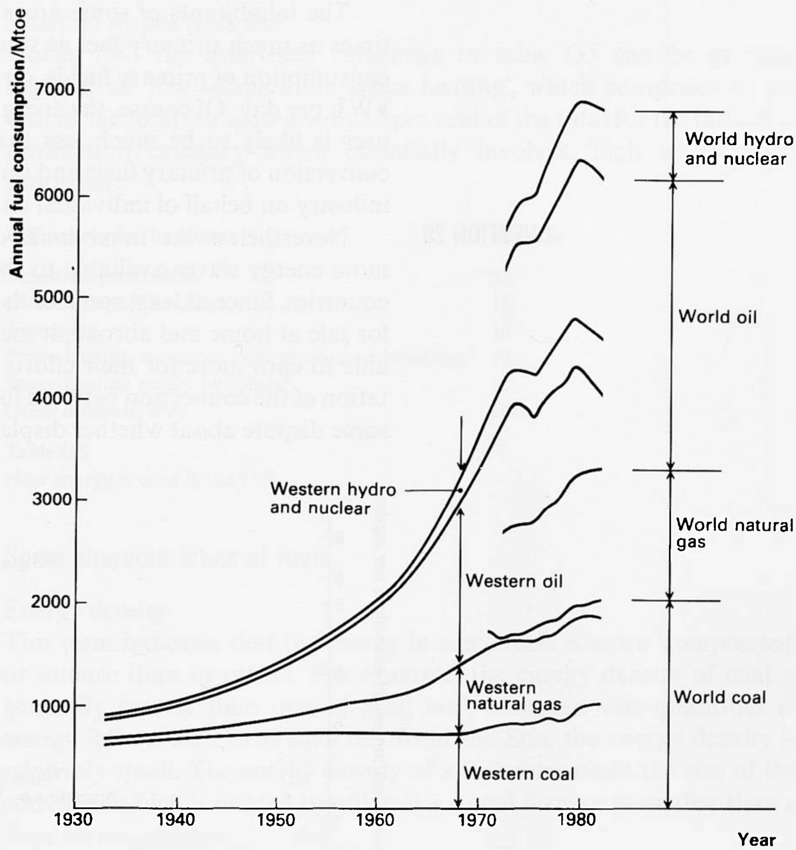

Figure G11

Variation of fuel consumption with time for the 'World', and for the World less U.S.S.R., Eastern Europe, and China – labelled 'Western'. Reliable data were not available for U.S.S.R., Eastern Europe, and China before 1971 hence 'World' graphs can only start at that point. Both sets of graphs are cumulative, that is, the data for gas are added to those for coal before being plotted.

Data from: BP statistical review of the World oil industry *and* BP statistical review of World energy. *British Petroleum Company P.L.C., 1963–1983.*

G

The dependence of different nations on fossil fuels

QUESTION 19

Some areas of the World are apparently even more dependent on fossil fuels than most, but how critical this is likely to be depends on the availability of resources to these areas. Table G11 (page 436) shows the consumption of various fuels by area, and table G13 (page 439) shows the production of fuels in these same areas. Although the fuel categories are a little different, it is possible by comparing the two tables to see which countries depend heavily on imports of particular fossil fuels.

Dependence of different peoples on fossil fuels

Table G11 shows that North America and Western Europe together consume about half the World's fuel each year. The fuel consumption *per capita* enables comparisons to be made without variations in population causing confusion.

The inhabitants of some areas of the World consume up to thirty times as much primary fuel as some others. In the U.K. the *per capita* consumption of primary fuel is, on average, the equivalent of about 100 kW h per day. Of course, the energy actually available to the individual user is likely to be much less than this because of the losses in the conversion of primary fuels and the fact that much of this fuel is used by industry on behalf of individual citizens.

QUESTION 20

Nevertheless the inhabitants of the developed world have many more energy slaves available to them than those in the less developed countries. Since at least some of the fuel is used in manufacturing goods for sale at home and abroad, it means that the developed countries are able to earn more for their efforts. Figure G12 is a common representation of the connection between fuel use and earnings; however, there is some dispute about whether displays like this are entirely fair.

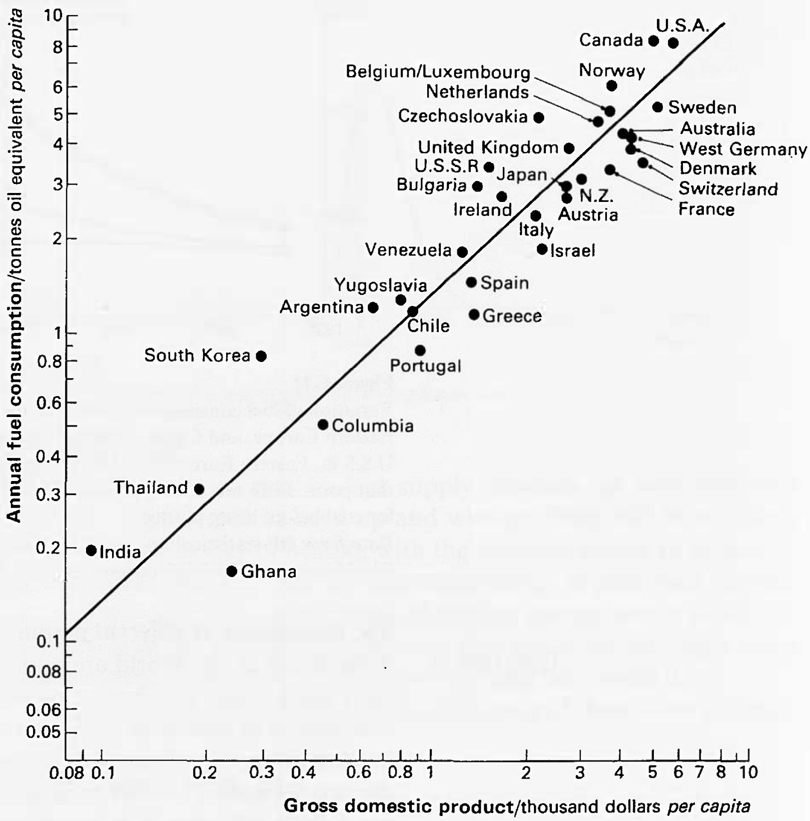

Figure G12
Fuel consumption against gross domestic product.
From: DARMSTADTER, J., DUNKERLY, J., and ALTERMAN, S. How industrial societies use energy. *Johns Hopkins University Press, 1977.*

What do we use fuels for?

QUESTION 22 Notice that the asterisked categories in table G5 can be grouped together as 'low temperature space heating', which comprises 41 per cent of the total compared with 33 per cent of the total for the industrial production category which essentially involves 'high temperature' processes.

U.K. energy use by sector	Per cent
Industrial production	33
Domestic space heating*	18
Transport	16
Space heating industrial and commercial buildings*	13
Space heating public buildings*	10
Other domestic uses	10

Table G5
How energy is used in the U.K.

Some characteristics of fuels

Energy density

QUESTION 21 This term indicates that the energy in some fuels is more 'compressed' or intense than in others. For example, the energy density of coal is generally greater than that of peat; and, although vast quantities of energy fall on the Earth each day from the Sun, the energy density is relatively small. The energy density of a fuel determines the size of the converter which is needed to utilize it – a coal furnace is smaller than a solar furnace of similar power.

Transportability

QUESTION 23 Fuels are not generally found in the places where they are going to be used. Some, like oil or gas, are relatively easy to move about while others, like wave energy, are difficult to move in that form though relatively easy to move if converted to electricity.

Time

QUESTIONS 24 and 25 Fuels may not be available when they are needed either because they cannot be turned on on demand – like solar power or wind energy – or because the demand varies over time. Figure G13 shows how demand for electricity varies on a typical daily basis in summer and in winter. Figure G14 shows a somewhat atypical day. The gas industry has a similar pattern of demand (figure G15, page 421).

Although some storage is possible, both the gas and electricity industries are generally faced with the need to meet the demand as and when it occurs. If they fail, they may cause inconvenience or worse, with consequent public outcry.

G

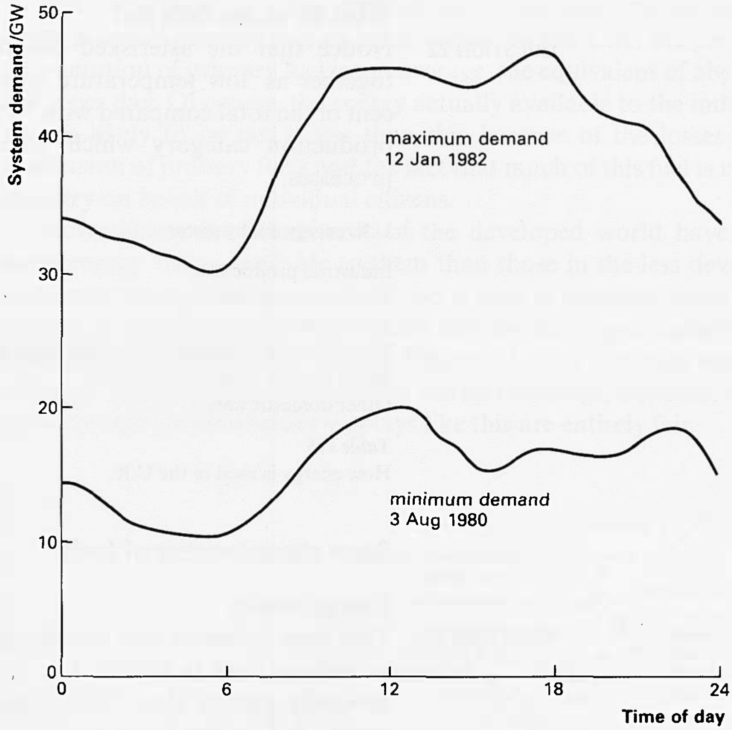

Figure G13
Variation in demand for power from Central Electricity Generating Board with time of day and time of year.
From: WADDINGTON, J. *and* MAPLES, G. C. *'The control of large coal and oil-fired generating units.'* CEGB Research, *February 1983*.

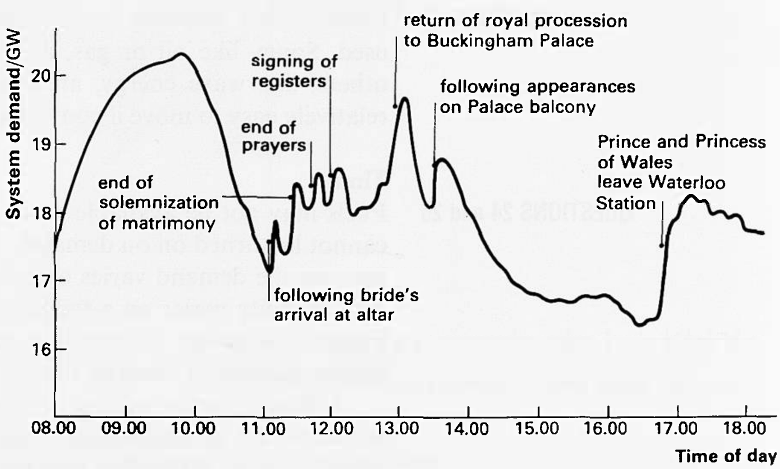

Figure G14
Variation in demand for power from Central Electricity Generating Board during an atypical day, 29 July 1981.
From: WADDINGTON, J. *and* MAPLES, G. C. *'The control of large coal and oil-fired generating units.'* CEGB Research, *February 1983*.

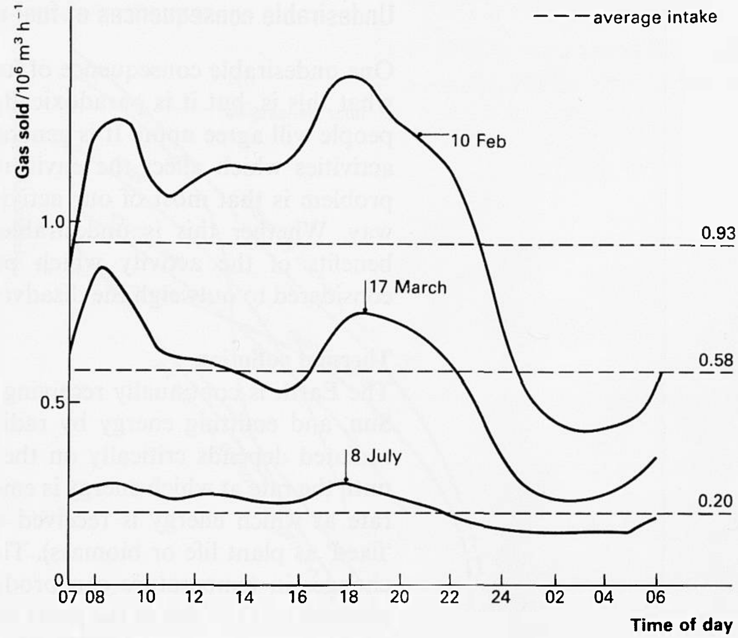

Figure G15
Gas sold in West Midlands Region of British Gas, for three days during a year.
Data: British Gas Corporation.

Feasibility

To use coal for heating does not require any elaborate apparatus, though maximizing the efficiency may involve some complication. On the other hand, using sunlight, waves, uranium, or deuterium may require complex devices which cost a great deal in terms of money and also of energy before they begin providing energy for use by our society.

A similar argument applies to any other form of energy conversion. The capital cost and the energy requirements mean that large energy converters cannot be built without a substantial amount of forward planning.

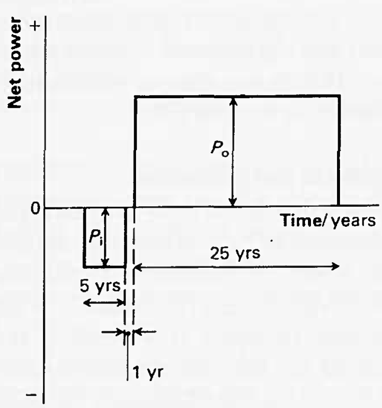

Figure G16
Schematic diagram showing how a power station requires energy while it is being built before it produces energy once it is running. P_i and P_o are the average input and output powers. The area under the graph is a measure of the energy involved.
From: CHAPMAN, P. Fuel's paradise. *Penguin Special, Copyright © Peter Chapman, 1975. Reprinted by permission of Penguin Books Ltd.*

Undesirable consequences of fuel use

One undesirable consequence of fuel use is pollution. Everyone knows what this is, but it is paradoxically difficult to define in a way which people will agree upon. It is generally taken to be the results of Man's activities which affect the environment in an undesirable way. One problem is that most of our activities affect the environment in some way. Whether this is undesirable depends in part on whether the benefits of the activity which produces the alleged pollution are considered to outweigh the disadvantages.

Thermal pollution

The Earth is continually receiving large quantities of energy from the Sun, and emitting energy by radiation. The rate at which energy is radiated depends critically on the Earth's temperature. This changes until the rate at which energy is emitted by radiation is just equal to the rate at which energy is received (less the small proportion which is 'fixed' as plant life or biomass). This is important because quite small changes in temperature can produce substantial changes in weather patterns and the size of the polar ice caps, for instance.

The consumption of fuels adds to the energy input to the Earth and its temperature must rise to enable the extra energy to be radiated away. This might not be a problem for the Earth as a whole for some time. But population, and hence energy use, tend to be concentrated in relatively small regions and in such regions the energy input from fuels can be a substantial fraction of the solar input.

QUESTION 26

Other sources of pollution

The burning of fossil fuels produces combustion products which can combine with water in the atmosphere to produce acid rain, which may well fall on a country other than that of its origin; some of the combustion products are radioactive because the fuel contains small quantities of radioactive material; quantities of solid matter may be produced as smoke or ash. Extraction of fossil fuels may also damage the environment.

Nuclear power stations require the manufacture and treatment of fuel and the disposal of waste which are polluting to different extents.

This is an area of continuing concern which will be considered further in Section G4.

Risks in fuel production

Just as human activity may be regarded as polluting, it also entails some elements of risk. Where the consequences of an activity frequently lead to injury or damage it is possible to determine statistically the risk involved to any individual. Where the activity almost never 'goes wrong' or where it is novel, it is possible to estimate what the risks might be, but this determination is different from the one described above. It is also necessary to consider whether the person at risk has knowingly agreed to take the risk. Figures G17 and G18 show the risks involved in a variety of activities. The risks attached to air crashes are

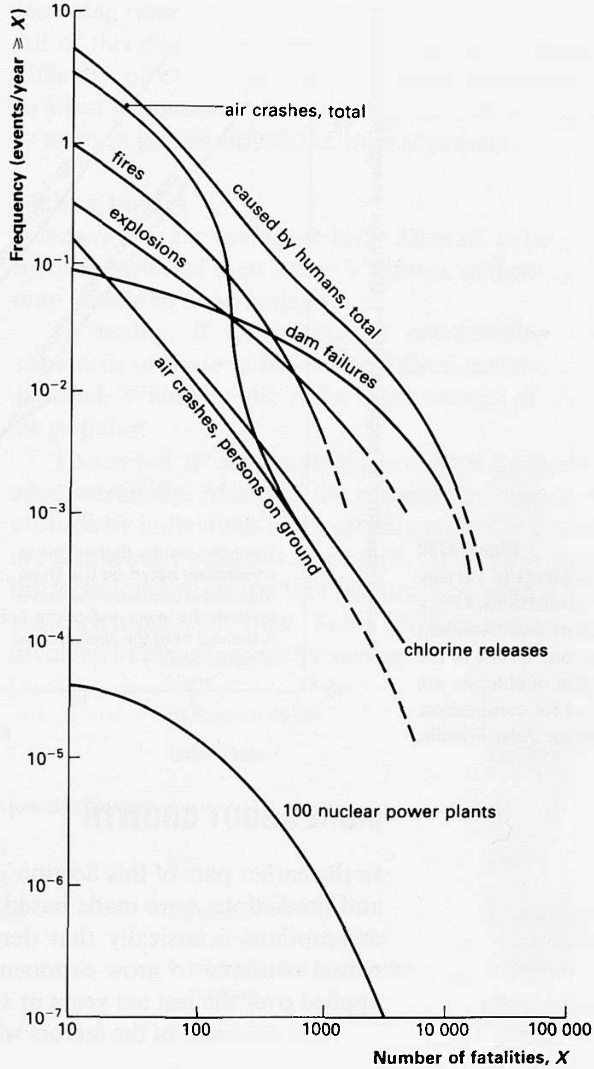

Figure G17
Probability of accidents, causing more than some number, X, of fatalities, against X for a variety of causes of accidents.
From: The Rasmussen Report, 1975. The reactor safety study. *NUREG 74/014. U.S. Nuclear Regulatory Commission, Washington.*

presumably calculated on actual evidence. Those who fly in aircraft probably accept the risk, but do all of us knowingly accept the risk of having an aircraft fall on us?

Nuclear power stations have been notably free from fatalities attributable to them, therefore the curves relating to their operation must have been calculated using the theory of risk analysis. There is therefore more doubt about whether these figures are reliable. Moreover, the population at large has no choice about whether it should be exposed to the risks involved in nuclear power stations, any more than it has about the risks involved in fossil-fuelled power stations.

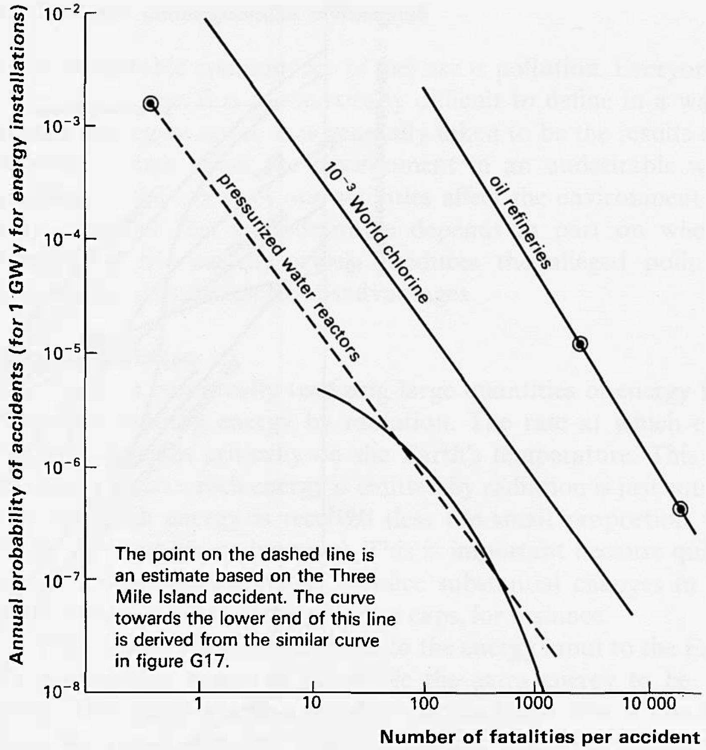

Figure G18
Probability of accidents of varying severity for energy installations. GW y means gigawatt year. Accident probabilities resulting from 1/1000 of the World's production of chlorine are included for comparison.
Data: Professor John Fremlin.

MORE ABOUT GROWTH

In the earlier part of this Section growth of fuel demand was examined and predictions were made based on some simple and probably naive assumptions – basically that demand would either remain static or would continue to grow exponentially at the average rate which has applied over the last ten years or so.

Here are some of the factors which may affect growth in demand.

Population growth

QUESTION 27

If World population continues to grow exponentially, then the demand for fuel will increase exponentially for this reason alone, assuming that fuel consumption per head remains at today's level.

Increasing expectations

QUESTIONS 28 to 30

It may be argued that the distribution of fuels is unfair and that more fuel ought to be available to each inhabitant of the less developed world (and that we in the industrialized nations ought to have no more, or perhaps less than at present).

It can be argued that centralized systems of both fuel consumption and production of goods may not be appropriate to the less developed countries. There is, however, evidence that fuel demand *per capita* is rising faster in the less developed world than elsewhere and this could produce a rapidly rising demand for fuel resources.

Reducing reserves

All of this discussion must be set against a background of increasing difficulty of extracting fuel and hence increasing cost. This is likely to affect the poorer countries more than the rest, and may possibly lead to an even greater imbalance than at present.

Making savings

It is easy to make savings in fuels. After all, in the past humans survived by their own and their animals' labour, and many people in the World may be said to do so today.

In reality, if we consumed considerably less fuel, the living standards of those in the industrialized nations would be substantially reduced. Whatever the rights and wrongs of this, it is not likely to be popular!

This is not to say that savings cannot be made, since much fuel is used wastefully. Many of the possible savings depend on changes of attitude by individuals or by governments; the problem is that attitudes are notoriously resistant to change. Other savings depend on developing techniques to ensure that resources are used less wastefully. Many of these changes take time. Table G6 summarizes some of the factors involved in planning energy savings.

	Savings made by: Individuals	Societies
Time required to put idea into practice/Years	1	10
Relative cost	low	high
Possible ways of making savings	Domestic heating; labour saving appliances; cooking Transport Communication	Choice of central or local power stations Choice of conversion method Choice of method of distribution
Possible sources of energy	Electricity Gas, coal Solar	Fossil fuels Nuclear Solar, wind, wave
Other considerations	Living standard Domestic safety Amount saved	Conformation to legislation Choice of site: environmental and economic factors

Table G6
Controlling factors for energy planning.

More speculative fuel resources, for example nuclear fusion, would take longer to develop, cost more, and present more technical difficulties than those mentioned in the table.

SECTION G2
POWER FROM NUCLEAR FISSION

ENERGY CAN BE RELEASED IN NUCLEAR REACTIONS

Unit F, 'Radioactivity and the nuclear atom' showed that the particles in an atom's nucleus (the nucleons) are strongly bound together by an attractive force. Since energy is required to separate the nucleons the binding energy is negative. Figure G19 shows how the average binding energy per nucleon depends on the number of nucleons and shows that if a massive nucleus were to split into smaller fragments (fission) the total binding energy would become more negative and so energy would be released. Similarly, if two light nuclei could be fused together to form one more massive nucleus, energy would again be released.

Fission – the basic ideas

Whilst fission can occur spontaneously, it is a very rare event and is more usually triggered by adding an extra neutron to a suitable nucleus. There are a number of possible nuclei but the one which is most commonly used is ^{235}U. Naturally-occurring uranium contains 0.7 per cent of ^{235}U, the rest being mainly ^{238}U. This can also undergo neutron-induced fission, as can ^{233}U and some plutonium isotopes. (Plutonium, proton number 94, does not occur naturally, nor does ^{233}U.)

A typical reaction is:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{236}_{92}\mathrm{U} \rightarrow ^{144}_{56}\mathrm{Ba} + ^{90}_{36}\mathrm{Kr} + 2^{1}_{0}\mathrm{n} + \text{energy}$$

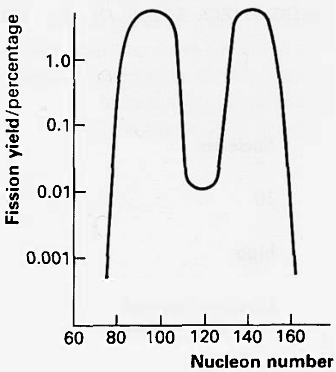

Figure G20
Yield of different fission products as a function of their nucleon number. Each fission produces two product nuclei, that tend to be of different size. *WINTERTON, R.* Thermal design of nuclear reactors. *Pergamon, 1981.*

but there are many possible products of the process as shown in figure G20.

Figure G19 shows that the energy released is about 1 MeV per nucleon, or a total of some 200 MeV. This gives a very high value of specific energy release (energy released per unit mass) compared, for example, with burning coal or other similar chemical reactions.

Most of the energy which is released is in the form of kinetic energy of the fission fragments. These are brought to rest in the fissionable material thus increasing its internal energy. This energy can be extracted by using a coolant.

QUESTION 31

Table G7
Energy released per thermal fission of ^{235}U. *Source*: Physics (Advanced) Notes for the guidance of teachers on the Nuclear physics option. *GRIFFITH, J. A. R., and TEBBUTT, M. J. J.M.B., 1977.*

Source	Energy/MeV
Kinetic energy of fission products	165
Kinetic energy of neutrons	5
γ-rays (immediate)	7
β-particles and γ-rays (from later decay of fission fragments)	13
Neutrinos/antineutrinos from decays	10
Total	200

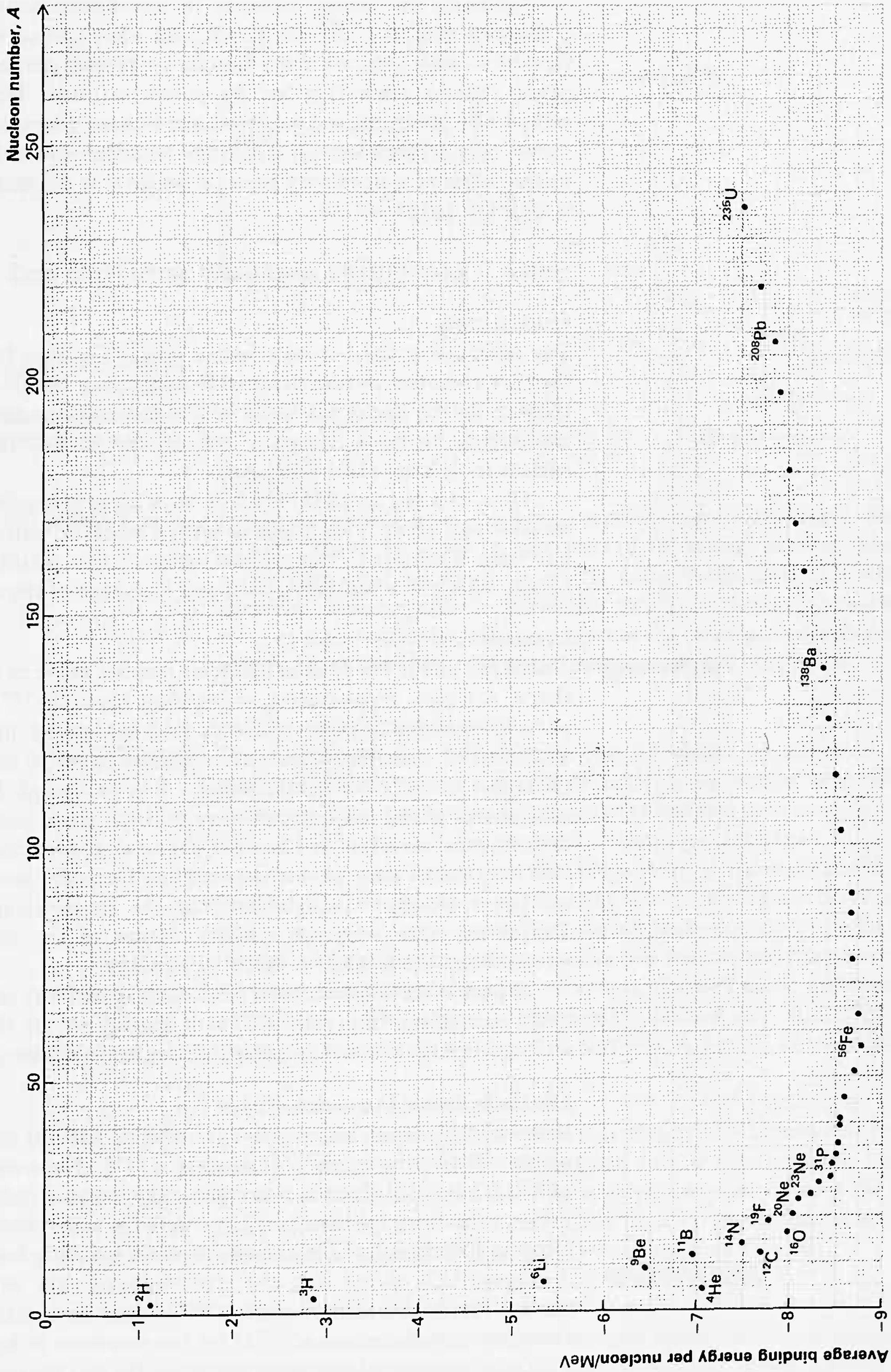

Figure G19
Average binding energy per nucleon against nucleon number.

G

Although the energy released in one fission event is very large compared with, say, the energy released when one carbon atom burns (combines with oxygen), many fissions are needed per second to provide power on the scale supplied by power stations. These fissions are induced by neutrons, and neutrons are emitted when the nuclei split. A rather neat arrangement in which the neutrons emitted induce enough further fissions to produce a reaction which is at the least self-sustaining is called a chain reaction.

Chain reaction

Fission – more detailed ideas about getting it to work

Critical mass

The neutrons emitted from a fission will not induce further fissions if they are absorbed by non-fissionable material or lost from the surface of the fuel. As the size of the block of fuel increases so does the chance of absorption by fissile material; the chance of loss decreases as the surface-to-volume ratio decreases.

Thus, as a fuel assembly is made up, a stage is reached when a chain reaction can occur. This happens when a so-called 'critical mass' of fuel has been assembled. The precise value of this quantity depends on factors such as the fuel being used and the shape of the assembly.

Cross-section

QUESTION 32

Nuclei are about 10^{-14} m in diameter and are more or less spherical in shape. A typical cross-section is therefore about 10^{-28} m^2. But not all particles passing through this area will necessarily interact with the nucleus. For example, some nuclei appear to allow neutrons to pass through them relatively unimpeded – it is as though they were semi-transparent to the neutrons. We say that the cross-section for absorption of neutrons is less, in this case, than the geometrical cross-section. Similarly nuclei may have cross-sections for some interactions which are larger, perhaps much larger, than the geometrical cross-sections. The actual value depends on the nature of the nucleus and the interacting particle, and on the latter's energy.

The cross-section for an interaction is a useful way of indicating how likely that interaction may be, but it should not be thought that the nucleus undergoes some peculiar swelling and shrinking process.

Obtaining fission in uranium-235

If the fuel for a nuclear reactor consists of natural uranium there is about 150 times as much ^{238}U as there is ^{235}U. Neutrons induce fission of ^{235}U, but ^{238}U absorbs them (generally without inducing fission). If the cross-sections for these reactions were the same there would be only a one in 150 chance of a particular neutron inducing fission.

QUESTIONS 33, 34

Figure G21 shows how the cross-sections vary with the neutron energy. The fission cross-section of ^{235}U must be at least 150 times the absorption cross-section of ^{238}U for the reactions to be equally likely. This only occurs for low energy neutrons. (In fact the detailed picture is much more complicated than this.) The energy of such neutrons is of the same order as the kinetic energies of the surrounding atoms – hence they are called thermal neutrons.

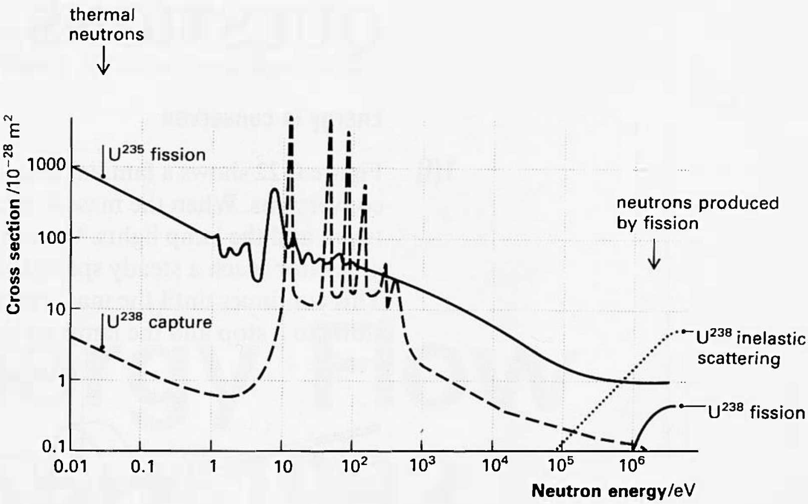

Figure G21
Principal neutron reaction cross-sections for uranium.
WINTERTON, R. Thermal design of nuclear reactors. *Pergamon, 1981.*

Moderation

QUESTIONS 35 to 39

Neutrons emitted from the fission process have high kinetic energies (about 2 MeV). In order to take part in the fission process described above they must be slowed down by being made to collide with the nuclei of a material called a moderator. But if too many are absorbed by ^{238}U or the moderator and other parts of the structure (or lost from the surface of the assembly) the chain reaction will not take place.

Enrichment and reprocessing

QUESTION 40

The likelihood of establishing a chain reaction is increased if the proportion of material capable of undergoing fission is artificially increased. Such material is called enriched fuel and may have, say, 3 per cent of ^{235}U instead of the naturally-occurring 0.7 per cent.

Once the reaction starts, this proportion is reduced as the ^{235}U nuclei split. The chance of maintaining the chain reaction decreases. In addition the fission products often absorb neutrons very strongly which further inhibits the chain reaction. For this reason, fuel may have to be withdrawn and reprocessed to eliminate the unwanted material. The extraction and treatment of this waste material is a matter of some concern which will be examined in Section G4, part C, Nuclear safety.

Control

QUESTION 39

Once a chain reaction is achieved it is unlikely to continue at just the required rate. In practice the reaction rate is set too high, and the process is controlled by absorbing enough neutrons to keep the chain reaction steady. Rods of a material with a large neutron absorption cross-section are moved in and out of the reactor to do this.

Different kinds of reactor

There are many different kinds of thermal fission reactor, in addition to fast fission reactors. Section G4, part A, Nuclear power stations, provides an opportunity to examine some of these in detail. The process of nuclear fusion and possible fusion reactors is studied in Section G4, part B, Nuclear fusion.

G

QUESTIONS

Energy is conserved

1(I) Figure G22 shows a familiar arrangement used to demonstrate energy conversions. When the mass is released it accelerates, the generator turns, and the lamp lights. Usually at some stage the mass and generator reach a steady speed and the lamp gives a steady output. This continues until the mass reaches the floor when the generator slows to a stop and the lamp no longer lights.

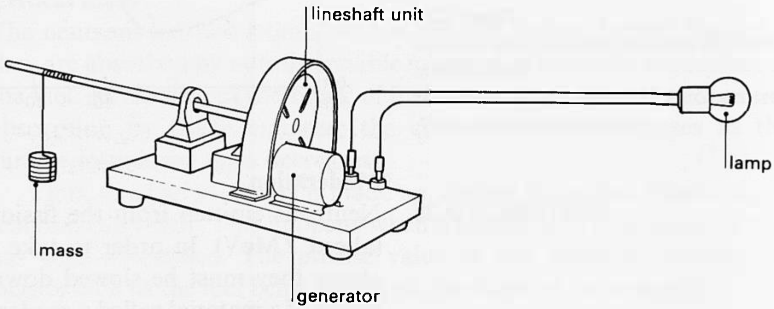

Figure G22
An energy conversion experiment.

Figure G23 attempts to show in a simple way how energy is transformed as it is transferred through the system. Notice the convention which has been used here: the part of the system being considered is represented by a labelled box and the 'form' in which the energy might be thought to exist is added as a label underneath each box.

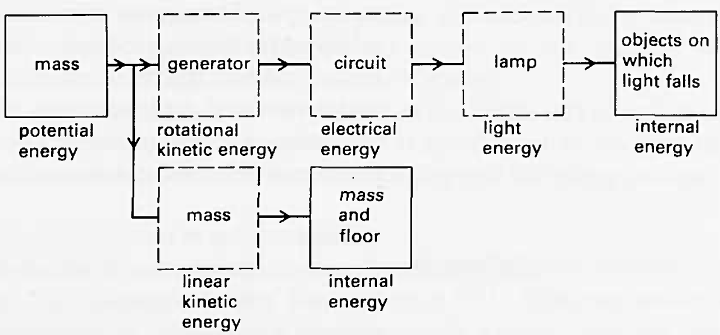

Figure G23
Energy transformations in the experiment shown in figure G22.

a *i* What is meant by 'the potential energy of the mass'? Is it strictly a property of the mass alone?
ii What is the process by which energy is transferred from the first stage in figure G23 to the second?
iii What is meant by saying 'the generator has a certain quantity (say 15 J) of rotational kinetic energy'?

A Publication of the
Government Statistical Service

Energy Flow Chart 1983

United Kingdom

DEPARTMENT OF ENERGY

JUNE 1984

UK ENERG

(THOUSAN

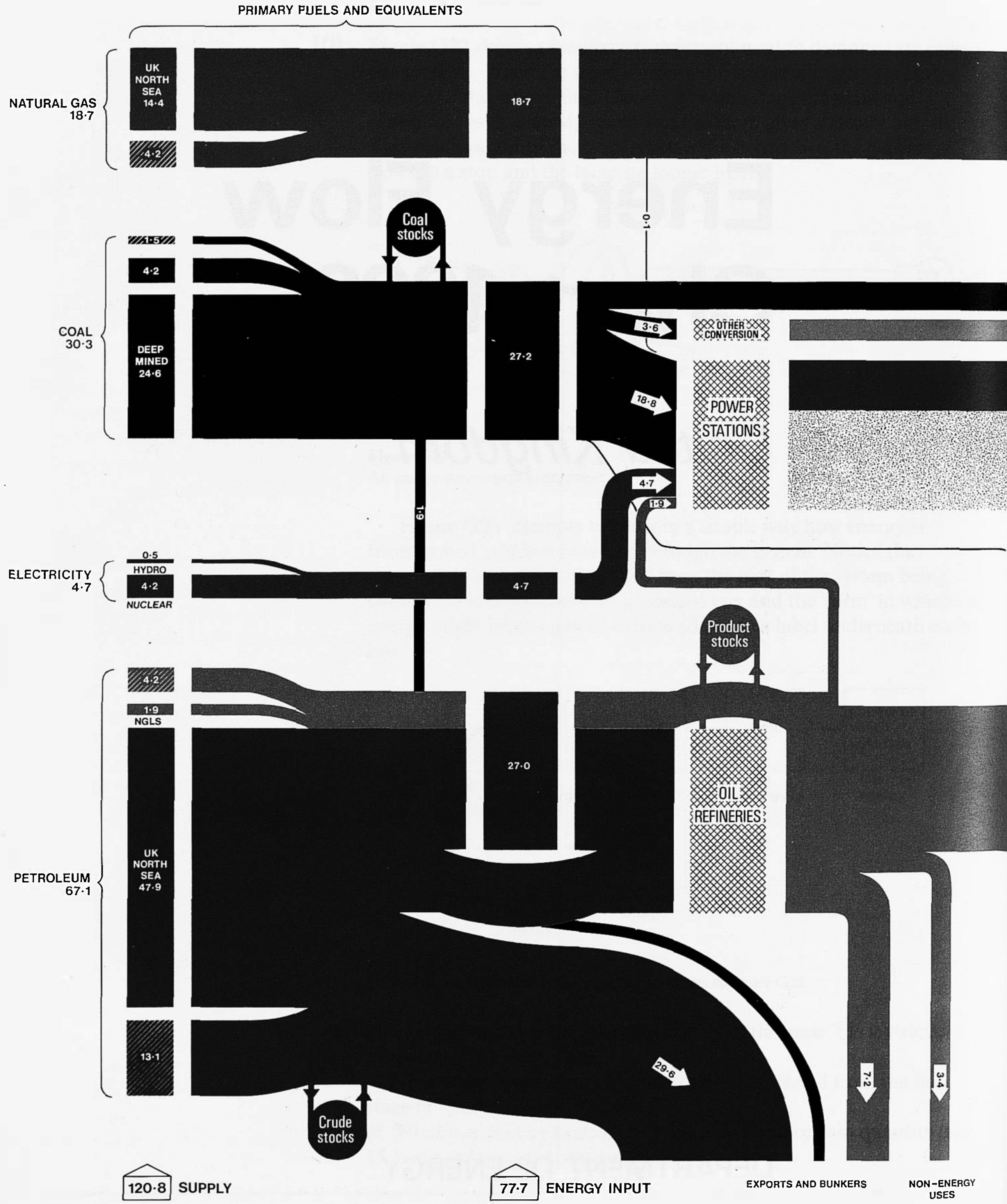

Figure G4

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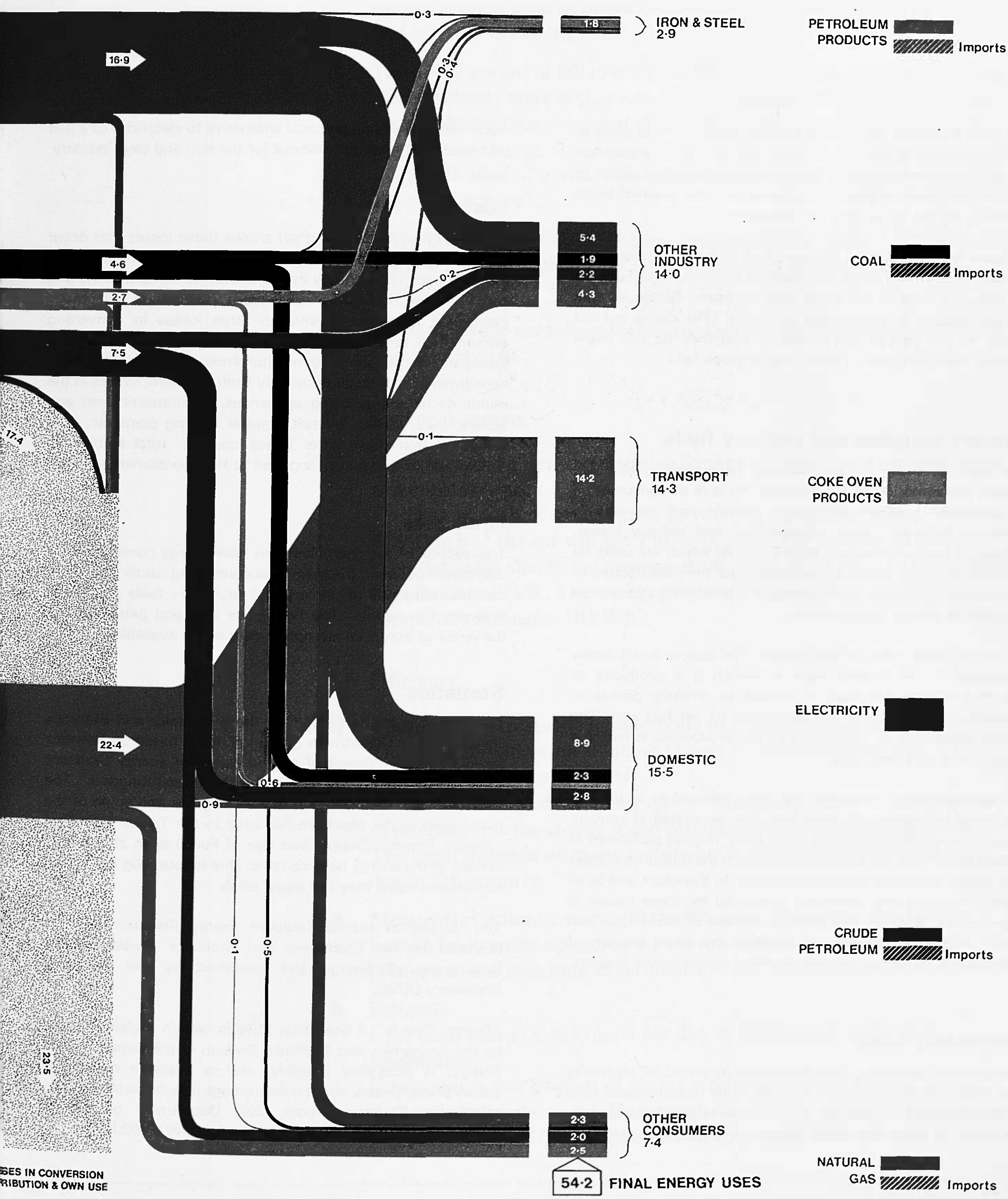

Energy Flow Chart 1983
United Kingdom

The chart illustrates the flow of primary fuels from the point at which they become available from (on the left) home production or imports to their eventual final uses (on the right), either in their original state or after being converted into different kinds of energy by the secondary fuel industries.

All flows are measured in thousands of millions of therms and the widths of the bands on the chart are roughly proportional to the absolute sizes of the flows they represent. Stocks of coal and petroleum are represented by circles. (The circles are not related to the size of the stocks — and they do not show whether there has been a stock rise or stock fall.)

Primary supplies and primary fuels

The chart is similar to the previous issue relating to 1980. Primary consumption of petroleum is the sum of consumption of petroleum products at power stations and gas works, *deliveries for other uses, refinery fuel and refinery losses.* Petroleum products derived from crude oil which are used for non-fuel purposes (eg as a raw material for the manufacture of chemicals and plastics, as bitumen for road making etc) are not included as energy consumption.

As can be seen, most of our primary fuel supply is not finally consumed in the original state in which it is produced or imported. Crude petroleum is refined to produce petroleum products (eg petrol, fuel oil, gas/diesel oil, jet fuel etc). The largest proportion of coal flows to power stations where it is transformed into electricity.

Nuclear and hydro electricity are often referred to as primary electricity to distinguish them from that generated at conventional power stations burning fossil fuels, ie coal, petroleum or natural gas. There are many ways in which the output of nuclear and hydro electricity can be measured. In the chart and in all related statistics the electricity generated by these means is expressed in terms of the notional amount of fossil fuels that would have been needed to generate the same amount of electricity at contemporary conventional steam power stations.

Secondary fuels

The principal secondary fuels are petroleum products, electricity and coke (which in the chart includes other manufactured solid fuels). Secondary fuels are in the main required for specific purposes for which the use of primary fuels is inappropriate. For many uses there is no practical alternative to electricity as a fuel and coke is an essential material for the iron and steel industry.

Losses

This large flow (in dotted grey) shows those losses that occur between primary supplies and deliveries to final users. Each fuel industry consumes energy in the course of its operations and some is lost during its subsequent distribution. Electricity generation in particular involves large losses in converting primary fuels to electricity. The chart does not show the further losses of energy which occur after energy is supplied to final consumers which result principally from the inefficiencies in the multitude of energy using appliances, eg domestic fires and boilers, cars, lorries, aircraft, central heating plant etc. It is estimated that these latter losses could in total amount to almost half of the energy supplied to final consumers.

Final uses

This section of the chart illustrates how energy consumption is distributed between the main final consuming sectors and how the different kinds of primary and secondary fuels are shared between the sectors. The figures for coal and petroleum are deliveries as actual consumption data is not available.

Statistics

The chart has been prepared by the Economics and Statistics Division of the Department of Energy and is based on statistics taken from the *"Digest of United Kingdom Energy Statistics 1984"*. (Table 6) 'Energy balance for the United Kingdom'. The flow chart is a simplification of these figures and some of the terms used in the chart are not used in the Table. Table 2 of *"Energy Trends"* (Supply and Use of Fuels) is an abbreviated version of the energy balance table. Due to rounding the sum of constituent items may not equal totals.

The *"Digest of United Kingdom Energy Statistics 1984"* is prepared by the Economics and Statistics Division of the Department of Energy and published by Her Majesty's Stationery Office.

"Energy Trends", a Statistical Bulletin, which is also prepared by the Economics and Statistics Division of the Department of Energy, is published monthly, and is available on annual subscription. Details about subscriptions may be obtained from Information Division, Room 1397, Department of Energy, Thames House South, Millbank, London SW1P 4QJ.

b Diagrams such as figure G23 are intended to show the main features of the conversion process, but they can easily include too much and become too complex, or they can miss out important points.

Some important boxes have been omitted which relate to energy 'losses'. Show where these boxes ought to be inserted and, following the convention used in figure G23, show what form the energy takes and where it is located.

c The initial period of fall involves all the boxes in the diagram. The later stage of steady state is characterized by the state of those parts of the system shown thus ---- not changing. Those shown thus ——— continue to change.

Which kind of border should the boxes you have inserted in answer to **b** have?

d What do you know about the total energy of this system? Why has the word 'losses' been printed in quotation marks in part **b** of this question?

e Construct your own diagram for another conversion chain with which you are familiar.

Sankey diagrams

2(L) Construct first a non-quantitative Sankey diagram for a car travelling at a constant speed.

Then make it as quantitative as you can using these data: At 90 km h^{-1} the car will use 7 litres of petrol per 100 km; in order to keep the speed steady the power output at the flywheel is 15 kW and at the wheels is 12 kW. The energy produced when 1 litre of petrol is completely burned is 10 kW h.

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Efficiency

3(E) This question treats the human body as a machine and as a heater. This oversimplification is intended only to give you some idea of the quantities of energy involved.

Human beings derive their energy from food. Hence one measure of food input is the energy it can provide in appropriate circumstances. The input depends on many factors but is normally about 10 MJ per day in this country.

a Measure (or estimate or recall) the mechanical power output of the human body when operating a pulley, walking, or running upstairs. (These are simple school experiments which you may have met before.)

b Estimate
i the total energy you expend per day in mechanical tasks and locomotion.
ii how much this amount of energy would cost if you bought it from the local electricity board. How much does it cost to buy food which will provide 10 MJ? (Examples: rice or mutton provide about

15 MJ per kg. Look at a recent electricity bill to find the cost of one unit, *i.e.*, 1 kW h.)

iii how efficient the human body is as a machine.

iv the power rating of the body as a heater. (*Hint:* estimate the power dissipated by the body assuming that all the energy obtainable from the food input can be used for heating.) Estimate the power delivered in, say, a theatre, by the 500 people inside it. Do such buildings need heating systems? If so, what for?

c The fuel (food) which a mammal's body takes in is used in a number of ways: for instance, doing external work, keeping the parts of the body working, building body parts, and keeping warm – some is wasted, of course.

i Construct a Sankey diagram to illustrate this. It is likely to be mainly qualitative but some of the information needed to quantify it is available from other parts of this question.

ii The overall energy input and output of a body must be balanced. Use the Sankey diagram to consider what might happen to the system when one of the components is changed. Useful changes to consider are: increasing or decreasing the food intake (which relates to dieting in human beings), increasing the external work which is done (perhaps by exercise), or changing the temperature of the environment (which has implications for battery farming of animals).

Energy units

4(P) This question provides practice in using various units. You will need to use data tables to find information for some parts, and in some cases you will have to make estimates.

How much energy (in J) is provided by the following?

a A 1 kW heater running for one hour.

b Burning 1 litre of petrol (enthalpy of combustion of petrol $= 4.7 \times 10^7\ \mathrm{J\,kg^{-1}}$, density $= 7400\ \mathrm{kg\,m^{-3}}$).

c Burning 1 tonne of coal (10^6 tonnes of coal produce 8×10^9 kW h).

d A 1000 MW power station in 1 hour.

e One kg of water falling through 100 m.

f A one tonne car in slowing from $80\ \mathrm{km\,h^{-1}}$ to rest.

g The total annual fuel consumption of the World (about 7000 Mtoe). (Use table G4, page 412.)

h A nucleus with a kinetic energy of 100 MeV coming to rest.

i One Btu (British thermal unit), which is the quantity of energy needed to increase the temperature of 1 lb of water by 1 °F (5/9 K). (You are expected to work this out, not just look up the answer in table G4.) (1 lb = 0.45 kg.)

5(L) This question tries to make clear the various ways in which the term 'energy slave' is used, taking domestic appliances as examples. Some of these use fuels just to do work, some for heating, and some for both.

a Make a list of all the machines used in your home for doing work (ignoring for the moment those just concerned with heating). How many are there? This is one use of the term – the actual number of machines which are available regardless of their power or how much they are used. Why may it be misleading to use this number?

b The second use of the term is to compare their power with that of a person. Add the power ratings of the machines (in W). How many 'slaves' is this equivalent to – assuming the average power output for useful work calculated in question 3? How could this be misleading?

c The third use of the term is to compare the energy 'consumption' over a certain time with the useful work which can be done by a person in that time. Estimate the average daily energy 'consumption' of all these machines. How many slaves does this amount to? (The word 'consumption' is in quotes to show that it is not quite correct. What is wrong about it and what would be a better way of putting it?)

d Many of the appliances which use fuel in the home are used for heating as part or all of their function (for example, washing machines or boilers). Repeat part **c** after adding devices which are used for heating and compare your previous answer with that from part **c**.

Efficiency

6(L) According to the data in figure G4 (between pages 430 and 431):

a What is the *operating* efficiency of power stations in the U.K.?

b What is the efficiency of the electrical *transmission* system?

c What is the *overall* efficiency of the electrical supply system?

d What is the *overall* efficiency of oil refineries?

e What is the efficiency of the gas supply system?

f Many of the machines you listed in your answer to question **5** are electrical. What, roughly, is the average daily consumption of primary fuel needed to run these machines?

Primary, end-use, and functional energy

7(L)a Householders are concerned about the cost of the fuel which they need to buy in order to provide each kW h for heating their home. How much more expensive can electricity be than gas, for example, for the cost of each kW h of internal energy to be the same? (Use table G1, page 409.)

b On the other hand, society ought to be concerned about the quantity of primary fuel needed to provide the kW h of internal energy. Estimate this quantity for electricity and gas.

8(P)a Check whether the population of the World (table G8) has been growing exponentially and, if so, determine its doubling-time.

b Table G9 gives data for World fuel consumption over the same period. Is the growth in population sufficient, on its own, to account for the growth in fuel consumption? If not, what deduction can you draw?

Year	Population/10^9
1925	1.890
1950	2.505
1955	2.726
1960	2.990
1965	3.281
1968	3.484
1970	3.609
1971	3.678
1972	3.747
1975	4.258
1981	4.528

Table G8
World population since 1925.

Year	Fuel consumption/10^{18} J	Year	Fuel consumption/10^{18} J
1925	47	1970	226
1929	55	1971	235
1937	59	1972	250
1949	73	1973	259
1950	81	1974	261
1953	92	1975	261
1955	105	1976	276
1958	118	1977	284
1960	131	1978	294
1963	146	1979	320
1965	169	1980	319
1968	200	1981	318
1969	214		

Table G9
World fuel consumption *per annum* since 1925.

9(L) Imagine a bacterium which breeds by dividing each hour. A colony of bacteria needs a volume of nutrient proportional to the number of bacteria present if all are to survive. One bacterium is placed in a jar of nutrient at the beginning of the day.

a Sketch graphs, using scaled axes, to show how the population and the breeding rate change with time.

b If the bacteria fill the jar after 24 hours, when will it have been half filled?

c If the bacteria were sensitive and intelligent when might they become alarmed at the idea that nutrient was becoming scarce?

d If they wish to continue to breed at the same rate after the first 24 hours, how much extra nutrient would be required for one hour's further life? And for another hour after that?

e If the original volume of nutrient had been underestimated, and in fact there was twice as much nutrient as had been thought, what difference would this make to the potential lifetime of the colony?

Questions **10** and **11** can be tackled by the incremental method and graph plotting technique used elsewhere in the course, or by means of a calculator or computer. If you have it available, the 'Dynamic modelling system' enables you to set up the equations and solve the problems very rapidly.

10(I) If you borrow money the interest charges are added to the original debt and the whole lot then attracts interest. If you neglect to pay back either the original debt or the interest the overall debt rises more rapidly than would be expected at first sight. Thus at a rate of interest of 2 per cent per month (a typical figure for a credit card) you might expect the doubling-time to be 50 months. How long is it in fact?

11(L) It is useful to know how the doubling-time (t_D) is related to the growth rate (g). If you are mathematically inclined you can find the relation quite easily by analytical methods.

Alternatively, repeat question **10** for growth rates of 4, 6, 8, and 10 per cent. (This is likely to be tedious to do unless a computer or calculator is available or the labour is shared.) Now process the results appropriately to find the relationship. (*Hint:* either plot a suitable graph or guess a relationship and check it by calculation.)

Making predictions

12(L) The term 'resources' is generally taken to mean the total of all the fuel stocks which could possibly exist. Since some of these are as yet undiscovered this figure must be an estimate. 'Reserves' or 'proved reserves' are the fuel stocks whose existence is based on much firmer evidence, such as geological surveys, drilling, etc. Reserves (or proved reserves) are thus a subset of resources. 'Recoverable reserves' are again a subset of proved reserves, since it may not be possible to extract all the proved reserves. Notice that the categories that are used in table G10 do not all correspond exactly with these.

Table G10
Fuel resources and reserves.
Source: U.N. Yearbook of World energy statistics, 1980

Fuel	Quantity/ millions of tonnes	Equivalent/ Mtoe
Bituminous coal/ anthracite		
Proved reserves in place	775 470	519 565
Recoverable	488 333	327 183
Additional resources	3 928 222	2 631 909
Sub-bituminous coal/lignite		
Proved reserves in place	544 958	179 836
Recoverable	394 074	130 044
Additional resources	5 994 062	1 978 040
Peat		
Proved reserves in place	57 022	14 256
Recoverable	15 819	3 955
Additional resources	261 618	65 405
Oil		
Crude reserves	80 633	80 633
Oil shale	41 137	41 137
Bituminous sands	40 001	40 001
Natural gas	($/10^9$ m^3)	
Reserves	77 109	69 398

a If the World were to continue to use the various fuels at the most recent rate listed in table G11, for how long could we be sure to have each fuel available? (For the purpose of this question you can add the recoverable reserves for the coals and for peat together.)

b Is there any evidence to suggest that consumption will continue at a constant rate?

13(L) Table G11 shows how the demand for different fuels changed over a 10-year period.

Region	Consumption/Mtoe *per annum* Oil	Natural gas	Coal	Water	Nuclear	Total 1982	Total 1972	% Change	(1981) Population /10^6	(1982) Consumption per capita/ toe
N. America	776.0	506.0	417.3	149.1	57.4	1 935.8	1 947.5	−0.6	252	7.7
Latin America	235.4	60.0	17.2	49.1	0.5	362.2	226.1	60	366	0.99
America – Total	1 011.4	566.0	434.5	198.2	87.9	2 298.0	2 173.6	5.7	618	3.7
U.K.	75.6	41.6	65.4	1.4	9.4	193.4	215.6	−10.3	55	3.5
West Germany	112.4	38.6	79.7	5.5	14.2	250.4	247.7	1.1	62	4.0
W. Europe – Total	601.1	174.3	264.3	107.1	70.5	1 217.3	1 168.9	4.1	372	3.3
U.S.S.R.	448.5	380.0	345.0	49.5	19.0	1 242.0	836.5	48.5	266	4.7
China	82.4	9.5	412.2	18.0	–	522.1	334.9	55.9	939	0.56
Japan	207.0	24.7	62.0	19.5	27.0	340.2	310.8	10.0	116	2.9
Africa	79.0	18.0	70.1	14.2	–	181.3	96.0	88.9	470	0.39
Rest of World – Total	1 807.4	746.1	1 606.7	247.8	128.7	4 536.7	3 457.1	31.2	3 381	1.3
World Total 1982	2 818.8	1 312.1	2 041.2	446.0	216.6	6 834.7			4 508	1.5
World Total 1972	2 592.4	1 045.0	1 629.4	325.5	38.4		5 630.7			
Change %	8.7	25.6	25.3	37.0	464			21.4		

Table G11
World primary energy consumption
In this table, 'America – Total' is equivalent to N. America and Latin America; but 'W. Europe – Total' is more than just U.K. and W. Germany; and 'Rest of World – Total' is more than just the total of the countries listed.
Source: *Based on* B.P. Statistical review of World energy, 1982. *British Petroleum Company P.L.C., 1983.*

a If the annual growth rate was one-tenth of the growth in ten years, what would be the doubling-time for the rate of consumption of each fuel?

b Is it reasonable to assume that the annual growth rate is the average rate over ten years? What are the problems in taking a shorter period?

c Use the growth rates for various fuels from **a** to calculate how long the recoverable reserves are likely to last if growth continues at these rates.

d What difference will the additional resources shown in table G10 make to your answers? Assume that all the additional resources are proved – but of course not all of these will be recovered. (Estimate this by assuming that the same proportion is recovered as in the table.)

e What, in general terms, would happen to the figures for recoverable reserves if the price of the fuel were to rise; or fall?

14(L) In question **9** it was assumed that the organism continued breeding at the same rate regardless of the impending lack of nutrient. In practice, the increasing scarcity of food would become steadily more apparent. This would increasingly inhibit the breeding rate.

The population–time graph might be something like the one in figure G24.

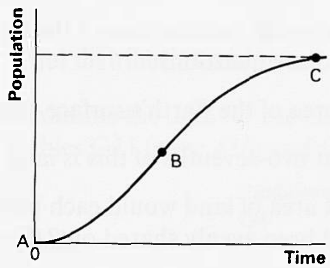

Figure G24

a What does the corresponding graph of breeding rate against time look like?

b What is the time corresponding to point B? (Conditions as in question **9**.) Is it 12 hours, somewhere between 12 and 22 hours, between 22 and 23 hours, or more than 23 hours? Why?

c What is the time corresponding to point C?

d What is the effect of the increasing shortage of food on the organism's life style or its 'intrinsic rights'?

Is there a crisis?

15(L) As fuels become more difficult to extract, the rate at which consumption grows will decrease.

a Sketch a graph showing how consumption (quantity of fuel used per unit time) varies with time for a constant growth rate of 2 per cent per annum. (Use a total time scale of about 4–5 doubling periods; it may help to refer back to question **10**.)

b On the same axes show how consumption varies with time when allowance is made for extraction becoming more and more difficult.

c What does the area under the consumption–time graph represent? It is possible to plot a range of such graphs with varying values of initial growth rate. What do you know about the total area under each one?

d If the maximum consumption is high what is the consequence for the lifetime of the resources?

e If we wish the limited resources of fossil fuels to last as long as possible when should we start reducing consumption?

16(E) At one time the number of horses in London was increasing exponentially and there were predictions that in a short time the whole city would be submerged in dung! Speculate first on the possible reasons why this did not happen, and then on whether there are parallels which can be drawn between that crisis and the present fuel crisis.

17(E) Make rough calculations of the following quantities, and answer any subsidiary questions:

a The area of the Earth's surface. (Radius of Earth $\approx$ 6370 km.)

b About two-sevenths of this is land – what is this area?

c What area of land would each person in the World have had in 1981 if it had been evenly shared out? (See table G8, page 434.)

d In fact, about one-fifteenth of the land area is cultivated. Grassland and forest account for a further quarter each. What accounts for the rest of the land area? What area would have been available per person in 1981 for food growing? Substantial numbers of people did not have enough to eat in 1981. This might suggest that the area available for food production is too small. Is this necessarily a reasonable conclusion? Why?

e What is the doubling-time for the World population? (See question **8**.) Which year is one doubling period after 1981; two doubling periods?

What would be the area available for food growing per person on these dates? Eventually this is likely to be too small, with obvious effects. It is not possible to decide when this will be without further data but the trend should be clear.

18(E)a 'Developed agriculture' seems to produce more food per unit area than less developed agriculture. Which columns of table G12 provide this evidence? Suggest two reasons for this.

Column Crop	1 Output/ $kW\,h\,ha^{-1}\,y^{-1}$	2 Ratio $\frac{\text{energy output}}{\text{energy input}}$	3 Output/ kg protein $ha^{-1}\,y^{-1}$	4 Input/kW h $(kg\ protein)^{-1}$
Rice, Philippines	797.3	16.5	11.4	3.17
Taro-yam, Tsembaga	388.9	16.4	5.6	4.25
Corn grain, U.S.A.	21 390.6	2.6	481.0	17.25
Wheat, U.K.	15 612.4	3.4	400.0	11.67
Bread (in shops)	12 223.2	0.53	350.0	67.51
Potatoes, U.K.	15 806.8	1.57	376.0	26.67
Pigs in gardens, Tsembaga	861.2	2.1	62.0	11.39
Battery eggs, U.K.	1 944.6	0.14	137.0	98.06
Milk (average), U.K.	2 778.0	0.37	129.0	57.78

Table G12
Efficiency and productivity of some food producing systems. ('ha' = hectare = $10^4\ m^2$).
Source: FOLEY, G. and NASSIM, C. The energy question. *Penguin, 1981.*

b Calculate, roughly, the energy input per hectare-year for rice growing, for growing corn in the U.S.A., and for battery farming. Are your answers consistent with those to part **a**?

c Is it reasonable to say that developed agriculture converts oil to food rather than converting sunlight to food? If so what are the implications for food production of depleting fuel resources?

Fuel distribution and use

19(E) Examine the data for the U.K., North America, U.S.S.R., and Japan in tables G11 (page 436) and G13.

Region	Production of fuels/Mtoe Oil	Natural gas	Solids	Water power	Nuclear	Total
N. America	557.5	514.3	502.7	149.1	87.4	1 881
Latin America	326.4	68.7	11.4	49.1	0.5	456
America – Total	883.9	583.0	514.1	198.2	87.9	2 267
U.K.	103.3	32.4	69.9	1.4	9.4	216
West Germany	4.3	14.6	89.4	5.5	14.2	128
W. Europe – Total	148.9	145.6	221.1	107.1	70.5	693
U.S.S.R.	612.2	451.3	355.5	49.5	19.0	1 488
China	101.7	10.0	417.2	18.0	–	547
Japan	0.3	1.6	11.6	19.5	27.0	60
Africa	222.4	30.4	87.8	14.2	–	355
Rest of World – Total	1 866.7	787.3	1 567.4	247.8	128.7	4 598
World (total)	2 750.6	1 370.3	2 081.5	446.0	216.6	6 865

Table G13
Fuel production by area, 1982.
Source: Based on B.P. Statistical review of World energy, 1982. *British Petroleum Company P.L.C., 1983.*

a Is each of these regions self-sufficient as far as total fuel requirements are concerned?

b What is the situation as far as individual fuels are concerned?

c What can be done in the short term to deal with any shortfall in supply?

d How is a continuing shortfall likely to affect decisions about fuel options?

20(E) Figure G12 (page 418) shows the relation between fuel use *per capita* and gross domestic product (GDP) *per capita.* It is sometimes claimed that the fuel use quoted ignores the fuel (food) consumed by the body and wood used for heating. The GDP allegedly ignores the food which is often the only product of less developed communities and which is ignored since it is used within these communities.

Make sensible, but quick and rough, estimates of:

a the energy equivalent of the food consumed per head per year;

b the energy provided *per capita* per year by burning wood (assume that the energy equivalent for burning wood is one-third of that for coal);

e transporting coal by small train from the U.K. coalfields to small, local power stations (80 km).

Method	Energy consumption/ $kW\,h\,tonne^{-1}\,km^{-1}$	Average speed/ $m\,s^{-1}$
Average pipeline	0.05	5
Barge	0.12	6
Cargo vessel	0.06	7
Supertanker	0.026	6
Local road delivery (petrol)	1.74	19
Road: 5 tonne diesel	0.81	25
Road: 20 tonne diesel	0.53	22
Road: 38 tonne diesel (articulated)	0.41	21
Rail: light train (1000 tonne)	0.25	33
Rail: heavy train (15 000 tonne)	0.10	21
Air: Boeing 707	6.86	225
Air: Boeing 747	4.88	230

Table G14
Energy needs in freight transport.
Source: RICHMOND, P. E. *and* FLOYD, T. *(eds)* Energy data sheets, *Southern science and technology forum, 1980.*
Original source: BOULADON, G. *Institute Batelle, 1974.*

24(L) Power stations are normally most efficient when running under full load. The variation in demand over a day means that there must be capacity to meet peak demand – but much of this will be out of use for most of the day. This is wasteful of capital equipment when it is standing idle and of the fuel needed to run the station up and down.

Assume that the demand shown in figure G13 (page 420) (upper line) is to be met from 'base load' stations which run for most of the time plus hydroelectric stations which provide for peak demand. The water for the hydroelectric stations is pumped into reservoirs at times when the base load station output is greater than demand. This is called a 'pumped storage system'.

a Estimate the 'base load' capacity which would allow this system to meet demand. How do you do this? What is the power output of the hydroelectric stations which will be required?

b If the water moves through 100 m difference in level, what volume of water would need to pass through the turbines each second to provide the peak power output? What area of lake 30 m deep would be required to provide the energy which the base load stations cannot provide? Is this a feasible arrangement?

c (*Hard*) The system considered in parts **a** and **b** would require more than half the 'base load' stations to be shut down in the summer. Would it be feasible to have a pumped storage system which kept base load stations running all the year round? (You will need to construct a possible demand curve over one year. For ease of calculation it should be a very simple shape. The values for system demand in figure G13 (page 420) can be assumed to apply to any year.)

25(P) Figure G15 (page 421) shows how demand for gas in the West Midlands Region of British Gas varied during three days at different times of the year. The North Sea gas producers require the supply rate to be nearly constant. Storage facilities must therefore be provided so that when the supply rate is greater than the demand, gas is stored, to be released when the opposite condition prevails. Some of this capacity is provided in a large storage ring main round the region consisting of 142 km of 0.6 m diameter pipe and 36 km of 0.75 m pipe. The minimum operating pressure is about 25×10^5 Pa (N m^{-2}) and the maximum normal operating pressure is about 55×10^5 Pa.

a What is the volume of the ring main?

b If gas is extracted from the main without being replaced what will happen to the main pressure?

c What volume of gas, measured at one atmosphere (10^5 Pa), would be needed to raise the pressure of the main to its minimum operating level?

d The uppermost line in the graph shows the most severe conditions for the system. From this graph determine roughly what volume of gas is available to be stored during the period when the supply rate is greater than demand.

e What pressure increase would storing this gas produce in the main? Is this acceptable? If not what proportion of the stored gas must be stored elsewhere?

26(E) Estimate or find out the fuel consumption of London and its area. What proportion of the insolation is this? (See question **30**.) What effect do you think the fuel consumption may have on the temperature in London? Does the evidence bear this out? Compare the ratio of fuel consumption to insolation for London with the average ratio for the Earth as a whole calculated in question **30**.

Population growth and rising expectations

27(E) How does the growth rate in population (question **8**) compare with the growth rate in fuel demand (question **13**)? Is it likely that the increase in fuel demand is due simply to rising population?

28(E) By what factor (approximately) would fuel demand rise on a global scale if *per capita* fuel consumption were to rise to Western European standards?

What would be the annual growth rate in demand due to this effect alone if this increase were to be achieved over 30 years?

29(E) (*Hard*) Using your answers to question **27** and **28**, comment on the combined effect of increasing population and increasing expectations on the growth in demand for fuels in the past. Is there evidence in table G11 (page 436) for expectations increasing faster in developing rather than developed countries?

30(E) The solar energy flux in space near the Earth is $1.4\,\mathrm{kW\,m^{-2}}$. Absorption in the atmosphere reduces this on average by 50 per cent.

Estimate the total solar energy delivered at the Earth's surface in one year. You need to take account of the day–night effect and of the fact that not all of the Earth's surface is at right angles to the radiation. (This can be done without using complicated methods.) (Radius of Earth $\approx 6370\,\mathrm{km}$.)

Compare your answer with the total global fuel consumption in the same time.

This may suggest that solar power is the solution to all our energy problems. Section G4 may 'throw some light' on this.

Energy from fission

31(L) Some idea of the energy available from the fission of a single nucleus of uranium can be obtained by considering either the interchange of mass and energy (using $\Delta E = c^2 \Delta m$) or the change of electrical potential energy as the fragment nuclei move apart.

This is just one example of fission:

$${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{236}_{92}\mathrm{U} \rightarrow {}^{96}_{37}\mathrm{Rb} + {}^{138}_{55}\mathrm{Cs} + 2{}^{1}_{0}\mathrm{n} + \text{energy}$$

a Look up the atomic masses of $^{235}\mathrm{U}$ and the neutron, preferably expressed in unified mass numbers, u. (Atomic mass of $^{96}\mathrm{Rb}$ is 95.932 58 u; atomic mass of $^{138}\mathrm{Cs}$ is 137.919 50 u.)
i Nuclear masses are needed for the calculation. Is it necessary to take into account the masses of the electrons in this case? (*Hint:* how many electrons would feature on each side of the equation?)
ii Find the change of mass per fission in kg.
iii Use $\Delta E = c^2 \Delta m$ to obtain a value for the energy released by this process.

b Assume that the total kinetic energy of the two fission fragments is equal to the loss of electrical potential energy as the two nuclei move apart.
i Write down the expression you would use to calculate the value of the electrical potential energy when the two nuclei are the diameter of a uranium nucleus apart (take this as $1 \times 10^{-14}\,\mathrm{m}$).
ii Assume that they move apart to a large distance so that all this potential energy is transformed to kinetic energy. Calculate the kinetic energy of the fission fragments.
iii Speculate on possible reasons for the difference between this answer and that for part **a***iii*.

c *i* Taking the value of 5×10^{-11} J per fission, calculate how much energy would be released by the fission of 1 kg of uranium-235. (You will need to calculate how many nuclei there are in one kilogram and to assume that they all disintegrate.)
ii How does this value compare with that for burning coal if about 1×10^{14} J are released by burning 4 000 tonnes of coal?
iii The 'natural abundance' of ^{235}U is 0.7 per cent. How much natural uranium metal would contain 1 kg of ^{235}U?
iv Assume that 0.1 per cent of uranium ore is uranium. How much ore would have to be mined to produce 1 kg of ^{235}U?
v Compare the masses of material which have to be mined to produce equivalent quantities of energy from fission of ^{235}U and burning coal.

Cross-section

32(L) Some idea of what is meant by 'nuclear cross-section', and how it might be measured for a particular element, can be obtained from the following simplified argument.

a Imagine a tube of the reactor core containing only one neutron, moving parallel to the tube axis, and one uranium target nucleus located somewhere in the tube (figure G25). Write down the chance of the neutron hitting the nucleus. (Assume that the neutron itself is of negligible size.)

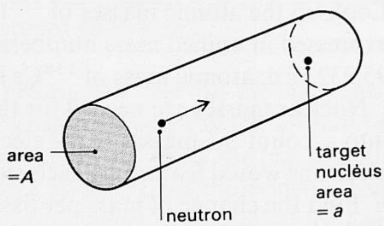

Figure G25
One neutron's chance of hitting a uranium nucleus.

b If the tube now contains n neutrons per cubic metre, travelling at speed v m s^{-1} how many neutrons could reach the target per second?

c If a thin slice of the tube, thickness d, contains N target nuclei per cubic metre, how many targets are there – and what is their total cross-sectional area? (Assume there is no 'shadowing'.)

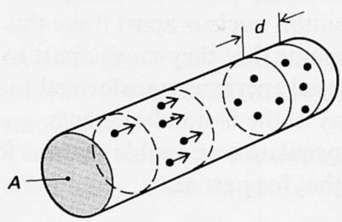

Figure G26
Many neutrons' chances of hitting target nuclei in a 'slice' of metal.

d What is the chance of a neutron hitting one of these target nuclei in one second?

e If R is the number of interactions (collisions) per second per unit volume of the target slice, obtain an expression for R in terms of a, n, N, and v.

f Now write an expression for the effective target area of a nucleus, a, usually called the nuclear cross-section.

g What sort of measurements would you need to make in order to calculate the nuclear cross-section for an element?

Fission of ^{235}U

33(R) The energy of a molecule in a gas at temperature T (kelvin) is, according to the kinetic theory of gases, $3kT/2$, where k is Boltzmann's constant. What is this energy in eV for a gas at 800 K? This is also the energy of a 'thermal neutron' at this temperature. How does it compare with the energy of a neutron released in fission of ^{235}U? (See table G7, page 426).

34(L) Figure G21 (page 429) shows how cross-sections of two nuclei vary with the energy of the neutrons which interact with them. This question is intended to show that the best conditions for fission of ^{235}U occur when the neutron energy is below about 0.1 eV.

a The neutrons arising from fission have energies of about 2 MeV. This is marked on the graph. (Notice that the neutron energy scale is logarithmic.)
i Read from the graph the cross-sections for neutrons with this energy for fission and for inelastic scattering with ^{238}U, and for fission with ^{235}U (notice that this scale too is logarithmic).
ii The figures give the relative probability of these interactions assuming that the nuclei are equally represented. For ease of calculation assume that there are one hundred times as many ^{238}U nuclei as ^{235}U nuclei. Obtain figures for the relative overall probabilities of the interactions.
iii In a similar way, find the cross-sections and the relative overall probabilities of fission of ^{235}U and capture by ^{238}U, for neutron energies of 10 keV, 100 eV, 1 eV, and 0.01 eV.
For which of these energies is a chain reaction most likely?
iv The uranium in thermal reactors has between 3 per cent (enriched) and 0.7 per cent (natural) of ^{235}U. What is the ratio of ^{238}U to ^{235}U for these extremes? Do these figures make any significant difference compared with the value of 100 used in parts *ii* and *iii*?

b While the neutrons are slowing down there is a substantial chance of them being captured by ^{238}U.
i How should the fuel elements and moderator be arranged in order to minimize the chance of capture by ^{238}U?
ii How might you explain the form of the ^{238}U cross-section curve between 10 and 200 eV? (*Hint:* critical potentials.)

G

Moderation and control

35(R) The neutrons emitted during the fission of ^{235}U have an energy of about 2 MeV and have to be 'slowed down' by a moderator for maximum probability of fission.

a Even if 3 neutrons are emitted (the average is about 2.5) this leaves most of the 200 MeV released on fission unaccounted for. What is the most likely form for the rest of the energy released?

b What happens to the highly-charged fission fragments and their energy?

c How are α-particles slowed down in a cloud chamber? Can we use that mechanism for neutrons? If not, why not?

d What process can be used to slow down the emitted neutrons from 2 MeV to about 0.02 eV needed for the best chance of fission?

(Questions **36** to **39** are about the details of this process.)

36(L) (*Hard*) A steel ball, A, of mass, m, and initial kinetic energy E_k has a head-on elastic collision with another ball, B, of mass M, initially at rest. Show that ΔE (the kinetic energy lost by A) is given by

$$\Delta E_{\mathrm{k}} = \frac{4mM}{(M+m)^2} E_{\mathrm{k}}$$

A more difficult problem is to show that this expression becomes

$$\Delta E_{\mathrm{k}} = \frac{2mM(1-\cos\theta)}{(M+m)^2} E_{\mathrm{k}}$$

if ball A is scattered through angle θ. Does this expression give sensible answers for glancing ($\theta = 0$) and head-on ($\theta = 180°$) collisions?

37(L)a For a neutron (mass m) colliding head-on with a nucleus of mass M assume that

$$\Delta E_{\mathrm{k}} = \frac{4mM}{(M+m)^2} E_{\mathrm{k}}$$

and derive an expression for the fractional energy loss of the neutron $\Delta E_{\mathrm{k}}/E_{\mathrm{k}}$.

b Taking the neutron's mass m to be 1 u, for what value of M does the neutron lose most energy?

c Draw up a table showing percentage energy lost by a neutron in collision with nuclei of masses 1, 2, 10, 12, 16, 112, and 238 u. Which elements are likely to have these nuclear masses?

d Which element seems to be the best moderator?

38(P) How many head-on collisions would be needed for a 2 MeV neutron to be slowed down to 0.05 eV (thermal energy) if graphite were used as a moderator?

(*Hint:* if the energy loss is 33 per cent then the energy of the neutron after one collision is $\frac{2}{3} \times 2 \times 10^6$ eV; after two collisions it is $\frac{2}{3} \times \frac{2}{3} \times 2 \times 10^6 \text{ eV} = (\frac{2}{3})^2 \times 2 \times 10^6$ eV. After n collisions it is $(\frac{2}{3})^n \times 2 \times 10^6$ eV.)

The actual number of collisions required is about 120. Suggest why your answer is less than this.

39(E) Steel balls always separate after a collision, but this is not so for neutrons and nuclei. The neutrons can be absorbed or captured to become part of the nucleus. The probability of capture is expressed as the neutron capture cross-section. Similarly, the probability of 'bouncing-off' a nucleus is expressed as the scattering cross-section.

a What cross-sections should a good moderator have?

b Choose two candidates for good moderators from table G15 and explain your choice. (The data relate to thermal neutrons.)

Nucleus	H	D	B	C	O	Cd	U
Scattering cross-section $\sigma_s/10^{-28}\,m^2$	20.436	3.390	3.6	4.75	3.76	5.6	8.90
Absorption cross-section $\sigma_c/10^{-28}\,m^2$	0.332	0.00053	759	0.0034	0.00027	2450	7.59

Table G15
Scattering and absorption (capture) cross-sections of various nuclei for thermal neutrons.

c If your choice does not agree with your answer to question **37d** explain why.

d Which of these elements would make good control rods?

Enrichment

40(L) Energy is released in the core of a particular reactor at a rate of 1000 MW. If each fission of uranium 235 produces 200 MeV:

a Calculate the number of uranium nuclei disintegrating per second.

b What mass of uranium 235 is 'used' per second?

c Estimate the mass of uranium 235 in the core if one-third of the fuel is changed each year.

d Assume that new fuel is enriched to contain 3 per cent of ^{235}U and that spent fuel contains only 0.8 per cent of ^{235}U. Estimate the total mass of the uranium in the core.

e Give reasons why a nuclear power station with an output of 1000 MW to the national grid would have a larger core than your estimate.

G

SECTION G3

CONSERVING FUEL IN THE HOME

The word ‘conservation’ is used in two different but linked senses in connection with energy. One has already been used earlier in this Unit – the physical law that says that energy cannot be created or destroyed but can be transformed from one form to another.

The other meaning is more colloquial, using as little as possible of a precious resource – in this case fuel. It is mainly this second meaning with which we will be concerned in this Section.

It is important to realize that we must keep on heating a house if we want the temperature indoors to remain higher than the temperature outside, even when we have warmed it to the temperature that we want. There is a flow of energy (associated with the temperature difference between inside and outside) through the walls, roof, windows, etc., and this energy has to be replaced in order to maintain the inside temperature. Once a steady temperature has been reached inside, the total rate of loss of energy from the house is equal to the rate at which it is being supplied inside. This energy is supplied by the central heating system or fires and heaters, as well as by other sources, such as cookers, lights, people, sunshine.

The following questions deal with a few of the things that must be considered when thinking about the flow of energy out of a house. The first question shows how the flow equations can be built up.

Thermal resistance

41(L) Thermal flow of energy through a block of material can be thought of as similar to flow of electric charge through a wire. Two similar situations are shown in figures G27 and G28.

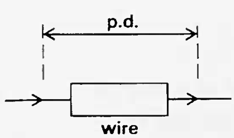

Figure G27

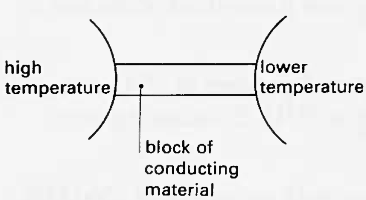

Figure G28

a What is the thermal equivalent of flow of charge?

b In figure G27 we might say that the potential difference drives current through the wire. What is it that drives energy through the block?

c The wire obeys Ohm’s Law, which can be written as:

Potential difference $\propto$ rate of flow of charge (provided the conditions remain the same).

i Rate of flow of electric charge may of course be referred to as ‘current’. Write down its units in two different ways.
ii What would be the analogous relationship for the block? Write down the unit for the righthand side of the relation in two different ways. What single word could replace the phrase on the righthand side of the relationship?

d What experiment could you do to try to justify the relationship for thermal flow of energy in c*ii*? In practice there are a number of difficulties with this sort of experiment. What difficulties can you think of and how would you try to overcome them?

e For a wire we can write:

$$V = IR$$

where V is the potential difference across the wire, I is the current through it, and R is the wire's electrical resistance.

For the block the analogous equation is:

$$\Delta\theta = \phi\mathcal{R}$$

where $\Delta\theta$ is the temperature difference across the block (*cf.* potential difference), ϕ is the rate of thermal flow of energy (power, *cf.* current), and $\mathcal{R}$ is the thermal resistance of the block. What are the units of $\mathcal{R}$?

The idea of thermal resistance is important. The greater the thermal resistance of the walls and roof of a house, the smaller the rate of loss of energy from that house, and so the lower the rate at which energy must be supplied to maintain a given difference between the inside and outside temperatures.

f In order to compare the electrical resistance of different materials we use the resistivity, ρ, which is the constant of proportionality in the equation:

$$R = \frac{\rho l}{A}$$

where R is the resistance of a wire of length l, and cross-sectional area A.

What are the units of ρ?

g Sometimes electrical conductivity, σ, is used instead of electrical resistivity, ρ. The equation for electrical resistance can then be written:

$$R = \frac{l}{\sigma A}$$

For thermal flow of energy the thermal conductivity, k, of a material is often used rather than its resistivity. Since resistivity = 1/conductivity the equation for thermal resistance becomes:

$$\mathcal{R} = \frac{l}{kA}$$

What are the units of k?

The following questions use the equations:

$$\mathcal{R} = \frac{l}{kA}$$

$$\Delta\theta = \phi\mathcal{R}$$

G

42(P) A furnace wall, 0.5 m thick, is made from brick of thermal conductivity $0.8\ \mathrm{W\,m^{-1}\,K^{-1}}$. If the internal temperature is 850 °C and the outside temperature is 30 °C find:

a The thermal resistance of a section of wall of area one square metre.

b The power loss through this section of wall.

43(L) Consider a brick wall measuring 4 m by 3 m containing a single-glazed window 1 m by 1.5 m. If the thickness of the brick is 0.1 m and the thickness of the glass is 4 mm, find:

a The thermal resistance of the glass. ($k_{\mathrm{glass}} = 1\ \mathrm{W\,m^{-1}\,K^{-1}}$.)

b The thermal resistance of the brick. ($k_{\mathrm{brick}} = 0.6\ \mathrm{W\,m^{-1}\,K^{-1}}$.)

c If the outside temperature is 0 °C and the inside temperature is 20 °C what is
i the power loss through the brick?
ii the power loss through the glass?
iii the total power loss through the wall and the window?

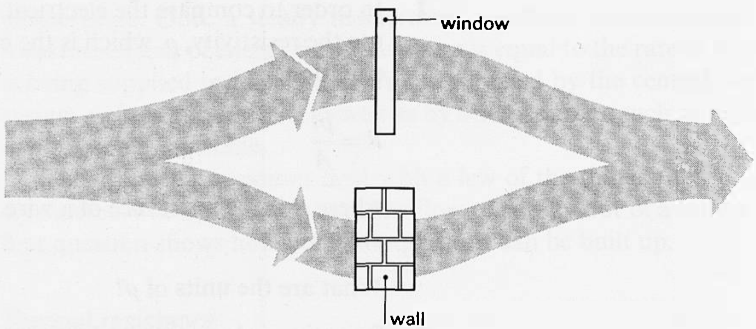

Figure G29
Paths of thermal flow of energy through a window in a wall.

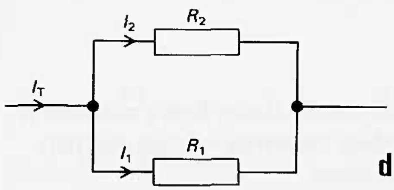

Figure G30
Electrical analogue of a window in a wall.

Notice that some energy goes through the window and *other* energy goes through the wall. Hence the brick wall and glass window represent thermal conductors in parallel.

d The electrical equivalent would be as shown in figure G30.
i What is I_T in terms of I_1 and I_2?
ii How does this compare with your calculation of the total power loss through the wall and window in part **c**?

44(P) The brick wall in question **43** is now insulated with a 10 mm layer of fibre board of thermal conductivity $0.05\ \mathrm{W\,m^{-1}\,K^{-1}}$.

a Calculate the thermal resistance of the fibre board.

b Using the analogy with electricity calculate the new power loss through the brick and fibre board section of the wall.

45(P) A cylindrical hot water tank is 0.5 m in diameter and 0.8 m high and made from copper 3 mm thick.

a If the water inside is at 70 °C and the outside temperature is 20 °C calculate the rate of loss of energy from the tank. (Thermal conductivity of copper = 385 W m^{-1} K^{-1}.)

(*Note.* The answer to this question will seem very large. The reason for this will become apparent later.)

b The tank is now insulated with a glass fibre quilt, 20 mm thick and of thermal conductivity 0.04 W m^{-1} K^{-1}. Estimate the rate of loss of energy now.

c Since the thermal conductivity of air is 0.024 W m^{-1} K^{-1}, which is less than that for the glass fibre quilt, suggest why it is beneficial to replace air with glass fibre. (Don't spend long on this now since it reappears later.)

Thermal resistance coefficient, *X*

Heating engineers find it convenient to use a quantity which is a property of unit area of a particular thermal conductor (for example, standard cavity brickwork or single-glazed window). One such quantity is the 'U-value' of the wall or window, which is the rate of thermal flow of energy per square metre for a temperature difference of one degree. If we think in terms of thermal resistance instead of thermal conductivity we can define an alternative property – the 'thermal resistance coefficient', *X*. This is the thermal resistance of unit area of the material. The area which matters in each case is, of course, the area perpendicular to the flow of energy. Because of the analogy between thermal and electrical resistance it is convenient to use thermal resistance coefficients rather than U-values in this course.

G

46(L)a If the thermal resistance coefficient of one square metre of a conductor is X, what is the thermal resistance $\mathscr{R}$ of 2, 3, or A square metres?

b Hence write down the relation between $\mathscr{R}$, X, and A.

c What are the units of X?

d By substituting for $\mathscr{R}$, obtain a relation between X, l, and k.

47(L)a What is the thermal resistance coefficient of a brick wall 0.1 m thick if the thermal conductivity of brick is 0.6 W m^{-1} K^{-1}?

b What is the thermal resistance coefficient of a timber wall 20 mm thick if the thermal conductivity of timber is 0.15 W m^{-1} K^{-1}?

c As far as thermal insulation is concerned which of the two walls above would be better?

48(P)a Calculate the thermal resistance coefficient for 4 mm thick glass and hence calculate the thermal resistance of a single-glazed window 2 m × 1 m with glass 4 mm thick. ($k_{\text{glass}} = 1$ W m^{-1} K^{-1}.)

b What is the power loss through this window when the outside temperature is 0 °C and the inside temperature is 20 °C?

c Using your general knowledge of the heating requirements of rooms, say whether your value is about right, much too high, or much too low.

Surface resistance

If the power loss found in the previous question were correct our heating systems would need to be much more powerful than they are.

HOME EXPERIMENT GH2
Surface layer

As you may know from experience the surface of a window pane is not at the general temperature of the mass of air near to it. In fact, the temperature variation near the window is rather as shown in figure G31.

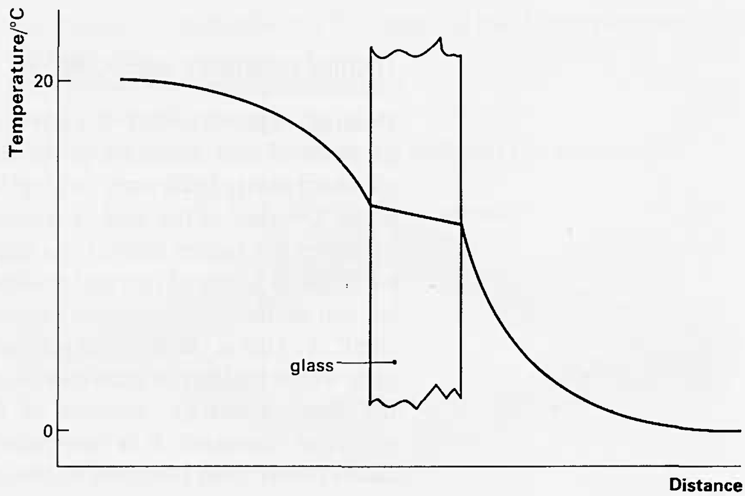

Figure G31
Temperature variation near a single-pane window.

Near each glass surface there is a surface layer of air with a temperature drop across it. This surface layer presents a resistance to the energy which must flow through it from the interior to the exterior. This resistance is called the surface resistance.

The way the temperature varies in the surface layer and the distance over which it varies depend on a number of factors, so it is usual to use measured values of surface resistances rather than to try to calculate them. Table G16 lists some values which are simpler than those which a heating engineer might use, because they ignore differences which might be produced by the textures of different surfaces (glass or brick), for example.

49(E) Suggest some factors which may affect the value of the surface resistance coefficient. Examination of table G16 should help with some of these, for instance compare **A** and **B**; **D** and **E**; and **C** and **D**.

Table G16
Some common values of surface resistance coefficients.

		Surface resistance coefficient/$m^2\,K\,W^{-1}$
A	Internal wall or window	0.13
B	External wall or window	0.06
C	External roof	0.05
D	Internal floor/ceiling (upward energy flow)	0.11
E	Internal floor/ceiling (downward energy flow)	0.15

50(E)a Do the thermal resistance of a glass window and its surface resistances act in series or in parallel?

b Calculate the total thermal resistance coefficient for the window in question **48**. By what multiple, approximately, is the power loss calculated in question **48** changed now? *Note:* the values of surface resistance coefficients for walls and windows are not usually the same but you should assume that they are for the purpose of the question.

c You can now use the electrical analogy to calculate the temperatures of the surfaces of the glass. (This calculation assumes a linear variation of temperature with distance. Figure G31 shows that this is not strictly so. The answers to this and similar questions must therefore be approximations.)

Cavities

Cavities between layers of brickwork (cavity walls) have been used in buildings for some time, though cavities between layers of glass in windows (double glazing) are more recent. Double glazing has the express purpose of reducing energy flow. Cavity walls were introduced to prevent water penetration but may have the added benefit of reducing energy flow. These next questions examine how effective cavities are in doing this.

Key

Hor ↑ means a horizontal cavity with energy flow upwards.
Similarly Hor ↓ means a horizontal cavity with energy flow downwards.

Faced means that one face of the cavity is covered with aluminium foil.

Orientation	Surface	Resistance coefficient/$m^2\,K\,W^{-1}$ Width of cavity d/mm			
		5	20	60	100
Vert	unfaced	0.11	0.18	0.18	0.18
Hor ↑	unfaced	0.11	0.15	0.16	0.16
Hor ↓	unfaced	0.11	0.18	0.21	0.22
Vert	faced	0.18	0.57	0.57	0.57
Hor ↑	faced	0.18	0.35	0.42	0.43
Hor ↓	faced	0.18	0.63	1.45	1.58

Table G17
The thermal resistance coefficients of cavities for various orientations, surface finishes, and thicknesses.

51(L)a Using the same axes, plot graphs of thermal resistance coefficient against cavity thickness for the different cavity orientations and linings (table G17).

b On the same axes plot a graph showing how you would expect the thermal resistance coefficient of air to vary with thickness if it behaved like a slab of insulator ($k_{air} = 0.024\ W\,m^{-1}\,K^{-1}$).

G

c What would be the thermal resistance coefficient of the cavity if it acted simply as two internal surfaces? (Use table G16.) How would this value vary with the thickness of the cavity? Plot this information on the same axes.

d Some patterns should now be apparent. Make some brief notes to describe them.

e Under what conditions does the cavity behave most like a slab of conductor?

f Compare the graphs for faced and unfaced cavities and say what effect lining one face of the cavity with aluminium foil has on the thermal resistance coefficient. Would this happen if the only mechanism for transferring the energy were conduction? Could another mechanism, such as convection or radiation, account for these differences in the graphs, and is this mechanism an important one? Explain your answers.

g Compare the graphs for horizontal cavities with upward and downward energy flows and suggest a mechanism of energy transfer which could account for the differences. How important is *this* mechanism?

h A cavity could be regarded as just two internal surfaces. Does a cavity behave like this?

i How do the measured values of thermal resistance coefficient compare with the value calculated for conduction only?

j If each mechanism of energy transfer is represented by its own resistance, how should the resistances be connected in order to produce this result?

Double glazing

It should now be clear that air in a cavity behaves quite differently from the same thickness of a solid, largely because it is a fluid in which convection and radiation take place as well as conduction.

Analysing such a system in detail would be very difficult because there are many factors which can affect the value of the cavity resistance. Fortunately, in practice, a limited number of types of cavity is used and it is possible to specify a definite cavity resistance for each situation.

Figure G32 summarizes the coefficients needed for calculations of thermal energy transfer across cavity systems. (There may, of course, be more than one cavity in the system, or none at all.)

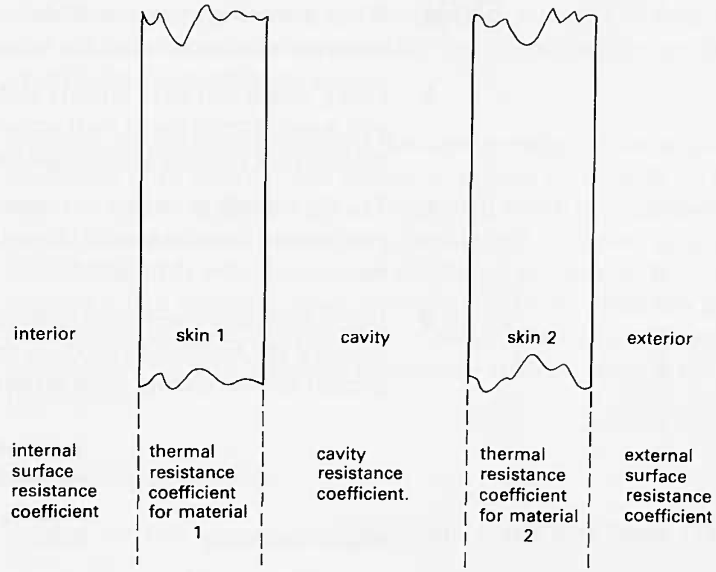

Figure G32
Summary of resistance coefficients required for calculating energy flows through cavity systems.

52(P) In questions **48** and **50** you calculated the power loss through a $2\,\mathrm{m} \times 1\,\mathrm{m}$ glass window, first without taking surface layers into account, then with. Now imagine that the window is double-glazed. Assume that the cavity resistance coefficient for a 15 mm air gap is $0.14\,\mathrm{m^2\,K\,W^{-1}}$.

a What other resistance coefficients need to be considered in describing this system?

b Calculate the power loss through the double-glazed window.

c Use the electrical analogy to find the temperature differences across all the resistances in the system.

d Draw a labelled graph, similar to figure G31 (page 452), to show how temperature varies with distance near this window.

e Is it permissible, using this analogy, to calculate the temperature at a distance, say, 3 mm from the outer surface of the window? Explain your answer.

53(E) Explain why a heavy curtain can increase the total thermal resistance of a single-glazed window by a significant amount.

54(E) Why do some householders have the cavities in the walls of their houses filled with glass fibre, foam, or insulating beads? Is this consistent with what you have learned?

You should now have a clearer understanding of why a glass fibre quilt is a better insulator than just the air around a hot water cylinder. (See question **45c**.)

G

55(E)a When answering question **49** did you suggest exposure as one of the factors which might affect the value of surface resistance coefficient?

b Using values you have already met, calculate the resistance coefficient you would expect for at least one of the types of window listed in table G18. Compare your answer with those in the table.

c The figures will probably not match exactly because you have ignored yet another factor – which? (*Hint:* does wood appear to be a better or worse conductor than glass?)

d If you were employed to advertise double glazing, how would you design a test window to produce results which, while correct, would present double glazing most favourably?

Window construction		**Total thermal resistance coefficient/$m^2\,K\,W^{-1}$** **Exposure** *sheltered*	*normal*	*severe*
Single glazed	wood frame	0.26	0.23	0.20
	metal frame	0.20	0.18	0.15
Double glazed	wood frame	0.43	0.40	0.37
	metal frame	0.33	0.31	0.29

Table G18
The total thermal resistance coefficients (including surface and cavity resistances where appropriate) of single- and double-glazed windows under different conditions.

56(P) A cavity wall measures $2\,m \times 6\,m$ and has a double-glazed window in it measuring $1.5\,m \times 3\,m$. Use the following data to answer the questions below.

External temperature $= 2\,°C$
Internal temperature $= 18\,°C$
External surface resistance coefficient $= 0.06\,m^2\,K\,W^{-1}$.
Internal surface resistance coefficient $= 0.13\,m^2\,K\,W^{-1}$.
Cavity resistance coefficient for the window $= 0.14\,m^2\,K\,W^{-1}$.
Thermal conductivity of glass $= 1\,W\,m^{-1}\,K^{-1}$.
Thickness of glass $= 4\,mm$.
Resistance coefficient for cavity brickwork of overall thickness 260 mm $= 0.67\,m^2\,K\,W^{-1}$.

a Draw a diagram (in a similar way to an electrical circuit) showing all the thermal resistances, whether they are in series or parallel, and their values.

b Find the total thermal resistance of the brick section of the wall.

c Find the energy flow through the brick section.

d Find the temperature on the inside surface of the brick section.

e Find the total thermal resistance of the glass section of the wall.

f Find the energy flow through the glass section.

g Find the temperature on the inside surface of the glass section.

h Suppose the air inside the room became very humid. Use your answers to **d** and **g** to say whether you would expect condensation on the brick or glass first. Explain your answer.

57(P) Building regulations specify that the average value for the resistance coefficient of an external wall should be at least $1.6\,m^2\,K\,W^{-1}$. (This includes surface resistance.) Consider a wall which just satisfies this requirement and comprises a brick section and a window section. The brick section has a total resistance coefficient of $2.0\,m^2\,K\,W^{-1}$. The window is of a wooden-framed, double-glazed type which has a total resistance coefficient of $0.4\,m^2\,K\,W^{-1}$ under normal conditions. If the total area of the wall including the window is $20\,m^2$, what is the area of the window?

The heating of buildings

58(L)a Suggest suitable temperatures for points A and B in figure G33.

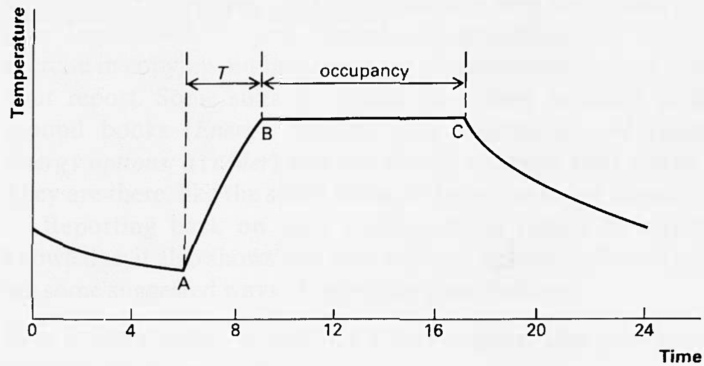

Figure G33
The temperature variation inside a building against time during a 24-hour period. The heating system is switched on at A and off at C.

b Compare the rate of loss of energy from the building at A and B.

c What physical factors concerning the construction of the building will affect the time lag, *T*?

d What is the disadvantage of constructing a building with a large time lag?

e We expect the temperature to continue to rise at B. This may be prevented by opening windows. Suggest a more economical way of achieving a constant temperature. Would your suggestion have any disadvantages?

f Redraw the graph to show how the temperature in the building would change if the heating system remained on at full power from A to C and no attempt was made to ventilate the building.

g Draw a graph to show how the temperature inside a lightly constructed hut, without ventilation, would change if a powerful electric heater were to be switched on.

h Suggest some of the factors an architect should consider when designing a building in order to heat it economically.

59(P) An occupied room requires ventilation. Calculate the power needed to warm air (from 0 °C to 20 °C) in a room 5 m × 6 m × 3 m which has two complete air changes per hour. Compare this with the loss of energy through a closed window as calculated in questions **50** and **52**. (Heating capacity of air per unit volume = $1.3\ \mathrm{kJ\,m^{-3}\,K^{-1}}$.)

60(R)a It is recommended that the air change in a school should be $0.014\ \mathrm{m^3\,s^{-1}}$ for each person in the room. Estimate the power required to maintain the air temperature in winter in your classroom, with this rate of ventilation.

b If each person converts the energy from food at a rate of 100 W, what percentage of the power needed is supplied by the occupants?

SECTION G4
ENERGY OPTIONS

In this course you can only touch on the large amount of information available on energy sources. But while you cannot study it all you ought to know something about its range and be familiar with at least part of it. This is what this Section is designed to achieve.

You are expected to find information by reading, and later report on your findings in some suitable way. To help you get started, each major option (for example, solar energy) has been divided into a number of smaller topics (passive devices; active devices; photovoltaic devices; biomass; wind energy). Some study items are suggested for each topic; you should tackle these items at least.

Part of the skill of reading for learning is to generate new questions (Do I understand this? What is it I don't understand? What is X (or Y or Z) that is referred to? Is it important? Where can I find out more about it?). Another important skill is to recognize new topics and to appraise their importance for you. You should remember that this is not an exercise in copying out large sections of material from your sources into your report. Some suitable articles have been included in the background books (*Energy sources: data, references, and readings* and *Energy options: a reader*), but you should not limit your search to these. They are there, like the study items, to help you to get started.

Reporting back on your findings helps others to broaden their knowledge; it also shows you how well you understand your topic. Here are some suggested ways of reporting your findings.

Give a short verbal report: not a very original idea perhaps, but it is effective if well prepared.

Give out a short duplicated report. You will probably also need to speak about the report or be able to answer questions on it.

Both of these would effectively be articles, written by you, presenting the most important findings. Much of the detail could be organized into appendices which you could keep for reference purposes.

You might make slides of appropriate features (wind turbines, power stations, etc.) or collect illustrations and use these as the centrepiece of your talk. This might even be extended to the production of a tape–slide sequence suitable for use in general studies, or for a younger class.

Display your report on a noticeboard alongside all the others from your group. This way of communicating is often used at scientific conferences and is called a 'poster session'. It is especially important to present the report well and highlight the main points clearly.

STUDY ITEMS

To save repeating items in each option, here is a list of the characteristics of energy sources discussed in Section G1 and which should be

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considered in connection with each option (except for the few where they are specifically excluded).

1 *Intensity or energy density* – how intense is the energy source? What implications does this have for its use?

2 *Time* – is the option available at any time it might be required? If not, what limitation does this place on its use?

3 *Transportability* – is the energy source available everywhere, is it easily transported, or is it limited to particular places? What implications does this have for its use?

4 *Feasibility* – can the option be implemented at once? Alternatively does it need new technology; or does it need a large input of money or energy before it can be implemented; or do people need to be persuaded about its value? What are the implications for its use?

To help to remind you that these four factors should be considered for most options, the other study items for each topic are numbered from 5 onwards. You may not find it sensible to tackle the study items in numerical order. Try reading them all through to start with, but come back to them every so often.

Finally, since the topics vary considerably in length and difficulty you may well need to attempt more than one.

NUCLEAR POWER

A Nuclear power stations

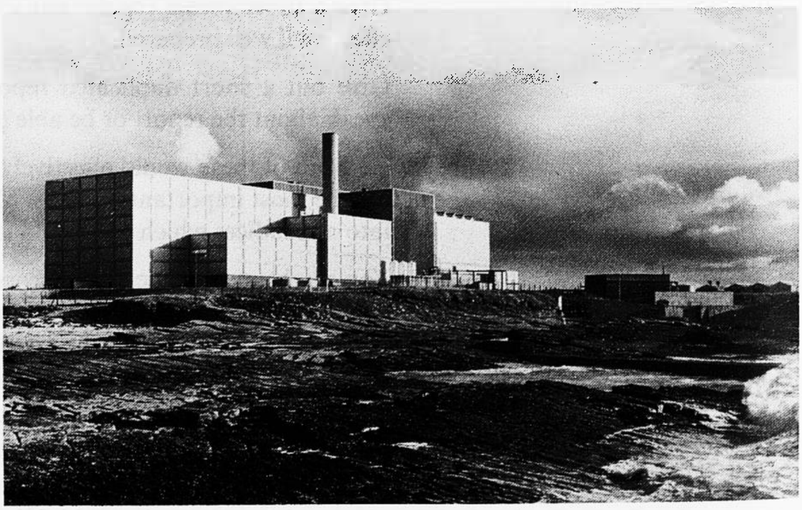

Figure G34
The UKAEA Prototype Fast Reactor at Dounreay, Caithness, Scotland.
United Kingdom Atomic Energy Authority.

5 Choose one particular type of reactor and study it in detail. There are many types to choose from, including: Prototype Fast (PFR) (figure G34), Advanced Gas-cooled (AGR) (figure G35), Pressurized Water (PWR), and Fast Breeder (FBR). How is energy extracted from the reactor you have chosen to study?

Figure G35
Work on Reactor 2 of the Advanced Gas-cooled Reactor, Hartlepool.
United Kingdom Atomic Energy Authority.

6 How are the principles described in Section G2 put into practice?

7 What is the moderator (if any), the coolant, the fuel? How are these arranged in the reactor?

8 What are the operating conditions and the efficiency?

9 What are the safety requirements?

10 What are the World's resources of suitable fuels?

11 What factors determine the cost of the energy produced?

12 What environmental implications does this option have?

B Nuclear fusion

5 Find out what the possible fusion reactions are. Which seems to be the most promising? Are there suitable fuel resources? How large are they?

6 The principal requirement for a fusion reactor is high temperature. Why? What is the Lawson criterion?

7 Choose one suggestion for a practical reactor and find out what you can about it. How is energy extracted from such a reactor?

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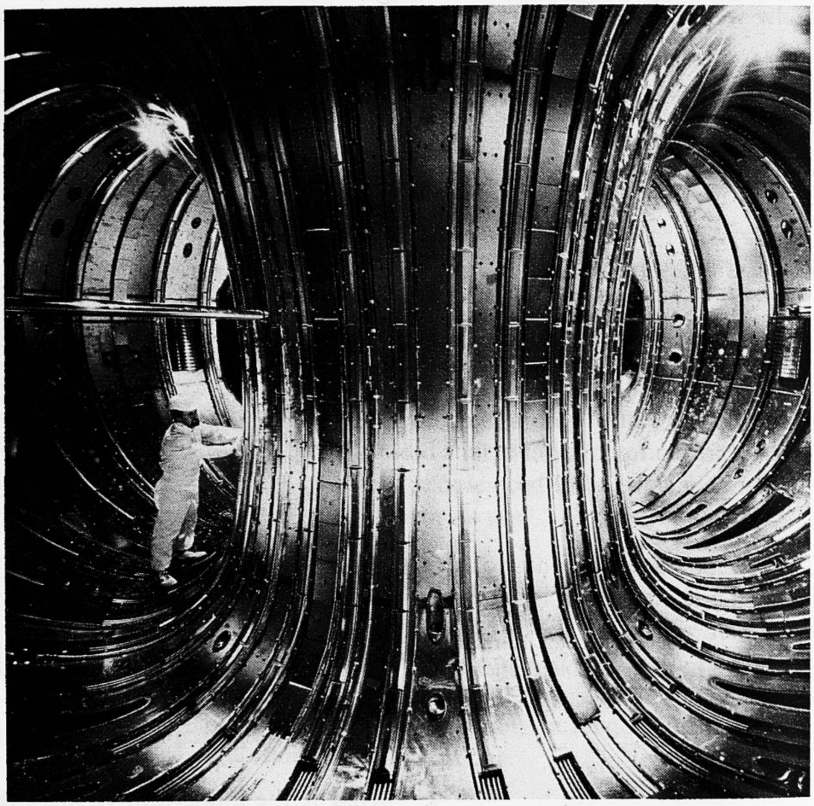

Figure G36
The interior of the vacuum vessel of the World's largest toroidal fusion experiment, the Joint European Torus. JET is staffed and funded by many European countries.
JET Joint Undertaking.

8 What safety precautions are necessary?

9 What factors will determine the cost of the energy produced?

10 What environmental implications does this option have?

C Nuclear safety

Since this is not an energy source, the four general study items are not appropriate to this topic.

1 What are the biological effects of various radiations? How are doses and dose rates measured, *i.e.*, what are the units and what instruments are used?

2 Trace the path of nuclear fuel from mining the ore to its use. (The nuclear fuel cycle.) What are the dangers at each stage?

3 What are the hazards in the operation of a nuclear power station? How are they minimized? How do they compare with other hazards – natural and Man-made?

Figure G37
Nuclear waste management.
If all the electric power, both domestic and industrial, that one person used in a lifetime were generated by nuclear power alone, the long-lived radioactive wastes that would be produced could all be incorporated in a piece of glass this size. *United Kingdom Energy Authority.*

4 The waste products from reprocessing the fuel present two main hazards: **a** radioactivity; **b** heating the environment. How does **b** arise from **a**? These are problems partly because of their intensity and partly because of their duration. Find out about them.

How are the hazards dealt with in the short, medium, and long term?

SOLAR ENERGY

D Passive solar devices

5 What is the common characteristic of passive systems? How large a contribution to our energy needs could this resource make?

6 How can buildings be designed to maximize the input of solar energy? (You need to consider at least the orientation of the building; the position and size of windows; the construction of walls.)

7 How can buildings be designed to conserve the energy which they gain? (You need to consider at least the position and size of windows; the construction of the building, particularly from the point of view of insulation and retention of internal energy.)

Figure G38
Roof panels to utilize solar energy.
United Kingdom Atomic Energy Authority.

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8 What factors determine the cost of the energy?

9 What environmental implications does this option have?

E Active solar devices

5 What are the common characteristics of these devices? How large a contribution to our energy needs could such devices make?

6 How is the input of energy maximized? You will need to consider at least the following: coating the collector; covering the collector with a transparent cover; orientation of the collector; use of reflectors; control systems.

7 What factors determine the cost of the energy?

8 What environmental implications does this option have?

F Photovoltaic devices

5 What are solar cells? How are they made? How do they work? How large a contribution to our energy needs might be made by photovoltaic devices?

6 What are the characteristics of solar cells? You will need to consider at least the following: e.m.f.; power output; impedance; spectral response; efficiency. It may be possible to investigate some of these experimentally.

7 What factors determine the cost of the energy?

8 What environmental implications does this option have?

G Biomass

5 What is biomass? By what process is it produced? How much fuel could be produced? What is the efficiency of the process?

6 Here are some possible systems: growing crops which are intended to be burned as fuel; growing crops which are processed to produce fuel; growing crops for food but using waste products for fuel; using naturally growing vegetable material (rather than cultivated crops) for fuel.

Give examples of these. Are there other systems? How successful are they? What is the potential size of the resource?

7 What are the advantages and disadvantages of this option?

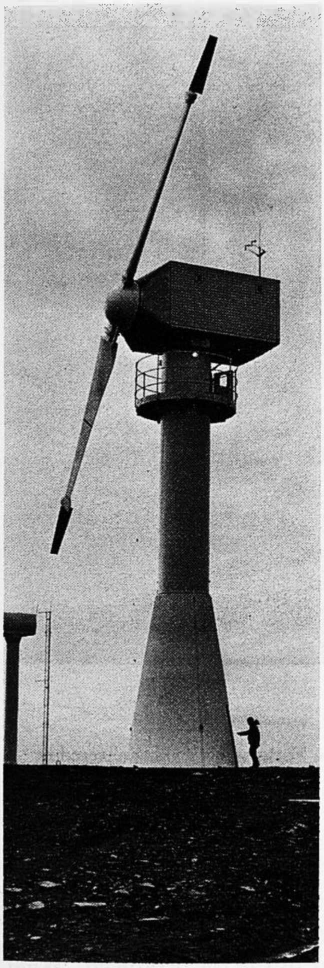

Figure G39
22 kW aero generator in Orkney.
Glyn Satterley.

H Wind energy

5 Prove that the maximum power which could be developed by a wind turbine $P = \frac{1}{2}\rho A v^3$, where ρ is the density of air, A is the area swept out by the blades, and v is the wind speed. Not all of this power can be extracted. Why not?

6 What does the equation in item **5** tell you about designing a turbine to produce as much power as possible? What are the practical limitations on this?

7 How does wind speed vary with place, time, and height above the land surface? What contribution could this resource make to our energy needs?

8 What are the advantages and disadvantages of the various designs of turbine?

9 What factors determine the cost of the energy?

10 What environmental implications does this option have?

FOSSIL FUELS

I Power stations

5 Make yourself familiar with the components of a fossil-fuelled power station and their functions. You should consider at least the following: boiler; cooling system; high- and low-pressure turbines. How do fossil-fuel and nuclear power stations differ?

6 What is done to maximize efficiency? What places limits on efficiency?

7 Find out about 'Combined heat and power'. How does the system work? What are its advantages and disadvantages?

8 What are 'the carbon dioxide problem' and 'acid rain'? Find out about them. How big a contribution do power stations make to these problems? What other environmental implications do power stations have?

9 Remind yourself of the size of the resources which are available. What are the advantages and disadvantages of using fossil fuels to generate electricity?

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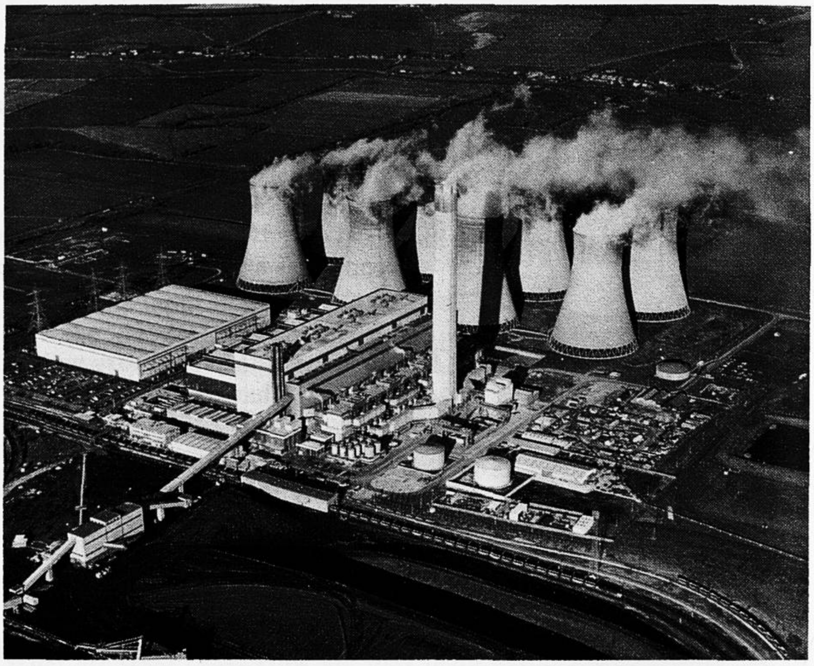


Figure G40
The 2000 MW coal-fired Eggborough power station in Yorkshire. Coal is delivered by rail on the merry-go-round system.
Central Electricity Generating Board.

J Alternative sources of fossil fuels

5 *Oil shale.* What is it? How big a resource of oil is it? How is the oil extracted? What factors determine the cost of the oil produced? What environmental implications are there in its use?

6 *Coal liquefaction.* How is it done? How big a resource of oil is it? What factors determine the cost of the oil? How would its use affect the lifetime of coal as a resource? What are the environmental implications of coal liquefaction?

7 *Coal gasification.* What methods are available? How big a source of gas is it? What advantages does it have? Does it have any disadvantages?

OTHER SOURCES OF INTERNAL ENERGY

K Geothermal

5 What is the origin of geothermal energy? What size is the resource? How does its availability vary with factors like geographical area, geological structure, and depth?

6 Is the energy used directly for heating or is it converted into electrical energy? What factors determine this choice?

7 What techniques are used for extracting geothermal energy? What factors determine the cost of the energy?

8 What are the arguments for regarding geothermal energy as a renewable or a non-renewable energy source?

L Heat pumps

5 What is the principle of the heat pump? What does 'coefficient of performance' mean?

6 What are common applications of heat pumps? Why is the coefficient of performance different for different applications? What factors determine the cost of the energy?

7 What are the advantages and disadvantages of heat pumps?

M Combustion of refuse

5 What size is the resource?

6 Is it a viable proposition? Have any countries used it? Which? How do they use it?

WATER POWER

N Hydro-power

5 What size of resource is available? How is it distributed over the Earth's surface?

6 Sometimes the power is utilized directly as it has been in the past. What methods are used and what conditions are required? What limitations does this place on the usefulness of the energy source?

7 Hydro-power is sometimes used for generating electricity. What factors determine the cost of the electricity produced?

8 What environmental implications does this option have?

O Wave energy

5 What size of resource is available? How is it distributed over the Earth's surface?

6 What methods have been proposed for utilizing this resource? Describe one of these in detail.

7 What factors determine the cost of the energy produced?

8 What environmental implications does this option have?

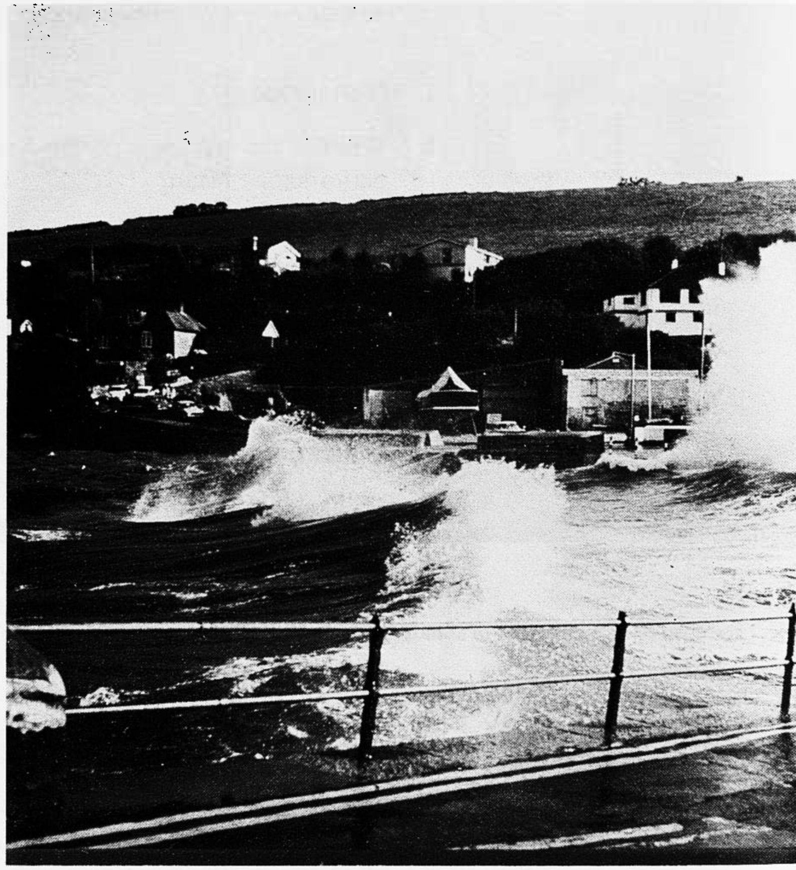


Figure G41
Waves.
Picturepoint – London.

P Tidal energy

5 How are tides produced? Why does their size vary in different places and at different times?

6 How is tidal energy utilized? What conditions are necessary for this? Study the installation on the estuary of the river Rance in France.

7 What determines the cost of the energy produced?

8 What environmental implications does this option have?

CONSERVATION

Conservation is not, of course, a source of fuel or of energy. However, saving a quantity of fuel which would otherwise be consumed is equivalent to discovering a new source for that quantity of fuel. The effect of conservation is to enable limited resources of fuel to last longer than they would otherwise.

It is therefore important to study conservation together with energy sources. However, since the four basic study items relate more to the fuel which is saved than to the process of conservation, these items will not be included here.

Q Domestic

1 What methods are available to save fuel in the home?

2 What savings in fuel cost can each produce in a year? Are there other advantages?

3 What does each cost to install? Are there other disadvantages?

4 What is the payback time? Is each method viable?

5 How can better control of heating systems save fuel?

R Industrial

1 How much fuel is used in industry in the U.K.? What proportion is used for process heating compared with space heating?

2 Compare the amount of energy needed to produce different materials, such as steel, aluminium, cement, and paper. How can savings be made?

3 Prepare a case study on process heating (breweries are an example).

4 How can the redesign of products or the re-cycling of materials lead to reduced energy costs? (One example is packaging for food and drink.)

5 What can be done to reduce the energy required for space heating and lighting in industrial buildings, while maintaining or improving working conditions (for example 'heat wheels')?

6 How have improvements in the efficiency of combustion helped fuel conservation?

S Transport

1 How much fuel is used for transport? What are the energy costs of different systems of transport?

2 How can improvements in design of vehicles help to conserve fuel (for example reduction in mass; drag; improved carburation; use of diesel engines)?

3 What alternatives to oil-based fuels for transport are possible? What are their advantages and disadvantages?

4 What are the implications of increased use of the more fuel-efficient transport systems (such as public transport, increased numbers of passengers in cars)?

T Storage

This is not an energy source so the four general study items listed on page 460 are not relevant here.

1 Why and where is energy storage important?

2 Here are some possible energy storage systems: chemical, for example, rechargeable cells; potential, for example, pumped storage hydroelectric schemes; kinetic, for example, flywheels; internal, for example, stores for solar energy; electrical, for example, capacitors; magnetic, for example, magnetic fields.

Find additional examples of the various categories given. Can you find other categories?

3 What is the energy density of a store? What values are obtained for the systems listed in 2? What other criteria are important in choosing an energy store?

HOME EXPERIMENTS

GH1 The cost of a 'cuppa'

Use the gas or electricity meter in your home to measure the quantity used to boil 1 kg of water. Work out the efficiency of the process (specific heat capacity of water is $4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$). Look at a recent gas bill for information about the total energy transformed when $1\,\mathrm{m^3}$ of gas is burned, and its cost; or look at an electricity bill to find the cost of one unit (1 kW h).

Compare the cost and time taken, as well as the efficiencies. Use the data given in figure G4 (between pages 430 and 431) to compare the quantities of primary fuel used in the two heating processes.

GH2 Surface layer

If there are surface layers near walls and windows, then the temperature at the surface will not be the same as that of the bulk of the air in the room. See whether you can detect any such temperature differences using your hands or face. How valid are your observations? For instance, are your hands reliable temperature sensors? (You probably know the simple experiment which involves placing a finger of one hand in cold water, a finger from the other hand in hot water, and, when they are acclimatized, putting both in tepid water, and seeing whether both fingers give the same temperature indication. Also, do you get the same sensation of hotness or coldness when you touch pieces of glass, metal, or expanded polystyrene which *are* at the same temperature? What causes these differences?) How might you make more reliable tests of the existence of a surface layer?

ANSWERS TO SELECTED QUESTIONS

UNIT A
Materials and mechanics

1 The graph is shown in figure Q1.

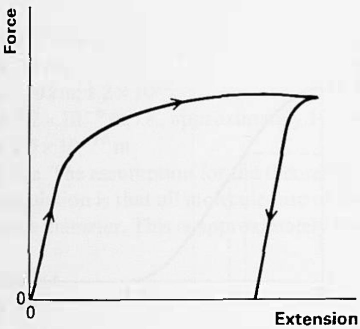


Figure Q1

2 Question **1** asked for a translation from words to a graph: this one asks you to reverse the process. Being able to 'read' a graph is an important skill to acquire. The four substances are: polythene strip, copper or aluminium wire, rubber cord, and steel or cast iron wire (this last one is too curved to be a thin glass rod).

5a $100\,\mathrm{N\,m^{-1}}$
b 0.02 m; $50\,\mathrm{N\,m^{-1}}$
c 0.005 m; $200\,\mathrm{N\,m^{-1}}$
d Halved; doubled.

8a Glass is stiffer than Perspex; it requires more force per unit cross-sectional area to stretch it the same amount.
b It is easier to flex wood across the grain – in the low Young modulus direction.
c Wood.

9a 10^{11} to $10^{12}\,\mathrm{N\,m^{-2}}$
b 100 to 1000 times greater.
c 3.4 mm

10a $\mathrm{M\,L\,T^{-2}}$
b $\mathrm{M\,T^{-2}}$
c $\mathrm{T^{-1}}$
d $\mathrm{T^{-1}}$, *i.e.*, frequency.
e $1/2\pi$ is purely a number. It does not represent a mass, length, or any other physical quantity, as k does.
f [stress] $=\mathrm{M\,L^{-1}\,T^{-2}}$; strain is simply the ratio of two lengths, so it has no dimensions; [the Young modulus] $=\mathrm{M\,L^{-1}\,T^{-2}}$.

12e,f If the force, F, were proportional to extension, x, so that $F=kx$, then the energy $\frac{1}{2}Fx$ would be $\frac{1}{2}kx^2$. The graph of potential energy against extension would be a parabola.

13a 1 N
b*i* 2 N to the left.
ii $2.5\,\mathrm{m\,s^{-2}}$ to the left.
c*i* 0.2 J
ii 0.25 J
d 0.05 J

17a*i* About 0.1 J.
ii About 0.07 J.
b The graphs are shown in figure Q2.

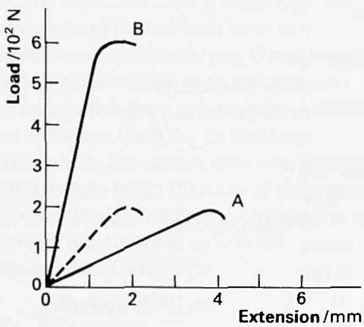


Figure Q2

c The longer cable can stretch more than a shorter one before breaking, and can store more energy. The longer one is therefore less likely to snap when there are sudden jerks.

18b $2.4\times10^{-9}\,\mathrm{m}$
c 10 %; 1 %; 30 %
d The layer might be more than one molecule thick.
e About 10.
f About 10^{19}.

23a 3
b Each bubble attracts other bubbles, and the result is that they cluster as closely as possible. The bubbles are round, and attract other bubbles equally in any direction, whereas, if the forces tended to act in certain special directions, some arrangement like **1** or **4** might result.
c By analogy, it might be true that the attractive forces between atoms in such metals were equal in all directions.

d Because the sizes of the Na^+ and Cl^- ions are very different.

24a $7.12\times10^{-6}\,\mathrm{m^3}$
b*i* $8.68\times10^{-30}\,\mathrm{m^3}$
ii $8.20\times10^{23}\,\mathrm{mol^{-1}}$
iii This is an upper limit for L because the spherical copper atoms cannot fill all the space in a crystal of copper.
c*i* $16.6\times10^{-30}\,\mathrm{m^3}$
ii $4.30\times10^{23}\,\mathrm{mol^{-1}}$
iii This is a low value for L because the assumption of a 'square' array over estimates the volume taken up by each atom.
d $6.15\times10^{23}\,\mathrm{mol^{-1}}$

27a The potential energy of the two atoms decreases as they move together with a net attractive force. 0Q gives the mean equilibrium separation, r_0. As the repulsive force is predominant for closer distances than r_0 the potential energy rises. It becomes positive for separations less than 0P.
b 0Q on the potential energy–separation graph = 0A on the force–separation graph.
c The sum of potential and kinetic energies must be constant if there is no external source or sink of energy. So as the kinetic energy increases the potential energy will decrease, and vice versa. The potential energy will vary by ΔE as the atoms oscillate between separations X and Y (see figure Q3).

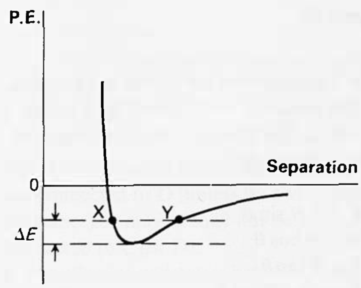


Figure Q3

d The depth of the potential well at Q gives the binding energy, E (see figure Q4).

28a A
b B
c B
d B

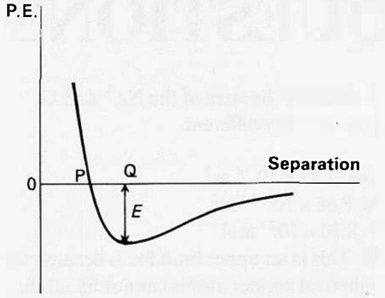


Figure Q4

29a 2 N
b 400 N m^{-1}

34 '...in the next picture the dislocation has moved on by one atom (See figure Q5.) In each succeeding picture the dislocation moves on by one atom until it reaches the edge'.

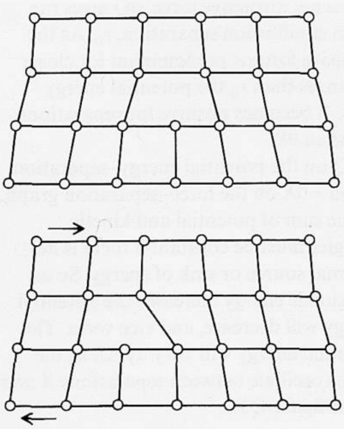


Figure Q5

37a The diagrams are shown in figure Q6.
b The tension $T = W/2\cos\theta$. As $\theta \to 0$ so $T \to W/2$. The tension is less in a long cord than a short one.

38a $F = N\sin\theta$
b $W = N\cos\theta$
c $F = W\tan\theta$
d $F\cos\theta = W\sin\theta$

41 Forces are 9.7 kN and 12.3 kN at left- and righthand supports respectively.

42a *mg*
b Greater
i $T = mg\cos\theta$
ii $F_e = mg\sin\theta$
c For small angles $\sin\theta \approx \tan\theta \approx \theta$, and $\theta \propto d$.

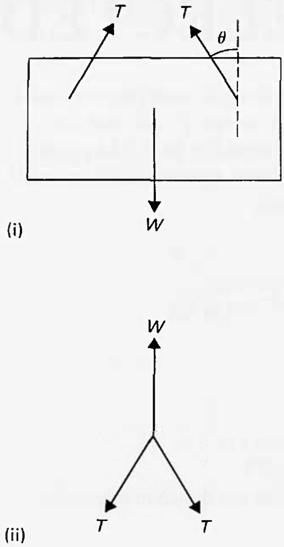


Figure Q6
(i) Forces on the picture.
(ii) Forces on the hook.

43ai 400 N
ii 346 N
iii 346 N to the right.
bi 200 N
ii 173 N
iii 173 N to the right; 100 N upwards.
ci 600 N
ii 520 N
iii 520 N to the right; 100 N upwards.
di 795 N
ii 521 N
iii 521 N to the right; 200 N *downwards*.

47a The weight of the earth above the pipe forces it into an oval shape. The breaks occur where the greatest (tension) stresses are.
b X is at a point of maximum compression rather than tension. Concrete is strong in compression and weak in tension.
c The concrete surface is under compression from the plastic. So when the pipe is buried the concrete remains in reduced compression or only low tension. The plastic cover also has to stretch.
d Concrete is porous, the plastic prevents leakage. Surface damage is also stopped.

50a 8 m s^{-1}
b 6 m s^{-1}
c 1.4 J
d 1.4 kg m s^{-1}
e 35 N

52a 6000 N s
b 6000 kg m s^{-1}
c About 10 m s^{-1}.
d The force at any instant is proportional to the slope of the graph of velocity against time. The graph is shown in figure Q7.

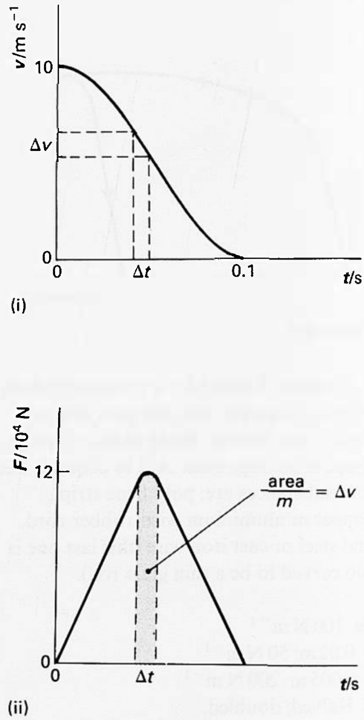


Figure Q7

54a The Earth.
b The pull of the Earth on the book, the push of the table on the book. The force pairs of Newton's Third Law are the two gravitational forces (Earth on book, book on Earth), and the two contact forces (book on table, table on book). Newton's Third Law does not require that the two forces acting on the book be equal. If they were not the book would accelerate (Newton 1).
c The Earth moves up towards the body with equal and opposite momentum.
d The train pushes on the Earth which moves 'backwards' as the train moves 'forwards'. The Earth and train make the closed system.

The ratio of masses ensures that the effect on the Earth's motion is negligible in both cases.

61a 42 moles
b 70 kg
c $6.0\,m^3$

65a 5.65×10^{-21} J
b 4.71×10^{-26} kg
c 5.65×10^{-21} J
d $1300\,m\,s^{-1}$

67a $210\,m\,s^{-1}$
b 2.4 ms
c 110 km; 1.2×10^{12}.
d 9.2×10^{-8} m, *i.e.*, approximately 10^{-7} m.
e 1.5×10^{-10} m
f Yes. The assumption for the theoretical calculation is that all molecules are of the same diameter. This is approximately true.

69a pA
b $pA\Delta l$
c $p\Delta V$
d $p\Delta V$
e No frictional losses at piston; change happens slowly.
f When air is compressed the work done on it increases its internal energy; when it expands it does work, its internal energy is reduced.

UNIT B
Currents, circuits, and charge

1a 0.16 C
b 10^{18} particles.
c 6×10^{22} particles per metre length.
d 1.67×10^{-5} m
e $1.0 \times 10^{-6}\,m\,s^{-1}$
f The algebraic answers to **a–e** become: $It, It/Q, nA, It/QnA, I/QnA$

The speed at which anything moves obviously does not depend on the length of time for which its motion has been observed.

g Reducing n increases v; fewer particles have to move faster.

Reducing Q raises v; each carries less charge, so they must move faster to carry the same total charge in a given time.

A molar solution is strong, but can be made, and contains 6×10^{23} solute molecules in one litre, and so 6×10^{26} molecules in one cubic metre. If every molecule provides an ion, there will be 6×10^{26} ions in each cubic metre. Water molecules are not by any means all dissociated into ions in very pure water while, say, paraffin would contain almost no ions.

3a A possible estimate can be reached by considering the time for which the cell could light a torch bulb (say 5 hours) carrying a current of, say, 300 mA.
b Estimate mass and cruising velocity and calculate $\frac{1}{2}mv^2$. An allowance for inefficiency is necessary (typically the overall efficiency of conversion of internal energy ('heat') into mechanical energy is about 30 %). An all-electric train itself will have a much higher efficiency *by itself* but the overall inefficiency is transferred back to the power station.
c With no losses, about 110 seconds.
d*i* 1000 A *ii* 100 A *iii* $(10)^2 =$ 100 times greater *iv* 100 times longer

4 B

7 The resistance of (a) is larger than the resistance of (b) and both seem to be linear components, obeying Ohm's Law. (c) might be a diode, as its resistance changes when the p.d. is reversed. (d) could be a filament lamp, for its resistance increases as the current increases. Initially, (d) conducts better than any of the others. You could also compare the forward and reverse resistances of (c) with the resistances of (a) and (b).

9a Only graph (b) obeys Ohm's Law as stated. Graph (a) is linear, but there is current when there is zero applied p.d. (perhaps the component included a source of p.d.). (c) shows a constant current, not a constant rate of change of current with change of p.d. Graph (d) is non-linear, though it might be said to obey Ohm's Law for p.d.s in one direction only.
b Yes. A graph of current against p.d. is a straight line through the origin. Alternatively, equal increases in current go with equal increases in voltage. The resistance at 2.0 mA, and at all other currents given, is $14.5 \times 10^3\,\Omega$.
c The answers here are partly matters of opinion. Certainly Ohm's Law is not applicable to all objects that conduct, but it does compactly describe the behaviour of some. Suggestion **3** avoids trouble by defining it away; sometimes physicists do this, but in this particular case they usually speak of 'ohmic' or 'linear' materials. **4** is wrong, for Ohm's Law requires the current–voltage graph to pass through the origin and a small section of the curve, though nearly straight, will not project back to the origin. Whether Ohm's Law is important and general enough to count as a law is a matter of opinion. It does not rank with those 'Laws of Physics' to which we know no exception (give an example).

11a*i* The resistance of A increases with applied p.d.
ii The resistance of B is constant.
b For 2 A, $V_A = 5$ V and $V_B = 10$ V, so the combined resistance is

$$\frac{(5+10)\,V}{2\,A} = 7.5\,\Omega.$$

c If they are in parallel, the p.d. is the same across each. A p.d. of 5 V produces $I_A = 2$ A and $I_B = 1$ A, *i.e.*, a total current of 3 A.
d The resistance of a parallel combination is less than the resistance of the smallest component. The parallel combination of A and B has a minimum resistance when that of A is as low as possible, *i.e.*, at a low value of p.d.

13a Their resistivities rise in constant ratio.
b The logarithm to base ten of the resistivity.
c More than 10^{20} km, over ten million light years; further from us than the nearest galaxy.

15a 1.6 A
b $2R$

17a $I/A = V/AR$
b $I/A = V/\rho l$
c $5.9 \times 10^7\,A\,m^{-2}$; current in a 1-$mm^2$ wire is 59 A.
d $v = 3.7 \times 10^{26}/n$
e 8.5×10^{28} atoms, or electrons, per cubic metre.
f About $4\,mm\,s^{-1}$.

18a A is connected electrically to B, B is not connected to D (unless by a very high resistance). A may or may not be connected to C (or D).
b The effective resistance between A and B has 1.5 V across it when a current of (4.5/1000) A passes, and is thus just over 330 Ω.
c The effective resistance between A and B is just over 330 Ω, in agreement with **a** and **b**.
d If there were resistors connected between A and C or between B and D, the meter would read more than 18 mA (but **a** suggested that B and D were not connected).

e If A and C were connected, the meter would give a reading when the switch was open. So they are not. The reading of *18 mA when the switch is closed confirms* earlier suggestions that the circuit is as in figure Q8.

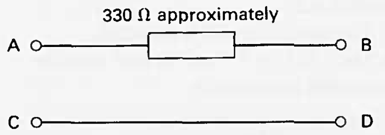


Figure Q8

20a 11 000 Ω
b 0.27 mA
c Three-quarters of the length of CD.
d The lamp resistance (15 Ω working) is so low compared with the resistance of the part of the potentiometer across which it is connected, that it effectively short-circuits the output, producing very nearly zero p.d. across it.

22 1 Ω. You should find, and perhaps be able to prove, that the power delivered has a maximum value when R is 1 Ω, that is, the resistance of the external circuit is equal to the internal resistance of the cell.

27a From the sliding contact into the meter.
b $I_1 - I_2$
c Three; only two are needed.
d 83 μA

28a The current is zero at first, rises sharply to 2 A, remains steady for 10 hours, and then drops sharply to zero at 6 p.m.
b 2 C
c 7.2×10^4 C
d The charge passed is represented by the shaded area in figure Q9.

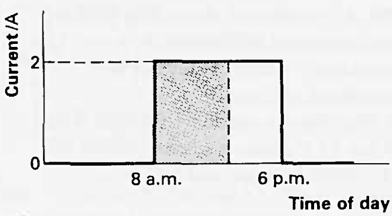


Figure Q9

e The charge passed is still shown by the area between the graph and the time axis.
f About 5×10^4 C.

29a The same as meter 1.
b 5 divisions to the left.
c None!
d Charging, anti-clockwise; discharging, clockwise.
e When the capacitor is being charged the equality of meter readings 1 and 2 shows that as much charge flows onto one plate as flows off the other.

30a As in figure Q10.

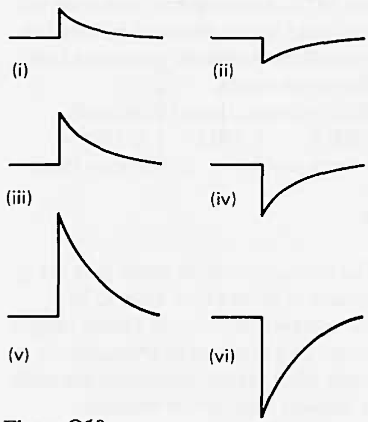


Figure Q10

b *i–iv* As for **a***i*. *v* As for **a***vi*.
c Equal changes in p.d. produce equal flows of charge (identical traces).

32a 10^{-3} C
b 6.25×10^{15}
c The resistance would have to be decreased at a steady rate.
d 10^{-4} A
e See figure Q11.

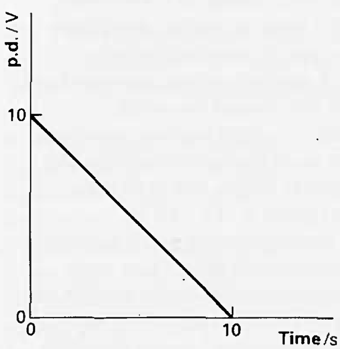


Figure Q11

34a 100 μC
b 200 μC
c 10 V
d 10 V
e 300 μC
f 30 μF
g Yes!

35a*i* No *ii* No
b No, from **a**. (Or consider the isolated part of the circuit consisting of the righthand plate of C_1 and the lefthand plate of C_2.)
c Q/C_1
d Q/C_2
f $C = Q/V$

37a 0.01 F
b 0.02 C
c 0.38 J. The taller shaded strip in figure Q12.

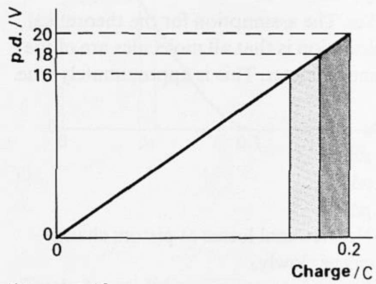


Figure Q12

d 0.34 J
e The two shaded strips in figure Q12.
f 0.38 J + 0.34 J + ... + 0.20 J = 2 J
g The total area of the triangle under the graph ($\frac{1}{2}$ × base × height) represents the energy transformed.

44 You could use a circuit with a capacitor discharging through a variable resistor, with the discharge current holding the relay on while it exceeds 0.1 mA. The time constant of this circuit determines how long the relay is closed and hence for how long the lamp lights. The variable resistor could be calibrated for the required lighting time. Sketch the curve of discharge current against time for these approximate values: resistance 10 kΩ, capacitance 700 μF, charging p.d. 10 V.

47 The current is the ratio of power to potential difference. An estimate of 10 W power gives a current of 5×10^{-4} A. This is the same as 3×10^{15} electrons per second, on an area of 10^5 mm^2 which, gives a density of 3×10^{10} electrons per mm^2. Notice that the number of electrons per square millimetre of screen must be large otherwise it might be possible to see random fluctuations in the brightness as the electrons arrived randomly.

48a QV
b Fd
c $QV = Fd$. If $F = W$, then $Q = Wd/V$.
d See table Q1.

V p.d. to balance drop/V	Q Charge on drop/C	n Multiple
470	4.75×10^{-19}	3
820	3.18×10^{-19}	2
230	6.44×10^{-19}	4
770	1.64×10^{-19}	1
1030	1.57×10^{-19}	1
395	7.83×10^{-19}	5

Table Q1

e The basic charge, e, is about 1.6×10^{-19} C.
f The values of Q and n in the table lead to the following values for e: 1.58×10^{-19} C, 1.59×10^{-19} C, 1.61×10^{-19} C, 1.64×10^{-19} C, 1.57×10^{-19} C, 1.56×10^{-19} C. Given that the true value of e is 1.60×10^{-19} C, the uncertainty in these calculated values is about 3% suggesting that the *sum* of the percentage uncertainties in V, W, and d is also about 3%. From the results, V, which is usually given to two significant figures only, seems to have the largest uncertainty, and d the least.

UNIT C
Digital electronic systems

1a Suppose the numbers are 1342 and 5768. Select the end digits of each number (2 and 8). Add them. The sum is 10. Store the digit 0 as the first digit of the answer, and carry the digit 1 to the next step. Select the next two digits (4 and 6). Add them, and the 1 carried over, getting 11. Store 1 and carry 1. Then continue.
A better system would have a stopping device that turned the adding process off when all of the incoming digits (carry as well) became zero. Without it a mindless computer would go on for ever, producing the answer

......0000000000000000000000000007110

b When the clock reaches a preset time it switches on the power to a heater immersed in cold water. When the water boils, the heater is switched off. The 'water boiling' signal also operates a device that pours the water onto the tea leaves. After a suitable delay for the tea to infuse, a 'tea ready' alarm is rung. The human part of the system then adds milk and sugar.
c Figures Q13 to Q15 show systems, or parts of systems, to do the suggested jobs.

5 There are only two digits in the binary system: 0 and 1. It is easy to define two voltage levels – high and low; supply voltage and zero volts – representing the digits 0 and 1. The binary system readily lends itself to electronic systems. Defining intermediate levels to represent more digits is much more complicated.

6i A simple on–off system, information easily transmitted by a digital signal.
ii Temperature is continuously variable, so it is difficult to use a simple digital code (although not impossible, as you may see later).
iii Depends on the number of directions. Two would be easy; more directions would require the use of more than one binary digit. For example, four directions could be coded using two 'bits'. If the directions are north, east, south, and west, then north could be 00, east 01, south 10, and west 11.
iv The magnitude of an electric current is like temperature: continuously variable.

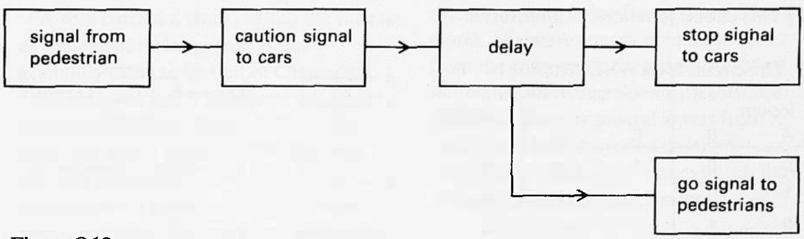


Figure Q13
Part of a system for controlling cars. Other parts should be added to operate go and stop signals for cars and pedestrians.

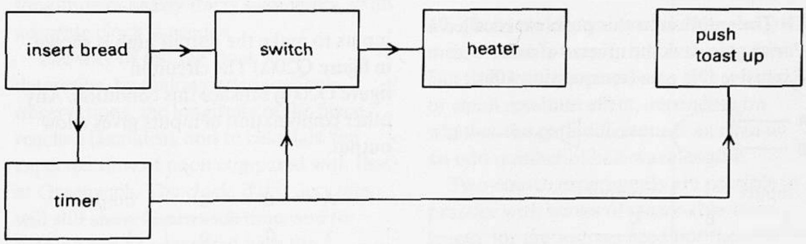


Figure Q14
A system for a 'pop-up' toaster.

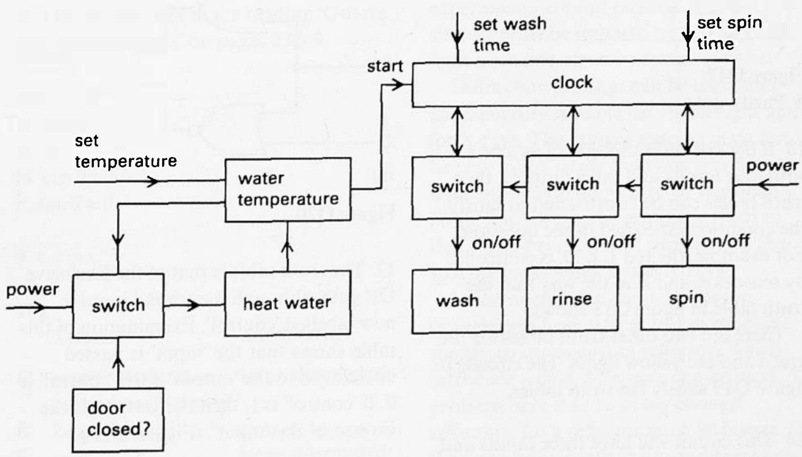


Figure Q15
A system for an automatic washing machine.

However, direction is likely in most cases to be easier to transmit with a binary code.

v The number of pages in this book can be expressed as a binary number which can be transmitted as a sequence of 1s and 0s. For example, if there are 76 pages: decimal 76 is binary 1001100.

vi The area of the page may also be expressed as a number and so can be expressed in binary form.

vii Again, this a number and can be transmitted in digital form.

8 Circuits (a) and (b) are both inverters.
(c) This circuit functions as an inverter.
(d) In this circuit the output is always low.
(e) In this circuit the output is always high.
(f) This circuit functions as an inverter.

9 The circuit is an AND gate.

inputs A	B	C	D	output E
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

Figure Q16

11 The solution to this problem is called a Parity gate. It is the inverse of an Exclusive OR gate (see question **10b**).

inputs A	B	output C
0	0	1
1	0	0
0	1	0
1	1	1

Figure Q17
A Parity gate.

13 If the system is thought of as a box with four inputs and three outputs, then truth tables can be constructed to satisfy the conditions specified in the question. For example, the red L.E.D. is controlled by sensors A and B in the way that the truth table in figure Q18 shows.

There are two other truth tables for the green and the yellow lights. The circuits in figure Q19 satisfy the truth tables.

14 This circuit will have three inputs and one output. Without showing the whole truth table, the necessary combination of

A	B	output red
1	1	1
1	0	0
0	1	0
0	0	0

Figure Q18

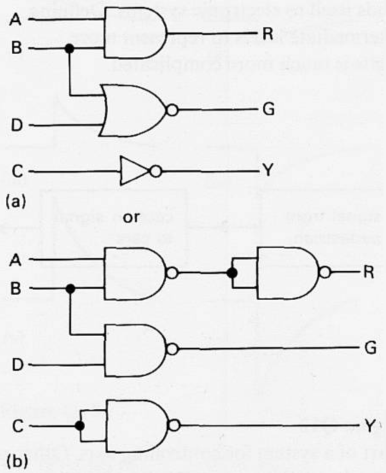


Figure Q19

inputs to make the output high is shown in figure Q20(a) The circuit in figure Q20(b) satisfies this condition. Any other combination of inputs gives a low output.

(a)

A	B	C	output
1	0	0	1

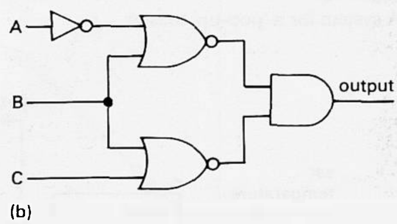
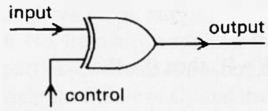


Figure Q20

17 The truth table is that of the Exclusive OR gate, although the second input is now labelled 'control'. Examination of this table shows that the 'input' is passed unchanged to the 'output' if the 'control' is 0. If 'control' is 1, then the 'output' is the inverse of the 'input'. (Figure Q21.)



input	control	output
0	0	0
0	1	1
1	0	1
1	1	0

Figure Q21

19a 5 volts. All the kits will work from a 5 V supply.

b When the output of one gate is connected to the input of another, the 'output low' of the first gate must be less than or equal to the 'input low' of the second gate, if the logic is to be preserved.

For the same reason, 'output voltage "high" is higher than 'input voltage "high"'.

c Basic unit and TTL gates have operating limits that are defined by fixed power supply limits. Examine the characteristic curves in figure C13 (page 167).

CMOS gates have a very sharp transition from high to low, but the level of this depends on the supply voltage. Since the supply voltage (V_s) can vary over a wide range, it is best to express the limits in terms of V_s.

21a Figure C72(b). As the resistance of the thermistor drops so does the potential difference across it and therefore the input to the inverter goes low. So the output of the inverter becomes high, lighting the warning lamp.

The circuit shown in figure C72(a) will warn against low temperatures.

b The circuit in figure C73 would be used to warn against decreases or increases in temperature. The indicator, which in this circuit is on when conditions are safe, will go out if either A or B goes high. The resistors are chosen so that the inputs are low when the temperature is within the safe range. If an OR gate were used instead, the indicator would be off when both inputs were low. It would then light when either input went high, that is, when the temperature went out of the safe range.

24a Figure Q22 shows a half adder using NOR gates.

b See figure C17 (page 169) for full adder from half adders.

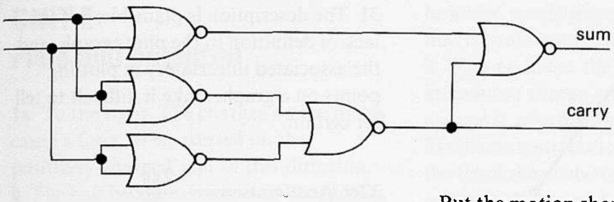


Figure Q22

c See figure C17. Only if A = 1 and B = 1 is the carry from the first half adder 1. But then the sum from the first half adder is 0, so there can be no carry from the second half adder.

25a A pulse producer – see figure C19.
b Input 1 goes from low to high; output 1 goes low, stays there for a while, and then returns to high. When output 1 goes high this triggers circuit 2, whose output then goes low, stays there for a while, and then goes high again.
c The process would continue indefinitely – just like an astable circuit.

27 The output D is high when either input A **or** input B are high.

A	B	C	D
0	0	1	0
0	1	0	1
1	0	0	1
1	1	0	1

Figure Q23

29a The output of the second also goes from low to high.
b Nothing happens to it next. It stays high. The output from the second keeps it high.

UNIT D
Oscillations and waves

1a It is probable that the car body has initially been set into oscillation, and the energy of oscillation is gradually being reduced due to damping. (This particular car has rather worn shock absorbers – the energy should disappear in only one or two cycles.) The energy is being distributed among the many atoms making up the surroundings of the car body. The reverse process – concentration of energy from many individual atoms into oscillations of one object – never happens.

But the motion shown could be reversed if the car body were being given a properly timed series of pushes by an external agent – that is, being set into forced oscillation.

3 The story of Harrison and his chronometers is told in a booklet obtainable from the National Maritime Museum.

A ship carried a clock, telling the time at some definite longitude, which gradually came to be that of Greenwich. The ship's unknown longitude was found by comparing local time (at, say, noon, when the Sun is highest in the sky) with the time indicated by the clock. If, for instance, local noon is six hours behind Greenwich, the ship is one-quarter of the way around the Earth West of Greenwich. At the Equator, a time difference of one minute corresponds to a distance in longitude of nearly thirty kilometres, so an accurate clock is needed.

The way to test the clock is to determine, by making astronomical observations, the longitude of the place reached (Jamaica), and to calculate the expected time of noon compared with that at Greenwich. The clock, if it is accurate, will still show Greenwich time, and its reading can be compared with the calculated time.

5 This is discussed in the reading 'Quartz and atomic clocks' on pages 237–9.

8a*i* 0.5 %
ii 0.006 %
iii 0
b*i* $\cos\theta \approx 1$
ii $\tan\theta \approx \theta$

9 $6.7\,\mathrm{m\,s^{-1}}$

10a

	Velocity	Acceleration
Q	zero (but about to move rapidly down)	large, downward
R	large, upward	zero
S	zero	large, downward

Table Q2

b*i* Tension in the Slinky (T) – a large tension will mean that for a given displacement there will be a large restoring force, accelerating displaced coils back towards their mean position more quickly.
ii Mass per unit length of the Slinky (μ) – a given restoring force acting on a large mass, will accelerate displaced coils more slowly towards their mean position.

In fact

$$c = \sqrt{\frac{T}{\mu}}$$

14c The amplitude would only be zero at a minimum if the amplitudes of the disturbances arriving from S_1 and S_2 were equal. The wave energy is spreading out from the sources, so the amplitude of an individual wave decreases with distance travelled. Since in general waves from S_1 and S_2 will have travelled different distances to reach a minimum point, they will not have equal amplitudes.

15 The path difference has increased by an odd number of half-wavelengths. So λ could be 2 m, or $\frac{2}{3}$ m, or $\frac{2}{5}$ m, or ....

17 There are many answers. Wavelength measurements usually involve using the fact that waves superpose to give a large or small resultant effect, depending on whether the path difference is an even or an odd number of half-wavelengths.

Two-source experiments are possible in practice with waves of reasonable wave-length, for the sources need not then be very close together. But the sources must emit in phase. For light, two-source experiments are still possible, but the two sources must be imitated by splitting the light from one narrow source.

Diffraction gratings can be used, and are especially suitable for visible light and for X-rays. The grating spacing must not be very much larger than one wavelength if the diffraction angle is to be reasonably large. For X-rays, the grating is tilted so that the X-rays graze its surface, and the spacing looks small to the X-rays.

In the microwave and v.h.f. region (wavelength from 0.01 m to 1 m roughly), simple arrangements of reflectors which introduce a path difference can be used. A problem here is to have big enough reflectors, for a reflector must be bigger than one wavelength in linear dimensions to reflect an appreciable amount of wave energy.

18b As S_2 is moving further from T, it is reflecting a decreasing amount of wave energy; and as R is also now further away, a smaller proportion of this amount reaches R. Hence the two superposing signals will no longer have equal amplitudes, which they must have to cancel out exactly.

20c The light following paths A and C superposes destructively – thus reducing the intensity reflected. B and D, however, give constructive superposition, increasing the intensity transmitted.
d There will be relatively little light intensity reflected in the middle of the visible band, and relatively more at the ends – that is, blue and red. These combine to make the reflected light appear purple.

21 156 m

27 (b) will oscillate. (Do not confuse the 'weightlessness' of the mass with lack of inertia. Its mass is the same as on the Earth, and it behaves as an oscillator with the same period, neglecting any relativistic effects.) You can decide about the others.

28c From diagram, *i* $\omega = 1100° \, s^{-1}$
ii $\omega = 19 \, rad \, s^{-1}$.

29a $-1 \, m \, s^{-2}$
b $-0.05 \, m \, s^{-1}$
c $-0.05 \, m \, s^{-1}$
d -0.005 m
e 0.095 m
f $-0.95 \, m \, s^{-2}$
g $-0.095 \, m \, s^{-1}$
h $-0.145 \, m \, s^{-1}$
Table Q3 shows the other values.

t/s	s/m	a/m s^{-2}	v/m s^{-1}	Δs/m
0	0.1	-1	0	
0.05			-0.05	-0.005
0.10	0.095	-0.95		
0.15			-0.145	-0.0145
0.20	0.0805	-0.805		
0.25			-0.226	-0.0226
0.30	0.0580	-0.580		
0.35			-0.283	-0.0283
0.40	0.0296	-0.296		
0.45			-0.313	-0.0313
0.50	-0.0017			

Table Q3

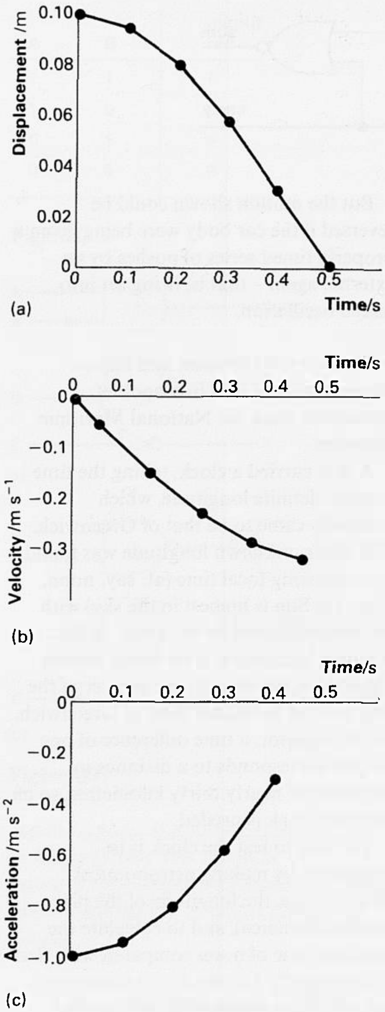


Figure Q24

i The graphs are shown in figure Q24.
j 2.0 s. It compares well (1.99 s).
k These answers would all be doubled; the period would be shorter.
l These answers would all be halved; the period would be longer.

31 The description is plausible; but the lack of definition in the photograph, and the associated uncertainty in plotting points on a graph, make it difficult to tell for certain.

32ci Acceleration $= -\dfrac{2xg}{l}$

ii Angular velocity $= \sqrt{\dfrac{2g}{l}}$

iii Periodic time $= \pi\sqrt{\dfrac{2l}{g}}$

33 $2\pi\sqrt{\dfrac{hd}{\rho g}} = 2\pi\sqrt{\dfrac{L}{g}}$

35c 1.14 p.m.

36b $400 \, N \, m^{-1}$
ci 2.8×10^{-20} J *ii* 0.18 eV

37a 0.1 m
b $10 \, rad \, s^{-1}$
c 0.16 s
d $1 \, m \, s^{-1}$

41a $8 \times 10^{12} \, m \, s^{-1}$
b $5600 \, m \, s^{-1}$

42b $\sqrt{\dfrac{E}{\rho}}$

46b 3.8×10^{-26} kg
c 1.2×10^{13} Hz (effective $k = 200 \, N \, m^{-1}$)
di 2.6×10^{-5} m

54c 1.7×10^{-8} s
d 5 m

56c $60 \, m \, s^{-1}$
d The system responds with large amplitude only at special resonant frequencies. The mass-on-spring has only one resonant frequency, however, whereas the string has a series of resonant frequencies: its fundamental frequency, and multiples of this. The resonance of the string depends on waves arriving back from the far end in phase with the new waves being generated: and this can happen when the path of the waves (4 m) is any whole number of wavelengths.

57 192 Hz; 320 Hz.

UNIT E
Field and potential

1a To the right. The charges on the plates cause a force to be exerted on the positively-charged ball in this direction.
b The ball becomes negatively charged. It moves to the left.
c Negative.
d Negative.
e Anticlockwise.
f Clockwise.
g So that charge can flow on and off the surface.
h The p.d., V, of the supply, and the separation, d, of the plates. The frequency rises with increasing V or decreasing d.
i Charge is displaced over the conducting surface of the ball; the larger force on the near side of the ball causes it to be attracted to the plate, whereupon it acquires the same charge as the plate and the process continues as before.

3a Fd
b QV
c $F/Q = V/d$
d $\mathrm{V\,m^{-1}}$
e*i* joules/coulomb
ii newtons × metres
iii $\dfrac{\mathrm{V}}{\mathrm{m}} = \dfrac{\mathrm{J}}{\mathrm{C\,m}} = \dfrac{\mathrm{N\,m}}{\mathrm{C\,m}} = \dfrac{\mathrm{N}}{\mathrm{C}}$
f*i* $10^4\,\mathrm{V\,m^{-1}}$
ii $10^{-9}\,\mathrm{C}$
iii $10^4\,\mathrm{N}$

4a Use $E = V/d$. $V \approx 2000\,\mathrm{V}$.
b Briefly, the higher the pressure the shorter the distance a particle moves before a collision with another. If in that distance the occasional ion or electron can acquire enough energy (about 30 electronvolts, or about $5 \times 10^{-18}\,\mathrm{J}$) from the p.d. it moves through, it may ionize the molecule it hits, and the ion or electron so freed may make another in the same way, leading to an 'avalanche' of ions, and a spark. At higher pressures, then, a greater p.d. is needed to start a spark, for the ions have to acquire a fixed energy in a shorter distance. The question suggests that the sparking p.d. in a car engine exceeds 2 kV, for the pressure in the cylinder is several times atmospheric pressure.

7a A large charge would affect the charges on the plates, completely altering the field between them. A small charge, however, would give such a small force that it could not be measured.
b With no flame, the probe may acquire an induced charge and thus affect the field around it, altering the potential at the tip. The flame contains ions which discharge the tip of the probe so that there is no potential difference between it and its surroundings. The electroscope gives little or no indication (unless a charged plate is touched) without the flame.
c Any source of ions would do, for instance a radioactive source giving out alpha particles. These are in fact used in probes in balloons in the upper atmosphere.
d Set up as described, it will acquire positive charge.
e Towards the electroscope.
f If the charge does not leak off the electroscope, then this current will drop to zero. In practice, it settles at a small steady value, enough to keep the electroscope 'topped up'.

8a Positive.
b To the left.
c Negative.
d Increase.
e The graph is shown in figure Q25.

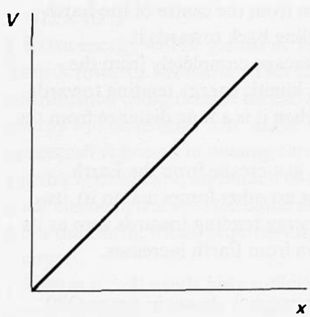


Figure Q25

f Positive.
g $E = -\dfrac{\Delta V}{\Delta x}$ or
'field strength = −potential gradient'.

9a The graph is shown in figure Q26.

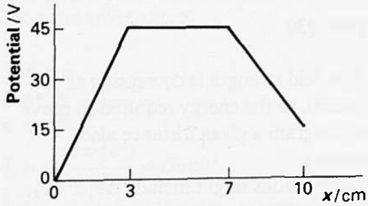


Figure Q26

b For CG_1 it is $-1500\,\mathrm{V\,m^{-1}}$;
for G_1G_2 it is $0\,\mathrm{V\,m^{-1}}$;
and for G_2A it is $1000\,\mathrm{V\,m^{-1}}$.
c Between C and G_1 there is constant acceleration up to maximum energy of 45 eV.
Between G_1 and G_2 there is constant speed.
Between G_2 and A there is constant deceleration down to energy of 15 eV.
d Since it has only 10 eV it cannot reach A, so will 'fall back' towards G_2.

12a $10^{-11}\,\mathrm{F}$
b Approximately $10^{-8}\,\mathrm{F}$.
c Approximately 0.1 %.
d The effective capacitance is now about $0.5 \times 10^{-11}\,\mathrm{F}$. Only 50 % of the charge would be transferred; nevertheless the p.d. across the voltmeter would be about 500 V, which could cause serious damage if an electrometer was being used as the high resistance voltmeter.

13a The statement is correct: $Q = 10\,\mu\mathrm{C}$ so $I = 0.5\,\mathrm{mA}$ at 50 Hz.
b $10^{-9}\,\mathrm{F}$

15a When it is rolled up, otherwise B and D will touch and short circuit.
b About 200 m.
c 2
d It would be halved, *i.e.* about 100 m.
e The cross-sectional area can be considered as πr^2, or as length (100 m) × thickness of sandwich (0.2 mm). This gives a diameter of about 0.15 m, rather larger than one might want of a $1\,\mu\mathrm{F}$ capacitor. In practice paper of this thickness would not be used.

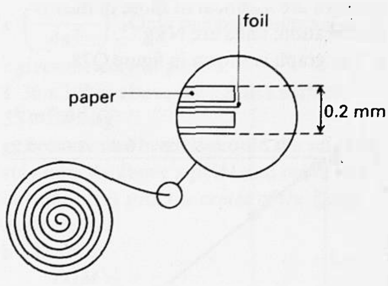


Figure Q27

16a*i* halved ***ii*** same ***iii*** halved
iv halved ***v*** halved.
b*i* halved ***ii*** same ***iii*** doubled
iv doubled ***v*** same.

c In **a** energy is transformed and heats the battery as half the charge flows 'the wrong way' through the battery. In **b** energy has to be supplied to move the plates apart.

19a $4\,\mathrm{m\,s^{-1}}$
b $4\,\mathrm{m\,s^{-1}}$
c*i* 8 J *ii* 8 J *iii* 640 J
d Approximately 1 m apart for 10 J intervals of energy; 100 m apart for 1000 J intervals.
e Contour lines, as on a map, joining points at the same potential energy.

20a*i* 50 000 J *ii* 200 000 J
iii −50 000 J (*i.e.* it *loses* potential energy).
b*i* 50 J *ii* 200 J *iii* −50 J
c*i* $50\,\mathrm{J\,kg^{-1}}$ *ii* $200\,\mathrm{J\,kg^{-1}}$
iii $-50\,\mathrm{J\,kg^{-1}}$
d*i* $150\,\mathrm{J\,kg^{-1}}$ *ii* $-100\,\mathrm{J\,kg^{-1}}$
e 120 000 J

23a The Earth's gravitational pull slows it down since it acts almost in the opposite direction to the motion.
b $7.65\times10^{-3}\,\mathrm{m\,s^{-2}}$
c $7.65\times10^{-3}\,\mathrm{N\,kg^{-1}}$
d $7.9\times10^{-3}\,\mathrm{N\,kg^{-1}}$
e $2.2\times10^{-4}\,\mathrm{N\,kg^{-1}}$. Its contribution is beginning to become significant, although still small ($\approx 3\,\%$ of the Earth's field strength).

24a

Mean distance $r/10^6\,\mathrm{m}$	Mean acceleration $g/\mathrm{m\,s^{-2}}$
27.7	−0.453
55.4	−0.122
96.5	−0.042
170.4	−0.013

b Numerical values of gravitational field strength are identical to those of the acceleration; units are $\mathrm{N\,kg^{-1}}$.
c The graph is shown in figure Q28.

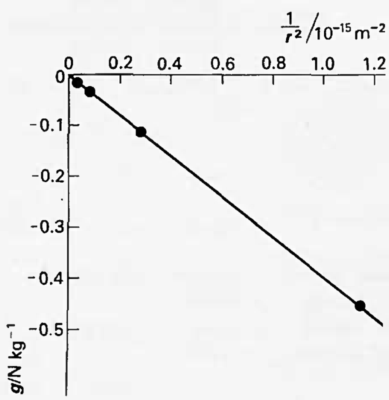


Figure Q28

25a One step. Assuming that g is constant over such a large interval leads to a poor estimate of the area under the graph.
b 1000 steps. This yields the best estimate of ΔV_g.
c One could use an even smaller step size.
d The program would take a very long time to run.
e 2 %
f Nearest the Earth, since this is where g varies most.

26a The graph is shown in figure Q29.

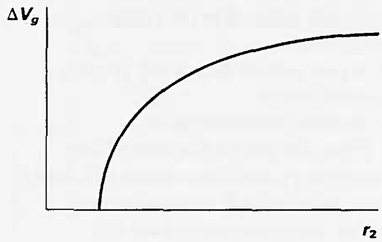


Figure Q29

b*i* $42.5\times10^9\,\mathrm{J}$
ii $10.6\times10^9\,\mathrm{J}$
iii $2\times10^6\,\mathrm{J}$
c 62.5 MJ
d*i* It will reach a distance of about $32\times10^6\,\mathrm{m}$ from the centre of the Earth before falling back towards it.
ii It will escape completely from the Earth, its kinetic energy tending towards 7.5 MJ, when it is a long distance from the Earth.
iii It will just escape from the Earth (assuming no other forces act on it), its kinetic energy tending towards zero as its separation from Earth increases.

27a*i* The graph is shown in figure Q30.

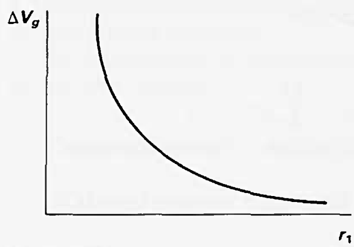


Figure Q30

ii The field strength is decreasing as r_1 increases, so the energy required to move one kilogram a given distance also decreases.
iii Suggestions might include $\Delta V_g \propto 1/r_1$; $\Delta V_g \propto 1/r_1^2$; $\Delta V_g \propto e^{-r_1}$. But note that whereas $\Delta V_g = 0$ when $r_1 = 50\times10^6\,\mathrm{m}$, ΔV_g will never become zero in any of the above relationships, however large r_1 becomes.
b*i* The graph is shown in figure Q31.

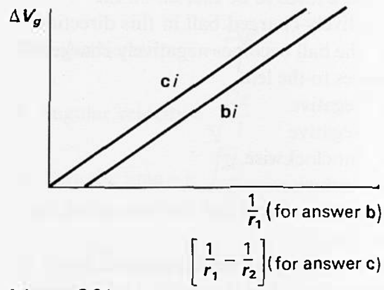


Figure Q31

ii Gradient $\approx 4.0\times10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$.
iii Intercept on $\frac{1}{r_1}$ axis is very close to the value of $\frac{1}{r_2}$, *i.e.*, $2\times10^{-8}\,\mathrm{m^{-1}}$.
iv $\Delta V_g = 4\times10^{14}\times\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$
c*i* See figure Q31.
d It would have had the same gradient but a smaller intercept.

28a Zero.
b No.
c ΔV_g now represents the amount of energy required to remove 1 kilogram to an infinite distance from the Earth, or 'as far away as one would wish'.
d Gravitational potential energy is lost.
e Negative.
f $-62.5\times10^6\,\mathrm{J\,kg^{-1}}$
g The graph is shown in figure Q32.

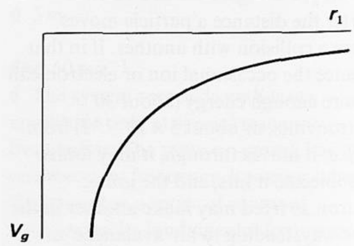


Figure Q32

29a $\Delta V_g \approx \dfrac{4\times10^{14}}{r_1}$
b $V_g \approx -\dfrac{4\times10^{14}}{r_1}$
c $GM = 3.98\times10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$. The two expressions are equal.
d $63\times10^6\,\mathrm{J\,kg^{-1}}$, using the approximate value for GM.
$62.5\times10^6\,\mathrm{J\,kg^{-1}}$, using the more accurate value.

30a They are equal in magnitude and opposite in sign, assuming that no other forces act.
b $-34.41 \times 10^6\,\mathrm{J\,kg^{-1}}$
c $34.41 \times 10^6\,\mathrm{J\,kg^{-1}}$
d Zero.
e $35.33 \times 10^6\,\mathrm{J\,kg^{-1}}$
f The graph is shown in figure Q33.

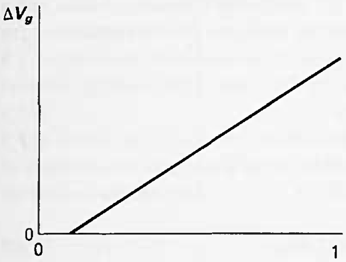


Figure Q33

g The gradient is very close to $3.98 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$, the value of GM_E.
h The intercept is approximately 2×10^{-9}, giving a value of r_0 of 5×10^8 m.
i Zero.
j $\Delta V_g = \dfrac{GM_E}{r}$

31a 10^5 J.
b 10 N. This is approximately the force at the Earth's surface, though it decreases with height.
c 61.5×10^6 J
d Because the force is much less than 10 N for most of the distance.
e 62.5×10^6 J
f $11\,200\,\mathrm{m\,s^{-1}}$
g The energy needed is $\dfrac{GMm}{r} = \frac{1}{2}mv^2$.
m cancels so a larger mass will require the same *velocity*, though of course it has greater *energy*.
h The graph is shown in figure Q34.

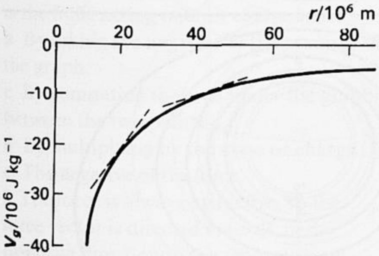


Figure Q34

i $1.0\,\mathrm{N\,kg^{-1}}$; $0.25\,\mathrm{N\,kg^{-1}}$
j They represent the magnitudes of the field strengths at those points.

k The ratio is 4:1. Since the ratio of the distances is 1:2, this is quite consistent with an inverse-square field.

32a $-62.5 \times 10^6\,\mathrm{J\,kg^{-1}}$; $-2.8 \times 10^6\,\mathrm{J\,kg^{-1}}$
b $59.7 \times 10^6\,\mathrm{J\,kg^{-1}}$
c 597×10^9 J
d*i* $-62.5 \times 10^6\,\mathrm{J\,kg^{-1}}$
ii $-4.02 \times 10^6\,\mathrm{J\,kg^{-1}}$
iii $-1.36 \times 10^6\,\mathrm{J\,kg^{-1}}$
iv $-1.32 \times 10^6\,\mathrm{J\,kg^{-1}}$
v $-1.35 \times 10^6\,\mathrm{J\,kg^{-1}}$
vi $-3.94 \times 10^6\,\mathrm{J\,kg^{-1}}$
e, f, g The graph is shown in figure Q35.

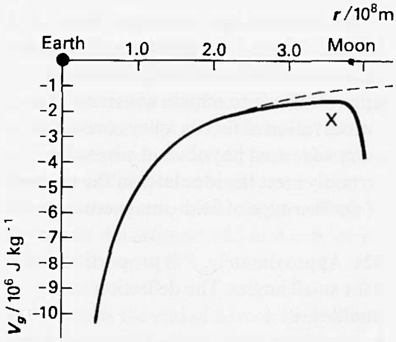


Figure Q35

h 612×10^9 J
i Extra energy will be gained in 'falling' from X towards the Moon. This must be transformed using reverse thrust. Also energy will have been 'lost' as the spacecraft is heated in passing through the Earth's atmosphere; and much of the fuel itself has been transported some distance from the Earth, which requires further energy.
j $2400\,\mathrm{m\,s^{-1}}$. It needs only sufficient energy to reach X, which is only about two-thirds of that required to escape completely. After X it will 'fall' towards Earth.
k Many molecules will have speeds in excess of the escape velocity (which is the same for all masses) and so will escape. However many molecules might have been present, a good proportion will always escape, so any atmosphere will soon dwindle to zero.

33a $r_1\theta$
b $\pi r_1^2\theta^2$
c $\sigma\pi r_1^2\theta^2$
d, e $G\dfrac{(\sigma\pi r_1^2\theta^2)}{r_1^2} = G\sigma\pi\theta^2$
f The result is of the same magnitude.
g Zero. The contributions are equal in size but opposite in direction.
h Zero. There is no field anywhere inside the hollow sphere.
i No.

37a*i* $\Delta t = \frac{1}{2}T$ *ii* $\Delta v = 2v$
iii $a = \dfrac{2v}{\frac{1}{2}T} = \dfrac{4v}{T}$
iv $T = \dfrac{2\pi r}{v}$ *v* $a = \dfrac{4}{2\pi}\dfrac{v^2}{r} \approx 0.64\dfrac{v^2}{r}$
b*i* $\Delta t = \frac{1}{4}T$ *ii* $\Delta v = \sqrt{2}v$
iii $a \approx 0.90\dfrac{v^2}{r}$
c*i* $\Delta t = \frac{1}{6}T$ *ii* $\Delta v = v$
iii $a \approx 0.95\dfrac{v^2}{r}$
d*i* $\Delta t = \dfrac{\theta}{2\pi}T$ *ii* $\Delta v \approx PQ = v\theta$
iii $a = \dfrac{v^2}{r}$
iv Acceleration is at right angles to motion.
e Centripetal acceleration $= \dfrac{v^2}{r}$
f Centripetal force, $F = \dfrac{mv^2}{r}$
g A negative sign, $F = -\dfrac{mv^2}{r}$

38a $\dfrac{2\pi r}{T}$
b $-\dfrac{v^2}{r}$
c $-\dfrac{4\pi^2}{T^2}r$
d $-\dfrac{GM}{r^2}$
e $\left(\dfrac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$. Only one particular value of *r* gives the correct period.
f 36×10^6 m above the Earth's surface. $53 \times 10^6\,\mathrm{J\,kg^{-1}}$.
g Because nowhere else could the satellite stay directly above a point and move in a circular orbit *with the centre of the Earth as its centre*.
h Shorter.
i $v = \left(\dfrac{GM}{r}\right)^{\frac{1}{2}}$. Faster.
j Briefly, energy is transformed during the impact so the satellite loses energy and cannot remain in its present orbit. In dropping to a lower orbit, it speeds up, as potential energy is now transformed into kinetic energy.
k A satellite in 'geostationary' orbit above the Equator can 'see' nearly a third of the

Earth's surface and so can be used for telecommunications over a very extensive area. However, with nearly 200 communications satellites in this orbit, space is becoming a little crowded, especially in the popular region over the Atlantic. (Signals from nearby satellites must be modified to prevent interference.)

Satellites in low orbits lose energy quickly as they encounter the upper levels of the Earth's atmosphere. Sometimes their short life is an acceptable price to pay for the better 'view' they afford, especially if they are designed for monitoring or military purposes. The more stable orbits further out tend to accommodate satellites serving more civilian purposes such as weather forecasting. Many satellites engaged in scientific research have extremely eccentric elliptical orbits which enable them to penetrate wide regions of the magnetosphere. All in all the first 25 years of space exploration have seen over 3000 satellites go into orbit.

41a Twice the distance.
b Twice the height.
c The same.
d The same.
e Four times the area.
f One-quarter.
g One-ninth.
h It is a constant (called the 'flux' of the light).
i In a parallel beam of light both intensity and area remain constant with distance, and so does the flux.

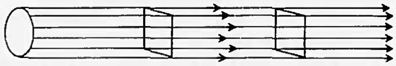


Figure Q36

But wherever light converges to or diverges from a focal point, the change in intensity with distance must be inverse square if flux is to remain constant. The 'conservation of flux' is a key concept in more advanced physics and you will certainly meet the idea later in the context of another type of field – magnetism.

42a Approximately, F is proportional to d for small angles. The deflection is doubled.
b The points on a graph of d against $1/r^2$ lie near to a straight line.
c $0.01\,\mu F = 10^{-8}$ F. At 0.5 V the charge on it is 0.5×10^{-8} C. This is nearly the charge that was on the ball. The fluctuations in charge could explain why the points plotted in **b** are scattered on either side of a straight line.
d, e The force constant is of the order $10^{10}\,N\,m^2\,C^{-2}$, and certainly lies between 5×10^9 and $5 \times 10^{10}\,N\,m^2\,C^{-2}$. You may think the limits are closer than this.

43a 10^{-4} N
b 10^{-9} C
c 10^{-11} A
d 10^3 V
e $10^{14}\,\Omega$

45a $Q/4\pi\varepsilon_0 r^2$
b $2r$
c $4Q$
d $4Q/4\pi\varepsilon_0(2r)^2 = Q/4\pi\varepsilon_0 r^2$
ei Upwards.
ii Small.
iii Approximately uniform.
f Approximately inverse-square.

49a You may produce a pattern something like that shown in Q37.
b See figure Q38.

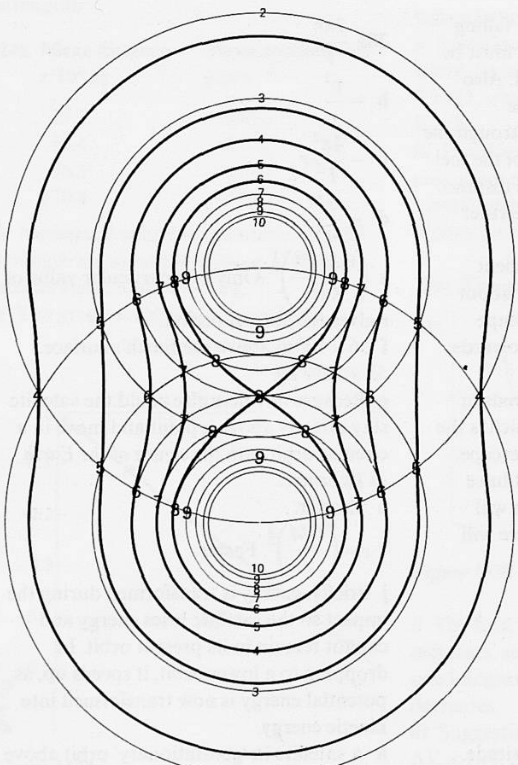


Figure Q37

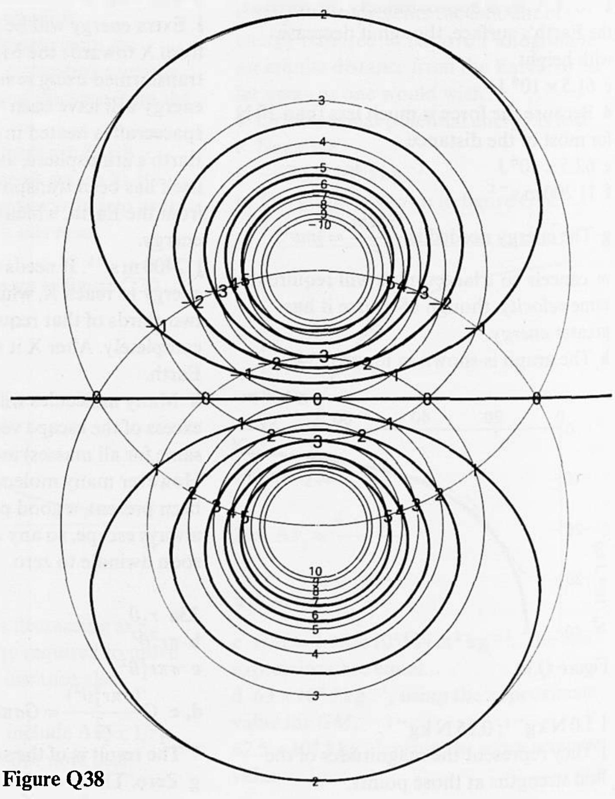


Figure Q38

c The field patterns should be similar to those obtained between two point electrodes with semolina particles in experiment E3.

51a 900 V
b 1200 V at X, 1080 V at Y; by simple addition of the potentials due to A and B.
c Yes, because there is a potential gradient across it.
d Charge would flow towards Y until the potential gradient fell to zero.
e No.
f The surface of a conductor must always be an equipotential, even if some parts of the surface are charged.

52a 10^{-47} N
b 10^{40}
c Both the electric force and gravity obey inverse-square laws so one force falls off with distance at the same rate as the other.
d Approximately 10^{36}.
e No.
f It falls off very sharply indeed and is quite insignificant outside the nucleus.

53a 1.4×10^{11} N C^{-1} (=1 unit)
b $\frac{1}{100}$ unit
c $-\frac{1}{9}$ unit at C; $-\frac{1}{144}$ unit at D.
d $\frac{8}{9}$ unit at C; $\frac{44}{14400}$ unit at D.
e $\frac{11}{3200} \approx \frac{1}{290}$.
f More rapidly (particularly if you consider that the distance of D from the middle of the dipole is only just over five times that of C). From a good distance the field strengths due to A and B almost cancel; indeed the dipole 'looks' almost like a neutral object.
g The array is neutral when 'viewed' from a distance and it is only very near any individual charge that the local field dominates significantly.

55a Electrical/gravitational field strength is the force acting on unit charge/mass.
b By taking the negative of the gradient of the graph.
c By computing the area under the graph between the two points.
d By multiplying by the mass or charge.
e The negative of the force.
f The force is always attractive, so the force vector is directed *inwards*, in the opposite direction to the separation or displacement vector.
g This is taken care of by the charges which may be positive or negative, giving both attractive (negative) and repulsive (positive) forces.
h Because the potential energy of a mass at all points is less than that at the agreed zero of potential (at 'infinity').
i A positive charge would have positive potential energy and a negative one negative potential energy.

UNIT F
Radioactivity and the nuclear atom

2a 10^6 V m^{-1}
b 1.6×10^{-13} N
c 1.6×10^{16} m s^{-2}
d 3.3×10^{-10} s
e 9×10^{-4} m $\approx$ 1 mm
f If $L = l$, the deflection y will be more than twice as big as s, because the straight path from the edge of the plates to the screen cuts the undeflected path through the plates (if projected back) somewhere between the plates, not at the far end of the plates. An estimate of 3 or 4 mm for y might be fair.

4a Towards the end of a track the particle has lost a lot of energy and is travelling more slowly. It is therefore spending more time in the vicinity of the atoms of the air; these atoms are more likely to be ionized.
b The number of ion-pairs is given by the area below the graph from the distance 50 mm back to the origin. One rough estimate put the number of ion pairs at about 200 000.
c The total energy of such an alpha particle is around 6 MeV.

6a 8.7×10^{-19} J
b 8×10^{-13} J
c 2.4×10^{12} J
d The energy released in the radioactive decay of radium is greater than that released on oxidation by a factor of over a million.
e Even with 1 mole of radium (226 g – a very large sample), the energy is dissipated over thousands of years. In the first 1600 years, 0.5 mole will have decayed.

7 The laboratory notes for this Unit are prefaced by a note about the precautions which you should observe when handling radioactive substances. You ought to read this.

Important points to make in explaining the precautions to be taken when using radioactive materials include:

The radiation from a radioactive substance can harm you in several ways if it is absorbed by part of your body. Because one kind of change it can produce is a change of the genetic information stored in the genes in cells, it is especially important to keep such radiation away from the reproductive organs. This does not mean that it is safe to allow the radiation to reach other parts of the body: it is not.

The simplest precaution, and perhaps the most effective, is to keep the source at a distance. Few alpha particles travel more than 50 mm or so in air, so an alpha source held in tongs is pretty safe, as long as the source itself is sealed and you cannot breathe or ingest any of the radioactive substance. Gamma rays spread out so that their intensity falls off as the inverse-square of the distance, so again distance helps, though it does not reduce the radiation received to zero. Lead blocks are used to absorb gamma rays from sources stronger than those in schools.

The only radioactive substances you are allowed to handle which are not in a sealed form are the naturally occurring salts of uranium, thorium, and potassium. These have long half-lives, and the radiation from them is small. The strength of the sealed sources you may use is also limited.

Everybody is continually exposed to radiation from both cosmic rays and the radioactive minerals in the Earth. The various regulations aim to make sure that you and your teacher cannot receive in a year more than a very small extra fraction of the dose you will in any case receive from this natural background.

It is very important that rules and precautions to protect people handling radioactive sources should be framed with care, revised from time to time, and, above all, obeyed.

8 197 g of gold contain 6×10^{23} atoms and occupy $\dfrac{197 \times 10^{-3}}{19.3 \times 10^3}$ m^3. 1 atom of gold occupies $1/(6 \times 10^{23})$ of this space, which is 17×10^{-30} m^3. The cube root of this gives an estimate of the diameter of a gold atom ($\approx 2.6 \times 10^{-10}$ m).

9 If the gold atom is about 2.6×10^{-10} m across, the number of layers, n, in a foil 6×10^{-7} m thick is 2.3×10^3 approximately.

If d is the diameter of a nucleus, then

$$\frac{d^2}{(2.6\times10^{-10})^2}\approx\frac{1}{8000n}$$

$$d\approx6\times10^{-14}\,\text{m}$$

The roughness of the data does not justify an accurate calculation, for example allowing for the way gold atoms pack together. The important thing is the order of magnitude of the answer, some 10^4 times smaller than the diameter of the atom.

11a 8×10^{-13} J
b As the alpha particle approaches the nucleus, this kinetic energy is converted to electrical potential energy. When the speed of the particle is zero, this conversion is complete and the energy stored in the system is 8×10^{-13} J.

The potential energy is given by

$$E_p=\frac{(2e)(79e)}{4\pi\varepsilon_0 r}$$

on the assumption that the gold nucleus acquires very little kinetic energy.
c $r=4.6\times10^{-14}$ m
d The electric potential at this distance is

$$V=\frac{79e}{4\pi\varepsilon_0 r}=2.5\times10^6\,\text{V}.$$

e Since the alpha particle carries charge $+2e$, its original kinetic energy in eV will be double this number, *i.e.*, 5.0×10^6 MeV.
f The electric field at this distance

$$E=\frac{V}{r}=\frac{2.5\times10^6\,\text{V}}{4.6\times10^{-14}\,\text{m}}=5.4\times10^{19}\,\text{V m}^{-1}$$

The force on a charge of $2e$ is about 17 N.

This is about the same as the weight of a mass of 1.5 kg – an enormous force to find exerted on a single atomic particle. The book was the best guess.

12a An attractive force which decreases with distance from A.
b A repulsive force which decreases with distance from A.
c No significant force at the distances shown.
d A repulsive force which suddenly affects P at a small distance from A.

14a*i* Not very good. It describes roughly what happens, but may give the wrong impression about why. The particles come in by chance, and if the chance of arrival stays the same, over a long time there will be a steady average rate.
ii We tried to make this a correct description.
iii Wrong. The time between arrivals may have any value, but not all intervals are equally likely. The average interval is most likely; much longer or shorter intervals are less likely.
b To say that in the kinetic model the molecules of a gas have random motion, *implies that a molecule may be moving in* any direction at any instant (not one of which is preferred). Molecular speeds are distributed randomly about a mean value; this mean value is the most likely to be found whilst far higher or far smaller speeds are unlikely.

16 A common answer is that a radioactive sample has an infinite lifetime 'in theory but not in practice'. Although not absolutely wrong, this seems to us to be a bit feeble.

The smooth mathematical model of exponential decay does not exactly fit the behaviour of radioactive atoms. As you may have found by experiment, the rate of decay of a sample fluctuates considerably. The average rate of decay is close to the value to be expected from the mathematical model, but need not be equal to it.

When the sample is reduced to only a few atoms, the smooth exponential model is a poor fit. Yet it is not enough to say that 'the theory breaks down'. The smooth exponential decay is a consequence of supposing that there is a constant chance of decay for each atom in each time interval, and also that there are very many atoms. When there are not many atoms left, the smooth decay is not to be expected. But so far as is known, the *chance of decay remains constant.* When one atom remains, that atom may decay in the next second, or in the next hundred years. It is possible to say how long it will last on average: that is, how long one atom will last in many trials. But it is not possible to say how long one particular atom will last.

19a Both are exponential. The lengths of the straws decrease in a fixed proportion.
b Plot the natural logarithm of y against x; or check whether y changes by a constant factor as x increases in equal steps.

21a Although the graph of ln (capacity in GW) against time does not follow a straight line, there are periods when it does so with reasonable accuracy. From 1951 to 1958 it was very nearly straight (and the growth exponential), and also from 1964 to 1970.
b The growth constant diminished in about 1962. During the 1970s growth ceased and the capacity remained approximately constant.
c The 'doubling times' were
i about 10 years, and
ii about 11 years.

24a The change in the number of families with video recorders.
b The rate at which the number of such families changes.
c The model $\Delta N/\Delta t=kN$ suggests that the rate at which the number of families with video recorders grows is directly proportional to the number of families already having video recorders. This would give an exponential growth. In real life this could be the result of 'keeping up with the Joneses'.
d There could be a limit on the supply of money available, costs might rise unexpectedly, or the supply of the equipment might become limited. Such changes would reduce the value of k after a time.
e The existence of television could increase the value of k.
f See figure Q39.

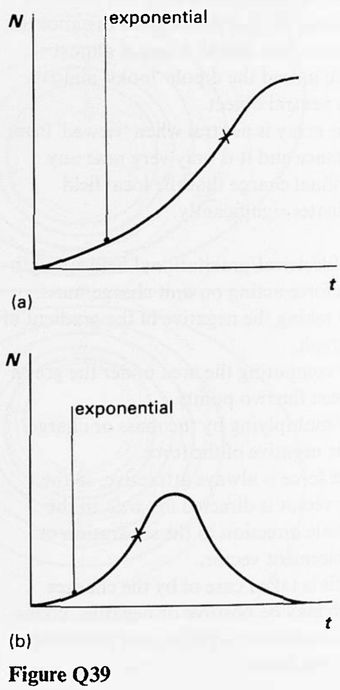


Figure Q39

25a Press, in order, the keys 2, x^y, 7, = (or the keys 2, log, ×, 7, =, 10^x; or multiply the logarithm of 2 by 7 and find the number whose logarithm is this product).
b N = 1.414
c, d The graphs start at $N=1$ because at $t=0$, $N=a^0=1$.

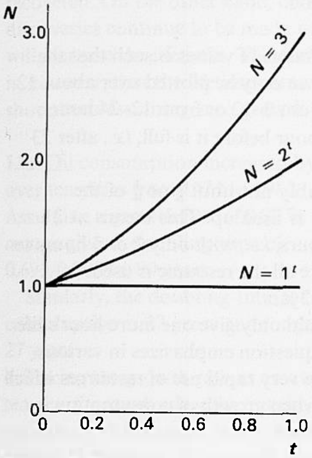


Figure Q40
Plots of $N=1^t$, $N=2^t$, $N=3^t$.

e At each step of 0.2 in t, the value of N is multiplied by the same factor, which is $2^{0.2}$, or 1.15.
f $2^{0.2}=1.15$; $2^{3.2}=1.15\times 8=9.20$.
g In the series a^t, a^{2t}, a^{3t}, etc., each exponent is equal to the previous one with the addition of t. Adding t to the exponent of a means multiplying by a^t; similarly, adding x to the exponent of a means multiplying by a^x.

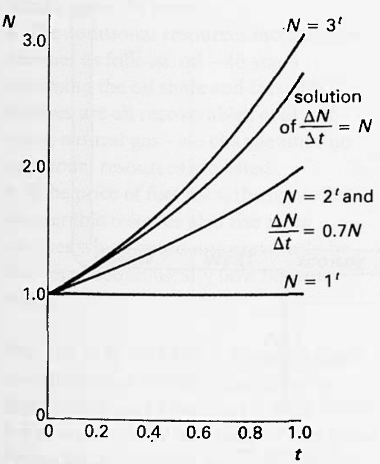


Figure Q41
Plots of $N=1^t$, $N=2^t$, $N=3^t$, and solution of $\dfrac{\Delta N}{\Delta t}=+(1.0)\,N$.

26a, b (See figure Q41.)
a ΔN is a little larger in the second 0.1 s interval because the starting value, N, is now larger (1.1 instead of 1.0).
b Growth is slower because the constant ratio is now 0.7, not 1.0.
c If $N=2^t$, then $N=2$ at $t=1$.
d $N=a$ at $t=1$.
e For $\Delta N/\Delta t=(1.0)N$, $N\approx 2.7$ at $t=1$, so $a\approx 2.7$.
For $\Delta N/\Delta t=(0.7)N$, $N\approx 2.0$ at $t=1$, so $a\approx 2.0$.

27a $e^{0.7}=2.01\approx 2.0$.
b Since $(e^{0.7})^t=e^{0.7t}$, the best speculation is $a=e^k$.

29a 1.993×10^{-26} kg
b 1.661×10^{-27} kg
c 1.007 u; 1.008 u
d 0.000 548 u

32a One gram of carbon contains 0.5×10^{23} atoms, since 12 g contain 6×10^{23} atoms. Of these, about 0.5×10^{13} atoms are the isotope ^{14}C. In one second, 4×10^{-12} of the ^{14}C atoms will decay, so in 1 g of carbon one may expect around 20 decays per second, an activity of 20 Bq.
b 11 000 years is close to two half-lives, so the activity might be around 5 Bq from one gram.
c The rate of decay is small, and the smaller it is, the harder it will be to measure the rate accurately unless one is prepared to measure it over a very long period. Even with counting times of about a day, the ^{14}C method is subject to pretty large errors for times above about two half-lives, and is not of great assistance much beyond three half-lives, say 20 000 years.

34a Mass of alpha particle $\approx 7\times 10^{-27}$ kg.
b Speed $\approx 7\times 10^{6}\ \mathrm{m\,s^{-1}}$. Note that to use the equation, kinetic energy $=\frac{1}{2}mv^2$, with m in kilograms, the energy must be in joules, not electronvolts.

36a These are the inert gases, which rarely form compounds with other elements. They have high ionization energies. It is not easy to remove an electron from one of these atoms.
b These are the alkali metals, all very reactive, readily forming singly charged positive ions. In a crystal of sodium chloride, the sodium atoms are all ionized, and the crystal is a vast assembly of Na^+ and Cl^- ions. All the alkali metals have low ionization energies, indicating the ease with which an atom loses an electron.
c Neon has a large ionization energy; sodium, with one more electron, has a low ionization energy; it seems that the 'last' electron in a sodium atom is rather loosely held. (In fact, it takes 47 eV to remove a second electron from sodium, so it is only the 'last' one which is nearly free.)
d Yes, each comes first on the rise following a trough. They are members of another family with chemical similarities.
e If francium is the element one before radium, it must be an alkali metal, like Li, Na, K, Rb, Cs. Their ionization energies are all seen to be low, from 3 to 5 eV. The value drops slowly as one goes along the list, so francium will probably have an ionization energy nearer the lower end of this range.

38a -2.18×10^{-18} J
b $-e^2/4\pi\varepsilon_0 r$
c 1.0×10^{-10} m
d The *total* energy of the electron is -2.18×10^{-18} J. Since it has some kinetic energy, which must be positive, the potential energy is more negative than -2.18×10^{-18}, *i.e.*, the electron is closer to the proton than this calculation suggests. (In fact, as will be seen in Unit L, 'Waves, particles, and atoms', the electron is not precisely located at one distance from the proton. But this calculation gives the correct order of magnitude for the proton–electron distance.)

42a F_e is repulsive if Q_1 and Q_2 are both positive or both negative; F_g is always attractive.
b $F_e/F_g=Q_1Q_2/4\pi\varepsilon_0\,Gm_1m_2$
c Both forces vary in the same way with distance (inverse-square law), so their ratio is the same at all distances.
d 1.2×10^{36}
e No; it is much smaller than the repulsive electrical force.
f Nuclei of neighbouring atoms would be attracted to each other: they would not stay about 10^{-10} m apart.

43a ^{12}C is the more stable.
b ^{2}H, ^{6}Li, ^{4}He, ^{235}U, ^{12}C, ^{208}Pb, ^{56}Fe.
c Helium is at a minimum in the curve. The average binding energy per nucleon is lower for some heavier nuclei (*e.g.*, ^{12}C, ^{16}O), but to form nuclei like ^{6}Li which are likely to be involved in intermediate steps

in the build-up of heavier nuclei requires an input of energy.
d C, O, Fe.

You can read more about the evolution of the elements in the article 'Our nuclear history' in the Reader *Particles, imaging, and nuclei.*

45a 0.001 kg (1 gram).
b 0.5 J (taking $k = 100\,\mathrm{N\,m^{-1}}$, and extension = 0.1 m).
c 6×10^{-18} kg
d 6×10^{-13} per cent.

46a $2m_p + 2m_n + 2m_e = 4.0330$ u.
The mass of an atom of $^4_2\mathrm{He}$ is 0.0304 u *less* than the mass of its parts.
b*i* -4.54×10^{-12} J *ii* -28.4 MeV
c -7.1 MeV
(Note that because the information refers to a neutral helium atom – nucleus plus electrons – the binding energy calculated includes the binding energy of the electrons as well as that of the nucleons. However, since electrons are so much more weakly bound than nucleons, the value obtained is still a good estimate for the average binding energy per nucleon.)

UNIT G
Energy sources

1a*i* The potential energy of the mass is the work which would be done by the gravitational field if the mass were allowed to fall to some reference point (often the surface of the Earth). The mass needs the presence of the Earth in order to have potential energy, which is therefore a property of the Earth–mass system.
ii The gravitational field acting on the mass transfers energy to the generator by working. The mass loses energy and the generator gains energy (to an equal extent in an ideal system).
iii This means that a quantity of energy (15 J in this example) would be transformed if the generator were brought to rest.
b Boxes should be inserted at every stage of conversion. They all represent internal energy of the surroundings.
c The boxes should have solid boundaries since they represent continuing 'losses' to the system.
d It depends what you mean by the system. If this is taken to be the 'Universe' then the energy is constant, though clearly it is not so if the mass–generator–lamp arrangement is regarded as the system. In this latter case energy is lost from the system. Because the 'losses' are only losses in this limited sense quotation marks are used.

2 The Sankey diagram is shown in figure Q42.

4a 3.6 MJ
b About 35 MJ.
c About 29×10^9 J.
d 3.6×10^{12} J
e 10^3 J
f About 250 kJ.
g 2.9×10^{20} J
h 1.6×10^{-11} J
i 1060 J

6a About 32 %.
b About 93 %.
c About 29 %.
d About 92 %.
e About 91 %.
f Roughly $3\frac{1}{2}$ times as much as the useful output.

7a For equal heating costs electricity can be $1\frac{1}{3}$ times as expensive as gas.
b 1 kW h of heating by electricity requires about $2\frac{1}{2}$ times as much primary fuel as using gas for the same purpose.

8a World population has been growing exponentially since about 1950 with a doubling-time of about 37 years.
b Between 1925 and 1981 world fuel consumption increased by a factor of about 7, whereas the population grew by a factor of about $2\frac{1}{2}$. Clearly the fuel consumption *per capita* increased during this period.

9a The range of values is such that the graphs can only be plotted over about 12 hours, from 0–12 or from 12–24 hours.
b One hour before it is full, *i.e.*, after 23 hours.
c Probably not until $\frac{1}{8}$ or $\frac{1}{4}$ of the nutrient is used up. This means at 21 or 22 hours, *i.e.*, with only 2 or 3 hours to go before all the resource is used.
d 1 jar; 2 jars.
e It would only give one more hour's life.

This question emphasizes in various ways the very rapid use of resources which occurs when growth of consumption is exponential.

10 Doubling-time is 35 months, to the nearest month.

11 $t_D = 0.693/g$

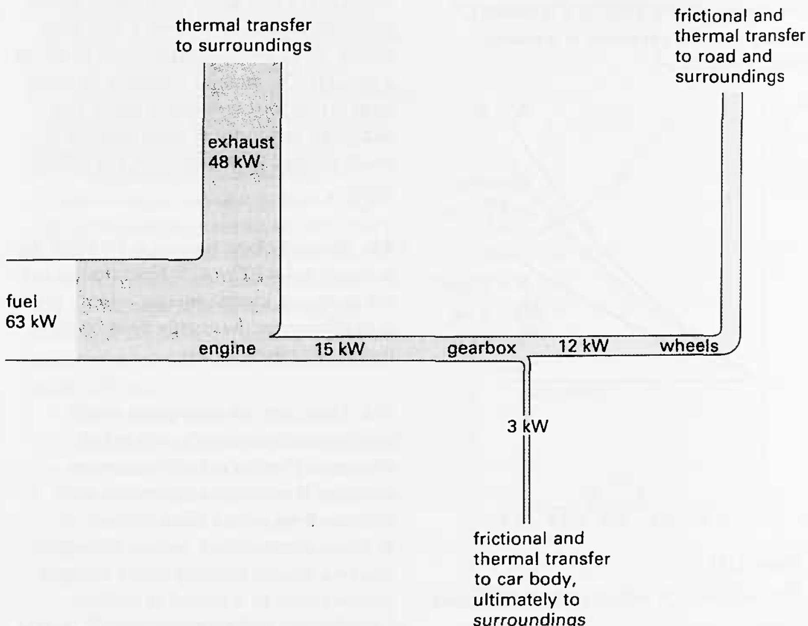


Figure Q42
Sankey diagram for a car.

12 Coal 226 years; oil, including shale, tarsands, etc., 57 years or 29 years for oil only; natural gas 53 years.

It is not clear in table G10 whether the reserves of oil and gas are proved reserves or recoverable reserves. If they represent proved reserves, the lifetimes would be less for the smaller proportion which can be recovered. On the other hand, oil and gas discoveries continue to be made which will increase the lifetimes. The main point is, however, that the lifetimes are very short, even assuming zero growth.

13a Oil consumption increased by 8.7 % over ten years from 1972 to 1982. Assuming a growth rate of 0.87 % *per annum* gives a doubling-time of $0.693/0.0087 \approx 80$ years.

Similarly, the doubling-time for coal consumption is 27 years and for gas also 27 years.

b Taking the growth rate as an average of ten years' growth smooths out the variations which occur over a shorter period. If, however, the growth is very rapid (as in the case of nuclear power from 1972 to 1982) the average value over ten years is not the same as the year-on-year value which applies throughout the period.

In fact, an annual growth rate of 0.87 % leads to growth in ten years by a factor $e^{(0.0087 \times 10)} = 1.091$, rather than 1.087, *i.e.*, using the 'average' growth rate overestimates the growth.

c Using 1982 figures the lifetimes of the fuels are: oil – 26 years; coal – 76 years; natural gas – 34 years.

d The additional resources increase the lifetimes as follows: oil – 46 years (assuming the oil shale and tarsands reserves are all recoverable); coal – 152 years; natural gas – no change since no additional resources are listed.

e If the price of fuel rises, the figure for recoverable reserves also rise since reserves which could not previously be recovered economically now become viable.

19a The U.K. and U.S.S.R. have a slight overall surplus, North America has a slight deficit, and Japan has a large deficit.

b For water power and nuclear energy the figures for production and consumption are identical. Only one set of figures is published, presumably because storage of electrical energy on any reasonable scale is not possible. The figures for coal, natural gas, and oil can be processed as in part **a**. The calculations are simple but the real point of the question is dealt with in parts **c** and **d**.

c Short-term shortfalls can be dealt with by importing from nations which have surplus resources. The importing nations must have some means of earning the currency to buy the fuel and the fuel-exporting nations may have some degree of control over the economies of the importers (for example, the Organization of Petroleum Exporting Countries (OPEC) in the late 1970s).

d Nations like Japan, with continuing shortfalls, are likely to wish to reduce their dependence on imported fuels. They may have to develop alternatives much more purposefully than do nations with fossil fuel resources. To some extent, what happens to Japan today happens to the World tomorrow.

21a*i* The area needed is $0.13\,\text{km}^2$.
ii 0.007 miner.
iii Area of panel needed $= 70\,\text{m}^2$.
b Roughly three times as much resource would be needed.

23a 0.04 %
b 4 %
c 5.5 %
d 7.5 %
e 0.25 %

24a A base load capacity of about 39 GW is required. About 9 GW of hydroelectric power will also be needed.
b $9000\,\text{m}^3$ of water would be needed per second to provide the peak power output. The total energy required is about 98 GW h. This would be provided by the contents of a lake of area about $12\,\text{km}^2$ and 30 m depth falling through 100 m.

25a Volume of the ring main $= 56\,000\,\text{m}^3$.
b The pressure falls.
c About $1.4 \times 10^6\,\text{m}^3$.
d About $3.9 \times 10^6\,\text{m}^3$.
e The increase in pressure which would result from storing all this gas in the main would be about 70×10^5 Pa (*i.e.*, from 25×10^5 to 95×10^5 Pa) which takes the main pressure above its limit. Over half ($\frac{40}{70}$) would have to be stored elsewhere.

28 About 2.3 %.

29 From question **27** increasing expectations seem to account for a rate of growth of 2.1 %. Table G11 shows much larger increases for less developed countries than for developed ones.

31a*i* No. The number of electrons is the same on each side.
ii Change in mass is 0.192 u or 3.18×10^{-28} kg.
iii 2.86×10^{-11} J
b*i* $\dfrac{Q_1Q_2}{4\pi\varepsilon_0 d}$
ii 4.61×10^{-11} J
c*i* 9×10^{13} J
ii About the same.
iii 143 kg
iv 143 tonnes
v About 30 times as much material needs to be used to provide the same amount of energy from coal as from uranium.

32a a/A
b $\dfrac{nAva}{A} = nav$
c NAd
d $nav \times NAd$
e $R = \dfrac{nav \times NAd}{A \times d} = nNav$
f $a = R/nNv$

33 0.1 eV. This is a factor of about 10^7 less than the energy of a neutron released in fission of ^{238}U.

37a $\dfrac{\Delta E_k}{E_k} = \dfrac{4_m M}{(M+m)^2}$
b M must be 1 u.
d Hydrogen.

38 54

40a 3.1×10^{19}
b 1.2×10^{-5} kg
c Total mass of $^{235}\text{U} = 1150$ kg.
d The mass of the core is 52×10^3 kg.

42a $0.63\,\text{K W}^{-1}$
b 1.3 kW

43a $2.7 \times 10^{-3}\,\text{K W}^{-1}$
b $16 \times 10^{-3}\,\text{K W}^{-1}$
c*i* 1.3 kW
ii 7.5 kW
iii 8.8 kW
d*i* $I_T = I_1 + I_2$
ii The two calculations are analogous.

44a $19 \times 10^{-3}\,\text{K W}^{-1}$
b 570 W

46a $X/2$, $X/3$, X/A.
b $\mathscr{R} = X/A$. If, in spite of being led to the answer, you have ended up with the relation $X = \mathscr{R}/A$, think again why this cannot be correct.
c $m^2\,K\,W^{-1}$
d $X = l/k$

47a $0.17\,m^2\,K\,W^{-1}$
b $0.13\,m^2\,K\,W^{-1}$

48a $2 \times 10^{-3}\,K\,W^{-1}$
b 10 kW

50a Series
b $0.194\,m^2\,K\,W^{-1}$. The power loss is reduced to about one-fiftieth of its previous value.
c Outer surface of glass is at 6.2 °C. Inner surface of glass is at 6.6 °C.

52b Power loss through double-glazed window is about 120 W, *i.e.*, reduced to 0.6 of the single-glazed value.
c, d See figure Q43.

56a See figure Q44
b $0.114\,K\,W^{-1}$
c 158 W
d 15.6 °C
e $0.074\,K\,W^{-1}$
f 244 W
g 11.7 °C

57 Area of window = $1.25\,m^2$.

59 Power needed to heat the air = 1.3 kW. This is substantially greater than the loss through either single- or double-glazed windows in questions **50** and **52**.

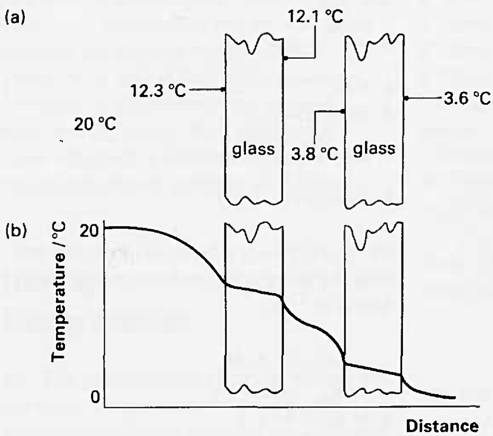


Figure Q43
(a) Temperatures at surfaces in a double-glazed window.
(b) Temperature variation with distance near a double-glazed window.

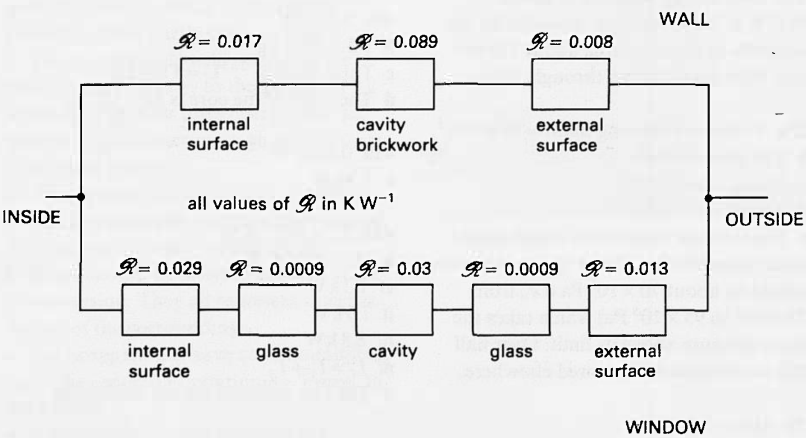


Figure Q44
Thermal resistances for wall with window.

REFERENCE MATERIAL

Textbooks and further reading

Textbooks that are useful throughout the course are listed here. Other books and references that are particularly relevant to specific Units are listed individually.

AKRILL, T. B., BENNET, G. A. G., and MILLAR, C. J. *Physics.* Arnold, 1979.
BOLTON, W. *Patterns in physics.* McGraw-Hill, 1974.
DUNCAN, T. *Physics: a textbook for advanced level students.* Murray, 1982.
OR
DUNCAN, T. *Advanced physics: materials and mechanics.* 2nd edn. Murray, 1981.
DUNCAN, T. *Advanced physics: fields, waves, and atoms.* 2nd edn. Murray, 1981.
WENHAM, E. J., DORLING, G. W., SNELL, J. A. N., and TAYLOR, B. *Physics: concepts and models.* 2nd edn. Addison-Wesley, 1984.

Unit A Materials and mechanics

In addition to the general list of textbooks above, these books are particularly useful for Unit A.

Strongly recommended for background reading

CRAC Hobsons Science Support Series, *Stress, strain and strength.* Hobsons Press, 1983.
CRAC Hobsons Science Support Series, *Gases and gas laws.* Hobsons Press, 1983.
GORDON, J. E. *The new science of strong materials.* Pitman, 1979. (First published by Penguin, 1968.)
GORDON, J. E. *Structures.* Pitman, 1979. (First published by Penguin, 1978.)
OGBORN, J. *Molecules and motion.* Longman, 1973. (Out of print.)
Materials. W. H. Freeman, 1967. (A *Scientific American* book.)

Textbooks for reference

AKRILL, T. B. and MILLAR, C. J. *Mechanics, vibrations, and waves.* Murray, 1974.
BOLTON, W. Study Topics in Physics *Volume 1: Motion and force.* Butterworths, 1980.
BOLTON, W. Study Topics in Physics *Volume 2: Materials.* Butterworths, 1980.
COLLIEU, A. MCB. and POWNEY, D. J. *The mechanical and thermal properties of materials.* Arnold, 1973.
MARTIN, J. W. and HULL, R. A. *Elementary science of metals.* Wykeham, 1969.
NUFFIELD REVISED ADVANCED CHEMISTRY *Students' book I.* Longman, 1984. (Topics 1, 3, 4, and 7.)
NUFFIELD REVISED PHYSICS *Pupils' texts.* Longman, 1978. (Especially Year 4.)

Further reading

BRONOWSKI, J. *The ascent of Man.* Futura, 1981. (First published by B.B.C., 1973.)
CLARKE, A. C. *The fountains of paradise.* Gollancz, 1979.
SCHOOLS COUNCIL Engineering Science Project, *Structures.* Macmillan, 1982.
SCHOOLS COUNCIL Engineering Science Project, *Dynamics.* Macmillan, 1982.
SCHOOLS COUNCIL Engineering Science Project, *The use of materials.* Macmillan, 1975. (Out of print.)
FARRER, R. A. Methuen Studies in Science, *The mechanical properties of materials.* Methuen, 1971. (Out of print.)
BLUNDELL, A., HAWKINS, R., and LUDDINGTON, D. Schools Council Modular Courses in Technology, *Structures.* Oliver and Boyd, 1981.
MCSHEA, J. Schools Council Modular Courses in Technology, *Materials technology.* Oliver and Boyd, 1981.
NUFFIELD ADVANCED PHYSICS *Physics and the engineer.* Longman, 1973. (Out of print.)
NUFFIELD CHEMISTRY Chemistry Background Book, *The structure of substances.* Longman, 1967. (Out of print.)
NUFFIELD CHEMISTRY Chemistry Background Book, *Plastics.* Longman, 1967. (Out of print.)
NUFFIELD REVISED CHEMISTRY *Handbook for pupils.* Longman, 1978.
NUFFIELD REVISED ADVANCED CHEMISTRY Special Studies, *Metals as materials.* Longman, 1985.

Unit B Currents, circuits, and charge

Textbook for reference

BENNET, G. A. G. *Electricity and modern physics.* 2nd edn. Arnold, 1974.

Background reading

CRAC Hobsons Science Support Series, *Instrumentation systems.* Hobsons Press, 1982.
MILLIKAN, R. A. Phoenix Science Series, *The electron.* University of Chicago Press, 1963.

Unit C Digital electronic systems

A large and rapidly growing number of books is available covering the work of this Unit. Some that you may find useful are listed below; try other books and magazines too.

General

ELECTRONIC SYSTEMS TEACHING PROGRAMME ESP 700 Book 3, *Processing systems.* 2nd edn. Feedback instruments, 1978.

The following books can be obtained in the U.K. from R.S. Components Ltd, but they must be ordered on school or college headed notepaper.

TEXAS INSTRUMENTS 'Understanding' series:
CANNON, D. L. and LUECKE, G. *Understanding communications systems.* Texas Instruments Inc., 1980.
CANNON, D. L. and LUECKE, G. *Understanding microprocessors.* Texas Instruments Inc., 1979.
MCWHORTER, G. *Understanding digital electronics.* Texas Instruments Inc., 1978.
TEXAS INSTRUMENTS LEARNING CENTER *Understanding solid state electronics.* 3rd edn. Texas Instruments Inc., 1978.

Computers

THOMPSON, D. L. *Inside the micro.* Unilab, 1982.

Practical circuits

MARSTON, R. M. *110 C-MOS digital I.C. projects for the home constructor.* Newnes, 1976.

The development and impact of microelectronics

BRAUN, E. and MACDONALD, S. *Revolution in miniature: history and impact of semiconductor electronics.* 2nd edn. Cambridge University Press, 1982.
Microelectronics. W. H. Freeman, 1977. (A *Scientific American* book of the September 1977 issue.)

Unit D Oscillations and waves

Textbooks for reference

As well as the general list of textbooks on page 491, these books are particularly useful for Unit D.

AKRILL, T. B. and MILLAR, C. J. *Mechanics, vibrations and waves.* Murray, 1974.
FEYNMAN, R. P., LEIGHTON, R. B., and SANDS, M. The Feynman lectures on physics, *Volume 1: Mainly mechanics, radiation, and heat.* Addison-Wesley, 1963.
PSSC *Physics.* 5th edn. Heath, 1981.
ROGERS, E. M. *Physics for the inquiring mind.* Oxford University Press, 1960.

Books for further reading

BISHOP, R. E. D. *Vibration.* 2nd edn. Cambridge University Press, 1979.
CHAUNDY, D. C. F. Longman Physics Topics, *Waves.* Longman, 1972.
DORLING, G. W. Longman Physics Topics, *Time.* Longman, 1973.
GRIFFIN, D. R. Science Study Series, No. 4, *Echoes of bats and men.* Heinemann, 1960. (Out of print.)
HOWSE, D. *Greenwich time and the discovery of the longitude.* Oxford University Press, 1980.
HUTCHINS, C. M. *The physics of music.* Freeman, 1978. (A *Scientific American* book.)
PROJECT PHYSICS Reader Unit 3. *The triumph of mechanics.* Holt, Rinehart, and Winston, 1971.
TRICKER, R. A. R. *Bores, breakers, waves, and wakes.* Mills and Boon, 1965. (Out of print.)
BASCOM, W. 'Ocean waves'. *Scientific American.* Volume **201**(2), Aug. 1959. (Offprint No. 828.)
BERNSTEIN, J. 'Tsunamis'. *Scientific American.* Volume **191**(2), Aug. 1954. (Offprint No. 829.)
BULLEN, K. E. 'The interior of the Earth'. *Scientific American.* Volume **193**(3), Sept. 1955. (Offprint No. 804.)
CRAC Hobsons Science Support Series, *Vibrations.* Hobsons Press, 1983.
CRAC Hobsons Science Support Series, *Waves and sound.* Hobsons Press, 1982.
GOULD, R. T. *John Harrison and his timekeepers.* 4th edn. National Maritime Museum, 1978.
GRIFFIN, D. R. 'More about bat radar'. *Scientific American.* Volume **199**(1), July 1958. (Offprint No. 1121.)
LYONS, H. 'Atomic clocks'. *Scientific American.* Volume **196**(2), Feb. 1967. (Offprint No. 225.)
OLIVER, J. 'Long earthquake waves'. *Scientific American.* Volume **200**(3), Mar. 1959. (Offprint No. 827.)

Note: Although *Scientific American* Offprints can no longer be purchased in Europe, the titles listed here may still be available in many schools and colleges.

Unit E Field and potential

Textbook for reference

ROGERS, E. M. *Physics for the inquiring mind.* Oxford University Press, 1960.

Further reading

ASIMOV, I. *The collapsing Universe: the story of black holes.* Corgi, 1978.
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The Library
Rutherford Appleton Laboratory
Chitton
Didcot
Oxfordshire
OX11 0QX

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DATA; FORMULAE AND RELATIONSHIPS; SYMBOLS

Data

(Values are given to three significant figures, except where more – or less – are useful)

Physical constants

speed of light	c	$3.00 \times 10^{8}\ \mathrm{m\,s^{-1}}$
permittivity of free space	ε_0	$8.85 \times 10^{-12}\ \mathrm{C^2\,N^{-1}\,m^{-2}}$ (or $\mathrm{F\,m^{-1}}$)
electric force constant	$\dfrac{1}{4\pi\varepsilon_0}$	$8.98 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$ ($\approx 9 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$)
permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{N\,A^{-2}}$ (or $\mathrm{H\,m^{-1}}$)
charge on electron	e	$-1.60 \times 10^{-19}\ \mathrm{C}$
mass of electron	m_e	$9.11 \times 10^{-31}\ \mathrm{kg} = 0.000\,55\ \mathrm{u}$
mass of proton	m_p	$1.673 \times 10^{-27}\ \mathrm{kg} = 1.007\,3\ \mathrm{u}$
mass of neutron	m_n	$1.675 \times 10^{-27}\ \mathrm{kg} = 1.008\,7\ \mathrm{u}$
mass of alpha particle	m_α	$6.646 \times 10^{-27}\ \mathrm{kg} = 4.001\,5\ \mathrm{u}$
Avogadro constant	L, N_A	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Planck constant	h	$6.63 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
molar gas constant	R	$8.31\ \mathrm{J\,mol^{-1}\,K^{-1}}$
gravitational force constant	G	$6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$

Other data

molar volume of a gas at s.t.p.	V_m	$2.24 \times 10^{-2}\ \mathrm{m^3}$
standard temperature and pressure		273 K (0°C), 1.01×10^{5} Pa (1 atmosphere)
gravitational field strength at Earth's surface	g	$9.81\ \mathrm{N\,kg^{-1}}$
mass of Earth		$5.98 \times 10^{24}\ \mathrm{kg}$
GM for Earth		$\approx 4 \times 10^{14}\ \mathrm{N\,m^2\,kg^{-1}}$
mass of Moon		$7.35 \times 10^{22}\ \mathrm{kg}$
average separation of Earth and Moon		$3.82 \times 10^{8}\ \mathrm{m}$
mean radius of Earth		$6.37 \times 10^{6}\ \mathrm{m}$

mean radius of Moon		1.74×10^6 m
the number e, the base of natural logarithms	e	2.718...

Conversion factor

unified atomic mass unit	1 u	$= 1.661 \times 10^{-27}$ kg

Formulae and relationships

Motion and forces

linear momentum $= mv$ (mass m, velocity v)

force = rate of change of momentum

$F = ma$ if mass is constant (force F, acceleration a)

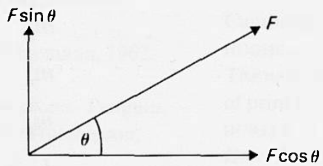
impulse $= F\Delta t$

translational kinetic energy $= \frac{1}{2}mv^2$

gravitational potential energy difference $= mgh$ (uniform field strength g, height h)

energy transformed (work) = component of force × displacement

components of force in two perpendicular directions:



moment of force about a point = force × perpendicular distance from point to line of action of force

static equilibrium conditions: $\Sigma F = 0$

Σ moments $= 0$

limiting friction $F = \mu N$ (coefficient of friction μ, normal force N)

circular motion $a = v^2/r$ (acceleration a, speed v, radius r)

$F = mv^2/r$ (centripetal force F, mass m)

Solids

For a material in tension

Hooke's Law: $F = kx$ (tension F, spring constant k, extension x)

stress = tension/cross-sectional area

strain = extension/original length

Young modulus = stress/strain

elastic strain energy $= \frac{1}{2}kx^2$

elastic strain energy per unit volume $= \frac{1}{2}$ stress × strain

Gases

Ideal gas equation

for n moles	$pV = nRT$	(pressure p, volume V, molar gas constant R, temperature T)
for one mole	$pV_{\mathrm{m}} = RT$	(molar volume V_{m})
Kinetic theory of gases	$pV = \frac{1}{3}Nm\overline{c^2}$	(number of molecules N, mass of molecule m, mean square speed $\overline{c^2}$)
	$p = \frac{1}{3}\rho\overline{c^2}$	(density ρ)

mean kinetic energy of translation of one mole of an ideal gas $= \frac{3}{2}RT$

Electricity

flow	$I = AvnQ$	(current I, area A, velocity of carriers v, carrier density n, charge Q)
resistance	$R = V/I$	(resistance R, potential difference V)
	$R = \rho l/A$	(resistivity ρ, length l, area A)
	$R = R_1 + R_2 + \ldots$	(resistors in series)
	$1/R = 1/R_1 + 1/R_2 + \ldots$	(resistors in parallel)
charge	$\Delta Q = I\Delta t$	(charge Q, time t)
capacitance	$C = Q/V$	(capacitance C)
	energy stored $= \frac{1}{2}QV$	
	$1/C = 1/C_1 + 1/C_2 + \ldots$	(capacitors in series)
	$C = C_1 + C_2 + \ldots$	(capacitors in parallel)

Oscillations

Simple harmonic motion

equation of motion	$a = -(k/m)s$	(acceleration a, force per unit displacement k, mass m, displacement s)
displacement–time relation	$s = A\cos\omega t$	(amplitude A, angular frequency ω, time t)
	$\omega^2 = k/m$	
	$T = 2\pi/\omega$	(periodic time T)
	$= 2\pi\sqrt{\dfrac{m}{k}}$	
	$f = 1/T = \omega/2\pi$	(frequency f)
	$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$	
	$v_{\max} = \omega A$	(maximum velocity $v_{\max}$)
	$a_{\max} = \omega v_{\max} = \omega^2 A$	(maximum acceleration $a_{\max}$)
	kinetic energy $= \frac{1}{2}mv^2$	
	potential energy $= \frac{1}{2}ks^2$	
	total energy $= \frac{1}{2}kA^2$	

Quality factor $\quad Q = = 2\pi \dfrac{\text{energy stored in oscillator}}{\text{energy lost per cycle}}$

Waves

Wave speeds		
for all waves	$c = f\lambda$	(wave speed c, frequency f, wavelength λ)
compression wave in mass-spring system	$c = x\sqrt{\dfrac{k}{m}}$	
sound in a solid	$c = \sqrt{\dfrac{E}{\rho}}$	(Young modulus E, density ρ)
transverse wave on string	$c = \sqrt{\dfrac{T}{\mu}}$	(tension T, mass per unit length μ)

Field and potential

All fields	$E = -\mathrm{d}V/\mathrm{d}r$ $(\approx -\Delta V/\Delta r)$	(field strength E, potential gradient $\mathrm{d}V/\mathrm{d}r$)
electric field	$E = F/Q$	(electric field strength E, force F, charge Q)
uniform field between parallel plates	$E = V/d$	(potential difference V, separation d)
	$\sigma = \varepsilon_0 E$	(charge density σ, permittivity of free space ε_0)
parallel plate capacitor	$C = \varepsilon_0 \varepsilon_r A/d$	(capacitance C, relative permittivity ε_r, area A, separation d)
point charges	$F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1 Q_2}{r^2}$	(charges Q_1,Q_2, separation r)
	$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$	(electric field strength E)
	$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$	(electric potential V)
gravitational field	$g = F/m$	(gravitational field strength g, force F, mass m)
	$F = -Gm_1 m_2/r^2$	(universal gravitational constant G, masses m_1, m_2, separation of centres r)
	$g = -GM/r^2$	(mass of Earth, or other body, M)
	$V_g = -GM/r$	(gravitational potential V_g)
	$\Delta V_g = GM(1/r_1 - 1/r_2)$	(gravitational potential difference ΔV_g)
uniform gravitational field	$\Delta V_g = g\Delta h$	(height h)

Atomic and nuclear physics

Radioactive decay	$\mathrm{d}N/\mathrm{d}t = -\lambda N$	(number N, decay constant λ)
	$N = N_0 e^{-\lambda t}$	(initial number N_0)
	$T_{\frac{1}{2}} = \dfrac{\ln 2}{\lambda} = \dfrac{0.693}{\lambda}$	(half-life $T_{\frac{1}{2}}$)
mass–energy relationship	$\Delta E = c^2 \Delta m$	(energy E, mass m, speed of light c)

Energy transfer

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

$$\text{efficiency of heat engine} = \frac{T_1 - T_2}{T_1}$$ (temperature of source T_1, temperature of sink T_2)

$$\Delta T = \phi \mathcal{R}$$ (temperature difference ΔT, rate of thermal transfer of energy ϕ, thermal resistance $\mathcal{R}$)

$$\mathcal{R} = l/kA$$ (area A, length l, thermal conductivity k)

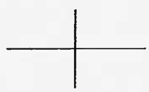
$$X = \mathcal{R}A$$ (thermal resistance coefficient X)

Electrical circuit symbols

Some of the symbols used for circuit diagrams are shown below:

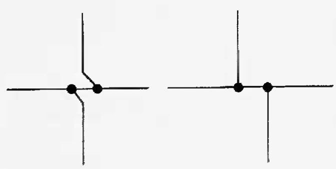
Wires, junctions, terminals

crossing of wires, no electrical contact

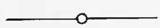


junction

double junction



terminal



aerial

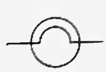
earth

frame or chassis connection

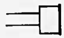
Lamps

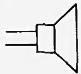
signal lamp

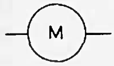
lamp for illumination

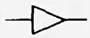


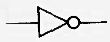
Transducers
microphone 

earphone 

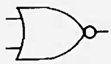
loudspeaker 

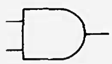
motor 

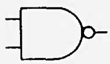
Amplifier
(or non-inverting gate) 

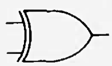
Logic gates
Invert or NOT gate 

OR gate 

NOR gate 

AND gate 

NAND gate 

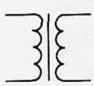
Exclusive OR
(XOR) gate 

Capacitors
general symbol 

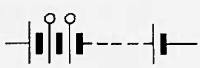
polarized (electrolytic) capacitor 

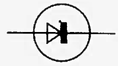
Inductors
general symbol 

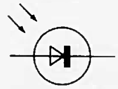
inductor with core 

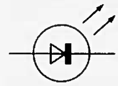
transformer with
ferromagnetic core 

Batteries
primary or secondary cell  *(The short thick bar represents the negative terminal)*

battery with tappings 

Diodes
diode/rectifier 

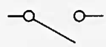
light sensitive diode 

light emitting diode
(L.E.D.) 

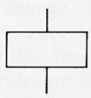
Measuring instruments
voltmeter 

ammeter 

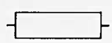
galvanometer 

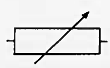
Switches, relays
normally open switch  

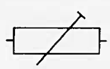
normally closed switch  

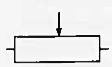
relay coil 

relay contact 

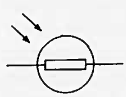
Resistors
general symbol 

variable resistor 

resistor with preset adjustment 

potentiometer 

resistor with inherent
variability (e.g. thermistor) 

light-sensitive resistor 

INDEX

X

Y

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This Students' guide **contains the first seven of the twelve Units in the A-level physics course.**

The Units are: Unit A, 'Materials and mechanics'; Unit B, 'Currents, circuits, and charge'; Unit C, 'Digital electronic systems'; Unit D, 'Oscillations and waves'; Unit E, 'Field and potential'; Unit F, 'Radioactivity and the nuclear atom'; and Unit G, 'Energy sources'.

Each of the first six Units follows the same pattern: a brief introduction is followed by *Summaries* **of the most important ideas in the Unit; there are then** *Readings*, **possibly extracts from specialist journals or other publications, followed by questions, which should help you to develop the skill of reading with a purpose; the** *Laboratory notes* **cover all the experiments and demonstrations; most Units have suggestions for** *Home experiments*, **practical activities you can do at home; finally, there is a large number of** *Questions*. **The structure of Unit G, 'Energy sources', is rather different. Among other things, you will be expected to look at one or more of the suggested topics in detail, and to report on your findings. This** Guide **ends with** *Answers to selected questions* **and some useful** *Data; Formulae and relationships; Symbols*.

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