Physics

Students' book Unit 7

Magnetic fields

Nuffield Advanced Science

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Magnetic fields

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator of the Nuffield Foundation Science Teaching Project

To the student

This book contains some of the things you need to help you to understand the work of this Unit, and some reading which we hope will help you to see how the work is relevant to the practical, everyday world. It does not contain all you need: you will have to consult textbooks and other more general books as well, working through theoretical arguments, reading about experiments, and finding out more about how the ideas can be put to practical use.

This book contains many questions; more than you will be able to do while working on this Unit. Later on, you may wish to use some of them for revision. You will find questions which take you step by step through the theoretical arguments in the course; students who took part in the trials have said that these questions are a good way to understand a piece of theory. You will have to pick and choose, according to your needs and tastes, amongst the other questions. A few give you simple practice in calculation. More invite you to argue about or discuss a problem, and some of these – usually marked *'For discussion'* – are not suited to formal written answers. They are meant to start off a discussion, which may then wander far from the question.

There are a few harder questions to challenge the clever, and you should not expect to be able to tackle every question easily. But most are meant for ordinary human beings, not for budding geniuses. If in doubt, try the obvious answer: usually there is no catch! Most questions have some kind of answer in the section headed 'Answers', though some of these suggest where you might find the needed information, instead of giving it. We have tried hard not to give wrong answers, but, being fallible like yourselves, may not have succeeded.

Some questions ask you to guess, speculate, or give your private opinion: obviously they have no one right answer.

What you are being asked to learn to do

This course aims to help you to become more like a physicist. Most of you will not become physicists, but will use physics or learn more of it in one of a variety of scientific jobs or in further education. Physics, and the world with it, are changing so fast that no one can tell what bits of physics you will use in, say, ten years' time; however, one can be pretty sure that there are some basic ideas that will be relevant to the new problems of tomorrow. We have tried to build the course around what we believe to be these basic ideas.

So one thing the course aims at is helping you to become able to learn, in the future, the new ideas in physics you may meet, and helping you to become able to use the physics you have learned. It does this because these are the tasks that will face you.

In the future, you will need to be able to learn from books and articles; that is why the course contains a good deal of reading (in a list at the end, you will find details of books referred to in the text). To use the physics you have met, you need to understand it – that is, to be able to use it in new kinds of problems. That is why so many questions in this book ask you to make up arguments about new problems, using what you know.

What is 'understanding'? That is, how does one recognize that someone understands a piece of physics? We think it is something like this. Suppose a group of people are talking about a problem in physics. Very rarely, even among research workers, will anyone immediately see an answer. More often, they each have some ideas which they try out in discussion with colleagues. Those who 'understand' their physics are the ones who can offer sensible, relevant ideas that would help towards clearing up the problem. A reasonably competent physicist expects himself and others to be able to draw on their knowledge and use it to make sensible contributions to the discussion of problems.

So to test whether you understand a piece of physics, it is asking too much to expect you to solve a new problem completely and correctly; few – if any – experts can do that. The test should be that of physicists talking together: can you produce sensible ideas that are relevant and would help a bit towards clearing up a problem? This is the test that will be used in the examination, and is the way to decide how well you have managed a question or problem in the work of the course.

The course also aims to show you what doing physics is like, and this is another reason for encouraging plenty of discussion of problems, for that is the way physicists work. It tries to show what kinds of questions physicists ask themselves and what sorts of ways they use to tackle them. We think this is important because to use physics successfully and to judge its claims and achievements you need to understand what it can, and what it cannot do. That is why several questions ask you about such things as how theories, models, experiments, and facts fit together. Physicists also guess, estimate, and speculate, so other questions ask you to do these things too, to find out what doing them is like and to become better at doing them.

There are a lot of misunderstandings about what physics is like. Some say it is all facts; others that it is all theory, having little to do with what happens in practice. Many are puzzled; asking whether what physics says is true or not, or how physicists arrive at their ideas. We hope you will find chances in this course to think about such matters, and that you will form your own views.

Some of the questions ask about how physics can be used in engineering and technology, and the articles in this book are also about that, because we think that you will rightly want to know when what you learn is of practical value.

Finally, one of the main reasons we want to offer you some physics is that we like the subject and get excited about it. So we hope you enjoy it too.

Summary of Unit 7 Magnetic fields

Part One

Forces on currents

In this Part, the magnetic field is regarded as something which produces forces on currents and on moving charges. This effect is used to measure the field, and finds practical use in particle accelerators and mass spectrometers.

Measuring magnetic fields

$F = BIL$, and $F = Bqv$ for the force on a current and on a moving charge.

A new effect

The Hall effect, arising from magnetic forces on moving charge carriers in a conductor, linking with earlier work on charge transport in Unit 2. Use of the effect as a tool for measuring B-fields.

Uses of forces on charges

Reading of books and articles about accelerators and mass spectrometers.

Part Two

Electromagnetic induction

This Part is almost all about the law of induction: *induced voltage* equals *rate of change of flux linking a circuit.*

Induced voltage in a moving wire

Use of $F = Bqv$ to understand the voltage induced in a wire moving across a magnetic field. Motors and dynamos.

A new sort of induction

Inducing a voltage without moving a wire: comparison of the two sorts of induction. One general rule for calculating the induced voltages produced by either method.

Investigations

Finding out about transformers: currents, flux, voltages; effects of changing the coils or the iron core. Learning to choose what experiment to do.

Self-induction

Reminder from Unit 6 of the voltage induced in a coil by the flux changes produced by changes of the current in that coil.

Part Three

Flux near currents

This Part is about the design of machines which use flux, and in particular, about what decides how much flux will be produced near a coil, by a current in the coil.

Quantitative investigations

Careful measurements of the variation of the *B*-field in the region around conductors of various shapes. Comparison of the results with formal, mathematical relationships.

Flux as like flow

The relationship *current-turns* = *flux* × *reluctance* as an analogy with other flow problems (such as Ohm's Law). Magnetic circuits using iron. Calculation of the field near a long wire. Ampère's loop law.

Absolute measurements

Choosing a unit of current. Measuring current without an ammeter.

New kinds of motor

Eddy currents. Making a motor without brushes. Induction motors.

Questions

Vectors

1 This question is intended to help you revise your knowledge of vectors.

a 2 kg plus 3 kg is 5 kg. When is 2 km plus 3 km *not* equal to 5 km? Suggest one other scalar quantity (like a mass) and one other vector quantity (like a velocity).

b Forces are vector quantities. Two forces, one of 2 N and one of 3 N, pull on a frictionless puck of known mass. What else must you be told, in order to work out the initial acceleration of the puck? Given the necessary information, how would you find the direction in which the puck begins to move?

c Figure 1, parts *a* to *e*, shows different directions in which the two forces pull. Make a sketch showing the direction in which the puck begins to move in each case.

d Why does the gravitational attraction of the Earth alter the orbital speed of a satellite only if the satellite is in a non-circular orbit?

Mark one place on the orbit, figure 1 *f*, where the speed is increasing, and one place where it is decreasing, because of the gravitational attraction.

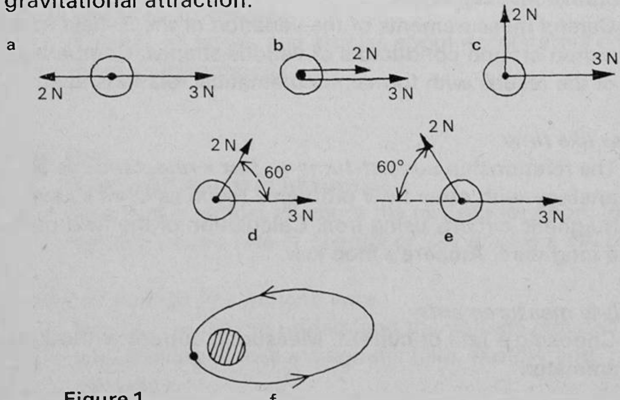


Figure 1

Questions 2 to 7 Motion in a circle
These questions are taken from Nuffield O-level Physics *Questions book V*. Further questions can be found in Rogers, *Physics for the inquiring mind*, Chapter 21, pages 295, 301, and 310.

Questions 2 and 3 are about the derivation of $\frac{v^2}{r}$ (central acceleration) and $\frac{mv^2}{r}$ (central force).

2 *First proof of* $\frac{v^2}{r}$. An object is moving in a circular path (or orbit) with a constant speed represented by *v*. The velocity is always changing in direction but is constant in magnitude. At one instant it is at A in figure 2, and its velocity, at a tangent to the circle, is shown by the arrow at A. A short time, *t*, later it is at B, and its velocity is represented by the second arrow. The radius of the circle is *r* and the distance between A and B is *x*. A chord is drawn from B at right angles to AC. As the time *t*, and therefore AB, are supposed to be very small, the length of the chord for all practical purposes is equal to 2*x*.

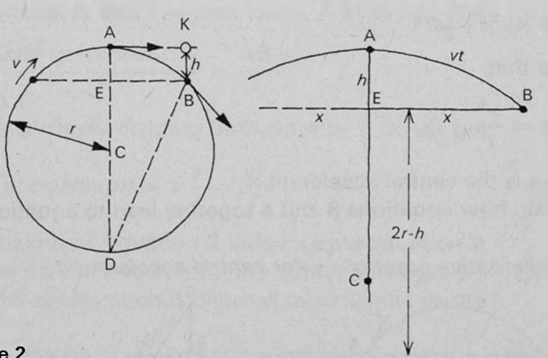


Figure 2

If the object had *not* been pulled round in a circle it would have travelled from A to K. Because of the force pulling it, it has fallen, in time *t*, through the distance KB, which we will call *h*. An equal distance, *h*, is marked off from A, towards C. As a first step towards obtaining the expression $\frac{v^2}{r}$ we write:

$$h(2r-h) = x^2 \quad \mathbf{1}$$

a Explain (as if to another student) why equation **1** is correct. We can write this,

$$h = \frac{x^2}{2r-h}$$

and then, without making any serious error, we can leave out h from $2r-h$ and write

$$h \approx \frac{x^2}{2r} \quad \mathbf{2}$$

b Explain why we can leave out the h in $2r-h$.
Since the object travels from A to B in time t with speed v, therefore $x = vt$, and we write

$$h = \frac{(vt)^2}{2r} = \tfrac{1}{2}\left(\frac{v^2}{r}\right)t^2 \quad \mathbf{3}$$

Now, h is the distance fallen towards the centre, and there was no initial velocity at A in that direction. There must have been an acceleration a towards the centre, and by using

$$s = ut+\tfrac{1}{2}at^2 \quad \mathbf{4}$$

we see that,

$$a = \frac{v^2}{r} \quad \mathbf{5}$$

where a is the central acceleration.
c Explain how equations **3** and **4** together lead to equation **5**.

3 *Alternative proof of $\frac{v^2}{r}$ for central acceleration*

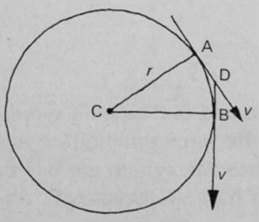
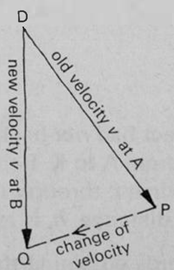


Figure 3



An object is moving with a constant speed v in a circular path (see figure 3). At one instant it is at A and its velocity, at a tangent to the circle, is shown by the arrow. A short time, t, later it is at B and its velocity is represented by the second arrow. The radius of the circle is r. The velocity line at B is projected backwards to meet that from A at D. On the right, a second diagram is drawn, also with arrows from D, but the circle is left out, so as to make it look simpler. DP represents the velocity at A. DQ, of equal length, represents the velocity at B. When the angle ACB is small, the sector ABC becomes a triangle, similar to triangle PQD.

a Why is PQ labelled 'change of velocity'?

b Why are PQD and ABC similar triangles?

We then say,

$$\frac{\textit{change of velocity}}{v} = \frac{\mathrm{AB}}{r} \qquad \mathbf{1}$$

or, $$\textit{change of velocity} = \mathrm{AB}\,\frac{v}{r}$$

c Why is equation **1** true?

We remember that $t =$ time taken, A to B, and write

$$\frac{\textit{change of velocity}}{t} = \frac{\mathrm{AB}}{t}\,\frac{v}{r} \qquad \mathbf{2}$$

which is simply dividing both sides by t. So we get,

$$\textit{acceleration} = v\,\frac{v}{r} = \frac{v^2}{r} \qquad \mathbf{3}$$

d Explain how equation **3** follows from equation **2**.

e How do you know, from the righthand diagram in figure 3, that the acceleration is directed towards the centre?

4 **a** If a body moving in a circle has a central acceleration, then there must exist a central force producing that acceleration. Mention three examples of an object moving in a circular path, and say in each case what provides the central force.

b The acceleration is v^2/r, so the force must be mv^2/r. Why?

c In the expression v^2/r, where did the v^2 come from? (Or why v^2 rather than v, or $\frac{1}{v}$, or v^3, etc?)

d In the expression v^2/r, where did the $\frac{1}{r}$ come from? (Or why $\frac{1}{r}$ rather than r, or r^2, or $\frac{1}{r^2}$, etc?)

5 A heavy ball rolls around inside a hoop resting on the floor. The mass of the ball is 0.25 kg, the radius of the hoop is 0.4 m, and the ball rotates through a full circle once in every second.

a What is the speed, v, of the ball?
b What force does the hoop exert on the ball, keeping it on a circular path (that is, what is the central force?). *Note:* you may assume $\pi^2 = 10$.
c Work out **a** and **b** again, this time for a hoop with a radius of 0.3 m.
d Your answers to **b** and **c** should be 4 N and 3 N. How do you account for the fact that the smaller circle requires the *smaller* force, although, from $\frac{mv^2}{r}$, you might think that, since r divides mv^2, the smaller circle would require the *larger* force?

6 You are inside a cage which is being rapidly rotated as in figure 4 – the sort of thing you might pay to have done to you at a fair. Afterwards you tell a friend that you were flung against the outer wall and kept pressed there. 'Rubbish,' he says, 'it was only that you tried to go straight on, as Newton said you must.' Explain what happened to you from your friend's point of view.

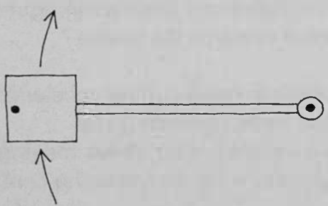


Figure 4

7 a *(Hard)* Explain how it is that a spin-drier can extract water from wet clothes, so that they are as dry as they would be if they had been put through a mangle.
b *(Easy)* A spin-drier has a tub which is 0.25 m in radius. It makes 5 revolutions per second. What is the value of the central acceleration (v^2/r) at the rim of the tub?
c Your answer to **b** should be in metres per second each second. The acceleration of gravity, g, is 10 m s^{-2}. How many times gravitational acceleration might your mother be using to dry clothes in her kitchen?

Part One

Forces on currents

Questions 8 and 9

These questions are about the force on a current in a magnetic field, and the use of that force to measure the field.

8 **a** Figure 5 *a* is a sketch of a 'current balance', which consists of a pivoted wire frame resting on knife edges through which current enters the frame. Which of the three forces X, Y, or Z will tilt the frame?

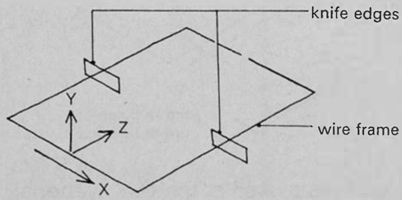


Figure 5a

b The force on a current is at right angles to the current and to the direction of the B-field at the current. Figure 5 b to g shows a series of attempts to tilt a current balance using a magnetic force. Which of them will produce a force on the wire which will tilt the balance?

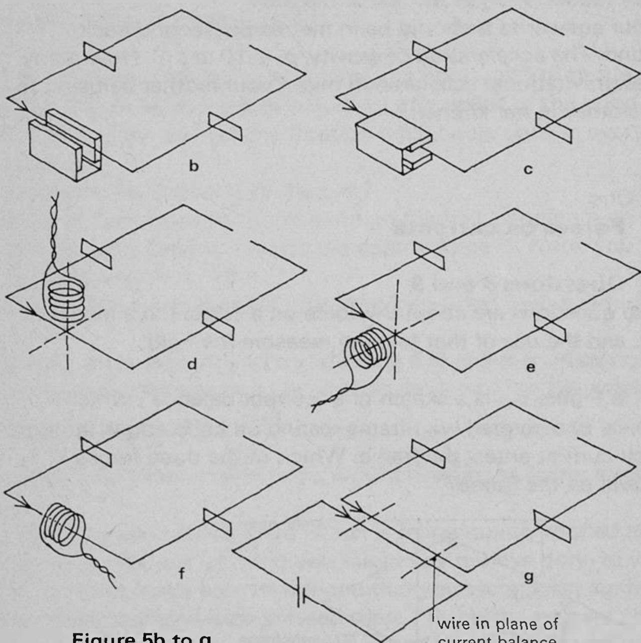
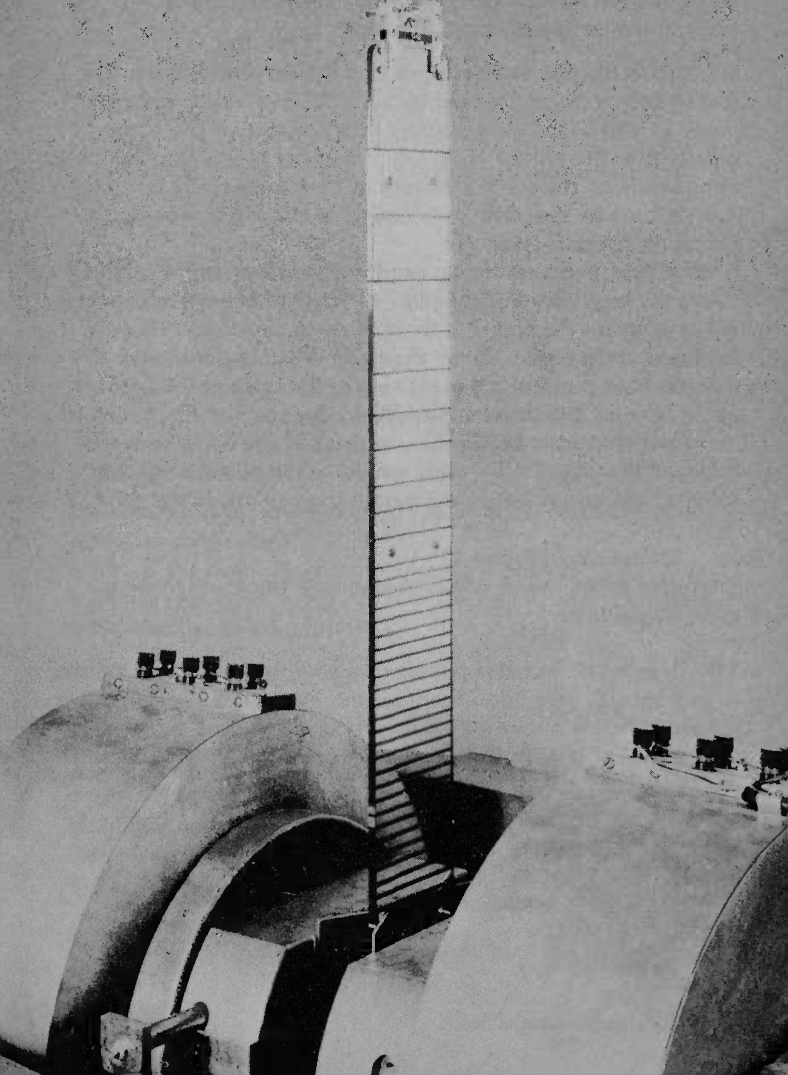


Figure 5b to g

9 Figure 6 shows apparatus used at the U.S. National Bureau of Standards to make an accurate measurement of the B-field in the gap of a powerful electromagnet, as part of a programme of precise determinations of fundamental constants in physics.

See Taylor, B. N., Langenberg, D. N., and Parker, W. H., 'The fundamental physical constants.' *Scientific American*, October 1970.

Figure 6
Accurate high field measurement of magnetic field. The field in the gap of the powerful electromagnet is being measured by hanging in it a coil wound on the edges of a glass plate suspended from a balance (not shown). The lines across the plate are threads wound round the plate to keep the coil in place along its edge.
Photograph, U.S. National Bureau of Standards.



In this apparatus a coil wound on the edges of a long glass plate is lowered into the gap of the magnet. The plate hangs from a balance (not shown) so that the magnetic force on the short, bottom end of the coil in the gap can be measured.

The horizontal lines across the plate in the photograph are *not* wires, but are threads wound around the plate to keep the wires along its edges exactly in place.

a Suppose the coil on the edges of the plate has 10 turns. The gap of such a magnet would be about 20 mm wide, which gives an idea of the dimensions of the apparatus. The B-field would be of the order 1 N A^{-1} m^{-1} or tesla, T. Estimate the force in newtons on the coil if a current of 20 mA passes in it. Could such a force be weighed fairly easily, or is it much too big, or too small, for convenience?
b What field direction would produce a force on wires running down the long edges of the plate which would tend to pull those wires off the plate? Is there likely to be a field in such a direction in the region above the gap? Why are there more threads wound round the plate near to the magnet than far away from it? Would it be possible to arrange that the forces on the wires in regions above the gap pressed the wires *onto* the edges of the plate? If so, what would be the direction of the force on the lower, horizontal part of the coil inside the gap?

Questions 10 and 11

These are about the direction of magnetic forces on moving charged particles.

10 A beam of particles passes in a straight line from a source X, to a spot, E, on a fluorescent screen. When the electromagnet shown in figure 7 is switched on, the beam hits the screen at one of the spots A, B, C, D, E. Where does it go if the particles are
a negatively charged?
b positively charged?
c neutral?
d What happens in each case if the B-field is reversed?

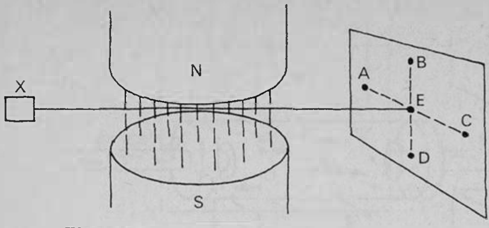


Figure 7

11 In one form of mass spectrometer, charged ions in a fanning out beam move in the paths shown in figure 8. Parts of the paths include a B-field whose direction is perpendicular to the plane of the paper.

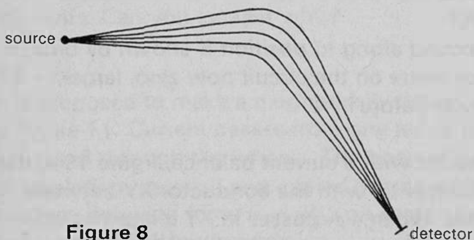


Figure 8

a Indicate places where there must be *no* B-field in the direction suggested.
b Shade the area where there *must be* a B-field.
c If the beam stays in the plane of the paper, why is the field perpendicular to this plane?

Questions 12 to 14

These are harder questions about forces on currents.

12 A circuit carrying a current has copper wires leading into and out of a tube of salt solution. In the wire, electrons are moving in the direction shown in figure 9. In the solution, Cl^- ions move in the same direction as the electrons, and Na^+ ions move in the opposite direction.

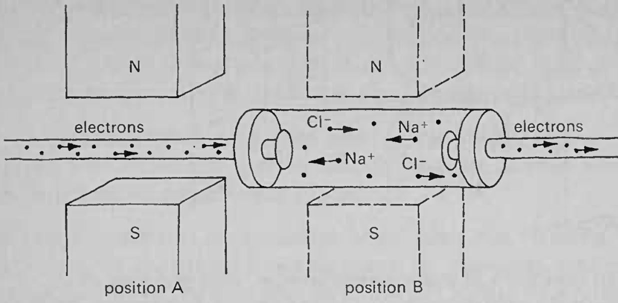


Figure 9

a In which direction is the force on the wire if a magnet is placed in position A?
b If the magnet is moved along to position B shown by broken lines, is the force per metre on the circuit now zero, larger, smaller, or the same as before?

13 **a** In an experiment with a current balance, figure 10 *a*, the balance is initially horizontal with the conductor XY between the poles of a magnet. When 4 A passes in XY it is deflected upwards, the field being horizontal. 0.10 m of the conductor is in the magnetic field. To restore XY to its original position 60 mm of paper tape are placed on it. The mass of one metre of the tape was 700 mg. What is the strength of the B-field? Earth's gravitational field at surface = 9.8 N kg $^{-1}$ (acceleration of free fall = 9.8 m s^{-2}).

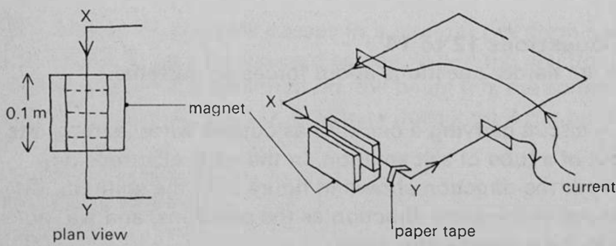


Figure 10a

$$B = \frac{F}{I\,L}$$

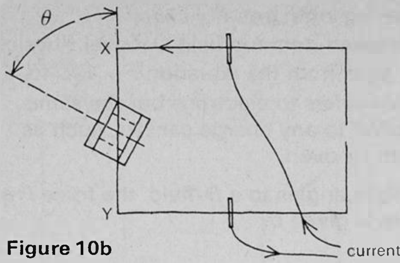


Figure 10b

b *(Hard)* The experimenter then turns the magnet so that the field is still horizontal but at an angle θ, less than 90°, to the wire AB. The force on the wire should be $BIL \sin \theta$, so a reduction in the force can be anticipated. In fact the force stays the same. Can you explain why?

14 *Hard*

It is proposed to make a magnetically driven vehicle as shown in figure 11. Current passes from one rail to the other across the axles and through the wheels. The body of the vehicle is made of insulating material, and carries magnets placed as shown, to produce a driving force $F = BIL$ on each axle. Do you think it will work? Explain your answer.

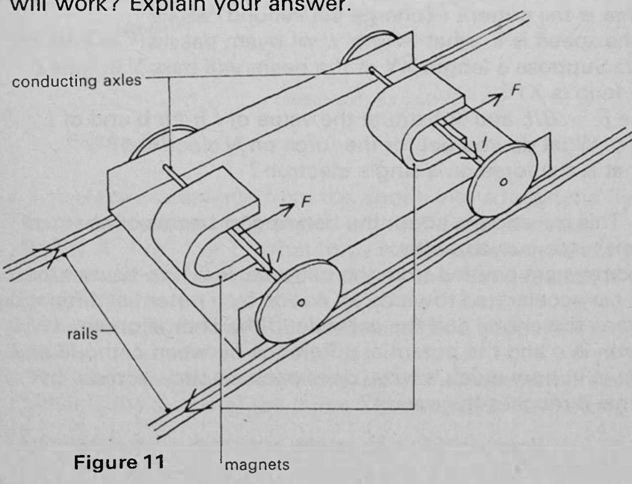


Figure 11

Forces on moving charges $F = Bqv$

15 This question is taken from Nuffield O-Level Physics *Questions book V*. It goes from the equation $F = BIL$ to $F = Bqv$. The question refers to electrons but the same argument can be applied to any charge carriers, such as sodium ions fired from an oven.

If a current I runs at right angles to a B-field, the force F on length L of the current is given by

$$F = BIL.$$

The question is about finding a similar expression for the force on an electron having a charge q and a velocity v. Suppose that, in a time t, N electrons pass any place in the beam, such as place Y in figure 12.

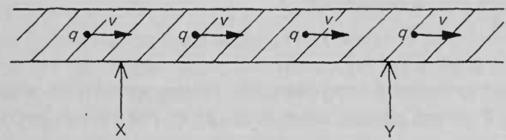


Figure 12

a What is the charge passing Y in time t?
b What is the current I (charge per second) at Y?
c If the speed is v, what length, L, of beam passes Y in time t? (*Hint:* Suppose a length XY of the beam will pass Y in time t. How long is XY?)
d Use $F = BIL$ and substitute the value of I from **b** and of L from **c**. What do you get for the force on N electrons?
e What is the force on a single electron?

16 This question is about the bending of beams of charged particles into curved paths.
a Electrons are emitted from the cathode, in a fine-beam tube. They are accelerated towards an anode by a potential difference between the anode and the cathode. If the charge on an electron is q and the potential difference between cathode and anode is V, how much energy does each electron acquire by the time it reaches the anode?

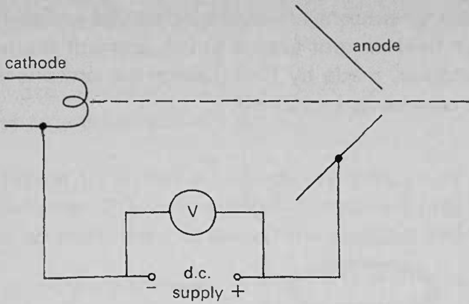


Figure 13a

b If the potential difference across the 'gun' is 100 V, how fast will the electrons emerge from the hole in the anode?

$$\text{charge on the electron} = 1.6 \times 10^{-19}\ \text{C}$$
$$\text{mass of the electron} = 9.1 \times 10^{-31}\ \text{kg}$$
$$\text{kinetic energy} = \tfrac{1}{2} mv^2$$

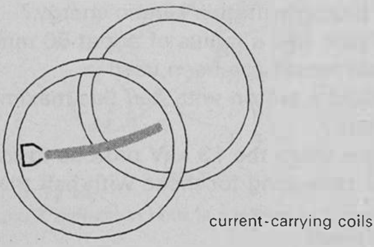


Figure 13b

c The electrons emerge from the anode into a magnetic field. They then travel at right angles to the *B*-field, strength 10^{-3} N A^{-1} m^{-1} or T. What force is exerted on the electron because it is moving across the field? In what direction is this force?
d What acceleration will the electrons have because this force acts?
e The electrons are accelerating. Does their *speed* increase?
f What is the radius of the curve the electrons move along?
(Acceleration towards the centre of a circular path $= \frac{v^2}{r}$.)

17 In a cyclotron, protons are kept moving in a circular path by a uniform *B*-field at right angles to the plane of the path. In the first cyclotron, made by E. O. Lawrence, protons were accelerated to an energy of 13 keV.

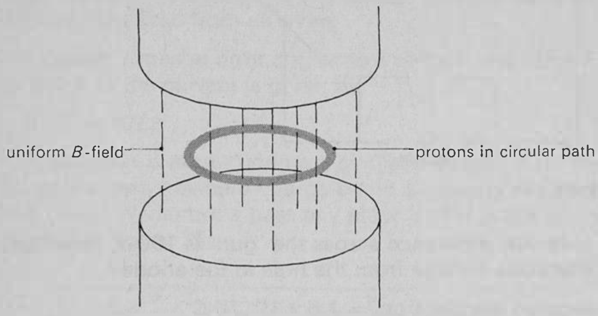


Figure 14a

a What velocity has a proton with this kinetic energy?
b The largest possible path had a radius of about 50 mm. What strength of *B*-field must have been used?
c What radius path would a proton with *half* this maximum energy follow in this field?
d How long must it have taken the 13 keV protons to travel once round their path? How long for those with half this energy?

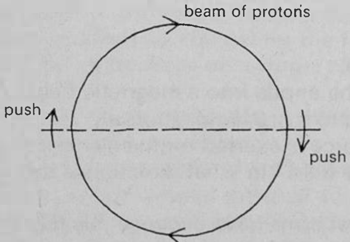


Figure 14b

e The cyclotron worked by giving particles of any energy, on their respective paths, a push every time they had completed half an orbit.

How was it possible for particles of differing energy all to be accelerated together?

mass of proton $= 1.7 \times 10^{-27}$ kg
charge on proton $= 1.6 \times 10^{-19}$ C

18 Figure 15 shows a cathode ray tube, of the sort used in a television set. The coils around its neck are magnetic deflection coils, used to sweep the electron beam to and fro.

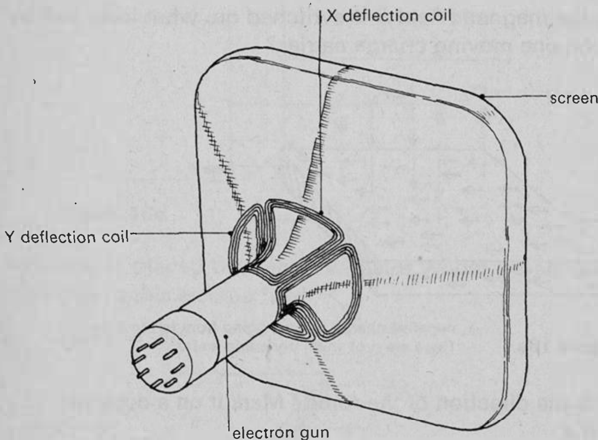


Figure 15
The X and Y deflection coils in a television cathode ray tube.

The beam has to paint out 625 lines to make a full picture every 1/25 second. Sketch a graph of the variation with time of the current:
a in the coils which move the beam in a horizontal direction;
b in the coils which move the beam in a vertical direction.

Your graph should cover the time needed to draw out, say, three complete horizontal lines. Label the graph as necessary to indicate the purpose of the current variations.

Questions 19 and 20

These are about the Hall effect.

19 A result from earlier work (Unit 2) is useful:
If I is the current when there are n charge carriers per unit volume, each with negative charge q passing through an area A with a velocity v, then:

$$I = nAqv.$$

a When the magnetic field B is switched on, what force will be exerted on one moving charge carrier?

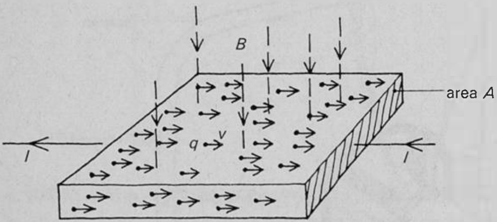


Figure 16a negative charge carriers going from left to right; there are n of them per cubic metre

b What is the direction of the force? Mark it on a copy of figure 16 *a*.

c After a time there will be higher density of negative charge near the front edge than near the rear edge, as shown in figure 16 *b*. Why?

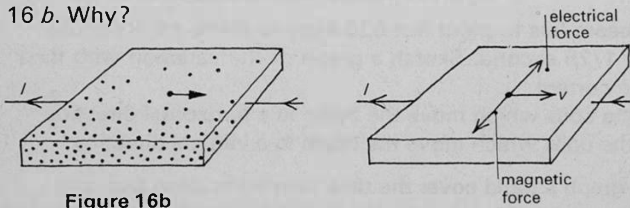


Figure 16b

d A charge carrier in the middle will still experience a force due to the magnetic field, but it will also be repelled by the extra negative charge near the 'front edge'. How big must the electrical force be for there to be no further change in the numbers of negatively charged particles near the edges?

e Which edge of the strip will be positively, and which negatively, charged?
f If, in another experiment, there were two parallel plates separated by a distance, d, and one plate was kept positively charged and the other negatively charged, then there would be an electric field, E, present between the plates, given by $E = V/d$ if V is the p.d. between the plates.

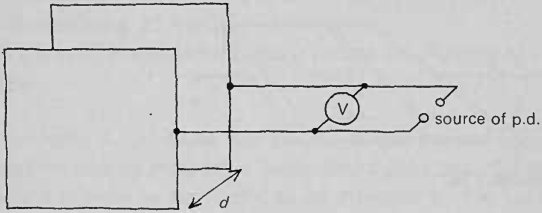


Figure 16c

A charge, q, placed between the plates would experience a force due to this electric field.

Force $= Eq$.

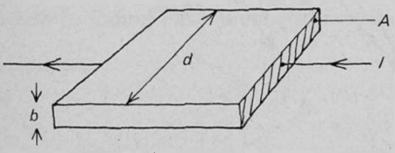


Figure 16d

The electric force acting on a charge moving in the metal strip has been given by your answer to **d**. What is the potential difference which has been produced between the front and rear edges of the strip?
g But $I = nAqv$. What is A if the strip has thickness b? An equation for the number of charge carriers is wanted which does not contain the velocity of the charge carriers. Eliminate v from your equation for the potential difference and obtain an equation relating the number of charge carriers per unit volume to the potential difference.

20 a How many atoms are there per cubic metre of copper? The Avogadro constant is 6×10^{23} mol^{-1}, the atomic mass of copper is 63.5, and its density is 8930 kg m^{-3}.

b *How many charge carriers (free electrons) are there per cubic* metre of copper, if it is sensible to guess that each atom provides one charge carrier?

c What are the velocities of these charge carriers if there is a current of 20 A in a copper strip 0.1 mm thick and 10 mm wide?

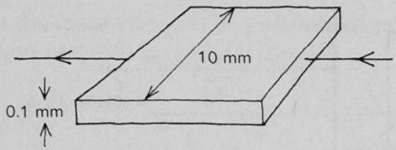


Figure 17a

d If a magnetic field of 0.2 N A^{-1} m^{-1}, perpendicular to the strip, is switched on, what will be the force on an electron in the copper?

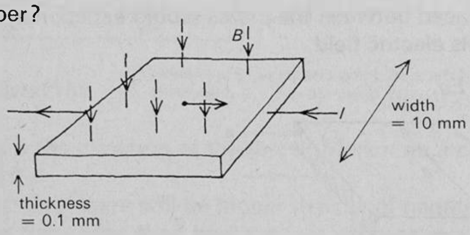


Figure 17b

e This force will cause the electrons to move and, after a time, there will be a higher density of negative charge near the front edge of the strip. The rear edge of the strip will be more positively charged. An electron near the middle of the strip will experience a force due to the magnetic field and an *electrical* force due to the higher density of negative charge near the front edge. What will be the electrical force if the electrons now *travel straight across the strip without deflection?*

f What is the electric field strength? (E = *force/q*.)

g An electric field is produced when there is a potential difference.

$$E = \frac{V}{d}.$$

What is the potential difference (Hall voltage) across the width of the strip?

Questions 21 to 25

These are further questions about forces on moving charged particles.

21 Student A has been wondering about the deflection of electron beams by magnetic fields and thinks that the electron beam in a television set ought to be affected by the Earth's magnetic field. He finds from a data book that the Earth's magnetic field, in this country, has a vertical component of 4.4×10^{-5} N A^{-1} m^{-1} and a horizontal component of 1.9×10^{-5} N A^{-1} m^{-1}.

Student B suggests that there should be an effect on the position of the picture if the set is turned from facing north–south to facing east–west (figure 18 *a*).

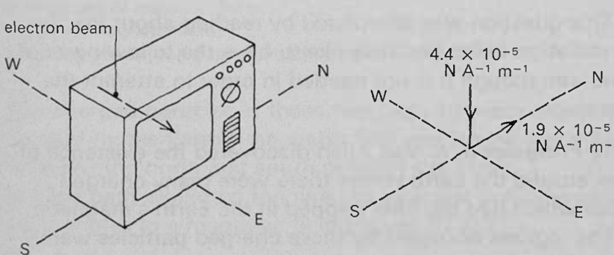


Figure 18a

Student C says that there will only be an effect if the television is on its back and is turned, still lying on its back, so that one edge moves from facing east, say, to facing south (figure 18 *b*).

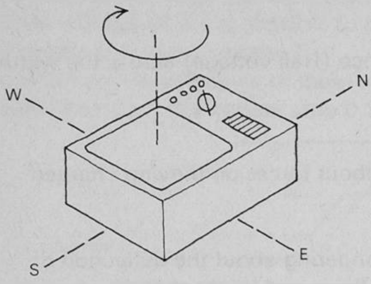


Figure 18b

A says that he thinks that any effect would be much too small to be detected. Will B's idea work, in principle? Will C's idea work, in principle? Is A right about the size of the effect?

$e = 1.6 \times 10^{-19}$ C
$m = 9.1 \times 10^{-31}$ kg
Typical acceleration voltage for a TV tube = 10 kV.

22 This question was stimulated by reading about the Van Allen radiation belts. You may like to have the following brief information, though it is not needed in order to attempt the question.

In 1958 Professor J. A. Van Allen discovered the existence of regions around the Earth where there were many charged particles which had become trapped in the Earth's magnetic field. The regions occupied by these charged particles were mapped in that year by Geiger tubes mounted in rockets launched from Cape Canaveral (now Cape Kennedy). The results indicated that there were two distinct, widely separated zones of high intensity. These zones are known as the Van Allen radiation belts.

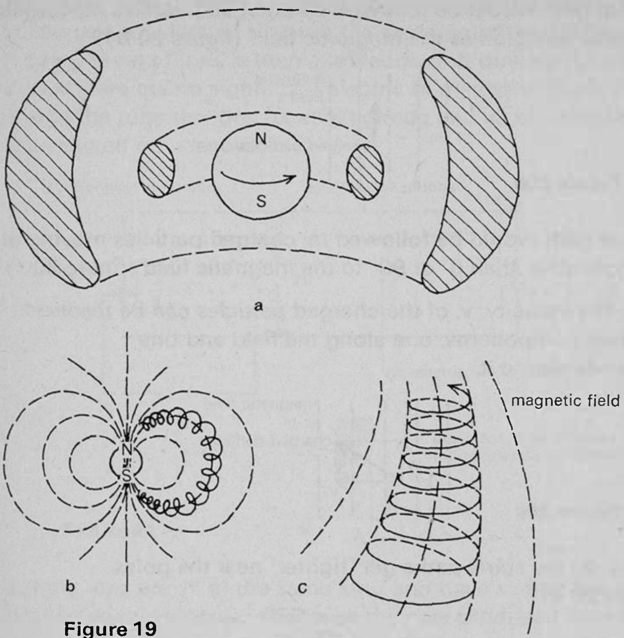


Figure 19
a Structure of radiation belts.
b Earth's magnetic field.
c Spiral path followed by charged particles in the belts.

The charged particles in these two high intensity regions are trapped by the Earth's magnetic field and follow spiral paths between the north and south poles of the Earth.
a What path would be followed by charged particles moving at right angles to a magnetic field (figure 20 *a*)?

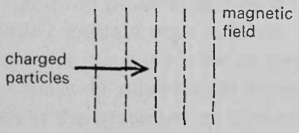


Figure 20a

b What path would be followed by charged particles moving in the same direction as the magnetic field (figure 20 *b*)?

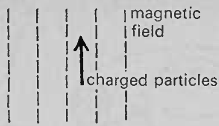


Figure 20b

c What path would be followed by charged particles moving at an angle other than 0° or 90° to the magnetic field (figure 20 *c*)?

Hint: The velocity, v, of the charged particles can be resolved into two components, one along the field and one perpendicular to it.

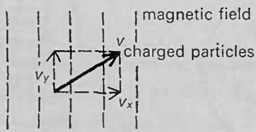


Figure 20c

d Why do the spiral paths get 'tighter' near the poles (figure 20 *d*)?

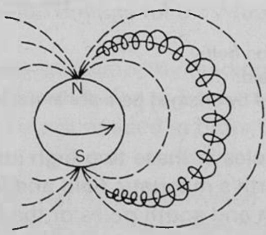


Figure 20d

(Background reading: J. A. Van Allen, 'Radiation belts around the Earth.')

23 In a 'time of flight' mass spectrometer all the ions from the source are accelerated through the same potential difference. A sharp burst of ions is then allowed to drift down a tube in which there are no significant electric or magnetic fields. At the end of the tube the ions hit an electrode and an electrical pulse is produced on a recorder.

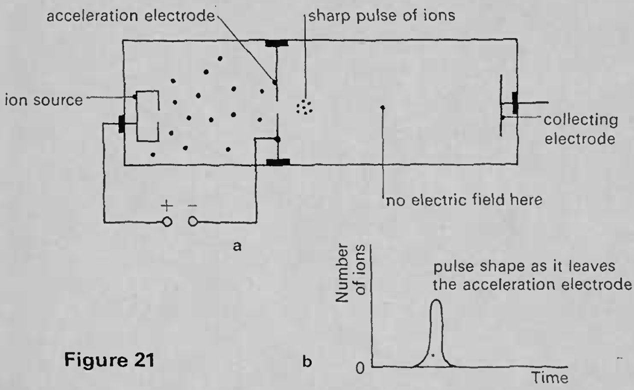


Figure 21

a If the ions are all of the same kind and have similar but not exactly equal velocities (because they are produced from the hot source with varying velocities though they are accelerated equally), show, on the same axes as figure 21 *b*, the pulse at the collecting electrode.
b How is the velocity with which the ions leave the acceleration electrode related to the acceleration potential difference if they leave the source with negligible speed?
c Does the velocity change as the ions move down the field-free tube?
d If the ion pulse contained ions of differing masses, say m and $2m$, would the pulse shape be different at the collecting electrode? Explain your answer.
e Would the instrument be as good at detecting the presence of ions of mass 41 mixed with mass 40 ions, as at detecting mass 21 ions in the presence of ions of mass 20? Explain.

24 The photograph, figure 23, shows the path of an electron beam in a bubble chamber when there is a magnetic field at right angles to the plane of the beam. Figure 22 is a sketch showing the apparatus.

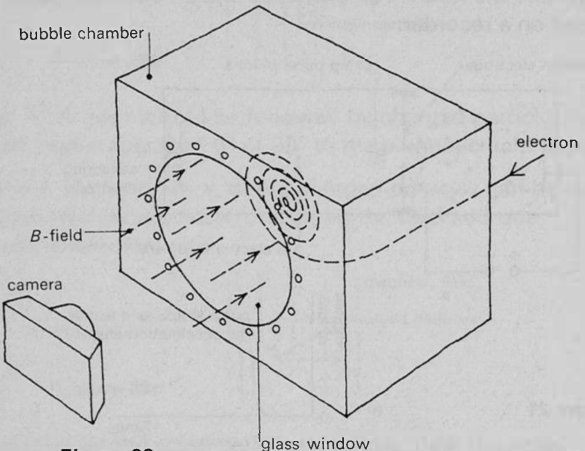


Figure 22

a The path is a spiral. Consider what could be changing as the electron moves in the bubble chamber, and suggest why the path is not a circle.
b The magnetic field was 1.2 N A^{-1} m^{-1}. Estimate the initial momentum of the electron using the scale shown.
c *Optional extra.* Plot a graph showing how the momentum of the electron beam changes as the distance covered in the chamber changes.

Note: In fact, the speed of the electron hardly changes, even though its momentum decreases. This is because its speed is near to the speed of light. The theory of relativity says that the momentum is not mv but $m_0 v/\sqrt{1 - v^2/c^2}$, where m_0 is the rest mass (the mass measured at low velocities).

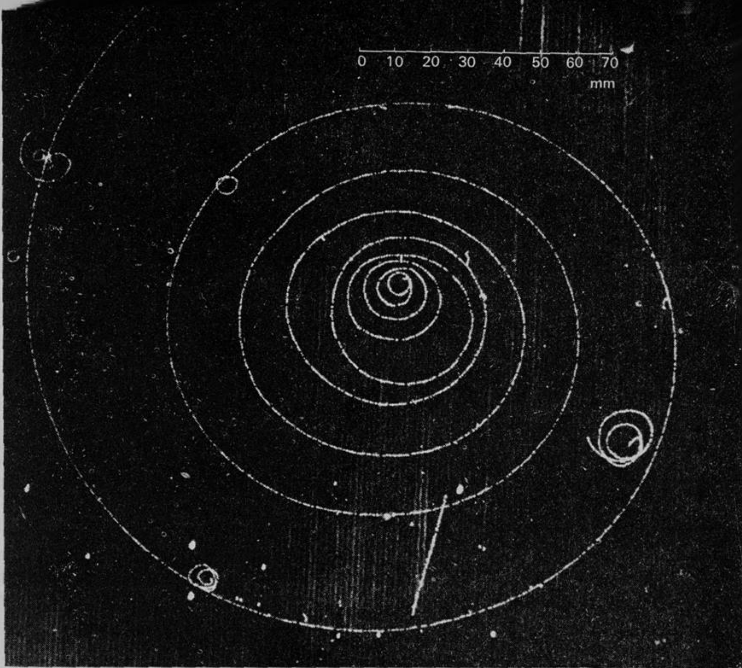


Figure 23
Photograph, Lawrence Berkeley Laboratory, University of California.

25 This is an exercise in scientific comprehension. **a** to **e** can be answered using ideas from this extract *(from Anderson, C. D. (1933) 'The positive electron'.* Physical Review, **43**).

Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the curvatures and ionizations produced require the mass to be less than twenty times the proton mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

Editor

On August 2, 1932, during the course of photographing cosmic-ray tracks produced in a vertical Wilson chamber (magnetic field of 1.5 N A^{-1} m^{-1}) designed in the summer of 1930 by Professor R. A. Millikan and the writer, the tracks shown in Fig. 1 [24] were

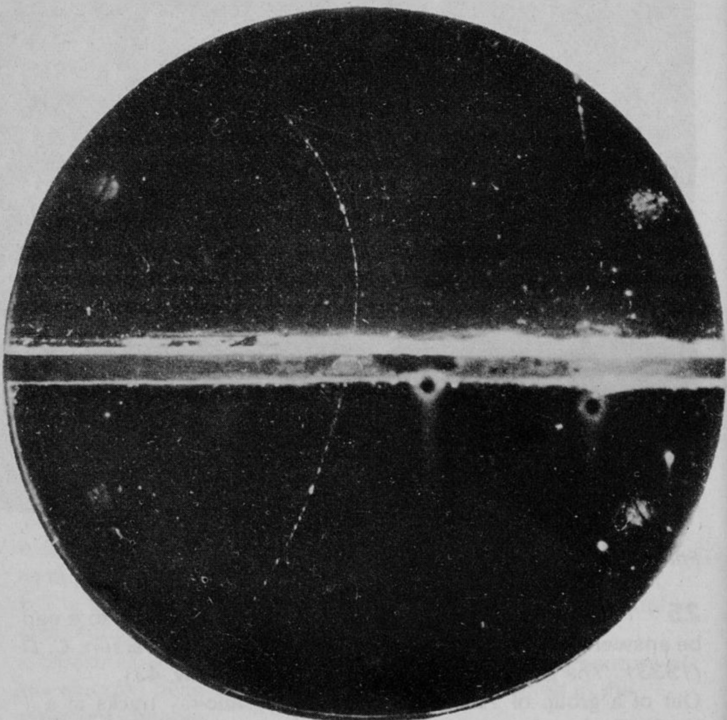


Figure 24
A 63 million volt positron passing through a 6 mm lead plate and emerging as a 23 million volt positron. The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.
Photograph, Science Museum, London.

obtained, which seemed to be interpretable only on the basis of the existence in this case of a particle carrying a positive charge but having a mass of the same order of magnitude as that normally possessed by a free negative electron. Later study of the photograph by a whole group of men of the Norman Bridge Laboratory only tended to strengthen this view. The reason that this interpretation seemed so inevitable is that the track appearing on the upper half of the figure cannot possibly have a mass as large as that of a proton for as soon as the mass is fixed the energy is at once fixed by the curvature. The energy of a proton of that curvature comes out 300 000 volts, but a proton of that energy according to well established and universally accepted determinations[1] has a total range of about 5 mm in air while that portion of the range actually visible in this case exceeds 5 cm without a noticeable change in curvature. The only escape from this conclusion would be to assume that at exactly the same instant (and the sharpness of the tracks determines that instant to within about a fiftieth of a second) two independent electrons happened to produce two tracks so placed as to give the impression of a single particle shooting through the lead plate. This assumption was dismissed on a probability basis, since a sharp track of this order of curvature under the experimental conditions prevailing occurred in the chamber only once in some 500 exposures, and since there was practically no chance at all that two such tracks should line up in this way. We also discarded as completely untenable the assumption of an electron of 20 million volts entering the lead on one side and coming out with an energy of 60 million volts on the other side. A fourth possibility is that a photon, entering the lead from above, knocked out of the nucleus of a lead atom two particles, one of which shot upward and the other downward. But in this case the upward moving one would be a positive of small mass so that either of the two possibilities leads to the existence of the positive electron.

In the course of the next few weeks other photographs were obtained which could be interpreted logically only on the positive-electron basis, and a brief report was then published with due reserve in interpretation in view of the importance and striking nature of the announcement.

[1] Rutherford, Chadwick, and Ellis, *Radiations from Radioactive Substance*, p. 294. Assuming $R \propto v^3$ and using data there given the range of a 300 000 volt proton in air S.T.P. is about 5 mm.

a What charge and mass would an object described as a 'positive electron' have?
b Would a proton have more or less mass than a positive electron?

c Why does the radius, r, of the path in a field, B, determine the momentum mv of the particle?

d *Kinetic energy* $= \frac{1}{2}mv^2$, *momentum* $= mv$

$$\therefore \textit{ kinetic energy} = \frac{(\textit{momentum})^2}{2m}.$$

Would a proton curving to the extent observed have more or less energy than an electron (or positron)?

e Where on the diagram in figure 24 would a proton of the right momentum have stopped?

Part Two

Electromagnetic induction

26 In a design for an electric car it is proposed to use the battery driven motor as a dynamo to charge the battery when the car is running downhill or slowing down.

Is this a sensible suggestion? It is further suggested that such an arrangement will help to brake the car. Is that right?

27 This question is about the voltage induced across a conductor moved in a magnetic field.

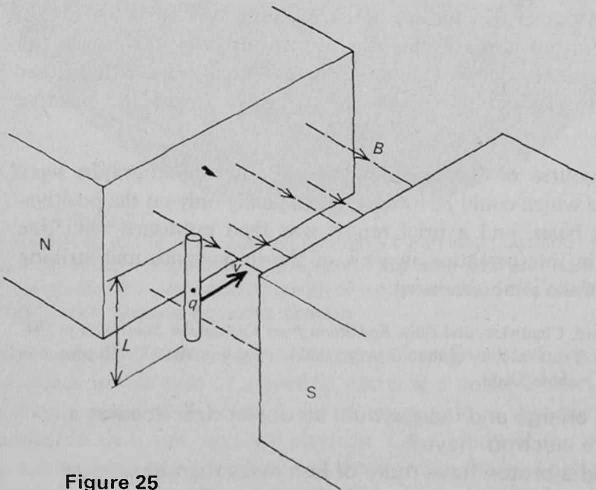


Figure 25

Figure 25 shows a conducting rod moving at right angles to its length L across a B-field. The rod, and all the charge carriers in it (each with charge q) are carried along at velocity v.

a What magnetic force will be exerted on one carrier? Will the force be parallel to the rod, the B-field, or the velocity?
b What will happen if a galvanometer is connected to the ends of the rod? (The galvanometer is stationary.)
c The carriers tend to pile up at one end of the rod, if there is now no external circuit (such as the galvanometer) for them to get out through. A voltage V develops across the rod. If V is steady, no further piling up of charge carriers can be going on. What electric field E exists across the rod, length L, when the p.d. across it is V?
d What force Eq acts on each charge carrier in the rod?
e When the carriers are no longer piling up more and more, the electric force (from **d**) must counterbalance the magnetic force (from **a**). Write an expression for V involving only B, L, and v.
f Will the magnitude and sign of the induced voltage V depend upon the magnitude and sign of the charge q on each carrier?
g Suppose now that the moving rod is connected to an external resistor (at rest) so that current I flows through resistance R. How much energy will be dissipated in R, in time t, in terms of I, R, and t?
h When the rod moves, there is an induced current I. When there is a current I, there is a force $F = BIL$ on the rod. Could this force be in such a direction as to increase the speed of the rod? Explain.
i What distance does the rod move in time t?
j Energy has to be delivered to the rod to keep it moving, since the moving rod is delivering energy to the resistor. Write down the work involved in pushing the rod for a time t, from previous answers for force and distance.
k Compare the answers to **j** and **g**. Express the voltage V in terms of B, L, and v.
l Suppose the rod were its own resistor (say a rod of the material used to make radio resistors), its ends being connected together by as short a piece of copper wire as possible. If this combination were pushed through the field, would any current flow?

28 *This question is about another – at first sight not very* helpful – way of writing down the voltage V induced across a rod of length L, moving as shown in figure 26, at right angles to field B, at velocity v. The induced voltage across the ends of the rod is given from question 27 by

$$V = BLv.$$

a Make up an argument to give a quantitative algebraic expression for the following verbal rule for V:

> The voltage V is equal to B multiplied by the rate at which the rod sweeps out area A perpendicular to B.

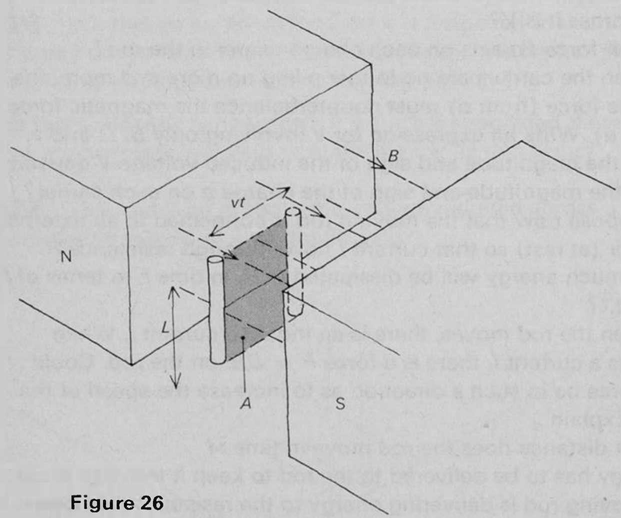


Figure 26

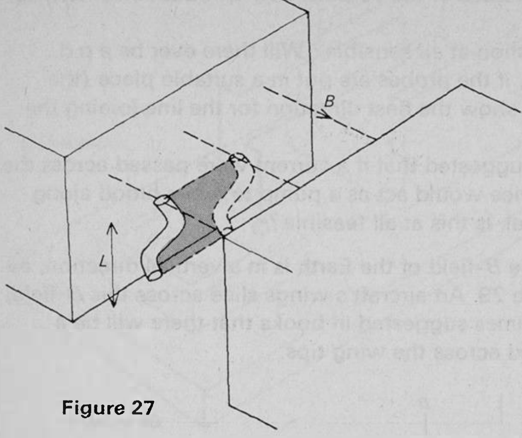


Figure 27

b *(Harder)* Suppose that the rod were a zig-zag shape, and its length were larger than the distance L between its ends, as shown in figure 27. Would the induced voltage be larger than $V = BLv$?

29 It has been suggested that a flowmeter to record the rate of flow of blood in the blood vessels of an animal could be made as indicated in figure 28. The idea is to produce a B-field at right angles to the blood vessel, and to record a voltage across the blood vessel between two probes attached to it.

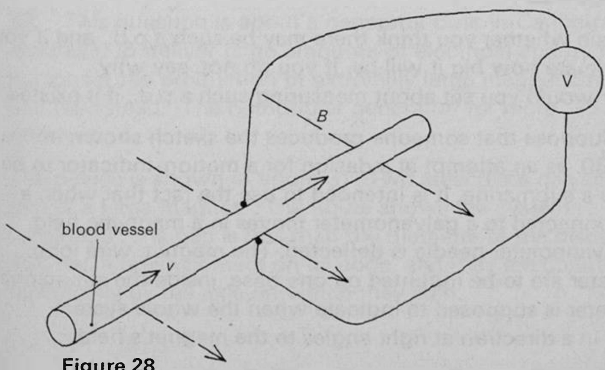


Figure 28

Blood, and the walls of the vessels, both conduct electricity to some extent.

a Is the suggestion at all sensible? Will there ever be a p.d., however small, if the probes are put in a suitable place (the figure may not show the best direction for the line joining the probes)?

b It is further suggested that if a current were passed across the probes the device would act as a pump to move blood along the blood vessel. Is this at all feasible?

30 Part of the *B*-field of the Earth is in a vertical direction, as shown in figure 29. An aircraft's wings slice across this *B*-field, and it is sometimes suggested in books that there will be a voltage induced across the wing tips.

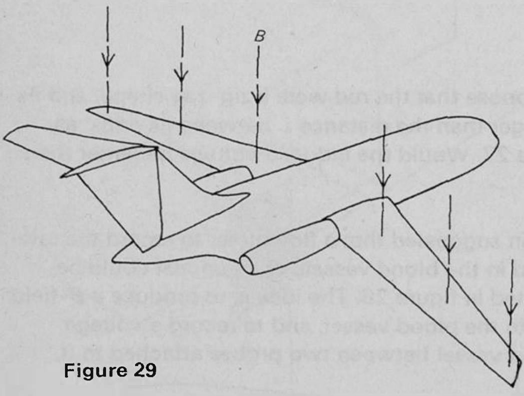


Figure 29

a Explain whether you think there may be such a p.d., and if you do, estimate how big it will be. If you do not, say why.

b How would you set about measuring such a p.d., if it existed?

31 *Suppose that someone produces the sketch shown in* figure 30, as an attempt at a design for a motion-indicator to be used in a submarine. It is intended to use the fact that when a wire connected to a galvanometer moves in a magnetic field the galvanometer needle is deflected. The magnet, wire loop, and meter are to be mounted on one base, inside the submarine. The meter is supposed to indicate when the whole system moves in a direction at right angles to the magnet's field.

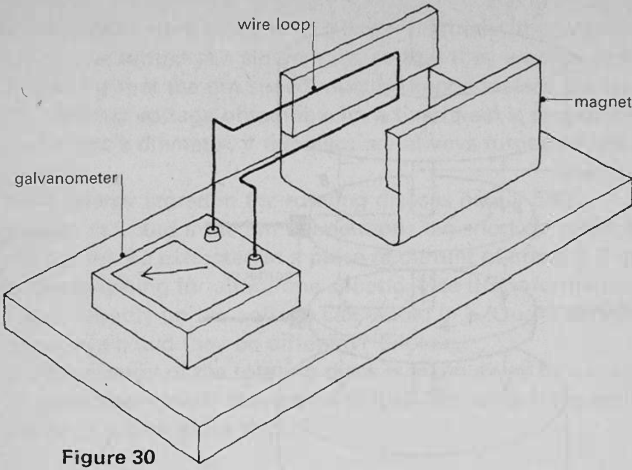


Figure 30

Explain as carefully as you can why it will *not* work at all. Under just what circumstances will a galvanometer connected to a wire loop show an indication, if the indication arises from *motion* and the *presence of magnets*?

In case you think this a silly question, you may like to know that it was the kind of question which Einstein said led him to the theory of relativity. You may be able to see that there is no chance of doing an experiment on Earth with magnets and wires to see if the Earth is 'moving through space'.

32 This question is about a generator built in Canberra, Australia, to deliver large amounts of electrical energy in short bursts, for the production of extremely large magnetic fields. See Newstead, 'The homopolar generator' for more information.

The device is shown in figure 31. Two steel discs can spin in the gap of a large magnet. (The figure shows the system 'exploded': the gap is less than the diameter of the discs.) Each disc has a brush contact on its edge, and they are connected electrically by the axle on which they spin.

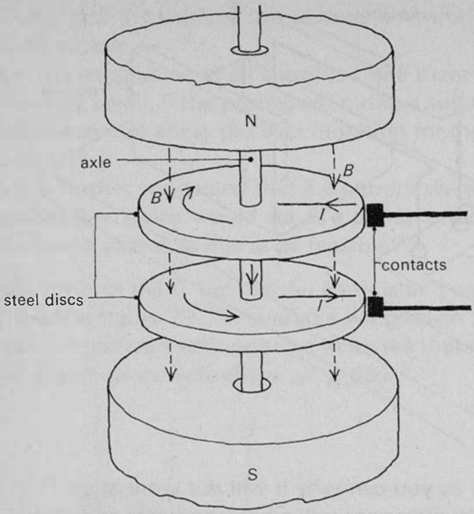


Figure 31

a *In figure 31, current is being passed from an outside source in along the radius of one disc, and out along the other. Why do the discs rotate, and why do they rotate in opposite directions?*
b Each disc in the machine is 3.6 m in diameter, and can safely be rotated at 900 revolutions per minute, when its rim is travelling at a velocity of about 170 m s^{-1}. Explain why the tensile strength and the density of steel set an upper limit on the speed of the rim of the disc. (*You need not make any calculations, though you may if you wish.*)
c Each disc has a mass of 40 000 kg. Make a *rough* estimate of the kinetic energy of moving steel at the rate of rotation which was given in **b**.
d *The discs can be made to rotate, using the device as a motor. When they are rotating, the device can be used as a direct current dynamo.*

When the discs spin, the voltage between axle and rim is the same as if the discs had spokes, being equal to the field B multiplied by the area swept out by a radius spoke in one second. Estimate the voltage between the two rims of the oppositely rotating discs, if the *B*-field is about 2 N A^{-1} m^{-1}.

e If the discs were made with a larger diameter, they would have to be turned at a slower rate, so that they would not burst. The rule is that the rim speed must be kept constant. Show that the induced voltage obtainable for a fixed field is proportional to the disc's diameter, if the discs are always turned as fast as possible.
f The energy stored in the rotating discs is nearly 580 megajoules, and if the rim connections are short circuited, this energy can be extracted in a pulse of current of about 1.5 million amperes lasting for about one second. Use this information to check roughly on the voltage calculated in **e**. Ought they to agree, or should they be different? Explain.
g If the energy of the rotating discs is taken away by using them as a dynamo, clearly they come to rest. But what is the source of the force which stops them?

33 In figure 32, there is a wire loop joined to a galvanometer G, and a U-shaped electromagnet connected to a variable d.c. supply. Suppose it takes 20 seconds to pass the loop over and into the U, without exceeding a galvanometer deflection of, say, 10 mm, the current in the electromagnet being 5 A.

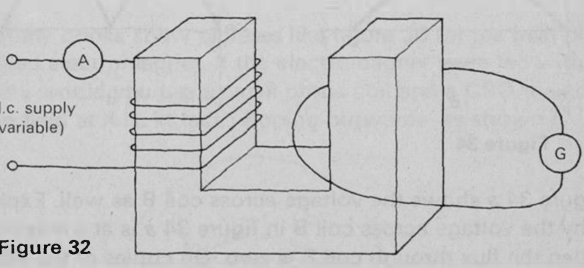


Figure 32

Someone now asserts that, 'It is not possible, by any means, to go from the state of affairs with the loop not within the U, and the current at zero, to the state of affairs with the loop within the U and the current at 5 A, in less than 20 seconds, without exceeding a 10 mm deflection.'

Do you think this assertion is true? What trials would you make to test its truth? Do all your trials involve carrying electrons in a wire at some speed through a magnetic field?

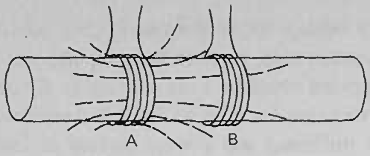


Figure 33

34 Figure 33 shows two coils A and B wound on a piece of iron. The current in coil A is varied so that the flux through coil B varies with time as shown in the graphs given in figure 34.

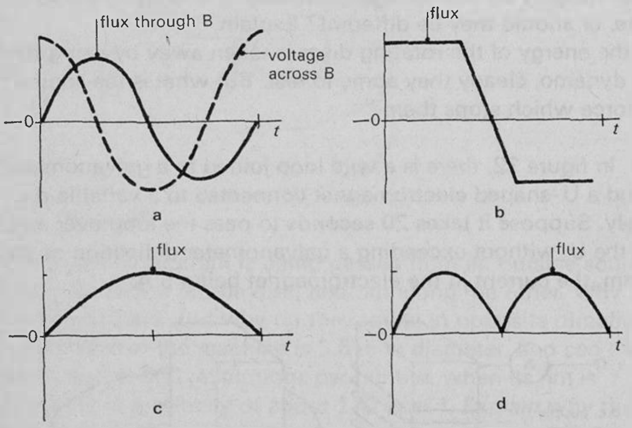


Figure 34

Figure 34 *a* shows the voltage across coil B as well. Explain why the voltage across coil B in figure 34 *a* is at a maximum when the flux through coil B is zero. On copies of the other graphs, sketch the variation of voltage across coil B. Each is drawn to the same scale.

35 A student tried to make an alternating current dynamo for use in the Earth's magnetic field. He wound ten turns of wire around a square frame 0.3 m by 0.3 m, and rotated this frame in the Earth's field ($B \approx 10^{-5}$ N A^{-1} m^{-1}). There was an alternating voltage across the coil, but it was small even when the coil was rotated as fast as possible.

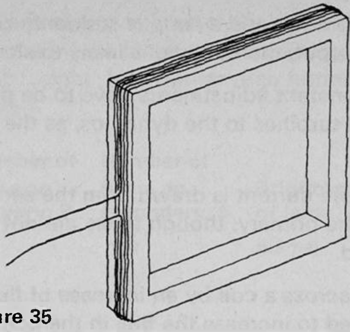


Figure 35

a Suggest two ways of increasing the induced voltage by a factor of four, other than by turning the coil faster.
b At first, the leads to the coil were just twisted wires. This worked, but the wires soon became too tightly twisted when the coil was turned, and broke. It was not hard to overcome this problem. How could it have been done?
c Another student, hearing of the device, tried turning it, but got absolutely no voltage. What way is there of rotating the coil which will give this result?

36 Many books show pictures like figure 36 for the field of a U-shaped electromagnet. If the electromagnet were fed with a.c., how would you use a small probe coil and a CRO to show that the field at X is, in fact, 'sloping outwards' as shown?

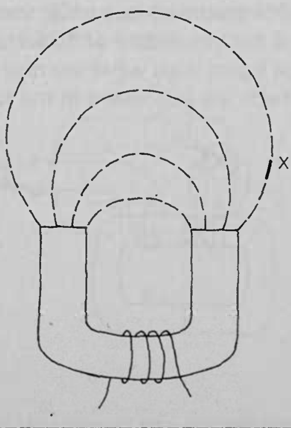


Figure 36

37 If a dynamo is turning, and a lamp is suddenly connected across the dynamo output, the dynamo is likely to slow down.

In a power station, constant adjustments have to be made to the power delivered from turbines to the dynamos, as the demand fluctuates.

In a transformer, if more current is drawn from the secondary, more is delivered to the primary, though these are not electrically connected.

The voltage induced across a coil by an increase of flux through the coil cannot be used to increase the flux in the coil by driving a current through it.

The current from the motors of a train, used as dynamos, can be used to stop the train but cannot be used directly to accelerate it.

Give all these separate facts one general interpretation, in terms of energy.

Transformers

38 A transformer for a toy railway set converts the 240 V alternating mains supply to a 12 V alternating voltage.
a What is the turns ratio?
b What do you think determines the actual number of turns?
c Could the transformer be used with a steady d.c. input?

39 In figure 37, P is a primary coil of 20 turns, joined to a 1 V alternating supply. S is a secondary of 50 turns joined to a small 2.5 V lamp, L, which lights with what we may call 'normal brightness'. These facts are expressed in the top line of table 1.

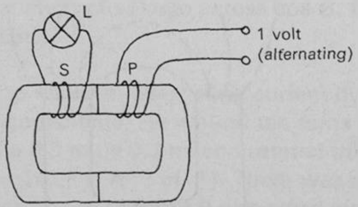


Figure 37

Fill in the remaining spaces of the table. For the third column, write 'normal' if the lamp is normally bright, 'dim' for dim or not alight, 'bright' for brighter than normal (including 'burnt-out').

Number of turns on primary, P	Number of turns on secondary, S	Brightness of lamp, L	Alternating p.d. across S, in volts
20	50	normal	2.5
50	20		
20	30		
40	100		
20	80		

Table 1

40 a Experiments show (as in question 39) that you can get 2.5 V from the secondary of a transformer by supplying 1 V to the primary. This, however, is not thought to contradict the 'conservation of energy' principle. Why not?
b In fact, you cannot get out quite as much energy from the secondary as you put into the primary. Suggest one or two ways in which energy may be wasted in the transformer.

41 Figure 38 shows three different arrangements of the same transformer. P is a primary coil of 20 turns, always connected to a one-volt alternating supply. S is a secondary coil of 50 turns, joined to a 2.5 V bulb. In **c** the secondary is wound on top of the primary.

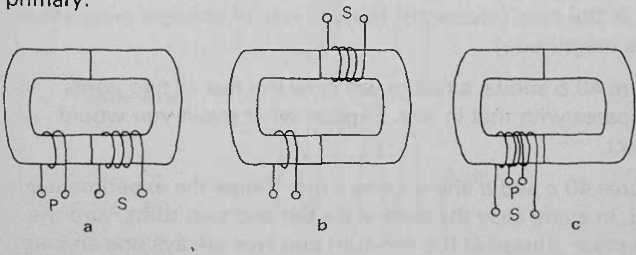


Figure 38

a There seems to be little difference between the three arrangements: in every case the bulb lights normally although the secondary is in different places. How do you explain this?

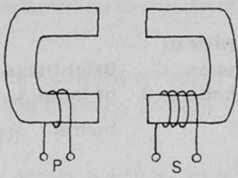


Figure 39

b Figure 39 shows arrangement *a* with the iron cores slightly separated. This gives a different result; the bulb lights only very dimly. Why is it that separating the two parts of the iron cores of a transformer causes it to work much less efficiently?
c If the iron cores in arrangement *c* are slightly separated, the lamp lights dimly. Why is this?
d Look again at figure 38 *a* to *c*. Suppose that all the three secondaries are placed on *one* transformer core with one primary. Each secondary is joined to a bulb, and all three bulbs are found to light normally. How do you explain this result?
e How would you expect the primary current in this case to compare with the primary current when there is one secondary with one bulb?

42 Figure 40 *a* is a sketch of an experiment in which a fixed alternating current was passed in a ten-turn coil wrapped round an iron core. The maximum voltage induced across a two-turn coil round the core was used to measure the maximum flux in the core (indirectly, from its rate of change, proportional to its magnitude).

Figure 40 *b* shows a test to see how the flux in two cores compares with that in one. Explain what result you would expect.

Figures 40 *c* and *d* show some other things the experimenter tried. In every case the coils were ten and two turns, and the maximum current in the ten-turn coil was always one ampere. Explain what you think was observed. If you are in doubt, guess an answer and then try it in the laboratory.

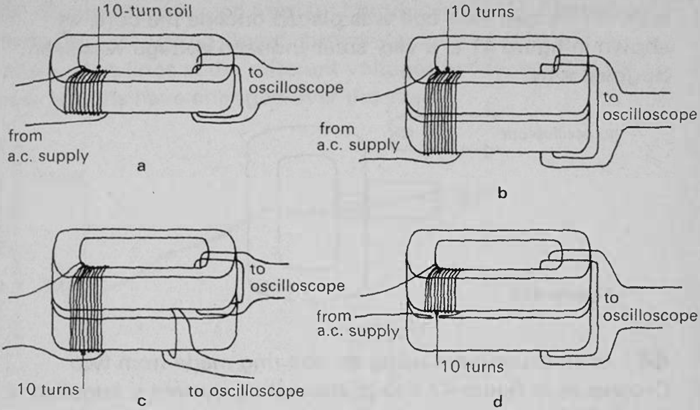


Figure 40

43 a In the experiment shown in figure 41 *a* to investigate the effect of turns on induced voltage, a student had a core with a 100-turn coil on one arm. He put a two-turn coil on the core and connected it to an oscilloscope, getting a trace like that in figure 41 *b*.

When the two turns were pushed round nearer to the 100-turn coil, the pattern became a little bigger, about 1.1 cm. Suggest a reason.

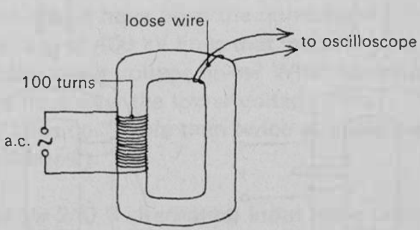


Figure 41a

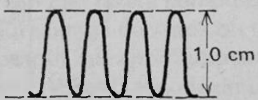


Figure 41b

b When his two-turn coil was placed outside the core, as shown in figure 41 *c*, a very small induced voltage was seen. Suggest why.

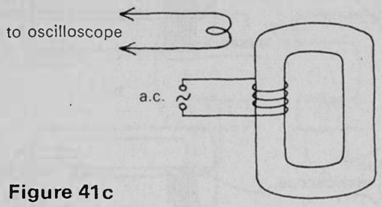


Figure 41c

44 In an experiment using an iron ring made from two C-cores as in figure 42 *a* to *d*, alternating current is supplied to coil B. The arrangement shown in diagram *a* gives an induced voltage of amplitude *V*. What amplitudes would you expect to find with the other arrangements?

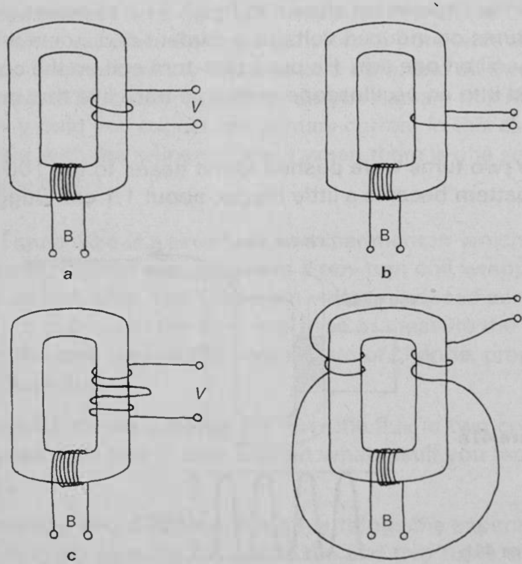


Figure 42

45 Figure 43, adapted from one produced by the Central Electricity Generating Board, shows the total lengths of transmission lines at the different voltages in use, and the way these lengths have changed over the years.

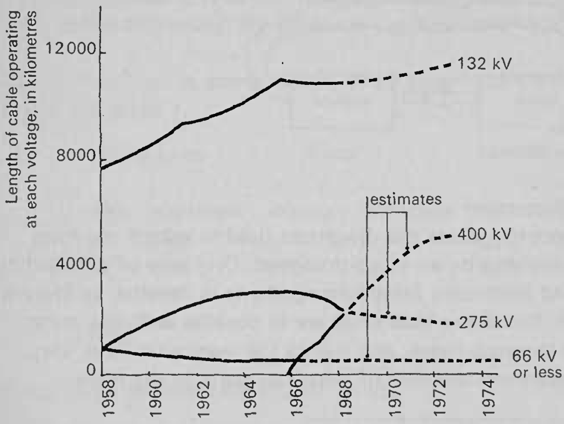


Figure 43
Data from C.E.G.B Statistical yearbook *1968.*

a Why are such high voltages used?
b Why do you think a number of voltages are used?
c What do you think might have been the difficulties associated with the use of 400 kV lines that made them a later development than the lower voltage lines? What advantages would 400 kV lines have over the lower voltage lines? (Overhead 400 kV lines cost more than twice as much per kilometre as 132 kV lines.)

46 A transformer for 240 V alternating input has a primary coil with 100 turns which can take a current of 2 A without overheating. The output voltage depends on the turns ratio. What determines the output *current*? Suppose the output load has a very low resistance. What is the maximum current that this transformer could possibly supply? (Assume that you can choose any kind of secondary coil.) What would the turns ratio and the output voltage be?

Electric motors and dynamos

47 'The electric motor drives the dynamo which supplies the electrical energy which keeps the motor turning, and power is produced for nothing.' Why won't this arrangement work?

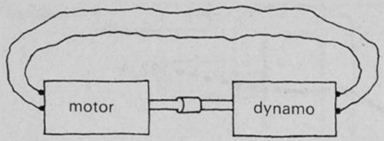


Figure 44

48 *For discussion*

In many electric motors the magnetic field in which the rotor rotates is provided by an electromagnet. One way of connecting the rotor and field coils (electromagnet) is in parallel, as shown in figure 45. Since the field coils are in parallel with the rotor, the current through them, and hence the magnetic field, stay pretty constant irrespective of what happens to the rotor.

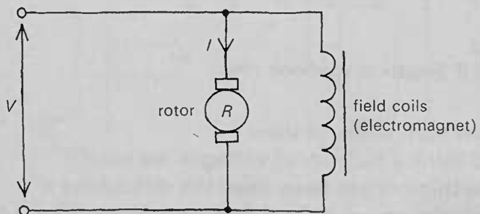


Figure 45

The rotor has resistance R, carries current I, and has a voltage V across it. In this question, the steady current drawn by the field coils is ignored.

a Write an equation for the current I if the rotor is held still.

b If the rotor turns, the flux linking its coils is changing. In effect, because of the commutator, a steady term $\mathrm{d}\phi/\mathrm{d}t$ can be written for this rate of change of flux. A voltage is needed to change the flux in a coil, and for the motor with a turning rotor, the equation from **a** becomes

$$V = IR + \mathrm{d}\phi/\mathrm{d}t.$$

If R is constant and the rate of change of flux is proportional to the rate of rotation, sketch a graph of I against rate of rotation.
c The torque produced by the motor depends on the current I. How, if at all, will the torque vary with speed of rotation?
d When a motor is required to drive a larger load, the current its rotor draws increases. By what means does this happen?

49 Table 2 gives some results of an experiment using a d.c. motor ('fracmo').

Field coils		**Rotor**		**Load**	**Rate of rotation**
V in volts	I in amperes	V in volts	I in amperes		in revolutions per second
5	0.35	5	1.0	zero	27
5	0.35	5	1.9	light	21
5	0.35	5	3.1	heavy	13

Table 2

Resistance of rotor (armature) $= 1\,\Omega$.

The field and rotor windings were supplied independently, from two different sources. Readings of applied voltage V and current I for both field coils and rotor were taken.

a The ratio V/I for the rotor varies, and is never equal to $1\,\Omega$. Why?
b Why is the rotor current larger when the motor is working against a load?
c How could the motor be made to turn the 'heavy' load at 27 revolutions per second?

Self-induction

50 When a current I flows in an air-cored coil, it produces a B-field, and so magnetic flux, within the coil.
a If the current doubles, what happens to the flux in the coil?
b In a particular coil, whose resistance was negligible, the current rose at an almost steady rate for 0.5 s, increasing from zero to 0.1 A in this time. What was the rate of increase of current?

c During this time, would the voltage V across the coil, needed to maintain this rate of increase, be zero, constant, rising steadily, or falling steadily?
d If the flux ϕ linking the turns of the coil is given by

$$\phi = LI$$

so that

$$V = L\,dI/dt$$

give a numerical answer to **c**, if the constant L, the inductance, is 5×10^{-3} V s A^{-1} (or henries, H).
e Why would one need to be careful if a current of 1 A were cut off suddenly (say in 1/100 s) in a coil of inductance $L = 10$ H?

51 Professor J. Henry wrote a paper in 1832 for the *American journal of science* (**22**, page 403) on the topic of self-induction. Here is an extract.

. . . When a small battery is moderately excited by diluted acid, and its poles, which should be terminated by cups of mercury, are connected by a copper wire not more than a foot in length, no spark is perceived when the connection is either formed or broken; but if a wire thirty or forty feet long be used instead of the short wire, though no spark will be perceptible when the connection is made, yet when it is broken by drawing one end of the wire from its cup of mercury, a vivid spark is produced.

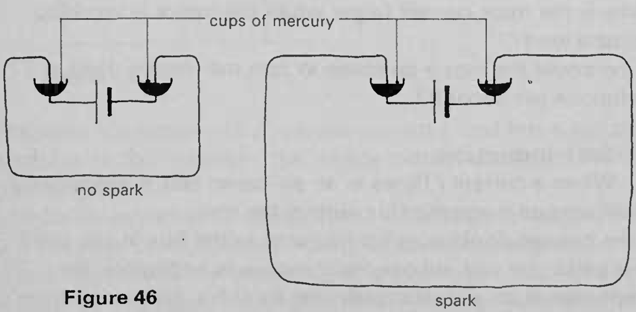


Figure 46

a What explanation can you advance for these observations?
b In a later paper (*Journal of the Franklin Institute*. March 1835, **15**, page 169) some further observations were presented.

'A thick wire gives a larger spark than a smaller one of the same length.'

'A wire coiled into a helix gives a more vivid spark than the same wire when uncoiled.'

Invent plausible explanations, or show the question to someone else, write down his or her explanation, and criticize it.

Part Three

Flux near currents

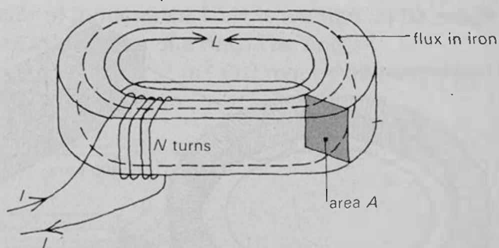
Magnetic flux, magnetic circuits, and reluctance

52 Figure 47 shows an iron ring, in which magnetic flux ϕ is set up by N turns of wire wrapped round the ring, carrying current I. As long as the current is not too big, the flux ϕ is given by

$$NI = \phi L / \mu_r \mu_0 A$$

where L is the length round the ring, and A is its cross-sectional area. The quantity $\mu_r \mu_0$ is more or less constant.

In a copper wire, the steady current flowing depends on the applied voltage and the resistance. The resistance depends on the length of wire and the cross-sectional area of the wire.



a Write an expression for the resistance of a wire in terms of length and area.
b Write an equation analogous to that for current-turns, flux, length, and area, but for the voltage driving current in a wire. Current is to be analogous to flux. What is the voltage analogous to in the 'magnetic circuit'?

c To what quantity does the combination $\mu_r\mu_0$ compare in the analogy?

d The general form of the relationship is

driving force = rate of flow × length/(conductivity × area)

What corresponds to each term in the case of energy leaking from a hot water tank through its lagging (the fibre-filled jacket over the tank)?

e The magnetic circuit equation is sometimes written

$NI = \phi$ *(reluctance)*

To what electrical quantity does reluctance correspond, in the analogy with Ohm's Law?

53 A solenoid is rather like a pipe which carries magnetic flux, and the flux in a solenoid can be calculated just as if one were calculating the flow of electric current along a copper bar, the flow of heat along a conducting bar, or (neglecting many of the real-life complications) the flow of fluid down a tube.

A calculation of the 'flow' of flux in a straight solenoid, such as that shown in figure 48 *b*, is complicated by the difficulty of working out what happens at the ends. One way of avoiding the difficulty is to regard the solenoid as part of a very long one whose ends are brought round to meet each other, as in figure 48 *a*.

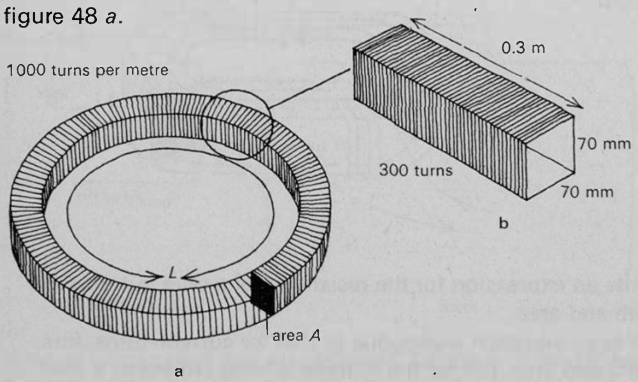


Figure 48

a The reluctance of the large, closed solenoid pipe shown in figure 48 *a* is given by $L/\mu_0 A$, where L is the length all around the pipe, A is cross-sectional area, and μ_0 is $4\pi \times 10^{-7}$ N A^{-2}. Calculate the contribution made to this reluctance by the shorter, almost straight section in figure 48 *b*.
b The flux 'going round' the large closed solenoid is given by

current-turns = flux × reluctance.

Explain why, if the length L is altered, keeping the number of turns per metre and the current the same, the flux is unaltered.
c Calculate the flux 'going through' any section of the solenoid, using the answer to **a**, if the solenoid carries one ampere.
d What is the value of B within the solenoid?
e Suppose that a steel rod having roughly the cross-sectional area of a retort stand rod were put in the middle of the solenoid. You may suppose that the rod is bent round so that the two ends meet. The reluctance of a steel rod is given by the same relationship as that in **a**, but with μ_0 multiplied by a large factor, μ_r, which you may take to be 500. Use a rough estimate of the rod's cross-sectional area to make a rough estimate of the ratio of the flux going through the rod to the flux going through the rest of the inside of the solenoid.

54 This question is about the design of an inductor.

Suppose an inductor of inductance 10 H is wanted, to be made from an iron C-core in the form of a ring. The ring's cross-sectional area A is 2×10^{-4} m^2 (10 mm by 20 mm) and its length L round the ring is 0.2 m.

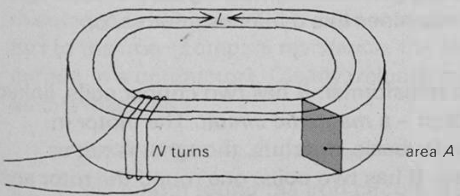


Figure 49

For the iron concerned, the reluctance is given by $L/1000\,\mu_0 A$, and μ_0 is $4\pi \times 10^{-7}$ N A^{-2}.

a Calculate the reluctance of the ring.

b Suppose the coil round the ring has N turns, and carries current I, in amperes. Use the answer to **a** to relate the number of ampere turns to the flux in the iron ring.

c The flux links each of those N turns. If $V = \mathrm{d}\phi/\mathrm{d}t$ is the voltage needed across one turn to change the flux through it, what voltage is needed across all N turns?

d If the inductance L is to be 10 H, then the voltage across all N turns must be given by $10\,\mathrm{d}I/\mathrm{d}t$. Use the answers to **b** and **c** to find N for an inductance of 10 H.

e Estimate the length of wire involved in the coil having N turns wrapped over the ring, using the value of N from **d**.

f Use table 3 to select the thinnest gauge of copper wire for which the coil's resistance does not exceed 10 Ω. Estimate roughly the bulk of the coil, and see if your previous estimate of the length of wire was reasonable.

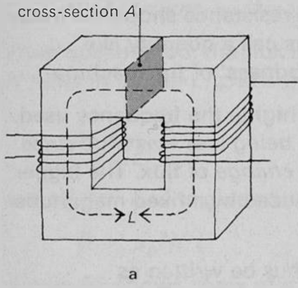
s.w.g.	Radius/mm	Resistance per metre/$\Omega\,m^{-1}$
14	1.0	0.005
18	0.60	0.015
22	0.35	0.043
26	0.22	0.105
30	0.15	0.30

Table 3
Relation between resistance and radius for copper wire.

55 *Optional, perhaps for discussion*

This question is about a general, engineering way of looking at the performance of machines like dynamos, motors, or transformers.

Figure 50 *a* shows a transformer. It has two copper coils, linked by a closed loop of iron – a *magnetic circuit*. The motor in figure 50 *b* has a similar basic structure, though it is not as obvious that this is so. It has two coils, one round the rotor and one round the iron core within which the rotor turns (the 'stator'). It too has a magnetic circuit: the iron core with curved pole pieces, the iron rotor which almost fills the gap between the poles, and the air gaps between rotor and stator.



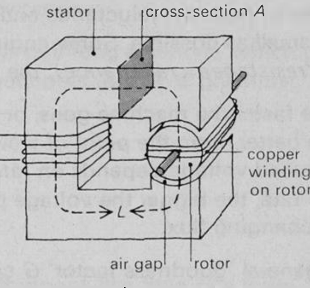


Figure 50

Clearly, resistance in the windings is a bad thing. Energy that could do something useful is used to warm up the machine. If the copper wires have length L_{copper}, and cross-sectional area A_{copper}, their resistance is given by,

$$\textit{resistance} = L_{copper} / \sigma A_{copper}$$

where σ is the electrical conductivity of the copper.

Both machines depend on the amount of flux in the iron. The more flux there is, the better. The motor will turn with a greater torque (force $\propto B$). The transformer will need fewer turns to operate at some specified voltage. The flux produced will be less the greater the *reluctance* of the magnetic circuit. The reluctance is the ratio of the current turns around the iron to the flux in the iron (compare resistance, the ratio of voltage to current in a conductor). Clearly, reluctance is a bad thing in such a machine.

$$\textit{reluctance} = L_{iron}/\mu_r \mu_0 A_{iron}$$

L_{iron} is the length round the iron circuit; A_{iron} is its cross-section. Air gaps, and varying thicknesses of iron, are taken into account by adding up the reluctances bit by bit around the circuit. See question 58. $\mu_r\mu_0$ might be called the 'magnetic conductivity'. For air, $\mu_r = 1$, but for iron μ_r may be as much as 1000, so air gaps increase the reluctance a lot, even if they are as short as possible.

Clearly, both the reluctance and the resistance should be made as small as possible. Some engineers call a quantity like 1/*(resistance × reluctance)*, the 'goodness' of the machine.

The faster the machine goes, or the higher the frequency used, the better, from the point of view of being less wasteful, since induced voltage depends on *rate of change* of flux. The bigger the rate, the bigger the voltage produced by a fixed magnitude of changing flux.

A general 'goodness factor' G can thus be written as

$$G = \text{rate}/(\text{resistance} \times \text{reluctance}).$$

a Use the expressions for resistance and reluctance to show that G has no units, but is a pure number. The unit of μ_0 is N A^{-2} (μ_r is a multiplying factor without units). The unit of resistance is Ω (which is also V A^{-1}). Remember that the voltage unit V is also J A^{-1} s^{-1}, and that a rate has unit s^{-1}.
b Imagine making a machine, say the transformer in figure 50 *a*, twice as big in every dimension. For the magnetic circuit, how does L_{iron} change? How does A_{iron} change? How does the reluctance change?
c If, as in **b**, the machine is scaled up by a factor two in every dimension, how does the resistance change? (Think of changes to L_{copper} and A_{copper}.)
d Is the 'goodness' of the scaled-up machine altered?
e Comment on the remark, 'A good electromagnetic machine is as big as possible, goes as fast as possible, and has its iron and copper circuits as closely entwined as possible.'

56 This question develops a theoretical reason for supposing that the B-field of a very long straight wire varies as the reciprocal of the distance from the wire.

The reluctance of a pipe or channel not filled with magnetic material is $L/\mu_0 A$, where L is the length of the channel, and A its (fixed) cross-sectional area. If flux ϕ 'runs' along the length of the channel, 'driven' by current-turns NI, then

current-turns = *flux* × *reluctance*

From question 53, the flux ϕ in a very long solenoid, which has N turns in length L, and cross-sectional area A, is given by

$$\phi = \frac{NI}{L/\mu_0 A}$$

and so $B = \phi/A$ is given by

$$B = \mu_0 NI/L.$$

Figure 51 shows some of the whirlpool-like field of a long straight wire.

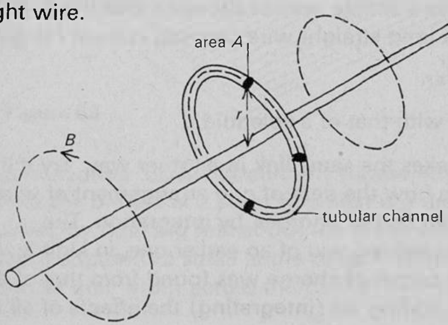


Figure 51

a What experiment could one do to show that the field has the shape of circles around the wire?
b Given that the field does not stick out from the wire like spokes, what *symmetry* argument can you offer in favour of the shape shown?
c Figure 51 shows a tubular channel following the field direction round the wire. What is the length round the channel, if it has radius r?
d The channel has, let us suppose, some small cross-sectional area A. Write down the relúctance of the path round this channel.

e The tubular channel encircles one wire once. If the wire carries current I, the flux round the channel is 'driven' by I current turns. Write an expression for the flux in the channel, using the reluctance found previously.
f The B-field at radius r is given by ϕ/A. Write an expression for the B-field of a long wire at radius r.
g The ampere is defined in such a way that if a long wire carried one ampere, and another long wire ran alongside it one metre away also carrying 1 A, there would be a force on the second wire of 2×10^{-7} N on each metre of it. But B *is* the force on a metre of wire carrying one ampere. What is B at one metre from the first long straight wire?
h Use the answers to **f** and **g** to write down the value of μ_0.

57 *Optional*
Question 56 gives a simple way of showing that the B-field at distance r from a long straight wire carrying current I is given by

$$B = \mu_0 I/2\pi r$$

linking this field with that of a solenoid.

This question makes the same link in another way. Try it if you would like to see how the field of one arrangement of wires can be calculated from that of another, by integration. The calculation might remind you of an earlier one, in Unit 3, where the field of a flat carpet of charge was found from that of a point charge, by adding up (integrating) the effects of all the point charges making up the flat carpet.

Now we shall consider what B-field there must be if a current is flowing uniformly across a sheet. In practice the effect is best measurable when the sheet consists of many wires laid closely side by side, each wire carrying the same current. It is possible to show experimentally that the field due to a single current-carrying wire is

$$B = kI/r$$

where k is a constant and figure 52 explains the other symbols. (r is the perpendicular distance from the point at which B is measured, to the wire.) The value of k will emerge from the calculation.

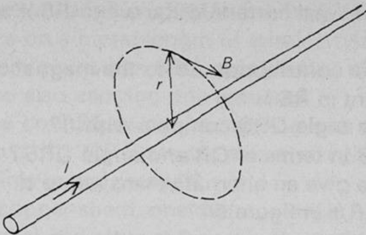


Figure 52

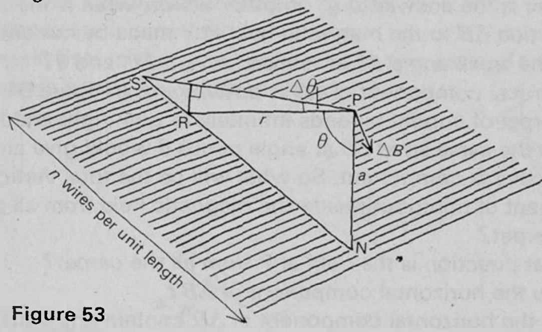


Figure 53

With a horizontal 'carpet of current' (figure 53) we can calculate the B-field at a point P, which is a distance a above the carpet, by thinking first about the wires within a small width RS which subtends a small angle $\Delta\theta$ at P (figure 54). There are n wires per unit width across the carpet.

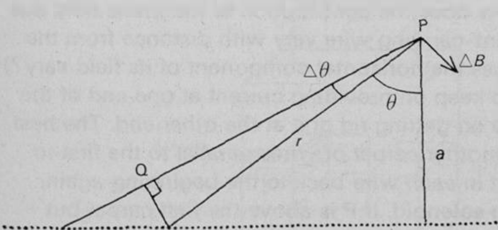


Figure 54

a Roughly what is the distance r of these wires from P, in terms of a and θ?
b How many wires are there in the small width RS of carpet?

c What is the total current in the width RS if each wire carries current I?
d What is the contribution ΔB to the magnetic field at P made by the current in RS?
e How does angle QRS compare with θ?
f What is RS in terms of QR and angle QRS?
g Use this to give an alternative answer to **d**.
h What is QR/r in figure 54?
i So what is the answer to **g**? In other words, what is the contribution ΔB to the magnetic field at P made by current within the small angle $\Delta\theta$ in terms of k, n, I, $\Delta\theta$, and θ?
j The vertical component of ΔB is *downwards* in figure 54 but if the carpet of current extends infinitely to both right and left, a ΔB from the same sized $\Delta\theta$ at angle minus θ would give an equal *upwards* component. So what will be the total vertical component of contributions to the magnetic field from all parts of the carpet?
k In what direction is the field at P from all the carpet?
l What is the horizontal component of ΔB?
m Does the horizontal component of ΔB contain any factor which depends on the direction in which $\Delta\theta$ has been chosen, that is, on which side of PN (figure 54) RS was chosen?
n What is the total of all the angles $\Delta\theta$ which the carpet subtends at P? (The carpet is big, and P is close to it.)
o What is the magnetic field at P?
p Does the magnetic field change in value with distance from the carpet? (How does the contribution to magnetic field due to a single current-carrying wire vary with distance from the carpet? How does the horizontal component of its field vary?)
q It is difficult to keep on providing current at one end of the carpet, and keep on getting rid of it at the other end. The best way is to have another carpet of wires parallel to the first to bring the current in each wire back to the beginning again, making a flattish solenoid. If P is above the first carpet but beneath the second, what is the total field at P?
r We can now relate the results to the result for the field in a solenoid, $B = \mu_0 nI$, where μ_0 is the magnetic force constant, $4\pi \times 10^{-7}$ N A^{-2}. What is k? ($n = N/L$ if N is the number of turns in length L.)

Optional extra questions which may be useful in Unit 8

s What is the force on a metre length of wire carrying one ampere if the wire is one metre from, and parallel to, another wire infinitely long also carrying one ampere?

t What is the force on 0.25 m of wire carrying one ampere inside a wide flat solenoid of 1200 turns per metre, carrying one ampere, all the wires being parallel?

u A long strip of copper sheet, one metre wide, is bent into two straight parts parallel and close to one another, and a current of 1200 amperes is passed along the strip (figure 55). What is the magnetic field in the space between the two parts, if the current is uniform across the width of the strip?

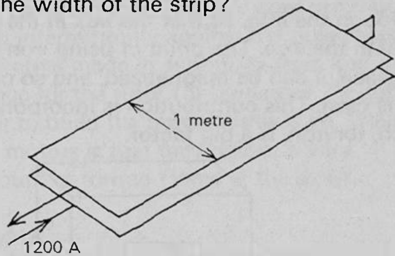


Figure 55

58 This question is about the design of an electromagnet, and about the importance of the air gap in such a magnet in deciding the B-field obtainable in the gap. The calculation is not rigorous or complete, but gives more or less sensible results.

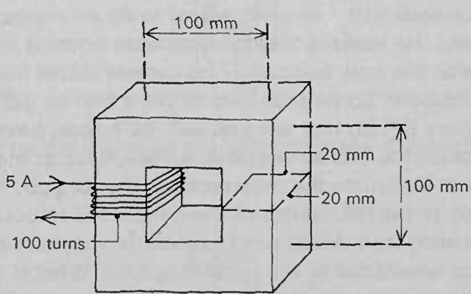


Figure 56

a Suppose the complete iron ring shown in figure 56 is available, and has the dimensions shown, with a mean total length round the iron of 400 mm, the cross-section being 20 mm by 20 mm. The ring has a 100-turn coil wound on it, which carries a 5 A direct current. Calculate the reluctance of the ring and the flux in the iron, if the reluctance of the core is given by $L/1000\,\mu_0 A$, where L is the mean length around the core, A is the cross-sectional area, $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}, and the factor 1000 is the value of the 'relative permeability' μ_r for this type of iron. The flux is given by *current-turns* = *flux* × *reluctance*.

Note. There is some flux in the air around the core, since the quantity $L/\mu_0 A$ (the reluctance) is not infinitely large for a path through air round the outside of the iron. But the extra factor 1000 for the iron makes this term larger than that for the path through the iron, so that the flux in the air is much less than that in the iron. The point of using iron is that, unlike empty space, it can be magnetized, and so contribute a lot of flux of its own. This contribution is incorporated in the factor μ_r, which, for iron, is a big factor.

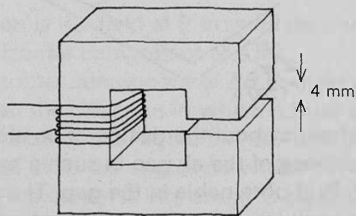


Figure 57

b Suppose that a student wants to do an experiment in a strong *B*-field. He realizes that the iron adds nothing to the field outside the iron, but clearly he cannot install his experiment in a solid core. So he proposes to cut a thin air gap in the core, as in figure 57. So that the gap will, he hopes, have as little effect on the flux as possible, he proposes to make it only 4 mm across. Calculate the reluctance of the air gap.

c Add to the reluctance of the air gap the reluctance of the remaining iron. Make any reasonable approximation to arrive at the reluctance of the remaining iron. What is the ratio of the reluctance now, to that for the closed ring in **a**?

d Calculate the flux in the magnetic circuit, with the gap introduced. How does it compare with the flux in the closed ring, from **a**?

e What is the *B*-field in the air gap?

59 *For discussion*

Use the results of question 58 to discuss the reasons for one or more of the following:

a The self-inductance of a coil provided with two C-cores which fit into the coil so as to make a closed ring of iron depends a good deal on whether the faces of the halves of the core are *flat, smooth, clean,* and *well pressed together*.

b The designer of a large power station dynamo goes to a lot of trouble to ensure that the rotor of his dynamo fits with the *narrowest possible gap* into the space between iron pole pieces, even though this gives him a lot of other design problems, such as ensuring that the rotor does not vibrate when it turns, and so rub intermittently against the pole pieces.

c Model electric motors made in school, or from kits, usually have plenty of space for the rotor (or 'armature') to turn in, because it is easier to build the model if this is so. The main trouble with such motors is that they deliver a very disappointing amount of torque (force at the axle).

Eddy currents

60 **a** A rotating metal ring is slowed down when placed between the pole faces of a magnet. Explain.

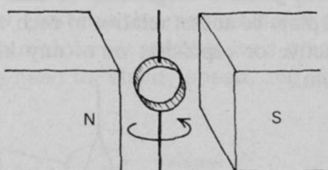


Figure 58a

b What would be the effect on the rotating ring if it were cut as in figure 58 *b*?

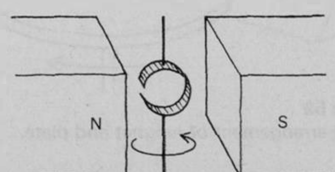


Figure 58b

c *(Hard)* What would happen if the ring were sliced many times as in figure 58 *c*?

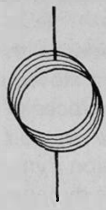


Figure 58c

d When the freely oscillating coil of some types of moving-coil galvanometer is short circuited, it stops very quickly. Why should this happen? (Try it with a light-beam galvanometer.)

61 Explain the following quotation from Faraday (1839, *Experimental researches in electricity,* Volume 1, article 81).

If a plate of copper be revolved close to a magnetic needle, or magnet, suspended in such a way that the latter may rotate in a plane parallel to that of the former, the magnet tends to follow the motion of the plate; or if the magnet be revolved, the plate tends to follow its motion; and the effect is so powerful that magnets or plates of many pounds weight may be thus carried round. If the magnet and plate be at rest relative to each other, not the slightest effect, attractive or repulsive, or of any kind, can be observed between them.

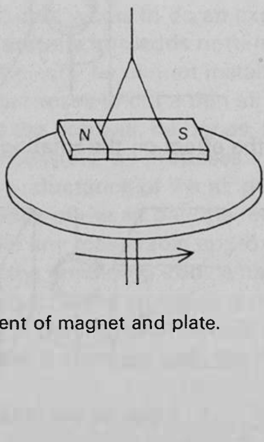


Figure 59
One possible arrangement of magnet and plate.

62 Some types of car speedometer have a magnet which is rotated by being connected to the wheels of the car, spinning just behind but not touching a copper or aluminium disc, to which the speedometer needle is fixed.

a An important component is missing from the above description, if the needle is to point to a steady speed on a dial when the magnet turns at a steady rate. What is it and how would it act?
b Sketch a suitable arrangement and explain how it works.
c *(Hard)* There is at least one way of rotating a bar magnet that will produce no effect at all on the disc. Suggest one way.

63 An electronic engine speed indicator might work by timing pulses of current in an electrical lead to a sparking plug in the engine. Suggest a means of detecting such pulses electrically without cutting the lead. (Your work on electronics may suggest to you ways of counting and timing the pulses.)

64 **a** A conducting plate carried on an air track vehicle (figure 60) can be moved along by moving a magnet steadily sideways. What happens if, once started, the plate happens to start going faster than the magnet (perhaps the magnet slows down)?
b Why is an induction motor very suitable for a gramophone motor? (Consider the need for steady speed, and your answer to **a**.)

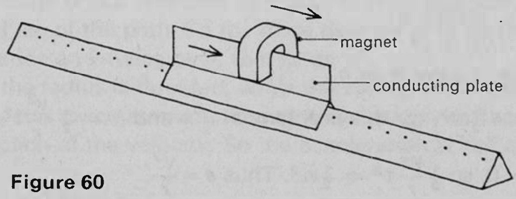


Figure 60

Answers

1 **a** For example: *scalar* – volume, density, energy; *vector* – displacement, force, acceleration.
b Directions of the forces are necessary. The puck accelerates along the direction of the resultant force.
c *a* Along the line of the 3 N force.
b Along the line of the forces.
c At an angle to the 3 N force whose tangent is $\frac{2}{3}$.
d At an angle to the 3 N force whose tangent is $\sqrt{3}/4$.
e At an angle to the 3 N force whose tangent is $\sqrt{3}/2$.
d If the force has no component along the path of the satellite, the speed of the satellite cannot be affected. A force at right angles to the path produces circular motion at a constant orbital speed. See PSSC *Physics*, page 344. If the satellite is in a non-circular orbit there is a component of the gravitational attraction force along the path. The speed of the satellite increases as it moves closer to the Earth and decreases as it moves away. Chapter 3 in Rogers, *Physics for the inquiring mind*, is useful.

2 **a** You need to explain that it is a matter of geometry. The little triangle AEB, with sides of length h and x at right angles, is similar to the bigger triangle BED, with sides of length x and $2r-h$ at right angles, so the lengths of these pairs of sides are in the same ratio.

$$h/x = x/(2r-h)$$

or $$h(2r-h) = x^2$$

b $2r$ is much greater than h.
c In $s = ut+\frac{1}{2}at^2$, for this problem, $s = h$ and $h = \frac{1}{2}\frac{v^2}{r}t^2$, while $ut = 0$, so $\frac{1}{2}\frac{v^2}{r}t^2 = \frac{1}{2}at^2$. Thus $a = \frac{v^2}{r}$.
Chapter 21 in Rogers, *Physics for the inquiring mind*, is useful reading on circular motion.

3 **a** Velocity is a vector quantity. For the old velocity to change to the new velocity, a velocity change in the direction PQ must be made.
b Angle PDQ is the same as angle ACB. Sides AC and BC are equal. Sides PD and QD are equal. In the limit, AB is almost straight.
c Equation **1** is a comparison of similar sides in two similar triangles.
d *acceleration = change in velocity/time.*

AB = distance travelled in time t

hence

$$\frac{AB}{t} = v.$$

e The direction PQ is the same as DC, along a radius.

4 **a** The Earth moves in an almost circular orbit round the Sun. The central force is provided by gravitational attraction between the two masses. A train rounding a curve has a force acting on the wheel flanges as they press against the track. In a centrifuge the force is provided by a tensile stress in the metal arm holding the test-tube.
b The force is the product of mass and acceleration, when the mass is constant.
c While the path swings through an arc, the velocity v changes direction. The vector change of v is proportional to v, for a given angle swung through. If v is larger, the change is larger and there is also *less time* to bring the change about within a fixed arc of the path. So the force rises on both counts, and depends on v twice over, that is, on v^2.
d If the radius is doubled, while the speed remains the same, the body has twice as much time to achieve a given change in the direction of the velocity. So the acceleration is half as much.

5 a 0.8π m s^{-1}.
b 4 N.
c 0.6π m s^{-1}, 3 N.
d v is smaller by the same factor as r is smaller. So the reduction in v^2 beats the increase in $1/r$.

6 See Rogers, *Physics for the inquiring mind*, page 306 for a fuller explanation.

If you are standing facing forwards in a train which then accelerates forward, you might say, 'I was thrown backwards until my back hit the wall of the carriage.' From outside the train it looks different. Your feet were carried forward, while the rest of you stayed in the same place (there being no force accelerating it) until the end of the carriage moved forward and hit you.

Going round in a fairground cage is similar, but a little more subtle. Suppose you were put down suddenly at X, close to Y on the wall of the cage, moving along at the same speed as Y. You would travel in a straight line until you hit the wall. When you hit the wall, it would be at Y′, which is where place Y will have got to. So it seems, inside the cage, as if you fall over backwards and hit place Y.

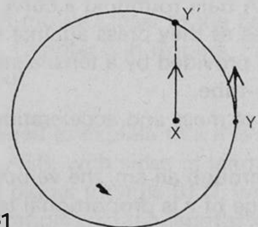


Figure 61

7 a See question 6. Water in wet clothes in a spin-drier is kept moving round in a circle by the inward force of the walls on the water. Where there are holes in the walls, the water is not pressed inwards and flies off *at a tangent*. If the holes are small, the pieces of fabric over them are prevented from following the water by the fibres joining them to nearby pieces, though a fast spin-drier with large holes could tear the fabric.
b 250 m s^{-2}.
c 25 times.

8 a Only Y.
b *b* Yes; *c* No; *d* No; *e* Yes; *f* No; *g* No (but yes if the wire were above or below the current balance wire).
See PSSC *Physics*, chapter 30, or *College physics*, chapter 28.

9 **a** Estimating the length L of the lower end of the coil as 50 mm, the force BIL on each of ten turns comes to 10^{-3} N; that is, 10^{-2} N on the whole coil. This is just about the weight of a one gramme mass, and it should be possible to weigh such a force with considerable accuracy.
b If the field crossing the gap pulls the bottom of the coil downwards as indicated in figure 62, the field outside the gap, in the same general direction, will pull the sides of the coil outwards. It would, therefore, be good experimental design to make the force on the lower part of the coil a *lifting* force, by sending the current the right way round the coil, so that the force on the wires along the sides of the coil presses them inwards, against the glass plate on which they are wound.

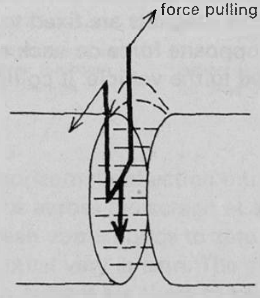


Figure 62

10 **a** To C.
b To A.
c To E.
d If the B-field is reversed **a** is to A; **b** is to C; **c** is still to E.

11 **a** Where the particles are moving in straight lines.
b The magnetic field is where the particles follow curved paths.
c If the beam stays in the same plane as the paper, there is no force perpendicular to that plane. The direction of a magnetic field is that direction in which there is no force on a charge moving in that direction.

12 a Out of the plane of the paper, towards the reader.
b The same. The current in the electrolyte is the same as in the wire. From $F = BIL$, the force per metre will be the same, even though the ions are not moving at the same speed as electrons, and are present in different numbers and even travel in the opposite direction, because the 'Cl^-' current plus the 'Na^+' current are together equal to the 'electron' current.

13 a 10^{-3} N A^{-1} m^{-1} approximately.
b The component of field causing rotation of the balance wire is $B \sin \theta$. The length of the wire in the field is however increased to $L/\sin \theta$. Thus the force remains the same.

14 It will not work. The force could bend the axles but not move the vehicle, for the magnets are fixed to the vehicle and there is an equal and opposite force on each of them. If the magnets were not fixed to the vehicle, it could move.

15 a Nq.
b Nq/t.
c $L = vt$.
d $NBqv$.
e Bqv.

16 a Vq.
b 5.9×10^6 m s^{-1}.
c 9.5×10^{-16} N.
d 1.04×10^{15} m s^{-2}.
e No.
f 35 mm.

17 a 1.6×10^6 m s^{-1}.
b 0.34 N A^{-1} m^{-1}.
c About 35 mm.
d Same time for both, 2.0×10^{-7} s.
e The time it takes a particle to go round does not increase as its energy increases, because it goes in a larger circle. Thus all the particles can be given a push at the same moment.

18 a See figure 63 *a*.
b See figure 63 *b*.

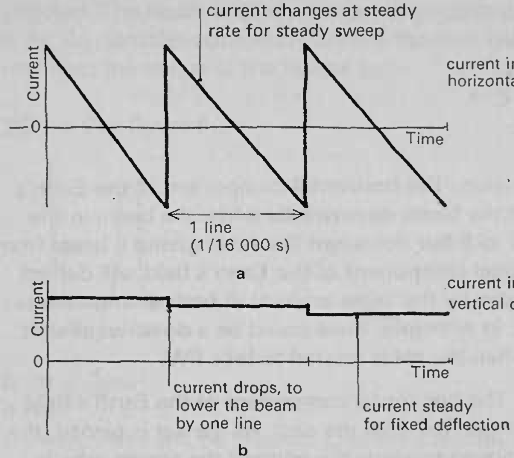


Figure 63

The field giving horizontal deflection must vary at a steady rate as the spot sweeps across the screen at a steady speed. If the middle of the screen corresponds to zero deflection and zero field, the current must vary in sign. The steps shown in the vertical deflection current are there to keep the spot at a fixed height for the whole of each line, dropping this height by one line spacing every time a line is completed. In practice, a very slowly changing current would do as well. Then each line would need to be tilted by an angle of about 1/625 radian. Such a steadily changing current would be much easier to produce than a series of small sharp steps.

19 a Bqv.
b Towards the front edge.
c The moving electrons are pushed towards this edge, so the density there will build up until there is an equally strong electrical repulsion pushing them in the opposite direction.
d $Eq = Bqv$ so that $E = Bv$.
e Front edge negative, rear edge positive.
f $V = Bdv$.
g $V = BI/nbq$ since $A = bd$.

20 a 8.4×10^{28}.
b As a guess, 8.4×10^{28}.
c 1.5×10^{-3} m s^{-1}.
d 4.8×10^{-23} N.
e 4.8×10^{-23} N.
f 3.0×10^{-4} N C^{-1}.
g 3.0×10^{-6} V.

21 *B's suggestion.* The horizontal component of the Earth's field will deflect the beam downwards when the beam in the set is moving W to E but not when the set is giving a beam from S to N. The vertical component of the Earth's field will deflect the beam sideways by the same amount in both arrangements of the set. Thus, in principle, there could be a downward shift of the picture when the set is rotated to face EW.

C's suggestion. The horizontal component of the Earth's field will deflect the beam towards the east. As the set is turned, the picture will be moved towards the edge of the screen which lies to the east. The vertical component has no effect on the beam in any orientation of the set. In principle, a shift in the picture could be detected.

A's suggestion. If the electrons are accelerated by a p.d. V over only a small distance, they could travel with a velocity v, given by

$$v^2 = \frac{2Vq}{m}$$

Using $\dfrac{mv^2}{r} = Bqv$,

we estimate r to be around 15 m.

The deflection over, say 0.3 m, from back to front of the set, would only be a millimetre or two, and rather hard to detect.

22 a The charged particles will be deflected in a circular path at right angles to the plane of the paper.
b No deflection.

c The v_x velocity component will give a circular path at right angles to the plane of the paper. The v_y component will not be affected. The result will be a helical (corkscrew) path.
d As the particles approach the pole the field becomes stronger, reducing the radius of the helical path.

23 a See figure 64.

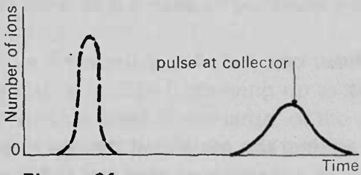


Figure 64

b $Vq = \frac{1}{2}mv^2$.
c No.
d Two pulses will be produced, as the particles of twice the mass have velocities $\sqrt{2}$ times smaller and take $\sqrt{2}$ times longer to arrive, supposing their charges to be the same.
e No, because the ratio of the two masses is not so large.

24 a The path is a spiral because the momentum of the electrons is changing.

$$Bqv = \frac{mv^2}{r}$$

$$mv = Bqr.$$

b About 2×10^{-20} kg m s^{-1}.
See Glazer, 'The bubble chamber'.

25 a Mass the same as an electron, 9.1×10^{-31} kg.
Charge $+1.6 \times 10^{-19}$ C.
b About 2000 times more mass.
c $\dfrac{mv^2}{r} = Bqv$

$\therefore\ r = mv/Bq$.
d Less energy.
e It would have travelled about one-tenth as far as the positron.

26 Yes, the motor could be used as a dynamo, to charge the battery. The connections would not need to be interchanged. If the motor, as a dynamo, delivers energy either to the battery or to a resistor, the car will be braked.

27 **a** A force Bqv parallel to the length of the rod.
b A current will flow in the galvanometer.
c $E = V/L$.
d Vq/L.
e $Vq/L = Bqv$ so $V = BvL$.
f No. The charge q cancels.
g I^2Rt.
h No. The force must pull against the motion of the rod. If it helped the motion, one would only have to start the rod moving for it to go faster and faster, giving itself energy without drawing on any supply.
i vt.
j $BILvt$.
k If $I^2Rt = BILvt$
then $IR = BLv$, and IR is the p.d. V across R.
$V = BLv$ which is the same as the answer to **e**.
l No. Charges will be pushed to one end of both conductors as they move together through the B-field, but after this initial movement of charge, the rod and wire travel along with one end of the pair positively charged with respect to the other end, but with no current circulating.

28 **a** In time t the rod travels distance vt, at right angles to its length L. The area over which it passes, shown shaded in figure 26, is Lvt. Now the voltage is $V = BLv$, from question 27, and this is B multiplied by the area worked out above, divided by t. That is, $V = BLv$ is the *rate* at which the area swept out by the rod increases, multiplied by B.
b The induced voltage across the ends would be just the same. Parts of the rod parallel to its velocity v contribute nothing to magnetic forces on charges within the rod. The area, perpendicular to B, swept out by the crooked rod is the same as the area swept out by the straight rod.

29 a The suggestion makes sense, though the effect may not be conveniently large. Ions travelling with the blood flow in figure 28 will, if they travel across a B-field, be given a push in a direction at right angles to the field. The electrodes in figure 28 are in the best position.
b In principle, the idea is sound. But it might not be a good idea to pass substantial currents through blood vessels. The idea has been used as a means of pumping hot liquid metals.

30 a If the wing span is L and the velocity is v, there should be a p.d. $V = BLv$ from wing tip to wing tip. The Earth's field B (the vertical part) is not large, of the order 4×10^{-5} N A^{-1} m^{-1}. For a wing span of 30 m, and a velocity of 1000 km h^{-1}, $V \approx 0.3$ V.
b To measure the p.d. might not be simple, not only on account of its modest size. A voltmeter in the fuselage connected by wires to the wing tips would always give a zero reading, since just the same p.d. would be induced in the wires as in the wings, with exactly zero voltage around the circuit. An electric field, of the order 10^{-2} V m^{-1} exists along the wing tip to wing tip direction. Such a field can be detected in principle by the force on a charge. In practice, its effect would be swamped by much larger stray electric fields from equipment in the aircraft, and from charges on people and things in the aircraft, produced by friction.

31 The device is quite hopeless. There will only be an induced voltage in a wire from a nearby magnet if the wire and magnet move relative to *one another.* Either may move relative to the outside world, but one of them *must* do so. If the device would indicate the motion of a submarine, it would also, set up in the laboratory, indicate the 'absolute' motion of the Earth through space.

32 a The current in the top disc, from rim to axle, is like a current in a wire in the same direction: there is a force BIL on it. (L is the disc's radius, in effect.) This force would be at right angles to the current, flowing along a radius, so it will be in such a direction as to make the disc spin. The current in the lower disc is in the reverse direction, the B-field is in the same direction, so the lower disc spins in the opposite sense. This has the practical advantage that the device as a whole transmits no torque to the structure which fixes it to the ground, while it is accelerating up to top spin-speed.

b If the rim has radius r and travels at velocity v in a circle, it is accelerated towards the centre with an acceleration v^2/r. For the values given, the acceleration is 1.6×10^4 m s^{-2}, nearly 2000 times the acceleration of free fall. The steel on the rim is pulled inwards by tension in the steel, and if the tension per unit area which is needed exceeds the breaking stress of the steel, the disc will break up. It is possible to show that, whatever the radius of the disc, there is a maximum rim velocity at which the disc breaks which depends on the strength and on the density of the material. The bigger the radius, the more slowly the disc must rotate, in revolutions per second.

c The steel in the middle of the disc moves slowly, while that at the rim goes fast. There is more steel at large radii than at small ones, though. From this one might guess that the energy would be about that of the total mass going round as fast as the steel at a radius of, say, 1.5 m. At this radius, the speed is about 140 m s^{-1}. This gives a kinetic energy estimate of 4×10^8 J, for one disc.

d About 600 V (*twice* the p.d. across one disc).

e If the maximum rim speed is v, a disc of radius r turns $v/2\pi r$ times a second. The area swept out per second is $\pi r^2 v/2\pi r = rv/2$, so the voltage across the radius of one disc is $Brv/2$.

f The voltage calculated from *energy/(current × time)* is just under 400 V. This ought to be less than the value of 600 V found previously, because the disc must slow down as it delivers energy, and so the voltage across it must fall from its maximum, top speed value of 600 V, to zero. The average voltage will be somewhere around half the maximum.

g The force on the current in the discs, flowing across a B-field.

33 In a rather trivial sense, the assertion is not true, because one could put extra resistance in series with the galvanometer, and so reduce its indication to as small a value as one wished. If the resistance of the galvanometer circuit is kept constant, the assertion seems to be true, though it depends on a proper interpretation of what 'the loop within the U' is supposed to mean. For example, the wire can be doubled and passed into the magnet, or through the gap, or be left in the gap while the magnet's current is changed, without there being any appreciable voltage induced across it. Figure 65 shows this arrangement.

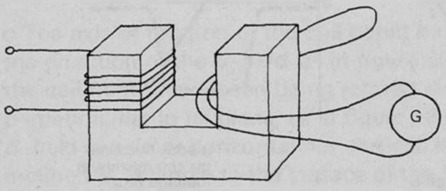


Figure 65

If the wire loop *encircles* the flux through the iron and across the gap as in figure 32, the assertion can be true. Things to try include:

- Lifting the wire out of the gap, trying both wide and narrow loops.
- Taking the *magnet* away from the wire loop.
- Reducing the magnet current to zero with the loop left in place.
- Combinations of the above, changing the current and moving the wire or magnet.

Note that the trials involve *two distinct effects.* If the wire is moved, electrons in the wire are carried by the movement across a B-field at velocity v, and there is a force Bqv on each, along the wire. But if the magnet's field is changed while the wire stays still, there can be no force Bqv on an electron in the wire, since v is zero. The remarkable thing is that the two effects can be described by one rule; that the induced voltage in one circuit is equal to the rate of change of flux through that circuit.

34 The voltage across coil B has its greatest magnitude (plus or minus) when the flux from A is zero, because the induced voltage depends on the *rate of change* of flux through B. The rate of change of the more or less sinusoidal variation of flux shown is largest when the flux is zero. Note that the voltage has been given a positive sign when the flux is increasing. This is needed in sketching the other voltage variations. See figure 66.

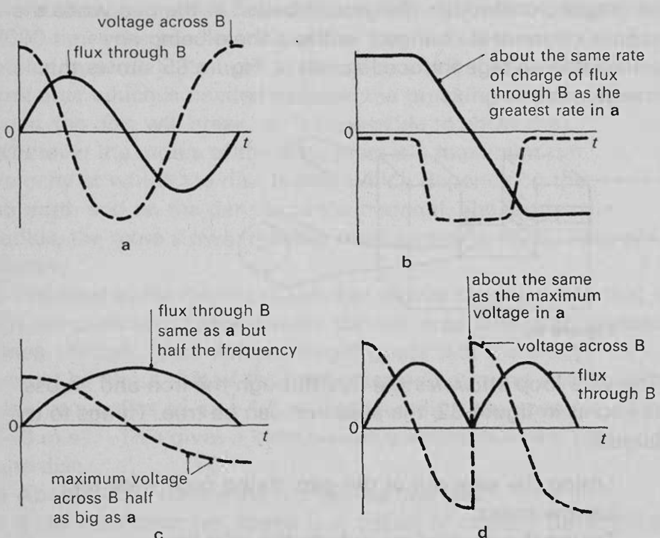


Figure 66

35 a Either multiply the number of turns by four, or double the linear dimensions of the coil, so quadrupling its area.
b Use slip rings, as in figure 67.

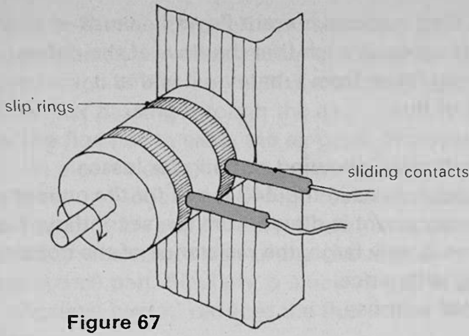


Figure 67

c The axis of rotation of the coil could have been pointing along the direction of the B-field, as in figure 68 *a*, or, less plausibly, the coil might have been being rotated about the axis perpendicular to its plane, as in figure 68 *b*. In fact, the Earth's B-field is neither horizontal nor vertical, but is in general inclined at an angle to the surface of the Earth.

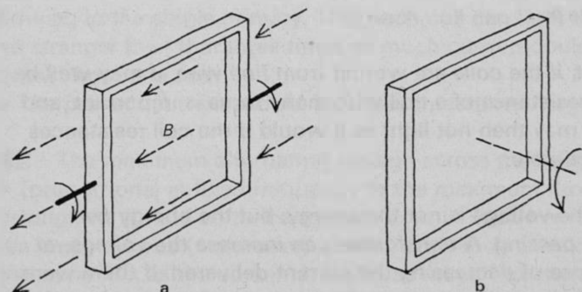


Figure 68

36 When the probe coil is connected to the oscilloscope the size of the trace depends on the size of the B-field component along the axis of the probe coil. The maximum trace will be obtained when the axis of the probe coil is aligned along the direction of the B-field.

37 In every case, if an induced current flows because of a change of flux, its direction is such that the flow of the current results in energy being taken from whatever it is that is causing the change of flux.

38 **a** 20:1 approximately, allowing nothing for losses.
b There must be enough turns in the 240 V coil for the current in it to be small when no current is drawn from the secondary. But if the number of turns is very large, the resistance of the coils begins to make a big difference.
c Not for the intended purpose.

39

Number of turns on primary, P	Number of turns on secondary, S	Brightness of lamp, L	Alternating p.d. across S, in volts
20	50	normal	2.5
50	20	dim	0.4
20	30	dim	1.5
40	100	normal	2.5
20	80	bright	4.0

Table 4

Note that, if the coils are wound from fine wire, it may well be that the resistance of a coil with many turns is important, and the lamp may then not light as it would if the coil resistances were negligible.

40 **a** The voltage is not the energy, but the energy per coulomb passing. A transformer can increase the voltage, at the expense of decreasing the current delivered. If there were no losses, the power (root mean square current multiplied by root mean square voltage) would be the same for primary and secondary.
b Energy is dissipated in the resistances of the two coils. Currents will be induced in the iron core; these can be reduced, but not completely eliminated, by laminating the core so as to provide thin, high resistance paths for such 'eddy' currents. Also, the process of magnetizing, demagnetizing, magnetizing in the reverse direction, etc., for the iron core dissipates some energy (called hysteresis loss).

41 **a** The flux through the secondary coil must be pretty much the same wherever it is on the core. The greater proportion of the flux must circle round within the core, with little flux passing through the air.

b The flux has to cross the air gaps. The system behaves for flux as an electric circuit behaves for current. The air gaps correspond to lengths of high resistivity material inserted into a circuit of thick copper wire. The current then depends more on the high resistance part of the circuit than on the low resistance part. Similarly, a small air gap in an iron filled 'magnetic circuit' reduces the flux in the 'circuit' by a large factor.

c The result **c** suggests that the flux is determined by the whole 'magnetic circuit'. If there are air gaps, it has a low value right round the circuit. (Similarly, one would have a low current in a thick copper bar if somewhere else in the electric circuit there were a high resistance.)

d The flux links each coil, and each has a voltage across it. Current can be drawn from each, a corresponding current flowing in the single primary. That each coil can light a lamp is no stranger than that three times as much current could be drawn from one coil.

e The primary current should be about three times larger.

42 The maximum alternating voltage across the secondary in *b* (proportional at fixed frequency to the maximum flux through it) should be twice as great as in *a*. Each iron core has ten ampere turns (maximum) around it, so each should carry the same flux as the core in *a*. Twice as much flux therefore links the secondary coil.

The arrangement *c* would test the above argument. If it is correct, each secondary wound round one core should have the same maximum voltage across it as the coil in *a*. In *d*, only one core has ten ampere turns around it, and the other core will have no flux in it. The secondary should give the same voltage as in *a*.

43 a Not all the flux was confined to the iron, some passing through the air. Near the primary, the flux in the air links the secondary, but less of it links the secondary when it is far from the primary.
b The coil would detect any flux in the air. It does not link the iron, and will not detect the flux in the iron.

44 a V.
b $\frac{V}{2}$.
c 0.
d $2V$.
(Simple rule: the voltage is zero if the wire could be removed without having to be cut.)

45 a Low voltage lines dissipate a higher percentage of power than high voltage lines. The power dissipated along a line is I^2R. The higher the voltage the lower the current and hence the lower the power dissipation for a given resistance.
b History is one reason: as technology improves, the cost of raising the voltage becomes less. Probably the voltage chosen is around that value at which the saving in power would be compensated by the extra cost of the higher voltage (or rather the cost of paying interest on the investment in higher voltages).

Also, the longer the power lines, the greater the power wasted. It may be acceptable to run short lines at lower voltages.
c Insulation of lines and transformers' windings; costly high voltage switchgear; troubles from ionization of air near high voltage lines.

For the same power, 400 kV lines carry $132/400 \approx 1/3$ of the current carried by 132 kV lines. For the same power I^2R dissipated, the resistance can be $(400/132)^2 \approx 9$ times greater, and the 400 kV lines can contain $(400/132)^2 \approx 9$ times less metal (for the same length), thus being cheaper in this respect.

46 The output current is determined by the output voltage and by the resistance connected to the secondary coil as well as the resistance of that coil itself. The voltage is proportional to the turns ratio. The current cannot be larger than that which flows when the secondary is short circuited, and the resistance of the secondary coil itself is the only resistance in the circuit. The resistance of the primary coil will also limit the current that can flow, because it reduces the effective input voltage.

The output current will be largest if the secondary has but one turn, when, neglecting losses, the turns ratio is 100 to 1. In these circumstances, if the primary carries 2 A, the secondary carries 200 A. The 'output voltage' cannot be given so easily. If there are 240 V across the primary, there are something like 2.4 V induced in the secondary, but if the 200 A current arose from short circuiting the secondary coil, and the whole voltage was used to drive current through the coil itself, then the voltage observed across the secondary would be zero.

47 Energy is conserved. The motor cannot produce power to spare. In practice, it won't turn at all, for neither motor or dynamo will be 100 per cent efficient.

48 **a** $I = V/R$.
b See figure 69.

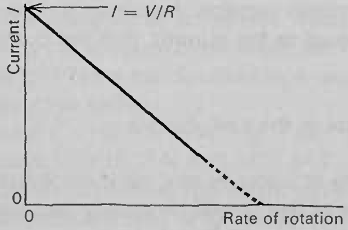


Figure 69

c The torque will fall as the rate of rotation increases.
d The rate of rotation must fall. If the motor is not producing enough torque to turn the larger load, it will slow down. As it does so, the rate of change of flux in its rotor falls, and less voltage is needed to maintain this lower rate of change of flux. The constant supply voltage can now drive a larger current through the rotor resistance, so the current rises. It rises until the motor has slowed down enough to deliver sufficient torque to turn the load. In this way, an electric motor is a self-adjusting device.

49 **a** In every case, the motor is turning, so that there is a rate of change of flux in the rotor. Part of the supply voltage is needed to maintain this rate of change of flux, so the current is always less than that which 5 V would drive through 1 Ω. The ratio of voltage to current varies because the rate of rotation varies.
b The field B in which the rotor turns is constant, so the torque is proportional to the rotor current. The larger the load, the larger the current must be. The motor slows down as much as is necessary for this larger current to flow, if that is possible without it stalling.
c Raise the voltage applied to the rotor, so that it exceeds the induced voltage at 27 revolutions per second (about 4 V) by as much as is needed to drive 3.1 A through 1 Ω (3.1 V). This assumes that the torque required remains the same, and so, since the torque is proportional to the current, that the current will remain 3.1 A.

50 **a** The field, and the flux in the coil, double.
b 0.2 A s^{-1}.
c The flux in the coil is rising at a steady rate, so the voltage needed to maintain this rate of rise would be steady, because it is proportional to the rate of change of flux.
d 10^{-3} V.
e If the current decreased at the rate 100 A s^{-1} in a 10 H coil, a voltage of 1000 V would occur across the coil. Such a voltage could deliver a nasty electric shock. The system is used for producing sparks in motor car engines.

51 **a** The induced voltage is proportional to the rate of change of $B \times area$ with time. (Strictly, the rate of change of $\int B\,\mathrm{d}A$.) The larger wire encloses a larger area and will produce a larger voltage. The current producing the field may be less, but not by as big a factor as the factor by which the area has increased.
b The thicker wire has a smaller resistance and carries a bigger current producing a bigger magnetic field. In a helix, more flux from one part of the circuit links other parts.

52 **a** *resistance* = *resistivity* × *length*/*area*.
b *potential difference* = *current* × *resistance*
= *current* × *resistivity* × *length*/*area*. If current is analogous to flux, potential difference is analogous to current-turns NI. The current-turns 'drive the flux round the circuit'.
c $\mu_r \mu_0$ is analogous to 1/*resistivity*; that is, to *conductivity*.

d	*Heat flow*
driving force	temperature difference
amount of flow	heat flow in joules per second
length	thickness of insulation
conductivity	thermal conductivity (the ease with which a material conducts heat)
area	surface area of hot water tank

e Reluctance corresponds to resistance.

53 **a** About $5 \times 10^7\ \mathrm{A^2\ m^{-1}\ N^{-1}}$.
b If L is doubled, so is the total number of turns, if the number of turns per metre is fixed. Both the reluctance and the number of current-turns are doubled as a result, so that the flux remains the same.
c About $6 \times 10^{-6}\ \mathrm{N\ m\ A^{-1}}$ or $\mathrm{T\ m^2}$.
d About $12 \times 10^{-4}\ \mathrm{N\ A^{-1}\ m^{-1}}$ or T.
e A steel retort stand rod might have a diameter of 10 mm, and a cross-sectional area of about 80 mm². The cross-section of the solenoid is about 5000 mm². If the steel 'conducts' flux 500 times better than the air, the ratio of the fluxes is about $500 \times 80/5000$, or about 8. (This estimate neglects the fact that the solenoid area not occupied by iron is only $5000 - 80$ mm², not 5000 mm².)

54 **a** 8×10^5 A^2 m^{-1} N^{-1}.
b NI = *flux* × *reluctance* = *flux* $\times 8 \times 10^5$ ampere turns.
c $N\,d\phi/dt$.

d $$10\,dI/dt = N\,d\phi/dt = N\frac{d}{dt}(NI/8 \times 10^5)$$

$$10 = N^2/8 \times 10^5$$

$$N \approx 2.8 \times 10^3 \text{ turns.}$$

e The 20 mm by 10 mm core has a perimeter of 60 mm. The wire length cannot be less than about 170 m. Because of the bulk of the wire, the actual length will be greater.
f 170 m of wire must have resistance less than 10 Ω, that is, less than 0.06 Ω m^{-1}. The 22 s.w.g. wire would serve. Wound in a coil 100 mm long along half the iron ring, there would be about 150 turns in one layer, so some 20 layers and a thickness of at least 15 mm would be needed to make up a 2800-turn coil. This allows nothing for insulation or gaps between wires, and is very much a minimum estimate. The actual coil would be quite a bit thicker. The coil could be up to 200 mm long, so needing fewer layers.

55 **a** The units of reluctance are those of *length*/(μ_0 × *area*), that is, m/N A^{-2} m^2 or A^2 N^{-1} m^{-1}.
The units of resistance are V A^{-1}, or N m A^{-2} s^{-1}.

Thus the units of 'goodness' G are
s^{-1}/(A^2 N^{-1} m^{-1}) (N m A^{-2} s^{-1}), and every unit cancels out. G is a pure number, whose magnitude does not depend on the units in which current, time, and length or area are measured.
b The length L_{iron} doubles; the area A_{iron} quadruples, so the reluctance is halved.
c The length L_{copper} doubles; the area A_{copper} is quadrupled, so the resistance is halved.
d Both reluctance and resistance are halved, so the goodness increases by a factor of four, as long as it is possible to keep the speed or rate factor the same (a big machine may not be able to turn as fast as a small one, for mechanical reasons).
e The bigger the linear dimensions of a transformer, motor, or dynamo, the bigger the goodness on that account; that is, the bigger the ratio of useful energy transformed to energy dissipated in resistance. For a given efficiency then, a big

machine will usually be more economical to make than a number of smaller machines which together produce the same total power. The goodness of a machine depends on its speed, because the induced voltage depends on the *rate of change* of flux. A fast machine does not need as much flux to work effectively as a small one. Thus, for example, a few turns of air-cored wire make an excellent inductor at the frequency 100 MHz used in v.h.f. radio sets, but at 50 Hz a lot of iron is needed.

Any space between the interlinked magnetic and electric circuits is wasteful. The electric circuits are longer than they need be, and have more resistance than necessary. The iron (or magnetic circuit) is longer than it need be, and less flux is produced for a given number of current-turns than could be achieved.

See Laithwaite, *The engineer in Wonderland*, Chapter 5, especially plates 5.3 and 5.4, for an excellent discussion of the place of 'goodness' in the design of electromagnetic machines. See also Laithwaite, *Propulsion without wheels*, Chapter 5.

56 **a** The shape of the field could be explored with iron filings, to give an impression of this shape. More accurately, one could use a Hall probe or a search coil to measure the field. At any place, the field direction is along the direction in which the measured value of B is largest. See question 36.
b A very long straight wire has the symmetry of a cylinder. There can be no reason for the magnitude of the B-field to differ at places equally far from the wire. The only two simple field shapes with cylindrical symmetry are a spoke-like shape (like a chimney brush) or a whirlpool shape, with the field going round the wire in circles.
c $2\pi r$.
d $2\pi r/\mu_0 A$.
e $I = \phi 2\pi r/\mu_0 A$
$\phi = \mu_0 IA/2\pi r$.
f $B = \mu_0 I/2\pi r$. (Replace I by NI if there are N wires.)
g 2×10^{-7} N A^{-1} m^{-1}.
h $\mu_0 = 4\pi \times 10^{-7}$ N A^{-2}.

57 a $a/\cos\theta$.
b nRS.
c nIRS.
d knIRS$/r$.
e Angle QRS is equal to θ.
f QR$/\cos\theta$.
g knIQR$/r\cos\theta$.
h $\Delta\theta$, in radians.
i $knI\Delta\theta/\cos\theta$.
j Zero.
k Horizontal, its projection onto the carpet being perpendicular to the wires.
l $\Delta B \cos\theta$ or $knI\Delta\theta$.
m No.
n π radians.
o πknI.
p No, if the carpet is as wide as you please, when the answer to **n** is always π. But with a real, finite carpet there will be some change, which can be neglected if P is close to the carpet.
q $2\pi knI$.
r $\mu_0/2\pi$ or 2×10^{-7} N A^{-2} (henry per metre).
s 2×10^{-7} N, from the way the ampere is defined (that is, the way the size of one ampere is specified).
t $1.2\pi \times 10^{-4}$ N; about the weight of a 40 mg mass.
u $4.8\pi \times 10^{-4}$ N A^{-1} m^{-1} (or tesla T).

58 a The reluctance is given by
$400 \times 10^{-3}/4\pi \times 10^{-7} \times 1000 \times 400 \times 10^{-6}$
$= 8 \times 10^5$ A^2 m^{-1} N^{-1}.
The flux in the iron is given by $5 \times 100/8 \times 10^5$, which is about 6.2×10^{-4} N m A^{-1}.
b The reluctance of the air gap is given by
$4 \times 10^{-3}/4\pi \times 10^{-7} \times 400 \times 10^{-6} = 80 \times 10^5$ A^2 m^{-1} N^{-1}.
c The reluctance of the iron is very nearly the same as in **a**, the length being reduced from 400 mm to 396 mm. Very nearly, then, the total reluctance is 88×10^5 A^2 m^{-1} N^{-1}. The reluctance with the gap is nearly 11 times larger than the reluctance without the gap.

d The flux with the gap introduced is given by $5 \times 100/88 \times 10^5 = 5.7 \times 10^{-5}$ N m A $^{-1}$. The flux is 11 times smaller than it was, because the reluctance is 11 times bigger.
e The B-field in the gap is given by *flux/area*, therefore it is about 0.14 N m^{-1} A^{-1}.

59 **a, b, c** In each example, the important factor is the size of an air gap in an otherwise closed ring of iron. Because iron has a reluctance 500 or 1000 times smaller than the same sized region of air, quite a small air gap can create a big increase in the total reluctance, and so cause a big decrease in the flux.
In **a**, if the surfaces of the core are rough or dirty, or are not held tightly together, there will be a small air gap (or 'dirt gap') between the faces. Even if this gap is only 1/500 or 1/1000 of the length round the whole core, its effect could be to double the reluctance, halve the flux, and halve the self-inductance.

In **b**, to obtain the same output from his dynamo, with a larger air gap, the designer would have to arrange for a greater current to flow in the coils which magnetize the iron in the dynamo. This would be wasteful, because the larger current would dissipate more energy in the resistance these coils are bound to have. The only way to improve matters, short of making the air gap narrower, would be to use thicker wire for the coils, which would cost more.

In **c**, if the motor is magnetized by a coil wound on an iron core within which the rotor turns, the flux in which the rotor turns will be rather small, because it has to cross a large air gap. The force on the rotor will be proportional to the flux (force proportional to B), and is likely to be small. If the motor is made from a permanent magnet, the design problem is not quite the same.

60 a As the ring spins, the flux linking it changes, and a current flows in the ring. Such currents are always in such a direction as to take energy from whatever is their cause; in this case, the spinning motion. Alternatively, electrons in the moving metal are carried across a magnetic field, and are pushed sideways, around the ring. As they flow round the ring, constituting a current, there will be a force on this current in the B-field. The force decelerates the ring, and the energy transformed must come from the energy of motion of the ring.
b No complete path for current; no current; no slowing down due to induction.
c If the ring is cut into n slices, each has the same voltage around it, but each has n times the resistance of the original and so a fraction $1/n$ of the original current. The total force on n rings should thus be the same. But if there is insulation between slices it will have some mass and the rate of change of motion of the ring may be less. Or the ring may be more bulky, so that the effect of air damping is different.
d When the moving coil is short circuited there is a current because of the voltage induced across the coil. The force on this current in the field slows down the motion of the coil.

61 If either magnet or plate moves relative to the other, induced currents will flow in the plate, and the forces on these currents in the magnet's field will tend to reduce the relative motion. So the two tend to turn together.

62 **a, b** A spring. Without it, the disc will rotate at a speed as nearly equal to that of the magnet as friction allows. The faster the magnet turns relatively to the disc, the bigger the torque on the disc. If the spring holds the disc still, it will do so at an angle such that the spring torque is equal to the torque on the disc, produced by eddy currents in the disc owing to movement in the magnet's B-field. See figure 70 *a*.

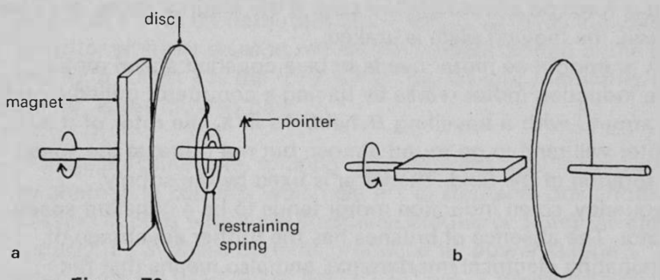


Figure 70

c There will be no effect if the bar magnet is rotated about its axis, as in figure 70 *b*. Try it.

63 A small coil could be placed beside the lead and the voltage pulses induced in that coil could be detected.

64 **a** When, and only when, the magnet and plate move relative to one another, are eddy currents induced in the plate. The currents are always in such a direction as to produce a force on the plate which tends to reduce the relative velocity. Thus if the plate is still, and the magnet moves, the plate speeds up. If the magnet moves at a steady velocity, the plate never quite goes as fast as the magnet, for if it did, the eddy current effect force on it would fall to zero. If the magnet stops, or slows down, the moving plate is braked.

b A gramophone motor needs to be a constant speed motor. The induction motor works by having a conductor pulled along or around with a travelling B-field. As in **a**, the rotor of the motor will tend to go round almost, but not quite, at the speed of rotation of the field. The latter is fixed by the supply frequency, so an induction motor tends to be a constant speed motor. The absence of brushes has the further advantage of eliminating electrical interference, and also means that the motor is very reliable and quiet.

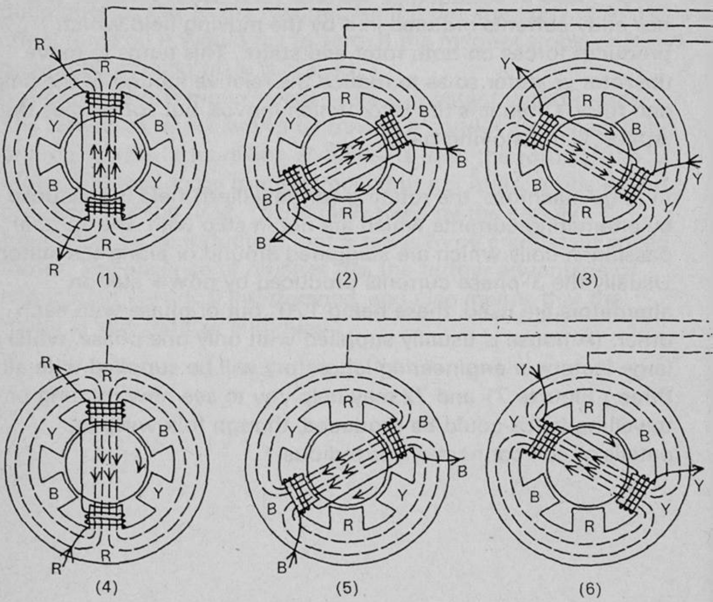
Induction motors

This chapter is for those who are interested in how large-scale induction motors work. In all induction motors a 'stator' wound with coils produces a rotating or travelling field and a 'rotor' made of conducting material (and usually iron as well) has eddy currents induced in it by the moving field which produces forces on both rotor and stator. This tends to move the rotor or stator so as to reduce the relative motion of the field and rotor. Often it is the rotor which moves, but sometimes the 'stator' is the moving part.

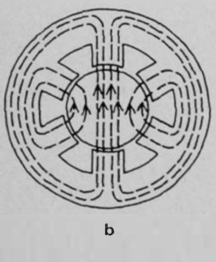
In large machines, the rotating (or travelling) field is produced by alternating currents which are not in step with one another passing in coils which are staggered around or along the stator. Usually the 3-phase currents produced by power station alternators are used, these being 120° out of phase with each other. (A house is usually supplied with only one phase, while a large factory or engineering laboratory will be supplied with all three.) Figures 71 and 72 may help you to see how rotating or travelling fields could be produced, though following the details of coil connections is tedious.

Two types of induction motor

Production of a rotating field using 3-phase currents (Rotary motor)



a



b

Time	Phase at maximun
1	Red (+)
2	Blue (–)
3	Yellow (+)
4	Red (–)
5	Blue (+)
6	Yellow (–)

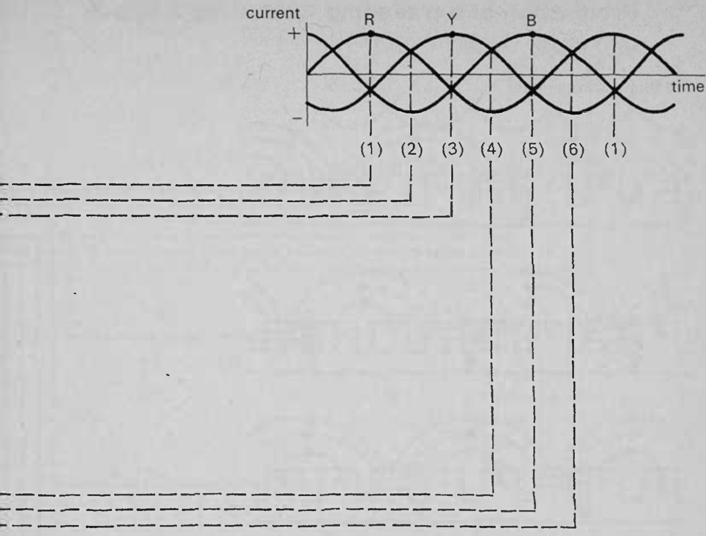
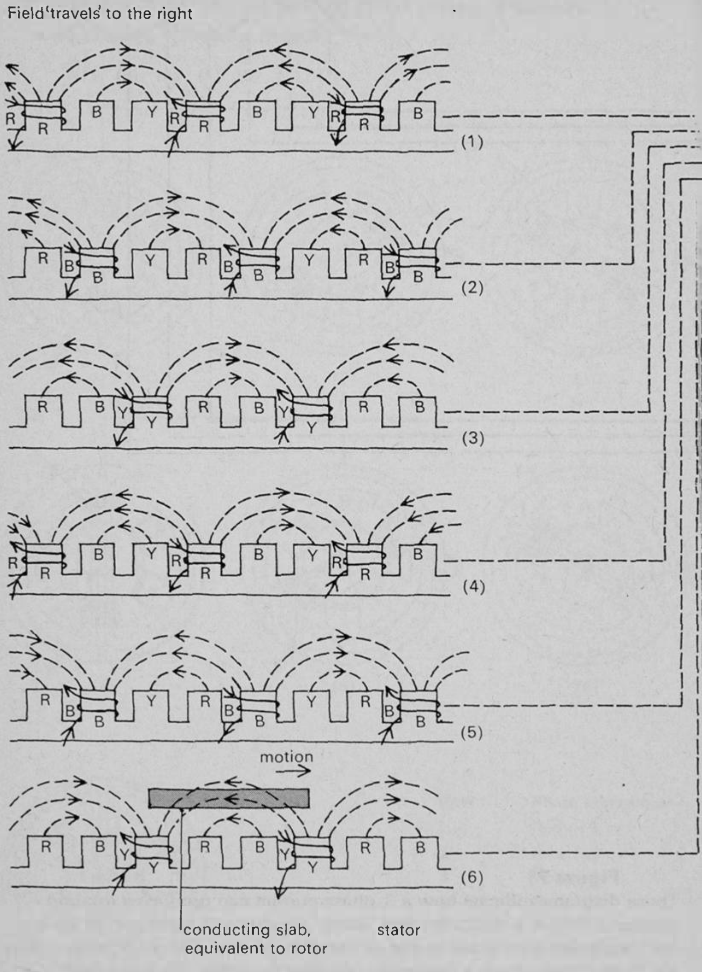
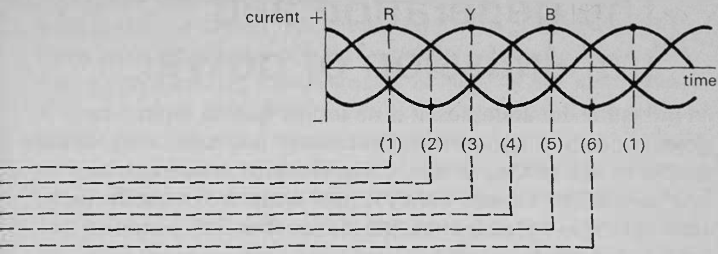


Figure 71
These diagrams indicate how a 3-phase current can produce a rotating magnetic field in a stationary iron 'stator' wound with three sets of coils, each set being connected to one of the three phases. The graph shows how the three phases reach a maximum one after the other. Six successive instants (1) to (6) are indicated. For clarity, only the winding carrying maximum current is shown at each time. The other windings carry smaller currents at each of these moments; the total field at time (1) for instance, is as in **b**. Induced currents in the rotor keep it turning with the field.

Production of a travelling field using 3-phase currents (Linear motor)





Time	Phase at maximum
1	Red (+)
2	Blue (−)
3	Yellow (+)
4	Red (−)
5	Blue (+)
6	Yellow (−)

Figure 72
These diagrams show how a 3-phase current can produce a travelling magnetic field above a stationary iron 'stator' wound with three sets of coils, each set being connected to one of the three phases. The graph shows how the three phases reach a maximum one after the other. Six successive instants (1) to (6) are indicated. The table shows which phase is at a maximum at each time, and the direction of its current. At each instant, a diagram shows only those coils which carry maximum current. The weak fields from the poles whose coils carry smaller currents are also shown. Notice that if this stator were rolled up end to end it would become like the rotary machine in figure 71.

The generation and transmission of power

In industrial communities, it is no longer dark at night. The great majority of people in Britain can sit and read, work, or play games as late as they please, using electricity costing only a few pence. The change from the time when a dim candle or rush light was the only available illumination has occupied several generations, and it is hard to know how to estimate its effect on the quality of life, especially as other changes have at the same time made at least as large an impact.

We can, if we wish, spend perhaps 10 to 20 per cent longer in good light than could our ancestors, and so have an 'extra' ten or so years of productive life. The way we live, and perhaps the kind of people we are, are influenced for good or ill by science and the technologies that grow out of it. The nationwide supply of electricity is a good example of such an influence.

It is arguable that the supply of electrical power and the availability of power-driven tools would now make it possible for some people to reject the values of a society based on industry, and live independent, self-sufficient, 'village' lives in a way not open to people in the past, for whom such an existence was not possible much above subsistence level. The question then arises of how the power and the tools are to be produced.

Demand versus amenities

Most of us seem to want electricity, for the demand doubles roughly every decade. But few want a power station in the view from their windows, and many are concerned at the effect on the countryside of pylons carrying the cables that bring electricity to their houses. Are underground cables the answer? Could electrical power not be produced by many small, independent generators, with a dynamo in every house? Why do we have large power stations, some producing over 1000 MW each, linked by miles of transmission lines using voltages up to 400 kV?

Why have large power stations?

At first sight, a million one kilowatt generators would seem to be as good as a single one thousand million watt station. But the effect of making things smaller or larger is not negligible. A large turbine (500 MW is now quite common) turns out to have a smaller capital cost per unit of power produced than the equivalent number of small ones. Large turbines can also be more efficient, if steam at higher temperatures and pressures can be used. (This will be mentioned again later in the course, in Unit 9, *Change and chance*.) Tables 5 and 6 show the trend towards large generating sets in Britain.

	Number of sets		
Age (years)	**Below 30 MW**	**100 MW, up to 500 MW**	**500 MW and over**
0–4	—	27	11
5–9	7	56	—
10–14	5	7	—
15–19	11	—	—
20–24	15	—	—

Table 5
Age and power of steam driven generating sets.
From C.E.G.B. Statistical yearbook *1968*.

		Efficiency	**Generator sets**	**Steam pressure $/\mathrm{N\,m^{-2}}$**
Old	Deptford West	14%	1×50 MW 3×35 MW 2×30 MW	24×10^5
New	Ferrybridge C	34%	2×500 MW 1×450 MW 1×320 MW	160×10^5

Table 6
Comparison of two power stations.
From C.E.G.B. Statistical yearbook *1968*.

It is not the size of the generator and turbine that decides efficiency, but the design and, in particular, the temperature at which they operate. Nevertheless, it would be very difficult to design a one kilowatt turbine with as high an efficiency as can be achieved for a one million kilowatt machine, if only because the smaller machine would have a larger surface area in proportion to its volume, so that heat losses (although smaller) would be a greater proportion of the energy transformed in the boiler and turbine. The graph, figure 73, shows how temperature is associated with efficiency for the stations in the C.E.G.B.'s South East Region.

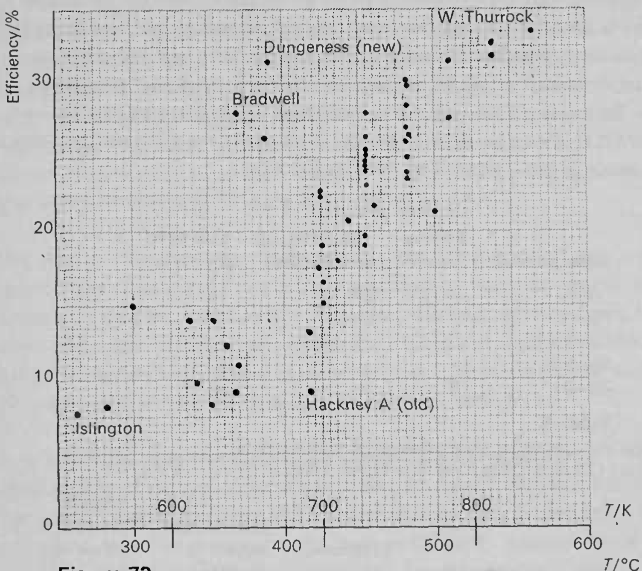


Figure 73
Efficiency and steam temperature for C.E.G.B. power stations (S.E. Region). *From C.E.G.B.* Statistical yearbook *1968*.

Electricity cannot be stored up

Unlike fuels, or water stored behind a dam, electrical energy cannot be stored in quantity. It must be produced on demand. This is another reason why the small domestic plant is not viable: the demand fluctuates widely, and to cope with the occasional use of a cooker, a capacity of ten or a hundred times the capacity needed at other times would have to be installed. A network of power stations, linked by a grid of transmission lines, can even out the fluctuating demands of many users. Even so, the task is not easy, as the daily demand curve (figure 74) shows.

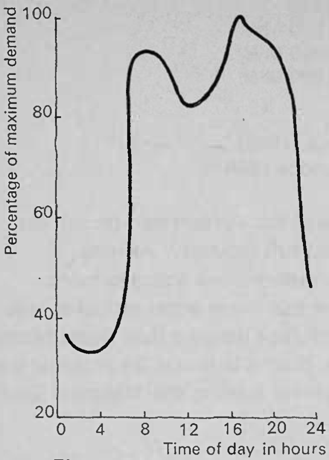


Figure 74
Daily demand for electricity.
From Weedy, B. M. (1967) Electric power systems, *Wiley*.

The nationwide demand is quite noticeably affected by popular television programmes, especially in breaks between programmes when large numbers of people boil kettles for tea.

The network system has its dangers too. A few years ago, much of the industrial east coast of the U.S.A. was blacked out for some hours when a major failure raised the demand on other parts of the system, causing further failure, which overloaded yet other parts which failed, and so on. The control engineer has an important role to play in preventing such failures.

In addition to enabling demand to be shared among many stations, the National Grid of transmission lines enables some power stations to be sited near the fuel they use, or away from centres of population, as has been felt to be desirable for nuclear powered plant. The availability of cooling water from rivers or the sea is another relevant consideration.

Number of stations	216
Maximum output capacity	42 000 MW
Maximum load met	36 000 MW
Average load	20 000 MW

Table 7
British power system, capacity and load (1968).
Adapted from C.E.G.B. Statistical yearbook *1968.*

By sharing out fluctuating demand the system can be run on average at as much as a half of its full capacity. As was suggested by table 6, the power stations we actually have differ widely in efficiency, so that it is more economical to run the most efficient stations twenty-four hours a day, transmitting their power over large distances, than it is to run all stations part of the time. This is another reason for having the National Grid linking stations together.

The National Grid

A generator usually operates at 10 to 20 kV, while users are supplied at 200 to 250 V. Power is transmitted at much higher voltages, of 275 kV or 400 kV on the Supergrid, or 132 kV on the old grid system. Figure 75 shows the Supergrid network.

The reason for using high voltages is simple: the power losses are smaller, as is shown below.

Two of the 400 kV Supergrid conductors, capable of handling 1000 MW, have a combined resistance of 0.034 ohm per kilometre. (They are made of aluminium on a steel core, cross-sectional area 2.6 cm^2.)

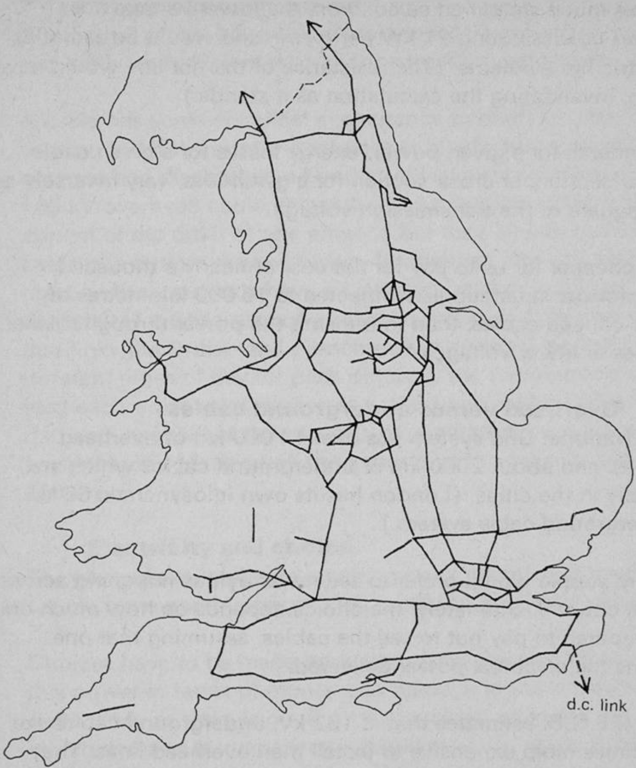


Figure 75
400/275 kV Supergrid England and Wales, March 1968.
From C.E.G.B. Statistical yearbook *1968*.

To make a simple calculation, we treat them as d.c. lines carrying 1000 MW at 400 kV. The current is 2500 A, and a power of 210 kW is dissipated in each kilometre of cable, a loss of 2.1 per cent in a hundred kilometres.

Had the voltage been ten times lower (40 kV), the current would be ten times higher (25 000 A) and the power loss (proportional to current squared) a hundred times greater, amounting to 21 MW in each kilometre.

In this much simplified calculation, the lower voltage lines would be dissipating 21 kW per *metre*, and would be as hot as electric fire elements. (The resistance of the hot line would have risen, invalidating the calculation as it stands.)

In general, for a given power, energy losses for a given cable cross-section, or cross-section for a given loss, vary inversely as the square of the transmission voltage.

It is cheaper for us to pay for the cost of nearly a thousand transformer substations, connected to 16 000 kilometres of high voltage cables, than to transmit the power through thicker cables at lower voltages.

Overhead versus underground cables

The National Grid system has over 14 000 km of overhead cables, and about 2000 km of underground cables which are usually in the cities. (London has its own idiosyncratic 66 kV underground cable system.)

Many people would prefer to see fewer pylons marching across open country. Ultimately, the choice depends on how much one is prepared to pay not to see the cables, assuming that one wants the electrical power delivered.

The C.E.G.B. estimates that at 132 kV, underground cables are ten times more expensive to install than overhead lines. They put the capital cost of underground cable for two of the 400 kV circuits mentioned above at about £500 000 per kilometre, compared with about £40 000 per kilometre for overhead lines.

The high cost of underground cables is mainly a consequence of the fact that soil does not conduct heat well. Overhead cables are cooled by the air, but buried cables can easily overheat. To avoid this they need to be made thicker, and are therefore more expensive. A 400 kV underground cable may be 20 cm^2 in cross-section compared with less than 3 cm^2 for an overhead cable. The electrical insulation round the cable, which also costs money, adds to the cooling problem.

Choice between a.c. and d.c.

Alternating power supplies are convenient because the voltage can easily and efficiently be changed with transformers.

A cable has some electrical capacitance to earth or other cables nearby, and if the supply alternates at 50 Hz, this capacitance is charged and discharged a hundred times a second. For a 400 kV overhead cable, this requires a charging–discharging current of the order of one ampere, but for a similar buried cable, the current may be nearer 40 amperes. For this kind of reason, direct currents have advantages at high voltages, especially for underground cables. In particular, the power line that links the British and French power systems, so that, having different times of day for peak demand, the two systems can feed each other spare power, is a direct current link. This use of d.c. also means that the two sets of generators do not have to be synchronized, as do all the generators in an a.c. linked system.

Electricity and choice

The general availability of electric power means that everyone can do more and for longer than would be possible without it.

Choices have to be made however, about the cost of supplying this power in terms of money and damage to the environment, particularly as there is no sign of a levelling off in the rate at which demand is increasing, a problem that is made harder by an increasing world population. If we choose to pay the cost of putting cables underground, we must also be prepared to choose from what other desirable things we divert resources.

There has long been concern over the pollution of the atmosphere by flue gases from fuel burning power stations, although domestic coal fires are worse offenders in this respect. Recently, the problem of disposing of radioactive waste from nuclear power stations has received attention. It may be, however, that the most serious pollutant in the long term is invisible; the warming up of the Earth which is the inevitable result of burning fuel.

The Earth's surface receives about 3×10^{24} joules per year from the Sun, while natural radioactive materials inside the Earth add a further 2×10^{20} joules per year. At present, man is adding another 2×10^{20} joules per year from all energy-using sources. Such an added energy will raise the temperature of the Earth by around 10^{-4} K per year. The large amounts of carbon dioxide produced by fuel burning may raise this rate by making the atmosphere more like a greenhouse, while smoke and dust may reduce it by producing clouds which reflect radiation from the Sun before it reaches the Earth. Quite small temperature changes would have a significant effect: a rise of a few degrees would melt the polar icecaps and raise the sea level by tens of metres, flooding most of the world's large cities.

Clearly, choices will have to be made. They will be better choices if they are made in the light of an understanding of the issues involved. We may have to balance the advantages of having larger scale energy resources for each person against the unintended consequences of providing them.

Railway electrification

By P. G. Holmes
Department of Engineering, University of Leicester

The railways were built to carry the Industrial Revolution across Britain, linking the heavy industries of the North and Midlands to the coalfields which supplied them with fuel, and to London, the centre of finance and government. In their heyday, railways were unbeatable as a rapid and efficient form of transport. The horse-drawn cart and the canal barge could not compete with them and the internal combustion engine had not been invented. Today, however, the railway is only part of a transport system that includes airways and roads, all of which compete both for freight and for passenger traffic.

The shape of Britain's rail network was formed in the period of fierce competition which characterized the Industrial Revolution. The outcome was a complex network of numerous branch lines serving small communities.

Figure 76
Photograph, British Rail.

Recently the railways have undergone drastic pruning, which reached its peak during the time when Dr Richard Beeching was Chairman of the British Railways Board. The social effects of line closures caused a rash of protest movements and led to the formation of the Transport Users' Consultative Council. Arguments were advanced to the effect that railways should be a social service, exempt from the profit and loss accountancy which governs competitive business. Resolving such a disagreement is a political, rather than an engineering, problem.

In addition to pruning, the railways had to undertake a vast modernization programme, involving new forms of motive power, track improvements, and radical changes in staffing. The human problems caused when staff changes result from new methods of working and line closures have their effect on the morale of railway workers. Perhaps in no other situation are the effects of advances in technology on human beings to be seen as clearly as on the railways; once a prestige industry whose workers formed a kind of labour aristocracy and now forced to fight hard against competition from the private car, the lorry, and the jet airliner.

In 1954, the British Transport Commission published its plan for the modernization of British Railways and capital was made available for this plan in 1955, ten years after the end of the war. One result of the modernization plan has been the electrification of the London Midland Region's main routes from Euston to Birmingham, Crewe, Liverpool, and Manchester at a cost of £160 000 000.

The change in motive power from steam to diesel and electric traction on our railways has come about for several reasons, but clear evidence exists from experience in Europe and America that greater efficiency results from diesel and electric operation. Steam locomotives, which first hauled passenger trains on the Canterbury and Whitstable Railway on 4 May 1830 and performed their last public passenger service on the London Midland Region on 4 August 1968, are romantic machines with many ardent admirers. Such locomotives are cheap to build, simple in design, robust, and long lasting, but they have severe disadvantages in modern terms. There is a shortage of large coal suitable for firing; the public demands cleanliness in trains and stations and resents being covered by smuts. It is also difficult to find men who will undertake the very hard manual work associated with cleaning, maintaining, and firing steam locomotives when more attractive conditions are available in other industries. The stark contrast between the comfort of the passenger, sipping iced drinks during a hot summer journey, and the lot of the fireman shovelling coal into the firebox of the locomotive, is inappropriate today. Modern technology assures the driver of a diesel or electric locomotive conditions comparable to those of his passengers.

The choice in motive power lies between diesel, diesel-electric, and electric traction. Diesel-electric locomotives offer many of the advantages of electric traction; cleanliness, rapid acceleration, high operating speeds, and a uniform standard of performance. The introduction of diesel-electric services does not call for major civil engineering works and the change-over from steam to diesel-electric can be effected as rapidly as such locomotives can be built. Line or overhead electrification, on the other hand, is a complex operation which involves heavy capital expenditure on the route involved. Nevertheless, on routes with a high traffic density, electrification provides the best economic solution, primarily because the electric locomotive can haul heavy trains at high speeds for long periods without needing to be withdrawn for maintenance and overhaul.

The London Midland Region's main route to Liverpool and Manchester is, for its length, probably the busiest main line in the world, and it was for this reason that it was selected for electrification. The investment of £160 000 000 of public money in such a venture must be based on a considered judgment as to the benefits to society which can be anticipated. Here we are on difficult ground because it is no easy matter to assess the public's need for inter-city transport. In 1968, the Department of Transportation and Environmental Planning at the University of Birmingham undertook a study of inter-city travel between London, the West Midlands, and the North-west of England. In this study they considered road, rail, and air routes and their report opens with the very significant sentence '. . . Very little is known about long distance travel . . .'.

What motivates a traveller to select a particular mode of transport for his inter-city journey? The value he places on such factors as speed, comfort, safety, cost, and convenience must all be weighed in the planners' decision. Speed is undoubtedly important, but it must be assessed for all the phases of the journey. Thus a high speed, technologically advanced inter-city link, whisking the passenger along at 150 km h^{-1} by rail or 800 km h^{-1} by air, has to be set against the often slow and difficult journey stages between home and station or airport and between the distant terminal and the passenger's ultimate destination. These link journeys are frequently tedious for travellers and account for the popularity of the private car which provides a high standard of comfort and convenience from door to door.

Electrification of the London Midland Region has reduced journey times from London to the West Midlands and North-west England by 25 per cent. The effect on the traffic pattern has been dramatic, resulting in a 65 per cent increase in the number of passengers using the route. Many of them have been won away from air services; the Birmingham study reveals no evidence of passengers transferring to rail from the private car, although British Rail claims to have attracted erstwhile road users. Fast rail links can compete attractively with air travel because of their city centre to city centre character, which is important for business travellers, and because they are far less vulnerable to delays and cancellations in bad weather. For journeys of over 300 km the time advantage lies with air transport, yet in spite of this many airlines lose money on internal inter-city routes which are also served by road and rail.

Motive power for fast inter-city services

A train leaving Euston in the morning en route for Manchester may be called upon to stop at Watford on the northern outskirts of London to pick up passengers from the commuter areas, so saving them a link journey to its starting point. It may then proceed to make a further stop in the Midlands or at Crewe for the convenience of passengers who have business there or who are changing trains for a journey further north.

If it is to make stops of this kind and yet accomplish the overall journey of 300 km in under $2\frac{3}{4}$ hours, the locomotive must be capable of rapid acceleration with a heavy load, sustained high speed running, and efficient braking to cope with stops and speed restrictions en route.

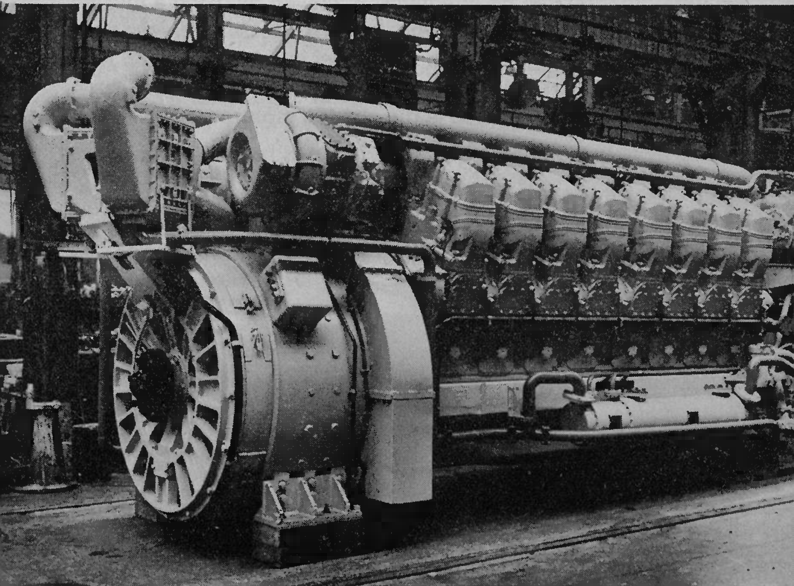


Figure 77
A 1.2 MW diesel engine, on the end of which is mounted an electric generator which supplies current to the traction motors. Two such engines and generators power British Rail's 'Deltic' locomotives.
Photograph, English Electric–A.E.I. Traction Ltd.

Two things are important from the technical point of view in all this; the type of traction motor employed and the adhesion between the wheels of the locomotive and the steel rail. The grip or adhesion between wheel and rail is severely reduced by contaminants, such as oil, and wheel slip reduces both acceleration and braking efficiency. Grip can be improved by spraying sand onto the rail ahead of the driving wheels; a crude method but one which is still used. For high speed services a better system is to clean the rail surface by a device mounted on the locomotive. Contaminants can be removed by heat, chemical action, bombardment of the rail with ions, or irradiation using radioactive sources or ultra-violet radiation. Current research being carried out by British Rail, in co-operation with the University of Leicester, has shown that decontamination can be effectively achieved by using a plasma torch mounted ahead of the wheel. The torch can increase the coefficient of friction between the wheel and the rail by up to 500 per cent. For rapid acceleration and high speed running, the most satisfactory device is the electric motor.

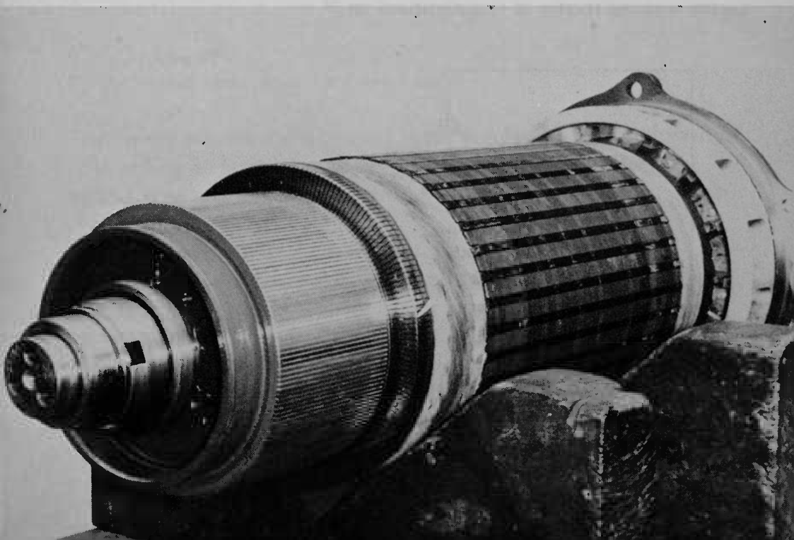
Figure 78
A '55' Class 2.5 MW diesel-electric locomotive designed and built for British Rail.
Photograph, English Electric–A.E.I. Traction Ltd.

The diesel-electric locomotive is (except for a few types employing hydraulic power transmission) an electric locomotive which carries its own power station with it. See figures 77 and 78. Inside the body is a diesel engine, coupled mechanically to a generator which produces electrical energy to be fed to *traction motors mounted on the axles of the bogies*. The electric locomotive, on the other hand, obtains the electrical energy for its traction motors from power stations in the National Grid system, picking it up either from a third rail laid alongside the running rails or from an overhead wire.

Electric generators and motors

The electric motor is a reversible device, both a motor and a generator. This is useful in traction applications because the motors can be made to act as brakes. If the current to the rotor is cut off and the kinetic energy of the moving train is used to turn the rotor, current will be produced which can be fed to a resistance (rheostatic braking) and the kinetic energy will be used up: hence the braking effect. The current produced can, in some cases, be fed back into the power supply (regenerative

Figure 79
Rotor of Brush traction motor.
Photograph, Brush Electrical Engineering Co. Ltd.



braking), thus reducing the net consumption of electricity. Sometimes, technical reasons make this process uneconomical because the saving in electricity does not justify the cost of extra equipment.

The diesel-electric locomotive can employ rheostatic braking but the only form of regenerative braking available to such locomotives is the flywheel. The kinetic energy of the moving train can be transferred to a flywheel in the locomotive and used later to assist with acceleration.

Engineers assess the ability of a motor to turn its shaft in terms of the torque. The torque is given by,

$$\text{torque} = \frac{\text{mechanical power delivered by rotor}}{\text{angular speed of rotor}}$$

The torque generated is proportional to the flux cut by the rotor coils and to the rotor current. The high torque needed to start a train, weighing up to 1000 tonnes, from rest can be achieved by using a series wound d.c. motor. In such a motor the same current, I, is supplied to both the rotor and field coils (figure 80). Since the flux produced by the field coils is proportional to I and the rotor current is also I, the torque in a series wound motor is proportional to I^2.

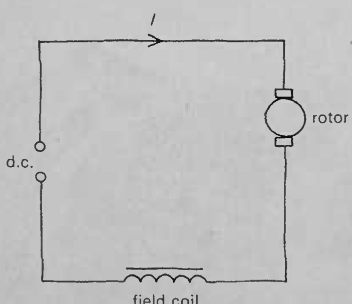


Figure 80
Series wound motor.

This is very helpful, because the current I is big just when the greatest torque is needed; when the motor is at rest or is just starting to turn. When the rotor is almost still, little voltage is needed to maintain a rate of change of flux in it, because the flux is not changing fast (the rotor is turning slowly). So almost all the voltage across the rotor is available to drive current through its low resistance. As long as the field coils also have a low resistance, the current, and so the torque, can be very large.

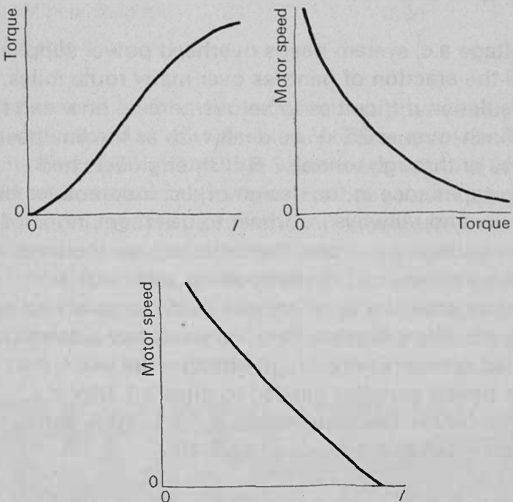


Figure 81
Torque, speed, and current for a series motor.

The series wound motor is just what is needed for a train which is to waste as little time as possible accelerating up to full speed after stopping at a station.

Electrification of the London Midland Region

All the electrification schemes laid down or proposed in this country up to 1956 were for direct current systems operating on voltages ranging from 600 to 1500 volts d.c. However, in France in the early 1950s an experimental electrification project was carried out on a 88 km section of route between

Aix-les-Bains and La Roche-sur-Foron. In this project 50 Hz alternating current was supplied to the locomotive from an overhead wire system at 20 kV. The traction motors were direct current machines operating on relatively low voltages, obtained by stepping down the 20 kV via transformers followed by rectification. Great success attended this experiment and the French went on to electrify with 50 Hz a.c. their Valenciennes to Thionville line which became the first main line to be electrified in such a fashion.

The high voltage a.c. system needs overhead power supplies which entail the erection of gantries over many route miles. There are insulation difficulties to be overcome – how can the problem of flash-over at 25 kV be dealt with as the line passes under bridges or through tunnels? British engineers had considerable experience in the design of d.c. locomotives but, since there were no railways in Britain to use them, none of building high voltage a.c. types. Nevertheless, on the basis of proved French success and the supporting technical and economic arguments, the decision was made to go ahead with high voltage a.c. electrification here. An overhead system can be constructed economically if high voltages are used. Consider the power supplies needed to drive a 1 MW d.c. locomotive. At 600 V, the requirement is for 1700 A, but at 25 kV, the current demand drops to only 40 A.

The overhead conductor supplying 40 A at 25 kV is much lighter than one needed to carry 1700 A. Hence there is economy in the conductor and, as a consequence, lighter overhead gantries became possible, which is an additional saving in materials.

The adoption of the 25 kV system has paid off in many directions. It has stimulated research into semiconductor rectifiers, high voltage insulation systems, wheel adhesion, and other allied fields. In addition it has put British manufacturers in a position to export equipment to developing markets in Europe and the United States. No one would have been interested in our old 600 to 1500 V d.c. systems.

The locomotives

The specifications laid down by British Rail for electric locomotives, the latest versions of which are known as the AL6 types, are given in table 8.

Duty	Speed/km h^{-1}	Mass of train/tonnes
Express passenger	160	500
Local passenger	120	500
Express freight	100	500
Mineral freight	88	1000

Table 8

These are met by 2.5 MW locomotives, powered by four axle-mounted d.c. traction motors. Current enters the locomotive at 25 kV through a pantograph and is fed to step-down transformers which provide lower voltages to the rectifiers. The present generation of locomotives has either mercury arc or silicon rectifiers. The semiconductor types have been developed as a result of advances in solid state electronics and they have many advantages over the mercury arc. For instance, they require less cooling and are much less heavy and bulky. The four traction motors are mounted so as to

Figure 82
An AL6 electric locomotive.
Photograph, British Rail.

drive each wheel axle of the locomotive's two bogies (figure 83). With some variation in type from one manufacturer to another, they are 6 pole motors, taking about 650 A at 900 V d.c. The driver controls the power supplied to each of the traction motors by tapping off from the main transformer of the locomotive a larger and larger voltage to be fed to the rectifiers. On his instrument display panel are two double ammeters which indicate the current being taken by each pair of motors. There is a lower green zone on the ammeter which covers the current range equivalent to the tractive efforts from 0 to 10^5 N force. Next comes an amber zone terminating at 2.5×10^5 N, and above this a red one indicating overload. The driver selects power settings which keep the locomotive in the amber zones when accelerating and in the green zones whilst cruising at steady speeds.

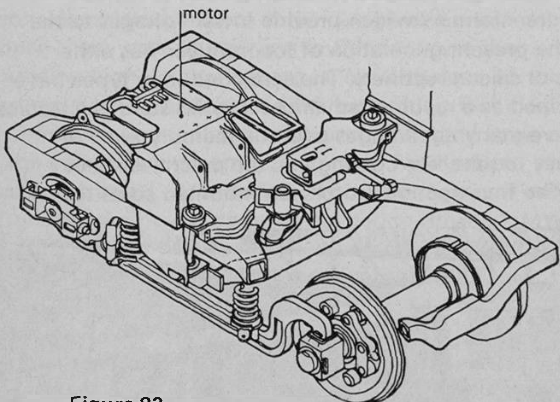


Figure 83
This drawing of one of a locomotive's bogies shows the location of one of its pair of traction motors.
By courtesy of British Rail.

Braking is accomplished either by vacuum brakes on locomotives with mercury arc rectifiers or by a combination of vacuum and rheostatic braking on those with semiconductor rectifiers. The saving in weight and space with germanium or silicon rectifiers makes room for the rheostatic braking device.

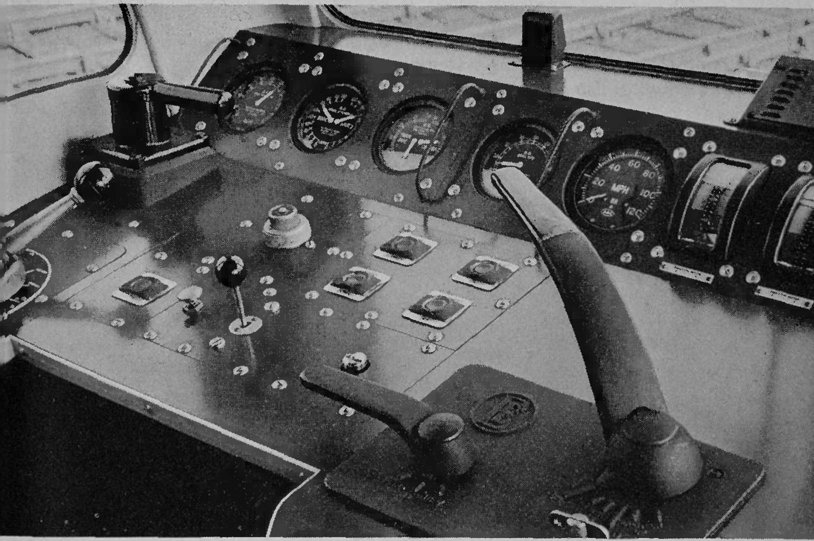


Figure 84
Control panel of electric locomotive, type AL6.
Photograph, British Rail.

The development of thyristors or silicon controlled rectifiers has greatly simplified the traction motor voltage control system. Large power levels can be controlled by small gate signals. This opens up the possibility of controlling trains with the assistance of computers. With the possible advent of trains running at up to 250 kilometres an hour (the Advanced Passenger Train being developed by British Rail, for example), the time for decision taking will be reduced, so that automatic control will become more and more desirable.

Figure 85
The Hawker-Siddeley 'Kestrel' diesel-electric, designed for use in computer-controlled railway systems.
Photograph, Brush Electrical Engineering Co. Ltd.


HS 4000
OZ 18
HAWKER SIDDELEY
BRUSH

Further reading

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* *Science journal* reprints are no longer available. The above titles will, however, appear in a collection of these reprints to be published by Penguin in 1972 as part of the Nuffield Advanced Physics publications, under the title *Physics and the engineer*.

Data and formulae

Data

permeability of vacuum (magnetic force constant, 'magnetic conductivity' of vacuum)	μ_0	$4\pi \times 10^{-7}$ N A^{-2} exactly
charge on an electron	e	1.6×10^{-19} C
mass of an electron	m_e	9.1×10^{-31} kg
mass of a proton	m_p	1.7×10^{-27} kg
charge to mass ratio, electrons	e/m	1.76×10^{11} C kg^{-1}
Avogadro constant	L	6×10^{23} mol^{-1}
Earth's gravitational field at surface (acceleration of free fall)	g	9.8 N kg^{-1} or m s^{-2}

One ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

Formulae

The force F on a conductor, length L, carrying current I in a field B at right angles to the conductor is given by

$$F = BIL.$$

The unit of B is N A^{-1} m^{-1}, abbreviated to tesla, symbol T. The force equation is modified to become $F = BIL \sin \theta$ if the field and conductor make an angle θ.

The direction of the force is given by the fingers of the left hand. If thumb, first finger, and second finger are held at right angles

thumb	gives	motion or force direction
first finger	gives	field direction
second finger	gives	current direction.

The force on a charge q moving with velocity v at right angles to field B is at right angles to both B and v, and is given by

$$F = Bqv.$$

When such a charged particle, mass m, moves in a circle of radius r, the acceleration towards the centre is v^2/r, needing force F at right angles to its motion given by

$$F = mv^2/r.$$

This is a useful result for magnetic fields, because the force can be supplied by a B-field acting on moving charged particles in a particle accelerator, a mass spectrometer, or in an experiment to measure the particles' charge to mass ratio.

If a current I is carried through a specimen of cross-sectional area A (at right angles to the current), by n carriers per unit volume, each carrier having charge q, and drifting at velocity v, then

$$I = nAqv.$$

Such carriers in a slice of material, thickness b, will suffer a force if there is field B at right angles to the current and to the slice. The result is a Hall effect voltage V_{Hall} across the slice, at right angles to the field and to the current, given by

$$V_{\text{Hall}} = BI/nqb.$$

Magnetic forces can give rise to electric forces. The electric field E is the ratio F/q, where F is the force on a small charge q. (Strictly, E is the limit of the ratio as the charge q tends to zero.) If there is a uniform field E over length L the p.d. across L is given by

$$E = V/L.$$

The voltage V induced in a single circuit, area A, perpendicular to field B is given by

$$V = \frac{\mathrm{d}}{\mathrm{d}t}(BA) \quad (BA \text{ is called the flux})$$

if B is uniform over the area A. Every linking of field and area must be added in, so that if the coil has N turns each of area A, the voltage is N times larger. If B is not uniform over the area, BA is replaced by $\int B\,\mathrm{d}A$ over the area.

This result summarizes two effects; the voltage induced by moving a conductor in a B-field (when there is a force Bqv on the charges carried along by the conductor) and the voltage induced by changing the flux linking the circuit.

In a coil, a change of current in the coil changes the flux through the coil, and a voltage V is needed, where

$$V = L\,\mathrm{d}I/\mathrm{d}t$$

L being the *inductance*, unit henries, symbol H.

The direction of an induced current is always such as to conserve energy.

The flux along a closed tube of length L, cross-sectional area A, which is encircled by N turns of current I and follows the flux direction, is given by

$$NI = \textit{flux} \times \textit{reluctance}$$

where

$$\textit{reluctance} = L/\mu\,\mu_0 A, \text{ if the flux and area are constant.}$$

μ_r is unity for vacuum, but may be as much as 1000 for iron, and varies with the applied field.

Around a closed path, the sum of $(B/\mu_r\mu_0)\cos\theta\,dL$ is equal to the current turns NI enlaced by the path. This may be written

$$\oint \frac{B}{\mu_r\mu_0}\cos\theta\,dL = NI$$

and is a valid result as long as the magnetic effects of the current and the magnetism of the material (e.g. iron) are parallel.

The B-field of a very long solenoid, with N turns in length L, is given by

$$B = \mu_0 NI/L$$

so long as there is no iron in the solenoid.

The B-field in air (strictly vacuum) around a long straight wire, at distance r from the wire, is given by

$$B = \mu_0 NI/2\pi r.$$

In a transformer, with a primary coil having N_p turns around a closed iron core, a small current I is needed to set up flux in the core, given by

$$N_p I = \textit{flux} \times \textit{reluctance}.$$

If the current is small, and the primary resistance is small, almost all the voltage V_p applied to the primary is used to maintain changing flux through it, with

$$V_p \approx N_p \times \textit{rate of change of flux through primary}.$$

If all, or nearly all the flux goes through the secondary coil, with N_s turns, and

$$V_s = N_s \times \textit{rate of change of flux through secondary}$$

then $V_p/N_p \approx V_s/N_s$

or $V_p/V_s \approx N_p/N_s$.

Since the transformer does not generate power, if current I_s is drawn, extra current I_p flows, and

$$I_p V_p > I_s V_s.$$

Index

Where significant information is contained in an illustration or diagram, the page reference is italicized.

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This *Students' book* contains a summary of Unit 7, *Magnetic fields*, and questions on its main work. The Unit is divided into three Parts: 'Forces on currents', 'Electromagnetic induction', and 'Flux near currents'. The book also includes answers to the questions, chapters on 'Induction motors', 'The generation and transmission of power', and 'Railway electrification', a list of background reading, and notes on data and formulae used in the Unit.

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