Physics

Students' book **Unit 4**

Waves and oscillations

Nuffield Advanced Science

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Waves and oscillations

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Waves and oscillations

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Contents

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people, in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator
of the Nuffield Foundation Science Teaching Project

To the student

This book contains some of the things you need to help you to understand the work of this Unit, and some reading which we hope will help you to see how the work is relevant to the practical, everyday world. It does not contain all you need: you will have to consult textbooks and other more general books as well, working through theoretical arguments, reading about experiments, and finding out more about how the ideas can be put to practical use.

This book contains many questions; more than you will be able to do while working on this Unit. Later on, you may wish to use some of them for revision. You will find questions which take you step by step through the theoretical arguments in the course; students who took part in the trials have said that these questions are a good way to understand a piece of theory. You will have to pick and choose, according to your needs and tastes, amongst the other questions. A few give you simple practice in calculation. More invite you to argue about or discuss a problem, and some of these – usually marked '*For discussion*' – are not suited to formal written answers. They are meant to start off a discussion, which may then wander far from the question.

There are a few harder questions to challenge the clever, and you should not expect to be able to tackle every question easily. But most are meant for ordinary human beings, not for budding geniuses. If in doubt, try the obvious answer: usually there is no catch! Most questions have some kind of answer in the section headed 'Answers', though some of these suggest where you might find the needed information, instead of giving it. We have tried hard not to give wrong answers, but, being fallible like yourselves, may not have succeeded.

Some questions ask you to guess, speculate, or give your private opinion: obviously they have no one right answer.

What you are being asked to learn to do

This course aims to help you to become more like a physicist. Most of you will not become physicists, but will use physics or learn more of it in one of a variety of scientific jobs or in further education. Physics, and the world with it, are changing so fast that no one can tell what bits of physics you will use in, say, ten years' time; however, one can be pretty sure that there are some basic ideas that will be relevant to the new problems of tomorrow. We have tried to build the course around what we believe to be these basic ideas.

So one thing the course aims at is helping you to become able to learn, in the future, the new ideas in physics you may meet, and helping you to become able to use the physics you have learned. It does this because these are the tasks that will face you.

In the future, you will need to be able to learn from books and articles; that is why the course contains a good deal of reading (in a list at the end, you will find details of books referred to in the text). To use the physics you have met, you need to understand it – that is, to be able to use it in new kinds of problems. That is why so many questions in this book ask you to make up arguments about new problems, using what you know.

What is 'understanding'? That is, how does one recognize *that someone understands a piece of physics? We think it is* something like this. Suppose a group of people are talking about a problem in physics. Very rarely, even among research workers, will anyone immediately see an answer. More often, they each have some ideas which they try out in discussion with colleagues. Those who 'understand' their physics are the ones who can offer sensible, relevant ideas that would help towards clearing up the problem. A reasonably competent physicist expects himself and others to be able to draw on their knowledge and use it to make sensible contributions to the discussion of problems.

So to test whether you understand a piece of physics, it is asking too much to expect you to solve a new problem completely and correctly; few – if any – experts can do that. The test should be that of physicists talking together: can you produce sensible ideas that are relevant and would help a bit towards clearing up a problem? This is the test that will be used in the examination, and is the way to decide how well you have managed a question or problem in the work of the course.

The course also aims to show you what doing physics is like, and this is another reason for encouraging plenty of discussion of problems, for that is the way physicists work. It tries to show what kinds of questions physicists ask themselves and what sorts of ways they use to tackle them. We think this is important because to use physics successfully and to judge its claims and achievements you need to understand what it can, and what it cannot do. That is why several questions ask you about such things as how theories, models, experiments, and facts fit together. Physicists also guess, estimate, and speculate, so other questions ask you to do these things too, to find out what doing them is like and to become better at doing them.

There are a lot of misunderstandings about what physics is like. Some say it is all facts; others that it is all theory, having little to do with what happens in practice. Many are puzzled; asking whether what physics says is true or not, or how physicists arrive at their ideas. We hope you will find chances in this course to think about such matters, and that you will form your own views.

Some of the questions ask about how physics can be used in engineering and technology, and the articles in this book are also about that, because we think that you will rightly want to know when what you learn is of practical value.

Finally, one of the main reasons we went to offer you some physics is that we like the subject and get excited about it. So we hope you enjoy it too.

Summary of Unit 4
Waves and oscillations

Units 1, 2, and 3 have been mostly about matter and about electricity, and have illustrated two sorts of concern that physicists share. These are a concern with links between things on the small scale like atoms and things on a large scale like lumps of steel, and a concern with fields, with the forces that particles exert on one another, and with the way such forces seem to act across empty space.

Unit 4 adds to these a third concern, with motion and with how things change. This has already entered the course in a small way, in the discussion in Unit 2 of the changing charge on a capacitor. Indeed, the mathematical methods used in that discussion will be developed further in Part Three of Unit 4 to help in understanding oscillators.

The ideas of Unit 4 will be used later on in the course, especially in Unit 8, *Electromagnetic waves*, and Unit 10, *Waves, particles, and atoms.* In both these Units, the ideas from earlier work will be used to build further understanding of new problems.

But waves and oscillations are of great practical importance, too. The designers of cars, bridges, blocks of flats, high fidelity record players, radar sets, radio navigation devices, and a host of other things, all need to know a good deal about waves and oscillations.

Part One

Waves of many sorts

The chief test for wave motion

Invisible waves; using superposition effects to test for waves you can't see. Investigations of radio waves, light, and sound waves.

The electromagnetic family

The speed of light and of microwaves. Infra-red and ultra-violet radiation. The purely experimental evidence that these waves may be of the same sort.

Part Two

Mechanical waves

Investigating wave pulses

Waves on springs and other things. Superposition again. What does the wave speed depend upon?

Predicting a wave speed

The speed of sound in a solid rod, as an example of how a wave speed can be understood using only basic physical principles. Examples of other waves and formulae for their speeds.

Part Three

Mechanical oscillations

What is time?

Does time flow evenly; how would one tell if it does? How is time measured?

The motion of oscillators

Investigations of many oscillators, looking for simple, common sorts of behaviour. What does the time of oscillation depend upon?

A mathematical model for harmonic oscillators

Using mathematics and basic principles to understand the motion of an oscillator and to obtain equations for the motion and the time of oscillation.

Resonance

Investigation of resonance, as an example of the experimental investigation of a problem.

Standing waves

Waves on strings, springs, metal plates, water surfaces, and many other objects, as examples of the variety of standing waves of interest to physicists and engineers. (Standing waves will be very important in Unit 10, *Waves, particles, and atoms.*)

Questions

Part One

Waves of many sorts

Questions 1 to 5 Superposition of waves

These questions are mainly for revision of the idea that wave motions superpose upon one another.

1 See figure 1. S_1 and S_2 are two sources of sound of the same frequency. An observer at A hears hardly any sound. How would you account for this? Sketch the path he would have to follow in order to keep the sound he hears at a minimum as he moves towards the sources.

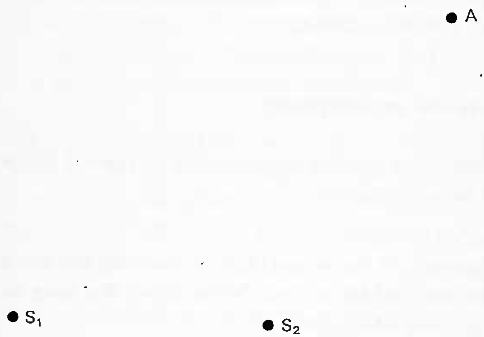


Figure 1

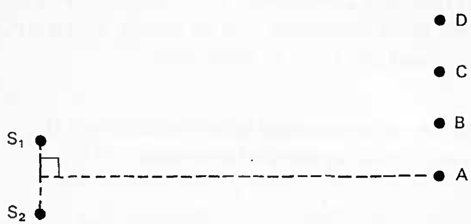


Figure 2

2 See figure 2. S_1 and S_2 are two water wave sources in a ripple tank. They are vibrating at the same frequency. There is a maximum disturbance at A, a minimum at B, another maximum at C, and so on.

a Write an expression for the wavelength of the ripples.

b How will the pattern of maxima and minima change if:

1 the two sources are moved closer to each other?

2 the frequency of vibration is increased?

3 the velocity of the ripples is decreased (by reducing the depth of water in the tank)?

Figure 3

3 See figure 3. S_1 and S_2 are two wave sources, vibrating in phase. M_n and M_{n+1} are two adjacent maxima in the interference pattern. Write an expression for the wavelength in terms of the distances from the two sources to M_n and M_{n+1}.

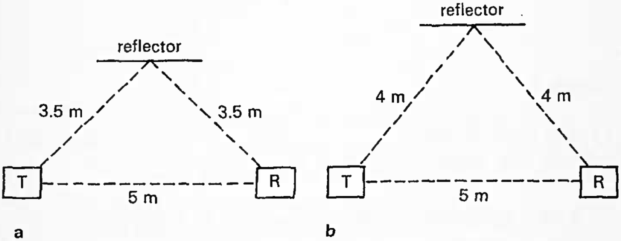


Figure 4

4 With the arrangement of wave transmitter T, receiver R, and reflector shown in figure 4 *a*, the signal strength received at R is a maximum. When the reflector is removed to the position shown in figure 4 *b* the signal reaches a minimum. Why? What might be the wavelength of the radiation? Why can you not be sure of the value?

5 See figure 5. A transmitter T emits a radiation, some of which is reflected from a screen S_1, and some of which carries on to be reflected from a second screen S_2. The radiation reflected back from S_1 and S_2 is detected by a receiver R placed alongside T. At a certain separation of S_1 and S_2 the detector records zero signal. S_2 is then moved away from S_1. As S_2 is being moved the detector records 2 signal minima and S_2 is stopped at the third minimum, a total movement of 120 mm.

a What is the wavelength of the radiation?

b At the original separation the signal detected was very nearly zero; but after S_2 had moved 120 mm the minimum signal was quite perceptible. Why?

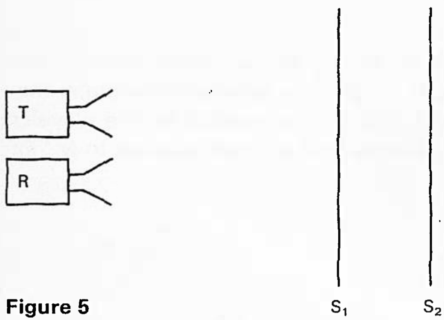


Figure 5

Questions 6 and 7

These revise the relationship $v = f\lambda$.

6 **a** Radio Luxembourg broadcasts on a wavelength of 208 m. What is its frequency?

b A television station transmits at 567 MHz. What is its wavelength?

c Green light has a wavelength of about 500 nm. What is its frequency?
d What is the frequency of X-rays with wavelength 0.1 nm?

Speed of electromagnetic waves in a vacuum (air) $= 3 \times 10^8$ m s^{-1}. 1 nm (nanometre) $= 10^{-9}$ m.

7 The lowest note that a clarinet can play is the E below middle C. The frequency of this note is 160 Hz. Sound travels in air at 340 m s^{-1}. What is the wavelength of this E? (About how long is a clarinet?)

Questions 8 to 18

These are all about waves. Questions 8 to 14 concern measuring the speed of waves or putting knowledge of the speed to use.

8 Figure 6 shows the main parts of apparatus used in a laboratory method of determining the speed of light.

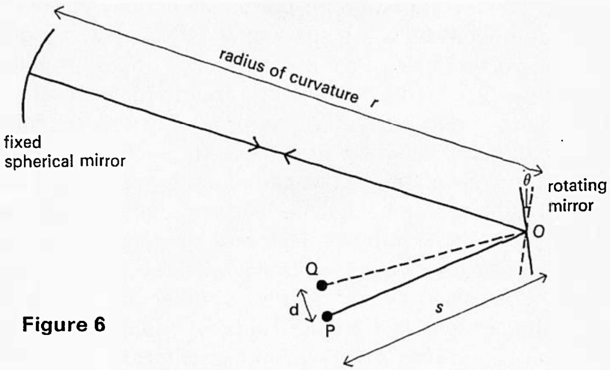


Figure 6

When the rotating mirror is stationary, an image of the illuminated slit at P is produced back at P. When the mirror is rotating, the image of P becomes displaced to Q. The diagram is not drawn to scale. *r* is greater than *s*, and *d* is very much smaller than both *r* and *s*.
a Why is the image displaced?
b How far does the light travel between reflections at the rotating mirror?

c If the mirror rotates at n revolutions per second, how long does it take to cover one revolution?
d How long does the mirror take to cover angle θ?
e The speed is the distance travelled divided by the time taken. Express the speed of light in terms of the answers from **b** and **d**.
f Now try to obtain a relationship between θ and the measurable quantity d. When the mirror rotates through angle θ then the angular movement of the image is POQ. What is this in terms of θ? What is θ in terms of d and s?
g Write down an expression for the speed of light in terms of d, n, r, and s.

9 Radio emission at about 11 metres wavelength from Jupiter can be detected on Earth. The radiation consists of a number of pulses, often occurring in groups. In any group the first pulse is always stronger than the second, which in turn is stronger than the third. This suggests an echo phenomenon in which the second and third pulses are echoes of the first. The time intervals between successive pulses within a group have three distinct sizes: a short interval of about 25 milliseconds, an intermediate interval of about a quarter of a second, and a long interval of the order of a second. This long interval corresponds to the time taken for a signal to travel once around the planet. It has been suggested that the short interval might be due to the source being above the surface of the planet. The intermediate interval could be explained if the planet had an ionosphere which was partially transparent to signals but also partially capable of reflecting signals. The pair of signals separated by 0.25 second could then be the direct signal and one which was reflected at the ionosphere down to the planet's surface and then back up again.
a Draw a rough sketch showing the possible paths of the radio signals.
b How high above the planet surface would the source have to be?
c How high above the planet surface would the ionosphere have to be?

10 a In some areas a television picture consists of a sharp image with a faint 'ghost' image just alongside. What could cause this 'ghosting' effect?
b When an aircraft passes overhead, a television picture may give 'ghost' images.

To 'paint' a television picture (u.h.f.) the electron beam crosses the screen so as to make 625 lines in $\frac{1}{25}$ second. The screen is about 0.5 m across. If a 'ghost' image is found to be 10 mm displaced from the main picture, when an aircraft is directly overhead, how high is the aircraft flying?

11 The distance of an object can be determined by measuring the time taken for a short duration signal to travel to the object and back again (see figure 7). This technique is used with many different forms of signal, including ultrasonics for the detection of shoals of fish, and radar to determine the positions of aircraft. Figure 8 shows how the

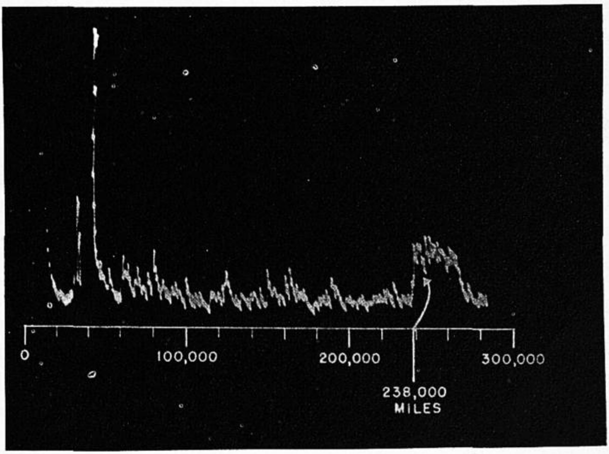


Figure 7
A cathode ray oscilloscope trace showing the first published radar echo from the Moon. The initial pulse is above the 0 mark, the echo above the 238 000 miles mark.
From Mofenson, J. (1946) 'Radar echoes from the Moon'; reprinted by special permission, from the April 1946 issue of Electronics; *copyright* © *1946 McGraw-Hill Inc., New York, N.Y., 10036*

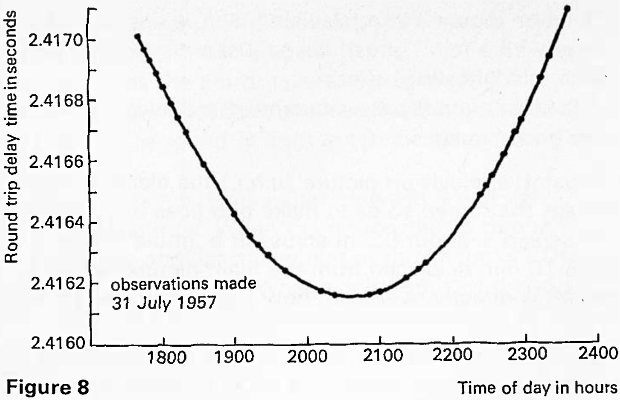


Figure 8

From Yaplee, B. S., Bruton, R. H., Craig, K. J., and Roman, N. G. (1958) 'Radar echoes from the Moon at a wavelength of 10 cm', Proc. I.R.E. **46** *No. 1, 293.*

time taken by radar pulses to travel from the Earth to the Moon and back varied during one six-hour period. Explain the form of the graph in figure 8. Obtain any information you can from it.

12 Comment on the following quotation from Galileo:

Everyday experience shows that the propagation of light is instantaneous; for when we see a piece of artillery fired, at a great distance, the flash reaches our eyes without lapse of time. But the sound reaches the ear only after a noticeable interval.

13 It is sometimes said that someone at home listening to his radio will hear the voice of the performer singing into a microphone *before* a member of the audience in the hall hears the sound directly. Can this be correct? Explain. What is the approximate time difference between the two listeners mentioned above? Make reasonable assumptions where necessary.

14 Lord Rayleigh is reputed to have said: 'If an observer moves with a velocity twice that of sound a musical composition would be heard backwards correct in time and tune'. Does this seem feasible? (The composition's sound has started out in space and the observer hurries after it.)

15 A researcher wants to measure wavelengths in **a** the visible region, **b** the microwave region, **c** the v.h.f. radio region, and **d** the X-ray region. How would he set about the task? In your answer be sure to give some idea of the order of size of the vital parts of the apparatus.

16 Arrange the following in order of increasing frequency:
a Middle C.
b Ocean waves.
c Medium wave radio waves.
d Long wave radio waves.
e Ultra-violet radiation.
f Visible light.
g Infra-red radiation.
h X-rays.

17 *For discussion*
Much of our knowledge of the world is brought to us through our ears and eyes by waves. How would our knowledge change if the wavelength ranges we can detect were to change?

18 *For discussion*
Here is a list of some uses of radio waves:
Ocean and coastal navigation.
Radio broadcasts covering whole nations from one transmitter.
Local city radio.
Radar detection of migrant birds.
Studies of the ionosphere.
International time checking.
Cooking.

Here is a short statement about radio waves:
'Radio waves can be generated in suitable circuitry with wavelengths ranging from a few millimetres to several kilometres. All travel at the same high speed, 3×10^8 m s^{-1}. All are electrical in nature, and are produced and detected by electrical means. All have the wave properties to be expected in waves of the particular wavelength involved.'

Choose some of the uses, and say what you can about which of the properties mentioned above are of particular importance for each use. It may be, for example, that the wavelength must be small or large, that the speed must be known, that its high value is of importance, or that the electrical nature of the waves is crucial.

Part Two
Mechanical waves

Questions 19 and 20
These are for revision of dynamics. Ideas from dynamics will be needed in Parts Two and Three in the discussion of the speed of a sound wave and of the motion of an oscillator.

19 Imagine a row of railway trucks, each with spring-loaded buffers in contact with those of the trucks on either end of it.

A truck at one end of the row is given a push by an engine, and the motion travels along the row of trucks.

a Think about a truck at a place which the motion has just reached, so that this truck is accelerating. What can you say about the forces in the spring-loaded buffers on either end of it?
b Think about a truck at a place which the motion has not yet reached, which is at rest. Is it necessarily true that there must be no force at all in the buffers on either end of it?
c Think about a truck which has been set into motion, and suppose that it, and all those which are moving, travel forward at steady speed. What must be true about the forces in the buffers on either end of it if wheel friction is very small?

d (*Hard*) The engine is pushing on the end of the row of trucks. Those which are moving at steady speed have no resultant force acting on any of them. Those which are still at rest have no resultant force on any of them, either. So why does the engine have to push on the end of the row at all? (*Hint:* it doesn't, if there is little friction, once *all* the trucks are set in steady motion.)

20 A student pushes a trolley against a spring. The other end of the spring is fixed.

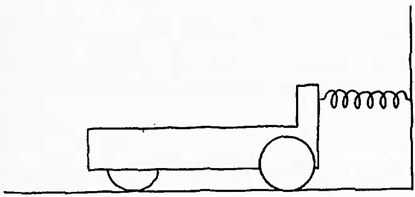


Figure 9

a A force of 5 N is needed to compress the spring by 20 mm. What force will be necessary to compress the spring by **1** 10 mm, **2** 30 mm? What assumption must you make to answer this question?
b How much energy is transferred to the spring when the spring is compressed by 20 mm? If the student lets go, what is the force applied to the trolley at the instant he does so?
c Describe the motion of the trolley after it is released.
d How could he change the time for which the spring pushes on the trolley after release, for the same compression?
e The product of the force and the time over which it is applied is known as the impulse. If the impulse applied to the trolley is 0.3 N s, what is the momentum of the trolley?
f A 1 kg trolley is released after the spring has been compressed by 20 mm. The experiment is repeated with a 2 kg trolley and the same compression. Which will acquire the greater velocity? Compare also the momenta and kinetic energies of the trolleys.

Questions 21 to 23

These questions are about the motion of parts of systems which are carrying mechanical waves.

21 a Figure 10*a* represents a wave kink travelling to the right along a taut rope. Copy the sketch and add arrows where appropriate, to indicate the velocity of the rope.

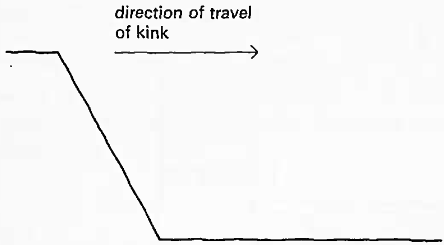


Figure 10a

Now add double arrows, in a similar way, to show the acceleration of the rope. Draw in the position of the rope a short while later.

b In figure 10*b* there is a 'double kink' pulse moving along a rope.

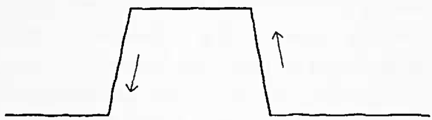


Figure 10b

The arrows indicate the velocities of those parts of the rope that are moving.

Copy the sketch, add the acceleration arrows, and decide which way the pulse is moving.

c Can you develop the same argument for a continuous wave on a rope?

Figure 10c

22 *For discussion*
A compression pulse travelling along a Slinky spring carries energy. Is it kinetic energy (because the spring is moving) or potential energy (because the spring is squashed by the pulse)?

What happens to the energy when a compression pulse going one way coincides with an expansion pulse going the opposite way, and the two superpose to give *no* net compression or expansion?

23 You are walking with a friend near a railway siding while an engine is shunting a row of trucks. To impress your friend you might listen to the shunting operation and say, 'Ah, the six trucks nearer the engine are empty and the following five are loaded'. How could you have known without looking?

Questions 24 to 26
These are about the speeds of waves in various things.

Question 24 takes you step by step through the explanation of the speed of a compression wave in terms of basic mechanical principles. This argument is an example of what can be done in a similar way for many other sorts of wave – waves on ropes or on guitar strings, on water, in air, or in the structures of buildings.

24 The diagram in figure 11 shows a row of trolleys linked by compression springs. At time zero (top picture) all the trolleys to the right of P are at rest, the springs between them being uncompressed. The trolley to the left of P is kept moving towards P at speed u.

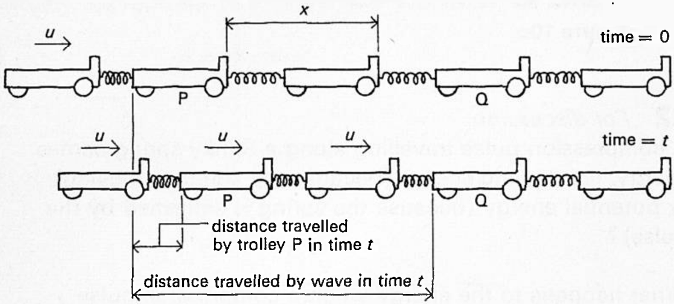


Figure 11

a What is happening to trolley P at time zero?
b A trial with a row of trolleys shows that after a time t all the trolleys to the right of P up to, say, trolley Q will be moving with steady speed u, if the lefthand trolley is kept moving at this steady speed.
What is happening to trolley Q at time t? (Bottom picture.)
c A wave of compression has travelled from P to Q in time t, for Q starts to move at time t in just the way that P did at time zero.
Which is the larger, the speed u of the moving trolleys or the speed v of the wave?
d What is the distance travelled by the wave in time t, in terms of v and t?
e If the distance between undisturbed trolleys is x, how many trolleys are set into motion in time t?
f How far will trolley P have moved at speed u in time t?
g By what distance has the row of trolleys that are now moving been compressed?
h Amongst how many springs in this row of moving trolleys has the compression found in **g** been shared? (See **e**.)

i Combine the answers to **g** and **h** to write down the distance by which each single spring in the moving row has been compressed.
j Suppose that the springs obey Hooke's Law, so that the force exerted by a compressed spring is a spring constant, k, multiplied by the amount by which the spring has been compressed. Write down an expression for the resultant force on trolley Q at time t, noting that the spring on its right is not compressed.
k Upon how many trolleys has such a force acted in time t? (See **e**.)
l What momentum has each of these trolleys acquired, if each has mass m?
m In the equation:

(force) (time) = (total change of momentum of all moving trolleys)

substitute expressions for the *force* on each trolley (see **j**), the *change of momentum* of each trolley (see **l**) and the *number of trolleys* now having this momentum (see **k**). The *time* is t, of course.
n Check that u and t cancel. Write as simple an expression as you can for the wave speed v.
o Now say in words why the wave speed would decrease if the trolleys were made more massive.
p Say in words why the wave speed would increase if the springs were made stiffer.

25 This question shows how the wave speed for a row of trolleys, discussed in question 24, can be applied to discussing the speed of sound in a solid.

The speed v in a compression travelling along a row of trolleys linked by springs is given by:

$$v = x\sqrt{k/m}$$

where x is the distance between the centres of successive undisturbed trolleys, m is the mass of a trolley, and k the constant relating the force causing the compression of a spring and the amount of compression.

Trolleys linked by springs might be compared to the atoms in a solid linked by interatomic forces. Table 1 gives some comparative values of x and m.

	Trolleys and springs	Atoms in steel
x	0.35 m	2.5×10^{-10} m
m	0.95 kg	9.3×10^{-26} kg
v	2.5 m s^{-1}	?

Table 1

k is roughly 50 N m^{-1} both for a pair of trolleys and a spring, and for a pair of atoms in steel.

a Suppose that, without changing anything else, the mass of a trolley were reduced by a factor of 10^{-25} (to about the mass of an atom in steel). What would be the speed of a compression wave in such an arrangement?
b Suppose, with this new arrangement, that the distance between trolleys is reduced by a factor of 7×10^{-10} (or 10^{-9} if you don't mind a rougher answer). What would be the speed of a compression pulse in such an arrangement?
c As the value of k is much the same for both steel and spring, the answer to **b** should be the speed of a compression wave in steel if it is permissible to scale down from the trolleys to atoms, and if the scaling has been done correctly. The measured speed of sound in steel is about 5100 m s^{-1}. What is your estimate?

26 This question shows how the speed of sound in a metal can be written down in terms of the Young modulus E and the density ρ.

The speed v of a compression pulse travelling along a row of trolleys linked by springs is given by:

$$v = x\sqrt{k/m}$$

where x is the distance between the centres of successive undisturbed trolleys, m is the mass of a trolley, and k is the spring constant in the equation

force $= k \times$ change in length.

Unit 1 *Students' book* question 32 showed, for a specially simplified case, that if E is the Young modulus and x the spacing between atom centres, the atomic bond spring constant k is given by $k = Ex$ if the atoms are in a simple cubic array. Consult that question again to see why.

a Suppose the density of the material is ρ and that each atom occupies a volume x^3. What is the mass m of each atom?

b Find a new expression for the speed v of compression waves (or sound waves) in a solid in terms of E and ρ, by substituting for k and for m.

c What is the speed of sound in an aluminium rod?

Young modulus $= 7.0 \times 10^{10}\ \mathrm{N\,m^{-2}}$
density $= 2700\ \mathrm{kg\,m^{-3}}$

Note: The atoms in aluminium are *not* arranged in a simple cubic array, but the answer to **b** *does* give a correct expression for the speed of sound in an aluminium rod. For arrays more complicated than cubical arrays, the relations between k and E and between m and ρ are both more complicated than above, but the spacing x still cancels out as it did in **b**, together with all the extra geometrical factors which allow for the more complex atomic arrangement.

d The speed of sound in aluminium is very nearly the same as its speed in steel, but steel is several times more dense than aluminium. What must be true of their Young moduli?

Questions 27 to 28

These are about calculating the speed of two new sorts of wave from basic mechanical ideas. Question 24 was a sample of how such calculations may be made. You may think that example quite sufficient. If not, try one of questions 27 or 28. You need not learn either of these arguments. Look at them if you want to see how such arguments go, or if you have an interest in waves on the strings of musical instruments (question 27) or water waves (question 28).

27 This question is about the speed of waves on the strings of musical instruments. It considers a special sort of wave, but the result is general.

Figure 12 illustrates the progress of a single kink wave travelling at speed v along a taut string under tension T.

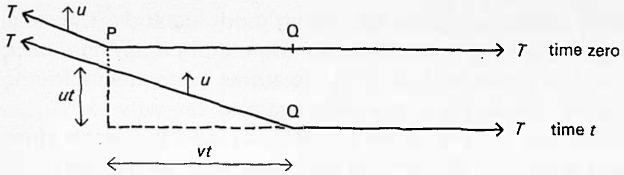


Figure 12

In the top picture, the kink is at P. The string to the left of P is moving upwards at steady small speed u; the string to the right of P is at rest. In the bottom picture, the kink has moved to Q in a time t so that *all* the string between P *and* Q *is* moving upwards at steady speed u. (Somewhere to the left of the picture there must be *another* kink, where the string stops moving. But that doesn't alter the speed of the kink shown.)

a Why, at time zero (top picture), is the bit of string at P being pulled upwards?

b Why, at time zero, is there no resultant force on the bit of string at Q?

c Why, at time t, is the bit of string at Q being pulled upwards?

d How far does the string at P move upwards in the time t during which the kink travels a distance vt?

e If u is much smaller than v, so that the kink angle is small, then the force pulling on the bit of string at a kink is Tu/v. Explain why.

f What mass of string is set in motion in time t if each metre has mass μ?

g What total momentum is given to the moving string in time t?

h Obtain an expression for the wave speed v by putting answers to **e** and **g** in the equation

force $\times$ time $=$ total change of momentum.

Check: t and u cancel.

28 This question is about the speed of one sort of water wave. Read the note above question 27 before trying it.

The speed v of a long wavelength wave in shallow water of depth h is given by $v = \sqrt{gh}$, where g is the Earth's gravitational field, the force in newtons on a one-kilogramme mass at the Earth's surface. (It is also the acceleration due to this force – prove it.) The tides are one good example of shallow water waves, as are also the waves made by a long boat in shallow water. For simplicity of calculation this question considers another sort of shallow water wave, a tidal bore in a river. Bores start in certain rivers because of the effect of the shape of the estuary on the tide. This question is about how the bore travels once it is started.

Figure 13 shows what a bore wave is like, if it travels up a channel in which the water ahead of the wave is still. (The argument still works if the water flows ahead of the wave, but is more involved.)

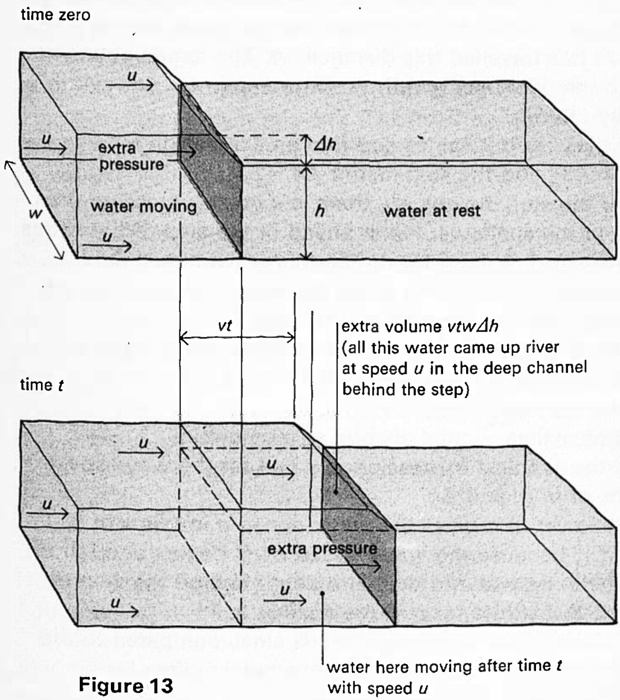


Figure 13

A bore wave has a step in the water surface, which travels at the wave speed v. Behind the step, the water flows in the wave direction at a speed u, smaller than v. Ahead it is still.
a In time t the wave travels a distance vt, and all the water within this distance now moves at speed u. If the channel width is w and the depth is h, what mass of water is set moving in time t? Call the density ρ. (The depth is h, not $h+\Delta h$, because the water in the extra height Δh has come up from behind and was already moving.)
b What momentum is given to newly moving water in time t?
c The moving water *must* be deeper than the still water; that is, there *must* be a step in the water surface at the boundary between moving water and still water. This is because the volume $utw(h+\Delta h)$ of water that flows up-river in time t must go somewhere, water being pretty well incompressible. So the newly arrived water makes part of the river deeper. If a length vt is made deeper by an amount Δh, the wave has travelled this distance, vt. The extra volume of water in the river over length vt, extra depth Δh, and width w, is simply $vtw\Delta h$.
Equate these two volumes and find an expression for u in terms of v, h, and the step height Δh.
d Under the step, height Δh, there is a greater pressure than there is in the shallower water ahead of the step. What is this extra pressure? Write g for the Earth's gravitational field.
e What extra force pushes water forwards over the area wh, separating moving water from still water?
f The force in **e** is what gives the newly moving water its momentum. Use the answers to **b** and to **e** to write an equation expressing the rule

$$\text{force} \times \text{time} = \text{total change of momentum}.$$

Obtain the simplest expression you can for the wave speed v in terms of u, g, and Δh.
g The answer to **c** gives the water speed u in terms of h, Δh, and v, because the water speed must be just enough to bring the extra water in the extra depth behind the step up the river. Substitute for u in the answer to **f**.
h Show that if the step height Δh is small compared to the depth h, the wave speed v is approximately given by

$$v = \sqrt{gh}.$$

Part Three

Mechanical oscillations

Questions 29 to 32

These are about clocks and the measurement of time.

29 Suppose that an inventor brings to a standards laboratory a clock that he claims will keep very accurate time. His clock punches dots onto a roll of moving paper tape, and he claims:

1 That it does so at very regular intervals.

2 That these intervals are accurately $\frac{1}{5}$ second.

The laboratory has a standard clock of its own which produces an audible 'pip' once every half second.

a How would the laboratory set about testing the two claims made by the inventor?

b Is it possible that claim **1** can be true, but claim **2** not true? Is the reverse possible?

c The laboratory finds that claim **1** is not true; it judges that the time-dots come irregularly. The inventor replies that it is the laboratory clock that is irregular, not his. Can the conflict of opinion be resolved?

30 *For discussion*

Two identical, very regular clocks are built, and compared. They are found to tick at the same rate to a high accuracy. One is stopped and taken to America and restarted, while the other remains in Britain. Outline a method of arranging that the clock in America should be set to read the same time as the one in Britain, to an accuracy of, say, $\frac{1}{1000}$ second. Both clocks can be made to emit short radio pulses at each 'tick', but the radio messages take about $\frac{1}{100}$ second to cross the Atlantic.

31 *For discussion*

Here are some (partly) philosophical puzzles about time. You may like to think about them by yourself or discuss them.

a Suppose *all* the clocks you could see, and *all* the processes that mark out time, like radioactive decay or the growth and death of plant and animal cells, *all* suddenly halved their rate.

Would one know it had happened?

b What would it be like if 'time' went backwards?

c Some theories say that the Universe – that is, everything that exists – had a beginning in time. Can one sensibly ask *when* it began? Can one sensibly ask questions about what happened *before* that?

See 'Suggestions for further reading', page 109, if these or similar problems interest you.

32 In 1761, the Board of Longitude, which had been set up to consider claims for a Government prize of £20 000 for a clock which could keep accurate time at sea, arranged a test of a clock made by John Harrison. This clock was more accurate and reliable than any other mechanical clock that had ever been made (except pendulum clocks, which will not work well on a rolling ship). Harrison and his clock were sent on a sea voyage to Jamaica, to test the clock. How could a test possibly be made, if there were no better clocks to compare it with?

33 *For discussion*

This question is about the practical uses of knowledge of vibrations.

An engineer concerned with the design of:

- cars
- high fidelity sound reproduction
- suspension bridges
- a factory using heavy machinery
- engines for ships
- engines for aircraft
- aircraft wings

would need to know about oscillations. He would need to know how they are caused; what decides their frequency; what influences the amplitude of oscillation: and other such matters, including how to damp out oscillations as well.

Explain for one such field why such knowledge would be needed.

Questions 34 to 42

These are about the motion of oscillators, graphs which may represent the motion, and reasons why some oscillators have a time of oscillation which does not depend on the size (amplitude) of the oscillation but does depend on the mass which is oscillating and on the forces which act upon it.

34 a A pendulum gives a 'time-trace' – the graph of displacement against time – which is a wavy, to-and-fro line, as shown in figure 14. Why can one be pretty sure that the lefthand side of figure 14 is earlier in time than the righthand side?

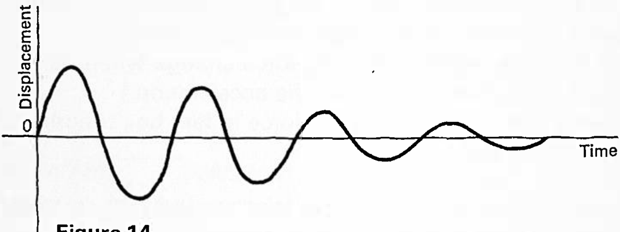


Figure 14

b What is the 'time trace' of a tennis ball's sideways motion (i.e. motion at right angles to the net) *in a rapid set of* volleys?
A sketch will do.
c Now try the 'time trace' of a yo-yo.

35 Comment on the following statement:
'*A sand pendulum can't be used to make* a record of harmonic motions, because, as sand comes out, the *pendulum's mass* changes and so will the period of its oscillations. Also, since you can't eliminate friction entirely its swings will get slowly smaller, which again will change its period of oscillation.'

36 This question is about reasons why some oscillators *have a time of oscillation that does not depend on amplitude.* Question 37 is an alternative version.

Consider a mass on the end of a vertical spring. If the mass is pulled down and then released it will oscillate. Observation of the oscillation shows that the frequency of oscillation is independent of the amplitude of the oscillation.

a *If the time for one quarter of an oscillation* (from an extreme displacement to the rest position) is T, what would be the corresponding time if the amplitude were doubled?

b How is the average speed of the mass changed by doubling the amplitude?

c What is the speed of the mass when it is at the extreme displacement?

d Your answers to **a**, **b**, and **c**, should enable you to tell how the average acceleration for the double amplitude oscillation compares with that for the single amplitude oscillation.

e How does the average restoring force change when the amplitude is doubled? (Consider the acceleration.)

f For what kind of spring will the force in fact be proportional *to the displacement?*

37 Figure 15 illustrates a simple kind of oscillator, for which the restoring force is proportional to the displacement.

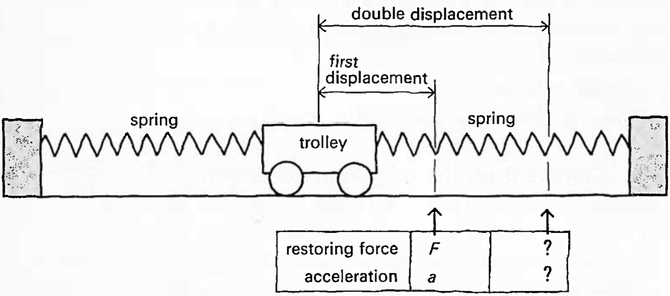


Figure 15

Suppose the trolley is given a small displacement and released. It oscillates, taking a certain time to go from the extreme position to the centre of the motion (one quarter of an oscillation).

Now suppose it is given twice that initial displacement.
a How has the average restoring force on it been changed?
b How must its average acceleration have changed?
c In the same time, how will the speed acquired compare with the first trial?
d How long will the trolley take to cover the double distance to the centre of the motion, by comparison with the first trial?

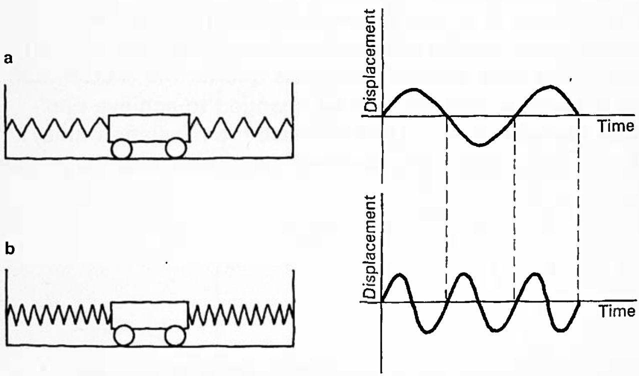


Figure 16

38 Figure 16*a* shows a trolley tied by two springs, and its displacement-time graph. Figure 16*b* shows the same trolley, with different springs. It oscillates twice as rapidly as before.

a Are the springs in figure 16*b* stiffer or weaker than those in figure 16*a*?
b In figure 16*b* the oscillation time is halved. For comparable positions, how does the speed of this second oscillator compare with that of the first oscillator?
c The speed of the second oscillator must be attained in half the time taken by the first oscillator. How do the accelerations of the two oscillators compare?

d $F = ma$, $F = ks$ where F is the force, m the mass, a the acceleration, k the spring constant, and s the displacement.
If the masses of the two trolleys are the same, how do the spring stiffnesses k compare?
e If T is the oscillation time, which relationship below agrees with the answer to **d**?

$$T \propto k^2;\ T^2 \propto k;\ T \propto \frac{1}{k^2};\ T^2 \propto \frac{1}{k}.$$

f To *decrease* the oscillation time, by changing the mass of the trolley, would one *increase* or *decrease* the mass?
g The answers to **a** to **c** above show that halving the oscillation time means quadrupling the acceleration, or that doubling the time means having one quarter the acceleration. How would the mass have to be changed to achieve one quarter the acceleration (and so twice the oscillation time)?
h Which of the following relationships agrees with the answer to **g**?

$$T^2 \propto m;\ T \propto m^2;\ T^2 \propto \frac{1}{m};\ T \propto \frac{1}{m^2}.$$

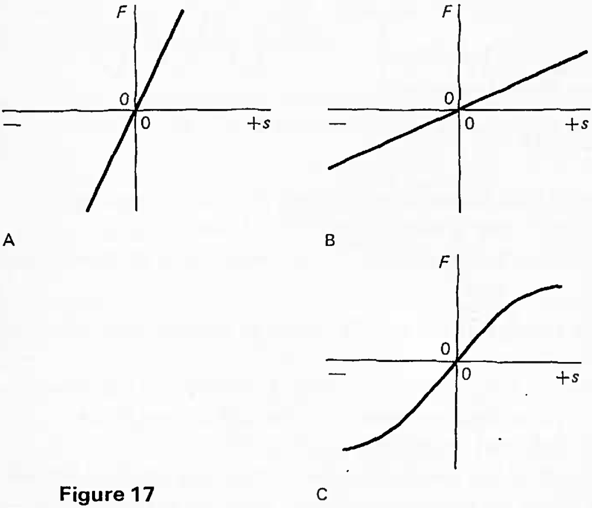
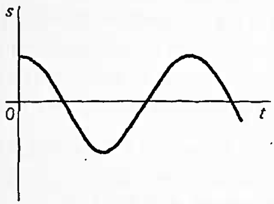


Figure 17

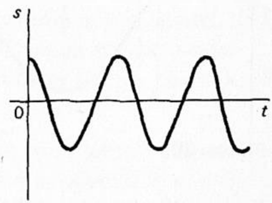
39 The graphs in figure 17 indicate how the force (F) necessary to displace a mass varies with displacement (s)

from the rest position for different cases. Each mass oscillates when it is released.

The second set of graphs (figure 18) show the displacement-time traces for the above cases. Which trace corresponds with which force-displacement graph?



D



E

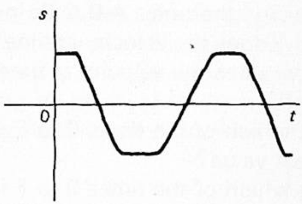


Figure 18

F

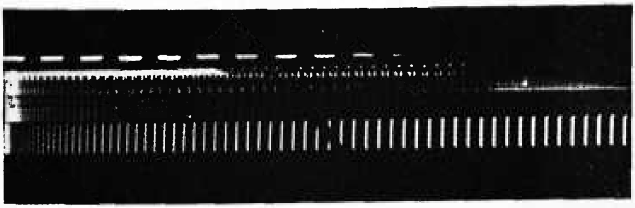


Figure 19

40 'Figure 19 is a stroboscopic photograph of the motion of a harmonic oscillator, released from rest at the extreme right. It is, however, detached from its restoring force when it reaches the mid position and as the restoring force is then zero, it goes on travelling at constant velocity.'

Check, by drawing a graph, whether the above description is plausible.

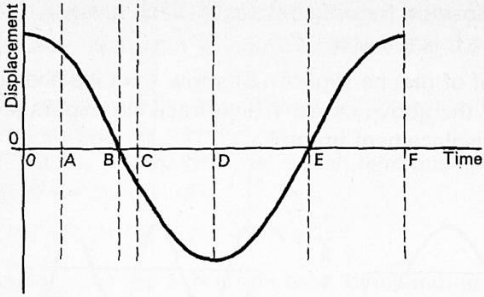


Figure 20

41 Figure 20 shows the displacement-time graph of an oscillator.

a Arrange the times A,B,C,D, in order of *decreasing speed.*

b How does the velocity at time B compare with that at time E?

c How does the velocity at time D compare with that at time F?

d At which of the times 0 to F is the acceleration at its largest value?

e At which of the times 0 to F is the displacement equal in size to the *amplitude of the motion*?

f Consider the time intervals 0B, 0D, 0F, BE, DF. If the periodic time of the oscillator is *T*, write down each interval in terms of *T*. ('0F = 3T' is the sort of answer expected, though this particular answer would be wrong.)

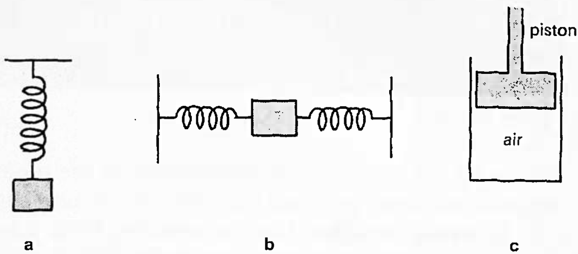


Figure 21

42 Figure 21 shows three things which would oscillate in a laboratory on Earth. Which, if any, would oscillate in a spacecraft going at steady speed a long way from the Earth and from any planet or star?

Questions 43 to 45

These questions take you through three approximate calculations of the distance moved by an object whose acceleration is given by some particular recipe. Question 44 is the most important one.

Question 43 is meant to show that the method suggested, called numerical integration, can be applied to cases which are rather hard to solve by means of writing down equations and solving them algebraically. Motion under constant acceleration can be tackled both ways, and maybe algebra is the easier. But question 43 concerns a rocket acted on by gravity and its own engines, which also loses mass because of the fuel it ejects. This is harder to do by algebra. Try it to get practice with the method of calculation, but don't worry if it seems too hard.

43 Imagine a rocket of mass 10 000 kg on the launching pad. After firing, it discharges 50 kg of gas every second at a velocity (relative to the rocket) of 2000 m s^{-1}.
a After t seconds what is the mass of the rocket in terms of t and the data above?
b How much momentum is delivered to the gas in a second?
c This rate of increase of momentum is equal to the force of the jet on the rocket. But the Earth pulls the rocket back with a force in newtons equal to approximately 10 times its mass. Write an expression for the net accelerating force in newtons on the rocket at time t.
d Write an expression for the acceleration of the rocket at time t.
e What is the acceleration at $t = 0$?

f See figure 22. Near $t = 0$, the rocket has not moved, and the velocity and acceleration are zero. Thus a flat section (AB) of the graph has been drawn in between $t = 0$ and $t = 5$ seconds.
What is the acceleration at $t = 5$ seconds? (Use a slide rule.)
g If the velocity remained zero, the graph would run on to C, at $t = 15$ seconds. But there is an acceleration, a, which in time Δt will carry the rocket a distance $a\Delta t^2$ further. Δt is 10 seconds. Calculate $a\Delta t^2$ for the time $t = 5$ seconds.
h In figure 22, the graph has been drawn from B to D, where CD is equal to $a\Delta t^2$ around $t = 5$ seconds.
What is the acceleration around $t = 15$ seconds?

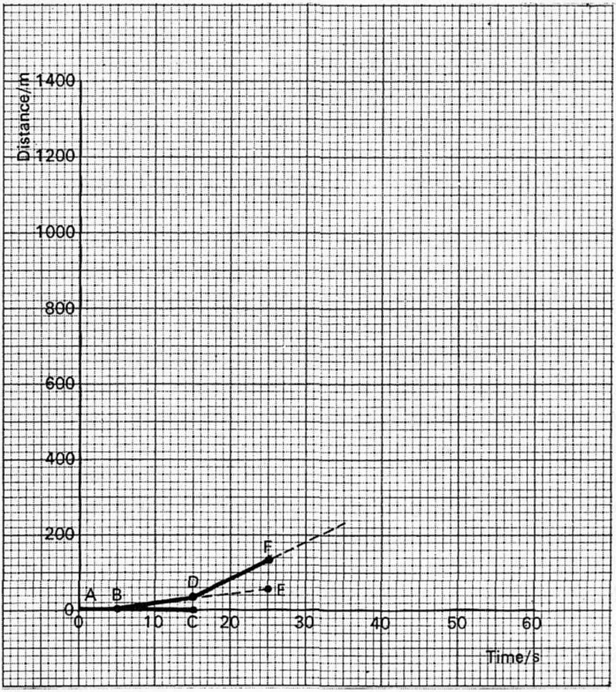


Figure 22
Distance travelled by a rocket.

i If it were not for this acceleration, the rocket would travel at the same speed, with the graph going in a straight line from B to D to E. But because it accelerates, the rocket will actually reach a distance F, where EF is $a\Delta t^2$, using the value of a at $t = 15$ seconds. Δt is 10 seconds. What is EF in metres?
j Continue the graph. Find the distance gone after 50 seconds.

44 This question is about the approximate graphical solution of the harmonic oscillator equation:

$$a = -\frac{k}{m}s$$

a1 What does m represent?
2 What does k represent?
3 What does s represent?
4 What does a represent?
5 What does the minus sign tell you?
b If $k = 20$ N m^{-1} and $m = 2$ kg,
1 What are the units of k/m?
2 When $s = 1.0$ metre, what is a?
c Suppose the *amplitude* of the oscillation is 0.10 metre and that a clock is started at the instant when the mass is at its extreme position. Then:
1 When $s = 0.10$ metre, what is t?
2 When $s = 0.10$ metre, what is a?
3 When $s = 0.10$ metre, what is the velocity v?
4 Would it be correct to say that the velocity after 0.5 second would be given by the acceleration multiplied by the time, that is, $v = 1.0 \times 0.5$ m s^{-1}?
5 Why?
6 Would the statement that the velocity after 10^{-6} second is given by $v = 1.0 \times 10^{-6}$ m s^{-1} be more nearly or less nearly true than the statement in **4**?
7 Why?
d It would be tedious to work out the velocity after each interval of 10^{-6} second, so assume instead of this that the acceleration will not change significantly during 0.1 second.

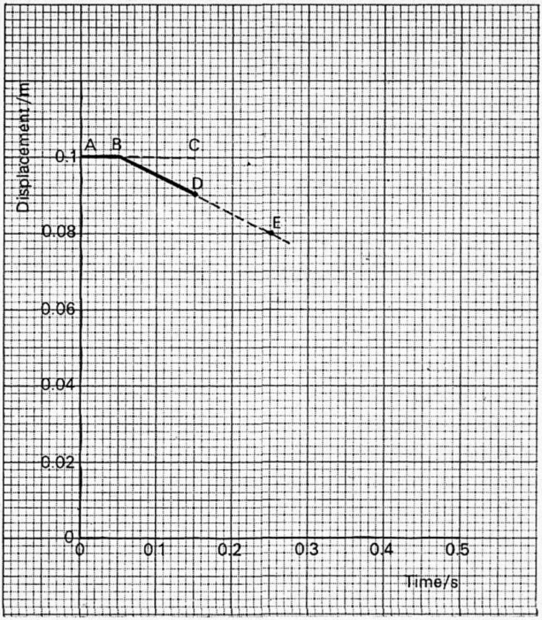


Figure 23

The graph, figure 23, has the first part (AB) of the displacement-time curve drawn in, for $s = 0.1$ metre and a time interval of 0.1 second around $t = 0$.

1 Why is the segment AB horizontal?

2 To what point on the graph would AB go if there were *no* acceleration?

3 There *is* an acceleration, of -1.0 m s^{-2} at this displacement (see **c 2**). Why is BD more likely to be the next bit of the graph than BC?

4 At an acceleration of -1.0 m s^{-2}, by how much does the velocity change in 0.1 second?

5 If there is a velocity v, in extra time Δt, a body must travel extra distance $v\Delta t$. If the velocity rises by Δv, it will in addition go an 'extra-extra' distance $\Delta v \Delta t$. Calculate the 'extra-extra' distance for the velocity change in **4** during the interval $\Delta t = 0.1$ second.

6 As $\Delta v = a\Delta t$, what must a be multiplied by to find $\Delta v \Delta t$?
7 CD on the graph is equal to 0.01 metre, being the answer to **5**. Check that CD $= a\Delta t^2 = 0.01$ metre.
e1 What is the displacement at D?
2 What is the acceleration at D?
3 What is $a\Delta t^2$ now?
4 If the acceleration were *zero* now, to what point would the graph go in the next 0.1 second?
5 Mark off a new 'extra-extra' distance below E, equal to the *present value of* $a\Delta t^2$ *calculated in* **3**. *Draw the graph on to* this point, say F. (F is not marked on figure 23.)
f1 What is the displacement at F?
2 Remembering that $a = -10s$, write an expression for $a\Delta t^2$, involving only s as an unknown quantity ($\Delta t = 0.1$ second).
3 Continue the graph, drawing each section of graph straight on, like BD drawn on to E, to find how far the oscillator would go if there were zero acceleration. Then allow for the 'extra-extra' distance $a\Delta t^2$.
4 At what time does the graph cross the axis?
5 After crossing the axis, the graph will curve *upwards*, 'extra-extra' distances being added *upwards* from each projected no-acceleration position. Why?
6 If you went on and on, your graph might look like figure 24.

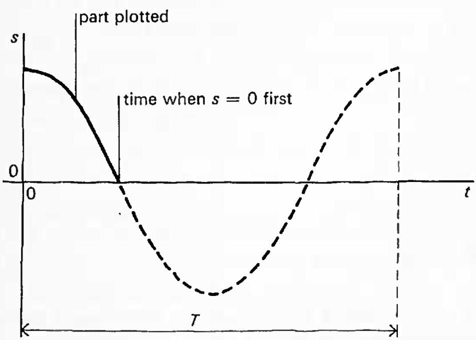


Figure 24

What fraction of the periodic time T is the time taken to first cross the axis from the start?
7 What periodic time T does your graph predict?

45 Question 44 produced an approximate displacement-time curve for a harmonic oscillator. The curve has a known mathematical form. It is a cosine function. This question is one way of showing that the curve has this form.

Question 44 considered the acceleration recipe $a = -(k/m)\ s$, with k/m given the value 10 N m^{-1} kg^{-1}. In this question, we take k/m to have the simpler value, 1.0 N m^{-1} kg^{-1}.

Then it is the case that s is given by the cosine equation:

$$s = A \cos t$$

where A is the amplitude and t the time, s being the displacement as before.

So if you produce a step by step solution for the acceleration recipe, with k/m equal to one, you should obtain, approximately, a cosine curve which can be compared with values taken from tables. You will also calculate π, as a bonus. The steps are given below. The method is the same as that used in question 44.

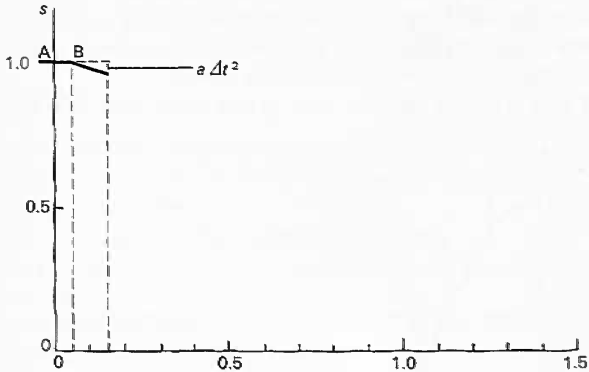


Figure 25

a Start with $s = 1.0$, and a graph whose time scale extends at least to 1.5 seconds. See figure 25.
b Start with zero velocity, as for the oscillator in question 44. That is, start with a horizontal section of graph like AB, spanning either 0.1 second or 0.2 second around $t = 0$.

(0.1 second intervals make for easy arithmetic; 0.2 second intervals will shorten the graph drawing process but will reduce the accuracy.)

c Previous graphical solutions have developed the rule shown in figure 26.

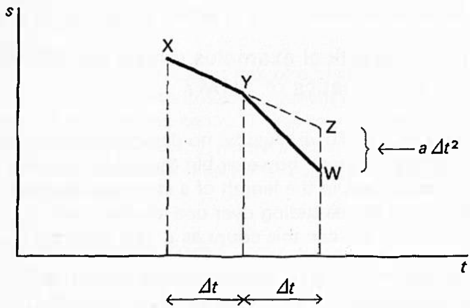


Figure 26

If the last section of graph drawn was XY, to obtain the next, first draw XY on straight to Z, time Δt further on.
Z is the point that would be reached at constant velocity, *no* acceleration.
Read off the displacement s at Y. Calculate the acceleration a (equal to $-s$ in this case), and mark in point W, an 'extra-extra' distance ZW $= a\Delta t^2$ below Z.
If $a = -s$, and $\Delta t = 0.1$, then ZW will always be 0.01 times the displacement at Y. (If $\Delta t = 0.2$, then ZW will be 0.04 times the displacement at Y.)

d Draw the graph, step by step, until it cuts the axis. If it *is* a cosine curve, it will cut the axis where $t = \pi/2$. What value of π do you get?

e If the graph is a cosine curve, the displacements at $t = 0$, 0.1, 0.2, 0.3 etc. second should equal cos t at $t = 0.0$, 0.1, 0.2, 0.3 radian. Check with tables.

46 *For discussion*

This question is about mathematical models. It is unusual in that answers are also given after each part of the question: you have to say what you think of the answers, each of which is one person's opinion.

a Could the idea of simple harmonic motion (SHM), and the mathematics that goes with it, be invented with pencil and paper without there being any actual examples of it?

Answer

Yes, for one might imagine a body tied to a spring which had a constant stiffness, and no damping, even if one had never seen a spring like that. But perhaps this is not a very likely state of affairs.

b Are there any real practical examples which are *accurately* described by the mathematics of SHM?

Answer

It is not easy to think of any. There must be no damping and the spring stiffness must be exactly constant, however big the extension. One might think of to and fro oscillations in the length of a steel rod, so small that there is no damping from atoms sliding over one another, and out in space away from air friction etc. But can this count as a 'real' example?

c Are there practical examples of oscillators which are *approximately* described by the mathematics of SHM?

Answer

Yes, plenty. The oscillations of a pendulum or of the atoms in a solid are two. Give some more.

d Does one study SHM because it is an elegant piece of mathematics, linking an imagined motion with sines, cosines, the geometry of circles, and the algebra of rates of change, or because it is widely, though approximately, applicable?

Answer

It probably depends on who you are and what you like to do. But perhaps most people would give the second reason.

e Is the mathematical model of SHM true, or false, or neither?

Answer

Neither. It is a description of the consequences of supposing that a certain motion occurs. But the statement that the two iodine atoms in the molecule I_2 oscillate in exactly this manner is either true or false. (Actually, false; the stiffness of the chemical bond changes as the separation alters.)

f It is possible to build up a mathematical model which might be better for the iodine molecule. The bond stiffness could be assumed to decrease in proportion to the displacement, for instance. This would not be SHM. Do you think this model would be a good description of things other than I_2?
Do you think that it would be as widely applicable as is SHM (approximately)?

Answer

The new model could probably cope with other molecules such as N_2, O_2. But it might be limited to that kind of problem, and be no better or be worse than SHM at describing the oscillations of car wheels or bridges.

g Is SHM so widely useful *because* it is too simple to be accurate in all cases?

Answer

I think so. The model leaves out lots of complications and only makes a few requirements – constant mass, constant stiffness, no damping, and force directed towards the centre. These few features appear in many cases *because* they are few. (There are many people with blue eyes, but not so many who have blue eyes, are over six feet tall, and have red hair and flat feet.)

Questions 47 to 57

These questions have to do with various practical problems involving vibration and resonance. You will need to use $T = \frac{2\pi}{\sqrt{k/m}}$ or $2\pi f = \sqrt{k/m}$ and to know that the energy of a harmonic oscillator is equal to $\frac{1}{2}kA^2$.

47 Estimate the spring constant of the suspension of a car. (Imagine a man sitting over one wheel; how much might the suspension deflect?) What frequency of oscillation might a wheel have, considered as a mass on the end of this spring? What sort of repeated ruts on a road would give trouble at, say, 50 kilometres an hour? Why does the car body move with smaller amplitude than the wheels?

48 Some high fidelity sound systems have the loudspeaker mounted in a 'bass reflex cabinet'. The cabinet in figure 27 is sealed except for a port, and the air in the port (shaded) behaves like a mass acted on by the springiness of the air in the cabinet. Suppose you made such a cabinet and found that it 'boomed' whenever notes of about 300 Hz were reproduced. How might it be improved? Can you find out any reason why the speaker is not just used on its own, without a cabinet?

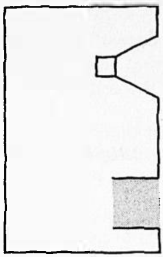


Figure 27

49 Diatomic molecules such as HF or HCl can vibrate by extension and compression of the bond between the atoms. They behave like a pair of masses held by a spring. For these two molecules, the H atom is much less massive than the F atom or the Cl atom to which it is bonded, and so, *roughly*, it will be good enough to imagine the H atom vibrating at the end of its bond. (Compare a small mass linked with a spring to a large mass – will the large mass move much?)

Now HF absorbs infra-red radiation very strongly at a wavelength of 2.4×10^{-6} m. The corresponding wavelength for HCl is 3.3×10^{-6} m.

Why can you say straight away that the HF bond is probably rather stiffer (larger force for the same extension) than the HCl bond?

Show that the bond stiffnesses are roughly:

1000 N m^{-1} for HF
500 N m^{-1} for HCl.

The mass of a hydrogen atom is nearly 1.7×10^{-27} kg. The velocity of light is 3×10^{8} m s^{-1}.

50 *Hard*

What have dyes to do with $T = \dfrac{2\pi}{\sqrt{k/m}}$?

51 Figure 28 shows that the amplitude of a strong vibration of a building, just sufficient to cause cracks in wall surfaces, *decreases* as the frequency *increases*? Why?

Hint: think about the accelerations.

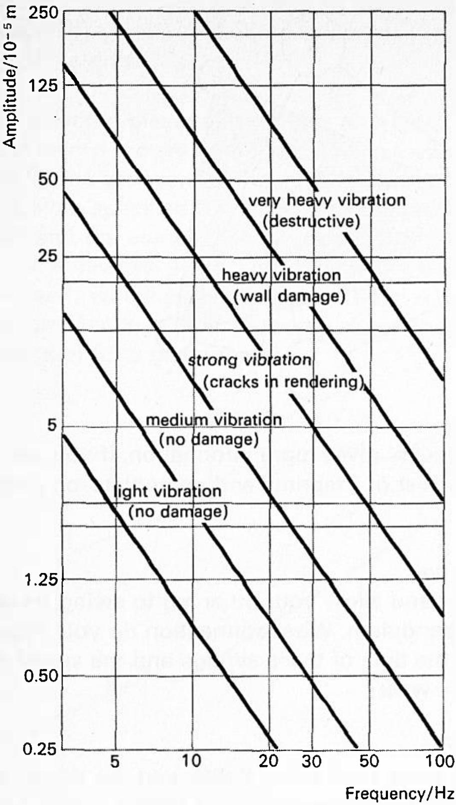


Figure 28

From the International Association for Earthquake Engineering; British National Section Symposium, ed. Skipp, B. O. (1966) Vibration in civil engineering, *Butterworth.*

52 The ammonia molecule NH_3 can vibrate with the nitrogen atom passing to and fro 'between' the three hydrogen atoms (figure 29). The frequency happens to be 23870 MHz. This vibration has been used as the basis of an 'atomic clock'. Would the rate of vibration be affected by using molecules of ND_3, with deuterium (heavy hydrogen ^2H) atoms in place of the hydrogen atoms? (A deuterium atom has a nucleus with one neutron and one proton.)

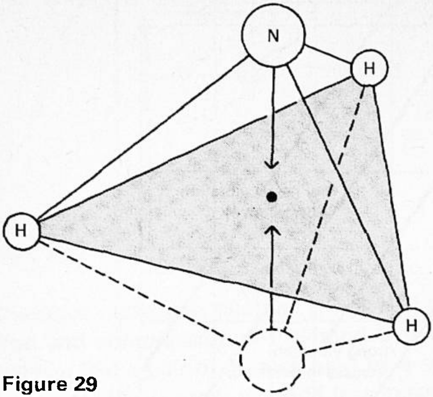


Figure 29

Lyons, 'Atomic clocks' gives more information, if you are interested. See the list of 'Reprints and pamphlets' on page 110 for details.

53 *For discussion*
Stand on one foot and allow your other leg to swing freely and easily like a pendulum. What connection do you think there is between the time of these swings and the speed at which you usually walk?

54 *For discussion*
A car accelerates away from traffic lights, and the driver notices that as the car accelerates past a certain speed, his view in the driving mirror goes 'blurred' and then becomes sharp again. Suggest a reason, and a practical way of reducing or eliminating the effect.

55 A violin, or a guitar, has its strings strung taut over a hollow wooden box which contains air and has holes in it (or a hole). Air in such a box can vibrate, the air near the hole acting as a mass driven in or out by the springiness of the air in the box.

What will be the effect on the sound produced when a string is sounded at a frequency near to that of the air in the body of the instrument?

What other parts of a violin or guitar can resonate to notes from the strings?

56 A record player pick-up has a stylus or needle that runs in the record groove and is oscillated sideways by the wavy walls of the groove. See figure 30. The performance of a pick-up is often specified by giving the *effective tip mass* of the stylus and the *compliance* or flexibility of the stylus when it is pushed sideways. The compliance is the reciprocal of the stiffness k, which would be measured in newtons per metre. The compliance is thus measured in metres per newton (deflection for a given force).

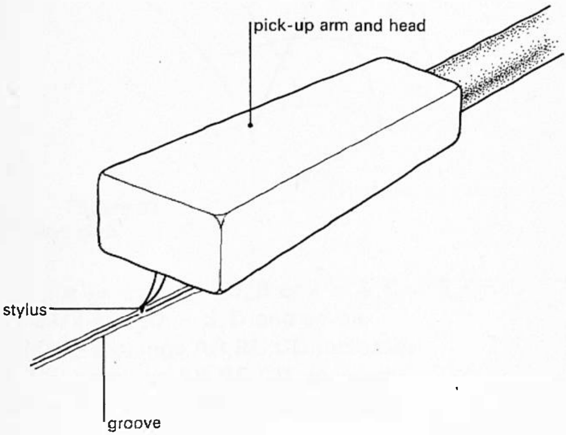


Figure 30

A suggested standard for high fidelity equipment (DIN April 1966) says that:

effective tip mass < 2 mg
compliance $> 4 \times 10^{-3}$ m N^{-1}

If the mass and compliances have these values, at what frequency will the stylus resonate? Would you regard this as satisfactory? You might go on to consider whether it would be desirable to have low or high values of tip mass and compliance.

57 *For private research*

Ultrasonic vibrations may be used to kill bacteria in liquids. How big are bacteria? Discuss the choice of a suitable ultrasonic frequency.

Answers

1 The difference in path between the waves from S_1 and S_2 at A is an odd number of half wavelengths. The observer must move so that the path difference stays constant.

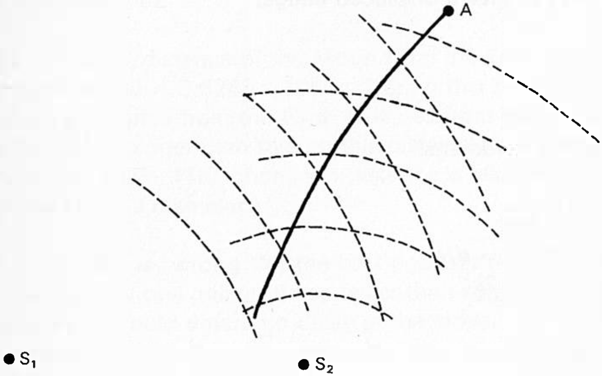


Figure 31
Typical path.

2 a $\frac{1}{2}\lambda = S_2B - S_1B$ or $\lambda = S_2C - S_1C$ or $3\lambda/2 = S_2D - S_1D$ and so on.
b1 The spacings AB,BC,CD increase.
2 The spacings AB,BC,CD decrease.
3 The spacings AB,BC,CD decrease.

3 $\lambda = (S_2M_{n+1} - S_1 M_{n+1}) - (S_2M_n - S_1 M_n)$

4 1 metre $= \lambda/2$ or $3\lambda/2$ or $5\lambda/2$ . . .

5 a 120 mm $= 3\ \lambda/2$.
b The amplitude of the beam via S_2 is less than that of the beam via S_1, the beam via S_2 having travelled further.

6 a 1.44×10^6 Hz.
b 0.53 m.
c 6×10^{14} Hz.
d 3×10^{18} Hz.

7 2.1 m. A clarinet is around half a metre long, suggesting that its length is about a quarter wavelength for its lowest note.

8 a Because the light takes a finite time to travel from O to the fixed mirror and back again. During this time the mirror at O has rotated sufficiently (to the position shown by a broken line) to give a displaced image.

b $2r$.

c $\frac{1}{n}$.

d $\frac{\theta}{2\pi n}$. if θ is in radians.

e $c = (2r) / \frac{\theta}{2\pi n}$.

f $POQ = 2\theta;\ 2\theta = d/s$.

g $c = \frac{2r \times 2\pi n \times 2s}{d}$.

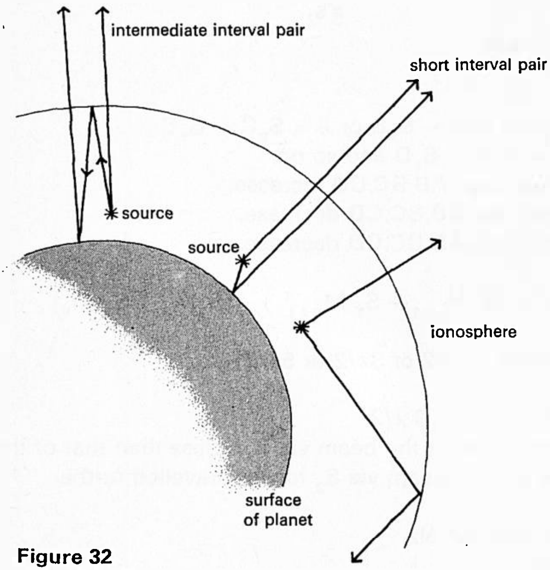


Figure 32

9 a See figure 32.
b 3.75×10^6 m.
c 3.75×10^7 m.

10 a A signal reflected from the aircraft that is received with a short delay after the direct signal.
b About 400 m.

11 Nearest distance of the Moon from the Earth during this 6-hour period $= 3.6243 \times 10^8$ m. During this time the Moon's distance from the Earth changes from being about 2.4×10^5 m bigger than this, to this distance, and then increases again. (The change in distance is less than a tenth of the Moon's diameter.)

12 Galileo is 'wrong' for the best possible reason. Light travels nearly one million times faster than sound, so it is no wonder he could detect no delay in the arrival of the light. He must be simplifying the story, for how would he know when the flash started out? Early experimenters tried flashing lamps at each other from distant mountain tops, one observer flashing his lamp when he saw the flash from the other, who could then see if this second flash came back to him much after the first went out. No delay longer than the human reaction time was observed, for light covers 3 km (a likely distance) in 10 microseconds. The important thing was to have thought of asking the question.

13 You shouldn't really need help with this. It involves putting down reasonable values for the distance of an audience from a concert platform (10 m?) and a likely distance over which the radio waves travel (100 km?). Radio waves travel at 3×10^8 m s^{-1}, sound waves at about 340 m s^{-1}.

14 Lord Rayleigh is right in an academic sense. Think of a train of oscillations at 1000 Hz lasting 1 second travelling along at the speed of sound. The observer catches up the train at twice this speed relative to the air, so passes the sound pulse at the speed of sound relative to it. He comes to the back end first, and passes the first oscillation in just 1/1000 second, so he begins to hear a note at 1000 Hz. It takes him just 1 second to pass the wave train, too, so it lasts for the right time. But in practice, Lord Rayleigh is less correct. The observer would have a high pressure shock wave around him as he moved at supersonic speed into the air ahead, and the chance of hearing anything would be negligible. His ears would not survive such treatment in any case.

15 There are many answers. Wavelength measurements usually involve using the fact that waves superpose to give a large or small resultant effect, depending on whether the path difference is an even or an odd number of half wavelengths.

Two-source experiments are possible in practice with waves of reasonable wavelength, for the sources need not then be very close together. But the sources must be in step. For light, two-source experiments are still possible, but the two sources must be imitated by splitting the light from one narrow source.

Diffraction gratings can be used, and are especially suitable for visible light and for X-rays. The grating spacing must not be very much larger than one wavelength if the diffraction angle is to be reasonably large. For X-rays, the grating is tilted so that the X-rays graze its surface, and the spacing looks small to the X-rays.

In the microwave and v.h.f. region (wavelength from 0.01 m to 1 m roughly) simple arrangements of reflectors which introduce a path difference can be used. A problem here is to have big enough reflectors, for a reflector must be bigger than one wavelength in linear dimensions to reflect an appreciable amount of wave energy.

16 **b, a, d, c, g, f, e, h.**

17 A sensible answer need not, probably could not, be complete. It is better to concentrate on one or two aspects and explain them carefully, with examples.

Suppose, for instance, that our eyes could 'see' radio waves, but not of wavelength less than 0.1 m. Radio and television masts would be invisible, in a totally dark world, except for a bright patch where the radio waves were emitted. One could use a radio 'torch' to find one's way about, but one would only see objects larger than about 0.1 m. A needle would be invisible, just as objects smaller than the wavelength of visible light are invisible to us, because of diffraction. The Sun would be a faint object (it emits some radio energy) and physicists might be arguing that indirect evidence suggests that it emits a lot of energy in the 'invisible' region, about wavelength 10^{-7} m.

18 Again, as with other discussion questions, we cannot anticipate all your answers. As usual, it is best to pick on one or two things and discuss them fairly fully rather than to talk very vaguely and superficially about many things.
For instance, the radar detection of migrating birds must depend on using a wavelength smaller than the linear dimensions of a bird, or the energy reflected will be inappreciable. But the wavelength could be too small, for at millimetre wavelengths reflections from water drops in clouds will confuse the radar echoes. Knowledge of the speed of the waves is essential for determining the range of the birds, though not for fixing their direction. It is important in this instance that the radio waves should be undetectable to the birds: an otherwise satisfactory 'sound radar' would obviously scare them all away!

19 **a** There must be a greater force in the buffers behind this truck than in those ahead of it, for there must be an unbalanced force on it to accelerate it.
b The buffers on either end of it could well be compressed, but they must exert equal and opposite forces on the truck if it is not accelerating.
c The truck going at steady speed is not accelerating, so, if there is negligible friction, the forces in the buffers on either end of it must be equal.

d While the 'wave' of motion-starting-up is still travelling along the row of trucks, a truck somewhere is being accelerated by a larger force in the buffers behind it than exists in the buffers in front of it. The engine provides this force, pushing on the far end of a row of moving trucks, between all of which the buffers exert this extra large force. But each of these moving trucks, except only the one that is *accelerating, has an extra large force on its front and on its* back, so they do not accelerate.

Actually, when the wave reaches the end of the row, if there is little friction, the final truck has nothing to push on and moves forwards, and a 'wave' of release-of-buffer-compression *travels back up to the engine. The trucks then move on at* steady speed, with no extra buffer compression, and the engine can stop pushing altogether.

20 **a1** 2.5 N.
2 7.5 N, assuming the force-compression graph is linear.
b 5×10^{-2} J; 5 N.
c It accelerates away from the wall. As the spring relaxes the acceleration of the trolley decreases. When the spring is no longer pushing the trolley, then, in the absence of friction, the trolley will move with constant velocity unless the ends *of the spring are secured.*
d Change the mass of the trolley.
e 0.3 kg m s^{-1} (or 0.3 N s).
f A 1 kg trolley moves faster. A 2 kg trolley has more momentum (because the same force acted on it for a longer time). Both trolleys have the same kinetic energy (5×10^{-2} J).

21 **a** See figure 33*a*.
b See figure 33*b*.
c See figure 33*c*.

22 Figure 34 shows where the potential energy is greatest, at places of maximum compression or extension. At these places the coils of the spring have no kinetic energy. But

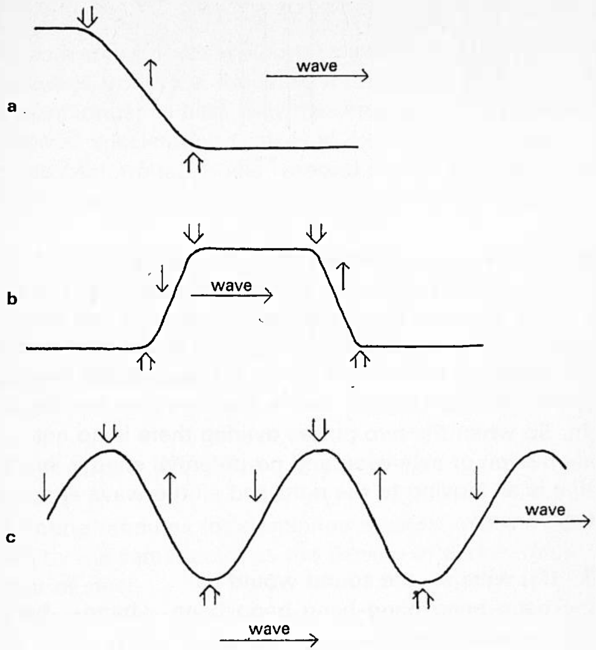


Figure 33

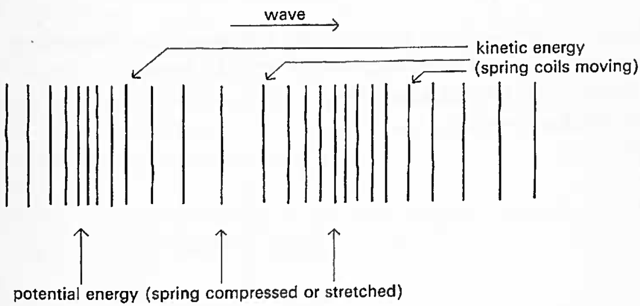


Figure 34

between compressions and extensions the spring coils must be moving to create the compressions and extensions in new places. So here the spring has kinetic energy.

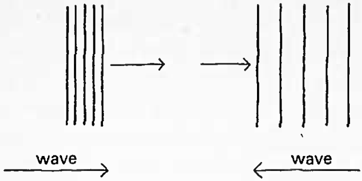


Figure 35

Figure 35 shows a compression pulse, going from left to right, about to meet an extension pulse, going from right to left. An arrow ahead of the compression indicates that the compression is moving the spring coils ahead of it to the right. Ahead of the extension, the spring coils are being pulled into the extended region, that is, they are also moving to the right. So when the two pulses overlap there is no net compression or extension and no potential energy, but the spring is all moving to the right and all the wave energy is kinetic energy.

23 If it were so, the sound would be:
bang-bang-bang-bang-bang-bang-bang---bang---bang---bang---bang. Why?
(The first bang is the sound of the engine hitting the first empty truck. The last truck has nothing to hit.)

24 **a** P is accelerating to the right, being pushed forwards harder from behind than backwards from in front.
b The same as to P at time zero.
c v is bigger than u.
d vt.
e vt/x.
f ut.
g ut.
h vt/x.
i ux/v.
j kux/v.
k vt/x.
l mu.
m $tkux/v = muvt/x$.

n $v = x\sqrt{k/m}$.
o The same spring force would take longer to accelerate the more massive trolleys at the wave front, so the wave front would take longer to pass over the same number of trolleys.
p The forces accelerating trolleys at the wave front would be bigger, so each trolley would respond sooner and the wave would pass over more trolleys in the same time.

25 a $\dfrac{2.5}{3.2 \times 10^{-13}}$ m s^{-1} since $\sqrt{10^{-25}} \approx 3.2 \times 10^{-13}$.
b $\dfrac{2.5}{3.2 \times 10^{-13}} \times 7 \times 10^{-10} = 5500$ m s^{-1}.
c Estimate, 5500 m s^{-1}.

26 a $m = \rho x^3$.
b $v = x\sqrt{k/m} = x\sqrt{Ex/\rho x^3} = \sqrt{E/\rho}$.
c 5100 m s^{-1}.
d The Young modulus for aluminium must be less than that for steel by the same factor as the density of aluminium is less than that of steel.

27 a To the left of P the taut string slopes upwards; to the right of P the string is horizontal. There is thus a net upward pull on P, though the sideways pulls more or less balance each other.
b Equal forces T both pull sideways on Q, and there is no resultant force on Q.
c For the reason given for P in answer **a**. The kink is now at Q, so the forces in the string on either side of it no longer produce zero resultant force on Q.
d ut.
e The force T to the left of the kink has an upward component approximately equal to $Tut/vt = Tu/v$.

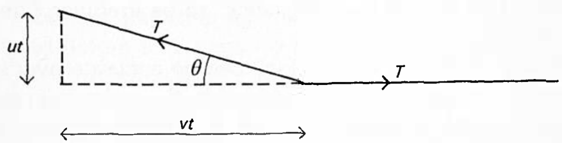


Figure 36

The vertical component is actually $T \sin \theta$, where θ is the kink angle shown in figure 36. But $\sin \theta \approx \tan \theta$ if θ is small, and $\tan \theta = ut/vt$. The force T to the right of the kink has no vertical component, so the resultant upward force is Tu/v.

(Notice that the sideways pull T on the right of the kink is just a little bigger than the sideways pull $T \cos \theta$ to the left of it, so the string is also pulled to the right. This must be so, if the string cannot stretch, for the same length of string occupies a smaller horizontal distance if it has a transverse wave on it. The end of the string must be allowed to move a little along the length of the string.)
f μvt.
g μvtu.
h $tTu/v = \mu vtu$; $v^2 = T/\mu$.

28 **a** $hwvt\rho$.
b $hwvt\rho u$.
c $u = v\Delta h/(h+\Delta h)$.
d $\rho g\Delta h$.
e $wh\rho g\Delta h$.
f $wh\rho g\Delta ht = hwvt\rho u$; $g\Delta h/u = v$.
g $v^2 = g\,(h+\Delta h)$.
h If Δh is small, $(h+\Delta h)$ is nearly the same as h.
If you are interested in water waves, try Tricker, *Bores, breakers, waves and wakes* or Barber, *Water waves*. (See page 109.)

29 **a1** To test for regularity, it would compare the number of dots punched out in fixed intervals of time as indicated by its own clock, and do so at several times of day.
2 To test for accuracy, it would need to count the number of dots produced over a long period as measured by its own clock, and see if there were, say, 5000 dots for every 2000 pips. There would be no point in doing so if the test for regularity had shown the 'dot-clock' to be irregular, compared with the 'pip-clock'.
b It is very unlikely that the clock can be accurate over long periods if it is not regular – that is, if it disagrees even with itself. But it can be regular, yet not punch dots at exactly $\frac{1}{5}$ second intervals.

c Only by *deciding* that some clock or other is going to be treated as a regular time marker. This standard time marker cannot be tested, though the choice would fall on one of a number of such clocks that show each other to be regular, and for which, if possible, there are theoretical reasons for thinking that the rate will not be affected by most kinds of laboratory disturbances. Thus, atomic clocks are preferred to clocks based on the oscillations of some lump of matter.

If the laboratory decides to trust its pip-clock, the issue will have been settled by decision, not by trial. The inventor could claim that the decision should have gone his way. Indeed, atomic clocks at present in different countries do show tiny disagreements, and there is no way to tell which one is 'right'.

30 This is the question with which Einstein begins his quantitative treatment of relativity in his paper of 1905.

In principle, one method works like this.

The British clock sends a pulse when it records time t_B.
The American clock receives the pulse when *it* reads t_A.
Arrange for the pulse to be sent straight back without delay.
Suppose the British clock reads $\frac{1}{50}$ second later than the time t_B, when the pulse comes back.

Then the time on the British clock at which the American clock received the pulse is $(t_B+\frac{1}{100})$ seconds. The Americans can be told by radio that this is what it should have been.

31 If this question has value, it is in the thinking, not in the answer. Here are some short answers.
a No.
b Try Butler and Messel, *Time* (see 'Suggestions for further reading', page 109 for details).
c Philosophers are rather inclined to think that to ask what happened before time began is to have asked a question, which, in terms of scientific inquiry, is one with no clear answer. But it may be all right to ask how long ago the Universe 'started', though it is worrying that this seems to require imagining a clock that has marked off time regularly

since the beginning of time, and it is not easy to see how one could choose the best clock for the purpose. The intense gravitational field of a highly condensed Universe could certainly upset all the oscillators one can think of for the job, even oscillating atomic nuclei.

32 The story of Harrison and his chronometers is told in a booklet obtainable from the National Maritime Museum. See the list of 'Reprints and pamphlets' on page 110 for details

The point of carrying a clock for navigation is to compare the time of noon (say) at the ship's unknown longitude with that, indicated by the clock, at some other definite longitude, which has gradually come to be that of Greenwich. If, for instance, noon is six hours different in time from that at Greenwich, the ship is one quarter of the way around the Earth from Greenwich. At the equator, a time difference of one minute corresponds to a distance in longitude of nearly thirty kilometres, so an accurate clock is needed.

The way to test the clock is to determine, by making astronomical observations, the longitude of the place reached (Jamaica), and to calculate the expected time of noon compared to that at Greenwich. The clock, if it is accurate, will still show Greenwich time, and its reading can be compared with the calculated time.

33 For ideas see Bishop, *Vibration*. See 'Suggestions for further reading', page 109 for details.

34 a In time, the motion of a pendulum will be damped out by frictional forces. The energy the pendulum had is then spread out among the many molecules of the air in which it was swinging, and the air is a little warmer than it was. This kind of change never happens the other way round.

But the motion shown could be reversed if the pendulum were being given a properly timed series of pushes and pulls by an outside force.

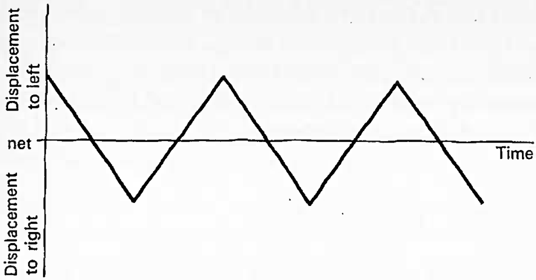


Figure 37

b See figure 37. The horizontal speed of the balls is steady, so the 'time trace' is a 'sawtooth' graph made up of sloping straight line sections. The faster the ball moves, the steeper the sloping lines. If the players are not equidistant from the net, the lines turn round at different distances from the net.
c A yo-yo accelerates downwards as it is released, but with an acceleration much less than the acceleration of gravity (it has to build up spin speed too). At the bottom, it is accelerated sharply upwards, so that it comes to rest and then starts to move upwards at considerable speed. On its upward travel, this speed diminishes, because there is a downward acceleration. The 'time trace' might look like that in figure 38.

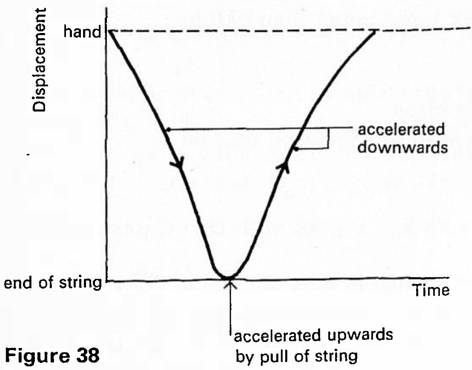


Figure 38

35 Wrong. The reduction in mass of the pendulum has no effect on its time of oscillation, except in so far as the leakage of sand changes the position of the centre of gravity. For small amplitude swings the time of oscillation does not depend on the amplitude either.

36 **a** T.
b It is doubled.
c Zero.
d Doubled: twice the speed to be acquired in the same time.
e It is doubled.
f A spring which obeys Hooke's Law. So if such a spring causes a mass to oscillate, the time of oscillation should not depend on the amplitude. See also Rogers, *Physics for the inquiring mind,* Chapter 10.

37 **a** The force is doubled.
b The acceleration is doubled.
c Twice the speed will have been acquired in any given time.
d The trolley goes twice as far, twice as fast, so it needs just the same time.

38 **a** The springs in figure 16*b* are stiffer.
b It is twice as great as before.
c That of the second is four times larger.
d They are four times larger than before.
e $T^2 \propto \frac{1}{k}$.
f Decrease the mass.
g It would have to be increased four times.
h $T^2 \propto m$.

39 A goes with E. B goes with D. C goes with F.

40 There are several graphs you could draw. A graph of distance against time should show that the speed is constant beyond a certain distance, but it will not be easy to decide if the earlier, curved part of the graph has the shape it should have if the motion is as described.

A speed-time graph may be better. It should have a slope (the acceleration) during the first part of the motion which is proportional to the distance from the start, but should then level off to a constant speed. Speeds can be measured, in arbitrary units, off the photograph by measuring the distances between pairs of images at equal times.

An energy graph can be instructive. The difference between the kinetic energy near the finish of the motion and that at some distance near the start when there is tension in the spring should be equal to the potential energy stored in the spring. A quantity proportional to the kinetic energy at a place can be obtained by squaring the measured speed at that place. The quantity obtained from these measurements, proportional to the potential energy, can be plotted against the square of the extension of the spring. Since the energy stored in a Hooke's Law spring is proportional to the square of its extension, this graph will be a straight line if the oscillator is harmonic. Actually a rubber band was used, and there may be detectable deviations from linearity.

41 a B C A D.
b Same size, opposite direction.
c Both zero.
d O,D, and F.
e O,D, and F.
f OB $= T/4$, OD $= T/2$, OF $= T$, BE $= T/2$, DF $= T/2$.

42 *b* will oscillate. Please don't confuse the 'weightlessness' of the mass with any lack of inertia. Its mass is the same as on the Earth, and it behaves as an oscillator with the same period (neglecting any relativistic effects). Now argue about the others.

43 a $10\,000-50t$ kg.
b 50×2000 kg m s^{-2}.
c $50\times2000-10\ (10\,000-50t) = 500t$ newtons.

d $\dfrac{500t}{10\,000-50t} = \dfrac{10t}{200-t}$.
e Zero.

f 0.26 m s^{-2}.
g 26 m.
h 0.81 m s^{-2}.
i 81 m.
j We make it about 1200 m.

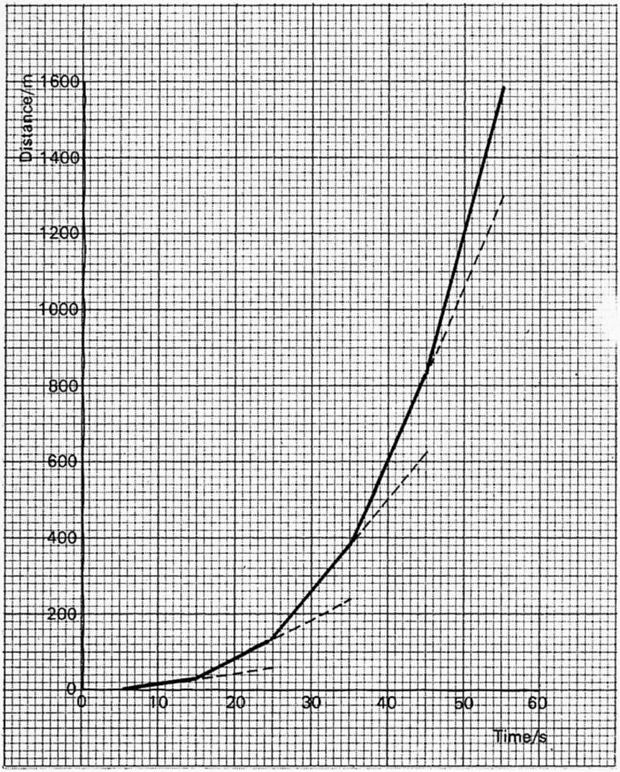


Figure 39
Distance travelled by a rocket.

44 **a1** The mass of the oscillating body.
2 The extra force pulling the body back when displaced by one metre, if that were possible.
3 The displacement from the equilibrium position.

4 The acceleration at the instant when the displacement is s.
5 When s is increasing the body is decelerating; when s is decreasing the body is accelerating. The acceleration acts towards the centre of oscillation.
b1 $\mathrm{N\ m^{-1}\ kg^{-1}}$, which is also s^{-2}.
2 $-10\ \mathrm{m\ s^{-2}}$.
c 1 $t = 0$.
2 $a = -1.0\ \mathrm{m\ s^{-2}}$.
3 $v = 0$.
4 No.
5 The acceleration is not uniform.
6 More nearly.
7 In a shorter time the acceleration changes less.
d1 The velocity is zero.
2 To C.
3 Because it slopes down, representing a velocity carrying the mass inwards.
4 $\Delta v = 0.1\ \mathrm{m\ s^{-1}}$.
5 0.01 m.
6 Multiply a by Δt^2.
7 $a\Delta t^2 = 1.0 \times (0.1)^2 = 0.01$ m.
e1 0.09 m.
2 $0.9\ \mathrm{m\ s^{-2}}$.
3 0.009 m.
4 To E.
5 $a\Delta t^2 = \mathrm{EF} = 0.009$ m.
f1 0.071 m.
2 $a\Delta t^2 = -10s \times (0.1)^2 = -0.1s$.
4 Nearly $t = 0.5$ s.
5 Because in SHM the mass accelerates towards the centre. (Acceleration is *minus* $10s$.)
6 $T/4$.
7 Nearly 2.0 s.

45 The graph should cut the axis at a point where t is a shade larger than the accurate value of $\pi/2$. The difference arises from the finite steps into which the time was divided, and can be reduced as much as one pleases by reducing the size of the steps, and having more of them.

The values of cos t should agree with tabulated values to within a few per cent. Figure 40 shows such a graph.

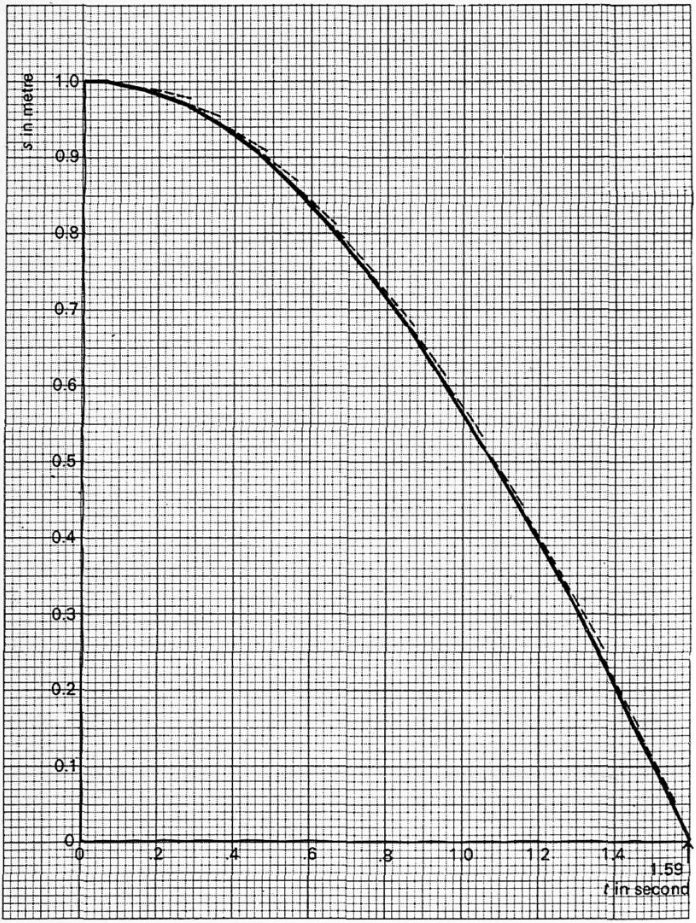


Figure 40

Solution of $\frac{d^2s}{dt^2} = -1.0$ s.

46 to 57 These questions are to help you to exercise your understanding of simple harmonic motion, the mathematics that describes it, and the uses to which it can be put. Most involve the important idea of resonance.

The Severn Bridge

A study of some of the aerodynamic problems associated with buildings and bridges.

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Aerodynamics Division, National Physical Laboratory

To build a bridge is a fine thing, for it links together communities and places which were separated by the lie of the land. Bridges carry people and commerce over valleys and ravines and across estuaries and rivers, and as they do so they become parts of the landscape. Most bridges are beautiful, but the suspension bridge in particular catches the eye as a slender and graceful thing. Such a bridge spans the Severn and the story of one aspect of the design of that bridge forms the main subject of this article.

Building a bridge, from the initial design study by the consulting engineers to the placing of the final bolt in its structure, is a matter imposing responsibility. Civil engineers accept that responsibility when they design and build bridges, dams, power stations, skyscrapers, and motorways. Bridges have failed, dams have burst, and high buildings have fallen, sometimes with loss of life. The men and women who build our cities and provide the routes between them take our lives into their hands as they do so, and take great care in design and construction. Yet, despite this care, buildings and bridges have failed. Usually this is a result of faulty design and construction, but sometimes a failure reveals hitherto unsuspected sources of danger and the lesson which has been painfully learned is applied to subsequent design. Every change in constructional materials and techniques may pose new problems that did not arise before, and the engineer must probe carefully into the possibilities his experience suggests.

One of the principal enemies of high buildings and of bridges is the wind. This is not only because it is likely to blow them down, but also because of more subtle effects in which the flow of air over and around the structure can induce vibrations in it. Such vibrations can be destructive, as we shall see, and the problem is magnified by the use of lightweight constructional materials. The Monument, in the City of London, which commemorates the Great Fire of 1666, was built under the direction of Sir Christopher Wren. It is a fluted column standing on a pedestal; its height is 61.6 m, and the diameter of the column is 4.6 m. These proportions more nearly approach those of modern slender structures than most buildings of its day, but here the resemblance ends. By modern standards, the Monument is very heavy indeed; except for the central spiral stairway it is solid stone with a total mass of about 6×10^6 kg. Nowadays it is not an economic proposition to build in stone, but relatively new methods of design and construction permit other materials with better load-bearing properties to be used. If the Monument were to be built today, it might be constructed in steel or reinforced concrete and its mass might be as low as 2×10^5 kg.

Figure 41 The Severn Bridge. *William Tribe*


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Wren's Monument has never suffered any wind damage, nor is it likely to. Both the mass and what is termed the structural damping of the Monument are more than enough to ensure safety in wind and, as will be explained later, both of these properties are important. However, one of the earliest major events which brought the problem of wind loading, as it is called, to the attention of civil engineers occurred in 1879 at a time when such heavy constructional materials as cast iron and wrought iron were common. On a stormy night a section of the railway bridge over the river Tay collapsed and fell into the river, taking with it a train full of passengers. This disaster, attended with heavy loss of life, was caused by the steady force imposed by the wind on the bridge. The Tay bridge did not oscillate, it simply collapsed under the pressure exerted upon it. This is described as a static effect of wind, the distortion of a structure under steady force. It can be seen in a less drastic form when a tree bends in a strong wind.

The wind can cause structures to oscillate. In 1939, the first Tacoma Narrows Bridge was built near Seattle in the U.S.A. It was then one of the longest suspension bridges in the world and was stiffened by two deep solid girders. In 1940 it oscillated to destruction in a wind speed of about 19 metres per second. For many months after its completion the bridge had oscillated at low wind speeds, with its road deck moving vertically up and down. This motion was not destructive and in fact was a source of pleasure to young motorists in the area who christened the bridge 'Galloping Gertie'! The destructive oscillation induced by the wind travelling at 19 metres per second caused, not a vertical displacement of the road deck, but a twisting motion, as shown in figure 43, which overstressed and eventually fractured the structural members and led to the complete collapse of the bridge. It is interesting to note that this sort of thing had happened earlier. In 1836, the Chain Pier at Brighton had partially collapsed in wind after exhibiting a twisting motion identical

Figure 42
The Monument.
Photograph, J. Allan Cash.

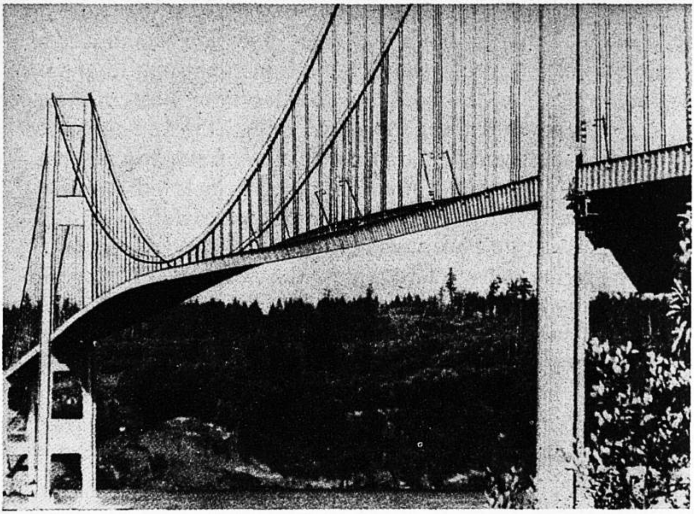


Figure 43
Wind-excited asymmetric torsional oscillations of the first Tacoma Narrows Bridge.
Photograph from Bulletin 116, *University of Washington Engineering Experiment Station.*

to that seen at Tacoma. The origin of the instability was not understood at the time and no lessons were learned that could have prevented the Tacoma collapse.

Tacoma marked a turning point in the history of civil engineering when it came to the design of suspension bridges. A thorough investigation into the mechanism of wind-induced oscillation began in the U.S.A. immediately after the Tacoma Narrows disaster, and in 1946 the National Physical Laboratory at Teddington in England carried out an investigation into the aerodynamic design of bridges. The primary object of the work at the National Physical Laboratory was to assist the consulting engineers responsible for the design of road suspension bridges to span the Firth of Forth and the River Severn. The designers of the bridges were Messrs Mott, Hay, and Anderson and Freeman Fox and Partners.

Before we proceed any further with the discussion of bridges, it is important to point out that many other types of structures are prone to wind damage which may cause their collapse or interfere with their function. Consequently, aerodynamic studies, at one time the preserve of those responsible for the design of aircraft or streamlined cars and trains, are now applied to buildings and in fact to whole groups of buildings as in town centres.

Enough has been said here to emphasize the importance of obtaining a thorough knowledge of the wind-loading and *aerodynamic characteristics of buildings during the design* stages. The techniques used are those which have been developed by years of research into the problems of flight.

The vibrations of buildings and bridges

A simple system which can vibrate consists of a mass m suspended by a spring. If we pull down the mass on the spring and let go, it will oscillate about its mean position. Figure 44 shows what a plot of the displacement of the mass against time looks like.

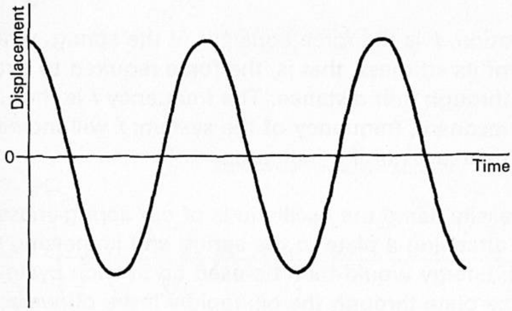


Figure 44

The mass-spring system is undergoing simple harmonic motion. The frequency of oscillation, f, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

A building or a bridge is more complicated than a mass on a spring, but we can draw some useful lessons from the simple system. First of all, the displacement versus time curve for the mass and spring shows deflections of equal amplitude all the time. This could only be so if we had hit upon perpetual motion. In practice the amplitude of the oscillation gradually decreases or decays, until the system comes to rest. The most important cause of the decay is structural damping. During an oscillation the energy of the system is changing from potential to kinetic and back again. In elementary theory the sum of the potential energy and kinetic energy is constant. However, in each cycle some energy is absorbed by the structure, and warms it up slightly. Let us go back to Wren's Monument; this has a large mass which keeps the resonant frequency low but, in addition, it has high structural damping, partly due to the properties of stone itself and partially to friction, or 'fretting', between the blocks as they move. The structural damping of buildings arises from other sources besides fretting. Some may come from movement of the foundations in the ground, for example. Unlike many properties of buildings the degree of structural damping cannot be calculated, and experiments are needed to determine it.

In this equation, k is the force constant of the spring, which is a measure of its stiffness, that is, the force required to extend the spring through unit distance. The frequency f is the natural, or resonant, frequency of the system; f will increase if either m decreases or if k increases.

We could easily damp the oscillations of our spring-mass system by attaching a plate to the spring and immersing the plate in oil. Energy would then be used up in each cycle in dragging the plate through the oil, rapidly if the oil were thick, giving heavy damping, slowly if light oil were used. The shock absorbers on a car work on this principle to damp out the oscillations of the car springs.

It is sometimes possible to obtain damping values for full-scale structures by direct measurement, though a great deal of energy is required to set them oscillating with enough amplitude for this purpose. An instrument to record the oscillatory motion is installed at the top of the structure. This is then pulled to one side of its equilibrium position at the top and released, and the decay of its motion is recorded. This method has been used on fairly light structures such as television aerial towers, which can be deflected with a rope and a winch. When the tension in the rope reaches a predetermined value, a mechanical fuse in the rope breaks and the tower oscillates. An alternative method of producing oscillations has been used on heavier structures. A number of rockets are fired sequentially at regular intervals equal to the period of oscillation of the structure so that it resonates and large amplitudes can be attained. The dampings of the Crystal Palace television tower and of a 130 m high concrete chimney at Ferrybridge power station were measured in this way.

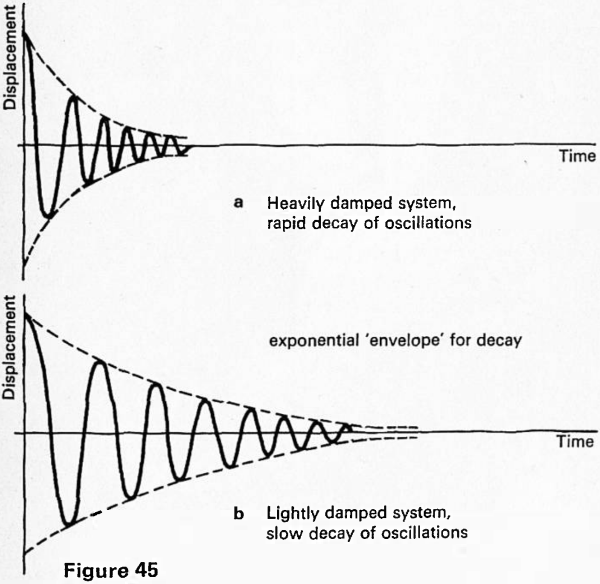
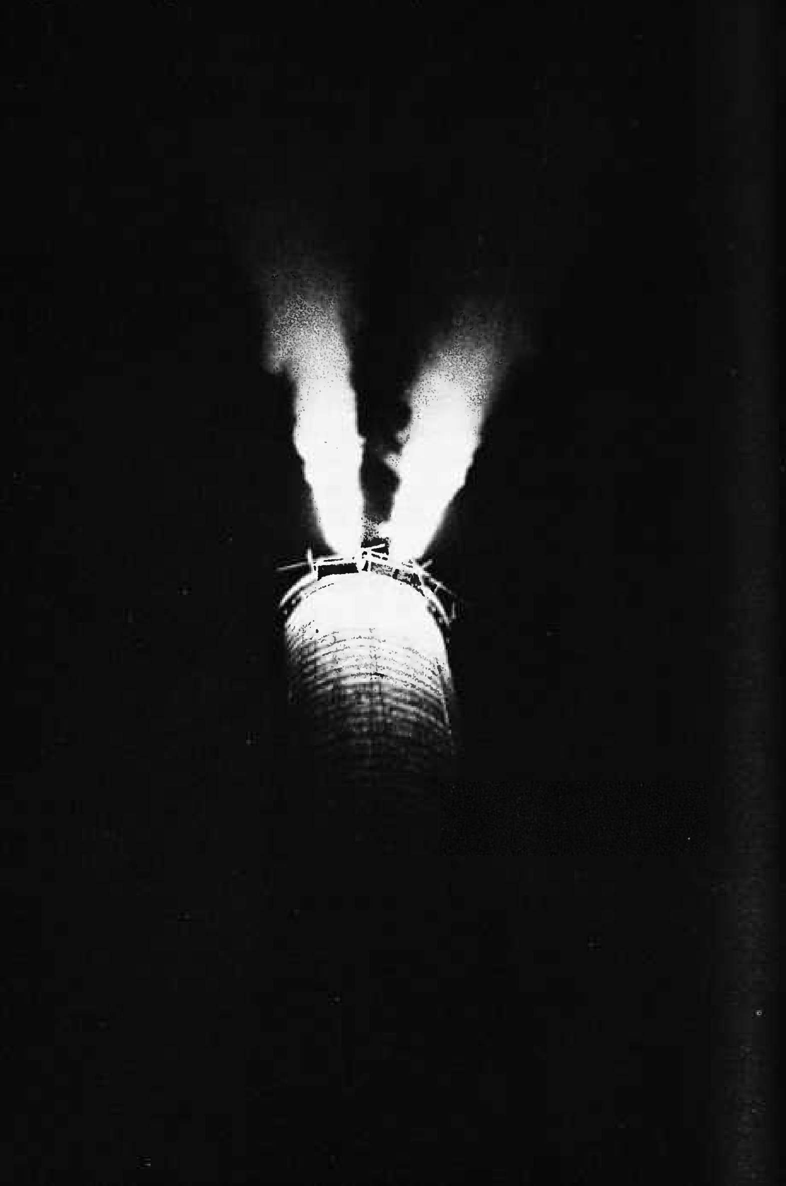


Figure 45

Note that the decay of oscillations in lightly and in heavily damped systems has the exponential form so often found in physics.

Figure 46
Rockets being fired sequentially at the top of the 130 m high reinforced concrete chimney stack at Ferrybridge to induce it into motion. When the rockets are stopped, the decay is measured.



In the simple example we have been discussing, all the mass was assumed to be in a single block at the end of the spring. A more realistic case is a chimney stack of constant circular section. The three properties, stiffness, mass, and structural damping, are all contained in the single structure.

For a more complex structure which is not uniform over its entire length (a lattice electricity pylon, for example), it is necessary to obtain a value of mass which, if uniformly distributed and built up into a tower of the same height and stiffness, would produce the same frequency of oscillation. This introduces the useful concept of *equivalent mass per unit length*. The complicated lattice tower is replaced by a simple mathematical model or analogue.

The equivalent mass per unit length of the analogue tower can be calculated by first writing down the energy of oscillation of each part of it in terms of the, as yet unknown, equivalent mass per unit length and the displacement of each part. The total or integral of all these terms is the total oscillation energy. If the analogue is to represent the real structure faithfully, this total energy must be the same as that of the real structure when it oscillates with similar amplitude.

The oscillation energy of the real structure, part by part, must then be written down, this time in terms of the varying mass per unit length at each place. The total, or integral, of all these terms is then set equal to that for the analogue, and the unknown equivalent mass per unit length in the analogue, at which analogue and real structure have the same total energy, can be found.

Usually the integrals can easily be solved numerically. The mass of the structure is lumped into a number of blocks for which the displacements have been calculated and the calculation of the equivalent mass is then quite simple. If the tower is oscillating, adding mass at the top will have a great effect on the equivalent mass, whereas adding it at the base will have no effect at all because there is no movement there.

Before leaving our discussion of vibrations, there is one further point to be made. If a structure is subjected to a force that alternates at the same frequency as the structure and if the structural damping is low, very large amplitudes of oscillation can be built up. Pushing a child's swing provides a common experience of this sort of thing, where the force and the natural frequency are said to be in resonance. If the forcing frequency is not in resonance with that of the structure, the amplitude will be very small.

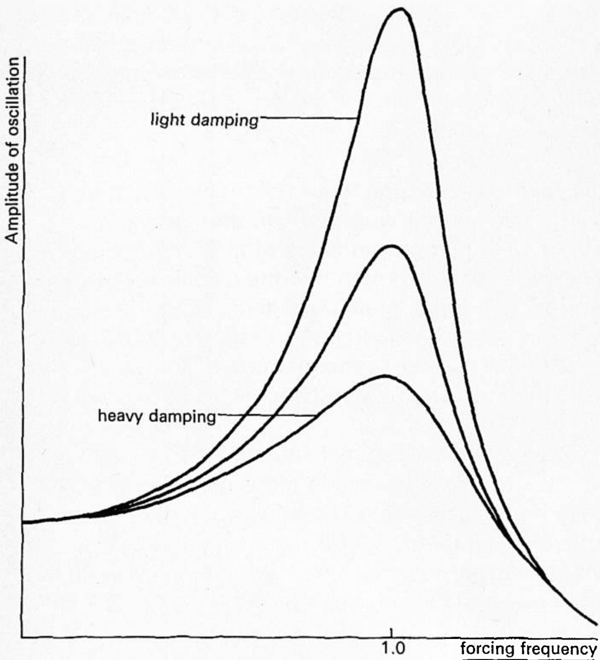


Figure 47

How maximum amplitudes of oscillation are built up at resonance and how damping can reduce the amplitude.

Some properties of fluid flow

Having looked briefly at buildings as vibrating systems, let us now turn our attention to the fluid in which we are all immersed, the air. The aerodynamicist is concerned with how air flows, particularly when it meets obstacles in its path or, conversely, when things are pushed through it as in the case of aircraft and other forms of transport. Our concern here is principally with the former situation.

Let us begin by looking at some of the forces which winds exert on bodies placed in their path and consider the flow characteristics round bodies of various shapes. Figure 49 (p. 80) shows examples of airflow patterns around long rod-like bodies which have different profiles but which all present a similar maximum width to the airstream.

Some shapes give greater drag than others. In such cases the shape of the body breaks up the airflow pattern, producing vortices which give a highly turbulent wake. Streamlined shapes, on the other hand, allow a smooth, or laminar, airflow over the body and do not give such highly turbulent wakes. Drag is the force needed to push a body through the air and this force will be greater if the body produces a highly turbulent wake, because energy is lost as the laminar airflow breaks down into vortices. In designing a car body, for example, especially if high performance is required, it is desirable to have a shape which causes as little disturbance as possible to the airflow over its surface. The penalty for breaking up the airflow is a need to provide higher power for a given speed.

Tall chimney stacks built of light materials can oscillate in the wind. Even if they do not fall, the oscillations can seriously weaken their foundations and give a need for extensive maintenance work. Chimneys are not very glamorous in themselves but they are important from an economic point of view. A manufacturing plant or a power station could be closed if its exhaust system were faulty. Radio towers and television masts are tall slender lightweight structures and they present problems of stability. Microwave radio links mounted on towers have highly directional beams. If the tower deflects violently in wind, the beam could miss the receiving aerial dish altogether. Radiotelescopes built on the dish pattern as at Jodrell Bank need to be aerodynamically stable for similar reasons. High tower blocks, which are becoming quite common, house large numbers of people and for such a block to collapse could be a large scale disaster, for the number of families in a single block could approach that living in a small village. If a block swayed appreciably in wind, the occupants of the upper storeys might experience something akin to sea-sickness.

A disaster resulting from a static effect of wind occurred in 1957 when the residents of a terrace of houses in Hatfield New Town, Hertfordshire, awoke in the early hours of November 4th to find themselves gazing, not at their bedroom ceilings, but at a stormy sky. A few seconds earlier the flat, sloping roof of the entire terrace had been lifted off by the aerodynamic forces of the strong wind. The photograph in figure 48 shows how neatly this scalping operation was carried out.

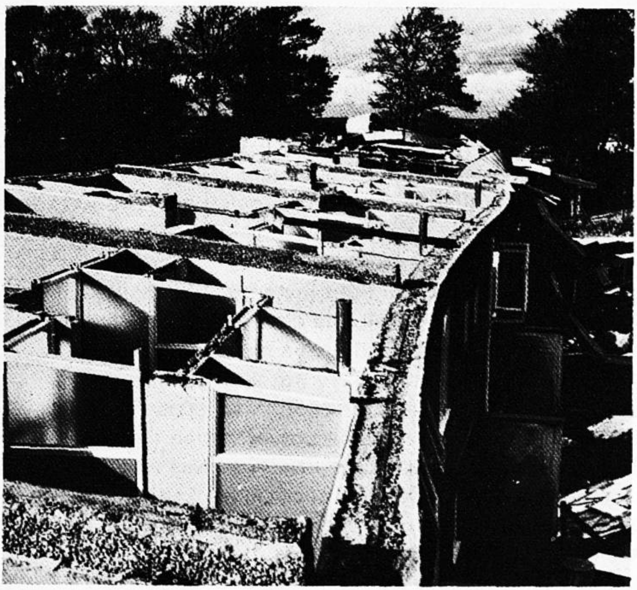


Figure 48
The terrace of houses in Hatfield after the roofs had been moved by wind forces.

As paraphrased in *The Times* of 14 November 1957, an architect's report in the *Architects' journal* stated:

> The circumstances were 'exceptional' – an exposed site facing the direction of the prevailing wind, a 95 m.p.h. gale, and a monopitch roof which any aeronautical engineer would recognize at once as possessing critical aerodynamic properties, but which current codes of practice and building research did not yet cover.

When the flow is not symmetric about a structure, the resultant aerodynamic force on the structure acts at an angle to the wind stream. This force can be resolved into the drag force acting in the wind direction and a lift force acting at right angles to the wind direction. To the aerodynamicist concerned with the design of aircraft, lift is a desirable thing, but this is far from the case when it comes to designing a structure we wish to remain on the ground in windy weather. Lift was responsible for raising the windward edge of the roofs of the houses in Hatfield, while drag forces completed the destruction.

As a vortex breaks away from the surface of a bluff (that is, not streamlined) body, the body experiences a force at right angles to the airflow direction. As vortices break away on either side of the body, the shedding sets up a fluctuating force. This force has a frequency, f, equal to that of the vortex shedding and, if the body is flexible, it will start oscillating when f is equal to its own natural oscillation frequency. Here, we have a condition of resonance, which will occur at a critical value of the airstream velocity.

We can calculate the wind speed at which a structure may be expected to oscillate. The vortex shedding frequency is proportional to flow velocity or

$$f \propto v$$

The shedding frequency is also inversely proportional to the size of the body, and if d is the dimension of the body across the flow,

$$f \propto \frac{1}{d}.$$

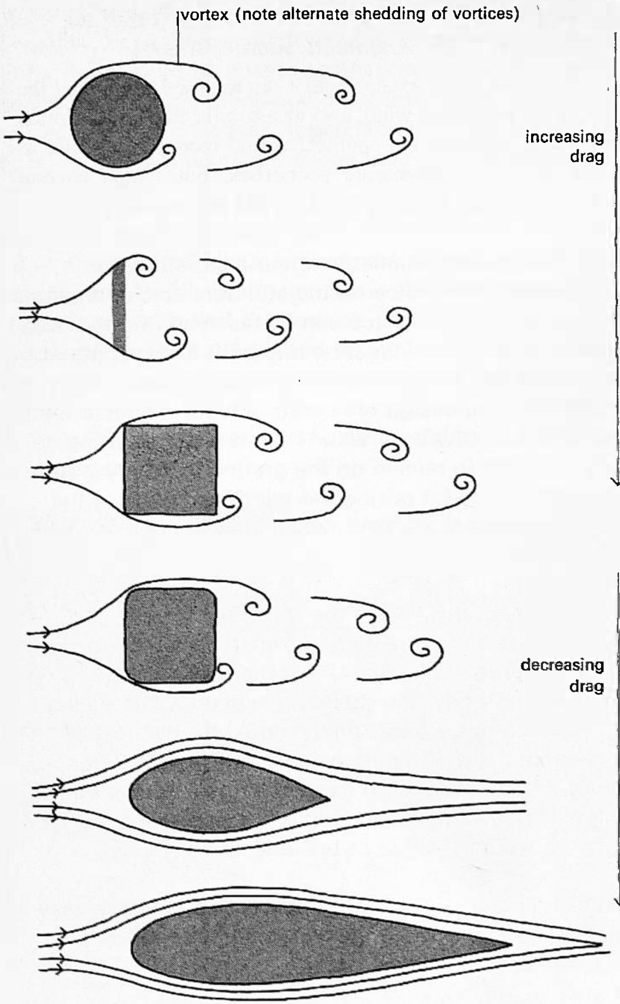


Figure 49
Airflow patterns around long bodies with different cross-sectional shapes but with the same maximum diameter.

We may write

$$f \propto \frac{v}{d},$$

or

$$\frac{fd}{v} = \text{constant.}$$

For a cylinder of circular section, this constant has been found experimentally to be about 0.2, so

$$v = 5fd.$$

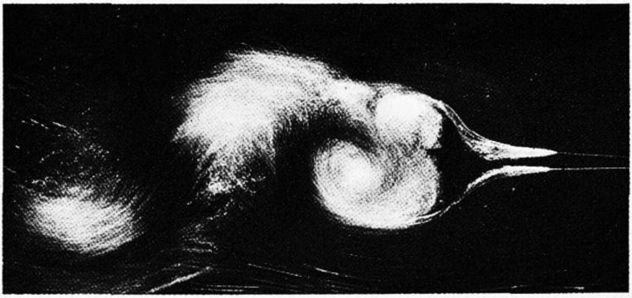


Figure 50
Flow past a flat plate showing vortex shedding.

Oscillations due to vortex shedding have been observed in many structures. For example, during the construction of the Forth Road Bridge, the towers oscillated with an amplitude at the top of about 1 m in a wind speed around 10 m s^{-1}. Whilst these oscillations were not catastrophic, they were very unpleasant for the constructors. The oscillations were stopped by increasing the structural damping of the towers. Chimney stacks are prone to oscillate due to vortex shedding and sometimes they and other thin-walled structures, such as storage tanks, oscillate in what is called an ovalling mode.

The effects of vortex shedding can be countered in several ways. One way is to increase the structural damping. Rockets on launch pads are susceptible to vortex shedding and, before launching, are fitted with impact dampers. These consist of chains hanging inside metal tubes, so that when the rocket starts to sway the chains collide with the tube walls and absorb the energy. Modifications can be made to the external

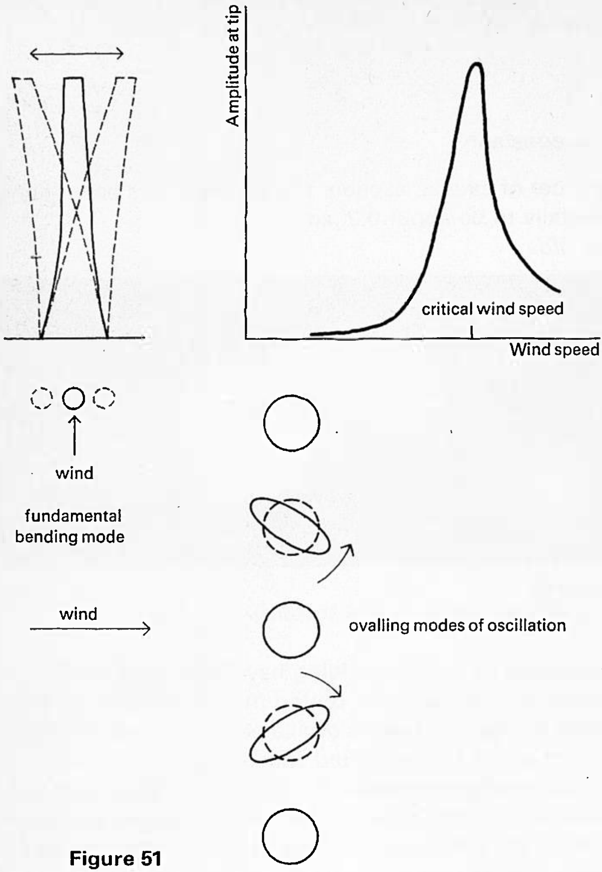


Figure 51
Oscillations of a chimney stack due to vortex shedding.

shapes of structures in order to reduce the power of the vortex shedding. 'Strakes' are widely used for this purpose. They consist of a helix fitted to the structure as shown in figure 52. An alternative scheme, not so widely used, is to fit a perforated shroud round the structure, as in figure 53. The purpose of the shroud is to feed air into the region directly behind the stack and prevent the formation of vortices.

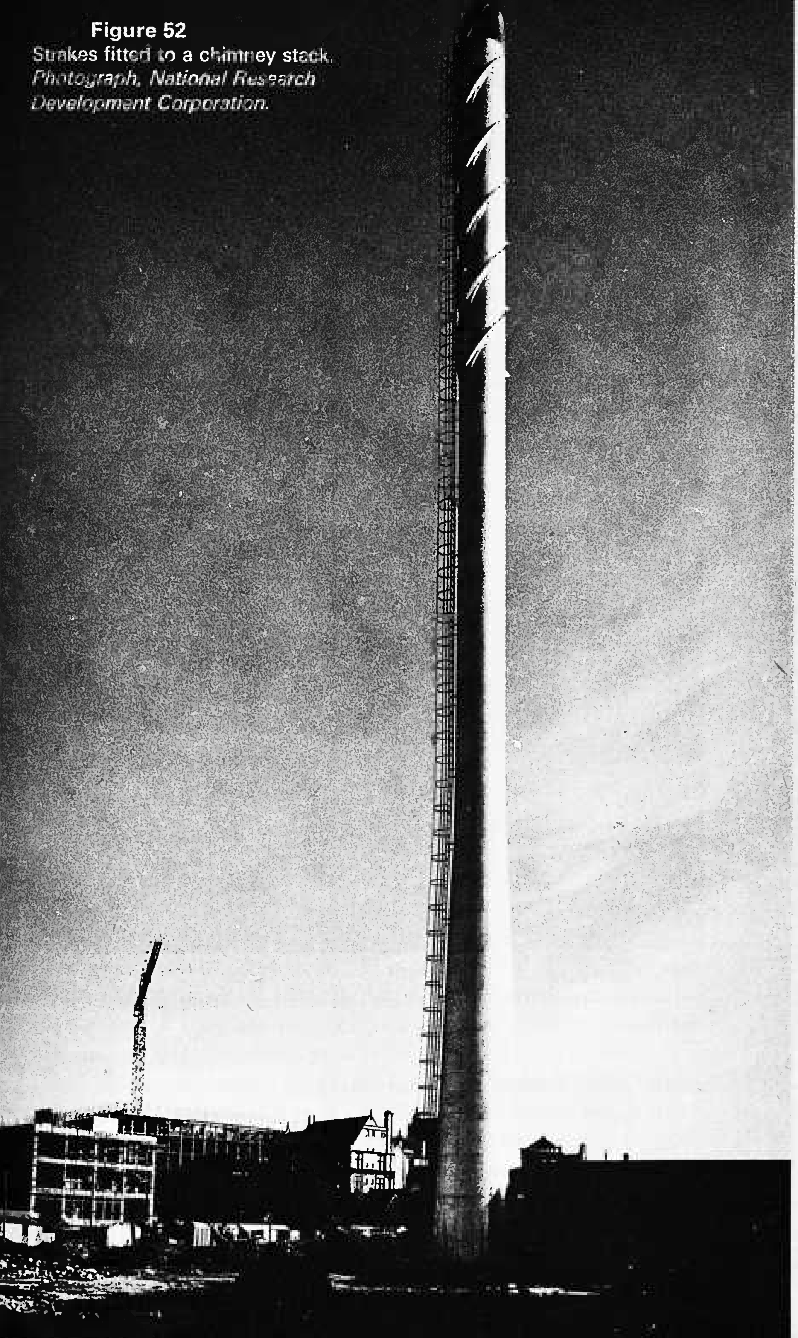


Figure 52
Strakes fitted to a chimney stack.
Photograph, National Research Development Corporation.

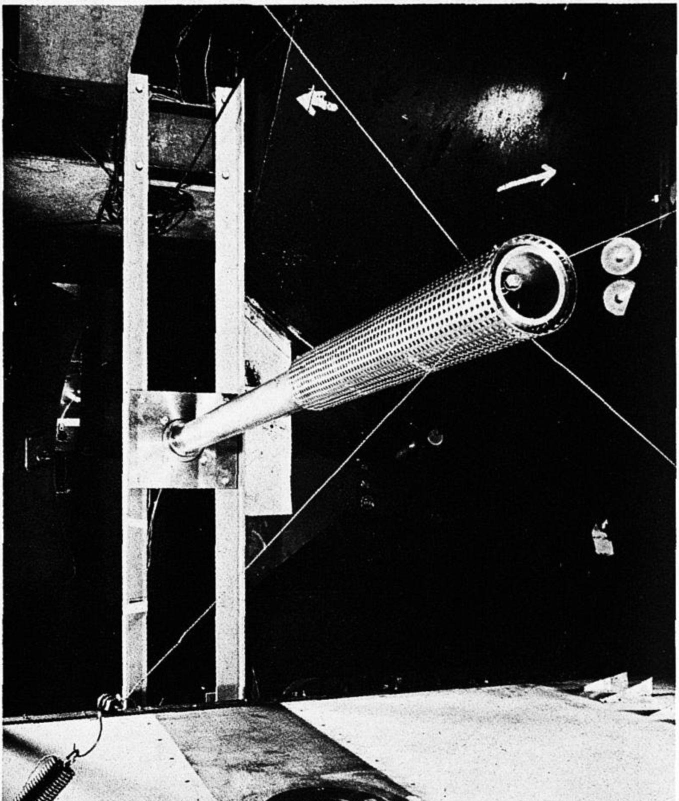


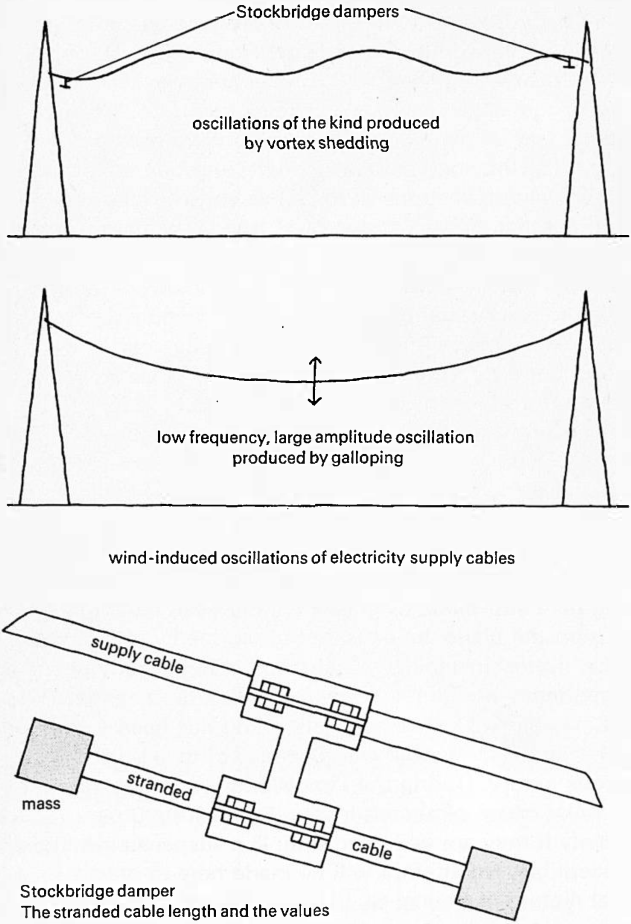
Figure 53
Experimental model of shroud to prevent vortex shedding, undergoing wind tunnel tests.

Other forms of wind-excited instabilities

Vortex shedding is not the only cause of structural oscillations. A large number of electric power lines have oscillated very violently, due to various phenomena, loosely known as 'galloping'. True galloping is a form of oscillation that requires an initial 'kick' to set it off. The structure is stable as long as it remains stationary but oscillates violently after being disturbed, for example, by a gust of wind.

An important example of violent oscillations in the United Kingdom occurred on the electricity supply cables crossing the River Severn, just south of the suspension bridge. These cables are made of stranded wire. They are over 1000 m long and are 35 mm in diameter. Oscillations occur in certain wind conditions and amplitudes of 10 m or so were obtained, with the result that the conductors clashed. Subsequent research in wind tunnels with sections of the cable showed that the cause of the instability was complex. It was the aerodynamic effect of the lay of the strands of wire, coupled with gusting. The remedy, however, was simple. The cables were bound with tape to form a smooth surface finish of approximately circular cross-section. Such a section produces vortex shedding, but the resulting aerodynamic forces are not sufficient to produce instability in this particular cable. However, vortex shedding has been known to cause instability of some power lines. The amplitudes are not so large as galloping oscillations, but can still cause cable failure.

Another form of instability is known as 'classical flutter'. Just about every schoolboy has created piercing whistles by blowing on a stiff blade of grass held between his thumbs. By blowing on the blade, he caused it to vibrate by inducing a 'classical flutter' instability which can be very clearly heard if the frequency of vibration is within the audible range. This type of instability is extremely violent and has been extensively studied in the aeronautical field because of its highly dangerous nature. During the First World War, classical flutter was a major cause of aircraft losses. Some structures, particularly if they are wide and thin, like suspension bridges, are susceptible. No attempt will be made here to give a physical picture of the cause, but for classical flutter to occur, the structure must have two degrees of freedom. Figures 68–70 (p. 101) show a cross-section of a suspension bridge. The structure has several degrees of freedom but the most important from the aerodynamic standpoint are the vertical and torsional motions depicted. When classical flutter occurs, the individual vertical and torsional motions combine so that the structure undergoes a rather complicated coupled motion under the action of the wind force.



wind-induced oscillations of electricity supply cables

Stockbridge damper
The stranded cable length and the values of the masses can be adjusted to give maximum damping in a given case.

Figure 54

Bridging the Severn

From Gloucester, the river Severn flows south-west for 80 km, separating England and Wales. Although it is only 40 km as the crow flies from Cardiff to Bristol, the road journey, via Gloucester, is 146 km long. Some 32 km up river from Cardiff and 16 km from Bristol is a ferry point, called the Old Passage, between Beachley and Aust. The service ran at thirty-minute intervals during daylight hours, each trip carrying 15 cars. Just below the Old Passage, the river Wye flows into the Severn from the north, and 5 km downstream, the Severn Railway Tunnel links England and Wales. The Severn is approximately 1.5 km wide at the Old Passage and 3 km at the point of the rail tunnel; opposite Bristol the width has increased to 8 km and by the time Cardiff is reached the river is 13 km wide. The Severn is a tidal river with a tidal range (the difference in water level between high and low tides) of some 14 metres for spring tides. This is said to be the second largest range in the world and the current is very fast.

At the beginning of the Industrial Revolution, communications to and from South Wales were similar to those existing in Roman times, 1700 years before. Between 1750 and 1830 main roads were improved by men such as Thomas Telford and John Macadam. The journey from London to Bath was cut from two days to fourteen hours, not without severe demands being made upon the horses which provided the motive power. South Wales, however, remained poorly served, but around 1850, with the coming of the railways and the development of the Welsh coalfields, the real need for a permanent crossing of the Severn was recognized. This was achieved when, in 1866, the Great Western Railway opened its Severn Tunnel. It was the largest underwater rail tunnel in the world and, apparently, one of the wettest—90 000 000 dm^3 of water had to be pumped out daily!

By the 1930s, road transport was growing rapidly and to reach South Wales involved a lengthy journey via Gloucester (see figure 55). The need for a road bridge was obvious. The Second World War intervened, but following it, the Welsh

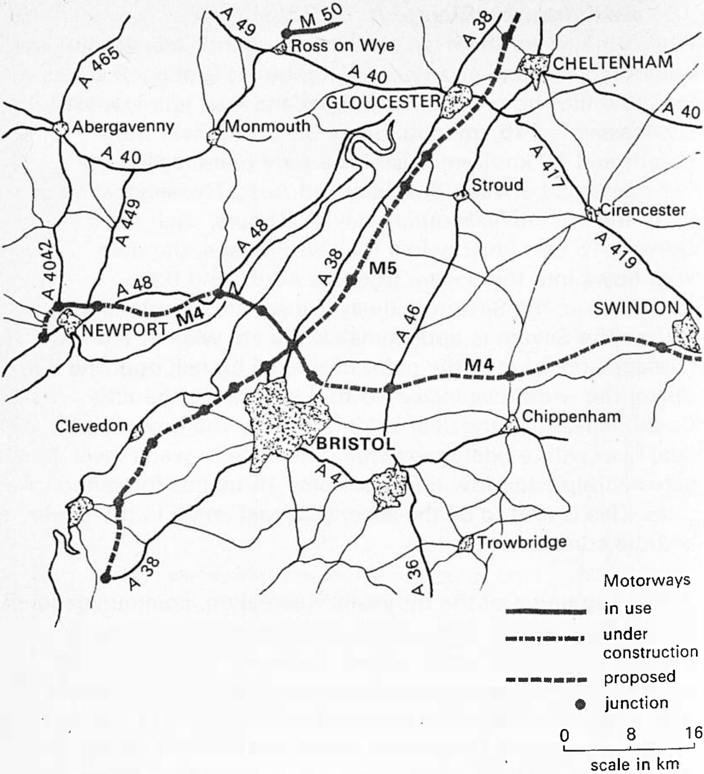


Figure 55
How the Severn Bridge joins the roads of Wales and the West Country.

Reconstruction Advisory Council recommended to the Government that, 'The construction of a Severn Road Bridge would be the greatest single contribution to the improvement of transport facilities in South Wales. We regard the provision of a bridge as so essential to that part of the Principality that we subordinate to it all other questions of the development of communications in that area.'

In May 1962, a contract for building the superstructure of the Severn Bridge, which was to span the river between Beachley and Aust, was placed with a consortium of engineers, Associated Bridge Builders Ltd. This same consortium of Sir William Arrol and Company, the Cleveland Bridge and Engineering Company, and Dorman Long Ltd, also built the Forth Road Bridge, which was completed in 1964.

Some comparison between the Severn Bridge and the Forth Road Bridge is inevitable. The same consortium of engineers built both, the same consulting engineers prepared the two designs, they are of comparable size, and their construction was phased so as to make use of some of the same methods and plant. The two bridges, however, differ in a very important way, namely the method by which the road decks are stiffened and protected from aerodynamic instability. In the Forth Bridge, which represented a big advance in bridge design, the road deck is an open, truss-stiffened structure. When they turned to the Severn Bridge, however, the engineers sought a road deck design which would be lighter than that of the Forth, yet would still be aerodynamically stable. Not only was the suspended structure more economical, but the foundations, towers, and suspension cables could be lighter and therefore cheaper. They wanted a road deck in the form of a shallow steel-plated hollow box which would have high stiffness and would be easy to fabricate in sections. The cross-sectional shape of this box structure was critical for aerodynamic stability. The final design for the deck consisted of a hollow box, 3.05 m deep and 22.9 m wide, with feathered edges. Another advantage was that its wind resistance was only about one-third of that on the Forth road deck, because of the excellence of its aerodynamic design.

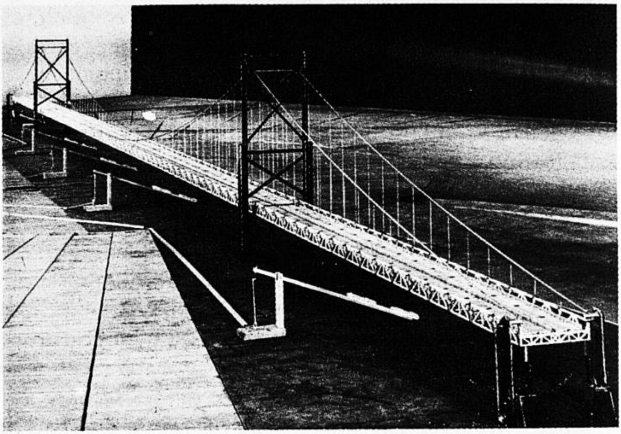


Figure 56
The full aeroelastic model used in initial investigations for the Severn and Forth Road Bridges.

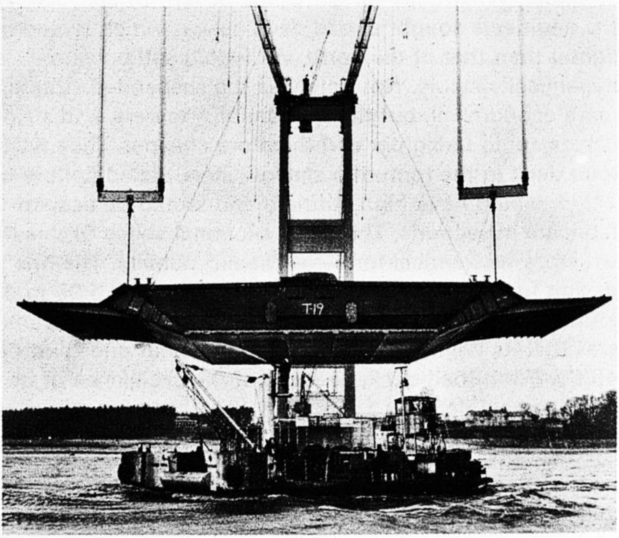


Figure 57
The *Severn Knave.*
Photograph, William Tribe.

Aerodynamic problems also arose from the method of construction adopted. Initially the towers were erected and the suspension cables spun between them. Sections of the road deck, 18.3 m long, were sealed by a bulkhead at each end and floated out into the river. Eighty-eight such sections were required for the complete road deck. Once in the river, each section was taken over by a special ship, the *Severn Knave*, which manoeuvred it into position, within an accuracy of about 1 m of a given point in the fast running river, beneath the suspension cables. Then the lifting of the section began; initially, it was lifted to a height of 10 m, after which powerful water pumps on the *Severn Knave* washed down its lower surface, freeing this from dirt accumulated in the river. Then began a slow hoist over some 30 m, taking about half an hour to accomplish. During this time the suspended section was exposed to wind forces, a problem which was coped with in a way discussed below. Finally, the section was attached to the supporting cables and welded into position.

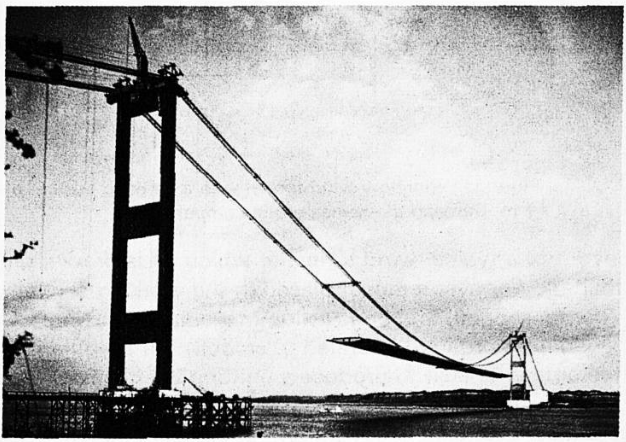


Figure 58
This photograph shows the road deck of the bridge in the course of construction. Notice how the construction has begun in the centre and decking sections have been added alternately, working out towards the towers.

Photograph, William Tribe.

So far, we have considered only the aerodynamic stability of the road deck, but the towers must be considered from this angle, as well, and they too received their fair share of attention in the wind tunnels at NPL.

Wind tunnels and model testing

The wind tunnel is the aerodynamicist's research tool in which he examines the behaviour of correctly proportioned models of full-scale structures. Let us begin by considering the features of such tunnels. Figure 59 shows the general

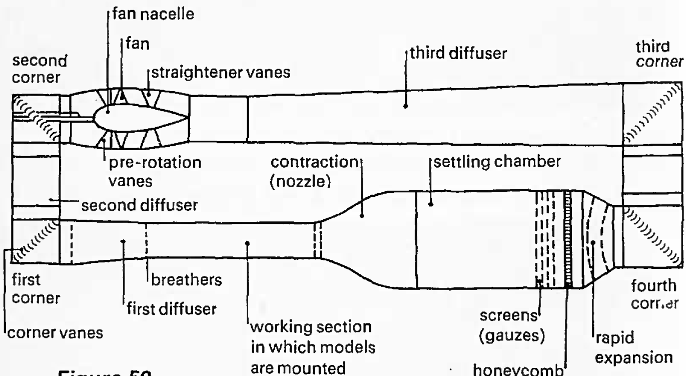
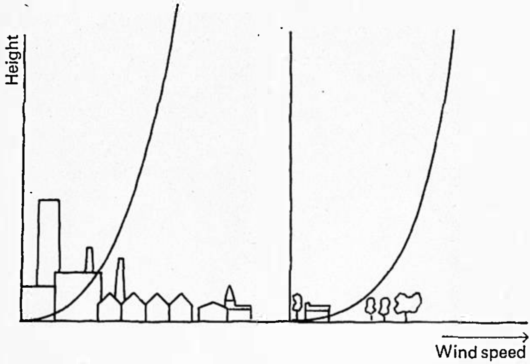
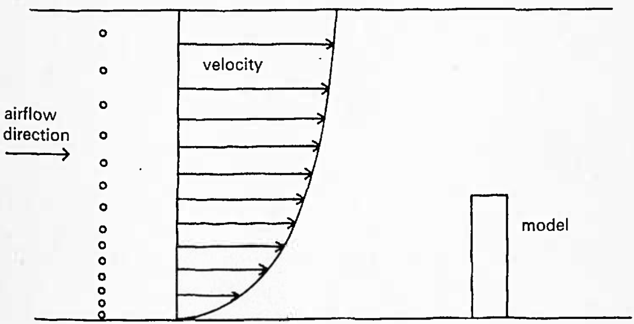


Figure 59
National Physical Laboratory wind tunnel with a working section of 1.1 m × 1.1 m, showing the names of the components.

layout of a typical wind tunnel in which air is drawn, rather than blown, over a model placed in the working section. The honeycomb and contraction immediately in front of the working section are intended to smooth out irregularities in the airflow and so to produce a uniform airstream. Of course, in many conditions which we try to reproduce in wind tunnels, the local airstream is not smooth at all. Atmospheric winds are gusty and generally the wind speed increases with height. The degree of gustiness and the variation of wind speed (the velocity profile) depend on the local terrain and the presence of buildings and even of trees. To try to duplicate natural wind conditions is an interesting challenge; sometimes a model of the geographical features associated with the structure is built into the working section, but no completely satisfactory way of duplicating all the natural features of a site in a tunnel has yet been found. A velocity profile, however, is easier to reproduce. It can be done by placing a grid consisting of variably spaced bars in the airflow upstream of the model (figure 60). An airstream with similar turbulence to that which occurs over cities can be reproduced by a grid such as that shown in figure 61 behind models of tower blocks proposed for Hong Kong.



Typical wind speed profiles over different terrains



How a velocity profile can be simulated in a wind tunnel by placing unevenly spaced bars across the working section

Figure 60

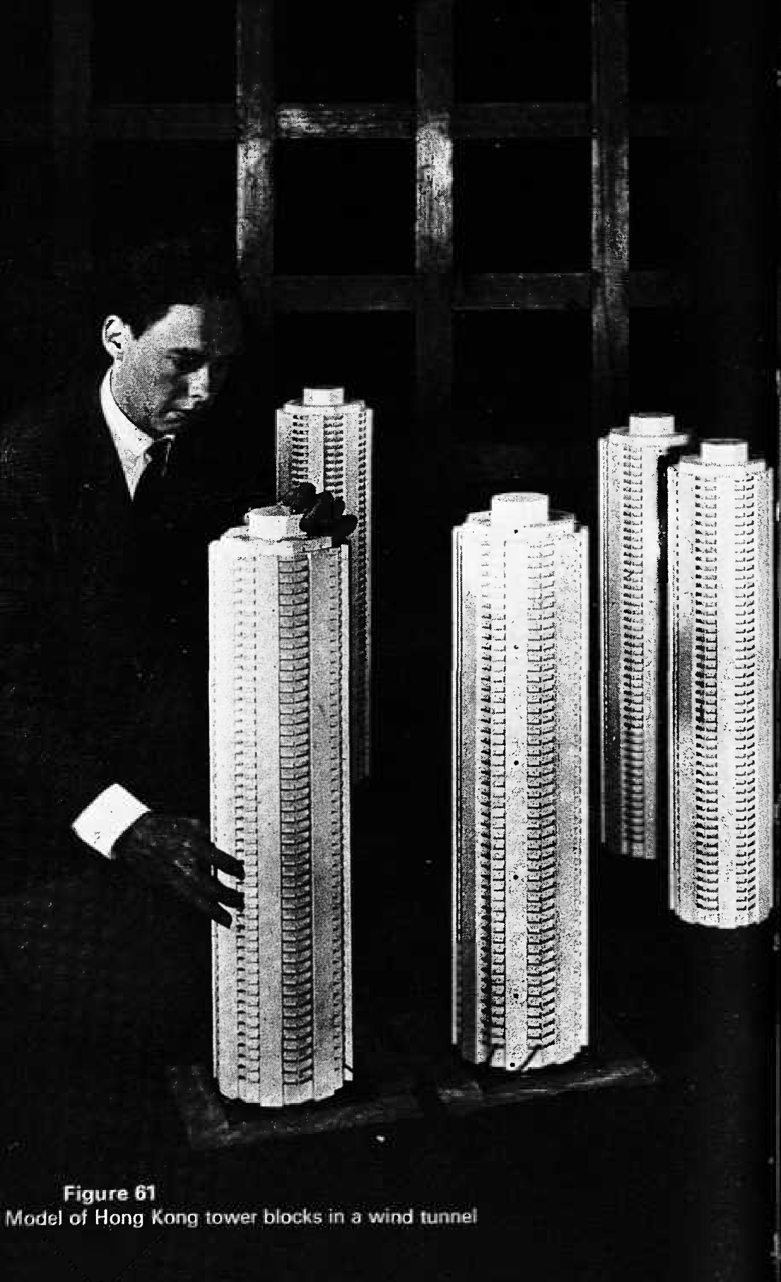


Figure 61
Model of Hong Kong tower blocks in a wind tunnel

There are several types of model which may be used in wind tunnel investigations. A full aeroelastic model is, as its name implies, a precision model of the complete structure, reproducing all the external features, and if possible made of the same materials so as to reproduce the mass and stiffness distribution. For the Severn Bridge towers, a full model was used, built to a 1:55 scale, complete, as shown in figure 62,

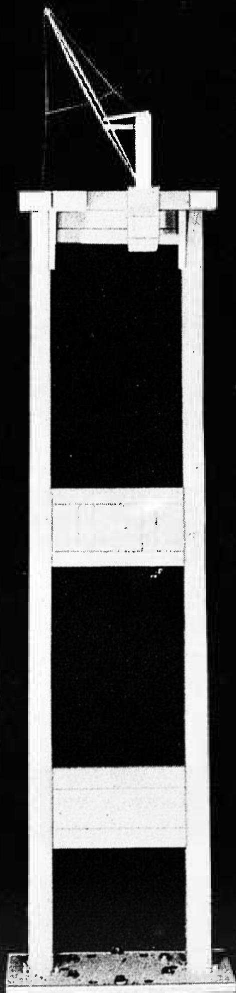


Figure 62
Full aeroelastic model of a Severn Bridge tower.

with a model of an erecting crane on top. Using this model it was found that the vortex shedding frequency at a wind speed of 15 m s^{-1} coincided with the natural bending frequency of the tower. At high wind speeds the bending oscillations decreased (moving away from resonance) and torsional oscillations set in at wind speeds above 34 m s^{-1}. These observations are illustrated in figure 63.

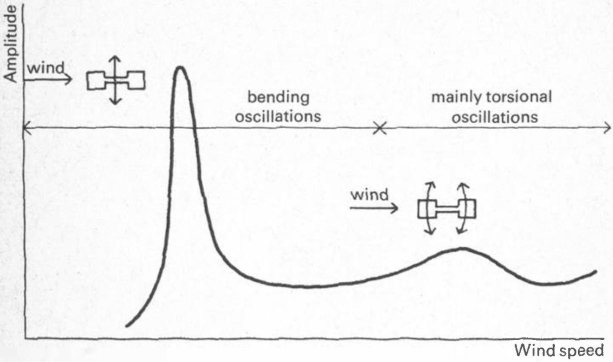


Figure 63
The oscillations of the model of the Severn Bridge towers. As the wind speed is increased from zero, the amplitudes of bending oscillations increase and reach a maximum when the vortex shedding frequency coincides with the natural bending frequency. Further increase in wind speed, and thus in shedding frequency, produces small amplitudes until the shedding frequency more nearly coincides with the natural frequency in torsion. The structure then responds in this mode.

One result of the tests was to provide the engineers with sufficient data on structural damping to enable them to prevent instability in the actual free standing towers. This they did by attaching cables between the tower tops and earth anchors, a method which served until the suspension cables were spun and the road deck attached. A full aero-elastic model will reproduce oscillations caused by all kinds of wind effects as well as vortex shedding. So if the bridge towers had been susceptible to galloping, that too would have shown up in the tests.

Full models are very expensive to build and it can be extremely difficult to alter their structural damping. Under some conditions they may be replaced by a much simpler linear-mode model (figure 64).

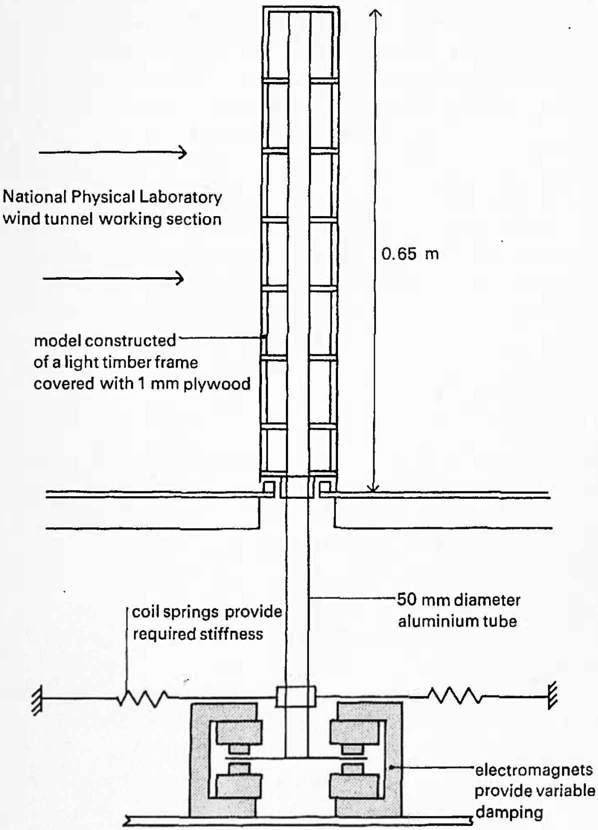


Figure 64
An example of a linear-mode model.

A *linear-mode model is a rigid structure of the same external* shape as the full-scale structure, mounted on a low friction bearing in the floor of the wind tunnel. Stiffness is provided by springs, as shown, and the dynamics of the system is based on calculations similar to those for a simple spring-mass system as described earlier. Structural damping is simulated by a copper plate attached to the model and placed between the poles of an electromagnet. Eddy currents produced in the copper plate when the model oscillates produce the required damping simulation; and the damping can be increased by simply increasing the current in the coils of the electromagnets. Linear-mode models also have other advantages. Because they can be made from wood placed over an aluminium spine, it is quite easy to alter the external shape to get improved aerodynamic profiles. The mass of the model can also be varied easily by simply adding weight to it. An important assumption is made when a linear-mode model is used. It is assumed that the full-scale structure will be free from all oscillations except those in the fundamental bending mode, and that this mode can be adequately represented by a deflection that varies linearly with height. This condition can often be met and the models of the Hong Kong tower blocks are linear-mode models.

The ultimate achievement in model building is to reproduce accurately in the wind tunnel the terrain and buildings ahead of the structure to be studied. A model of the centre of Sydney, Australia, was made when the Qantas Centre (figure 65) was being designed. A similar model provided information about the flow velocities at the base of the proposed towers so that the design modifications could be made to reduce pedestrian discomfort due to windy conditions. Linear-mode models of the structures to be tested are useful in studies in which local terrain is reproduced. They can be made to a smaller scale than full aeroelastic models and so more surrounding terrain can be reproduced in the working section of the wind tunnel.

Finally, there is the sectional model, which is a rigid geometrical copy of a typical length of a structure. This type of model is used to represent a sample length of a long structure such as a suspension bridge. It is found in practice that air flow around the ends of the full-scale bridge have an insignificant effect and can be ignored in tests. (Figure 66.)

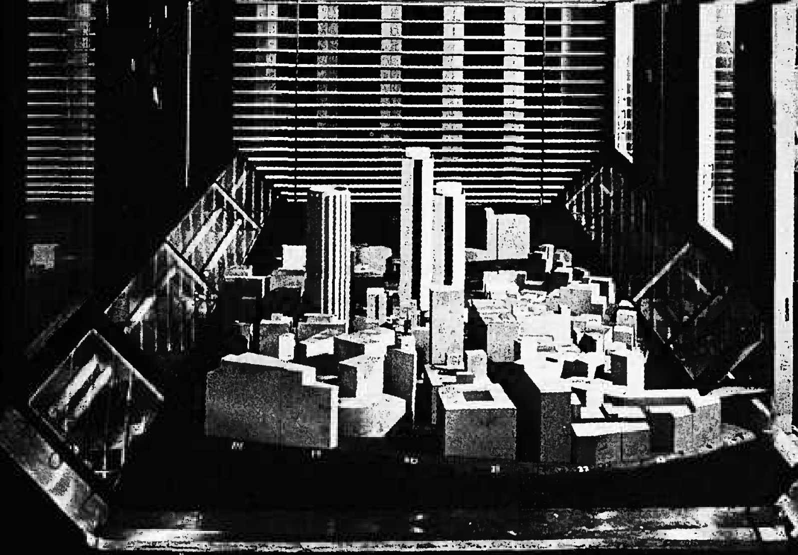


Figure 65 The Qantas Centre model.

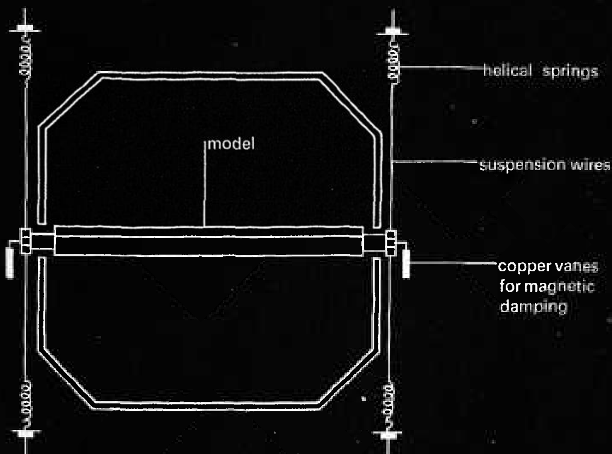


Figure 66 A sectional model is mounted in the wind tunnel as shown in this diagram. Springs provide stiffness and eddy currents produced in the copper vanes simulate structural damping effects just as in linear-mode models. The suspension of the model permits both vertical and pitching oscillations to occur.

The results from the wind tunnel experiments are used to predict the behaviour of the full-scale structure. For example, we have already seen that the critical wind speed is given by

$v = Cfd$, where C is a constant.

The critical wind speed on the full-scale structure is given by

$$\frac{v_{\text{structure}}}{v_{\text{model}}} = \frac{(fd)_{\text{structure}}}{(fd)_{\text{model}}}$$

An important assumption has, however, been made in the above discussion. This is that the flow over the model is precisely the same as that over the full-scale structure and hence that C is the same in both cases. Often, this is true, but it cannot always be assumed. The task of relating results from models to full-scale structures is a vital and difficult part of the work of the aerodynamicist.

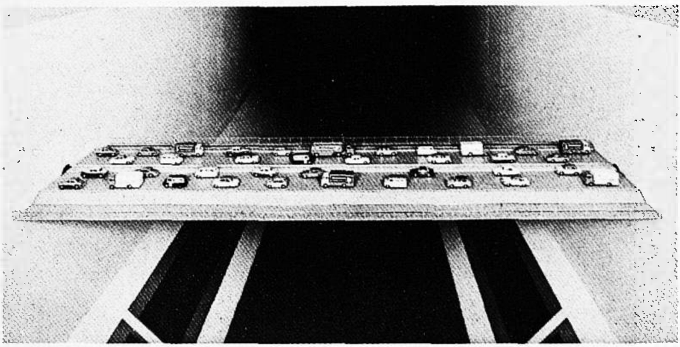


Figure 67
The sectional model used in wind tunnel tests on the Severn Bridge.

Modes of oscillation of suspension bridges

Tests on full aeroelastic models and experience of full-scale structures reveal three main types of road-deck oscillation. Figures 68 to 70 show these types of oscillation in diagrams of the bridge cross-section.

The simplest form of oscillation is shown in figure 68.

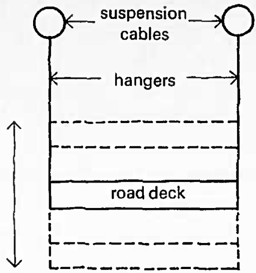
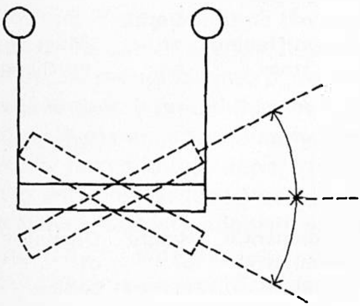


Figure 68
The 'galloping Gertie' type of oscillation.

Alternatively, a torsional oscillation can take place (figure 69)

Thirdly, the torsional and vertical oscillations may couple to give the type shown in figure 70.



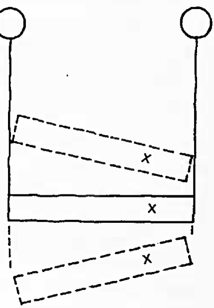


Figure 69 (above left)
A torsional oscillation.
Figure 70 (above right)
The centre of torsional motion moves vertically.

These vertical and torsional oscillations can occur in a number of *modes of oscillation* of the bridge as a whole, of which two examples are shown in figures 71 and 72.

The tests carried out in the wind tunnels of the National Physical Laboratory resulted in an aerodynamic design for the cross-section of the road deck which would ensure the stability of the bridge under all anticipated wind conditions.

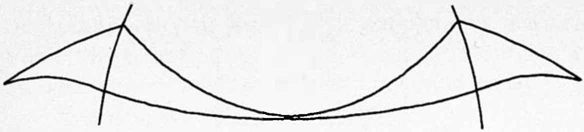


Figure 71
Fundamental symmetrical mode of oscillation of a suspension bridge.

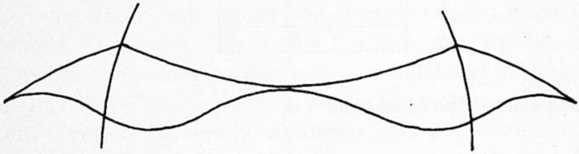


Figure 72
Another symmetrical mode of oscillation.

Bridge	Length of main span /m	Length of each side-span /m	Depth of stiffening truss /m	Depth of box /m	Mass per unit length /10^3 kg m^{-1}
Forth Road	1006	408	8.38		20.9
Severn	988	305		3.05	11.7

Bridge	Fundamental symmetric mode N_θ /Hz	N_z /Hz	Fundamental asymmetric mode N_θ /Hz	N_z /Hz	Height of towers /m	Diameter of cables /mm
Forth Road	0.263	0.150	0.351	0.127	156	600
Severn	0.374	0.143	0.510	0.128	122	533

Table 2
Some dimensions and values of mass and frequencies of oscillation for the Forth Road and Severn Bridges

N_θ refers to the torsional oscillation, N_z to the vertical oscillation.

A comparison between the Fourth and Severn Bridges, in table 2, emphasizes, in terms of mass per unit length and suspension cable diameter, the savings effected by the new approach adopted in the design of the Severn Bridge.

We mentioned earlier that the method of construction in which sections of road deck were hauled up from the river gave rise to aerodynamic problems. The lifting system originally proposed for the job allowed a pitching motion, analogous in some ways to the swinging of a pendulum, to occur in wind speeds of about 5.5 m s^{-1}. Tightening the stay ropes increased the natural frequency of oscillation and hence the wind speed for instability, but the instability was finally overcome by using a system of diagonal restraining wires.

It was the original intention not to weld the units rigidly together immediately after the units were hoisted into position, but to wait until the span was complete. At this stage, the cables would adopt their design sag. The suspended structure would fall into place and the bottom of the units would abut. It would then be a comparatively simple matter to weld the units together. However, until the units were rigidly linked, the torsional frequency of oscillation would be low and close to the vertical motion frequency. Further aerodynamic tests, coupled with a theoretical analysis, showed that with this frequency ratio near unity, the suspended structure could oscillate with a classical flutter instability. The engineers devised a suitable means of connecting the units as the span progressed and thereby avoided the conditions for flutter.

In 1966, the Severn Road Bridge was opened to traffic. Like many twentieth century structures, it is functional and awesome in its striking simplicity of design. Beside its approach roads, industrial sites will grow, across it will roll the commerce between England and Wales, with all that that implies for the development of the area. The men who built the bridge will leave something of value and beauty to succeeding generations.

Mathematical models in physics

In this Unit, mathematics has been used to describe the motion of oscillators. You have done experiments which suggest that the mathematics adequately describes *some* oscillators to *some* extent.

What can truly be said about the relation between mathematics and experiment? Must what the mathematics says be true in some way or other? – mathematics does seem to be designed not to leave loop-holes for doubt. Or should experiment be the final court of truth? Yet why should one suppose that an experiment must come out the same way twice running?

The first of the extracts that follow, from Poincaré, *Science and hypothesis* (1905), discusses 'experiment' and 'generalization'. By 'generalizations' Poincaré means those laws, cast in the form of equations, which describe what happens. He argues that, without mathematics, the facts of science would be like a heap of rubble with no shape or order.

The role of experiment and generalization. Experiment is the sole source of truth. It alone can teach us something new; it alone can give us certainty. These are two points that cannot be questioned. But then, if experiment is everything, what place is left for mathematical physics? What can experimental physics do with such an auxiliary, one moreover, which seems useless, and may even be dangerous?

However, mathematical physics exists. It has rendered undeniable service, and that is a fact which has to be explained. It is not sufficient merely to observe; we must use our observations, and for that purpose we must generalize. This is what has always been done, only as the recollection of past errors has made man more and more circumspect, he has observed more and more and generalized less and less. . . . Cannot we be content with experiment alone? No, that is impossible; that would be a complete misunderstanding of the true character of science.

The man of science must work with method. Science is built up of facts, as a house is built of stones; but an accumulation of facts is no more a science than a heap of stones is a house. Most important of all, the man of science must exhibit foresight. Carlyle has written somewhere something after this fashion. 'Nothing but facts are of importance. John Lackland passed by here. Here is something that is admirable. Here is a reality for which I would give all the theories in the world.' . . .

That is the language of the historian. The physicist would most likely have said: 'John Lackland passed by here. It is all the same to me, for he will not pass this way again.'

We all know that there are good and bad experiments. The latter accumulate in vain. Whether there are a hundred or a thousand, one single piece of work by a real master – by a Pasteur, for example – will be sufficient to sweep them into oblivion. . . . A fact is a fact. A student has read such and such a number on his thermometer. He has taken no precautions. It does not matter; he has read it, and if it is only the fact which counts, this is a reality that is as much entitled to be called a reality as the peregrinations of King John Lackland. What, then, is a good experiment? It is that which teaches us something more than an isolated fact. It is that which enables us to predict, and to generalize. Without generalization, prediction is impossible. The circumstances under which one has operated will never again be reproduced simultaneously. The fact observed will never be repeated. All that can be affirmed is that under analogous circumstances an analogous fact will be produced. To predict it, we must therefore invoke the aid of analogy – that is to say, even at this stage, we must generalize. However timid we may be, there must be interpolation. Experiment only gives us a certain number of isolated points. They must be connected by a continuous line, and this is a true generalization. But more is done. The curve thus traced will pass between and near the points observed; it will not pass through the points themselves. Thus, we are not restricted to generalizing our experiment, we correct it; and the physicist who would abstain from these corrections, and really content himself with experiment pure and simple, would be compelled to enunciate very extraordinary laws indeed. Detached facts cannot therefore satisfy us, and that is why our science must be ordered, or, better still, generalized.

The second extract is from R. P. Feynman, R. B. Leighton, and M. Sands, 1963, *The Feynman lectures on physics* Volume 1, Addison-Wesley. Feynman talks as a physicist about how mathematics is used in dynamics and about the problem of understanding how the ideal world of mathematical laws is connected with the real physical world of experiment.

Newton also gave one rule about the force: that the forces between interacting bodies are equal and opposite – action equals reaction; that rule, it turns out, is not exactly true. In fact, the law $F = ma$ is not exactly true; if it were a definition we should have to say that it is *always* exactly true; but it is not.

The student may object, 'I do not like this imprecision, I should like to have everything defined exactly; in fact, it says in some books that any science is an exact subject, in which *everything* is defined.' If you insist upon a precise definition of force, you will never get it! First, because Newton's Second Law is not exact, and second, because in order to understand physical laws you must understand that they are all some kind of approximation.

Any simple idea is approximate; as an illustration, consider an object . . . what *is* an object? Philosophers are always saying, 'Well, just take a chair for example.' The moment they say that, you know that they do not know what they are talking about any more. What *is* a chair? Well, a chair is a certain thing over there . . . certain? how certain? The atoms are evaporating from it from time to time – not many atoms, but a few – dirt falls on it and gets dissolved in the paint; so to define a chair precisely, to say exactly which atoms are chair, and which atoms are air, or which atoms are dirt, or which atoms are paint that belongs to the chair is impossible. So the mass of a chair can be defined only approximately. In the same way, to define the mass of a single object is impossible, because there are not any single, left-alone objects in the world – every object is a mixture of a lot of things, so we can deal with it only as a series of approximations and idealizations.

The trick is the idealizations. To an excellent approximation of perhaps one part in 10^{10}, the number of atoms in the chair does not change in a minute, and if we are not too precise we may idealize the chair as a definite thing; in the same way we shall learn about the characteristics of force, in an ideal fashion, if we are not too precise. One may be dissatisfied with the approximate view of nature that physics tries to obtain (the attempt is always to increase the accuracy of the approximation), and may prefer a mathematical definition; but mathematical definitions can never work in the real world. A mathematical definition will be good for mathematics, in which all the logic can be followed out completely, but the physical world is complex, as [in the case of] ocean waves and a glass of wine. When we try to isolate pieces of it, to talk about one mass, the wine and the glass, how can we know which is which, when one dissolves in the other? The forces on a single thing already involve approximation, and if we have a system of discourse about the real world, then that system, at least for the present day, must involve approximations of some kind.

This system is quite unlike the case of mathematics, in which everything can be defined, and then we do not know what we are talking about. In fact, the glory of mathematics is that *we do not have to say what we are talking about*. The glory is that the laws, the arguments, and the logic are independent of what 'it' is. If we have any other set of objects that obey the same system of axioms as Euclid's geometry, then if we make new definitions and follow them out with correct logic, all the consequences will be correct, and it makes no difference what the subject was. In nature, however, when we draw a line or establish a line by using a light beam and a theodolite, as we do in surveying, are we measuring a line in the sense of Euclid? No, we are making an approximation; the cross hair has some width, but a geometrical line has no width, and so, whether Euclidean geometry can be used for surveying or not is a physical question, not a mathematical question. However, from an experimental standpoint, not a mathematical standpoint, we need to know whether the laws of Euclid apply to the kind of geometry that we use in measuring land; so we make a hypothesis that it does, and it works pretty well; but it is not precise, because our surveying lines are not really geometrical lines. Whether or not those lines of Euclid, which are really abstract, apply to the lines of experience is a question for experience; it is not a question that can be answered by sheer reason.

In the same way, we cannot just call $F = ma$ a definition, deduce everything purely mathematically, and make mechanics a mathematical theory, when mechanics is a description of nature. By establishing suitable postulates it is always possible to make a system of mathematics, just as Euclid did, but we cannot make a mathematics of the world, because sooner or later we have to find out whether the axioms are valid for the objects of nature. Thus we immediately get involved with these complicated and 'dirty' objects of nature, but with approximations ever increasing in accuracy.

Textbooks and further reading

Textbooks

Arons, A. B. (1965) *Development of concepts of physics.* Addison-Wesley.

Holton, G. and Roller, D. H. D. (1958) *Foundations of modern physical science.* Addison-Wesley.

PSSC (1968) *College physics.* Raytheon.

PSSC (1965) *Physics.* 2nd edition. Heath.

Rogers, E. M. (1960) *Physics for the inquiring mind.* Oxford University Press.

Suggestions for further reading

Books

Barber, N. F. (1969) *Water waves.* Wykeham.

Battan, L. J. (1962) Science Study Series No. 18. *Radar observes the weather.* Heinemann

Bishop, R. E. D. (1965) *Vibration.* Cambridge University Press.

Butler, S. T. and Messel, H. (eds) (1965) *Time.* Pergamon.

Graham Smith, F. (1966) *Radio astronomy.* Penguin.

Griffin, D. R. (1960) Science Study Series No. 4. *Echoes of bats and men.* Heinemann.

Hurley, P. M. (1960) Science Study Series No. 5. *How old is the Earth?* Heinemann.

Sanders, J. H. (1965) *The velocity of light.* Pergamon.

Tricker, R. A. R. (1965) *Bores, breakers, waves and wakes.* Mills & Boon.

Reprints and pamphlets

Bascom, W. (1959) 'Ocean waves.' *Scientific American* Offprint No. 828.

Bernstein, J. (1954) 'Tsunamis.' *Scientific American* Offprint No. 829.

Bullen, K. E. (1955) 'The interior of the Earth.' *Scientific American* Offprint No. 804.

Deevey, E. S. (1952) 'Radiocarbon dating.' *Scientific American* Offprint No. 811.

Frischmann, W. W. (1965) 'Tall buildings.' (From *Science journal*; not obtainable separately but due to be re-published as part of the Nuffield Advanced Physics publications in 1972 in *Physics and the engineer*, a collection of *Science journal* reprints.)

Gould, R. T. (1958) 'John Harrison and his timekeepers.' National Maritime Museum.

Griffin, D. R. (1958) 'More about bat "radar".' *Scientific American* Offprint No. 1121.

Heeschen, D. S. (1962) 'Radio galaxies.' *Scientific American* Offprint No. 278.

Hutchins, C. M. (1962) 'The physics of violins.' *Scientific American* Offprint No. 289.

Lyons, H. (1957) 'Atomic clocks.' *Scientific American* Offprint No. 225.

Oliver, J. (1959) 'Long earthquake waves.' *Scientific American* Offprint No. 827.

Westerhout, G. (1959) 'The radio galaxy.' *Scientific American* Offprint No. 250.

Information, formulae, and data

The information below contains some things you should know.
We have added some other things you may find interesting or useful.

Things you should know

Wave motions *superpose*. That is, if two or more wave motions arrive at one place, the resulting motion is the sum of all the motions each wave would have produced on its own, each superposed on the others. (This is not quite generally true, however, for if the resulting amplitude is large, new effects can arise. Two sea waves near the shore, each on its own small enough to be smooth, can add together to produce a wave that is tall enough to break.)

Interference is the name usually given to cases of superposition where waves arrive at a place from two sources, or from one source by two paths, and combine to form zero resultant (destructive interference), or a resultant larger than either wave alone (constructive interference).

Diffraction, the effect on a beam of waves produced by making it go through an aperture, is also the result of superposition. Here the wave motions from different parts of the aperture combine to produce larger or smaller effects, at particular places, than are observed if the wave is not limited by an aperture.

There will be more about these matters in Unit 8, *Electromagnetic waves.* Unit 4 merely introduces some of the ideas.

The *frequency* f and the *wavelength* λ of a wave motion are related to the *speed* v by $v = f\lambda$. This is always true, but v need not be constant, if λ or f vary.

Other formulae you will need

The equation which governs the motion of a harmonic oscillator is

$$a = -(k/m)s$$

where a is the acceleration of a mass, m, restrained by a spring of force constant, k; s is the displacement of the mass from its rest position. The minus sign indicates that the force is opposite to the displacement; that is, it acts towards the rest position. In calculus notation, the equation is

$$d^2s/dt^2 = -(k/m)s$$

A solution of this equation describes just how s changes with time t.

One solution is

$$s = A \cos \omega t$$

where A is the amplitude and ω has the value $\sqrt{k/m}$. The frequency f of the oscillator is given by $2\pi f = \sqrt{k/m}$. The quantity $2\pi f$, equal to ω, is called the angular frequency. The periodic time T, equal to $1/f$ is

$$T = 2\pi/\sqrt{k/m}.$$

Wave speed formulae

You may find these interesting, and perhaps helpful for problems or investigations.

Sound in solid rods	$v = \sqrt{E/\rho}$	E = Young modulus ρ = density
Compression waves along masses linked by springs	$v = x\sqrt{k/m}$	x = spacing between mass centres k = spring constant of each spring m = mass of each lump
Sound in a gas	$v = \sqrt{\gamma p/\rho}$	p = pressure ρ = density γ = numerical factor, between about 1.2 and 1.7, varying from gas to gas. (When compressed or expanded, if no heat flows in or out, the gas follows the rule pV^γ is constant.)
Transverse waves on taut strings	$v = \sqrt{T/\mu}$	T = tension μ = mass per unit length
Deep water waves (wavelength much less than depth)	$v = \sqrt{g\lambda/2\pi}$	g = Earth's gravitational field (acceleration due to gravity) λ = wavelength
Shallow water waves (depth less than wavelength); examples: tides and river bores	$v = \sqrt{gh}$	h = water depth g as above
Water ripples (wavelength only a few millimetres)	$v = \sqrt{2\pi\sigma/\lambda\rho}$	σ = surface tension λ = wavelength ρ = density

Frequencies of some mechanical waves/Hz	10^{-4}	10^{-2}	1	10^{2}	10^{4}
	earth-quakes	sea waves	ripples	sound	ultra sound

Speeds of some mechanical waves/m s^{-1}	10^{-1}	1	10	10^{2}	10^{3}
	ripples	flexing waves on mast stays	wind raised sea waves	sound in air waves on guitar strings, 'tidal waves' (tsunamis)	sound in metals

Electromagnetic waves

Frequency/Hz		10^{18}	10^{16}	10^{14}	10^{12}	10^{10}
	gamma ray	X-ray	ultra-violet	infra-red		radar television
Wavelength/m		10^{-10}	10^{-8}	10^{-6}	10^{-4}	10^{-2} 1

So far as we know, all the waves in the above 'spectrum' travel at the same speed, close to 3×10^{8} m s^{-1}. There are theoretical reasons for expecting there to be such a family of electrical waves travelling at this speed. Unit 8, *Electromagnetic waves*, will outline some of these reasons.

The speed has been measured at many different wavelengths. High accuracy is easier to achieve at some wavelengths than at others. Here is a collection of the experimental evidence that these waves all have the same speed, taken from French, A. P. (1968) *Special relativity*, Nelson.

The speed of electromagnetic waves

Wavelength/m	Speed/10^{8} m s^{-1}
6.4	2.9978 ± 0.0003
1.8	2.99795 ± 0.00003
1.0	2.99792 ± 0.00002
1.0×10^{-1}	2.99792 ± 0.00009
1.2×10^{-2}	2.997928 ± 0.000003
4.2×10^{-3}	2.997925 ± 0.000001
5.6×10^{-7}	2.997931 ± 0.000003
2.5×10^{-12}	2.983 ± 0.015
7.3×10^{-15}	2.97 ± 0.03

Speed of sound in various substances

	Speed/m s^{-1}
Air (dry, temperature 273 K)	331.46
Air (dry, temperature 90 K)	181
Carbon dioxide (dry, temperature 273 K)	258, rising towards 268.6 at high frequencies
Water (pure, temperature 293 K)	1482.9
Sea water (3.5 per cent salinity, temperature 288 K)	1507.4
Aluminium	
(thin rod)	5102
(bulk material)	6374
Steel	
(mild, thin rod)	5196
(bulk material)	5960
Copper	
(thin rod)	3813
(bulk material)	4759
Brick ('rod' form)	3650
Granite (bulk material)	5400
Sandstone (bulk material)	2920
Perspex (thin rod)	2177
Wood	
(oak, with grain, thin rod)	4100
(pine, with grain, thin rod)	3600

Note The speed of sound in a rod, which is thin compared to the wavelength, is given by $\sqrt{E/\rho}$, as on page 112. The speed of sound in bulk material, with wavelength less than the thickness, as in many earthquake waves, is larger.

Index

Where significant information is contained in an illustration or diagram, the page reference is italicized.

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This *Students' book* contains a summary of Unit 4, *Waves and oscillations,* and questions on its main work. The Unit is divided into three Parts: 'Waves of many sorts', 'Mechanical waves', and 'Mechanical oscillations'. The book also includes answers to the questions, chapters on 'The Severn Bridge' and 'Mathematical models in physics', a list of background reading, and lists of relevant formulae and data

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