REVISED NUFFIELD ADVANCED SCIENCE

PHYSICS

DYNAMIC MODELLING SYSTEM

MODELS

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o Software

Preface

This computer assisted learning unit has been produced by Professor Jon Ogborn supported by the Computers in the Curriculum Project at Chelsea College. It was designed to be used in the Revised Nuffield A-Level Physics course as well as other secondary science courses.

The program has two parts: the operating system and a Physics models disk. The program is supported by a comprehensive guide on how to use the system with examples of how to create a model. A guide to the models disk is also provided, which describes each of the Physics models on the disk. The user is also able to create and save new models, thus building up a library of models to meet individual needs.

The system has been designed to meet the needs of different teaching methods, student abilities and class groupings. Additional guides and models disks are being developed for other subjects, for use in further and higher education as well as in the secondary school curriculum. This system has a wide range of applications in science teaching and can also be used in many other areas of the curriculum.

The Dynamic Modelling System is part of a wide range of computer assisted learning materials developed by the Computers in the Curriculum Project for use in education.

Margaret Cox
Project Director
1985

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Acknowledgements

This system has been developed in conjunction with the Computers in the Curriculum Project, part of the Educational Computing Section, Chelsea College. The principal author is Professor Jon Ogborn (Institute of Education).

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Margaret Cox
Project Director
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1 Introduction

These notes describe the models on the models disk designed for use with the Dynamic Modelling System, in the Revised Nuffield Advanced Level Physics course. Most of the models described will be useful in any Advanced Level Physics course.

The booklet *Dynamic Modelling System* gives general information about the use of the system and technical information about its facilities. It also gives examples of some dynamic models and their uses, together with a worked exercise for those new to the idea.

It would be impossible, and undesirable, to provide on the models disk all the potentially useful models, and all the variations upon them. Instead, the models provided are intended as a starting point. They include representatives of important kinds of models together with useful parts of models which can be used in building up new ones.

2 Using the models with different microcomputers

One of the models which is concerned with radioactive decay, RDECAY, and the models concerned with optics, SLITS, 1SLIT, 2SLIT, use programming features which are only available in the BBC microcomputer version of the Dynamic Modelling System. The models which can only be used with the BBC microcomputer version are clearly marked in the text.

The BBC, RML 380Z and 480Z, and Apple IIe microcomputer versions all allow the use of lower case symbols to represent variables and constants. The Apple II microcomputer version will not allow the use of lower case variable and constant names. In order to make the models in the text compatible with all the implementations of the system, only upper case symbols have been used. Users may wish to change variable names to include lower case symbols, for example to represent a time increment as dT rather than DT, if this is possible.

Models disks other than those provided with the system can be used. A formatted disk can have some of the models supplied transferred to it, or be used to save newly developed models.

3 Descriptions of the models provided

The models on the disk are grouped below broadly by subject to bring out general similarities and differences. Thus models in any one group will not necessarily be used close in time to one another, but it may be useful to recall an earlier use of one model in a group when developing a later model.

Decay, growth and resource models

a Decay of charge on a capacitor

The decay of charge on a capacitor (*Unit B, 'Currents, circuits and charge'*) is likely to be an early use of the system. A model CAPDIS is stored on the disk.

CAPDIS

Model	*Values*	
Q = C*V	V = 10	(V)
I = V/R	C = 500 E–6	(F)
DQ = –I*DT	R = 100 E3	(Ω)
Q = Q + DQ	T = 0	(s)
V = Q/C	DT = 2	(s)
T = T + DT		

On each loop through the model, Q = C*V calculates the charge on the capacitor from the present potential difference across it, and I = V/R calculates the current through the resistor. The change of charge DQ = –I*dT is found and the charge Q is then altered by this amount. (The single line Q = Q – I*DT would do instead.) Then the new potential difference V = Q/C is found, T is increased by DT, and the loop begins again.

It may be useful to begin with simpler models of the same kind. Thus

```
Interest = Rate  *Cash
Cast     = Cash + Interest
Year     = Year + 1
```

will give compound interest, or approximately exponential growth, whilst

Spend = Fraction *Cash
Cash = Cash − Spend
Year = Year + 1

will give approximately exponential decay, as a fixed fraction of total cash that is spent each year. Alternatively, one could try

Births = Fertility*Population
Deaths = Mortality*Population
Population = Population + Births − Deaths
Generation = Generation + 1

which gives growth if fertility exceeds mortality, or vice versa.

To explain the capacitor discharge model it may be easiest to start with the change of charge DQ = I*DT. The current I has to be found first, so I = V/R must be inserted earlier in the model. To change the charge, the present charge has to be known, so one puts Q = C*V in at the beginning. Finally, after the new charge Q is obtained, a new potential difference V has to be found from V = Q/C.

In the capacitor discharge model, R and C can be varied to see the effect of the time-constant R*C on the discharge. It may be useful to change the way in which the values are expressed to give R and C from

RC = 50
R = (assign a value)
C = RC/R

In this way it is easy to show that the same value of R*C always gives the same rate. (Note that the system reads 'RC' as one variable, not as R times C. It may be better to write TIMECONST in place of RC.)

The model on the disk is set to plot potential difference V against time T. This can be changed to show charge Q against time or current I against time. Logarithmic graphs can be shown by asking for a plot of LOG(V). (LOG is log to base e on the RML and Apple microcomputers, but log to base 10 on the BBC microcomputer, for which log to base e is LN). For a maximum of 10 V the maximum on the logarithmic axis can be set to 1 (base 10) or 3 (base e).

b Radioactive decay

Radioactive decay (*Unit F, 'Radioactivity and the nuclear atom'*) comes later in the Nuffield course but is conveniently described here. The stored model, DECAY, is simply:

DECAY

Model	*Values*	
DN = −1*N*DT	N = 1000	(particles)
N = N + DN	L = 5/1000	(s^{-1})
T = T + DT	DT = 2	(s)
	T = 0	(s)

where L is the decay constant, that is, the probability of decay in an interval of one second. This model is unrealistic, in that the random nature of the decay is not modelled. The model RDECAY shows how this feature can be included:

RDECAY
[*BBC microcomputer version only*]

Model	*Values*	
D = 0	N = 100	(particles)
P = L*DT	L = 0.05	(s)
FOR I = 1 TO N	T = O	(s)
IF RND (1) < P	DT = 1	(s)
THEN D = D + 1		
NEXT I		
N = N − D		
T = T + DT		

Here the loop inside the model (FOR ... NEXT) gives each of the N particles a chance to decay, such that if the random number RND(1) in the range 0 to 1 is more than the probability P = L*DT a decay is added in that time interval, and the next particle is tested. (The FOR loop takes less time as N diminishes: it may be possible to use the computer's internal timer to make the loop occupy the same time.)

The effect of the random selection is that the decay fluctuates around the exponential form. The smaller N is made, the larger the fluctuations become, relatively. Very large values of N cannot be used easily because the computation is too slow.

c Resource models

Another use for models of growth and decay is found in *Unit G, 'Energy sources'*, when the depletion of energy resources is discussed. The simplest model of this type stored on the disk is GROWTH, which shows an unlimited growth of consumption:

GROWTH

Model	*Values*
If R < = 0 THEN C = 0	R = 2000
DC = C*G*DT	C = 10
C = C + DC	G = 0.1
R = R − C*DT	DT = 1
T = T + DT	T = 0

R represents the magnitude of the resource, C is the rate of consumption, and G is the growth rate of C.

This model gives exponential growth of consumption, up to the point where the resources R are exhausted, when consumption falls directly to zero. Graphs of C and R against T can be plotted, as can logarithmic graphs.

One reason for such growth is that consumption can stimulate further consumption, by increasing economic activity. However, the assumption that consumption grows as a constant fraction G of present consumption in each time interval is unrealistic, if the resource consumed is vanishing. One attempt to represent this is given in the model RESRC:

RESRC

Model	*Values*
F = (R − U)/RO	RO = 100
DC = C*G*DT*F	R = RO
C = C + DC	U = 0
R = R − C*DT	G = 1
U = U + C*DT	C = 1
T = T + DT	DT = 0.1
	T = 0

RO is the initial resource, R is the current resource, and U is the resource used up.

Here the fractional increase in consumption is effectively the constant growth G times the fraction F which falls as the amount of resource left falls and the amount used rises. It is defined so that the consumption rate starts to fall when U exceeds R, that is when half the original resource is used. The outcome is a rise in consumption, followed by a drastic fall to zero. The shape of the curve is sensitive to the initial rate of consumption, and to the growth rate G.

There is no particular reason for the factor F to have the form it is given, and a more serious model would need some economic analysis of the nature of variations in the growth rate. Nevertheless, a fraction F which falls as the resource is used, and changes sign when a substantial part is used, is not unreasonable.

d Models of thermal conduction, thermal losses and home heating

Closely related to models of decay are models of thermal conduction and thermal losses. No particular models are provided, but amongst the obvious possibilities are:

- cooling of an insulated body;
- warming up of an electric kettle;
- warming up of a room to an equilibrium temperature.

More complicated models of warming a room could include heat transfer to the ground, daily temperature changes or the temperature changes in a room with the heating controlled by a thermostat. This last example also points to a number of control applications, relevant to *Unit I, 'Linear electronics, feedback and control'*.

Models of motion

A large class of models suited to the Dynamic Modelling System is concerned with dynamics. This includes models of free fall, projectile motion, orbital motion, trajectories of alpha particles and oscillators. A selection of models have been stored on disk together with the component parts of models which can be used to construct further models.

a Free fall

After doing a graphical solution for uniformly accelerated motion by hand, stressing that the acceleration only alters the position via changing the velocity, FALL can be constructed:

FALL

Model	*Values*	
F = −G*M	G = 9.8	($N\,kg^{-1}$)
A = F/M	M = 2	(kg)
V = V + A*DT	Y = 4	(m)
Y = Y + V*DT	V = 0	(ms^{-1})
T = T + DT	T = 0	(s)
	DT = 0.02	(s)

The first and second lines could be omitted and the third line could be written as V = V − G*DT.

A graph of Y (height) against T (time) will give a parabola, of course. It is useful to ask the class what a graph of V (velocity) against T would look like: that it is a straight line – because the velocity is changing uniformly – may be less obvious than it should be. This is where the modelling system can bring out very important matters in a simple way: the line V = V + A*DT must increase V by the same amount in each interval if A is constant.

It is no great trouble to insert lines to calculate kinetic and potential energies:

KE = (1/2)*M*Y↑2
PE = −G*M*Y
E = KE + PE

and plot how they and their sum vary with time or height.

It is of particular importance in the modelling system that any variables can be plotted quickly, as well as the 'obvious' ones, and that new variables (such as energy) can be introduced. Asking what the plot should look like, and then, when one has it, why it has the form it does, is frequently a useful teaching device.

For a little interesting realism, resistive forces can be added. For a body falling in air, the most important resistive force comes from setting air in motion in front of the body. The mass of air set in motion per second at a velocity (of the order of) V is the density of air multiplied by the area of cross-section and the velocity. If it were this simple we would have:

Resistance = (air density)*area*V*V

In fact, the shape of the body can be allowed for by multiplying this expression by a form factor F between 0 and 1. Computationally, to get the resistive force to oppose the velocity, one needs a term −SGN(V) where SGN(V) is +1 for positive and −1 for negative velocities.

In this way a model of a freely falling body can show the body reaching a terminal velocity, with quite realistic values.

b Component parts for models of motion

An inspection of FALL shows that it is made of several parts. The force of gravity is specific to the problem, but the rest is not. Newton's second law (second line) will always be needed to calculate accelerations from forces. The calculation of velocity from acceleration (third line), and position from velocity (fourth line), will always be needed too. Thus, it is useful to think of dynamics problems as made up of these steps and of their models as made of the corresponding components. For this reason, generally useful parts of models are stored on the disk, and these are now described below. For generality, they have been written for working in two dimensions.

One component part is NEWTON, which is simply:

```
NEWTON

AX = FX/M
AY = FY/M
```

giving X and Y components of acceleration AX and AY from X and Y components of force FX and FY. Knowing the acceleration, the velocity change can be found, as is done in the kinematics component part KINEM:

```
KINEM

VX = VX + AX*DT
VY = VY + AY*DT
 X = X + VX*DT
 Y = Y + VY*DT
 T = T + DT
```

In the kinematics model, after VX and VY are known, new positions X and Y are calculated. Taken together, NEWTON and KINEM are sufficient to model all kinds of dynamics problems if expressions for the relevant forces FX, FY are also given.

The forces of a uniform gravitational field are given in GRAV1:

```
GRAV1

FX = 0
FY = -G*M
```

Thus a model for a projectile moving in the Earth's gravitational field near the surface can be constructed by reading into the system in turn:

```
GRAV1
NEWTON
KINEM
```

The resulting model will have the form:

FX = 0
FY = −G*M
AX = FX/M
VX = VX + AX*DT
VY = VY + AY*DT
X = X + VX*DT
T = T + DT

The model PROJ on the disk has this form, except that the forces also allow for viscous resistance proportional to velocity. The equations describing these forces are shown below:

FX = −FR*VX
FY = −G*M − FR*VY

Viscous resistance forces (proportional to velocity) are stored as a separate component in DAMP1. More realistic resistive forces for the projectile are proportional to the square of velocity. These are stored in DAMP2:

```
DAMP2

RX = FR*SGN(VX)*VX↑2
RY = FR*SGN(VY)*VY↑2
FX = FX − RX
FY = FY − RY
```

(The term SGN(V) makes the direction of the resistive force oppose that of the velocity. For systems without this function, IF statements will have to be used.)

c Orbital motion

Unit E, 'Field and potential' presents an opportunity to develop models of motion in an inverse square gravitational field. The component parts NEWTON and KINEM are needed, as above, but X and Y components of force also have to be calculated from the radial distance R and the angular position. The radial distance is just:

R = SQR(X↑2 + Y↑2)

The radial force F is:

F = −G*M1*M2/R↑2

G is the gravitational constant here, not the gravitational field strength as in FALL and PROJ.

The force components are F cos (angle) and F sin (angle). Since cos (angle) = X/R and sin (angle) = Y/R, FX and FY can be expressed as:

```
FX = F*X/R
FY = F*Y/R
```

The model ORBIT stored on the disk is in fact:

ORBIT

Model	*Values*	
X = X + VX*DT	M1 = 6.6E24	(kg)
Y = Y + VY*DT	M2 = 100	(kg)
R = SQR (X↑2 + Y↑2)	G = 6.7E–11	(N m^2 kg^{-2})
F = – G*M1*M2/R2	R = 2E7	(m)
FX = F*X/R	V = 5000	(m s^{-1})
FY = F*Y/R	A = 0	(radians)
VX = VX + FX*DT/M2	B = PI/2	(radians)
VY = VY + FY*DT/M2	DT = 500	(s)
T = T + DT	X = R*COS(A)	
	Y = R*SIN(A)	
	VX = V*COS(B)	
	VY = V*SIN(B)	

For convenience, the radius R and resultant velocity V are given as initial values, and then the X and Y components of position and velocity calculated from these and angles A and B. A and B are the initial angular directions of R and V respectively. This enables initial distance and velocity to be changed rather more simply than by giving the components.

Orbital calculations are computationally rather tricky. The simple method of solution used in all the dynamics models so far, the 'Euler' method, does not in fact conserve energy or angular momentum for discrete intervals of time, so it is possible to obtain orbits which – quite incorrectly – spiral in or out, or which precess. The placing of the calculation of position at the beginning of the loop is quite helpful, as the force is then calculated for an updated radius, but is still not precise. Various devices can be used, the best adjusting the values so as to conserve energy and angular momentum, are likely to be too complex to be helpful where the goal is understanding the mechanism, not calculating with precision. However, it will be important to stress that nature does not work in discrete time intervals, delivering gravitational tugs in short bursts, as the model makes it do. Computational models can be made better, but may never be perfect.

The values in the model can be altered to consider, for example, a satellite much nearer the Earth, the Moon going round the Earth, or the Earth going round the Sun. Very elliptic orbits such as those of a comet can be produced, though computational errors are likely to be large in the part of the orbit near the gravitating body.

The values in the stored model are arbitrary, but have been chosen to illustrate a mildly elliptical orbit, such that changes of velocity can clearly be seen. The Earth is used as the gravitational body. As well as plotting a plan of the orbit (that is, the plot of Y against X) it is interesting to plot V = SQR(VX↑2 + VY↑2) against R.

d Alpha particle scattering

No model for alpha particle scattering is included, though one will be useful in *Unit F, 'Radioactivity and the nuclear atom'*, primarily to stress that it is identical in form to that for a gravitational orbit, with the force made repulsive instead of attractive. A suitable form for the electrical inverse-square law is provided, in the component part ELINVSQ:

ELINVSQ

Model	*Values*	
K = 1/(4*PI*EO)	PI = 3.142	
F = K*Q1*Q2/R↑2	EO = 8.85E^{-12}	(F m^{-1})

The value of PI does not need to be specified in the BBC version of the Dynamic Modelling System.

These lines substituted for the inverse-square gravitational force complete the conversion. To do so, erase the gravitational force, open a line where the new force is wanted and put the cursor on the screen on the gap, and read in ELINVSQ. It will be necessary also to remove the values of G and M1 and put in values of EO, Q1 and Q2. The distances and velocities will need altering too, of course. It might be valuable to give the energy of the alpha particle in (say) MeV, and calculate the velocity from that using V = SQR(2*KE/M) with a suitable energy conversion factor from MeV to J.

e Oscillations

Unit D, 'Oscillations and waves', offers an opportunity to make models of oscillators. The only difference from FALL, described previously, is the force law. The model OSCIL is:

OSCIL

Model	*Values*	
FX = −K*X − FR*VR	K = 8	(N m^{-1})
AX = FX/M	M = 2	(kg)
VX = VX + AX*DT	X = 0.5	(m)
X = X + VX*DT	VX = 0	(m s^{-1})
T = T + DT	FR = 0	(kg s^{-1})
	T = 0	(s)
	DT = 0.05	(s)

The force law says that the force is proportional to the displacement X, with force constant K. In addition, viscous damping is included. In the stored model, the coefficient FR is set at zero so that the oscillator is initially undamped.

This model could be developed from free fall just by changing the force law, and later adding damping. It has been stored written in terms of an *X*-component of force to suggest the extension to two dimensions, and so to Lissajous figures.

Another important development is to introduce a forcing term. The force computation could be replaced by using the equation below:

FX = −K*X − FR*VK + FO*SIN(W*T)

A sinusoidal driving force of amplitude FO and angular frequency W has been added. W can be defined as W = 2*PI*f values if that is convenient. With W close to the natural angular frequency, SQR(K/M), the oscillations build up to a large value. Transients can be observed, especially with low damping (note that with zero damping the transients persist indefinitely). An example of such a driven oscillator is stored on the disk, but as an electrical example (See below, for a discussion of the model RESON).

Models of inductive and capacitative circuits

Unit H, 'Magnetic fields and a.c.' and *Unit I, 'Linear electronics, feedback and control'*, both provide opportunities for models involving circuits with capacitance and inductance. The analogy of electrical with mechanical oscillations can be effectively illustrated by the modelling system.

A number of sample models are provided on the disk, to suggest the range of possibilities. One is driven by a single step voltage, while another is driven by a square wave. Such input voltage variations can clearly be applied to any case, as can the sinusoidal variation used to drive the model of LRC resonance.

a RC circuit

This model represents a circuit with R and C in series; the input is across both components and the output (VO) is across R. If the output across C is required, a plot of the potential difference across C instead of that across R is required. The model RC is:

RC

Model	*Values*	
IF COS (W*T) > O		
THEN V = VO	VO = 100	(V)
IF COS (W*T) < 0		
THEN V = 0	R = 1000	(Ω)
VC = Q/C	C = 5 E–6	(F)
VR = V − VC	Q = 0	(C)
I = VR/R	T = 0	(s)
DQ = I*DT	W = 2*PI*20	(rad s^{-1})
Q = Q + DQ	DT = 0.001	(s)
T = T + DT		

The input is a square wave, alternating between VO and zero. It is easy to alter the model to give a square wave going from +VO to −VO and back. The potential difference across R shows the differentiating possibility of the circuit, with the square wave converted into spikes. By contrast, the potential difference across C shows the circuit acting as an integrator. Clearly, values of R and C can be varied to study the behaviour of the circuit. It may be best to start with a model whose input potential difference is just a single step, as in the RL example which follows. In addition, a sinusoidal input voltage can be used, when the variations of potential difference across the input, R and C will show phase differences, if plotted in succession.

b RL circuit

This model represents a circuit with R and L in series. There is a single step voltage going from 0V to 10V after T = 0.1. The model RL is:

RL

Model	*Values*	
R = 0:IF T >0.1 THEN V = 10	R = 2	(Ω)
VR = I*R	L = 0.5	(H)
VL = V − VR	T = 0	(s)
DI = (1/L)*VL*DT	DT = 0.01	(s)
I = I + DI	I = 0	(A)
T = T + DT		

The current is initially zero. When the input voltage rises to 10V, the current, and the potential difference across R, slowly increase. The crucial point is that the increase of current DI in interval DT is limited by the inductance L, such that a potential difference VL across L is needed to produce a rate of change of current with VL = L*DI/DT.

As with the RC circuit, square wave and sinusoidal inputs can be used, and phase relations examined in the latter case.

c LRC circuit

The LRC circuit modelled is a parallel circuit, so that VL + VR + VC = 0. The model assumes that the charge Q on the capacitor is known (it is calculated in the initial values), so that VC can be found. From the current I, initially set to zero, VR is found. From these the potential difference across L can be obtained, which gives the change DI in the current, and thence the change DQ in the charge. The model LRC is:

LRC

Model	*Values*	
VC = Q/C	C = 0001	(F)
VR = I*R	R = 10	(Ω)
VL = −VC − VR	L = 10	(H)
DI = (1/L)*VL*DT	VC = 10	(V)
I = I + DI	Q = C*VC	(C)
DQ = I*DT	T = 0	(s)
Q = Q + DQ	I = 0	(A)
T = T + DT	DT = 0.005	(s)

With these values, the model gives slowly decaying oscillations at about 5 Hz. Clearly values of L, R and C can be altered to study their effects. In addition, the oscillator can be driven by a step, square or sinusoidal input. An example of the last is stored as a resonant circuit RESON, identical to the above but starting with

V = VO*COS (W*T)

and with the potential difference across L calculated from:

VL = −VC − VR + V

This model gives a slowly rising amplitude of oscillation, since W is set at 2*PI*5. Varying W permits transients to be seen, and the differences in the final amplitude characteristic of resonance. By plotting V, VR, VL and VC the phase differences in the circuits can be inspected.

d Control and feedback

As mentioned earlier, control and feedback problems offer considerable opportunities for dynamic models. They may involve RC circuits, as in dealing with operational amplifier circuits, or they may be more general. Those which are generally called first order are analogous to growth and decay models; those which are called second order are close to oscillatory models in dynamics or electricity.

Interference and diffraction models

The modelling system can be used for spatial as well as for temporal changes, as can be seen from examples below of interference and diffraction arising in *Unit J, 'Electromagnetic waves'*. The basic idea associated with these models is to add contributions of waves from separate sources. These sources can be two or more discrete sources or waves originating from points across a slit. Three models are provided which can readily be extended or modified.

a Two or more sources

The model SLITS allows the waves combining from two or more slits to be studied. It is set up for Fraunhofer diffraction, by using approximate formula for calculating angles which assumes that the screen is very distant compared with the slit spacing. The model is:

SLITS

[*BBC microcomputer version only*]

Model	*Values*	
Y = Y + DY	A = 1	
AR = 0:AI = 0	X = 1000	(mm)
B = A/N	Y = −10	(mm)
P = 2*PI*(Y/X)/L	DY = 0.1	(mm)
H = 0:FOR J = 1 TO N	L = 5 E–4	(mm)
H = H + S	N = 5	
PH = H*P	S = 0.1	(mm)
AR = AR + B*COS (PH)		
AI = AI + B*SIN (PH)		
NEXT J		
I = AR↑2 + AI↑2		

X is the slit to screen distance, and Y the position on the screen. Y is varied in steps, increasing by DY. For each position the loop inside the model adds contributions from each of N sources spaced a distance S apart. With wavelength L, the phase difference P per unit distance across the sources is 2*PI*(Y/X)/L. As the position H across the slits is increased by S at each step, the phase PH is given by H*P. The calculation is done like this to reduce the amount of time-consuming computation inside the loop. The waves are added by taking two components, which may be thought of as the real and imaginary parts AR and AI of the amplitude. When the loop ends, the intensity is found from the sum of the squares of the amplitude components.

The amplitude from each source, B, is made a fraction 1/N of the fixed amplitude A, so that graphs with different numbers of slits N have the same maximum intensity.

It may be best to start with N = 2, or perhaps to write the rather simpler model for just this case. As N is increased, the maxima become sharper, and there are small variations of intensity close to each.

b Single slit

Clearly it would be useful to look at the diffraction pattern from a single slit. The attempt to model this produces an unexpected subtlety: the job reduces to adding waves across a gap divided into a number of sources, and so is in principle identical to the model for a large number of multiple sources, such that if the slit width W is divided into slices of width DW, there are W/DW sources. A consequence of this is that the central maximum appears as one expects, with the usual side lobes, but one can be sure, by analogy with the multiple slit case, that at larger angles there will be similar maxima, which with a true single slit do not exist. The reason is the true single slit is not divided into discrete slits, but is continuous, a feature which a discrete computation cannot model. As N = W/DW increases, the other maxima move to larger and larger angles, vanishing as N tends to infinity.

The single slit model is:

1SLIT

[*BBC microcomputer version only*]

Model	*Values*	
Y = Y + DY	X = 1000	(mm)
AR = O:AI = 0	Y = −2	(mm)
B = A*DW/W	DY = 0.05	(mm)
P = 2*PI*(Y/X)L	L = 5 E–4	(mm)
H = 0:FOR J = 1 TO W/DW	W = 0.5	(mm)
H = H + DW	DW = W/10	(mm)
PH = P*H	A = 1	
AR = AR + B*COS (PH)		
AI = AI + B SIN (PH)		
NEXT J		
I = AR↑2 + AI↑2		

This models the situation with a 0.5 mm slit shining light of wavelength 5 E–4 mm onto a screen 1000 mm from the slit. The central maximum occupies a few mm on the screen (Y starts at −2 mm). It is easy to alter the slit width, wavelength and screen distance. The precision can be improved (though not by much) by making DW a smaller fraction of the slit width W than 1/10.

c Double slits of finite width

The ideas of the last two models can be combined to model a pair of slits, with a spacing of S and a finite width W. The model has to contain two loops: one as above to add effects of waves across the widths of the slits, and one to take waves alternately from each slit. The model on the disk, 2SLIT, is:

2SLIT
[*BBC microcomputer version only*]

Model	*Values*	
Y = Y + DY	A = 1	
AR = 0:AI = 0	X = 1000	(mm)
B = A*DW/(2*W)	Y = −4.5	(mm)
P = 2*PI (Y/X)/L	DY = 0.05	(mm)
FOR J1 = 1 TO 10	L = 5 E–4	(mm)
FOR J2 = 0 TO 1	W = 0.1	(mm)
PH = P*(J1*DW + J2*S)	DW = W/10	(mm)
AR = AR + B*COS (PH)	S = 0.5	(mm)
AI = AI + B*SIN (PH)		
NEXT J2: NEXT J1		
I = AR↑2 + AI↑2		

This models slits 0.5 mm apart and 0.1 mm wide, with other dimensions as before. The counter J1 steps across the slit widths in ten slices, while J2 alternates between the two slits. The result is the expected interference pattern within the envelope of the diffraction pattern of one slit.

Other applications could clearly include multiple slits of finite width, and such problems as diffraction at circular or other shaped apertures.

The value of the modelling system in these cases may be twofold: showing something close to the theoretical expectations, which might otherwise have to remain empirically based, and more important, showing in their structure how the results all come from the same process of adding up waves with phase differences.

Should one wish, Fresnel diffraction could be studied, using values for which the small angle approximation fails, and replacing the approximation in the model. However, beware of subtracting large distances to find small path differences, when rounding errors may be serious.

Function models: the Boltzmann factor

Another kind of use of the modelling system, to display functions, arises in connection with *Unit K*, *'Energy and entropy'* where the Boltzmann factor is of interest. The model stored as BOLTZ is simply:

BOLTZ

Model	*Values*	
T = T + 1	T = 272	(K)
BF = EXP (−E)/(K*T)	K = 1.38 E–23	(J K^{-1})
	E = 10*K*T	(J)

This simply plots out the Boltzmann factor, starting at T = 273 K, showing how it increases with temperature. Note that a plot can be started from a 'false origin', here T = 273, by asking to plot not T but T–273. Further study of the function could include looking at how it varies with energy E, making E variable instead of T.

The modelling system can be used to display a wide range of functions of interest, such as EXP (−T/T0), COS (W*T) and the resonance curve. Note that, as above, the initial value of the independent variable should be set at one increment less than is required, so that the first increment brings it to the required initial value.

Atomic models: the hydrogen atom

Unit L, 'Waves, particles and atoms' concerning the structure of atoms, includes the possibility of solving the Schrodinger equation for a hydrogen atom using numerical methods.

The case studied is the radial wave function for the hydrogen atom, that is, the wave whose $(\text{amplitude})^2$ gives the probability of finding the electron at a certain distance from the nucleus, for spherically symmetric states. The Schrodinger equation can be regarded as a differential equation for standing waves:

$$d^2A/dr^2 = (2\pi/\lambda)^2 A$$

in which the wavelength λ is a function of radius r. Since

$$\lambda = h/mv$$

and the momentum mv can be found from the kinetic energy, itself found from the difference between total and potential energy. The equation determines the curvature of the wave function at every point. The stored model HATOM to solve it is:

HATOM

Model	*Values*	
A = A + B*DR	Q = 1.6 E − 19	(C)
R = R + DR	K = 9 E9	(N m^2 C^{-2})

V = −K*Q*Q/R	R = 0	(m)
KE = E − V	DR = 5 E–12	(m)
C = −A*KE*1.65 E38	B = 0.2 E12	(m^{-1})
B = B + C*DR	A = 0	
	E = −2.185 E–18	(J)

Q is the charge on the electron or proton, and $K = 1/(4\pi\epsilon_0)$ is the electrical force constant. The initial amplitude A is taken as zero, since the radial wave function must go to zero as the radius R goes to zero. The initial slope B is arbitrary: its value is chosen so that B*DR = 1, which makes plotting simple. Note that the choice of B, in effect, decides the arbitrary scale of A. The quantity C is related to the curvature. This is large where the wavelength is small and the wave function is strongly curved. The constant 1.65 E38 contains the Planck constant and the mass of an electron.

The model supplied is set up to plot the amplitude A against radius R. A helpful display is obtained by plotting on the same axes a horizontal line corresponding to the selected total energy E and a curve corresponding to the variation of potential energy V.

Different total energies E can be tried. Only certain values give a bounded solution asymptotic to the *R*-axis at large R. The allowed energies E are found to follow the rule $E = E_0/n^2$, where E_0 is close to the initial energy chosen above.

The discrete step-by-step solution is difficult to implement with precision, and at 'large-enough' R all solutions will diverge. One can only estimate the energies of states by interpolating between values which give solutions which diverge in different directions. Exact solution would require more advanced numerical techniques which fit a suitably shaped 'tail' to the wavefunction. The difficulty is that the very exact matching of amplitude and curvature needed to bring the solution asymptotically to the axis, cannot be achieved numerically in any simple way.

A similar model of some interest would be the harmonic oscillator, changing essentially only the form of the potential energy V in the above model.

4 Teaching suggestions

Classroom use

There are broadly three possible ways of using the Dynamic Modelling System in class teaching:

- call up a prepared model already stored on disk, run it, and discuss with the class how it works;
- start with a prepared model or with parts of a model and build up a new model, after discussion, by revising or adding to what is already there;
- start with nothing, and build up the desired model from scratch, during discussion with the class.

The first is close to using the system as a simulation program, with the difference that the model here is visible, and can be changed. The second is useful in helping students to see what parts of Physics are used in a given problem, and how different problems relate to one another. The third works on the assumption that programming the solution to a problem is a useful problem-solving exercise in itself.

The system can of course also be used individually by students, for example in modelling a phenomenon being studied in an investigation.

Dynamic models and step-by-step integration

In solving problems such as capacitor discharge or the motion of a projectile, the Dynamic Modelling System uses the same step-by-step method of solution as is recommended in the Nuffield course for producing graphical solutions. It would seem best to start with the graphical solutions, done by hand, and then move on to the computer system for its greater speed and flexibility.

A first step – what Physics is needed?

A useful starting point may be to ask a class to list all the relevant ideas in Physics for a given problem. Having sifted out the genuinely relevant, one can begin writing equations relating variables.

A distinction will need to be drawn between equations which represent basic steps in the physical process, and those which are in fact the result being sought. Thus in tackling free-fall, it is likely that $s = \frac{1}{2}at^2$ will be proposed as needed for the solution. It is worth asking if there is a direct physical connection between distance and time such that this relationship is basic to the problem. Of course, $s = \frac{1}{2}at^2$ is the answer, rather than part of getting to the answer.

Programming and problem solving

A possible value of the use of the system is that in general each step in the solution corresponds to some simple real process. Examples of this are given below:

DQ = I*DT	expresses the meaning of current;
Q = Q + DQ	follows from the conservation of charge;
V = V + A*DT	Together show how acceleration
X = X + V*DT	changes velocity and thus position.

One approach, exploiting this fact, may be to build up sets of cards, on one side of which is the physical principle expressed in words and equations, and on the other side of which is one or more lines of BASIC program which shows how computations using the ideas would translate.

Solving a problem might then be tackled by selecting suitable cards, arranging them 'Physics-side-up' in a suitable order to work out step-by-step what happens, and then turning them 'BASIC-side-up' to convert the sequence into a program.

Bugs and debugging

As anyone who uses computers knows, errors of various kinds, 'bugs', invariably creep in. Some, such as mis-spelling a variable name and leading the computer to expect a different variable, are little more than irritating, but some are more fundamental and so more interesting.

An example might be the choice of signs. 'Friction' with the wrong sign will produce spontaneous acceleration and absurd results which may be more obvious in a graphical display than they would be by merely having the wrong sign in the solution of a differential equation. It may even be worth introducing such errors on purpose to see their effects.

A significant computational malady is 'try it and see'. When unexpected results are obtained, some people try tinkering with axes, scales and values, or with the model, trying to make the visual effect closer to what they think it should be. This is often a disaster, giving completely artificial effects unrelated to the real problem. The slope of a graph may be too small, for example, not because the scale is wrong but because the model is fundamentally at fault.

Graphs, scales and tables of results

One of the hardest aspects of using the system is the appropriate choice of graph scales. It is rather easy to choose values which plot right off the screen so that nothing appears to happen. Beware also of values which plot along an axis – with one variable always zero – which may not be visible.

One answer is first to get a table of results, to see the range of values covered, and then choose scales appropriately. The tabulating facility is also a useful debugging tool, showing how results are departing from those which are expected.

Another answer to choosing suitable scales is to think! Indeed, working out approximately how large a variable may be is a valuable test of understanding. For example, if one has an oscillator showing displacement against time and wants to show velocity against time, the magnitude of the velocity can be estimated from the slope of the trace already on the screen. Choosing the right scale of time in the first place is perhaps harder: either one needs to know how the frequency is related to spring constant and mass, which is part of the answer being sought, or one has to guess or work from a table of results.

It is very tempting to give completely arbitrary values, but it is much better to insist on choosing values which mean something. It would be sensible to pull an actual spring in the hands, and estimate its force constant, so that physical intuition has a chance of guessing how fast that spring would make a given mass oscillate.

Values of constants

In setting up a model, it is frustrating not to be able to recall values of relevant constants: for example an orbit model can be held up for lack of values for G, the mass and radius of the Earth, and so on. One possibility would be to store a 'model' with a set of convenient values written in – a set of constants for orbit calculations, for instance. It will be necessary to edit them, perhaps altering variable names and deleting some, but this may still be worth the trouble.

Physics and computing

Some may fear that the modelling system will be used as a substitute for thought and for the ability to solve problems on paper. If it were to become so, that would indeed be wrong, so it is important to form a clear view of its proper roles.

An essential distinction here is that between the formulation and the solution of a problem. The formulation of a physical problem in computer terms is often rather simple and illuminating, and may bear little relation to the mathematical difficulty of getting a solution. Thus the ingredients of a simple growth model, such as:

Babies = Fertility *Rabbits
Rabbits = Rabbits + Births
Generation = Generation + 1

are rather transparent, while the exponential function, which is the solution, would be considered rather difficult. Similarly, an oscillator damped by various kinds of frictional force presents considerable mathematical difficulties, while rising only slowly in computational difficulty.

Thus, one role for the modelling system is to emphasise the basic physical processes at work in a given situation.

Another distinction is between the analytic solution of a problem, if there is one, and its computational solution. The great advantage of an analytic solution is that one may be able to work out general properties of families of solutions, instead of having to generate them and observe or speculate on relationships. For example, if the displacement varies as sin (ωt) we know that the velocity varies as cos (ωt), if we know how to differentiate. On the other hand, it is important to remember that functions such as cos, sin, exp, or log are themselves computed functions. The tables of their values have to be calculated in some way essentially similar to that used by the modelling system. It is salutary to reflect that if one asks BASIC to calculate COS(X) it must use an internal routine like that written in the system for an oscillator.

A consequence of this argument is that it will often be valuable to use the system to generate, not one solution, but whole families of solutions, by altering variables systematically. In this way, students may be able to see visually how the properties of the solution depend on its parameters.

It is to be hoped, therefore, that the system will be used to deepen, not to trivialise, students' mathematical and physical understanding. Being a program with an empty space waiting for whatever one wants to put there, it can be used well or badly. It is merely a tool, and does not have an educational agenda of its own, so it is up to the teacher and student to judge how best to use it.