Physics

Students' book Unit 8
Electromagnetic waves

Nuffield Advanced Science

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Physics Students' book **Unit 8**

Electromagnetic waves

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Nuffield Advanced Science
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Contents

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator of the Nuffield Foundation Science Teaching Project

To the student

This book contains some of the things you need to help you to understand the work of this Unit, and some reading which we hope will help you to see how the work is relevant to other, wider issues. It does not contain all you need: you will have to consult textbooks and other more general books as well, working through theoretical arguments, reading about experiments, and finding out more about how the ideas can be put to practical use.

This book contains many questions; more than you will be able to do while working on this Unit. Later on, you may wish to use some of them for revision. You will find questions which take you step by step through the theoretical arguments in the course, and students in trials have said that these questions are a good way to understand a piece of theory. You will have to pick and choose, according to your needs and tastes, amongst the other questions. A few give you simple practice in calculation. More invite you to argue about or discuss a problem, and some of these are not suited to formal written answers. They are meant to start off a discussion, which may then wander far from the question.

There are a few harder questions to challenge the clever, and you should not expect to be able to tackle every question easily. But most are meant for ordinary human beings, not for budding geniuses. If in doubt, try the obvious answer: usually there is no catch! Most questions have some kind of answer in the section headed 'Answers', though some of these suggest where you might find the needed information, instead of giving it. We have tried hard not to give wrong answers, but, being fallible like yourselves, may not have succeeded.

Some questions ask you to guess, speculate, or give your private opinion: obviously they have no one right answer.

What you are being asked to learn to do

This course aims to help you to become more like a physicist. Most of you will not become physicists, but will use physics or learn more of it in one of a variety of scientific jobs or in further education. *Physics, and the world with it, are changing* so fast that no one can tell what bits of physics you will use in, say, ten years' time; however, one can be pretty sure that there are some basic ideas that will be relevant to the new *problems of tomorrow.* We have tried to build the course around what we believe to be these basic ideas.

So one thing the course aims at is helping you to become able to learn, in the future, the new ideas in physics you may meet, and helping you to become able to use the physics you have learned. It does that because these are the tasks that will face you.

In the future, you will need to be able to learn from books and articles; that is why the course contains a good deal of reading. To use the physics you have met, you need to understand it – that is, to be able to use it in new kinds of problems. That is why so many questions in this book ask you to make up arguments about new problems, using what you know.

What is 'understanding'? That is, how does one recognize that someone understands a piece of physics? We think it is something like this. Suppose a group of people are talking about a problem in physics. Very rarely, even among research workers, will anyone *immediately see an answer. More often,* they each have some ideas which they try out in discussion with colleagues. Those who 'understand' their physics are the ones who can offer sensible, relevant ideas that would help *towards clearing up the problem. A reasonably competent* physicist expects himself and others to be able to draw on their knowledge and use it to make sensible contributions to the discussion of problems.

So to test whether you understand a piece of physics, it is asking too much to expect you to solve a new problem completely and correctly; few – if any – experts can do that. The test should be that of physicists talking together: can you produce sensible ideas that are relevant and would help a bit towards clearing up a problem? This is the test that will be used in the examination, and is the way to decide how well you have managed a question or problem in the work of the course.

The course also aims to show you what doing physics is like, and this is another reason for encouraging plenty of discussion of problems, for that is the way physicists work. It tries to show what kinds of questions physicists ask themselves and what sorts of ways they use to tackle them. We think this is important because to use physics successfully and to judge its claims and achievements you need to understand what it can, and what it cannot do. That is why several questions ask you about such things as how theories, models, experiments, and facts fit together. Physicists also guess, estimate, and speculate, so other questions ask you to do these things too, to find out what doing them is like and to become better at doing them.

There are a lot of misunderstandings about what physics is like. Some say it is all facts; others that it is all theory, having little to do with what happens in practice. Many are puzzled; asking whether what physics says is true or not, or how physicists arrive at their ideas. We hope you will find chances in this course to think about such matters, and that you will form your own views.

Some of the questions ask about how physics can be used in engineering and technology, because we think that you will rightly want to know when what you learn is of practical value.

Finally, one of the main reasons we want to offer you some physics is that we like the subject and get excited about it. So we hope you enjoy it too.

Summary of Unit 8 Electromagnetic waves

Part One
Looking through holes

Difficulties in seeing distinctly

Consequences of the wave nature of light for waves going through apertures. Diffraction effects at a hole, contrasted with image defects not due to diffraction.

Adding waves

Explanation of the diffraction effects at a slit, as an *example of the general method of calculation with waves:* adding amplitudes, taking account of phase differences, to find the resultant amplitude.

Practical problems involving diffraction

Direction finding with optical and with radio telescopes. Investigating the structure of small, distant sources of radiation. Radio astronomy.

Part Two
Spectra

Spectra as a source of information

Measuring spectra (spectroscopy) at many different wavelengths, as a way of obtaining a wide range of kinds of information about materials, from the radiation they absorb or emit.

The grating as a tool

The grating equation $n\lambda = d \sin \theta$. Why a grating produces sharp spectra, giving accurate wavelength measurements.

Part Three

Electric waves

Radio waves and electrical signals

Radio waves, and the possibility that they are electrical in nature. Electrical signals on cables, and their speed of travel.

Explaining the speed of electrical signals

The speed $1/\sqrt{LC}$ of signals on a chain of inductors and capacitors. Comparison with one special case of a pair of long conductors, along which signals travel at the speed of light. A description of the wave that travels in the space between the conductors: similarities with a radio or light wave in empty space. The speed $1/\sqrt{\varepsilon_0\mu_0}$ of electromagnetic waves. Electromagnetic waves as travelling fields.

A consequence of the nature of the waves

Polarization of light and of radio waves, understood as a consequence of the transverse nature of the waves.

Part Four

Relativity

A strange fact about light

The fact that, although the speed of electromagnetic waves is a speed relative to an observer, the speed does not alter even though the observer moves.

Strange consequences of the constant speed of light

Using radar for distance, velocity and time measurement, and a consequence: 'moving clocks run slow'. An example where the effect is not small. The Doppler shift. Effects on forces between things: how magnetism and electricity might be related.

Questions

Part One

Looking through holes

Questions 1 to 7 are general questions about waves and light, and about seeing things distinctly.

1 Figure 1 shows a printed photograph, together with a small part of the photograph greatly enlarged, showing what you would see if you looked at the printed photograph under a microscope. (Try looking at photographs in magazines or books under a microscope – colour photographs are particularly interesting.)

Why does it not matter that the photograph is made up of a multitude of dots which, when looked at in detail, are unintelligible? Can you think of any artists who have used a 'dotting' technique in their painting?

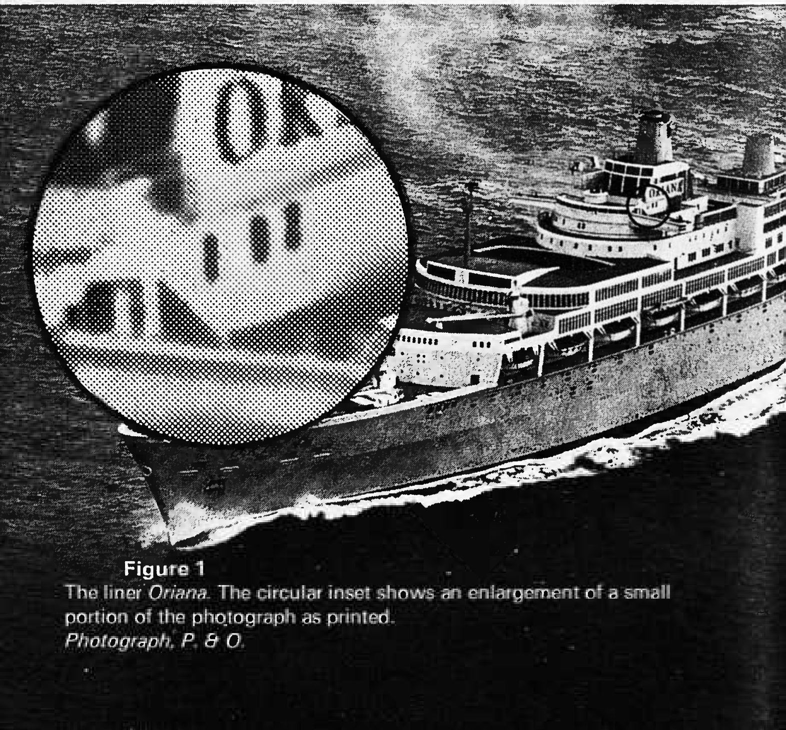

Figure 1
The liner *Oriana*. The circular inset shows an enlargement of a small portion of the photograph as printed.
Photograph, P. & O.

2 Figure 2 shows circular ripples on water, showing where drops of water have fallen.

In what sense do the ripples *not* affect one another?
In what sense *do* the ripples affect one another?
Explain what a physicist means when he says that waves 'superpose' on one another. You may be able to think of an example where water waves do *not* superpose, at least not in the simple sense intended by a physicist when he uses the term.

Figure 2
Ripples on water.
Photograph, Barnaby's Picture Library.

3 Figure 3 is an aerial photograph of a narrow harbour mouth formed by two angled jetties, facing the open sea. Why are there waves inside the harbour, well within the 'shadow' of the jetty which runs parallel to the waves in the open sea? Make a sketch showing what you expect to see in a ripple tank if plane-parallel waves reach a barrier which runs parallel to the waves, and blocks off half the tank.

Figure 3
Morro Bay, California.
Photograph, University of California.

4 Hold your first and second fingers together so that there is a narrow crack between the middle parts of the two fingers, although they touch at the top. Look at a distant small lamp through the crack, and press the two fingers gently together. As the crack closes, the lamp gets dim (obviously), but when the crack is very narrow, the lamp also seems to get wider. Why?

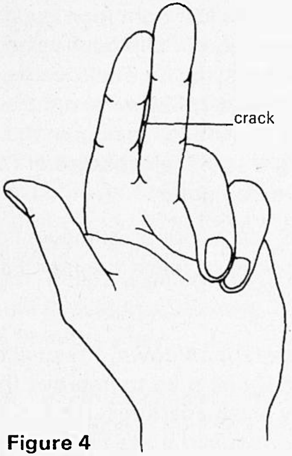

Figure 4

5 Watch a car driving away from you, and note where it is when the individual letters or numbers on its registration plate can no longer be seen distinctly although they are still visible as separate 'splodges'. Estimate, or pace out, how far the car was from you at that moment. Use this estimate to make a further estimate of the *angle* at your eye between two things which it just cannot distinguish from one another. If you can, use the angle to estimate a value for the wavelength of light, assuming either that the effect is the result of diffraction at the pupil of your eye, or that human eyes have evolved so as to distinguish objects only a little worse than the wave nature of light will allow.

6 A camera is marked with *f*-numbers, which are a way of indicating the diameter of the open aperture of the lens, as a fraction of its focal length. Thus an aperture marked *f*/8 is twice as wide as one marked *f*/16, has four times the open area, and lets in four times as much light in the same time. *f*/8 means that the lens diameter is $\frac{1}{8}$ of its focal length. Suppose a photographer has worked out, or found by experimenting, that an exposure at *f*/16, for 1/30 s is just right for a good picture of a sunny garden, with the sort of film he is using. Then he takes a picture of two children playing on a seat in the garden, and uses an aperture of *f*/8 at 1/120 s. Next he is asked to photograph the children having a race down the garden. A friend suggests using a 1/500 s exposure at *f*/4, but he says that that might not be much good.

Suggest why he changed to *f*/8 and a shorter exposure time for the second picture. Was he right to reject the friend's suggestion?

7 As the lens of a camera is stopped down, the quality of the picture improves. However, if one tried to improve the quality still more, by using very small apertures, the quality would start to deteriorate again. Explain these two effects.

Questions 8 to 13 are revision questions about interference, mainly simple two-source effects such as Young's fringes.

8 Light from a coloured filter is used to produce Young's double-slit fringes. The slit separation is 0.4 mm. The distance between the slits and the screen on which the fringes are formed is 1.4 m, and the distance between successive dark spaces (or bright fringes) is 1.7 mm.

a Find the average wavelength of the light used.

b Why 'average'?

9 Light of wavelength 5×10^{-7} m from a very small source falls on a pair of slits 0.1 mm apart, and forms fringes 2.5 mm apart on a photographic film, 0.5 m from the slits (figure 5)

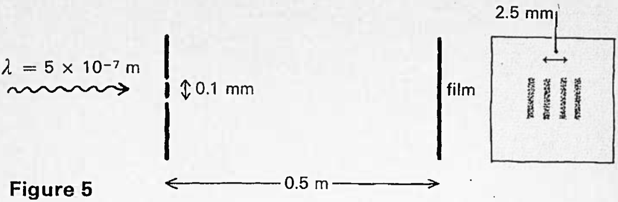

Figure 5

a The wavelength of the light is halved. What will the fringe spacing become? Would the fringes be visible if you looked at a screen placed where the plate was held?
b The slit spacing is halved, still using the shorter wavelength. What is the fringe spacing now?
c An experiment has been done to test whether electrons have wave properties, using this arrangement. The expected wavelength was about 10^{-10} m. The fringes could be detected if they were at least 10^{-2} mm apart. Approximately what slit spacing would have to be used if the film were still 0.5 m from the slits?

10 **a** Two loudspeakers are stood on stools out-of-doors, on a rough, grassy, non-reflecting surface. They are mounted about a metre apart, and each produces the same musical note. How will the sound pattern heard from the two speakers differ from the note heard from one?
b Using one loudspeaker in a room, you can get effects similar to those obtained with two speakers outside. How do you explain this?

11 An inventor claims to have found the perfect device for hecklers at political meetings. He says it has a microphone, a phase reversal circuit, and a loudspeaker. The sound is taken in and broadcast out again, but every compression is turned into a rarefaction, and vice versa. Then in the air of the room the loudspeaker gives out sound which cancels out the sound of the speaker, and so nothing is heard. Do you think this device will work?

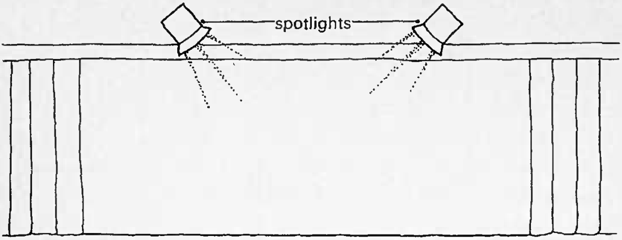

Figure 6

12 A student who was interested in stage lighting effects proposed to mount two spotlights so that the area where both beams met would be dark because of the interference between them. It wouldn't work. Why not?

13 *For discussion*
If it were not for diffraction, Young's double-slit interference fringes could not be produced. Why not?

Questions 14 to 24 are about diffraction at a narrow aperture, and the effect this has on the ability to resolve sources; that is, to see distinct sources as distinct.

14 The world's largest optical telescope has a 5 m diameter mirror, but the mirror of a large radio telescope can be as much as 80 m across. Why is the mirror of the radio telescope so much bigger?

15 Why do you think astronomers sometimes photograph stars through a blue filter?

16 This question is about an effect which has nothing to do with diffraction, but looks as if it might, when you see it.

Most simple lenses focus blue light more strongly than they focus red light, as shown in figure 7. If the lens is forming an image of a narrow, straight filament emitting white light, whereabouts would a screen be if the image were a bluish-white line with fuzzy red edges? Where would it be if the image were a reddish-white line with fuzzy blue edges?

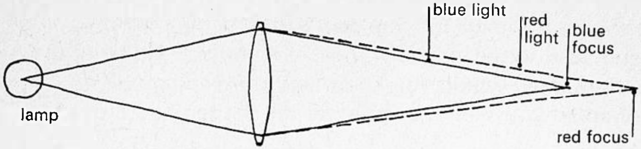

Figure 7

17 Light with a wavelength of 500 nm (5×10^{-7} m) from a very narrow slit-like source is focused by a high quality lens onto a screen 10 m from the lens. The source is 0.1 m from the lens, and is on the axis of the lens. See figure 8.
A long slit only 1 mm wide is put over the lens, with the centre of the slit also on the axis of the lens.
A pattern of light now appears on the screen, some parts of the pattern being a good deal dimmer than others.

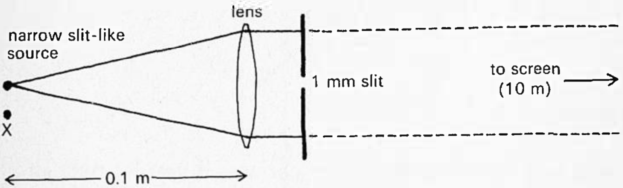

Figure 8

a Sketch the pattern on the screen. You needn't do any calculations at this stage.
b Explain why the part at the centre is bright.
c What is the path difference between the edges of the 1 mm slit and a point on the screen 1 mm from the centre of the pattern?
d How many wavelengths is this?
e Do you think this part of the screen will be *1* almost as bright as, *2* about half as bright as, or *3* much less bright than the centre?
f How far out from the pattern centre must a point on the screen be for the path difference to be 1 wavelength?
g Waves from the edges of the 1 mm slit then reinforce one another, yet this point is dark. Why?

h Where will the next dark point be?
i The source is moved 0.1 mm down to X, being moved perpendicular to the length of the slit. If you were looking at the pattern, how would you know this had happened?
j How could you work out how far the source had moved?

18 This question is about the angle θ, given by $d \sin \theta = \lambda$, over which parallel light is spread if it goes through a slit of width d.

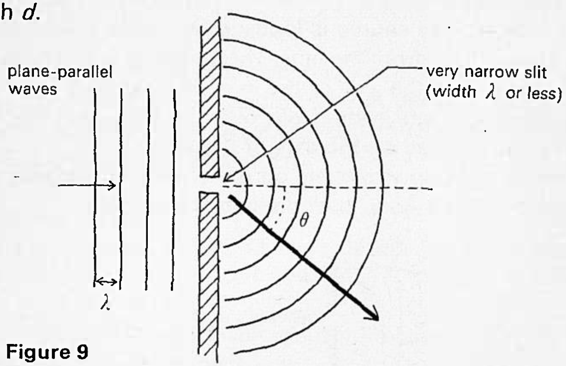

Figure 9

a Figure 9 is a sketch of what you can see if you let parallel waves in a ripple tank fall on a slit which is a wavelength or less wide. Is any wave energy travelling in the direction shown by the arrow, at angle θ to the direction of the waves before they reach the slit? If the slit were removed, would the answer be the same?
b Why is it very hard to do this experiment with light?
c Figure 10 is a sketch of parallel light falling on a slit AB, which is more than a wavelength across. If you try such a slit in a ripple tank you will find that the wave is only spread out a little, as suggested in figure 11. The amount of this small spreading can be calculated if the slit is imagined as being composed of a row of sources, all very close together, each emitting radiation as shown in figure 9. If the slit is ten wavelengths wide, at least how many such sources must be imagined? (You could test the idea by comparing the wave pattern from a row of closely spaced single dippers with that coming through a gap as wide as the row is long, between

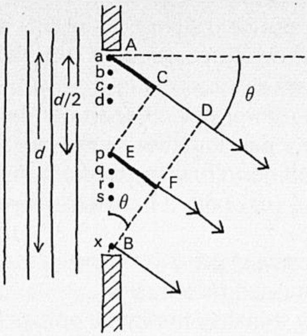

Figure 10

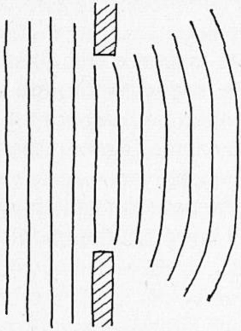

Figure 11

two barriers in a *ripple tank*.)

d Imagine the wide slit, figure 10, made up of a row of point-like sources a, b, c, etc. Each may send some wave motion in a direction making an angle θ with the original direction of the plane-parallel waves falling on the slit. What difference in path (AC) along such a direction, is there between waves from a and from p, if p is in the middle of the slit?

e If plane-parallel waves fall on the slit, all the imaginary sources representing the slit will emit in phase. Will radiation from any pair of them, reaching a screen a long way away, coming at angle θ from the slit, always be in phase?

f If AC (figure 10) is half a wavelength, what combined effect will waves from a and p have at angle θ, on a distant screen?

g What will be the combined effect of waves from b and q, the imaginary sources next door to a and to p respectively?
h Complete the following passage: 'If the slit AB is divided into many imaginary sources, there is an angle θ at which sources half the width of the slit apart can be paired off, such that each and every pair together send waves to a distant screen which combine to have zero effect. For this to be so, the path difference $(d/2)\sin\theta$ between waves from each pair must be . . .'
i In terms of the wavelength λ how big is the distance AD, at the angle θ discussed previously?
j Give a reason for thinking that at *all* angles less than θ, there will be *some* light on the screen. At what angle will there be most light?

19 A telescope is set to look at a star. The image at the focus of its main lens, which is inspected through an eyepiece or is photographed, is not a sharp point of light, but is blurred out into a disc surrounded by some fainter rings. Figure 12 *a* suggests the way the intensity varies across the pattern. A line from point X on the pattern to the middle of the lens makes an angle θ with a line from the middle of the pattern, at Y, to the lens.

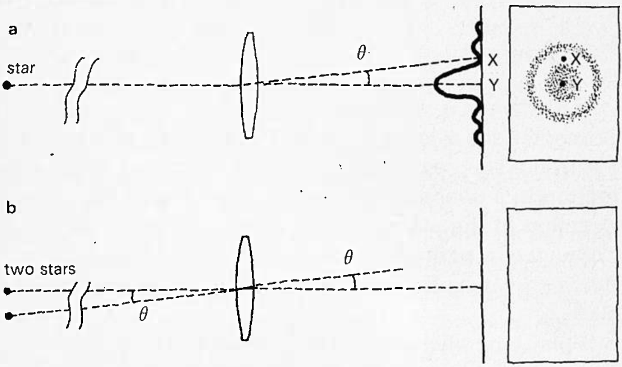

Figure 12

a If the pattern shown were obtained with red light, what difference would it make, if any, if a blue filter were used instead of a red one?
b The same telescope is pointed at a pair of stars which happen to subtend the angle θ mentioned above, at the lens, as indicated in figure 12 *b*.
Make a rough sketch of what the pattern of light at the focus of the telescope would now look like, supposing that the stars are equally bright.
c The telescope is pointed at a planet, whose diameter subtends this same angle θ at the telescope. Would the pattern shown in figure 12 *a* be altered much?

20 Figure 13 shows a dish-shaped reflector with an aerial at its focus (it is the radio telescope at Dwingeloo, in the Netherlands). It was used to detect radiation at a wavelength of 0.21 m which is the wavelength on which radiation is emitted by hydrogen atoms in space, and in this way to plot the concentrations of hydrogen gas in our galaxy. The results confirmed that the galaxy is a spiral-shaped structure, the Sun being in one spiral arm some two-thirds of the way from the centre of the whole flattish spiral.
a The diameter of the dish is 25 m. Supposing that a circular dish is equivalent to a slit of the same width (actually it is equivalent to a slit whose width is equal to the diameter divided by 1.22), over what angle away from the direction of one place in the sky which emits radio waves at wavelength 0.21 m will the telescope detect an appreciable intensity?
b A part of our galaxy being studied might be roughly 20 000 light-years from the telescope (1 light-year is about 10^{16} m). At least how far apart must two parts of the galaxy be for the telescope to be able to distinguish them as separate sources?
c What arguments might be given in favour of, and against, building a similar telescope with a diameter ten or a hundred times bigger? You will see from figure 13 that this telescope can be steered so as to point at different places in the sky.

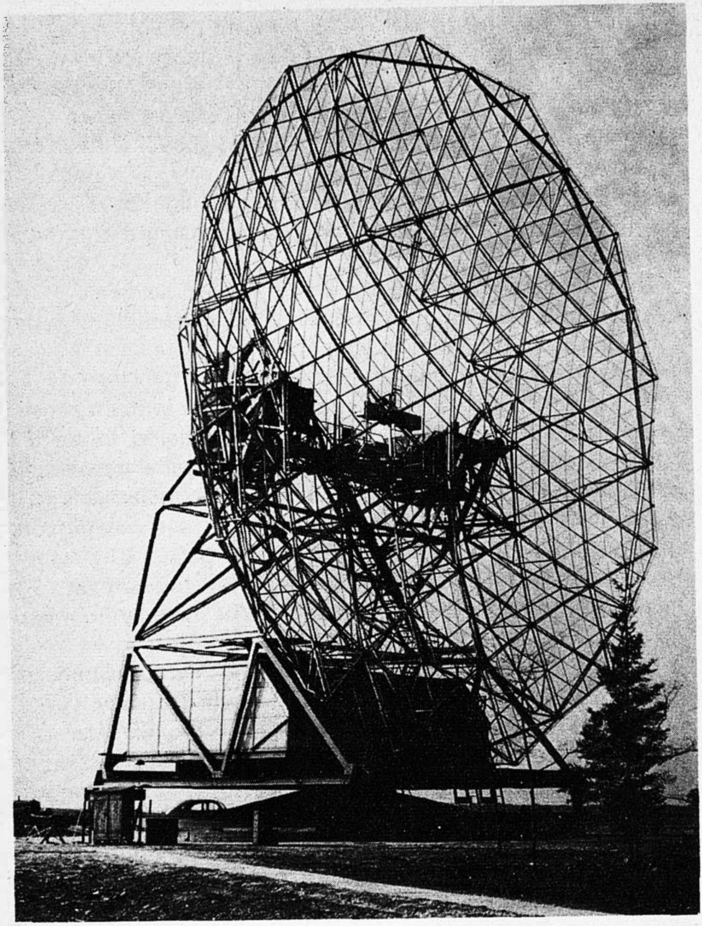

Figure 13
The radio telescope at Dwingeloo in the Netherlands.
Photograph by courtesy of the Royal Netherlands Embassy.

21 The planet Venus, when it appears as the Morning Star, is about 150 million kilometres from the Earth. It looks perceptibly different from a true star, perhaps because the unaided eye can see it as a disc rather than as a point of light. The diameter of Venus is about 12 000 km. Using rough estimates of the wavelength of light and of the diameter of the pupil of your eye, do you think that the eye can detect that Venus is not a star? Is this plausible, or does the difference in appearance between Venus and a typical star require some other explanation?

22 In the aerial photograph, figure 3, details of objects more than about 3 m across are just visible. If the photograph was taken from a height of 3000 m, what can you say about the diameter of the camera lens?

23 The ability of a radio telescope to resolve pairs of sources is limited by its aperture, because of the relatively long wavelength of the radio waves being detected by the telescope, but it is impracticable to make a telescope much bigger than about 100 m across. Even one this size is much less able to distinguish between sources than the human eye.
One way around this problem is to use pairs of small telescopes a long way apart, when, like pairs of slits in a Young's fringes experiment, they produce an interference pattern whose spacing is finer than the fringe spacing of the diffraction pattern from either small telescope on its own. Such a device is called a stellar interferometer.

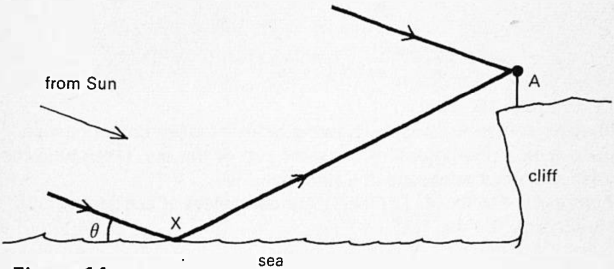

Figure 14

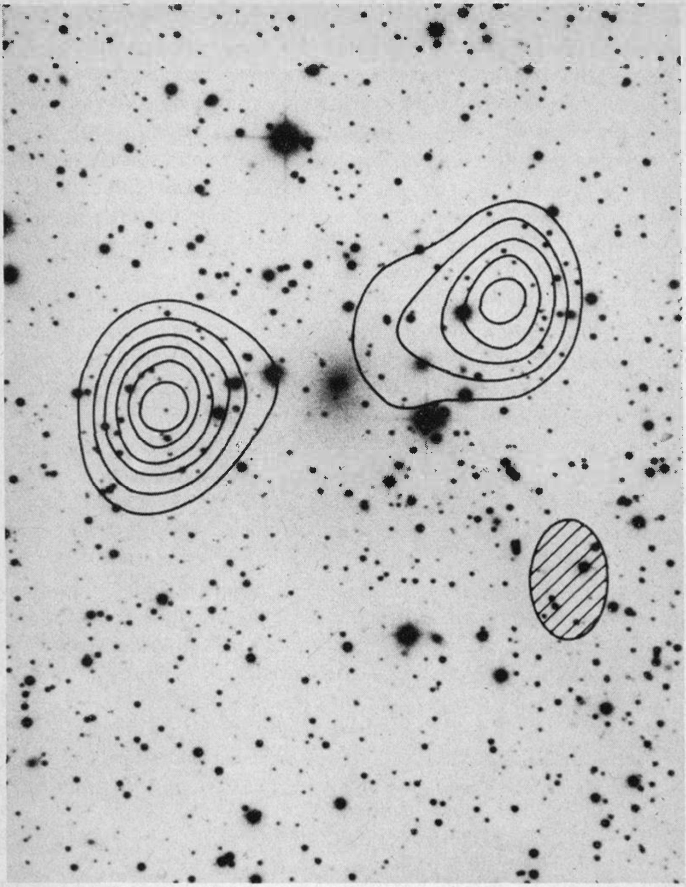

Figure 15
The radio source Cygnus A, showing radio intensity contours superimposed on a photograph of the same part of the sky, taken with the 5 metre Palomar telescope in California.
Photograph, Moffet, A. T. (1966) Annual review of astronomy and astrophysics, ***4***, *149*.

In Australia a stellar interferometer has been constructed by placing an aerial array on top of a 100 m high cliff near the entrance to Sydney Harbour. The system was designed for the study of 1.5 m wavelength radiation emitted from the Sun.
a If figure 14 shows the Sun's rays when the intensity at A is at a maximum, mark a point Y on a copy of the figure, such that AX–AY must be an odd number of half wavelengths. (The phase is reversed on reflection by the sea.)
b As the Sun sets, the angle θ changes. Explain how and why the intensity at A varies as the Sun sets.

24 Figure 15 illustrates the value of good resolving power in a radio telescope. The radio source Cygnus A seems to be located near the blurred object at the centre of the photograph, which may be an exploding galaxy. It is believed to be 500 million light-years from the Earth.

A sufficiently good radio telescope can show that the radio source corresponding to this visual object is actually a pair of sources. The figure shows contours of radio intensity, obtained with a radio telescope at Cambridge University. The two sources are some 300 000 light-years apart.
a What angle do the two sources subtend at the Earth?
b The ellipse in the lower righthand corner of the picture shows the size of the area within which pairs of radio sources cannot be resolved by the telescope at the distance of Cygnus A. Estimate the mean diameter of this region in light-years, and the angle subtended by this diameter at the Earth.
c Estimate the diameter of the dish aerial (such as that in figure 13) which would be needed to resolve sources as well as the telescope used to obtain the result in figure 15, at a wavelength of 0.15 m. Explain why a dish aerial was not used.

Spectra

Questions 25 to 31 are about diffraction gratings, and the spectra they produce. Questions 25 and 31 are the ones which deal with essential pieces of theory.

25 Figure 16 illustrates some effects of a diffraction grating on plane-parallel light falling on the grating, at right angles to it. The grating can be imagined to be made of a number of equally spaced transparent slits cut in an otherwise opaque material, though a real grating is not usually so simple.

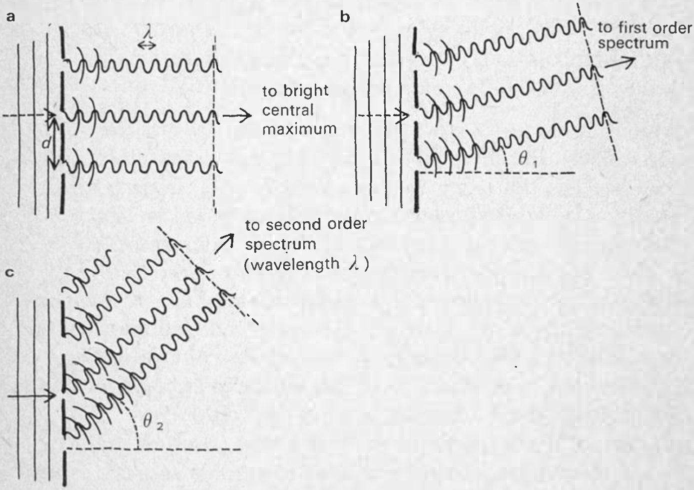

Figure 16

a Use figure 16 *a* to explain why light going in the same direction as the light falling on the grating, combines to give a bright central band on a distant screen. If the incoming light is white, why is this band white?
b Use figure 16 *b* to explain why light going at an angle θ_1 given by $\lambda = d \sin \theta_1$ also combines to give a bright patch on a screen, whose colour is that corresponding to the wavelength λ.

c Use figure 16 *c* to explain why there is another bright patch at another angle θ_2, given by $2\lambda = d \sin \theta_2$. Explain why radiation having wavelengths of either λ or 2λ can give a bright patch at this angle.
d Write a general equation for the angles θ at which radiation of wavelength λ will give a maximum intensity, coming from a grating with spacing d (in the case of parallel light falling at right angles onto the grating).

26 A grating composed of many narrow slits ruled on an otherwise opaque sheet gives, with a monochromatic source of light, a set of bright lines. Then every other slit is blocked out. What happens to the positions and intensities of the bright lines given by the grating?

What do you think might happen if half the slits were blocked out but selected at random rather than regularly? (Guess – don't spend too long on it.)

27 A long-playing gramophone record (or a 45 r.p.m. extended play record) can be used as a diffraction grating for light, even though the grooves are very many wavelengths apart. The trick is to reflect light from the record at a shallow angle, so that the light grazes the surface, across the groove direction.
a Look at a record and estimate how many wavelengths of visible light would fit into one groove spacing.
b Explain why the 'trick' explained above works.
c The same 'trick' is used to measure the wavelength of X-radiation, using a grating ruled for use with visible light. Why is it necessary to use the grazing angle method for this purpose?

28 To do this question you need a diffraction grating, a darkened room, some lavatory paper, and a box of matches. It would be a good idea to have a bowl of water handy, too, for safety.
Light a match, and look at it at arm's length with the grating over your eye. You should see at least one, maybe more, pairs of spectra of white light split into colours, on either side of the

flame. (If they are above and below the flame, how can you make them appear on either side of it?)
Now light a spill of lavatory paper. The flame looks similar to the match flame, but through the grating, you should see a clear, sharp, yellow image of the flame 'on top of' the colours. What does the existence of a clear, sharp 'first order flame' tell you about the light from the burning paper?

29 At roughly what frequency would a slatted wooden fence (or, if you prefer, iron railings) be a good diffraction grating for sound waves? The speed of sound is about 340 m s^{-1}; make any other estimates you need.

30 Criticize the following statement.
'Diffraction by a diffraction grating is not the same as diffraction at a single slit. In the second case, the light just spreads out; in the first, the radiations from many slits are superposed and can interfere with one another, as can the radiations from a pair of slits.'

31 This is about why the spectral lines produced by a diffraction grating in light of one wavelength are *sharp.*

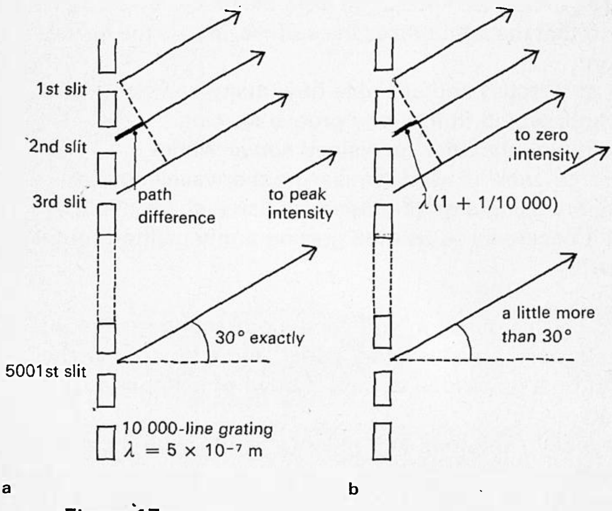

Figure 17

Figure 17 shows some of the slits in a grating which contains 10 000 slits, each one 10^{-6} m from the next. Light of wavelength 5×10^{-7} m falls on the grating (the light is a parallel beam at right angles to the grating).
From $n\lambda = d \sin \theta$, the first maximum of intensity, with $n = 1$, comes at $\theta = 30°$, where $\sin \theta = 0.5$.
a If figure 17 *a* illustrates this first maximum, what is the path difference between waves from the first slit and from the second, when they reach a distant screen?
b What is the path difference between light from the first slit and light from the third?
c What is the path difference between light from the first slit and light from the five thousand and first?
d Figure 17 *b* illustrates the situation at an angle a shade more than 30°. Why is the path difference between light from the first slit and from the second now a little larger than it was before?
e If the path difference in **d** now happens to be $\lambda\,(1 + 1/10\,000)$, what does the answer to **b** become?
f What does the answer to **c** become, at the slightly larger angle?
g What will be the resultant intensity at a distant screen of light from the first slit and from the five thousand and first?
h What will be the resultant intensity at a distant screen of light from the seventh slit and the five thousand and seventh?
i Imagine pairing each slit with one 5000 slits further along the grating. What will be the net intensity at the screen from the whole grating?
j The new larger angle, at which the intensity is zero, has a sine which is larger than 0.5. Write down its sine.
k Write down the wavelength of radiation which would give a *maximum* ($n = 1$) at the angle whose sine is given in the answer to **j**.
l It may seem reasonable that if light containing wavelengths of both 5×10^{-7} and the wavelength in **k**, shone on the grating, it would just be possible to see the two maxima as a pair and not as one blurred maximum. What is the resolving power of the grating, expressed by the fraction, perceptible difference in wavelength divided by wavelength used?

Questions 32 to 37 are general questions about waves, spectra, and spectroscopy. A number of them show how the wave ideas developed in this Unit can be used in other problems outside the scope of the Unit.

32 This question is about an interesting difference between what your ears and eyes do to signals containing more than one frequency. A physicist would say it shows that your ear is a 'Fourier analyser', but that your eye is not.

a Get someone to play you three or four notes on a piano, one at a time. Give them arbitrary names, say P, Q, R, etc. Then get your helper to play two at once, and see if you can say which two notes are being played.

b Now supply your eye with two colours of light simultaneously, as follows. Stand in front of a pair of electric lamps with a diffraction grating held over your eye, and move about until part of the lefthand first-order spectrum from one lamp seems to fall over part of the righthand first order spectrum from the other lamp. Suppose red overlaps blue: do you see 'red and blue', or some other, single colour?

Both eye and ear were supplied with a complex oscillation made up of at least two frequencies. Your ear and brain, at least partially, separate the complex oscillation into its constituent frequencies. A diffraction grating does the same for light, sending different frequencies off at different angles. Both *analyse* the input sent to them. Your eye is not an analyser, however, and sees one colour for each complex light input.

33 For this question you either need a simple hand spectroscope, or you will have to make one from a diffraction grating, a posting tube, and a slit. Figure 18 suggests how you could make one.

Light from the hot outer surface of the Sun has to pass through the Sun's cooler (though still hot) atmosphere. This atmosphere contains many different chemical elements, and the atoms of these elements can absorb radiation at the same frequencies as those at which they would emit radiation. As a result, the Sun's white light spectrum has light missing from it at a number of frequencies. If the spectrum is formed from a slit, the spectrum seems to be crossed by narrow dark lines. These lines were found by Fraunhofer, and are often called Fraunhofer lines.

To see the Fraunhofer lines, look through a hand spectroscope at a white screen on which sunlight is falling. Several thicknesses of good white paper laid on the ground make a suitable and simple screen.

Warning: *Under no circumstances should you look at the Sun directly, even for a moment.* You could easily blind yourself doing so.

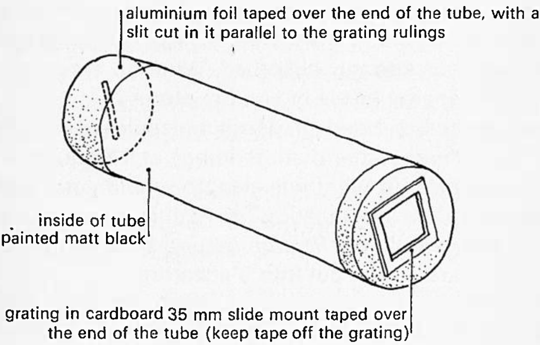

Figure 18
A home-made hand spectroscope.

Angle the spectroscope so that the very bright, white, 'zero-order' image is cut off by the side of the tube. Try small adjustments to the slit width, to find a good compromise between brightness and sharpness. A millimetre is about right.

What information about the Sun's atmosphere could the lines be used to give?

34 Measuring the frequency or wavelength at which a material emits or absorbs radiation can yield information about the material. By analogy with the inspection of a visible spectrum, in which the wavelengths present are spread out into a band of colours, making such measurements is called observing a spectrum. Spectroscopy – looking at spectra – is used by very many kinds of scientists and engineers, at almost all of the wavelengths of the whole electromagnetic 'spectrum'. This question is about some of those uses. Remember that the energy of a photon of frequency f is given by $E = hf$, with $h = 6.6 \times 10^{-34}$ J s. The speed of light is 3×10^{8} m s^{-1}.

a Sodium street lamps emit yellow light, of wavelength about 5.9×10^{-7} m, from individual sodium atoms in a vapour inside the lamp. If white light is passed through sodium vapour, in which most atoms are in the lowest energy level, light at just this wavelength is strongly absorbed. What do these facts tell you about the energy levels of sodium atoms?

b The carbon–carbon bonds in petrol molecules oscillate in and out (stretching, rather than bending) at a frequency of just about 10^{14} Hz. At what wavelength would you expect petrol vapour to absorb radiation? How might you detect such radiation? Why would a *reflection* grating be a good way of spreading such radiation out into a spectrum?

c Protons – that is, hydrogen nuclei – are little magnets, and in a magnetic field, they can accept energy from radiation, turning them from one orientation in the field to another. In a field of 0.94 tesla, radiation of frequency 40 MHz is just right to flip the little proton magnets in CH_3 groups in ethanol over from one orientation to the other. What is the energy difference between the two orientations? Suggest a way of measuring the wavelength of radiation which has this frequency. This sort of spectroscopy is of interest because it makes it possible to measure the strength of the little proton magnet. Its effective strength depends upon what electrons are nearby, and on where they are, so it is not the same for every proton in, say, ethanol. This molecule gives absorption at three distinct frequencies (or three different fields, which comes to the same thing), corresponding to protons in OH, in CH_2, and in CH_3 groups. The pattern of peaks, or nuclear magnetic resonance spectrum, can be used to analyse compounds, just as visible spectra can be used.

d The energy of a gamma ray photon can be measured, by – in essence – measuring the intensity of the flashes of light gamma ray photons produce when they go through a suitable material which scintillates when gamma rays fall on it. How could you work out the frequency and wavelength of a gamma ray photon of energy 1.76 million electronvolts (2.82×10^{-13} J), which is one of the gamma rays emitted by naturally occurring uranium? Why would it be impracticable to measure either directly?

Gamma ray spectroscopes have been used for uranium prospecting by aircraft. The aircraft carries a slab of the scintillating material (crystals of sodium iodide) and a detector and energy analyser is set to look for the 'spectrum line' of uranium gamma rays. Others have used gamma rays to tell them something about the energy levels of nuclei, much as visible spectrum lines do for the energy levels of atoms.

35 In this Unit, light has been treated as if it were a wave motion, even though, in Unit 5, you may have seen evidence that light energy is delivered in 'lumps', called photons.

Say what you think of one or more of the following statements. Comment in particular on whether the writer's confidence or caution seems justifiable.

a 'When we consider . . . the very great speed with which light is propagated in all directions, and the fact that when rays come from different directions, even those directly opposite, they cross without disturbing each other, it must be evident that we do not see luminous objects by means of matter translated from the object to us, as a shot or an arrow travels through air. Light is propagated in some other manner, an understanding of which we may obtain from our knowledge of the manner in which sound travels through the air.'
Christiaan Huygens (*1690*) Treatise on light.

b Having shown that a theoretical explanation of the diffraction patterns of slits and obstacles can be given, using the hypothesis that the brightness of light on a screen is to be

obtained by adding together wave contributions from parts of the wave front at the obstacle, Fresnel writes:
'Our [wave] theory rests upon a hypothesis which is at once so simple and so inherently probable, and which besides has been so strikingly verified by many varied experiments, that one can scarcely doubt the truth of the fundamental principle.'
Augustin Fresnel (1819) Memoir on the diffraction of light.
c 'If the mechanism of light is imagined to have a periodicity in time, determined by the colour . . . then a convenient review of optical phenomena may be obtained with this aspect as a starting point.'
Ernst Mach (1913) The principles of physical optics *translated by J. S. Anderson and A. F. A. Young.*
Pay special attention to what Mach *avoids* saying. He was a 'positivist', and his creed might be crudely summarized in the words, 'Don't say what you don't *know*'.

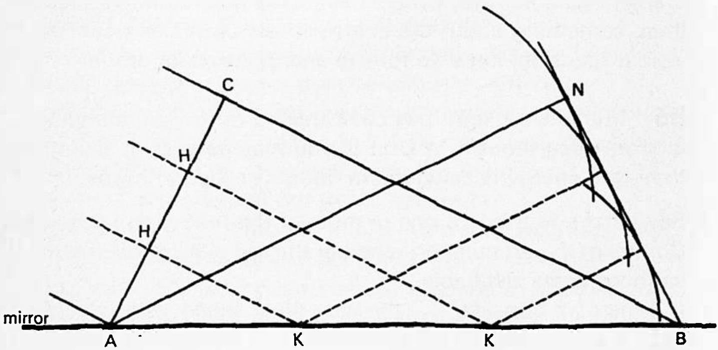

Figure 19

36 **a** Figure 19 comes from Huygens's *Treatise on light* (1690), and is part of his explanation of how a wave theory of light could explain the fact that light is reflected from a flat, mirror-like surface, at an angle equal to the angle at which the light strikes the surface.
How do you think Huygens used this diagram to make his explanation?

AB is the flat mirror, seen edge on. AC is a plane wave front of light travelling in the direction HK towards the mirror. NB is the reflected wave front, being the common surface of a lot of waves from places K on the mirror. Distance CB is drawn equal to distance AN.

b Figure 20 is part of Huygens's explanation of the refraction of light as it passes from, say, air into water or glass. How do you think that Huygens explained refraction, particularly the fact that the ratio of the sines of angles DAE and FAN is a constant for any one pair of materials? You will need to know that light travels more slowly in water or glass than it does in air.

AB is the flat surface of, say, the water. AC is a plane wave front of a wave in air, travelling along the direction HK towards the water surface. NB is the refracted wave front in the water, travelling along the direction AN, and is the common surface of many waves starting from places K on the surface.

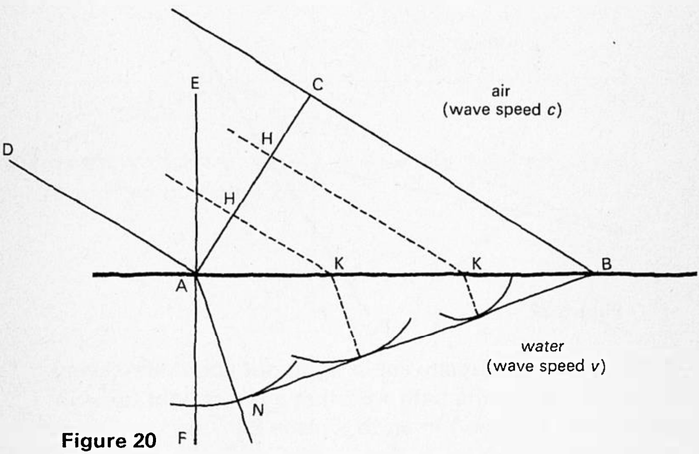

Figure 20

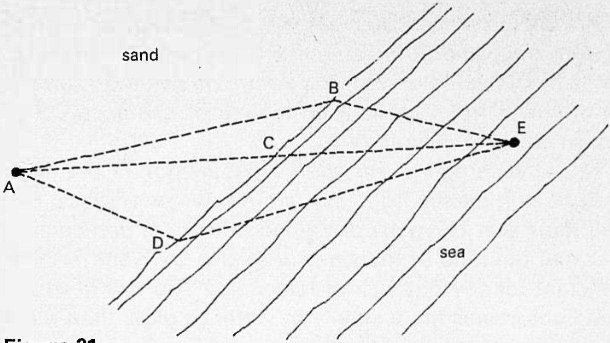

Figure 21

37 **a** Suppose you were sitting on the beach, at place A in figure 21, and needed to get as quickly as possible to a person in distress in the sea, at place E. Which sort of path is the best one to take: 'straight there' along ACE, or along one of the angled paths ADE or ABE? Remember that you can run on sand faster than you can swim.

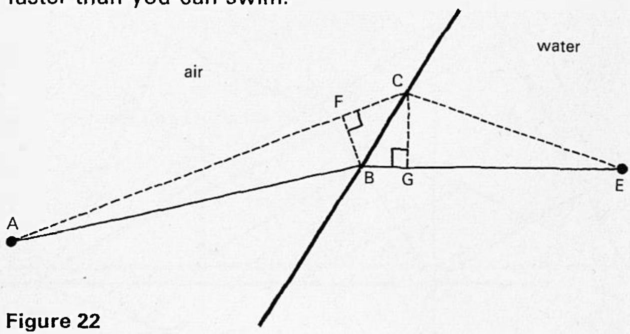

Figure 22

b This question is really about light, not about life-saving. Figure 22 shows the path ABE that a ray of light takes in going from a place A in air to a place E in, say, water, within which light travels more slowly than it does in air. The light path is similar to the 'least-time life-saving path'.
Just suppose that the light went along the nearby path ACE, and that the two paths are near enough for AF to be very nearly equal to AB, and for EC to be nearly the same as EG. (Angles AFB and EGC are right angles.) By how much would

the time taken to go along ACE differ from the time taken to go along ABE? Write c for the speed of light in air, and v for the speed of light in water.

c Now suppose that the true path ABE is in fact the one which takes the least time. If it is, then a small shift in path will make almost no difference to the time taken. This is a general property of minimum (or maximum) values: a graph of time taken against path angle will have a trough, and a small shift at the bottom of a trough hardly alters the time taken, just because any graph must be flat at the bottom of a trough. See figure 23.

What is the ratio FC/BG, if path ABE is the least-time path?

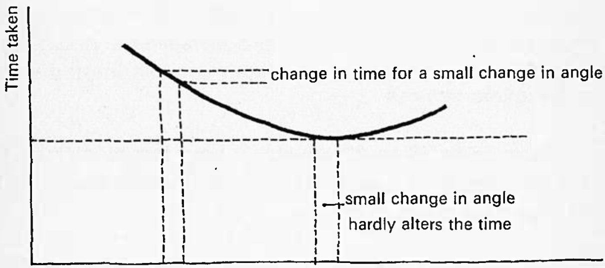

Figure 23

d Now show that $\sin i/\sin r = c/v$, where i and r are the angles marked on figure 24.

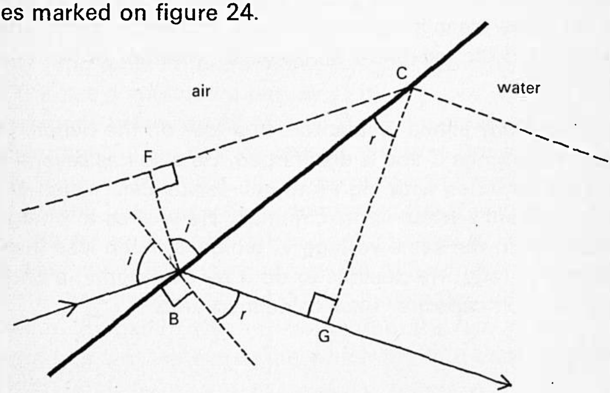

Figure 24

The ratio $\sin i/\sin r$ is called the *refractive index* of the pair of transparent materials concerned. As you have seen, it can be understood simply as the ratio of the speeds of light in the two materials.

The idea of turning optical problems into least-time problems was suggested by Fermat. It turns out that the principle is of very general use, though it needs to be enlarged to include maximum values as well as minimum values. Interestingly, similar methods work for dynamics too, though it is not normally the time that is a minimum or maximum, but quantities related to the energy. Look up 'the principle of least action', if you are interested.

Part Three
Electric waves

Questions 38 to 42 are about how fast electrical waves propagate. Question 40 is the one which develops the most important piece of theory.

38 This is a long question about the speed of electric waves along a row of capacitors and inductors. It is not essential that you finish it, especially as it isn't easy. But it will be worth seeing how far you can go, if only because it enables you to revise some ideas about capacitance and inductance. You may like to read the tail-piece on page 38 whether or not you try the rest.

Bill has many similar capacitors in a row on the bench. Each has capacitance C and is discharged. He also has several high voltage batteries with high internal resistances, which give a small current I when short-circuited. He wishes to charge each capacitor to the same voltage V, which is much less than the battery voltage. He decides to do it by connecting a battery across each capacitor for a calculated time.

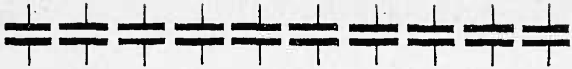

Figure 25

a How much electricity will enter a capacitor each second while this is happening?
b By how much will the p.d. of the capacitor increase each second?
c How many seconds will it take for the p.d. to rise to V?

If Bill uses only one battery it will take him a long time to charge all the capacitors. So he goes along the row at u capacitors per second, connecting a battery to each capacitor, and he gets Ben to go along the row at the same speed shortly afterwards, disconnecting them and handing him the battery as each capacitor reaches the voltage V.

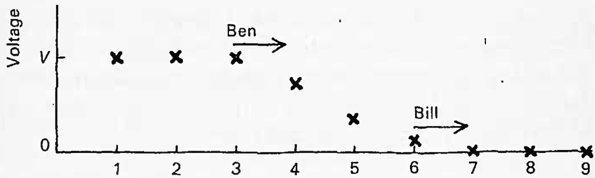

Figure 26

At a *given moment* the *voltages* of the capacitors might be as shown in figure 26.
d How long a time must Ben leave each capacitor to charge after Bill has connected a battery to it?
e How many more capacitors will Bill have connected up in the meantime? That is, how many capacitors does Bill connect up in the time it takes one of them to charge fully?
f How many batteries will be connected up at any one time?
g What is the total current taken from batteries at any one time?
h By how much is the p.d. across the fifth capacitor in figure 26 more than the p.d. across the sixth capacitor while both are being charged? (This can come from the rate at which the voltage rises and the extra time which the fifth capacitor has had.)

David watches this eccentric process going on. It occurs to him that the process is like a 'wave of charging' going along the row of capacitors, much as a 'wave of starting to move' can be sent along a row of trolleys linked by springs. (See Unit 4, *Student's book*, question 24.) He would like to make such a wave move along the row of capacitors more automatically, as a result of connecting a battery to one end of the row (much as he would only need to push on one end of a row of trolleys and springs to send a wave along.)
He realizes that if he just connects the capacitors in parallel and applies the current at one end, the voltages of all the capacitors will rise together. So he thinks that if all the negative sides of the capacitors are connected to one another, the positive sides must be separated by some sort of new component, shown by circles in figure 27. He wants to apply the same total current (VCu) as before, but this time it is all to enter at the lefthand side. If his new components are to have the right effect, then the voltages at a particular time are represented by the earlier graph (figure 26), assuming that he increases the current at the correct rate until the first capacitor is charged to p.d. V. The next questions refer to this new plan, using results from questions **a** to **h**.

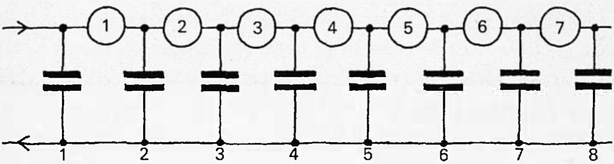

Figure 27

i What p.d. must be across the ninth of these new components?
j What current must pass through the ninth of these components?
k What p.d. must be across the first of these components?
l What current must pass through the first of these components?
m What p.d. must be across the fifth of these components, that is, any one which links two capacitors which are in the process of being charged? Use your answers to **a** to **h**.
n How many new capacitors per second is the current through the fifth of these components flowing into?

o How much current flows onto each newly charging capacitor?
p At what rate is the current through the fifth of the new components increasing?
q Would the answers to **m** and **p** be the same for any other of the new components through which the current is increasing?

David thinks about these questions, and decides that each new component must have no p.d. across it when a steady current passes through it, but must have a p.d. of I/Cu across it if the current is changing at Iu (in amperes per second). He would like the speed at which the increase of voltage travels to be independent of the particular value of current I, so, because both the required p.d. and the rate of change of current are proportional to I, he decides that the voltage across his new component must be directly proportional to the rate of change of current through it.

r What must be the ratio of the voltage produced across one of the components to the rate of change of current in it? Note that the ratio does not depend on the current.

An inductor is a component with the required behaviour. The p.d. across an inductor of inductance L is $V = L\,\mathrm{d}I/\mathrm{d}t$. If $\mathrm{d}I/\mathrm{d}t$ is one ampere per second, the p.d. is numerically equal to the value of L in henries.

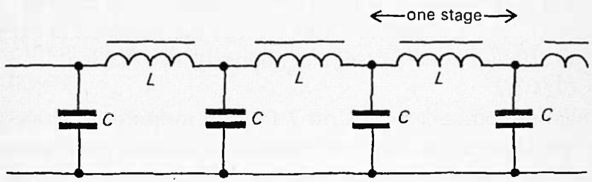

Figure 28

s What value of L must David's inductors have?
t From the answer to **s**, what is the speed u in stages per second of the electrical disturbance, in terms of the inductance L and capacitance C of each stage?
u If there are n inductors and n capacitors per metre length of bench, what is the speed v of the disturbance along the bench, in metres per second?

v If the symbols are changed so that L^* means inductance per metre of bench and C^* means capacitance per metre of bench, (i.e. $L = L^*/n$, $C = C^*/n$) what is the speed v of the disturbance in terms of L^* and C^*?

w If a dry cell is connected across one end of a coaxial cable 200 metres long, of 4×10^{-7} henry per metre and 6.25×10^{11} farads per metre, how long will it take for the cell's voltage to get to the other end?

This is the end of the question. The 'story' has one last part, however. Yet another acquaintance of Bill, Ben, and David said, when he saw the results of all their efforts and calculations, that he had a quick way of guessing the answer. His idea was that the row of capacitors and inductors was very like a row of masses joined by springs, along which compression waves could be sent. He remembers, from Unit 4, that if there are masses m spaced at intervals x, with springs between them having spring constant k, the speed of such a wave would be

$$v = x\sqrt{k/m}.$$

He remembers from Unit 2 (and Unit 6) that a capacitor is rather like a spring, the one needing a voltage to charge it, and the other needing a force to compress it. The analogy to the spring constant k is the reciprocal of the capacitance, $1/C$, because a spring with a large spring constant is hard to compress, but a capacitor with a large capacitance is *easy* to charge (only a small voltage is needed). $F = kx$ and $V = (1/C)Q$.

He also remembers from Unit 7 that an inductor is analogous to a massive object, the one needing a voltage to increase the current in it, and the other needing a force to increase its velocity. The inductance L is the electrical analogue of mass m. $F = m\,dv/dt$ and $V = L\,dI/dt$.

Arguing like this, he guesses that the speed of the electrical wave on the row of capacitors and inductors in figure 28 might be

$$v = x\sqrt{1/LC}$$

putting L for m and $1/C$ for k in the compression wave equation.

Finally, he points out that if v is the speed in metres per second, v/x is the speed in stages per second, since each stage occupies distance x, giving $\sqrt{1/LC}$ for the speed of the wave in stages per second.

David, Bill, and Ben thought that this was all pretty dubious, though they agreed that anything which helps you to guess the answer you want to prove, might make proving it a lot easier.

39 *Optional*
You may like to see how the result obtained in question 38 can be used to show that electricity travels along at least one kind of electric circuit at the speed of light, accompanied by electric waves in the space near the circuit. Question 40 does the same thing another way, and if you prefer to do only one of them, question 40 may be the better one to choose.

David has calculated (see question 38) that if a set of inductors and capacitors is connected as shown in figure 29, a pulse of electricity can pass along the line at a speed v metres per second, which is equal to $1/\sqrt{LC}$ where L is the total inductance per metre along the line and C is the total capacitance per metre. Measurements with a set of big inductors and capacitors agree that the speed is $1/\sqrt{LC}$. David also found that when he measured the speed of a pulse along a coaxial cable it was the same as $1/\sqrt{LC}$, L and C being the values per unit length given in the makers' catalogues.

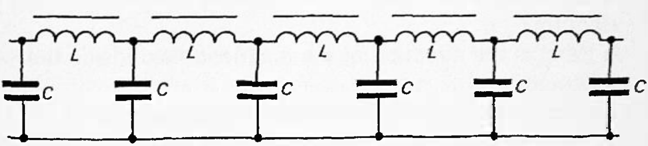

Figure 29

He has now thought of a situation in which he can calculate the inductance and capacitance per unit length of conductor from its size and shape. He wishes to test his theory. He has some pairs of square metal plates which can be used as capacitors if they are separated by small insulating spacers. He sets up the capacitors in a line, their plates horizontal and touching one another at the edges, with a cell at one end and a resistor at the other so that a current I flows out along the bottom plates and back along the top ones. Each plate has width a, area a^2, and the top plates are a very small distance d above the bottom ones. David thinks that the current is distributed evenly across the plates (except near the ends of the line) and that in the space between a pair of plates in the middle of the line there is a magnetic field, just as there is in a solenoid. He imagines a rectangular solenoid of length a and *cross-section ad, having N turns and carrying a current of I/N.* See figure 30.

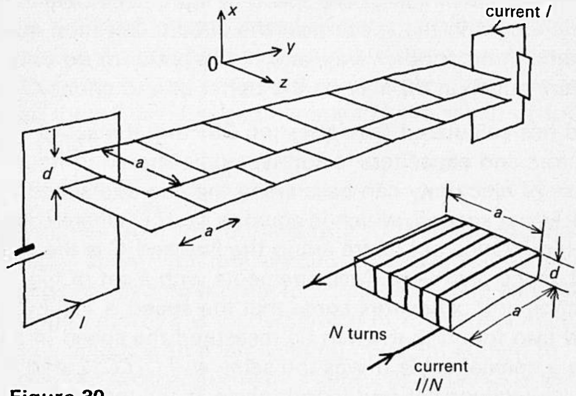

Figure 30

a Is the magnetic field inside the solenoid (not near the ends) uniform?

b *What is the direction of the magnetic field inside the solenoid?*

c Is there any magnetic field outside the solenoid?

d How many turns per metre does the solenoid have?

e How many ampere-turns per metre produce the magnetic field in the solenoid?

f What is the B-field inside the solenoid?

g Does a point inside the solenoid differ, in the way electric current goes round it, from a point between the top and bottom plates in the middle of the line which David has set up?

h What is the B-field between the plates?
i What is the area, cutting the magnetic field at right angles, between the plates in one metre length of their path?
j What magnetic flux is between the plates in one metre length of their path?
k If the current I is made to increase at one ampere per second, how fast does the flux between the plates increase, per metre of their length?
l If the current I increases at one ampere per second, what is the voltage across the plates (needed to increase the flux between the plates at the rate found in **k**)?
m What is the inductance (voltage induced per unit rate of change of current) of one metre length of the plates?
n Is the top or the bottom set of plates positive with respect to the other?
o In what direction is the electric field between the plates?
p Is there any electric field outside the space between the plates, where the magnetic field is zero?
q The capacitance between a pair of parallel plates of area A, distance d apart is $\varepsilon_0 A/d$, so what is the capacitance of the plates David has arranged, per unit length of plates?

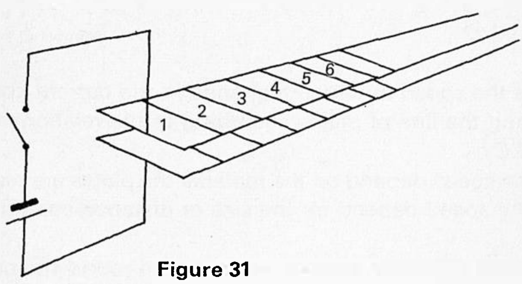

Figure 31

David thought about the short row of capacitors shown in figure 30, in which the current is everywhere the same at any particular moment, so that he could calculate L and C per unit length. But what really interested him was the case of a very long line of capacitors (figure 31), with no connection at all between the plates at the far end. After switching on, the top and bottom lines of plates could not charge instantaneously, because this would mean momentary infinite current, and consequently infinite rate of change of current through the plates near the cell. Instead, because each pair of plates had inductance as well as capacitance, he expected to get what he had in question 38. For example the p.d. between each of the first three pairs of plates might be V, between pairs 4, 5, 6, and 7 the p.d. might be progressively smaller, and all the rest of the pairs might have no p.d. at all. In this case plates 1, 2, and 3 would all carry the same current I, plates 4, 5, 6, and 7 would carry progressively smaller currents because the electricity was being used to raise the p.d. of these pairs of plates, and the remaining plates would carry no current. So a graph of V, I, E, or B against distance along the line, would have the same shape (see figure 32).

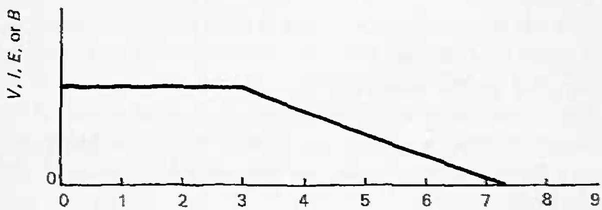

Figure 32

r What is the speed at which a change in the current should travel along the line of plates according to the relationship $v = 1/\sqrt{LC}$?
s Does the speed depend on the material the plates are made of?
t Does the speed depend on the size or distance apart of the plates?
u Would the speed be different if the space round the outside of the plates, but not between them, were occupied by some additional new material?
v Would the speed be different if the space between the plates were occupied by some additional insulating material?

David decides to set up a pair of conductors and measure the speed, using an oscilloscope to time how long a pulse takes to travel 200 m and back. He finds that 200 m of 6 mm thick aluminium capacitor plates 0.25 m square would cost about

£5000, so he buys rolls of aluminium foil half a metre wide and supports two sheets 10 mm apart, fixing them by their edges. The oscilloscope shows that the pulse takes 1.3×10^{-6} second to travel 200 m and back.

w At what speed does the pulse travel?

x Look up the values of μ_0 and ε_0 on page 116. What pulse-speed ought David to expect?

40 This question is about the speed of electrical signals along an electric circuit. To make the calculations simple, a special shape of circuit has been chosen, but the result has a more general value, since the speed turns out to be the same for all long, straight circuits, as long as there is no insulating material around or between the wires. The speed is also the speed of light.

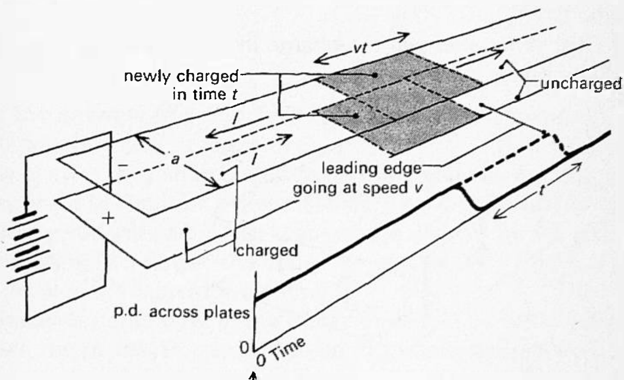

Figure 33

Figure 33 shows the particular circuit chosen for discussion. Instead of a pair of wires, it consists of a pair of flat conducting plates, with a gap between them which is much smaller than their width, so that any part of the plates is like a parallel plate capacitor.

Experiments with wires and cables indicate that after a p.d. is switched on across one end of a long pair of conductors, the sudden rise in voltage sweeps along the circuit at a high speed. In this question, it is assumed that the same kind of thing happens here. Behind the leading edge of the travelling voltage, the plates must be left charged (top negative, bottom positive, with the battery connected as in figure 33).

a If the leading edge of the travelling increase in voltage goes at speed v, it goes a distance vt in time t, as in figure 33. This new length of the plates is charged in time t. Where did the necessary extra charge come from?

b Why must there be electric currents I flowing in the top and bottom plates between the battery and the leading edge of the travelling voltage?

c If the current is I, what charge flows down the plates in time t?

d Over what area will the charge in **c** be spread, if the width of the plates is a?

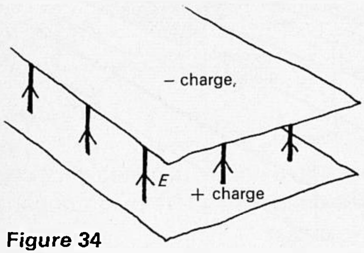

Figure 34

e As indicated in figure 34, there will be an electric field E between charged parts of the plates, given by

$$E = \textit{charge density}/\varepsilon_0$$

Use the answers to **c** and **d** to write an expression for the field E in terms of the current I. Why does the expression not contain the time t?

f As indicated in figure 35, there will also be a magnetic field B between charged parts of the plates, because they carry current I whilst the leading edge is propagating onto uncharged distant parts of the plates.

The plates are somewhat like a flattish solenoid, in which the many wires usually wound round a solenoid have become one wire – the plates themselves. In figure 35*a*, if the solenoid has N turns in width a, the B-field is given by

$$B = \mu_0 NI/a.$$

What will the B-field be in figure 35*b*, where there is *one* turn in width a?

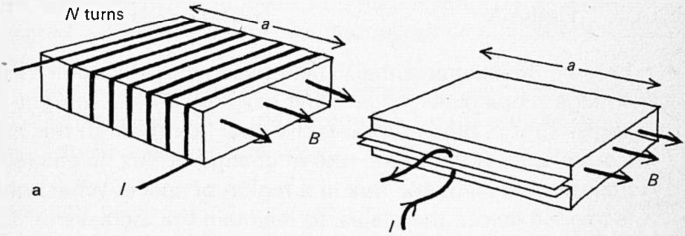

Figure 35

g Use the answers to **e** and **f** to write as simple an equation as you can, giving B in terms of E.
h Your answer to **g** should show that the larger the velocity v, the larger the B-field for a given E-field. The E-field is fixed by the battery voltage and is just that voltage divided by the plate spacing. Why is B larger, the larger the velocity v? (Think about how much current must flow.)
i Wherever currents flow in the plates there will be a B-field between the plates. Currents flow up to the present position of the travelling leading edge of voltage. Thus the B-field extends further and further along the plates, just as the E-field does, while the voltage is propagating along the plates. Into what new length between the plates does the B-field extend in time t?

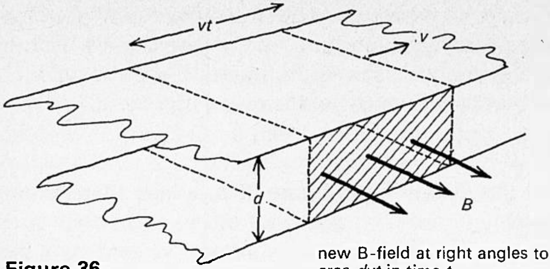

Figure 36

j Figure 36 illustrates the B-field occupying new area as the voltage propagates. An area dvt has new B-field at right angles to it in time t. What is the new flux through this area?
k A voltage, equal to the rate of change of flux, is needed to increase the magnetic flux in a region of space. What voltage is needed across the plates, to maintain the increase of flux found in **j**?
l If there is a voltage V across the plates, there is an electric field E between them, equal to V/d. Write an expression for this electric field E in terms of B, using the answer to **k**.
m From **g**, the electric field between the plates was related to the B-field between the plates by

$$B = \varepsilon_0 \mu_0 E v.$$

The larger the speed v, the bigger the B-field, because a greater current would be needed to deliver the same charge per unit area at a faster rate to new parts of the plates.

From **l**, the E- and B-fields are related by the second condition

$$E = Bv.$$

The bigger the speed v, the faster the magnetic flux must be produced in new area. v is limited by both equations. Eliminate B between these two equations, and obtain an expression for the velocity v.
n Use $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2} and $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2 C^{-2} to obtain the value of the speed v.
o Show that the units of $1/\sqrt{\varepsilon_0 \mu_0}$ are m s^{-1}, using the units given in **n**.

p If the plates were twice as wide, half as far apart, and the battery voltage were doubled, why would the speed v be the same?

41 Advances in solid state electronics, in particular the production of integrated circuits, have made it possible to design computers which can process data in times of the order of a nanosecond.
a One limitation on the speed of operation of a computer is connected with the physical size and layout of the circuits. Can you explain this?
b In a certain experiment it is necessary to delay the passage of an electrical pulse by 2 ms. Suggest a simple way of doing this and give some practical details.

42 **a** Puck said:

> 'I'll put a girdle round the Earth
> In forty minutes.'

Thanks to Maxwell and Marconi, how long does it take nowadays?
b When you switch on the light, about what time elapses before current starts to flow in the lamp? Why would it be no use using a photocell 'looking' at the lamp to detect the delay?
c At what distance from the Earth would a planet be whose occupants must wait until the year 2000 to discover that we have world-wide television? What is the earliest date that we on Earth could learn that they have picked up some of our television signals?
d Electric power is distributed at 50 Hz. Assuming that the voltage variations travel along the power lines at $3 \times 10^8\ \mathrm{m\,s^{-1}}$, what is the wavelength of the alternating voltage signal transmitted by the power lines? In view of the size of the country, what can you say about the voltages on the lines at any instant, at different places?

Questions 43 to 46 are general questions about electromagnetic waves.

43 The output power of a space probe's radio might be about 100 W. Its signal can be detected from as far away as the Moon's distance from Earth (380 000 km) by a dish aerial about 10 m in diameter.
a Use this information to set a limit on the power that modern receivers can detect.
b If you assume that the power follows an inverse square law, what are you supposing about the space probe's aerial?
c The receiving dish aerial is strongly directional. Does this fact make the use of an inverse square law in **a** invalid?

44 *For discussion*
In 1968, there were 75 v.h.f. stations in Britain, transmitting BBC Radio 4 at a frequency of about 100 MHz. Fifteen of these were high power stations (60 to 120 kW), and the rest had powers from 2 W to 20 kW with 30 stations of power up to only 100 W.
For medium-wave broadcasting of Radio 4, at about 1 MHz, there were fewer stations. There were 28 stations in all, with 9 stations of high power (100 to 150 kW), and the remaining 19 all with powers of over 1 kW, (with one exception, 250 W).
a Explain why there are more v.h.f. than medium-wave stations, and why there are more v.h.f. stations of low power. Remember that the national coverage of good reception is no worse on medium wave than on v.h.f. (though the quality may be poorer).
b Make some estimate of the total sound broadcasting radio frequency energy emitted over the whole world, in a day.

45 *For discussion*
A television programme may be received in one of two ways:
1 the signals are transmitted from an aerial, and 'picked up' in a receiving aerial connected to a receiver.
2 Signals are transmitted along a solid-dielectric coaxial cable which is connected directly into the television receiver (there may be an amplifier somewhere in the transmission line).

Describe how each system conveys the programme information and explain how it is that the final results are very similar.

46 Here is a simple experiment to determine the power and the electric field of the Sun's radiation.
Next time the Sun shines, 'look at it with your eyes closed'. (Your *must* keep them closed or you may injure them badly.) Your eyelids will feel warm, and you will see a redness through your closed eyelids. Try to get the 'feel' of this warmth and redness; then go indoors and try to match the sensation with an ordinary incandescent lamp (say a 100 W clear-glass lamp), by adjusting the distance between your face and the bulb.

The intensity is then $S = \frac{P}{4\pi R^2}$, in watts per square metre where P is the power of the bulb (100 W), and R the distance to your eyelids, in metres.
If you want to compare your result with a published value look up the 'solar constant' in a data book. This is the rate of arrival of energy at the top of the Earth's atmosphere.

If this energy from the Sun (and from the light bulb) is really electromagnetic, it ought to be possible to work out the values of the electric and magnetic fields in the radiation.

The average power available from one square metre of electromagnetic wave front is about $2.65 \times 10^{-3} E^2$ in W m^{-2}, where E in volts per metre is the root mean square of the electric field. The B-field is given by $E = Bc$.
a Estimate the root mean square E-field and B-field in sunlight.
b Estimate the root mean square E-field in radiation intense enough to cook 100 g of steak in one minute. Assume that the steak absorbs all the radiation that falls on it.
c For **b**, what is the advantage of choosing the wavelength of the radiation so that the oven walls are almost perfect reflectors of the radiation?

Questions 47 to 50 are about polarization effects.

47 Figure 37 shows how much light 'ideal' polaroid transmits. In figure 37 *a*, the electric field of the incoming light is at right angles to long polymer chains, to which iodine atoms are attached. The presence of the iodine molecules allows electrons to migrate along the chains, so that the chains behave like a very fine grid of conducting wires. With the electric field perpendicular to the grid, all the incident light is transmitted.

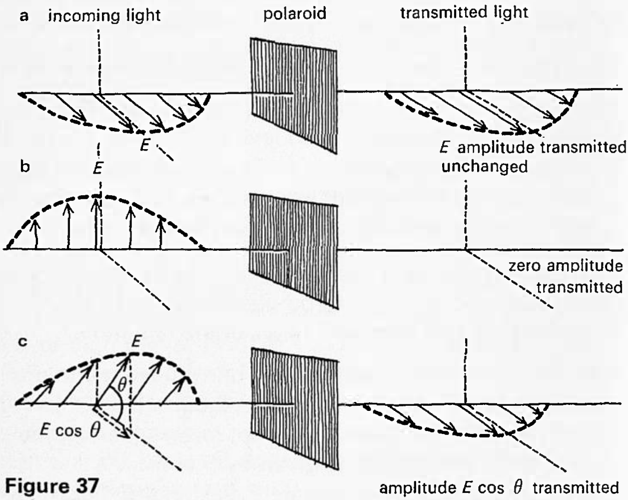

Figure 37

In figure 37 *b*, the electric field is parallel to the grid, and no light is transmitted.

In figure 37 *c* the electric field makes an angle θ with the direction it had in figure 37 *a*. A component $E \cos \theta$ lies along that original direction, and is transmitted. (A component $E \sin \theta$ lies along the direction in figure 37 *b*, and is not transmitted.).

The amplitude of the transmitted electric field oscillation in figure 37 *c* is $E \cos \theta$, if the amplitude of the incoming wave is E. The *brightness*, or intensity, of the transmitted light is proportional to the square of the amplitude, and so to $\cos^2 \theta$.

a Sketch a graph of the variation in brightness of the transmitted light if plane polarized light falls on a sheet of ideal polaroid, which is rotated through 360°.
b If unpolarized light, containing a random assortment of polarization directions, with no direction favoured, falls on ideal polaroid, by what factor is the brightness of the light reduced by polaroid? (You should recall something about an average value of $\cos^2 \theta$ from work on alternating currents in Unit 6.)

48 Look at shiny reflections from the smooth surfaces of a knife, a piece of smooth aluminium cooking foil, glass (a tumbler or a pair of spectacles), and a piece of polythene wrapping film, through a piece of polaroid. Rotate the polaroid. What kinds of materials usually, at least to some extent, linearly polarize light that they reflect, and what kinds do not? (The essential difference might be an electrical one, if light is an electromagnetic wave.)

49 Stick a piece of clear adhesive tape (Sellotape) onto a piece of glass, put it between 'crossed' polaroids, and look through the sandwich. Try turning the polaroids together, keeping them crossed. Make whatever suggestions you can about what the tape does to polarized light passing through it. You may like to go on to try several thicknesses of tape, and to try rotating one polaroid.
You may also recall, from Unit 1, that materials like polythene sheet which do not behave like the tape, can be made to do so if they are strained. Try cutting a V-shaped notch in the side of a long strip of polythene, and pull it longways between crossed polaroids.

50 On a sunny day, stand so that the Sun is on your right or on your left, and look at a patch of blue sky through a piece of polaroid. Choose a patch of sky from which the light to your eye is at right angles to the direction from it to the Sun.
a Rotate the polaroid, and try to explain what you see.
b *Harder.* Refer to question 47, and try to decide along which direction in your polaroid sheet the iodine-loaded, long-chain polymer molecules generally lie.

Part Four

Relativity

Questions 51 to 56 are about relativity, and about other things like magnetic effects and the Doppler effect, which are connected with relativity.

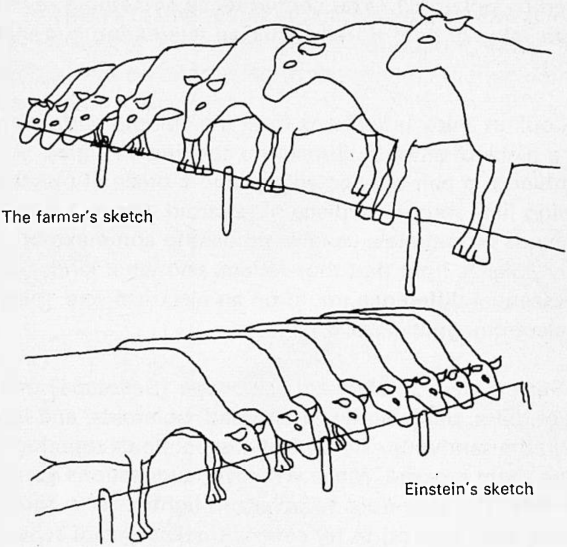

Figure 38
'Einstein's dream.'

51 This question is about a dream Einstein might have had. It is about one of the ideas leading up to the theory of relativity, though relativity itself is not involved in this question.

Einstein dreamed that he was in a field with seven cows of an unusually agile breed. Fearing they might get bloated, their owner had controlled their grazing by an electric fence which he moved forward a little every day. The box energizing the fence produced a high voltage pulse every few seconds, but at the time of Einstein's dream the battery had run down, and the cows had become accustomed to touching the fence gently, and were grazing right up to it. The owner was replacing the battery and Einstein was standing near the other end of the fence watching. When the owner switched on the voltage Einstein saw the cows jump back.

The farmer came across, and Einstein said to him, 'Your cows react very quickly. They all jumped back from the fence the instant you switched on, and they kept in a line perfectly parallel with the fence.'

The farmer replied, 'They are better than that. The first had jumped back before the electricity had got to your end of the fence. Although they kept in a straight line, the line was at an angle to the fence after they had started to move.' He took some paper out of his pocket and sketched what he had seen. Einstein drew a different sketch, and the farmer looked at it. 'Is this how you saw them at the same moment my sketch describes?' he asked.

How would you expect Einstein to have answered and explained it all?

52 Astronauts A and B pass each other, and note down the time at which they pass. They move apart at a steady velocity. Both have radar equipment and A uses his to measure the steady speed with which they are separating as follows.
First, A sends a radar pulse towards B, 98 s after they passed.
a Will B find that the pulse arrives more than 98 s after they passed, less than 98 s after they passed, or exactly 98 s after they passed?
b We shall write the time t_B after which B notes the arrival of the pulse at B as $t_B = k\,98$ (k is more than 1).
Does the value of k depend on the velocity with which A and B are separating?
c Does k depend on the velocity of the radar pulse? ($c = 3 \times 10^8$ m s^{-1}.)
d The pulse is reflected from B (when his clock records time t_B) and A notes its arrival back at his space ship 102 s after they passed. What is the total time of flight of the pulse for the round trip from A to B, and back to A? How long will A reckon the pulse took to reach B?

e How far will A reckon that B was from him when the pulse reached B?
f At what time, still counting from the time at which the two astronauts passed, will A calculate that the pulse reached B?
g A now works out B's velocity from the distance obtained in **e**, and the time since they passed from **f**. What value does he get?
h If B had done the same experiment, using the same value for c in his calculations, is there any reason why he should get a different answer for their relative velocity?
i If B sent a pulse to A at time t_B, would it reach A after, before, or at time t_B?
j B did send (or at least reflect) a pulse to A at time t_B. It arrived back at A at time 102 s after they passed.
If we write

$$102 = k\, t_B \ (k \text{ more than } 1)$$

k depends only on their relative velocity, and on the velocity of the pulse. Will the k in this equation be the same as that in **b**? (Consider the answers to **h** and **i**.)
k Questions **b** and **j** give two equations:

$$t_B = k\,98 \qquad \mathbf{1}$$
$$102 = k\, t_B \qquad \mathbf{2}$$

Show that:

$$t_B^2 = 98 \times 102 = 9996$$
$$t_B = 99.98 \text{ s}$$

l Commonsense – which is wrong – says that t_B, the time when B records the pulse arriving, is the same as the time, 100 s, when A calculates that it arrives (see **f**). If you put $t_B = 100$ s in equations **1** and **2**, can the value of k be the same for both equations?

Conclusions

If A and B agree on the same time, 100 s, for the pulse arrival, they must disagree on their values of k. But k depends only on the time for a pulse to travel between them because they have moved apart; that is, on the relative velocity carrying them apart and on the radar's velocity, both of which are the same for both A and B.

If A and B agree about k, and so about their relative velocity, they *disagree* about the time of an event. A thinks the pulse got to B after 100 s; B thinks it arrived after 99.98 s. As A finds that B's clock gives a lower time than his, he must think that B's clock is running slow. Of course this is quite symmetrical: if B did this experiment of sending a radar pulse to A and back, he would conclude that A's clock was running slow. This difference in the clock rates is the famous *time dilation*.

53 Think of a pair of copper wires 1 metre long and 10 mm^2 in cross-section. Each contains about 10^{24} conduction electrons.

a What is the order of magnitude of the total charge on the electrons in each wire?

b Of what order of magnitude would the electrostatic force between such charges be, if they were 1 metre apart? Treat them as small lumps of charge, if you like.

$(\frac{1}{4\pi\varepsilon_0} = 9\times10^9$ N m^2 C^{-2}, $e = 1.6\times10^{-19}$ C.)

(If you want to do it more accurately, the electric field a distance r at right angles to a long wire carrying charge q per unit length is $q/2\pi\varepsilon_0 r$.)

c Why do two such copper wires not fly apart under this enormous force?

d If a current of 1 ampere flows in each, in the same direction, the wires are attracted by a magnetic force of 2×10^{-7} newton, from the definition of the ampere. (The wires must be long and this is the force on each metre of either wire when they are a metre apart.)

What is the ratio between this magnetic force, and the electrical force estimated very roughly in *b*?

e The electrons move at velocities v of the order of 10^{-5} m s^{-1}. v could be found from the answer to **a**, the current being 1 A.

What is the ratio v^2/c^2? ($c = 3\times10^8$ m s^{-1}.) Compare the ratio in **e** with that from **d**.

Note This is essentially a calculation in relativity, which says that the electrical force between electrons, moving past you at velocity v, will be diminished by a factor v^2/c^2 of its normal value. The force between the positive ions, which are not moving past you, is unchanged. The result is that the tiny relativistic correction for electrical forces between moving things (a factor of 10^{-27}) shows up as an observable 'magnetic' force, the nearly equal and huge electrical forces almost cancelling out.

Many people think that relativity is only important for velocities near to the velocity of light. It is interesting that the only relativity experiment that can easily be done in a school laboratory, uses velocities slower than a snail's pace.

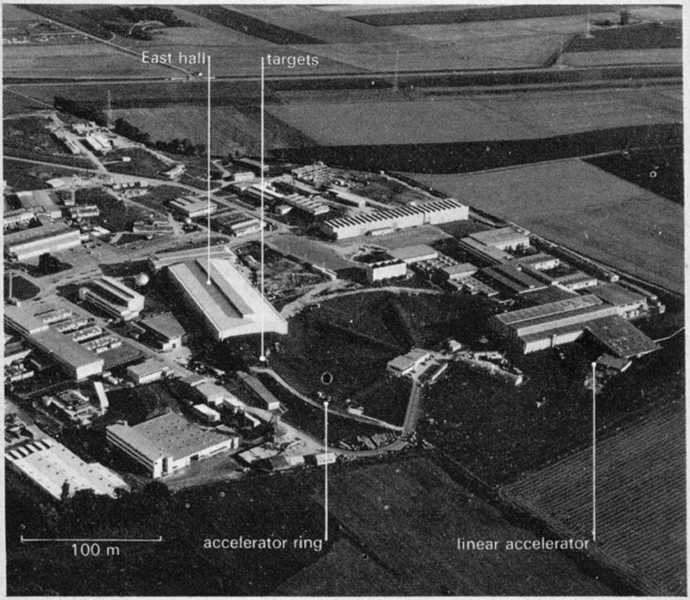

Figure 39
An aerial photograph of the CERN accelerator. *Photograph, CERN.*

54 Figure 39 is an aerial photograph of the proton accelerator at Geneva, run by CERN (the European nuclear research laboratory). The ring, most of which is below ground level, is the circular accelerator tube, while the buildings near the ring contain services and halls in which experiments are

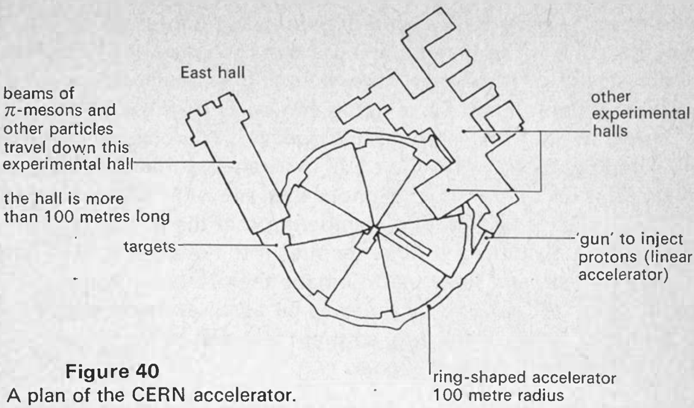

Figure 40
A plan of the CERN accelerator.

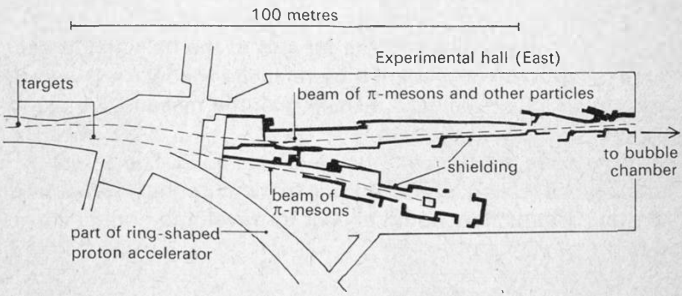

Figure 41
A plan showing two of the beams set up in the East Hall of CERN. The beams are shown by broken lines. The many magnets which bend and focus the beams are not shown.

done on beams of particles coming from targets placed in the proton beam in the ring. Figure 40 is a plan of the accelerator and the layout of the buildings.

Figure 41 is a plan of the layout of experiments in the East Hall (identifiable on the left of the ring in figure 39, and marked in figure 40). The plan shows one beam of π-mesons (nuclear particles with a mass in between that of a proton and an electron), running right through the experimental hall to a bubble chamber beyond the hall. (The chamber is not shown.)

π-mesons are unstable, and live on average for only 2.5×10^{-8} s (the lifetime being measured when the particles are at rest, or nearly so). How far will a π-meson which lives for this time travel, if it is going as fast as possible, at the speed of light (or nearly at that speed)? You can see that the experimental hall is at least 100 m long, and that the path laid out for the mesons is longer than that. (To check the scale, look at figure 39, remembering that the accelerator is 200 m in diameter.) It does seem as if the designers have been very foolish, and have tried to make the mesons travel between ten and twenty times as far as an average meson could go before decaying, so that there will be very few reaching the bubble chamber.

In fact, the set-up is not a very expensive way of wasting a good beam of mesons. Because they are travelling so fast, they seem to us to live longer, and the effect is big enough for most of them to get to the far end of the hall. This is the time dilation effect predicted by relativity theory. In this case, the effect is by no means a small one: the mesons live *much* longer than they 'ought' to.
(The mesons don't 'think' they live over-long. From their point of view, the length of the experimental hall seems to be the few metres they could expect to travel in a normal lifetime.)

55 To find out if a child has discovered the Doppler effect, you can ask him or her to 'make a noise like a racing car'. Why does a racing car seem to make a sound which falls in pitch as the car goes past?

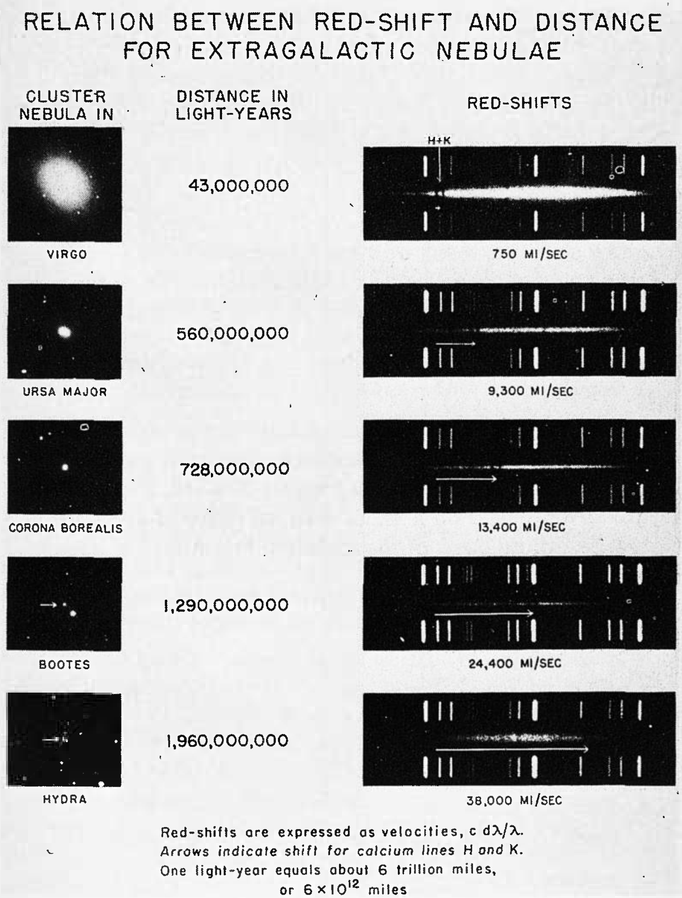

Figure 42
Doppler shifts of calcium absorption lines, for several galaxies.
Photographs, Hale Observatories.

56 Figure 42 is a famous set of photographs of some distant galaxies, together with their spectra. It is very difficult to photograph the spectra, because there is so little light, but each contains a perceptible pair of dimmer patches, the result of absorption of light by the element calcium. The wavelengths at which the absorption occurs are shifted to the red, long-wave, end of the spectrum by varying amounts, indicated by the arrows. If the shift of wavelength is a Doppler shift, the result of these galaxies moving away from us, the velocity of recession can be calculated from the shift. Note: If the frequency is divided by a factor k, the velocity v is given in terms of the velocity of light c by

$$k=\sqrt{(1+v/c)/(1-v/c)}.$$

Table 1 gives the velocities calculated on this assumption, together with estimates of the distance of each galaxy. (The distance estimates are sometimes revised, those given below being based on a major revision made in 1958. They are made on the basis of the apparent brightness of each galaxy.)

Galaxy in constellation of	Distance/light years	Speed/km s^{-1}
Virgo	0.4×10^8	0.12×10^4
Ursa Major	5.6×10^8	1.40×10^4
Corona Borealis	7.3×10^8	2.14×10^4
Boötes	13.0×10^8	3.9×10^4
Hydra	20.0×10^8	6.1×10^4

Table 1
Distance and speed of recession of some galaxies.

Plot a graph of speed against distance. What might the graph make one suspect about the Universe as a whole?

Answers

1 Photographs to be reproduced by printing are converted into a pattern of tiny dots. From a reasonable distance, your eye does not distinguish individual dots, but sees a pattern of light and shade which depends on the density of dots in the different areas of the picture. The process exploits the lack of resolution of the eye. Some of the later French Impressionists, particularly Seurat, painted in tiny dots of colour, using only a few different colours which, from a distance, give the impression of a wider range of colours. More recently, some artists have found themselves interested in the process of mechanical reproduction, and have painted pictures as if they were large scale reproductions, making paintings of printed paintings, as it were.

2 The ripples do not affect one another, in the sense that a ripple which on its own would spread in a certain way does so even though other ripples cross part of it. The ripples do affect one another in the sense that the motion of the water at a place where there are waves from two sources is the sum of the motion from either wave alone. To a physicist, it is this simple adding together of motion from two or more waves at the same place which is implied by the use of the term 'superposition'. In shallow water, a wave may not have too great a height, or it will collapse into a 'breaker'. It can happen that two waves, each too low to break, add together to give a wave which will break. Then the combined effect of the waves is not the same as the sum of their separate effects.

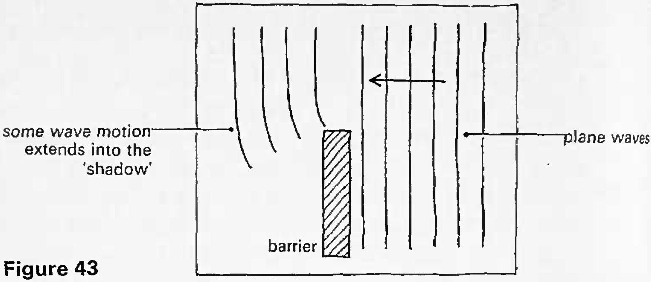

Figure 43

3 *Figure* 43 shows what the waves might look like. A similar effect can be seen in the aerial photograph, figure 3.

4 The widening of the lamp seen through a narrow crack is the result of *diffraction*. Any wave motion going through a hole not too large compared with the wavelength spreads out into the region which would normally be in shadow. Because the waves spread out, they seem to your eye to come from a wider source than they really do.

5 The letters on a number plate are of the order 0.1 m apart, the detail in the numbers being on a scale of the order 0.05 m, or less. The details can certainly not be distinguished at 100 m, but they can be distinguished at 10 m. At 50 m, detail on a scale 0.05 m subtends an angle of 10^{-3} radian at your eye.

As a rough estimate, the wavelength of light will be some 10^{-3} of the diameter of your eye pupil (see questions 17 and 18). Actually, this is something of an overestimate, because the ability of the eye to see detail is limited by factors other than diffraction, such as the coarse-grained structure of nerve endings in the retina, and the optical defects of the eye itself.

6 In the second picture, the children might move, although if they are playing on a seat, they are not likely to move very quickly. At 1/30 s, the picture might be blurred, but at 1/120 s, it should be sharp even if they move a little during the exposure. The exposure lasts for a quarter of the previous time, so to let in as much light altogether, and expose the film correctly, the

An exposure time of 1/500 s would be perfectly adequate for photographing running children. (At 1 m s^{-1} running speed they would only move 2 mm in this time.) For a sharp picture, it would be best to photograph the race with children running towards the camera, rather than across the field of view. The aperture suggested is about right, too, for a correct exposure. But at *f*/4, the *depth of focus* of the camera would be limited, so that the others in the race might be out of focus.

7 At first, as the lens opening is reduced, the effect of the outer parts of the lens, which do not usually focus light at the same place as the inner parts, is reduced. Since the depth of focus increases, more of the picture or image is sharp. However, if the aperture is small enough not to be a very large multiple of the wavelength of light, diffraction effects start to be important, so that the image of a point of light is a fuzzy circle of appreciable size. (At large apertures, the diffraction effect is still there, but is too small to see in a photograph.)

8 **a** Nearly 5×10^{-7} m.
b The first few fringes are alternately bright and dark. Because of the range of wavelengths present, outer fringes look blurred, or coloured if white light is used. If the spacing of bright fringes is estimated, the wavelength obtained is a rough average over the range of wavelengths present.

9 **a** The fringe spacing will be halved. The wavelength, now 2.5×10^{-7} m, is in the ultra-violet part of the spectrum, so the fringes would be invisible to the eye, though a photographic film could record them.
b The fringe spacing now returns to its previous value. In interference or diffraction, the *angle* at which an effect is seen depends only on the ratio of the wavelength to the dimensions of the slit or aperture.
c The electron wavelength is 5000 times smaller than the wavelength of the visible light used, so to get the same fringe spacing, the slits would have to be 5000 times closer. But

fringes 250 times closer together are said to be acceptable, so the slits need only be 20 times nearer together, with a spacing of about 0.005 mm.

Such an experiment has actually been done. See Unit 10, *Waves, particles, and atoms.*

10 **a** Figure 44 illustrates the regions in front of the loudspeakers where the sound heard will be loud or quiet. The sound will be quiet at places whose distance from one speaker is an odd number of half-wavelengths different from its distance from the other speaker.

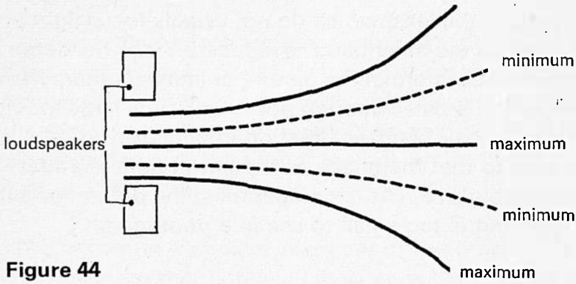

Figure 44

b The experiment only works well out of doors, on a sound-absorbing surface, because if the sound is reflected from walls or other surfaces, still more paths become available, and further interference patterns can arise. One loudspeaker in a room can give interference effects, as there are quiet spots at which two waves, one reflected from walls, floor, or ceiling, arrive out of step. In general, there will be waves coming from several directions at once, but unless the room has a very irregular shape, there will still be quiet spots where a number of these waves superpose to give little net effect.

11 No, the anti-heckling device will not work. The only hope of success would be with sounds so low-pitched that their wavelength is large compared with the distance between the speaking man and the anti-heckling device. Even so the device would have to be carried close to the heckler, by which time he would have made his point, very probably. Because the lowest frequency of speech tones is about 300 Hz, wavelength about 1 m, the idea is hopeless.

12 There is a lot wrong with this simple-minded plan. The sources of light are quite independent, their oscillations having no fixed phase relationship. Indeed, each is composed of many more or less independent atoms in a hot filament. In fact the problem is how interference is ever achieved by combining light from a filament lamp. One way to do it, as Young did, is to split the wave front with a pair of slits, and to use a source so narrow that the interference patterns from different parts of it fall pretty well in the same place. Finally, the small wavelength of light means that any effects will be small, much too small to produce a decent sized dark area.

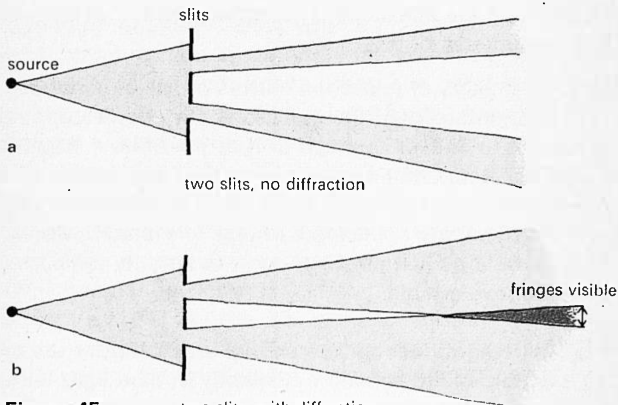

Figure 45

13 Figure 45*a* shows light going through two slits, but travelling in straight lines, with no diffraction effects. The beams never overlap, so interference is impossible. But because the slits are narrow, light is diffracted in going through them, and at a sufficient distance from the slits there is some overlap, where interference fringes can be seen. The design of a good two-slit experiment is a matter of careful compromise between slit spacing and slit width, source width, and the two large source-to-slits and slits-to-eye distances, so as to obtain clear, bright, distinct fringes.

14 Any telescope, looking at a distant point source, gives a response over an angle of the order of the ratio *wavelength/aperture diameter*, the angle being in radians. Radio waves have a much bigger wavelength than light waves, and a radio telescope must therefore have a bigger aperture than an optical telescope, if it is to approach the optical telescope in direction-finding ability. (In practice, radio telescopes are still much worse than optical telescopes in this respect.)

One of the factors limiting the size of a telescope mirror is the accuracy of its reflecting surface. The surface must not have irregularities bigger than a fraction of a wavelength. This is a particular problem for optical telescopes, and to grind and polish an accurate, smooth mirror, 5 m across, is no mean feat. It is also a problem for designers of radio telescopes, especially if the telescope is to operate with wavelengths of a few centimetres or less.

Large diameters of telescope mirrors are also demanded by the very small intensity of the radiation being detected, so that it is important to collect as much of it as possible. A few years ago it was estimated that all the information then obtained from radio telescopes throughout the world was based on an amount of energy collected by these telescopes which, in one lump, would be just about enough to let a fly jump its own height above a table.

15 Blue light does have a shorter wavelength than red light, so it is possible to see more distinctly in blue light with a telescope. But the advantage is not very great, since the visible range of wavelengths varies by less than a factor of two. Perhaps more important is the need to know more about what kind of light is being emitted by a star, and photographs in blue and in red light together give some of the information that would be obtained by the slower process of obtaining a spectrum. For instance, hot stars emit light which has more blue in it than the light from cool stars. A good proportion of spiral galaxies have bluish spiral arms and reddish centres, and it has been suggested that the arms contain hot, young stars, while the centre contains older, cool stars.

16 A screen held near the blue focus (figure 7) will tend to carry an image with a bluish-white centre and fuzzy reddish edges. The image will be reddish-white with bluish edges, if the screen is at the red focus.

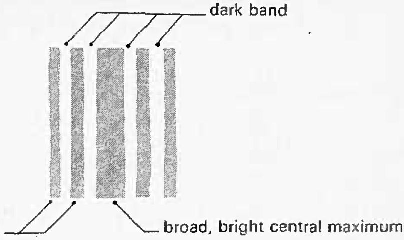

Figure 46

17 **a** Figure 46 shows what you might see on the screen: a central white maximum, with darkness on either side of it, and fainter bands of light beyond that. The outer bands would be coloured in white light, but in light having just one wavelength, would be bright or dark. Notice that the simple arguments used in this Unit about diffraction assume that the light going through the slit is parallel, or nearly so. That is why the screen was put so far off in this question. The theory for light which may converge or diverge follows the same general lines, but is harder.
b The centre of the pattern is bright because, at the centre, light from each part of the slit has travelled just as far as, and is in step with, light from any other part. Notice how the simplifying assumption that the light is parallel is being used.
c 10^{-7} m. See figure 47.
d One-fifth of a wavelength.

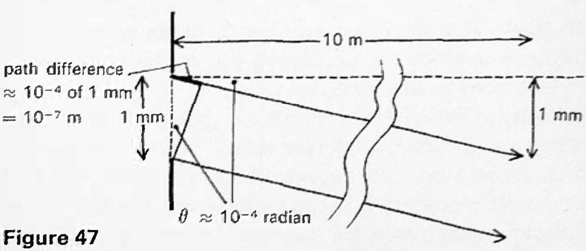

Figure 47

e *1* – almost as bright as the centre.
f 5 mm.
g Waves from the top part of the slit are out of step with waves from the middle of the slit. For every part of the slit, there is another part, half the slit-width away, from which the waves are out of step with those from the first. See figure 48.

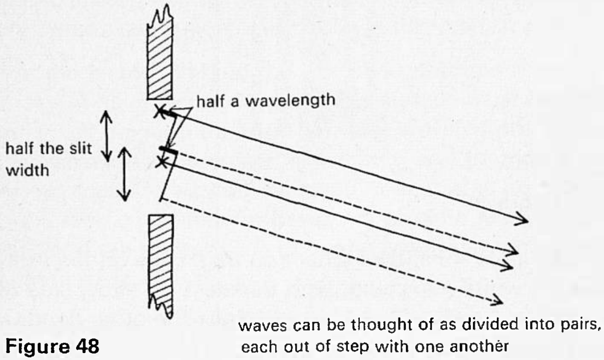

Figure 48

h 10 mm from the centre.
i, j The pattern would move upwards, by a hundred times as far as the source was moved down, the factor one hundred being the ratio 10 m/0.1 m.

18 **a** Wave energy is travelling in the direction of the arrow in figure 9. If the slit were removed, little or no energy would go in this direction.
b The wavelength of visible light is about 5×10^{-7} m, or half a thousandth of a millimetre. Making a slit about a thousandth of a millimetre across is not easy, but the hard thing would be to detect the tiny amount of light it would let through. The detection problem is made worse by the effect of diffraction, since the spreading out of the light means that even less goes in any one direction.
c At least ten imaginary sources, so that each is a wavelength or less from the next, and light would spread out very widely from any one alone. (It happens that, however narrow the source, light does not go equally in all directions, but this is a complication that must be left out of a simple theory.)

d $(d/2) \sin \theta$.
e No. There are path differences such as AC, or EF, in figure 10, between waves from such pairs of sources.
f The two sets of waves will superpose trough on peak and peak on trough, giving zero net amplitude. The screen will be dark in this direction.
g The same as in **f**.
h . . . the path difference $(d/2) \sin \theta$ between waves from each pair must be just half a wavelength (or an integral multiple of $\lambda/2$), for every pair, so that the angle θ at which there is first a dark patch is given by $\sin \theta = \lambda/d$ (or, in general, $\sin \theta = n\lambda/d$).
i AD $= \lambda$.
j At all angles less than the angle θ at which AD $= \lambda$, if the parts of the slit are paired off into pairs with half a wavelength difference in path to the screen, there will be some parts of the slit left unpaired. At the centre, no pairs can be found, since the light from all parts of the screen travels equally far from the slit to the screen, and at this angle $(\theta = 0)$ there will be most light.

19 **a** The pattern would have the same shape, but would be on a smaller scale. The reduction is in proportion to the reduction in wavelength.
b Figure 49 is an attempt at a sketch of the two-star pattern. Remember that you wouldn't know which star was responsible for each part of the pattern, so the rings would not keep a distinct identity, and a sketch showing clear, overlapping rings must be wrong.

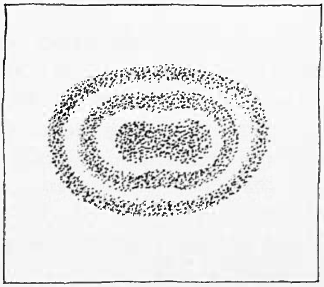

Figure 49

c The pattern would not be very different in size or general form. It would be hard to tell from the pattern produced by light from a point source.

20 a $\sin\theta \approx 0.21/25 \approx \theta$
$\theta \approx 0.008$ radian.
b About 160 light-years.
c *In favour:* greater resolving power; greater sensitivity to feeble radiation. *Against:* difficult to steer a large object accurately; difficult to hold it still in a wind; difficult to keep its shape accurate under deforming forces of its own weight and of wind; expense.

21 If the wavelength of visible light is 5×10^{-7} m, and the pupil of the eye is 5 mm in diameter (it is usually a bit less), the eye could resolve objects subtending an angle of 10^{-4} radian. This must be an estimate of the best it could do, and an actual eye might be worse by a small factor. Venus's diameter subtends an angle of rather less than 10^{-4} radian at the Earth, so it is not very likely that the eye could directly see that Venus had a finite size. Try looking up an explanation of the twinkling of stars if you are interested.

22 If objects 3 m across are detectable at a distance of 3000 m, the camera is able to resolve things subtending an angle θ of 10^{-3} radian. As a rough rule (neglecting a factor 1.22 for a circular aperture), the camera lens's diameter must be greater than d, given by $\sin\theta = \lambda/d$. Since θ is small, $\sin\theta$ is little different from θ, in radians, so it follows that the camera lens was at least 1000 wavelengths across.

Clearly, it was larger than this, since 1000 wavelengths is only about 0.5 mm. It seems very probable that the detail visible in an aerial photograph is more seriously limited by haze and by turbulence in the air between the ground and the camera, than it is by diffraction effects.

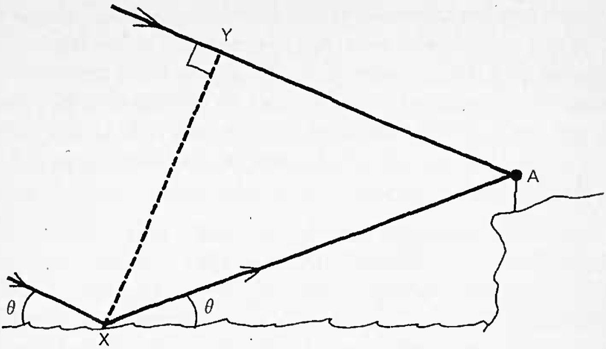

Figure 50

23 **a** Figure 50 shows the point Y. Waves from the Sun just reaching A are in phase with those at Y. AX–AY is an odd number of half-wavelengths, and reception at A would be at a minimum if it were not for the reversal of the phase of the wave as it is reflected at X.
b As the Sun sets, θ decreases, and so AX increases, so that the angle at which the wave is reflected at X is the same as that at which the waves reach A. AY also increases, since the angle at Y is a right angle. As the Sun sets, the *path difference* AX–AY increases. If it is, say, three half-wavelengths in figure 50, it will soon be four half-wavelengths, and the signal at A will be a minimum. The signal at A will alternately be a maximum and a minimum as the path difference progressively becomes larger, according to whether it is an odd or even number of half-wavelengths.

24 **a** 0.6×10^{-3} *radian.*
b We estimate the 'diameter' of the ellipse to be about 50 000 light-years, subtending an angle of 10^{-4} radian at a distance of 500 million light-years.
c The ratio of the wavelength to the diameter of the telescope must be of the order 10^{-4}, from **b**. (See question 22 and its answer.) At a wavelength of 0.15 m, the diameter comes to 1.5 km; not far short of a mile. In fact, the plot shown in *figure 15 was made with the 'one-mile radio telescope' at*

Cambridge University, but this telescope is *not* a dish a mile across. It consists of smaller, separate dishes which are moved around over a region about a mile across, the signals from which are combined so as, in total, to simulate the presence of a much larger single telescope. A dish one mile in diameter would not be a practical proposition on mechanical grounds, leaving aside the question of its expense.

Cygnus A is the second 'brightest' radio object in the sky, despite its great distance. It was at first thought that the optical source, midway between the two patches of radio intensity seen in figure 15, was a pair of colliding galaxies. When high resolution radio measurements showed that the source consisted of two opposite regions, well away from the visual object, views changed, and at the time of writing, the view held by most people is that the galaxy in the middle of the picture has exploded, emitting a pair of rapidly moving clouds of material in opposite directions, and that these are the radio sources. But it would not be surprising if, next year, another hypothesis were put forward.

25 **a** The paths of all the waves from all the slits are equal. If the light falls as a plane wave on the screen, the slits are like sources in phase, so the amplitude at the screen (if it is far off) from every slit is simply added to that from every other slit. This is true for every wavelength present, so the central patch is white.
b At the angle θ_1, for light of wavelength λ, the light from each slit travels a distance λ further than the light from the previous slit. All waves from the slits start in phase, so they all arrive in phase, each being just one cycle behind or in front of the wave from the slit next door.
c There can be a bright patch at angle θ_2, for two reasons. If the path difference at angle θ_1 between adjacent slits was one wavelength, at the larger angle θ_2, it can be two wavelengths as in figure 16*c*, giving again a bright resultant intensity. But θ_2 could also be the angle, corresponding to θ_1, but for a longer wavelength, such that the path difference between light from adjacent slits is one longer wavelength.
d $n\lambda = d \sin \theta$, where θ is any angle at which the intensity

has a maximum, λ is the wavelength, n is an integer, d is the slit spacing, and $d \sin \theta$ is the path difference between light from adjacent slits, such that it is equal to a whole number n of wavelengths.

26 When every other line is blocked out, the slit spacing doubles, so the maxima appear at angles whose sines are half what they were previously. Half as much light goes through the grating, so the maxima will be less bright. (They may not be half as bright, because they may not have the same angular width as before, and because there could be more of them in the range of angle from 0 to 90°.)

If the *regularity* of the grating is lost, because slits are blocked out at random, the whole cooperative effect of the grating is lost, and the light from each slit no longer combines additively with the light from every other slit at certain sharp angles. In fact, the most likely thing to see would be the broad, hazy diffraction pattern of a single slit, brighter than that from one slit in proportion to the number of slits left open. If there were some mean spacing not too large compared with a wavelength, the 'grating' could produce a diffuse halo at an angle corresponding to that spacing. You should be content with having thought of any point like this; especially if you noticed that losing the grating's regularity would be important.

27 **a,b,c** You should have decided that the grooves are rather a large number of wavelengths apart. Tilting the record so that the light grazes its grooves is a way of making the grooves look effectively closer together. X-rays have a wavelength of the order of 10^{-10} m, several thousand times shorter than the wavelength of visible light. To use an optical grating with X-rays, it is necessary to use angles of only a small fraction of a radian.

28 The fact that there is a *clear, sharp, yellow* flame image in the first order spectrum tells you that the burning paper, unlike the match, contains a lot of light of one sharp wavelength, in the yellow. The wavelength is actually one emitted by sodium.

29 If the slats repeat every 0.1 m, the wavelength should be of that order, so the frequency will be 3400 Hz or lower.

30 Here are some points you might have mentioned. A full discussion could be a very long one.

In diffraction at a single slit the light does not 'just spread out', but has maxima and minima at angles where waves from different parts of the slit combine constructively or destructively. The pattern from two slits contains extra bright and dark bands which are the result of combining the light from the two slits, but it also involves diffraction, because the light from the two slits must spread out enough to overlap.

If one wished to distinguish interference and diffraction, it might be best to reserve the term interference for cases like a pair of small loudspeakers, whose effect is the sum of waves from point sources, the waves having a finite phase difference. Diffraction could be kept for the case where, as for a wide single slit, waves with a continuously varying phase difference are to be combined. Young's two-slit experiment involves both, and so does a diffraction grating. Both are special cases of the principle of superposition: the principle of calculating the combined effect of waves by adding their amplitudes, taking due account of their phases.

31 **a** One wavelength, 5×10^{-7} m.
b Two wavelengths.
c 5000 wavelengths.
d The path difference is $d \sin \theta$, and if θ increases, so does this length of the path along a tilted line, since d is constant.
e $2\lambda(1+1/10\,000)$.
f $5000\lambda(1+1/10\,000) = 5000\lambda+\lambda/2$.
g Zero.
h Zero.
i Zero.
j The angle whose sine is $0.5\ (1+1/10\,000) = 0.500\,05$.
k $5 \times 10^{-7}\ (1+1/10\,000) = 5.000\,5 \times 10^{-7}$ m.
l 10 000.

32 **a,b** You should find that you can tell which of two notes are playing together in a composite sound, though you may . . have difficulty for some pairs, and people differ in their ability to analyse sounds. But everyone sees just one colour (normally purple or mauve) when red light and blue light enter the eye from the same direction and fall together in the same place on the retina.

Another example brings out the difference even better. It is possible to find yellow filters (car direction indicators are quite good) which transmit light that a grating will analyse into red and green, but which cannot be distinguished by eye alone from sodium light, which consists of (very nearly) just one wavelength. The two sorts of light are different, but look just the same.

The process of analysing a complex waveform into constituent, single-frequency components is called Fourier analysis, after the man who showed that the process is always a possible one. Your ear, and a diffraction grating, are both Fourier analysers of different kinds: your eye is not. We mention this because the idea of Fourier analysis has proved to be very useful in physics and engineering, in most problems where something oscillates.

33 The lines, which are there because elements in the outer part of the Sun (or in the Earth's atmosphere) absorb radiation at those wavelengths, can be used to find what elements are present. The spectrum of each element is used like a fingerprint, by matching it to one of the patterns of lines in the Fraunhofer spectrum.

34 **a** There is an energy level of sodium at an energy about 3.3×10^{-19} J above the lowest energy level.
b At a wavelength of about 3×10^{-6} m, which is in the infra-red. In principle, such radiation can be detected by its heating effect, and there do exist some rather sensitive detectors for this purpose. A glass transmission grating would be likely to absorb such radiation (though glass transmits some infra-red, mostly of rather short wavelength), but a reflection grating would not suffer this disadvantage whatever the wavelength.

c Using $E = hf$, the energy difference is close to 2.6×10^{-26} J, a little less than 2×10^{-7} electronvolt. The wavelength is 7.5 m, so the wavelength could be measured by a large-scale interference method, using a pair of transmitters, or by setting up standing waves between a reflector and a transmitter. Actually, the frequency is not too high for the oscillations to be simply counted, and that would be by far the most accurate way of finding the frequency.
d $f = E/h$; $\lambda = c/f$, giving $f = 4.3\times 10^{20}$ Hz, $\lambda = 7\times 10^{-13}$ m. Measuring the frequency directly would mean counting the oscillations, which in this case is out of the question. Measuring the wavelength is not so hopelessly impractical, but requires the gamma rays to be diffracted from an object or objects with dimensions of about 10^{-12} m. It would be necessary to use atomic nuclei. In practice, measuring the energy is much the best way, in general.

35 This is not the sort of question with one 'right' answer. There are many aspects to discuss, and there is no reason to expect you to be equally interested in them all. We hope you will notice that there is room for disagreement among scientists about how secure their ideas are, and about the extent to which the information they have should be treated with caution.

Huygens, in a manner common among those who seek to persuade others that a new and largely unexplored idea is worth pursuing, sounds remarkably confident. Notice how he argues from simple phenomena (the light beams do not affect one another) to a general point of view. This is frequently found in the arguments of scientists as they begin to study a new field. Their problem is to find what sort of model to employ; only later on do the details seem to matter very much.

Fresnel is also acting partly as a persuader. Nearly every argument in favour of a physical theory claims that it is 'simple', but just what 'simple' means in some of these contexts takes some unravelling. Is the idea of adding multitudes of wavelets from different parts of a source, taking account of the phase of each, a simple one? You might also doubt whether *any* theory can decently be said to be 'probable' in any ordinary sense, when tomorrow it may be completely overturned and replaced by a new theory which gets nearly the same results in a quite different way. You might think that, whatever the weight of evidence, every theory is vulnerable, so even the likelihood of it lasting until next year cannot easily (or perhaps at all) be estimated from the weight of evidence that seems to support it.

Mach is an interesting example of extreme caution. You may notice how he merely claims that the wave model is *convenient* for *summing up* what happens. There is no place for 'true' or 'certain' or 'likely' in Mach's vocabulary. Some people still agree with this general position; others think it leaves no scope for theories to be found wrong and so to be rejected; and some object that Mach's view hides or distorts the immense predictive power of theories, especially when those predictions surprise the creator of a theory himself.

Debates about such matters still go on, especially in parts of science where people are still wondering what *sort* of theory it might be sensible to try. But there are those who think it all rather sterile and pointless. We expect you will want to form your own views.

36 **a** Briefly, little waves start out from the places such as K on the mirror (figure 19) when the incoming wave reaches those places. One starts from A and travels out in a circle, radius AN, in the time the part of the wave front at C takes to get to B, so that CB = AN. The reflected wave front is the flat surface common to all the little waves from different places on the surface. Because CB = AN, the angle of reflection is equal to the angle the incoming wave makes with the surface (angle ABN = angle BAC).

Huygens introduced the idea of imagining a wave as the net result of many little waves from imaginary point sources along the wave front, an idea which has also been used in discussions of diffraction in this Unit. The little waves are often called 'Huygens's wavelets'.
b The explanation follows the same lines as **a**, except that in the water the waves go more slowly than in air. As the

wave front arrives along the boundary, wavelets start out one after the other into the water. The wavelet starting at A travels a distance equal to AN, in the time the part of the wave at C takes to get to B, along the longer distance BC in air, where the speed is greater. Clearly CB/AN = c/v, where c and v are the two speeds. Since CB/AB is the sine of angle CAB, and AN/AB is the sine of angle NBA, the ratio c/v is also the ratio of the sines of these angles. That the ratio of the sines is constant for a given pair of materials is Snell's Law.

37 **a** A path like ABE is the quickest.
b $FC/c - BG/v$.
c $FC/c = BG/v$
$FC/BG = c/v$.
d $FC = BC \sin i$; $BG = BC \sin r$; $FC/BG = c/v = \sin i/\sin r$.

38 **a** I	(coulombs per second).
b I/C	(volts per second).
c VC/I	(seconds).
d VC/I	(seconds).
e VCu/I	(capacitors).
f VCu/I	(batteries).
g VCu	(amperes).
h I/Cu	(volts).
i Zero.	
j Zero.	
k Zero.	
l VCu	(amperes).
m I/Cu	(volts).
n u	(capacitors per second).
o I	(amperes).
p Iu	(amperes per second).

q Yes, if there were many of them, except for the first and last one or two.

r $1/Cu^2$	(volts per ampere per second).
s $1/Cu^2$	(henries).
t $u = 1/\sqrt{(LC)}$	(stages per second).
u $v = 1/n\sqrt{(LC)}$	(metres per second).
v $v = 1/\sqrt{L^*C^*}$	(metres per second).
w 10^{-6}	(second).

39 **a** Yes. The magnetic field well inside any shape of solenoid is uniform, if it is evenly wound and is long enough.
b In the direction 0Z.
c No, except near the ends.
d N/a.
e I/a.
f $\mu_0 I/a$.
g Hardly at all. Only the short edges are missing, and if $d \ll a$, their absence has little effect at the edges.
h $\mu_0 I/a$.
i d.
j $\mu_0 Id/a$.
k $\mu_0 d/a$.
l $\mu_0 d/a$ (volts).
m $\mu_0 d/a$ (henries).
n The bottom set of plates is positive with respect to the top.
o Vertically upwards, along 0Y.
p No, except near the edges.
q $\varepsilon_0 a/d$.
r $1/\sqrt{\varepsilon_0 \mu_0}$.
s No.
t No, so long as d is small compared with a and the length of the plates.
u No, unless the material were a conductor, so that the plates were in effect connected together around the outside.
v Yes, because the constants ε_0 and μ_0 refer to empty space, and become $\varepsilon_r \varepsilon_0$ and $\mu_r \mu_0$ respectively when the space is filled with material. For materials like polythene, ε_r is a small number between 1 and 10. For iron-like materials (ferrite, for example), μ_r can be as large as 1000, but for most substances it is very little different from 1.
w 3.1×10^8 m s^{-1}.
x 3.0×10^8 m s^{-1}.

40 **a,b** The charge must have come from the battery. If the parts of the plates behind the leading edge keep a steady charge, current must flow along the plates so as to deliver fresh charge to the leading edge, taking charge from the battery (at one terminal) at a rate equal to the rate of delivery. With the battery connected as in figure 33, a (conventional) current flows from the battery to the leading edge along the lower plate, which acquires positive charge. The upper plate acquires negative charge, and a (conventional) current flows from the leading edge to the battery in this plate.

c It.

d avt, since the leading edge travels a distance vt in time t.

e $E = I/\varepsilon_0 av$.

The time t cancels. Physically, t is not involved because in twice as long a time, twice as much charge covers twice the area, giving the same charge density and the same field E. It is this which makes it convenient to argue in terms of E.

f $B = \mu_0 I/a$.

You could think of the one-turn solenoid in figure 35*b* as part of a longer one with one turn in each distance a. Or you could argue that a current I/N in the N-turn solenoid ought to give the same field as a current I in the one-turn solenoid.

g $B = \varepsilon_0 \mu_0 Ev$.

h For a given E-field, the charge density on the plates is fixed. But a faster speed v requires this same charge to be delivered more rapidly, so a bigger current is needed. A bigger current implies a bigger B-field.

i vt.

j $Bdvt$.

k Bdv.

l $V/d = E = Bv$.

m $v^2 \varepsilon_0 \mu_0 = 1$

$v = 1/\sqrt{\varepsilon_0 \mu_0}$.

n $v = 3 \times 10^8\ \mathrm{m\ s^{-1}}$.

o The units of $\mu_0 \varepsilon_0$ are $\mathrm{NA^{-2}/N\ m^2\ C^{-2}}$.

Cancelling the unit N, and writing $\mathrm{C\ s^{-1}}$ for the unit of current A gives units $\mathrm{m^{-2}\ s^{-2}}$, the unit of charge C also cancelling. Thus $\mu_0 \varepsilon_0$ has the units of the reciprocal of the square of a velocity, and $1/\sqrt{\varepsilon_0 \mu_0}$ has the units of a velocity.

p The speed would be the same, whatever changes were made to the dimensions of the plates, or the battery voltage, because all these quantities cancel out in the final expression for the speed. It would be correct to take this as a hint that the particular shape of plates chosen is rather unimportant; indeed, so long as the fields in one part of the plates do not affect the state of affairs in other parts, which means using long straight conductors, the speed is the same for any such shape of conductors, whether they are pairs of wires, or coaxial cables, etc. The speed is changed if insulating material is introduced between the conductors, because the charge on the conductors is then different, for a given voltage across the conductors, or *E*-field between them. Polythene, for example, in the space between the conductors in a coaxial cable, reduces the speed from 3×10^8 m s^{-1} to a somewhat smaller value, though still of the order 10^8 m s^{-1}.

41 **a** In a computer, numbers are represented by chains of electrical pulses. To add numbers (or perform other operations) the pulses must arrive together at the same time at the right place. There will be trouble if one set of pulses is delayed because it travelled further. But the pulses cannot propagate along wires faster than the speed of light, so if the electrical connections in the computer are some metres in length, there will be delays of the order 10^{-8} s. This may not seem much, but it is important, because computer designers would like to be able to use pulses recurring as often as 10^9 every second if they could, so that complex sequences of operations would take as little time as possible.
b A long length of coaxial cable could be used, but it would need to be about 100 km long, so this is not a very practical suggestion. A row of capacitors and inductors, connected as in figure 29, might serve, but would be likely to distort the pulse. To get a delay of 2 ms within a reasonable-sized piece of apparatus, one needs a propagation speed of the order of 10^3 m s^{-1}, which suggests using sound waves in water or in a solid. This method has been used, employing a loudspeaker and a microphone in a tank of water or other liquid.

There is, of course, no need to use wave propagation to obtain the delay. If the recording and replay electromagnets of a tape recorder were 10 mm apart, the tape would need to travel at 5 m s^{-1} to give the desired delay. This is perfectly practical, especially if the tape is replaced by the surface of a rapidly rotating drum or disc. You may well be able to think of other ways of doing the job.

42 a About 0.1 s.
b If the wires are 10 m long, the time delay is about 3×10^{-8} s. The filament takes at least 0.1 s to warm up, so it would be no use looking for this delay, in the light emitted by the lamp.
c World-wide television began in the late 1940s, so a planet about 50 light-years from us would not know until the year 2000. There are a fair number of stars within this distance, though not so many that it would be reasonable to expect one of them to have a planet which had intelligent life on it. We could not expect a reply before the year 2050 or so.
d The wavelength is 6000 km. As Britain is about 1000 km from north to south, at most there is a phase difference of less than a sixth of a cycle between voltages at different places.

43 a Guessing the power at 100 W, and assuming an inverse square law, at the distance of the Earth from the Moon there is about 6×10^{-17} watt per square metre.
A dish 10 m in diameter has a collecting area of about 80 m^2, and so it collects a power of some 5×10^{-15} W.
b The use of an inverse square law implies that the space probe's aerial is not directional. Since it is almost certainly directional, the estimate above had better be revised upwards by perhaps a factor of 100.
c No.

44 a At 100 MHz the wavelength is 3 m, which means that the distance from transmitter to receiver cannot be very large, both because of the curvature of the Earth, and because of hills that might lie in between them. At 1 MHz, the wavelength is 300 m, and the range can be much greater, so fewer stations are needed, though each will generally need to have a higher power so as to achieve a greater range.

b Your guess is as good as ours. Britain contains some 100 transmitters, with an average power of perhaps 10 kW, making 1 MW of power. The U.S.A. and Canada must amount to at least ten 'Britains-worth' in power, and so must Europe as a whole. Perhaps 100 'Britains-worth' of power is about right for the world as a whole, giving an estimate of 100 MW for the total radio broadcasting power. This ignores television altogether, and it seems probable that powers emitted for television and radio are comparable in order of magnitude. In a day, the energy emitted comes to some 10^{13} J.

45 In both cases the signal is carried on a rapidly oscillating 'carrier' signal, the amplitude or frequency of which is modified to convey the programme information. In both cases the signal is conveyed by an electromagnetic wave; in *1* the wave goes through space from the transmitter to the receiver, in *2* it goes in the space between the conductors of the cable. The second signal travels a little slower than the first, but only because the cable has a solid insulating material in it.

Anybody in the radio beam can pick up the signals in *1*, but only a set connected to the cable can obtain the signal in *2* (indeed, outside the cable there is no evidence that a wave is going along it). The final results are very similar, because the propagation of the signal is similar in the two cases. The sending of a signal along a wire is also the sending of an electromagnetic wave, though if one is thinking more in terms of switching on a light or of using the telephone, this aspect may escape attention. The currents in the wires can be thought of as running along at the edges of the electromagnetic wave. Indeed, for waves sent over the conducting surface of the Earth, there are currents in the ground below the wave.

46 a Strong sunshine may amount to as much as 1 kW m^{-2}. This gives $E \approx 600$ V m^{-1} and $B \approx 2 \times 10^{-6}$ T.
b We guessed that the steak would have an area of 0.01 m^2, that it must be warmed by 200 K to cook it, and that 1 kg of steak needs as much energy as 1 kg of water to warm it by 1 K, that is, about 4 kJ. The power needed then comes to about 1400 W, which if it is to fall on the steak, requires the radiation to deliver about 1.5×10^5 W m^{-2}. The root mean square E-field then comes to about 7 kV m^{-1}.
c The walls of the oven do not become hot, and only radiation-absorbing materials are cooked in the oven. At infra-red wavelengths, much radiation is absorbed by the oven walls, and re-radiated into the kitchen. But if microwaves are used, the oven itself remains quite cool, and only things put in it are heated. A person's hands would absorb the radiation as easily as would a plate of food, but the oven, because it isn't hot, appears to be quite safe. For this reason a microwave oven must be provided with safety locks to prevent the cook's hands going into it when the radiation is switched on. You may have thought about suitable circuits for safety locks in Unit 6.

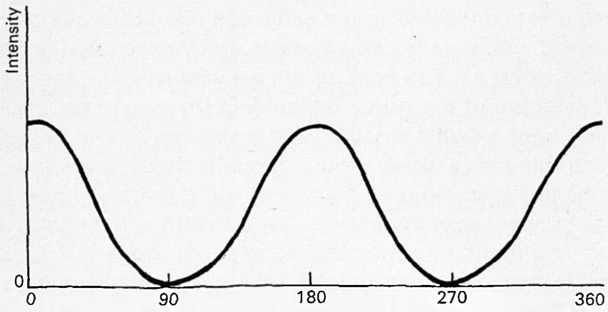

Figure 51

47 Figure 51 indicates how the intensity varies with angle. Like the energy dissipated in a resistor carrying alternating current, over one cycle, it is a graph like $\cos^2 \theta$, which has the same shape as a sinusoidal graph of twice the frequency, displaced upwards on the axes so that its least value is zero.
b The average value of $\cos^2 \theta$ is 1/2, so one polaroid

transmits half the light energy that falls on it, if it is 'ideal' (that is, non-absorbing apart from its effect on polarized light).

48 *You should find that reflections from electrical* conductors are not linearly polarized, or are less markedly so, but that reflections from insulating materials are linearly polarized. (Metals in general give 'elliptically polarized' reflections.)

49 The point here is for you to try inventing explanations of your own. If you want some guidance, or if you want to know if your suggestions are at all like anyone else's, look up *'double refraction' in a book.*

50 **a** The light from the blue sky, scattered through 90°, is at least partly polarized.

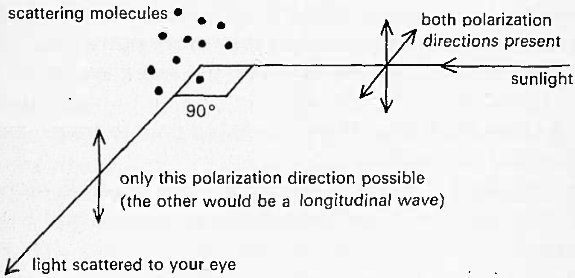

Figure 52

b *Figure 52 illustrates how you might decide on the orientation* of the polaroid. Light from the Sun passing at right angles to the line of sight can contain an electric field oscillating along the line of sight (at right angles to the light), but light scattered to your eye, through 90°, cannot contain this light with this direction of polarization, for such light would have a longitudinal electric field. It is assumed here that the scattering is by electrons responding to the electric field of the *sunlight, and emitting* scattered light as they oscillate. Knowing the direction of the *E*-field in the scattered light, you can determine the orientation of the fine grid of polymer molecules in the polaroid (parallel to the *E*-field when the *amount* transmitted is small).

51 The light reflected from each cow, and the electrical pulse along the wire, travel at the same speed. Einstein and the farmer were equidistant from the middle cow, and sketched it in the same position, so the sketches are simultaneous. The cows are unusually agile, for they seem to have reacted with essentially no delay. Einstein sees them jump back in a row parallel to the wire because the light from each cow is delayed by as much as the signal which makes it jump. The farmer sees them at an angle, because the time delay before the nearest cow is seen to jump is small, while there is a delay equal to the time for the electrical pulse to go along the wire and for light from the cow to come back to him, in the case of a distant cow.

This perhaps rather trivial question does not involve relativity, but it is like a relativistic argument in one way. Like such arguments, *because* light has a finite speed, two people seem to see *different* things, but when they work out what happened, it turns out that they saw the *same* events.

52 **a** More than 98 s. The pulse takes time to travel, and B has moved away from A.
b Yes. The time of travel of the radar pulse depends on how far it has to go, which will be bigger the faster A and B are separating.
c Yes.
d The pulse took 4 s for the round trip, giving half that time, 2 s, for it to go from A to B. This is what A would deduce from his clock readings alone.
e 6×10^8 m is what A would calculate from this 'radar' measurement.
f 100 s.
g 6×10^6 m s^{-1}.
h He should get just the same value.
i It should arrive after time t_B, since light goes at a finite speed.
j The signal delay factor is k, which depends only on how fast the travellers are separating and on the speed of light, from which they can find their relative velocity. If they are to get the same relative velocity, k must be the same for both.

k Divide the two equations, giving

$$t_B^2 = 98 \times 102.$$

l If you put $t_B = 100$ s in both equations, A's value for k will differ from B's.

53 **a** The charge is of the order of magnitude 10^5 C.
b If the charges were in small concentrated lumps, the force between them would have the order of magnitude 10^{20} N. (The more accurate method only alters this estimate by a factor of two.)
c The wires do not fly apart, because each contains exactly the same amount of positive charge as negative charge, carried on protons in the nuclei of its atoms.
d The ratio is of the order of magnitude 10^{-27}.
e The ratio v^2/c^2 is also of the order 10^{-27}.

54 Going at 3×10^8 m s^{-1}, a π-meson which lives for 2.5×10^{-8} s will travel 7.5 m. Not every π-meson lives for the average time, but the fraction living for a large multiple of the average time is small. In fact, a fraction $1/e$ (where e is the number 2.718...) lives for 2.5×10^{-8} s; a fraction $1/e^2$ lives for twice that time, and so on. The hall is about 150 m from target to bubble chamber, which is twenty times the distance 7.5 m. The fraction which would reach the bubble chamber if the lifetime were 2.5×10^{-8} s would be about $1/e^{20}$, or $1/10^9$, out of those which leave the target. Because of the time dilation effect, which is here large, not small, a good proportion of the original mesons does reach the bubble chamber.

55 As the car comes towards you, the peaks of the sound oscillation reach your ear more frequently than the rate at which the car emits them, because each has less far to go to your ear than the one emitted before it, since the car has moved forward in the interval between their emission. Similarly, as the car recedes, the note seems to fall in pitch.

The Doppler effect with sound differs from the Doppler effect with light, because sound travels at a speed which is fixed relative to the air, and to a moving listener sound would seem to travel more rapidly or more slowly depending on which way the listener was moving relative to the sound wave. But light seems to go at the *same* speed relative to anyone who observes it, however he or she is moving. The result is that the Doppler effect for light depends *only* on the relative velocity of source and observer, while the Doppler effect for sound has a part due to the source's motion through the air and another part due to any motion of the listener through the air.

56 The graph points lie reasonably close to a straight line, the speed of recession increasing linearly with distance. If it is fair to suppose that such a motion relative to us would be found in all galaxies (apart from small extra random motions), the inference is that the Universe of galaxies is expanding steadily. It does *not* mean that we are in an astonishingly privileged, if cosmically unpopular position at the centre, with every galaxy receding from us. Every one of a cloud of particles in a volume which is expanding steadily will seem to be receding from any other at a rate proportional to the distance between them.

To find out more about the expansion of the Universe, try Sandage, 'The red-shift'. Remember that when he wrote it in 1956, the scale of distances was not the same as is now believed to be correct. The estimated distances were increased by a factor of about two in 1958, as a result of further studies of distant galaxies.

Radio astronomy

Introduction

Fifty years ago astronomy was confined to observations in the visible region of the spectrum. Today it extends over most of the electromagnetic spectrum (from wavelengths of about 10^{-10} m to metre wavelengths). The existence of radio noise emission from outside the Earth was first detected in 1932 and significant developments in radio astronomy began to occur during the Second World War, due to the techniques developed in radio for war purposes. Now radio telescopes probe the heavens and play a significant part in the solution of problems in cosmology. One of the most spectacular recent discoveries has been that of some remarkable sources of radiation, called quasars, in 1961. Debate still continues about what sort of thing quasars are, how far away they are, and what could be going on inside them. By the time you read this article, no doubt there will have been further changes, as more evidence is obtained and interpreted.

Atmospherics

The development of radio astronomy has seen the construction of large bowl-shaped aerials, like that shown in figure 53 (and in figure 13 on page 18), used to collect and focus as much radiation as possible. These aerials, and others like them, are not only capable of being used for radio astronomy, but they can also be used for sending or receiving man-made signals. The Jodrell Bank telescope, shown in figure 53, has been used to track and to take messages from space probes, and similar though smaller dish aerials are used to transmit and receive television signals across the oceans through telecommunications satellites. It happens that radio astronomy and telecommunications have been connected since the birth of radio astronomy, for the first radio source outside the Earth was detected by a radio engineer, Karl Jansky. Jansky was investigating the general background noise detected by radio receivers. This noise is known as atmospherics or static.

Figure 53
The Mark I radio telescope at Jodrell Bank.
Photograph, The Guardian.

Karl G. Jansky

The paper which begins on the next page, first appeared in December 1932 in *Proceedings of the Institute of Radio Engineers,* **20,** 12; figures 55 to 57 are also taken from this paper. At the time when it was published, Jansky was working in New York at the Bell Telephone Laboratories. He was only 27 years old.

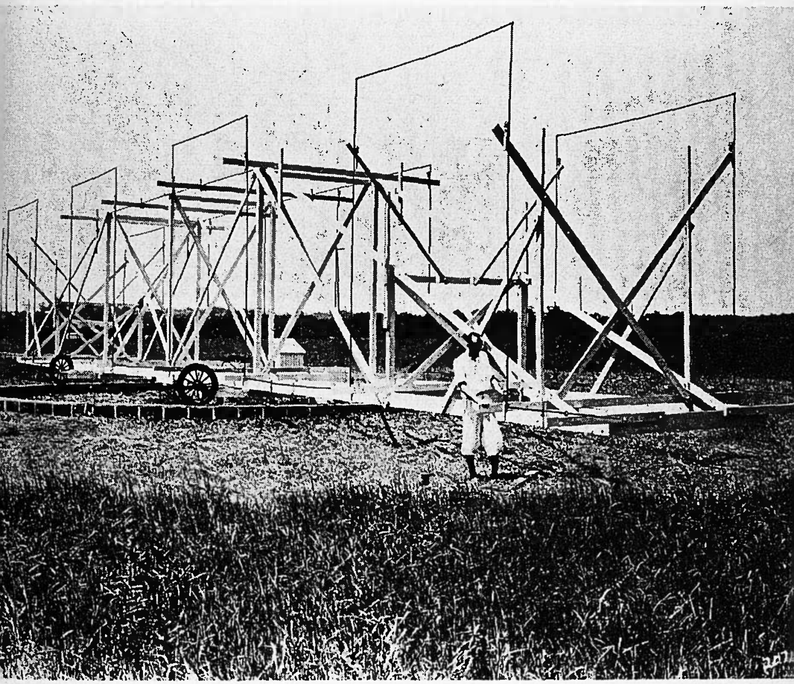

Figure 54
Karl G. Jansky and his 14.6 metre rotatable directional antenna.
Photograph, Bell Laboratories.

Directional studies of atmospherics at high frequencies

Description of apparatus

Since the middle of August, 1931, records have been taken at Holmdel, New Jersey, of the direction of arrival and the intensity of static on 14.6 metres. Fig. [55] shows a schematic diagram of the recording system. It consists of a rotating antenna array, a short-wave measuring set, and a Leeds and Northrup temperature recorder revamped to record field strengths. . . .

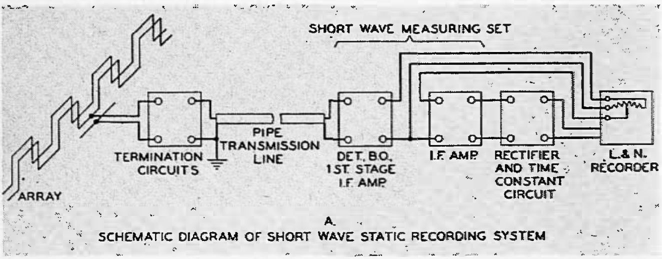

Figure 55
Schematic diagram of short wave static recording system.

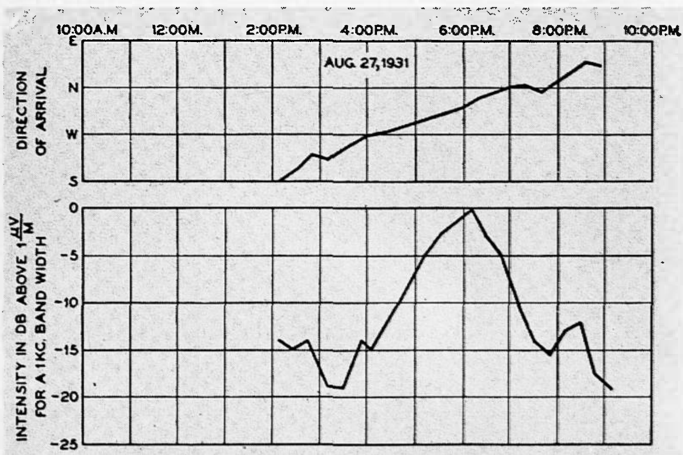

Figure 56
Direction of arrival and intensity of local storm-type static on 14.6 metres.

Results

From the data obtained it is found that three distinct groups of static are recorded. The first group is composed of the static received from local thunderstorms and storm centres. Static in this group is nearly always of the crash type. It is very intermittent, but the crashes often have very high peak voltages. The second group is composed of very steady weak static coming probably by Heaviside layer refractions from thunderstorms some distance away. The third group is composed of a very steady hiss type static the origin of which is not yet known.

The static of the third group is also very weak. It is, however, very steady, causing a hiss in the 'phones that can hardly be distinguished from the hiss caused by set noise. It is readily distinguished from ordinary static and probably does not originate in thunderstorm areas. The direction of arrival of this static changes gradually throughout the day, going almost completely around the compass in twenty-four hours. It does not quite complete the circuit, but in the middle of the night when it reaches the northwest, it begins to die out and at the same time static from the northeast begins to appear on the record. This new static then gradually shifts in direction throughout the day and dies out in the northwest also, and the process is repeated day after day. Fig. [57] shows the direction of arrival of this static for three different days plotted against time of day. Curve 1 is for January 2, 1932, curve 2 is for January 26, 1932, and curve 3 is for February 24, 1932. . . .

This type of static was first definitely recognized only this last January. Previous to this time it had been considered merely as interference from some unmodulated carrier. Now, however, that it has been detected it is possible to go back to the old records and trace its position on them.

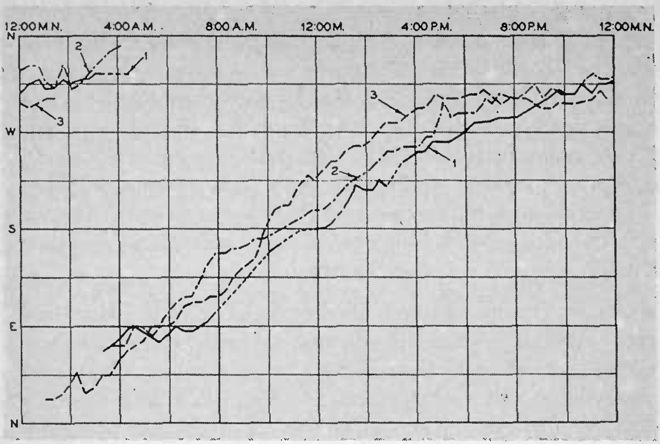

Figure 57
Direction of arrival of hiss-type static on 14.6 metres.

During the latter part of December and the first part of January the direction of arrival of this static coincided, for most of the daylight hours, with the direction of the Sun from the receiver. (See curve 1, fig. [57].) However, during January and February the direction has gradually shifted so that now (March 1) it precedes in time the direction of the Sun by as much as an hour. It will be noticed that the curves 2 and 3 of fig. [57] have shifted to the left. Since December 21, the Sun's rays have been getting more and more perpendicular at the receiving location causing sunrise to occur at the receiver earlier and earlier each day. It would appear that the change in the latitude of the Sun is connected with the changing position of the curves. However, the data as yet only cover observations taken over a few months and more observations are necessary before any hard and fast deductions can be drawn.

The following year Jansky wrote to *Nature* (8 July 1933). Referring to the paper quoted above, he described the way further data which he had collected suggested that he had been wrong in drawing the conclusion that the static he had detected was associated with the Sun.

The following extract reproduces that letter.

In a recent paper on the direction of arrival of high-frequency atmospherics, curves were given showing the horizontal component of the direction of arrival of an electromagnetic disturbance, which I termed hiss type atmospherics, plotted against time of day. These curves showed that the horizontal component of the direction of arrival changed nearly 360° in 24 hours and, at the time the paper was written, this component was approximately the same as the azimuth of the Sun, leading to the assumption that the source of this disturbance was somehow associated with the Sun.

Records have now been taken of this phenomenon for more than a year, but the data obtained from them are not consistent with the assumptions made in the above paper. The curves of the horizontal component of the direction of arrival plotted against time of day for the different months show a uniformly progressive shift with respect to the time of day, which at the end of one sidereal year brings the curve back to its initial position. Consideration of this shift and the shape of the individual curves leads to the conclusion that the direction of arrival of this disturbance remains fixed in space, that is to say, the source of this noise is located in some region that is stationary with respect to the stars. Although the right ascension of this region can be determined from the data with considerable accuracy, the error not being greater than ± 30 minutes of right ascension, the limitations of the apparatus and the errors that might be caused by the ionized layers of the Earth's atmosphere and by attenuation of the waves in passing over the surface of the Earth are such that the declination of the region can be determined only very approximately. Thus the value obtained from the data might be in error by as much as $\pm 30°$.

The data give for the co-ordinates of the region from which the disturbance comes a right ascension of 18 hours, and a declination of $-10°$.

A more detailed description of the experiments and the results will be given later.

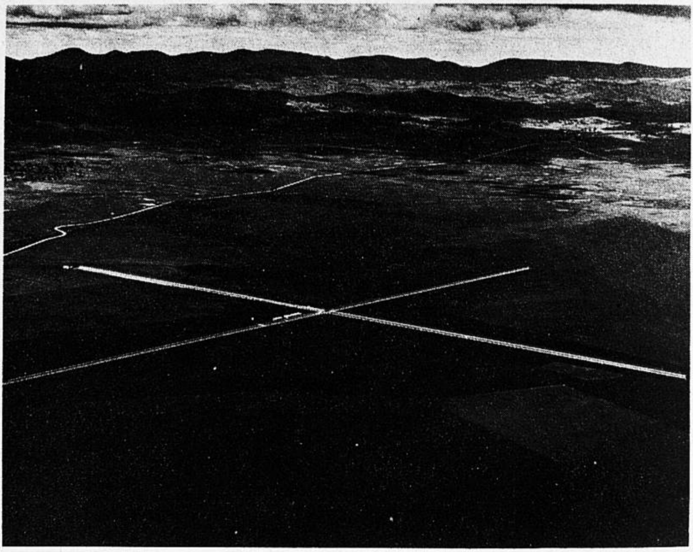

Figure 58
The one-mile Mills Cross pencil beam interferometer, Molonglo Radio Observatory, University of Sydney.
Photograph, Professor B. Y. Mills.

Radio maps of the sky

Jansky found that there was a broad band of radio emission coming from the general direction of the Milky Way, that is, from our galaxy, and it soon became clear that the radiation was most intense from the direction of the centre of the galaxy.

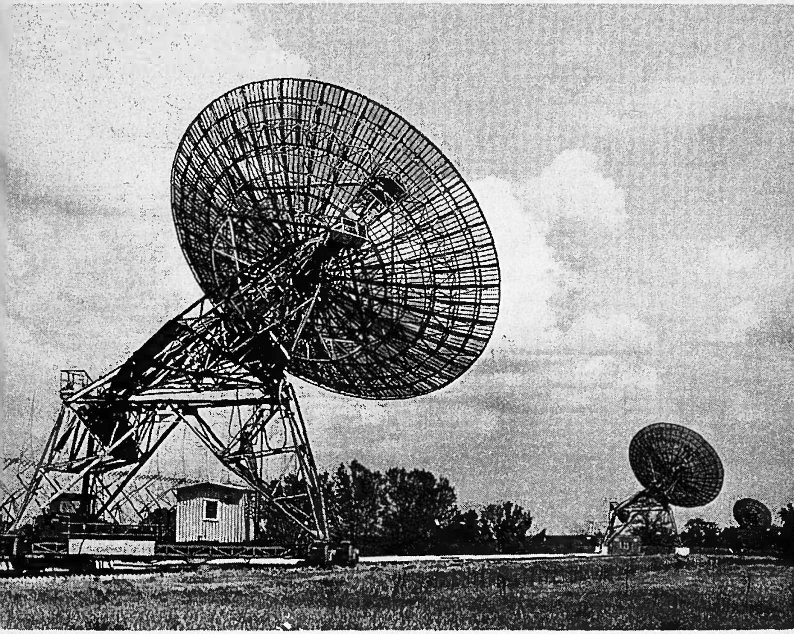

Figure 59
The Cambridge three-telescope system (known as the 'one-mile' radio telescope).
Photograph by courtesy of Sir Martin Ryle, Mullard Radio Observatory, Cambridge.

After 1945, at first with wartime radar equipment, and later using specially built radio telescopes such as those shown in figures 58 and 59, radio astronomers began to map the 'brightness' of the radio sky in greater detail.

The task was not an easy one, because radio wavelengths are large, and no telescope can pinpoint an object to within an angle (in radians) less than the ratio of the wavelength to the largest dimension of the telescope. Telescopes built so as to resolve individual sources as well as possible, for the least cost, took several forms, two of which appear in figures 58 and 59. The design of such telescopes is discussed in greater detail later in this chapter.

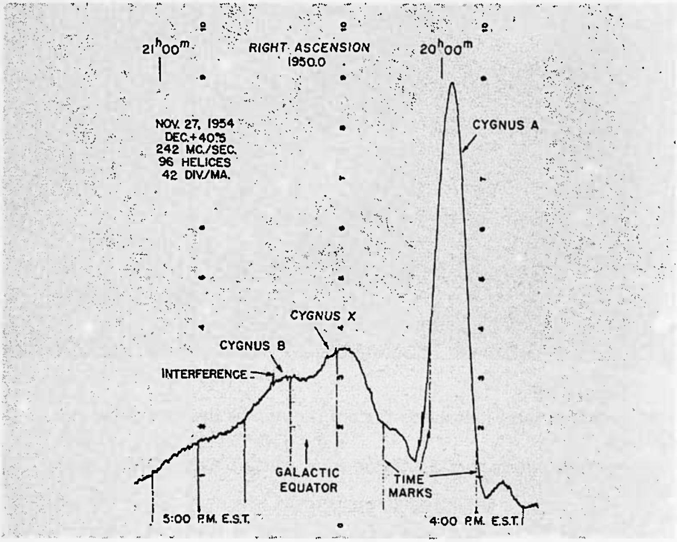

Figure 60
Sample profile taken with the Ohio State University 96-helix radio telescope in the Cygnus region. The deflections due to the transit of radio stars Cygnus A, Cygnus X, and Cygnus B are shown superposed on the gradual rise due to the general background radiation.
From Ko, H. C. (1958) 'The distribution of cosmic radio background radiation'. Proceedings of the Institute of Radio Engineers, ***46***, *1*.

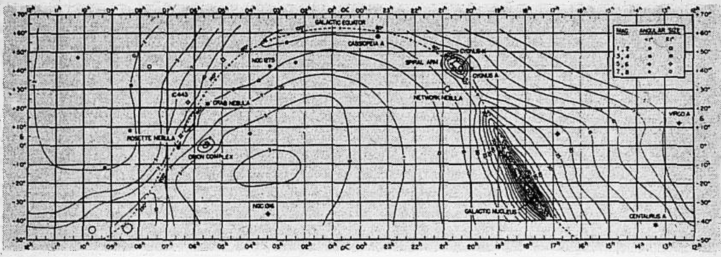

Figure 61
Radio map of the sky in celestial co-ordinates made with the Ohio telescope at 250 MHz. The contours show the radio brightness of the background radiation of the sky, while the small circles indicate the location of radio stars.
From Ko, H. C. (1958) 'The distribution of cosmic radio background radiation'. Proceedings of the Institute of Radio Engineers, ***46***, *1*.

Some of these telescopes, like the Jodrell Bank dish shown in figure 53, can be steered so as to point at any desired place in the sky. A cheaper solution is often to build a fixed aerial and let the Earth's rotation carry its 'beam' past different places in the sky in succession. Figure 60 shows a trace from the recorder attached to the amplifier connected to such an aerial, and figure 61 shows a map published in 1958, made with the same telescope.

As successive radio maps were made, it became clear that the radio sky has these general features:
1 A narrow belt of radiation along the plane of the galaxy.
2 A strong, diffuse region of emission near the centre of the galaxy.
3 A number of localized, discrete sources of various sizes.

The first maps made by different astronomers, particularly by groups at Cambridge and in Australia (the latter using the Mills Cross interferometer, figure 58) did not agree with one another. Indeed, out of 2000 positions of localized radio sources listed in the second Cambridge catalogue, only about 500 are now thought to be reliable.

Figure 62
The Crab nebula.
Photograph, Hale Observatories.

When the precision with which the various sources were located was improved, it became possible to begin to identify some of them with objects visible in optical telescopes. The sources turned out to be very varied in nature, and the old term 'radio stars', for the localized radio objects, has been dropped. Some are stars which have exploded, like the Crab nebula (figure 62), which flared up in the year 1054, and now appears in optical photographs as a cloud of violently moving, glowing gas. The 'brightest' looking discrete source in the radio sky, Cassiopeia A seems to be a similar remnant of a stellar explosion, of which only a few wisps of gas can now be seen in an optical telescope.

Some of the discrete sources turned out to be distant galaxies, and these, despite their optical faintness, are among the more prominent objects in the radio sky, so it must be that they produce an unexpectedly large amount of radio energy emission.

Identifying the discrete radio sources

Some of the problems which face the radio astronomer, the optical astronomer working with him, and the theoretician trying to explain their evidence, are illustrated by the work on the radio source called Cygnus A (see figure 61) which appeared on the earlier maps of the radio sky.

In 1951, after a precise position for it had been obtained at Cambridge, a search was made for a corresponding visual object, using the large Mt Palomar telescope in California. Figure 15 (with question 24) shows the result: the source seems to be an exploding galaxy some 500 million light-years from the Earth. (It was at first thought to be a pair of colliding galaxies, and articles written near the time of the identification refer confidently to it as such.)

The view that Cygnus A is an exploding galaxy is supported by the radio map of its structure, shown in figure 15, which reveals that the emission comes from a pair of regions on either side of the visible galaxy.

Martin Ryle

The following extract is from a paper given in 1967 to the International Astronomical Union by Sir Martin Ryle, Professor of Radio Astronomy in the University of Cambridge, and Director of the Mullard Radio Astronomy Observatory. The paper later appeared in Perek, L. (ed) (1968) *Highlights of astronomy*, Reidel, Netherlands.

Radio galaxies and quasars

If we observe the sky with a radio telescope operating at metre wavelengths, we find first a continuous background mainly due to the Milky Way – which we observe undimmed by the obscuration which spoils our optical view. In addition we find compact sources, a few minutes of arc or less in extent. About 8000 of these compact sources have now been discovered using large instruments, but only a few hundred of the most intense have been studied in any detail; a few are within our Galaxy and represent the emission from the remnants of supernova explosions, but many of them are found to be associated with faint galaxies. From their distances we can conclude that their radio emission is very great, in some cases more than a million times greater than that from our own Galaxy or the Andromeda nebula. These powerful sources are known as 'radio galaxies'.

Observations with instruments of high resolving power show that about 60 per cent have a double structure, with radio emission from two components, one on each side of the related galaxy; the two components are frequently of unequal intensity, and they may be located at unequal distances from the galaxy.

Their appearance and other evidence . . . suggest that the radio-emitting material has been ejected by an explosion within the galaxy.

The maps shown in figures [63] and [64] were made with the one-mile telescope at Cambridge – which has a resolution of about 25 seconds of arc when operating at a wavelength of 21 cm.

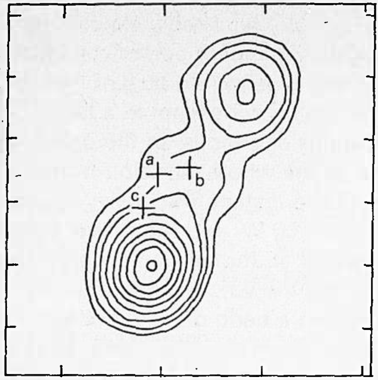

Figure 63
3C 103 consists of two compact components separated by 88″ arc; the cross at a represents the position of a 16th magnitude galaxy with which the radio source is probably related.

Some sources show a more complex structure, in which a number of components can be distinguished, sometimes linked with bridges of emission (figure [64]). Such sources might be the result of a series of lesser explosions.

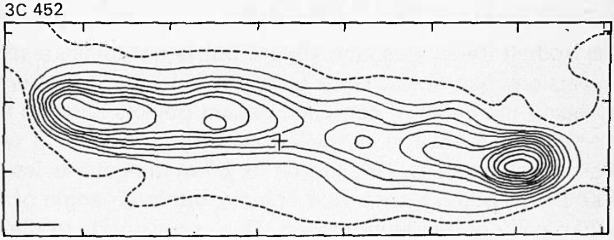

Figure 64
3C 452 contains a number of peaks in a long ridge of emission, suggesting that more than one release of energy has occurred. The cross marks the position of a 16th magnitude galaxy, which may be the source of the energy.

Radio telescopes

If you could look at the night sky with radio eyes, Cygnus A would be the second brightest compact object you could see. Yet it is not a nearby star, but a galaxy so distant that its visible emission is too faint to detect except in a large telescope. Its radio emission is enormous, of the order 10^{37} W (1960 estimate), as large as the whole emission from a normal galaxy. Because it is so distant, the power falling on the whole Earth is only about 10 W, and the power collected by a telescope as large as that at Jodrell Bank is only about 10^{-10} W. This figure includes the whole range of radio frequencies. The power within a band of, say, 1 MHz close to a typical frequency of reception of about 80 MHz, is as small as about 10^{-13} W. Remembering that Cygnus A is a 'bright' object, as astronomical radio objects go, it is clear that not only must a radio telescope have a large area, but also an extremely sensitive amplifier system.

Not only does the radio astronomer have to detect very weak sources, but he has also to cope with the problem of obtaining good resolution despite the long wavelength of the radiation involved. For example, the pair of sources which constitute the source Cygnus A subtend an angle of less than one-thousandth of a radian at the Earth.

The Jodrell Bank telescope (figure 53) is not unlike a scaled-up version of an optical reflecting telescope, but its performance in seeing the detail of compact, distant objects is much the worse of the two. The shortest wavelength at which it will work efficiently is 0.3 m, so that its 80 m diameter is less than 300 wavelengths across, and objects within an angle of 1/300 radian will not be distinguished. This corresponds to the width of a car at a distance of 600 m, or to a fly seen across a room. The naked eye can do much better than that (with visible light, naturally). Figure 65 shows the variation of the response of such a telescope, as it sweeps past a single, small source. It is, of course, just the usual single-aperture, diffraction pattern.

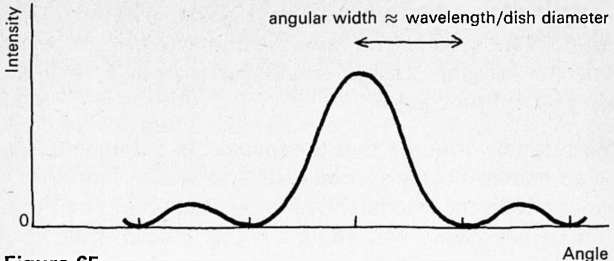

Figure 65
Intensity variation from a single-aperture telescope (single, point source).

The pattern can be made narrower by making the telescope wider, but to achieve a resolution of, say, 10^{-4} radian, would require a Jodrell Bank type of dish 10 000 wavelengths across, which is a diameter of 3 km at a wavelength of 0.3 m. This is scarcely practicable, but fortunately it is not necessary.

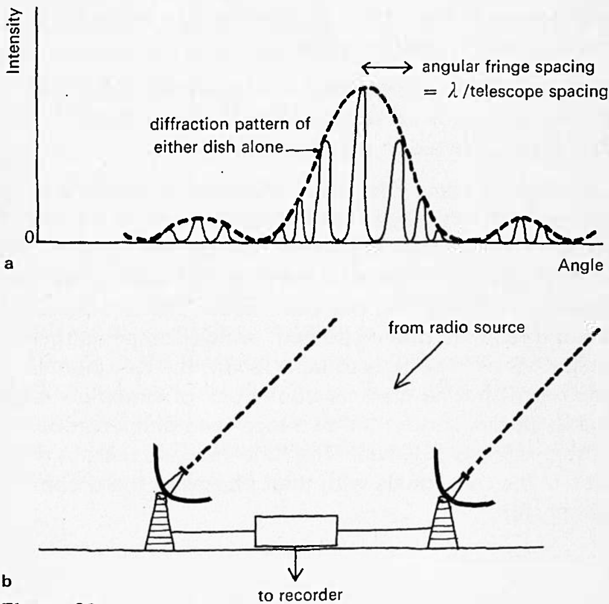

Figure 66
a Intensity variation from a two-aerial interferometer (single, point source).
b A pair of dishes connected together.

A pair of dish aerials placed a large distance apart will do instead. Figure 66 shows how the intensity from such a pair of aerials varies as a single small source crosses the line along which they point.

If the signals from the two telescopes are combined, the record shows closely spaced rises and falls of intensity, exactly analogous to the interference fringes that would be produced if the aerials were transmitting a signal. Because the fringes are much narrower than the diffraction pattern, the central fringe identifies the direction of a single source much more accurately than either dish can do on its own.

The other fringes do not, of course, indicate other sources: all are produced by one source. When the telescope is used to look at a region of the sky filled with sources of various sizes, at various angular distances from one another, it is very hard to tell whether a peak of intensity represents a genuine but weak source at that angle, or whether it is a side fringe from a strong source at another angle.

Such confusion between real and 'telescope-produced' sources was the main reason for the deficiencies of the early radio maps, referred to previously.

A baseline of a few kilometres is about the largest which can be used with telescopes connected together, but clearly there would be advantages in placing the telescopes much further apart, at opposite sides of a continent, or even of the Earth, if possible. Recently, this has been done. The two signals cannot be fed directly to one instrument which compares their phases. What is done is to tape-record the two signals, together with time-markers from a pair of extremely accurate atomic clocks, so that the two tape recordings can be combined later, at leisure. The time-markers make it possible to combine the signals with their phases in the proper relationship.

This method has been used in the U.S.A., with a baseline of 4000 km across the North American continent. It has also been done with a pair of dishes, one at Algonquin in Canada and the other at Parkes, in Australia, which gives a baseline almost as long as the Earth's diameter.

A similar process, of combining records from separate telescopes, has been used to enable a set of small movable aerials to simulate a single much larger one. The Cambridge 'one-mile' telescope (figure 59) is of this type. The aerials are moved about over a large area, records of the signals they receive in different sets of positions being combined afterwards, and used to compute what a telescope covering the large area would have 'seen'.

The quasi-stellar objects, or 'quasars'

Not only can an interferometer accurately locate a pair of close point sources, but it can also measure the angular size of an extended source. As we have seen, some of the radio sources in the sky turned out to be extended clouds of gas from exploded stars, while others – more distant – were identified with whole galaxies, often galaxies in turmoil.

When measurements of the angular size of the radio sources began to be made in considerable numbers, a proportion of them were found to be puzzlingly small. Even using the long baseline technique discussed previously, some could not be distinguished from point sources. As Smith explains in the following extract, the puzzle deepened when the first identifications of these small sources were made, since the visible objects in the place where the radio signal came from appeared at first sight to be ordinary, faint stars. These objects, called quasi-stellar objects, or quasars, have turned out not to be ordinary at all, and at the time of writing debate still goes on as to their nature.

F. Graham Smith

This extract is from 'The problem of quasars' (*Nature*, **213**, 5080). The author is at the Nuffield Radio Astronomy Laboratory, Jodrell Bank.

The angular diameters of radio sources can be measured by the use of interferometers with sufficiently long baseline, and it is fairly easy to achieve this for most radio galaxies. Many of the radio sources in the 3C Cambridge catalogue were found in this way to have angular diameters of the order of 1 minute of arc. In 1960 an attempt at Jodrell Bank to resolve the smallest angular diameters by extending the measurement to a baseline of 32000 wavelengths was surprisingly unsuccessful; some radio sources were apparently less than 3 seconds of arc across. It was known that three of these radio sources were coincident with three bright blue star-like

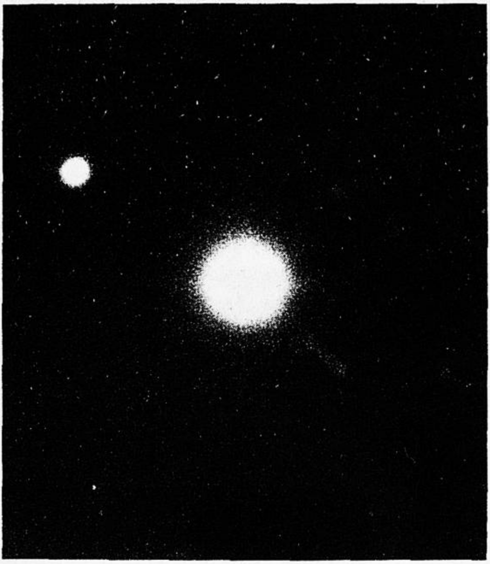

Figure 67
Optical photograph of the quasar 3C 273, one of the first to be identified. This quasar has a 'jet' extending from it, and there is radio emission from both the jet and the round object.
Photograph, Hale Observatories.

objects, but it was not until accurate positions were available from a combination of measurements at Cambridge and Owens Valley, California, that the coincidence was taken seriously. Sandage then measured the spectrum of one of them, 3C 48, and found unusual and unidentified broad emission lines.

Then came, in 1963, the identification of 3C 273 with another 'star', even brighter, with a position determined to 1 second of arc by the observation of a lunar occultation at Parkes, Australia. This 'star' had a wispy cloud extending 19 seconds of arc from it, and a part of the radio source was located in the wisp.

The optical spectra of the three stars had shown unidentifiable emission lines and an exceptionally bright blue continuum. In 3C 373 the emission lines were identified by Schmidt as the Balmer lines from hydrogen, shifted to the red by a factor 1.158. The spectra of the other three were re-examined and clearly fitted the same pattern but with different Doppler shifts. About 100 of these quasi-stellar objects, the quasars, have now been identified, and it seems likely that a third of the 10000 catalogued radio sources may turn out to be quasars. There are probably many more visible, but radio-quiet, quasi-stellar objects. Observational evidence is mounting rapidly, but we still do not know for certain what they are, where they are, how they were made, or how long they last.

If the red-shift is indeed a Doppler shift conforming to Hubble's Law, the quasars are the most distant objects known. Several values of the red-shift parameter greater than 2 are already known, while radio galaxies have values only up to 0.46. The emitted power is then calculable from the distance and the observed intensity over the radio spectrum: it amounts to 2×10^{40} W for 3C 273. A lifetime of 10^5 years is plausibly assumed, and the total energy emitted is reckoned to exceed 10^{52} J. This energy could be obtained by converting into helium a mass of hydrogen equal to 10^9 solar masses. There are only about 10^{11} stars in a whole galaxy and even if we involve a whole galaxy it is difficult to find a sufficiently efficient physical process of energy conversion. So much energy can only have come from gravitational potential energy, or from a nuclear process, or a combination of both. But the explanation would be easier if the energy were less, which means either a shorter life or a smaller distance. Both are possible.

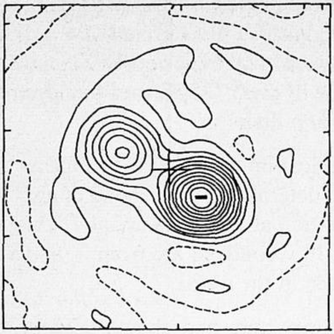

Figure 68
Shape of the radio source 3C 47 determined by the 'one-mile' interferometer at the Mullard Radio Astronomy Observatory, Cambridge, on a wavelength of 21 cm. The optical quasi-stellar object is marked by a cross. The radio source consists of two components 62 seconds of arc apart, set nearly symmetrically either side of the optical object, with flux densities in the ratio of 1.8:1. The optical object has a red-shift of 0.452.

Are the quasars very distant objects?

The only evidence that the quasars are very distant, as far away as or further away than the most distant galaxies, is the large shift towards the red end of the spectrum of wavelengths detectable in their spectra.

If the red-shift is the result of the Doppler effect produced by enormous speeds of recession (up to four-fifths of the speed of light), then it is possible to suppose that the recession is, like that of the distant galaxies, a consequence of the expansion of the Universe (see question 56).

On this interpretation, because their red-shifts are so large, the quasars would be the most distant objects known. But this interpretation raises acute difficulties. The more distant we suppose a quasar to be, the more energy it must emit in order to seem as 'bright' to us as it does.

The trouble is that the energy they must each radiate, if the expansion hypothesis is accepted, is comparable with the energy normally available from an entire galaxy.
For example, the gravitational potential energy of a pair of

stars, each like the Sun, with mass $M = 2 \times 10^{30}$ kg, at a distance R apart roughly equal to the dimensions of our galaxy, 10^5 light years, or 10^{21} m, is given by

$$-\frac{GM^2}{R} \approx -2 \times 10^{28} \text{ J}.$$

If there are 10^{11} such stars in the galaxy, each can be paired off with 10^{11} others. The number of pairs, not forgetting to divide by two to avoid counting each pair twice over, is about $(10^{11})^2/2$. Thus the total gravitational potential energy in a galaxy is about -10^{50} J. If the galaxy contracted to half its former size, this amount of energy would become available.

It is clear that the energy, 10^{52} J, presumed as a figure for the emission by a quasar on the assumption that quasars are very distant objects, will take a considerable amount of explaining. The difficulties are made worse by evidence that quasars are much smaller than galaxies. It seems that their emission alters over quite short periods of time. Figure 69 shows some results for the quasar 3C 273, suggesting that the intensity of its emission has increased appreciably over only a few years.

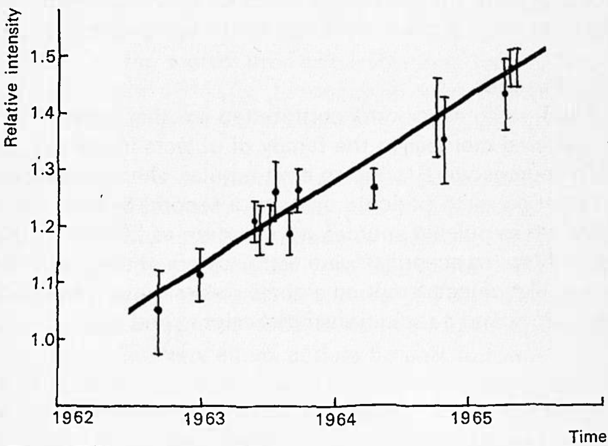

Figure 69
From Graham-Smith, F. (1966) Radio astronomy. *Pelican.*

If this is so, a quasar cannot be more than a few light-years across, because no change could occur in it as a whole, faster than light could travel across it. On the hypothesis that quasars are very far away, the energy of a whole galaxy is being radiated from regions each 10^5 times smaller than a galaxy.

It may be that quasars are not very distant, but are close to our galaxy. Except that they are found equally in all directions, and not mainly in the plane of the Milky Way, only the interpretation of the red-shift stands between this hypothesis and the one which leads into such difficulties. It is not easy to see how to explain the huge red-shifts of quasars if they are relatively near us in space, however.

Some recent evidence (early 1971), as yet unconfirmed, looks as if it might destroy the 'distant quasar' hypothesis. A study of the quasar 3C 279, with a long baseline interferometer between Massachusetts and California, used as described previously, suggests that this quasar consists of two pieces which are moving apart. The rate at which the angle seems to be increasing indicates that if 3C 279 is as far away as its red-shift implies, the pieces are moving apart at ten times the speed of light. Similar results for other quasars would force us to assign them a place much nearer to our galaxy.

Pulsars

In 1968, radio astronomy contributed another quite unexpected member to the family of objects in the sky. Some radio sources were found to emit regular, sharp, rapid pulses of radiation, with periodic times of a second or less. For a while, these pulsing sources were known as LGM, for Little Green Men, so sceptical were astronomers of the possibility of star-like objects emitting signals so like those a man with a good clock and a radio transmitter might send out.

A. Hewish

The following extract comes from an article in *Physics bulletin* **19** (October 1968), by Anthony Hewish, the Cambridge radio astronomer.

Seldom have astronomers been presented with such an intriguing problem as that of the pulsed radio sources, revealed just over six months ago during a survey made at Cambridge using a new radio telescope of high sensitivity. The difficulty is to find a satisfactory theory to account for celestial bodies of planetary size which emit flashes of radio energy with the regularity of a broadcast time service.

The observational evidence

The first four sources to be discovered are known as CP 0834, CP 0950, CP 1133, and CP 1919 (Cambridge pulsar at 08 hours 34 minutes, etc.). These have since been confirmed at Jodrell Bank and at observatories in the USA, Australia, USSR, and Italy. The radio pulses have been detected at frequencies ranging from 40 MHz to 2700 MHz; the signal strength is greatest at low frequencies and decreases rapidly above 1000 MHz. Accurate timing of the pulses has shown that for at least two of the sources the periodicity is maintained to an accuracy which exceeds one part in 10^8, and it is probable that all the sources are comparable time keepers. The actual values of pulse period for the sources, in order as above and rounded to four figures are 1.274 s, 0.2531 s, 1.118 s, and 1.337 s.

The mystery now has a generally accepted explanation. In 1970, it was possible for a writer in *Nature* (Volume **227**, 29 August 1970) to say, '. . . nobody is doubting that pulsars are rapidly rotating neutron stars'. The idea is that there could be whole stars in which matter is compressed to a density comparable with that in the nucleus itself. Such a star could be very massive but very small, and so be able to rotate rapidly. If it has a magnetic field, it becomes possible for it to send a beam of radio waves out into space, which sweeps round as the star rotates, like the beam of a lighthouse. The pulses would then represent the beam sweeping rapidly across the Earth.

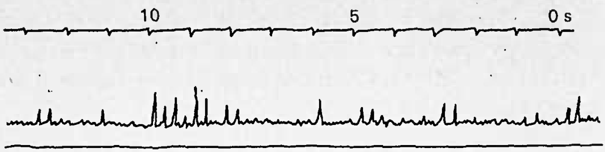

Figure 70
A typical pulse train emitted by CP 0950 as observed at 81.5 MHz. On this trace, fine detail within each pulse is smoothed by the time constant of the pen recorder.

Theories of stellar evolution suggest that just such a star would be likely to be the remnant of a stellar explosion like that which produced the Crab nebula, figure 62. And a faint star at the centre of the Crab has been found to be a pulsar. Recently, this star has also been found, to the surprise and delight of astronomers, to be emitting rapid flashes of visible light, as well.

No account of radio astronomy can, at present, tell a complete story. By the time you read this, ideas will doubtless have changed again, in the light of fresh evidence. But it is clear that Jansky's investigation of the nuisance of radio interference was the beginning of an important chapter in scientific history. It will be a long while before that chapter is closed.

Books and further reading

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Data and formulae

Data	symbol	value
Permittivity of a vacuum (electric force constant)	ε_0	8.854×10^{-12} C² N⁻¹ m⁻²
	$1/4\pi\varepsilon_0$	9×10^{9} N m² C⁻² very nearly
Permeability of a vacuum (magnetic force constant)	μ_0	$4\pi \times 10^{-7}$ N A⁻²
Speed of light	c	3×10^{8} m s⁻¹ very nearly

Formulae

Optics

When parallel light is diffracted at a long, narrow slit, width d, the intensity is first zero at an angle θ given, if the wavelength is λ, by

$$\sin\theta = \lambda/d$$

At a circular aperture, the angle is given by

$$\sin\theta = 1.22\,\lambda/d.$$

When parallel light falls on a diffraction grating, the directions in which the intensity has a sharp maximum are given by

$$n\lambda = d\sin\theta$$

where d is the distance at which the rulings recur, and n is an integer, the 'order' of the spectrum.

A perfectly ruled grating will produce distinct maxima from light containing wavelengths λ and $\lambda+\Delta\lambda$ if

$$\Delta\lambda/\lambda > 1/nN$$

where n is the order of the spectrum and N is the total number of lines in the illuminated part of the grating.

Electromagnetic waves

The speed of an electrical signal on a chain of capacitors connected across a pair of wires, and joined by inductors as in figure 28 is given by

$$v = 1/\sqrt{LC}$$

where v is in sections of line per second if L and C are the inductance and capacitance of the components in a section, or is in metres per second if L and C are respectively the capacitance and the inductance *per metre* of the chain. Such a chain is a form of transmission line. The result above is restricted to the case where any change in the signal is spread over many sections, when it is unimportant that the line is made of discrete components. Faster oscillations may propagate at a different speed, or not at all.

The speed of electromagnetic waves in regions of space not occupied by materials is given by

$$c = 1/\sqrt{\varepsilon_0 \mu_0}$$

For long straight conductors, again without any insulating or other materials nearby, the speed of electrical signal is the same as the speed of electromagnetic waves in empty space, given above. The expression $1/\sqrt{\varepsilon_0 \mu_0}$ can be shown to be equal to $1/\sqrt{LC}$ where L and C are respectively the inductance and the capacitance per metre, of the conductors. The speed of a signal along an air-spaced coaxial cable would be $1/\sqrt{\varepsilon_0 \mu_0}$; but if the cable is filled with polythene, the speed is lower, ε_0 being replaced by $\varepsilon_r \varepsilon_0$, where the relative permeability ε_r is a property of the polythene.

Relativity

A source emitting signals spaced at time t, and moving relatively to an observer at velocity v along the line of sight, seems to the observer to be emitting signals at an interval kt, where k, the Doppler shift factor, is given by

$$k = \sqrt{\frac{1+v/c}{1-v/c}}.$$

k is greater than one if the two stations are moving apart. If the stations are approaching one another, the sign of v is reversed, which, for the same magnitude of the velocity, gives a factor $1/k$.

If one observer looks at a clock carried by another observer, moving relative to him, the Doppler shift is what is *directly observed* (for velocities along the line of sight). When due allowance has been made for time delays in the signals, one observer will *calculate* that the clock moving relative to him is going slow by a factor $\sqrt{1-v^2/c^2}$. This, and not the Doppler shift, is the relativistic time dilation effect. It is seen whatever the direction of motion relative to the line of sight.

The time dilation, together with laws of motion, implies also that other quantities will, as calculated from measurements by a moving observer, differ from those made by another observer not moving in the same way. If m_0 is the 'rest mass' measured by an observer at rest relative to a massive object, $m = m_0/\sqrt{1-v^2/c^2}$ gives the larger mass m measured by an observer for whom the mass is moving at speed v.

Measurements of forces are also influenced by relative motion. In particular, *the electrical repulsion between a pair of charges is reduced by* approximately the factor $(1-v^2/c^2)$ if both charges move at velocity v perpendicular to the line joining them. The small term $-v^2/c^2$ multiplied by the original 'electrical' repulsion, can be interpreted as a 'magnetic' attraction between the moving charges.

Index

Where significant information is contained in an illustration or diagram, the page reference is italicized.

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This *Students' book* contains a summary of Unit 8, *Electromagnetic waves*, and questions on its main work. The Unit is divided into four Parts: 'Looking through holes', 'Spectra', 'Electric waves', and 'Relativity'. The book also includes answers to the questions, a chapter on 'Radio astronomy', a list of background reading, and notes on data and formulae used in the Unit.

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