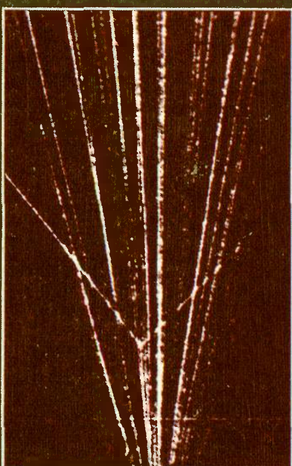
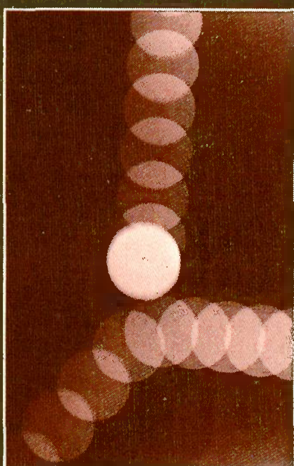
PHYSICS

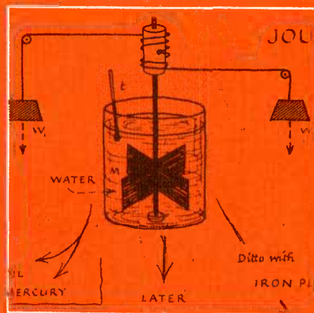
Teachers' guide IV

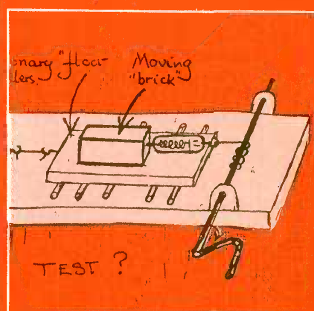


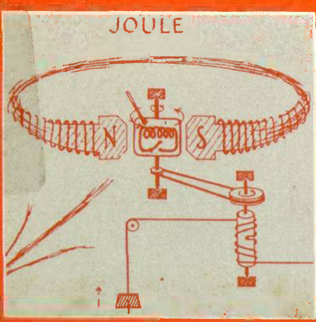
A Nuclear Collision
Tracks of α - particles passing through helium

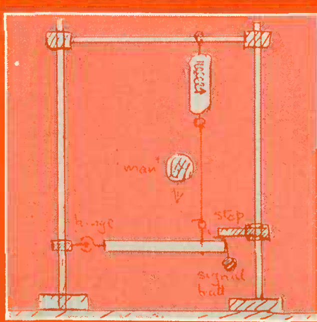


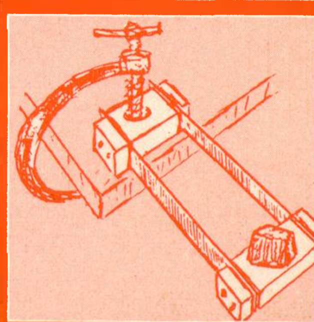
Elastic Collision
A ring-magnet 'hits' an equal magnet at rest











NUFFIELD PHYSICS TEACHERS' GUIDE IV

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FOREWORD

This volume is one of the first to be produced by the Nuffield Science Teaching Project, whose work began early in 1962. At that time many individual schoolteachers and a number of organizations in Britain (among whom the Scottish Education Department and the Association for Science Education, as it now is, were conspicuous) had drawn attention to the need for a renewal of the science curriculum and for a wider study of imaginative ways of teaching scientific subjects. The Trustees of the Nuffield Foundation considered that there were great opportunities here. They therefore set up a science teaching project and allocated large resources to its work.

The first problems to be tackled were concerned with the teaching of O-Level physics, chemistry, and biology in secondary schools. The programme has since been extended to the teaching of science in sixth forms, in primary schools, and in secondary school classes which are not studying for O-Level examinations. In all these programmes the principal aim is to develop materials that will help teachers to present science in a lively, exciting, and intelligible way. Since the work has been done by teachers, this volume and its companions belong to the teaching profession as a whole.

The production of the materials would not have been possible without the wholehearted and unstinting collaboration of the team members (mostly teachers on secondment from schools); the consultative committees who helped to give the work direction and purpose; the teachers in the 170 schools who participated in the trials of these and other materials; the headmasters, local authorities, and boards of governors who agreed that their schools should accept extra burdens in order to further the work of the project; and the many other people and organizations that have contributed good advice, practical assistance, or generous gifts of material and money.

To the extent that this initiative in curriculum development is already the common property of the science teaching profession, it is important that the current volumes should be thought of as contributions to a continuing process. The revision and renewal that will be necessary in the future, will be greatly helped by the interest and the comments of those who use the full Nuffield programme and of those who follow only some of its suggestions. By their interest in the project, the trustees of the Nuffield Foundation have sought to demonstrate that the continuing renewal of the curriculum – in all subjects – should be a major educational objective.

Brian Young
Director of the Nuffield Foundation

CONTENTS

ESTIMATED ALLOCATION OF TIME

YEAR IV

If it is assumed that a school year includes 30 weeks and that each week includes 3 physics periods, each of which lasts for 40 minutes, then a very rough estimate of the number of periods suggested for each section of this Year would be:

Chapter 1	22
Chapter 2	20
Chapter 3	10
Chapter 4	8
Chapter 5	20
Chapter 6	10
	90

Although these estimates are rough they will, nevertheless, provide some guidance as to weight to be placed on the various parts of the programme. It should be noted that the relative amounts of printing are not proportional to the teaching time required. Where subject matter is new and unfamiliar, it has been dealt with at length in order to help any teacher who may wish to experiment with it. On the other hand, more familiar subject matter has been dealt with quite briefly.

KEY TO MARGIN REFERENCES

C = Class experiment

D = Demonstration experiment

H = Suggestions for optional experiments at home

F = Film

T = Teaching of material (lectures, discussions with pupils, etc.)

P = Problem

*** = Commentary (notes on methods, aims, etc., offered to teachers)

‡ = Reference to footnote

§ = Reference to a comment made by a teacher during trials.

(The experiments are numbered serially through the Year, irrespective of the classification C, D, F or H. The same numbers will be found for each experiment in the *Teachers' Guide to Experiments and Apparatus*. Where (a), (b) ... are added to the number these refer in some cases to separate parts of the same group of experiments, in other cases to alternative versions of an experiment.)

PREFACE TO YEAR IV

Year IV is a stage for more serious and thorough study – still with enjoyment and delight, we hope, but now with more reasoning and co-ordinating thinking – a Year of learning to 'make a noise like a physicist', as Rutherford once put it.

We might think of pupils in Years I and II as taking a journey in a bus across some of the foreign countryside of physics: making acquaintance with things seen from the window, exploring a town here and there in a carefree way, and learning, unconsciously, some of the vocabulary of the inhabitants. In Year III, the boys and girls were making a more serious journey to visit a foreign family and learn how people live, to develop a vocabulary of phrases as well as words – but still rather a visit to explore and make acquaintance than an expedition for serious study. Now on their journey in Year IV, our young pupils should do much more; they should read the timetables, plan with maps, and look forward to reading the literature of the land they visit with sufficient care and appreciation to gain a sense of knowledge.

Thus, we should present the physics of Year IV as a much more logical and closely knit programme than that of earlier Years. If the previous work has been satisfactory, pupils will have had sufficient experience and enjoyment to wish to undertake the work, and will continually ask questions – which at this age will be calling for a more highly organized intellectual structure for the subject. However, it is important that this change of treatment should not be made the excuse for a degeneration into a purely mathematical exercise: theory, as always, must be soundly based on experiment – and that experimental work should remain largely the pupil's own class experiments, not the teachers' demonstrations.

Preparation provided by Year III

Preparation by Year III is essential. It will have provided some foundation for this Year's work in dynamics and in electricity and magnetism. Pupils should be able to handle the ideas of acceleration and velocity comfortably, and have some operational knowledge of inertial mass and force – but it is doubtful whether they will have a grasp of any formal version of Newton's Second Law; and although they will have seen collision experiments, the idea of momentum will not have been mentioned as such.

Year III will have built on the electric-circuit work of Year II by pupils' experiments with the electromagnetic kit: magnetic fields due to currents and due to permanent magnets; the motor effect; a working electric motor; electromagnetic induction.

Some pupils may have continued to try simple experiments with a transformer; and some fast groups may have tried using a voltmeter – we hope only empirically – but we should not rely on the preparation having gone as far as that.

Year IV and Examinations

In most programmes of teaching physics in school, there is some major stress on learning new material in Year V, because pupils are then more mature and have a well-prepared background, and face examinations. In our programme, we suggest bringing some of that attitude back from Year V to Year IV, where we can give some formal treatment more comfortably, without the immediate shadow of external examinations. The knowledge of physics which we have been exploring in earlier Years should now be consolidated, so that by the end of this Year pupils have some sense of physics as a connected science and are ready to explore important continuations in Year V. For example, Year I's introduction to ideas of atoms and molecules now links with kinetic theory of gases and the idea of electrons with atoms of charge joins in. The introduction to energy in Years I and II broadens into general conservation and kinetic energy becomes a quantity that can be calculated. Pupils see how energy plays an essential part in defining voltage. Laws, first seen as a simple Hooke's Law in Year I, now include great generalizations such as conservation of momentum and conservation of energy.

We trust Year V can follow its programme without teacher or pupils feeling that only then, at last, have they reached the solid examination material. We hope much of that material will have been reached already, both in matters of information and in attitude towards understanding science.

Thus we hope to avoid making Year V a servant of examinations partly by the construction of the programme in earlier Years, partly by asking for examinations that require a general understanding of physics rather than factual material that can be crammed or coached in a final year.

Year V Ahead

Year V will deal with electron streams crossing magnetic fields, the mass spectrograph, and experimental studies of radioactivity. Those studies will, we hope, make a good contribution to pupils' knowledge of 'atomic physics' and give many a pupil a keen interest that will carry him into further reading on his own.

In Year V we shall conduct a historical study of planetary astronomy and apply Newton's Laws of Motion and of Gravitation to it, so that pupils appreciate the grand conceptual scheme which Newton himself set forth. Our aim in including that in our physics programme is not to teach astronomy but to let pupils see for themselves how good theory is developed. That will give them, we hope, a lasting feeling for the use of theory in science.

Year V will continue the story of atom models, using a description of alpha-particle scattering to lead to a nuclear atom model. And, with faster groups, we hope to continue to other topics in modern 'atomic' physics such as wave-particle behaviour; photo-electric effect and 'photons'.

The Work of Year IV

With those thrilling developments ahead in Year V, we must consider the necessary preparations to be made in Year IV. We can make Year IV a very interesting Year, with some of its pleasure that of growing discipline as a scientist, and a very fruitful one in preparing some material for examinations.

In Year IV we should give pupils a good opportunity to explore Newton's Laws of Motion by their own experimenting – not to discover those laws, but to make measurements that illustrate them. Out of those measurements should come an increasing sense of 'mass' as a very important tangible property of matter – and, as pupils will find later, of energy. They should consider Newton's Second Law in terms of momentum changes; and, with the Third Law, arrive at a clear knowledge of Conservation of Momentum.

Pupils should apply Newton's Laws of Motion to the qualitative kinetic picture of gases that we have already built up and arrive at a simple kinetic theory prediction of gas pressure. That will at once lead to a numerical prediction of the speed of molecules from simple measurements. There should be further discussion of gas molecule behaviour, such as diffusion, in terms of this more quantitative kinetic theory. Some pupils will even arrive at the size of an individual gas molecule, from a simple experiment and some imaginative reasoning – but that will be only for those with courage to follow a difficult line of thought.

Electrical studies will continue: first with voltmeters in circuits; then with streams of electrons observed in class experiments.

Then we shall discuss Millikan's great experiment to show that there is a universal 'atomic charge' – that electrons are all alike.

In this Year, results of experiments can be stated more formally, and records of experiments should be somewhat fuller – though still not a tedious piece of unnecessary copying and drawing. The idea of putting together knowledge from several parts of physics should be emphasized when there is opportunity. This should be a Year of interesting experimenting, some reasoning, clever thinking, and the building of a more solid sense of knowledge.

Teachers' Planning

Since Year III gives pupils practice with tickertape and other devices for measurement in exploring Newton's Laws of Motion in an informal manner, any teacher coming new to the programme with Year IV should first look at the work of Year III in mechanics, and for that matter in electricity and magnetism too. And he will need to look carefully at the introduction to energy and work in Years I and II.

Teachers, with a slow group, will find that the burden of establishing links with the Year III material threatens to give them more than can be done in the Year – unless the teacher spoils the whole point of the programme and hurries through the material by giving lectures and demonstrations and notes to be learnt. There would be nothing wrong with that condensed treatment if our aim were to cover the material for future training and for immediate examinations. However, one would not then be giving the Nuffield programme a useful trial.

So, we offer teachers the following comments about speed:

> The electricity and magnetism can be hurried considerably and some of it can be postponed to Year V.
>
> The kinetic theory treatment cannot be hurried; but if teachers consider the material very carefully in the light of the ability of their pupils, they will be able to plan a treatment which will not take very long – the abler pupils who deserve the whole of the discussion of molecule size, etc., will be able to take even that quite fast.
>
> The energy-conservation discussion – the work of Joule and others – should not take long, particularly as we shall offer some sketches of diagrams and a table of results for pupils. However, we hope that every teacher will give that discussion full emphasis.
>
> The class experiments on dynamics – force, mass and motion and momentum conservation – should be hurried gently. It would be very unwise to hurry them much, particularly as they come at the beginning of the Year, but they should not be allowed to drag.

In looking at that commentary on the timing of the programme, given above in reverse order, teachers will see that a careful, almost leisurely, start of studies of motion will be justified. And they can look forward to considerable elasticity towards the end of the Year, when some experiments with alternating currents could be postponed to the following Year. That would be better than any attempt to compress or economize the modern physics of electron streams and Millikan's experiment, which come at that time and form a very important new part of our physics programme.

Since the treatment in this guide for Year IV inclines towards a more formal approach than in earlier Years, teachers may find that a glance at the guides for Years I and II will give them a clearer idea of our underlying aims, which were set forth more fully there for teachers beginning the programme. We have the same aims in Year IV.

As an example of our attitude: we should like to see things like the following printed on the front page of examinations to provide information for pupils, who will then be asked to make good use of them.

$$PV = \tfrac{1}{3}Nmv^2 \qquad F = ma \qquad Ft = \text{change of } mv$$

and even (without explanation) the rubric 'volts = joules/coulomb'.

This is intended to be a Year that gives greater knowledge, with clearer foundations and a growing sense of being a capable scientist.

Note on Shortened Programmes

Some schools find they would have to shorten our programme for Years III+IV+V to two years before they could adopt it. This note explains the difficulties of forms of shortening that have been suggested; and urges schools who need a two-year programme to make a fresh start in constructing one.

Even if our present programme gives little detailed help in such new planning, we believe our general guiding principle will still be fruitful: that we should *try to bring the spirit of our sixth-form physics teaching – the genuine doing of science and learning for understanding – to our younger pupils.*

Could a Class begin the Programme at Year IV? It is not possible to make a good beginning as late as Year IV. We have tried that in preliminary trials and have found that it damages the programme very seriously.

In Year IV pupils need the preparation with trolleys given in Year III; otherwise, the work on Newton's Laws becomes too long and boring, and its extension into kinetic energy experiments does not get a fair treatment with class experiments.

Electric circuit experiments in Year IV should follow those of Year III; and if pupils have to go back and do the long, important, series of class experiments of Year III, their progress will indeed fall short in Year IV. If we attempt to save time by teaching the Year III preparation in electromagnetism by demonstrations, we lose much of the point of the series of class experiments with the electromagnetic kit. Furthermore, the work with that kit in Year III presupposes a series of class experiments on electric circuits in Year II; and pupils who missed those would need some special, extra preparation, even if they had seen some electric circuit experiments in a different programme.

Our strong treatment of kinetic theory in Year IV draws upon acquaintance with a picture of molecules in motion in gases – and some pictures for solids and liquids too – built up in Years I, II and III. Pupils who began at Year III could perhaps replace the preparation of earlier years by watching several demonstrations; but pupils who started at Year IV would find that those added to their burdens too seriously.

In preliminary trials classes starting at Year IV had so much earlier preparation to fit in that they had a difficult, crowded year. That influenced Year V in turn, making revision for examinations more anxious than it need be. And the examinations, dipping back necessarily into Year III, imposed serious hardships on candidates who had missed it.

In the light of the structure of our suggested programme and of experience in trials, we urge schools very strongly to embark on our programme only at Year I or Year III; and not to start at Year IV. That does not mean that we think the contents of Years III +IV+V is an ideal minimum programme. Doubtless, a good programme of teaching physics with the spirit and methods we suggest could be constructed to occupy two years. Such a shortened programme could not be made by picking parts from our present programme without losing much of its value. We have tried to make our present programme a connected scheme in which some topics are introduced early and treated again with increasing sophistication from year to year and other topics are treated early to give preparation for later work. To make a similar, connected scheme to be taught in two years instead of three, it will be necessary to plan from a fresh start. That is certainly possible but we do not know whether that would satisfy Examining Boards as a sufficiently full programme for an O-level subject.

Could a Class omit Year V? Another suggestion for a two-year course based on our programme, is that pupils should start at Year III, proceed to Year IV but omit Year V. With our present structure, that would indeed present the course as a headless body. Without its uses for astronomy and for electron streams in Year V, the work on Newtonian dynamics in Years III and IV would seem much nearer to sterile drill. Without the study of light and interference in Year V, the ripple tank experiments of Year III would seem a pleasant sideshow. And without the topics of atomic physics in Year V our early interest in atoms and molecules would lose a considerable amount of its value. Of course, pupils who continue with physics and shift our Year V treatment into A level will lose nothing; and they may gain by being in a fast group that can use special mathematical tools. But others who leave physics after O level would miss the proper drawing together of old topics and the fruitful look at present-day physics which we feel the programme owes them in Year V.

We hope that schools who must compress what we call Years III+IV+V into two years will make a fresh start in planning a new programme to fit their needs and will not try to compress our programme or upset its structure by picking items from it, while hoping the result will serve the same ends. We hope our examinations, both those in schools and those set by Boards, will continue to look for understanding – particularly in their marking. So we must warn schools who plan a two-year version taken from our present programme that their pupils are likely to find our examinations ill suited.

We express these doubts from no feeling of unkindness or of false pride in our programme, but from our knowledge that we have carried out our original instructions, which were to make a concerted five-year programme leading to O level. We do not think our programme is unique or ideal; but we do believe it is a very good programme for pupils who can give it the time for which it has been planned. A shortening of time brings temptations towards quick memorizing. Also, as in most teaching for understanding, a spread of time for digestion is one of our strongest aids.

Note to Teachers: Uses of Newtonian Mechanics

Before embarking on a serious study of Newton's Laws, all of us who teach might profitably reflect on their importance and uses. The Laws are great guiding summaries, consistent with the behaviour of things in our world.‡ Even if they are not based fully

‡ They are very important in modern physics – we need them in detailed studies of atoms, rockets, stars.... But we do not now regard them as simple experimental laws: to us they are a mixture of experimental knowledge and definitions which we assume to organize our science.
The experimental basis *is* there: Newton's Laws *do* fit our world. In relativity terms, we *do* live in a world that is approximately an inertial frame. If we had lived in an accelerating frame, we might never have arrived at those laws.

or directly on experiment, they provide a basis for examining nature, relating one natural event with another, and predicting other events. In that role, they seemed very important laws to Newton's contemporaries and to physicists ever since, including Einstein:

> 'No one must think that Newton's great creation can be overthrown by "Relativity" or any other theory. His clear and wide ideas will forever retain their significance as the foundation on which our modern conceptions of physics have been built.'
>
> Albert Einstein (1948)

However, we cannot expect a pupil to see the importance and glory of these Laws unless he understands what they summarize, why they are drawn as summaries, and what use is made of them. (We may see a similar difficulty when young pupils just starting on Latin or Greek are urged to appreciate the glories of the Classics.)

Motor cycles and railway engines can be understood more easily by common sense than by use of Newton's Laws. To understand the firing of a rocket, a qualitative knowledge of Newton's Laws of Motion is valuable; but the proper quantitative form that we deal with in Year IV is of little use until quite difficult studies of rockets with their changing mass are undertaken.

Kinetic theory of gases makes good use of Newton's Laws, but that is still ahead of us. And we make some use of the Laws in our studies of electrons and atoms in Years IV and V.

Satellites are of great interest to pupils and an understanding of them does need Newton's Laws. In fact, the first plan of an artificial satellite was made by Newton himself. In his addition to later editions of the *Principia*, he sketched the Earth with a mountain on it and a gun firing projectiles from the top of the mountain. For faster and faster projectiles, the path landed farther and farther out, until it reached an orbit that would just encircle the Earth. Newton himself pointed out the obvious difficulty of air resistance; and he did not expect artificial Earth satellites to be practicable.

Except for kinetic theory and satellites, our possible uses of the Laws lie in the future and the need for them is not very clear. So, unless we can show a more appealing need, we must not expect our pupils to be thrilled with the power that a knowledge of Newton's

Laws can give them. They should study Newton's Laws carefully but rather quickly, leaving further emphasis and extension and revision until need arises.

We certainly should not follow our experimental studies and short theoretical discussion with artificial problems constructed to test pupils' use of Newton's Laws; not even artificial problems constructed to find out whether pupils understand Newton's Laws – understanding will come later when uses and needs are clear.

If this discussion of our teaching plans seems depressing, reflect on Newton's own work. Newton gathered a good deal of knowledge of force and motion from the writings of Galileo (who himself copied much of his work from a few earlier writers who had been quietly reforming knowledge of motion). Newton applied this knowledge and his own thinking to the great problem of his day – the motion of planets. A revolution of astronomical knowledge was 'in the air'. Galileo's teaching and writing had spread the Copernican picture of the planetary system across the reading world. Jupiter's moons, seen through the newly invented telescopes, even provided a model of the system. Kepler had disentangled three astounding laws of planetary motion from the observations of Tycho Brahe, and had announced them in a profusion of mystical writing. The Copernican theory was being accepted, but Kepler's Laws raised interesting questions: what kind of machinery could account for these experimental laws that described the planetary motions so well? Kepler himself had mentioned magnetic forces and even gravity spreading from the Sun, but not with any understanding of the relations of force and motion that would make a consistent story. Questions were being asked across Europe. There were suggestions of a force like gravity, emanating from the Sun and growing weaker with distance, perhaps with an inverse-square law.

Thus, in the 1660s, more than a century after Copernicus's book appeared, there was a ferment of discussion. It was clear from Galileo's writings that one need no longer look for a force to propel a planet *along* its orbit – motion continues if left alone. The problem instead was one of finding the force that pulled a planet *in*, to a circular or elliptical orbit instead of a continuing straight line.

Some members of the Royal Society of London knew that an inverse-square law of gravitation would account for Kepler's Third Law, relating planetary years and circular orbit sizes. However, Kepler's elliptical orbits were too difficult for them; no one could show that an inverse-square law of gravitation would require such an orbit, or that Kepler's other law, the Law of Equal Areas, should hold. An appeal to Newton produced the astonishing answer that he had already solved the problem of all three laws and knew that they are necessary consequences of universal inverse-square-law gravitation.

Newton was finally persuaded to publish his work in greatly expanded form, in which he set forth not only his proofs of Kepler's Laws but a whole study of force and motion and universal gravitation in which he linked together: the gravity of falling bodies, the Moon's circular motion, planetary motions (Kepler's Laws), motion of comets, the tides, the shape of the Earth and ensuing differences of gravity, precession of the equinoxes, disturbances of the Moon's simple motion and planetary perturbations – all as parts of one tremendous structure of theory. As he himself wrote of his intentions:

> '... from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; ... the motions of the planets, the comets, the Moon and the sea. ...'

This was success beyond all expectations. No wonder Newton's work was written about and expounded in every civilized country; and no wonder it came to be taught in school, generation after generation, to the present day. Yet, somehow, the astronomical problems that called for his work and received his magnificent solution have been crowded out of present-day teaching for the majority of school pupils. We teach the Laws of Motion and make some use of them, in engineering and in atomic physics, but we miss the great drive felt by Newton's contemporaries and successors. How then are pupils to develop appreciation?

In planning the Nuffield Physics Programme, we wish to restore the balance to some extent, *not as a move towards historical teaching*, but to give our pupils a chance to appreciate Newton's Laws. And, far beyond that, they should see for themselves the part that good theory plays in science. We do not feel that we should interrupt the programme in Year IV with astronomy when we are about to study Newton's Laws. This would be a most fitting place for the historical study leading to Newton's work, but we are not sure that it would appeal to all pupils and we are anxious not to delay progress into kinetic theory and work with electrons, etc. So, we suggest that teachers should give at that point only a brief statement, that Newton, in stating his Laws, was clearing up man's ideas about motion so that he could clear up and extend our whole knowledge and understanding of the solar system. And then we should promise to return to that in Year V.

In Year V we shall suggest a historical study of knowledge and theories of the planetary system, from early information and empirical rules to Greek theories, on to Copernicus and Kepler and Galileo, so that pupils then feel the force of a great body of knowledge waiting for a concerted explanation and can thus see the full glory of Newton's work. We shall do that, not as a piece of history of science, but as an example of the building of physical theory which is sufficiently simple for pupils to follow and understand as part of their own knowledge. Pupils in school should see some example of good theory being built up, in a way that they can understand. Then, if it is some thinking that they have seen and worked with themselves, it will be something to be remembered, and theory will not be said to be 'mysterious, difficult thinking'.

Atomic theory is often suggested as an example of theory for pupils. However, we should have to provide, at this stage, so many ready-made results, that we would hand out the story without sufficient justification. It would form a poor prototype for this particular use. Instead, we suggest that Newtonian gravitation theory should be taught first, after building up a clear history of need – and then atomic theory can be described, almost ready-made, without harming the understanding of theory that we wish to give to all educated people.

If we now take a realistic look at the needs for Newtonian mechanics in Years IV and V we see that pupils will need to:

understand and measure momentum-changes, for kinetic theory and later for atomic collisions

know conservation of momentum as a general law; and use it in atomic collisions

use $F = ma$ with $s = \frac{1}{2}at^2$ to calculate deflection of electron streams, etc.

obtain the expression $\frac{1}{2}mv^2$ for kinetic energy (this is perhaps the most important single use that we shall make)

use $F = ma$ with $a = v^2/R$ for motion in circular orbit, for planets and for electrons, etc., in Year V

understand the meaning of a *newton* as a unit of force for use in pressure measurement in kinetic theory and for use in atomic physics; and understand *joule*, *watt*, and *volt*

and we hope they will also profit from seeing a great theory being built up and bearing fruit.

Note to Teachers on Apparatus for Dynamics

We have a healthy tradition of illustrating Newton's Laws of Motion by experiments with trolleys. Various forms of Fletcher's Trolley are in use for demonstrations in many a school, and there is often some pupil participation. Yet the arrangements for precise timing and uniform behaviour make the experiments seem complicated, or even confusing, to all but the ablest pupils. Sometimes that is a question of interest: it is not very exciting to watch a complex experiment and its analysis, when one does not feel personally involved. The laws being 'proved' do not strike the beginner as vital, essential, knowledge he has been waiting for; so he is not likely to be intensely interested. Yet we could build interest more easily if the experiments were clear and pupils did them themselves.

Therefore in our programme we suggest using simple, robust, well-behaved apparatus for *class* experiments in dynamics. We have chosen designs that allow pupils to carry out their experiments easily and even make some ingenious modifications of their own.

And we hope to lessen worries over *mass* and *weight* by avoiding gravity pulls for the accelerating forces.

That is why we give an important place in Years III and IV to simple trolleys with good ball-bearing wheels, running on plane runways. We suggest that schools should provide a long runway and two trolleys for every pair of pupils – not one set for three or four pupils, which will harm the strong sense of doing one's own experiment. Our object is not so much to provide a convincing demonstration of Newton's Second Law as to give pupils a personal feeling for forces and masses and their connection with motion – through their own experimenting. Two pupils sharing equipment can carry out their experiment with little instruction, feeling they are really exploring motion; but with more pupils crowded round, the experiments lose that flavour.

If a trolley is pulled by a thread running over a pulley to a hanging load, we run into questions of gravity, weight and mass at too early a stage. The underlying story of that scheme is:

$$[\text{gravitational mass of pulling load}] \times [g]$$

$$= [\text{inertial mass of trolley} + \text{inertial mass of load}] \times [a]$$

and however we simplify that – usually by concealing some of the distinctions – we give pupils difficulties.

Instead of that we apply forces by stretched elastic threads in parallel. A pupil stretches one thread by an agreed amount and uses it to pull a trolley (after allowing for friction). Then the other pupil helps by pulling at the same time, with another identical stretched thread in parallel. We let them take it for granted that those provide twice the pulling force. Instead of adding a third puller, pupils evolve the idea of one pupil pulling all the threads in a parallel bunch.

We can prepare for 'newtons' as units for force, by calling our unit stretched-thread-pull a 'Smith' or a 'Brown'. This 'Smith' unit is reproducible and of a suitable size for pupils' experiments. It produces accelerations that make it possible for short-legged pupils to maintain a fairly steady stretch. However it does mean we must use friction-compensated runways, since our force unit is relatively small. For the same reason our runways must be very nearly straight. Although buying and storing long metal-edged runways will be troublesome, schools should regard them as necessary parts of the trolley equipment.

Measurements of speed and acceleration must be made by some method that seems obvious and sensible to pupils. Complicated timing devices will overload the experiment with explanations. That is why we recommend timing by 'tickertape' – a long strip of paper drawn by the trolley past a simple vibrator that marks the strip every $\frac{1}{50}$ second. Not only is the scheme of timing obvious, but pupils can *see* speeds by cutting the strip into sections, say, 10 vibrations long. By pasting those speed-strips side by side they can form a chart which makes acceleration obvious.

Trolleys and tickertape will provide well for pupils' own experiments on force, mass and acceleration, on momentum changes, on conservation of momentum, and on kinetic energy.

However, when we treat conservation of momentum we also want to show and analyse collisions in two dimensions. Our trolleys will not provide for that. Furthermore we would like to reduce friction to insignificance in this very important test. So we suggest a demonstration experiment with 'pucks' moving with practically no friction on a level lake of glass. The pucks carry a store of solid carbon dioxide which produces a gas-bearing to support the puck almost without friction. The glass table must be very carefully levelled and then we can see collisions in a closed system. The picture is done as a demonstration, though some pupils can try experiments for themselves afterwards.

The *linear air track*, a novel combination of trolleys and frictionless pucks, has simple trolleys that ride on an air-bearing along a long straight girder. Compressed air fed to a duct in the girder, issues from tiny holes in its surface and supports the riders. This wonderful demonstration is such a delight to watch that each of us on first meeting it wants to adopt it. However, for the present programme, it is not recommended – even as an adjunct – for several reasons. The main reason: it is a demonstration device, and we feel that experimenting with motion and momentum should consist chiefly of class experiments for close personal experience. Minor reasons: it is expensive; it needs careful storage and careful adjustment for use, and minor damage is apt to spoil its working. Some experimenters dislike the continual hissing of escaping air. And some have a general feeling that this apparatus is too 'special', too remote from everyday machinery to give simple everyday teaching – but in an age of hovercraft that feeling may disappear. A two-dimensional *air table* (pierced with hundreds of minute holes) will come soon. Driven by the exhaust of a vacuum cleaner, it may supplant the glass table where dry ice is not available for pucks.

To analyse motions of pucks, we need a timing device. We suggest using a camera to take a photograph with several exposures, at regular time intervals, on one picture. We place a motor-driven disc with a radial slit in front of the camera lens and use that instead of the normal shutter, to take a 'multiflash' picture. That must be developed and projected on a screen for public measurements. The distance from one exposure to the next shows how far each object has moved, and thus gives a measure of velocity.

As a luxury version, the rotating shutter may be replaced by intermittent illumination: a flashing xenon tube, pulsed at a regular rate, lets the camera, with its shutter held open, take a multiflash picture.

Or a flashing illumination may be produced by shining light from a small bright lamp filament through slits in a disc spinning in front of the lamp. For sharp flashes it is essential to use a lens to *form an image of the filament on the disc* in the region of the slits. This does well for showing an audience a repetitive event 'frozen', as in the demonstration of pulsed water drops. With a camera it gives sharp pictures without requiring the camera lens to be limited to a slit; but there are dangers of spoiling the picture by stray light and of overexposure of white backgrounds.

Once a teacher has multiflash equipment and has gained some practice in using it, he will be tempted to try it for many other demonstrations. He certainly should put it to several uses, in Year III as well as Year IV; yet such demonstrations should not be allowed to crowd out the trolley experiments with which pupils gain strong personal experience.

Equipment for both trolley experiments and multiflash pictures with frictionless pucks will be expensive. But in considering the cost schools should remember that these are not still more complicated experiments for proving Newton's Laws: they are ways of simplifying the story to give convincing understanding.

Note to Teachers: Multiflash Pictures and Stroboscopes

Our tickertape and vibrator equipment provides an easy way of recording distances covered in short, equal, intervals of time, and we want pupils to use it for a variety of class experiments.

However, we need some other measurements of times and speeds. Pupils should be able to make sure that the vibrator with their tickertape is vibrating regularly, marking constant intervals of time. And they may need to measure that interval – or the equivalent, the frequency of the vibrator. They also need to measure speeds of objects which collide and recoil – where the tickertape would certainly get entangled. And they should measure the wavelength of rapidly travelling water ripples. To meet these additional needs, we suggest two devices:

a simple hand stroboscope that each pupil can operate for himself;

and a form of photography that quickly yields pictures with several exposures, at equal time intervals, on the same print.

In the latter case, a moving object appears in several places, and measurements of the distance between them gives us its velocity. Various schemes for taking those photographs, which we shall call 'multiflash' pictures, are discussed below.

We also suggest that the laboratory should make a device that shoots out a pulsed stream of water drops, 50 per second. That is described in the apparatus guide for Year III.

Stroboscopes

Hand stroboscopes were introduced for class experiments with ripple tanks in Year III. See the note on stroboscopes in the General Introduction at the beginning of the *Teachers' Guide* for Year III.

Flashing Lamp: Stroboscopic Illumination

It is a short step from the hand stroboscope with intermittent viewing, to stroboscopic illumination. If the laboratory is lucky enough to have a xenon or krypton flashing lamp that is driven with a regular frequency of flashes, pupils who have used a hand stroboscope will understand the working of that device easily. We can use it for looking at a repetitive motion and 'freezing' it, or for taking photographs with several exposures on the same picture (see below).

Xenon lamps are now available with an oscillator to make flashes at regular intervals over a wide range of frequencies. The flash is of extremely short duration: so multiple-exposure pictures, taken with a camera with shutter held open, are very sharp.

These devices are delightful for photography, but, in classroom use, we are apt to find that we need a brighter flash than the one we have, and then there is disappointment. Lamps that give enough light for all the varieties of multiflash picture we desire are likely to remain expensive. Furthermore, the alternative motor-driven strobe disc is needed in any case for some other experiments. So we recommend schools that do not already have a flashing lamp to use the motor-driven disc instead (see below). The lamp is an expensive alternative, a very convenient luxury if funds allow.

Multiflash Pictures

It is useful to have a way of recording precisely the position of a moving object at several instants of time on the same picture. Then if the instants are spaced at equal intervals in time, measurements of the picture will yield velocities. Even if we do not know the absolute scale of the measurements, the relative speeds will be almost as useful for our teaching. We expect to derive great benefits in teaching from any device that will take such multiple pictures, provided the pictures can be produced for use by pupils fairly quickly.

We would not suggest the use of any such device were it not for the enormous gain in teaching clarity and speed. The conservation of momentum, for example, is one of the most important fundamental principles of physics. In the past, few pupils have ever been able to see a convincing demonstration of it, or even a reliable test; but multiflash pictures and 'frictionless pucks' of one form or another afford an easy convincing test of momentum-conservation *in two dimensions*. We suggest that as an essential experiment in Year IV.

To take such 'multiflash' photographs, we must either place a rotary shutter in front of a camera or illuminate the scene with a regularly flashing lamp. The latter can be a xenon lamp or a powerful steady source whose beam is interrupted by a rotating shutter.

Teachers will find it easier to use the rotating shutter in front of the camera. That consists of a light disc with 5 or 6 slits cut in it, spun by a small electric motor. Small synchronous motors that are used for clocks do well.

An ordinary camera using 35 mm film is quite suitable – provided that it focuses down to 1 or 2 metres, has a lens with an aperture of f4 or better and has a shutter with a 'B' setting. The detailed techniques of exposure and development are explained in the Experiment Guide for the Year. A Polaroid camera is equally suitable, but is expensive and uses expensive materials.

Once pupils have seen a picture made and analysed their own copies, we may give them printed copies of photos of other events taken by a similar process. It would, however, be very poor teaching to use such printed copies if pupils had not first seen a real experiment done.

The 'object' for a camera with a rotating shutter should be a small electric lamp (such as a toy lamp attached to a falling stone) or a small polished steel ball attached to the moving object and illuminated by a floodlight far away behind the camera. The latter arrangement gives an excellent record, because the ball forms a small virtual image of the lamp which makes a tiny spot on the photograph. If the spinning shutter has a narrow slit, the spot is small even if the object is moving fast – provided the lens aperture is also made narrow. If the shutter has a wide slit, the spot is drawn out into a streak which indicates speed by its length.

Various 'pucks' have been tried: some use a supply of compressed air, others use evaporating CO_2, to provide a 'gas bearing' of flowing gas under a massive moving disc. The Nuffield Physics Group have now found a very good form that is easily provided with its gas supply: a ring magnet with an aluminium lid, under which one places a little solid carbon dioxide (manufactured by letting some CO_2 out of a cylinder). The ring will coast along on a plate of glass – or very flat aluminium – with very little friction. It will make an elastic collision on approaching a similar ring; so, by taking multiflash pictures, we can measure the velocities and look for conservation of momentum – as a vector. We recommend that form of pucks for our experiments.

Various methods of taking these photographs and producing prints have been investigated with the aim of making the processes as simple and reliable as possible: the recommended procedure is as follows.

After the photographs have been taken, the exposed film is transferred to a developing tank, using a daylight loading tank, or a changing bag – operations which to some teachers seem fraught with difficulty but which when practised once or twice become part of one's normal demonstration technique. The film is then developed and fixed in one operation using a monobath solution.

The film is then washed briefly, a suitable frame selected, mounted in a simple transparency mount and projected, magnified on to a screen in negative form, using a normal 2 inch by 2 inch slide projector. This enables discussion to take place and measurements to be made.

The class thus has the thrill of seeing the whole process from start to finish in a normal class period. With practice, taking photographs, developing, and projecting can be done in 20 minutes and it is hoped that further refinement and simplification of these operations will lead to an even shorter process.

A paper which can be handled in artificial light (both tungsten and fluorescent) can be used to produce prints for the class to take home and examine or analyse. The paper is held in a simple holder and the negative is projected on to it, using the slide projector as before. The paper is then developed and fixed using standard techniques and once again the whole process can be carried out in full view of the class.

We urge teachers to experiment in expanding the uses of stroboscopic devices and multiflash photographs. This is a region of physics teaching where one's natural reaction is to avoid a technique that seems to all of us unfamiliar and likely to be uncomfortable for discipline. Those of us who have disregarded that plea of unfamiliarity find that the technique is so rich in its possibilities, and the results so quick and satisfying that we feel sorry not to have used it before; and pupils appreciate it so fully that it does not raise the discipline problems that we anticipate.

Note on Scientific Explanations

Year IV is a time for growth in the meaning of 'explanation'. In earlier Years we 'explained' things in science by giving extra information. Secretly, without even saying so – and certainly without being comprehended if we tried to say so – we were explaining in the proper scientific way: attaching unfamiliar or difficult things to things already known. That is, after all, what 'explanation' means in science.

Children hope to find us explaining by giving the 'really true cause of things'; but in fact we only link one thing with another. For example, we 'explain' a lightning flash by saying 'it is a big electric spark'. That tells us nothing about a spark but it reduces the number of unknowns. It offers to decrease the hold of superstition. In that, as Lucretius said, 'Science [ratio] frees man from the terror of the gods.'

We can, later on, 'explain' an electric spark by saying that is an event in air in which there are many ions, driven so hard by an electric field that they make more ions by collision, and so on. All that, if we examine it carefully, does little more than link a spark to things which we have seen in a lot of other experiments on gases being bombarded, flames conducting currents, etc.

If we refuse to be disappointed and claim that our explanation goes deeper than that, we find that we are linking the spark to some models of gases and things in them: a picture of molecules, a picture of an electron and a model of an electron being ripped out of the molecule; and that leads us back to explanation as a linking – this time, linking through our models to our knowledge of collisions of billiard balls and things like that.

One of the best examples is the explanation of the Moon's motion around the Earth. We say the Moon is just falling under the action of (diluted) gravity, like a cricket ball, and we feel satisfied. Yet we have there no ultimate explanation of gravity.

Though many of us enjoy scientific explanations like that and wish to endow them with special virtues beyond mere linkage with the more familiar, we should be wise in our teaching to think of explanation as a linkage – connecting 'new', unfamiliar, knowledge to 'old', accepted, knowledge.

Note to Teachers: on Proportionality

Much of our knowledge of physics is expressed in the form of proportionalities. (See also the Note on 'Constant' in the General Introduction at the beginning of Year III.) Most of us in teaching physics give pupils no preparation for dealing with proportionality but wait until an important case arises. Then we expect pupils to understand the relationship which has appeared: and, when we find that some of them have considerable difficulty in understanding proportionality or making use of it, we are surprised and disappointed and blame our colleagues who teach mathematics. We embark on curative measures of blame, exhortation and explanation, but with only moderate success – the stumbling-blocks often remain.

It is suggested by some wise critics that we have the good examples in physics with which to make a fresh start and teach proportionality successfully and that we should therefore not assume previous knowledge or skill. Instead we should start by explaining very carefully what proportionality is and how to use it, before we use it to codify our knowledge of physics. For teachers who wish to experiment with such a preparation before using it for *force*, *mass* and *acceleration*, we offer the following comments.

Start with simple examples of proportionality as a relationship, in which A doubles, triples, etc., when B does. For example:

cost of a basket of eggs versus number of eggs;

weight of potatoes eaten per week versus no. of men in an army camp;

weight of copper wire versus length of wire;

area of a square versus $[\text{side}]^2$.

In each case, we should emphasize the essential characteristic that one thing increases just as the other does, the two keeping step. Illustrate that by a graph with a straight line through the origin.

Then, with the help of the graph point out another view: that if A varies directly as B the fraction A/B keeps the same value – it is the slope of the graph line. In many uses in science it is the *constancy* of A/B rather than the particular value of that constant quotient that is important. It is the constancy that tells us an important law of nature; while the value only gives us information relating to a particular example – Ohm's Law is true for a great variety of wires but the value of the resistance applies to a particular wire.

Since we are aiming at using proportion in science, we should avoid trick methods that may serve as temporary props when it is taught prematurely in arithmetic – such as reducing a problem about men digging ditches to a unit form that tells us how many weeks it would take one man working one hour a day and one day a week to dig a ditch one yard long, one foot deep and one inch wide – that result to be built up by mystical multiplication into the required answer for the time needed by many men to dig some huge ditch. That method, which often failed to produce clear-headed skill, carries pupils far away from a simple feeling for proportion. Instead of that, we might move to more informal versions of our first descriptions and say, as physicists do, 'A goes as B'. Then we can say that the stretch of a spring goes as the load; the area of a circle goes as its radius squared; the volume of a cube (or sphere) goes as the linear dimension cubed.

Then we should take a look at inverse proportionality, expressing it in two forms: PV is constant, and P varies directly as $1/V$. Pupils who word that as 'P goes as $1/V$' are likely to have a clear feeling for this relationship.

Note that in our first discussions we have not emphasized the value of the proportionality constant, the value of A/B or of PV. Sometimes pupils are taught to start by working out the value of the constant from one set of data, then to use that value of the constant to calculate another value of B for some given value of A. That will yield the right answer without any doubt, but it diverts attention from the structure of the relationship and it probably does not help a clear understanding – so we should avoid it as far as possible.

Returning to problems about men digging ditches, we suggest that pupils should attack them with a commonsense feeling for proportion, such as: 'The time needed *goes as* the length of ditch, so 200 feet instead of 50 feet multiplies the time by 200/50; ... the time needed *goes inversely as* the number of men, *goes as* (1/number of men) so, 4 men instead of 12 men makes a factor of 12/4, ...'

Non-proportional Examples. Simple proportionality is common in elementary physics teaching, partly because we choose those easy relationships for our pupils, partly because they are the important beginnings of physical science, chosen or sought out by man in an attempt to find the simple relationships first. There is, therefore, a danger of pupils thinking that every physical relationship is likely to be – or, worse still, ought to be – one of simple proportionality. We should give them some examples to the contrary, even flippant ones. For instance:

a. A spiral spring of steel wire is hung up and loaded. Its length with no load is 10 inches, with a 1-pound load its length is 12 inches, with 2 pounds 14 inches, with 10 pounds 30 inches. Is the spring's whole length directly proportional to load?

b. An army camp (unlike the simple one mentioned earlier) needs:

2,200 lb potatoes per week for 100 men
4,200 ,, ,, ,, ,, ,, 200 men
6,200 ,, ,, ,, ,, ,, 300 men

Why is the potato supply not directly proportional to the number of men?

(The answer is not spoilage, which is likely to be a constant fraction, but the silly story, 'We have forgotten the cooks, who need 200 lb per week themselves.')

c. A spiral spring of heavy steel wire is placed in a vertical tube (like a gas jar) and a piston of negligible weight is placed on top, so that experiments can be carried out on the compression of the spring. The spring is 20 inches high with no load. With 5 pounds on the piston the spring length is 15 inches, with 10 pounds the spring length is 10 inches. Having learned from an earlier problem not to use the whole length of the spring in looking for proportionality, we ask: 'Is the *change* of length proportional to the force?' (Yes.)

Now the spring is removed and the piston is made airtight (but we imagine it remains frictionless and of negligible weight). The air enclosed in the tall jar is now the 'spring' to be experimented on. With no load the piston is 20 inches above the bottom, with load 5 pounds 15 inches above and with 10 pounds 12 inches above the bottom. We ask the same question.

(This is, of course, a Boyle's Law story for a tube of cross-section about 1 square inch so that atmospheric pressure provides the equivalent of 15 lb extra load on the piston all through. This should not be used to divert a discussion of proportionality into Boyle's Law – unless it happens to crop up at the right time. If Boyle's Law is discussed, this problem could take an interesting form by asking for the height with load 15 pounds, both for steel spring and for air.)

d. If 1 barking dog can keep 5 people awake all night, how many people can be kept awake by 2 barking dogs?

e. Henry VIII had 6 wives. How many wives did Henry IV have?

f. A fence consists of light wire netting with a thick wooden post every 10 feet. The fence along the side of a field has 10 posts. How many posts would a fence twice as long have?

g. A bank notifies the police that banknotes numbered 1262 to 1272 inclusive have been stolen. They then ring up again and say that twice as many notes have been stolen, beginning with 1262. What should the end number be?

h. A current of 5 amps driven through a certain resistor immersed in water delivers 3 kilocalories in 1 minute. How much would a current of 10 amps deliver in the same time?

Note to Teachers on M.K.S. Units

We hope the decision to use M.K.S. units will not seem difficult or annoying. In O-level physics it is a simple change that has the advantage of making the practical electrical units the natural ones.

Its disadvantage lies in the large sizes of the kilogram and the metre. We ameliorate that by lapsing into the smaller units that feel sensible when we are describing sizes and not embarking on calculations. For example, we say: 'Join the balls with a thread about 5 centimetres long' (not 0·05 metre!).

The strong feelings that many physicists have about M.K.S. units arise at a later stage, in much more advanced electricity where the move to new units has been used as an excuse to change the formal structure, inserting factors such as 4π. Those changes are advocated with missionary zeal, as if they provided a truer description of nature. We cannot alter the facts of nature by a change of units or formula factors – a 4π pushed underground, for convenience, in one place will reappear from another burrow.

It is we who have strong feelings, because we dislike the confusion of having several systems for something as trivial as units. We should expect young people to accept whatever system we suggest, and not worry.

We do not press for M.K.S. with missionary zeal in this programme; but we have decided to use them in dynamics so that we can treat energy simply in electricity. We make our absolute unit of force one newton (a shorthand name for one kilogram.metre/sec^2). Our unit of energy is one joule (shorthand for one newton.metre). Then power is measured in watts (shorthand for joules/sec). And a potential difference of V volts means an energy-transfer of V joules for every coulomb of charge passing through the region concerned.

In Year I, we suggested pupils should use feet and inches at first until centimetres and metres became familiar. And pounds at first, then grams and kilograms. Not until we meet quantitative dynamics in Year IV are we forced to settle on a consistent set of units of mass, force and acceleration that fit our chosen form of Newton's Second Law, $F = Ma$. We hope that teachers will try using the M.K.S. units mentioned above, so that they can see the benefits that then appear in dealing with electricity.

Therefore, in Year IV we need to insist on all lengths being expressed in metres, whenever a dynamical question is involved. Similarly all masses must be in kilograms. That will necessitate some irritating drill; but we hesitate to suggest insisting on M.K.S. units at earlier stages just to avoid that drill now.

KEY TO MARGIN REFERENCES

C = Class experiment

D = Demonstration experiment

T = Teaching of material (lectures, discussions with pupils, etc.)

F = Film

H = Suggestions for optional experiments at home

P = Problem

*
* = Commentary (notes on, methods, aims, etc., offered to
* teachers)

‡ = Reference to footnote

§ = Reference to a comment made by a teacher during trials

† = Reference to Year I, Year II and Year III material needed now

(The experiments are numbered serially through the Year, irrespective of the classification C, D, F or H. The same numbers will be found for each experiment in the *Guide to Experiments and Apparatus*. Where (a), (b) ... are added to the number these refer in some cases to separate parts of the same group of experiments, in other cases to alternative versions of an experiment.)

SYNOPSIS OF PROGRAMME FOR THE WHOLE OF YEAR IV

The programme for this Year consists of five sections:

1. Physical basis of Newtonian mechanics

2. Kinetic theory of gases (continued)

3. Investigation of heat as a form of energy and discussion of general conservation of energy

4. Electricity (continued) with use of voltmeter, potential difference and e.m.f. defined in terms of energy-transfer, voltage-current characteristics, power line and power problems, (transformer)

5. Properties of electron streams, Millikan experiment, positive rays, cathode ray oscilloscope, simple atom model.

These could be taken in a different order, but that would necessitate some retracing of steps.

The first section on Newtonian mechanics provides a clear meaning, at least, for the newton as a unit of force, and the joule as a unit of energy and arrives at the expression for kinetic energy, $\frac{1}{2}mv^2$, that we have awaited so long: and these are useful things in dealing with energy in each of the other sections. Newton's Second Law is expressed in the form that connects force with change of momentum, and that is essential in the kinetic theory section.

Chapter 1

PHYSICAL BASIS OF NEWTONIAN MECHANICS

Force; Mass; Acceleration; Weight and Gravitational Field; Inertia; Momentum; Kinetic Energy

PROGRAMME: SECTION 1: NEWTONIAN MECHANICS

(A general Note on the teaching of Newtonian Mechanics in the Nuffield Physics Programme is appended to the Preface to this Year.)

Multiflash Pictures. Start with two multiflash picture experiments to re-open problems of motion and show the technique in a real experiment.

Revision of Tickertape. Pupils may revise the tickertape technique by recording and analysing the motion of a trolley running down an incline. (Any pupils who missed the preliminary work with tickertape in Year III should go through that carefully now.)

Rough Experiments on Motion. Pupils try rough, quick, class experiments on force, mass and motion, to prepare for more precise measurements.

Force, Acceleration, Mass. Pupils investigate carefully the relationships between force and acceleration and between mass and acceleration, or mass and force. They use tickertape and make careful analyses and graphs.

Formulae for Constant Acceleration. Geometrical descriptions of accelerated motion lead to formulae such as $s = ut + \frac{1}{2}at^2$ for constant acceleration. We use these for measurement of g and for simple problems.

Newton's First Law. Experiments to illustrate Newton's Law I. We discuss the meaning of inertia.

Discussion of $F = ma$. Discussion: mass; force; weight; Earth's field strength, g. We lead from experiments to $F = Kma$ and then $F = ma$, with absolute units (newtons) for force.

Weight: Gravitational Field Strength. We treat weight as a force, the pull of the Earth on any mass; and we treat g as the strength of the Earth's gravitational field (9·8 newtons/kilogram).

Momentum Changes. Alternative discussion of the experimental story of Newton's Law II, using momentum:

Ft = change of $[mv]$.

Conservation of Momentum. We take Newton's Law III as an axiom (following Poincaré) and predict conservation of momentum. We test that and give a number of experimental illustrations.

Kinetic Energy. Return to energy: derive K.E. $= \frac{1}{2}mv^2$ from the momentum form of Newton's Law II. Illustrations and applications of conservation of [K.E.+P.E.].

Power. Short discussion of power, with some class measurements.

Note to Teachers: Speed and Depth of Dynamics Teaching this Year

We made some informal studies of force, mass and motion in Year III and in Year V we shall use Newton's Laws of Motion for astronomy and for atomic physics, so our studies of $F = ma$ in this Year should not be expanded into long investigations or driven home by many problems. All we want is that pupils should have a confident acquaintance with Newton's Laws as part of our scientific heritage. The sooner they can finish dealing with $F = ma$ the better, because they still have to deal with momentum – which is more useful for kinetic theory this year and astronomy in Year V – and with kinetic energy, before proceeding to the other parts of this Year's work.

So just before starting this teaching, it would be a wise precaution for teachers to review for themselves the perspective of all the dynamics teaching ahead, right up to the kinetic theory of gases.

For the teaching of $F = ma$ we suggest the following should suffice:

A quick rough qualitative experiment.

Careful measurements in a class experiment with tape and vibrator. (See also Note on Trolley Experiments in the Preface to this Year.)

One demonstration with multiflash and a frictionless puck, to serve as introduction for later uses.

Primitive measurement of acceleration with scaler. (Much better moved to Year III.)

Only with pupils who have special interests should we extend these experiments to other methods.

FORCE, MASS, ACCELERATION

Preparatory Work for Pupils who missed Preparation of Year III

In the course of Year III, pupils following the Nuffield Course will have gained considerable skill in using tickertape and vibrator to measure speeds and accelerations. And they will have gained some informal ideas about the relationships between force and motion, and even the beginning of an idea of mass.

In Year III, pupils are asked to use tickertape for simple measurements of motion, pasting up lengths of tape each 'tenticks' long for a chart that is really a velocity-time graph. They use a 'tentick' as a unit of time: ten oscillations of the vibrator. And they may have seen demonstrations of multiflash photographs.

If any pupils starting Year IV have not used tickertape, *they should now go through those introductory experiments and chart making, as class experiments with plenty of time* – because if they do not understand what this new technique is giving them, they will have difficulties and delays with the more precise measurements ahead.

Teachers who have any pupils who missed Year III, are advised to look very carefully at the Teachers' Guide for Year III and take suggestions of preparatory teaching from that. The instructions for use of tickertape are given in the Guide for Year III, and are not repeated in this Year IV Guide.

Experiments to start the New Year: Multiflash

We start with some multiflash demonstrations to re-open the problems. (See Note on Multiflash in the Preface to this Year.)

1. **Free Fall.** Make a multiflash picture of a freely falling object, a small lamp or some object with a small steel ball lit by a remote flood lamp; and give pupils prints of it to analyse. (See the Note on Multiflash.) This is not the time for exact measurements but just for a quick look to show what the picture tells us. (However, the picture may be used later for an estimate of g, if it carries the data needed. For that, the picture should include a vertical metre rule, and the frequency of the flashes should be recorded.) D1

2. **Diluted Gravity.** Take a multiflash picture of a bright ball rolling down an incline, or of a trolley running down an inclined plank. This, too, is for a quick look. D2

Revision of Tickertape Technique (*Optional*)

Remind pupils of the tickertape experiments of Year III. If they do not remember them clearly, ask them to run a tickertape class experiment with a trolley running down a sloping plank. Then they should paste up a chart of tentick lengths. They should do that quickly, and be ready to discuss the chart fairly soon. C3. OPT

We explain that this chart is only a preliminary example; and that we are going on to deal with different forces pulling a trolley along a level table, and different chunks of matter (several trolleys) being pulled along, so that we shall know more about rocket motion, satellite motion and the general problems of modern transport engineering, astronomy and 'atomic physics'. T

Preliminary Look at Newton's Laws of Motion

Before pupils embark on systematic experiments on Newton's Laws, we should raise the questions that lead to those Laws and ask pupils to make a rough investigation. Otherwise pupils will work blindly in an uninspired way that propels no scientist – or else, if we have insisted on the outcome to be expected, they will just collect results for a foregone conclusion. T

Constant force: constant acceleration? We ask: T

'What kind of motion do you get if you pull on something with a steady force?' Try this, unless you are already sure.

If pupils are not sure of the Year III experiment, they should drag a trolley along a friction-compensated runway and measure its motion with tape and vibrator, to see whether the acceleration is constant. (Of course, the acceleration will not be constant unless the pupil pulls with a constant pull. At this stage we should ask for great care, and suggest practice, to ensure a steady pull. Unless the runway is plane and the friction compensation is carefully made and tested, this experiment will be disappointing. It is essential to use a plank with metal girder edging.) C4

Acceleration which is not Constant (*Optional*). With a fast group, we might give a quick experiment with a case of changing acceleration. That would anticipate a warning which will be given later: that acceleration does not have to be constant, and in fact many motions in nature have changing acceleration. D/C5 OPT.

We lay a length of chain on a smooth table, perpendicular to one edge of the table. We pull the end of the chain a little way over the edge and watch the motion as the hanging portion of chain pulls the rest, with increasing acceleration. The other end of the chain may be attached to a length of tickertape running under a vibrator; and in that case the changing acceleration may be extracted from quick measurements. Until it is all falling freely, the chain moves with changing acceleration. (Unfortunately, the motion in which the chain starts in a *loose pile* at the edge of the table, which is easier to arrange, is one with constant acceleration, $g/3$.) The motion of the bob of a long pendulum will also show changing acceleration. Either of these can be analysed by dragging a tape through a vibrator; and, done as a demonstration, would not take long.

Rough Experiments: Relationships between Force, Mass, and Acceleration (*a*) With trolleys (no tape) or (*b*) With playground trolley. We suggest: C6a, b

> 'Just to remind yourself, try pulling a trolley without measurements, with a pull, double pull, treble pull, and watch the accelerations.
>
> 'Then try the same pull on more "stuff-to-be-pulled" by pulling several small trolleys piled on top of each other (or by piling more people on a playground trolley). Again, just feel and watch. You will return to careful measurements in a moment.'

All we want from the first experiment is a clear statement that a bigger force makes the thing accelerate faster; and perhaps a suggestion that doubling the force doubles the acceleration. Again, for the second experiment what we want at this stage is a clear idea that with more stuff to be accelerated we either have to use a bigger force or expect a smaller acceleration. * * * * * *

All that may have been clear in Year III, but it is wise to have it quite clear now as the problem before the class is to be investigated by careful measurements. We can point out that in designing rockets for a given job or studying planets or atoms we need to know just what acceleration will be produced by the force that we measure on the test bench or the force we calculate from other knowledge; and just how the rocket will change its motion as it loses material through its exhaust. T

Careful Measurements. So we now ask for really careful measurements. If possible, we should discuss plans towards the end of some class period, so that pupils have time to think about the careful experiments ahead of them next time. T

Newton's Second Law: Force and Acceleration

Class Experiment with Vibrator and Tape. Pupils pull a trolley with tickertape along a friction-compensated plank. To make that compensation, place books or blocks of wood under one end of the plank until its slope is such that a trolley will just keep moving steadily downhill once it is given a start. With good wheels on the trolley and a smooth, flat plank, the slope for this is very small, so friction compensation like this may seem unnecessary. However, it is probably wise to ask pupils to arrange this, as a matter of principle. Later on, in momentum conservation experiments, even small effects of friction may spoil the story seriously. C7

For a standard pulling force, pupils pull with a rubber thread stretched to some agreed length (see Year III for details). They should paste their strips of tentick lengths of tape in their notebook to make a chart; but they should *also* draw a new form of graph.

Tape Chart once more. We need to make sure that the idea of our samples of velocity is clear. Tape strips are more basic, material, reminders of distance-travelled than graphs which are artificial extracts. Graphs are soon to be our standard scheme, but pupils should make a tape chart now – perhaps for the last time.

To many a professional physicist this distinction between a tape chart and a graph appears trivial; and there is a strong temptation to omit the chart and proceed straight to adult graphs. But to young pupils who cannot know clearly until they see where their studies are leading, the tape charts still offer some valuable teaching – which we shall use in our formula-making soon. Some teachers in trials have reported pupils more than ready for graphs and have omitted trying a further tape chart. Since we are not sure whether such reports arise from pupils' impatience or teachers' own skilful habit, we make this plea for one more chart now.

The tape chart teaches the concept of 'distance given by area under a velocity-time graph'; and it provides for some useful O-level examination questions.

Of course if pupils themselves insist, unprompted, on proceeding *
to graphs, they should do so. If pupils do plot a graph instead of *
making a tape chart, they must, at this stage, plot it with many *
points – equivalent to many strips of tape – so that they can see *
whether the acceleration really is constant. *

Graphs. Then pupils should draw graphs. The new graph can C7
have just two plotted points, *if the chart has already shown that the acceleration is constant.* Pupils can take a speed, represented by a length of tape which is distance travelled in tenticks and then another tentick length a known time later, say 50 ticks. They plot these two speeds upward on a graph against the time along, measured in ticks. Then they draw a straight line through their two plotted points. Although this misses the advantage of averaging, it makes a simple graph that is easier to look at. They repeat the experiment with double pull, by two loops in parallel; and then with treble pull.

If the plank along which the trolley runs is not really smooth and *
flat, local variations of motion will make this use of two points *
quite unreliable. Therefore, it is important to use a plank of thick *
plywood held flat by angle girders along each edge. Otherwise, *
many points must be plotted and some kind of average arrived at *
for the motion. *

Each pupil should plot graphs on the same sheet of paper for these C7
experiments. Pupils should be left on their own to draw conclusions from their graphs. It is much less valuable, though much quicker, for the teacher to impose a well-taught conclusion. What pupils find out for themselves from the slopes of these graphs (without even being told to look at the slopes) will remain in their minds as one of their discoveries in physics – particularly if we can then tell them that they are finding out part of the story of Newton's great Laws of Motion.

Alternative Methods

This is the time for varying the method of experiments and showing *
pupils other ways of 'removing friction' and other schemes of *
timing, as alternatives to tickertape. (See Note on Multiflash pic- *
tures in Preface to this Year and in General Introduction at *
beginning of Year III.) It may be obvious to us that changing from *
tickertape to a gas-supported puck will tell the same story; but it *
was not obvious to the medieval scientists, and it is not obvious to *

many a pupil. That is one of the delights for young people learning science: to share with Galileo the making of the distinction between essentials like velocity, force, distance and the non-essentials such as the colour of the trolley or the musical squeak of its wheels.

The Nuffield Physics group have tried a number of methods; and ingenious teachers are likely to construct some more of their own. The traditional 'Fletcher's trolley' is available in one form or another in many schools. It can be used with some pupil participation so that it is more than a demonstration, almost a class experiment. But that would only repeat in a more difficult form the simple class experiments with trolleys, on which we have decided to place our main emphasis.

We have tried special trolleys with meters to measure velocity and even acceleration; and other trolleys carrying a spring balance to measure the pulling force visibly. Those are only demonstrations and we do not think they should be included in our programme because they compete unfavourably with class experiments; so we shall give only a brief note of them later.

However, demonstrations with frictionless pucks and multiflash pictures are so striking and offer such a different look at the phenomena, that we urge teachers to include some of those demonstrations. Furthermore, they will be needed when we come to collisions in two dimensions.

We also suggest a primitive demonstration measurement with a scaler and pulse generator used as a millisecond timer. That makes a good, direct, introduction to acceleration – so we hope it will be presently transferred to Year III – and the method leads to interesting measurements, such as the speed of a rifle bullet, later on.

Apart from a few multiflash pictures with frictionless pucks and the use of the millisecond timer, we urge teachers not to burden the work of this section with demonstrations – the simple class experiments with trolleys should suffice. However, demonstrations might offer 'buffer extensions' for a very fast group.

Frictionless Pucks. We should experiment with pucks on a *
smooth table of glass. The puck slides along on a bearing of gas in *
the tiny space between the puck and the glass. The gas may be CO_2 *
from solid CO_2, or air from a reservoir that has been pumped up *
with a bicycle pump. The latter form sounds easy and economical *
but it is very irritating and disappointing in use, because its air *
supply runs out so quickly. We do not recommend it. *
*

Large pucks such as those used in the P.S.S.C. films work well – *
their large mass enables us to use pulling forces of the same size as *
for trolleys – but they are expensive to make and use. *

Ring-Magnet Pucks. The Nuffield Physics group has devised a D8
simple puck which seems better than all others for our use. It is a small metal ring with a lid of cardboard or aluminium. We place a small piece of solid CO_2 under the lid and, as the CO_2 evaporates, it maintains a layer of gas under the lower surface of the ring, which then coasts with practically no friction on a glass table. A thin post placed on the lid acts as marker for measurements, and a long piece of elastic thread can be used to provide a small constant force. Pulling such a puck with a rubber thread is no easier than pulling the trolley, but at least we have no friction here; and pupils have the excitement of a different method of measurement.

We take a multiflash picture and project it with a slide projector, or pass the print around, or, best of all, make a print for each pupil to analyse.

Millisecond Timing of Trolley with Scaler

It is good to make an estimate of acceleration with measurements D9
that follow its basic definition:

acceleration = [gain of speed]/[time taken for the gain].

For that, we need to make two measurements of speed, one early in the accelerated motion and one some time later. Then we subtract to find the gain of speed and divide that difference by the time between the two speed-sampling measurements. For each speed sampling we need to time the travel of the moving body over a short distance – we must not use a long distance or time, or pupils themselves will object that the speed which we are trying to measure changes too much during the sampling.

With a trolley pulled by a pupil keeping an elastic thread at constant stretch, we may take as our speed-sampling distance the length of the trolley, and therefore measure the time that the trolley takes to pass a fixed observation post. We take two observation posts, one near the beginning of the run and the other near the end, and we measure the transit time at each. The transit times are likely to be of the order of 1 or 2 tenths of a second, too short to measure with an ordinary stopwatch. (We could measure them with tape and vibrator, but that is just what we have already been doing.) We need a clock that measures milliseconds: the scaler gives us that.

Use of Scaler as Clock. Schools following the Nuffield programme will have a scaler (Panax or similar) which has a built-in pulse generator giving 1,000 pulses per second.

The scaler is simply an electronic counting device which counts small pulses of electric charge or voltage fed into it and registers the total on three decimal dials. It is the 'counter' of a 'Geiger counter'. It can easily be converted into an electrical clock – one of the two essential ingredients of every clock is a counting device, to count swings of a pendulum, cycles of alternating current, etc. Since scalers are often used to count particles emitted by radioactive substances, they usually have, as a built-in accessory, a high voltage supply to drive the avalanche of electrons triggered by a particle passing through the Geiger tube. We do not need that high voltage when we are using the scaler as a clock. But we do need instead a source of regularly spaced pulses for the scaler to count and a suitable switching mechanism – equivalent to the balance wheel of a stopwatch and its control button. The scaler designed for teaching use contains a pulse generator making 1,000 pulses per second, whose frequency can be checked against an ordinary clock.

On their way from the generator to the counting mechanism, the pulses can be brought out to an external switch and back, so that they are not counted unless the switch contact is made. There are also connections which stop the counting as long as another switch is closed – in effect, by short-circuiting the generator.

Thus we can arrange the scaler to count the time, in milliseconds, between two switching events, each of which may be the closing of a switch or the opening of a switch. The switches may be knife switches or other metal-to-metal contacts, or they may be solid-state devices such as photo-diodes, which have a low resistance when illuminated and a high resistance in the dark.

Primitive Acceleration Measurement with Scaler. For our primitive measurement of acceleration, we have two photo-diodes in series, one at each of our sampling stations. Each tube is strongly illuminated by light from a small lamp, so the tubes have low resistance and act as a closed switch to stop the scaler from counting. If the light reaching either tube is cut off this becomes an open switch and the scaler counts the time during which the illumination is obstructed.

We make our moving trolley carry a strip of cardboard which interrupts the light to the photo-diode while the trolley is passing each 'station'. Then the scaler counts the number of milliseconds in the time taken for that length of cardboard to pass the station. The (average) speed during that is given by [length of cardboard]/[time]. Thus, for different samplings, the speeds are proportional to the reciprocals of the times. With our arrangement of two photo-diodes in series some distance apart the scaler will measure the time for transit at the first station and then *add* the transit time for the second station to that; so that pupils must be appointed to observe and record the first reading before the trolley gets to the second station. (The scaler has a 're-set' switch which can be pressed to return the counting dials to zero before the second reading; but it is easier to leave that alone and subtract.)

For any measurement of acceleration, we also need the time that the trolley takes in travelling from one station to the other. That should be measured with a stopwatch operated by hand. (To try to measure *that* time by the scaler would be to misunderstand this rough experiment, which only uses the scaler to estimate the short transit times. A crude measurement of the total time between stations will suffice to give the same order of accuracy.) The trolley should carry some mark at its mid-point for that timing. (For precision, we should take the time between the *mid-times* of the transits, not the mid-points; but that is too difficult and not necessary for this teaching experiment.)

This sounds a complicated scheme for doing something relatively simple, and not doing it very accurately. Yet, when pupils see it, they recognize it as a clear way of finding acceleration from two separate measurements of speed and a measurement of the time taken to make the change of speed.

This simple measurement belongs more properly in Year III, and we hope that when schools have scalers (which are listed as necessary for Year IV but not for Year III) they will move this experiment to Year III.

Teachers giving this demonstration will find they need to emphasize T
the fact that the velocities at the sampling stations are not proportional to the scaler readings but to the reciprocals of those readings.

Use of Gravity Pulls in Acceleration Experiments to be Avoided. In any of these experiments, the force should be applied by rubber threads (or spring balances) 'in parallel'. We should at all costs avoid making the force by using a calculated pull of the Earth on a load hung on a thread. Although that is simple and obvious, it leads to some confusing troubles.

Special Trolley with Meter (*Buffer option*). If the teacher wishes D 10a
to manufacture it, an interesting trolley can be made which offers OPT.
readings of velocity and of acceleration on a millivoltmeter. One wheel of the trolley is made to drive a small permanent-magnet d.c. dynamo. The output from the dynamo is fed to a millivoltmeter which will indicate speed.

If, instead, the output is fed to the primary of a small transformer, the secondary of that transformer will drive, through a millivoltmeter, a current roughly proportional to the rate-of-change of the primary current; so the meter gives a measure of acceleration. This ingenious trolley will repay the trouble of manufacture; but for use with pupils it would have to be tested to show that it does measure those quantities faithfully.

Trolley with Spring Balance and Sandbag (*Buffer option*). We D 10b
may use the pull of a hanging load provided it is in the form of a OPT.
sandbag of unknown weight, which merely does the job for us like a trained dog. Then we must make a direct measurement of the pull that the thread applies to the trolley. We must install a spring balance on the trolley.

It is possible to construct a simple spring balance with easily visible readings in arbitrary units. (We have coined the name 'strang' for such a unit and we may call the whole instrument a 'strangmeter'. For satisfactory use, the strangmeter must have good damping, so that pupils can watch a steady reading during the run of the trolley. The damping device must not add mass which the spring fails to measure; but that can be arranged by wrapping the pulling thread round a rod which carries a vane in a dash pot of oil. This method does not require any discussion of gravity pulls, nor does it raise the difficulty of some of the weight of the sandbag being used to accelerate the sandbag itself. Though it is a simple demonstration but we do not consider the trouble of constructing it or the time taken to show it are worth while.

A trolley with a strangmeter can have its pulling thread wound once or twice round a pulley-wheel attached to a small d.c. dynamo which will then indicate the speed of the trolley on a millivoltmeter. D10c OPT.

Mass and Motion

Before pupils start on experiments with different masses, we ask them to think about the problem: T

> 'Suppose you have a force that gives a trolley a certain acceleration. Now put another trolley on top of your trolley and pull them both. What force would you expect to have to use if you wanted to get the *same* acceleration as with one trolley?'

That suggests the simplest form of test: one trolley pulled by a force, two trolleys with double force, three trolleys with treble force. We look for the same acceleration in each case. C11

Although that looks like trickery – prearranging to get a simple answer – it is in fact a clear illustration of the basic story, that masses do not interact, and that forces do not interact, so that when we have two masses side by side each needs its own share of force to accelerate it. One mass does not affect the other one's force requirement; and one force does not affect the ability of a neighbouring force to accelerate things. * * * * * * *

For clear, convincing results, it is important to adjust the friction compensation anew for each new mass. We might expect the proper tilt to be the same for several trolleys as for one, but in practice it often changes and unless the compensation is made the experiment C11 & C12 does not go well. The amount of compensation necessary depends enormously on the smoothness of the runway – hence our insistence on thick plywood and metal edges. It also depends on the design of the trolley and on its state of preservation.

(Optional for slower groups.) Ask pupils what they would expect if they kept to one single standard force all through and applied it to one trolley, two trolleys and so on. Let them guess: do not comment on their answers. Ask them to try both versions of the experiment. To many a young pupil, the second experiment is quite different from the first, partly because of its mathematical difference, and partly because it now shows the essential property of inertia more clearly to him. C/D12

Trolleys and Tape. Pupils should now try the two demonstrations described above with tickertape. C11 & C/D12

Multiflash (*Optional*). Now that we have multiflash photography available, we may also show that with some form of puck being accelerated. In each case, we should increase the mass by piling identical items on top of each other. We should *not* compare masses by weighing at this stage. D13 OPT.

Careful Experimenting

These experiments on force, mass and acceleration are not easily done with great precision unless pupils have time to practise techniques and make careful measurements and have a flat board for the trolley to run on. By giving them sufficiently clear and detailed instructions and helping them whenever they are uncertain, we could reduce the time taken for these experiments enormously; but, if we do, we are missing the point of these experiments. These are intended to be investigations made by pupils on their own. We hope for the full satisfaction that young people can get when their own measurements reveal natural behaviour clearly. * * * * * * * * * *

If some pupils experiment too roughly and do not arrive at convincing results, we should not blame them or hurry on to other things but we should offer them a chance to try again: T

> 'Are you yourself satisfied with this? Knowing what you now know, how long would it take you to try this over again very carefully?'

Even as early as Year IV, self-respect is waiting to push its owner *
towards a careful second trial, if we will only give suggestions and *
approval. Time taken for such further trials will be justified – *
there are other things this Year which could be cut shorter. On the *
other hand, this investigation should not drag on into a long and *
tedious business of repeated measurements by pupils who do not *
clearly understand what it is driving at. This is not one of those ex- *
periments which a first year's trial shows to be so fruitful that *
teachers decide to expand it in a second round. Experienced *
teachers are more likely to carry this investigation through a little *
faster with next year's class, by some coaxing and some more care- *
ful preparation. *

These experiments will give some pupils a strong, satisfying sense *
of being knowledgeable. They feel that they have got the better of *
the apparatus – that they are apprentices who have grown expert *
with it – and that they are finding out things about nature by *
making accurate measurements. Although these investigations may *
well take several periods, they are the pupils' own experimental *
work and they are worth the time. *

If convenient, one complete set of trolley apparatus should be kept C11 &
available in the laboratory for several weeks, in case some pupils C/D12
wish to try things again – or both teacher and class may want to go back to part of the experiment when there is discussion.

Discussion of Force, Mass and Acceleration

When pupils have finished their experimental investigations and T
have some form of graphs in their notebooks to show the results, the teacher should hold a general discussion. Pupils should be encouraged to offer suggestions and argue with each other – it would not be wise for the teacher to issue his own clear summary at this point. Now, or in Year III, pupils should have seen for themselves that:

1. a constant force makes the trolley accelerate, with constant T
acceleration;

2. doubling the force doubles the acceleration, and so on. The accleration 'produced' is directly proportional to the force.

At this stage we should remember the confused views on 'Propor- T
tionality' that cloud the minds of many pupils at this age – goodness knows why. (See the Note on Proportionality in the General Introduction at the beginning of Year III.) Whenever we mention proportionality we should always avoid that long word at first and say:

> 'When you double the force, you get double the acceleration, when you use three times the force you get three times the acceleration, and it goes on like that. *Acceleration* "goes as" the *force*. They both go up in the same proportion. In fact, acceleration is proportional to the force.'

(Note that here we are measuring force by the number of equal *
pulls in parallel, assuming that forces simply add, and do not *
interact, that is, they do not 'frighten each other'. See the Note on *
'Interaction' in the General Introduction at the beginning of Year *
III.) *

3 (*a*) The force needed for a chosen acceleration is proportional to T
the mass, which is measured by the number of equal items piled on top of each other. We can carry that down to the atomic scale and say that mass is a measure of the number of atoms in a body, provided they are all of one kind.

And if we like to lump protons and neutrons together under the *
name 'nucleons' and regard them as the fundamental constituents *
of atomic nuclei, we may even say that mass is to a rough approxi- *
mation a measure of the total number of nucleons in a body, what- *
ever the mixture of atoms in it. *

3 (*b*) When a chosen force acts upon different masses the accelera- T
tions are inversely proportional to the masses. (We should expect a much more naïve wording of this from pupils; and we should be unwise to give the inverse proportional version as our own way of stating this result.)

At this point, some teachers like to expand the discussion of mass *
and show some additional experiments. Others prefer to postpone *
that for a short time until some more work on acceleration has pre- *
pared the ground for quantitative measures. Teachers planning *

their work for this may wish to look ahead and decide whether to bring experiments 24, 25, (26), 27 back to this place in their teaching. *
*

Discussion of Newton's First Law of Motion

Although we regard Newton's First Law as a special case of the Second Law, pupils do not recognize it as that – any more than did the philosophers of Galileo's and Newton's times, who found in it a startling change of view. So we should be wise to discuss the First Law with pupils as a separate topic. *
*
*
*
*

How much we have to explain, and whether we show the experiments described below, will depend on the treatment in Year III. Pupils who followed Year III of our programme fully should have seen demonstrations with a block of solid CO_2 and with a ring puck. If they missed either of those they should certainly see them now. T

We ask pupils about an object moving straight along without changing its speed. What force does that steady motion need to maintain it? If pupils say no force, we at once ask them to push a chair or table along a rough floor and ask them whether they really call that no force. In other words, we must still respect the common-sense view which Aristotle investigated: that steady motion requires a steady force. That is true when the motion pushes against some invisible opposition such as that provided by friction.

Motion Against Variable Friction. If the friction is 'fluid friction', the motion takes on a very interesting characteristic: since fluid friction increases as speed increases,‡ a body being pulled against fluid friction speeds up until the friction drag reaches a size that exactly balances the pulling force, and then there is a constant 'terminal velocity'. T

We remind pupils of that behaviour, which they have met in earlier Years. We show again a demonstration of some object falling in viscous liquid. We let a small paper tray fall in air. We show small steel balls falling in thick oil or glycerine, or polystyrene balls falling in water. Pupils should see once again that the object falls with constant speed, after a short initial stage of acceleration. Soon we are going to interpret that constant-velocity motion as due to D/C14

‡ Though not proportionally, in most cases. The commonest form of fluid resistance varies as v^2.

friction forces, upward, just balancing the pull of gravity (less the buoyancy of the surrounding fluid) downward.

Frictionless Motion. Then in contrast we show a demonstration of motion continuing with no force either way along the motion. *If pupils have not seen it*, we show a big block of solid carbon dioxide coasting to and fro across a carefully levelled glass sheet on a table. (As was pointed out in the Guide for Year III, this is a demonstration well worth the trouble of sending specially for the block of CO_2.) D15a

Or, we show a ring magnet coasting with practically no friction on any smooth table. The ring is given a lid of cardboard or metal and a small quantity of solid carbon dioxide is placed under the lid. As the CO_2 evaporates, it provides a 'gas bearing' on which the magnet slides like a hovercraft. The small quantity of solid CO_2 needed can be obtained from a cylinder of carbon dioxide released into a bag.‡ D15b

Simpler still, but much less satisfying, show a small disc sliding on a glass sheet covered with small, polystyrene beads. D15c

Ask pupils whether the Moon is slowing down appreciably, as it goes round and round the Earth, and how they can know whether it is; ask whether a space traveller has to keep his rockets going when he is far out in space. And ask whether a molecule of air moves slower and slower although there is no driving force to keep it going. (If pupils say that molecules of air do move slower and slower, through some mysterious friction, ask where the air will all be in a few minutes from now.) T

Force and Constant Velocity: discussion continued. Now we must face the conflict between a steady push needed to keep a chair sliding along the floor and no force needed to maintain some other steady motions. Or we may think about riding a bicycle on a level road, in contrast with coasting on a frozen lake – but the bicycle involves more complex forces. By now pupils should be T

‡ The easy way to obtain the small quantity of solid carbon dioxide that is required, from a cylinder, is as follows. Fold a piece of closely woven cloth (preferably of dark colour to make the product easy to see) in the form of a bag. Hold this bag tightly round the nozzle of this cylinder and open the valve at full blast for 5 to 10 seconds. If the cylinder is a syphon type it should be kept upright; but if it is an ordinary cylinder of carbon dioxide it should be tipped upside down during this process.

ready to point out that in the case of the chair there is more than just the force exerted by the chair pusher. There is also the force of the friction dragging backward. We ask:

> 'How do you know friction is acting on the chair? Oh, yes, you know that you have to push the chair and you think you are pushing because friction is dragging back; so you think you are feeling the force of friction. How do you really know the force of friction is there?'

(Note that we are still asking questions, respecting pupils' reactions, *
and discussing matters; we are most anxious *not* to begin our formal *
study of Newtonian mechanics with strong assertions that must be *
swallowed unthinkingly.) *

Feeling Friction Forces. There are two ways in which we can experience the friction for ourselves:

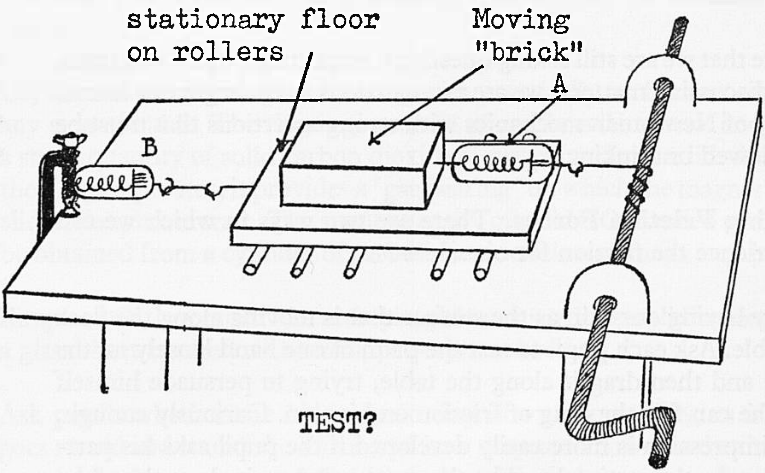
a. By having our skin as the surface that is moving along the floor or table. Ask each pupil to rest the palm of one hand loosely on the table and then drag it along the table, trying to persuade himself that he can feel the drag of friction on his skin. Curiously enough, this impression is more easily developed if the pupil asks his partner to take the resting hand by the wrist and drag it along the table. Then the victim's thoughts are concentrated on the forces at the surface. C16a

b. We use our own skin as the 'floor'. The pupil places a heavy load, such as a brick, on his upturned hand held at rest on the table and asks his partner to drag the load along. C16b

The pupil who points out that the experiments (*a*) and (*b*) are the same, is already developing a sense of Galilean relativity, and deserves immediate praise.

Teachers may want to comment on the heat developed when friction forces drag along surfaces. Some teachers feel that this heat which can be felt should be the starting point of the study of heat, and suggest we should diverge on a discussion of heat at this point. In any case, if pupils raise this question of heat, we should certainly pursue it here. T

Demonstration of Balancing Forces. We then offer a demonstration of a rough object being pulled along on a floor which is itself on rollers, so that we can measure the drag on the floor. This is a comforting experiment‡ even if it is very rough. The demonstration is performed thus: A brick or some other rough block is dragged along a plank of wood which itself rests on metal rollers so that there is very little friction between the plank and the table. D17a



We drag the brick with a spring balance,‡ A, which measures our pull. The plank which acts as floor under the brick is kept from being dragged along with the brick by a cord which tethers it to a post at the end of the table. A second spring balance,‡ B, interposed

‡ However, if we take a stern philosophical view and think out what is happening in terms of Newton's First Law, with Newton's Third Law treated as an accounting rule, we come upon grave doubts. We grow less and less sure what we are really measuring or demonstrating. We can even convince ourselves that the whole demonstration is a swindle. It is not, because it is a demonstration of a real event in the physical world and it does supply some information to people who might have expected a different result. Therefore, it does convey some knowledge and it should be shown.

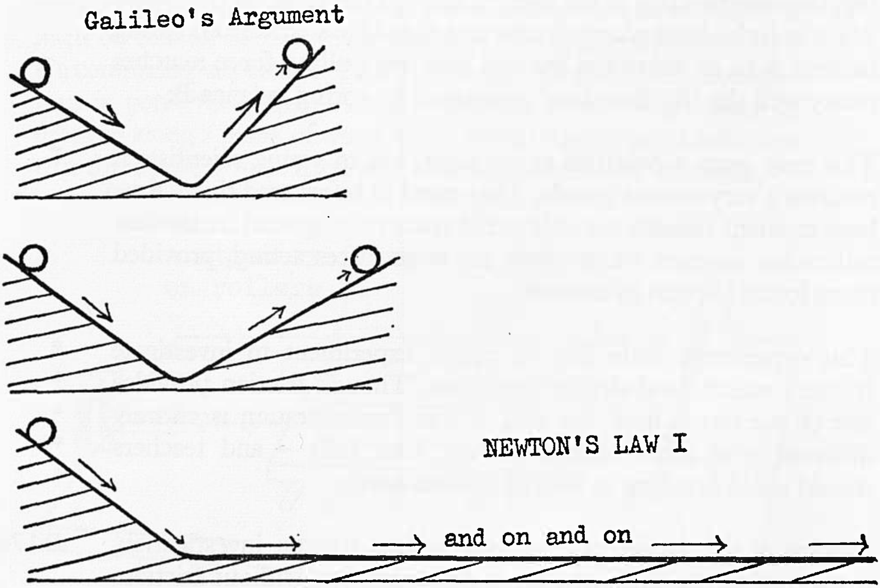
‡ For this demonstration to be worth anything, the spring balances must be ones whose readings show clearly at a considerable distance. The usual small spring balance with a little pointer on a straight scale is no use here – the teacher might just as well teach the experiment by asserting its result. Large, light, *dial* balances should be provided. A good form already in use has a dial 6 inches or more in diameter, a black face with clear white figures and a brightly coloured pointer. That enables pupils to see whether the two forces *are* the same, providing two properly calibrated balances are used.

between plank and post measures the tension in that cord, or as we say, the counter-drag of the friction force of the brick on the plank. We winch the brick along at constant speed by a cord from spring balance A to an axle; and we find that our pulling force matches pretty well the 'friction drag' registered by spring balance B.

This may seem a pointless experiment, but to young scientists it resolves a very serious puzzle. They need to be assured that we can have constant velocity not only out in space or on special frictionless rollers but in cases where there are large forces acting, provided those forces happen to balance. T

This experiment *looks like* an earlier experiment to investigate friction, which used similar apparatus. Though friction provides one of the forces here, the aim of this demonstration is entirely different – to show Newton's First Law fully – and teachers should avoid bringing in laws of friction here. * * * * *

In view of the danger of confusion with a friction investigation, it may be worth while to give a demonstration without friction. Load up the plank on rollers to make a large, massive 'trolley'. Install a spring balance, with a clear pointer and dial, on the plank at each end. Run a cord from one spring balance to the end of the table and over a pulley to a load, which will pull the plank one way. Run a cord from the other spring balance to a winch (or over a pulley to another pulling load), to pull the plank the opposite way. Pupils watch both balances when the opposite pulls are arranged to maintain constant velocity. (A further trial with unequal pulls producing acceleration will promote fruitful discussion.) D17b



Galileo's Argument leading to Newton's First Law. We remind pupils of Galileo's downhill-and-uphill argument (described in the *Guide* for Year III). Galileo assured himself, by drawing upon his commonsense knowledge of nature, that a ball rolling down one hill and up another would, apart from friction troubles, reach the same height on the opposite hill as its starting height. He argued that that must happen whatever the slopes of the hills might be. Then he considered the special case of a ball rolling down one hill and meeting another 'hill' consisting of a level plane, a hill that would never reach the same height. He concluded that the ball would never stop moving. In this way, Galileo arrived at Newton's Law I by a 'thought experiment'. T D18

Formal Statement of Law I? If we give pupils a formal statement of Newton's First Law, we should at least insert the essential word 'resultant'. When we say *resultant force* in that law, the law makes sense. Otherwise, it has that rarefied form which seems to apply only to outer space. T

We need a name for the *vector sum* of all the forces acting on the body. Law I applies to cases where that vector sum is 0; and the force in Law II is that vector sum when it is not 0. The name 'total force' is not suitable, because that has another technical meaning. The name 'unbalanced force' does not seem good, because it suggests there is something a little strange or inferior about that force. The names 'net force' and 'resultant force' are suitable; and here we choose the old-fashioned word 'resultant'. We use it to mean the effective sum; the single force which would, for most purposes, replace all the actual forces together.

In discussing the other aspect of Newton's First Law, the inertial property of matter, we should be very careful never to speak of a force 'overcoming inertia', as if inertia were a sort of internal armed guard, which once vanquished allows a frictionless life.

To a pupil with a good feeling for Newton's First Law of Motion, two experiments illustrate the idea clearly: an object coasting along a level table on some form of frictionless bearing, and an object falling with terminal velocity in viscous fluid.

Comment to Teachers: Newton's First Law. It looks as if we could substantiate Newton's First Law of Motion by experiments such as the one described above with spring balances. However, a very careful examination of the underlying logic involved suggests that all we really do when we state Newton's First Law is describe a force. We say, when there is no force we see uniform velocity, when there is a push or a pull we see acceleration. When we cannot see whether there is a (resultant) force, we look at the motion and decide whether there is any acceleration. That is how we know when there is a force and when there is no force. Although that seems a rather miserable analysis, there is still some practical knowledge of nature therein; because we can link 'no force' in our common knowledge with 'leave a thing completely alone' and expect, therefore, a space traveller to continue to move with constant speed in a straight line if he is far away from any disturbing influences that we can see or think of.

And there is some knowledge of real nature in the inertial property of matter: the property of opposing change-of-motion, of continuing to move along when we leave it alone. Therefore, many of us prefer to keep Newton's First Law as a separate law and not just a case of Law II. It is now fashionable to call Newton's Law I a

special case, but in our teaching we, like Newton, may find Law I *
worth stating clearly. Newton himself was trying to establish an *
entirely new way of treating mechanics; a treatment that was partly *
built by Galileo from the work of a few earlier scientists, and then *
came out into the open as a reforming influence on the whole of *
medieval physics. *

Constant Acceleration: Discussion of Charts, leading to Graphs and Formulae

We point out that the chart which pupils make for accelerated T
motion is really a graph, with speed plotted upward and time plotted along. We have plotted speed in queer units, centimetres per 10 ticks, or centimetres per tentick; and time in queer units, one tape width representing a tentick from the half-time of one strip to the half-time of the next. Instead of that we might plot, in imagination, a true speed-time graph like this:

> 'Imagine that speed is sampled at regular intervals of time, say 0, 10, 20, 30 seconds from the start and plotted upward on some suitable scale and that the time is plotted along. We might have a graph of a car accelerating, in which case the speed would be in miles per hour and we could imagine reading the speedometer every 10 seconds.'

As revision of Year III work, plot such a graph on the blackboard showing speeds 0, 15 mph, 30 mph, 45 mph, 60 mph – this is a car speeding up to 60 mph in 40 seconds from the start.

> 'Now we can look at the acceleration. We can see how much speed the car gains every second; here it is moving 45 mph, a gain of 15 mph in how long? ... Yes, 15 mph gain of speed in 10 seconds, that is $1\frac{1}{2}$ mph gain of speed in every second. We call that the acceleration.
>
> 'We say that the acceleration is $1\frac{1}{2}$ miles/hour per second. That doesn't mean its speed was $1\frac{1}{2}$ mph any more than it was 15 mph at any time between that 30-mile-an-hour stage and that 45-mile-an-hour stage.

'This is our way of saying how it's speeding up, how it is gaining speed – rather like somebody's statement about pocket money going up: if you get 5*s* a week this year and 5*s* 6*d* a week next year and 6*s* a week the next year, your acceleration of pocket money rate is how much? ... Yes, 6*d* per week per year.'

The teacher should draw the complete graph-line sloping up through the points and ask if that looks like a fair graph of the motion we are imagining. He can draw a little triangle for each 10-second period, showing the 15 mph increase, like steps of a staircase, step after step all the same size. He can ask whether the graph shows that the acceleration stays the same, and how one can always tell a constant acceleration if one sees the graph. The straight slanting line is the scientist-detective's clue. (We do not at this point introduce the technical term 'slope' unless that is already well known in mathematics.)

We can ask what the graph should look like if the car had twice that acceleration, or half that.

We should ask what pupils would expect for the same car loaded up with a great many passengers and pulled with the same force. We should also ask whether pupils think that this graph would go on for ever with a real car, continuing with a straight line.

The last question will lead to the commonsense answer that air resistance will change the story; and we must make it clear that the different motion that we then expect is not 'wrong' but is just what does happen in nature.

Accelerations are often Not Constant. In fact, we must be careful to avoid giving pupils the idea that the only right kind of accelerated motion, or the only common kind, is one in which the acceleration is constant. We should certainly *show* some motion in which the acceleration is not constant. As a good example show the accelerating chain described earlier in D/C5. T

Graphs of Velocity versus Time Formulae. We now return to the idea that we built up and used in Year III, that on a graph of speed against time, the area under the graph gives the distance travelled. In Year III we made such graphs informally by pasting strips of tape from the recording system, so the area under our T 'tape chart' was obviously, automatically, distance travelled, provided we reckoned it by adding lengths of tape. Now we point out – if that has not been discussed already – that if we take lengths of tape for a one-second period and make the chart by spacing the strips one second apart the area of the chart certainly gives distance travelled. And the chart is now a graph on which we plot speed upward and time along. We illustrate that by treating the problem of the car above, as follows:

'I am going to redraw that graph just as before except that I am going to write the speeds in different units, in feet per second instead of miles per hour. I shall do that because I want to deal with time in seconds along and speed in feet per second upward.

'What is this speed 30 miles/hour in feet/second?'

(The teacher should then work it out and arrive clearly at 44 feet/sec.)

'Then I mark this 44 feet per second. The next point that we had was 45 mph and that is 66 feet per second, a gain of 22 feet/second. These two points are 10 seconds apart. I am going to imagine we divide that time into ten periods of one second. (*Teacher makes marks.*) In each second the gain is 2·2 feet/second.

44 .... 46·2 .... 48·4 .... and so on to 66.
gain 2·2 gain 2·2

Here are two times, just *one* second apart. What is the speed just here? Well, at the beginning of that one-second period, the speed was 44 feet per second and at the end a little more, actually 46·2. How could you find out how far the car travelled in that one-second time; what was its speed then? Oh roughly ... Yes, 45 feet per second. Then how far did it travel in a second? Yes, 45 feet, just this height. How far did it travel in the next second? This height, and in the next second this bigger height. What I am really doing is multiplying each speed by a one-second piece of time: if I wanted to know how far the car went in a half second I should just take a half second along, like this, and multiply it by the height up to the line and that would be the speed times half a second.‡ [Speed] multiplied by [time], that's the area of the strip.

‡ Even with quite able pupils we should be careful not to say: '*Of course* distance is speed multiplied by time.' That, which is so obvious to us, is still in the foggy region of unfamiliar thinking in many a young mind. We are reminded of that when we see quite capable pupils scribble on the edge of an examination paper some mnemonic for remembering 'distance equals speed × time'.

The area of this strip shows how far the car goes in that time. Then how can you tell the total distance the car travels between, say, here and here?'

We try to elicit the statement – already asked for in Year III – that the *area* under the graph between the two points gives the distance travelled between those two instants of time.

Formulae. We should then put this idea into algebraic form. We draw the graph with v (velocity) upward and t (time) along and show a motion with uniform acceleration starting from rest. We end at some place on the line which we mark v_{final} and t_{final}. T

It will confuse pupils if we label a particular chosen point on their graph v and t, when they are already using v and t for their general coordinates. We should use either v_{final} or v_1 or something of that kind.

We ask how far the object has travelled. Pupils will tell us now that it is the area of the triangle, of base t_{final} and of height v_{final}. So the distance s is $\frac{1}{2}[v_{\text{final}}][t_{\text{final}}]$.

Now we go back to acceleration, which is defined as gain-of-speed in each second. a is the acceleration (a feet/second gain of speed in each second), how much speed shall we have gained at t_{final}? That will be $a[t_{\text{final}}]$. For a car starting from rest, that is the speed then.

$$v_{\text{final}} = at_{\text{final}}$$

and $s \quad = \frac{1}{2}[v_{\text{final}}] \times [t_{\text{final}}] = \frac{1}{2}a[t_{\text{final}}]^2$

We should remember that to pupils this is not only unfamiliar, but at the moment pointless. We can give it a little point by asking pupils to calculate how far a racing car will go in certain times with certain accelerations and we can put it to some use at once to measure g.

Instead of being impatient, we should offer some comforting medicine:

'Suppose you drive a car for two hours at 30 mph. How far will you go? How did you get that answer, by multiplying or dividing? Give yourself a simple obvious problem like that whenever you are not sure.

'If you are given speed and distance and asked for *time* say to yourself, "I drive at 30 mph and go 60 miles. That takes me 2 hours. How do I get that *time* of 2 hours? By dividing 60 miles by 30 mph. Therefore time equals distance/speed." There is no need to remember this as a formula. Just use common sense and think about driving a car.'

Measuring g

g is an interesting property of nature. Unless pupils measured g in Year III, they should do so now. If we make some quick measurements they will enjoy it and be impressed to find that they can all agree fairly closely with each other. If we labour the measurement we shall spoil things at this stage. We should be careful not to over-emphasize the need for accuracy, by following the tradition of earlier times when measurements of g were considered specially good training and were in fact needed for engineering and other purposes.

The acceleration of a freely falling body is as important a thing as ever – in geophysical prospecting, engineering, rocketry – so pupils should understand what it is and should have the experience of measuring it. But if they need a very precise value they can obtain it from other people's work. If precise measurements of g were something that pupils could carry out easily and well (and thus achieve a great sense of success in precision) we should certainly urge it on all teachers; but the only available methods for high precision are pendulum ones, which do not give pupils their result of g by a fully understood route, as we have to supply a formula without much support.

Even the formal claim that nevertheless pupils should try the pendulum measurement, since it is the ultimate standard used by professional physicists, is put out of date by the fact that the most precise measurements of g are now being made on freely falling objects by using an interferometer with electronic frequency measurements for timing.

Pupils should measure g by some method that gives an answer that can be trusted within a few per cent, a method they feel they understand perfectly; but first they should make a rough, simple estimate.

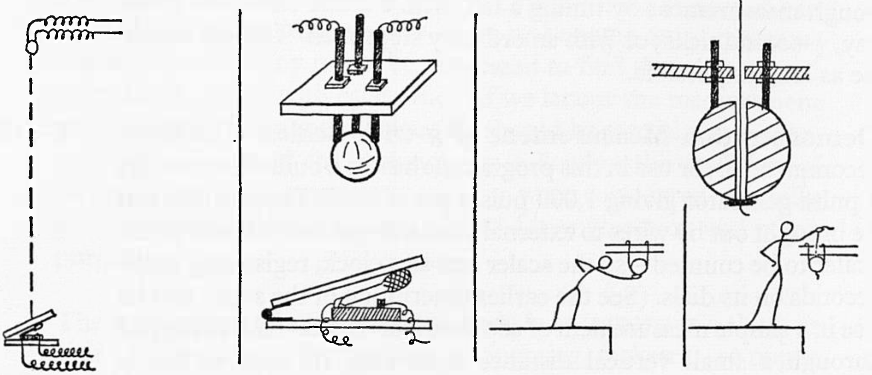
Rough Estimate. So, first of all, pupils should make a very rough measurement by timing a fall, with a crude clock that gives, say, $\frac{1}{2}$-second ticks; or with an ordinary stopwatch. The fall should be as long as possible. C19

Demonstration Measurement of *g* with Scaler. The scaler recommended for use in this programme has, as a built-in accessory, a pulse generator giving 1,000 pulses per second. These pulses can be brought out by wires to external switches and carried back to the scaler to be counted – so the scaler acts as a clock, registering milliseconds on its dials. (See the earlier description of the scaler and its use in a simple measurement of acceleration.) A steel ball is dropped through a small vertical distance from rest, its time of fall is measured by the scaler, and its acceleration is calculated on the assumption that it fell with constant acceleration. C–D20

At the starting point, the ball is held by the experimenter against three metal pegs (held by hand or a string). The ball completes an electric circuit through two of the metal pegs, thus keeping the 'clock' from running until the ball is released.

(If an electromagnet is used instead, there is danger of a delay in releasing the ball when the magnet is switched off, so it is important to place a small piece of paper between the magnet core and the ball. And there are difficulties in arranging for the prompt switching of the scaler so we do not recommend an electromagnetic release, tempting though it sounds.)

When the ball arrives, having fallen a distance of, say, 1 or 2 metres, it stops the clock by hitting a switch. A micro switch does well but a simple knife switch is better because pupils can see it. It should be placed on the floor with a small hinged platform on its handle so that when the ball lands on the platform it pushes the handle down, closes the switch and makes the circuit that stops the clock. This is a very good demonstration if the scaler is used as a familiar object in the teaching; but it is a bad demonstration – for our purpose of well-understood physics – if the scaler has to be brought out specially and carefully adjusted and presented as something strange. So we urge teachers to explore the uses of the scaler.



If possible this should be a class-demonstration in which the teacher sets up the arrangement and then each pupil in turn makes his own measurement. C20

Make this a Quick Experiment. This class experiment should *
be a quick experiment for the fun of measuring *g*. Pupils should not *
do this until they have some confidence in the relation $s = \frac{1}{2}at^2$. It *
should not be an experiment that drags that relation in as a mysteri- *
ous 'formula', simply to get the right answer. Nor should it turn *
into a whole series of measurements to 'get a good average' or to *
explore the relationship between height and time of fall. At this *
stage we should be anxious to get on to new topics in dynamics and *
it would be easy to delay progress by developing an enthusiasm for *
a set of measurements that would not have a very important out- *
come. *

Measuring *g* with Electric Stopclock (*Optional de luxe alternative*). If the laboratory has a demonstration electric stopclock, that should be arranged so that pupils take it in turns to use it for a fairly precise measurement of *g*. They should time free fall of a ball from rest, for a distance of several feet. Before release, the ball is held against three pegs, as in the Panax measurement above, holding the clock stopped by making contact between two of those pegs. If the construction of the clock is such that this use of the pegs in- C–D 21a OPT.

volves any danger of shock, the ball may be held by a piece of string threaded through a hole or hook in it; or a relay may be interposed.

(Or, if the construction of the clock allows it, the ball may be released by a small electromagnet, arranged to stop the clock when the current through the coil of the magnet is switched off. However, that should not be used if it involves a switching arrangement, such as a relay which looks complicated, or any double switching operation with unknown delay. Such arrangements might seem to us admirable in their ingenuity but they would divert pupils' attention from the simple important measurement. If an electromagnet has to be used to release the ball, a small piece of thin paper should always be placed between the ball and the core of the magnet – otherwise, residual magnetization is likely to cause uneven delays in the release.)

At the end of a measured fall, the ball should stop the clock by hitting a small plate on the handle of a knife switch resting on the floor – thus closing the switch, as in the scaler measurement. If this arrangement matches the Panax scaler arrangement closely, all the better: pupils can then do a class experiment, taking turns with the electric stopclock, to make their own measurement of g by a method they have already seen demonstrated.

Teachers might prefer to hold up the scaler measurement until after this class experiment and then produce it as a measurement with improved precision.

Pulsed Jet of Water Drops: Estimate of g (*Buffer option*). A fast group or a pupil with special interests might make an estimate of g from measurements of the drops of water in a pulsed water stream viewed stroboscopically. That experiment is intended primarily to demonstrate the constancy of the horizontal velocity of a projectile; but when a grid of horizontal wires (or some other vertical measuring scale) is placed in the plane of the jet the vertical fall from drop to drop can be measured and plotted, yielding an estimate of g. D21b

Special Experiments for Measuring *g*. Many ingenious schemes have been suggested for measuring *g* with simple apparatus: such as a swinging pendulum that hits a dropping ball or a rotating gramophone turntable that catches a falling dart, but in general these are to be avoided here because the ingenuity of the method is likely to swamp the essential sense of measuring something.

There is one important exception: where a teacher has invented and constructed his own special device for measuring *g*, he should certainly use it with his own class: that sense of research which is transmitted automatically to his pupils is of enormous value in giving them a true sense of science. Yet here, as elsewhere in the programme, other teachers should hesitate to adopt someone else's special scheme – however ingenious and successful – unless they are satisfied that it will not delay progress and that the essential measurement will shine through the details of the operation of the apparatus.

Multiflash Picture for *g* (*Optional*). At the beginning of this Year, it was suggested that a multiflash picture be taken of a freely falling body. If that was done, pupils will probably ask, now or earlier, if they can obtain the value of *g* from that. The answer to that depends on whether they know the frequency of flashes and whether they have a scale of distances in the photograph. So we hope that a metre ruler was included in the photograph and a record made of the flash frequency. If not, it is probably worth while to repeat the experiment now. Or, if the equipment takes too much time to arrange, the teacher might now distribute printed copies of such a picture.

D22a (=D1) OPT.

Printed Copies of Multiflash Pictures (*Optional*). To distribute printed copies to pupils who have never seen a real multiflash experiment carried out, would be confusing to pupils and a very bad mistake in our teaching. In spite of obvious economy, we hope that no school in the Nuffield Physics Programme will ever yield to temptation and do that.

C22b OPT.

However, when pupils have taken part in a real multiflash experiment, they should be ready to use printed copies of someone else's picture of a similar experiment without much confusion or damage. Again and again in each area of natural science, we all of us have to receive secondhand evidence, review it and accept it with only moderate reservation. But we should not accept such evidence unless we understand clearly how it was obtained and feel that we have some knowledge of its reliability. In the case of multiflash pictures those assurances are best provided by doing a real experiment as a prototype.

In making an estimate of g from a multiflash picture of free fall, pupils can also test the constancy of that acceleration; so we hope that such pictures will be used, where the speed of the class permits.

AIMS AND PLANS

Pupils of this age want to know where they are going in a programme of study. They ask, 'Where does this lead?'; or there may be a more unhappy question – coming from either pupils or teachers – 'Why are we doing this?' Sometimes such questions come only from a general uneasiness over a new programme and call upon the teacher's feeling of confidence and enjoyment, rather than needing a specific answer.

Sometimes the questions are voicing a complaint over some of our demands which make the work seem difficult and uncertain compared with learning science by memorizing facts and rules. Those demands are important matters in our policy: we ask for reasoning to be done; we give problems to be thought out; we pose questions to be kept partly answered for some further thinking; and all those are of the essence in our teaching. We have to meet complaints of difficulty and feelings of uncertainty very gently, because habit is there from other teaching and – particularly with slower pupils – our new demands do feel harder to meet.

We should not reply by making fun of parrot-learning; but we should point out the advantages of doing one's own experiments and some of one's own thinking, so that one understands well and can keep that knowledge.

And we should reply both to that complaint and to the simple question of our aims by giving examples of the physics we are dealing with and by outlining, from time to time, the work that lies ahead.

Problems with Unexpected Difficulties: help or hindrance?
Sometimes we suggest, intentionally, problems that involve knowledge that is yet to come in the course, problems that must, therefore, be left unfinished for the present. For example, see problems 3 and 4 below. The rocket problem leads to two accounts of the rocket's propelling force: both of them involve Newton's Law III, both involve an understanding of momentum changes, and one requires a fuller discussion of kinetic theory than pupils have yet met. In the racing car problem pupils have no sooner started than they find they must wait and learn about absolute units for force. So these problems are offered to teachers for *possible* use to promote interest, to ask questions, to lead towards further studies – to help, perhaps, by creating a demand.

We consider our work in teaching physics to be far away from the ruthless ways of the advertising man: and yet we can learn much from his skill in 'creating a demand', in putting the customer in the frame of mind to want what he is going to be offered. For example, he does not always say, 'Go and buy Sunflower Soap; you must have it.' He sometimes asks more delicately, 'Have you tried Sunflower Soap?', or even more delicately, 'Can Sunflower Soap really make you more beautiful?'

Unfinished arguments and unanswered questions sometimes have a constructive action. Yet we hesitate to start a problem in physics and leave it unfinished, or ask a question and leave it unanswered for a long time. Our hesitation is well founded because both we and our pupils expect the teaching to go straight through to a definite result, and leaving things unfinished is likely to make all of us feel confused and doubtful. So we should not often leave things unfinished *unless* we can do that with the light touch of the advertising man: present our questions as an interesting puzzle, or come to a dead stop in our argument, with pupils feeling that they are co-operating in the search and are glad to have gone so far, even though they and we now find a barrier.

The use of unanswered questions or unfinished problems must depend strongly on the frame of mind which the teacher can encourage. Given the right reception, they can be very powerful, the essence of teaching pupils to learn for themselves.

We need questions which relate to real life or appeal to pupils' own interests, questions that bring in some *application* of the physics that is being taught. It is difficult to suggest examples of applications that will suit all classes. Furthermore, such examples come best with the full force of the teacher's own interest; so we hope teachers will keep an eye open for interesting problems or applications and make use of them. However, we suggest a few examples later below.

Views of Science and Unfinished Questions. Most of our pupils have a clear, almost rigid, view of science as completely 'right'. They believe that science tells the true story of real nature (which has been waiting to be discovered); science can give the correct explanation of every phenomenon. Scientists, as pupils see them, find out and *know*, and they can *tell*: they set forth the facts and reveal the scientific explanation. There is considerable truth in that view; but it is too rigid; it is out of tune with modern scientists' views of their work.

Today, we are more humble about 'knowing all the facts'. In our experimenting we can only go by what our instruments tell us; and we now know that there are severe limitations – inherent in nature – on our experimental work. When we interpret our experiments – even while we are conducting them – we make assumptions; we plan and infer in terms of a model, in the light of our picture-of-the-moment of nature.

Science today is not just a pile of measured results like a table of densities or some values of g; nor is it a set of formulae connecting measurements, like $s = \frac{1}{2}gt^2$. We try to build a connected frame of knowledge, in which models of nature – sketched by imaginative thinking based on experimental results – enable us to think and plan, to check the models themselves to some extent, to predict, to guide more investigations, and over all to discuss our own knowledge. It is the growing edge that most scientists enjoy, the doing and thinking to extend the framework of knowledge, rather than the wealth of facts and rules already accumulated. Unless young people understand something of that attitude they will be out of tune with modern science.

For that reason, we want pupils to have glimpses of theory – in the making and in use – at this O-level stage, and not wait for sixth-form theory that may never come. In Year III we offer a simple magnetic theory whose fruitfulness is obvious to beginners. That comes at the end of Year III; and we trust no teacher trying our programme will omit it through lack of time or equipment or – as one might think – pupils' interest. The interest will certainly be there, if we put the purpose clearly; the material can be bought cheaply or prepared at home; and the time is well worth saving from an overdose of trolley practice or a study of gas expansion – an early understanding of theory will last longer and be of greater value. Nor should that be postponed. We rely on it to have sown seeds of interest some time before we tackle kinetic theory in Year IV and gravitational astronomy in Year V. In Year IV, we trust teachers will develop a theory of gases as fully as pupils' skills and interests permit. Here again, successful learning will bring rich rewards. The rewards increase exponentially with the depth of understanding and variety of uses that we can give for the theory.

So we need to prepare the ground for speculation. That is why we hope teachers will try some problems that promote discussion. We need to soften our pupils' picture of the scientist as the man who knows, having found out with ultimate precision by experiments. If they can think of the scientist as the man who has some knowledge but knows its limitations, as the man who is finding out, who enjoys discussing questions as much as solving formal problems, as the man who *thinks*, we shall bring our pupils much nearer to modern science.

Discussing Plans with Pupils. As a look into the future of the programme, teachers should say now that we are investigating force and acceleration because the Laws of Motion are very important parts of our knowledge of science: useful in analysing atoms and parts of atoms such as electrons, because we make those tiny things move and we change their motion, when we are trying to find out about them; useful in solving some great problems in astronomy (for example: What keeps the Moon moving? Why does it move in a circle?); of great use in engineering in dealing with trains and cars and planes; and absolutely essential in arranging to fire rockets and satellites successfully.

(To a very fast group we might say: 'You have probably heard of $E = mc^2$. If you want to understand that, you must know what mass is really like – the m in that relation is the difficult thing: the E and the c are comparatively easy to understand. If your work with force, mass and motion gives you a good clear idea of mass you will have learned something difficult and important.')

Telling Pupils the Programme Ahead. Teachers should now tell pupils what they already know from their own plans – that this year they will go on from the present work on force and motion:

to a theory of gases with molecules treated with Newton's Laws, to make remarkable predictions and increase our understanding;

then to a new discussion of energy and conservation of energy, with heat really understood at last;

then new work with electric circuits and voltmeters and power transmission;

and finally experiments with electron streams in a vacuum and even an experiment to measure the electric charge of a single electron.

Each of those later developments requires a knowledge of force and motion and an understanding of kinetic energy which itself uses a knowledge of force and motion. In Year V, we shall again use knowledge of force and motion to make measurements on electron streams being pulled into an orbit by a magnetic field; and we shall deal with the orbits of planets and the laws that describe them; and we shall use our knowledge of force and motion to describe some of our experiments in radioactivity.

In this Year and the next, we shall describe the building of increasingly successful models of atoms and the insides of atoms. The present work on force and motion is a preparation to enable pupils to understand how we get the information which helps to build our models.

USEFUL PROBLEMS

These examples are placed here but teachers who wish to use them T
should move them to appropriate places in their teaching.

1. **Pushing a Car.** 'A family car has run out of petrol and is P
standing outside the house some distance from the garage. Fortunately the road to the garage is smooth and level. Two boys, both equally good at pushing a car, offer to get the car into the garage. When one boy pushes alone, with the car brakes off and the gear in neutral, the car will not move. A second boy gives the car a push for a short time to help start it: then one boy, pushing alone, can keep the car moving steadily along with his (maximum) push. What forces are acting on the car now? What can you say about those forces?

'Now suppose that two boys push together (each with the same maximum push), what kind of motion would you expect the car to have, if both boys keep up a steady push?

'Suppose that, with both boys pushing fully, the car travels 40 feet in 10 seconds starting from rest. How far would you expect it to travel, from rest, if the two boys pushed for 20 seconds, that is, twice as long?

'Now suppose a third (equal) boy arrives. All three boys push together. How far would you expect the car to travel, from rest, in 10 seconds? In 20 seconds?

'Now suppose that, owing to a misunderstanding, two boys are pushing forward at the back of the car while the third boy is at the front of the car, pushing it *backward.* If the boys are silly enough to continue this arrangement, how far will the car go, from rest, in 10 seconds? (Answer: 0.)

'Now suppose one more (equal) boy joins them. Three boys push forward and one pushes backward. How far will the car go from rest in 10 seconds?

'Finally suppose that two of the four boys climb into the car and the other two push the car forward. Will the car travel the same distance from rest in 10 seconds as when two boys push it with no extra boys inside? If not, will it travel a greater distance or a smaller one and why? (This last question is of course intended to help build the idea of mass.)

2. **Damage.** 'Suppose you know the force, F, needed to change the motion of a certain object in a known time, from some given speed, v, to rest, what force would be needed to make the same change of speed in half the time? In $\frac{1}{10}$th of the time? In one hundredth of the time? P

'Any object pulled by its own weight (*the pull of the Earth on it*) falls with an acceleration 9·8 metres/second^2. Its weight tells us just the size of force needed to give it that acceleration. What size of force would be needed to give it 10 times that acceleration? Or if it is already moving fast, what force would be needed to give it a negative acceleration (a deceleration or retardation) 10 times 9·8 metres/second^2? Tell me an easy way of giving some moving object a big negative acceleration ... yes, let it fall and when it is moving fast let it hit a hard floor. Or put it down abruptly on a table.'

Give examples of a watch, a wine glass, a scientific instrument, a baby, estimating forces (as multiples of weight) for sudden decelerations. T

3. **Rocket.** (The following question asking about rocket motion is a difficult one. Teachers will probably wish to postpone it. Before using it, give a demonstration of a rocket – see Experiment 23 below.) * * * *

'Suppose a large rocket has its rocket motor running – that is, blasting out gas downward – so that the rocket is pushed with enough force to give it a considerable upward acceleration. Later in its flight, the rocket is to be turned from that direction to a "horizontal" direction and given enough speed to make it an Earth satellite. When the rocket has turned, will the same blast from its motors give it the same acceleration as before?' P

This is raising a question of force, although much of our attention has been on acceleration. That is intentional, of course, because we want to move on from studies of motion to dynamics. This does ask for new, difficult thinking; but we hope teachers will experiment with its use. With some classes it will bring out in discussion the fact that the upward thrust by the motors is *not* the resultant accelerating force in the vertical case, because we must subtract from that the pull-of-the-Earth on the rocket. In the horizontal case, the full thrust *is* the accelerating force.

While the rocket motors are running, large quantities of exhaust gas are being blasted out from the tail of the rocket. Ask whether this changes anything about the rocket; and what effect that change will have. (This raises the question of the mass of the rocket becoming smaller and the acceleration thereby being increased.)

Demonstration. Give a demonstration of either a water rocket or D 23a
a small rocket-car driven by CO_2 from a capsule. or b

Ask whether, in the flight, anything else changes as the rocket gets farther away from the Earth. (This hopes for a suggestion of gravity decreasing at greater distances. We have not taught anything about that yet; and we should neither expect a ready answer nor give a ready-made answer. This is the time for raising an eyebrow of enquiry. We might possibly ask an extreme question such as: 'If somebody far away from the Earth, out among the stars, released a rock would it fall towards the Earth just like a stone?) T

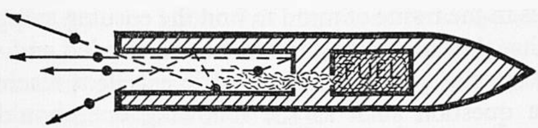
Ask: 'How do the rocket motors push the rocket to make it accelerate?' T

This last is a leading question that we ask with our eye on the next *
section, kinetic theory of gases, with molecules moving very fast. *
These gases are not pushed out of the tail of the rocket by some *
mysterious set of springs called gas pressure! They simply travel *
out of the tail of the rocket with their own high-temperature speed. *
Any molecules that are travelling backward go out from the tail *
of the rocket. Any molecules that happen to be travelling forward *
hit the front wall of the rocket motor inside the rocket, bounce *
off it and are then travelling backward and will probably escape. *
All the molecules that escape carry away momentum:‡ so we can *
examine the rocket's action from that overall point of view. *

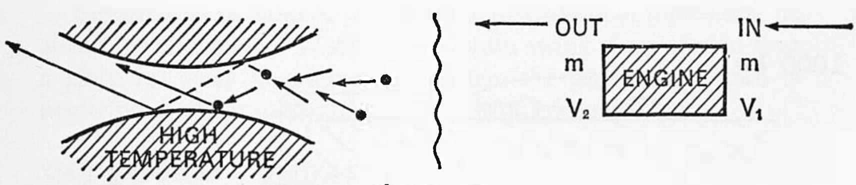
‡ When the rocket is moving faster than hot gas molecules, will the escaping molecules still help to propel it? Of course they will. They are still hitting the front wall of the rocket before rebounding and travelling out to escape. Since they emerge with smaller speed, relative to the rocket, than the rocket's speed, an outside observer will see them moving forward, after the rocket. But they will be moving forward slower than the rocket, with less forward momentum than they had a little earlier when they were part of the rocket. Therefore the escaping molecules have lost some forward momentum and the rocket has gained some.

So we have two stories about the force with which the rocket is propelled:

a. The detailed mechanism by which the hot gas molecules propel the rocket is their bombardment of the front wall of the rocket motor inside the rocket. In the case of some jet motors there is not 'a front wall' like that but there are very hot side walls which are rough – to a molecule's view – and present the equivalent of many bits of front wall. The side walls heat up gas molecules that hit them. Those gas molecules approach a side wall slowly (V_1) and leave it much hotter, therefore moving much faster (V_2). As gas flows through the jet engine, those impacts are oblique, the molecules having a general motion towards the rear; and since that motion increases the side walls must give molecules a backward push and the molecules must give the side walls a forward push.



Rocket engine

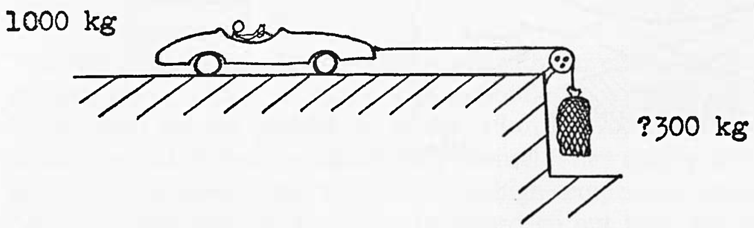


Jet engine

b. The overall momentum treatment. Since the hot gases leave the tail of the rocket moving backward, relative to the rocket, they are carrying backward momentum, momentum that they did not have before the fuel was fired. If we trust conservation of momentum, it tells us that the rocket gains an equal amount of forward momentum.

(The rocket, including its fuel, does not form a closed system because it is attached to the Earth by a gravitational field; but we take account of that when we subtract the weight of the rocket from the upward thrust produced by the motors. Then we can apply conservation of momentum to the rocket and gases. As explained above, this discussion of rockets goes far beyond our pupils' present stage. It is given here for teachers to store up for possible use.)

4. **Problem on a Racing Car.** (This is a problem that has to be left unfinished for the present. It is only offered here in case teachers wish to present an unfinished problem to promote interest. At a stage when pupils do not have $F = ma$ clearly in mind for use with absolute units of force, we have to set very simple problems that are little more than qualitative discussions, or we have to start problems and then leave them unfinished for want of knowledge of force units. In some classes pupils will be infuriated when they find a question has to be left unfinished; but other classes will be intrigued. This is a matter for skilful judgment on the part of the teacher. Such problems should not be presented unless the class is in the frame of mind to find the ensuing stoppage interesting, so that they are willing to see what is needed and wait for it – in that case the question may provide excellent teaching; but otherwise a question such as the following one should be avoided.)



'A 1,000-kilogram racing car is being tested. When it is running at 50 miles/hour, with the throttle fully open – "the accelerator on the floorboard" – it can just keep going at 50 if the brakes are on at half pressure. To find out the force the brakes were exerting then, the testing people stop the car, stop the engine, put the gears in neutral and then drag the car forward with the brakes still at half pressure. They keep the car moving forward by a rope which runs from the front of the car to a pulley wheel over a pit, over the wheel, and down to a load of scrap iron hanging on the end. The car continues to creep ahead as the load falls. P

'What acceleration would that car have, with the throttle open and brakes off, when driving at 50 miles/hour? With that acceleration how long would it take to speed up from 50 to 60?'

This is a question that needs considerable discussion and teaching. Pupils will find that one piece of data is missing: the force pulling the rope. The purpose of giving this problem now is to raise the problem question of force-units in $F = Ma$. At first some will use the 1,000 kilograms as a pulling force; but that is only the mass of the car; and anyway pupils should see that the downward pull of the Earth on those 1,000 kilograms of car is balanced by the push of the road upward on the car. We have 1,000 kilograms of stuff to accelerate but we do not know the accelerating force. We ask for some guesses about the scrap iron. In fact, 100 kilograms would be reasonable for an ordinary car and 500 kilograms would be unreasonably big for any car. We might suggest 300 kilograms of scrap iron as a high testimonial to that racing car. This brings us to another difficulty: *we* know that the force must be expressed in absolute units, in newtons; the pupils do not yet know this. So at this point we have to leave the problem and explore our knowledge of force, mass and motion more fully.

5. **Pendulum in Space.** To promote discussion of topics that lie ahead we might ask, 'Would a pendulum swing if we carried it to a place far away in outer space (where the gravitational pull is negligible)?' That may lead to a discussion of mass. P

THE CONCEPT OF MASS

Mass. In our programme, the concept of mass receives more and more attention from year to year. If we can succeed in giving pupils some feeling for that concept we shall have made an important contribution to their education. Mass may have been mentioned in passing in Year I; again, in Year II, faster groups may have found the word being used carefully, in contrast with 'weight'. In Year III mass may have been described as a measure of how much stuff is piled together when we try to accelerate several trolleys.

This is a difficult, strange, sophisticated concept; but it is now so important in science that we should do our best to build up a sense of understanding mass.

Pupils should have seen demonstrations in Year III to illustrate the idea of mass, or give meaning to the word inertia. Whether we can take for granted an understanding of mass from Year III or must start afresh and discuss the idea carefully, will vary greatly from class to class and pupil to pupil. Because the concept is a difficult but important one, we are probably wise to discuss mass fully now, even giving some of the old demonstrations, though maintaining a reassuring claim that this is revision. On the other hand, we shall not make this difficult idea, mass, clearer by routine repetition of the work of Year III. Therefore, discussion and experiments are only given below so that teachers may choose what seems to them most suitable.

Descriptions. Experiments show that the more trolleys we have piled together the bigger the force we need for a given acceleration; or, the less acceleration we get for some standard pull. There is something about these chunks of matter that makes them 'difficult' to accelerate – not difficult in the sense of a rough backward drag of friction to be opposed, but a sluggishness, a slowness to get moving. The effect of a small resultant force is slow but sure; we get any amount of motion if we wait for a long enough time. (That last is an important point for some schemes of rocket-propulsion in outer space.)

It may help to coin some slang description of mass such as 'unaccelerability', 'difficultness of getting goingishness', and perhaps even 'massiveness'. (The origin of the word *mass*, a *lump of dough*, is suggestive.)

Professional scientists use the word 'inertia' to describe this property, but in teaching boys and girls that is no more than taking refuge behind a pompous long word. Pupils soon learn to avoid answering 'friction' and to be very careful not to mention 'weight' and to use 'inertia' as a magic word that will get good marks. That has little to do with understanding this difficult but important concept of mass.

(Incidentally, to say that 'mass is energy' or something to that effect, in a bow to relativity, will not help at all here; and anyway many a physicist would consider that wording a misleading version of the safer statement, 'energy has mass'.)

Note that many an engineer in earlier generations could afford to ignore mass. He was concerned with the weight of the bridge and the weight of the load on the bridge, which increased the stress in the members of the bridge. He could afford to measure forces in pounds-weight, and use them in his own version of Newton's Laws of Motion. In this age of nuclear power and space flight, the modern engineer needs mass in its own right. He learns, as every physicist must do, to regard mass as an important fundamental property of matter and energy, quite different from the pull of the Earth on everything that has mass.

Newton himself described mass as 'quantity of matter'. It is the fashion today to laugh at Newton's statement, saying that it only referred 'mass' to another word 'matter'. However, Newton was a wise man, writing first for himself and then publicly for his contemporaries; and his description was probably only a teaching device to make something clearer to people who were struggling with a new and unfamiliar idea. We might try that phrase with our pupils too. (As mentioned in an earlier note, when we consider atoms we may well think mass is a measure of 'quantity of matter', particularly if we work it rather carelessly by counting nucleons.)

A Useful 'Thought Experiment' for our teaching here is to imagine experimenters in a space ship in outer space free from a gravitational field, trying to pull a trolley along 'horizontally', on a frictionless table. They use a spring to exert the pull. Then they hang the trolley 'vertically' on the same spring, holding the top of the spring in one hand. They try again accelerating the trolley by pulling it upward with the spring. For the same stretch of spring in each case, what differences will they notice between the two?

By drawing a 'leading diagram' on the blackboard one can mislead pupils temporarily into believing that vertical and horizontal have a real meaning and are different. When they realize they have been tricked, they will be left with a feeling that the mass is still there needing a force to accelerate it and giving the same acceleration with the same force whatever the direction of pull.

Experiments to Illustrate the Concept of Mass

At this stage, we should not just talk about mass as a theoretical concept; but we should repeat some experiments from Year III.

Comparison Pendulums. Hang two large tin cans by long strings from the ceiling, one empty, the other full of sand. Ask pupils to give each a short, sharp, push, to *feel* how easily each can accelerate. If we have a toy gun that shoots ping-pong balls or peas or something like that, we might try bombarding each tin can in turn. We should point out that there is little question of resistance by friction in this case: it is the mass of the can of sand that makes it slow to get moving – not impossible as friction might make it, but slow. C–D24

Mass Exhibit. The laboratory should have a 'mass exhibit' on a friction-free table. This consists of a level surface covered with glass or other smooth material, with a sprinkling of small ball-bearing balls. A kilogram of brass on a plywood disc slides about freely on that. It should be clearly labelled 1 kg. There should also be a 1-pound mass. This table should remain there for weeks, and pupils should be free to push these standard masses with a finger and feel and see the results. (A layer of small beads of polystyrene is cheaper but does not do so well under large loads like 1 kilogram. Steel ball-bearing balls $\frac{1}{8}$ inch in diameter cost less than £1 for 2,000 from the makers.)‡ C–D 25

Towing a Barge (*Optional*). Give a similar demonstration with a small barge in a tank of water. Various masses can be put on the barge, which is towed by a gentle finger. Although fluid friction plays some part, pupils can feel and see the effect of mass in opposing change of motion. Only with a very able group should we suggest making measurements. D26 OPT.

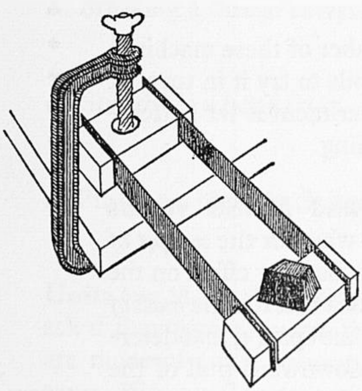
'Wig-wag' or Inertia Balance. Pupils should play with a 'wig-wag' machine, preferably as a simple class experiment. This consists of a platform carried by two springy steel blades in such a way that it can oscillate to and fro horizontally without much damping. C27a

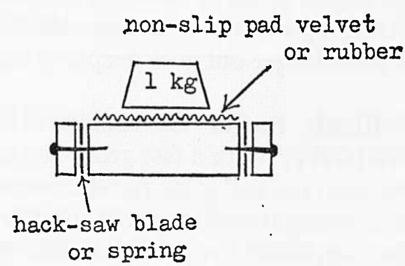
When a mass is placed on the platform as a load, the time of oscillation is increased; the forces provided by the bending blades shove the platform with its load to and fro, and the larger the load the smaller the accelerations the forces produce and the longer the platform and load take for a complete oscillation. We let pupils try

‡ Teachers who find that steel balls get carried away too freely as souvenirs might like to try the following experiment: appeal to pupils not to take these as souvenirs but at the same time give every pupil one or two balls to take home and keep – a shilling's worth of insurance.

various loads on the platform and watch the motion. We want them to gain a clearer feeling for mass by watching the machine, rather than try to use it as a scheme for measuring mass. We may say it is a shoving device and the slowness of the motion to and fro tells us something about the 'unshovability' of the load (including the platform itself).

For our own interest, rather than for our teaching of pupils in this Year, we might ourselves try guessing at the relationship between load and period by some general thinking. The device has mass and springiness, so it is equivalent to a load attached to a spring which obeys Hooke's Law. Therefore we predict S.H.M. and we expect to find the period proportional to the square root of the load. Experiments with the device show that this is the correct relation, provided we ascribe to the platform itself a certain equivalent mass. Plotting T^2 against the mass placed on the platform gives a straight-line graph. Gravity plays no part whatever in the operation of this machine.





Homemade Wig-wag Best. A wig-wag machine may be bought, but a homemade one is even better provided its springy blades are clamped properly. In fact this is a case where apparatus that is bought may give an unfortunate impression of being specially made and polished up to carry out one teaching trick. It is not an instrument like a voltmeter, which is put to many uses – in which case we should buy one that is made with full professional skill.

Making a Wig-wag. For a homemade wig-wag, we use two hack-saw blades or two lengths of clock spring (of that size or larger) and place a block of wood between them at one end, to serve as anchorage to be fixed to the table. We place another block of wood between them at the other end, to act as the platform to carry the loads. It would seem simplest to attach the blades to the blocks of wood by driving screws through holes in the blades into the blocks. But that would leave some play between the blades and the blocks of wood as the blades bend to and fro, and that would lead to considerable damping. To avoid spoiling the machine's behaviour by damping, it is essential to clamp each blade very firmly on both sides. Where the blade is clamped against a wooden block, another small block of wood or metal should be placed outside the blade, flush with the main block, so that the blade emerges as if from the well-matched jaws of a vise. Then screws may be driven through small block and blade into the big block. Or, if the machine is to be put together only temporarily, a large G-clamp may be used to clamp small blocks, both blades, and the large block together in a multiple sandwich, with a similar clamp at the other end.

We hope that teachers will either have a number of these machines for pupils to use or establish one and ask pupils to try it in turn. It makes a good demonstration, but a class experiment is far better – it takes longer but goes deeper in understanding.

Difficult Buffer Extension: Wig-wag and MASS versus WEIGHT? T
With a fast group we might ask whether the *weight of the load*, the *pull of the Earth* downward on it, has any effect on the time of motion to and fro. We encourage the idea that it is the *mass of the load*, which has to be shoved to and fro by the spring, that determines the time of the motion, and not the downward pull of the Earth. Pupils may make a test of that. The experimenter places a C27b
considerable load on the platform of the wig-wag and pulls it upward with a long thread, maintaining enough pull to remove most of the weight of that load from the wig-wag without removing any of its mass! (He must move his hand with the top of the thread in tune with the wig-wag's motion, to keep the thread vertical.) He times the motion. Then we ask what a wig-wag would do in outer space.

This technique of 'relieving the wig-wag of weight' takes practice, but a pupil, once he understands the purpose of the experiment, can develop considerable skill. Then he will want to take the whole device home and explain the difference between mass and weight to his family – nothing could be better.

Inertia Tricks.‡ (*Optional*). In discussing inertia, we may want to show the demonstration with card and coin, or remind pupils, for fun, of the quick pulling of a table cloth from under crockery. But teachers are advised to look at the note concerning these demonstrations in Year III. The following experiments, described there, are easily done without special apparatus, but we do not recommend them strongly here either: * * * * * * *

a. Coin on card on tumbler D 28a OPT.

b. Breaking a thread above or below a suspended weight D 28b OPT.

c. Breaking a thread carrying a very small weight by jerking it D 28c OPT.

d. Snatching a book from a pile of books D 28d OPT.

e. Pushing a block back into a pile of blocks (inverse of *d*). D 28e OPT.

Units for Mass. We hold two brass weights each labelled 1 kg and ask if they have the same mass. Pupils will probably say that they are made of equal volumes of the same material so they are the same. We explain that any serious enquiry must be referred back ultimately to the world's standard kilogram kept at Sèvres in France. Since we cannot borrow that, we have to use a substandard which, by many stages, has been tested against the standard. D 30a

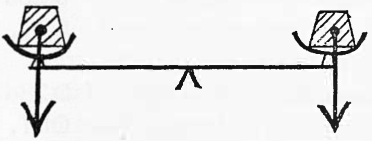
‡ See also Inertia Demonstrations with playground trolley in *Science in Secondary Schools*, H.M.S.O. for the Ministry of Education, Pamphlet 38. D 29 OPT.

We produce a kilogram labelled 'standard kilogram mass'. We ask how we can compare any other 'kilogram' with our standard. Since we are talking about units for mass as a measure of inertia or unaccelerability, the only legitimate experiments that we could use to compare two masses are ones in which we try accelerating them: either

1. with a trolley loaded with each kilogram in turn or D30b

2. with a wig-wag carrying each kilogram on its platform. D30c

Although we might try one of those experiments as a demonstration, we know that it will be difficult to make an accurate comparison in this way. So at this point we offer a much easier method, in spite of the danger of confusion. We place the standard kilogram and our other 'kilogram' on the two sides of an equal arm balance. If they balance, we say that the Earth pulls equally on the two lumps; therefore they are the same amounts of material. D30d



EQUAL PULLS $\therefore$ EQUAL AMOUNTS OF STUFF TO BE PULLED

Note to Teachers: Inertial Mass and Gravitational Mass

In making this claim, we have made an enormous jump, from *
inertial mass to *gravitational mass* – from the amount of stuff to be *
accelerated to the amount of stuff to be pulled on by a gravitational *
field. However, we are in good company: Newton did that too. *
*

It was only in the last hundred years that physicists realized the *
extraordinary significance of the common property of falling bodies *
– the result of the experiment that Galileo didn't do‡ – that a large *

‡ Most historians of science say that Galileo did not give a public demonstration of dropping a large object and a small one from the top of the Leaning Tower of Pisa. They point out that, if he had done that, there would have been letters carrying the news all across Europe: and no such letters have been found. Of course he knew quite well what such a demonstration would show; and he quoted, in one of his dialogues, the small difference of fall that would be observed.

mass and a small mass fall with the same acceleration g. The Earth's gravitational field pulls with a much bigger force on the larger mass but the material to be moved is bigger in the same proportion, so the acceleration is the same for both. The pull of the Earth's field is proportional to the amount of matter-to-be-pulled. The mass to be accelerated is proportional to the amount of matter-to-be-accelerated. We have no guarantee – except from that vital experiment which shows g the same for all – that these two kinds of 'amount of matter' are equal, or proportional, for all different materials. We have no guarantee that a large chunk of aluminium and a small chunk of lead which are shown by trolley experiments to have the same inertia will also be pulled by the Earth with equal forces – until we let them fall.

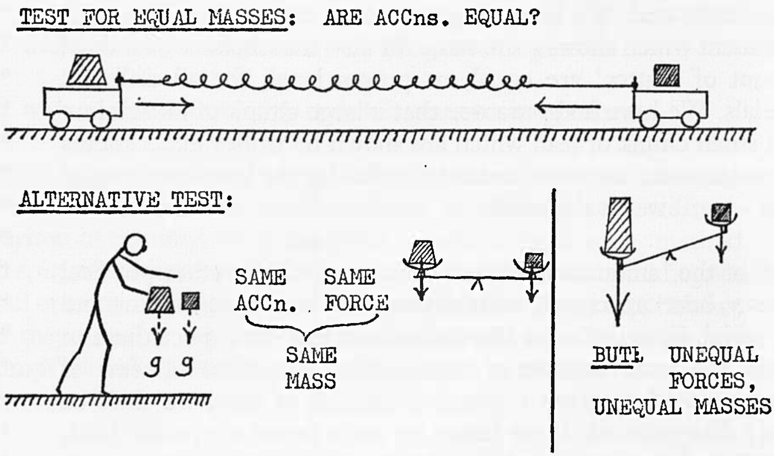
Think of the 'amount of matter' concerned with another physical property, heat capacity. If we take these two lumps, aluminium and lead, which have the same inertia and say that we expect them to contain the same amount of matter from the point of view of warming-up when given a standard amount of heat, we shall be gravely disappointed. If we insert no extra factor of specific heat, we shall find that the aluminium block seems to have six times as much matter as the lead block. However, it does happen that in the construction of our universe the mass which responds to gravitational fields and the mass which is involved in Newton's Second Law of Motion are proportional: and in fact we can use a kilogram as a unit for each.

We shall not mention this distinction between inertial mass and gravitational mass to our pupils at all – with the possible exception of a brilliant pupil at a later stage in this course – but we should keep this discussion in the back of our own mind in teaching because it will help us to avoid certain logical troubles.

Equal Masses of Different Materials. We then use a balance to provide two equal masses of, say, 1 kilogram each, of different materials, such as lead and aluminium. (These are, to our private knowledge, equal gravitational masses, two chunks of matter which will be pulled with equal forces by the Earth's gravitational field.) D31a

We now want to make sure by experiment that we have two equal amounts of inertia (to our private thoughts, equal inertial masses). We put each of these in turn on the wig-wag and show that the machine's time of shoving to and fro is the same when loaded with each of them. We conclude that they are equally 'unshovable', that they have equal (inertial) masses. D31b

Or we might put each of these in turn on a trolley and pull it with a standard force and make some measurement, such as the time of travel for a measured distance from rest, therefore indicating acceleration. We find the masses are equal in that sense also. D31c OPT.



Or we might do this experiment by using two equal trolleys, each bearing one of these lumps of metal and pulling those two trolleys towards each other by attaching a stretched rubber thread between them. Although that demonstration ending with an exciting collision is basically a good test, it probably distracts attention and had better wait until we discuss momentum changes in collisions. D31d OPT.

Instead of a test with trolleys, we can perform a test that is much quicker and neater but very puzzling to some pupils: we simply drop the two kilograms side by side and see that they both fall with the same acceleration. Then, since the balance already told us that the Earth pulls those two lumps of metal with equal forces, we now see equal forces producing equal accelerations (each of them g) and therefore we conclude the masses are equal. D31e

In a way, this is equivalent to pulling two trolleys side by side with equal forces. If we then notice that the trolleys have equal accelerations we say that they have equal masses. The peculiarities here are that we use no carrying trolleys but observe the single kilograms themselves and that we assure ourselves by a separate experiment that the forces are equal. That separate experiment is the weighing in which we show that the Earth pulls the two masses with equal forces. T

If pupils think that this is a deceptive experiment which tells us something that is necessarily true instead of testing anything, we should repeat it, using a 1-kilogram lump and a 2-kilogram lump. True, these two lumps will fall with the same acceleration when we release them in the first part of the experiment; but, when we compare the pulls of the Earth on them with a balance, the 2-kilogram lump obviously outweighs the 1-kilogram lump and therefore we cannot say their masses are equal because in their case unequal forces produce the same acceleration.

'Weight $= mg$'

Notice that when we hold an object and feel the pull of the Earth's T
gravitational field on it, that object is not falling freely. It is not accelerating downward with an acceleration g. And therefore we talk nonsense when we find the pull of the Earth on it by multiplying the mass by an acceleration g. The g that we use for this is the Earth's field strength which we shall soon discuss again.

FORCE

We have adopted a standard kilogram for mass and can manufac- T
ture copies of it in any material we like either by using a balance for equating Earth pulls or by using the clumsier but philosophically safer method of comparing inertias by means of a trolley or a wig-wag.

We know from experiments that the force needed to accelerate some body – such as a trolley – varies directly as the acceleration we want and varies directly as the mass of the body. We can write the conclusion of our experiments $F = K\,ma$. This combining of two pieces of behaviour: (1) *force proportional to acceleration*, and (2) *force proportional to mass* for the same acceleration, is a difficult algebraic matter for many pupils. It seems obvious to us but it is not obvious to young people, even after considerable explanation. We should write down the final story straight away, $F = K\,ma$, and then show that when we keep to a constant mass Km is a constant so we have F proportional to a. And for constant acceleration, Ka is constant, so we have F proportional to m. (Some pupils will find it much easier to understand the rather more wordy momentum story which follows in a later section here.)

Force Units. Then we tell the usual story about units. We start by explaining that we are going to invent new units for force, to save trouble and later confusion. We have a kilogram for unit mass, one metre/second per second for unit acceleration; and for our own convenience we choose the size of the force unit so that K will be 1. This is rather like the attempt that was made to choose the size of one gram so that it would make the density of water 1 in C.G.S. units. That may be a clever device for ease in calculations, but it is not always a clever device for teaching, because the constant, having the value 1, becomes concealed.

(We who are physicists are quite used to having K equal to 1 in $F = ma$ and having density of water 1; yet most of us are slightly shocked when we find relativity experts taking the speed of light equal to 1 in order to 'simplify things'. The effect of that latter change is to make m and mc^2 indistinguishable, and for a light-quantum the momentum, mc, also has the same value. Those of us who have a qualitative feeling for mass, momentum, and energy as essentially different concepts are offended by this high-handed treatment – although the experts may persuade us to swallow some of our annoyance.)

Pupils may think it strange to decide on a new unit of force now, when we have taken force as the obvious well-understood thing all along. Force has been, in our treatment, a push or a pull measured by counting the number of stretched springs or rubber strings in parallel. But mass is still emerging as a strange concept. It might have seemed more suitable if we had chosen to name a new unit of mass and taken some familiar unit of force. However, there are two objections to that: (1) the decisions were made long ago and are now too widely accepted for a change to be feasible; and (2) we do not have a *reliable* old-fashioned unit of force. The pound-weight is not reliable, because its actual size *as a force* varies over the surface of the Earth and would vary still more if we took it on a trip farther out, or down inside the Earth.

Note to Teachers, on 'Engineering Units', etc. Until recently, engineers were only concerned with building things on the surface of the Earth, and the small changes of a pound-weight from place to place did not worry them; so they were content to use that unit. Nowadays, we all wish to avoid such a variable unit.

From time to time, however, there is talk of a 'universal pound-force' such as a 'standard-pound-weight-at-London' to be transferred by use of some spring balance to all other places in the world. That is a perfectly feasible constant unit favoured by some scientists, but likely to look rather foolish when we go to the Moon. Pupils will certainly use the pound-weight as a force unit in their early days of physics, because that seems natural from experience in ordinary life; but we do not think it advisable to try to crystallize it into a standard pound-force. Pupils will have to meet absolute units such as newtons in physics and in other sciences – and, much more important, they will meet joules, watts and volts which derive from the newton. So we shall resist any temptation to use a universal 'pound-force'.

We shall also resist any temptation to manufacture a special unit for mass, such as the slug. There is nothing wrong with these alternative unit schemes; but there is nothing world shaking about them either – a change of units will not alter the facts of nature. We hope that teachers busy making their first trial of the Nuffield Physics programme will be able to avoid spending energy and long arguments with those enthusiasts who believe that a change of units will make a profound change in physics. Here we simply offer a decision and then hope to get on with the real physics.

Making Absolute Units of Force. With most pupils, we should T
not labour the business of making $K = 1$. We should simply say we are going to use force units which are called newtons and which are the same size of force everywhere; and with newtons the relation is $F = ma$. If pupils feel the need for some justification of that we may say:

> 'Instead of having force $= K\,ma$ we want to have $F = ma$, with K disappearing because it has the value of 1. We can get what we want by taking what we want and then paying for it. We write $F = ma$ and then find what a force 1 must mean.

'Take 1 kilogram for the mass, m. Take 1 metre/second per second for acceleration, a, and ask what force produces that.

'Then Force = [1 kilogram] multiplied by [1 metre/second per second].

'Then F is 1.
That means: a force 1 is the force that will give 1 kilogram of matter an acceleration of 1 metre/second per second. We could call that unit a kilogram.metre/second per second and that would be a good descriptive name for the unit; but it seems to us too long, and so we have chosen a single word to mean that. We use a word that honours the name of Sir Isaac Newton, whose laws we are dealing with, and call it 1 newton.

'Then 1 newton is the force which will give one kilogram an acceleration of 1 metre/second per second.

'1 newton *is* just another name for 1 kilogram.metre/second per second.'

New Force Units: a newton

'Now I want you to try an important experiment; to feel what a force of 10 newtons is like. Hold a kilogram in your hand. Can you feel the force of the Earth pulling down on it? ... How big a force is that? ... Yes, it is a force of one kilogram-weight; but we are not going to use those units any more. We must not measure forces in kilograms-weight if we are going to use $F = ma$. We must express forces in newtons (otherwise K won't be 1). Can you tell me how many newtons there are in the pull that you can feel now? Even without a newton balance you can tell. We do an experiment and calculate the force by $F = ma$. C32a

'Suppose you let the pull of the Earth act on the kilogram with nothing else there. Start by holding the kilogram yourself. Let it fall. ... It fell too fast for you to measure its motion; but you have made that measurement before. How does that kilogram fall? ... Yes, it fell with an accelerated motion; with an acceleration which is the same for all falling things, 9·8 metres/second per second.'

'Now we can use $F = ma$ and calculate the Earth's pull that made it do that. The mass m is 1 kilogram, the acceleration a is 9·8 metres/second per second. So the force must be ma, that is, $1 \times 9{\cdot}8$ kilogram.metres/second per second. And we call these "newtons"; so the pull that you feel is 9·8 newtons. Hold the kilogram and feel a pull of almost 10 newtons. Hang the kilogram on your newton balance and see if the balance was correctly marked.'

'Feeling' a newton: Forces Box. Then we provide a 'forces box' for pupils to try pulling with a force of 1 newton, also with a force of 1 kilogram-weight. This box, which was provided for an earlier Year, has a hole in the front from which a cord emerges. There is a label saying 'pull the cord and feel a force of 1 newton'. The cord runs into the box and over a pulley and carries a load on its other end to make the tension 1 newton. The cord is limited by stops, 1 metre apart, which enable the pupil to 'transfer 1 joule of energy' by pulling the cord out as far as he can – the transfer being from his food energy to gravitational potential energy. There are similar cords for a force of 1 kilogram-weight and 1 pound-weight, with provision for an energy-transfer of 1 kilogram.metre; and 1 foot.pound. This box should be left available for some weeks; because it provides a very useful sense of the sizes of these units of force and energy-transfer. C32b

'Feeling' a kilogram: Mass Exhibit. At the same time we should bring out the 'mass exhibit' again: 1 kilogram on a disc of plywood which slides freely on a bed of small steel balls on a glass table. This should remain available for pupils to try for some weeks. It helps to provide a feeling for mass. C–D32c (=C–D25)

Test of Balance Marked in newtons. If time permits we show that the newton balance was correctly calibrated by a more direct test that does not involve gravity. We use it to pull a known mass in kilograms along a level table; we measure the acceleration and we calculate by $F = ma$ the actual pulling force in newtons. The result is compared with the balance-reading to see if the balance is correct. D33a

This is a difficult experiment to do with any precision; and it is more likely to be important to us for our own sense of justification than to most pupils, so it is probably best to do this quickly as a demonstration.

We build up the mass on a trolley to several kilograms (by adding kilogram loads and some fractions). We pull the trolley along a friction-compensated plank keeping the spring balance pointer at some agreed mark. We time the motion from rest for a measured distance, calculate the acceleration, and thence the [mass] $\times$ [acceleration] and compare that with the spring balance reading that we used.

The test is much more dramatic if we place a piece of paper on the spring balance to conceal its scale and make an ink mark at a suitable place on the paper and keep the pointer at that mark during the experiment. Then, when we have calculated the actual force in newtons, by $F = ma$ (necessarily right, by definition), we compare that with the balance reading under the mark on the paper.

Rough Giant Test (*Optional*). A giant version of this experiment can be done if we have a spring that will read a few dozen newtons. We use a playground trolley (or table on roller-skates‡) and load it up with pupils; or we use a pupil on roller-skates. We then pull this big trolley with a constant pull, keeping the balance pointer at some chosen mark, make measurements and calculate ma and compare that with the reading of the balance. This is great fun but it is troublesome: we have to find the total mass of [trolley+pupils] in kilograms;‡ we have to allow somehow for friction. The most satisfactory way of allowing for friction is to arrange for a separate pull to pay for friction, as described in Year III. Unless that is done, we must start with an experiment in which we maintain constant speed. Then, in the later experiments the force necessary for that must be subtracted from the pulling force. Although the friction force is great the experiment then goes well. D33b OPT.

If teachers feel that pupils need something to do themselves at this point, they might use a spring balance marked in newtons to pull a trolley of unknown mass. They should use tape to measure the C33c

‡ See the sketch for D 53, in this *Guide*, and description in Year III.
‡ It is not easy to weigh a large trolley or table. It should be weighed once and for all, and its mass written on it like the tare of a wagon. To weigh pupils, it might be good to return to the simple experiment in Year I and use a large beam of wood balanced on a fulcrum at its centre. The pupil sits one or two feet away from the fulcrum and we balance him with a pile of kilograms far out on the other side. This serves to revise the idea of moments, which we treated informally in Year I without any definite rule.

acceleration and then calculate the mass, using $F = ma$ and compare it with the total of the trolley and its loads which are then revealed. In practice this proves to be an experiment which pupils enjoy.

The newton as a Unit of Force

A Strange Unit for a Familiar Concept. Since force is the familiar concept, it may seem strange to invent a new, artificial, unfamiliar unit of force at this late stage. In doing so, we are not casting doubt on force. We shall, in this course, continue to regard forces as pushes and pulls, to be measured by counting the number of standard stretched springs or rubber threads in parallel. Mass is the new concept and still a strange one; but, for mass, we already have a good old-fashioned unit, the pound, and now the kilogram. The reversal of choice of units – a strange unit for the old familiar concept and old unit for the new concept – need not worry us. To pupils, the units that we choose for measurement are just the things in which the scales are marked. They find an ammeter graduated in amps and learn to use them for current measurements – and they even develop a sense of size – one amp is a small current and 100 amps a very big one. It is only much later that they learn about an absolute definition of an amp in terms of two wires 1 metre apart.

Uses of $F = ma$

As the warning in the Introduction suggested, we shall not make a great many uses of Newton's Laws of Motion at once. And we should certainly not put pupils on a diet of artificial problems such as Atwood's machine calculations.

However, we should give pupils some examples of calculating accelerations from forces and forces from accelerations. Problems relating to car driving, swimming races, and rockets, are probably fairly real to pupils. Our examinations should offer a number of such problems. Pupils should be given such problems in homework – a few at a time – to build confidence without delaying the progress of teaching.

Although problems on the motion of atoms and electrons involve very large or very small numbers – and compel us to provide data without explaining their origin as yet – we shall do a lot of good if we prepare for future work with atoms by using them as the moving things in problems. If pupils complain over the difficulty of handling the numbers in these problems, we should return to the special Problem C in Year I. * * *

Problems‡

Specimen Problem 1. We can ask simple force and acceleration problems such as: A boy wants to pull a 5-kilogram cart‡ loaded with 95 kilograms of bricks. He can pull with a force of 200 newtons. (*a*) Neglecting friction, estimate his acceleration. (*b*) How far can he pull the cart, starting from rest, in 2 seconds? (*c*) Trusting your 'formula' calculate how far he could pull the cart, from rest in 10 seconds, and find how fast it would then be moving. Then say whether those answers are impossible. (*d*) To see whether the boy in this problem has reasonable strength, suppose he pulls with the same force, 200 newtons, horizontally as before, but on a rope that runs to a pulley, up to another pulley and down to a basket of rocks. Calculate how heavy a basket he can raise. P

Specimen Problem 2. The same boy pulls the same loaded cart but this time assume that friction drags the cart back with a constant force of 50 newtons. Repeat the calculations. P

Specimen Problem 3. A 20,000-kilogram goods wagon is at rest on a slightly inclined railway which runs from east to west. The railway slopes downhill just enough to compensate for friction, for a wagon moving westward. A child pushes the wagon steadily westward with a small force of 1 kilogram-weight. Having nothing else to do, the child continues to push for 5 minutes (300 seconds). What speed will the wagon acquire in those 5 minutes? How far will the child walk in the 5 minutes? (We have to suggest that the child gives a small extra push to deal with static friction at the very beginning.) P

‡ When we supply the data for a problem, it is difficult to specify masses without involving the confusing word 'weigh', which will worry beginners, or at the other extreme giving the show away by saying at greater length, 'The mass is ...' We can avoid those difficulties and leave pupils to make their own choice if we word the data like this:
'A 5-kilogram cart. ...' 'The 20-kilogram trolley is pulled by a 4-kilogram load hung on a string. ...' That device is used in our suggested problems here.

This question is useful as a reminder that F in $F = ma$ must be in absolute units, because the data are obviously ones that will lead to slow motion and progress of a few dozen metres at most. A mistake over units will make a very clear difference.

Specimen Problem 4. The brakes of a car in fairly good order can exert a retarding force of $\frac{1}{4}$ of the *weight* of the car, that is one-quarter of the pull of the earth on the car. How long would such brakes take to stop a 1,600-kilogram car moving 12 metres/second (about 35 miles/hour)? To find out, answer the following questions: P

What is the pull of the earth on 1,600 kilograms, in proper units for use in $F = ma$?

What is a quarter of that pull of the earth, in proper units?

What (negative) acceleration would that braking force give to the 1,600-kilogram car?

With that (negative) acceleration, how long would the car take to slow down from 12 metres/second to rest?

Specimen Problem 5. A 1,500-kilogram car moving 12 metres/second (about 35 miles/hour) crashes into a wall and comes to rest. The whole collision takes 0·10 second. Calculate the collision force involved as follows: P

Write down the final velocity after the crash, the initial velocity just before the crash, the change of velocity: and, using the time taken for that change, calculate the acceleration. Use that acceleration in $F = ma$, with the mass 1,500 kilograms. The force will emerge in newtons; and to gain a feeling for that we should ask how large a lump of metal is pulled by the Earth with that force. To calculate that remember that the Earth's gravitational field strength is 9·8 newtons per kilogram. (Answer about 18 tons-weight.)

Specimen Problem 6.‡ 'A 60-kilogram boy jumps off a window ledge 1·25 metres above the floor to a hard floor. Estimate the force exerted on him by the floor while he is stopping, by answering the questions below. Suppose that he foolishly forgets to bend his P

‡ This is a specially useful problem: it applies to ordinary life. We illustrate it by a demonstration and we go through it again to show the momentum-change method.

knees while landing so that the total 'give' of his feet, etc., is only 0·025 metre (1 inch), in compression of floor, shoes, feet, ankles, spine, etc., during the stopping process. (The height 1·25 metres is chosen to make the calculation easy; because free fall through that height, starting from rest, takes $\frac{1}{2}$ second.)

a. Calculate his time of fall ($\frac{1}{2}$ second).

b. Calculate the speed of the boy at the end of his $\frac{1}{2}$ sec fall, just before landing (5 metres/sec).

c. To calculate the time taken by the landing process we must find the boy's average speed during the landing process. Write down his speed just before he lands and his speed when he has finished landing. Take the average. Use that average speed to find the time he takes for the process of landing; *that is, the time he takes to travel 0·025 metre.*

d. You know his speed before landing (5 metres/sec) and his speed (0) after landing, so you know his change of speed; and you now also know how long he took to make that change of speed. Calculate his acceleration (negative) during landing.

e. Using $F = ma$ calculate the force the floor exerted on him during landing. (Answer about 30,000 newtons, or 3 tons-weight, reckoning 1,000 kilograms to a ton.)

This provides a good chance to discuss the sensible use of round numbers: the time-of-stopping is 0·025/2·45. To work that out carefully would be to show one had not understood that this is a rough calculation to make an estimate: the best thing to do is to call it 0·025/2·5 in which case the time is $\frac{1}{100}$th of a second. This problem can lead to a discussion of bending one's knees on landing! This can be illustrated by dropping a ball of plasticine on to a kitchen scale. D34a

The problems above are only suggested as possible guides for teachers making up or choosing problems to illustrate this work with Newton's Laws. We hope that teachers will use some such problems; but we hope they will not spend much time making sure that every pupil can solve Newton's-Law problems, because at this stage those problems are likely to seem artificial, far away from the real life of cars and satellites. * * * * * * *

Weight and Earth's Gravitational Field Strength

In some problems we shall have to deal with the weight of a pulling T
load. We also have to face the fact that spring balances, etc., are usually marked in kilograms or pounds rather than in absolute force units, newtons or poundals. We may want to ask pupils to change some force which they have calculated from 'good' units such as newtons to 'bad' units such as kilograms-weight, so that they have a feeling for the size of the force. Again and again, as in earlier Years, we must insist that the weight of an object is the pull-of-the-Earth on the object.

> **'Weight is a force that can stretch a spring or make a trolley accelerate, just like any other force. The only peculiarities of weight are that it is vertical and unavoidable.'**

We should continue to say, 'The pull of the Earth on the object' all through our discussion. We should not reduce that to the name 'weight' until pupils are quite sure of it.

We regard weight as due to the effect of the Earth's gravitational D36
field which spreads out all around the Earth waiting to pull upon matter. We do one important experiment again and again to measure the strength of the Earth's field. We allow one kilogram to fall; measure (in our imagination) the acceleration, and calculate the force [1 kg] × [9·8 metres/second per second], then: *The Earth pulls with a force of 9·8 newtons on 1 kilogram.*

Or, we can do the experiment with 5 kilograms: the acceleration of T
5 kg is again 9·8 metres/second per second; so, while it is falling, the force on it is: [5 kg] × [9·8 metres/second per second], or 49 newtons. The pull on each kilogram is 49 newtons/5 kilograms, or 9·8 *newtons/kilogram.*

The field strength is 9·8 newtons/kilogram whatever type of matter we choose as our object for experiment.

Use of Field Strength. This idea of a field strength will be use- T
ful in other parts of physics. We shall find that both the number, *9·8*, and the unit 'newtons/kilogram' are very useful. Whenever we wish to know how much the Earth pulls on some piece of matter,

we ask how many kilograms of stuff we have (the mass). Then we remember that the Earth applies its gravitational field to that, at the rate of 9·8 newtons pull on each kilogram, and we can calculate the weight. In all this, we should continue to call the weight 'the pull-of-the-Earth on the object'.

(If, when we want the weight of some object, instead of using a field strength of 9·8 newtons/kg, we multiply the mass of the object by an *acceleration* 9·8 metres/second per second, we make physics look rather foolish to pupils. They can see that in most cases the object is not falling with that acceleration; so the operation seems to be a nonsense rigmarole – whereas multiplying by field strength seems reasonable, if we have described field strength well. With this use of field strength, we do not meet that difficulty experienced by some beginners of not knowing whether to multiply or divide by g when working out a problem.)

If we give this idea of field strength considerable importance, we shall find it helpful when we come to electric fields later on. Remember that the concept of a field plays a very important part in modern physics. We should give field strength all the emphasis we can, both for immediate help with force problems and for later understanding of physics.

Problems with Things Pulled by Weights

If we want to give pupils practice with problems in which *the force is provided by gravity*, we should start with one in which the mass of the pulling agent may be neglected. For example: T

Specimen Problem 7. A 50-kilogram boy on roller-skates stands on a long, smooth table. He is pulled forward by a cord round his waist which runs, horizontally, over a pulley to a ½-kilogram load. Calculate his acceleration and how far he will travel from rest in 2 seconds. In this case the force is 4·9 newtons (say 5) and the moving mass is 50 kilograms, so the acceleration is 0·1 metre/second^2. (We have made two quite different errors, one of 2 per cent, one of 1 per cent, in the same direction.) P

Specimen Problem 8. Again imagine a 50-kilogram boy on roller-skates, as above, but instead of the ½-kilogram pulling load have another 50-kilogram boy hung on the end of the rope. That will do some good teaching of mass. P

Outcome. In all this, we want pupils above everything else to feel that they are exploring the way in which a force changes motion, makes things go faster and faster, and finds it difficult to do that if there is a great deal of matter to be speeded up. This is more important than ability to calculate the force in newtons that will do a particular job, or to calculate the acceleration produced by a given load hung on a string and pulling a given trolley along a level table. Pupils should emerge with a clear feeling that they have explored and learned a good deal about force and mass and motion. *

Note to Teachers. Philosophy of Newton's Laws. There are some definitions and assumptions interwoven with all experimental illustrations of Newton's Laws of Motion: for example, the description of force, and the definition of our system of force measurement, which we encourage pupils to take for granted; and rules that forces acting side by side are additive, and that masses piled up together are additive. (See Note on Interaction in the General Introduction at the beginning of Year III.) These are not philosophical weaknesses in Newton's Laws: they are simply proper parts of the structure of his description of nature. We should not worry pupils or even ourselves about these philosophical matters; and yet we should keep in the back of our mind just enough hint of doubt to prevent us telling pupils that they have proved that Newton's Laws are wholly right. In fact, the Laws *are* right (subject to some relativistic modification) but they are partly right by definition and partly right because they do describe nature – and the pupils' experiments have illustrated the latter connection. *

Algebra with Constant Acceleration. Perhaps the clearest way T
to sum up and reinforce the knowledge that pupils have gained with these experiments is to express it in terms of momentum – Newton's own choice. In many cases a problem is easier to think about, and may be easier to work out, by saying that [force] × [time] is change of momentum. And then when we combine that with Newton's Law III we arrive at Conservation of Momentum as a very important general rule for dealing with mechanical systems.

The traditional way of changing to this form is to write:

$F = ma = m(v-u)/t$

therefore $Ft = mv-mu$, the change of momentum.

Descriptive Method. For some pupils a slower, less algebraic T
introduction does better, with momentum appearing as the thing which must be provided by a force acting for a certain time. Here is a suggestion for a new start along those lines:

'You have seen that a force makes things go faster. I do not mean just one force pulling one way while other forces are pulling back. The other ones may be enough to hold the object at rest. I mean that if, after adding up all the forces forward and backward, the total is a resultant (net) force in one direction then you get increasing motion in that direction.

'I shall call the *net force*, or the *resultant* of all the pulls and pushes on a moving object, "*the* force". When the force is there the object it acts on moves faster and faster, gaining velocity. The bigger the force, the more velocity the object gains; and the longer the time the force acts for, the more velocity the object gains. So we might say the total gain in velocity goes up in proportion to force multiplied by time.

'But if we have several trolleys all piled on top of each other – a lot of stuff being accelerated – we do not have such a big gain in velocity. So [force] × [time] does not tell us just the gain in velocity. If we have twice the mass we only get half the gain in velocity. So, when we look at [force] × [time] and ask what we get from it we have to consider not only how much velocity we gain but also how much mass is there, gaining velocity.

'Suppose we push one kilogram for a certain time and it gains 10 metres/sec in velocity. We could push two kilograms with the same force for just as long a time but it would gain only 5 metres/sec. Or 5 kilograms (pushed for the same time with the same force) would gain in velocity only two metres/sec.

'For the same force acting for the same time we get in every case the same gain of [velocity] × [mass]. We call that momentum. We say that gain of momentum, gain of mv, is proportional to [force] × [time]. Your experiments have not proved that that is true. They have illustrated it: they fit with it.

'Then we can turn that round and say

[force] × [time] = change of momentum.

Ft = change of (mv).

'We have chosen our units for force, newtons, for use in $F = ma$. We chose to make K equal to 1 in $F = K\,ma$ by choosing our units of F. The same choice applies here. We choose the newton so that one newton acting for one second will produce a change of momentum of 1 kilogram increasing its speed by one metre/second.'

This will seem to some pupils a long roundabout form of $F = ma$. But, to others, it may seem a more natural approach. And, in any case, we shall be glad to have this form for use in our later studies.

Momentum is a difficult, strange concept for beginners, compared with acceleration which seems more obvious to many. That is why we treated the example of the jumping boy first by $F = ma$. Now we should try it again, using change of momentum. But, for momentum to become really important, we must wait for collisions, in which the various momentum-changes will always add up to 0. Then conservation of momentum will become so important, as a universal rule in physics, that we are justified in starting now to build up a feeling for momentum.

As soon as we introduce momentum, we should warn pupils that there is another very useful quantity, kinetic energy. They must learn to distinguish clearly between mv which is a measure of [force] multiplied by [time] and $\frac{1}{2}mv^2$ which is distinctly different and which is a measure of [force] multiplied by [distance]. * *

Pupils should look on momentum as a good way of specifying 'motion'. It tells us how much material is rushing along, combined with how fast it is rushing along. If we wish to stop a body moving we have to take away its momentum; we have to apply a force for a certain time and the momentum that is to be removed tells us [force] $\times$ [time]. T

Examples of Changing Momentum

We should give pupils examples to show how, when a given amount of momentum has to be lost (or gained), we may have a choice between using a *large* force for a *short* time and a *smaller* force for a *larger* time. T

Car hits Wall: Discussion of Problem. Suppose a car, driving fast, collides head-on with a wall. If its bumpers are soft or springy the car will take some time as the push of the wall brings it to rest, taking away its momentum to be shared with the earth. Even then the force will be quite large. But if the bumpers are very rigid, the impact takes much less time and the force is much larger. Example: T

> 'A 1,000-kilogram car moving at 20 metres/second (about 40 mph), hits a wall head-on and comes to a stop. Suppose the front bumpers are specially built for safety and are very squashy, so that the process of the car coming to a stop takes $\frac{1}{10}$th second. Then the momentum of the car before the crash was [1,000 kilograms] $\times$ [20 metres/sec] and its momentum afterwards zero. That change of momentum is produced by the force exerted by the wall acting for 0·1 second. Therefore
>
> $F \times 0{\cdot}1 = 1{,}000 \times 20$
>
> therefore Force = 2,000 newtons = about 20 tons-weight.'

Even that will seem an enormous force, but if we calculate the distance the car must travel while the crash is happening, taking the average speed to be 10 metres/sec we see that 1 metre of the front of the car must crumple up.

'Now suppose the bumpers are very strong and very rigid so that the crash takes only $\frac{1}{100}$th of a second. Carrying out a similar calculation, we find that the force will be 2,000,000 newtons or about 200 tons-weight.'

In that case the car would only move forward about 4 inches during the crash – no wonder the force is so huge and the crash so improbable. We should suggest when this calculation is done that the second crash would break the wall down.

Landing after a Jump. In landing on the floor after a jump, we T can decrease the force by lengthening the time-of-landing. Here is the earlier problem of the boy jumping from a window ledge, changed to a smaller fall from a table and discussed in terms of momentum:

'Stand on the table and step off it and fall to the floor. Just before you reach the floor you are falling with a speed of about 4 metres per second. If you weigh 110 lb that is about 50 kilograms. Then your momentum downwards is [50 kilograms] × [4 metres/sec] or about 200 kilograms-metres/second. When you land on the floor you lose all that momentum; it is shared out with the great world.

'A force must act on you to take that momentum away from you (and an equal counter-force acts on the Earth to give it some momentum instead). That force multiplied by your time-of-stopping must come to 200. If you were very squashy and took a whole second to stop, then [force] × 1 = 200; and the force would be 200 newtons.

'If you bend your knees only a little‡ and stop in one-tenth of a second, then the force is 2,000 newtons.

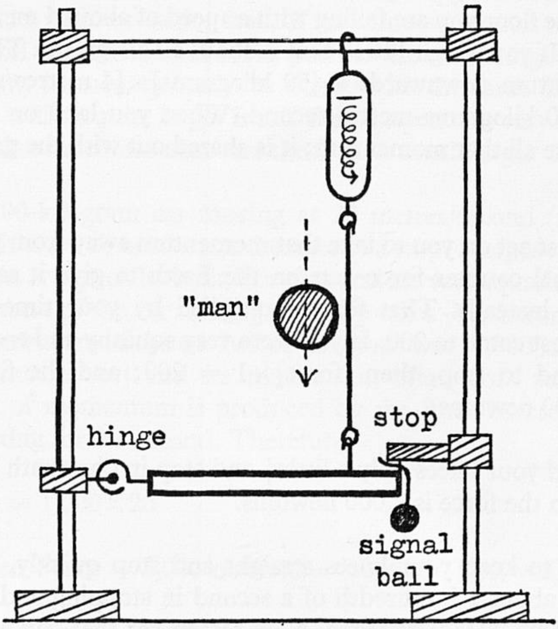
'If you try to keep your knees straight and stop quickly, you would take about 1 hundredth of a second in stopping and the force would be 20,000 newtons. Now remember that you are a 50-kilogram boy and the Earth pulls with a force of 9·8 newtons on each kilogram of you. So, the pull of the Earth on you is about 500 newtons.

‡ 4 metres/second to rest. Average velocity = 2 metres/second. In 0·1 second, stopping distance = 20 centimetres.

'If somebody wants to feel a force of 500 newtons on his hand, all he has to do is let you stand on his hand.

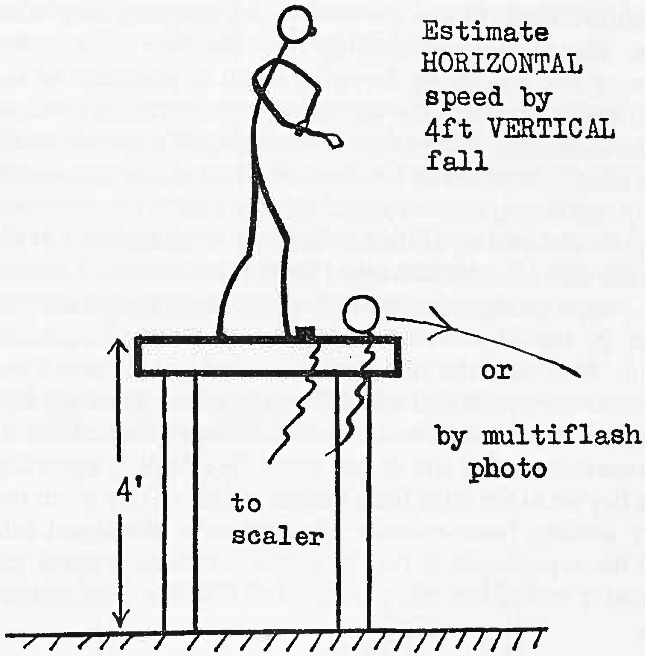
'Now think of those forces that stop you when you are landing on the floor: 200 newtons if you could take a whole second to stop, 2,000 newtons if you stopped in a tenth of a second, 20,000 newtons if you took only 1 hundredth of a second to stop – as you would if you kept your knees straight. What would those forces feel like? (50 pounds-weight; $\frac{1}{4}$ ton-weight; $2\frac{1}{2}$ tons-weight.) What would they do as they drove up through your feet, legs, knees, spine?'

This is the time for some demonstration experiments that illustrate our calculations of forces by momentum changes (or by the earlier method, using $F = ma$):



Experimental Test: Force that Stops a Jumping Boy when he Lands. We can illustrate the large force the floor must exert to stop a moving boy quickly by dropping a ball of plasticine on to a household weighing scale. Or we can give a more intricate demonstration as follows. We arrange a wooden shelf to represent the floor. The shelf is attached to a horizontal pivot at one end so that it can tilt up and down, and it is pulled up by a wire at the other end. The wire pulls that end up against a rigid stop so that as long as the tension in the wire is maintained the 'floor' is horizontal. The wire runs up to a large spring balance which shows that the upward pull in the wire is, say 40 newtons. A small bit of thread squeezed between the 'floor' and the stop carries a metal ball, so that if the floor is pushed down the ball will fall to the table. Thus the ball acts as a signal to tell us when a downward force greater than 40 newtons is exerted on that end of the 'floor'. We then try dropping a plasticine boy on to the table from various heights. Only when the momentary landing force exceeds 40 newtons is the signal ball released. This experiment is fun to watch – though it gives no clear qualitative test of our calculation – but it needs considerable explaining. D34b

Force Used to Kick a Football. We can estimate the force exerted by a player's boot on a football by using the scaler as a clock. We strap a covering of thick aluminium foil on the toe of the boot and we cover the football with foil. We arrange the scaler to record the time-of-contact between toe and ball. We shall estimate the force from $F.t$ = change of mv. D35



Before we can estimate the force during that time of contact, we must know the momentum imparted to the ball. We weigh the ball to find its mass; and we must somehow estimate its velocity after impact. The simplest way to do that is to place the ball on a table and let the boy who kicks it give it a kick that starts it out horizontally. Then by measuring how far away the ball is when it reaches the floor, some distance below, we can estimate its time of fall, which is also its time of flight horizontally. (If the ball falls 123 centimetres (4 feet) it takes just half a second. If we have a table as high as that it will simplify the calculation.) As the ball is likely to acquire a horizontal velocity of 10 to 20 metres per second, we cannot time its flight by a stopwatch. The scaler is already being used to measure the much smaller time of contact; so some rough estimate like the one suggested above should be used. Alternatively we take a multiflash picture of the ball in flight.

Bernoulli Paradoxes as Examples of Newton's Law II

We should show some 'Bernoulli paradoxes' and then explain how all such effects are really only examples of Newton's Law II. T

Although the Bernoulli effects are of interest in discussing aeroplane flight and a number of other things, they form a small part of physics which we could well leave out. So far as factual knowledge is concerned, we do not mention them here for any special interest in the principle itself, but as an example of linking strange phenomena with a simple law which is now well known to pupils – Newton's Second Law of Motion. * * * * * * *

Bernoulli Demonstrations. We can show a number of amusing demonstrations of 'Bernoulli effects':

Air from a jet supports a ping-pong ball; the jet may be tilted over to a considerable angle from the vertical and still hold the ball. D37a

Water from a jet supports a ping-pong ball. D37b

If a supply of compressed air is connected to the small end of a conical funnel, the funnel can be used to pick up a light ball. D37c

We can show a spinning ball taking a curved path by throwing a light cork ball with a cardboard tube: we swing the tube with outstretched arm, making the ball roll out to the mouth of the tube, thus giving it considerable spin. D37d OPT.

A pupil holds two thin pieces of paper a short distance apart and blows strongly into the space between them; they move together instead of apart. C38a

A pupil holds a thin sheet of paper horizontally just below his lips and blows out over it; the paper is lifted. (This shows the mechanism of 'lift' for an aeroplane.) C38b

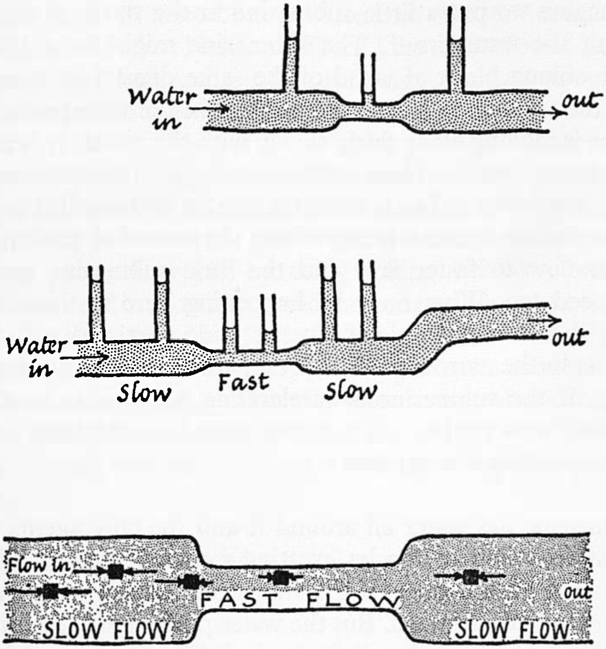
These, and other surprising effects, can all be 'explained' by appealing to the qualitative form of Bernoulli's principle: that in streamline flow of fluid the pressure is smaller where the flow is faster. Although this is an 'explanation' that links together several phenomena which look dissimilar, it is not a very good scientific explanation in this form because it drags in a new, unexpected * * * * * *

principle. If we could link this principle in turn to something *
familiar, we should feel we are much more powerful scientists. We *
should be farther from superstition and nearer to the assurance of *
Lucretius that 'Science frees men from the terror of the gods.' We *
can do that. *

First, we must show one more experiment to illustrate the principle D 39
itself: we drive a rapid flow of water through a wide glass tube $\frac{3}{4}$ inch or, better still, 1 inch in diameter, about 2 feet long. For a length of several inches in the middle, the tube is narrowed to about half the original diameter. Three vertical standpipes rise from the horizontal tube, one from the central narrow part, the others from the two wider parts. In each case the standpipe must be attached by careful glass-blowing so that the inner surface of the main tube is not distorted, but is merely pierced by a small hole that leads up into the standpipe – otherwise the flow may be disturbed. The tube is connected to the water supply by a large rubber hose at one end and water spouts out from the other (open) end into a sink.

To avoid air entering the large open end, the tube near that end is given a crook, so that it rises an inch or two higher and then runs horizontally for the last two inches at a higher level. (See figure at end.) For a large class, the water should be coloured by dye from a reservoir near the inlet, and backed by a translucent screen lit from behind.

Even with fast flow, there is little difference of pressure, as shown by water in the standpipes, between the two wide sections of tube, because all the tube is so wide. However, the pressure in the central portion is much lower. This demonstration shows the behaviour summed up in Bernoulli's principle; but it cries out for some explanation of that paradoxical behaviour. (If the demonstration does not work well, it is because the glass tubes are not wide enough – and in that case fluid friction plays too big a part – or because the flow is not fast enough.)



BERNOULLI DEMONSTRATION

Where the pipe is narrow, so that the water moves fast, pressure is lower than in the region of slower flow. With a wide pipe (1″ bore not ½″), fluid friction is comparatively unimportant, as illustrated here by the pairs of stand-pipes. Note the crook in the exit pipes to make water pile up to visible heights in the stand-pipes.

We can link that paradoxical behaviour with other knowledge as follows: T

'In this wide part the water is flowing along quite fast. What must it be doing in the narrow part here? Remember that the same amount of water has to get through the narrow part as the wide part. ... Yes, the water must be moving faster in the narrow part. Then when it gets to the next wide part it must move slower again.

'Now imagine we put a little submarine in the water to move along with the water itself. The submarine might be a little square or oblong block of wood of the same density as water. Suppose the submarine has flat ends. Here in the wide part the submarine is moving along fairly slowly with the water. It is not changing its motion. And here in the narrow part the submarine is moving much faster, but is not changing its motion. But here in the knee, where the tube is narrowing, the water has to change from slow flow to faster flow and the little submarine must change speed too. What must be happening here to the submarine if it was moving slowly in the wide part and will be moving fast in the narrow part? ... Yes, here in the knee it must accelerate. If the submarine is accelerating what must be the story about forces for it? ... Yes, there must be a resultant forward force pushing it to accelerate it.

'The submarine has water all around it and the only agents to push on it are water molecules exerting a pressure on it. The water pushing on the sides of the submarine cannot very well help it forward or backward. But the water pushing on the hind end of the submarine pushes it forward, and the water pressing on the front end of the submarine pushes it backward. When the submarine is in the knee are those two pushes equal? ... They must be unequal. The submarine must feel a bigger pressure of water on its hind end than on its front end. Then the pressure must be bigger here in the wide part of the tube than here in the narrow part of the tube.

'Now we can forget about the submarine and think of the water itself. In going from wide to narrow it changes from slow flow to fast, it *accelerates*. There must be a force to make it accelerate and that force is provided by water pressure. The water pressure must be bigger here behind than here in the narrow part.'

Thus, the Bernoulli effects are only a matter of force being needed *
to speed up the fluid flow from the slow-moving regions to the fast- *
moving regions: $F = ma$. *

(The more usual account of the effects and the derivation of the *
Bernoulli principle by energy considerations is much more artificial, *
because it makes us invent 'pressure energy' and that would not be *
helpful here.) *

Newton's Law III and Conservation of Momentum

(We took a brief look at Newton's Third Law of Motion in Year III. *
We suggest that, before discussing it further, teachers should look *
at the comments in the *Guide* for Year III.) *
*
The great French mathematician Poincaré wrote a strong, con- *
vincing commentary on Law III, in one of his general books on *
Philosophy of Science.‡ He showed that the experiments usually *
quoted to prove Law III by using spring balances, etc., can- *
not really succeed in showing that action and reaction are necessarily *
equal. He decided that Law III is really only a matter of definition, *
an 'accounting rule', chosen by us for our own convenience in *
expressing our knowledge of nature. We certainly should not men- *
tion this discussion to our pupils, but we should keep it in mind *
because it may restrain our comments somewhat when we are *
teaching. *

Action = Reaction. We ask a pupil to pull on a metre rule while T
we hold the other end:

'Which way am I pulling you? ... Yes, towards the blackboard. D40
And which way are you pulling me? Towards the clock at the back of the room. Those two pulls do not cancel out and come to no pull at all. I can feel your pull quite well, and you can feel my pull. Only one of those pulls acts on you, the pull towards the blackboard. And only one of those pulls acts on me, the pull towards the clock.

'The reason you do not accelerate towards the blackboard is because another, quite different, force is shoving you in the opposite direction. Your rough shoes on the rough floor stop you moving. The floor pushes you away from the blackboard with a friction force which happens to balance my pull.

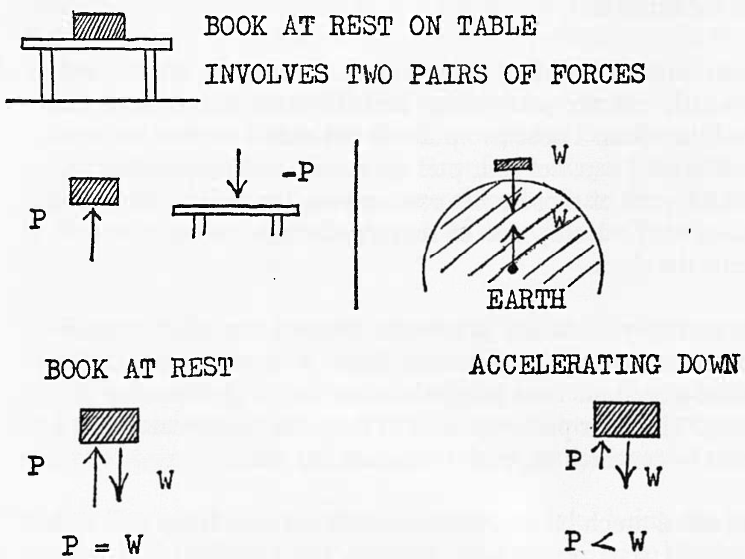
'If I pulled much harder, friction could not match my pull and you would start accelerating towards the blackboard. If you were on roller-skates you would be accelerating and I would have to run towards the blackboard if I wanted to keep up that pull. But even while we were running, my pull on you would be just matched by your pull on me.

‡ H. Poincaré, *Science and Hypothesis*.

'The fact that I pull you towards the blackboard *does not constitute a force on me* towards the blackboard. The only force on me is your pull that you exert on me, towards the clock. Newton decided that every pair of pulls like that are equal and opposite: my pull on you is exactly equal in size and opposite in direction to your pull on me. That does not sound very interesting; but if it is true, and we adopt it as a true working rule, it leads to a surprising prediction.' T

False Examples of Law III. We should be careful to avoid any *
attempt to point at a case of equilibrium as an example of Law III. *
Occasionally some textbooks suggest that a book at rest on a table *
is an exhibit of Law III: *

'The book remains at rest: therefore the push-up of the table is *
equal and opposite to the weight of the book.' *



(This is, of course, correct). Therefore, in this case, action = re- *
action (nonsense). There are two action and reaction pairs in this *
story: the push-up of the table on the book and the push-down of *
the book on the table – which are equal and opposite – and the *
weight of the book and the pull of the book upward on the whole of *
the Earth – which are also equal and opposite. Each of the 'equal *

and opposite' remarks remains true whatever happens, but if we allow the table to accelerate upward or downward (which we can easily do by putting the book on our hand instead and moving our hand appropriately) a force of the first pair is no longer balanced by a force of the second pair; the push-up of the table is no longer balanced by the pull down of gravity on the book. The true statement that those two forces balanced when the book was in equilibrium was a statement belonging to Newton's Law II and not Law III.

We need to say clearly, all over again, that the push-up of the table is a force on the book; *but the fact that the book also pushes down on the table does not constitute a force on the book.* Of those two forces, only one acts on the book, the upward force – and the book would accelerate if there were not some other force acting on it.

We should not now labour discussion of Law III with pupils who do not understand it. However, we should be careful not to store up any misunderstandings which would be troublesome in future studies.

CONSERVATION OF MOMENTUM

'Here are two things which are going to collide, two trolleys, or two rocks or anything else. During the collision A pushes B to the right. And B pushes A to the left with an equal and opposite force. What happens to the momentum of each of those things? B receives that force F from A and has a change of momentum [force] $\times$ [time for which the force acts] or $(F)t$. And A has a change of momentum given by this force $(-F)t$.

'But the time during which A pushes B must be just the same as the time during which B pushes A. Think about that: ... can I push you for a longer time than you are pushing back against me? Then in these changes of momentum the time t is the same for both; so we have changes of momentum Ft and $-Ft$. What is the total change of momentum? ... Yes, it is 0.'

We discuss this further, making it clear that we expect to find the total momentum always the same because in *any* kind of interaction whatsoever, the changes of momentum will happen in equal and opposite pairs. We point out that this is a remarkable new Law which will help us to keep track of any kind of event in which one thing exerts a force on another: 'The total momentum after the event is the same as the total momentum before.'

Of course, all the participants must be included in that. When a moving car comes to rest it loses a lot of momentum, and that momentum seems to disappear altogether, but we believe that the Earth itself gains an equal amount of momentum. We can draw pictures of two things colliding and exchanging momentum without any gain or loss in the total, but in most experiments there is some friction which carries off momentum to the Earth, and then we find less momentum afterwards than before.

We need to give demonstrations in which there is no serious loss of momentum to the Earth; otherwise our story of conservation seems unconvincing. We believe the principle is just as true when momentum *is* lost to the Earth, but the account-keeping is harder to demonstrate experimentally.

Although we should not start by insisting on it dogmatically to pupils, we should keep in mind ourselves the universal nature of the Conservation of Momentum. In *every* kind of action, in any kind of closed system, the total momentum remains the same. If we have a system which is not closed, one which has connections by forces to other bodies including possibly the Earth, the total momentum of the system may change, but then too we expect to find conservation completely true when we keep account of the momentum changes in the other regions to which the system is connected. (We must include momentum of electromagnetic fields.)

Experiments on Conservation of Momentum

The experiments to illustrate or test conservation of momentum should be class experiments as far as possible. Pupils should now have such skill with tickertape that they can keep track of the motion of two trolleys by using two tapes at the same time. We should also take multiflash photographs, and give each pupil a print to analyse; and we should extend those to events in two dimensions.

The experiments are of three kinds:

a. Collisions in which one of the objects is already moving before the collision;

b. collisions in which both bodies start at rest and are given equal and opposite lots of momentum by a spring or some other 'explosion';

c. collisions in which both bodies start at rest and after developing some motion both come to rest again.

Of these, the first type seems to pupils a much more genuine exhibition of conservation of momentum: there is some visible momentum at the beginning and the same amount is there at the end. The other two types of experiment have the advantage of simpler measurements, but are somehow obvious to pupils without being convincing: symmetry seems to distract attention from the essential conservation.

Choice of Experiments. Teachers will find a considerable variety of experiments suggested in the pages that follow. They should not try to carry out all these experiments, or momentum will seem a boring troublesome business instead of a very interesting story. The more teachers have succeeded in shortening the earlier studies of $F = ma$, the more time they can now safely give to momentum experiments, without risk of boredom, and the better. We do not want to use so many experiments to 'prove' that momentum is conserved that we never have time to show the principles being used. We shall put the principle to use in measuring the speed of an air gun bullet and then check the speed by an alternative measurement that does not assume momentum conservation.

A. **Head-on Collision between Moving Trolley and Trolley at Rest.** C41a This is a difficult but convincing experiment. Two trolleys are used, each carrying a tickertape. The two tapes run under the same vibrator, but with a separate carbon paper for each.

One trolley, loaded to have a large mass, is given a shove by hand and allowed to run at constant velocity along a friction-compensated plank until it hits the hind end of another, lighter trolley. The two trolleys do not stick together on collision, but the lighter trolley bounces forward with greater speed while the original moving one proceeds more slowly after it.

The vibrator is placed some distance behind the starting point of the first trolley. There are two discs of carbon paper that run under its hammer. A tape from the hind end of the first trolley runs back to the vibrator and under one piece of carbon paper and out beyond to a pupil who guides it. A tape from the hind end of the other trolley also runs back under carbon paper to the guiding pupil.

Some teachers have found that using two vibrators, one for each tape, is much easier than running both tapes under one vibrator.

Pupils arrange the collision and analyse the traces.

Here they are dealing with the momentum of each trolley which should be constant before collision, and constant with a different value after collision. Therefore there is no need here to paste up charts of tentick lengths of tape. Pupils simply measure the length of tape for a large, round number of ticks and work out the speed in centimetres per second or in centimetres per tentick – any unit will do as long as the same one is used throughout. Pupils require the masses of these two participating trolleys; and they should be allowed to find those in this case by weighing. (Far better: if the loading is done by stacking trolleys, they just count trolleys.)

Some pupils may like to attach magnets to the trolleys and examine the momentum changes in a 'silent' collision with magnets. They will probably hope to find different changes in this case; so when they find the same story here as for other collisions it will be an important reinforcement of the general validity of momentum conservation.

In each experiment, pupils are seeking an answer to the very important question, '*Is momentum conserved in this collision?*' We, as physicists, believe that momentum is always conserved; so that if we find momentum failing to show the same total before and after collision we simply blame friction for giving some momentum to the Earth. With pupils, we should be careful not to be too glib with this excuse but to discuss the possibilities very carefully.

B. Inelastic Collision between Trolleys. We can arrange a similar head-on collision in which the first trolley hits the second and sticks to it so that the two of them proceed as one unit. That requires some locking device for holding the trolleys together such as a sharp needle on the front of the first trolley that sticks into a cork on the second one. In that case, a single tape will suffice if the target trolley starts at rest because it will show the record of the speed of the first trolley before collision and the speed of the combination after. C41b

C. Adding Mass to a Moving System. A trolley carrying a tickertape is given a shove so that it moves with constant velocity along a friction-compensated track. At some point in its travel, we drop a brick on to the moving trolley from just above. That is certainly an inelastic change. The pupil should analyse the tape in the usual way and see whether momentum is the same before and after arrival of the brick. We should remind pupils that, although the brick brings in, and at once loses, some vertical momentum, it contributes no horizontal momentum but only changes the mass of the moving system. C42

D. Multiflash Photos. We make the taking of the photograph a demonstration; and, if possible, we issue a print to each pupil to analyse. We use small ring-magnets with a cardboard lid and solid carbon dioxide under the lid (the solid CO_2 is obtained from a special cylinder). The ring-magnets slide over a level glass table. We can change from the mass of one ring to double mass by adding another magnet, or a brass ring of equal mass, on top. The motion is practically frictionless until the glass table is cooled so much by the carbon dioxide that water vapour condenses on it. It is important to avoid that, if possible. Opening the windows generally helps. D43

We take pictures of some of the following events:

a. One ring moving on a glass table. (We hope to see constant velocity: Newton Law I.) D43a

b. A ring moving with constant velocity makes a head-on collision with another ring of the same mass. D43b

c. A ring moving with constant velocity makes a head-on collision with another ring of double mass (arranged by piling another magnet on top). D43c

And very important collisions in two dimensions.

d. A ring moving at constant velocity makes a collision with a stationary ring and the two move off in different directions. This will need analysis of the momenta as vectors and will raise important new questions. D 43d

e. A collision between two rings which are already moving. This is both difficult and grand, requiring careful graphical analysis. If there is time for this to be done and discussed carefully, it will prove to be a rewarding experiment. D 43e

Vectors. The last two demonstrations involve velocities, and therefore momenta, in different directions. We have to explain that, like velocity, momentum is a vector, that is, something to be added by geometrical construction. We may need to discuss vectors and addition of vectors quite carefully at this point. But that should be very brief. If we present simple geometrical addition as obvious, pupils will accept it. There has probably been some simple discussion of vectors in Year III, but this is the first time we make important use of them and we shall need them even more strongly in Year V. T

For the vector analysis of oblique collisions, we may either draw a line to represent the total momentum – drawing the line in each case in the proper direction – or simplify matters by resolving the momentum into components; preferably components along the original ring's momentum and perpendicular to that.

These are experiments that a teacher will find delightful if he has *
had time to try them out carefully on his own so that the techniques *
of taking the photographs and the tricks for the analysis are not a *
worry to him at a time when pupils are struggling with new ideas. *

E. **Colliding Pendulums** (*Optional*). If the laboratory has facilities for hanging long pendulums from the ceiling, pupils should observe directly (or measure with multiflash) elastic collisions between steel balls, and some inelastic ones when the balls are made sticky with wax or plasticine. D 44 OPT.

We should show head-on elastic collisions between equal masses, and between a mass and a larger mass. D 44a D 44b

Pupils should see oblique collisions between a moving mass and an equal mass at rest; and if the collision is sufficiently elastic, the angle between the two paths after collision will surely be 90°. We hope that some pupils will notice that. If they do, we should either promise that they will later on see a cloud-chamber photograph which shows that, or produce the picture at once. (Alpha particle hits helium nucleus.) D 44c

If we have easy means of taking multiflash pictures of these pendulum collisions, we should certainly try some. Pupils will probably regard them as more genuinely and obviously frictionless than some of our experiments with trolleys. They will understand and forgive the obvious deceleration as the pendulums swing out and upward.

One very interesting type of collision that can be shown clearly with long pendulums is a head-on elastic collision between a very large mass and a small one. If the large mass is at rest, the small one will move up to it and bounce back with almost its original speed. That seems obvious enough; and pupils will predict it if we ask for a guess beforehand. But if we then reverse the story and ask what will happen if the large mass is moving and the small mass at rest, pupils will find it hard to guess. We try the experiment and find that the small mass moves on in the same direction as the large mass, with twice the original speed. Here again, if we have multiflash equipment ready, a picture of such a collision shows the story clearly. D 44d

Theoretical Discussion of Head-on Collision of Large Mass and Small One (*Buffer extension*). With a fast group of pupils we might even show that the last result – double speed – can be predicted by a 'theoretical' argument. Of course we can no more predict the facts of real nature by argument than the medieval Aristotelians could; but our argument contains a concealed piece of experimental information: a very general one that pupils accept unthinkingly – the principle of Galilean relativity. That says: Newton's Laws of Motion and the mechanical events that they describe are independent of uniform motion of the observer or apparatus. We observe the same laws of mechanics in a steadily moving railway train as we do in a laboratory at rest. T

Here is the argument:

'Let us consider a collision between a ping-pong ball and an elephant. First we throw a ping-pong ball straight at the stationary elephant's forehead at 20 feet/second. The ball will bounce back with a speed almost 20 feet/second. If the elephant is on ideal roller-skates he will recoil very, very slowly, barely noticeably.

'Now suspend the ping-pong ball by an imaginary thread in mid-air and let the elephant rush towards it at 20 feet/second. When the elephant's forehead hits the ping-pong ball, what motion will the ping-pong ball take? It seems quite difficult to answer that until we try the following trick. Imagine the elephant surrounded by fog, so that you, who are riding on his shoulders, have no idea how fast he is moving along the road. Pretend the elephant is moving so smoothly on his roller-skates that you know nothing at all about his motion. Then, in the fog, you see a ping-pong ball. What will you think the ping-pong ball is doing? ...

'Yes, you will think the ping-pong ball is moving towards you at 20 feet/second. You still do not know that you and the elephant are sliding along through the fog; and, seeing the ping-pong ball rushing towards you at 20 feet/second to hit the elephant's forehead, you know what it will do. It will bounce away 20 feet/second from the front of the elephant.

'Now let the fog clear away and, standing on the ground, watch what is happening from the outside. You see the ping-pong ball bounce away from the elephant's forehead (at 20 feet/second relative to him) but now you also see the elephant himself is moving 20 feet/second; so how fast will you see the ping-pong ball move if you are standing on the ground?'

We might comment to pupils that this is what is called a 'thought experiment', a very useful method in theoretical physics.

This result has an interesting application in sports, and in kinetic theory of gases. Whenever a massive bat or piston hits a stationary object of smaller mass, making an elastic collision, the victim moves away at double the speed of the bat. That applies roughly to the head of a golf club hitting a ball, or a tennis racket hitting a ball in serving or an engine shunting a light wagon. When gas molecules hit a stationary piston head-on they rebound, on the average, with equal speed in the opposite direction but when they hit a moving piston that is approaching them they rebound, on the average, faster in the opposite direction with a *gain of speed* of twice the speed of their piston. (This explains why diffusion pumps use heavy particles, atoms of mercury or molecules of oil as their moving 'pistons' – and why those pumps are so much better at pumping hydrogen than at pumping heavy gases such as xenon.)

F. **Collisions between Rolling Balls.** A length of curtain rail is used as a track along which steel balls can roll. A sloping section of track leads by a smooth curve to a level section. One ball rolled down the launching slope meets one or more balls on the level section; and pupils watch the result of the collision. D45a

This can be done as a demonstration or in a rougher form with marbles as a class experiment. It forms an amusing part of a pupil's widening acquaintance with collisions; but we should not spend long on it.

Behind the simple story of the momentum of the impinging ball travelling through a line of balls and sending the front ball of the line forward, there is a complicated story of a propagation of a compression wave through a ball. The complete transfer of momentum from one ball to the next is even more surprising when one thinks of it in terms of compression waves. * * * * * *

An alternative (optional extra) demonstration with a line of steel balls each hung up by bifilar suspension is also good. D45b OPT.

G. **'Explosion'.** As a class experiment, pupils arrange two trolleys on a level table (not friction-compensated) and push them together until the buffer rod has compressed the spring and the catch holding the rod in is latched. The trolleys are placed so that when the catch is unlatched and the spring pushes them apart they will travel out in opposite directions towards the ends of the table. C46

Since there will now be some slowing of the trolleys by friction, it is probably unwise to use tickertape to record the motions. Instead, pupils should place a metre stick or a lath of wood across each end of the table to act as a stop. These stops should be placed so that each trolley runs the same distance from its starting point to the stop. Then, with equal masses, trolleys are released and pupils watch to see whether the trolleys arrive at the stops simultaneously.

This experiment is repeated with one trolley having double the mass of the other. We do not tell pupils how to rearrange the stops for that, but let them argue it out or try it for themselves.

H. **Inverse Explosions.** As a class experiment, pupils hold a pair of trolleys some distance apart and attach a large rubber band or a spiral spring to pull them together. The trolleys start at rest and are released simultaneously. Pupils watch to see the motion that is left over when the trolleys have met and stuck together. Some device is necessary to make the collision completely inelastic. For example, a sharp needle on one trolley sticks into a cork on the other. C/D47

I. **Collision with Marbles** (PSSC) (*Optional extra*). (Reports from trials of this experiment differ greatly. Some teachers who have practised both the experiment and its teaching consider it a very good, clear experiment which pupils find successful. Other teachers report that the adjustments are tricky, the ideas of the analysis are difficult to teach and therefore they do not consider this experiment worth doing.) C48 OPT.

In this experiment we use some earlier knowledge, that a marble projected horizontally takes the same time to reach the floor as a marble dropped vertically from the same height. Therefore, this is a specially good experiment because it illustrates the way in which a physicist uses one piece of knowledge in investigating the next.

A marble rolls down a sloping curved channel and arrives at the edge of the table moving horizontally with a speed which will be the same each time that launching channel is used. The channel may be made of plastic or metal curtain rail, bent to make a suitable curve. A marble is released, rolls down the channel, rolls horizontally along the last half inch of the table before the edge and then travels as a projectile to the floor where it hits a piece of carbon paper resting on white paper. Or pupils may use a piece of soft cardboard, such as the corrugated packing material with its smooth face upward, and locate the 'hit' by the dent the marble makes.

Then a marble is held just beyond the edge of the table and dropped vertically on to the carbon paper. The distance between the two marks gives a measure of the speed of the projected marble.

In analysing the records made by the marbles, we assume that both marbles fall for the same time. That is not true unless they fall through the same distance and start with no vertical velocity. It is essential to make sure that the latter condition is fulfilled. The balls must be arranged carefully to start at the same level and in the later part of the experiment we must make sure that neither ball is given any vertical velocity during a collision. Furthermore, this careful adjustment needs to be made beforehand or the whole experiment appears to pupils to be an artificial arrangement.

Then a marble is allowed to roll down the launching ramp from the same starting place and collide with another marble just at the edge of the table. Both marbles fall (for the same time) and hit the carbon paper and make marks. The pupil then has measures of the velocity of the original ball before impact and the velocities of both balls after impact. He can find out whether momentum, treated as a vector, is conserved.

Nuclear Collisions. We should tell pupils that those collisions are models of nuclear collisions which we can infer from cloud-chamber photographs. Such photographs should be posted up for all to see while this work is being done and should remain on view. (The Nuffield Physics Group has gathered a collection, for use in schools, as pictures or transparencies.) D49a

† The expansion cloud-chamber was shown first in Year I. If pupils missed that, it should certainly come out now, although the explanation of its action will come in Year V. Show an actual cloud-chamber. D49b

J. **Collisions of Coins** (*Optional*). (This ingenious suggestion is due to an American physics teacher who uses it as a homework problem that can be done without a laboratory.) It is more difficult to make much of this experiment with English coins: the United States coinage has the advantage of five-cent pieces, made of nickel, without a milled edge which seem to behave unusually uniformly in this collision experiment. H50 OPT.

A coin is allowed to slide down a launching ramp made of a curved sheet of stiff paper. Arriving on a horizontal plane of smooth paper, the coin travels some distance, decelerating, before it is brought to rest by friction. Then the experiment is repeated with another coin placed at the bottom of the ramp, so that there is a collision and both coins move along the paper and come to rest.

In this case, the distance travelled is not proportional to the original velocity of the coin. That poses a new problem for teacher and pupil to investigate.

K. **Electrostatic Forces** (*Optional, difficult*). Since one of our uses of collision studies will be in interpreting cloud-chamber pictures, we should show a collision where electrostatic forces are the controlling ones. Two metal- or carbon- coated ping-pong balls hung by long nylon threads will serve for this. These should be given large charges of like sign. One should be pulled aside and allowed to swing towards the other, making a gentle collision. In this, as in other collisions with equal masses, we hope that pupils will discover the 90° angle between the paths after collision. That enables us to make an important inference from the 90° angle seen in cloud-chamber pictures when an alpha particle hits a helium nucleus. D51a OPT.

L. **Electrostatic Model of Alpha-particle Scattering.** At some stage in our programme, probably in Year V, we should show a collision involving electrostatic repulsion between a small mass with a small charge and a large mass with a large charge: this is a model of alpha-particle scattering. The small object should be a ping-pong ball or pith ball, coated to make it conducting, hung on a very long nylon thread. The large object should be the collecting ball of a small Van de Graaff machine. Unless there is considerable time to spare with a fast group this should probably be postponed till Year V. D51b OPT.

M. **Magnetic Forces.** Pupils should see a demonstration of a head-on collision between toy train trucks on a piece of straight track carrying strong horseshoe magnets to give repulsive forces. (If necessary, the trolleys should be confined between sidelines, such as metre rulers clamped to the table, so that they cannot slew sideways and let the magnets cling.) D52

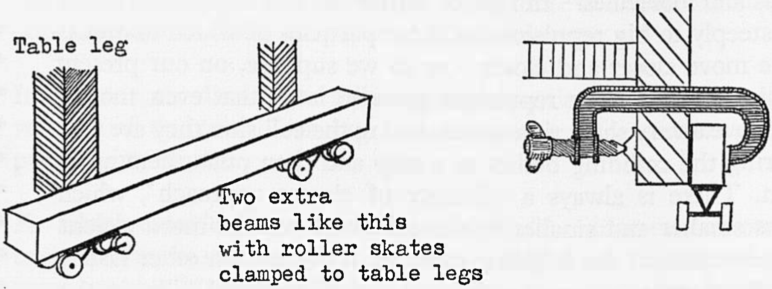
'Contact'. This demonstration is important as one more reminder that in collisions we do not have to have 'contact'. At this stage the teacher with a fast group should raise the question, 'What is contact? Is there ever real "contact" down at the microscopic level of atoms and molecules?' In fact, of course, all that happens is forces rise steeply to big repulsions as those particles of which matter is made move closer and closer – or so we suppose, on our present physical models. The repulsions grow so large that even though they have a very short time to act during the collision they are able to bring the colliding bodies to a stop and then push them apart again. There is always a 'distance of closest approach', which grows smaller and smaller as the collisions become more violent when we project the colliding particles towards each other faster. See the sketches of graphs of force and of potential hills, in the *Teachers' Guide* for Year I.

Comment on the Programme above

Neither teacher nor pupil should carry out all the experiments in the long series described above. We have offered suggestions and hope that teachers will select what seems possible for their equipment. In general, we hope that class experiments will predominate, since it is a personal feeling for momentum exchanges and momentum conservation that we want pupils to gain.

This work should continue as long as the variety of experiments gives that feeling of exploration, but we should not let it drag on until it becomes boring or organize it into a careful study in which precise measurements obscure the general understanding of natural behaviour.

Those teachers who themselves went through the work of a well-organized university practical course in mechanics experiments may remember themselves questioning, every now and then, the purpose of the series. One timed a simple pendulum, and then a compound pendulum and then a bifilar pendulum, and then still more pendulums. ... At first one calculated g from the measurements, and then a more reliable value of g, but then in later experiments it was not quite clear what one should calculate. If it was to be a value of g one would not expect it to be very reliable; and yet one had a strong sense that one ought to be proving something. One was, of course, acquiring a repertory of techniques. Many of us look back on such practical work as a valuable part of our training; and yet we may well remember that sense of doubt about the purpose. Here in Year IV, with young pupils, we certainly should not let our series of momentum experiments give that feeling.



As a demonstration to emphasize Newton's Law III, we might return to the experiment of Year III that used two tables on wheels (e.g. roller-skates‡) loaded with pupils who pull the tables together with a rope. We mark definite starting points with the tables far apart and ask pupils on one table to hold the rope while the pupils on the other table haul the rope in towards them until the tables collide. We mark the collision, start again and ask the other group of pupils to pull. Then, in the third experiment we ask both groups of pupils to pull. D 53

If the wheels of the roller-skates play fair, the collision will occur at the same place in every case. Moral: it is not possible for pupils at one end of the rope to pull without developing an exactly equal and opposite pull at the other end.

In an A-level discussion of this, we should raise the question of the mass of the rope being appreciable. Then the three cases are not identical in effect; and one may even imagine the rope to be so long that we have problems of wave propagation when pupils start pulling.

‡ See the sketch and instructions in *Teachers' Guide* for Year III, for a simple, temporary scheme. (Playground trolleys should *not* be used for this.)

Uses of Momentum Conservation

We shall not do much for the good name of science if pupils merely arrive at the conservation of momentum and then make no use of it. We can suggest conservation by a theoretical discussion and encourage pupils to test it by experiments but then they must make some use of it. There are two kinds of use which could justify our teaching: *

1. Pupils add this great general rule to their collection of laws or rules of nature that are either extracted from experiment or at least found to fit in with the behaviour of the natural world. If pupils make this addition without really understanding it – just one more butterfly in the box – that is not a good use. But if pupils have seen, by their own experiments, that this simple rule, 'total momentum remains constant', applies over a wide variety of events and promises to be a universal guiding rule that describes natural behaviour irrespective shorn of the decorations of local circumstances, then it is a worthy part of their collection of natural knowledge. T

2. Pupils should see at least one example of the conservation of momentum being trusted and put to use for a practical measurement, as follows.

Measurement of Speed of Rifle Bullet

This is a genuine application of conservation of momentum. It is not difficult or dangerous; so we hope that teachers who find it unfamiliar will try it and will not omit it. It forms a simple practical example. D 54

We can measure the speed of a bullet, *assuming* the conservation of momentum. We can then measure the same speed by a time-of-flight method which does not assume conservation of momentum.

If, as we hope, there is reasonable agreement between the two estimates we have some practical support for conservation of momentum as well as a demonstration of a method that puts it to use.

We use an air rifle‡ for safety, and fire a small lead slug into a block of plasticine on a toy train wagon. The wagon is initially at rest on a length of track which is tilted enough to compensate for friction. The slug, fired horizontally along the direction of the track, embeds itself in the plasticine sharing its momentum with the plasticine and wagon. We measure the speed of the wagon (with bullet in it) by timing its motion over a short distance of track with a stopwatch. This is a fairly rough estimate which does not justify the use of the

‡ By using an air rifle instead of some more dangerous firearm, we avoid technical doubts of danger.

According to the 'Air Guns and Shot Guns, etc., Act, 1962' we may use an air gun provided the person using it is over 21 years of age. Persons under 17 but not under 14 may use an air weapon without supervision, but may not be in possession of an air weapon in a public place unless it is covered so that it cannot be fired. Persons under 14 may use an air weapon under the supervision of a person over 21; but may not be in possession of an air weapon in a public place unless it is covered so it cannot be fired and it is under the supervision of a person over 21.

The experiment is more than well within the rules, if the firing is done by the teacher, providing the following obvious precautions are taken:

The air rifle is securely clamped. (It should *not* just be held by a G-clamp.) It should be turned over on its side, and attached to a table or horizontal board by two bolts that run through the stock and the table or board. Then, the loading mechanism can be pulled out sideways. In order to give easy play to the loading mechanism, and to raise the barrel to the right line of fire for the toy train wagon, blocks of wood of suitable thickness should be interposed as spacers between the stock and the baseboard. These blocks should be chosen so that with the baseboard horizontal the barrel of the air rifle is also horizontal. Then the toy train track can be placed on the baseboard (and so can the arrangements for holding the paper sheets and metal strips in the second experiment); and the baseboard can be clamped firmly to the table.

There must be a safety stop at the end of the range to catch slugs. This may be a large block of plasticine or clay or a large block of polystyrene foam with wooden backing. Just for appearances, there should be a metal plate behind that.

Pupils should be in a safe region out to one side.

According to the 1962 Act, any person carrying or using an air weapon 'outside the curtilage of his home' must be in possession of a gun licence obtainable from any post office at a cost of ten shillings. It is doubtful whether this is necessary for the use of the air rifle for this experiment, but it is probably wiser to obtain a licence.

We find this goes well with an air rifle such as the B.S.A. 'Merlin'. With 0·22 inch bore, taking the larger slugs (mass almost 1 gram), the muzzle velocity is about 350 feet/second.

scaler as a millisecond timer for the measurement of the wagon's speed. It could be used, of course, with a photo-diode arranged to show the time taken by a piece of cardboard on the wagon to pass some fixed observation post. However, since the scaler is going to be used for the alternative measurement, it is better to make a simple direct measurement with a stopwatch here.

To make use of conservation of momentum, we must know the mass of the bullet and the mass of [wagon+plasticine+bullet after impact]. We find those masses by weighing beforehand. D54

Then we assume conservation of momentum and write:

$mV = (M+m)v$ and solve for V.

We should make fun of worrying about m in the total mass $(M+m)$. 'Why worry about the odd needle when you are dealing with a haystack?'

We can return to this demonstration after pupils have learnt about kinetic energy and know the expression $\frac{1}{2}mv^2$ for it; we can ask them to calculate the initial kinetic energy of the bullet and the final kinetic energy of [bullet+target] afterwards. They will find that almost all the kinetic energy of the bullet disappears. We ask where it has gone to. * * * * * *

Measuring Bullet Speed with Scaler. The 'ballistic method' above is one of the methods really used for measuring bullet speeds. Nowadays, we have electronic devices which can do the timing more easily; and in fact we can show pupils a modern method for bullets. We measure the time-of-flight of a slug in milliseconds, for a flight of a few feet. This will not be a very accurate measurement, because the time taken over the distance that we can use is a few dozen milliseconds at most. Nevertheless this gives an interesting example of a modern method; and since it does not depend on assuming conservation of momentum it gives a valuable check. It is probably as accurate as the first method that uses conservation of momentum, with likely errors about 10%. D55

As we explained for earlier uses of the scaler the connections from the pulse generator in it (making 1,000 pulses per second) to the scaler which counts the pulses are brought to two pairs of terminals on the front. If we connect together one pair of terminals the counting starts; and if we connect together the other pair of terminals, the counting stops, whether the first pair are connected or not.

We set up two strips of metal foil‡ a measured distance (1 or 2 metres) apart in the line of fire of the air rifle. The slug breaks the first strip when it passes through, thus breaking any electrical connection that included that strip, and, a very short time later, breaks the farther strip. We connect the farther strip to the first pair of terminals of the millisecond timer, and the nearer strip to the second set of terminals. Then the scaler will record the time of flight of the slug between the two strips.

This might seem a difficult experiment to arrange and conduct successfully. It would be discouraging if slug after slug missed one or both strips. However, it is easy to ensure success by firing a preliminary slug through sheets of thin paper to find the proper places for the strips of metal foil. We clamp the air rifle firmly to the table with its barrel horizontal. A short distance from its muzzle, we place a sheet of paper to catch the blast of air that follows the bullet. Just beyond that we place another sheet of paper, held in a frame of wood or cardboard, in the place where we expect to have the first strip of metal. Some distance farther along the line of fire we place another sheet of paper in a cardboard frame, where we expect to place the second strip of metal. Then we fire a preliminary shot from the rifle so that a slug passes through these sheets of paper and marks the places for the metal strips. We keep the first sheet there in future firings, to catch the air blast.

We place a thin strip of aluminium foil across the bullet hole in each of the other two sheets of paper. We leave the paper there with the metal strips covering the bullet hole; and we connect the strips to the scaler. Then we fire another slug. We measure distance and time and calculate V.

It is easy to spoil this experiment by doing it too roughly so that both pupils and teacher feel that it is unreliable and insecure. But it is easy to do it well if one arranges the paper sheets and metal strips carefully in secure clamps beforehand. If the air rifle is clamped loosely so that its aim changes when it is reloaded, or if the metal strips are held precariously by loose wire or plasticine, or if the

‡ Thin pencil-leads are better. They snap easily.

installing of new strips for a second trial appears difficult and fidgety, we shall lose the value of a delightful experiment.

To arrange this experiment to go well, the three sheets of paper should be attached with drawing pins to their frames which are held in firm clamps on retort stands, or attached to a board that carries the rifle and everything else. These sheets of paper should not be small scraps that flutter in the wind but should be pieces about 4 inches square to give plenty of margin.

To make it easy to attach the strips of foil, each should be pasted on a thin sheet of paper which is then fixed on top of the trial sheet with its bullet hole. One can see, by looking through the bullet hole, when the metal strip crosses the hole. The frames carrying these sheets of paper should each have terminals or spring clips of metal to hold the sheet of paper and make contact with the metal foil.

Cloud-chamber Pictures

† Pupils should have seen and used cloud-chambers in Year I; and they will meet them again in Year V and use photographs of alpha-particle tracks as important evidence for a nuclear atom model. Since we are discussing collisions now, we might well show some photographs of alpha-particle tracks at this stage.

† If pupils have not seen an expansion cloud-chamber we should give a quick demonstration. We should explain that physicists obtain a great deal of valuable evidence concerning sub-atomic particles from such pictures. In many cases a track continues across the picture with no major change (though it may follow a curved path if the chamber is in a magnetic field). However, we occasionally see evidence of violent collisions and even 'explosions'. We are most anxious to find out all we can about the masses and energies of the particles involved in such events. D 49

Whenever we attempt such an analysis of a fork in a cloud-chamber photograph (or a bubble-chamber photograph) we assume that *momentum* is conserved in every such nuclear event. (We could, of course, test conservation of momentum if we trusted other estimates. Sometimes we can make independent estimates of speed or momentum but usually we assume conservation of momentum.) T

When a uniform magnetic field is applied to a cloud-chamber, tracks of charged particles moving perpendicular to the field are bent into a circle. Measurements of the radius of that circle enable the experimenter to estimate the momentum of the moving particle. We cannot mention this without its seeming a puzzling mystery, until Year V.

Alpha-particle Tracks: Energy

We cannot carry pupils through a detailed discussion of such events *
until they have met kinetic energy. However, our earlier teaching *
should have given a picture of kinetic energy as motion energy, and *
in discussing cloud-chamber pictures we are not likely to need a *
quantitative measure of K.E. So discussion such as the following *
could come now, or it could be postponed until after kinetic energy *
has been treated, or even until Year V. In any case it would be a *
great pity not to turn our studies of momentum to this important *
use in discussion with pupils. *

Where an alpha-particle hits some other nucleus effectively, that T
was at rest originally, we see the tracks of two moving particles proceeding after the collision. Again we assume that the momentum of the alpha-particle before collision is equal to the total of the two components of momentum in the same direction after collision. And we assume the two components of momentum perpendicular to the original motion of the alpha-particle are equal and opposite. Knowing the gas that was used, we can guess what the target nucleus was, and thus know the relative masses of alpha-particle and target nucleus. That enables us, assuming momentum conservation, to find the velocities before and after collision, on some arbitrary scale. Then we can find out whether kinetic energy was conserved in the collision.

If we find K.E. was *not* conserved we have a choice: either we *
have come across a rare but interesting case of an inelastic collision; *
or we guessed wrongly about the identity of the target nucleus. The *
latter mistake may well occur, because whatever gas is used in the *
cloud-chamber there is also water to make the water drops. How- *
ever, a skilful reader of cloud-chamber pictures soon learns to *
recognize the alternative targets and can then scan a large number *
of pictures successfully for the very rare unusual inelastic events. *
The latter are nuclear transformations effected by an alpha-particle; *
and in those cases we do not find kinetic energy conserved – we may *
well find that the particles that emerged from the collision have *
more kinetic energy than the alpha-particle that went in. *

'Weightlessness'

At some stage in our discussions of force, mass and motion or of projectiles, the question of 'weightlessness' in a satellite or rocket will crop up. We must meet it fairly every time it is raised. The question is partly one of fact and partly a semantic one.

The Fact. In a satellite, or in an ordinary lift falling freely, which is not acted on by forces other than gravity, all objects appear to have no weight. Something placed in mid-air will just float there.

(If the lift cable is pulling it, if the rocket's motor is blasting and propelling it, if the satellite is encountering air resistance there *is* another force and objects inside will not appear to be completely 'weightless' – though their 'weight' may seem to take a peculiar direction.)

Wherever gravity alone acts, all objects are pulled by forces that are proportional to their masses – as shown by the Leaning-Tower experiment that Galileo did not demonstrate, or by Newton's guinea-and-feather experiment – and they all fall with the same acceleration. Therefore, inside the satellite, etc., any object that is given some motion just keeps it – Newton's Law I – and any object at rest relative to the satellite just remains so. D56

Note that the Earth is such a satellite of the Sun. Therefore we never notice the Sun's pull on us because, like all our surroundings, we are falling towards the Sun with an acceleration which keeps *us* in the same yearly orbit as all the rest of the Earth. Except, that is, for minor differences due to differences of distance from the Sun between different places on the Earth. These tiny differences build up the Sun's contribution to great ocean tides.

The Semantic Question. What is the meaning of 'weight'? Does that word mean the pull of the Earth as seen by an outside observer, or as judged by an inside one? Children with a hard-headed commonsense view, aided by newspaper writing and encouraged by a love of romance, may incline to the latter view. There is nothing wrong with it, except that we must then be careful to reword our dynamical statements to conform with that choice.

However, it will probably make our teaching clearer if we make the former choice and say: T

> ' "Weight" is the word we shall use for the pull of the Earth (and Sun, etc.) which *always* acts on any mass. If you are busy measuring that pull (say, on a brick) with a spring balance and you jump out of the window and fall, then you and the spring balance and the brick will all fall together. Remember the guinea-and-feather experiment. Then you can unhook the brick from the spring balance and leave it beside you. Except for air resistance slowing it (or you) both will fall together. You can avoid some of the effects of air resistance by climbing into a big box and arranging to let that fall freely. Then, inside the box, everything will seem to have no weight. We shall say things have no "*apparent* weight".'

We suggest that teachers should clarify the factual story by going over it again every time a question arises, but ask pupils to compromise on the semantic story by using the word 'apparent'. * * *

Local Effects in a Satellite. Of course, a large enough satellite will reveal local differences inside; and a long enough time might reveal tiny local gravitational attractions between bodies inside. Both these effects will seem natural enough when the time comes – all the easier to understand if we do not mention them now. * * * * *

Experiments to illustrate 'Weightlessness'. We place a load such as a small bag of sand, on a spring weighing scale. We ask pupils to watch the reading of the scale when we let scale and load fall freely. Of course we need for this a scale that we do not mind dropping – but we can avoid most damage by catching the scale in a blanket held by four pupils. Or pupils may carry out a small version with light spring balances.‡ D 57a

Or (optional) we might exhibit a photograph of this event. D 57b OPT.

Or (optional) we might show a short film taken in an aeroplane flying on the correct curve for its speed to show 'no gravity'. D 57c OPT.

‡ It is difficult to carry this out fairly. When they release the system the spring in the balance produces unwanted local motions.

EQUATIONS OF MOTION WITH CONSTANT ACCELERATION

With pupils who appreciate the neatness and power of simple algebraic statements, we should certainly obtain the equations for motion with uniform acceleration, and give some problems that make use of them. However, these should not become an end in themselves, nor should they represent a major item in examinations – either as a ready-made basis for routine arithmetic or as a piece of formal algebra that is to be learnt by heart and reproduced.

In general, *they should only be offered to those who can use them with confidence*; and avoided, or at least given little emphasis, with slower groups. One useful criterion is the attitude that the pupils will take towards these equations when they have learnt them. If they understand that the equations are relationships argued out from an assumption of constant acceleration and expressed simply, they should know how those relationships are deduced and they should be ready to use them. If, however, they regard the relations as mysterious magical pieces of physics which necessarily reveal the real world, they have missed the point and would be better without them.

We should tell pupils that problems using these formulae will not play an important part in examination. And then we must be careful not to let them play a large part in homework or our own tests.

It would be very easy to let habit encourage us to set a series of problems, ranging from easy to difficult, which are really no more than 'putting numbers into formulae' – and then we should have moved far away from our present objective of teaching for understanding. Even pupils who understand these formulae and their meaning very well will not find many uses for them in our O-level programme. Other pupils who do not understand the formulae well will not be at a great disadvantage because of that.

In any case, we hope that teachers will always print these 'formulae' on the front of every test or examination paper, both for the sake of fairness to different groups of pupils, and as a public policy statement that we do not consider learning formulae by heart a necessary part of understanding physics. Those of us who are used to teaching this part of physics carefully and then setting a considerable number of problems on it may feel a sense of dismay that we are 'giving the show away' by providing the formulae in examinations. But we

are likely to feel reassured when we find how much deeper we can *
go into modern physics if we hurry on, and how little we then *
regret that public announcement when we look back on it. *

Acceleration

We define acceleration as: T

[change of velocity]/[time taken to change the velocity]

Even though pupils are quite familiar with units of acceleration, a return to mixed units may be a helpful reminder: e.g. miles/hour per second for a car or metres/second per century for some piece of matter so far out in space that it feels hardly any force.

We may describe that definition by saying that acceleration is the gain-of-velocity made in each second. The formal description 'rate of change of velocity' requires a more sophisticated level of understanding; it should be given later, and only to those pupils who have a taste for it.

$$\text{Acceleration} = \frac{\text{Gain of velocity}}{\text{Time taken}}$$

That is all we can say about acceleration (without calculus) unless we know that it is *constant* acceleration. Even then we have only described some average acceleration.

Acceleration may be Variable. We should not labour this *
point with pupils; yet we should not lead them into thinking that *
accelerations are always of the constant kind. In many motions in *
ordinary life the acceleration is not constant; a car starting and *
speeding up, a car stopping and skidding, a gas molecule colliding *
with another one, the simple harmonic motion of a pendulum bob *
or of a tuning fork. We deal with constant acceleration in elemen- *
tary physics *

1. because it is easy to deal with, *

2. because it represents a first attempt at analysis of motion, and *

3. because it occurs naturally in a very common form of motion, *
the first to be investigated mathematically by Galileo and his pre- *
decessors. *

Relations for Constant Acceleration

For constant acceleration, $a = \dfrac{\text{Gain of velocity}}{\text{Time}} = \dfrac{v-u}{t}$ T

Then algebra gives us $v = u + at$.

In the discussion of examination questions in the general introduction, we gave an example in which the candidate is asked to explain what u stands for. In reading answers to that question, many of us find that pupils learn the formulae and learn to use them without a clear sense of meaning of the symbols. It is worth while spending a moment to put this to the test and then explain again – that u is simply 'the velocity the thing had when the clock was started at 0'. It is not some mysteriously important 'original velocity', a dowry with which the object was endowed by heaven. We might illustrate the meanings of u and ut by asking pupils to imagine a small clockwork toy placed on a rug and allowed to accelerate forward from rest with constant acceleration, a. Then its travel along the rug is given by $s = \frac{1}{2}at^2$. Now suppose we drag the rug along the floor with constant velocity u in the direction of the motion of the toy. An observer standing on the floor will see the toy's progress increased by ut. Or, we might think of that in a different way: let the toy crawl along the rug and let the observer walk at constant speed $-u$ along the floor (the opposite way). These are just models of what happens in space when the moving object simply keeps any initial motion, u, and adds the effect of acceleration to that.

We might follow that by an example such as:

> 'A motor cyclist leaves home in a hurry, starting his stopwatch at the instant he leaves. He travels with constant acceleration; and 15 seconds from his start he is moving at 30 miles an hour. For the motor cyclist, what is u for that 15-second trip; what is v for that trip, and what is his average acceleration? P
>
> 'A little later, when the motor cyclist is still accelerating, a policeman in a high-speed police car decides to accompany him. Just 30 seconds after the cyclist left home, the policeman catches up and starts his stopclock at that instant: and he stops it 10 seconds later. He also stops the motor cyclist and has a discussion with him. Assume that the motor cyclist maintained the same acceleration all through this story until the policeman

stopped him. From the policeman's point of view, with his stopclock, the experiment lasted only ten seconds. For that ten-second stretch of time (recorded by the police stopclock) what is the value of u for the motor cyclist, and the value of v at the end, just before the cyclist was told to stop? What speed did the policeman accuse the cyclist of reaching?

'Now look at it from the point of view of the motor cyclist and *his* watch. What is the motor cyclist's value of u? (It is 0, as it always was, because he is still using the same watch.) What is the motor cyclist's value of v at the instant the policeman joins him for that 10-second trip? What is his value of v at the end of the 10-second trip just before he is told to stop?'

Distance Travelled, *s*

Graph. If the acceleration is not constant we can still calculate *
distance travelled by the graphical method in which speed is *
plotted upward and time is plotted along. We used an informal *
version of that in Year III with strips of tickertape pasted up to *
form a chart; and for faster groups that may have been carried on to *
the formal graphical method. Unless pupils remember that clearly, *
we should start that afresh now and show a graph with speed or *
velocity upward and time along; and discuss the meaning of a small *
vertical strip of area. We shall use that in Galileo's method *
described below. *

Simple Algebra. If we know that acceleration is constant we can *
construct expressions for calculating distance travelled from other *
measurements. The simple algebraic method assumes that the *
average velocity for covering distance is the arithmetic mean, *
$\frac{1}{2}(u+v)$. Pupils will accept that happily; but we should feel uneasy *
about it, as we know that such an average applies only to cases of *
constant acceleration. We should feel uneasy because in using that *
method we conceal the fact that we are assuming constant accelera- *
tion – merely announcing that when we are about to take the average *
should not remove our uneasiness. Fortunately Galileo's geometri- *
cal method (see below) avoids that trouble entirely, because the *
assumption is visible. *

Using the arithmetic average we have
$s = [\text{average velocity}] \times [\text{time}] = \frac{1}{2}(u+v)t$
This is a simple useful formula. We shall do great good if we can make pupils anxious to show this averaging at once in the geometrical form because then they will see what it means.

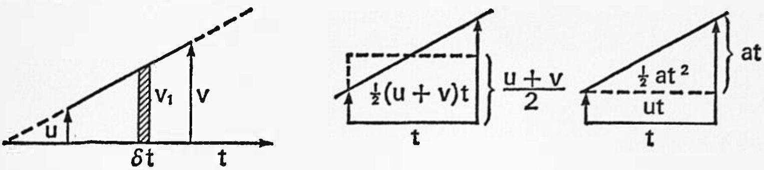
Then we substitute in the usual algebraic way:

$$s = \tfrac{1}{2}(u+v)t = \tfrac{1}{2}(u+u+at)t = ut + \tfrac{1}{2}at^2.$$

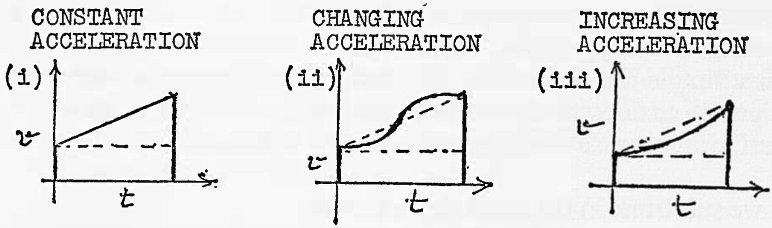
Geometrical Form

Galileo was one of the world's greatest teachers and we should be wise to look at the methods he used. He would have had a great deal of trouble at first with the Nuffield Physics Programme, because he was such a brilliant expounder that he would have found it very difficult to restrain his clear descriptions and emphatic teaching and give young pupils time to learn for themselves. Yet as a teacher he knew very well the value of the pupil's own thinking: he drove his audience into doing some thinking by his powerful dialectical method. We should not be wise to follow him in the latter; since it more often produces annoyance than a sense of understanding; but we should indeed be wise to watch the schemes he used when he hammered conviction into people's minds.

Following Galileo, we draw a graph with (time) along and (velocity) up. At the origin, we draw a vertical arrow representing u. Farther along, at the time at which we stop our measurement, we draw another vertical arrow representing v. We join the tops of those arrows by a slanting line. We ask pupils what they know about the motion when they see that the slanting line we have drawn is a straight line. In many versions of physics teaching, pupils would have considerable difficulty in thinking out the meaning of that, and expressing it clearly. But pupils who have made charts from tickertape are likely to say at once: 'Oh, that means that the velocity is going up and up climbing at a constant rate', or 'That means it has a constant acceleration.' If they do not see that at once, we should point it out to them.



Area $v_1\delta t$ gives distance travelled in time. Total area gives s



$s = u_0t + \frac{1}{2}at^2$ belongs to CONSTANT acceleration. In that case, v grows steadily with t; and area of triangle leads to $\frac{1}{2}$. If the line is curved, the upper area is not a triangle and the $\frac{1}{2}$ no longer applies—except for some particular region, as in (ii).

That is a graph or chart for one particular kind of motion, motion with constant acceleration. In drawing the chart we have stated that we are at the moment considering that kind of motion. We have not stated that we are considering the motion of a free fall or the motion of a particular trolley drawn along by a constant force; we have merely stated what we are going to think about – whether it is common in nature or not.

We then go very carefully through the argument which shows us that the area under such a graph gives the distance travelled. It may seem obvious to us, but it is a new idea – or *was*, in Year III – which pupils will enjoy using if they really understand it. They are more likely to understand it if they think it is going to have a good use – so this is the time for careful teaching to make the new idea seem clear and worth learning.

A careful mathematician is likely to take considerable trouble over *
the small triangles of area where vertical rectangular strips fail to *
meet the slanting line or exceed it. To worry about those and place *
the slanting line itself as a limit sandwiched between two staircases *
is probably very wise mathematical teaching: but it will probably *
spoil our present teaching of physics entirely if we labour it. We *
hope that – except with very unusual pupils – physicists will leave *
that part of the discussion to mathematicians and proceed rapidly *
without worrying. *

We now change from the trapezoid area to an equal rectangle to T
show that $s = \frac{1}{2}(u+v)t$. Instead of marking the mid-point of the slanting line and showing pupils how to change from the area under the line to the area of a rectangle, it might be a good idea to let them first have a shot at finding that scheme for themselves. We explain the problem; and we put it as a puzzle:

> 'Here is the area that we want, for the distance travelled between the starting instant (clock at 0) and the instant at which we finish the measurement (clock at t). This is a clear shape of area to deal with. Can you find an equal area of a shape that is easier to work out? If you can, you know that this arrow is u, this arrow is v and this distance is t. Use those to find what the area is. You should call the area s, which stands for distance travelled.'

This is a problem for pupils to take home between one physics class *
and another. Reflect that the answer itself is not one that is going to *
be used so constantly in our physics course that we must ensure *
every pupil knowing it in a hurry. The pleasure of finding it for *

oneself is a small share of the pleasure that Galileo and others have enjoyed in building science. We owe our pupils a chance of that. * * *

Then, for those who do not find the answer, we must mark the mid-point of the slanting line, draw a horizontal line through it, point out the height of that line is $\frac{1}{2}(u+v)$ and the width of that rectangle is t. Then we have: T

$$s = \tfrac{1}{2}(u+v)t$$

We could proceed to the other form of s by a mixture of algebra and geometry: substitute for v in the expression we have just reached geometrically. Then we are likely to miss the full force of the geometrical representation in exhibiting our assumption that the acceleration is constant – so we should stick to geometry. In this case it is probably better to give a hint:

'Draw a horizontal line of height u. That divides the area for s into a rectangle and a triangle. The area of the rectangle needs a further step: we must find its height in another form.

'As the slanting line shows, the velocity grows greater and greater. What shows the total *increase* of velocity on the diagram? ... Yes, it is this height, at the end of the triangle.

'So the height of the triangle is the gain of velocity during that time. The height is [acceleration] × [time].

'The height is at. And the area of the triangle is $\frac{1}{2}at^2$.'

Then pupils will find that $s = ut + \frac{1}{2}at^2$.

So far our discussion has been purely one of motion which we *assume* to have constant acceleration. Now, however, we should point out that the work of Galileo on projectiles, and the work of generations of physicists and engineers ever since, have shown that T motions are additive‡: and if some agent gives a body one motion and then some other agent gives the body another motion the two motions do not influence each other but just add up, as vectors, to give the resultant motion which we observe. Therefore, if we have a body moving with velocity u and then give it additions of velocity by imposing an acceleration, it will keep that velocity u and cover some ground as a result, as well as any ground covered as a result of its acceleration. So ut tells us how far the object would go if it had no acceleration. And $\frac{1}{2}at^2$ tells us how far the object would go if it started from rest and had that acceleration, a. In fact it travels the sum of those two distances.

Distance and Velocity. We are accustomed to proceeding from *
these relationships, by algebra, to $v^2 = u^2+2as$. For a fast group it *
is probably just as well to show the derivation of this expression, *
but we should adjust our treatment to the ability of the class, *
remembering that we are aiming for a sense of understanding this *
physics; and that we do not expect to ask for the algebra of this *
derivation to be returned to us in an examination. *
*
There is one very strong reason for arriving at $v^2 = u^2+2as$: we *
need it to show that kinetic energy is $\frac{1}{2}mv^2$. Without that latter ex- *
pression, our physics will seem weak. However, we shall suggest a *
different method of arriving at the kinetic energy expression with- *
out using $v^2 = u^2+2as$. (See later section.) *

Formulae and Experiments. When we make use of these for- *
mulae in dealing with experimental measurements on a freely *
falling body or an example of diluted gravity, we should be very *
careful of logic. *

‡ Long ago practical experience assured us that motions are additive over a wide range, from a crawling toy to an express train; and we generalized that into a commonsense rule that *all* motions are additive. But now experiments have forced us to adopt the geometry of special relativity, in which motions are not simply additive. For speeds of toys, trains and jet planes the modification is imperceptible. But for speeds near the speed of light simple addition fails seriously to predict what an observer would find. A man on a station sees a train pass at 90 ft/sec; and the passengers in it see a boy running forward in it 10 ft/sec. To the observer on the station, the boy's speed is (90+10) ft/sec by simple addition. In relativity geometry it is:

$$(90+10)/[1+(90).(10)/(\text{speed of light})^2]$$

The denominator is not observably different from 1 for those speeds; but for speeds near c it is large.

If *experiment* shows that for free fall from rest [distance] is pro- T
portional to [time]^2, we should *not* say that this proves that [distance] is proportional to [time]^2 for a case of constant acceleration. We do not need an experiment to prove the latter, we need logic. Starting from the assumption of constant acceleration we arrive by safe logic of geometry or calculus at the prediction $s = \frac{1}{2}at^2$ for motion starting from rest. If, in an experiment, we find a case of s proportional to t^2 then we can say we think we have found a case of constant acceleration. That is the conclusion to draw from comparison between experiment and the formula which we have arrived at in a safe mathematical manner.

Although we should not labour such logical discussions with our *
young pupils, we should be careful how we talk. This is not a *
matter of hair-splitting, but a place where we should make a clear *
distinction between a theoretical relationship derived from an *
assumed simple behaviour and the experimental relationship *
derived from measurements. When the two agree, we link the *
behaviour of nature to the assumptions of our theory. *

Mathematics the Honest Servant

In some problems where a fast group calculates the time taken for T
some accelerated motion, a quadratic equation may yield two answers. We should be careful not to let pupils throw away the second answer as irrelevant or wrong. Algebra is our servant in this work, giving us answers to definite questions that we have asked; and if there are two answers each of them is a feasible answer to the questions that we put to the algebra.

For example, consider the following problem:

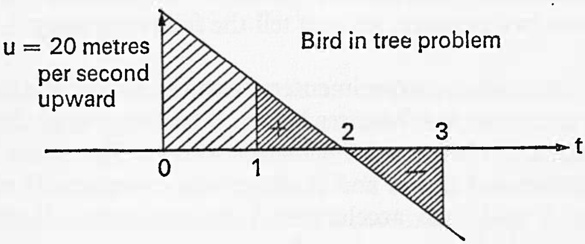
A bird sits in a tree, 15 metres above the ground. A man on the ground vertically below throws a stone vertically up at the bird with initial velocity 20 metres per second upward. How long will the stone take to reach the bird? (Take $g = 10$ metres/sec per sec.)

This problem will yield an answer $t = 1$ second *or* 3 seconds. We should not tell pupils what the 3-second answer means. We should ask them what they think it might mean, and leave them to consider it. But, later on, we should discuss the matter fully.

Some pupils will realize that the 3-second answer is the time for the stone to fly up, missing the bird, to a much higher place and fall down again, hitting the bird on the return. A few pupils will give answers which are not consistent with the data, such as the '3-

second time is the time taken for the bird to fall back to the ground'.

We point out that both the 1-second answer and the 3-second answer are quite sensible and possible; and then we explain why there are two answers. The reason is this: we did not ask the algebra the question we thought we were asking. We did *not* ask how long after being thrown the stone would reach the bird, while travelling upward, and hit it. There was no sign in our equation of feathers flying, nor any sign of the velocity having to be upward at the instant we were seeking. The question that we actually put into the algebra was '*at what instant of time will the stone be at the bird's height above the ground?*' And the algebra, being a faithful but rather wooden-minded servant, gave us two answers, both literally correct.



(**Distance *s* is Net Distance.** The basic story here is that *s* is not *
the total length of path travelled but is the *net* distance, the (vector) *
resultant displacement. We get an interesting but difficult illustra- *
tion of this if we plot a 'Galileo's geometry' diagram for this. We *
draw a large *u* upward then a line slanting downward from the top *
of that, to show the deceleration, continue that slanting line down *
across the axis to negative velocities. Then the lightly shaded patch *
on the diagram here shows, by its area, the distance travelled from *
man to bird; and the two heavily shaded areas, which are equal but *
opposite in sign, show the further travel up to the vertex and back *
to the bird.) *

Another 'Double Answer' Problem. If that proves too difficult an example for such a discussion, teachers might try the following, provided their class knows the relation $v^2 = u^2+2as$: T

A stone is dropped from rest. What is its velocity after falling 4·9 metres? If we use common sense and a knowledge of the acceleration of free fall we arrive at the answer $v = 9{\cdot}8$ metres/second *downward*. But we may use $v^2 = u^2+2as$ and substitute the conditions

$u = 0$ at $t = 0$; $a = 9{\cdot}8$; $s = 4{\cdot}9$; and $v = ?$

Then we obtain $v = +9{\cdot}8$ or $v = -9{\cdot}8$.

Both those solutions are consistent with the 'boundary conditions'. The second solution, $v = -9{\cdot}8$ metres/sec, is the velocity that a body moving *upward* would have had at $s = +4{\cdot}9$ at time $t = -1$ sec, so that it would rise to $s = 0$ and have velocity $v = 0$ at time $t = 0$. To interpret these two answers, we may tell the following story:

> Suppose there are two experimenters A on the roof of the house and B on the ground, 4·9 metres below. B on the ground throws a stone upward with speed 9·8 metres/second. The stone flies upward slower and slower and comes to rest momentarily when it reaches A and then accelerates downward again. When it passes B, just before it reaches the ground again, it is moving *downward* with speed 9·8 metres/sec. A, who has charge of a stop-watch, does not start it until the stone reaches him and starts it making $t = 0$ at that instant. B, having charge of a velocity-measuring device, measures the velocity of the stone when it is near the ground; and he obtains two answers (reckoning a *downward* velocity as *positive*).
>
> $v = -9{\cdot}8$ metres/sec (at a time 1 sec before stone reaches A)
>
> $v = +9{\cdot}8$ metres/sec (at a time 1 sec after stone reaches A)

We told the algebra nothing about the time at which the velocity measurement was to be made. We only asked for the velocity when the stone was 4·9 m. below A (and we stated that at the instant $t = 0$ when the stone was at A, the velocity v was 0; and s was to be reckoned 0 at A).

Such problems are artificial and do not in themselves deserve much attention; but as an example of the part played by mathematics in physics, this discussion is probably worth while for able pupils. They should see mathematics as capable and logical and powerful, but unimaginative. Our equation is our servant in thinking, as a compact (but not very complete) 'model' of the motion. We could tell a fuller story in words. In more advanced physics, the mathematical description may be more complete and powerful than a verbal one – it is the prime model. At a still more advanced stage, mathematics provides the only statement we can make. It is *the* model. Yet, even then, its meaning and its limitations need to be understood.

But at this stage it is better for pupils to see that mathematics is not mysterious than for them to meet the imaginative, almost mystical value that we put upon mathematics in modern theoretical physics. That should come presently; and before it comes we shall need to show mathematics as a reliable machine; so that, where we cannot give the full mathematical derivation of something, pupils will not feel that what we then have to omit is itself an impossible mystery.

In dealing with a projectile problem like this, we should not let the mathematics be a black box. We should have the box wide open and show that it contains ordinary machinery – gears and levers in our logical minds – machinery which is able to produce clear answers from the information and rules that we put in. Then, at a later stage, when a piece of mathematics is too difficult,‡ so that we cannot explain the mechanism – perhaps cams have replaced the gears as calculus takes charge – the box can remain closed without being talked of, or thought of, as containing mysteries. It contains clever machinery, not a concealed wizard. And pupils may, on that basis, be willing to accept the result of that piece of mathematics without

‡ The prime example, which we can neither avoid, in our programme, nor solve comprehensibly, is Kepler's First Law. In Year V's unrolling of Newtonian theory, we shall claim that inverse-square gravitation and Newton's Laws of Motion combine to predict planetary ellipses for planetary orbits. But pupils will have to take the mathematics on trust.

doubt and put it into their growing knowledge of physics. In that way we may be able to carry our teaching of physics farther.

PROGRAMME

Continuing with Dynamics: Kinetic Energy. *If Year III provided good, simple preparation for measurements to test or illustrate $F = Ma$, and if the class has worked through the study of momentum rather quickly, though not too heavily, we should now proceed to a study of kinetic energy – theoretically, and experimentally with trolleys.*

Alternative Order, for a Change. *If, on the other hand, the experiments with tapes have proved burdensome and repetitive, the class will need a change now. In that case, we should bring forward some material from later in the Year. The main topics that follow the study of kinetic energy are:*

quantitative Kinetic Theory of gases
heat and the Conservation of Energy
electric circuits and p.d.; power
electron streams
electric charges and Millikan's experiment.

Unfortunately, each of these involves a discussion of energy, with at least some reference to kinetic energy; so it would be better to treat kinetic energy first. Apart from isolated topics, such as demonstrations of electron streams or class experiments in electrolysis (if they are planned), the only main topic that can be brought forward comfortably is the first part of kinetic theory. Pupils are ready to treat gas pressure as due to molecular impacts. So, if a change is needed, we suggest starting on the kinetic theory treatment outlined in a later section of this Year. That should not be carried beyond a prediction of $PV = \frac{1}{3}Nmv^2$ *and an estimate of molecular speed. Then we should take up kinetic energy – with that '*$\frac{1}{3}Nmv^2$*' offering an interesting introduction. That would also have the advantage of making a break in the long theoretical treatment of kinetic theory.*

As we do not know what teachers will decide in this choice of order, we shall continue dynamics here, taking up kinetic energy.

† Note to Teachers of Classes who missed Earlier Teaching of Energy in this Programme

This note outlines some of the treatment of work and energy suggested in the Nuffield Physics programme for Years I and II. Since some schools begin at Year III instead of Year I, we offer this summary as a suggestion of teaching that will now be necessary for such classes. Teachers are also advised to consult the notes on: 'Work', 'Conservation of Energy' and 'Perpetual Motion' in the General Introduction at the beginning of Year III.

If the teacher who has taken over this class is himself not familiar with the discussion in Years I and II he should be urged to read the discussion in those *Teachers' Guides*. The discussion below is only a short summary of the earlier material.

† **Early Energy Teaching and the Concept of Work.** At this point, some special teaching of work and energy changes will be essential if any pupils of the group missed the full discussion of 'work' in Year II. Then, our particular meaning of work was explained and pupils made measurements and did calculations. If our pupils missed that, they would find a sudden plunge into the kinetic energy discussion below very puzzling – and if they met a different view of work in some other science teaching, for example as a name for potential energy, they would be confused unless we gave a careful discussion of our view now.

† **Energy, Jobs, Fuel.** We consider energy to be something we get from 'fuels' (such as coal, oil, food, also sunshine), whose change to other forms provides for 'useful jobs' such as hauling up a load to the top of a building, speeding up a car or a bullet to higher speed – and presently we shall include heating up bath water.

Such useful jobs were described in Year I as those jobs that need fuel, jobs that cannot be done without fuel. Though that looks like talking in a circle, it actually provides a clear criterion: they are jobs that man can only get done at a cost, by drawing on his food supply or using an electric supply from a power station or using a supply from a waterfall, etc. They *are not* jobs (such as maintaining a big force at rest) that could be done by a stationary paperweight or by tightened G-clamp. They *are* the jobs for which we have to pay money to provide the fuel or food. (True, sunshine brings us free fuel sometimes, but we have only to examine the needs for fuel on a cloudy day to find whether we are dealing with a fuel-needing job or not.)

We then discuss the amount of fuel demanded by a job. We take the simple job of raising a load of, say, 2 pounds 3 feet. We imagine that job broken up into six separate jobs, each of raising one pound one foot. We assert that the fuel demands for each of these 'unit jobs' are all the same. That is not an unreasonable assertion because we can picture the man doing the whole job by using a cord and pulley and standing on the ground to pull down one foot of cord with a pull of one pound-weight in just the same way for each unit job.

He pulls hand over hand, with his back straight, so that each unit job costs him the same amount of 'food energy'. Calculating on that basis, we see that we can use [force] $\times$ [distance] to measure the total job.

That 'job' involves an energy transfer, FROM chemical energy (food) TO potential energy. We measure the job by [force] $\times$ [distance], which we name work. We do not say that the work we calculate belongs to either chemical energy or potential energy alone. It simply tells us the amount of energy transferred. It tells us the energy lost by our muscles; and it tells us the energy gained by the raised load.

† **Energy Forms.** Meanwhile, we build up an informal feeling for 'energy' by describing various forms of energy, always with an underlying, but hitherto unspoken, assumption that energy is something universally conserved, something changed to another form or moved from one place to another but never manufactured or destroyed.

We describe the energy given to a wound-up clock spring or a stretched spiral spring as 'potential energy or strain energy' stored in the material of the spring. (We called that, at first, 'springs energy', for simplicity.)

We describe the energy gained by a load that we raise higher up as 'gravitational potential energy', probably stored in the gravitational field. (We called that, at first, 'uphill energy'.)

We see that when we have given the spring or the load extra potential energy it could do a useful job for us by hauling up some other load as it loses that potential energy; or in losing some of that potential energy, it could make some object move faster. And, in return, a body that is moving fast can provide potential energy by winding up a spring or raising a load, providing it is allowed to lose some of its motion in doing that. So, we build up a qualitative idea of 'energy-of-motion', which we name kinetic energy. (We called that, at first, 'motion energy'.) We illustrated this by experiments in which a small load hung on a thread pulled a trolley along, increasing the trolley's motion energy at the expense of the potential energy lost by the load; and by experiments in which a moving object, such as a trolley, gave up its motion energy and came to rest while compressing a spring or raising a load.

We should show such demonstrations again now (still carefully avoiding too precise an account-keeping over the exchange between K.E. of a trolley and P.E. of a pulling load, because there are losses at the inelastic impact each time a pulling thread is jerked taut).

† **A List of Energy Forms.** In earlier Years we described the following forms of energy:

strain energy, the potential energy stored in a stretched spring, wound-up spring, bent beam, etc. ('springs energy').

gravitational potential energy, which is increased when a load is raised ('uphill energy').

kinetic energy ('motion energy').

molecular energy. In addition to the strain energy of a bent spring, etc., we must imagine energy, stored in intermolecular or interatomic fields – which changes when melting or evaporation occurs. Until we have studied heat and linked it fully with other forms of energy, we cannot say very much about this; but we should point clearly to the extra energy that steam has in comparison with water at the same temperature.

chemical energy, stored in fuels and food, involved in chemical reactions. (This could be called molecular energy, or atomic energy, using 'nuclear' for the energy released in radioactivity.)

electrical energy (mentioned without clear description).

radiation energy (which pupils met in the 'radiation circus' of class experiments in Year II. See *Teachers' Guide* for Year II, Experiments

90–98, for knowledge pupils should bring from those experiments. No clear description of the nature of radiation was given then).

nuclear energy (mentioned in Year I when a cloud-chamber and a simple spark-counter were shown. No explanation).

† **Heat and Conservation of Energy.** We also *mentioned* the idea of taking heat (measured by mass-of-water multiplied by temperature-rise) as probably a form of energy. We certainly made it clear that, when some kinetic energy disappeared, unaccounted for by a gain of P.E., we notice that heat appears. In Year III we talked qualitatively of the kinetic energy of gas molecules and may have suggested that the heat-content of a gas is related to that energy. But at no time so far have we faced the conservation of energy fully and openly. Yet we have had to talk as if energy is something that does not get lost, that does not appear from nowhere, that can be changed to other forms.

We are only now approaching a full discussion both of the many forms of energy and interchanges between them, and of the precise account-keeping which leads us to believe in universal conservation of energy.

† **Transfers of Energy from Form to Form or Place to Place.** We discussed, in Years I and II, many kinds of energy transfer, giving examples and showing experiments, for example:

1. A boy kicks a large box along the floor. The box comes to rest. (FROM chemical energy of muscles TO kinetic energy TO heat.)

2. A toy steam engine driven by a Bunsen burner raises a load on a string. (FROM chemical energy of coal-gas + oxygen TO heat energy in flame TO heat and molecular energy in steam TO gravitational potential energy.)

and many more. (See *Teachers' Guides* for Year I, C/D 74 and Year II, C/D 61.)

In describing each change we avoided any wording which would give the impression that energy is ever created or destroyed. We insisted on saying, for example, energy is transferred 'FROM chemical energy in the boy's muscles TO energy-of-motion of the box'. We pointed out that it is not *creation* of energy that benefits mankind, but the *transfer* of it to some other form. It is helpful to transfer energy from muscles to the P.E. of a load of bricks raised to the top of a building, because we need to have the bricks up there. It is helpful to transfer the chemical energy of some coal and oxygen to heat in some bath water because we want a hot bath.

† **Work as a Measure of Energy Transfer** (see Note on Work in General Introduction, at the beginning of Year III). In all those cases where transfer of energy from one form to another (or just from one place to another as with a see-saw) involves a force pushing through a distance, we can estimate the transfer. As mentioned above, we can break up the job of raising 2 pounds 3 feet into 6 'unit jobs' of 1 foot.pound. The result of multiplying [force] by [distance] tells us how many such unit jobs are required: it tells us the amount of energy transferred. Often, it indicates the amount of fuel that we need to draw upon that energy transfer. So, the product [force] times [distance] is a very useful one as a measure of energy transfer. Therefore, we give it a name, 'work'.

This is a return to the old-fashioned use of the word, *work*, for energy transfer. In our programme, we suggest that *work* should be used with that meaning throughout. Teachers will find it makes the discussion of energy changes clearer.

So we urge teachers to use work as a measure of energy transfer and not as a name for mechanical energy, etc. The Nuffield Chemistry Group is following our practice regarding 'work'.

† **Units for Work and Energy.** Since work, measured in foot.pounds(-weight) or in newton.metres, measures the amount of energy transfer, the same units must apply to measures of energy itself. At first we used foot.pounds for work (and, therefore, for energy itself). In the first two years, we introduced a newton as an arbitrary, little understood, unit of force which we said would be found to be universal. With faster groups we suggested teachers might measure work in newton.metres. But most pupils are unlikely to have met any units other than foot.pounds(-weight) in serious use until now – in Year IV – when newtons have come into full use.

† **Machines.** At an early stage in Year I, pupils did an open experiment with a simple 'see-saw' to look for a lever law; but we did not suggest extracting a formal rule of moments. Then when

work and energy were discussed in Years I and II, we looked at the energy changes involved when a balanced see-saw is tilted. Using [force] × [distance] or work as a measure of energy transfer, we compared the work given to the see-saw when one side moved down with the energy given out on the other side as it moved up. That might be a transfer of potential energy of one load sinking to potential energy of another load rising; or it might be a transfer from chemical energy in muscles of someone pushing down one end (unloaded) to potential energy of a load raised on the other end. Using a simple appeal to similar triangles, we demonstrated the ideal behaviour of such a see-saw: output energy is exactly equal to the input energy.

We mentioned the practical fact that a little less energy appears at the output than goes in at the input: but we left unanswered the question of the fate of the missing energy.

Pupils also experimented with a simple set of pulleys with an ideal mechanical advantage of 3 to 1. Looking at the ideal force-ratio and at the distances moved by 'effort' and 'load', they found that here again the ideal output energy is just equal to the input energy.

'Machines' like this do not manufacture energy. They do appear to lose a little energy between input and output and pupils are likely to suggest themselves that the difference goes into heat through some mechanism of friction.

† **Power.** Pupils may have made some estimates of rate-of-transfer of energy in earlier Years and may have met the name 'power'. But in any case this is the time for a fresh start.

KINETIC ENERGY

We now start our Year IV teaching of energy. We remind pupils that energy belonging to motion is something very important, which we need to know more about for moving rockets, moving gas molecules and lots of other moving things. We named that energy informally 'motion energy' in Years I and II. By now, we should call it kinetic energy.

We are now going to treat kinetic energy as a measurable quantity. We shall arrive at an expression for calculating it, and pupils will do class experiments. But the theoretical argument and practical measurements will not prove easy at this stage unless pupils have a clear picture of kinetic energy and a good feeling for it from the start. So we suggest teachers should begin with some simple qualitative demonstrations. These should be shown quickly without any attempt to measure or calculate. Pupils will presently repeat some of them as class experiments before proceeding to measurements, but demonstrations seem best now for a clear introduction.‡

In each case, we follow our practice of describing the energy change as a transfer FROM ... such and such a form ... TO another form.

Kinetic Energy: Qualitative Demonstrations

a. **FROM Chemical (Food) Energy TO Kinetic Energy.** Put a trolley on the table; give it a shove. (Or put a pupil on roller-skates and give him a shove.) D 58a

b. **FROM Gravitational P.E. TO Kinetic Energy.** Put a trolley on the table and give it some motion by letting a falling load pull it. Run a thread horizontally from the trolley to the edge of the table and over a pulley to a small load hung on the other end. Let the load fall a short distance to the floor. The thread runs slack and the trolley is left moving at constant speed. D 58b

Show the reverse: start the trolley moving with a push away from the pulley and let it pull the thread taut and raise the load as it comes to rest. (Here too one should recite the changes as they happen: 'FROM chemical energy TO kinetic energy TO gravitational potential energy.')

This trolley experiment too could be replaced by a much more dramatic one in which a pupil on roller-skates is pulled by a horizontal cord that runs to a pulley and down to a load.

c. **FROM Strain Energy TO Kinetic Energy.** In this case, a trolley is set in motion by a spring. It is good for pupils to see the strain potential energy being stored up in the spring first. One end of the spring, A, is anchored at the end of the bench. The other end, D 58c

‡ We are so familiar with the concept of interchanges between kinetic energy and potential energy that we think of the experiments described here as obvious and would be satisfied with thought experiments: imagining men getting cars going, weights accelerating things, a spring buffer bringing a moving thing to rest. But we should be unwise to think that these are equally obvious to pupils who are learning about energy. Although we could persuade pupils to imagine these changes, it is better to show them.

B, is pulled along until the spring is stretched a lot, and then attached to the trolley, which is held at rest. The trolley is released and gains kinetic energy.

There should be a long piece of thread between the end of the spring B and the trolley so that after the spring has finished contracting the trolley has some way to run at constant speed without getting entangled with the spring.

As in the previous demonstration, this can then be reversed: the trolley is given kinetic energy by a shove and then spends it in stretching the spring.

Here, too, the trolley on the table may be replaced by a pupil on roller-skates, pulled by a much stronger spring.

d. **FROM Strain Energy TO Kinetic Energy TO Strain Energy Again.** (This is an important experiment that pupils will soon do quantitatively. Teachers are advised to give a qualitative demonstration either now or just before the class experiment.) D58d

A trolley is given kinetic energy by a catapult at one end of the table; it runs along the table, meets another catapult at the other end, and comes to rest storing up strain energy in the second catapult. The message of this experiment is likely to be more telling if the trolley runs on a friction-compensated runway; and that will be essential in the quantitative experiment.

At this stage, we only comment on the energy changes: we ask how the initial and final stores of strain energy compare, just from the look of the catapults. We make no attempt to measure the stretches of the catapults, still less to calculate estimates of strain energy.

Some teachers prefer to show this preliminary demonstration now; others prefer to leave it until pupils are setting it up as a class experiment.

Catapult Designs. The catapult consists of a large rubber band or loop of rubber thread stretched, just taut, between two massive retort stands, one each side of the trolley's runway, higher than the top of the trolley. When the trolley is pulled back to the starting end of the runway a vertical post firmly fixed on the trolley hits the rubber band and pushes the middle part of it back, stretching it.

The stretched rubber band acts as a catapult accelerating the trolley for a short distance until the post is clear of the band. The trolley then runs at constant speed to the other end, where the post meets a similar rubber band catapult.

(The form of catapults suggested seems the simplest and most suitable. Other forms have been tried, in which a rubber band is pushed by a specially shaped 'snow-plough', or a steel blade is made to bend and store strain energy; but we do not recommend these. True, the bending blade stores energy more reliably – it does not suffer from the lags and fatigue effects of rubber – but it does not seem to pupils such an obvious device as the rubber band. They know that energy is stored in a stretched rubber band or coiled spring; but the bending of a blade is an unfamiliar form.)

(There is a variant of the rubber band catapult in which the band is installed on two posts *carried by the trolley* and pushed and stretched by a fixed post which it meets at each end of its run. Though that has the advantage of using the same rubber band for energy storage at both ends, it does not seem quite so clear a demonstration as the one suggested with two matched rubber bands, one at each end.)

Any of these arrangements will project the trolley, giving it considerable speed, starting from rest, then leaving it to travel with constant velocity to the other end of the runway. There a similar device brings the trolley to a stop. This can be done as a qualitative experiment to illustrate energy changes, as suggested above, or, if pupils make measurements of the force-characteristic of the catapult, it can turn into a quantitative experiment to test whether $\frac{1}{2}mv^2$ is a good measure of the energy gained from stored strain energy – (see below).

e. **Collision with Magnets.** This is a grand experiment, very valuable as a qualitative demonstration. It would probably be a nightmare as a class experiment. Strong horseshoe magnets are installed on two trolleys so that they act as repelling buffers. Catapults, as in the previous experiment, are set up at the ends of a level runway. The trolleys are pulled back each to a catapult stretched by a post as before. The trolleys are released, run towards each other, are brought to rest and pushed away again by the magnets and return to their catapults. D58e

During the collision, when the trolleys are momentarily at rest the kinetic energy they had has disappeared and is stored as some form of 'potential energy'. Magnetic fields are stores of energy, and in this case we have increased temporarily the amount of energy stored in the fields of the magnets.

Calculating Kinetic Energy

We can make a body move, or make it move faster, at the expense of chemical energy from fuel or some other supply. We need to be able to calculate the energy transfer to kinetic energy in such cases. And we would like to know how much energy we could get back from a moving body, into some other form, by bringing it to rest.

In general, we imagine that a moving body has a store of energy 'because of its motion'. And now we want to calculate the amount of that energy, with the help of Newton's Laws of Motion.‡ By combining $F = ma$ (or $Ft =$ change of mv) with our use of work or [force] $\times$ [distance] as a measure of energy transfer, we find the kinetic energy is given by $\frac{1}{2}mv^2$.

Is K.E. Real? A Note to Teachers. In discussing kinetic energy with our pupils we should not throw any doubt on its reality; and yet we should remember that our estimate of a body's kinetic energy does change when we change our own frame of reference. If we stand at rest and look at a moving body, we endow it with a certain amount of K.E.; but a moving observer endows the moving body with a different amount of K.E. If he is moving along beside the body with the same velocity, he considers the body has no K.E. Only in the case of the mixed random motion of gas molecules

‡ *Note to Teachers: Possible different approach to Dynamics.* It has been suggested that, as an entirely different approach to Dynamics, we might study kinetic energy empirically: measure the mechanical jobs that a moving trolley can do in coming to rest, and arrive at the following properties of kinetic energy by experiment:
K.E. varies directly as [moving mass] and K.E. varies directly as [speed]^2.
Thus we could make some estimates of kinetic energy without having to derive the expression $\frac{1}{2}mv^2$ from Newton's Law II. That does not seem such a fruitful line of teaching, even if somewhat easier, unless we make a wholesale change of policy over the teaching of conservation of energy and the treatment of heat, and adopt heat as our fundamental form of energy. If we did that, we could start with heat, and proceed through kinetic energy (measured by the heat developed when K.E. is destroyed) to potential energy and thence to a study of momentum and Newton's Laws. That would produce an interesting and tenable scheme of teaching; but at present we think it would prove difficult. So, we do not suggest that in our programme.

(heat) do we find that a change from one observer to another who is moving with a different constant velocity makes no difference to our calculation of energy content.

Finding a 'Formula' for Kinetic Energy

Before we show pupils how to arrive at the expression $\frac{1}{2}mv^2$, we should talk to them about the way in which kinetic energy is given to a moving body.

A force must push the body along in the direction of its increasing velocity. We calculate the transfer of energy from some other form to kinetic energy by calculating the product [force] $\times$ [distance], the *work.*

When we introduced energy in Year I we gave qualitative descriptions of energy forms and energy transfers; but in Year II we measured the transfer of energy from one form to another by that product and we called it work. Thus, in our treatment in this programme, work is not a form of energy; it is not a name for potential energy or mechanical energy. Work is simply a calculated value of the energy transfer from one form of energy to another.

Pupils who have forgotten our use of [force] $\times$ [distance] or have not met energy-teaching with work in this form, may confuse it with the momentum changes which we have just been calculating by [force] $\times$ [time]. We should point out that in contrast with change of momentum which is given by [force] $\times$ [*time*], we are now going to deal with something different, that is given by [force] $\times$ [*distance*].

Algebraic Method

We imagine a force pushing an object along, making it move faster. That force must be the RESULTANT (or net) force on the body. Then we can use that as the force F in $F = ma$. To find the expression for kinetic energy we calculate the work, [force] $\times$ [distance], which tells us the energy transfer to kinetic energy. If we are dealing with constant acceleration we can say:

$$\begin{aligned}
[\text{force}] \times [\text{distance}] = Fs &= F\tfrac{1}{2}(u+v)t \\
&= m\left(\frac{v-u}{t}\right)\tfrac{1}{2}(v+u)t \\
&= \tfrac{1}{2}m(v-u)(v+u) \\
&= \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 \\
&= \text{gain of } (\tfrac{1}{2}mv^2)
\end{aligned}$$

With average or slower groups it is easier to start from rest and simply show, by the method above, that [force] $\times$ [distance] $= \frac{1}{2}mv^2$.

Example. If the moving body is being pushed forward with a force of 30 newtons and dragged backward with a force of 10 newtons (perhaps friction), then the force we must use in calculating K.E. is 20 newtons and not 30. That force is the F in $F = ma$. Then we could use $F = ma$ and $v^2 = u^2+2as$ and find that the work, Fs, is equal to $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$, which is the gain in the quantity $\frac{1}{2}$(mass).(velocity)^2.

Recommended Alternative: Geometrical Method

However, the proof above is likely to seem too long to pupils to make sense. Also it appears to refer only to acceleration by a constant force, which is unfortunate when we want to treat kinetic energy as a very important quantity that is independent of the particular way in which the moving body acquires its motion.

Therefore we suggest teachers should try the following method themselves; and then, if they approve, with a group of pupils. It is a derivation that is much easier to do with a blackboard and talk than in print – unless one is allowed to use calculus notation for small quantities. We can *talk* about 'a little bit of distance travelled ahead' or 'this little bit of extra momentum' but, in *printing* an outline of the method here, that would be too clumsy; so we shall use calculus notation, δs and $\delta(mv)$, on the understanding that *there is no suggestion whatever of giving the explanation to pupils with that notation.* It is only a shorthand in the present communication with teachers.

'We want to find out how much energy we transfer to a moving thing when we make it move still faster; how much energy it stores up with its motion as "motion energy" or "kinetic energy". We shall show that you can calculate the total kinetic energy of an object of mass m kilograms moving with speed v metres per second by working out the value of $\frac{1}{2}mv^2$. Presently you will get quite used to saying kinetic energy $= \frac{1}{2}mv^2$; and you will find that very useful in dealing with moving electrons and other particles in atomic physics. T

'Suppose a moving object is already moving with some speed v and we are pushing on it with a (resultant) force F. Suppose the force pushes the object ahead a short distance δs. Then the work which tells us the transfer of energy *from* our muscles, or whatever else is pushing, to the moving object is [force] $\times$ [distance] or $F\,\delta s$. That is the increase of kinetic energy in that short space. If we know the speed v at that stage in the growing motion, we can calculate δs.

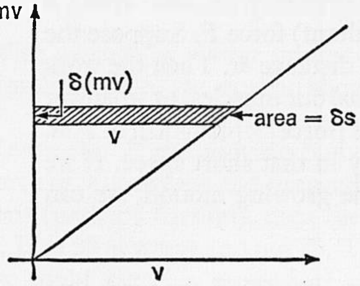
'Distance = [speed] $\times$ [time]. So $\delta s = v\,\delta t$. Then the work that shows the increase of kinetic energy is $F\,\delta s$, or $F\,v\,\delta t$. But you already know what $F\,\delta t$ is.

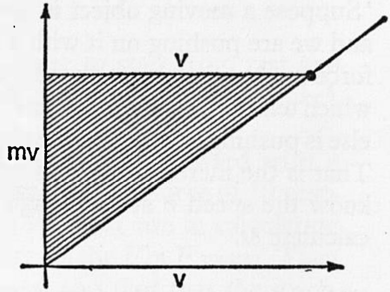
'It is [force] $\times$ [time for which it acts] on this body which is moving and accelerating, so $F\,\delta t$ is the object's [gain of momentum].

$$\begin{aligned}\therefore \text{ Increase of kinetic energy} &= F\,\delta s = F\,v\,\delta t = F\,\delta t\,v \\ &= (\text{increase of momentum})v \\ &= \delta(mv)\,v\text{'}\end{aligned}$$

Note to Teachers. The part above is the difficult part of the story. It may be better to postpone it until after the strange graph described below has been sketched and discussed.

Draw on the blackboard a strange graph, showing v plotted along and mv plotted upwards. Ask pupils what shape that graph line must have.‡





‡ This is an unfamiliar kind of graph. Pupils who think of a graph as a way of showing experimental results may object that this is not an experimental graph but only a geometrical game of drawing a line which is necessarily a slanting straight line. When they realize that our argument will still give us the same answer for kinetic energy, $\frac{1}{2}mv^2$, even if the point on that line which shows a particular velocity and momentum moves up and down that line with quite irregular motion (if the velocity changes in a quite irregular way), they may feel still more uneasy. To comfort them, we should give two examples:

1. We show them how we can derive $s = \frac{1}{2}at^2$ by drawing a graph of t along and at up. This applies *only* to constant acceleration, because with a constant, the graph is a straight line through the origin. This is, of course, a simple case of the 'Galileo's geometry' method which we used before to arrive at $s = ut + \frac{1}{2}at^2$; but in this case we are labelling the graph differently, so that its coordinates have the same silly property of being necessarily proportional (if a is constant). A strip of width δt and height at has area $at \,.\, \delta t$, which gives the distance travelled, δs. Then the whole distance, s, is given by the area $\frac{1}{2}at \,.\, t$.

2. We offer to show that the area of a circle is πr^2. That is likely to be amusing and worth seeing, provided they already know the definition of π in the form 'circumference of circle $= \pi \times$ diameter'. Many a pupil just learns $C = 2\pi r$ and $A = \pi r^2$ without knowing whether one leads to the other; so we must start by explaining the basic meaning of π. We might illustrate that by making a rough experimental estimate of π with a tape-measure and some round object, such as a wastepaper basket, whose diameter is easily measured. And then we point out the use of that in reverse to estimate diameter where only the circumference is easily available – for example, the diameter of a man from a tailor's measurement.

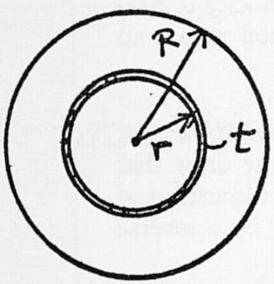
Give them plenty of time to think about it. To accomplished mathematicians the answer is obvious, although even they may be surprised when they ask themselves what kind of motion this graph applies to – it applies to *any* motion, however irregular.

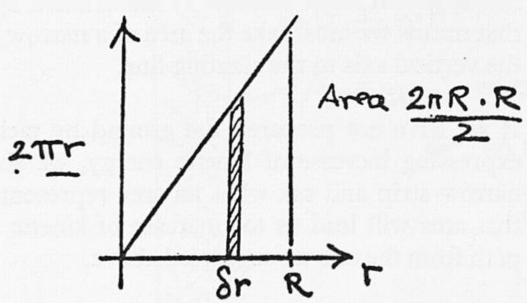
The shape is not obvious to beginners, and needs careful introduction, by plotting point after point:

> 'Here is one speed that object might have; and we multiply that by m (which might be 2 kilograms) and plot m times that speed, upwards. Here is another speed which the object might have and we multiply that by the same m (still 2 kilograms) and plot that mv upwards; and so on.'

Pupils see that the graph must be a slanting straight line running through the origin.

We move a pencil or piece of chalk up the line to show how it would be plotted by various kinds of motion. For motion with unchanging speed, the only part of the line visible would be a single point for the right velocity. For uniform acceleration from rest, the characteristic point would move steadily up the line from the





Then we offer to obtain $A = \pi r^2$ *from* $C = 2\pi r$. We draw a large circle of radius R and mark on it a circular band of radii r and $r + \delta r$ (where δr is the width of the band). We ask what the area of the band is ... $(2\pi r)(\delta r)$. We plot a graph, like the mv–v graph under discussion, with r along and $2\pi r$ up. On that we draw a vertical pillar of height $2\pi r$ and width δr. Pupils will see that the area of the strip gives the area of the ring $(2\pi r)(\delta r)$. Then we ask for the area of the whole circle, made up of such rings all the way from the centre out to radius R. Teachers who prepare the ground by asking the question about πr^2 as an intriguing offer, will find that this gives considerable help in making the momentum graph sensible and clear.

origin to the final speed. For the motion of a pendulum bob, the *
characteristic point swings up and down the slanting line, with *
simple harmonic motion, between extreme points corresponding *
to $+v_1$ and $-v_1$. *
*
(We need not give that latter example; but we should somehow *
make it clear that the characteristic point can move in lots of *
different ways. And yet if it starts at the origin and ends at a par- *
ticular speed v_1 the kinetic energy that the object has gained is, as *
we are now going to see, $\frac{1}{2}mv_1{}^2$.) *

When pupils understand the behaviour of this strange graph, we T
take two points on the slanting line quite close together and draw horizontal lines back to the vertical axis, up which we have plotted momentum mv. We mark the corresponding two points on that vertical axis and ask what the distance between them shows. Pupils should tell us that it shows the small gain of momentum $\delta(mv)$. We ask:

> 'What must you multiply that gain of mv by to find the work: the transfer of energy into kinetic energy?'

We have already discussed the way of expressing increase of kinetic energy in terms of change of momentum. Pupils will tell us we must multiply that small increase of momentum by the velocity v. And that means we must take the area of a narrow horizontal strip from the vertical axis to the slanting line.

If we have not prepared the ground by tackling the problem of expressing increase of kinetic energy, we should now draw that narrow strip and ask what its area represents. And discussion of that area will lead us to 'increase of kinetic energy' by a reverse path from the one we suggested above.

Area of strip $= v\,.\,\delta(mv) = v\,.\,F\,.\,\delta t = F\,.\,\delta s =$ work

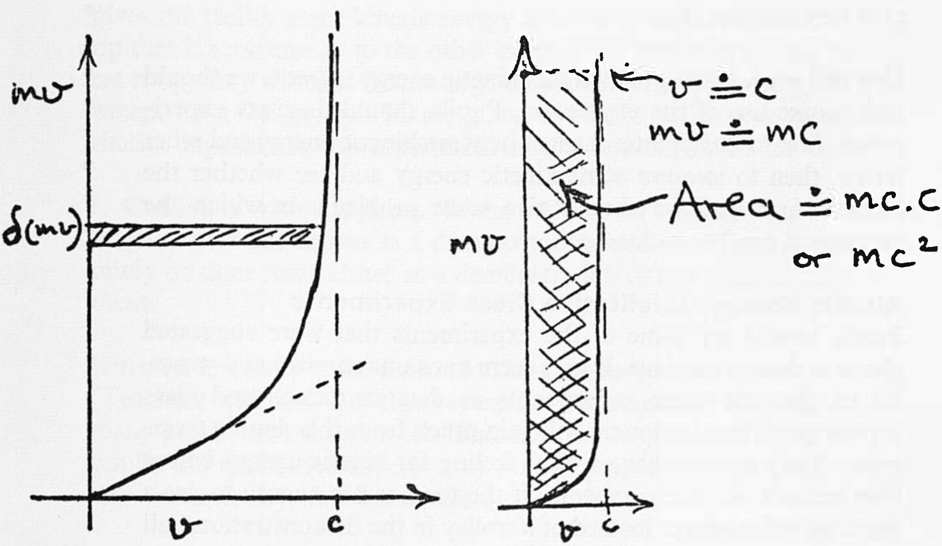
Then if some object starts from rest and ends at any particular velocity v_1, its total kinetic energy is the total of all the transfers shown by the little bits of work, and is therefore the area of a certain triangle on the graph. That triangle is half of a rectangle of area $mv_1 \times v_1$. Therefore kinetic energy $= \frac{1}{2}mv_1{}^2$ when velocity is v_1.

Seen for the first time, this seems an abstruse method; but we trust teachers will experiment with it.

Note to Teachers: Extension to Relativistic Energy. Incidentally, at A level this method can be extended to an interesting commentary on mass in special Relativity. Suppose we make the following assumptions:

1. [Force] × [time] = change of (mv), as in ordinary Newtonian dynamics *but m may not be constant.*

2. As suggested by some experiments on high-speed charged particles, m does in fact increase with increasing speed – not noticeably at the ordinary speeds of a car or an aeroplane or even a rocket, but very noticeably at the higher speeds that we can give to electrons or charged atomic particles. Mass piles up to higher and higher values as a moving body approaches the speed of light. We shall assume that mass approaches an infinite value as the speed approaches c. If so, we see at once that we have no hope of accelerating any material body to the actual speed of light by any finite force.



3. We continue to calculate kinetic energy as the total of all the pieces of work $F.\delta s$ from rest to the speed in question, so it will be given as before by an area to the left of the line on our strange graph.

Now we plot the same graph, of momentum (upward) against velocity (along) but extending to much greater speeds. We run along the horizontal axis from rest to the speed of light, c, as a practical limit. We run a vertical axis from the origin up to an enormous value of mv when the mass has increased greatly when the speed approaches the speed of light. The graph-line is a straight slanting line from the origin at first, and presently curves up more and more steeply until it is asymptotic to the vertical line given by velocity $= c$.

As before, we look at the area to the left of the graph to obtain the expression for kinetic energy. At low speeds we obtain $\frac{1}{2}mv^2$ as before. However, if we go up to some high speed, v_1, close to the speed of light, c, the area becomes a tall strip of height mv_1 or almost mc. And it is a rectangle, of width v_1, or almost c, except for a small 'triangular' piece at the bottom – which for present purposes we may neglect if we have proceeded to high enough energies. Therefore, as an approximation for *very* high energies, we have a total kinetic energy $mc \times c$, or mc^2. This is not the proper relativistic expression for kinetic energy. We should subtract m_0c^2, the rest energy of the mass. That piece to be subtracted is actually given by the 'triangular' area that we neglected, but there is no easy way to show that.

Use of $\frac{1}{2}mv^2$. Having shown that kinetic energy is $\frac{1}{2}mv^2$, we should make some use of the expression. Pupils should do class experiments, first to *look at* interchanges between kinetic energy and other forms, then to *measure* some kinetic energy and see whether the formula fits. And we should give some problems in which the expression can be used.

Kinetic Energy: Qualitative Class Experiments

Pupils should try some of the experiments that were suggested above as demonstrations. It may seem to us unnecessary and wasteful to give the same experiments as demonstrations and class experiments, but beginners will gain much from this double treatment. They need to have a clear feeling for kinetic energy before they embark on measurements. If the teacher has chosen to use a pupil on roller-skates instead of a trolley in the demonstrations, all the better, because now trolleys will be used.

We suggest the following, which correspond to the suggestions above for demonstrations:

a. **K.E. from Muscles.** Pupil places a trolley on the table and gives it a shove. C59a

b. **K.E. from Falling Load.** Pupil adjusts his runway so that it is compensated for friction, places a trolley on it and arranges a thread to pull the trolley. The thread runs horizontally to a pulley at the end of the runway, over the pulley to a small load. This is used both ways: the falling load accelerates the trolley and gives it kinetic energy; and the trolley, given some kinetic energy by a backward shove, raises the load as it comes to rest. C59b

c. **Catapult gives Trolley Kinetic Energy.** Pupil arranges a long stout rubber band across the runway, above the level of the trolley, between two massive retort stands. He installs a vertical post on the trolley, to meet the rubber band and push it back when the trolley is pulled back to the beginning of the runway. With the catapult thus loaded, he releases the trolley and sees it gain kinetic energy. He also tries reversing this. C59c

d. **Trolley with two Catapults** (*Optional*). This was suggested above, D58d, as an important demonstration, in which one catapult gives the trolley some kinetic energy and the trolley later on gives up that kinetic energy to the other catapult. If pupils have time to set it up, they will find it amusing and profitable to play with: but except for a fast group who are going to return to this as a quantitative experiment, this experiment may be omitted if time is short. C59d OPT.

e. **Collision with Magnets.** If the collision of trolleys with two magnets was not shown as a demonstration earlier, it should certainly be done now, either as a demonstration or as a class experiment. D/C59

Kinetic Energy: Quantitative Class Experiments

(Teachers will find that these seem more strange and difficult to many pupils than they expect. Therefore we urge teachers to give pupils the qualitative experiments first as preparation, preferably on a separate day, well before the quantitative ones.)

Measurements with P.E. changing to K.E. The potential energy lost by a falling load which pulls a trolley is calculated from measurements and compared with kinetic energy acquired by the trolley. C60a

Pupils arrange a trolley on a friction-compensated runway, pulled by a thread that runs horizontally to a pulley and down to a small hanging load. The load should be small, so that there is not a large error in forgetting that the load itself acquires some K.E. The trolley is released from rest. The load is allowed to fall a measured distance to the floor, while its fall accelerates the trolley to a final speed which then remains constant. That constant final speed, after the thread has run slack, can be measured either by timing the trolley with a stopwatch over a measured distance, or by the use of tickertape. In the latter case, we must be careful to see that the tape does not exert a pull comparable with the pull of the pulling load; in fact, we must arrange the friction compensation to include the effect of the tape.

Pupils calculate the kinetic energy, $\frac{1}{2}mv^2$, for the trolley. They must measure the mass in kilograms, and the speed in metres per second to obtain the K.E. in newton.metres or joules. Pupils calculate the P.E. lost by the falling load, [weight of load] $\times$ [height fallen]. They must remember that weight is a force, the pull of the Earth. If they want to calculate the potential energy in joules, they must express the weight in newtons; and for that they should use the mass of the pulling load in kilograms and the strength of the Earth's gravitational field in newtons per kilogram.

Pupils should compare their calculated results: [kinetic energy gained by trolley] with [potential energy lost by load]. They are likely to be disappointed when they find these do not agree. It would be wise to prepare them for that disappointment by asking them beforehand whether they consider the experiment reliable and easy.

Even if we have conscientious scruples about this calculation *
because it is really repeating our derivation of the expression $\frac{1}{2}mv^2$ *
in the reverse order, we should be wise to give it to pupils, since it *
forms a preparation for potential-energy calculations in discussing *
the work of Joule and others on general conservation of energy. *

This form of the experiment, with the load falling, will probably seem simpler to pupils, and make measurements easier, than the reverse form with the load rising. Teachers might add the latter experiment as a buffer option. For that, the trolley starts near the pulley, with the thread slack and the pulling load resting on the floor. The pupil gives the trolley a sharp shove to set it moving. A C60b OPT.

tape record is taken to measure the velocity before the thread pulls taut; and then the rise of the load is measured.

Measurements with Strain Energy changing to K.E. A loaded catapult projects a trolley along the table. The K.E. gained by the trolley is measured and compared with the strain energy lost by the catapult. The latter is estimated by measuring the work involved in loading the catapult. C61a

This is a good class experiment for giving pupils a confident, quantitative, understanding of kinetic energy – provided they do not find the reasoning and the manipulation too difficult. With a fast group this will prove well worth while. But with average or slower groups it may well do more harm than good, so teachers should consider very carefully whether the abilities and interests of their class justify this experiment. It should not be drawn out into a series of attempts and long discussions of the meaning of the graph. The time would be better saved for other topics that lie ahead.

The catapult should be a long rubber band stretched between two massive retort stands one each side of the runway. The trolley carries a vertical post which pushes the rubber band back and stores energy when the trolley is pulled back. The pupil arranges a friction-compensated plank with the catapult near the starting end. The catapult must be placed so that the trolley is pulled back a considerable distance in 'loading' it because the pupils must make measurements of that distance, and of the catapult's force, for their calculation of its strain energy.

If pupils have not tried the catapult in a qualitative class experiment they should start by playing with it and watching its working and gaining some skill in manipulating it.

Then we discuss measurements with them. We ask them how they could make measurements to find how much energy is stored up in the loaded catapult. This is difficult, because, as the pupils will soon find, the force changes as the trolley is pulled back and the catapult is stretched. To calculate the strain energy stored in the loaded catapult, pupils will need to make measurements and plot a graph of [force] versus ['distance-pulled-back']. (An unfamiliar word such as distortion or displacement would be unwelcome here.)

The Graph. Show pupils carefully that the area under such a graph represents the *work*. That work measures the transfer of energy FROM chemical energy in our muscles as we push the trolley back to load the catapult TO strain energy in the catapult. Take a small part of the total-distance-pulled-back and draw vertical lines up from that to the curve that has been plotted from measurements. The area of that pillar is the work,

[the force at that stage] × [that small distance moved].

The total area under the graph – from the beginning of the loading process, when the post on the trolley first touches the catapult to the position where we hold the trolley ready to launch it – gives the energy stored in the catapult before launching.

It would be helpful to give a homework problem a week or more beforehand on calculating energy transfer by the area under a graph. The problem should start with a case of constant force, when energy is being stored by hauling up a load. Then the problem should ask about a case of steadily increasing force, as when the spring is stretched from no load to some load. Finally the problem should ask about a case where the force changes in a complicated way, making a curved graph. That preparation would prevent the dismay of meeting a strange shape of graph for the first time in the actual experiment with the rubber band – whose geometry is not simple.

The measurements for the graph will need considerable care. One pupil should pull the trolley back with a spring balance and read the force for various stages of loading the catapult while another pupil measures the distance the trolley has been pulled back from an initial mark. It is best to use a spring balance graduated in newtons – that will save one possible confusion.

Often, experimenters make a set of measurements and then when they plot a graph are disappointed to find that there is some region where they need more points. If the teacher has discussed the use of the graph very fully with pupils beforehand, he can ask them to plot a very rough preliminary graph as they work, simply to see where they need to take more measurements.

When the trolley has been launched and is travelling, free from the catapult, pupils measure its speed, roughly with a stopwatch and metre rule, or more carefully with tickertape.

Pupils then compare their results for strain energy lost and kinetic energy gained. Here, too, it would be kind to draw their attention to the difficulties beforehand, so that they do not expect tremendously close agreement. Even rough agreement should give comforting support in our present discussion of energy changes.

Energy Changes with a Trolley started and stopped by Catapults. Pupils arrange two rubber band catapults, one at each end of the runway. The first catapult shoots the trolley along the runway and the second catapult brings it to rest. Pupils compare the strain energy lost by the first catapult with the strain energy gained, finally, by the second catapult. If they wish to turn this into a grand experiment of measurements, they also measure the speed of the trolley so that they can calculate its kinetic energy at the intervening stage. C61b

Since this experiment is conducted in one direction, from catapult no. 1 to catapult no. 2, the runway should be tilted to compensate for friction. We certainly should not 'rig' the experiment to give right results for the wrong reason, but we should give it every chance to provide a fair comparison.

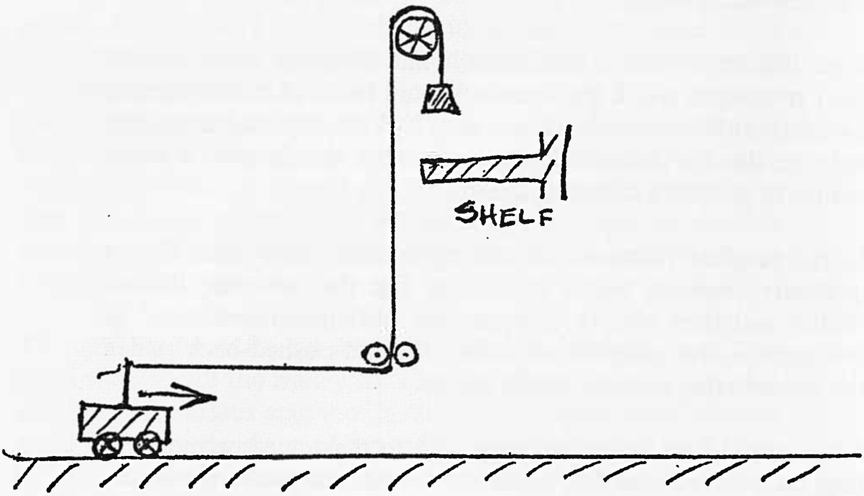
In the simplest treatment of this experiment, little more than a qualitative version, pupils make sure that the two catapults are similar and then simply compare the 'distance-pulled-back' in loading the first catapult with the 'distance-pushed-back' when the second catapult receives the trolley.

Some pupils in a fast group may wish to make measurements of both catapults so that they can make a closer comparison, with the help of graphs to calculate the work for each.

Temporary Use of K.E. to Move an Object across the Table. Living in a world where there is plenty of friction, pupils think that energy must always be supplied to move an object from one place to another at the same horizontal level. Our demonstrations with solid CO_2 deny that; and our experiments with two catapults and a trolley should support the ideal story of no energy being needed for a move across the table. Pupils will say dutifully that the potential energy of a brick at one end of the table is the same as its potential energy near the other end. They will agree that if the brick is at rest in the first position and again at rest in the second it D62 has gained neither K.E. nor P.E., yet they feel uneasy when we say the voyage from one position to the other requires no energy.

In fact, unless we are prepared to allow infinite time for it, the voyage does require some energy, temporarily, on loan from some store. To get across in reasonable time the brick must move quite fast; it must have some kinetic energy. So to transport it we must give it some kinetic energy from somewhere; but at the end of the trip we can get that energy back.

Although the catapult experiments illustrate that, a demonstration in which a small falling weight provides the loan of energy seems to make things much clearer to many pupils.



A trolley is placed at the beginning of a friction-compensated runway; a thread runs horizontally from the trolley to a small pulley above the mid-point of the runway, then vertically up and over another pulley to a small weight hung on its end. A platform is arranged below to stop the weight when it has fallen, say, 1 foot. The trolley is released, pulled by the descending weight for its first foot of travel, and then proceeds at constant speed along most of the runway. But the small pulley over the mid-point of the runway is arranged to hold the thread so that when the trolley nears the end of its run it pulls the thread taut again and raises the load as it comes to a stop.

We should like to see the load hauled up just as far at the end as it fell at the beginning. Then we could say 'the small load delivered some of its potential energy to the trolley; the trolley ran along with the kinetic energy it had been given; then the trolley gave up its kinetic energy to potential energy as the load was raised to its original height. Except for that loan, no energy has been needed to get the trolley from one end to the other.' Unfortunately there is not only friction but an unavoidable inelastic impact, when the thread pulls taut. And, unless the mass of the falling load is a very small fraction of the mass of the trolley, the load itself will waste an appreciable amount of kinetic energy when it hits the platform. Altogether we must expect, and shall see, a much smaller rise than the original fall of the load. Nevertheless this seems to teach the general idea successfully.

Of course we must avoid the trolley hitting the pulley and we must make some arrangement for the thread to be kept on the pulley. Therefore we install a post on top of the trolley and run the thread from that to the pulley placed a little higher still. And instead of one pulley wheel we have two wheels, one after the other like the wheels of a bicycle, with the thread caught in the narrow space between them.

Discussion and Demonstration with Pendulum

We point out that when we pull a pendulum to one side we haul T
its bob higher and it gains some gravitational potential energy. There is no way in which energy can be lost through the thread of the pendulum, unless its support is insecure; so we expect that all potential energy lost while the pendulum bob swings downward will turn into kinetic energy. Therefore, if we know the height of rise of a pendulum from lowest point to its starting point at rest, we can calculate its K.E. at lowest point, and thence its speed there.

We point out that this also applies to a frictionless trolley running down a hill of any shape; a marble rolling to and fro; a car on a scenic railway.

Newton knew this property (though he did not discuss energy) and *
he used it to calculate the speeds of pendulums at their lowest *
point when he was investigating conservation of momentum. He *
did some very ingenious experiments with colliding pendulums, *
making clever allowances for air friction. He calculated the momen- *
tum of each of his two colliding pendulums before and after *

collision and assured himself of momentum conservation. He tried pendulums of many materials – including one with wheat inside, in case organic materials had a different behaviour. This is not something that we need to discuss with pupils but it is an interesting matter to have at the back of our minds for teaching; Newton's own account in the *Principia* is impressive.‡ * * * * * *

Long Pendulum: Changes of Energy. If the laboratory has a very massive pendulum that hangs from the ceiling, preferably on a steel wire, we should let it swing to and fro commenting on the changes from potential to kinetic energy and so on. Then we ask pupils whether they expect the pendulum to come back to the same height when it returns. If it does not, where has the energy gone to? D 63

If it is a very massive pendulum, and a long one so that it never moves very fast, air friction will be relatively unimportant and we should expect the pendulum to return to the same place. We pull it out until it just touches our own head, then let it go and stand still and wait for its return. It is more comfortable, and just as entertaining for the class, if we stand with our back to the swinging pendulum and wait for it to return and just miss brushing against our hair. This seems an obvious demonstration to any mature physicist – barely worth trying – but it is not just entertainment for pupils; it preaches a definite moral.

Pendulum Measurements (*Optional*). Pupils might measure the speed of a pendulum bob at its lowest point and compare that with the value predicted by assuming that the potential energy lost is equal to the K.E. gained.

a. The speed could be measured by any of a number of methods. With a long massive pendulum, tickertape can be attached to the bob, and skilful pupils can run the tape and read the record. C/D 63a OPT.

b. The bob can be arranged to interrupt a light beam from a lamp to a photo-diode, the duration being measured in milliseconds by the scaler. The bob needs to be large for that; and it should be round so that if it rotates it does not alter the measurement. It may be necessary to install a large cylindrical collar round the bob to act as the obstacle for light. D 63b OPT.

‡ Isaac Newton, *Principia* (ed. Cajori), University of California Press (2 vols.). W. F. Magie, *A Source Book in Physics*, McGraw-Hill.

c. The method used for measuring the speed of a bullet with the scaler could be adapted to this. A spike attached to the bob of a massive pendulum is arranged to pass through two strips of metal foil, one a little before the lowest point and one soon after, breaking each in turn. The connections are the same as for the rifle bullet measurement. This would be an amusing demonstration if teacher and pupils are already familiar with the method; but it will not be very precise. (Suppose the pendulum is 2 metres long, and the bob is pulled 1 metre out to the side so that the pendulum swings 30° either side of the vertical. Then if the strips of metal are placed 10 centimetres apart near the bottom of the swing the scaler will only count about 40 pulses.) D 63c OPT.

Multiflash Picture of Pendulum. As a mixture of demonstration and class experiment, we take a multiflash photo of a moving pendulum and each pupil gets a print to analyse. This gives such an interesting detailed exhibit of the motion that we ought to make sure that every pupil sees it. The object photographed may be the complete bob of the pendulum, or it may be a small steel ball attached to the bob and illuminated by a distant floodlamp. D 63d

Galileo's Pin-and-Pendulum. At this stage, pupils should certainly see again Galileo's famous 'pin-and-pendulum' experiment, described in Year III. A long pendulum is allowed to swing to and fro and then a firmly supported peg is interposed some distance above the bob at the lowest point. The pendulum which has been swinging down in the shallow arc has to climb up a steep arc; and pupils look to see whether the heights are the same for both arcs. This experiment fails miserably unless both the pendulum and the peg are very firmly supported, so that energy does not leak out through the supports. This was Galileo's answer to the problem of friction's spoiling his very simple downhill-and-uphill story, which he was convinced was correct. C 64

Galileo's Downhill-and-Uphill Experiment. Pupils should see the direct downhill-and-uphill experiment, of letting a steel ball roll down a hill of curtain-rail, along the level and up another hill of different slope. If they are genuinely willing to blame the failure to reach the original height on friction, well and good; but if they have to accept our excuse to that effect as a matter of duty rather than their own conviction, we must doubt whether this is a good experiment. However, pupils enjoy this experiment with rolling balls. They sometimes combine it with a collision with D 65 (=D 18) another ball at the bottom – leading to no very clear result. An ingenious pupil may even wish to bend the curtain-rail so that the ball goes up and down many hills, like a scenic railway; and we should encourage that.

A further extension, in which the curtain-rail is bent into a shape that makes the ball 'loop the loop', is a delight to see, and it will arouse questions that lead into discussions of circular motion in Year V. However, it is probably better postponed. D 66 OPT.

'Kinetic Energy Disappears'

We take two trolleys connected by a thin rubber thread or a weak spring and start both of them moving apart. When they have come to rest, we seize them and hold both still in our hands, and ask where the kinetic energy has gone to. Pupils will reply that it has been stored up in the strain energy of the spring. We ask 'Can we get it back?' Then we release the trolleys and get the kinetic energy, or most of it, back. D 67a

Then we start with a spring stretched between two trolleys held at rest, far apart. We let it pull them towards each other and give them plenty of kinetic energy; and we wait until they meet with a bang. If the trolleys are arranged to stick together on meeting, there will be no more motion and we have lost the potential energy of the spring and lost the ensuing kinetic energy of the trolleys; and we ask where the energy has gone to. D 67b

Then we arrange the two trolleys to collide elastically with spring buffers, starting with a stretched spring or rubber thread between them, arranged so that when the trolleys meet there is still a little tension. We pull the trolleys apart and let them go. The trolleys accelerate, collide, bounce away, return, collide several times, and finally come to rest. D 67c

Again we ask where the energy has gone to. There will be some who maintain that the energy has become sound and heat; so we may proceed to the question, 'What makes you think heat is a form of energy?' – or this topic can be introduced by some other route according to the path pupils or teacher assume. We mention now, if not before, the heating of car brakes, and the heat developed as one slides down a rope. T

Heat as Energy? We are moving towards a new and careful enquiry about heat as a form of energy, which undercuts our earlier practice of quietly taking it for granted that heat is at least something like energy. The point of view that we might take now is that we find it reasonable, satisfying and *tidy* to think of energy as always conserved; and we cannot do this unless we are allowed to consider heat as a form of energy in the same terms as other forms that we have been using. T

Universal Conservation

Later this Year, we shall discuss very carefully the great series of *
experiments which convinced scientists that heat must be counted *
among the other forms of energy and measured interchangeably *
with them. That is one of the best examples we can give of building *
up a tremendous body of evidence and then, feeling so sure that the *
conclusion is right, that we are willing to extend it still further. We *
should not at this stage or earlier say anything about heat and *
energy that will make that historical discussion look stupid or *
pedantic; yet we can profitably lead them to a qualitative dis- *
cussion of heat as molecular motion at this point and take up *
kinetic theory of gases again before we discuss the great nineteenth- *
century experiments on conservation of energy. *

SPECIAL PROGRAMME: AN EARLY LOOK AT PLANETS NOW

Preparation for Astronomy in Year V. *In Year V we shall use the development of planetary astronomy as a great example of the building of theory. Newton's theory of gravitation pulls together a great deal of knowledge, giving explanations and making predictions. We shall take pupils through the early history of planetary astronomy, to a stage where they can appreciate the value of Newtonian theory and see how Newton's Laws of Motion were used in that grand scheme.*

We shall not be teaching astronomy for the sake of factual knowledge of stars and planets, but shall have to begin with some descriptions. Pupils will need to know a little about the daily and yearly motion of the Sun, and the Moon's monthly motion. And they will need to know about the planets and their strange motions through the star pattern. We must teach them such things or they will not know what the theories made by scientists, from Greeks to Newton, aimed at explaining.

We hope the pupils will make some simple observations of the Moon, to see its rapid shift through the star pattern from night to night; and we hope that they will look at the two brightest planets, Venus and Jupiter. The weather next autumn when they are studying astronomy may not offer good seeing at the right time. Therefore, we suggest teachers should encourage a little amateur astronomy during the present year. Now or at any convenient time they should ask pupils to watch the Moon, and perhaps bring in notes to mark positions on a communal map. And they should point out Venus and Jupiter and encourage pupils to look at them with field glasses, or better still a small telescope. That early preparation will be of great help later.

Newspapers give a star map from time to time that will tell us the season to choose for looking at Jupiter and Venus. Amateur astronomical societies will give help willingly. Teachers should not feel that they have to be amateur astronomers themselves. Finding a planet is not difficult, and a first look at it through the telescope is as great a delight to a teacher as to pupils.

Chapter 2

KINETIC THEORY OF GASES

Models; Simple Derivation and Predictions; Estimate of Molecular Diameter

MOLECULAR PICTURE: SOLIDS, LIQUIDS, GASES

We remind pupils of our earlier picture of the three states of matter; in every case made up of small particles – atoms or molecules – but with different amounts of arrangement ('order') and different types of motion. (If we feel we are well-in the thermonuclear age, we may want to describe one more state of matter: a 'plasma' of electrons and atomic nuclei.) T

Solids. In solids we picture atoms or groups of atoms arranged in a regular 'lattice', like the patterns on a wallpaper, but in three dimensions. They are held in that lattice by strong forces – attractions at short distances and repulsions at very short distances – which give the solid its strength. We now know that both forces are electrical forces arising from the electric charges in atoms, charges of electrons and charges of nuclei, subject to the ordinary inverse-square law of forces between charges, but disciplined by some quantum rules as well. T

The atoms in a solid are in constant motion, vibrating in various directions with vibrations of characteristic frequency, and of amplitude determined by the temperature.

We should show a model of atoms in a crystal, represented by massive balls joined together in a cubic 'lattice' by weak springs. (If the model vibrates as a whole, we have made the springs much too strong and used 'atoms' that are not massive enough. Golf balls placed 4 or 5 inches apart, held by springs like those suggested for the 'springs investigation' in Year I, would make a good model.) D 68

That model, rather like a three-dimensional spring mattress, illustrates the structure of a solid but it does not show the complexity of vibrations very well.

At higher temperatures, vibrations of atoms in solids may carry them so far out in the range of atomic forces that the ties are too weak to hold them in their well-ordered arrangement, and the solid melts. T

A teacher who does not mind considerable confusion, can make a very good model of this with the whole class of pupils seated regularly in desks. He asks the pupils not to pretend to be at absolute zero (at which, on simple classical theory, they would have no vibration at all – though on modern theory they would have D 69 a little vibration) but to warm up to room temperature by vibrating to and fro. In order to provide a good model of a solid we should not rely on the desks alone but each pupil should put an arm on to the shoulder of a neighbour so that there are strong forces of linkage. This may go better still if the pupils are not in desks but are standing in regular array on an open floor. Then we ask pupils to rise to a higher and higher temperature; and the 'crystal' will come to pieces. We say, 'The class has melted.'

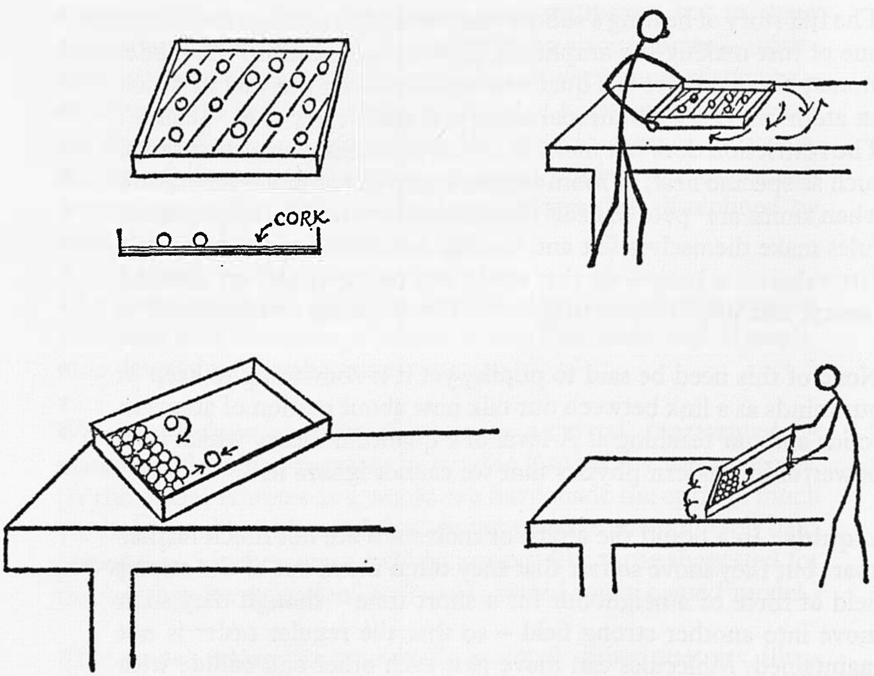
The full story of heating a solid is more complicated than the simple one of just making the amplitude grow as we warm up the model atoms; because there is a quantum-restriction on the way in which an atom can carry vibrational energy; it must carry it in 'quanta'. The restriction does not make itself felt in ordinary measurements, such as specific heat, at room temperature. But at low temperature when atoms are 'poor in heat' the currency restrictions of quantum-rules make themselves felt and specific heats drop to unexpectedly low values – a behaviour that could not be explained on classical theory, and itself helped to point to the quantum restrictions.

None of this need be said to pupils; yet it is something to keep in our minds as a link between our talk now about motion of atoms in solids and our teaching at A-level of a quantum theory which is so powerful in modern physics that we cannot ignore it.

Liquids. In a liquid the atoms or molecules are not much farther apart but they move so fast that they often move out of the strong field of force of a neighbour for a short time – though they soon move into another strong field – so that the regular order is not maintained. Molecules can move past each other and collide with neighbours so that the liquid behaves in its characteristic fluid way.

(If we think about this carefully, we may come to doubt whether the liquid can have lost all sense of order and regimentation. If it has, it is really only a very highly compressed gas, and we know that liquids are different from that, because we can have a bottle containing liquid in the lower part and vapour above – a two-phase system which we could hardly imagine if the liquid itself were an entirely disordered random system. In fact, liquids do show some local short-range order; but they do not have the completely organized state of a crystal, so they form, *as seen through our macroscopic eyes*, disordered fluids).

Class Experiment: Model of Liquid. We return to the tray of marbles, used as a two-dimensional model of a gas. We ask pupils to hold the tray slightly tilted so that all the marbles run down to one end (without running on top of each other), and then add more marbles until the tray is, say, one-third full. They should agitate the tray gently and watch the motion of the marbles, looking for 'diffusion' and for 'evaporation'. C70



Treated as a silly toy, the tray will fail to show much in this experiment; but if pupils want to use it for a good model they will find that with careful adjusting of the tilt and the amount of agitation they can make it very fruitful. It will help when we wrestle with an important question in our measurement of molecular size later on: 'How close can molecules be crowded together and yet show the fluid behaviour of a liquid?' We shall need a very rough guess.

Energy and Fusion. We should certainly expect to have to spend some energy tearing the atoms of a solid away from the strong attractions of neighbours and making a liquid. That is the latent heat of fusion, the heat that we have to supply to make melting occur without any temperature rise. T

Yet we should hardly expect that demand of energy to be as big as the demand for tearing the molecules of a liquid away from each other into vapour, because in melting the molecules do not move out of the range of attraction of neighbours, while in vaporization they move far out of range, moving farther against attractive binding forces. Compare 80 kilocalories per kilogram of ice to be melted with 500 or 600 kilocalories per kilogram of water to be turned into vapour.

Gases

In our picture of vapours and gases we must imagine that there are enormous spaces between one molecule and the next, *unless* the change from liquid has somehow made molecules swell up (or else made them multiply). The volume change of something like 1 to 1000 from liquid to gas tells us that the spacing apart must have increased enormously. We should ask pupils how far apart the 1:1000 expansion from liquid where molecules are practically 'in contact' – therefore, 1 diameter apart centre to centre – would carry them. We should not at this stage tell them the answer, but, in fact, some teachers find that pupils suggest it easily: the cube root of 1000. We should give praise for their answer but ask those who know it not to 'give the show away' to others.

That kind of thinking-out of new knowledge from some piece of information is good science; and we want to do everything we can to have pupils know that it is good science. 'Science tells the answers' is a very dangerous view, if people once adopt it. It is not a description of real science. 'Science finds out. Science does experiments. Science does some thinking. Science arrives at more knowledge by hard work.' Those are more useful, if we value the reputation of science.

In gases, molecules are far apart, and are probably moving fast, if they have considerable energy. We might, of course, picture a stationary set of molecules exerting some strange field of force upon each other – and that was in fact what Newton himself suggested for air – but pupils who have seen the Brownian motion are likely to favour a kinetic theory of gases in which the molecules are moving about rapidly and can buffet a small speck of smoke ash. So, when we ask: 'How can we put energy into a gas?' pupils will say, 'Heat it'; and we encourage them to interpret that as meaning an increase of the molecular motion.

Unless pupils are fully familiar with the three-dimensional gas D 71
model and the two-dimensional model with marbles in a tray,
we should show that demonstration again now and give pupils the
class experiment apparatus for further trials. Both these experi- C 72
ments throw more and more light on the molecular story as pupils
grow more mature and ask more tricky questions of the model.

We suggest, for the moment, that all the increase of energy when T
we churn up a gas, or compress it, or pass it over a heated wire, etc.,
can be accounted for as extra energy of motion of the molecules.
With this preliminary view we proceed to develop our kinetic
theory of gases.

Energy and Evaporation. We should not spend much time dis- *
cussing vapours, evaporation, boiling, etc. Those are interesting *
descriptive parts of physical science but our pupils' progress into *
atomic physics and other modern topics will not be seriously hurt *
if we do not treat change of state in detail. But now that we are dis- *
cussing energy and molecules we should certainly ask what happens, *
from the point of view of molecules, when a liquid evaporates. *

We ask pupils how they know that molecules in a liquid attract T
each other. We hope for the answer 'surface tension'. Unless a
molecule in liquid is attracted by near neighbours, we could hardly
expect a liquid to hold together and show the tight 'skin effect' of
surface tension. Pour some mercury, with a splash, on a clean glass
table and ask pupils to see how little gravity distorts the smallest
drops. In a gas the molecules are too far apart (unless it is very
highly compressed) to feel the attraction of their neighbours
except for a negligibly short time when they are near a collision.

Physicists know that, because real gases at ordinary densities fit *
so closely with Boyle's Law, which theory predicts for molecules *
with negligible interaction. (Put twice as many molecules in a box *
and, provided they do not interact, we expect double rate of *
bombardment, therefore double pressure – Boyle's Law.) We can- *
not give that argument to pupils at this stage; but we should tell *
them we have reason to believe gas molecules do not attract *
noticeably. *

When a liquid evaporates, molecules leave its surface. A molecule T
which has just left the surface and is still very near to neighbours
in the liquid must feel some attraction from those neighbours

pulling it back into the liquid. In fact, many a molecule that does leave the surface falls back in – much as many a projectile flung up from the Earth falls back to it (see Experiment 70). Only a few, hurled out extra fast, have enough energy to get away altogether. Those are the molecules that evaporate.

There is no mysterious influence pulling molecules out from a liquid towards the space outside. All that happens is some molecules nearer the surface gain enough outward speed from some chance bombardment by neighbours to carry them out beyond attractions.

For molecules in a liquid there is an average kinetic energy of their *
jostling motion which is much the same as the average kinetic *
energy of a gas molecule at the same temperature. There must be *
many a molecule with average kinetic energy in the surface layers, *
and some of those must be moving outward at any instant. Those *
do not have enough energy to get away – in modern terms, their *
velocity is smaller than the escape velocity. They move out a very *
short distance away from the surface but then they fall back in. *
Only some molecules which happen, at the moment, to have *
much more than average K.E. can escape. We know that for two *
reasons: *

1. If the average energy of a liquid molecule were sufficient for *
escape, we should expect the whole liquid to come to pieces: the *
liquid would behave almost like a gas. That does not happen until *
we heat a liquid up to a certain high temperature which we call its *
critical temperature. *

2. When there is evaporation, the remaining liquid is left colder. *
Evaporation is always a cooling process. From that we infer that *
the molecules that do escape take away more than an average share *
of kinetic energy. Then since 'extra rich' molecules have left, the *
remaining population of molecules in the liquid have less kinetic *
energy, on the average, than before. *

That second comment on evaporation is so important that we T
should discuss it with pupils. They know from surface tension that liquid molecules do attract each other. We should discuss evaporating molecules escaping, and lead to the idea that only 'extra fast' molecules will escape successfully. And those will carry away an extra large share of kinetic energy, leaving the liquid cooler.

Then we point out the importance of evaporation for human beings. Our open skin is slightly moist and evaporation enables us to avoid overheating: it plays a very important part in maintaining an even temperature. Drops of perspiration that drip from our brow do no cooling; but if the surrounding air carries away water molecules that evaporate we are cooled. In a very crowded room, the air soon becomes so damp, so full of water vapour, that water molecules return to a damp brow as fast as they leave. Then there is no cooling, our temperature goes up, and we feel uncomfortable. The stuffy feeling of a crowded room is *not* due to increased concentration of CO_2: it is due to damp air preventing successful cooling by evaporation, so that we develop a slight 'temperature'.

Rapid evaporation from our skins provides much more rapid cooling than, say, a cold bath. That is why one can catch a chill so easily by walking about in damp clothes. A macintosh on top of the damp clothes, to stop the evaporation, can greatly lessen the chilling, though it is very uncomfortable.‡

Programme

We should start by talking with pupils about their own programme ahead. They are now in a position to think about molecules in a gas quantitatively. They can arrive at an expression to predict the pressure of a gas in terms of mass and speed of molecules. And they can reverse that, to estimate molecular speed from sample measurements of density and air pressure. They can see that the 'constant' in Boyle's Law is proportional to the total kinetic energy of all the molecules in the sample. From that they can at least see that it is reasonable to use the kinetic energy of a molecule as a measure of temperature. They can see, from the predicted form for pressure of a gas, that the molecules of dense gases must move slower than those of light gases. Having predicted that, they can see it illustrated by diffusion demonstrations!

This should make a thrilling expansion and consolidation of earlier, much simpler studies of gases. It should not turn into a discouraging drive through algebra that is too difficult for slower pupils, or into a period of memorizing things that seem too complicated to teach for understanding. In that case it would be better to leave kinetic theory of gases in the simpler form of earlier Years. However, we find that pupils who see that they are making a very im-

‡ For evidence see Wells, Huxley, Wells, *The Science of Life*; section on 'Air of a stuffy room'.

portant exploration in the world of molecules can take the trouble, and want to take the trouble, to surmount the difficulties of algebra. They themselves think that this is a mountain worth climbing.

We can even offer pupils a chance to find the size of a single gas molecule, and then the number of molecules in a known volume, if they have the courage to follow the quite difficult reasoning and imaginative thinking that go with the simple experimental measurements. For that, we not only show the rapid spread of bromine vapour in a vacuum but also bromine diffusing through air; and from that arrive at the mean free path of gas molecules from which we estimate the diameter of a single molecule.

Note to Teachers: 'Equipartition of Energy'

In the discussions that follow, we have to neglect one very important part of a professional physicist's kinetic theory of gases: equipartition of energy. Statistical studies, combined with the assumption that every molecular collision conserves momentum and conserves kinetic energy – that is, that the collisions are elastic – lead to the conclusion that all gases at the same temperature have the same *average* kinetic energy of molecular motion.

The full form of this theorem states that each 'degree of freedom' will have the same energy on the average. For the linear motion of molecules as they fly through the space of the container, there are three degrees of freedom, x, y, z. For a gas whose molecules are single atoms, such as helium or neon, those three degrees of freedom, each with an equal share of kinetic energy on the average, are the only ones concerned. For a molecule of more than one atom, for example oxygen or carbon dioxide, there are other possibilities too: the molecule can rotate in various ways, and the atoms of the molecule may vibrate.

The Quantum Restriction. At the beginning of this century, it seemed clear that equipartition should apply to the energy of rotational motions and vibrations. However, certain experiments – such as those measuring the specific heat of gases over a wide range of temperature – threw increasing doubt upon this. That doubt reflected doubt in turn upon the actual laws of motion and simple statistics which had made equipartition seem inescapable. Finally, it was realized that these doubts, combined with inconsistencies in other parts of physics, all pointed towards quanta. We were forced to modify our view of nature and develop a quantum theory, by the failure of equipartition among other things.

Equipartition holds for Linear K.E. However, that failure in no way affects the kinetic energy of linear motion of molecules. We are as sure as ever that all gas molecules, at a given temperature, have the same *average* kinetic energy of linear motion. Since we still trust equipartition for that, we can write:

$$\text{average } \tfrac{1}{2}mv^2 \text{ for molecules of one gas} = \text{average } \tfrac{1}{2}mv^2 \text{ for molecules of any other gas,}$$

at the same temperature. This at once tells us that we can compare molecular speeds if we know relative molecular masses. Chemistry provides us with ratios of molecular masses. But then we find we do not need even the ratio of molecular masses: our kinetic theory enables us to use gas densities instead.

Also, as we shall see, $\frac{3}{2}PV$ tells us the *total* of all the $\frac{1}{2}mv^2$ values for all the molecules of a sample (that is, the number of molecules times average $\frac{1}{2}mv^2$); therefore equipartition tells us that in equal volumes of two different gases at the same pressure and temperature, the numbers of molecules must be equal. Thus, *if* we trust equipartition, we can deduce Avogadro's rule and then the ratio of gas densities gives us the ratio of molecule masses. Otherwise, if we do not have that assurance about the rule we must look very carefully at the chemical evidence which, as Avogadro himself felt, practically forced him to adopt his rule.

Unfortunately, equipartition would seem to pupils at this stage an outrageous statement about some unseen mathematics imposed from outside, so we cannot profitably use it, and that limits the uses we can make of our simple kinetic theory.

That discussion is nothing that we should give to our pupils but is inserted here as a reminder of something that we should keep in the back of our minds in teaching.

INTRODUCTION TO KINETIC THEORY

We first discuss the qualitative picture, show models and look again at the Brownian motion.

Models. We show the model of a gas once again, with visible 'molecules' driven by a vibrating piston at the bottom of a tall transparent tube. If pupils comment that this 'gas' seems to need a continual supply of energy to maintain its motion, we reply: D 71 (again)

> 'Yes, it does, because the walls of its container are dead cold. The walls of the container of a real gas are as hot as the gas itself. The molecules of the walls are in constant vibration, and when hit by a gas molecule they "give as good as they get". T
>
> 'If you put a lot of hot gas in a room with very cold walls the molecules would indeed lose a lot of their energy. And that is what happens here with the model's "molecules"; so we have to supply energy from outside if we want to keep them as hot as they are. (Also, when the model "molecules" hit the rubber sheet at the base of the tube they make a rather inelastic collision, wasting some of their kinetic energy as heat.)'

If pupils raise the question of the uneven distribution of molecules we say that that is characteristic of a real gas and is what one would expect from chance collisions. If they raise the question of decreasing density with height we ask:

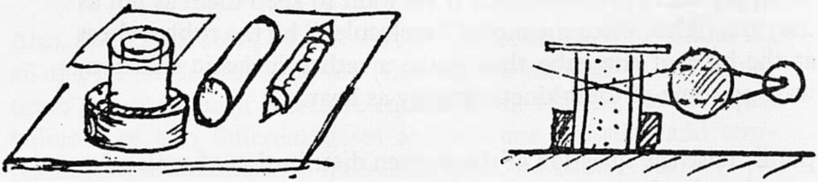
> 'Why don't air molecules all fall down to the bottom of the container? Think of a small section of air half-way up, why doesn't it just fall down? The air below it must push it up with a bigger force than the air above it pushes it down. There must be more bombardment upward on the bottom of that imaginary chunk than downward on the top. And that means there must be a more crowded population of molecules below than above. An individual molecule must have a greater chance of being hit by another molecule moving upward than by one moving downward.'

We 'raise the temperature' of this model of a gas by feeding more power to the driving motor. We ask what really determines the temperature of a gas; and move towards the idea that average kinetic energy would be a good measure. D 71 (again)

At this point ask pupils to return for a short time to the class- C72
experiment model of a two-dimensional gas: marbles in a tray with (again)
vertical sides. A larger marble is added, to illustrate the Brownian C73
motion.

Seeing Brownian Motion. An experiment to see the Brownian motion of smoke specks *in air* was an important *class* experiment in Year I. We listed it again in Year III for pupils entering the programme then. Since it is now essential, we repeat, below, the listing from Year III.

Brownian Motion. Any pupils who have not seen the Brownian C74
motion of smoke particles should now see the essential experiment, not as a demonstration, nor as a film, but as an individual observation with a microscope. The motion of particles in water will *not* suffice as a substitute. Whatever the cost in trouble and time, we should borrow microscopes, use the smoke cell‡ designed for Year I,



WHITLEY BAY SMOKE CELL

and encourage pupils to see specks of smoke ash dancing about as they are bombarded by air molecules. In our suggested programme for Year I, we urged schools to buy microscopes for pupils to look at various things; and urged them to make sure that the instruments have sufficient aperture to show the Brownian motion of smoke particles clearly. Now, for older pupils, teachers are likely to find colleagues in the biology department more willing to lend microscopes for this special use. A considerable number of microscopes will be needed, so that this does not become a quick demonstration but is something which pupils see for themselves. We consider this is such an important part of learning about molecules and atoms that it should be a personal experiment.

‡ The Whitley Bay smoke cell (shown in the sketch) is so much easier for pupils to handle than other forms, and makes it so much easier for pupils to see the Brownian motion for themselves, that we hope teachers will put these into use. They are not difficult to make: a small festoon lamp, a glass rod as a cylindrical lens, and a short vertical 'well' of glass tubing into which one pours smoke. A microscope cover glass serves as lid. The optical distances must be arranged carefully. The light should slant upward so that the brightly lit field of view is just under the lid to minimize convection.

Teachers may find that pupils who have not seen this before take as much as 15 minutes to get used to their microscope and look at the smoke and really understand what they are seeing but it is worth that. However, the very simple Whitley Bay smoke cell with lamp attached makes it much easier for pupils to see the Brownian motion for themselves. We hope that schools will use this or some equivalent design, rather than one of the older designs which provide too little light or which look too complicated to make sense to pupils when they use it themselves.

Discussion of Teaching Policy

This is now a stage at which young physicists should not just collect data and give interesting descriptions and formulate descriptive models, this is a time for more definite knowledge and some use of mathematics, the necessary tools of physics. So we should somehow, at this stage, show how physicists treat the vast horde of molecules that they imagine in a gas with Newtonian mechanics with some simple algebra and some simple averaging, to arrive at a definite prediction which can be turned to good use. We wish to show pupils how physicists predict $PV = \frac{1}{3}Nmv^2$.

That is not a result to be memorized, nor should the full argument of 'deriving' it from simple assumptions be learnt by heart as something that can be produced in an examination without the candidate knowing quite what he is doing. Somehow, we want every pupil to feel he knows what he and the teacher are doing when they arrive at that statement. That suggests for one thing that we should use different methods for classes of different ability; and for another thing that we should stimulate and encourage pupils to follow such a story through with the teacher without expecting it to be fully reproducible afterwards.

'Ah, but a man's reach should exceed his grasp, or what's a heaven for?'

From time to time a pupil should be able to say: 'I have seen that done, I have followed it through, understanding as far as I could. It was worth the trouble and time for the experience of knowing that scientists can do that and feeling that for the moment I was with them.' We do not expect a young pupil to put it in that emotional way – yet that is the general conviction that we hope he will keep.

Most experienced teachers say that it is impossible to teach kinetic theory of gases to pupils at the stage of Year IV with any derivation of the formula. Most experienced teachers say that it is possible to teach anything provided one takes enough time.

The first statement probably refers to teaching-schemes in which the pupil is expected to reproduce the material in examinations. Here where we want the pupil to make the work part of his general experience, we do not aim at facility in repeating the calculation in examinations: we believe in a more general understanding and we take assurance from the second statement that that can be achieved. Of course the second statement has a concealed condition, that the time needed goes up out of all proportion when teaching younger children. On the other hand 'where there is a will there is a way' applies in this matter to both children and teachers. If only we can build up *a wish* to guess the size of a single molecule or atom, we shall find pupils anxious to accept rough methods and special schemes instead of being suspicious of them. This deserves some skilful advertising, the teacher somehow placing an intellectual temptation before his pupils. If pupils have been taken once through the argument and seen what a wonderful result can emerge, many of them will be quite anxious to see it once more – particularly when assured that this is not something that has to be copied out in an examination – but something to be done for the sake of their own power of knowledge. The second time through will build it into knowledge of something seen with a sense of understanding.

A Choice of Versions. Teachers will find below three suggested versions of the essential argument. It is hoped that they will look at all three and make their own choice for their particular group of pupils.

I. SIMPLEST VERSION OF KINETIC THEORY

The simplest kinetic theory story of all is the one given in Year III: we imagine that molecules exist, and are in motion, and we suppose that the pressure of a gas on the walls of a box is produced by the impact of bombarding molecules. Suppose we employ a microscopic demon to put more molecules into the box one by one through a trap door. When he has put in so many that there are twice as many molecules as before, what pressure would we expect? There are twice as many molecules to bombard the walls and therefore we should expect double pressure.

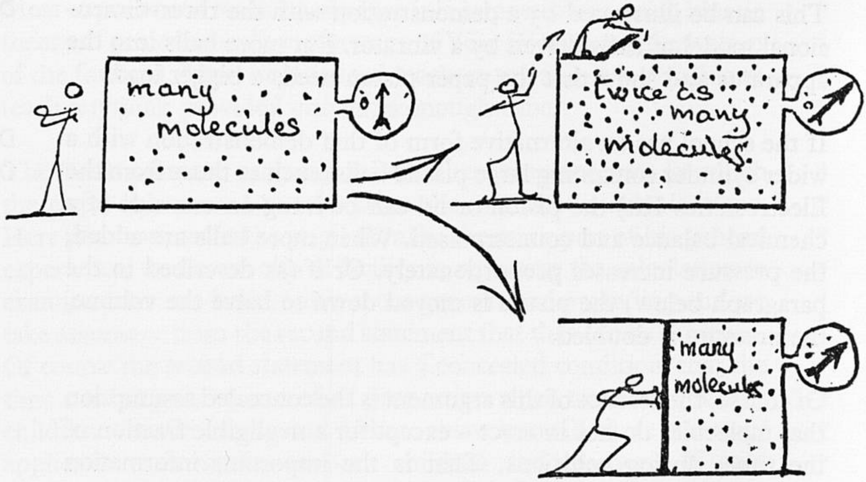
This can be illustrated by a demonstration with the three-dimensional model of balls driven by a vibrator. Put more balls into the apparatus and show that the paper piston needs a bigger load. D 75a

If the school has an alternative form of that demonstration with a wider cylinder containing large plastic balls (such as those from the Electrostatics Kit) the piston or lid can be hung on one side of a chemical balance and counterpoised. When more balls are added, the pressure increases proportionately. Or if (as described in the paragraph below) the piston is moved down to halve the volume, the pressure is doubled. D 75b OPT.

Of course, the essence of this argument is the concealed assumption that molecules do not interact – except for a negligible fraction of the time, during collisions. That is the important information about real gas molecules that Boyle's Law gives us when we compare experiment with theory.

With more neighbours to collide with, an individual molecule will not travel so far before it makes a collision, but in a collision the two molecules that collide and rebound simply exchange jobs with each other; so we should not expect the more frequent collisions to upset our prediction. We might again give a demonstration of a head-on elastic collision between equal masses: let a moving steel ball hit a stationary one head-on. The stationary one takes on the full job of moving ahead. Then repeat the demonstration with both balls moving and making a head-on collision. Whatever their velocities, the balls simply exchange motions. Of course the collisions between real gas molecules are seldom head-on: however, we are discussing and illustrating a simplified picture in which we deal with motion in one dimension.

As explained in Year III, the story of the two cars that meet on a mountain road which is too narrow to allow passing is an illustration that helps pupils to understand this: the solution in an emergency is to exchange cars and drive backwards. If we use this story, we must start by warning pupils that it is only an illustration. And after that we must sketch real molecules moving towards each other and just missing each other, and then in another case moving towards each other hitting and rebounding. Then we can ask the important question about them. Some teachers prefer to show this with two pupils running from side to side of the room, sometimes just missing each other and sometimes colliding and rebounding.



Instead of making a gas of double density by having the demon put more molecules into the same box, we could push a piston in until the volume is halved, and then we have the double density without needing any extra molecules. Then the pressure on the walls should be the same as if we had the full box and had added more molecules (because the walls can hardly know what is near them except a gas of double density). So we now conclude that when we halve the volume we should expect double pressure: we expect Boyle's Law, and are delighted to find it.

We should give a quick demonstration of Boyle's Law (not a long series of measurements, but enough to show that doubling the pressure halves the volume). We suggest using the apparatus described earlier in this *Guide*: a tall, wide tube with a piston of oil confining a sample of air, the volume of air being measured by the height of the enclosed column, visible from the back of the class, and the pressure shown by a Bourdon gauge attached to the oil reservoir and reading *absolute* pressure – also visible from the back of the class. D 76

With the simple model of marbles in a tray, it is obvious that putting in more marbles will increase the 'pressure'. Pupils should try crowding the area occupied by moving marbles to a smaller area by using a ruler as a fence. Pupils might also try adding a great many marbles to the tray, so that they can see how the behaviour is modified when the free area for motion is greatly reduced. C77

(**Real Gases: Modifications.** We can even speculate about modifications. Suppose the demon goes on packing molecules into the box until there are so many that the molecules themselves reduce the space available for motion appreciably. Then the to and fro motion of bombarding molecules has a shorter path than we would expect from our simple picture of molecules as points. Bombardments will happen a little more often, the pressure will be a little greater than we expect: and that is what we find when we push our Boyle's Law experiment to very small volumes (at high temperatures). Again, if molecules attract each other when they are fairly close – as surface tension shows – we should expect to find mutual attractions pulling the molecules, so to speak, towards a central clump so that they would not press on the walls so much. That effect would decrease the pressure when molecules are crowded close together, and moving slowly, at low temperatures.‡

In real gases we find both these effects at high pressures and small volumes; and we can distinguish them from each other because they change in different ways with temperature. At low temperatures the effects of attraction make themselves felt strongly: gases even liquefy.)

2. SIMPLE CALCULATION OF VELOCITY, WITHOUT ALGEBRAIC DERIVATION OF $PV = \frac{1}{3}Nmv^2$

(This is a strange method that will seem unpleasantly artificial to some teachers; and an interesting simplification to others. If the teacher himself tries this method and does not like it he should certainly avoid it. On the other hand, some teachers who have tried it have been very pleased to find that pupils of medium ability understand it well enough to enjoy the result, where a longer method would have been too hard.)

‡ The usual explanation – that *all* bombarding molecules are slowed by that attraction and therefore exert less pressure – is misleading because it would suggest a cooler layer of gas near the walls. The real mechanism is this: some slowest molecules are *weeded out* by the attraction and never reach the wall. Thus there is a *less dense* layer near the walls, and that exerts a smaller pressure.

We can make a crude estimate of the speed of air molecules by a thought experiment that continues a suggestion made in Year I. Now, in Year IV, pupils have the knowledge needed to finish the calculation. We tell a fanciful story, but it leads to a sensible answer. We suggest that teachers should try this if they do not intend to try the full derivation of $PV = \frac{1}{3}Nmv^2$ (and the ensuing calculation of average speed) with their class. Here it is: T

We explain that we are going to make a rough guess at the speed of the air molecules which make air pressure by bouncing on every surface. We remind pupils that we live at the bottom of an ocean of air.

We set up a barometer to measure that air pressure and measure the height of the mercury column, about $2\frac{1}{2}$ feet. Then we say: D 78

'The air pressure on the mercury outside in the bowl pushes the mercury up inside the barometer, and the column of mercury just balances the air pressure. Now imagine, just for fun, that we lived in an atmosphere of mercury instead of air. How high would the mercury have to be from the floor to the top of the atmosphere if that was all there was to make the pressure that we actually live in down at the floor? ... Yes, the height would have to be the barometer height, $2\frac{1}{2}$ feet of mercury.

'Now think about the real atmosphere. How high would that have to be if it went on up and up just as thick as the air is in this room and then stopped at the top and there was nothing more above? How high would that atmosphere of air have to be to make the pressure we measure here?

'An atmosphere of *mercury* would have to be $2\frac{1}{2}$ feet high, the same height as the mercury barometer height; because that is the height of mercury which can press on the base of anything with the same pressure as the whole atmosphere. How high would a *water* atmosphere have to be? Remember: mercury is much more dense than water. In fact, it is $13\frac{1}{2}$ times as dense as water; so a water atmosphere would have to be $13\frac{1}{2}$ times as high as the mercury height, $13\frac{1}{2} \times 2\frac{1}{2}$ feet; about 34 feet.

'Now what about an atmosphere of air – not air that gets thinner and thinner all the way up, but air that stays just as thick as it is here in this room?'

That brings us back to an estimate of the density of air. If that was done in Year I, it may have been forgotten by now; and it would be better to repeat it.

We can use the standard method of weighing a 1- or 2-litre‡ flask before and after pumping the air out. Then we let water in to fill the volume of air removed. Or we can use the method suggested for Year I.‡ D 79a D 79b

We show a comparison of densities of mercury and water: we weigh three equal bottles, one full of air, one full of water, one full of mercury, preferably on a family spring scale. If possible these bottles should also be available for pupils to try lifting them: they should be placed on a felt mat in a tray on a bench available to pupils. D 80

Density measurements show that air has a density of about 1·2 grams/litre. Mercury has a density of 13,600 grams/litre. Thence arithmetic gives a density ratio of about 11,300 : 1. Therefore, if we had a uniform atmosphere of air, its height would have to be [$11{,}300 \times 2\frac{1}{2}$ ft] or somewhere between 25,000 and 30,000 ft. T

We must warn pupils, if we carry them through this story, that the real atmosphere gets thinner and thinner as we go higher and higher. (We may show the demonstration model again.) So the story we are telling is an artificial, simplified one‡ in order to arrive at an interesting guess. 'Desperate measures for desperate needs.' D 81 (=D 71)

‡ The 500 cubic cm flask often used for this is disappointingly small. The class deserves to see a difference of at least 1 gram. We hope teachers will not use a flask smaller than 1 litre.

‡ The method used in Year I was to pump air into a large plastic container, weighing it first full of atmospheric air, and then after much air had been pumped in. Then the extra air that has been pumped in was released under water into a rectangular plastic box full of water. The air that bubbled out was seen and measured with a ruler. This gives a sufficient difference of weight for a lever balance to be used. See *Teachers' Guide* for Year I.

‡ Teachers may need to discuss the importance of rough estimates with pupils. Suggestions for that were given in the *Guide* for Year I, and in Problem C in the *Guide* for that Year.

Then we ask:

'Suppose the air is made up of little molecules, tiny things far apart; then one of them which happens to be at the very top of the atmosphere all alone, at rest, would start to fall, faster and faster and faster – like anything else that falls. It would not flutter down like a sheet of paper, fluttering against air resistance, because it *is* just a molecule of air itself, falling through spaces between other molecules. Then it will be moving very fast indeed when it reaches the ground and bounces against the floor, or the top of your shoe, or anything like that. No wonder air molecules make a big pressure.

'Let us pretend that molecules do fall down from the "top of the atmosphere" like that, bounce on the floor; and rebound, losing no energy, up to the top of the atmosphere; come to a stop there; fall down again and so on.‡ This is a very rough-and-ready model of what does happen in the atmosphere, but it may help us to make a guess at the speed.

'How fast will a molecule be moving if it has fallen from the height that we have just worked out? (Suppose we worked out 28,000 ft.) The molecule will fall with acceleration g. Starting from rest and falling 28,000 ft, the *time* it takes will be given by $s = \frac{1}{2}at^2$, so that $t^2 = 2s/a = 2 \times 28{,}000/32$. Then, t = about 42·5 seconds.

'Falling from rest for that time, gaining 32 ft/sec in each second the molecule would reach a speed of $42{\cdot}5 \times 32$ or 1,360 feet/second.'

The expected result is 20 per cent below the proper value, but we might still welcome it as a rough guess by a risky, imaginative method.

If a critical colleague tells us it is not only imaginative but wrong, we should tell him that this method was in fact used by Boltzmann to arrive at the Maxwell distribution, with a modification of the

‡ We are assuming that model molecules are perfectly elastic: so are real air molecules. We are assuming that they fall all the way without collisions: we are pretending they are infinitely small in size – but that does not change simple kinetic theory much. We are confining the motion to one direction, the vertical: that is artificial and we should admit the defect.

'uniform atmosphere' assumption. This is the children's version *
of a famous adult method. *

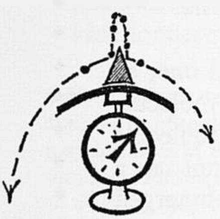
Programme

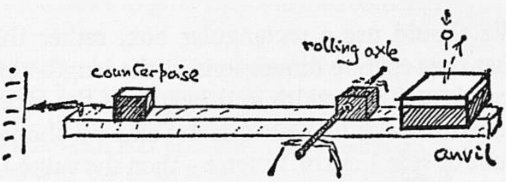
With faster groups, we hope teachers will try the formal treatment *
3, described below. But whichever treatment is chosen, we should *
show some model of a rain of molecules arriving at a surface and *
making a 'fairly constant' force by their bombardment. *

Model to show Bombardment Force

a. **Simple Model.** Pour a stream of balls on to the pan of an D82a
open, equal-armed balance, or, better, a domestic spring scale. In order to avoid upsetting the balance by an increasing load of accumulated balls, place a wedge on the pan so that impinging balls bounce off. The curved pan of the kitchen scale will serve instead of the wedge if it is inverted.

If we pour marbles (or large steel balls) one by one, in irregular succession, the balance will show a sort of Brownian motion. If we pour a stream of much smaller balls (e.g. the $\frac{1}{8}$ inch steel balls used for the 'mass exhibit', or even dry sand) the balance will show a steady force.





b. **Anvil and Beam** (*De luxe option*). We construct a see-saw D82b
with a massive elastic anvil at one end. Then we pour a stream of OPT.
balls from some distance above on to the anvil. As the stream continues to arrive, the see-saw shows there is a deflecting force on

the anvil. Although the massive see-saw 'smears out' the force – averages it – we can also see the little random motions which show that the force is made up of individual impacts.

A very good elastic anvil is made by placing a sheet of Perspex ($\frac{1}{2}$ or $\frac{3}{4}$ inch thick) on top of a massive block of steel, with a thin layer of glycerine between them. The Perspex can be screwed into the steel near the edges. This massive anvil is then placed on a large wooden beam which rests on a pivot near that end, the other end of the beam carrying a counterpoise and a pointer to indicate deflections.

(The small steel balls recommended for the 'mass exhibit' are too small for this, even in a copious rain; so we suggest dropping a stream of common marbles on the anvil from several feet above.)

Unless a very careful release system is arranged the balls will not fall vertically: then they will bounce off the anvil in many directions. This experiment is not easy to set up and it is only suggested in case some teachers consider it would be worth constructing.

3. FORMAL DERIVATION OF $PV = \frac{1}{3}Nmv^2$

Choosing Conditions that look 'Honest' to Beginners. In deriving the kinetic theory expression for pressure, we should avoid using specially simplified conditions as far as possible.

We should use a rectangular box, rather than a cubical one, so that the separate dimensions of the length along which a molecule travels and the width and height of the face which the molecule bombards can be seen clearly; and we should certainly not use a cube of side 1 cm or 1 metre – then the value 1 becomes submerged and invisible. Just as we do not want pupils to think that Newton's Laws only apply to motion where there is no friction, we do not want them to think that the kinetic theory of gases is limited to a gas in a unit cube.

We should, of course, also avoid the delightfully ingenious but artificial scheme of imagining molecules in a spherical container in which a given molecule is said to pursue chord after chord of constant size around the interior – even the more realistic version of that scheme seems oddly artificial to the beginner.

Making the Difficult Job Worth While. 'A thing that is worth doing is worth doing well' and for those who can manage it this kinetic theory derivation will be rewarding. The argument is long, but it can be left posted on a wall or it can be printed in a booklet for pupils. The only really difficult part of it is the use of rate-of-change-of-momentum to measure the force. To get over that difficulty, we should take two precautions:

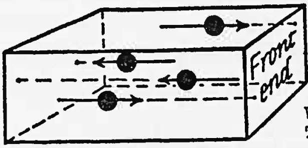
Precaution 1. We give a general description *beforehand* of what we are going to do:

> 'We are going to imagine a box full of gas molecules flying about in all directions at random. We shall pretend we know the speed of molecules, the mass of each molecule and the number of molecules in the box. We are going to try to calculate how big the pressure would be on the wall of the box, if that pressure is simply due to the molecular bombardments.
>
> 'We shall take for granted the picture of gases that we have been talking about in earlier years. We shall assume that *the molecules are very small* and travel a long way between one collision and the next, and take very little time in colliding, so that most of the time they're flying along on their own inside the box. We shall assume that the molecules are "elastic": that is, *they do not lose kinetic energy in collisions*. (After all, where is there for the kinetic energy of two gas molecules to go if they did lose some?‡)'

‡ *Note to Teachers.* Of course, gas molecules *can* collide inelastically, with a disappearance of some K.E., though they have to be enormously more energetic than gas molecules at room temperature. In an *inelastic* collision a molecule may be torn apart into separate atoms, or a molecule may be raised to an excited state, or an electron may be knocked completely out of an atom or molecule leaving it ionized; or there may be a loss of energy by radiation. All these are hopelessly improbable events among molecules of gases at ordinary temperatures as we know them.

Gas molecules at room temperature bounce against each other with perfect elasticity. And, of course, such collisions cannot 'manufacture heat by friction' – an entirely mistaken idea that some pupils conceive. The heat-energy of a gas is there *in* its molecular motion, and if in a collision a gas did lose some motion and generate heat, that would mean it was losing some of its motion and thereby increasing some of its motion!

The assumptions in building kinetic theory are mentioned in the suggested talk above; but in many cases it is better to proceed with the argument – using those assumed properties without discussing them – and then look back at the assumptions we have made after the result is obtained.



'Think about one molecule which is moving along the length of the box and hits one end like this, bang; it bounces back and flies along towards the other end of the box and hits that, bing; rebounds from that and flies back to the first end: bang ... bing ... bang ... bing ... bang ... and so on.

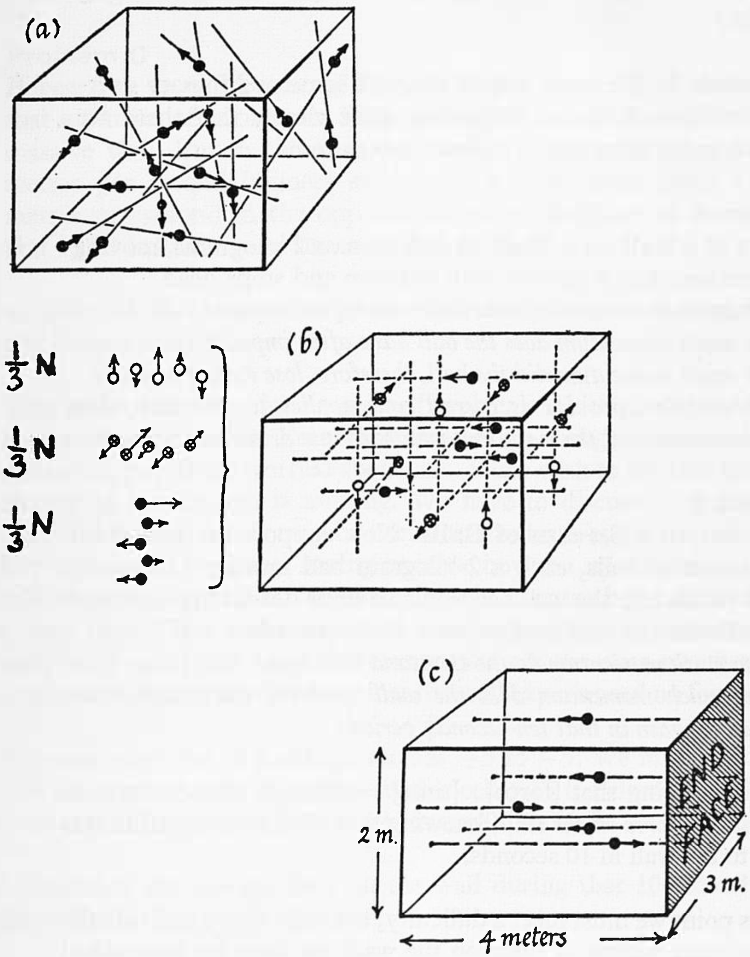
'The front end receives a small bang from a molecule every so often: bang, bang, bang, bang and so on. Each bang is a little tiny blow but if there are millions of millions of molecules in the box, these blows coming one after another in rapid succession will even out to something that feels like steady pressure.

'We are going to calculate how much one bang will do to the wall, how big a force it will exert; and then we shall calculate how big the force will be for all the molecules in the box. And then we shall divide that force by the area of the wall to find the pressure. That will give us a prediction of pressure.

'This is theory: the thinking that clever scientists‡ do tells them

‡ We suggest this description because the making of good theory *is* clever work. However, we do not suggest it as the right phrase for teachers to use with pupils: each teacher will need to modify the wording, if he uses it at all, to fit the interests or vocabulary of the class. In some cases, a phrase like 'clever scientists' is irritating; and the idea should be put differently. This kind of commentary is important in building up an understanding attitude to theory, but the proper choice of wording depends here, as everywhere in these suggestions, on the tastes of the teacher and his class.

what to look for and helps them to build more knowledge. Watch how we do it. You will find it difficult the first time but much easier later on and it will be something that you will enjoy understanding. You will not have to write all this out in an examination but you will be expected to know what we did, and why we did it, and what we found.'



Note that in this introductory discussion we did not touch the *
question of motion in different directions. If pupils ask about that, *
we should at once meet it and say that we are going to pretend that *
we can divide the molecules into three regiments, one flying along *

the length, the other … etc. Although that is very artificial, we *
had better use it here – for all but an exceptionally bright pupil – *
because the resolving of velocities into vector components will *
certainly obscure the whole business. (There is a comfortably *
easy way out of this difficulty: we say that the box has six faces and *
we divide by six at an appropriate place in the story. *Unfortunately*, *
this loses its appeal unless we keep to a cubical box; and, anyway, *
we should probably be concealing the difficulty rather than meeting *
it openly.) *

Precaution 2. We carry pupils through some preliminary exer- T
cises, Problems A, B, … explaining quite clearly that these are
preparation for success in a difficult job to come.

Problem A

Impact of a Ball on a Wall. A ball of mass 2 kilograms, moving P
12 metres/sec, hits a massive wall head-on and stops dead.

a. How much momentum did the ball have before impact?

b. How much momentum does the ball have after impact?

c. How much momentum did the ball, therefore, lose during impact?

d. If Newton's Law III is correct and applies to this case, how much momentum did the wall, and whatever it is attached to, gain?

Problem B

Force due to a Stream of Balls. Now suppose the wall is hit P
by a stream of balls, each a 2-kilogram ball moving 12 metres/
second which hits the wall head-on and stops dead. One thousand
such balls hit the wall in the course of 10 seconds.

a. How much momentum do the thousand balls lose?

b. How much momentum does the wall (and the Earth which it's attached to) gain in that ten-second‡ period?

c. Remembering that [force] × [time] = [change of momentum], *calculate the force on the wall*, knowing that all that momentum was given to the wall in 10 seconds.

At this point we must meet a difficulty; the balls arrive individually T
and so exert bumps of force on the wall; yet here we have asked
for a force over the 10-second period. We must explain that what

‡ Professional physicists proceed at once to calculate momentum change *per second*; but, for beginners, that imposes a jump which is strange and unnecessarily sudden. Using a ten-second interval makes it easier for pupils to see that they are using *force* × *time* = change of momentum.

we are asking for is not the big force that appears momentarily during a bump but a 'smeared-out average force'.

If the wall were a light target of plywood hung from a tree it would certainly show something like Brownian motion under these impacts. But as the wall is a very massive one its Brownian motion is too small to notice. A device for measuring the force on the wall would register the smoothed-out value.

Problem C

Force due to a Stream of Elastic Balls. Suppose, as above, that a stream of 2-kilogram balls moving 12 metres/second hit a massive wall. But in this case each ball arriving with speed 12 metres per second bounces straight back with equal speed 12 metres per second in the opposite direction. Suppose as before that 1,000 balls arrive at the wall in 10 seconds: P

a. Calculate the change in momentum when one ball arrives at the wall and bounces away.‡

This is the difficult point. In spite of problems which should have been given earlier in the Year when momentum changes were being discussed, pupils are worried about this. Some wish to say that the change in momentum is nothing. We have to discuss this very gently and carefully. It may be useful to illustrate by placing one boy against the wall and having another boy run up to him and collide with him head on. The first time the running boy comes to a dead stop. Then this is repeated but the running boy bounces away with equal and opposite velocity. We ask the class whether in the second case the victim against the wall experienced no force.

We need examples of a change such as +5 to −5. We might ask how far a boy climbs if he goes from a cellar 15 ft below ground to an attic 15 ft above. The answer is not 0 but 30 ft.

b. Calculate the average force on the wall during that 10-second period.

Problem D

Pressure on a Wall. Suppose, as in the problem above, a stream of 2-kilogram elastic balls arrives at a massive wall head on. In ten seconds 1,000 balls arrive travelling 12 metres per second, bounce straight back with equal speed 12 metres per second in the P

‡ See previous page for footnote.

opposite direction. Suppose that these 1,000 balls hit various places on the wall, distributed over an area 2 metres high by 3 metres wide. *Calculate the average pressure.*

We go through this with pupils repeating the whole calculation, and then dividing by area, 6 square metres, to find the pressure.

Problem E

Pressure on Wall of Box. Now suppose that we have a closed box containing just one elastic ball moving to and fro between the ends of the box. It is a 2-kilogram ball moving 12 metres/second parallel to the length of a box 4 metres long. *Calculate the average force on one end.* P

In this case instead of using the number of balls hitting the wall, we must use the number of hits made by the one ball on its repeated returns to one end.

Again, take pupils through this calculation letting them try each step in turn themselves first, giving help when necessary. Pupils should first calculate the distance the ball must travel between successive hits *on one end* of the box, and then the number of hits the ball will make in some chosen time, say, 10 seconds. T

As explained in a footnote above, we should avoid choosing the magic time of 1 second, which then disappears in the calculation. It is much better to choose some round-number time such as ten seconds and calculate the change of momentum during that period, and then divide by that period to find the force. If we choose a period of only 1 second, pupils think there is a sudden switch of attention from the [change of momentum in the chosen time] to [rate of change of momentum], and then to [force] – and that confuses them.

When they know how many hits the ball will make on the front of the box in the chosen time, pupils calculate the change of momentum for one hit, the change of momentum for all the hits and then divide by time to find the average force.

Molecules in a Box: Algebraic Version. Now we are ready to try the same kind of reasoning with gas molecules in a box. If we have the kind of class that likes algebra and wants to use it to deal with the problem of gas molecules we proceed in the usual way for kinetic theory.

We arrive very carefully at the change of momentum $2mv$ at each impact.

We call the length of the box a. We take the total distance, vt, that such a molecule will travel in a time t, and find how many return trips the molecule will make in that time t. That is $vt/2a$ trips.

We find the total change of momentum at one end face of the box in the time t. That is $(2mv)(vt/2a)$.

Then we must multiply by the number of molecules available for this, which we call $N/3$. And we divide by the area of the end face, say bc. That gives us the pressure, $\frac{1}{3}Nmv^2/abc$.

$$P = \tfrac{1}{3}Nmv^2/abc$$

or $PV = \frac{1}{3}Nmv^2$ since abc is the volume V.

For many pupils, giving the algebra straight away will lessen the chances of full understanding and it is better to start with an arithmetical example, although the big numbers involved will make it seem heavy. The arithmetical version below uses some data which are reasonably close to the actual data for air at room temperature and 1 atmosphere pressure.

(The suitability or success of this version is not easily judged without trying. Teachers will find that some class groups derive a much fuller understanding if the arithmetical version is used first, despite pupils' own claims that they would like to proceed at once to adult algebra. We urge teachers not to let their own professional skill – or the dignified assurances of the class – persuade them to omit this arithmetical introduction. Of course, it contains an unfortunate inversion of the logic of discovery because we provide data concerning mass and speed of molecules, although it is really the completed theory itself that enables us to obtain such data.

Yet pupils who proceed from this arithmetical story to the algebraic story will then be able to see how the latter is used; so the inversion will only be temporary.) * * *

Molecules in a Box: Arithmetical Version. A box 4 metres long by 3 metres by 2 metres contains about 6×10^{26} molecules. (This is a 'kilo-mole'.‡) At room temperature air molecules move with average speed 500 metres per second. T

The average mass of an air molecule is 5×10^{-26} kilograms.

We have to carry pupils through a long piece of arithmetic. As a preliminary, we discuss the problem of motions in different directions and decide to pretend that the molecules are arranged in three regiments, each of 2×10^{26} molecules. Therefore to calculate the pressure on one end of the box we take only 2×10^{26} molecules moving to and fro between the ends.

We calculate the length of one trip to-and-fro from one end to the other end and back.

We calculate the total distance a molecule travels in, say, 10 seconds, and thence the number of hits it makes on *one* end in 10 seconds.

Then we carefully work out the change of momentum that one molecule makes in one hit on one end. That is the momentum given to one end of the box at one hit. Thence we calculate the momentum given to one end of the box in 10 seconds.

Then we divide by the area of the end of the box to find the pressure. The answer will be just about 1 atmosphere. From barometer measurements, one atmosphere is just over 100,000 newtons per square metre. We point out that the data were *chosen* to give this answer. They are truthful data for ordinary air.

Algebraic Sequel. After that – preferably a week later, when pupils have had time unconsciously to consider the whole story – we give the algebraic version, and we end with the expression $PV = \frac{1}{3}Nmv^2$. T

‡ The volume of the box is 24 cubic metres and that contains 1 kilo-mole of air, at room temperature, 28·8 kilograms. [24 cubic metres] × [measured density 1·2 kg/cu. m] = 28·8 kg.

Uniform Pressure. It is often profitable to repeat the calculation for the pressure on *another face* of the box of area *ab* instead of *bc*. Although it is obvious to us that the answer will be the same, pupils find it satisfying to see the same answer emerge; and this emphasizes our knowledge that pressure is the same in all directions – even though we have really assumed that by using the fraction $\frac{1}{3}$ for every pair of faces. T

Boyle's Law?

We can point out that this expression predicts or suggests Boyle's Law *providing* we have some assurance that molecules keep the same speed, at the same temperature even when crowded closer together. Many an honest physicist would say that at this stage of the discussion he does not have any such assurance; so we should *not* make a tremendous celebration about arriving at Boyle's Law. Instead we should go straight on to a remarkable estimate: the speed of air molecules. T

Yet we should comment on the relation of our result to Boyle's Law and unless pupils have recently seen a clear demonstration of Boyle's Law, we should give one now. For many pupils, the Law is not shown clearly and simply by the traditional demonstration apparatus with columns of mercury, which involves careful discussions of adding the difference of levels (or is it subtracting?) to the barometer reading. We recommend instead a demonstration in which the sample of gas (dry air) is enclosed by a 'piston' of oil in a wide glass tube, closed at the top. By applying pressure to an oil reservoir at the side, the experimenter drives oil up the glass tube, compressing the sample of air to various volumes. Meanwhile, the pressure is shown by a Bourdon gauge (reading *absolute* pressure‡) attached to the oil reservoir. Thus, pupils see clear readings of pressure and volume. A bicycle pump may be used to apply pressure to the reservoir by driving in air at the top of it. That does not in any way change the amount of air trapped in the experimental tube but it does change its pressure. The tube should be wide enough for pupils to see the oil easily, at least $\frac{1}{2}$ inch bore. A translucent screen placed behind the tube, and illuminated from behind, D 83 (= D 76)

‡ Commercial gauges read *pressure excess* over atmospheric and professional physicists add atmospheric pressure to the reading almost without thinking. For pupils, that extra detail is confusing enough to spoil the whole experiment. It would be tragic to let the lack of an absolute gauge do that.

will make the oil easy to see in silhouette. The tube should have a coarse scale of divisions to show the volume, with zero at the closed end at the top.‡

SPEED OF MOLECULES

If we trust the expression we have just arrived at, we can calculate the speed of molecules from simple measurements. In that way our theory will produce a piece of information about one of its assumptions. We *assumed* that gas molecules are small, numerous, *speedy*, elastic, and make the pressure by their impacts on the walls. Now we can calculate the speed with which we must endow them. We can take a measured sample of gas, and knowing P the pressure, V the volume, $\frac{1}{3}$ and Nm (which is the total mass of gas in that volume), we can calculate v^2, and thence an average speed. T

Pressure. We set up a mercury barometer and discuss the measurement of atmospheric pressure all over again. Long ago, in Year I, some pupils arrived at a pressure of 14·7 pounds-weight per square inch. Now we must make our measurements in other units and arrive at the pressure in newtons per square metre. We explain to pupils that a pressure must emerge in absolute units such as newtons per square metre because we have been predicting gas pressure with the help of Newton's second law using change of momentum: therefore the force must be in the 'absolute units' that make $K = 1$ in $F = Kma$; e.g., in newtons. And since we use kilograms and metres/second the force will be newtons. D 84 (=D 78) T

We know the height of the mercury column in the barometer that balances atmospheric pressure. We point out that we might have a very wide barometer tube with a cross-section of 1 square metre and the mercury in it would still have the same height, about 0·76 metre. We calculate the total mass of mercury in that barometer sitting on the square-metre base. It would be

[0·76 metre] $\times$ [1 square metre] $\times$ [the density of mercury].

For the density of mercury, pupils may go back to a measurement in Year I, or we may take a more accurate value from tables. Mercury is 13·6 times as dense as water. As a quick demonstration of that, we may show the weighing of three equal bottles, one of air, one of water, and one of mercury. D 85 (=D 80)

‡ If the two or three readings that are taken fail to fit Boyle's Law, the fault lies in the gauge or in the placing of the zero for volume.

A cubic metre of water contains $100 \times 100 \times 100$ cubic centimetres of water or 1,000,000 cubic centimetres.‡ A cubic metre of water, therefore, contains 1,000,000 grams of water or 1,000 kilograms. So the density of water is 1,000 kilograms per cubic metre.

Therefore the density of mercury is 13,600 kilograms per cubic metre.

(This is the detailed account that we must give to pupils very *
carefully because they will meet measurements in centimetres and *
grams elsewhere. Yet if we use centimetres and grams here and *
try to calculate the force in newtons, we shall encounter mysterious *
factors like 1,000 which arise from a careless change of units. *
Teachers who prefer to use centimetres and grams will need to *
make a complete conversion to dynes instead of newtons, or else *
they will find themselves introducing factors which confuse pupils. *
We decided early in our project to advocate, and use, metres, *
kilograms, newtons, joules, watts, for the sake of *simple practical *
electrical units*: volts, amps, etc.‡ And we have already asked *
teachers to change to the new set of units that use metres and kilo- *
grams when we adopted newtons as our units for force.) *

Therefore the atmospheric pressure is T
[0·76 metre] × [1 square metre] × [13,600 kilograms/cubic metre]. This will give the pressure in kilograms-weight/square metre. To convert to the absolute units we want, we remember that the Earth pulls on each kilogram with a force of 9·8 newtons; so we multiply by the Earth's gravitational field strength, 9·8 newtons per kilogram. That makes the pressure just over 100,000 newtons/square metre.

Density. We need a weighing of air. (This is discussed here although it was discussed before, at D 79, because the earlier discussion was part of the primitive method (II) which may not have been used.)

‡ Pupils who are used to saying the density of water is 1 may find it difficult to remember that the density of water is 1,000 kilograms per cubic metre. We can convince them that it cannot be 1 kilogram per cubic metre if we show them a complete metre cube. It is well worth while to construct a cardboard cube of side D86
1 metre and have that available. We should place beside it the transparent cubical box of plastic of side $\frac{1}{10}$ metre and show that the water needed to fill it weighs 1 kilogram.

‡ See note on M.K.S. units in Preface.

We should either use the traditional method of pumping out a 1-litre or 2-litre flask‡ and weighing it full of air and empty, or return to the method‡ of Year I. The traditional method requires a special balance and would have to be done as a demonstration experiment. Some pupils will enjoy returning to the method of Year I and repeating it as a class experiment now that they are going to make an important use of it. Such measurements will show that one cubic metre of air weighs 1·2 kilograms. D87a (=D79) D87b

Then pupils calculate v from $PV = \frac{1}{3}Mv^2$. With these data pupils will find that the speed of air molecules is about 500 metres/second. This is an astounding result: over 1,600 feet/second – faster than a small rifle bullet – over one thousand miles an hour. If pupils have arrived at that with some of their own measurements they will not only be astounded but they will be impressed with it as an 'atomic' measurement that they have made. T

An Incredible Speed. What has gone wrong? Can gas molecules really be travelling as fast as that? Of course atmospheric pressure *is* astonishingly high – think of all those pressure demonstrations in Year I. Actually it is true that air molecules are travelling at that kind of speed. Of course, some molecules are travelling faster than that and others slower just as in a large group of people some are richer than average and some are poorer than average; but – unlike rich and poor, who often stay like that – a gas molecule that is moving slowly now may be moving much faster than average after the next collision and a faster one may be slowed down. T

The reason why gas molecules have a great variety of speeds is that they are frequently colliding with each other and exchanging kinetic energy in collisions so that a molecule sometimes moves faster and sometimes slower. Of course, the whole lot keep the same *total* K.E. all the time; therefore (dividing by the number of

‡ The 500 cubic cm flask often used for this is disappointingly small. The class deserves to see a difference of at least 1 gram. We hope teachers will not use a flask smaller than 1 litre.

‡ The method used in Year I was to pump air into a big plastic container. Then the extra air that had been pumped in was released under water into a plastic box full of water. The container was weighed when full, and again after the 'extra air' had been released. The air that bubbled out was seen and measured with a ruler. This gives a sufficient difference of weighings for an ordinary balance to be used.

molecules) the same average kinetic energy per molecule all the time.

With a fast group, the teacher should sketch a histogram or chart showing the velocity distribution of molecules and marking the average speed near the hump. (See comment later: Velocity Distribution.) *

Direct Test of Molecular Speed. We should tell pupils that it is possible to measure the speed of gas molecules directly; but the experiment is difficult. For most pupils we should merely give the assurance that measurements agree very well with this prediction that we have just made. For those who want to enquire further, a pupils' guide or a film will give a short description of a 'chopper' experiment in which a bunch of molecules are timed as they travel across a measured distance in a vacuum. The experiment itself is far too difficult for a demonstration. T

A film to show a chopper experiment, and its result, would be a great help. A film that only showed the real apparatus would be disappointing – the profusion of electrical meters and pumping equipment divert attention from the essential chopper. If the film also showed an explanatory model, or perhaps an animated sketch, the apparatus would then be comprehensible. But the apparatus should be shown in motion, and a specimen result exhibited. (An animated cartoon alone – to show what *ought* to happen – would be bad teaching; and we should avoid that.) F88

Demonstration of High Speed of Molecules

We say that, although we cannot show a direct test, we can show that gas molecules do have high speeds, by using some visible molecules: bromine vapour. We offer to do that. D89 & 90

Diffusion of Bromine: Experimental Arrangements

Bromine is a dangerous substance. If liquid bromine splashes on to skin it makes a bad blister. Bromine vapour attacks skin and will produce a sore throat easily if we are careless. In general, bromine attacks skin, finger nails, metals, plastics, rubber – almost everything except glass and paraffin wax. We urge teachers to show the diffusion of bromine and to use it for some measurements described below; but we urge them to use it with great care. The only other brightly coloured vapour or gas, at room temperature, that would serve these demonstrations is NO_2, which is also poisonous and more difficult to use for our demonstrations. Fortunately, bromine is now obtainable in small sealed capsules or 'ampoules' of glass, which can be smashed inside the apparatus to release bromine. (The ampoules contain 1 cubic cm of liquid bromine. Their wall is as thin as the shell of a small bird's egg.) We have devised a scheme, described below, that uses these ampoules safely.

In our preliminary trials, several members of the Nuffield group devised ingenious schemes for releasing bromine; but each of those schemes seems to involve some risk. Where the bromine is fed in through an open funnel and taps, it has first to be pipetted from a stock bottle. Although teachers in trials were skilful and had no trouble, this is in general a dangerous method: the vapour pressure of bromine is high and rises with slight warming, so there is danger of squirting out bromine, or splashing it out, during the pipetting. That method should not be used. There have been ingenious suggestions for using the ampoules in a plain glass tube with a rubber stopper at each end, the ampoule being smashed inside that. That is simple and economical, but we do not believe the economy is worth the risk. Bromine attacks rubber, and to have liquid bromine released in contact with the large rubber stopper at the bottom is courting trouble. The stopper will work the first time, but we should not trust it after that. Furthermore, there is too much danger of the stopper loosening and releasing some *liquid* bromine.

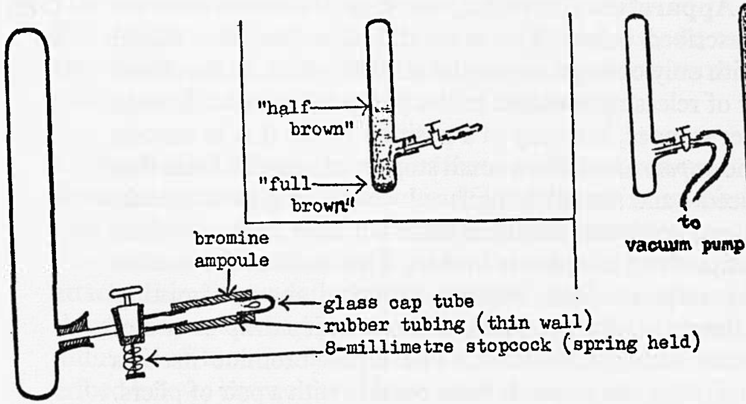
Suggested Apparatus. Therefore, we urge schools to use the apparatus described below. The main diffusion tube is a closed glass tube with only one opening to the outside world, so that there is no danger of releasing bromine to the pump by mistake. It does have a rubber stopper, but only in a position where it is in contact with bromine *vapour*; and it is a small stopper of standard size that can be replaced – and should be replaced – often. A glass stopcock, with large bore, separates the main diffusion tube from the place where the ampoule of bromine is broken. That enables the teacher to break the ampoule first, without worrying about admitting bromine to the main tube; and he can then admit a drop or two of liquid bromine with the stopcock. The liquid bromine itself is released by crushing the ampoule from outside with a pair of pliers. For that, the ampoule must be housed in a short piece of flexible rubber (or plastic) tubing attached to the outside of the stopcock. Bromine will attack that tubing, but the tubing remains safe for further use during the same day and should be replaced after that. (Obviously, the bromine ampoule could be smashed in a glass container by hitting it with, say, a steel ball controlled by a magnet. But the risk of some unexpected breakage there seems greater than the small cost of spare pieces of rubber tubing.) D 89 & 90

So, we recommend the following apparatus; and we hope every teacher will try it because, once tried, it will prove too important an experiment to miss.

The main tube in which diffusion is shown is about 2 inches in diameter and 18 inches in height. It is held firmly with clamp and retort stand in a vertical position and a translucent screen, backed by a lamp, is placed behind it. Near the bottom, a side tube about 1 inch diameter emerges at a slight slant. That is closed by a rubber stopper bored to take the tube of an 8-millimetre stopcock.‡

‡ The stopcock should be the ordinary quality. It need not be the high-vacuum quality. However, it should be a good brand with interchangeable taps, with a screw and spring to hold the tap in. It is not worth the economy to use a tap with small bore that gets clogged up with grease or one with no spring to hold it in – in which case there is a risk of the tap getting pushed out and liquid bromine being released unexpectedly. The reason for using a brand with interchangeable taps is that the apparatus needs to be taken to pieces and washed and there is a danger of exchanging parts.

Teachers should insist on having an identical tap on the tube that is used for diffusion in air without pumping, although that might seem an unnecessary luxury. The increase in comfort and safety is worth the cost: the tap enables the experimenter to release the liquid bromine into the main tube with full control. These taps will add about £1 each to the cost, and in this case the extra cost is well worth while.



The glass tube that continues out from the stopcock carries a short piece of rubber tubing which must be flexible and wide enough to admit the ampoule, or at least its snout. (If plastic tubing is chosen, for the advantage of visibility, it must be warmed to make it expand and attached to the glass tubing in such a way that it fits very tightly.) The other end of the rubber tubing is attached to a short piece of glass tubing, closed at the far end. This glass 'cap-tube' is used to hold the ampoule until the experiment is performed.

The cap-tube is tilted and tapped until the ampoule slides into the rubber tube. The experimenter squeezes the rubber tube with pliers and crushes the ampoule. (Therefore, the rubber tube should not be so short that there is danger of pulling it off the glass tube when one squeezes it with pliers. Some teachers in trials advise wiring the rubber tube on to the glass tube at each end.) During this release of liquid bromine, the stopcock is kept closed. Then the stopcock is turned, and bromine is admitted to the main diffusion tube.

Vacuum. If diffusion in vacuum is to be shown, the main tube must be pumped out through the side tube and stopcock before the cap and ampoule are attached – thus, there is no danger of getting D90 bromine vapour into the pump. Then, when the ampoule is attached and broken, there will be a little atmospheric air round it, beyond the stopcock; but that will prove trivial when the main tube, with a good vacuum in it, is connected to the bromine by turning on the tap.

Safety Precaution. Before and during the experiment, the D89 & 90
experimenter should have a beaker of strong ammonia solution at hand. Ammonia combines with bromine to form harmless ammonium bromide. Strong ammonia solution, '0·880', diluted to quarter-strength, provides an excellent safety precaution. If bromine splashes on table or skin, pour ammonia solution on at once. Of course, ammonia should not be used near eyes, for which plenty of cold water is the treatment. (We have experimented with an alternative suggestion of using photographic 'hypo', but find that ammonia solution acts more quickly and surely.)

Washing the Apparatus. After the experiments, the apparatus D89 & 90
should be taken to pieces under water which contains ammonia. It is more comfortable to wear rubber gloves for that.‡ Prepare a plastic bucket half full of dilute ammonia solution, plunge the lower end of the whole apparatus into solution, remove the cork from its neck, and then disassemble the stopcock, etc. It is best of all to do this out of doors, but that is not necessary.

(It is possible to give a 'poor man's version' of the diffusion of *
bromine into air by using two gas jars placed mouth-to-mouth. *
The upper jar contains air, the lower jar is filled with bromine *
vapour, and a plate of glass separating the two is removed to allow *
diffusion to start. However, this raises considerable difficulty over *
filling the gas jar with bromine vapour, therefore it is not as safe *
as the method suggested above; and nor is it so clear for measure- *
ments. Furthermore, it cannot be used for the vacuum case.) *

Diffusion of Bromine: The Experiments

Diffusion of Bromine into Air. Release liquid bromine (from D89
a smashed ampoule) at the bottom of the tall diffusion tube. Leave it for some time so that pupils watch the diffusion.

‡ Some experimenters like to wear rubber gloves during the main experiment, but that invests the experiment with an air of danger which it does not deserve.

The bromine diffuses slowly into the air. Give pupils plenty of time to consider this and wait for them to suggest that the disappointing slowness of the motion of the brown 'gas' is probably due to air molecules getting in the way. We ask:

'Suppose I threw this ping-pong ball, to represent a bromine molecule, in among your heads, representing air molecules, as you sit there. What chance do I have of hitting a head? A very good chance to hit one in the first few rows. T

'Now suppose instead that your heads were very small, just pin-heads. Would the ball have the same chance of hitting them or would it go a lot farther before it made a collision and bounced away in some other direction?

'If the bromine molecules spreading into the air found the air molecules mere pin-heads they would nearly always rush on past, missing the pin-heads, and travel a long way and fill the whole tube almost at once. This experiment shows that air molecules cannot be utterly small; they certainly are small, but they cannot be points.'

Later we shall offer pupils a difficult piece of measurement and reasoning which will lead from that last remark to an actual estimate of the size of an air molecule. But now we ask:

'What experiment would you like to see next? Can you suggest an experiment with bromine molecules which might show us their great speed?'

Bromine Diffusion in Vacuum. We expect the suggestion that bromine should be allowed to diffuse in a vacuum. Then we show that, using a duplicate of the apparatus just used for diffusion in air. (Waiting while the apparatus is washed out and dried and set up again for this would make a delay that would spoil some of the value of this experiment. And it is better to have both together for comparison. So we consider the cost of duplicate apparatus well worth while.) D90

The main diffusion tube is connected by pressure-tubing to a motor-driven pump and exhausted. The stopcock is turned off. The pump is disconnected, and the cap-tube, with the ampoule in it, is attached to the stopcock instead. The ampoule is broken, and the stopcock is then turned to admit bromine to the main tube. (The rubber tube where the liquid bromine was released collapses when the stopcock is opened, but this does not affect the entry of sufficient bromine to the main tube.) The translucent screen behind the main tube should be well illuminated so that the rapid spreading of the brown vapour is seen very clearly.

When the tap is opened and the bromine released in the vacuum, the motion of the brown vapour is apparently instantaneous. There is no need to labour the point. If the pupils have been (*a*) impressed by the magnitude of the calculated velocity of gas molecules, (*b*) puzzled and worried by the apparent failure of the test with bromine in air, and (*c*) startled and delighted by the dénouement, they have got the point.

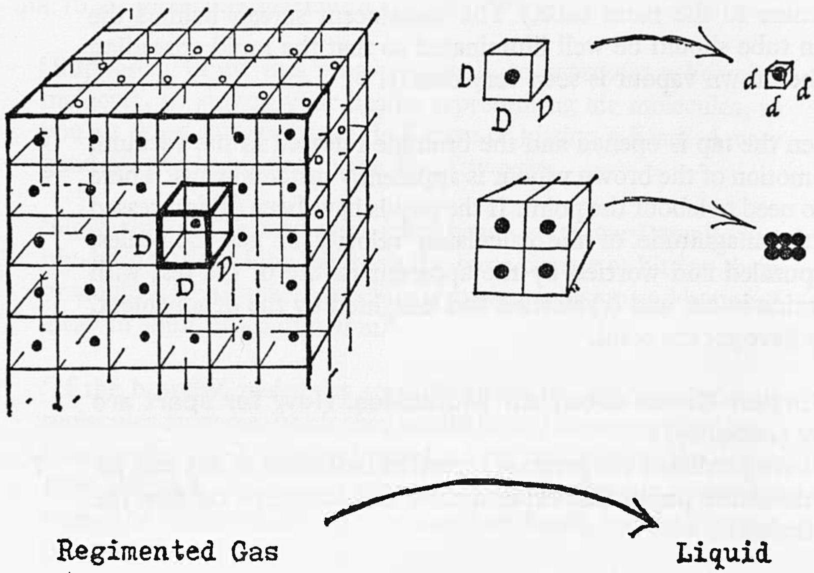
A Further Guess about Air Molecules. How far apart are they (roughly)?

We have predicted the (average) speed of molecules of air, and we should assure pupils that experimental measurements confirm the prediction. T

We can also say something about the spacing of molecules – their distance apart in air. An instantaneous snapshot of molecules in some layer of air would show them spaced at random; but we might reorganize them into a regular spacing by imagining each molecule placed in a cubical prison cell, the whole room being divided up in a cubical array of such cells, each with one molecule in it. Then the length of the side of one cell, D, would tell us the distance from molecule to neighbour to neighbour in such an array – it would tell us some kind of 'average spacing-apart' of molecules in common air.

The teacher should make a sketch of air molecules represented by chalk marks on the blackboard and then reorganize the sketch to a version with an array of cubical cells. Then he should ask what that array would look like if the air were changed to liquid air. For any gas turning to liquid, there is an enormous change of volume. The array will grow much smaller, but the molecules in themselves will not change in size appreciably. How closely crowded are the molecules in a liquid? They cannot be jammed tightly together, in closest packing, because the material would then behave like a solid. To maintain fluid properties characteristic of a liquid, one molecule must be able to wander among others.

Yet the molecules must be so close that the forces we meet in surface tension can hold the material together.



As a crude picture that will lead us simply to a rough estimate, we pretend that each molecule in the liquid is in a little cubical box of side d, the diameter of a molecule (of course, real molecules are not hard lumps like billiard balls, and certainly not spherical: but this is part of our simplifying assumption). The teacher should sketch molecules in the liquid, enclosed in little cubical boxes, stacked in a cubical array. At a glance, this picture seems to have placed the molecules too close together for liquid behaviour; but the volume of space occupied, d^3, is almost twice the volume of the sphere itself, and such a spacing would have liquid behaviour.

In our closely packed array, which we imagine for liquid, the spacing for molecule to neighbour to neighbour is d, one molecule diameter. How much greater is the spacing in gas, common air? We can find the answer to that if we know the volume-change from liquid air to air. Since that answer would be a very interesting piece of information, part of our kinetic-theory picture of gases, we should measure or estimate that volume change if possible. We can make a direct measurement, letting some liquid air expand and become ordinary air, or we can make a comparison of densities. In either case, it is essential to have a considerable quantity of liquid air, at least one pint. This measurement plays such an important part in the work ahead in this Year, that we hope schools will somehow obtain enough liquid air to carry it out. It is not just a matter of giving entertaining demonstrations with the material; it is a matter of making a vital molecular measurement.

Volume Change, Liquid Air to Air (*important*). With liquid air, the best, and simplest, demonstration of the volume change is done as follows: Fill a very small container of known volume with liquid air, then quickly attach a plastic tube to the outlet of the container and lead the air that evaporates to a pneumatic trough where that air is collected over water, in jars or boxes of known volume. Very small volumetric flasks are obtainable, with a narrow neck on which PVC tubing will fit snugly. A flask of marked volume, say, 5 cubic centimetres, is lowered on a piece of wire into a vacuum flask of liquid air. As soon as the little bottle has cooled down to liquid-air temperature, it fills with liquid air. The little bottle is withdrawn quickly, and the plastic tube slipped on to its neck. The plastic tube runs over to a large trough of water and ends up under water with its end pointed up so that the air bubbling out will fill the inverted measuring vessels placed over it. Even a small volume of liquid air like 5 cubic centimetres will produce several litres of air. So the catching and measuring of the air that is produced requires some practice. D91a

As an alternative, less thrilling and less direct for teaching, we may measure densities. Weighing a sample of ordinary air will have shown that its density is, at room temperature, 1·2 grams per litre or 1·2 kilograms per cubic metre. A rough weighing of some liquid air will show that its density is about ·9 gram per cubic centimetre, or 900 grams per cubic metre. 900/1·2 gives the volume change, 750 to 1. D91b

It is not necessary to use a special double-walled container for that. One just pours liquid air into a tall measuring jar; and when the bubbling has almost stopped, one tops up to a known mark. Then one can weigh the jar quickly.

If liquid air cannot be obtained, the teacher should announce the result to pupils. Giving the final result, volume change 750 to 1, would look very poor in our scheme of teaching, a weak link in the chain of knowledge we are now building. It may be easier for pupils to accept the knowledge if the information is given them in the form of density measurements. They know the density of common air and the teacher should simply tell them that liquid air is found to have about $\frac{9}{10}$ of the density of water. (When liquid oxygen is poured into a beaker of water, it sinks in large globules, which then rise repeatedly as they are buoyed up by gas; but liquid nitrogen floats on top. In the proportions of common air, the measured densities of liquid oxygen and liquid nitrogen lead to a specific gravity of 0·9.)

Volume Change for CO_2 (*Optional, easy*). If liquid air is not available, teachers may want to illustrate the volume change with some other material. Solid carbon-dioxide thrown into water will turn into gas which bubbles out from an increasing shell of ice. The carbon-dioxide snow that we make in small quantities for pucks will not suffice for this. It is necessary to obtain a piece of dense 'dry ice' and saw out a small brick. We measure the brick and quickly place it in water under an inverted measuring jar full of water. The gas is collected and measured. D91c OPT.

Volume Change for Steam (*Optional, difficult*). We might even use water itself as our simplest alternative, but the experimental arrangements are difficult. For water turning to steam, the volume change is 1 to 1,600. That can be demonstrated qualitatively at least with a large glass hypodermic syringe. The piston is pushed in to remove all air, and a small (measured) volume of water is inserted by using a small hypodermic syringe to drive the water in through a rubber cap on the nozzle of the large syringe. The large syringe is then held in a beaker of salt water or motor oil heated just above 100°C. Unfortunately, the heating of the syringe takes considerable time and it has been suggested that it is better to do the heating by a coil of wire in air instead. It would probably be best of all to use brine heating, but to preheat the syringe in boiling water beforehand. (The piston must be treated with Vaseline to discourage salt water from getting into the very narrow space between it and the barrel, otherwise the syringe will jam.) D91d OPT.

Spacing of Air Molecules. The volume change from air to liquid air is 750 to 1. Other gases give similar changes, some of them much greater. In general, we may say the change of volume from gas to liquid is about 1,000 to 1. We ask pupils what they think that will tell them about the distance apart of gas molecules. Teachers who have tried this with pupils find that the next step seems fairly easy. Pupils themselves suppose that molecules are practically 'in contact' in liquid, one molecule diameter from centre to centre. And when they think of the gas spread out to a thousand times that *volume* they jump to the conclusion that average spacing apart in gas is the cube root of 1,000 – namely 10 molecular diameters. That seems to be a harder piece of reasoning for us than for pupils. But those who do find it hard should be given help so that they see clearly that this is a reasonable statement about gas molecules.

Speed of Sound and Molecules' Speed

We suggest looking at the speed of sound in air as a support for the reasonableness of the large speed that we have predicted for air molecules. If we think about the mechanism of a sound wave, in terms of air molecules, we may find we expect sound to travel almost as fast as molecules. (In fact, the speed of sound in air is about 340 metres/second, while the average molecular speed, according to our prediction, is 500 metres/second. After the following discussion, pupils may consider that reasonable agreement.) T

We shall not resume serious studies of waves till Year V; but we may mention a picture of sound waves.

To help pupils to think about sound waves, we ask them to think of a 'wave' travelling along a line of railway wagons at rest on a siding. An engine gives one truck a sudden push and then leaves it: the 'compression' travels along the line. Some pupils find this easy to picture straight away, because they have watched shunting operations; others find it easy to picture only after we have drawn the trucks on the blackboard with little connecting springs between them. We also remind pupils of a longitudinal wave travelling along a slinky; and we bring that out and show it again. D 92a

Then we think about air molecules. We sketch a line of molecules spaced ten diameters apart and ask how we could send a sound wave along that line, when there are no connecting springs between one molecule and its neighbours. (We know that, because if there were springy forces we should not find Boyle's Law holding.)

We may illustrate the line of gas molecules by a line of ring-magnet pucks, each with some solid CO_2 under its lid, placed on a glass table with wooden laths as side walls to keep the motion to a straight roadway. We place the pucks some distance apart, then start a 'wave' by pushing the end puck so that it crowds the next few pucks. Or we can do the same thing with a long train of trolleys but the connecting springs modify the analogy. D 92b

To help us think about a sound wave, we imagine we give a few molecules at one end of a line of molecules a push, to crowd them together. We ask how that crowding, that compression wave, can travel along. The only sensible answer seems to be that we must wait until those molecules with their natural motion arrive in a crowd among neighbours further along and crowd them in turn, and so on. In other words, a sound wave should travel with something like the speed of molecules or, rather, a bit slower since there would be a lot of random incompetence among these molecular messengers.

As a useful analogy, ask pupils to think of relay runners carrying a baton (momentum) and handing it on from runner to runner in turn. A compression in a gas carrying a sound wave is a region of extra momentum in the direction of travel of the wave. The place where a runner hands over the baton might correspond to a compression. We point out that the speed at which the baton (compression) travels along is likely to be slightly below the maximum speed of runners.

Measuring the Speed of Sound. If pupils have not met a measurement of the speed of sound, we suggest making a rough estimate by timing echoes (a method Newton used). This is best done out of doors, though it is possible in a long corridor. D 93a

The experimenter stands as far away as possible from a large reflecting wall, and claps his hands (or hits a gong) rapidly at a regular rate. He tries to adjust the rate until each clap just coincides with the return of the echo of his predecessor. The adjustment can be made by moving to a different distance instead of changing the rate. Or, probably easier, he and the class find the rate (and distance) for the bangs and echoes to be equally spaced: 'bang-echo-bang-echo-bang-echo ...' Then a stopwatch measurement of the rate of clapping and a rough estimate of the distance to the wall and back will yield an estimate of the speed of sound.

The traditional apparatus for this is a metronome; but the expense is not justified, and anyway that would introduce an undesirable element of 'specialness' of apparatus in a simple experiment.

Schools which are teaching Year I of our programme and therefore have the simple 'broomstick pendulum' should certainly try using that since it is simple and available and not 'special'. It will need a pivot nail much nearer the bricks, to make its period short enough for the echo timing. For example, if the wall is 300 feet away, the pendulum which makes a tick every half oscillation must have an *equivalent length* only just under 1 ft. It might be better to start afresh and make a small version with a short wooden stick and one brick; but the same crude design is all one needs. To make the 'ticks' audible a loudspeaker could be used, in series with a battery and a simple electric contact at the end of the pendulum; or a pupil could be appointed to make hand-claps in time with the ticks. The simplicity of such a method is a great advantage – even at this late stage in our course – in giving a genuine picture of physics.‡

Speed of Sound: Elaborate Method (*Buffer option*). If the teacher is interested in setting up a more elaborate arrangement for measuring the speed of sound and if some form of microphone is available, there are a number of interesting possibilities. The cathode ray oscilloscope used later in this Year can be arranged to exhibit pulses from the microphone with its horizontal time base sweeping the spot regularly across in a few milliseconds. That time base can be calibrated by millisecond spaced pulses from the scaler. Then the microphone is used to pick up a pulse of sound which has made 0, 2, 4, or more trips along a measured distance. That can be done out of doors, as in the hand-clap method above, or we can use a long glass tube or section of plastic drainpipe. One end of the tube is closed by a metal drum which will start a pulse of sound when we hit it from outside, and will reflect the returning pulse when it comes back to that end; and the other end of the tube is closed by a massive stopper to act as reflector. The microphone is placed in the tube, near one end, well cushioned to give it a poor impedance-match (acoustically) with the glass tube, which also conducts sound at its own speed. The record shown on the oscilloscope will probably be rather complicated, including stray noises, a series of reflections, and signals made by pulses conducted by the wall of the tube itself. D93b OPT.

‡ Despite the intricate ingenuity and complexity of controls of modern research machinery, simplicity is still characteristic of some of our modern probes: a diffusion cloud-chamber, and a scintillation counter are essentially simple. Others are not – the *programming* of magnet and oscillator in a racetrack accelerator is tricky and complex; and the *mechanism* of a transistor needs some advanced knowledge for its understanding – but in all the apparent complexity of his equipment, the physicist prefers simplicity where it is possible. It is when he has to redesign his apparatus for use by others who are not specialists in his field, that he makes ingenious elaborations for the sake of another kind of simplicity: easy mechanical use.

Standing-wave Methods to be avoided. There are traditional methods that use standing waves formed by continuous sound waves from a small loudspeaker and their reflection by a plane wall. The incident waves and reflected waves form a stationary pattern in front of the wall. A microphone moved along this region will show the antinodes for pressure changes, half a wavelength apart. Then we still have to measure the frequency and we have to teach pupils the story of standing waves. Furthermore, stray sound waves reflected from the walls of the room are apt to spoil this experiment; so we do not recommend this method now and suggest it should not be done – or should be postponed till A level.

An easier form of the last experiment uses continuous waves from a small loudspeaker instead of standing waves. We arrange the loudspeaker to emit sound of known frequency. We connect the microphone to the oscilloscope; but we pick up an *external* synchronizing signal for the oscilloscope from the loudspeaker, not from the microphone. That will lock the oscilloscope pattern to the wave as it starts from the loudspeaker. Then as we move the microphone farther and farther away from the loudspeaker, it shows a trace of the sound wave with smaller and smaller amplitude and changing phase. We look at the phase, mark one point on the time-base and move the microphone until the pattern has travelled one whole wavelength past that point. The distance moved by the microphone for that gives us the wavelength *if we know the frequency*, and then we can calculate the speed. D93c OPT.

We do not suggest any of these more elaborate methods for an average group. Where a teacher has a very able group and wishes to spend the extra time trying out one of these measurements, he may find it worth while.

Comment on Measuring Sound Speed. Unless the group is a fast one, we should not spend much time or trouble measuring the speed of sound. That is something which could be drawn from common knowledge, thus saving us from interrupting the kinetic theory discussion. With an average group, we should not expect to stop and 'prove' everything in this course, or we shall give a poor picture of physics as always worrying about detailed 'proof' and never reaching the interesting science of this century. We should aim at giving experimental support and illustration frequently, but we need not extend that to 'logical' completeness. If the teacher enjoys setting up an elaborate measurement and showing it quickly, well and good; but otherwise it is best to make do with no measurement at all, or possibly a rough one.

In any case, the traditional scheme of holding a tuning fork over a glass tube partly filled with water and looking for resonance should be avoided. That hangs on a discussion of standing waves which is seldom well understood and would be quite out of place at this point when we are busy thinking about molecules handing on a compression through air.

Sound Waves and Atmospheric Pressure. We can ask able pupils an interesting question, 'Would the speed be bigger or less if you could send a sound through a much thinner atmosphere, say at half the pressure but at the same temperature?' T

Unfortunately, the answer, which is surprising, is difficult to test in a school demonstration; though teachers who set up the elaborate measurement of the speed of sound, using a microphone in a long glass tube and an oscilloscope, could arrange to pump the tube down to, say, half an atmosphere and show that the speed of sound is the same. (Mountain climbers who try a hand-clapping estimate will find the same speed at 5,000 feet, though the pressure will be 25 per cent smaller – the change of temperature makes only a small difference. If they carried a recorder and a tuning fork on their climb, the agreement of their tuning would not change much.) D94 OPT.

Yet pupils probably enjoy reasoning out the conclusion that since molecules would travel at the same speed, the sound would probably travel at the same speed. To many a pupil, thinking about this picture of air molecules handing on the clumping of molecules in a sound wave as they move along, and make collisions, and so on, is an interesting business of putting theory to work. What seems obvious to us is new thinking for them. T

Gas Diffusion and Kinetic Theory

We now go back to our general prediction, $PV = \frac{1}{3}Nmv^2$, and ask about molecular speeds for other gases. Suppose we have samples of gas, each of them of the same volume, each at atmospheric pressure. Then P and V are the same for our different samples and the only things that can differ are v^2 and Nm ($= M$ the total mass). So, to keep our equation true, v^2 will have to be smaller for a gas which has bigger M. T

If we change from air to a denser gas which has a bigger mass in the standard volume, it will have a smaller speed for its molecules. So we expect carbon dioxide, which is much denser, to have slower molecules, and hydrogen, which is much less dense, to have faster molecules. (About 1 mile a second for hydrogen is worth remembering.)

We should show that these two gases do have markedly different densities from air. With a small compact source of light we cast a shadow of carbon dioxide being poured downwards like water from one beaker into another, and of hydrogen being poured upwards. D 95a D 95b

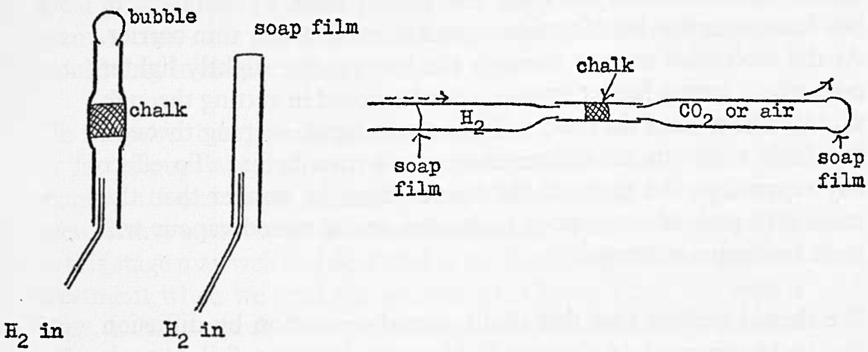
In the case of carbon dioxide we might also give the usual chemistry demonstration of pouring CO_2 into a beaker in which there is a lighted candle.

We should also hang balloons of air, carbon dioxide and hydrogen on a simple metre-rule balance to show qualitative differences. (Measurements would take a long time and interrupt the present interest. Also, pupils might get entangled with Galileo's amusing paradox that if we weigh a bladder full of air and then let the air out and weigh the balloon again there is no change in the weighing – because of buoyancy. With a rubber balloon, there *is* a small change, on account of the extra pressure needed to inflate the balloon.) D 95c

Only with a very fast group should we give densities of other gases and ask for rough estimates of the molecular speeds, using the result of the air calculation as our starting point. With them, this forms a useful test or it may be given as a homework problem. T

We then show diffusion demonstrations. The usual ones with a porous pot of white porcelain are amusing but too indirect to be fully impressive. D 96 OPT.

Simpler Diffusion Demonstration. So we suggest a new demonstration that is more direct. The porous barrier is a short piece of soft blackboard chalk. (It must be the soft kind often used for coloured chalk, not the hard, dense 'dustless' kind.) Take a piece of PVC tubing about 6 inches long, slightly too small to admit the chalk, warm it so that it can be stretched and push a $\frac{1}{2}$-inch length of chalk a little way into the tubing. Hold this tube, D 97a with its porous barrier, upright, and make a soap film at the top end by smearing soap solution across it. Feed hydrogen in through a fine tube pushed into the lower end of the PVC tube. The hydrogen in the lower part diffuses up through the chalk faster than the air above the chalk diffuses down; and therefore a small soap bubble grows at the top of the tube.



Pupils will show a good feeling for scientific care if they raise the objection that simple buoyancy of hydrogen may well be the essential agent in bubble-blowing. If they do not raise that objection, the teacher should do so. Then we should show a control experiment, the same arrangement without any chalk. The buoyancy effect is barely noticeable. (When the chalk has become wet with soap solution in repeated trials, its efficacy can be renewed by scraping it with a screwdriver, while it is still in the tube.)

Watching Two Gases Diffuse. As a more elaborate version, some teachers may wish to try the following. A piece of soft chalk is used for the diffusion barrier as before, but the PVC tubing holding it is horizontal and horizontal glass tubes are attached to its ends. Before the glass tubes are joined to the ends of the PVC tube, we 'seal' the outer end of each with a soap film. We fill the glass tubes with samples of two different gases, such as air and hydrogen, and attach their open ends to the PVC tube. Thus we have two samples of gas, each enclosed by a glass tube, a soap film at one end and the chalk barrier at the other end. We watch the progress of the soap-bubble indicators to see the effects of diffusion. D97b OPT.

Uranium Separation by Diffusion. We may mention the separation of uranium isotopes by a diffusion method. Uranium extracted from ore is converted into uranium hexafluoride, which when heated by steam becomes a dense vapour. That vapour contains molecules of two slightly different masses, because uranium has two kinds of atom one slightly lighter than the other, and the lighter, rarer, kind is fissionable in ordinary circumstances. To obtain fissionable material it is necessary to separate that lighter uranium from the rest; and that is done by diffusion of hot dense uranium hexafluoride vapour through a very thin barrier. As the molecules stagger through the barrier, the slightly lighter ones which have a higher average speed succeed in getting through slightly faster than the others. Then the mixture seeping through is a little richer in the lighter component than before. To effect any separation, the pores of the barrier must be smaller than the mean free path of the vapour molecules or the mixed vapour will gush through unchanged.

We should explain that this slight partial separation by diffusion has to be repeated in thousands of stages before a full enough separation is obtained. That process is presumably still in action as the major way of separating uranium, which is useful for making small power reactors as well as for warlike purposes.

ESTIMATE OF THE SIZE OF A MOLECULE OF AIR

We can offer to measure the size of a molecule and to find out how many air molecules there are in the room. That is quite difficult, but with pupils who have the ability and interest we should do it: and we hope that most pupils will have both.

For the essential experiment, we show bromine diffusing in air again; and pupils estimate the progress of diffusion in a measured time. From that, we calculate the mean free path – the distance from collision to collision – of one molecule wandering among others. We look at the volume change from liquid air to air; and that links our estimate of mean free path in open air with the size of an air molecule crowded into liquid. A rough estimate of molecule diameter emerges, which we can use to estimate the mass of a molecule, the number of molecules in a known volume, or the Avogadro number of molecules in a 'mole'.

Preliminary Discussion with Teachers

This work offers a great extension of our kinetic theory discussion: an argument that enables us to obtain a microscopic estimate of molecule size from macroscopic measurements. Although we shall now spend considerable space describing the argument and the experiment, we trust that teachers will not infer that this is an absolutely essential part of our scheme, without which our suggested programme would somehow fall to pieces. It would be a great pity to give this discussion such extreme importance, because many a class that can follow most of our programme successfully would find it discouragingly hard.

On the other hand, we hope that every teacher who has a medium or fast group will try out this discussion at least once. At a first glance, it looks to us unfamiliar and difficult, and its arguments seem risky; so those of us who are familiar with teaching pupils at this stage may well feel doubtful of the feasibility of the suggested treatment when we read the account of it here. Tried out with a class, it has a much better chance of proving feasible, because it offers pupils a very exciting chance to join in an atomic measurement. What seems difficult to us on account of our background of knowledge, may seem much easier to pupils.‡

Difficulties and Doubts. The success of this part of our course depends very strongly on the attitude and feeling of each of us teaching it. We have a chance here to delve into the atomic world with our young pupils, and let them feel that they themselves have measured molecules and counted their number. Pride in that achievement will not be spoiled by the honest knowledge that the results are very rough estimates. But that sense of living in the atomic world is delicate: it can easily be killed by doubt, by worry, even by well-meaning care in precise teaching. The teacher needs to be a guardian against worries as well as a skilful guide.

The essence of a good scientific method is here – this is one of the early classical estimates of molecules in the last century – but in dealing with the geometry of collisions and the statistics of a chaos of a myriad molecules, we must take many short cuts, round off some numbers, settle for a simpler average than the 'proper' one, and forget a dozen refinements of professional treatment. Yet if we can turn a blind eye to those shortcomings of the simple treatment, we shall find the essence of the estimate is there, and our pupils will understand it.

‡ The following pages contain a number of discussions with teachers. Some of those discussions are long, and we do not want them to bulk too large or delay readers who are anxious to proceed to the experiment. Therefore we have printed them in small type. Since they are clearly marked in that way, we have omitted the * * * in the margin.

This is a rare case in our teaching where concern about short cuts will not so often come from pupils as from our own pride in careful teaching. Yet this is a case where we could afford to be lighthearted, so that we carry our pupils with us. We should perhaps feel Browning's warning in 'A Grammarian's Funeral':
'This man decided not to Live but Know.'
Can we leave our own doubts in the background for once? If so, we, or rather our pupils, can make rough guesses and leave out correcting factors, yet emerge with knowledge to be proud of.

Since this treatment is unfamiliar to many teachers – at least in its simplified form – it would be unreasonable to ask teachers to accept it unthinkingly; and very unwise, because they would soon meet doubts. Therefore, as well as outlining the suggested teaching we shall give private notes to teachers, showing where our simplification deviates from the proper treatment. These notes are not intended to be given to pupils – crowding in such details would spoil the chances of success. The notes are there because we who teach do need a fuller account for our own satisfaction – a background for reassurance in our teaching.

Of course if pupils do raise doubts, we must reassure them with honest answers.

In making our estimate, we shall leave out factors like $\frac{1}{2}$ or $\sqrt{\pi}$. We shall be careless in some of our steps. It would be better not to embark on this topic unless one feels ready to take those short cuts in a holiday spirit.

All through this experiment, and the reasoning that goes with it, the overall assurance to teachers and to pupils is this:

> 'This *is* a rough estimate, but it is worth while because it gives us an actual measurement of the size of a molecule. We do have to make rough guesses and to leave some details out; so it is not accurate. But it follows a real method of measuring molecules and will show you how that is done. And its result will not be a wild guess but a rough estimate, well worth having.'

The General Scheme. This is a general description for teachers. It is followed by a summary, and a brief outline, also for teachers. After that, we give a detailed account of the suggested teaching.

Diffusion and Molecule Size. We explained earlier that the slow diffusion of bromine in air tells us that gas molecules are not infinitely small. Instead of flying up the tube, the bromine molecules are stopped by collision after collision with air molecules. If both bromine and air molecules had no size at all ('pin-heads'), the bromine molecules would see no targets to hit: they would travel straight on. The bigger the air molecules (and the bromine molecules), the bigger the targets to hit, the shorter the path between collisions, and the slower the net progress of diffusion. Arguing backwards from that, we hope to extract an estimate of molecule size from the slowness of diffusion.

Mean Free Path and Molecule Size. If we could somehow measure the length of path from one collision to the next, we could calculate the target size of a molecule – its area – then r^2, and thence its diameter. Then we know the size of a molecule.

We can measure the length of a bromine molecule's path from one collision to the next, or rather the *average* length, for a great many bromine molecules; we call that mean free path. We can do that by timing the spread of brown bromine up through the air in our tall tube, and using a statistical rule.

Mean Free Path estimated from Diffusion. We can see how fast the brown bromine seems to travel as a crowd: we can time its advance. But that advance is really the result of an enormous number of short staggers from collision to collision, in all kinds of directions. A simple timing of the crowd's advance will not tell us the mean free path, unless we know the connection between a random stagger of many free paths and the overall advance.

Random Walk. That raises the mathematical problem of the 'random walk'. In statistics, that problem is posed and discussed with many special conditions and both the result and the calculation that lead to it look very complicated. Fortunately, the result we need here is simple, and the Nuffield Physics Group have found a simple experimental way of letting pupils test it.

We describe a random walk to pupils and tell them the result: if we take a walk of N equal steps, in succession, in random directions, and measure the resultant from start to finish, the average resultant for many trials, each of N steps at random, will be close to $\sqrt{N}$ steps. For example, a walk of 100 steps, with every successive step in a new direction chosen at random, we are likely to end up about 10 steps from our start in the average of *many* walks.

We tell pupils this rule, and let them test one example of it.

Then, we explain that the brown bromine molecules diffusing to air execute a random walk of an enormous number of steps; and we use the rule to extract the length of a single step – the mean free path of a bromine molecule in air – from a timing of the progress of diffusion.

The Diameter of a Molecule. The mean free path of an air molecule in air will be much the same; and we calculate the diameter of an air molecule from that, either by the traditional method or by a crude guess about the crowding of molecules in liquid. In either case, we use the volume-change from liquid air to air as a connecting factor.

Summary: The Essence of the Method

1. *Experiment: Diffusion of Bromine.* We allow bromine to diffuse in air and we estimate its (average) progress in a measured time.

2. *Argument: from Diffusion Measurement to Mean Free Path.* We appeal to the 'random walk' statistics to help us calculate the mean free path of a bromine molecule among air molecules. We assume that that is much the same as the m.f.p. of an air molecule in air.

2*a*. *Experiment: Empirical Test of Random Walk Rule.* The rule says that the *average* resultant of a walk of N equal steps, taken in random directions, is $\sqrt{N}$ steps. Since even a crude algebraic derivation of that would be too confusing, if not too hard, for pupils, we announce the rule and ask pupils to give it a practical test by drawing a random walk on paper. We then average the results of all pupils.

3. *Experiment: Volume change from Liquid Air to Air.* To obtain an estimate of air molecule size from the m.f.p., we need another piece of information: we use the volume change.

4. *Argument: From m.f.p. and Volume Change to Molecule Diameter.* We picture molecules as crowded fairly closely in liquid. The volume change enables us to link a guess about that closeness in liquid with the mean free path in open air. An estimate of the 'diameter' of an air molecule emerges.

5. *Other Results:*
a. Using that diameter for molecules crowded in liquid, we expand by the measured volume change and estimate the number of molecules in a cubic metre, or a roomful, of air.
b. Thence we could calculate the mass of an air molecule.
c. We can calculate the number of molecules in a 'mole', the Avogadro number.

Note for Teachers: Outline of the Method

1. *Bromine Diffusion in Air.* We start the clock as we release liquid bromine at the bottom of a tall glass tube of air. After a measured time, say 500 seconds, pupils estimate the average distance the bromine has diffused up the tube. They do that by guessing by eye how far up the tube the vapour looks 'half-brown'.

We are using brown bromine molecules as markers to show how one gas travels through another. The diffusion is so much slower than the spreading of bromine in vacuum that we infer each bromine molecule makes a huge number of collisions with air molecules on its way. The path between collisions must be very short.

2. *Random Walk.* We suppose that after each collision a bromine molecule moves off in a new direction at random. We assume (as a simplification) that the path from collision to collision is always the same length, y. (Or, rather, we use the *mean* free path, y, throughout.) We appeal to measurements of density and pressure of bromine vapour to give us the average speed of molecules, 200 metres/sec. Then in 500 seconds a bromine molecule would travel altogether 500×200 metres. That is the *straightened-out* path. The number of paths it would follow from collision to collision would be $(500 \times 200)/(\text{mean free path } y)$. $N = 500 \times 200/y$.

We appeal to the random walk rule: average resultant (net progress from start) of N steps, each of length y, is $\sqrt{N}$ steps.

We say that the diffusion distance measured in (1) is a measure of that average net travel of bromine molecules.

$$\therefore \text{ distance measured for 'half brown'} = \sqrt{N}\, y$$
$$= \sqrt{500 \times 200/y}\,.y$$
$$= \sqrt{500 \times 200 \times y}$$

We solve for y, the mean free path of a bromine molecule in air. We assume that that is much the same as the m.f.p. of an air molecule in air.

From this point on, we use that estimate for air, and deal only with air and liquid air. The bromine molecules have served their purpose as visible markers.

2*a*. *Test of Random Walk Formula.* Although we offer teachers a short algebraic derivation, we suggest pupils should be told this surprising rule and asked to test it. They throw dice to indicate the directions of 25 successive steps on paper. The average of the resultant walks (start-to-finish distances) is compared with the predicted value of 5 steps.

3. *Liquid Air to Air.* When liquid air changes to air the volume change is shown by experiment to be about 1 to 750.

4. *Argument.* We assume each air molecule is a ball of diameter d. We imagine molecules are closely crowded in liquid, though not in closest packing.

We link our estimate of mean m.f.p. in air with d by either of two methods; each of which uses the volume change.

1. *Risky, Simple Method.* We imagine a piston compresses ordinary air in a cylinder till it is as crowded as liquid. The m.f.p. is squashed to a value 750 times smaller. We ask for guesses of the m.f.p. in liquid: a whole molecule diameter? $\frac{1}{10}$ of a diameter? or what?

If the class agrees on a very rough guess of say $\frac{1}{2}d$, we know that

$\frac{1}{2}d = (\text{mean free path}, y)/750$

Hence we know d, as roughly as our guess is rough.

2. *Traditional Formal Method.* We picture an air molecule travelling from collision to collision. We enlarge it to radius d and reduce all target molecules to points. Thence we show that $\pi d^2 \times (\text{m.f.p.}, y) =$ volume containing one molecule, in air.

We use the volume change 1:750

Then $\pi d^2 \times y = (750 \times \text{vol. containing 1 molecule in liquid}) = 750 \times d^3$

Here we have assumed, as a rough guess, that each molecule in liquid occupies space of volume d^3 as if in a cubical array of cells of side d. Then $d = \pi y/750$, which gives a rough estimate of molecule size.

Note to Teachers on the Random Walk

The rule we have given is correct for a random walk in two dimensions or in three dimensions. But, like any statistical rule, it only tells us the probable result of averaging a large number of trials. The result of a single trial may be far away from the $\sqrt{N}$ steps given by the rule; and the average of a small number of trials may be quite far away, or it may be close – the averages of groups of trials ranging either side of the value suggested by the rule.

Furthermore, the simple rule requires us to take a special kind of average of the many trials. Suppose pupils make a test: each pupil plots a random walk of 25 steps and measures the distance from start to finish, in steps. Suppose 100 pupils do that (or 33 pupils carry out the walk three times each). When we average the 100 start-to-finish distances measured by the pupils, we hope to emerge with the square root of 100, or 10 steps. With only 100 trials there is a considerable chance of deviation from 10. As likely as not the final average will be 8 or 12. Even so, the result will be far away from the extreme 100 steps, and perhaps near enough to 10 to lend some support.

However to get even that close to the ideal value, we should have to take the *root mean square average*. That is: we should have to take each pupil's measured start-to-finish distance, square it, add the squares for all 100 pupils, divide by 100 to find the mean square, and take the square root of that. Not only would that involve considerable extra work for the teacher, but it would make the test itself seem complicated and confusing to pupils. Suppose, instead, we take the *simple arithmetical average* of the 100 distances. In that case the ideal result is about $\frac{8}{10}$ of the previous one. Instead of $\sqrt{N}$ we expect $\sqrt{N} \times \sqrt{2/\pi}$ or about $\sqrt{N} \times 0{\cdot}8$.

Since pupils will certainly want to take the plain average, we had better give them the rule in the less simple form, that we expect the average of the start-to-finish distances to be $0{\cdot}8 \times \sqrt{N}$. In practice with only 100 trials, the 20 per cent change in our formula competes with the kind of deviation, say 20 per cent, that we may expect as a result of taking so few trials. We could reduce the latter trouble to 10 per cent by taking 400 trials – but that would certainly take too much time. Instead, we must secure the goodwill of our pupils in making a very rough test of the statistical rule.

We suggest that each pupil should obtain his instruction for each step by throwing a die. The steps are then plotted on triangular grid paper. This two-dimensional walk is thus restricted to six directions, instead of all directions at random. That does not impose any serious modification, unless the number of steps in the walk is very small. For any number of steps, however small, the *root mean square average* of a large number of trials will approach $\sqrt{N}$. (It is amusing to try that for walks of 1 step, 2 steps, 3 steps, by thinking out, with pencil and paper, all the possible results and taking the root mean square average.) However the *plain arithmetic* average is not expected to approach $0{\cdot}8\sqrt{N}$ for a walk of a small number of steps. The ideal result for a large number of trials will be a little greater than that. For a large number of trials each of many steps, the restriction to six directions loses its effect and we may expect an arithmetical average $0{\cdot}8\sqrt{N}$.

When we consider the diffusion of bromine molecules up the tube, the mean free path is so small that to most molecules the sides of the tube are infinitely far away; so we should think of each molecule making a random walk in three dimensions. And yet we measure progress in one dimension, the vertical. That suggests that we should not use the full mean free path but only a component of it. However once we begin considering such modifying factors, we are worrying about refinements which we are not justified in pursuing here.

The random walk rule applies to a walk in one dimension. One can simulate that by taking instructions from a coin that is tossed to give steps $+1$ or -1. Then if the series of tosses shows equal numbers of heads and tails the walker returns to his starting point. If there are two more cases of heads than tails, the walker will end up two steps to the right of his starting point. If there are three more tails than heads in a set of throws, the walker will end up three steps to the left of his starting point. The average of many such walks is likely to be 0. The root mean square average of many such walks is likely to be $\sqrt{N}$. And the plain average of all the *positive* walks (or *all* the walks with negative signs ignored) will approach $0{\cdot}8\sqrt{N}$. However, such a one-dimensional walk will seem a less realistic model of the bromine diffusion.

Alternative Forms of Test. As an alternative to the random walk test on triangle ruled paper, with directions decided by throwing a die, pupils may use *squared* paper and limit their equal steps to four directions, up, down, left, right. The instruction for choosing the direction must be provided by a random sample of, say, 25 objects drawn from a large collection of such objects labelled in equal numbers up, down, left, right. We might use white dried beans sprayed with four different colours to mark those instructions. Or we might have a large collection of the cardboard discs with metal edges that are used for labelling keys, and mark them with numbers or words to give the four kinds of instruction. (A statistician who has used the latter in teaching reports that they last well and are easy to mix by shaking in a large basket.)

Each pupil takes a handful from a well-mixed collection and makes sure he has the right number, such as 25. He follows the instructions given by the items in his handful, plotting the random walk on squared paper. Then he throws his handful back into the pool and takes a fresh one. Some pupils will prefer to give their walk the fullest random look by plotting it in the order they happen to find among their items. Others will soon see the advantage of organizing the items in their handful into four groups before they plot the sums of those.

A simpler version still, in which a *one*-dimensional random walk is plotted by tossing pennies or by using beans or key-tags, is likely to seem unimpressive or even unreal. So we do not recommend it. We ourselves might think it a very good model for the progress of bromine molecules in a single direction, upward. Wherever the result is negative, a net move downward, we could neglect it, on the ground that it represents a bromine molecule that has returned to the liquid at the bottom of the tube. Here again, the prediction for the root mean square average of many trials is $\sqrt{N}$.

Teaching the Experiment and Estimate

We point out that crude, large-scale measurements have enabled us to find out something about the *motion* of molecules in the microscopic world. And by looking at the volume change from liquid to gas we have found out something about the *spacing* of molecules in air. We ask pupils if they think we can get even more knowledge: could we even find the size of a molecule or atom by some large-scale measurements? T

The point here is that pupils must *want* to see this done if they are to persevere with the argument that we shall need. If they do not want to know, it is better not to attempt it. But if we have been asking this question about the size of atoms ever since Year I, the possibility of an answer now should be attractive.

> 'Well, put on your super-microscopic spectacles and have another look at this bromine wandering its way through air.
>
> 'Can you see the bromine molecules, travelling very fast for a short distance, colliding with another molecule, rebounding in a different direction, then striking another ... ?
>
> 'How far can a molecule go with that sort of movement? The molecule's progress is rather like yours if you tried to work your way through a rush-hour crowd in a busy station blindfolded so that you had no memory for direction. We call that a "random walk".‡ Imagine you take one stride from your starting point.
>
> Then choose a new direction – any direction at random, not knowing or minding which – and take one stride in that direction. Again choose a new direction at random and take one stride. Go on like that until you have taken a large number of equal strides, say 100. How far are you now from your starting point, *as the crow flies*?
>
> 'A bromine molecule makes a wandering path from collision to collision, starting out in a new direction every time. Obviously the more "strides" we give a molecule time to make, the farther from its start it is likely to end up; but as the direction of motion after each collision is random we cannot just add up all the steps in a straight line. All the same, we *can* make predictions

‡ Teachers will find an impressive sketch of this, described as 'the drunkard's walk' in *One, Two, Three, ... Infinity* by G. Gamow. (Macmillan, 1956; paperback edn., Muller.)

about the distance travelled in a "random walk". After a given time, one molecule will have got to a place a long way away from the start, another will have got back near to its start, many will have got to some middling distance.

'We could even make a chart of numbers of molecules at various distances. We could make the chart by imagining a "random walk" or sketching it on paper, and if we made hundreds or even thousands of trials of that and took an average, we should find a definite average "crow-flies" distance from start to finish.

'There is a definite rule for predicting that average distance; but it is a surprising one. It is this: suppose a molecule takes 100 steps from collision to collision, bouncing away each time in just any new direction (the teacher sketches this on the blackboard) and we catalogue its net progress from start to finish and take the average of many trials of 100 steps, the result is not 100 steps which is the maximum, or no steps, which is the minimum, but 10 steps. That is because 10 is the square root of 100 and the general rule says that for N steps the average distance travelled is $\sqrt{N}$ steps.

'You need quite a lot of algebra, adding and averaging all those wild wanderings, to predict this strange rule with the square root of N in it. So instead of trying the algebra you shall play a game and see if you can at least test the rule for yourselves.'

We give each pupil a die, or a pair of dice. Every pupil draws on a sheet of paper six spokes each making 60° with the next, to represent six directions, and labels them 1, 2, 3, 4, 5, 6. (It is much easier if we provide paper ruled with a 60° triangular grid, 'isometric rulings'.) C98

Then, the rule of the game is:

'When you throw your die, the uppermost face tells you the direction for your next step. Start in the middle of a sheet of paper and throw the die again and again and again, obeying its "blindfold" instructions for successive steps that you draw on the paper. Each step should be one centimetre or one triangle-side.'

The alternative form (suggested on page 233) with squared paper and beans instead of dice is simpler and just as good. The treatment that follows applies equally well to it.

We ask pupils to make their imaginary 'molecule' take a random walk of 25 steps all of the same length. Then they measure the distance, as the crow flies, from start to finish. In order to have a reasonable hope of approaching the predicted average of 5 steps, we need to average the results of many more than 30 pupils. So each pupil will have to make several 25-step walks so that we have a large number of walks to average.

While this should not turn into a frantic continued throwing of dice in order to 'get the right answer', this is the only form of support – short of statistical algebra – we can give at this stage to the statistical story we so strongly wish to use. If pupils complain that this statistical averaging seems unreliable, we should point out that the real game is being played not by a few dozen pupils plotting a wandering path but by millions upon millions of brown bromine molecules carrying out random walks on a profuse scale.

Averaging the Results. Before we take the average of all the resultant crow-flies walks, we must explain about the two kinds of average.

If the class would find the idea of a root mean square average a confusing extra burden, we should not even describe it. We should simply say that the $\sqrt{N}$ rule is the prediction for a special kind of average. For the ordinary average that we shall use, the prediction is 20 per cent smaller, $0{\cdot}8\sqrt{N}$. We ask the class to see whether our average of all their walks (each 25 steps) comes out anywhere near to $0{\cdot}8\sqrt{25}$ or 4. If the result is far from 4, we have to say regretfully that we are playing our test game with far too few trials. That may encourage some pupils to continue the trials on their own – if so, we should welcome any results they bring in, and include them in a new average. Interest in this may even lead to a feverish business of running more trials – and, since statistical ideas are so important in science and other studies, that particular feverish interest is probably one to be encouraged.

Some classes could easily understand the idea of a root mean square average, though they would not welcome it for use in this experiment. They may see that some such average may be necessary in dealing with alternating currents. With such a class we should describe the average briefly, say the prediction for it is $\sqrt{N}$; and then proceed to the story above and try the plain average against

$0{\cdot}8\sqrt{N}$. Any pupils who are keen to try the root mean square average will enjoy doing that on their own.

If the class takes easily to algebra and statistics and would welcome the use of a root mean square average, as 'advanced', we should certainly take that average and compare it with $\sqrt{N}$. A large poster of a table of squares will help.

Note to Teachers. Preliminary trials have shown that this kind of experimental support for the random walk expression goes well with pupils. However, some difficulties sometimes appear in the discussion of this test, and misunderstandings cloud the issue. So we shall now discuss some difficulties and offer teachers the algebraic discussion, before proceeding to the main demonstration experiment with bromine.

The real diffusion occurs, of course, in three dimensions; and because the throwing of dice offers a choice of a backward or forward stride in each of *three* directions, pupils sometimes jump to the mistaken conclusion that they are mapping a random walk in three directions. They are *not* doing that. They are only trying a simpler experiment *in two dimensions*; and the use of paper with triangular rulings arises only because cubical dice are easily provided – if we provided polyhedral balls with twenty faces, pupils could make moves forwards or backwards along ten different directions, still only in two dimensions, obtaining a 'walk' that would look more genuinely random. However, the limitation to three directions of moves does not change the essential story in two dimensions, providing one takes a sufficiently large number of throws.

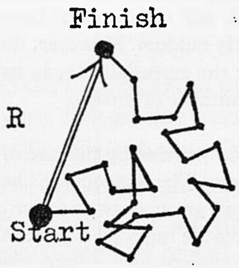
Pupils, themselves, are not likely to be worried by this use of a two-dimensional model for a three-dimensional problem. The dangers are likely to present themselves more strongly to teachers. Pupils are anxious to arrive at the result and get on with the experiment. Even if doubts are raised, it is probably better to proceed fast than to meet doubts by long discussions here – long, since this is a difficult matter, perhaps the highest peak in our climb from the plain of simple kinetic theory.

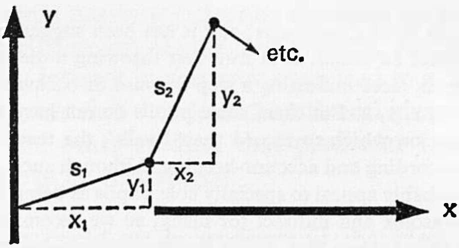
Yet the bromine diffuses in three dimensions; and it has been suggested that a three-dimensional test would be better. It is true that throwing a die could be used for that, each of the six faces indicating a step forward or backward along one of the three directions, x, y, z. But then, since pupils do not have a three-dimensional sheet of paper on which to record their 'walk', the teacher must supervise a good deal of recording and account-keeping. Although such a three-dimensional test would probably appeal to specially able pupils as being fairer, it is likely to become burdensome and indirect for many, so we recommend the two-dimensional version strongly – in fact, where the three-dimensional version has been mentioned it has led to considerable confusion and has even brought the experiment to a stop.

The alternative form (suggested on page 233) with squared paper and beans instead of dice works well and avoids that confusion with three dimensions.

With able pupils who question the extension to three dimensions, we can suggest a theoretical attack. We show them the alegbra of two-dimensional averaging which depends upon Pythagoras, and then point out that the Pythagoras method for calculating a resultant vector from components in two directions also applies to components in three directions. The algebraic version runs as follows:

The Random Walk Calculation with Algebra. We need to know the answer to the random walk problem, sometimes called 'the drunkard's walk'. If a man takes a large number (N) of strides, each of the same length (s) in succession but in random directions, what is his resultant distance (R) of travel? Obviously this will vary from one batch of N strides to another; and may often be zero (in the cases when he comes back to the starting point) and may be as large as $N.s$ (in the rare cases when he happens to take all strides in the same direction). We want the average distance from start to finish, averaged over many batches of N strides.





We observe a walk of N strides and find the resultant travel-distance R from start to finish. We observe a large number of such walks starting afresh each time and find the average value of R for all those walks. Because it leads to the simple result, we find the average value of R^2 and take the square root, obtaining a root mean square (R.M.S.) average. We can show that this average should approach the value $\sqrt{N}.s$. Here is a *two*-dimensional proof. The three-dimensional one is similar.

Sketch the first few strides of a random walk. Choose a set of perpendicular axes, x and y, arbitrarily. Using x- and y- coordinates, resolve stride no. 1 into components x_1 and y_1, stride no. 2 into x_2 and y_2 and so on. Then the resultant of that walk, R, has

x-component $(x_1+x_2+\ldots+x_N)$

and y-component $(y_1+y_2+\ldots+y_N)$

and

$$
\begin{aligned}
R^2 &= (x_1+x_2+\ldots+x_N)^2+(y_1+y_2+\ldots y_N)^2 \\
&= x_1{}^2+x_2{}^2+\text{etc.} \qquad +2x_1x_2+2x_1x_3,\ \text{etc.} \\
&\quad + y_1{}^2+y_2{}^2+\text{etc.} \qquad +2y_1y_2+2y_1y_3,\ \text{etc.} \\
&= s^2+s^2+\text{etc.} \qquad +\text{ZERO} \\
&= Ns^2
\end{aligned}
$$

The 'cross terms', such as $2x_1x_2$, add up to zero in averaging over many walks, because those terms are as often negative as positive, and they range similarly from 0 to $2s^2$. Similarly for the y- 'cross terms'.

Then average value of $R = \sqrt{N}.s$

The proof is better if we use trigonometry and resolve each stride, s, into horizontal and vertical components $s\cos\theta$ and $s\sin\theta$. Then the cross terms in the expression for R^2 form $2s^2\cos(\theta_1-\theta_2)$, etc., and we argue that the cosines are as often positive as negative.

The Diffusion Measurement. We repeat the demonstration of bromine diffusing up through air. This time pupils estimate by direct observation the 'average distance' travelled in a measured time such as 500 seconds. D99 (=D89)

When 500 seconds have elapsed, we ask pupils to judge where the tube looks 'half brown'. That is the best we can do to specify an 'average' distance of travel. That may seem an impossibly difficult thing to judge, but the teacher should first point to the bromine vapour just above the liquid, where it is practically saturated, and say 'We call that full brown' and then point to the air high up in the upper jar and say:

> 'Here it is practically clear. And down there it is "full brown". Somewhere you can see a place that you could call "half brown". I will take votes on it.'

Then he should run a horizontal pencil up in front of the tube and watch the points at which pupils call 'stop' for half brown. He should run the pencil downward and again take votes. The votes will probably range 9, 10, 11 centimetres if the diffusion has proceeded for 500 seconds. If we choose the most popular vote, pupils who voted differently can always calculate from their own estimate afterwards. This is obviously a matter for the teacher to rehearse quite carefully beforehand, though he will find that pupils' voting gives him a feeling of confident progress which will be a real help. In the following example we shall assume that the 'half brown' point is voted to be 10 centimetres above the liquid.

> 'Then in 500 seconds the average walk of a molecule from start to finish was 10 centimetres, or 0·10 metre.
>
> 'Suppose we call the length of each stride taken by a molecule (or rather the *average* of its various strides) y metres, and suppose the molecule takes N strides in 500 seconds. Then the 10-centimetre average walk must be equal to $\sqrt{N}\,y$.
>
> 'Therefore 0·10 metre $= \sqrt{N}\,y$ metre.
>
> 'We can now get help by another line of attack. We ask how long the total (straightened-out) path of a molecule would be in 500 seconds of travel.
>
> 'Air molecules travel 500 metres per second. Bromine molecules are much more massive and if you worked out the speed of bromine molecules from a barometer reading of pressure, and a weighing and measuring of a sample of bromine gas, you would find the result for bromine only 200 metres per second.

'How far will a bromine molecule travel *altogether* (*straightened-out path*) with speed 200 metres per second travelling for 500 seconds? That total distance will be 500×200, or 100,000 metres.

'Isn't that amazing, a hundred thousand metres, 60 miles in 500 seconds. But then they don't get very far – you can see they don't – only 10 centimetres on the average, because they make so many collisions. Each molecule must make an enormous number of collisions if its *net* progress is so small in that time.

'We can calculate just how many collisions, or, rather, just how many strides between collisions, a bromine molecule makes. That number will be 100,000 metres divided by the length of one stride, y. But that is the number which we have called N. So:

$$N = 100{,}000/y \quad \text{or} \quad y = 100{,}000 \text{ metres}/N$$

'Now we can put that value for N into our earlier equation.

$$\therefore\ 0{\cdot}10 \text{ metre} = \sqrt{N}\,y = \sqrt{N}.(100{,}000 \text{ metres}/N)$$

$$\therefore\ 0{\cdot}10 = 100{,}000/\sqrt{N}$$

$$\therefore\ (0{\cdot}10)^2 = 10^{10}/N$$

$$\therefore\ N = 10^{10}/10^{-2} = 10^{12}$$

'A million million collisions in 500 seconds.'

Now we can find the length of one stride from collision to collision. We divide the total straightened-out path by N, which we have just calculated.

$$\therefore \text{ one stride, } y = 100{,}000 \text{ metres}/10^{12} \text{ collisions}$$

$$= 10^{-7} \text{ metre from collision to collision.}$$

(Some teachers may prefer to carry out the calculation in a different order, writing:

$$0{\cdot}10 \text{ metre} = \sqrt{N}.y = (\sqrt{100{,}000/y})\,y$$

And then solve for y without first obtaining N; but the method shown above is probably easier.)

This gives us the average length of a molecule's stride in its wanderings. It is the stride of a brown *bromine molecule* wandering among air molecules; but that is not far off from the stride of an *air molecule* among air molecules.

Note to Teachers. Teachers will realize that in this very rough calculation any attempt to make a correction for bromine molecules being bigger would place unscientific emphasis on precision in one particular place where we know we are being imprecise overall. Judging from relative densities of liquids and relative molecular weights, bromine molecules have a diameter about 1·2 times that of air molecules. That makes the 'average diameter' for a bromine molecule hitting air molecules 1·1 times an air molecule diameter. Then a bromine molecule probably has a mean free path among air molecules about $1/(1{\cdot}1)^2$ or $1/1{\cdot}2$ times the mean free path of an air molecule among air molecules.

And in this very rough calculation, we have made no attempt to decide what kind of average should be used for the resultant in a random walk treatment. Does the estimate of 'half brown' fit best with a root mean square average of the random walk of bromine molecules, or should we use the plain arithmetical average? Since we estimate progress in a vertical direction alone, should we take some component of velocity, or of mean free path? Unless we give up our simple experiment, in which pupils make a guess, and resort to colorimetry and density measurements, these questions must remain unanswered, as matters of choice and taste rather than definite knowledge. Nor would it be sensible to try to answer them here – that would miss the point of proceeding quickly in a simple story so that we do not lose our pupils on the way.

So we say an air molecule moves about 10^{-7} metre between one collision and the next at atmospheric pressure. In our measurements of molecules and atoms, we shall probably find Ångström units more comfortable. One Ångström unit is 10^{-10} of a metre (easily remembered by its old name 'tenth-metre'). Here the average path of a gas molecule between collision is 1,000 A.U. T

Value of Mean Free Path. The estimate we have used is one of *
the great classical approaches of the last century. There are other *
methods, depending on measurements of vircosity, estimates of *
Van der Waals' constants, etc. The result differs somewhat according *
to the method chosen. For air at room temperature, older estimates *
gave 800 to 1000 A.U. but the modern value (from modified *
statistics) lies between 600 and 700 A.U. In the following discussion *
we shall take the round number 1000 A.U. though a compromise *
value of 800 A.U. might be fairer. *

Model to illustrate Mean Free Path. At this point, pupils C100
should return to a simple, practical model as a class experiment, and look at the progress of a single marble in an agitated tray of marbles. They should have the same tray and marbles as before, but one of the marbles should be of a different colour from all the rest so that its progress can be watched.

(It might be interesting to supplement this by a film of a crowd of people taken from above. If the crowd is not moving as a whole as in a crowded party, the film might reveal the random motion of one person. Or, better still, we could show a film of marbles agitated in a tray. If all the marbles but one are of a dark colour, the film will show the wandering path of the one bright marble.)

Check against Oil Molecule Estimate. Our measurement of T
the length of an olive oil molecule in Year I gave about 16 A.U.; and, taking a hint from chemistry that the molecule is about a dozen atoms long, we say one atom is about $1\frac{1}{2}$ A.U. in diameter. That is an estimate for a carbon atom in the chain of the oil molecule, and oxygen and nitrogen atoms may be bigger, so we might expect 3 or 4 A.U. for an average diameter of an air molecule.

Distribution of Velocities

At this stage a teacher hardly dreams of discussing with pupils the *
distribution of velocities of molecules in a gas. The Maxwell distri- *
bution is something that one thinks of as belonging in university *
physics or perhaps in some A-level discussions: and yet it is of *
enormous importance in real life. The molecules of water that *
evaporate from laundry drying on the line are molecules with *
much more than average kinetic energy. They are out on the tail *
of the Maxwell distribution. The sodium atoms in a salted flame *
emitting the characteristic yellow light are a very small fraction of *
the whole group of sodium atoms, they are far out of the tail of the *
Maxwell distribution.

In fairness to the realities of physics, we should be wise to sketch T
a humpy curve as a chart or histogram of molecular velocities and point out that while there are many molecules moving with speeds close to the average speed, there are some moving slower and faster and just a few are moving very fast indeed, and are very much richer in kinetic energy than the average – just for the moment. It is those very rich ones, made rich by chance collisions, that have enough energy to escape by evaporation. And in many a

case of chemical activity it may be those atoms which have at the moment much more than average energy that are able to partake in a reaction.

The molecular community is not democratic, with a uniform share *
of kinetic energy for everyone. Nor, in a molecule's random walk, *
are all the steps of the same length. A statistical survey would show *
that the steps between collisions have a histogram with a similar *
hump. Of course, that is not a direct consequence of the assortment *
of velocities, but it is another symptom of the molecular chaos. *

Photograph of Marbles (*Buffer option*). If we return to the two- D 101
dimensional model with marbles in a tray, we can take a photograph OPT.
from above, and catch a glimpse of the assortment of molecular velocities. The exposure must be chosen so that it shows the motion of each marble for a fraction of its path between collisions. Then rough estimates of the lengths of blur made by marbles will enable us to make a tally of velocities. This demonstration is not recommended except to an enthusiast who wishes to experiment with exposures, etc.

Knowledge of Air Molecules

Speed. Pupils already know quite a lot about the gas molecules T
which we have imagined to exist. They have seen tiny specks of smoke ash jigging about in an unceasing dance, which may well be due to chance bombardments by yet smaller air molecules. Since we see the smaller specks dancing more than the larger ones, we guess that those still smaller air molecules are moving faster than any specks. Now our imaginative theory of gas molecules in motion tells pupils that air molecules move with an average speed some 500 metres per second.

Space. And a measurement of volume change from liquid air to T
air, 1 to 750, suggests that air molecules are some 9 or 10 diameters 'apart' – that being the distance between a molecule and its nearest neighbour if we could arrange them in a regular array.

Mean Free Path. Now our estimate of progress in bromine T
diffusion suggests that the 'mean free path', the average distance of travel between successive collisions, is about 1,000 A.U., or 10^{-7} metre.

Collisions. We can at once go back from that estimate for air molecules to the number of collisions that an air molecule makes each second. That will be different from our experimental estimate for bromine molecules because O_2 and N_2 molecules move faster. We combine our estimate of molecular speed with our estimate of mean free path. In one second a molecule of air travels 500 metres of *straightened-out* path, and that distance contains $500/10^{-7}$ mean free paths. So the molecule makes 500×10^7 or 5 thousand million collisions in one second. T

Size? We still do not know the size of a molecule, or how many molecules there are in any sample of air, such as the air in a room. But we already know that a room full of air is also, so to speak, a room full of collisions. There is no question of these collisions being anything but elastic, because if any energy disappeared in a collision, say by some mysterious radiation, the air in the room would be down on the floor in a fraction of a second. T

Time for a Long Journey. We can also calculate how long, on the average, the molecules on one side of the room would take to wander across to the other side of the room if there were no convection currents to sweep them across as a crowd. (Diffusion, as we now see it, is not a motion of the whole crowd. Even with the strong driving field provided by hunger, a passenger in a train diffuses slowly down a crowded corridor to the dining car. But the train carries the whole crowd in a very rapid 'convection current'.) T

In T secs, the total straightened-out path of a molecule is 500 T metres. The number of collisions it makes in that time is 500 T metres/(mean free path), or $500\ T/10^{-7}$, or $5 \times 10^9 T$. The average progress from start to finish will be $\sqrt{N} \times$(mean free path), or $\sqrt{5 \times 10^9 \times T} \times (10^{-7})$ metres.

Suppose the room is 6 metres wide (a large room, some 20 feet wide). Then, for molecules to get across the room,

$$\text{average net travel, 6 metres} = \sqrt{5 \times 10^9 \times T} \times 10^{-7}$$

$\therefore\ T = 720{,}000$ seconds, or over one week for a molecule to get across a room of completely still air.

A similar story holds for neutrons diffusing out from inner regions of an atomic reactor, or for quanta (photons) of radiation cannoning their way out through the inner layers of the Sun.

Estimate of Molecule Size: Choice of Two Methods

We can show pupils how to make an estimate of the diameter of a molecule from the estimate of the mean free path of an air molecule. There is no easy way of making this step. We offer a choice between two methods:
(1) a quick method that involves an interesting guess; (2) the standard method, given in textbooks. As an elementary teaching scheme, (1) seems to appeal to many physicists who speculate carefully in modern physics – taking a risk, making a guess, but always remembering the ensuing doubt – but it is not welcomed by those who prefer to maintain formal reasoning and geometry even in a first approach. Its appeal to young pupils is likely to depend strongly on the teacher's own feelings. We hope teachers will try it with classes because it is much shorter and avoids a stretch of argument that will delay the development of our molecular story even if it does not interrupt it completely.

Whichever method the teacher chooses, he will find that young pupils *can* follow the argument if he can succeed in giving them a glimpse of where they are going to and a rough idea of how they are going to get there, because they will then want to follow the detailed story. (As practice for that, teachers will find that trying the argument out on another adult is very helpful.)

Volume Change. Both methods use the volume change from liquid air to air, 1 to 750; and both methods assume that, in a liquid, the molecules are crowded very close together.

We have already discussed the measurement of the volume change, and have shown that it leads to a picture of air molecules spaced 9 or 10 diameters apart in ordinary air.

In Experiment 91, we urged teachers to demonstrate the volume change with liquid air; or, failing that, to use solid CO_2 or show the change from water to steam. That volume change *for air* plays an essential part in the arguments ahead; and pupils must have it clearly in mind, as something they have seen and know. Even if it was too difficult to obtain liquid air before (D 91), we hope teachers will obtain some now‡ and show the volume change.

The best method is that of D 91(a): fill a tiny volumetric flask, say 5 cubic centimetres, with liquid air, quickly push a flexible plastic tube over the neck, and collect the air that bubbles off, over water in a large trough. D102

If liquid air *cannot* be obtained, it will probably be best to show a short film of that experiment – a very poor second best when we hope that pupils will see a real demonstration offering essential evidence directly. F103

As a crude assumption to enable us to make an estimate easily, we assume that in a liquid each molecule has a space d^3, where d is the diameter of the molecule, imagined to be spherical. Then the volume change tells us that in ordinary air the space containing one molecule has on the average a volume $750d^3$. T

We now proceed to make an estimate of molecule size from our measurement of mean free path and our knowledge of the volume change. The alternative methods are described below, the 'quick method' first and then the usual formal method.

1. **Estimate of Molecule Size: 'Quick Method'.** We draw a picture of molecules in a gas: dots of chalk spaced far apart on a blackboard. We may add arrows to show the random motion, not all speeds the same, but speeds around an average. We say: T

> 'Here is a snapshot of air molecules in this room with the camera focused for just one distance.'

(For some pupils that snapshot takes on even more significance if we do not add arrows but pretend that the exposure was just long

‡ The manufacture of liquid air is now such a large industrial business that it seems to be more difficult than it used to be for schools to obtain small quantities.

However, most universities use large quantities of liquid air (or liquid nitrogen) in their physics and chemistry departments. They regard it as a common commodity. If teachers consult those departments in the nearest university, explaining that their need is serious, they are likely to find them ready to help.

Liquid oxygen is considered a dangerous material to transport. Liquid nitrogen and liquid air are easily carried safely in open thermos flasks. If there is a university or a commercial research laboratory within reach, the school should be asked to provide transport and equip the laboratory with two or three thermos flasks.

enough to let an average molecule move, say, one inch. Then we draw little lines to show the blur we imagine made on the picture by each molecule.)

We sketch one small molecule moving through space, passing many a neighbour before it hits another molecule. We do not yet know how the mean free path that we have estimated, 10^{-7} metre, compares with the average spacing between molecules, which we already found is 9 or 10 diameters. But presently we shall find that the latter spacing is some 30 times smaller than the mean free path; so we should sketch our molecule moving past many neighbours before it hits another.

> 'Here is one molecule travelling through air missing others until it makes a hit here. If the molecule were a living creature, such as a bird, that could look ahead as it flies, it would see neighbouring molecules as very small targets and would not be surprised at missing many of them before it hits one target molecule.
>
> 'Suppose it has just finished a collision here and travels on and on until it makes a collision there. We call that one free path. At that collision it bounces off in some new direction – which we can hardly predict – perhaps like this. It travels some way before it makes another hit; and then another hit; and then another ... the average value of those travel-distances between hits is what we call the mean free path. You and I have estimated that mean free path for air in this room: 10^{-7} metre, or 1,000 Ångström units.
>
> 'Now imagine we have some air in a tall cylinder with a piston, like this. (Blackboard diagram.) Suppose we push the piston in and crowd the air molecules into smaller and smaller volume until we have reached the volume for *liquid* air.'

Pupils already know the amount of compression necessary, from the measurement of volume change discussed above. The piston must compress the volume in the proportion 750 to 1. The compressed air would be as dense as liquid air if we have made the right change of volume, but it would not be liquid unless we then cooled it down to a suitable temperature, nearer to $-200°C$ than room temperature. However, if we kept the volume the same, that temperature change would have no effect upon the crowding. So, although we may want to mention the cooling necessary for liquefaction, we should not let it play any part in the ensuing discussion.

'How far should we have to push the piston down for that? How big is the volume change from ordinary air to liquid air?

'How much must we compress ordinary air, squeeze a sample down, until it occupies the same space as it would if it were liquid air? ... Yes, we must squeeze it down in volume 750 to 1.

'What will that 750 to 1 squeezing do to the mean free path? Think of a flying molecule, like a bird, looking at target molecules that it misses, until finally it hits one at the end of a free path. What have we done to the chances of hitting a target molecule by crowding the molecules together, 750 to 1?

'From the point of view of our flying molecule, we have moved all its targets up closer. We have shortened all their distances 750 times. Putting it another way, we have crowded the targets closer as if we had piled in more molecules to make the array of targets 750 times as dense. The flying molecule will hit another target molecule after a much shorter flight; in fact, after a path 750 times shorter.‡

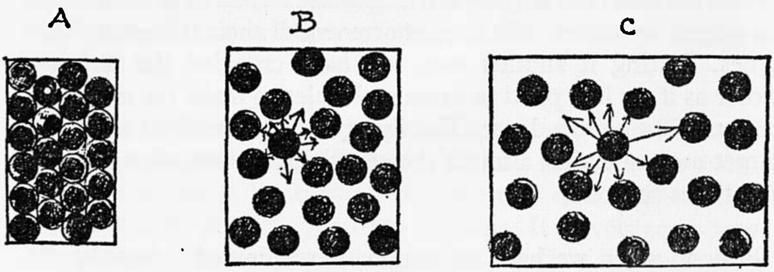
'So now, when we have in imagination squeezed a sample of ordinary air down until it is crowded as close as liquid air, we have changed the mean free path from 1,000 Ångström units to 1,000/750 Ångström units.'

'Now let us think about the crowded air. How close should we T
find the molecules if we could actually see them? Let us pretend that the crowded air is actually *liquid* air. (That would only need some cooling, without any further change of volume, to turn it into liquid air.) If you can guess how crowded the molecules would look in that liquid, you will be able to find out the size of a single molecule.

‡ **Note to Teachers.** That may appear to be the obvious change of mean free path when the volume is changed 750 to 1. Or, looking at it another way and thinking about motion in one dimension, we might be tempted to expect a factor of cube root of 750; but that is incorrect. The mean free path is a matter of hitting a target. And the volume change tells us directly the change in population-density of targets. With 750 times as many targets per unit volume, the average travel for a molecule before it hits a target is shortened by the factor 750.

'This is where you will first have to make a guess. But you should be just as good at guessing – as amateur scientists – as professional scientists could be. We shall draw the molecules as round balls and imagine they are hard, round balls like billiard balls, although real air molecules must be oblong and perhaps a little squashy.

'First, suppose that our crowding 750 to 1 has pushed them together until they are all touching, packed as closely as possible, as in Picture A. Do you think they *could* be that close? Could molecules arranged like that behave as a liquid which can be poured easily? What would Picture A be a sketch of? ... Yes, a solid crystal – and a very strong one too. In a liquid which can be poured and move around easily, molecules are probably a little farther apart than that.



'Now look at Picture C, where we have taken out half the molecules. Does that look to you like a liquid? ... Yes, it might be, but the spaces do look large. It looks as if a molecule could easily move quite a long way among its neighbours. Diffusion should be fairly fast; but actually diffusion is very slow in liquids. (Try putting copper sulphate crystals at the bottom of a tall jar of water; and wait and see how slowly the blue solution diffuses.) With the spacing in Picture C, it looks to me more like a gas. Think of a crowd of people when it is behaving as a liquid – a crowd that can flow through corridors to a railway station or a football match – but is too dense to allow individual people to move far among their neighbours.

'Look at Picture B, as an intermediate guess somewhere between the extremes of no distance between molecules, which would lock them tight like a solid, and spaces as big as one whole diameter in Picture C. Look at one molecule, X, in Picture B, and guess at its mean free path. Draw an arrow from X to show how far it can move in any direction you choose. Start the arrow at the *surface* of X and continue it until you meet the *surface* of another molecule. Do that in many different directions and look at the length of the arrows. Take an average of those lengths. (This is only a flat picture, in two dimensions, but we shall use this to make guesses about the picture in three dimensions.) Picture B is drawn so that the average length of such arrows is about $\frac{1}{3}$ of a diameter. On that basis, a molecule would have a mean free path of something like $\frac{1}{3}$ of its own diameter.

'Now it is *your* turn to guess. Do you think that Picture B shows molecules too close together or too far apart to be a good picture for molecules in a liquid? You can only guess. This is a wild game of just making the best guess you can, knowing that the result it leads you to may be wrong; but also knowing that that result will still be very interesting and valuable.'

In the teacher's description of the problem, sketches may suffice; but when pupils come to make their own guesses they should have tangible models to play with. The simplest model is a collection of pennies placed on the table. Starting with the pennies far apart, the pupil sweeps them closer with a ruler and tries various close spacings. A collection of *small* ring magnets instead of pennies may make things easier for the averaging glance, because the rings will tend to avoid clumping – but their repulsive forces make the model even less realistic. As a luxury demonstration the small ring magnets could be fitted with lids and given small charges of solid CO_2 – but that would probably divert attention unprofitably at a busy stage.

What happens here is obviously unpredictable. The outcome depends on both the teacher's feelings about the problem and the pupils' feelings. Guesses may run as high as $\frac{9}{10}$ of a diameter, and as low as $\frac{1}{10}$ of a diameter. But yet that whole range covers only one order of magnitude and even that is forgivable. We are trying to enable young pupils to make a guess at the microscopic diameter of a molecule. An uncertainty or error of half an order of magnitude, a factor of 3, should not frighten us. We should still welcome the microscopic estimate. Teachers who carry out the standard formal method will find that the guess in this crude method which will lead to the 'right answer' is about $\frac{1}{3}$ of a diameter. However, we certainly should not drive pupils to accept that guess – or we might just as well announce the molecular size itself and drive them to accept *that* without any experimental basis. If pupils understand that we accept whatever guess they make, they will have some pride in the result.

Note to Teachers. Of course, this geometrical guessing is unreliable and unrealistic because, at this close crowding, the usual meaning of mean free path is modified by the question of the volume occupied by each molecule itself. We might doubt whether the calculated reduced mean free path is really the average distance travelled surface-to-surface, instead of some centre-to-centre distance (which would be greater by a whole diameter). Nevertheless, this is sane guesswork which does enable us to estimate a molecule's size within an order of magnitude without having too serious doubts.

In the discussion that follows, we should accept some kind of vote T
from the class on the fraction of a diameter to be taken as the mean free path. In the example below, we shall use the 'right' fraction $\frac{1}{3}$, but that is not what we suggest teachers should enforce in their class discussion.

'All right, we agree on $\frac{1}{3}$ of a diameter for mean free path at liquid crowding. Then, when we have squeezed the mean free path down by 750 from that in ordinary air, it is only $\frac{1}{3}$ of a diameter, $d/3$.

'Therefore, 1,000 Ångström units/750 = $d/3$.

$$\therefore\ d = \frac{3{,}000}{750} \text{ Ångström units, or about } 4\times10^{-10} \text{ metres.}$$

'We have found the diameter of a single molecule of air. An atom is probably about half that size. This is certainly a very rough estimate because our measurements were difficult and we have had to make a very risky guess. But yet this is a good estimate for many working purposes. It is in the right county, it is of the right "order of magnitude".

'We have made an atomic measurement here ourselves, and you can see how small atoms and molecules are.'

This is the end of the quick method. Teachers who use this method should now proceed to the discussion of molecular sizes and numbers, which follows the description of the 'Formal Method'. * * *

2. **Estimate of Molecule Size: Formal Method.** This is the method given in standard textbooks of Properties of Matter.

We draw a picture of molecules in a gas: dots of chalk spaced far apart on a blackboard. We may add arrows to show the random motion, not all speeds the same but speeds around an average. We say: T

> 'Here is a snapshot of air molecules in this room with the camera focused for just one distance.'

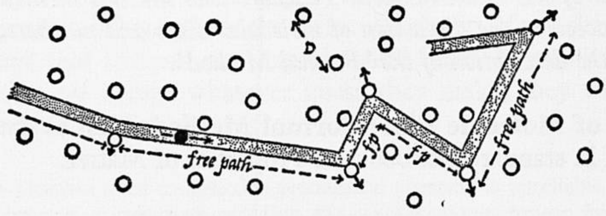
We point out that to find how one molecule would move through this vast array of moving neighbours is too difficult a business. Instead, we shall pretend that we freeze all the molecules except one and watch that one molecule go hurtling through the crowd.

Now we redraw the picture, showing each molecule as a small round blob, without any indication of velocity. We take a piece of chalk of *length equal to a molecular diameter* and draw the path of our chosen molecule, holding the chalk sideways so that it makes a white strip one molecular diameter wide. When at last this path meets one of the other molecules, there is a collision; we bend the path and proceed in a new direction until there is another collision. (As we shall find, the mean free path is many times longer than the average spacing between molecules; so we should draw our missile molecule passing by many neighbours before it makes a hit.)

We now move off to a separate preparatory discussion, looking at such a collision in detail. We draw a large round molecule bouncing against another large round molecule and ask how far apart they are, centre to centre, at the collision. The answer is (radius + radius), or one diameter apart. Then we say:

> 'I am going to show you a trick for finding out how far a molecule goes before it hits another. This is a trick which has been invented by scientists, and it is not what really happens; but I think if you watch carefully you will see it will give good results and presently you will be glad to be able to use it.

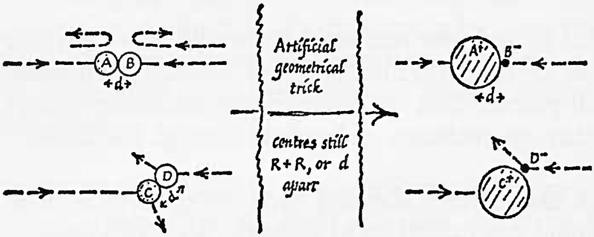
1



Mean Free Path of a Gas Molecule

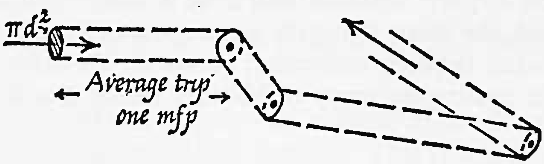
At atmospheric pressure, the mean (= average) free path is much longer than the spacing, D. (The shaded tube shows the volume swept out by one molecule moving through others.)

2



Simplifying Geometry for Mean Free Path

3



$[\pi d^2] \times$ [mean free path] is the volume that holds one target molecule (on average).

'When two molecules collide they must be (1 radius + 1 radius) apart, 1 diameter apart. Instead of drawing the collision like that I could pretend that the molecule flying along to make this collision is much bigger and any other molecule that it hits is much smaller – we shall get the same result as long as we have the centres of the two a molecule-diameter apart at the collision. I am now going to push this to the limit and make the travelling molecule have double size, so that its *radius* is just one molecule *diameter*. Then I must make the other molecule that is hit have no size at all: I must draw it as a point.

'Now we can start the story all over again. Here is the molecule flying along, an artificial molecule with radius one molecule diameter. It flies along marking out this broad strip two diameters wide. I am showing all the other molecules just as points. Where must one of these be if it is to be hit by the flying molecule? ... Yes, that point must be anywhere on this wide chalk track that I am making. Then there will be a collision. So here is the story: the flying molecule travels along until here, where it hits a molecule that is on its path, so it bends its path, flies along like this; and here it hits another molecule; and so on.

'Now let us think about the path swept out by this flying molecule which is possessively patrolling its "share" of the volume of the containing box. We are pretending that this extra big molecule is a round ball of *radius* d, the diameter of a real molecule. What shape in space does it sweep out, as it flies along? ... Yes, it sweeps out a cylinder. What is the cross-sectional area of that cylinder ...? Yes, not πr^2 but in this case πd^2, where d is the diameter of a real molecule. How long is that cylinder between one collision and the next? You already know that. That is the distance from collision to collision, y, which we call, on the average, the mean free path. We have measured that with our bromine experiment. In ordinary air it is 1,000 Ångström units, or 10^{-7} metres.'

In describing this cylindrical path which makes a bend at every collision, we might say 'like a bent gutter pipe', or we might call it a 'bent sausage', to make the picture clearer to pupils. * * *

'We are nearly there; we have nearly arrived at the actual size of a molecule. But first we must bring in some more information. T

'We shall need to know the volume of space that belongs to one molecule in the air in this room.

'Remember that the volume change from liquid air to air is about 1 to 750. If we say, as a rough guess for liquid air, that each air molecule of diameter d occupies a cubical box of side d, volume d^3, then in ordinary air the space for each molecule is 750 d^3, on the average.'

(Curiously enough, that spacing, which sounds much closer than the $d/3$ mean free path we guessed at in the quick method above, does fit with the latter guess, when we view it in three dimensions.) * * *

Note to Teachers. All through our discussion we used a very crude picture of molecules in a liquid. It is not a good picture for a liquid – it looks more like a picture of atoms in some cubic crystal – but it is close enough for our present rough estimate. We should say to pupils 'Remember that this is a case of desperate measures for desperate needs.'

'Now look back at the place where we left our calculation from the mean free path. We have one extra-fat molecule tracing out a "bent sausage" cylinder through the gas in which all the other molecules are just points. From one collision to the next, the missile molecule sweeps out a cylinder of area πd^2 and of length one mean free path, 10^{-7} metres. The volume of that length of sausage between one bend and the next is the volume of space that contains just one target molecule for the missile to hit. So that volume is $750d^3$.

'Therefore $750\ d^3 = \pi d^2 \times 10^{-7}$

'Therefore $d = \pi \simeq 10^{-7}/750 \times 4 \times 10^{-10}$ metre,

or 4 Ångström units.

'We have found the diameter of a single molecule of air. An atom is probably about half that size. This is certainly a rough estimate because our measurements were difficult and we have had to make all kinds of risky moves in carrying through our calculations. But yet this is a good estimate for many working purposes. It is in the right county, it is of the right "order of magnitude".'

† Teachers who have not taught the material of Year I, are advised to look back at the *Teachers' Guide* for Year I and make use of the remarks regarding rough estimates, also the example suggested in Problem C in Year I. T

'Precise' Values for the Diameter of an Air Molecule?

When either the quick method or the formal method has been used to arrive at a rough estimate of air molecule diameter, both teacher and pupils are likely to ask how big an air molecule 'really is'. The diameter that is given by precise measurements is about 3·7 A.U. Careful measurements give different diameters according to the experiment chosen and method of interpretation used. After all the diameter of a molecule is not as definite a thing as the diameter of a steel ball or even the diameter of a man's head. Not only is an air molecule 'oblong', but it behaves as something 'squashy', so that more violent collisions are likely to reveal a smaller effective diameter. See the note following the oil – molecule estimate in Year I *Teachers' Guide.*

All we are really measuring is some distance of approach at which inter-molecular forces grow large enough to make noticeable effects in our experiments.

Of course, extremely violent nuclear collisions point to a far smaller diameter. Alpha particles passing through air usually make only trivial collisions in which they pull an electron off an atom and continue straight ahead. Very rarely, they make a violent collision, shown by a fork in the track, where the change of momentum tells us they have hit something massive. Those collisions are so rare that we conclude that most of the mass of an atom is concentrated in a very small 'target' region, a nucleus. We can calculate the alpha particle's closest approach in those rare nuclear collisions and we find a distance some 10,000 times smaller than the atomic size we have just been guessing at. By using more energetic particles, we find that the nucleus itself is still smaller, say $\frac{1}{100,000}$th of the whole atom – so far as we can assign a definite 'size' to a nucleus.

Numbers of Molecules

After using the quick method or the formal method to reach an estimate of the diameter, of an air molecule – with a result somewhere between 1 and 9×10^{-10} metre – we can proceed to estimate the number of air molecules in, say, the classroom.

Now that we know the diameter of a molecule, we can go back to the space, d^3, 'occupied' by a molecule in a liquid, and then the space in open air.

Then we can find out how many molecules there are in the room. We assign to one molecule a space of volume d^3 in liquid; and therefore we must assign to a molecule in ordinary air a volume $750\ d^3$. We know d, so we know d^3 and $750\ d^3$. We can calculate how many molecules there are in a room full of air by dividing the volume of the room by $750\ d^3$.

Example (taking $d = 4\times10^{-10}$ metres). A small room 4 metres $\times$ 3 metres $\times$ 2 metres has volume 24 cubic metres and the number of air molecules in it is about

$24 \div 750\ (4\times10^{-10})^3$. That is, 5×10^{26} molecules.

Estimate of the 'Avogadro Number'

The last number is in fact our estimate of the 'Avogadro number', for a kilo-mole. A gram-molecule, or mole, of any gas contains 6×10^{23} molecules. It occupies 22·4 litres at 0°C or about 24 litres at room temperature and atmospheric pressure. If we use M.K.S. units, we should for consistency deal with a kilo-mole, which contains 6×10^{26} molecules and occupies 22·4 cubic metres at 0°C, or 24 cubic metres at room temperature. Where does that number, 6×10^{26}, come from? From measurements and arguments just like those we have carried through, though in fuller and more careful form.

Our pupils will have that number given to them (or the number for the ordinary mole) in chemistry and we owe it to them to show them that they now know how it was obtained.

Either in class, or in a problem for homework, pupils should repeat the calculation just above, to find the number of molecules in 24 cubic metres (or in 24 litres).

When pupils point out that the 'volume in chemistry is 22·4, not 24', we should reply that we are working in a warm room, where the 22·4 measured at melting ice temperature will have expanded – always at atmospheric pressure – to 24.

Note to Teachers on the Avogadro Number. We should rejoice over good luck if our estimate of the Avogadro number comes as near as 4 or 9 to the expected 6.

The essential measurement, the pupils' judgment of the distance for 'half brown', comes into that final result to the sixth power; so a change of 10% in that measurement will make a change of a factor of 2 in our result! Teachers who try this experiment with pupils will find that the general voting does not range more than about 10% either side of the average vote. * * * * * *

Comparison of Molecule Size with Oil Film Measurement. T
Of course we should now compare our estimate of air-molecule size with the measurement made with an oil film spreading. In Year I, the olive oil seemed to have a molecule about 16 A.U. long. If we make use of chemical knowledge about the oil molecule, and divide by a dozen atoms, we get about 1·3 A.U. for a carbon atom's diameter; so it is not surprising to find 3 or 4 A.U. for an air *molecule*.

General Comment to Teachers. This has been a long journey to travel from the first experiments with bromine diffusion to the final knowledge of the number of air molecules in a room. Without question, young pupils cannot follow this long argument in a single day. We should prepare the ground, by talking about the kind of thing we are going to do and by doing some simple experiments on the random walk story; and then we should cover part of the story at a time, always pointing to the top of the difficult mountain that we are climbing. It is clear from trials with fairly young pupils that success is possible. * * * * * * * * * *

General Comment to Pupils. Our knowledge built on assumptions about molecules, but reinforced by some cross-checks, has now grown to a much more definite picture: we think of air molecules as 3 or 4 A.U. in diameter; moving 500 metres per second on an average, with an average spacing, in common air, of 30 A.U. or more from their nearest neighbours; and travelling about 1,000 A.U. between one collision and the next. We know how many there are in a large room, and we know the mass of a single molecule. T

We might post up these pieces of molecular knowledge in a chart. D104

All this came from imagining a theoretical picture – guided by things we know about nature, such as Newton's Laws of Motion – and then making measurements. T

Chapter 3

UNIVERSAL CONSERVATION OF ENERGY

Heat as a Form of Energy; Universal Conservation

HEAT AND MOLECULAR MOTION

We have developed our idea of the kinetic theory of gases – a 'grand conceptual scheme' – and we have seen that it covers a good deal of our experimental knowledge of gases and enables us to predict certain phenomena.

Now it gives us a convenient link with ideas of mechanical energy. The notes that follow offer suggestions to teachers for a word about heat and molecular motion. It should only be a brief word now, before we go back to heat measured with water and discuss its relation to mechanical energy. Looking back on the kinetic theory expression for PV, we see that clearly it contains the kinetic energy of motion of all the gas molecules. Now that we know their speed, we can in fact calculate that total kinetic energy for a sample of gas of known size at known pressure. Then if we imagine the gas heated up, without change of volume, so that the pressure becomes greater, we can calculate the new value of kinetic energy. We may well imagine that the heat that we have put in to warm up the gas to the new temperature has gone to increase the kinetic energy of motion of gas molecules.

Note to Teachers: Changes of Energy in a Gas. When a confined gas is heated, it gains thermal energy in the form of increased molecular motion. Some of that is kinetic energy of random motion of molecules; but with many kinds of gas molecule there is also a gain of rotational energy and sometimes of energy of vibrations as well. That is all the heat energy of the gas consists of; there is no store of potential energy, *except* the P.E. of the (P.E. + K.E.) of vibrational motion.

(Pupils sometimes think of a gas at high pressure as being like a compressed spring with 'strain energy'. If we compress a gas by pushing a piston quickly into a cylinder, the gas grows hotter, and all the energy transferred from us to the gas goes into thermal energy of molecular motion. If we let the compressed gas cool back to the original temperature, it loses all the thermal energy that it gained; so all the energy that we transferred to the gas has now escaped to the outer world. The compressed gas, back at the room temperature, has no extra energy by virtue of being compressed. Yet we can make it transfer energy to other things by letting it push a piston out with its high pressure. True, but the energy it then supplies will be taken from the gas by the gas cooling down below room temperature.)

Heat Energy given to a Gas. We can even calculate, in joules, how much that increase of molecular kinetic energy is. We can also measure it experimentally – a specific heat measurement. A measurement of the specific heat of a gas is too difficult to demonstrate, but we can tell pupils the result if we wish, and we can imagine an idealized way of obtaining it by heating a sample of gas electrically, measuring the energy input with voltmeter and ammeter. We might hope to find that the energy taken from an electric supply to heat a sample of gas agreed with our calculated increase of kinetic energy of motion of the gas molecules. We should find that true for a gas such as helium‡ or neon, in which the molecules are single atoms that do not indulge in rotation or vibration that can be increased by heating. However, for other gases, such as air or carbon dioxide, we find that the electric supply has to deliver more energy than goes simply into the kinetic energy of molecules flying about in the gas. The extra energy goes to provide for rotational motions and vibrations of molecules. All this is private knowledge that a teacher keeps at the back of his mind in returning now to a discussion of heat and mechanical energy.

† Revision of Energy Teaching

We explain that we shall go back to very early discussions of energy and question them. We know how to measure an energy change when a force pushes something along for a measured distance. [Force] × [distance] tells us something very useful: it tells us how much fuel must be used to do that job; it often tells us how much money must be paid for fuel; it tells us something about the job which is inescapable – we cannot get a job which involves a force pushing along a measured distance done without paying for fuel or without taking some energy from some other store.

† We started by measuring the 'work' which tells us the amount of energy changed from one form or place to another in foot.pounds (meaning foot.pounds-weight) also in kilogram.metres (meaning kilograms-weight.metres), and now we measure work in newton.metres, which we call joules.

Conservation of [P.E.+K.E.]. We now know how to account for energy which goes into a moving body. A mass m moving with speed v has 'kinetic energy' $\frac{1}{2}mv^2$. Since we arrived at $\frac{1}{2}mv^2$ by

‡ But very difficult experimentally because helium molecules carry heat away so fast to the walls of the container.

working out the value of [force] $\times$ [distance] for a force that is accelerating the mass m, we expect to find that kinetic energy $\frac{1}{2}mv^2$ and potential energy add up to a constant total in many cases, for instance in a gravitational field. A stone dropped over the edge of a cliff starts with a rich store of gravitational potential energy, and it gains more and more kinetic energy. We certainly should not be surprised to find that, at all stages of the fall (forgetting air friction), the sum of K.E. and P.E. remains the same. That is because we found the expression $\frac{1}{2}mv^2$ by asking for [force] $\times$ [distance], and the latter is exactly a statement of P.E. that is lost when that kinetic energy is gained! There we have a very simple form of conservation of energy.

Note to Teachers, on Conservative Systems. When we are storing up potential energy, we push with a force through a certain distance, against the push of a spring (in which we are storing up strain energy); or we push or pull against the pull of gravity on some object, storing up gravitational potential energy as we raise the object. The counter-force exerted by the storage system, the push of springs or the pull of gravity, is completely reversible. The spring exerts the same push when we are letting it expand again after compressing it, as it did during compression (at the same stage). Gravity exerts the same pull when the object it acts on is moving downward as when we are pulling it upward.

In general, such forces have the very important property that they 'are the same on the way out as on the way in'. Then the same characteristic holds for the force we, or some other engine, must apply in pushing against those forces and storing up potential energy. The change of potential energy for a given distance moved is the same 'on the way out as on the way in'. We get back from the store all the energy that we put in. And when the energy that we get back is all turned into kinetic energy, we find that the total of K.E. and P.E. remains constant.

However, there are other forces such as those due to friction which are not 'the same on the way out as on the way in'. If we push a load up a rough sloping hill, we increase its potential energy; but we also produce some heat, pushing against friction forces. If we then let the same object move down the hill, gravity still pulls downward, and its downhill component is still the same in size and direction as before – so we regain just as much energy from the P.E. store as we put in, but that does not apply to friction. When the

object moves downhill friction turns round and drags uphill *
against the motion. Some of the energy supplied by the agent that *
moved the object uphill went into heat; but that heat does not get *
reconverted into mechanical energy on the way down. Instead of *
that, still more heat is produced. Then, although all the gravita- *
tional potential energy that was stored up is returned, we do not *
get it all back as kinetic energy, but some of it is delivered, inevit- *
ably, as heat. In this case, we cannot say that (P.E.+K.E.) keeps a *
constant total. Considering the forces, we see that the agent pushing *
the body uphill has to exert more force than the component of *
gravity, while on the downward trip the body exerts less than that *
component on any external agent. The external force is not the *
same on the way down as on the way up. The essential criterion for *
a conservative system is that the force should be 'the same on the *
way out as on the way in'. *

Machines. Again if we transmit mechanical energy from one place to another by a 'machine', such as a lever or set of pulleys, we find that the energy that we gain at the 'output' where the lever or pulley raises a load, is at best exactly equal to the energy we supply at the input, where we push the other end of the lever down or pull down on a rope of the pulleys. In practice, the output is less than the input. No machine ever manufactures energy. No machine puts out more energy than it takes in. Perpetual motion is impossible. T

(Teachers are advised to look at the Note on Perpetual Motion, in *
contrast with Perpetual Movement, in the General Introduction at *
the beginning of Year III.) *

Energy 'Disappears'. Heat Appears. In practice, a machine puts out less mechanical energy than it takes in. Does that mean that energy is disappearing? Is energy conserved in that case, and is it conserved in other more complicated transactions? When we put mechanical energy into a body, for example, fire a bullet at a suspended target, we can account for some of the energy clearly. The target is raised as it swings back and we say it gains potential energy. But if we calculate the kinetic energy of the bullet and the potential energy of the raised target, we find that the latter is smaller. T

As in the case of practical pulleys and levers, we account for the difference by saying that the remaining energy of the bullet has been transferred to the particles of the target (and of the bullet's own material) so that they are now moving about more rapidly or vibrating more violently – and that the temperature has risen accordingly. But how do we know that? Is it just wishful thinking? In some of the cases there certainly is measurable temperature rise. As practical demonstrations or class experiments:

bore a piece of metal with a blunt drill, and pass the drill around to be felt; D/C105a

let each pupil hammer a piece of lead‡ violently and feel it; D/C105b

push the piston of a bicycle pump in quickly while holding a thumb on the outlet, and feel tangible heating. (And there we have a very interesting question to ask pupils: '*How* does the bicycle pump heat up? What really happens to the molecules?' We can give them a hint by asking the difference between blocking a ball with a stationary bat and hitting a ball with a big swipe.) D/C105c

Molecular Stores of Energy. We also know cases where we put energy into a body and its temperature does not rise. As we melt a bucket of snow, we are certainly taking heat from the gas flame but the slush grows no warmer. There we might say a few words about the transfer of heat energy to potential energy in the force-fields of molecules, in the course of pulling molecules apart against the forces of mutual attraction that hold them in a solid crystal. T

Extended Discussion of Thermal Energy (*Optional extra*). T
If we assume that all the energy supplied to a gas becomes energy of molecular motion, do we expect it all to contribute to the molecular motion that maintains pressure on the walls of the surrounding box? No. In fact, some of it may become energy of vibration or rotation of molecules, and that does not contribute to the overall motion of the molecules which is responsible both for the bombardment pressure and for the thermometer reading of temperature. Thus thermal energy of materials includes not only the K.E. of straight line motion of molecules, but also kinetic energy of rotation and vibration, and some potential energy of atomic interactions in vibrations.

‡ A small piece of sheet lead ($\frac{1}{2}$ ounce to 1 ounce at most) is wrapped round a piece of stout iron wire, near one end. The experimenter holds the other end of that handle, places the lead on an iron kilogram or some other anvil and hits the lead violently with a hammer. The most impressive way of feeling the temperature-rise is to try the lead on one's cheek.

With a very fast group we might mention – as a look forward into *
A-level physics – this matter of molecules spinning and vibrating *
and needing some heat energy for those interactions; but in general *
we should not mention them. *

Note to Teachers. Some pupils will suggest a strange idea that *
collisions among molecules absorb some of the energy we give to a *
gas. We must explain that unless collisions are completely elastic *
and there is no loss of kinetic energy, the gas would quickly *
collapse – and real gases do not. If we once allow pupils to believe *
that perhaps just a little energy disappears somewhere in a col- *
lision, we shall soon get lost in an underworld of unknown measur- *
ables, which is not the world of science. *
*
True, if we raise a gas to a high enough temperature some energy *
may be used in tearing molecules apart – dissociation – but then *
we are well aware that we have changed to a different gas. And at *
very high temperatures we may 'excite' some molecules by raising *
one of their electrons to a higher energy level – but that is an *
exception which will be obvious when we meet it (and that should *
not be mentioned now.) *

†Revision of Simple Heat Measurements

Heat as an Experimental Concept. If pupils do not remember T
their earlier measurements of heat clearly, we should offer quick revision now, by some class experiments. In these, we treat heat as 'something that makes things hotter' (also melts solids, etc.), something whose measurement is defined by [mass of water] × [temperature rise].

We are in a hurry to get on to a great discussion of Conservation; and we should not spend long on thorough teaching of heat measurements. It may even be good to cut corners and teach by a method reminiscent of the old, uncouth 'unitary method' in arithmetic. Instead of first building a scheme for heat measurement and then choosing a unit, we define our unit and make the whole scheme appear to come from that. If a problem asks 'How much heat is needed to warm up 40 kg of water from 20°C to 50°C?', we do not start by saying:

Heat = [mass of water] × [temperature rise] = ..., etc.
Instead we say:
1 kilocalorie warms up 1 kg of water 1 C°
∴ 40 kilocalories warm up 40 kg of water 1 C°
and 40 × 30 kilocalories warm up 40 kg of water 30 C°
That looks harmless, and much like the formal method. Yet look at the following excerpt from an answer to a question on electric currents:
1 amp passing through 1 ohm generates 1 joule of heat per sec.

∴ 10 amps passing through 1 ohm generate 10 joules of heat per sec.

Simple faith in universal proportionality can trip up the unwary.

We also point out briefly that when the material being heated is not water we multiply [mass] by [temperature rise], as for water, and then – if we wish to measure the heat in our standard units – we multiply by a 'special factor for the substance', the factor we call specific heat. We do not say much about specific heat in this * course or make any systematic measurements because we have * agreed to leave thermal measurements mainly in the Nuffield * Chemistry Programme. *

We must also mention the heat needed for changes of state. When T we have defined our units, kilocalories, we may tell pupils that we measure the heat needed for melting a solid in kilocalories per kilogram of solid converted to liquid, with a similar statement for the change from liquid to vapour. There again we shall leave detailed studies to the chemistry course.

Our object in these revision experiments is simply to show heat as * something measured by [mass of water] multiplied by [temperature * rise] and to suggest that when hot and cold things share heat be- * tween them the total heat is usually conserved – apart from the * practical losses which are so troublesome. *

a. **Demonstration Experiment. Measuring Heat Exchanges between Hot and Cold Water. Conservation.** We use a large light container so that the thermal capacity of the container itself may be neglected in this rough demonstration. Weigh out a quantity of cold water, say 2 kilograms, in a separate beaker and take its temperature. Weigh a quantity of hot water, say 3 kilograms, place D 106a it in the light container and take its temperature; then without delay pour in the cold water. Stir the water and take the final temperature.

We tell pupils we would like to find something which stays the same during this mixing. After all, that search for constancy or conservation runs all through the development of physics. (See Note on 'Constant' in the General Introduction at the beginning of Year III.)

We look at the temperature-changes of the two lots of water that we have mixed and ask if they are equal and opposite – in which case they would hold the key to something that is conserved. In general they are not equal.

Then we elicit the suggestions that we can calculate a quantity which stays (almost) the same by multiplying [mass of water] by [change of temperature].‡

We explain that because this product is something that stays the same in many exchanges – and because it seems to measure something that we think worth paying for – we give it a name, 'heat'; and we give a name to its units; kilocalories as a short name for [kilograms of water] . [C°]. Of course the heat lost by the hot water will not turn out to be exactly equal to the heat gained by the cold water, as we calculate those quantities from our experiment. We should forestall worries or complaints about this disagreement by warning pupils beforehand that we cannot prevent some of the heat that we are trying to transfer escaping to the air, etc.

Pupils will ask if some heat is also taken by the container in which we do the mixing; but we should forget our own skilful training in dealing with such matters of calorimetry, and not allow ourselves to be sidetracked into that.

‡ In dealing with temperature-rises, some teachers find it an advantage to express them in C°, rather than °C. They make a distinction between 10 °C, the temperature of a cold day, and 10 C°, the temperature-rise from a tepid bath at 30 °C to a hot bath at 40 °C. In 10 °C, the C (for Celsius or Centigrade) is like an adverb, showing *where* on the thermometer. In 10 C° the C is like an adjective telling us the *size of the degree* we are using. Other teachers consider this an irritating trivial distinction and prefer to avoid it, using °C throughout. Teachers who make the distinction report that pupils, taught it from the start, rather enjoy it and find it useful. We suggest trying it here.

In order to minimize the last trouble – the effect of the thermal capacity of the container – some teachers do the mixing in a bag of thin Polythene sheeting. That certainly has negligible thermal capacity, but it makes the demonstration an anxious one, because the bag is easily broken. Some teachers use a large glass beaker with very thin walls. To use insulated calorimeters or vacuum vessels – admirable though those are in courses where calorimetry is given a full treatment – would be a mistake here.

Demonstration Thermometer. This experiment is rendered almost worthless as a demonstration if the temperatures have to be read by the teacher alone. It deserves either a crowd of pupil observers round the apparatus or some form of demonstration thermometer. However, it does not seem justifiable to buy a big demonstration thermometer with steel tubing running from mercury bulb to pointer and dial, quite apart from the question of thermal capacity. The Nuffield Chemistry and Biology Groups have developed electrical thermometers that show temperature differences clearly on a larger meter; and those are excellent where the demonstrations are concerned with *heat measurements* rather than a discussion of *the meaning of heat*. Here, we should have to calibrate such instruments or else convince pupils that our electrical thermometers follow the essential definition of temperature. So, unless the laboratory already has a demonstration thermometer, we suggest a group of pupil observers.

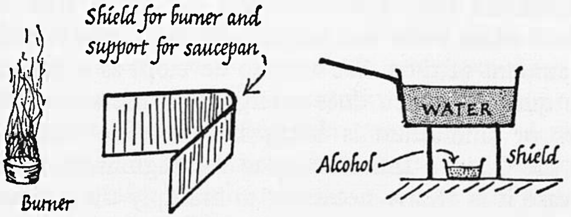
Simple bi-metal strip thermometers, with dials large enough for a small audience, are available and cheap. Although these really need calibration just as much as electrical thermometers, to show that their scale agrees with the mercury scale (and its modern Kelvin sponsor) pupils accept them as familiar instruments. We urge teachers to use one here.

Simple Measurements in Calories. This simple mixing experiment has suggested a way of calculating something that is conserved in such mixing processes and has provided us with a unit, the kilocalorie. We now ask pupils to make some measurements in those units. T

b. **Class Experiments: Measuring Heat.** Pupils use an electric heater, run safely on a 12 volt a.c. supply, to heat water in an ordinary aluminium saucepan. (This should not be a special beaker, still less a complex 'calorimeter' used for physics, but an C106b ordinary saucepan used in cooking. One that is somewhat deeper than the usual shape is better, but the very tall metal cans supplied for physics do not permit a thermometer to be read so easily – and some pupils even take it out of the water to read it!) We say:

> 'Find out how much heat your electric heater will give to water in a saucepan if you run it for 5 minutes.'

Pupils should try that rough measurement with, say, 1 kilogram of water; and then repeat it (after cooling the saucepan) with, say, 0·5 kilogram. This is intended merely to show the idea of measuring heat.



c. We ask pupils to find out how much heat the burning of 1 cubic centimetre of alcohol (methylated spirits) will give to a saucepan of water. The saucepan is supported on a strip of metal, say 8 inches by 3 inches, bent into a vee to act as both windshield and support. They place a small aluminium cup, or a very small glass beaker, under the saucepan and the teacher spoons out to each pupil 1 cubic centimetre of methylated spirits with a dipper (like a toy version of the dipper used by milkmen in earlier days). One cubic cm of alcohol burnt fully will yield about 4·5 kilocalories *to the saucepan of water*. C106c

Of course both experiments (*b*) and (*c*) above are rough and inaccurate. There are large losses to the air; in fact they are only measurements of the amount of heat received by the water. Pupils who wish to allow for the heat received by the aluminium pan could do so with the help of the rough measurement of specific heat suggested below, but that would still leave these experiments very rough – though they suit our purpose well – so we do not consider the next two experiments important.

d. **Specific Heat of Aluminium: Rough Estimate.** Using a block of aluminium drilled with a hole to receive the compact electric heater of (*b*) above, and a hole for the thermometer,‡ pupils deliver 'the same amount of heat' to the metal block as to the water in the saucepan. They do that by running the heater for the same total time, and *assuming* that it delivers 'heat' at the same rate whatever its surroundings. Thus they do not need a voltmeter or ammeter. C106d

(Pupils who understand the use of one or both instruments could connect them in the supply system but we suggest it would be better to hurry through the experiment.)

Pupils weigh their aluminium block and multiply mass of aluminium by temperature rise. The result will *not* agree with the product obtained when water was heated with the heater running for the same amount of time. We want to develop, as a measure of heat, some quantity which does emerge with the same value whether water or aluminium is being heated. (See Note on 'Constant' in the General Introduction at the beginning of Year III.) In that case it is clearly necessary to multiply the product, [mass].[temperature rise], by an additional factor characteristic of aluminium. We say that is called the specific heat and ask pupils what value it should have for aluminium.

The value obtained from this very rough comparison may well be quite far away from the 0·2 yielded by more careful experiments; but we urge teachers not to pursue precision at this point.

e. **Specific Heat of Aluminium: very rough estimate using alcohol flame** (*Optional*). We can repeat (*c*) above with the block of aluminium instead of the saucepan. Since the alcohol flame is sensitive to air currents, etc., we are unlikely to supply the same amount of heat to the aluminium block as to the saucepan. So this estimate is an even more unreliable one than the previous one. Yet it has a certain primitive simplicity, so we think some teachers may wish to let pupils try it. C106e OPT.

‡ Note that it is important to provide good thermal contact by pouring oil into the hole before inserting the thermometer.

GENERAL CONSERVATION OF ENERGY

Heat is one form of energy; and energy is universally conserved, never manufactured or destroyed. This is *our* present view, based on a century's work of building the Principle of Conservation of Energy with such sure supports that we now rely on it unthinkingly. It is time we gave our pupils strong reasons for joining us in that view.

Time and again in our teaching from Year I until now, we have talked about energy as if it is something that is conserved, never manufactured, never thought of as appearing from nowhere, never destroyed, but often changing from one form to another – that *change* being very useful to mankind. We have shown little evidence for our strong belief in this very important principle – only a look at input versus output for levers and pulleys. Our pupils have now reached an age for much deeper questioning. In many parts of their education we are encouraging these young people to look at their knowledge more carefully, to examine the structure of grammar in language, to think about the reasoning in algebra and geometry, to look for human emotions in great plays. It should not appear as a guilty afterthought that we now worry about our belief in Conservation of Energy, but rather as an adult enquiry about a complex matter that seems too important to be taken for granted.

We must talk to pupils about our present aim and explain that we are going to conduct that enquiry.

The comments to pupils suggested here assume that pupils followed the energy teaching in Years I and II as well as III. If not, a much fuller introduction is needed.

> 'For some time, in learning physics you have met energy as something which we get from fuels, and which in changing to other forms allows us to get very useful jobs done. We cannot get those jobs done, such as raising a load, or making a car go faster, without drawing upon some supply of stored energy.

'You know how to measure the transfer of energy from one form to another when you see the transfer being carried out by a force pushing along. You multiply [force] by [distance moved in the direction of the force]. We call that "work" and now measure it in newton.metres or joules.

'Sometimes, when some mechanical energy (potential energy or kinetic energy) disappears and cannot easily be accounted for, you say: "Oh, that has just turned into heat."

'You may even have a *picture* of the heat you have given to something as being really the energy of random motion of molecules. Yet how do you know that heat is really a form of energy, just as good as the mechanical energy that you get from some fuel? Do you really know whether we get the same amount of heat from a certain chunk of mechanical energy or sometimes more heat, and sometimes less? So far you have not seen any tests or proofs. You are not really entitled to say "Heat and all kinds of energy add up to a total which never changes." The only reason you can give for believing that very important statement – which goes behind all engineering and all atomic physics and all rocket-making and astronomy – is to say "Well, my teacher (or my book) told me so." That is not first-class science.'

Examining Scientific Evidence. This is a point at which teachers may want to talk with their class about the way in which the scientist accepts evidence for new knowledge. What to say, and how much to say, must depend upon the abilities and interests of the class and the taste of the teacher. When a scientist is looking at the evidence for the construction or testing of a new theory he examines the written accounts of the experiments very carefully and if possible he deduces consequences which can be tested. But most scientists, most of the time, have to take the work of their colleagues on trust, they have to trust the original experiments of others. They review the process of deduction, but they have again to take the subsequent tests on trust. Only a few experts with time and facilities for the necessary experiments subject the evidence to really careful examination. Yet the knowledge that this examination is being carried out, at each stage in the development of science, makes it reasonable to accept new results. Although at this point our pupils are being invited to examine the historical evidence, to see how assurance was gained, later on there will be many things T which, for lack of time, they will have to accept with much less scrutiny.

'In science you do have to accept many things as right when you are taught them, but you should certainly see how some of the biggest pieces of science are built up. You should see or know about the experiments which make professional scientists believe they are true or at least sensible knowledge.

'We believe that Energy is never created or destroyed. Is that true? Is it true that if we keep track of the *total* of all forms of energy we shall find that the total never changes, even in the most violent or strange events: machines running, fireworks exploding, atoms colliding, atomic nuclei disintegrating? Of course we should have to keep track of any energy escaping from our apparatus or of energy arriving from outside – we must do our accounting for what scientists call a "closed system". Then, for a closed system, we do believe the total of all the forms of energy remains constant. We consider that so powerful and useful a general rule that we give it a special name "the Principle of Conservation of Energy". And we hold it is true universally.'

The Court. 'How have we come to believe in the Principle of Conservation of Energy so fully? Now is the time to ask for evidence. I shall call you together as Jury in a Court to look at the evidence. This time you will not go out into the lab. and gather the clues,‡ but I shall give you the clues which were

‡ In teaching classes at different stages and with different traditions or attitudes, teachers may find that some groups are irritated by this pretence of a Jury and Court trial. It works well with older students who see that this is a tremendous 'proving of the case' in one of the greatest developments of science; and they take part wholeheartedly and critically but appreciatively in the trial. Where the teacher feels that pupils will not welcome this description of the discussion as a court trial, he should of course describe it differently to suit the interests and tastes of his pupils.

Again, we suggest here speaking of 'clues' as if scientists were detectives. In that, as explained in notes in earlier Years of this *Guide*, we are following the advice of a very able lawyer who points out that much of science, as young people should see it, is the collecting of clues (data) and the wise assessing of the reliability of those clues and the inferences drawn from them – an important part of the making of theory. Where teachers feel that this description would not fit well with the stage of interest or attitudes of their class group, they should of course change it. In any case, such expressions are only offered as suggestions.

gathered over the last 100 years in great experiments, and you shall be Judge and Jury. Before the court is gathered to sit, I want you to go out as detectives and gather one clue, just to see what clue-gathering is like in this matter.

'Think of the water in a waterfall. At the top the water has a lot of potential energy. On the way down the water loses potential energy and gains an equal amount of kinetic energy; so, near the bottom of the fall, the water is moving very fast. Then the water falls with a great splash into a pool at the bottom and comes to rest. Where has its kinetic energy gone? The water has not rushed up another sloping hill and regained potential energy. Nor is it moving any more, once the whirlpools at the bottom have settled down. Either energy has disappeared altogether, or the kinetic energy that the water had has turned into some other form. What form do you suspect? ... Yes, the only form I can think of is heat, that is just the thing we want to investigate. Is there heat at the bottom of the waterfall; is the water really a little warmer at the bottom than at the top? And if so, is the heat which is gained always the right amount to account for kinetic energy lost? You could not do that experiment at all easily, even if the school had a waterfall nearby, because the people who have tried that all tell us the temperature rise is very small and needs a very sensitive thermometer to show it.

'The experiment was done about 120 years ago by James Joule, as a young man on his honeymoon. He carried a huge thermometer – with a big bulb so that it would show very small temperature rises – up to a waterfall in Switzerland and measured the temperature at the top and the bottom of the fall. The tiny temperature difference agreed with some ideas about heat and energy that he had already begun to build from other experiments.'

We might ask pupils in homework about the ways in which this experiment could fail: 'Would wind do any harm? A hot spring underneath the pool? A curved bowl at the bottom of the waterfall that allows the water to continue as a rapid river?'

'Instead of a waterfall of water I can give you a "waterfall" of lead shot.‡ Let a handful of lead shot fall from top to bottom of this cardboard tube and take its temperature before and after. Try that and find out what happens and come back and tell me about it.' C107

We provide each pair of pupils with a handful of lead shot, a long cardboard tube corked at each end, a paper cup and a thermometer.

We simply show pupils how to take the temperature of the shot by pouring it out of the tube into the paper cup and carefully plunging the thermometer into it. Then we let them experiment without any explanation about taking many falls of shot, or any suggestion that it will be unnecessary to know the mass of shot, or any question about specific heat: they just try it.

We shall learn a great deal about our pupils from the way in which they react to this simple experiment. Some will need encouragement, others will need careful argument to disentangle sense from nonsense. Still others will be angry because we did not suggest the obvious improvement.

Presently we make sure that each in turn understands that he will have a better chance if he lets the handful of lead shot have many falls, so that the temperature rise is larger. We also explain to each that this is an experiment which is likely to be quite unreliable in its result. We give it to pupils as a very rough experiment to show them the general idea of the kind of experiments that we are going to describe. Thus, it is an 'experiment of principle'.

When the cardboard tube of lead shot is suddenly turned over, some of the shot does not fall through the full distance; and if the experimenter cushions the end of the tube with his hand some of the potential energy may go to his arm instead of into heat in the shot.

‡ We use lead shot rather than water because of the low specific heat of lead, nearly $\frac{1}{30}$th of that of water. Thus, we expect a temperature rise 30 times as great as with water. We choose lead rather than some other metal because it is inelastic, not springy, so that all the energy of the falling shot turns rapidly into heat as the atoms in the lead are dragged this way and that in the impact of stopping.

It is well to remember that most metals have approximately the same thermal capacity per unit *volume*; so, for some uses, elasticity is the important criterion. Here, however, it is the thermal capacity per unit *mass*, or the specific heat, that determines temperature rise; and we want as large a temperature rise as possible.

On the other hand, if he carries the tube down with a grand swing of his arm and bangs it on the table he may give the shot more heat than the simple fall provides. Add to these minor difficulties the two great difficulties, that heat leaks away to the room and that we are heating the tube to an unknown extent as well as the shot, and we have an almost hopeless experiment, at least with ordinary thermometers.

One could probably choose the material and length of the tube, and adjust the instructions so that pupils obtain a result that looks fairly good. But experimenters who have tried varying the conditions have emerged very doubtful of the possibilities. And they remind us that if the number of falls is extended further and further the temperature rise approaches a limit at which heat is wasted to the room at the same rate as it is generated by successive falls. In that series, the resulting value for 'J' grows larger and larger as we increase the number of falls. It seems clear that this is *not* an experiment that one should try to perfect by being ingenious or by carrying out careful corrections. To take that trouble, or to encourage pupils to do so, would be to misconceive the purpose of the experiment.

Joule and the other experimenters who carried out the great experiments were not just clever enthusiasts with a new idea: they were, nearly all of them, experimenters of extraordinary skill in designing apparatus and making observations. This rough class experiment represents no attempt to ask pupils to copy Joule's work. It is intended to provide an example of the type of experiment and, we think, to prepare pupils to sympathize with Joule and admire his work. So, in this case, the teacher should not suggest a series of researches with different numbers of falls, etc., but should now carry pupils through the experiment rather quickly.

When they have turned the tube over a round number of times, C107
say 20 or 50, pupils should pour the lead shot out into the paper cup and take its temperature again. They make a rough estimate of the distance fallen by the shot each time – a careful discussion of the proper height to be measured in view of the bulk of shot would distract attention from the main issue here.

'Now you know that your shot *did* get warm after those falls. Can you work out how much potential energy your shot lost? How do you work out its loss in one fall? ... Yes, it will be [weight] × [height] but remember weight is a *force*. You had better put that force in "good" units – newtons. Suppose you have one kilogram of lead shot. What is its weight, what is the pull of the Earth on it? ... Yes, 9·8 newtons. If you have some other mass you take the mass in kilograms and then remember that the Earth pull is 9·8 newtons per kilogram, 9·8 newtons pull on each kilogram of shot. You had better find out how much shot there is there. Work out how much potential energy your shot lost in newton.metres.'

It is probably better for pupils to work out the loss of potential energy first and not discuss the calculations of heat until they have got a clear statement of the amount of energy that they consider was lost from the supply of potential energy. We should ask each pupil individually where that potential energy came from, considering that the lead shot made many falls. (At each turning over of the tube, he gave potential energy to the shot from the chemical energy in his muscles. The shot converted that potential energy into kinetic energy which then 'disappeared'.)

'Now you know how much potential energy your lead shot lost, how much potential energy turned into kinetic energy which then disappeared when the shot landed in the bottom of the tube.

'We want to find out whether the heat that appeared really is equivalent to that potential energy or kinetic energy that was lost. We can only find that out by trying many different experiments and seeing whether we always get the same amount of heat for each newton.metre of mechanical energy that we put in and lose. You know how much mechanical energy disappeared.

'Now find out how much heat appeared in your experiment. Measure heat just as you did in the early experiments with water. There you multiplied [mass of water in kilograms] by [temperature rise in centigrade degrees] and that gave you the heat in kilocalories. But remember that when you tried an experiment with aluminium you found that you would not get a story that agreed with the water story unless you also multiplied by the special number called the "specific heat" of aluminium.

'Here you are going to need the specific heat of lead. I will tell you what it is, although you could measure it yourself if you wanted to. This specific heat is quite small, nearly 30 times smaller than the number 1·0, which is the specific heat of water. It is 0·035. Since you know how many kilograms of lead you have, and how much its temperature rose in centigrade degrees, and the specific heat 0·035, you can find how much heat was gained by the lead. You need to multiply those three things together. Go ahead and calculate that.'

When that is done the teacher should gather everybody round and explain that the next move is to calculate how much heat appeared for every newton.metre of mechanical energy that disappeared.

The Result of the Simple Experiment. Very careful modern experiments show that one kilocalorie is equivalent to just under 4,200 joules. Results of this class experiment will run from 3,000 (surprising) through 4,000 (lucky) to 6,000 (usual errors) and even to 10,000 (careless in one way or another). The teacher should write the result obtained by every pupil on the blackboard and ask everyone to survey the collection of numbers. Then he should ask whether it is an easy experiment. That is the point at which we should leave that experiment without much comment, without any attempt to average its results or to excuse the great divergencies.

However, a wise teacher will be tempted to ask two last questions: 'If you had lots of time, several weeks and all the apparatus you liked to ask for, do you think you could overcome some of the difficulties of this experiment?' And 'If you did, what do you think results would look like when you compared yours with those of other people who had also taken a lot of trouble to design new apparatus and use it very carefully?'

'THE COURT WILL SIT'

We then tell pupils some of the story of the history of heat. (The Nuffield Physics Group expect to produce a Pupils' Guide which will give this story to pupils for their own reading and save teachers from having to give a rather long historical account.) Here is a short summary; also a chart of results which gives the essential testimony for the court – a condensed form of what we might say to pupils.‡

Early History

A long time ago people began to think about atoms and atoms in motion, and even wondered whether the atoms of hot things were moving faster. Greek philosophers and the Latin poet Lucretius talked about moving atoms 2,000 years ago; but although their suggestion of the idea may have been useful in helping science to develop much later, it was just an imaginative idea and they had no experimental evidence at all. T

About 200 years ago some scientists were suggesting quite seriously that heat is motion of atoms (John Locke the philosopher, Lavoisier the French chemist, and Laplace the French mathematician). Newton thought about the possibility of heat being atomic motion.

But another group about 200 years ago did very careful experiments to measure the heat lost by hot water and the heat gained by cold water when hot and cold water were mixed, and other experiments like that, all of which tempted them to believe that heat is something that is never lost, a mysterious fluid, 'caloric', that can flow from one material to another and even lie hidden between the atoms of materials. Of course those people had an easy explanation for the fact that you burn your hands if you slide down a rope: you are squeezing this mysterious fluid heat out of the rope.

When people did more and more experiments on mixing warm things and cold things they grew more and more fond of the simple idea that 'caloric' could not be made or destroyed but passed from one thing to another, just as you might squeeze water out of one sponge into another. That idea seemed simple and clear and it fitted very well with most experiments; so people thought it was true and began to forget the earlier more complicated ideas about heat being connected with moving atoms.

Then, two centuries ago, scientists developed clear ideas about potential energy and kinetic energy, and conservation of energy when something falling lost potential energy and gained an equal amount of kinetic energy. At first it was not clear to them that these

‡ The summary which follows is worded as one might put it to pupils but, since it is long, it is not printed in a narrow column like the other 'suggested comments to pupils'.

kinds of energy could turn into heat. Heat seemed to them so different and they did not welcome that idea.

Then about a century and a half ago a very clever man began to think that the idea of heat as an indestructible fluid was wrong: heat must be connected with motion and energy. He was Benjamin Thompson, whom we now speak of as Count Rumford, a strange, ingenious, very capable man. Born in America, he found himself as a young man in the American Revolution and chose to take the British side. That proved a dangerous choice and he had to leave America in a hurry and escape to England. He was well received there and elsewhere in Europe and was unusually good at making friends with the right people in the Government. He made a wonderful impression of his powers and managed to get himself appointed to various important jobs – which he carried out extremely well. He was a magnificent organizer and he had tremendous interest in scientific work. In London he designed new stoves and improved chimneys for the heating of London houses. He founded the Royal Institution for helping scientific research and encouraging public scientific lectures. He was restless and set out to travel across Europe. In Bavaria he made so good an impression that he was appointed Minister of War and asked to reorganize the army. For his success there, he was made Count Rumford.

In that post, Rumford was concerned with the arsenal where brass cannon were being made. Each cannon was cast as a solid cylinder and the barrel was then drilled out. He noticed that when a very blunt borer was used it turned out very little of the metal but produced an enormous amount of heat. He even boiled a kettle sitting on the cannon while a team of horses drove the very blunt borer round and round. He came to the conclusion that the supply of heat was inexhaustible, that more and more heat would go on appearing as long as the horses continued to work. The important thing about that was not that Rumford just noticed the heating, which may seem obvious to you and me, but that he set out to expound his new idea far and wide. He said very clearly that he considered the heat being developed in his cannon was not being squeezed out from among the atoms but was a new motion of something, that was manufactured at the expense of mechanical energy. He did not put his conclusion in clear words but he certainly meant that heat is a form of energy.

This suggestion was not welcomed by scientists: people do not like to give up a simple theory that seems to fit the ordinary facts. So most scientists argued against the idea that heat is some form of motion of atoms, something connected with mechanical energy. But one or two enthusiasts took it up, among them a very able young Manchester brewer, an amateur scientist who put his heart into proving that heat *is* just mechanical energy in another form. He was James Prescott Joule, in honour of whom we have now named the energy unit 'joule'. T

Joule was fired by a belief that heat is just a form of energy and he was determined to spend all his spare time doing experiments to prove the case. He did a very great many different experiments all with the same general aim as your experiment with the lead shot. He did every experiment he could think of in which some mechanical energy (potential energy or kinetic energy) disappeared and some heat appeared.

In every case Joule measured the amount of mechanical energy that had disappeared. He expressed that in foot.pounds where nowadays you and I might express it in newton.metres. And he measured very carefully how much heat appeared – making every allowance he could for the heat that was lost by accident – expressing that in some ancient British units of heat where you and I would now use kilocalories. He was a genius at careful experimenting, and he employed an instrument-maker who could make specially delicate thermometers for him; so he was able to work with small temperature rises and avoid the big losses of heat that occur when apparatus gets much hotter than the surrounding room. He went farther than that, he devised schemes for allowing for the small losses of heat that crept away from his apparatus.

And Joule used *large* apparatus because that makes loss of heat less important. The heat is lost from the surface of apparatus and if you make your apparatus 10 times as big, the surface area which allows heat loss is 100 times as big, but the volume of water, etc., to be warmed up in your apparatus is then 1,000 times as big, so that the heat leakage becomes a much smaller fraction of the total heat that you have to measure. Big apparatus is an advantage here.

In one of his first experiments he developed heat with water driven with a piston through very fine pipes, so fine that the water came out from the other end a little warmer from rubbing its way through the pipes. The pipes were actually thin holes in the moving piston itself. Try pulling a piece of rope through your hand while you clench your fist round it. He drove the piston by a falling weight and multiplied [weight] by [the height it fell through] to find how much mechanical energy was lost. He found the heat produced by multiplying the [mass of water] by [temperature rise].

In another experiment he developed heat in water by churning it up with a paddle-wheel driven by falling weights. The paddle-wheel churned up water in a large bucket, and by multiplying [mass of water] by [temperature rise] Joule could calculate the heat developed. The weight of the falling load multiplied by distance fallen gave the loss of mechanical energy. Joule used a large quantity of water so that the surface heat losses were less important. And he made very careful allowances for those losses that he thought did occur, by watching his apparatus cool after the experiment and working backwards from that. And he had a thermometer graduated so that he could estimate temperatures to $\frac{1}{200}$ of a degree.

Models. Teachers should remember that although they are familiar with Joule's paddle-wheel experiment, pupils have no picture of the apparatus and may easily emerge with no very clear idea of what it did. Therefore it is worth while to show a model, even if it does not yield any measurable temperature rise. The simplest model of Joule's paddle-wheel apparatus is just an egg beater driven by hand in a beaker of water. Or teachers may like to devise a model nearer to the real apparatus, with a paddle-wheel driven by falling weights. One can make a working model by using two small turbine assemblies with opposing pitches, from air blowers; but that is rather far from Joule's own model. D 108a

Lest critics should claim that his results were a peculiarity of water, Joule then changed to other materials and churned mercury with his paddle-wheel, and then churned whale oil. Then he changed to rough iron plates chattering and grinding against each other under mercury so that he was making heat by rubbing solids. T

Again and again Joule got more or less the same answer, the same amount of mechanical energy in foot.pounds (or newton.metres) disappearing for every Calorie of heat developed. This made him feel confident that he was on the right track because if heat really is just a form of energy (rather as half-crowns are a form of money) we must expect to pay the same amount of mechanical energy for every Calorie of heat manufactured, whatever the method of manufacture – just as we expect to get 8 half-crowns for every pound, whatever kind of money-changing business we meet.

Joule did not only churn up water or other materials. In some of his earliest experiments he used a roundabout set of energy changes that included electric energy. At that period, in the 1840s, electric circuits were being investigated for the first time and people were making the first electromagnets and dynamos and motors. They were doing just the kind of thing that you have been doing with the electromagnetic kit, but often on a big scale and always in a rather clumsy way. Joule made a big primitive dynamo and drove it by falling weights. Then he used it for two experiments: in one experiment he short-circuited the spinning coil of the dynamo, surrounded it with water and measured the heat developed. In the second experiment he turned the switch off so that the dynamo could generate no current. Then it was much easier to drive, so he used smaller weights, just sufficient to keep the dynamo running as they fell – to pay for friction and air resistance. He calculated the mechanical energy lost by the falling weights in each case and subtracted the result for the 'light' experiment from the result from the 'heavy' experiment, thus getting rid of the mechanical energy used to pay for friction, etc. That told him how much mechanical energy disappeared when the heat was produced in the water in the 'heavy' experiment. And he could measure that heat by weighing the water and measuring its temperature rise.

When we describe that to pupils it might be worth while to show a model of it. This should be only a mock or token experiment. It D108b
would be a mistake to interrupt the story with attempts at realistic models. We attach a large wooden axle to a fractional-H.P. motor, which we use here as a dynamo. Unfortunately, the field of the machine must be excited by a battery. We set up the motor with its axle vertical, wind cords around the wooden axle and run them out and over pulleys to large loads which will drive the armature as they fall. Instead of making the dynamo generate heat in a coil in

water (its own short-circuited armature as in Joule's own experiment) we might connect it to a small electric lamp. We show pupils the difference between a 'heavy' run with the lamp being lit, and a 'light' run with the lamp turned off.

Joule used his 'electromagnetic engine' in another way – as a motor. Again, he did two experiments. First he drove it with a battery and made it haul up weights. Next he let the battery drive the same current through the motor when he kept its axle fixed and prevented it from rotating. In the second case the current did not provide energy to haul up weights, but heated some water instead. Joule allowed the battery to use up the same amount of chemicals in each experiment but in one case he obtained mechanical energy of raising loads, and in another case he obtained heat, and again he calculated how much mechanical energy he got in one way for each calorie of heat he got in the other way. T

Again we might show a token model. We drive an electric motor with a battery and make it haul up loads. We cannot show the second part of Joule's experiment clearly because he measured the heat developed in the armature and we have no easy means of immersing the armature of our motor in water. Furthermore, Joule had to make a careful calculation – difficult and almost mysterious in those early days of electric circuit knowledge – to convert the measurements of his second experiment, in which he had to use a different voltage and/or current, to what he would have found with the same demand on his battery as in his first experiment. D108c

Other Experimenters

A century ago many other people were doing experiments, in a most ingenious variety of methods. Hirn, a French engineer, produced heat by hammering lead. He did not do it on the small scale we tried. He let a 700-pound hammer, moving 15 feet per second, smash into a 6-pound block of lead held against a 1-ton stone anvil. The lead warmed up about 5 C°. Hirn measured the distance fallen by his hammer to gain its kinetic energy and multiplied that by the weight of the hammer to find the amount of kinetic energy it had gained from potential energy. Then he knew the energy that had disappeared and, as he thought, appeared as heat. He measured the heat much as we tried to do with lead shot, multiplying [mass] by [temperature rise] by [specific heat]. T

Hirn did another experiment; a very unusual one, because the change was carried out in the opposite direction – from heat to mechanical energy. He borrowed the use of a steam engine in a large commercial mill for the weekend and ran it, keeping very careful track of the heat supplied by the fuel, the heat wasted up the chimney, etc. He found some heat definitely disappeared while the engine was working hard hauling up a heavy load. He calculated the potential energy gained by the load and so could work out how much mechanical energy seemed to be provided by each Calorie that disappeared. T

It would be quite unsuitable to drag in a toy steam engine to illustrate Hirn's method – our pupils are quite old enough to imagine one. A picture of Hirn's real engine, if it could be found, would be impressive; but a modern toy would not even indicate the essence of the energy story. It would only amuse and waste time here. * * * * *

The Long Tale. And so the story ran, over a long time, from just before 1800 when Count Rumford started his rough experiment of boring cannon and made his enthusiastic suggestion, on all through the last century while Joule worked at experiments from 1840 to 1870, trying to find different methods and improving his great water-churning apparatus with the result that they agreed more and more closely with each other. Other scientists all across the world tried their versions of such experiments and added their testimony. T

The Court will Sit

'You shall be Judge and Jury. You shall see the record of these great experiments which started when scientists believed that heat was something quite different and rather uninteresting, and ended up with all mankind convinced of the conservation of energy. T

'Please do not think of these experiments as a series of attempts to measure some important constant called "J". That measurement can be carried out accurately enough today once and for all and we should not bother you with a series of earlier attempts just to give you a useful number.

'The earlier attempts are spread out on this table because they are the witnesses, each witness brings one clue: the clue is the number calculated out, the number of newton.metres that went to make one Calorie in that experiment.

'If in some famous case in court the witnesses seemed to be brothers of the same family, all with red hair and a shifty eye, the jury might not consider their combined testimony very strong. But if there are many different witnesses from different families with different outlooks on life who all agree with increasing certainty on the guilt of the defendant, the jury will find him guilty. Now look at the evidence yourself.'

The Demonstration Chart of Results, and Sketches

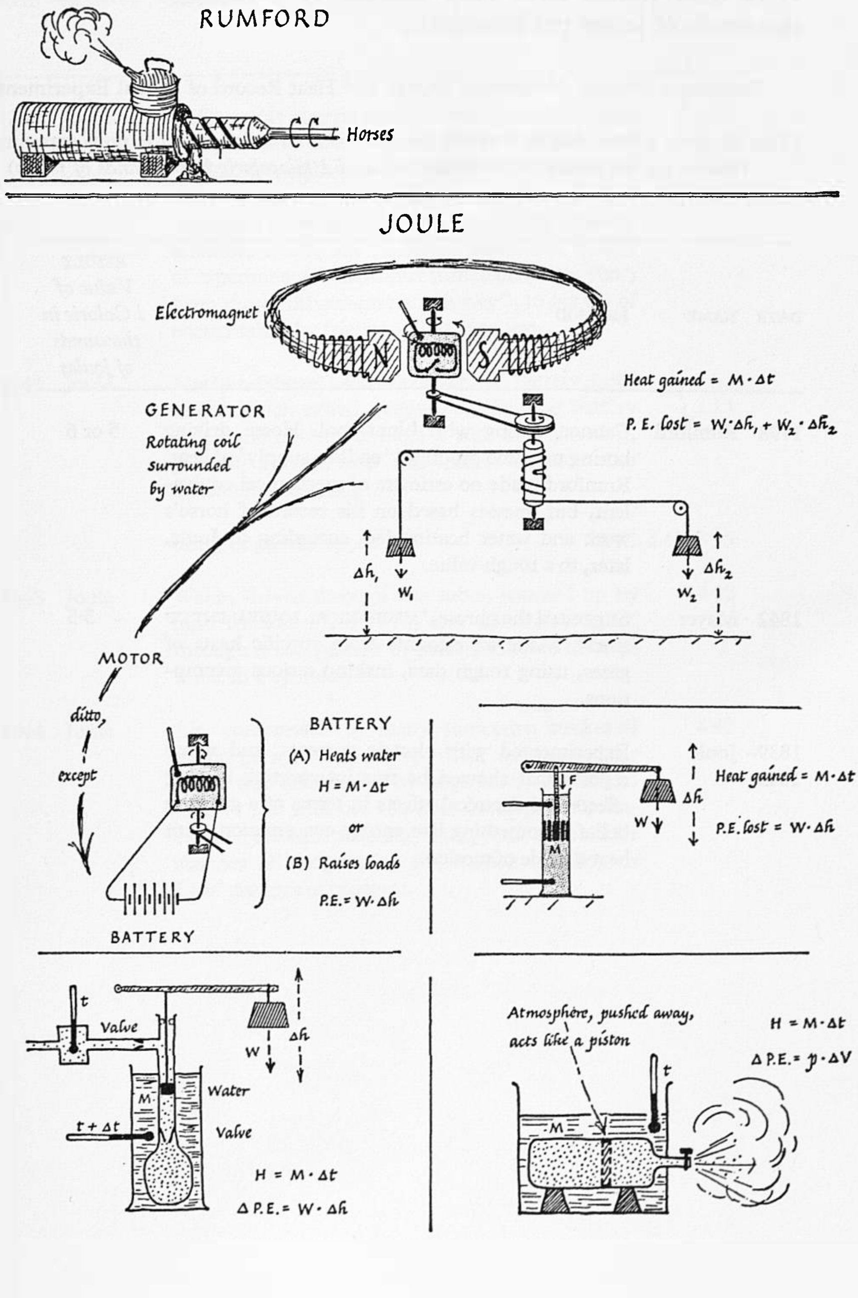
The chart‡ which follows here gives more experiments than most D 109
teachers will want to describe in surveying the evidence with a class. On the other hand, it gives less detail of the working of each experiment than young pupils probably need in order to be convinced. We suggest the teacher should describe a few methods very carefully, showing how the P.E. lost could be estimated and how the heat produced could be measured.

Then he should show a great list, like this chart either by a large poster or by lantern slides. He should run right through the table reading the account of the method and giving the resulting number. As the numbers progress, he should comment on the way they move nearer to constancy around a value of about 4200 joules per kilocalorie. All that is not for 'revision' but to convey the tremendous evidence: blow after blow.

Teachers need to be very careful to avoid any sense of clumsy early *
experiments failing to give the right value and the later accurate *
methods giving the right value at last. For our purpose the great *
variety of methods and the general trend of values are the important *
things. *
*

This long list will soon seem dull and difficult unless there are *
some pictures to explain some of the methods. The sketches‡ that *
follow are offered as an aid to teaching. They are, of course, just *
fanciful illustrations to make the scheme of the method clear. They *
are *not* sketches of the real apparatus. *

‡ Table and sketches are reproduced from E. M. Rogers, *Physics for the Inquiring Mind*, Oxford University Press, 1961.


RUMFORD
Horses
JOULE
Electromagnet
N
S
Heat gained = M · Δt
P. E. lost = W₁ · Δh₁ + W₂ · Δh₂
GENERATOR
Rotating coil surrounded by water
Δh₁
W₁
Δh₂
W₂
MOTOR
ditto, except
BATTERY
(A) Heats water
H = M · Δt
or
(B) Raises loads
P.E. = W · Δh
BATTERY
F
Δh
W
M
Heat gained = M · Δt
P.E. lost = W · Δh
t
Valve
M
Water
t + Δt
Valve
W
Δh
H = M · Δt
Δ P.E. = W · Δh
Atmosphere, pushed away, acts like a piston
H = M · Δt
Δ P.E. = p · ΔV
t
M

The numerical results in the table are not the ones that the original experimenters published. They used a variety of strange units. Here their results have been reduced to a standard form of thousands of joules per kilocalorie. * * * *

Exchanges between Mechanical Energy and Heat Record of Actual Experiments

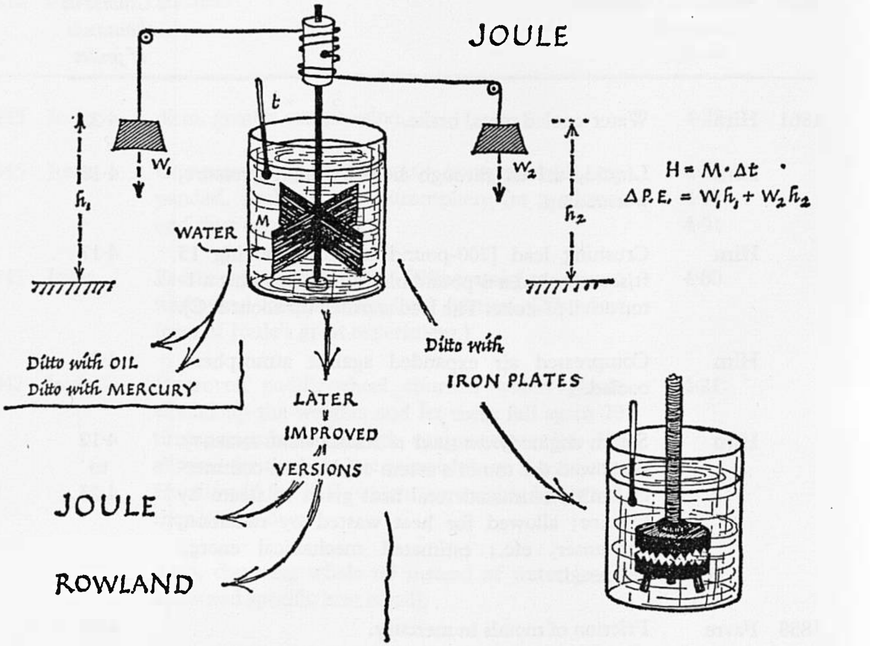
(This list gives a short description and the result of some of the most famous experiments. Results are expressed in the form: value of *1 kilocalorie in thousands of joules*)

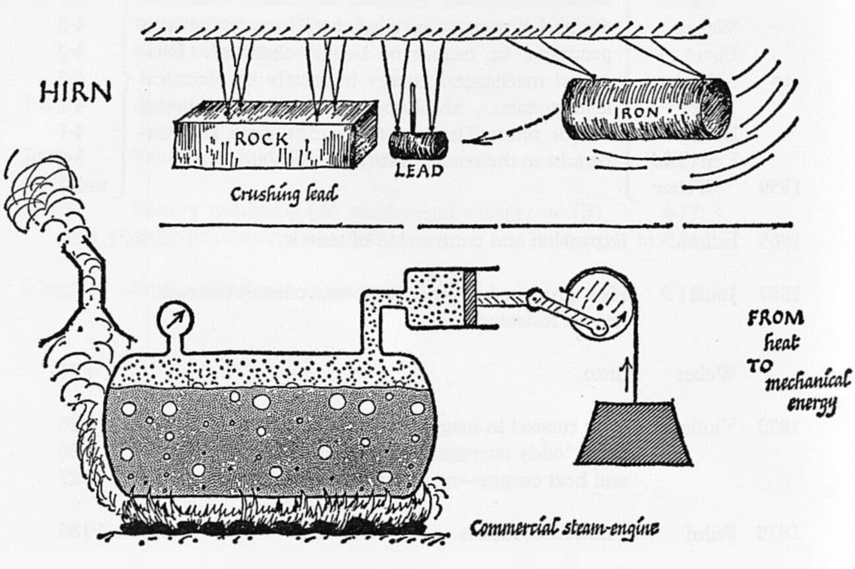
DATE	NAME	METHOD	RESULT *Value of 1 Calorie in thousands of joules*
1798	Rumford	Cannon boring with blunt tool. Horse driving boring machine produced 'endless supply' of heat. Rumford made no estimate of mechanical equivalent, but guesses based on his record of horse's work and water heating led, according to Joule, later, to a rough value.	5 or 6
1842	Mayer	Suggested the phrase, 'MECHANICAL EQUIVALENT OF HEAT'. Made an estimate from specific heats of gases, using rough data, making serious assumptions.	3·5
1839–1843	Joule	Experimented with electric currents, and wrote reports that showed he was interpreting heating effects and chemical effects in terms of a growing belief in something like energy-conservation, with heat a mode of motion.	

DATE	NAME	METHOD	RESULT *Value of 1 Calorie in thousands of joules*
1843	Joule	Built simple electric machine which could be used as a generator or as a motor. Drove it as generator by falling weights, and measured heat produced when generator drove a current through coil immersed in water. (Coil was actually the rotating armature-coil of the machine.) Subtracted results of experiments with magnet turned off ('light run') from those with magnet on ('heavy'), to get rid of energy taken by friction of bearings, etc.	4·76 5·38 5·60 4·90
1843	Joule	Machine (above) used as a motor. (A) Battery drove motor which raised weights: *or* (B) The battery sent same current through a wire and heated it. [Actual arrangement was more indirect, but essentially like this.]	5·51 3·15
		ditto, improved apparatus.	4·62, 4·62, 3·95
1843	Joule	Water, driven through fine tubes, warmed up by fluid friction. Piston with very fine holes drilled through it was pushed by measured force through water in a cylinder.	4·22
1844	Joule	Air, compressed by many successive strokes of piston-pump, warmed up. The compressed-air bottle was surrounded by large mass of water to remove and measure the heat developed. In calculating mechanical energy used, Joule allowed for changes of compressing force made by 'Boyle's Law' changes of pressure.	4·42

DATE	NAME	METHOD	RESULT *Value of 1 Calorie in thousands of joules*
1845	Joule	ditto, greater compression.	4·27
1845	Joule	Compressed air, from bottle in water bath, expanded, pushing away atmosphere (as a piston) and thus cooled.	4·08 4·37 4·91
1845	Joule	Paddle-wheel, driven by falling weights, stirred water and heated it by fluid friction. [The first form of Joule's great experiment.]	4·80
1847	Joule	Improved paddle-wheel churned water. [Joule wound up the weights and let them fall again 20 times, to obtain enough temperature rise. He allowed for the heat lost meanwhile to the air, etc. He allowed for K.E. which the weights had when they hit the floor.]	4·21
		ditto, churning whale oil instead of water [used measured specific heat of oil].	4·22
		ditto, churning mercury.	4·24
1848	Joule	ditto, churning water. Forty more experiments with greater care. [Joule believed this result reliable to 0·5 per cent.]	4·15
1850	Joule	ditto, churning mercury.	4·16
1850	Joule	Friction of iron plates rubbed together.	4·21
1857	Favre	Battery produced (A) mechanical energy, or (B) heat, for same current and time.	4·17 to 4·54
1857	Hirn	Boring metal with blunt borer.	4·16

DATE	NAME	METHOD	RESULT *Value of 1 Calorie in thousands of joules*
1861	Hirn	Water-cooled metal brake.	4·23
	Hirn	Liquid, driven through hole by high pressure, warmed up.	4·16
	Hirn	Crushing lead [700-pound hammer moving 15 ft/sec smashed a 6-pound block of lead against a 1-ton anvil of stone. The lead warmed up about 5°C].	4·17
	Hirn	Compressed air expanded against atmosphere, cooled.	4·31
	Hirn	Steam engine (*from* HEAT *to* MECHANICAL ENERGY). Borrowed the use of a steam engine in a commercial mill; estimated total heat given to steam by furnace; allowed for heat wasted by radiation, condenser, etc.; estimated mechanical energy delivered.	4·12 to 4·23
1858	Favre	Friction of metals in mercury.	4·05
1857	Quintus Icilius	Indirect electrical methods. Measured heat produced by current in a wire or battery, or heat produced in beaker of battery-chemicals. Estimated mechanical energy indirectly by electrical instruments: absolute ammeter, voltmeter, and/or ohm. The electrical units were still uncertain so the results were not reliable.	3·9
	Weber		4·2
	Favre		4·2
to	Silberman		4·2
	Joule		4·1
	Boscha		4·1
1859	Lenz and Weber		3·9 to 4·7
1865	Edlund	Expansion and contraction of metals.	4·35, 4·21, 4·30
1867	Joule	Heat produced by known electric current through known resistance.	4·22
	Weber	ditto.	4·21
1870	Violle	Disc rotated in magnetic field was heated by electrical 'eddy currents'. Measured mechanical drag and heat output—no electrical measurements.	4·26 4·26 4·27
1875	Puluj	Friction of metals.	4·167 to 4·180
1878	Joule	Water churned by paddle: improved apparatus [weighted average of 34 experiments].	4·158(5)


JOULE
t
W_1
h_1
WATER
M
W_2
h_2
$H = M \cdot \Delta t$
$\Delta P.E. = W_1 h_1 + W_2 h_2$
Ditto with OIL
Ditto with MERCURY
LATER IMPROVED VERSIONS
Ditto with IRON PLATES
JOULE
ROWLAND

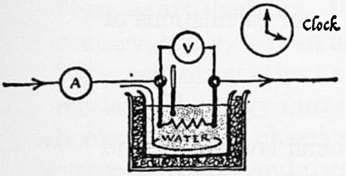

HIRN
ROCK
LEAD
IRON
Crushing lead
FROM heat TO mechanical energy
Commercial steam-engine

Suppose the court has been sitting to review that long line of witnesses and their testimony. The experiments that provide the testimony have ranged over a great variety of methods and materials. And it is clear that the final testimony of successive experiments – the number that each yields – has come nearer and nearer to some value about 4·2. The testimony has converged on a convincing answer. By the end of the list above, we might say that the case was proved. We might say that the remaining question was only 'the exact length of sentence' – meaning the value which the final constant would have if one could do an ideal experiment.

That value, 'the mechanical equivalent', or 'J', was by that time being measured so accurately that a careful measurement of g had to be used; and the value of 1 Calorie depended on whether the water was weighed against a brass kilogram in air or in vacuum without the buoyancy of air. And it had become clear that water does not take quite the same energy to heat it up from 10° to 11°C as from 17° to 18°C. If we make the specific heat of water 1, by definition, around 20°C (temperature of a very warm lab.), it is slightly bigger at lower temperatures. So, for statements accurate to 0·1 per cent and better, we must state the temperature region used to define the Calorie.

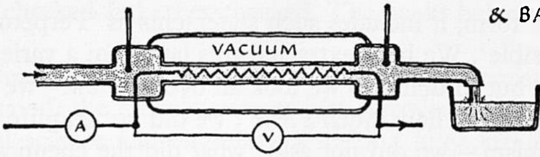
There have been many careful determinations of 'J' in the last eighty years. A few are given below, with vacuum weighing, for 20° Calorie (1 kg of water 19·5° to 20·5°C).

DATE	NAME	METHOD	RESULT *Value of 1 Calorie in thousands of joules*
1878	Joule (England)	Water churned. Result of experiment above reduced to weighings in vacuum and corrected to gas thermometer.	4·172
1879	Rowland (U.S.A.	Water churned by paddle-wheel driven by steam engine. Tremendous care over apparatus design and thermometer corrections.	4·179
1892	Micelescu (France)	Water churning.	4·166
1899	Callendar and Barnes (England)	Continuous flow of water heated electrically. Temperature rise measured electrically.	4·183
1927	Laby and Hercus (Australia)	Water churned by paddle.	4·1802±0·0001
1939	Osborne *et al.* (U.S.A.)	Electrical heating of water.	4·1819



(MANY EXPERIMENTERS)

CALLENDAR & BARNES



Continuous flow of water heated by electric current

EXPERIMENTS ON ENERGY CONVERIONS
Indirect methods using electrical measurements. Ammeter standardized by comparison with apparatus that weighs the force between coils carrying currents. Voltmeter standardized by comparison with primitive generator that provides an e.m.f. which can be calculated from geometry, a measured current, and spin-speed.

The Position Now

Finally, heat, chemical energy, electrical energy were established as interconvertable with P.E. and K.E. But energy was still being measured on two distinct systems: P.E. and K.E. in work units, such as newton.metres; and heat in [kg of water] $\times$ [C°] or kilo-calories. Chemical energy was measured *indirectly* in heat units. Electrical energy could be measured in either. We have been using the ratio of those units (1 kilocalorie):(1 newton.metre) as witness of our trial of 'caloric'. If we now agree that Heat is Energy, that ratio must be universally the same; but we still need to know its value accurately. T

Taking a weighted average of the most careful measurements, we may say,

The 20° Calorie = 4180 joules
The 15° Calorie = 4184 joules

Hence our rough value, 4200 joules/kilocalorie for calculations of interchanges.

Note to Teachers

Thermodynamics. We can now state a general law: 'Heat and mechanical energy are interchangeable at a fixed rate of exchange.'‡ We call this the 'First Law of Thermodynamics'. In its most general form, it includes such statements as 'Perpetual Motion is impossible'. We have extracted this law from a variety of experiments, but in doing so we took an overall view – we asked: 'how much heat?', 'how much P.E.?'; we did not enquire into detailed mechanism – we did not ask: 'what did the chemicals do in the battery?', 'are the atoms of hammered lead vibrating?' This overall view treatment is characteristic of thermodynamics, in contrast with the approach of atomic physics that investigates detailed mechanism before stating general results.

A similar overall survey of heat engines yielded the Second Law of Thermodynamics: 'Heat does not *of its own accord* flow from cold to hot.' This simple platitude combines with the First Law to produce a powerful theoretical science. Thermodynamics provides the Kelvin scale of temperature, the basic theory of heat engines – from steam turbines to rocket motors – the basic theory of refrigerators and heat pumps. It provides a great variety of useful predictions – such as a connection between battery-e.m.f. and chemistry, or the relation 'radiation flow $\propto T^4$'. Its foundation on overall views makes it all the more powerful; no change of detailed mechanism can upset its conclusions.

When molecular details are added, we develop a 'statistical mechanics' that treats the probabilities of chaotic motion and makes new predictions, and recently, when applied to bits of information instead of molecules, offers to reform communication in theory and in practice.

‡ Once we believe that the rate of exchange is absolutely fixed, we may use the same units for heat energy and mechanical energy. Then the rate of exchange would be 1 joule per joule. That would be like saying the conversion rate is £1 per £, for conversion between one edition of pound notes and another. Yet we may also say that the rate of exchange is 20 shillings per pound; and in the same way we can say in physics the rate of exchange is 4200 joules per Calorie.

Nineteenth-century Physics. At the beginning of the last century, energy was an idea without a clear name. In the hands of Joule and many others the scheme of conservation was built up: mechanical energy into heat, and heat to mechanical energy: the books balanced; chemical energy to heat, or chemical energy to electrical energy and then to heat, or electrical energy to chemical energy and then to heat – all these were tried in a host of measurements that were checked and cross-checked. The books balanced.

It was a tremendous scientific century: at its beginning, chemistry growing to manhood, the electric current just discovered; in the middle, electrical science and engineering making huge strides; and at its end atomic physics just beginning to open up. The conservation of energy was perhaps the greatest development of all; it was the conceptual scheme that tied the others together.

We hope that teachers will experiment with this 'Court Trial of the Conservation of Energy' to find out the best way of teaching it. §

Enthusiasm is better than profusion of technical detail. We need somehow to persuade the class to share in a great adventure which changed the whole reputation of energy in just over a century, from a new name for a mechanical concept to an enormous family within which conservation appears to be preserved without exception.

Nowadays we find conservation of energy so useful that if physicists discovered a case of some energy disappearing without any other form of energy appearing instead, they would probably manufacture an imaginary new form of energy to keep the balance sheet true. That would not be wrong science, provided they always remembered that the new form had been *invented.* In fact that was, in a way, done earlier this century when the little particle called

§ One teacher in preliminary trials reports a pupil saying, 'If you believe it, sir, we will.' This delightful remark epitomizes a serious difficulty in modern science teaching. General talk, newspapers, and some of the more traditional teaching besides, have taught pupils to regard science as material knowledge that is issued by authority and should be taken on trust. Although in our programme we shall offer pupils knowledge – which we hope will be trustworthy knowledge – we are just as much concerned with giving them an understanding of the basis of that knowledge and helping them to maintain a critical, questioning attitude. Here, our Court Trial is no mere historical game: it offers a glimpse of building true scientific knowledge.

the *neutrino* was invented by theoretical physicists. In certain radioactive changes, some of the released energy was unaccounted for; so the neutrino was suggested as a tiny, invisible, perhaps undetectable, particle that carried away the extra energy. At the same time some momentum and some angular momentum were unaccounted for, and the same small particle could be employed to keep the balance of those physical qualities true as well. Therefore, even if it was not real, the neutrino was useful in cataloguing what happened in those nuclear events.

Physicists described the events in terms of a neutrinos as agents quite happily for years without having much hope of ever being able to observe the particle. Since it has no charge and practically no mass, it is a much more evasive creature than the large neutron. In recent years, however, experiments have provided strong evidence for the existence of real neutrinos and we now believe in them as well as using them as an idea.

Nevertheless, if need arose again we should probably invent one more form of energy or carrier of energy, rather than give up a principle which has proved so valuable. Thus, in modern days conservation of energy has changed from being a general principle which we have found by experiment to hold in the natural world to being a part of, so to speak, the parliamentary constitution of our physical knowledge.

We should not discuss this development of the idea of energy with our pupils at this stage. That might mislead them into thinking that we do not really understand energy or that we are not quite sure about conservation of energy or perhaps that we are prepared to tell lies about it. None of those are true.

Heat as Energy

'From now on we shall consider heat as a perfectly respectable ordinary useful form of energy. We shall measure heat in joules as far as possible and forget about kilocalories. If for any reason a chunk of water is warmed up through a measured temperature rise and we calculate the heat that it has been given in kilocalories we shall take results of the latter experiments on exchanges between mechanical energy and heat and say that one kilocalorie is worth 4200 joules.

'If you would like to consider yourself a present-day scientist making sure that that number 4200 is the right one to use, you may do an experiment in which a falling load or the equivalent loses some mechanical energy and some heat is produced which you could measure. It will be an accurate form of your lead-shot experiment, and we shall offer it to you next year.'

If we do offer pupils a 'good' experiment on the 'mechanical equivalent of heat', we are offering them an interesting, difficult experiment that is also dangerous: there is a danger that they will think the aim of the experiment is to get 'the right answer', 4185 joules/kilocalorie. The nearer their result is to that right value, the more contented they will be, and the more they will expect us to call their work very accurate.

In fact, a trustworthy measurement of 'J' is very hard to come by: the work is beset by errors, some known and some unknown. Earlier generations of physicists, aiming at training pupils to follow standard experiments, have with great ingenuity designed apparatus that does give the right result. At worst, look at the simple lead-shot experiment: by choosing the right amount of shot and encouraging pupils to give the right amount of bang to the downward-slung tube, we can produce a surprisingly good result. None of us in teaching physics would dream of doing that; but we may easily be persuaded by someone who has made a more elaborate apparatus that it does provide for a genuine, accurate measurement and we might eagerly lead our pupils to it, without realizing the risk. When some apparatus gives a 'good' result easily and consistently in pupils' hands, that success is likely to come about through compensating errors rather than with true accuracy – which, as we know from Joule's work, is hard to attain. Joule's own increasingly consistent results were obtained by using very small temperature rises and in most cases large apparatus. Even then he had to make meticulous allowances for heat losses. In modern measurements of 'J' the care in carrying out the experiment will amaze any reader who follows the full published records.

If, however, a simple apparatus gives honest results that differ by a few per cent from the ideal value, we should let pupils use it carefully, provided we can persuade them not to make the experiment a game of trying to get the right answer. So, at best, we should offer pupils some apparatus in which heat is developed in a comparatively short time and the essential measurements are simple enough to be clear to the experimenter. The second requirement seems to most of us to rule out any series of 'cooling corrections'.‡

Many of us who value precise experimenting very highly are tempted to say that such experiments do more good when they teach pupils the difficulties of obtaining reliable results than when, by some happy chance of design or use, they give an appearance of great accuracy.

‡ School laboratories often have a device in which a band brake rubs against a metal drum which is rotated. If the drum is made of solid metal, the result will hang upon a value for its specific heat, which is usually simply told to pupils. In the design with a larger drum containing water – and a special bent thermometer to get into the water – there is still a problem of thermal capacity of the container, and cooling corrections are necessary if a trustworthy 'good' value is to be obtained. In all such devices the pupil needs to understand fully the calculation of the transfer of mechanical energy to other forms by a stationary band brake rubbing on a rotating cylinder. Though it is obvious to us that we should simply multiply the difference of tensions by $2\pi r$ and number of revolutions, that usually adds an element of difficulty and unreality even for quite able pupils.

Chapter 4

POWER

Measurements of Pupils' Power

Power seems to us more usefully discussed now than earlier when it would have interrupted discussion of mechanical and thermal energy. Some experienced teachers, however, consider that *power* strikes beginners as a more obvious and useful concept than *energy*; and they advocate teaching power first. We hope that some teachers trying our programme will experiment with that change of order, perhaps on a second round of teaching Year IV.

Two small electric motors can be made to raise *equal loads*, one faster than the other. At the end of the operation, pupils will readily state that the *work*, the energy transferred from electrical to (mechanical) potential form, is the same in the two cases, but one motor is more powerful than the other. Labels on the motors may reveal the horse-power or wattage. (If pupils ask whether those labels give the power that the motor always delivers, we must at once explain that a motor adjusts itself to its load, and that the power ratings on a motor usually indicate the maximum rate-of-transfer of energy which the motor is built for. If we demand more the motor will overheat. Also its efficiency is likely to be smaller. Animals (including man) have a somewhat similar ability to adjust to the load. D110

Two electric light bulbs of different wattage present a harder problem. They are emitting radiation at different rates. The labels on the bulbs will raise the question of the meaning of 'watts'. We explain that one *watt* is just a shorthand word for 1 *joule per second*. Therefore watts are not units of energy: they are units of 'power', which tell us how much energy is transferred from one form to another in each second. (The phrase 'rate-of-transferring-energy' seems much harder to young pupils: we might be wise to avoid it at present or they will learn it by heart, carefully, without understanding.) D111

By now pupils are familiar with joules as 'lumps of energy', and through them will come to grasp the watt as a rate. A mention of knots as 'nautical miles per hour' helps pupils to recognize our use of a single word for a rate. T

There are many occasions when we want to know how fast energy is being transferred from one form to another; a steam engine using fuel to raise a load; an electric motor driving a sewing machine or a lathe; an immersion heater in a water tank warming up the bath water; sunlight concentrated by mirrors on a boiler to produce steam; a loudspeaker changing electrical energy into sound wave energy; our own body converting chemical energy into mechanical energy and heat. In each of those cases we may want to know how much energy has been transferred from one form to another in the course of a whole day, so that we know how much fuel has been used, or how big a bill we will have to pay. But we often ask a different question: *how quickly* is energy being transferred in that process *at any instant*? The latter is called power. We may say that energy and power are related rather in the way that gallons-of-water are related to a flow from a tap of so many gallons-of-water per second.

Watts and Horse-power. Note that many of the examples mentioned above are non-electrical. A watt is not just an electrical unit. Mechanical engines are rated in watts or kilowatts on the Continent as well as electric motors. We still use horse-power instead in many cases; and the word, with its good history, is likely to continue. Before the time of James Watt himself, railways and other machinery were run by horses. When Boulton and Watt started offering their early steam engines to mine-owners, they met the question: 'If I buy your engine, how many horses will it replace?' So Watt experimented with a Cornish farm horse raising a load in a mine shaft and decided that 550 ft.lb per sec was a reasonable estimate for 1 horse-power. We should tell our pupils the equivalent: 1 horse-power = 746 watts. T

We should spend very little time on power, because we can always extend knowledge later. However, pupils should do at least one class experiment. * * *

Class Experiment Measuring Useful Power

Pupils should time each other running up a flight of stairs. Every pupil should try this. Those who have a weak heart should walk upstairs at whatever rate is considered safe; all others should calculate the power at which they are transferring energy when running. They should calculate it in horse-power and in joules per second (which they may then call watts). A very short staircase taken with a flying start will give astounding values of power which are quite misleading from the point of view of any consideration of food and normal life. To be fair, we should insist that each child runs up a long flight of stairs. (For an adult man, *useful* power output ranges from 0·1 hp for continuous labour, through 0·2 hp for short pieces of hard work, to over 1 hp for a short spurt. C112a

Children will probably show smaller power values for climbing – because they have to raise their own smaller weight at about the same speed.)

If we want to carry out a further useful study, on which the Nuffield Biology Project will throw some light, we should ask pupils to repeat the experiment climbing the stairs at the rate they think they would be prepared to maintain if they were obliged to continue for an 8-hour working day. This will give them some indication of the power involved in medium manual work. Although we live in a civilization where the push-button-controlled electric motor is replacing human muscles more and more, there are still many parts of the world where human muscles do most of the jobs. C112b

Efficiency of Human Beings. We should tell pupils that when they convert so much chemical energy into mechanical energy the human muscle system does not do that with complete efficiency, but converts a lot more chemical energy into waste heat.

At best, the human body is about 25% efficient for doing mechanical jobs. For every 1400 joules of mechanical energy our muscles supply they also produce three times that, about 4200 joules of heat (1 kilocalorie).

So, when the pupil estimates the useful transfer from his chemical energy to useful mechanical potential energy by climbing stairs for an 8-hour day, he should multiply that by 4 to find the total demand on his food. That will tell him, in joules/day (or kilocalories/day) – a unit of power – the increase in his daily diet that would be needed to provide for that job. An adult man takes over 1800 kilocalories per day from his food for plain living: keeping his lungs, heart, etc., going. See the Nuffield Biology Project material for data, for people of various sizes and ages.

Diet. We might post up the chart of food needs and food values suggested for Year I. If pupils know someone who is busy with a special diet that will provide less than 2,000 kilocalories a day they may ask how it is possible for that person to do any manual work without breaking the Conservation of Energy. The answer is that there are only three possibilities: D112c

1. the person is not doing any manual work, but is spending a lot of time resting and keeping warm with plenty of blankets or sitting near a fire;

2. or that person is doing manual work and is drawing for his needed energy on extra stores of fat which can be burnt as fuel;

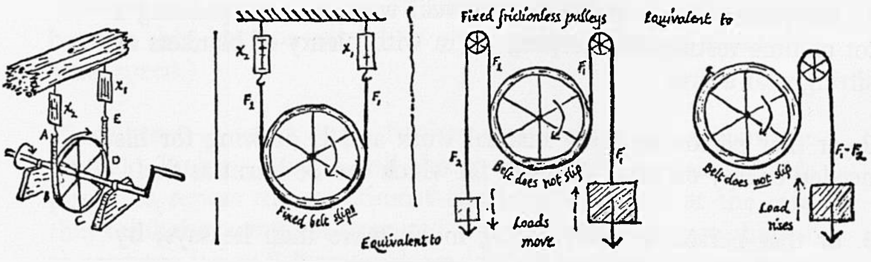
3. or that person is really eating much more than he says, by eating extra sweets, etc., which 'don't count'.

We turn from that to a word about the people in large sections of the world whose diet provides 2000 kilocalories a day or less. Even in hot climates where little fuel is needed to keep man's body warm, we cannot expect heavy manual labour on such a diet; such people are not lazy but unfortunate.

Working against a Band Brake (*Optional*). If the laboratory has a wheel with a band brake against which pupils can work (or a 'bicycle ergometer', the same thing on a bicycle frame), it should be brought into use. We should encourage pupils to find the power at which they can convert their chemical energy to mechanical energy in the wheel which is in turn converted to waste heat in the band brake. This raises questions over calculating the work where the belt slips against the surface of the wheel. We have to convince pupils that D/C112d OPT.

$$\begin{bmatrix}\text{difference}\\ \text{of tensions}\end{bmatrix} \times \begin{bmatrix}\text{circumference}\\ \text{of wheel}\end{bmatrix} \times \begin{bmatrix}\text{number of}\\ \text{revolutions}\end{bmatrix}$$

tells us the work. Some pupils find that easy to understand when we change it to a story of a load (whose weight is the difference between the tensions) being hauled up from a well by a rope which is winding up on the circumference of the wheel. Others find the whole idea very difficult; and for them we should postpone it. An alternative in which the wheel raises a load is easier to understand; but that needs a deep well, or else the imagination to convert it as above to a belt with two loads.



If any pupils try to make an accurate measurement of 'J' in Year V, they will need to understand the calculations for a band brake.

Later this Year, pupils should use voltmeter and ammeter in measuring the power at which an electric lamp transfers electrical energy to heat and radiation. They should also do some experiment with an electric motor. T

A Feeling for the Size of Units

A watt is a joule per second.
A joule is a newton . metre. The newton is roughly $\frac{1}{4}$ pound-weight. 1 watt is the useful power at which a small animal transfers chemical energy to gravitational potential energy if it is a $\frac{1}{4}$ lb animal that climbs a tree at the rate of 1 metre/second, or a $\frac{1}{2}$ lb animal that climbs $\frac{1}{2}$ metre/second. This seems to be about 1 'rat power'. We can profitably tell pupils this, because it helps to make the meaning of power a little clearer.

ENERGY

'Energy Box'. We should show an 'energy box'. This is the 'forces box' that was introduced in Year I and used again this Year to enable pupils to feel a force of 1 newton. The strings which emerge from the box run to various loads inside; so that a pupil pulling a string out feels the weight of that load. But those strings are arranged with limiting stops so that they can be pulled out a measured distance, 1 foot in the case of the load of 1 pound, 1 metre in the case of the load that pulls with 1 newton. Thus, by pulling the string out for the full distance allowed, a pupil can transfer 1 foot . pound-weight, or 1 newton . metre, from his food C113 energy to gravitational potential energy. This box, which sounds like a mere toy, is, in fact, a valuable teaching tool which we should keep available in the laboratory for several weeks.

Kilowatt.hours. Some simple calculations of the number of *
joules concerned in certain transfers should be done before kilo- *
watt.hours are introduced. We should mention kilowatts as useful *
units for a big flow-rate of 1,000 watts and then we should mention *
kilowatt.hours. This is going backwards in the evolution of units *
but it is too common and too important to be neglected. Yet we *
should not labour kilowatt.hours: if we have been careful with *
units throughout the course kilowatt.hours will practically look *
after themselves. *

The Fate of Energy

Most of the fuels that we use release their stored energy into the T
form of heat. We can use that heat to do a job such as raising a load, by giving the heat to a steam engine, but after we have got the job done we notice that there is a good deal of 'unused' heat left around. Cars and power stations need cooling systems to get rid of waste heat; and aeroplanes lose a lot of heat, though they do not have to make special arrangements for it to be carried away. Although we find that in every case of change from heat energy to mechanical energy one kilocalorie provides 4200 joules of mechanical energy, we also find *we can change only a fraction of the heat that we would like to use into useful mechanical form.* There is a tendency for energy changes to be lopsided, with some of the heat energy from the high-temperature furnace running down to low-temperature heat. Low-temperature heat is definitely not so useful, in fact not so much of it is available for our engineering use. A kettle of boiling water can run a steam engine; but emptied into a tub of cold water, it will only provide a tepid bath which could not run any ordinary steam engine. Pupils should realize that the same quantity of heat is there but it is less available, less useful.

At this point, we should confer very carefully with chemical *
colleagues who are now distinguishing strongly in their teaching *
between 'thermal energy' and 'free energy'. *

We should return to the question asked in an earlier Year about the T
fate of the chemical energy stored in the petrol (+oxygen) used by a small aeroplane that leaves the ground, goes on a flight and returns to the starting point – or a car on a similar trip on the

roads. Ultimately, all the chemical energy that is transferred becomes heat. It is strange to think that the whole fuel-burning business of a vast system of airways ends up in doing nothing more than make the Earth's atmosphere slightly warmer.

Newtons, Joules, Calories (Review of Units)
(This is just a review of the status of those units.)

† In Years I and II we measured forces in pounds(-weight) and in kilograms(-weight); but we also provided spring balances marked in *newtons* and simply said that those were units of force. Pupils could try hanging a pound or a kilogram on such a balance and would find that 1 newton is about $\frac{1}{4}$ pound-weight or about $\frac{1}{10}$ of a kilogram-weight. Those who never tried that should try it now. T C114

† When we discussed work as a measure of transfer of energy from one form to another, we defined it as [force] $\times$ [distance] and said that a force 1 newton pushing along a distance of 1 metre made 1 *newton.metre* of work, which we called 1 *joule*. T

Both in Year I and Year II we talked of heat as (*probably*) one of the forms of energy without raising any question about the validity of that view. That was a very dangerous thing, if we are going to raise the whole problem of heat being a form of energy in Year IV. And yet we did not consider that we could ask pupils to hold their breath for three years in a course that proposes to use energy changes as a very important binding thread. Therefore, we took the risk of using an unenquiring attitude at those earlier ages and trusting to pupils and teachers to re-examine the whole question with sincere enquiry in Year IV. It is quite clear that the enquiry will fall flat and seem merely to be a complaining investigation of something that was long ago settled, unless teachers take special care to present it as a major problem in the growth of scientific knowledge.

In Year II, we used the term heat to describe the result of multiplying [temperature rise] by [mass of water] and we measured the result in *kilocalories*.

The newton. This was introduced in Year I and Year II by spring balances marked in newtons and has had to remain an arbitrary unit like that until Year IV.

We have now defined a newton as the force which will give 1 kilogram an acceleration of 1 metre/second per second; and we have given at least a token demonstration that this new kind of newton is the same size as the one marked on our spring balances.

We have avoided confusing that justification with the weight of various objects. Although experiments with gravity pulls provide a much easier method of linking spring-balance readings to dynamic values of forces, we want to avoid confusion over weight: so we have put measurements that involve the Earth's gravitational field strength in a secondary position.

A kilocalorie becomes 4200 joules. From now on we need not use kilocalories any more, but can measure any form of energy in joules. On those occasions when some water is heated we shall either calculate the heat in kilocalories and convert it at once into joules (at the rate of 4200 joules/kilocalorie), or take the 'specific heat' of water to be 4200 instead of 1.

Programme

We should not spend long on the study of heat and energy or on the ensuing problems, and we should treat power very briefly, because we need to save enough time for the long and important continuation of electricity.

We now proceed to more work with the electromagnet kit: transformer; potential difference and use of voltmeter; model power line with a.c.; experiments with materials that do obey Ohm's Law and with some that do not.

Continuing our extension of 'non-Ohm's Law experiments' pupils try a diode tube and find it acting as an 'electron gun' and as a valve.

Pupils then see demonstrations of streams of electrons casting shadows, being caught and shown to have a negative charge, and being deflected by electric fields.

Then comes Millikan's experiment: an ESSENTIAL experiment to show that 'electricity is atomic', that electric charges come in multiples of a universal, basic, unit, the electron charge. This is not a very difficult experiment for pupils to understand, but it is an exceedingly difficult experiment for pupils to do and even a difficult one for teachers to demonstrate. Whether it is shown by film or given as a demonstration, we feel sure that we must show it, to do some justice to the 'atomic nature of electricity' at this stage. Otherwise, all our demonstrations of electron streams, including the usual demonstrations of photo-electric effect, can equally well be interpreted in terms of a continuous stream of negative electric 'juice'. Even measurements of e/m in Year V, and the corresponding measurements for ions in electrolysis and in gases, only suggest *atomic charges: it is Millikan's experiment that* forces *us to believe in them.*

Therefore in planning our timetable we should leave plenty of room for Millikan. It is tempting to postpone that teaching till Year V; but we shall have much to do in that Year, and if we are to do justice to some modern physics then we shall want to find the idea of atoms of charge already established. Then we can start at once on measurements of electron streams with magnetic fields to bend their path to yield a value for e/m. *So it is very important to tackle Millikan this year.*

We mention, without giving details of the analysis, the phenomena of discharge tubes: electrons – all with the same charge and mass – and negative gas ions, driven one way; and streams of 'positive rays', or positive ions driven the opposite way. We should show something of those phenomena if we can at this stage (although we can do little with them until we can talk about the effects of magnetic fields in Year V) because they give us a chance to paint a simple picture of atoms. We picture atoms as solid blobs containing positive electricity anchored in their mass with light removable electrons. We are giving 'early acquaintance' without justification. However, we shall be glad in Year V that pupils have met this picture, because we shall proceed from it to other models.

If there is time, pupils should explore the use of oscilloscopes, preferably as a class experiment.

After Millikan's experiment, we should discuss the idea of an electron 'falling through' a potential difference of 1 volt, and introduce the 'electron-volt' as a unit of energy. However unfamiliar now, that unit will be useful in Year V. At A level it will be essential; but non-scientists should also know it, since it appears in common literature.

Chapter 5

ELECTRICITY

Voltage, Current and Power

P.D and Voltmeters. Discussion of Approach

We return to electric circuits. Now that we are well equipped to deal with energy changes we describe potential difference clearly and define a volt as a joule per coulomb.

This is a direct, definite, fruitful way of dealing with potential difference. To some teachers it is the way they have always used; but others may find it unfamiliar and wish they could use a different approach. Yet we recommend this approach strongly because it provides such clear teaching of power and energy in electric circuits.‡

In modern formal treatment of electricity, we often choose unit current as the fundamental quantity, defined in terms of the force between parallel currents; and unit resistance as the other fundamental quantity, defined by a standard that can be preserved easily. Then the unit of potential difference is derived from those two fundamental units.

However convenient that scheme may be, it leaves the nature of potential difference itself without a clear description. Certainly young pupils will find voltage a mysterious concept – at best vaguely described as an electrical pressure – if they are taught that it is something calculated by multiplying current by resistance. And when we extend our use of potential difference to cases where there *is* no current, or cases where there *is* no Ohm's-Law resistance, it remains very puzzling.

There is, here, a danger of confusion between several different purposes in building knowledge of electric currents, etc. There is the matter of careful definition of fundamental units and the deriving of secondary units – that is a matter for professionals and specialists, which should not concern us heavily here. There is the matter of describing and defining the physical quantities to be measured in those units. There we need to know the physical relationship, which we extract from experiment, such as [heat] varies as [current]^2, or [rate of copper plating] varies as [current]. We need 'operational' definitions – in the technical sense of that

‡ This was set forth very clearly in a textbook of *Electricity and Magnetism* by C. L. Reynolds, who used it at Dartmouth Naval College (G. Bell, rev. edn., 1949).

word – which describe our scheme of measurement in terms of actual apparatus that could be used.

In earlier days, scientists sometimes used concepts that could not be given an operational definition – for example, the exact position of an electron on a sharply defined orbit. Nowadays we are more careful and try to define, or at least describe, our concepts of physical quantities in terms of possible, or at least conceivable, methods of measuring them. Such definitions should yield a clear knowledge of the concept; but they do not always lead to the most convenient *unit* in which to measure the physical quantity. The unit may be defined quite separately – and we often find it has already been chosen, earlier in the history of the subject.

There is no logical objection to defining unit current in terms of mass of copper deposited per second in electrolysis while we define our system of current measurement in terms of the force between wires or coils carrying currents. That may sound unnecessarily complicated, but we can lessen that feeling of uneasiness by paying less attention to the unit itself and treating it as something already given us by earlier generations.

In our present teaching we find it wise to deviate from modern theoretical practice, and describe current as a flow of charge, measured in coulombs. We then describe and define a coulomb in terms of copper plating.

We can even state our unit current, one amp meaning one coulomb per second, in terms of copper plating (0·000000329 kilogram of copper carried across every second, in a copper plating bath). Although that does not agree with the present fashion of defining currents by forces, it gives pupils a much easier way of picturing currents. They already have, from common knowledge, a strong feeling for currents as streams of little electrons; and if we bunch those electrons into large coulombs of charge, we can easily persuade them to think of currents being measured in coulombs per second.

Again, we find it easier – after the initial difficulty of introducing new ideas – to treat potential difference as a fundamental measurable quantity, described as energy transfer, from electrical energy to heat, etc., for each coulomb that passes through the region in question.

It is, of course, unscientific fantasy to picture coulombs carrying loads of energy on their backs and disgorging some of the load in each part of the circuit – then gathering a fresh load each time they pass through the battery. Yet if we warn pupils from time to time that this is an artificial picture, with unrealistic details, they can use it to develop a clear working knowledge of potential difference.

Then resistance, which may well be more convenient in developing a professional scheme of electrical units, takes a secondary place as [potential difference]/[current], with one ohm merely defined as a name for one volt/amp.

With our descriptions and definitions of potential difference and current, it is obvious that [potential difference] × [current] gives us the power, the rate at which electrical energy is changed to some other form such as heat or mechanical energy. It is obvious in slang terms that 'volts × amps = watts'.

And when we generate an e.m.f. we can give a clear description of that concept.

The following discussion describes the kind of teaching we suggest.

Before dealing with volts and voltmeters we must give pupils a clear understanding of coulombs. Some pupils will not have met this unit of charge at all; whilst others will have met a description of it in Year III.

Coulombs and Ampères. We define one coulomb as one ampère.second but that is of little value in giving pupils a picture of it. We should talk of coulombs as the things that go round the circuit, the things that we might count flowing past any point we might choose on the circuit, much as a policeman might count 'cars per minute' for traffic flow, or a hydraulic engineer 'gallons per minute' for water flow. We count coulombs per second and call them amps.

If pupils ask us how we know the size of a coulomb, the best reply at the moment is that we simply read the ammeter, which tells us the rate at which coulombs are passing, and multiply by the time in seconds. That puts the blame for definition on the ampère. And the ampère, we may say for the moment, is defined by the reading of a standard ammeter kept at some standardizing laboratory in each country of the world. It is obvious enough to pupils how one ammeter can be compared with another and that in turn with a standard ammeter.

Worries about absolute standards and units belong in A level and not now. Both we and pupils accept the metre and the kilogram as well-understood units, when they are no more than copies of some chosen standard. We may just as well do the same with the ampère.

In a much later Year, we can define 1 amp as that current which when flowing in each of two parallel wires one metre apart produces a force of 2×10^{-7} newtons on each metre of either wire. That definition has two virtues: it reduces the number of *arbitrary* standards that we have to maintain, and it enables us to carry out some very important calculations; e.g. the force on a current-carrying coil placed in a region near another current-carrying coil. However, our pupils at their present stage would not be able to carry out those calculations and they are not worried about the number of absolute standards that we find necessary in physics – nor should we worry them.

Coulombs and Electrons. Pupils may ask if a coulomb is the same kind of thing as an electron. We should say that we think of the electron as a very tiny particle, which has a mass like any other piece of matter, but also carries a charge. A coulomb is the charge of a vast collection of electrons. In fact, we can measure the size of an electron charge in coulombs – that means comparing the two sizes, one electron charge and one coulomb, by an experiment. We shall describe that experiment soon and we may give the result now:
One electron charge $= 1{\cdot}60 \times 10^{-19}$ coulomb
That comparison tells us:
One coulomb is 6×10^{18} electron charges

A coulomb is *always* 6×10^{18} electron charges, but in many cases of currents, in conducting solutions, in gases, and in some semiconductors, some of those charges are negative and are moving one way, and the rest are effectively positive and are moving the opposite way.

If we were beginning the development of the science of Electricity all over again in the twentieth century, we might well take the electron charge as our basic unit,‡ instead of the coulomb. But, as we might explain to pupils who ask, electricity is too well established for us to make that change comfortably now. (Also, it would prove far too small for convenient use in many practical applications.)

Coulombs in Circuits. The essential need here is to make pupils think of electricity travelling round the circuit as bunches of electrons or pieces of charge, or a stream of 'electric juice', or what you will, but as measured in chunks called coulombs. T

Instead of saying *what* things are travelling in a circuit – electrons, or electric charges – we should rather, at this stage, emphasize a cruder view, just saying something travels that we measure in chunks called coulombs. Coulombs travel along a wire in a circuit and we can count them as they go by with an ammeter and a clock.

Pupils need to be equally confident in expressing some energy changes in joules per coulomb. We can price oranges in shillings per dozen, or milk in pence per gallon; and we know quite well what kind of things a dozen and a gallon are. So, we should almost make a coulomb too real, by tracing it round the circuit, pushed by the coulomb behind it and pushing the coulomb in front (by means of electrostatic fields), slipping smoothly through a low resistance, banging its way through a high resistance, shoving against the edges of the armature-wires in a motor, carrying the material of chemical ions across with it in electrolysis.

Even before a clear feeling for coulombs is established, we describe our current measurements in terms of coulombs. We say, 'A current of 2 amps means that 2 coulombs of electricity (or charge) pass each point in the circuit every second.' Or, 'If a lamp takes 2 amps, 2 coulombs pass through it every second' – a slightly less

‡ In fact the P.S.S.C. programme for teaching school physics did that. Some teachers, and many pupils, considered that a good, realistic, modern scheme; but it led to uncomfortably large numbers. For example, unit potential difference is then 1 joule per unit charge (one electron charge) and that is $6{\cdot}25 \times 10^{18}$ volts. We use the M.K.S. scheme, which leads directly to the practical units, such as 1 joule/coulomb. As a result we shall need a Millikan-experiment film to fit our teaching, later in the Year.

truthful statement. Then, whenever we give pupils data that include a current in amps (or when they measure a current) we ask them to interpret it thus:

'The current is 2 amps; *that means 2 coulombs per second.*'

Then we should give simple problems that ask pupils to interpret currents like that. (For example: The current through a lamp is 2 amps. What does that mean, in other words? How much electricity (charge) flows through the lamp in 10 minutes?) This is one of the rare places in our kind of teaching where routine drill seems necessary and wise. We need to build familiarity quickly.

We draw a circuit that carries five amps and branches at one place into a 2-amp branch and a 3-amp one: and we picture the coulombs drifting 5 a second past any point in the main circuit, then 2 a second in one branch and 3 a second in the other. We sketch a water circuit with a pump for battery, tap for switch, flowmeter for ammeter and thin pipe for resistance.

We say that a gallon of water corresponds to a coulomb, a flow of a gallon per minute corresponds to a flow of a coulomb per second or one ampère. We should be very careful in using that analogy. The proper line of argument is this:

1. We know from simple experiments that something is the same all round the circuit, because lamps light equally all round and an ammeter changed from one place to another gives the same reading in the same circuit. We call that something which is the same all round the 'current' because that experimental property reminds us of a similar property for water in pipes.

2. With the water analogy in our mind, we think of the wires of the circuit as full of something that can be made to move once a battery is applied – rather like starting everyone seated round a tea table on a continuous shifting. We call that something, which we suppose fills the wires and is ready to move, 'electricity', or 'electric charge' – and we measure that in coulombs.

Demonstration Circuits

Simple Electric Circuit. We should show demonstration circuits: first, a simple circuit with several lamps and one or two ammeters in it; then a circuit with branches, with several ammeters at interesting places. D115a D115b

Water Circuit. We should also show a simple hydraulic model of an electric circuit, a 'water circuit'. This should have a small pump arranged to drive water round through a circuit of polythene tubes, including a flowmeter to represent the ammeter, a tap for switch, and a section of thin pipe to represent high resistance. D116

The circuit is best shown without branches or interchangeable 'resistances'. We want to show the *general idea* simply and not build a complicated analogue. Then we use our simple model to *illustrate the idea of potential difference* – its main benefit.

The Worcester flowmeter is an ingenious device in which the water flows downward from a break in a vertical tube in the circuit into a glass funnel. As the water swirls around in the funnel and passes on through the neck to the rest of the circuit, the rate of flow is indicated by the speed at which a small float is carried round by the swirling water.

Where the school has facilities for simple glass-blowing, we hope that teachers will also make a Rotameter flowmeter. That consists of a vertical tapered tube (connected in the water circuit) whose cross-section increases from bottom to top. A coloured marble or a small spinning-top placed in the tapered tube indicates the rate of flow by the height that it maintains in the tube. The faster the flow, the higher the marble has to be to allow the water to pass it at that rate. Commercial Rotameters are far too expensive to be justifiable for school use in a model. Their great cost lies chiefly in the carefully made tapered tube. For our use, a piece of glass tubing roughly drawn out to a conical form does very well indeed. The instrument would have to be calibrated by independent measurements if we wanted a scale, but its value here is in showing a device where, as in an ammeter, a steady flow seems to produce a steady reading by exerting a steady force on an indicator; and a faster flow produces a bigger force and changes the reading.

Pressure Gauge in Water Circuit. The water circuit should be shown first without any pressure gauge. Then we add a pressure gauge to represent a voltmeter.

Coulombs of Charge Taught by Electrolysis. To some pupils the feeling for coulombs comes most easily from electrolysis. If pupils have studied electrolysis in chemistry and remember it, we may be able to use that knowledge. In schools which are following the Nuffield Chemistry Programme, electrolysis will be so fully treated that it would be wise to omit the experiments below from our physics course. Otherwise, we should show simple electrolysis now. Chemistry will certainly have provided background knowledge which makes the experiments quick to do and easy to interpret.

Electrolysis Experiments

Electrolysis of Copper Sulphate, to illustrate Coulombs. D117
Unless we foresee having a good deal of spare time, we should do these as demonstrations. We pass a current, measured with an ammeter, through copper sulphate solution and weigh the copper deposited on the cathode. The weighing should be quick and clear – we should borrow the quickest and easiest chemical balance that will show the weighings clearly.

We do that for a current flowing for a measured time; the same current for twice the time; twice the current for the original time. From those experiments we extract the idea that copper carried across is proportional to [current] and to [time], or to [current] × [time].

Electrolysis of Water. If possible, we should do a similar experiment with the electrolysis of water, and calculate the mass of hydrogen liberated by a known current for a known time and compare that with the mass of copper. (Note that a measurement for hydrogen will be essential in Year V, so that pupils see and understand the measurement of e/M, which we compare with e/m for electrons. So an electrolysis experiment in which hydrogen is evolved will have to be done then whether or not we do it now.) D118

Chemistry will certainly have provided a clear knowledge of the relative atomic weights of hydrogen and copper. Copper experiments suggest that if the copper carried across consists of some tiny atoms, T all alike, then *probably* the electric current is moving across in tiny atoms of positive charge,‡ all alike – 'an electric charge as rider on every copper horse'. The comparison with hydrogen suggests that only half as many copper atoms have to travel across for the same amount of [current × time].

In modern chemistry teaching, we are likely to find valency given *
much less attention, so we should avoid mentioning chemical *
equivalents and stick to whole atoms. We emerge with the sug- *
gestion of a double charge for copper ions. *

Even this demonstration will not be so convincing to pupils brought up on 'amps' and 'currents' as to those of a much earlier generation brought up on electrostatic charges, which led into the idea of charge in a circuit and thence to current. However, the latter method had serious disadvantages of making electrical studies in schools seem remote from the ordinary electric currents that ring bells or make motors work. As it is, we shall have to say again and again that [current × time] tells us how much electricity, how much 'charge' has passed by. And again and again we illustrate that by pointing out that

[water flow, in gallons per minute] × [time in minutes]

tells us how many gallons have flowed out of the tap. (The water circuit may be shown again here.)

Electrostatic Charges and Current Charges

Teachers who are skilful in an approach to electrostatics will be D115
tempted to link up these charges in currents with the electrostatic *
charges that repel each other with the inverse-square law. True, *
they are charges of the same kind of electricity, but the phenomena *
seem so different that the linking of them is not very helpful to *
beginners. Above all, the moving charges in an electric circuit are *

‡ The facts of electrolysis make ions plausible but not necessary. The facts of chemistry known in the nineteenth century made molecules and atoms plausible, but not necessary. By the beginning of this century all scientists felt forced to believe in atoms; and then it was easy to accept the view that all atoms of a given element must be identical. It came as an uncanny surprise when experiments showed the existence of isotopes with different masses for the same element. That might make us more cautious about assuming that all ions have the same charge (or multiples of the same basic charge). However, we are assured by Millikan's experiment that not only does electricity come in small atoms of charge, but those atoms are all the same size.

moving through a lattice of opposite charges, so no electrostatic forces of attraction or repulsion are noticeable.‡ Pupils can be persuaded to pay lip-service to the identity of the two kinds of charge, but it does not seem to help their understanding of coulombs travelling round in electric circuits.

Nor can we repeat as a demonstration the famous experiment by Rowland. He managed to spin an insulating disc with charges on its rim so fast that he could detect the tiny magnetic field near the centre of the disc due to the moving charges acting as a current. One can easily satisfy oneself by a rough calculation of the magnitudes involved that there is no chance of obtaining a big enough magnetic field to demonstrate in an ordinary experiment.

(In some recent discussions of physics teaching, Rowland's experiment has regained considerable importance because it has become clear that, without it, we are likely to teach some things about charges in electrostatics and other things about moving charges in electric circuits and assert that these two kinds of charge are the same thing without any basis for that assertion at all. There is that risk – but there are similar risks in our teaching of Newton's Laws and many other things. It would be a great pity to divert time and energy into performing a Rowland experiment at all costs. In fact, nearly all the demonstrations that look successful are really showing other effects. For example, a compass needle held near the moving belt of a Van de Graaff machine will often be deflected when the belt is running; but an investigation will show that the deflections are arbitrary and variable and due to electrostatic inductive effects: the result of charges on the belt inducing charges on the compass needle.)

Thus we shall not be able to show that electrostatic charges kept moving mechanically are equivalent to a current from a battery, by any demonstration of their magnetic field. However, we can pile up electric charges at rest on the plates of a capacitor, drawing them equally well from a battery or from an electrostatic device, and then show that those charges have both 'electrostatic' properties and 'electric current making' properties. The demonstrations described below suggest that the electric charge that we measure by

‡ Since there is a potential difference between the ends of any ordinary wire when a current is flowing, there must be an electric field along it and a tiny distribution of net charges which will exert minute forces – far too small to be considered here.

(current) × (time) is the same kind of thing as the electric charge that *
we gather by rubbing plastic with wool or pile up with the help of *
a Van de Graaff machine. *

Demonstrations with Capacitors

We use a large capacitor, several microfarads. We explain to pupils D
that it is simply a pair of metal plates, separated by an insulator, rolled up and housed in a box. We do various experiments to show charges running to those plates, and from them, and the charges accumulated on those plates producing sparks or making an electroscope leaf rise.

For our present purpose, we must obtain charges from two kinds of source:
Current sources: batteries, power-packs‡ (which we explain are equivalent to batteries), dynamos.
Electrostatic sources: a piece of plastic rubbed with wool, an electrophorus, a Van de Graaff machine.‡

And for our present purpose, we must in each case make the charges show two kinds of behaviour:
Electric current behaviour: moving the pointer of a sensitive moving-coil meter, lighting a lamp; and possibly chemical effects, detected by feeling a shock.
Electrostatic behaviour: sparks, attracting small pieces of metal leaf, deflecting an electroscope.

It is not necessary to show all the experiments suggested below; *
it is desirable to make any that are shown relevant by using both *
kinds of source and demonstrating both kinds of effect. That is *
the ideal, but in most cases we can only show part of the story. *

‡ Whenever we use the high voltage power-pack we must show that it is not a disguised electrostatic device, but can perfectly well drive a steady current through a high resistance, just like a large battery. We run a wire from one terminal to a high resistance, on through a microammeter, to the other terminal. The microammeter shows a deflection; and then we make it show a similar deflection when we connect it to an ordinary 6-volt battery through a much lower resistance (for example, our own body).

‡ If we use a Wimshurst instead of the Van de Graaff machine, we must be more careful not to produce spurious effects by sudden pulses of charge. We should connect the output from the Wimshurst to one plate of a big air-capacitor, with the other plate *earthed*, to act as a smoothing device.

Charging and Discharging a Capacitor. *a.* We first show currents flowing to the plates of the capacitor. We connect a battery to the two plates of the capacitor through *two* galvanometers or microammeters, one on each side of the capacitor. When the switch is turned on, pupils see momentary pulses of current carrying, say, positive charge to one plate and negative charge to the other – or some equivalent version. These currents last only a very short time and then there is no more current (yet, as we shall see, there are charges on the capacitor, which can run out of it again). D119a

Here we have seen the 'electric current' type of coulombs running from a current source to the plates of the capacitor. We imagine they may be resting there.

We remove the battery and complete the circuit without it: pupils see momentary currents in the opposite direction.

Unfortunately the charges stored on the capacitor plates by an ordinary low voltage battery will not make sparks or affect an electroscope noticeably. Therefore we have to extend this first experiment to higher and higher charging voltages.

b. We add a high resistance in series, and repeat the experiment with the same battery. The charging currents are smaller, the charging process is slower, and we can see the currents dying exponentially as the charges on the capacitor rise to their full value. Then, having established the effect of the high resistance in slowing the process, we replace the battery by an E.H.T. supply. Pupils see larger currents flowing in and the currents may be noticeable for a longer time. When the capacitor is charged, now to a much higher voltage, we may use it to show both kinds of effects. D119b

We show current effects by removing the E.H.T. supply and letting the capacitor discharge through the high resistance and the meters.

Having charged our capacitor again, we show that the charges which we say are resting on its plates can produce 'electrostatic effects'. We bring a wire from one terminal of the capacitor round to the other and pupils see a small spark.‡

We also show that connecting the plates of the charged capacitor to a not-very-sensitive electroscope's leaf and case produces a deflection. An ordinary electroscope gives appreciable deflections for voltages in the range from a few hundred to a few thousand volts. Therefore the capacitor should be charged by an E.H.T. supply – or, better still, with a real battery giving many hundreds of volts.

c. If possible, we should then try charging the same capacitor with a Van de Graaff machine, through a very high resistance, such as a piece of wet string, to avoid overstraining the capacitor. If we succeed, we can then show that the charged capacitor can produce similar sparks when it has been charged from an electrostatic source. (To charge the capacitor with an electrophorous would take far too long; here we are dealing with charges of thousands of microcoulombs, and one microcoulomb is about the largest charge one can put on a toy balloon.) D 119c

Immediately after that we should if possible show an electrostatic source such as a Van de Graaff machine driving a current through a similar microammeter to earth. If a pupil then says, 'I am muddled, that seems to me just the same thing all over again. What is the point of these experiments?' our answer should be: '*You have got the point.* These are all versions of the same story.'

d. It would be more convincing if we could show current effects by lighting a lamp instead of just moving the pointers of meters. That can be done if we charge our capacitor to a more moderate voltage, say 240 volts from d.c. mains. When it is charged we connect it through a medium resistance to a pea-lamp. D119d

‡ Both the length of spark and the energy dissipated impress people. The length of spark is roughly proportional to the voltage to which the capacitor is charged; and we are limited there by the construction of the capacitor. The energy dissipated when the capacitor is discharged increases in proportion to the capacity and varies as the square of the voltage. The bigger the voltage, the more impressive the spark; but if we are limited in voltage we may compensate by increasing the capacity by placing several capacitors in parallel.

Remember that if we charge *any* capacitor to a potential difference of 240 volts and then connect it suddenly to a lamp, it will *start* by driving that current through the lamp which 240-volt mains would drive. However, even a capacitor as large as 10 microfarads, discharging through a lamp as small as a 10-watt mains lamp, would drop to half voltage in about $\frac{1}{20}$th second; so we must not expect startling results. A mains lamp could barely flash. However, a pea-lamp can be arranged to give one flash. That is worth while, because to pupils the lamp shows there is an 'ordinary current' – while a momentary spark would be something quite different.

Capacitor with Vibrator as Repeating Switch (*Optional extra*). The tickertape vibrator can be converted on to a vibrating switch to charge and discharge a capacitor 50 times a second. Two contacts must be installed one above, one below the end of the vibrating blade, so that it hits them alternately. The blade is connected to one plate of the capacitor. One contact is connected to the charging battery, the other contact to a suitable meter to measure the 'current' of the repeated discharges. D119e OPT.

Capacitor with a.c. (*Optional extra*). With a fast group who understand alternating currents well enough to appreciate the demonstration, we should show the effect of connecting an alternating voltage to a capacitor. We insert a lamp in the circuit to show that the capacitor appears to 'conduct' current. A capacitor of a few microfarads will show that well with a mains lamp of low wattage. D119f OPT.

High Resistances for Capacitor Experiments. For a very high resistance we may use a wooden stick, or a piece of damp string. Cotton thread or string will remain damp enough to conduct reasonably for a considerable time in humid weather. For a high resistance with better behaviour, we draw a pencil line on a strip of paper. We should show pupils how to make a very high resistance like that. With a sensitive galvanometer and an E.H.T. supply, we can show the very slow charging of a capacitor when such a high resistance is inserted. That leads to a method of measuring small intervals of time, by collecting the charge that trickles through a high resistance in that time and then measuring the voltage to which it has charged the capacitor.

Other Experiments with Charges (*Optional*). A crude electrostatic experiment is just possible with a 5,000 volt E.H.T. supply: we hang two very small, very light conducting balls on long nylon threads, charge them from the E.H.T. supply, and show that they repel or attract, according to the way we have charged them. If pupils agree that the supply is 'just like a large battery', this forms a good illustration. The light plastic balls in the electrostatics kit will just show this, but larger or heavier balls, even on very long threads, will not show a sufficiently clear deflection. D119g OPT.

Another (*optional*) version of the last experiment offers some preparation for discussing Millikan's experiment. We place a metal plate on the table and another horizontal metal plate, insulated from it, a few inches above. We connect the two plates to the E.H.T. supply, thus establishing a vertical electric field in the space between them. We place small scraps of aluminium leaf in the space between. The scraps acquire charges when they touch either plate and are driven to the other plate; so they dance up and down in the space between the plates. We can then show that such dancing up and down can be maintained by an electrostatic source; we replace the upper metal plate by a sheet of plastic which we charge by rubbing and produce much the same dance. D119h OPT.

POTENTIAL DIFFERENCES: VOLTS

Demonstration. We show two lamps which take the same current but obviously give quite different amounts of light, a low voltage lamp and a high voltage lamp that take the same current – perhaps the most striking form of this is to put the two lamps in series on the mains. That shows clearly that the ammeter reading, the current in coulombs/second, is not enough to tell us how much light or heat to expect. D120a D120b

We should, if possible, demonstrate the same thing with two electric motors taking the same current, one a toy and one a big one. Again, there must be some other measurement before we can find out how fast electrical energy is changing to other forms. D121

Any energy that we receive, in any form (radiation, or heat, etc.), can be measured in joules. As long as the dynamo or battery maintains the electric current through a lamp or motor, we continue to obtain a stream of energy in some other form. And we can measure the energy produced in that other form (in joules), by catching the radiation in ink, by giving the heat to water, by measuring the T mechanical energy delivered by the motor, etc. So we can find out how many joules are being delivered, per second, FROM electrical form TO some other form.

Experiments show that the rate at which we obtain the output energy in (joules/second) goes in direct proportion to the rate at which coloumbs are passing in (coulombs/second), which the ammeter tells us. We can demonstrate that by having one, then two, then three, etc., lamps in parallel all fully lit. An ammeter shows that the currents run in proportions 1 : 2 : 3 : ... Common-sense tells us that the rate of radiation goes up in proportion 1 : 2 : 3 : ..., etc. D 122

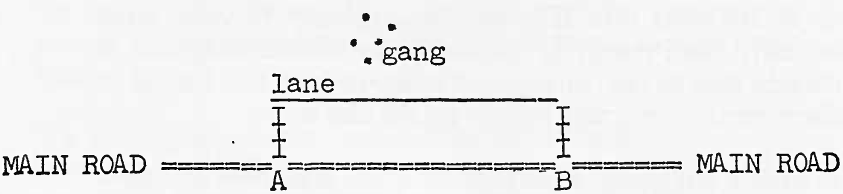
Instead of discussing how fast energy comes out, we might ask how much energy comes out in a given time, such as one second or one minute. Then we see that the output energy is directly proportional to 'charge' or (current × time) or the 'number of coulombs' of electricity that pass – measured by ammeter and clock. But, once again, we see that 'charge' cannot be the only factor: we can have a low voltage lamp and a high voltage lamp that take the same current and give out quite different amounts of light. We also need a device that tells us how many joules of energy are delivered *by each coulomb* going through the lamp or motor. That is the device we call a voltmeter.

Just as a mass of one kilogram can transfer more energy if the cliff it falls off is 100 metres high than if it is only 10 metres high, so a coulomb of electricity delivers more energy if it falls through a p.d. of 100 volts than if it falls through only 10 volts. Also, obviously, more energy is transferred by several kilograms or coulombs than by one. Analogies of helter-skelters, coal wagons on railway circuits, etc., may help to get the idea across. T

For a fairly fast group, some discussion like that above will serve to introduce voltmeters. For a slower group it is probably better to take a crude operational viewpoint and just show pupils how to connect a voltmeter across a lamp or motor; and then assert that the voltmeter is a device for counting how many joules each coulomb delivers as it travels through the lamp or motor. Teaching by assertion is a weak method; and if we indulge in it much we shall give science a poor name. Yet occasionally we may save pupils from confusing arguments for which they are not yet ready; and then we may be wise to make assertions. These are D 123 (slow groups only) not so damaging if we ourselves remember clearly that we have made them.

Voltmeters play a very important part both in scientific labora- *
tories and in engineering and commerce. And voltage, as a *
familiar name for potential difference, is an essential concept in *
discussions of modern physics. Therefore we consider it essential *
to make our pupils familiar with the idea of voltage and the use of *
voltmeters, even if that familiarity is one of habitual use rather than *
full understanding. Here, in fact, is a good example of a place *
where we might relax our plea for clear understanding in favour *
of working familiarity. We suggest that by the end of Year IV *
pupils should know very well that they must connect a voltmeter *
across the item being investigated; and should know quite well how *
to use its reading in calculating power. That familiarity should *
take precedence over the knowledge that potential difference is a *
measure of energy-transfer coulomb; and it should also take pre- *
ference over formal experiments to prove Ohm's Law. *

Analogy for Voltmeter. It is sometimes helpful to liken the current flow of coulombs to a flow of cars along a main road. We suppose that every driver arrives at a point A on the road with the same amount of money in his pocket and has spent all of it by the time he reaches a point B. T



To find out how much money that is, we do not hold up every car and examine the drivers' finances, but instead arrange to divert just a few cars out to a side road at A and along a small lane and to the main road at B. Somewhere on the lane we have installed a hold-up gang who empty the pockets of each driver in the small diverted stream. That is, of course, a model of the working voltmeter which is a milliammeter in disguise.

The more rational analogue, a pressure gauge attached to a water circuit, raises quite a difficult detailed picture for pupils to understand if they think of a gauge with (apparently) only one inlet. We should now show a model water circuit with a pressure gauge that has *two* connections to the flow-line – for example, a U-tube of mercury. D 124a

We suggested a water-circuit demonstration earlier because we wanted to prepare for this very useful demonstration. It is much better to show the earlier demonstration without a pressure gauge and now install one, pointing out that it corresponds to a voltmeter. This demonstration is well worth constructing as a piece of teaching apparatus.

Whatever explanation we give, we end up by saying that a voltmeter measures the energy transferred FROM electrical form TO heat or mechanical form, by each coulomb passing through part of the circuit across which the voltmeter is connected. We then give pupils several experiments to do. T

Class Experiment with Lamps and Voltmeter. Pupils arrange to light a small lamp and measure the current through it. *After* the circuit is connected, they insert a voltmeter. D 125a

It may be better to give such vague instructions that most pupils put the voltmeter in series at first. If so, wait and see what happens, and help them; but do not at once tell them what to do.

Then, when the pupils have discovered or been shown how to connect the voltmeter across the lamp, we should warn them that it is always easier to connect voltmeters if one connects up the whole circuit without the voltmeter first and then adds it.

To show why voltmeters are connected 'across' the part of circuit concerned we return to the water analogy. We show a demonstration water circuit and find out how a pressure gauge is applied to find the pressure *difference* used to maintain the water-flow through some part of that circuit. D124b

The class experiment should be supplemented by a demonstration with a lamp being lit by the mains. D125b

In discussing such experiments, we say:

> '5 volts means 5 joules per coulomb: *volt* is just shorthand for *joules/coulomb* which is itself an abbreviated form of *joules of energy transferred from electrical form to another form in that part of the circuit for every coulomb passing through it.*'

Soon, we should give pupils problems that ask them to interpret voltages like that. This, too, is one of the rare places in our kind of teaching where routine drill seems necessary and wise. We need to build familiarity by use.

Overall Test of Voltmeter's Action. Young pupils will not gain much insight into the working of a voltmeter if we carry them through a detailed investigation of its action, or a calibration to prove that it measures what we claim. But they should try a short test. They should see a voltmeter applied to successive batteries: one cell, two cells, three cells ... in series. C126

This is an overall ('thermodynamic') test. If the voltmeter is a proper device to show energy transferred per coulomb (and if the imaginary coulomb passing through a battery gains in electrical energy at the expense of chemical energy) we should expect three cells in series to give each coulomb three times as much electrical energy as one battery. And we should expect to find the voltmeter across three batteries reading three times as much as across one. In that expectation we are trusting that the conservation of energy extends to electrical energy – and Joule investigated that as well as heat.

This test with batteries only tests a *necessary* condition. It does not prove that voltmeters measure energy-transfer per coulomb; but a failure of that test would disprove our contention.‡ It supports the excellent preliminary description of a voltmeter as a 'cell counter'. T

Calibration of Voltmeter (*Advanced buffer option*). We could of course proceed to test one point on a voltmeter's scale by an absolute measurement. We could send a current through a coil of wire immersed in water, measure the current, measure the time, and measure the heat developed in water. Then, if we kept a voltmeter connected across the coil during the experiment, we could compare its reading with the measured energy transfer (from electrical to heat) for every coulomb passing through the coil. That would compare a reading in (volts) with a measurement in (kilocalories/coulomb), which we should then convert into (joules/coulomb). For that conversion, we should rely on our assurance that heat is a form of energy equivalent to mechanical energy; and we should need to know a conversion rate between kilocalories and joules. And we should have to make sure that our conversion factor, 4200, was not drawn from *electrical* experiments in the later work of Joule and others – because we should be arguing in a circle. However, there are plenty of good mechanical measurements of 'J', and we can rely upon them.

That experiment will test whether our voltmeter does measure joules/coulomb at some chosen point on its scale. It may be very important in our logical development of physics, but it is a difficult, rather messy experiment that pupils are likely to find dull. C127 OPT.

Teachers who are skilful in running this as a class experiment, without too many detailed instructions, but maintaining an enthusiastic sense of the voltmeter being on trial, might try this with a fast group. Others, who consider that the difficulties of thermal measurements make this a rather confusing test, may prefer to leave it out but keep it in mind for those pupils who ask about it. This is a place where we should ask many pupils to take our word for the instrument's action – as we do with stopwatches, balances and (quite often) ammeters.

‡ The position here is rather like that of an experimental test of a theory. An experiment cannot prove that a theory is right; but it may prove a theory is wrong.

(Note that this treatment of voltmeters does *not* involve us in any *
logical difficulty or threat of dishonesty if we use such a voltmeter *
later on to investigate the Ohm's Law behaviour of a metal wire. *
We have not opened up the voltmeter; we have not worked out the *
resistance necessary to convert its internal milliammeter into a *
voltmeter; so we have not assumed what we want to prove; instead *
we have applied *external* tests. See Note on Logic and Voltmeters *
in the General Introduction at the beginning of Year III. *

Pupils' View of Voltmeter. Whatever tests we make, we should T
now urge pupils to take a very simple, clear view of a voltmeter:

> 'Here it is, an instrument designed, and manufactured, and carefully marked, to do an important job, just as a stopwatch is. A volt is a joule (of energy transfer) per coulomb and this instrument is marked to measure volts.'

Hill Diagrams. Talking about voltmeters and the thing that T
they measure ('voltage' or potential difference, or p.d.) we should speak of a coulomb 'falling through' a certain voltage and we should sketch diagrams of hilly terrain – uphill through the battery, downhill round the circuit. (See the discussion of e.m.f. which follows.)

E.M.F.

We encourage pupils to think of potential differences as 'electrical T
pressure difference' between the ends of some part of a circuit where electrical energy is transferred into heat, etc., and voltmeters as devices to measure that energy transfer, for each coulomb passing through that part of the circuit.

Then, presently, we apply the voltmeter to the battery itself. Of D128
course, that is the same as applying the voltmeter to the whole of the outside circuit when there is one; but there may be no outside circuit and yet the voltmeter gives a reading.

We discuss this with pupils and suggest that the voltmeter con- T
nected across the battery tells, in one sense, the energy transfer per coulomb FROM electrical TO other forms all round the circuit. But in another sense it tells the energy transfer FROM chemical form TO electrical form *in the battery*. We call that the e.m.f. of the battery. There it shows the uphill push given to the coulomb which then slides downhill round the circuit.

We may picture a 6 volt battery giving 6 joules to every coulomb with the instruction, 'Remember you must spend *all* this energy on your trip round the circuit.'

We may liken the battery to a lift or moving ramp – such as the machine used to raise coal or gravel to the top of a tower for sorting, or like an escalator for people, or like a chain of sand-scoops in a toy. Electric charge, measured in coulombs, is hauled or pushed by the battery up to the top; then as it travels round the rest of the circuit it is 'running downhill', changing electrical energy (which it has gained from the battery) into heat, etc., as it makes some kind of collisions in the wires. Then as each piece of electricity, each coulomb, reaches the battery again, it is raised up again with a new dose of energy, at the expense of chemical energy in the battery.

With a fast group, we may give a demonstration with a battery which has internal resistance. (See Discussion in a later section on e.m.f. and 'Lost Volts'.) * * *

OSCILLOSCOPES

Oscilloscope. Drawing Wave-form of Voltage. We hope that oscilloscopes, both the large demonstration one and smaller ones for use in class experiments, will be given as much use as possible. Here are some examples: T

1. Show the wave-form of the mains a.c. D/C129a

2. Show the shape of the 1,000-per-second pulses from the scaler used earlier in this Year. D/C129b

3. Measure a short time by sweeping rapidly and showing two sharp pulses on the trace. D/C129c

4. Illustrate the valve action of a diode. D/C129d

5. And, of course, give pupils the pleasure of an 'acoustic mirror' by enabling them to look at the wave-form of their own voice. D/C129e

Somewhere in the course, in Year III for a few, in Year IV if there is time, but otherwise in Year V, pupils should play with a C.R.O. and learn to use its controls capably. Until they have done that we must give help in arranging a suitable trace. Nevertheless, pupils should use their own C.R.O. as early as possible if it can be provided. * * * * * *

Oscilloscope as Voltmeter. A C.R.O. is also a good voltmeter: the electron-stream obeys instructions to move up or down practically instantaneously and the 'input resistance' of the device is very high. For alternating voltages, it is not necessary to sweep the trace across. The spot can be left to run up and down and the total height, twice the amplitude, measured. For d.c. voltages the spot should be kept sweeping across so that the screen is not damaged. D/C129f

The C.R.O. acts as an uncalibrated voltmeter. We must either apply a known e.m.f. from a cell, treated as our 'standard cell', or compare the C.R.O. with an ordinary voltmeter.

Now that oscilloscopes are in such common use in testing laboratories and research laboratories, this example of using a C.R.O. as a voltmeter is one that pupils should see and, if possible, come to regard as an ordinary simple use.

TRANSFORMERS AND A.C.

Demonstration Experiment: Building up Voltage with a Transformer. On one leg of the core of a demountable transformer, place a coil of many turns, suitable for connecting to the a.c. mains. Then, with that primary switched on, wind a secondary of loose insulated cable round the other leg of the core. There should be enough cable to make many turns round the core, and the ends of the cable should be connected to a small lamp from the start. We begin with one turn round the core, then wind turn after turn after turn while pupils watch, hoping the lamp will light. This proves to be a very useful teaching demonstration. D130

We may place the yoke across the top of the core when there are enough turns to light the lamp. The effect is impressive but not particularly helpful.

Class Experiments with the Westminster Electromagnetic Kit continued. Using the kit, pupils make a simple model transformer. We discuss current and voltage relationships. We ask whether the conservation of energy applies to the transformer. If we wish to hold on to conservation, then since the product of current and voltage is what is involved in the measure of energy or energy flow, it appears, roughly speaking, that we can have a low voltage and high current or high voltage and low current – the product is conserved. C131

Use of Cathode Ray Oscilloscope with Westminster Kit. We hope the pupils will use a C.R.O. in class experiments, to show the alternating voltages – either a few large C.R.O.'s transported round to each table or small ones for class use. * * * *

Properties of a.c.

There are simple experiments with alternating current that will enable pupils to build up a clear idea of alternating currents and their behaviour. Some of these are done with ordinary a.c. meters; some are done with very slow a.c., about one cycle per second; and some are done with a C.R.O. to show what happens on an expanded scale of time. * * * * * * *

There is no reason why these should not be done in Year IV, except for the crowding of time. Year IV is rich in class experiments but Year V will be relatively poor. Therefore it may be better to postpone them to Year V. * * * * * *

At most, we suggest the following now as buffer options. *

Class or Demonstration Experiments with a.c. (*Buffer options*). Pupils try connecting the bicycle dynamo to each of the following: *a.* a small lamp; *b.* a d.c. voltmeter; *c.* an a.c. voltmeter; and *d.* a cathode ray oscilloscope. C/D132 OPT.

Then they connect to the C.R.O. a low voltage a.c. supply from a transformer (attached to the mains), so that they can see an alternating wave-form again. C/D133 OPT.

On the C.R.O. trace for a.c. pupils can see the meaning of peak voltage and we can ask pupils what they would expect for average voltage (0) and what they would expect for another type of average, R.M.S. average.

Note to Teachers: R.M.S. Average. We should not labour the idea of 'root mean square' averages here, but we might point out to a fast group that they already calculated such an average when they worked out the average speed of air molecules. In that, kinetic theory provided a total of all the v^2 values for all the air molecules and then when we divided by the total number of molecules (in the total mass of air), we arrived at an average value of v^2, and then we took the square root of that. Though pupils did that and accepted the speed as an ordinary average, a detailed examination of the idea of 'root mean square' averages would give most pupils great difficulty and would cloud the present teaching.

Programme

Important Uses of Potential Difference. *By now, pupils should have a general acquaintance with simple electric circuits, and it is chiefly a question of making certain things explicit. We have settled on a definite, but arbitrary, amp (and coulomb), and a definite volt, defined in terms of energy transfer per coulomb, but measured in practice by a voltmeter which has simply been marked reliably.*

We now make two uses of voltmeters and the p.d.s they measure. We look for relationships, in the behaviour of various conductors, between p.d. (thought of as a driving force) and current. And we calculate power and energy transfers (treating p.d. as energy transfer per coulomb, and current as rate of flow of coulombs).

These two important topics are interchangeable in order. There is a slight advantage in dealing with p.d.: current characteristics first, because Ohm's Law can then be brought into power calculations at once. There is a slight advantage in dealing with energy and power first because they follow straight on from our introduction to voltmeters. Here we adopt the former choice.

P.D.-Current Characteristics. Ohm's Law and other Relationships

How are current and potential difference related? Speaking in slang, how are amps and volts related, when we make measurements with some sample of material?

Note to Teachers on Ohm's Law. We urge teachers to spend much less time than is customary on Ohm's Law; and to make sure that pupils meet other things which do not 'obey' Ohm's Law. As a practical rule, Ohm's Law assumed great importance in the last century when telegraph lines were being designed, installed and balanced, and electrical engineering was growing up. Nowadays Ohm's Law does not receive so much attention. We still regard it as an important description of the behaviour of some materials, and we still use it for some calculations; but the training of modern scientists and engineers does not lay much stress on it. Series and parallel resistances are used; an engineer must be able to calculate them but he learns that quickly when the need arises, so the traditional emphasis in school teaching is hardly necessary. Calculations of arrays of cells in series and parallel are quite unnecessary: they date from the early batteries in the last century, when internal resistance required such arrays.

Modern electric circuitry makes transistors and other non-linear devices as important as the linear ones that 'obey' Ohm's Law; and modern measurements are made with multi-meters rather than bridges. The Wheatstone bridge fell out of use generations ago, except in teaching laboratories.

So our work here with Ohm's Law should be treated lightly and not made the corner-stone of a great electrical edifice. It is only a particular case, applying to *some* conductors; and other behaviours should be investigated.

In this century it is the non-linear devices that have enabled us to make great advances: valves, transistors, rectifiers, etc. Of course, Ohm's Law has also been given importance in teaching because it is a simple law. In developing physical science all of us look for simple linear relationships first and are pleased if we find them. But we then take a wider view.

Introductory Class Experiment. As a class experiment (or, C138
if necessary, as a demonstration) pupils set up a series circuit of five or seven dry cells or accumulators, a resistance of 25 ohms, and a milliammeter. They record the current reading with seven cells, then with one of the cells reversed, so that there are effectively five in the circuit, then with two reversed, then with three reversed, making effectively only one cell in the circuit. The total resistance of the circuit is constant throughout. The currents are seen to be proportional to the effective numbers of cells.

We ask pupils what instrument we could use instead of counting the cells; and we hope to elicit the answer at once, 'a voltmeter: that *is* a cell-counter'.

We then ask pupils to add a voltmeter to their circuit, arranging it to act as a cell counter that keeps track of what is happening in the resistance alone. We then offer them various ways of changing the current. They may disconnect some cells, reverse some cells, put a rheostat in the circuit outside the voltmeter and resistance of the test. (Note the voltmeter must have a much higher resistance than the circuit.) This experiment shows the essential behaviour described by Ohm's Law.

Ohm's Law. We give pupils a sample of alloy wire and ask them to take readings of an ammeter in series and a voltmeter across the sample, and plot a graph. They can also find, both by arithmetic and from their graph, the ratio p.d./current. We explain that that is called the resistance of the object across which we have connected the voltmeter. (See general Note on Resistance in the Preface.) C 139

We lead to the definition of the ohm as 'one volt/amp'. We insist that the use of 'ohm' is only a piece of dictionary work, a short way of saying 'volt/amp'. T

Graphs of p.d. – Current Characteristics. Pupils should draw graphs, following the usual practice of plotting p.d. upward and current along. If time is short, these relationships could be demonstrated, and then pupils could draw their own graphs from the demonstration measurements. T

Questions. Some questions should be raised (but not answered): T

1. What is the meaning of Ohm's Law, in terms of something like water flow, or in terms of a crowd being driven through a corridor?

2. What would happen to the heating in an Ohm's Law wire if we doubled the current? (Of course, we should not just give a formula for the heating effect. We should encourage pupils to argue their way through to the idea that doubling the current would double the voltage, and therefore the heat would be quadrupled.)

3. We may even ask pupils what they think happens to electrons as they are driven through the wire and what the Ohm's Law behaviour tells us in terms of electron motions – though that is liable to lead to difficult arguments.

Temperature Changes. Some things which appear not to 'obey' Ohm's Law would do so if we could keep their temperature constant. That is true, for example, of pure metals such as the tungsten of a lamp.

As an optional class experiment for a fast group, we might give pupils a small open coil of copper wire which, they will find, gives a curve. But, if they keep the copper wire cool in a large bath of water, very well stirred, they may at least be prepared to believe that the deviation from the simple straight line may be due to temperature changes. (See the Class experiments C 143.) C/D 140a OPT. C/D 140b OPT.

Pure metals do 'obey' Ohm's Law but their resistance is roughly proportional to absolute temperature. The wires that we use to demonstrate 'Ohm's Law behaviour' are special alloys, often 60% copper and 40% nickel, with exceptionally small temperature coefficient. We should feel rather guilty if we based an extensive investigation on experiments with such special materials and claimed a far-reaching result. * * * * * * *

Volt-Amp Relationships for Electrolytes (*Buffer option*). Pupils might investigate the following, or see demonstrations of them:

1. a solution of copper sulphate, with copper electrodes; D/C 141a OPT.

2. a water-electrolysis apparatus. (For water electrolysis do not use a rheostat or potential divider to vary the p.d. but try 1, 2, 3, cells in series, in turn, without any rheostat. Using a rheostat leads to troubles with polarization; but the method suggested will yield a graph which is almost a straight line. The line will miss the origin, cutting the axis at about 1·5 volts, suggesting that there is a 'back e.m.f.') D/C 141b OPT.

Volt-amp Relationship for a Gas. We should also give a demonstration of a current passing through a gas. The easiest thing to show is probably a neon lamp, which we should run on a d.c. supply. This does not sound so grand as a 'discharge tube', D 142 yet the latter is only a qualitative show, offering no chance of useful measurements. We should not stress the complexity of affairs inside the lamp. Here we simply want to show that a gas *can* carry a current in some circumstances.

Other Materials: Other Behaviours?

Programme. Pupils should measure p.d. and current for some *
other things, which do not show Ohm's Law behaviour: such as *
some form of transistor and a hot-cathode diode. We should let *
them make their own discoveries and not warn them of the peculiari- *
ties to be expected. As an introduction, we suggest simple class *
experiments on effects of temperature changes but these should be done quickly – or omitted.

Class Experiment: Effect of Temperature Changes (*Optional, except for fast groups*). C143
Pupils should try simple *qualitative* experiments to see how various materials change their conductivity with change of temperature. This is intended to be an 'open' class investigation, in which some materials will present technical difficulties which a pupil must surmount unless he wishes to miss the experiment.

This is *not* suggested as a basis for systematic notes on temperature effects – those effects are not very important in themselves at this stage and any formal study of them should be left till A level. So, unless these experiments can be class experiments (in which pupils are given encouragement but not much help) they should be omitted.

Pupils arrange to measure the current driven by a battery through each of the samples suggested below. In most cases any kind of ammeter, or even just a lamp, will indicate current, and changes of current. In some cases more sensitive meters should be offered if they are available, so that the investigation can be pressed farther.

With each material pupils should try the effect of cooling with ice (better still, with solid CO_2) and warming with hot water or a flame:

a. a coil of copper wire;
b. a coil of alloy wire with low temperature-coefficient (such as constantan);
c. a thermistor;
d. a block of salt. (We must provide a crucible);
e. paraffin wax in a small test-tube;
f. a rod of glass.

When pupils have tried these, the teacher may have to give demonstrations in which the salt and the glass rod are heated much more, though if pupils can do that safely the gain is much greater.

Here are some notes on the behaviour of these materials:

The resistance of copper is clearly less at room temperature than when red hot, also when very cold its resistance is less than at room temperature. We ask pupils what use might be made of this effect, hoping that someone will suggest thermometry.

With the thermistor, the reverse is true. (*de luxe buffer option:* demonstrate its use in a spectrum.) D144

Sodium chloride seems to behave as an insulator until it is nearly at its melting point, but then it readily passes a current. (Mention the use of this in the extraction of sodium.) D145

On the other hand, paraffin wax fails to pass a current even when melted.

The glass rod (which must be made of soft *soda* glass) should be tried as a demonstration, with at least 200 volts applied to a piece one or two inches long, while the rod is being heated almost to melting point with a Bunsen flame. This is rather dangerous but startling. The sodium ions in the glass carry a current when the glass is fluid enough to let them move, and the heat developed as that current is driven through the resistance of the rod (which is still quite high) is sufficient to carry the glass up to melting point and make it drip. D146

These experiments might suggest that, instead of our original division of solids into two groups: conductors and insulators, we now may have to include two other groups: semi-conductors, which appear to pass no current at low temperatures, but readily conduct at higher temperatures; and solid electrolytes, which do not appear to pass a current until they are almost melting. (Our previous experiments showed us that salt, in this group of solids, dissolves easily in water and then enables water to conduct a current.) T

In which group should we include glass? Is it a semi-conductor or is it an electrolyte?

Thermistors are made of a mixture of metal oxides, such as those of copper manganese and nickel. They are classed as semi-conductors and their resistance decreases with temperature rise.

Demonstration with Germanium (*Optional*). Pure germanium D147
belongs to a type of semi-conductor which behaves exactly like an insulator except that the temperature at which it becomes a conductor stops suddenly and is much lower than with a true insulator. These are termed intrinsic semi-conductors. A simple circuit consisting of battery, lamp and a specimen of germanium can be used to show this behaviour; the lamp only lights when the germanium is heated (this is simple enough for a class experiment, but may have to be a demonstration until specimens of germanium are readily available).

We have now moved away from the simple Ohm's-Law behaviour to that of semi-conductors, which asks for a more complicated explanation and offers great possibilities for use in transistors, etc. Most physicists expect that within a few years transistors will have taken the place of valves that use electrons emitted by hot filaments. In making suggestions for a new programme of teaching, we should be careful to look ahead and not tie our teaching unnecessarily to devices, or even phenomena, that are passing out of common practice. We shall let pupils use a diode tube, because it is the prototype of an electron gun, and electron guns are likely to be with us – in our homes if nowhere else – for a long time to come. However, we should also give pupils some chance to see transistors in action.

Transistor: Class Experiment. Our present view is that pupils should certainly do at least one experiment with transistors; but that we should not attempt any explanation of the mechanism of transistors unless pupils ask for it – and even then we should give only a simple picture. C/D148

POWER IN A CIRCUIT

Power should be taken up again here, for the electrical case. We have defined potential difference in volts by saying it is a measure of energy transfer, in joules, for each coulomb passing through the part of the circuit concerned. T

We sketch a simple circuit that includes an ammeter and a lamp. We add a voltmeter across the lamp.

'Suppose the ammeter reads 3 amps. The current through the lamp is 3 amps. What does that mean? ... Yes, *that means 3 coulombs pass through the lamp each second.*

'Suppose the voltmeter reads 10 volts. What does that mean? ... Yes, that means each coulomb passing through the lamp delivers 10 joules.

'How fast is energy being delivered to the lamp? Try multiplying:

$$3\ \frac{\text{coulombs}}{\text{second}} \times 10\ \frac{\text{joules}}{\text{coulomb}} = 30\ \frac{\text{joules}}{\text{second}} \text{ or 30 watts}$$

'We call a joule/second a watt, just as a shorthand name.

'If we multiply the p.d. (in joules per coulomb) by the current (in coulombs per second) we find the power in joules per second.'

Pupils who find this difficult may be offered a very simple analogy:

> 'Suppose a big block of flats is supplied with bread by a number of bakers' boys each delivering several loaves. Suppose we know the number of bakers' boys passing through the block of flats per day – that is the current in boys per day. Suppose we know the amount of bread delivered by each boy, in loaves per boy – that is like the potential difference. Now we multiply the two together and we have, for example, [6 loaves per boy] × [10 boys per day], which tells us 60 loaves per day delivered at the flats.'

(The equivalent story of the country doctor whose wife makes up pills for his patients has an amusing ending. She makes up 6 pills per patient and he sees 10 patients per day, so the total output rate of pills is [6 pills/patient] × [10 patients/day], or 60 pills per day. If by a misunderstanding of arithmetic, we divide instead of multiplying and try [6 pills/patient] divided by [10 patients/day] we obtain a useless number, 0·6, with units that make nonsense, pill . days per square patient. And the example above leads similarly to 0·6 loaf . days per square boy.

Moral: work out the units as a check on what you have done.)

We should somehow encourage pupils to work with this game played among the units. Remember the dangers of the corresponding game at an earlier stage in arithmetic when men were paid wages to dig ditches of various widths and depths. Pupils were easily confused about handling the data: whether to multiply or divide by a number. Some damage from that confusion will remain now, unless we offer pupils the kindly discipline of keeping track of units. If the data give numbers of shillings per hour, hours per day, men, feet of width, feet of depth, hours per cubic foot excavated, days per working week, and then we ask pupils how many weeks 50 men will take to dig a ditch of a given depth and width and length, they can easily end in confusion, unless they keep the units attached to the numbers. It would be almost worth while to go through such a silly problem, just to show pupils how easy such things are now to work out, and how particularly easy they are with the units attached. Then we go back to electrical problems, reminding pupils that changes of volts into joules per coulomb, amps into coulombs per second, and watts into joules per second, are mere dictionary work, mere changes of name and not physics.

Class Experiment: Power Measurements. Pupils should do a real class experiment to obtain data for a power calculation, using a voltmeter and ammeter. Then from their record they should calculate the power used by the lamp, not just by putting numbers in a formula but with a full explanation, at this stage. They should reword the voltmeter measurement like this: C149a

> 'The potential difference across the lamp is 12 volts (or the voltage across the lamp is 12 volts): *that means that each coulomb passing through the lamp delivers 12 joules.*
>
> 'The current through the lamp is 3 amps: *that means that 3 coulombs pass through the lamp each second.*
>
> 'Therefore, in each second 3 coulombs pass through the lamp, each delivering 12 joules.
>
> 'Therefore in every second 36 joules are transferred from electrical energy to heat and radiation.'

Pupils should make a similar measurement with a small electric motor. C149b

Demonstration: Power taken by Commercial Motor. The teacher should show a small fractional horse-power d.c. motor hauling up various loads, and he and the pupils should measure the input power for various jobs. If the output power is also estimated, the efficiency of the motor (likely to be about 60 per cent at best) can be found. D150

Units. No experiment is needed to show that 100 watts is the same as 100 joules per second. Watts is merely a shorthand word for joules per second. Comparison with knots as sea-miles per hour will help. T

Alternative View. We might tell pupils that the potential difference as a measure of energy transfer in joules per coulomb is the same thing as a *current-rate* of supplying power, in watts per amp. Actually, this is a legal definition; and some pupils find it conceptually easier because both the terms [rate-of-delivery-of-energy in joules per second] and [rate-of-flow-of-charge in coulombs per second] are 'continuous' and feel more familiar. To illustrate that, we point out that a 100-watt bulb that is taking a T

$\frac{1}{2}$-amp current is running at a power/current rate of 200 watts per amp. Therefore, the potential difference across the bulb must be 200 volts.

Formulae for Power

Pupils should now express the rule for power in the form: T

power $= V.I$

Then they should use Ohm's Law in the form:

p.d./current = a constant called resistance,

$$V/I = R$$

and arrive at the statement:

power $= RI^2 = V^2/R$ as well as the form $V.I$

We should, of course, tell pupils that these 'formulae' will appear on the front of their examination paper, so that there is no point in memorizing them.

USES OF OHM'S LAW

Some Traditional Uses may be Omitted. Pupils should do C151
some class experiments in which they use a voltmeter and ammeter to measure the resistance of something, but it should be a useful measurement and not just a formal exercise in measuring 'the resistance of the unknown coil'.

Remember that resistance measurements were very important fifty years ago in the management of telegraph systems, telephone lines and even power systems. Wheatstone's Bridge was a telegraph engineer's instrument for balancing lines (very important for multiple message systems) also for fault-finding. Nowadays engineers use much more compact instruments; and Wheatstone Bridges have been relegated to practice in school laboratories. In standardizing laboratories, most resistances are 'four-terminal' devices, to be used in potentiometer measurements. Therefore, a Wheatstone's Bridge should *not* be offered as a practical example of using Ohm's Law in everyday life – we should *not* keep it in our school programme.

We certainly should not give pupils practice in measuring 'the given pair of coils in series and in parallel'. There is nothing wrong with those experiments and many a pupil rather enjoys making four measurements and then doing some interesting arithmetic with them to verify a formula. However, that will not advance our

programme into modern physics, so we urge teachers to omit it. *
In general, calculations using Ohm's Law should not be allowed *
to take charge and obscure experimental interest. *

Useful Resistance Measurements. Pupils should measure the resistance of a lamp, using voltmeter and ammeter, so that the lamp is measured at its running temperature; and they should measure the resistance of a resistor used in electronics, just to see how such measurements are made. They might measure resistance of a motor's armature or its field coils; or the resistance of a real electric fire. C151a-d

We might ask able pupils to measure the resistance of their own voltmeter – and that will indeed open up a very interesting squabble among able pupils. C151e OPT.

That may lead into a discussion of making voltmeters and ammeters from the basic milliammeter. For pupils with the ability to follow the argument, we may go into details, but with others we should give simply a qualitative story. D/C151f OPT.

Fault-finding (*Buffer option*). We may offer some fake telephone lines in which the wires are crossed at some intermediate point between 'London' and 'Edinburgh'. Pupils can make measurements of the resistance at the London end and at the Edinburgh end and infer the position of the fault. These should be simple homemade models, made by running a pair of insulated wires of resistance alloy underneath a wooden board about 2 feet long by a few inches wide. Terminals on the top of the board lead to the wires underneath at each end. Somewhere between one end and the other, the two wires have been soldered together. C152 OPT.

When pupils have worked out where the 'fault' must be, they turn the board over and see if they are right.

Many pupils will then ask about the obviously more likely and difficult case of a fault which is an open break in one wire. We have to tell them that such a break is much harder to locate – though it can be done by capacity measurements – so that many a broken cable is just left in the ground while another cable is buried instead.

Practical Example. Pupils should both work out and try some practical case where a resistance has to be calculated. Example: a small carbon arc that runs at 5 amps will give excellent light for casting shadows and other demonstrations in physics. Running at 10 amps, small carbons will be consumed too fast and the arc will wander; and big carbons would give a great big arc. Running at 1 amp, the arc is likely to be unsteady. So, for many purposes we use a 5-amp arc. We demonstrate a 5-amp arc running, and put a voltmeter across it. Such an arc, running comfortably, takes 50 to 70 volts. However, the arc is unstable if it is fed by a battery or dynamo of e.m.f. only 70 volts. We must feed the arc from a 240-volt supply or a 100-volt supply or anything in between. We ask pupils to calculate the resistance that must be inserted in series with the arc. T D/C153

We give pupils a sample of nichrome wire and ask them to measure the resistance of a measured length, say one foot of wire, and calculate how much they need to make a suitable resistance for running the arc on the mains.

The answer will be several yards – too much for a roomful of pupils to measure out and wind on insulators. At this point the teacher will have to produce the required coil, already wound. A 1,000-watt conical heater element wound with coiled nichrome wire does well. To avoid making this seem a disappointing imposition, the teacher must have a duplicate heater, and unwind its coil and stretch it out to show that it has the right length of wire. (Replacement elements of ready coiled nichrome are available cheaply.)

Then the teacher should set up a small arc (preferably an open-air one with pencil-lead carbons) and show that it works. Although arcs run more steadily on d.c., the a.c. mains will suffice.

This sounds an artificial game, but with pupils it carries a sense of reality and gives them a chance of growing more familiar with carbon arcs – ancient things but still useful in physics teaching.

Model Power Line

This is the same experiment as in Year III – a very important one for understanding power distribution – now extended to alternating supplies. The whole point of the experiment is that pupils should see for themselves (rather than just be told in a demonstration) the importance of using high voltages for long-distance power lines. C154

We hope that this experiment will be given plenty of time for each class in our programme, because we think it will give considerable insight into power distribution. The experiment has three parts:

a. A qualitative class experiment to see a d.c. line showing very low efficiency with low voltage supply. This is followed by a demonstration with high voltage to show good efficiency.

(That class experiment and demonstration should have been done in Year III, to set the stage for later experiments with the power line and to keep the d.c. case well clear of the a.c. version. Those of us who have tried the d.c. one find that it is quick, impressive and quite easy for young pupils to do, and we suggest strongly it should be included in Year III.)

b. The d.c. power line as in (*a*) but with voltmeters and ammeters to make measurements. (That should be done this year by faster groups.)

c. An a.c. power line experiment, first with low voltage without transformers; then with small transformers so that the power lines themselves have a high voltage between them and show good efficiency.

If pupils have tried the first part (*a*) in Year III, they should now know how to set up the power line and use it for some measurements. If they have not done that, here is the description of the power line for part (*a*), as in Year III.

D.C. Power Line: Qualitative. Pupils are given two wooden dowels as pylons, to erect on their bench, to carry a pair of 'power cables', consisting of thin eureka (or other resistance) wire. The resistance of these power cables should be chosen so that when a low voltage lamp is attached to the far ends of that power line, the lamp only just glows a very faint orange when a power pack or battery of the right voltage is attached to the other end of the C154a power line. With two lamps at the far end there should be no visible glow!

We give pupils two such lamps, one for the 'village at the far end', the other for the 'town where the power station is'. These might be 12-volt lamps with a 12-volt supply, but they should be of sufficient wattage for us to be able to change to 240-volts lamps of about the same wattage.

It is well worth while arranging dowels to support the power lines with sturdy clamps to hold them. A ramshackle model power line is dangerous and unimpressive.

We ask pupils to set up the power line and try it. The important thing is for them to see what happens. Then they insert an ammeter in the supply line but they should not use a voltmeter.

Then we change to 240 volts. Teachers will need to do that as a D154b
demonstration; but it should be done with an identical power line. The lamps should have about the same wattage but must of course be designed for the high voltage.

If there is an ammeter in the power line each time, pupils will themselves comment on the difference between the currents in the two cases.

The importance of this experiment lies not in the measurements of *
power but in the clear change from a highly inefficient line to a *
highly efficient one. Teachers who have not tried this experiment *
will be amused themselves by the impressive demonstration. *

b. (*Optional for slower groups.*) Unless pupils used a voltmeter with C154c
their power line in Year III, they should now add voltmeter measurements.

We remind pupils that the voltmeter measures the energy transfer, in joules/coulomb, from electrical form to other forms *in the region across which the voltmeter is connected*. So, if they connect a voltmeter across the power line at the 'power station', it will measure the energy delivered by the power station to the complete power line and village, for each coulomb passing round the circuit. That is, it will measure the electrical energy converted to heat, etc., *in the power line and in the village*. If they connect a voltmeter across the power line at the 'village' end, it will measure the electrical energy transferred to heat, radiation, etc., *only in the village* by each coulomb passing through.

We should ask pupils a question and leave it unanswered. 'Could you connect the voltmeter in still another way and what would it tell you?' We hope, of course, that some pupils will suggest connecting the voltmeter across one wire of the power line itself. In the case of a real power line, that would require voltmeter leads many miles long and no electrical engineer would consider doing that when he could do it more simply by using two voltmeters, one at the power station, the other at the village, and subtracting their readings. T

Some pupils will ask whether there is some way of using a voltmeter to measure not '*what is used*' in (power line+village) but '*what is supplied*' by the power station. They are feeling their way towards the idea of e.m.f. Except for the loss by heating in the power station itself – even the best of dynamos has some Ohm's Law resistance, and wastes some heat when supplying current – the voltmeter connected across the power line to embrace [power line+village] *is* also a voltmeter connected across the power station to measure the 'e.m.f.' (See the previous discussion of e.m.f. and the later discussion of 'lost volts'.)

When pupils have made and recorded measurements with ammeter and voltmeter they should calculate T
the power delivered to the complete power line and village
the power delivered to the village
and, if they have separate measurements, the power wasted by each half of the power line.

They should then do a general accounting of power, remembering that their measurements are not likely to be precise enough to give a complete agreement.

Efficiency of Power Line. We explain that one might reckon the efficiency of the power line as a system for conveying power to the remote village by the fraction:

$$\frac{\text{Power used by village}}{\text{Power supplied by power station}}$$

They should calculate the efficiency of their low-voltage line (for one lamp at the village).

They should see and record similar measurements for the high-voltage power line and work out the efficiency of that.

Those measurements with voltmeter and ammeter and the ensuing calculations of power are less important than the general moral of the qualitative experiments (*a*) and (*c*). We suggest that those pupils who are now able to use a voltmeter well should try this experiment (*b*) but teachers should not labour that experiment if pupils find it hard.

c. **A.C. Power Line.** We now return to a simple qualitative C154d
experiment with low voltage a.c., feeding one lamp at the power station and one lamp at the village. This will show the same disastrously low efficiency as the d.c. power line. Then we provide small transformers – cheap filament transformers, or transformers made from the electromagnetic kit – one to act as a step-up transformer at the power station and the other to act as a step-down transformer at the village. These should be arranged so that the low voltage a.c. supply, 6 or 12 volts, is stepped up to one or two hundred volts between the power lines, and then down to the low voltage for the lamp at the far end.

If it is essential on account of safety rules to show this as a demonstration, the teacher should be very careful to make it clear that exactly the same kind of power lines *and low voltage lamps* are used in the demonstration.

Measurements with the low-voltage and high-voltage a.c. power lines are of course possible but these should not take emphasis away from the qualitative effects. When the transformers are used, the voltage between the power lines themselves can be demonstrated with a high-voltage lamp. Except for that, high-voltage lamps are not used in this experiment.

Electrical Measurement of Specific Heat of Aluminium (*Buffer option*)
Pupils could now make a much more precise measurement of the specific heat of a metal, by using voltmeter and ammeter to measure the power input to their electric heater. If suitable meters are available, pupils will find it a quick, clean experiment. The metal block can be insulated with plastic foam. D/C155 OPT.

Investigation of Lamp Efficiency (*Buffer option*)
If a photographic light-meter is available, pupils may make quick measurements of power taken by an electric lamp and light emitted by it. The graph of light against voltage is a very important one in commerce. D/C156 OPT.

Discussion of e.m.f. and 'Lost Volts' (*Buffer option*)
(This is advanced optional material. Teachers should not offer it to any but a very fast group. *

This treatment of e.m.f., etc., goes much farther than the usual description with which we begin, of e.m.f. as 'the potential difference on open circuit'. That tells us the way we measure e.m.f. but advanced pupils will find it profitable to think of the thing that is being measured as an energy transfer per coulomb TO electrical form – in contrast with potential difference when transfer is FROM electrical form. Discussions involving internal resistance may be interesting to able pupils, but they are not so generally important because we now have batteries with trivial internal resistance.) *

A voltmeter connected across the whole of some external circuit – such as power lines and village – is also a voltmeter connected across the supply to measure e.m.f. Many pupils do not see that until we point it out to them. T

To illustrate this change of viewpoint, we draw a battery supplying a simple circuit containing a resistance, an ammeter, a lamp, all in series. We sketch in a voltmeter across the lamp, another voltmeter across the resistance, and another across the ammeter. Then we sketch in one 'master voltmeter' to embrace all those items. Then we draw that master voltmeter in another position up near the battery and move the leads round the circuit until the voltmeter now appears to embrace only the battery: and we point out that we have made no essential change of connections in making that last move.

Yet, we have made an essential change in attitude. When the voltmeter embraces all the items of the external circuit it measures the energy transferred, for each coulomb, FROM electrical form TO heat, mechanical energy, etc., in the outside circuit. When we regard the voltmeter as embracing the battery, it measures the energy transferred FROM chemical energy TO electrical energy in the battery.

Teachers will certainly object to the latter view as being inaccurate – and some pupils may do so also – since some of the e.m.f. is wasted as 'lost volts' in the battery as long as there is a current flowing. We can tackle that objection by redrawing the battery as a few cells of no resistance, then a resistance to show the internal resistance of the battery, then more cells with no resistance. We apply three voltmeters, one to the first part of the battery, one to the 'internal resistance' and one to the last part of the battery. These should be centre-zero meters, in our drawing. A discussion of what they read, and what one overall voltmeter would read, will prove illuminating.

If some pupils find that too difficult we should not insist on clear understanding, and we should certainly not dictate notes on e.m.f., potential difference and lost volts, but we should leave those ideas to develop in a later Year.

The Excelsior Cell (*Buffer option*). Some teachers may find it preferable to make and discuss the 'excelsior cell' designed long ago to illustrate this matter. It is an accumulator (with negligible resistance) with an external resistance attached to it to represent internal resistance. The combination is placed inside a black box with terminals and labelled Excelsior Cell. This cell will give simple, clear behaviour which we can analyse. We connect a voltmeter across the cell and then draw various measured currents from the cell. When the reading of the voltmeter drops from 6 to 4, we say, 'There are only 4 joules available for every coulomb to spend in the outside circuit. But the cell is still making the same D157
amount of chemical change for each coulomb that goes round the OPT.
circuit and through the cell, so we expect the cell itself to be supplying each coulomb with 6 joules as always. So there are 2 'lost volts', 2 joules wasted by every coulomb, but *not in the outside circuit*. Each coulomb must waste 2 joules in the battery itself, producing heat.

The phrase, 'lost volts', proves useful in teaching. When we then open up our special cell and reveal the built-in resistor, we can connect the voltmeter directly across that resistor and show that our claim of lost volts is a true claim of the potential difference across that resistor, the measure of energy-transfer in joules for every coulomb passing through that resistor. We can then discuss power supplied to outside circuit and power wasted in the cell.

Chapter 6

ELECTRONS

Properties of Electron Streams; Millikan Experiment

ELECTRONS

We introduce electrons by a class experiment with a diode tube. This follows directly on Ohm's Law experiments with currents in wires, etc. So we do not have to begin with any special stories about discharge tubes; we do not have to use dangerously high voltages; we simply let pupils find a new, strange behaviour, with delight.

Once pupils have discovered the strange, negative stream from a hot filament, we do have to bring in other knowledge; but they will feel they have made their own start on electrons.

The Diode

There are two good ways of introducing the diode:

1. We offer it as one more device, like a sample of semi-conductor, for which pupils should make measurements of p.d. and current. We let them try it (guiding them to connect to the plate and cathode) first with the filament cold and then with a heating current supplied. We give no description or explanation except that, when pupils ask, we tell them what there is in the diode, mentioning a vacuum.

2. We tell pupils that we believe that metals contain some very mobile carriers (which *they* will tell *us* are electrons) and we ask them to picture those carriers in rapid random motion, somewhat like molecules of a gas. (In fact, the statistics of electron motion in a metal are not the same as those for gas molecules in a gas and we may mislead pupils somewhat by this description.) We try to picture what happens if we make the metal hotter and hotter: if this makes the carriers in the metal move more and more energetically, as it would with gas molecules, we might expect to see some of the carriers escaping from the surface of the very hot metal – and then they might be able to travel across to another place, their motion constituting a current.

In the latter treatment (2), we suggest to pupils that they should heat up the filament of the diode and see if any carriers evaporate.

In treatment (1) we do not make any such suggestion but let pupils use the diode first with cold filament and find that for whatever voltages they apply there is no current. Then we suggest trying the same thing with a hot filament, still with a vacuum intervening between cathode and plate.

The choice between these two is obviously a matter of taste for teachers and pupils and we hope that where a teacher has two groups of pupils he will try each approach. (For most pupils the idea involved in (2) is likely to seem rather forced; while (1) leads to a thrilling discovery.) * * * * *

The essential story that the diode can carry current only one way, when the hot filament supplies carriers, will be lost unless we somehow arrange with pupils beforehand that they should take measurements 'both ways'. In order to ensure that, without spoiling the diode experiment by extra hints, it is advisable to preface the experiment with one in which pupils use a 'mock diode' consisting of an ordinary radio resistor, and are asked to try running current through it *each way in turn*. They should plot a graph that shows negative and positive values for that. Then we ask them to do the same for the diode. T

We hope that simple hot-cathode diodes will be available for pupils to use them as a class experiment; that class experiment should precede any demonstration. We should let the phenomena of one-way conduction and saturation come as surprises, and we should encourage pupils to suggest why those occur. (We suggest such a diode here, rather than a solid-state diode, because we want to lead up to the idea of an electron gun.) C158

This should be a class experiment of seeing (and discussing) what a diode *does*, and not a formal experiment to plot a characteristic – that would miss the point of the discovery.

We tell pupils that this is part of the evidence for (negative) electrons in metal. It is, of course, teaching by assertion to say that pupils are seeing 'electrons boiling off the surface of the hot filament', and we should be careful to do as little of such teaching as possible in Years IV and V, where the microscopic nature of modern physics makes that tempting. Yet here there is evidence of some negative electricity coming out from the filament. A triode tube can offer some strong circumstantial evidence for the electricity to be travelling from filament to plate, in which case it must be negative. It is only a guess, but a good one, to jump from that to the idea of electrons in metals – electrons conducting currents through wires – and leaking out by evaporation from a hot enough surface.

We ask whether the carriers that seem to travel across the vacuum from hot filament to plate are negative or positive in their charges. This can be great fun, but it easily turns into a rather confusing argument with pointing fingers and anxious trial with labelled batteries. If we can keep it lighthearted and cheerful, we should certainly lead pupils to clear conviction that they have a stream of negatively charged particles, or perhaps negatively charged juice, travelling across. T

Valve. Then we ask why the tube is called a valve; and ask what uses can be made of its valve action. This is an interesting practical aspect; but we should fulfil our duty to the suggestion of electrons first.

Oscilloscope and Rectification. When pupils have had another look at an oscilloscope and tried applying an alternating voltage they should arrange to show rectification on it. An alternating voltage is applied to a diode (with filament heated) in series with a resistor; and the p.d. across the resistor is applied to the vertical deflection terminals of the C.R.O. C/D159

Before pupils look at the pattern, we should encourage them to *guess* what it will be like. This is a difficult intellectual question, but a stimulating one. If possible, we pose the question a week before the experiment will be shown and let it brew.

Demonstration with Electron Streams

This is the time to show several demonstrations of electron *
streams. Until recently, most of those demonstrations had been *
done with tubes which had been pumped out to only a moderate *
vacuum, leaving enough gas to provide some electrons when a *
high voltage was applied, without producing enough positive *
ions to make a confusing glow. Such discharge tubes, which used *
to be called 'high vacuum' discharge tubes, produce a stream of *
electrons that seem, most of them, to originate at the cathode so *
that the stream was called cathode rays. Such tubes contain a little *
residual gas and much of the fall of potential is in a short space *
outside each electrode, so the cathode is a likely origin. Those *
tubes were complicated in action and rather uneven in working *
(with X-rays, electron bombardment, ion bombardment, collisions, *
all taking part in the mess inside the tube). If we use them now, *
our demonstrations are more difficult to run and less clear in their *
showing than those offered by the new tubes with a hot cathode *

that emits a flood of electrons in a really good vacuum, and we might feel that we are letting the historical development of half a century or more ago influence our teaching too much, now that the new tubes are available. * * * *

So we hope that teachers will show the following demonstration, each with a glowing-cathode tube:

1. stream of electrons coming straight out from cathode, making a shadow of an obstruction on a remote screen ('Maltese Cross'); D160

2. stream of electrons coming from the cathode through the slit to make straight splash along a screen; D161a

3. deflection of stream of electrons (in tube above) by electric field; D161b

4. at this point we should show the Leybold fine beam tube again. (Some of our Nuffield group feel that it might spoil the fun to bring this tube out year after year. But others regard it as a very important general piece of apparatus which should come out just as often as a voltmeter. This tube should not be solely a matter for wonder in this electronic age, but should be an essential tool for further learning; and on that score we suggest it should come out again now.) We should show the beam first with a low voltage on the gun so that it loses all its energy in a short distance in the dilute hydrogen in the tube; then with bigger gun voltage so that the beam hits the glass of the tube. D162a

5. We apply deflecting *electric* fields to the stream in the fine-beam tube. We do that by attaching a battery to the pair of small plates just outside the nozzle of the gun. D162b

We explain that the gun itself has an accelerating voltage in it, equivalent to the voltage accelerating electrons in the tubes that have been shown before. The electric field that we now apply between the plates is a transverse field pulling the electrons sideways, giving them some sideways momentum in addition to the forward momentum provided by the gun.

Then we try slow, alternating voltages on those two little plates and also rapidly alternating voltages. (For the slowly changing voltage, we use a potential divider and a d.c. supply.)

6. The next effect to show is deflection of stream by magnetic field. This is an unwise one to show at this point because we would really like to keep it until Year V when we can discuss circular motion. However, it would be inhuman to expect any teacher to restrain himself or pupils from trying a magnet. So let's try it. D163

With the fine-beam tube, we show the effect of a magnetic field, first with a magnet, then with some coil carrying a current, held near the tube. Only after that do we use the pair of Helmholtz coils.‡

We should tell pupils that they will meet this experiment again in Year V when they will be able to combine measurements of the wonderful circular orbit with knowledge of the mechanics of satellites (meaning, of course, $F = mv^2/R$) to obtain an important measurement: the charge/mass proportion for electrons.

7. Pupils should see the heating effect of a stream of electrons hitting a target. (This should be done with electrons from a hot cathode, as in the rest of these demonstrations. A simple wireless valve over-run will show this.) D164

8. (*Optional extra.*) If the apparatus is available, we may show Perrin's experiment which was a crucial one in the very early studies of electron streams. D165

We deflect a stream of electrons into a collecting 'Faraday cage' connected to an electroscope, to show that electrons have a negative charge. In testing the charge given to the electroscope, we should *not* trust to the signs of charges, associated with particular rubbing materials, but we should charge the electroscope by means of a battery or power-pack which has a definite red knob for the positive.

‡ Otherwise a strange saga will build up – that a magnetic field for the deflection of electron streams must always be produced by this special pair of coils. Their use is a piece of tradition which is not necessary. One *large* circular coil would do better. It is true Helmholtz devised this pair of coils with a spacing between them arranged to give a very uniform field over a very wide region, but since the calculation of that magnetic field is quite outside the range of our present course we have no special need of it here.

Note to Teachers: Discharge Tubes. A century ago high voltages were applied to residual gas in glass tubes of many shapes and produced wonderful coloured glows. Ever since then, we have been showing 'discharge tubes', producing delight and mystification. The full story of the events when air is slowly pumped out of that long tube is extremely complicated: among the glows that we see there are patches rich in positive ions, streams of electrons, neutral molecules; and visible light and ultra-violet light and X-rays are flying to and fro. At the end of the last century, a well-pumped discharge tube was the only available source of streams of electrons for the early experiments that led towards atomic models. Now we have hot cathodes in a good vacuum to provide copious streams of electrons of easily controllable energy. So in modern teaching we should not attempt to use 'gas' (discharge) tubes for demonstrations of electron streams.

'Positive Rays.' Discharge tubes, which produced electron streams when the pumping was good enough, more often showed the coloured glows characteristic of gaseous ions and excited atoms recovering from bombardment. By drilling holes in the negative electrode, the early experimenters obtained streams of 'positive rays', particles much more massive than electrons – atoms in fact – carrying one or more positive charges.

Measurements were difficult, but they yielded important hints regarding the structure of atoms. The positive ions showed a variety of masses, following chemical expectations, but the negative 'ingredient' proved to be particles of much smaller mass and (probably) all the same mass and charge, whatever the material they were derived from. More than half a century ago, those hints, gleaned from early experiments with discharge tubes, enabled J. J. Thomson and others to construct atom models. Presently improved techniques led to mass-spectrographs – complicated in design at first, and nowadays much simpler – which could measure the masses of positive ions of all kinds with considerable precision.

The 'Discharge Tube': a historical model, better avoided. Looking back on that history, we can see why there has been a tradition in physics teaching of showing a discharge tube being pumped out, as part of elementary teaching about atoms. But while it is still an interesting historical demonstration we cannot recommend it now as leading to very useful knowledge. We might use it to show that gases will conduct electricity at low pressures; but there is no point in going further.

The various stages of the beautiful glow that one sees are too complex for any interpretation that we could make with pupils at this stage.

Thus we consider such tubes are antiques that should not now be shown. We advise teachers not to continue the tradition; and we certainly advise schools not to spend money on special discharge tubes and diffusion pumps for such a demonstration. Showing an ordinary neon lamp will suffice instead – all they need to see is that it glows and conducts a current. We need to tell pupils something about the atom models that evolve from studies of discharge tubes, but neither the traditional tube nor a neon lamp provides any convincing measurements, or even evidence.

If we do show a discharge tube being pumped out, we should say clearly that it is just for the joy of an amusing picture and not for useful knowledge.

Positive Ray Tube (*Optional luxury extra*). We may show a prepared 'positive ray tube' where there is enough residual gas to show the glow due to gas atoms being excited or ionized by bombardment. (We explain that the glow does not come when the atom is being damaged but in the process of recovery from damage.) By bringing a magnet near the tube we show that some parts of the glow move very little compared with others. Parts which move a lot are glows made by electron streams (with very high ratios of charge to mass). The parts that move very little are made by positive ions (with a much smaller ratio of charge to mass). D166 OPT.

The deflection made by an ordinary magnet is so small that it is doubtful whether it affords a convincing demonstration of the positive charge of those moving ions.

Like the antique discharge tube, the demonstration of this tube should be optional. We doubt whether pupils at this stage would find it convincing. (They do not know that magnetic fields produce deflections proportional to the particle's momentum, and electric fields produce deflections proportional to its K.E. On that basis, since we expect both + and − particles to have much the same range of K.E.s, the magnet's smaller effect shows the positive particles have much greater mass.) * * * * * * * *

A film of a real experiment, showing an actual demonstration of positive rays and their deflection by a very strong magnet, would be valuable. But an animated cartoon to show what 'ought to happen' would be poor teaching – concealed assertion. F166

Atom Models

The teacher, however, should give some account of the way in which early studies of discharge tubes have led to a model of atoms. Whatever the gas put in the tube, electrons seem to be ripped out of atoms. By drilling a hole in the anode, one obtains a stream of electrons passing through which have the same charge/mass proportion in all cases. By drilling a hole in the cathode, one obtains a stream of positive rays whose charge/mass proportions are entirely different, but agree closely with the charge/mass values for chemical ions in solution (and for ions in gases, made by radioactive bombardment, etc.). T

We have reason to believe, from Millikan's experiment, that the charges on electrons and on ions are of the same size, except that some ions have double or treble charges, and some are positive.

If the charges are the same size, then experiments which measure charge/mass give us a hint about masses. We conclude from experiments which will be discussed in Year V, that the mass of an electron is about 1840 times smaller than the mass of an ion consisting of a hydrogen atom that has lost its only electron (that is, a proton). Then from those ingredients, we build in our imagination a 'model' of an atom.

All we know is that we can get electrons from atoms and then what is left over has a big mass and a positive charge. So, we can only picture an atom, at this stage, as a massive lump that has a positive charge, with some small electrons embedded in it, like raisins in a pudding. These electrons can be removed by a bombardment, if that is sufficiently violent, or by electric fields, if those are sufficiently strong.

Ions in Air. With a simple atom model in mind, we should give a description of ions as atoms or molecules that have had an electron (or two) chipped off or added on – charged particles that can act as carriers for currents. Pupils need some knowledge of ions now, as preparation for Millikan's experiment. T

† Pupils should think of gases as insulators, yet in earlier Years they should have seen special cases of gases conducting – a spark counter in Year I, a candle flame and a neon tube in Year II – but without any discussion of mechanism. Now we should describe the making of ions. In a very violent collision between molecules, an electron may be torn off; in some chemical changes (e.g. in a flame) electrons may be exchanged, leaving atomic particles with + and – charges; or electrons may be whipped out of gas molecules by ultra-violet light or X-rays – the action of 'photons', yet to be discussed in Year V. Radioactive materials emit particles with such great energy that in passing through air each of them can make many thousands of pairs of ions in coming to rest.

The fact that alpha and beta particles are electrically charged is usually mentioned as an essential reason for their effectiveness in making ions. Actually a neutral atom, given the same great energy (say a million e.v.) would be just as effective – whether it kept its own electrons or lost them during the encounter would make only a trivial difference – but a *neutron* would be different. * * * * * *

In all those cases, where an electron is detached from some atom it is likely to be collected up quite soon by a neighbouring atom to form a negative ion as massive as an atom or molecule. When a gas such as air contains a great many + and – ions, made by such processes, it will carry a small current if an electric field is applied.

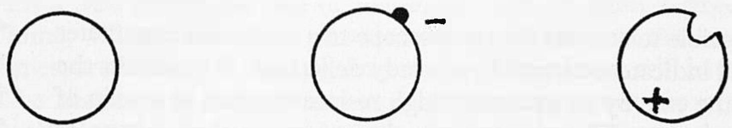
Pupils should see again the two streams of hot gas formed when a candle flame is placed in an electric field: positive ions from the chemical changes carrying a streamer of flame one way, negative ions the other. D167

We suggest that teachers should give some of the following story of ions and ionization. Pupils do need a simple picture of ions in gases, but it should not expand into a long description of complicated processes. * * * *

Teachers will find that pupils understand ions more easily if an atom (or a molecule) is sketched as a round blob. T

That is all we know of atoms or molecules at this stage, except that electrons can be detached from them, leaving them positively charged.

Then a negative ion is sketched as a round blob with a tiny round electron stuck on its rim; and a positive ion as a round blob with a tiny round bite out of its rim.



Driving Ions through a Gas. The field may be applied by connecting a battery or E.H.T. power-pack to a pair of insulated metal plates in the gas. Then in the space between the plates positive ions are driven one way and negative ions the opposite way. D168a OPT.

In a weak field (say 1,000 volts/cm) the overall motion of the ions is a slow drift, as they make myriads of elastic collisions with neutral gas molecules. Each ion, staggering along, driven by the field, is in constant jeopardy of losing its charge by meeting an ion of opposite charge. So the current of ions is driven by the applied field in competition, so to speak, with recombination. With weak driving fields that competition leads to a linear relationship between current and applied voltage – Ohm's Law.

With stronger and stronger fields, the slanting straight line of the graph flattens to a constant plateau (saturation) when ions are swept across as fast as they are formed, with little chance to recombine.

We apply a few hundred or a few thousand volts to the space between two vertical metal plates on good insulating stands. An ionizing agent is held in the space, or near it, a tiny current is driven across between the plates. The ionizing source may be a lighted match, a small Bunsen flame, a strong radioactive source, or an X-ray tube nearby.

To detect the current we need a far more sensitive meter than an ordinary microammeter. We need a micro-microammeter, or something more sensitive still. That should be a *simple* device; so we just let the tiny current charge up an ordinary electroscope. We disconnect one plate from the battery and join it to the leaf of an electroscope, and connect the case of the electroscope to the battery. Pupils watch the electroscope leaf rising slowly when a flame is brought near the space between the plates. After the battery is removed, the flame produces no such effect. But if the electroscope is charged up and the battery is then replaced by a wire, pupils see the electroscope losing its charge when a flame is brought near the space.

It is possible to convert the electroscope to a micro-microammeter that will indicate a current by a steady deflection. We connect the leaf to the case by an extremely high resistance, such as a strip of ordinary paper. The electroscope charges up to that voltage at which the leakage current through the resistance exactly matches the current being fed to it. (This is, in a sense, the inverse of our way of making an ordinary voltmeter from a milliammeter. Here we are making our current-meter from a true voltmeter.)

D.C. Amplifier to Measure Ion Currents? The current carried by ions from a flame (driven by 100 or 1,000 volts to sweep them across) is too small to show on an ordinary meter; but there are now electronic meters, under the name of d.c. amplifiers, that have enormous sensitivity. The device takes tiny input currents – as small as 10^{-14} amp – and amplifies them to be read on an ordinary moving-coil meter. It can even drive a large demonstration voltmeter as a 'slave' indicator.

Some of these are housed in a box with an assortment of input terminals for a wide range of measurements of charge, current and voltage. The cost of such a box would not be justifiable here. And to many a critic its use would not be good teaching. These devices can hardly rank as 'familiar black boxes' like stopwatches; and their behaviour is somewhat temperamental, so they are not likely to achieve that status at present.

However, simpler forms – essentially a triode used upside down, electrically – are becoming available, at a price like that of an ordinary microammeter – and teachers who like to set one up will find it can measure tiny streams of ions provided by a candle flame.

Demonstration: Radioactivity and Ions (*Optional extra*). If the laboratory has a suitable source, the teacher might show now – in anticipation of Year V – one example of radioactive material producing ionization. (Though the nuclear changes involved in radioactivity now seem more thrilling, the ionization is the property by which radioactivity was discovered, the basis of its measurement today, and the effect that makes it hazardous. And it provides the mechanism for cloud-chambers and spark-counters which pupils have met.) D168b OPT.

With a safe source the rate of ionization, even in quite a large volume, will be too small to provide an ionization current that can be measured on any ordinary galvanometer. The current will have to be shown as a leakage current carrying charge away from a simple electroscope as described for the flame ions. That makes the demonstration look unfortunately different from demonstrations of ordinary, larger currents.

It would *not* be wise to use a special oscillating-leaf electroscope (a Zeleny or Wulf type). For one thing, that would burden the teaching with the weight of special explanations. Also, such instruments can be very misleading when we come to show radioactivity. The regular pulsing of the leaf, faster for stronger radioactive sources, may be confused by pupils with the random pulsing of a counter by individual particles. It would be most unfortunate, when we are trying to show a few examples of the particulate nature of the microphysical world if we ran any such risk of confusion. If a simple electroscope will not suffice for the ionization available, it is better to show no experiment.

When a *cheap* d.c. amplifier is available, it might be possible, with several alpha sources collected together, to obtain a big enough current to show. Then we could prepare the ground for demonstrations of range and of ions carried by a puff of air.

Inelastic Collisions: Multiplication of Ions. Suppose we apply an electric field to a region where ions are formed. We picture an ion, driven by the force exerted by the field on its charge, accelerating, gaining kinetic energy until it hits a neutral molecule, where it gives up its gains in an elastic collision. Thus its motion is a series of accelerated runs, each of length one free path.

In a *very strong* electric field an ion can gain enough K.E. between one collision and the next to make an *inelastic* collision and knock an electron off the next molecule it hits, making a new pair of ions. That process of ionization by collision multiplies the number of carriers, so the gas conducts easily; and the ionization may mount up in a chain-reaction pattern – then we have a spark. The actual mechanism is much more complicated than that but we should give pupils a simple glimpse of the picture. (Where there are ions there will be photons of ultra-violet light and perhaps X-rays; the negative ions will be electrons part of the time, and therefore much more mobile; and excited atoms will play a part.)

Air Molecules Make Elastic Collisions. We should make it clear that collisions between air molecules at room temperature are quite unable to produce ions: they will be perfectly elastic every time. At the end of this Year we shall mention the use of electron.volts as very useful units for energy in atomic events. For a collision to detach an electron and make a pair of ions, the colliding particle must bring in, say, a dozen electron.volts of K.E. – ranging from a few e.v. for some metals, to 20 or 30 e.v. taken from a fast particle to make a pair of ions as it rushes through air.

The average K.E. of air molecules at room temperature is about 0·03 e.v. and their collisions are completely elastic.

Since K.E. varies as absolute temperature, we should expect an ionizing collision to be rare indeed even at ten times room temperature, 3,000°A. On the other hand, a simple electron gun, like the one in the fine beam tube, shoots out ionizing projectiles with ten times the K.E. needed to make ions – and that is why the hydrogen in the tube shows the path of the beam by a glow.

THE ELECTRON: MILLIKAN'S EXPERIMENT

The Importance of this Experiment. Electrons are common objects in modern physics, and even young pupils talk glibly of them. Yet *none* of the experiments with cathode rays or thermionic effects provide convincing evidence that electricity comes in atomic particles.

Measurements of e/m, which will come in Year V, merely show a constant proportion of charge/mass for all cathode ray streams. And thermionic experiments merely show that hot filaments emit negative electricity which can travel across a vacuum. Even the photo-electric effect fails to show individual electrons in the usual demonstrations.

The only experiment that really shows 'atoms of electricity' is Millikan's experiment. Since the idea of universal electron charge is essential to any picture that we are now building in atomic physics, we regard that experiment as a very important part of our teaching. It should come now and not be postponed till Year V, because a well-digested understanding of this part of our atomic knowledge will be needed at the beginning of Year V. Teaching Millikan's experiment then, even if only by a quick film, would add too serious a burden to our new work on circular orbits and e/m. However much we are tempted to postpone it, we should not do so.

Methods of Showing Millikan's Experiment

Individual Viewing. A number of forms of Millikan's apparatus have been manufactured and brought into use in school and university laboratories. Again and again there is a rumour of a still simpler form that 'really works easily' in pupils' hands. Although there are improvements, the new forms still require one or two pupils to work with them undisturbed for some time.

Even in demonstrations, the most well-behaved apparatus, set up by the teacher and kept running with a visible droplet, can only be seen by one pupil at a time. The thing to be seen, a tiny moving or stationary speck, is even more difficult for a beginner to see quickly – if he is to appreciate its importance – than the Brownian motion glimpsed in a hurry. The difficulties are intrinsic: however much the apparatus is simplified and improved there will remain the problem of individual watching. If our pupils were growing physicists with full appreciation of the importance of this experiment, the large delay for such viewing might be worth while; but that should come at earliest at A level. The demand of time for pupils to look, and time and care for the teacher to set up the apparatus, are far too great for use of the real apparatus in classes at this O-level stage. * * * * * *

(Where a school already has apparatus for a Millikan experiment we suggest it might be used by some pupils as a luxury option in Year V if specially keen pupils can give it the time it needs.) * * *

Closed-circuit TV. There are possibilities of demonstrating a real Millikan experiment by closed-circuit TV. That can be done at considerable expense; and if it is done with sufficient teaching it will be slightly better than showing a film. Pupils can see the apparatus; they can come up to it afterwards and look at details; and a few keen pupils can use the apparatus at other times. But this again seems too expensive a way of teaching this one part of our course, however important. It is not only expensive in money for equipment: it also makes considerable demands on the teachers' time. Therefore, we do not recommend this, except where a school already has the Millikan apparatus and already has closed-circuit TV for other purposes. * * * * * * * * * * * *

Films. And yet the experiment is vital. It is the key experiment in the development of an atomic story. So many other evidences of atomic nature – of matter, of electricity, of radiation – are things that we have to quote as assertions at this stage or, as in the case of e/m, go through a long calculation that threatens to spoil the sense of clear conviction in our pupils. Millikan's experiment can show young people clearly that electric charge comes in 'atomic' form. Therefore, we resort to film. That is far less direct form of teaching than an individual experiment, but far better teaching than description and assertion in a book. * * * * * * * * * *

So our main hope lies in films.

The Millikan Experiment Itself

The Reason for Film. As discussed above, a direct demonstration arranged by the teacher and shown to a few pupils at a time is possible but very difficult because the lighting is tricky and the 'droplet' is easily lost. To enable each pupil in turn to see the droplet and watch its motion *and understand what he is seeing*, would take an enormous amount of time with a large class. Closed- * * * * * *

circuit television offers a possible way for pupils to watch the Millikan droplet while the teacher controls its motion with an electric field. But that still involves considerable cost, and trouble for the teacher. That is why we have prepared a film‡ to show the real experiment in a simple way. The method used is described below. * * * * * *

The Experimental Method. In our version of the experiment, the oil-drop is replaced by a tiny plastic sphere of known size and density. The sphere is allowed to fall, in air, in a shallow space between two horizontal plates. A battery connected to the plates maintains a vertical electric field in the space. The tiny floating sphere is given an electric charge, and the field between the plates is adjusted, by changing the potential difference with a potential divider, until the upward pull of the electric field on the charge on the sphere exactly balances the downward pull of the gravity. The sphere is held poised in mid-air in the field of view. F170

‡ There is also a film made by the P.S.S.C., to show Millikan's experiment. It is a very fine film; but it adopts a different scheme of teaching and we do not suggest its use in our programme.

The small plastic sphere is allowed to fall under gravity against air friction, with constant 'terminal' velocity. The relation between the drag of air friction on such a small sphere and its velocity is investigated in the film.

Then the experimenter in the film measures the distance that the sphere falls or rises in a standard time, such as five seconds, under gravity alone, and with a standard electric field.

For the first charge on the sphere, the electric field is adjusted to hold the sphere poised, as in our method. But when the charge on the sphere is changed the *same* electric field is used; so the sphere is no longer poised, but moves up or down with a speed determined by the charge it has gained. That speed is measured, in the film, after each change of charge on the sphere.

Then the speed of the sphere *in the standard electric field* is used as a measure of the *extra* charge it has gained since the original charge with which it was poised.

Thus changes in charge are measured by measuring speeds directly; and they all prove to be small integral multiples of a basic charge.

In that film, no voltmeter is used – for reasons connected with the method of developing electricity and magnetism used in the P.S.S.C. course. In the Nuffield film with a voltmeter, we change the electric field and poise the droplet for each successive charge that it acquires. Therefore, the total charge varies inversely as the field, inversely as the voltage. That makes the reasoning a little more difficult but it avoids all question of the relationship between air friction and speed.

Then the charge on the sphere is changed, and the voltage applied to the plates is varied until the sphere is again poised. That is repeated after several successive changes of charge. Thus pupils see a set of different voltages applied to the plates, as the sphere takes on different charges, each time just the voltage needed to hold the sphere poised.

The charge on the sphere is changed by ionizing the air around it and letting it pick up some charges from ions. If we bring a radioactive source near the apparatus, we can make many ions in the region between the plates (or, instead of the source we ionize with a discharge from a small Tesla coil – a 'vacuum tester'. If the electric field is maintained those ions are quickly swept away and we may have to wait some time before the sphere can pick up a charge. If the electric field is turned off, the sphere falls slowly through air containing ions and soon picks up a charge. It often changes its total charge by several electron charges.

In each case we assume that when the sphere is poised, the upward pull of the electric field on the charge exactly balances the pull of gravity downward. T

Therefore, weight of sphere = (charge).(field strength)
because the field strength is the force on unit charge there.

In deriving the essential atomic nature of electricity from Millikan's experiment we do not need to teach or use the concept of field strength explicitly. With a slow group we can talk more loosely about forces, charges, and voltage and convey the main argument successfully:

> 'The battery drives equal and opposite charges on to the two plates, + above and – below if the little balloon has a – charge. Those charges on the plates together push and pull the balloon upward (while gravity pulls it down). *Two* batteries in series, *twice* the voltage, will drive twice as big charges on to the plates. Then the upward force on the balloon's charge will be twice as big. The voltmeter tells us how much charge we are putting on the plates, how big the force on the balloon will be.

'But if we change the charge on the balloon, that will alter the upward force too: doubling *that* charge will double the upward force exerted on the balloon. In every case we shall make the upward force balance the weight of the balloon, the downward pull of the Earth on it.

'There are two factors that affect the upward force:
The charges on the plates, which we can measure by the voltage V;
the charge on the balloon (which we want to find out about).

'We multiply those two factors in finding the upward force, and the result must always be the same if the upward force balances the balloon's weight.

$$\text{(Voltmeter reading V)}.\text{(Balloon charge)} = \text{same answer every time.}$$

$$\therefore \text{ Balloon charge} = \text{(constant)}.(1/\text{V}).\text{'}$$

Such a discussion needs the help of a large model, and teachers should try various wordings. It is close to the method used in the P.S.S.C. teaching of 'Millikan', which succeeds with pupils over a considerable range of ability.

With average and faster groups we should deal with field strength fully, as below. (That will also be useful in discussing, qualitatively, the effect of electric fields on electron streams.)

The Field Strength. The field strength is the force on unit charge placed in the field, measured in newtons per coulomb. We used that definition of field strength in the statement above, when we calculated the weight of the drop by multiplying charge on the drop by field strength. T

We can measure the field strength between the plates by using a voltmeter and a metre ruler. Imagine a tiny charge q coulombs in the space between the plates. If the field strength is X newtons per coulomb, the force on q is Xq newtons. That is the force with which the electric field drags q from one plate to the other.

Attach an imaginary thread to q, pulling in the opposite direction and arranged to raise a tiny imaginary load. Then if we let the electric field pull q from one plate to the other, the thread will raise the load a distance d, the distance from plate to plate. We can calculate the *work* which measures the energy transfer FROM the electric field (and the battery maintaining it) TO the potential energy of the load that is raised.

The work is (force).(distance), or $Xq.d$.

But in making that transfer the electric field has dragged a charge q from one plate to the other. If the potential difference between the plates is V volts, every coulomb dragged from plate to plate transfers V joules of energy. And q coulombs transfers Vq joules of energy. This is another statement of the work.

Therefore, $Xq.d = Vq$. Therefore, $X = V/d$.

Thus by connecting a voltmeter across the plates and measuring the distance between them we can calculate the field strength V/d.

Since the field strength is directly proportional to the voltage between the plates, we can use the voltmeter reading as a measure of the field strength.

Each time the sphere is poised:

the weight of the sphere, which remains constant,
$$= [\text{charge}].[\text{field strength}] = [\text{charge}].[V/d].$$

Therefore, $[\text{charge}] = [\text{constant weight}]/[V/d]$.

Analysing the Measurements. Therefore, the charges on the sphere in successive poisings are proportional to $1/V$.

Thus we can calculate the numbers which represent the total charge on the sphere – on an arbitrary scale – by taking the reciprocals of the voltages. We do that, and then discuss with pupils the information about electric charges that we can extract.

To lead from those data to the strong suggestion that electric charge *
comes in uniform basic units, we need to give a strange new argu- *
ment. We should prepare pupils for that: we should show a large- *
scale model; we should give a homework problem using $1/V$ to *

measure charges, and a very useful fable to prepare for the important analysis. *
*

Large Model of Millikan Experiment. Before any demonstration or film, the teacher should show a working model, a 'macroscopic Millikan experiment' in the form of a light plastic ball, coated with metal, hung in the electric field between a pair of horizontal plates. D169

The ball should be held midway between the plates, because if it is much closer to one plate than to the other 'image forces', due to induced charges on the nearer plate, will complicate matters.

This is a qualitative demonstration in which we give different charges to the suspended ball and show that there are different forces on it.

Homework Problem. We do not wish to make Millikan's experiment seem heavy and difficult by giving too much preparatory drill. Yet, a homework problem leading to the idea of measuring charges, when they are held poised by an electric field, by values of $1/V$ would be of considerable help. P

The Fable of the Eggs in the Paper Bags. As in Millikan's original experiment, we cannot see straight away from the measurements that we have discovered universal electrons. We have to find out whether our measures of charge are all multiples of some universal unit. That seems obvious to us, who already know the problem and the solution, but it is a difficult scheme of investigation for pupils to understand. So we tell them a story: T

> 'Suppose I have some eggs in a paper bag. And another lot of eggs in another paper bag. I have several bags of eggs and I do not know how many eggs there are in any of the bags. To find out, I weigh each bag. One bag weighs 14 ounces, another 12 ounces; others 18 ounces, 8 ounces, 10 ounces. Can you make a guess of how much an egg might weigh – taking it for granted the eggs are all the same? ... Yes, 2 ounces is a good guess: it will fit all the facts.
>
> 'Now suppose I have one more bag of eggs and that weighs 13 ounces. What happens to your guess now? ... Yes, that will force you to climb down to 1 ounce for each egg.

('Suppose I had one more bag which had eggs in it also and weighed 13·2 ounces. What happens now? ... In that case, you will probably have to give up the idea that the eggs all weigh the same.)

'Now forget about that last, strange bag and go back to the other bags. You would guess that each egg weighs one ounce, and then you would know how many eggs there are in each bag. Of course you might still be wrong; but if you continued the weighing and guessing you would be pretty sure to be right in the end.

'That is what Millikan did with his experiment and that is what we have to do with the experiment shown in the film. We know the total charge the droplet had to begin with. That is like weighing one bag of eggs. We know the charge the droplet had later on when we had changed the charge on it by letting it catch some extra charge from air molecules that had been ionized nearby. That is like the weighing of another bag. Now we look at all those charge values to see whether, like the weights of bags of eggs, there is any basic number which fits the lot.

'Perhaps the droplet started with a lot of basic charges to begin with and then gained or lost one or two, etc. So it may be better to start by looking at how much it gained or lost from one time to the next. That gives us a hint of small numbers. In fact you were doing that with the bags of eggs because the first thing you did was not to think about the whole weighing, such as 14 or 16 ounces, but to subtract 14 from 16 and think about 2 ounces, and 13 and 14 and think about 1 ounce, etc. Then, with a guess for the weight of one egg, you could deal with a whole bag.'

With that kind of illustration, pupils are likely to be ready to *
join in the analysis of the film. The film shows many spheres, with *
different charges, moving under gravity alone and with electric *
fields applied. Then one sphere is singled out and poised by ad- *
justing the voltage between the plates.

The charge on that sphere is changed again and again, by letting *
it collect ions, and after each change the voltage is adjusted to *
poise the sphere. The reciprocals of those voltages are then taken *
as measures of the total charge on the sphere and, like the eggs *
in the bag, they suggest strongly that in every case we have whole- *
number multiples of some basic charge. *

Our principal objective here is to let pupils see that all the charges on the specimen sphere could be treated as made up of a few 'electrons' that are all alike. We want to give them the idea of a universal basic charge. We are *not* at the moment trying to measure that charge. In fact, pupils will have to take our word for the absolute measurement.

Note on Use of Plastic Spheres for Millikan's Experiment. In any teaching version of the Millikan experiment we should take advantage of a modern development. We can use extremely tiny spheres of solid plastic instead of the droplet of oil used by Millikan.

Some years ago, chemical manufacturers discovered that they could make batches of very small spheres of uniform size. These are now supplied to the users of electron microscopes, for calibration. We can obtain a large quantity of these that are all the same, known size. Thus, if we lose the 'droplet' we can start again with another; and we can use the known size of the 'droplet' to calculate its weight without having to appeal to Stokes's Law.

The size of these spheres can be measured by comparing them under an electron microscope with something larger and that with something larger still at lower magnification, and so on by 'bracketing' into the optical field; so that ultimately we know their size compared with a centimetre ruler. That size, for a uniform batch, is supplied by the makers.‡

Electrons Themselves

In watching Millikan's experiment pupils have seen evidence for the electron *charge* as an atomic unit of electricity. What have they seen of electrons themselves, as particles of minute mass carrying such a charge?

The sphere or oil-drop in Millikan's experiment collected its charge from ions – air molecules nearby that had gained or lost an electron or two. The sphere may have acquired its original charge by contact ('friction') as it entered the apparatus. Millikan

‡ A rumour arose some time ago that the use of these plastic spheres in a Millikan experiment is deceitful 'because that is how the spheres are measured'. It would be possible to measure the spheres by their terminal velocity in air – though one would hardly bother to charge them and impose electric fields – but this is not the standard method. Nor would such method invalidate our form of Millikan's experiment, because we aim at showing the existence of universal atomic charges of electricity, and not at measuring the absolute value of *e*.

himself also changed the charge on his oil-drop by ejecting photo-electrons with ultra-violet light or X-rays. All these ways of providing charge agree with the idea of a universal unit of charge. But they do not tell us about the electron's mass; and at most they make only a vague suggestion about the part played by electrons in atomic structure.

To learn about electrons themselves we must experiment on streams of electrons; and we should look at the tracks of electrons in cloud-chambers and bubble-chambers.

Pupils have now seen a heated wire emitting a stream of something that is negatively charged. They have seen such streams cast shadows, generate heat when stopped, make certain substances glow, provide negative charge when collected, and they have seen them deflected by electric and magnetic fields – the deflection being in a direction consistent with the idea that it is a stream of negative charges. Except for the Millikan experiment – which we fear will have to be shown solely by film at this stage – pupils have no evidence that the electron stream consists of very small unit charges of electricity all alike. For all they can tell it might be a stream of negative electric juice.

We should not keep on insisting on the latter agnostic view, yet we should maintain a warning flag. It is our duty in teaching science to wage a gentle warfare against the tremendous decorations of unsupported detail that popular writing builds around modern physics. However, we can tell pupils that later experiments, as well as Millikan's experiment, do suggest that this stream of 'cathode rays' is a stream of small particles, all alike, carrying a basic electric charge which we have never found subdivided. We should say that the compelling, though rather indirect, evidence will come in Year V. We shall measure e/m for a stream of cathode rays, and find it huge compared with e/M for hydrogen ions. Combining that with knowledge from Millikan's experiment we guess that cathode rays are streams of tiny particles, all with the same negative charge, all with the same tiny mass – a fraction of an atom's mass. We are sure, then, we have discovered chips from atoms; and we call those chips electrons.

Then we see more evidences of electrons. If we give them enough energy we can count them one by one with a Geiger counter or the equivalent. We can observe the tracks of individual high-energy electrons in a cloud-chamber. These are demonstrations in Year V.

Electrons in Year V: Mass and Speeds. Looking forward to those assurances next year, we should tell pupils that their e/m experiment will also tell them the speed of electrons issuing from a 'gun'. T

Although an electron carries a small charge, its mass is so minute that we can give it enormous accelerations by using the ordinary electric fields at our command. Pupils will be able to calculate the speed that, say, a 100-volt battery can give electrons.

We might warn them that they will need to remember some things they already know:

that kinetic energy is calculated by $\frac{1}{2}mv^2$
that a p.d., in volts, is the energy transfer per coulomb, in joules/coulomb

They will need to learn about the effect of a magnetic field on a stream of charged particles. Then they will be ready to measure e/m for electrons, and their fantastic speed. They will see for themselves why we think electrons are only tiny chips off atoms.

Currents in Wires. We can, if we like, describe in terms of electrons the events inside a wire that is carrying a current. In our picture, most of the electrons in a metal wire are anchored firmly to atoms in the crystal lattice; but some of them are detached and live a life rather like molecules in a very hot gas, moving to and fro in the crystal lattice at fantastic speed. (We should be careful not to say the electron gas is quite like a gas of ordinary molecules at room temperature. The statistics are different, and the speeds are unexpectedly high.) T

The electric field along the wire, produced when we attach a battery to the wire, drives that hurtling horde of electrons with a slight general drift, which makes the current.

Evidence of Electrons in Wires? If a wise pupil asks us how we know the electrons are there, we should tell him the evidence is indirect and the arguments are sophisticated.

We may remind him he has heard about electrons 'boiling off a hot filament'. We might even tell a very able pupil about the 'Hall effect' in which a magnetic field applied across a wire carrying a current produces a very small voltage across the wire in the third direction perpendicular to current flow and magnetic field. That voltage is due to crowding of electrons by the catapult push of the magnetic field. It is in a direction which shows that we have a flow of negative charges. The effect is far too small in metals to demonstrate easily. Unfortunately, there are exceptions, which modern atomic physics can explain, in which the Hall effect takes a reverse direction. There are many exceptions among semiconductors, where the moving things may be 'holes' instead of electrons, the holes behaving like a positive charge. Therefore we advise against quoting this evidence. Although a Hall-effect demonstration with semiconductors is fairly easy, it would be most unsuitable here where we are discussing the motion of electrons in metals.

Electrostatics and Mobile Electrons. We can describe electrostatic behaviour in terms of the same electrons that are free to move in metal. This time, with charges at rest, the charges reside on the outer surface of metal objects (unless we have parked one on an insulating stand inside some hollow metal thing). This description in terms of electrons moving, with great care not to have the positive charges move, makes electrostatics a little more complicated, though much more real to many a pupil, and much more exciting for those pupils who picture electrons running about like beetles all over the surface. We may give such descriptions but we should raise a warning flag very clearly. (See Note on 'Teaching Electrostatics with Electrons' in the General Introduction at the beginning of Year III.)

Positive Charges in Atoms. We know that atoms are electrically neutral. They experience no set force in strong uniform electric fields. (Of course *ions* do experience forces, but they have a whole electron charge.) Therefore we believe that whatever positive charges there are in an atom must exactly balance the negative charges of the electrons there.‡ When, in later studies, we decide a hydrogen atom has one electron, a helium atom 2, ... an oxygen atom 8, ... and so on, we decide that the rest of the atom, the positive nucleus, must have exactly 1, 2, ... 8, ... times an electron's charge, but of opposite sign.

Electrons at Work: Cathode Ray Oscilloscope

If there is time at the end of this Year, pupils should have cathode ray oscilloscopes to work with on their own for class experiments. The reward for skill in manipulating the controls and making the trace do what one wishes should be a small microphone, such as an old telephone mouthpiece, to be connected to the oscilloscope so it will show voice waveforms. Musicians should be encouraged to bring their own musical instrument. Pupils learning French might practise the light 'u' as in 'tu' and derive some phonetic advantage from watching the results of their efforts. C171

Diode as Rectifier with C.R.O. Pupils should return to the demonstration of a diode as a rectifier and see its action on the oscilloscope now, if not before. D172 OPT.

Pupils who are interested may be encouraged to construct a 'full wave rectifier' and try that on the oscilloscope.

‡ The exact quality is vouched for by the failure of electric fields to exert *any* NET force on small uncharged objects, which consist of a vast number of atoms. A 'Millikan-Experiment' test on a collection of oil drops of many different sizes shows some that are charged. We know they are charged because an electric field affects their fall; and our measurements show the charges are all multiples of e. But some drops are uncharged, and even a very strong electric field has no effect on their fall, showing that they have not even a small fraction of e as net charge. Since we find that for many sizes of uncharged drop, the neutrality cannot be due to a chance adding-up of fractions to an integer, so the equality of + and − charges in those drops, and thence in atoms, is vouched for.

The argument here is easier to understand if we apply it to comparing thicknesses of two types of coins, or, say, dominoes. We make a pile of 50 of type A. If we can match that exactly in height with a pile of type B, we think the B items *may* have exactly the thickness of A items. But the B pile *might* contain 49 items each 1·02 times as thick as an A item – and there are other possibilities. But if we *always* secure exact matching of A and B piles for many different total heights we are forced towards the conclusion of equality.

As an alternative, insulate a closed metal box. With the box initially uncharged, generate charges inside (e.g., by friction) and look for effects outside. In simple form, this traditional ice-pail test is much less sensitive.

Electron Gun

When pupils first used a diode and found negative electricity streaming across from hot cathode to plate, they had, without knowing it, discovered an electron gun. If a hole is drilled in the plate a stream of electrons will go out through the hole; and we have a 'gun' that can fire a stream of electrons into other apparatus.

Electrons boil off a hot filament, in a vacuum, emerge with a little kinetic energy‡ and collect nearby in a cloud which repels the continuing supply of evaporating electrons back to the filament. To fire a stream of high-speed electrons out from our gun, we apply an electric field to accelerate them in the space between filament and plate – which we may now call the gun muzzle. If the electric field is strong enough it sweeps electrons across to the plate as fast as they evaporate and there is no discouraging cloud round the filament. With a weaker field, there is a cloud, from which we can pull a fast stream (of uniform energy) by interposing a grid with a strong accelerating field beyond.

If the battery maintains a p.d. V volts between filament and plate, an electron with charge e coulombs which just escapes from the filament and is dragged across to the plate gains energy (e coulombs)$\times$(V joules/coulomb) from the field. It arrives at the plate with energy Ve joules.

If such an electron reaches the plate where we have drilled the hole that serves as gun muzzle, it flies on through the hole into a region where it finds no accelerating field. (We may connect the plate to the far end of the tube or any other apparatus at which our gun is firing.) So the electrons in the stream emerging from the gun continue with constant velocity thereafter – Newton's First Law – unless we apply further fields to the stream.

‡ Experiments to plot the 'characteristic' of a diode often seem to show that electrons boil off the filament with energies of a few electron.volts, because a negative p.d. of a few volts must be applied between plate and filament to reduce the plate current to zero. That is a mistaken inference: the electron current when no voltage is applied is due to a driving voltage from one end of the filament. Where the cathode is a directly heated filament, the battery that maintains the heating current keeps its ends at different potentials from the mid-point where the 'cathode' connection is attached; so the region outside one end of the filament has a small concealed accelerating potential that accelerates electrons.
The kinetic energy with which evaporating electrons really emerge is only a small fraction of an electron.volt. To see that, one must use an indirectly heated cathode and make very careful measurements.

Energy of Electrons from Gun. In what form do those flying electrons carry the energy that the accelerating field has given them? As seen by us, stationary observers, all the energy they have gained is just kinetic energy $\frac{1}{2}mv^2$. As the electrons travel from filament to plate in that electric field, they gain more and more K.E. The energy with which they leave the filament is trivial, so we say that their final K.E. when they emerge from the gun muzzle is the energy given them by the field. For one electron, that is Ve joules. Therefore if each electron, of mass m kilograms, emerges from the gun with speed v metres/sec (constant thereafter):

$$\frac{1}{2}mv^2 = Ve$$

We picture the electrons in the stream from the gun flying on with constant velocity, given by the equation above, until they hit a target or are pulled by some applied field. When they hit a target and are brought to rest, their kinetic energy is nearly always converted into heat in the target – only occasionally is the energy radiated as a photon of light or X-rays.

Pupils cannot at present calculate the speed v, because they do not know m. That is obtained from measurements of e/m combined with Millikan's value for e. We shall measure e/m in Year V, with the help of a magnetic field to deflect a beam of electrons into a circular orbit that can be measured. Before that, we should have to announce the value of m (about 9×10^{-31} kilogram) if we want pupils to calculate v – and that would seem an unfortunate anticipation.

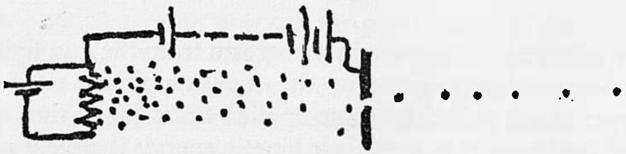
If we do insert that value and the value for e, we obtain amazing results: v for electrons from a 100-volt gun is about 6 million metres/second, about $13\frac{1}{2}$ million miles per hour.

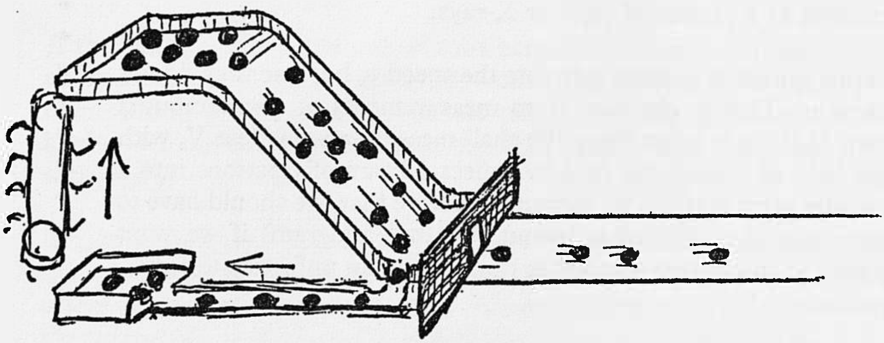
Although we do not advise teachers to announce the value of m and ask pupils to calculate v, we suggest it would be good preparation for Year V to lead up to the equation $\frac{1}{2}mv^2 = Ve$ and point out the need for one more measurement before we can find the speed of electrons from a gun of known voltage. (Direct measurements of v are possible, by a time-of-flight method like our estimate of a bullet's speed with the scaler; but those are difficult.)

We might start by showing an analogue model to illustrate the idea of electrons 'falling through' a potential difference.

Model. We could make an analogous toy gun for marbles by using a sloping plank to represent the region of electric field from filament to plate. We provide a reservoir of marbles at the top of the plank and let them run down to a wall at the bottom – where they stop and collect in a pool from which some sort of escalator might act as a battery and return them to the top. D173

We make a hole in the wall, so that some of the marbles arriving at the bottom of the slope meet the hole and go straight on through, out along a level table in a stream at constant speed.





With that picture in mind, we speak of electrons as '*falling through*' *the p.d.* between filament and gun muzzle just as the marbles in the model *fall through the height h* and gain K.E. – given by $\frac{1}{2}mv^2 = mgh$.

Speeding Up Electrons. Then we might give a suggestive description: T

'Picture electrons released from a hot filament, oozing out, finding themselves in an electric field which accelerates them towards the muzzle of the gun, moving faster and faster and gaining kinetic energy. When they come out of the gun muzzle, they are no longer in an electric field, they are not accelerating; and they have a definite kinetic energy which they carry with them until they hit a target – which may be a solid glass wall of the tube or possibly a molecule of gas left over in the tube.

'That kinetic energy is given by $\frac{1}{2}mv^2$ for an electron as for anything else.‡ It has been given to the electron by the electric field dragging it.

'Suppose we apply 100 volts to the gun, by connecting a 100-volt battery to filament and gun muzzle. There is a transfer of energy, of 100 joules for every coulomb, from chemical energy in the battery to electrical energy in the electric field in the gun, and thence to kinetic energy of the electron. The electron moves faster and faster until it emerges from the gun with amazing speed. Then it has kinetic energy given by

(electron charge) $\times$ (gun voltage, 100 joules/coulomb)

or 100 e joules, where e coulombs is the electron's charge,

or $100 \times 1{\cdot}6 \times 10^{-19}$ joules.

'If we knew m, we could calculate the speed, v. Next year we shall measure e/m for electrons and that will enable us to calculate v.

'Here we have expressed the energy of each electron in joules. It seems a very tiny number; but then electrons are very tiny particles. That much kinetic energy is an appreciable load for an electron to carry – though we can pile on a million times as much, and even more, with modern electron accelerators.

‡ K.E. $= \frac{1}{2}mv^2$ is not quite true for the very high speeds that we can easily give electrons. As the speed approaches the speed of light, we have to use a relativistic expression for the kinetic energy. But we need not use that for electrons from guns with applied voltages of a few hundred volts. For electrons from a gun with 50,000 volts, m has increased 10% and with 500,000 volts m has doubled and this increased mass goes into the relativistic expression $(m - m_0)c^2$ for K.E.

'To make it easier to picture the amount of K.E. we have given an electron, we express it in different units, "electron.volts". Here the electrons emerge from the gun muzzle with energy 100 electron.volts or 100 e.v.'

Energies in Electron.volts. In all atomic physics, we need the electron.volt again and again as a very useful unit of energy. Before this Year ends, we should give pupils that unit so that they are ready to meet it in Year V. We say:

'An electron that has "fallen through" a potential difference V volts has acquired energy Ve joules; but we sometimes call that amount of energy V electron.volts – in other words, we lump the electron charge, e, in with the unit.

'One electron.volt is the energy gained by 1 electron charge falling through a potential difference 1 volt. Physicists know from the full version of Millikan's experiment that 1 electron charge is $1{\cdot}60 \times 10^{-19}$ coulomb. Therefore, 1 electron.volt is $[1{\cdot}60 \times 10^{-19}$ coulomb$] \times [1$ joule/coulomb$]$, or $1{\cdot}60 \times 10^{-19}$ joule.'

Examples of Energies in Electron.volts. An excited atom that radiates 1 quantum of yellow light loses about 2 electron.volts. An electron that stops with a great jolt in an X-ray tube driven by a 30,000 volt power-pack, may lose as much as 30,000 electron.volts (but no more) and give 1 quantum of X-rays with that energy.

The average energy of an air molecule is between $\frac{1}{20}$ of an electron.volt and $\frac{1}{40}$ of an electron.volt. (The uncertainty in that statement arises from the variety of ways in which one can do the averaging to specify the value.) Chemical reactions involve energy changes, per atom, of a few tenths of an electron.volt up to a dozen electron.volts.

Year V Ahead

We hope that during this year pupils will have enjoyed investigating force, mass and motion; will have learnt to include kinetic energy and heat in universal conservation of energy; and will have built up pictures of molecules, atoms and electrons – all this knowledge to be used and extended in the coming year.

SUBJECT INDEX

Schools collaborating in the trials

The Abbey School, Ramsey
Ashfield County Secondary School

Banbury Grammar School
Baptist Mills School, Bristol
Barnard Castle School
Bartley Green Girls' Grammar School
Batley Grammar School for Boys
Batley High School for Boys
St Brendan's College, Bristol
Bridgnorth Secondary Modern School for Boys
Bromsgrove County High School

Calder High School, Mytholmroyd
Chatham House Grammar School, Ramsgate

Dame Allan's School, Newcastle
The Downs School, Colwall

Elliott School, Putney
Erith County Grammar School

Forest Hill School

The Grammar School for Boys, Cambridge
Godolphin and Latymer School

Harborne Hill School, Birmingham
Hinckley Grammar School
Huddersfield New College
Huntingdon Grammar School

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Maidstone Grammar School
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Whitley Bay County Grammar School
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NUFFIELD FOUNDATION
SCIENCE TEACHING PROJECT
PHYSICS SECTION

The physics programme was inaugurated in May 1962 under the leadership of Donald McGill. It suffered a severe setback with his tragic death on 22 March 1963, but those who were appointed to continue the work have done so in the spirit in which he initiated it, and in the direction he foreshadowed.

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Other Nuffield Physics publications

Teachers' guide I
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Teachers' guide III
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Guide to experiments I
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