Physics

Teachers' guide Unit 3

Field and potential

Nuffield Advanced Science

Physics Teachers' guide **Unit 3**

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Nuffield Advanced Science

Nuffield Advanced Physics team

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Field and potential

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people, in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator of the Nuffield Foundation Science Teaching Project

The Teachers' guide

This volume is intended to contain whatever information and ideas are required for the day-to-day teaching of the Unit. Not every teacher will need all of it all of the time; sometimes the summary and list of experiments will come nearer to meeting the need.

The main text contains, on the righthand pages, a detailed suggested teaching sequence, which teachers can adopt or adapt. The facing lefthand pages carry practical details, suggested questions, references, and background information for teachers in the form of a commentary on the text. This commentary also indicates aims of the teaching, and points out links with other parts of the course.

At the end, there are some appendices containing material needed on occasion only and lists of apparatus and teaching aids for the Unit. These include details of books and articles referred to in this *Guide*.

Introduction

Unit 3 is concerned with the ideas of field and potential, developed for the electric and the gravitational fields. It can follow naturally from Unit 2, *Electricity, electrons, and energy levels*, especially if, as in the suggested teaching sequence, a start is made with the parallel plate capacitor.

Experience in the trials suggests that Unit 2 followed by Unit 3 may be too long a stretch of electricity for some classes. If so, Unit 4, *Waves and oscillations*, which was originally planned as a break from electricity, could be taken between these Units.

It is one aim of Unit 3 to suggest that the ideas of field and potential are rather general and are very useful. Their generality and usefulness are brought out, we hope, by switching attention from the uniform electric field in Part One to the gravitational field in Part Two, and back to the electric field of a point charge in Part Three. The gravitational field, for which an inverse square law situation is the natural problem to discuss, is used to develop the idea of variations of potential in regions where the field is not uniform. These ideas are taken over in Part Three for the electrical inverse square law, illustrating the value of a formal analogy. The gravitational discussion illustrates the practical value of field and potential by centring around data from a real space flight.

The usefulness of the ideas will also, we hope, be illustrated by the work of Part 4, when the concepts of field and potential are applied to a discussion of the bonding of an ionic crystal, sodium chloride, whose structure has previously been mentioned in Unit 1, *Materials and structure*. Part Four also provides an opportunity for students to practise the moves back and forth between field and potential and between force and energy. The idea of a potential well, developed in this Part, will be used again in Unit 10, *Waves, particles, and atoms*.

These ideas are also used in Unit 5, *Atomic structure*, to discuss the scattering of alpha particles by nuclei. It would be possible to omit Part Four ('Ionic crystals') and make Unit 5 contain the first uses of field and potential. Such a strategy would better fit into a sequence in which Unit 4 came before Unit 3. But experience in the trials did not suggest that the work on ionic crystals is as daunting or as long as it may seem, and many students have responded favourably towards it.

The order suggested within Unit 3 will not suit all teachers. Some will prefer to start with the inverse square law, either for electricity or for gravity. Those who start with the electrical inverse square law will need to modify Part Three, which starts with an experimental exploration of the potential around a charged sphere, using ideas from Part Two and a tool (the flame probe) from Part One. Our view, in choosing to start with the uniform field, was that the relation between field and potential is difficult enough without the added complication of space variations of field.

Kinetic theory of gases

Besides illustrating the usefulness of electric field and potential concepts, Part Four shows how the behaviour of matter can be discussed in terms of basic principles. In the Nuffield O-level Physics course, this job is done by a discussion of the kinetic theory of gases. Of the two examples, the kinetic theory is the more significant for physics as a whole, and ought not to be missed. Students who missed it at O-level might do better if the time for Part Four were given to a brief account of the kinetic theory. See Nuffield O-level Physics, *Teachers' guide IV*. We hope that the O-level Pupils' Guide, *Molecules and motion* (shortly to be published), will also be helpful.

Summary of Unit 3

Time: about 4 weeks.

(Numbers in brackets refer to suggested experiments, listed on page 6.)

Part One
The uniform electric field

Time: about a week.

This Part develops the ideas of field and potential for the uniform electric field between the parallel plates of a capacitor, continuing work on capacitors begun in Unit 2. The constant ε_0 is introduced, later to be compared with the constant $1/4\pi\varepsilon_0$ in Coulomb's law.

Suggested sequence

Electric field; force on a charge between charged plates (3.1); uniformity of field (3.2). $E = V/d$ from energy argument. Shapes of electric fields (3.3). Relationships between field, charge, area, spacing, and potential difference. Experiments with reed switch and parallel plate capacitor (3.4), electrometer and parallel plate capacitor (3.5) to test $Q \propto V$, $Q \propto A$, $Q \propto 1/d$. Tests of reed switch, and series and parallel combinations (3.6). The constant ε_0 (3.7). Introduction of flame probe for investigating potential variations (3.8).

Part Two
Gravitational field and potential

Time: less than a week.

This Part starts with a general discussion of the field concept and of action at a distance, and then goes on, using data from a space flight, to show how the ideas work, and to develop the concept of the potential at a point and its variation as $1/r$ in a $1/r^2$ field of force.

Suggested sequence

Field and action at a distance. Revision of Newton's gravitational field equation, and the value of G (3.9). Use of Newton's Laws to calculate velocity and energy changes of a spacecraft, giving a rough check on the inverse square law.

Gravitational potential introduced by considering changes of kinetic energy of a coasting spacecraft. Field $= -\mathrm{d}V/\mathrm{d}x$. The $1/r$ variation of potential, checked by space flight data. The field and potential near a large sphere. The concept of potential at a point, referred to zero potential at infinity (3.10).

Part Three

The electrical inverse square law

Time: about a week.

This Part uses the results of Part Two in the analogous electrical problems, starting with variations of potential near a charged sphere. The inverse square law is used to explain the uniformity of the field found in Part One, and the Coulomb's Law constant $1/4\pi\varepsilon_0$ is introduced.

Suggested sequence

$1/r$ variation of potential around a charged sphere (3.11); value of constant in $V = kQ/r$ (3.12). Formal analogy with gravitational field. Coulomb's Law, and tests of the Law (3.13). Relation between uniform and radial fields; the constants ε_0 and $1/4\pi\varepsilon_0$. Comparison of size of electrical and gravitational forces. Mapping fields.

Part Four

Ionic crystals

Time: 2 double periods.

This Part shows how the ideas developed so far can be used in a problem of microscopic explanation. The work is offered as a detailed sequence of questions in the *Students' book*, and is summarized in this *Guide* on page 93.

Choosing one's own path

We hope and expect that teachers will find their own ways of using the material in this Unit. The detailed teaching programme laid out in the following pages represents as good a way of handling the material as we have been able to find in the light of experience in the trials, but should not be thought of as more than a possible, fairly well tested way of achieving the aims we decided upon. No doubt others can and will do better.

But teachers will know that it is the detail that counts in successful teaching, and so the *Guide* is full of particular teaching suggestions and practical details. We hope that these will help those who are uncertain how to handle either new material or old material taught in a new way for unfamiliar aims.

The summary and list of experiments will, it is hoped, assist those who have taught the course a few times and no longer need to refer to all of the detailed teaching suggestions, as well as those who feel confident that they can make up their own teaching programme out of their previous experience. We also hope that the summary will provide an overall view of the work suggested. Such a view is necessary for keeping a sense of perspective and direction, both when one is immersed in particular detailed teaching suggestions and comments, and when students lead the teaching off in an unpredictable direction by contributing their own ideas.

It seems fair to add that the summary, taken on its own, could mislead. It cannot easily indicate the aims of pieces of work in any precise way, or find words to express the relative seriousness or lightness of particular episodes. Nor should a phrase one might find in a current examination syllabus always be taken here to imply the same work as it would imply there.

Experiments suggested for Unit 3

Part One

The uniform electric field

Time: about a week

Part One deals only with the uniform electric field between parallel plates, linking earlier work on capacitors with new ideas, particularly with $E = V/d$, and then introducing ε_0 and the capacitance of a pair of plates. We think that the relation of field to potential difference is hard enough without the added complexity of a field that varies with distance.

It is suggested that the idea of the electric field between two charged plates be introduced briefly in the first lesson or two, to be followed by a series of experiments with capacitors.

Millikan experiment

A student may have started this long experiment in Unit 2 (experiment 2.24). Others may have tried question 69 in Unit 2, which discusses the force on a charged drop between charged plates. The work of Part One of Unit 3 can help to summarize and complete the earlier discussion.

Practical applications

Appendix A gives some information about capacitor microphones, an electrical seismograph, electrical spraying, 'static' charges, electrostatic loudspeakers, and electric potential prospecting methods. Some could be mentioned now, while others can find a place later when they are more directly relevant. See also Felici, 'Electrostatic engineering' (list of books, page 114).

Students' book

This contains an article on 'Thunderstorms' which shows how ideas of charge and field can be used to gain some understanding of how thunderstorms and lightning occur.

Questions 1 to 3 discuss charges on capacitors in circuits, bridging Units 2 and 3. They could be used at the start of Unit 3, or delayed until experiments 3.4 to 3.6, which need the results, especially for the sharing of charge between capacitors in parallel.

Electric fields

Forces between electric charges are important in physics, not least because matter seems to be made of electrically charged particles. In Unit 1, *Materials and structure*, there was a suggestion that salt crystals are made of positive sodium ions and negative chlorine ions nestling together. Unit 3 develops the ideas needed to enable students to think further about the forces between these ions so as to understand how ions cling together in a salt crystal.

In Unit 2, *Electricity, electrons, and energy levels*, electrons were accelerated by a potential difference between metal electrodes in gas-filled tubes. In the Millikan experiment, charged drops were held poised in the space between charged plates. In Unit 3, there is a closer look at the forces on charges in such situations.

The study of electric fields and forces between charges has practical importance too, in such matters as the design of cathode ray tubes for television sets, or in understanding 'static' charges on gramophone records or aircraft, as well as in investigating nature's most dramatic electrical effect: the thunderstorm. It is also a start on the job of understanding radio and television.

Forces on charges between charged plates

A start can be made by following up the problem of the force on a charge between metal plates. The plates can also be thought of as a capacitor. A series of brief demonstrations illustrates aspects of the problem.

Demonstration

3.1 The shuttling ball

14	e.h.t. power supply
65	metal plate with insulating handles *2*
57L	table tennis ball coated with Aquadag
57K	reel, nylon sewing thread
1001	galvanometer (internal light beam)
51G	polythene strip
503–6	retort stand base, rod, boss, and clamp *3*
1000	leads

See figure 1 and Nuffield O-level Physics, *Guide to experiments III*, experiment 96. Connect the plates to the e.h.t. supply, with 50 MΩ in series on one side and with the galvanometer in series with the other side, which *must* be earthed.

The coated table tennis ball hangs from an insulating strip on about half a metre of nylon thread. Place it between the plates, so that it clears each by about 10 millimetres. When the p.d. is raised to about 4 kV, the ball shuttles backwards and forwards and the galvanometer indicates a current.

Students' book

Questions 4 and 5 discuss the shuttling ball and charged foil experiments.

Demonstration
3.1 The shuttling ball

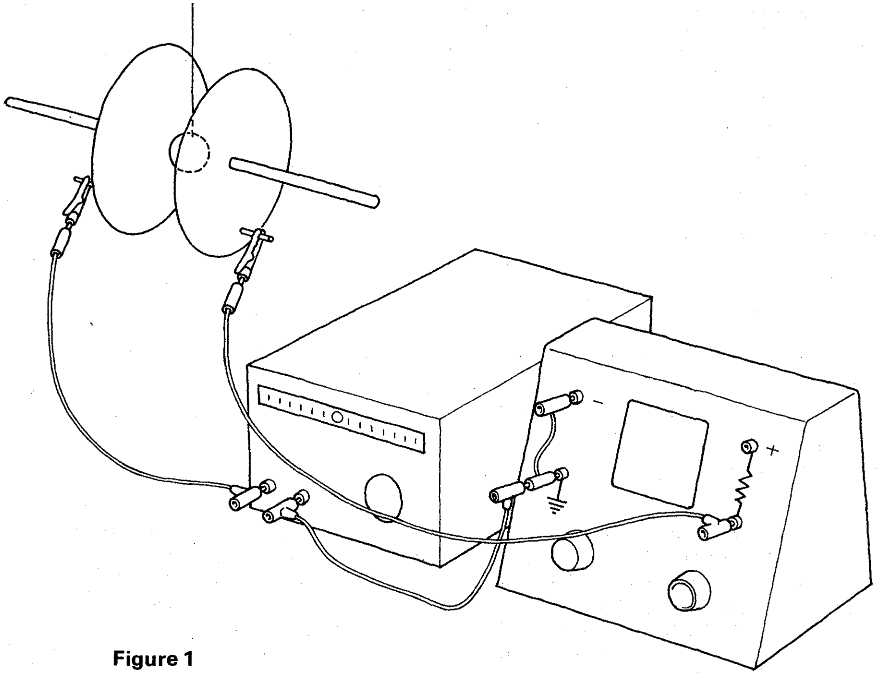


Figure 1
The shuttling ball.

A conducting table tennis ball hangs between plates connected through a galvanometer to a source of high potential difference.

The ball shuttles back and forth, being pulled first one way and then the other, the direction depending on the sign of the charge it acquired from the plate it last touched.

The galvanometer shows a current, supporting the view that the ball carries charge. The fact that the current has the same direction when the ball carries positive charge one way as when it carries negative charge the other way needs a little discussion.

It is clear that there is a force on such a charged ball, the direction of the force depending on the sign of the charge.

Demonstration

3.2 Force on a charged strip of foil

1025 capacitor plates *1 pair*

30 slotted base *2*

51M square polythene tile *2*

51G polythene strip

1054 foil (see below)

1053 adhesive tape

1053 razor blade

14 e.h.t. power supply

Optional: means of projection

1000 leads

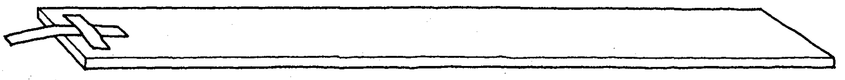


Figure 2
Foil attached to insulating strip.

A narrow strip ($\frac{1}{2}$ mm × 20 mm) of aluminium foil is cut, picked up on adhesive tape, and stuck to the polythene strip with about 10 mm projecting. The best foil to use for this is aluminized plastic film (25 gauge), but it is not easy to obtain. Very thin plastic sheet (sold as 'cooking film') coated with Aquadag forms an acceptable alternative. Aluminium leaf (item 10G or 58A) may be used but is difficult to handle. The foils used in the packing of cigarettes, and as chewing gum wrappers have been used successfully. Lurex – sold for knitting – contains a metallized plastic foil which can be 'unpicked' and which works well.

Normal aluminium cooking foil is too thick, while the foils sold for gold leaf electroscopes are too thin and too delicate.

The plates are held vertically, each in a slotted base resting on a polythene tile. Put the plates about 0.5 m apart, and parallel.

Connect the plates to the e.h.t. power supply, earthing the negative terminal. Charge the foil by touching it onto the plate connected to the positive side of the supply, and place the foil between the plates. It bends, as shown in figure 3, and the voltage can be adjusted until it bends by about 45 degrees.

The foil can be used to explore the field, moving it sideways, parallel to the plane of the plates, showing that there is little change in the force on the foil until it is quite near the edges of the plates. Moving the foil from one plate to the other shows that the field is also uniform in this direction. It is usual for the foil to increase its deflection a little as its tip approaches a plate, because the charge on the foil pulls extra opposite charge onto that region of the plate. If the effect is marked, the foil is too thick or too stiff, and carries too large a charge for a given deflection.

Following an argument that the force should depend on the ratio of the potential difference to the spacing of the plates, make a rough test by halving (or doubling) both the spacing and the potential difference. The foil hangs at the same angle as before, showing that the force on it is unchanged.

It may help to project an image, or a shadow, of the foil on a screen.

Demonstration

3.2 Force on a charged strip of foil

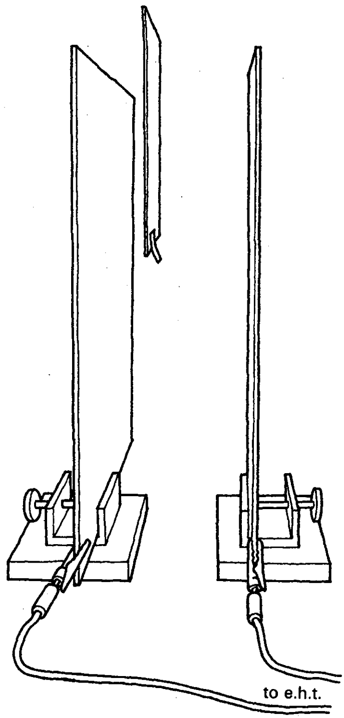


Figure 3
Exploring the field between two parallel plates with a charged strip of foil.

A smaller charged object, such as a sliver of foil on an insulating handle, can be used to find out where the force is big, where it is small, and how it varies from place to place. (Figure 3.)

The class may be asked to guess what will happen when the foil, charged by touching it onto a plate or onto the terminal of the e.h.t. supply, is put into the space between the plates.

There is a force on the charge on the foil, and the foil bends. It does not bend when held outside the plates, nor when it is held so that its direction of easy bending lies parallel to the plates. The force on it is directed from one plate to the other.

If the foil is moved parallel to the plates, then it bends by much the same amount wherever it is, unless it comes close to the edges of the plates. The name 'electric field' can be introduced, to describe a region where there is a force on a charge. The field, or the force on a charge, is constant (uniform) across much of the width of the plates.

Students' book

Question 6 looks again at the energy argument for the force on a charge in a uniform field, supplementing Unit 2, question 69.

Note to teachers

The foil strip bends equally well after it has been touched onto the earthed plate, since the field induces a charge on it. But it is best to touch it only onto the plate connected to the terminal of the power supply that is not earthed, to avoid having to discuss induced charges.

Optional further demonstration
Uniform field (Millikan apparatus)

The Millikan apparatus (see Unit 2, experiment 2.24 and *Students' laboratory book*) can be used to show with greater accuracy than the rough experiment with the foil that the force on a charge between parallel plates is constant both across the width of the plates and, a fact which is crucial to the argument opposite, from plate to plate.

Show that a charged drop balances at the same p.d. wherever it is along a line joining the plates. The drop can be moved up or down by raising or lowering the p.d. To show that the drop is balanced at the same p.d. even if it is moved sideways, tilt the apparatus about the line of sight through the plates, when the drop will drift sideways.

Use a large brightly illuminated drop which falls rapidly (not more slowly than 1 mm in, say, 5 seconds) so that the experiment is quick, easier to see, and not confused by Brownian motion.

The demonstration is not easy with a large class, but offers a good chance to discuss the experiment in outline with the whole class.

Students' book

Question 6 could replace some of the discussion of the measurement of field either in $V\ m^{-1}$ or $N\ C^{-1}$. Questions 7 to 11 deal with some applications of electric fields.

$$\boldsymbol{E} = \lim_{q \to 0} F/q$$

Strictly, the field is the limit of the ratio F/q as q becomes as small as one pleases. It is defined in this way because the electric field due to a conductor carrying a charge will be modified by the test charge q, which, if positive, will pull extra electrons into regions of conductor close to it. The ratio F/q then *correctly* measures the electric field at the test charge *in the presence of the test charge*, but this field is not the same as that which existed before the test charge was introduced.

Occasionally, it is also wrongly supposed that any change in the field due to the test charge is the result of the field of the test charge being 'added' to the original field. If the original field were due to charges which could not move, the test charge could have any magnitude and the field due to those charges would be correctly given by F/q.

Moving the foil from one plate to the other shows that the force, and the field, are constant (uniform) in this direction also.

Increasing the p.d. or decreasing the spacing between the plates raises the force on the foil. An argument, possibly already used in Unit 2 in connection with the Millikan experiment, now suggests why.

Energy argument for the dependence of force on p.d. and spacing

Suppose the foil, or a charged oil drop in the Millikan experiment, moves from one plate to the other, with a steady force F acting on the charge q on the foil or drop. If the force is constant, as it seems to be, the energy transformed by the electrical force is Fd, if d is the distance between the plates.

But there is a p.d., V, between the plates. Ask again for the meaning of potential difference (energy transformed per coulomb passing), so that the energy transformed is also seen to be qV.

Thus, $Fd = qV$
or $F = qV/d$

If the argument is right, the force on the foil, and so the amount it bends, will be unchanged if both V and d are changed by the same factor, say, halving both. This can be tested.

Measuring electric fields

The force on the charge is, on this argument, proportional to the charge. Indeed, in the Millikan experiment, the force, together with the ratio V/d, is used to measure the charge, for

$$F/q = V/d$$

The ratio F/q, which happens to be V/d in this case, enables one to calculate the force on a known charge, as one might do in designing an accurate Millikan experiment or an 'electrostatic' loudspeaker. Or it enables one, having measured the force, to calculate the charge.

The ratio has a name, the electric field E, and is a measure of how large the force-on-a-charge effect will be at a place, if a charge is put there to feel the force. Between the plates, the field E has the same value at many places. In general, at a place where there is a force F on a charge q, the size of E is specified by:

$$F = qE \quad \text{or} \quad E = F/q$$

The field is in the direction of the force.

For practical purposes, the relation $E = V/d$ for parallel plates is very convenient, enabling the field to be found with a voltmeter and a ruler and avoiding the difficult business of measuring forces on charges.

Textbooks

$E = V/d$ between parallel plates. See:

Bennet, *Electricity and modern physics,* Chapter 10.4.
Rogers, *Physics for the inquiring mind,* Chapter 33, page 548.

Photographs of electric field patterns appear in:

PSSC, *College physics,* Chapter 24.
PSSC, *Physics* (2nd edition), Chapter 27.

Demonstration
3.3 Electric field patterns

14 e.h.t. power supply
or
60/1 Van de Graaff generator

149 electric field apparatus

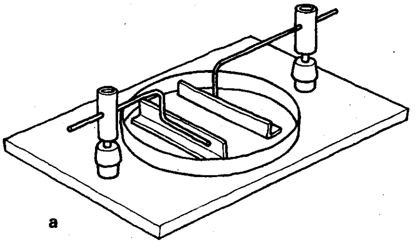
1053 castor oil

1053 semolina
or
chopped hair (see below)

1056 carbon tetrachloride

1054 copper wire (bare, 14 s.w.g.) for electrodes if necessary

1000 leads



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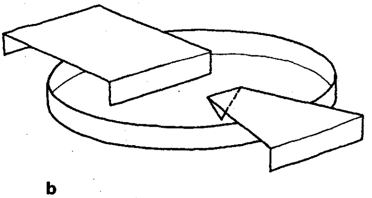


b

Figure 4
a Electric field apparatus.
b Improvised electric field apparatus.

See Nuffield O-level Physics, *Guide to experiments III*, experiment 95a.

The demonstration is improved if it is shown on an overhead projector. The apparatus can be improvised from a Petri dish and electrodes made from suitably shaped metal strips. See figure 4*b*.

Semolina floats on carbon tetrachloride, so the demonstration can be done with semolina floating at the interface between a layer of carbon tetrachloride upon which floats a layer of castor oil. The latter prevents the carbon tetrachloride from evaporating. The semolina particles are then mobile enough to be oriented under a p.d. of 5 kV.

Note that carbon tetrachloride vapour is particularly dangerous if inhaled with tobacco smoke.

In experiment 3.2, the potential difference may have been 4 kV and the spacing 40 mm, giving a field E of 10^5 volts per metre (symbol $V\ m^{-1}$).

That means that a charge of 1 coulomb in the space would have a force of 10^5 newtons on it. Actually, so large a charge would push or pull electrons about on the plates, and alter the field out of all recognition. Indeed, the presence of the 4 kV supply would barely be noticeable any more. 10^5 N is close to the weight of a 10 tonne mass.

But clearly the charge on the foil was much less than one coulomb. If the force were equal to the weight of a fly (say a mass of one milligramme, weight 10^{-5} N), the charge would be 10^{-10} coulomb.

Such an example serves also to bring out that the field measured in volts per metre also gives the force on a charge in newtons per coulomb.

'Seeing' electric fields

Iron filings make it possible to 'see' the field of a magnet. The electrical forces on needle-shaped fragments of insulating material make it possible to 'see' electric fields in a similar way. Good physicists and engineers can often sketch the probable shape of the field produced by the electrodes of, say, a proposed design of electron gun, using a mixture of experience and intuition.

Demonstration

3.3 Electric field patterns

The class should see the shape of the field between a pair of straight conductors. Note the straightness of the field in the middle, where its strength, as shown in 3.2, is uniform. Note also the curvature near the edges.

The non-uniform fields between two point conductors, between a point and a straight conductor and around a point conductor inside a ring, are worth seeing, to emphasize the special nature of the uniform field. In general, there is said to be an electric field wherever there is a force on a charge, but usually the field changes magnitude from place to place, and also changes direction.

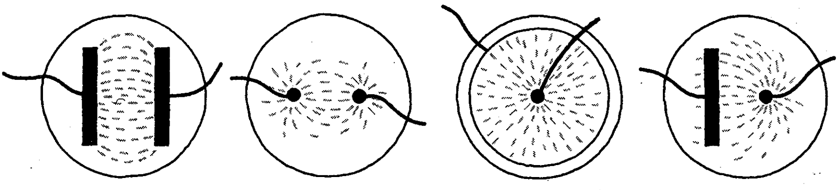


Figure 5
Electric field patterns.

Semolina sinks in castor oil, but if the oil and semolina are stirred before applying the p.d., the demonstration works with the oil only, especially if the potential difference is increased by taking it from a Van de Graaff generator. The potential difference should be removed as soon as the particles are orientated.

Very short chopped hairs, cut from a clean dry paint brush, are a good substitute for semolina in castor oil. Again, the Van de Graaff generator is the best source of potential difference. The hairs should be less than 1 mm long.

Castor oil seems to absorb moisture and become conducting if it is left exposed to the atmosphere. The particles, and the oil, may then move about in an interesting but, for the present purpose, confusing way.

Field – abstract or 'real'?

We suggest 'showing' the field after a less visual, more abstract introduction to emphasize that a field is something people thought of, not something to be caught hold of. But some classes may be happier the other way around. See also Appendix B, on lines of force.

Superposition

It has been supposed that E might be proportional to Q, and so that V might be proportional to Q. At root, this is a question about the superposition of electric fields. If one lot of charge (spread over some plates) produces a field E, and if another lot of charge that would, by itself (on the same plates) produce field E' is added to the first, will the new field be $E+E'$? It is, as it happens. Fields superpose linearly. Not many students will appreciate so abstract a discussion.

Test of $Q \propto A$ for fixed E (or fixed V/d)

Conceptually, it would be easier to vary the area and measure how much charge had to be put on the plate to produce a certain field. Practically, it is easier to control the field (using fixed p.d. and spacing) and measure how much charge flows onto plates of varying area. In the end, the test is the same; ultimately a test of whether the field is determined by the surface density of charge on the plates.

Reasons for speculating about area and spacing

Two artificial teaching attitudes are, we think, best avoided. One is, 'We will investigate what factors affect . . . and the factors we shall choose are . . .', which announces the vital features of the problem and leaves only the more trivial to be discovered. The other is, 'Although the answers to this problem are to be found in books, we will pretend they aren't and set out on a voyage of discovery . . .', which is embarrassing. The speculations opposite about spacing and area should, if possible, be seen as attempts to limit inquiry to things it would be reasonable to investigate or, perhaps, to suggest why books discuss area and spacing, not colour and plate thickness.

Speculation: the field and the charge that produces it

The charged plates used previously are also one type of capacitor, and the plates carry charges $+Q$ and $-Q$ when there is a potential difference across them. Why is there a field, that is, a force on another charge q, between the plates? Presumably the charges $+Q$ and $-Q$ push or pull on the 'test' charge q.

One could ask whether the field would be proportional to the charges Q on the plates.

Is V/d $(=E)$ $\propto Q$?

Other questions can follow. Was $Q \propto V$ for other capacitors? (Yes.) What is Q/V? (The capacitance C.) So if $Q \propto V/d$ or, the same thing, $C \propto 1/d$, the electric field is proportional to the charge that produces it.

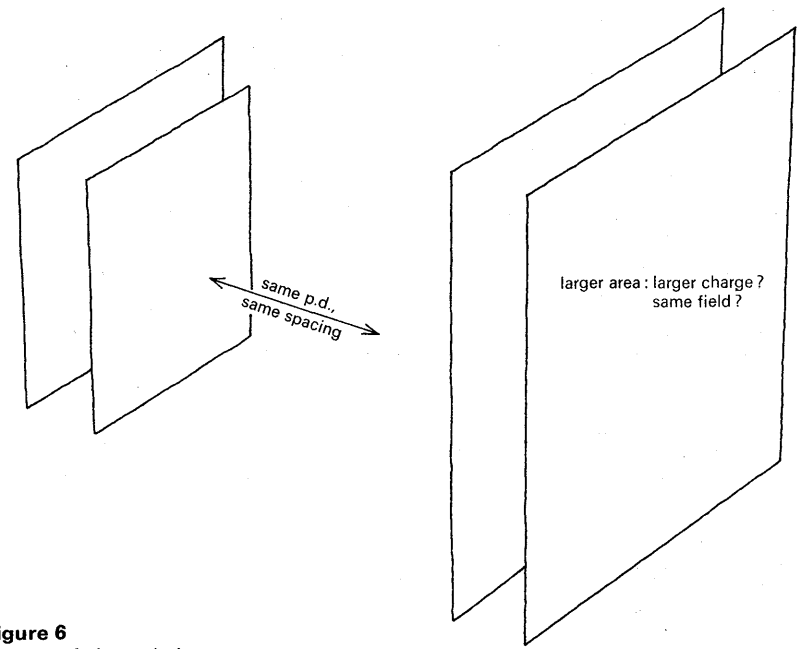


Figure 6
Changing the area of charged plates.

What about the size of the plates? Suppose, as in figure 6, that the plates were increased in area but kept at the same potential difference and spacing. Would the charge stored be larger? If so, would charge be proportional to area? Would the field, presumably still equal to V/d, be the same?

Organization of a group of experiments

It is suggested that a number of experiments with parallel plate capacitors, experiments 3.4, 3.5, and 3.6, should be assigned to individual groups of students, who would later present their findings to the rest. These experiments probably deserve written reports.

Because of the variety of tasks and of apparatus, notes for all the experiments are provided in the *Students' laboratory book*. With adequate briefing beforehand, the practical work should not run over more than two practical sessions.

The arrangement of this group of experiments is balanced between complication and economy. It would be simpler but expensive to have parallel plates and reed switches for all students, for example, though this might also make the reed switch seem more important than it is.

With the quantities suggested for a class of 16 (see *Teachers' handbook*), two groups can use the large square parallel plates and reed switches (experiment 3.4), while two more groups can use other metal plates and an electrometer (experiment 3.5) in less accurate versions of the same experiments. Two further groups can test the reed switch on simpler, commercial capacitors and go on to connect capacitors in series and in parallel (experiment 3.6).

If students work in pairs, and there are more than twelve students, some would thus be left unoccupied. Larger groups of students could work together on this occasion. Or some could start or continue one of the long experiments; the Millikan experiment (experiment 2.24) being especially appropriate. It may be that a group could try the flame probe demonstration, experiment 3.8, being given suitable instructions beforehand.

Alternative route to field $\propto$ charge density

Teachers may prefer to treat the experiments as investigations of the variation of capacitance with area and spacing. Then given $C \propto A/d$, and writing $C = \varepsilon_0 A/d$, recall that $C = Q/V$.
Thus, $Q/V = \varepsilon_0 A/d$ or $Q/A = \varepsilon_0 V/d$
that is, charge density $= \varepsilon_0 \times$ field.

The important thing is to make it clear that the discussion is about charges on plates and the field they produce, and not to lose sight of this in a chain of algebra.

The last point is a little subtle. If the field is uniform, it ought to be equal to V/d and to be unchanged by an increase in area of the plates. But a student might reasonably expect a larger charge on a larger area (correct) and a larger field from this larger charge (not correct). It may be best to bring out in discussion how field depends on charge *density* after some experimental tests, rather than before, keeping now to simple speculation about factors worth the trouble of investigating.

One possible trial would be to overlap a pair of plates by a varying area A, with V/d kept constant, and to see if Q is proportional to A. This amounts to testing whether the capacitance is proportional to area.

Experiments with capacitors

Experiments to investigate the suggestions about the interdependence of charge, p.d., spacing, and area can now be shared among the class, together with others on related themes.

The points in question are:

Is $Q \propto V$ for parallel plates?
Is $Q \propto 1/d$ at constant p.d., and to V/d?
Is $Q \propto A$ for constant p.d. and spacing, that is, for constant field E?

Or, the same question, does a certain value of charge density Q/A determine the size of the field E, that is, V/d?

If all these results held, they could be put together as

$$\frac{Q}{A} = \varepsilon_0 \frac{V}{d}$$

replacing the proportionality by a constant, ε_0. The name and value of ε_0 can be discussed later. It need not be mentioned at all at this point, keeping to a proportionality for the time being.

Another way to summarize the possible relationship would be to write

$$C = \varepsilon_0 \frac{A}{d}$$

or $$C \propto \frac{A}{d}$$

But there is no need to do so yet if the extra algebra would confuse.

Reed switches

The reed switch is a device to enable small quantities of charge to be measured by repeatedly passing the charge through a sensitive ammeter. It is used in the circuit shown in figure 7 in experiments 3.4 and 3.6. The 'tools' section of the *Students' laboratory book* gives practical details, which teachers who are unfamiliar with the device may find useful.

The switch may be vibrated at up to 400 Hz by an alternating current, so that a capacitor may alternately be charged from a battery and discharged through a meter. A 50 Hz vibration rate is effective for many purposes, though not when the charges are very small. The current I at frequency of switching f then indicates that a charge q was on the capacitor, q being found from

$$I = fq$$

The voltage across the reed switch contacts is limited to about 25 V for most switches, though some can be bought which operate at higher voltages. These, however, only work at up to 50 Hz. The instructions below assume that the low voltage, high frequency switch is being used.

In each experiment, the protective resistor should have as high a value as is consistent with complete discharge of the capacitor in each switching operation. An oscilloscope can be connected across the resistor to check the point.

Experiment
3.4 Reed switch and a parallel plate capacitor

1010	reed switch	
1009	signal generator	
1025	capacitor plates with 16 polythene spacers ($10 \times 10 \times 1.5$ mm)	*1 pair*
15	h.t. power supply (for 25 V smooth d.c.)	
	or	
59	l.t. variable voltage supply	
	and	
1064	low voltage smoothing unit	
1004/3/2	voltmeter (100 V) and (10 V)	
1001	galvanometer (internal light beam)	
1017	resistance substitution box	
158	class oscilloscope	
501	metre rule	
32	1 kg weight	
1000	leads	

See *Students' laboratory book* for fuller details.

Small spacers, 10 mm square and about 1.5 mm thick, are used to separate the plates.

The signal generator is used to drive the reed switch at as high a frequency as possible, up to 400 Hz, using the circuit of figure 7. The protective resistor should be about 100 kΩ.

$Q \propto V$: record values of the galvanometer current for various voltages. The 100 V range meter is needed for voltages from 10 V to 25 V.

$Q \propto 1/d$: the spacing d can be varied by stacking spacers. It is convenient to vary d from one to four spacer thicknesses.

Introduction of the reed switch

There may be a need to introduce the reed switch, referring back to experiments in Unit 2 (experiment 2.14) in which high value capacitors were discharged in bursts through milliammeters. The notes on the reed switch in the *Students' laboratory book* may, however, be sufficient by themselves. The class as a whole can consider the action of the switch when those who have done experiment 3.6 report their work.

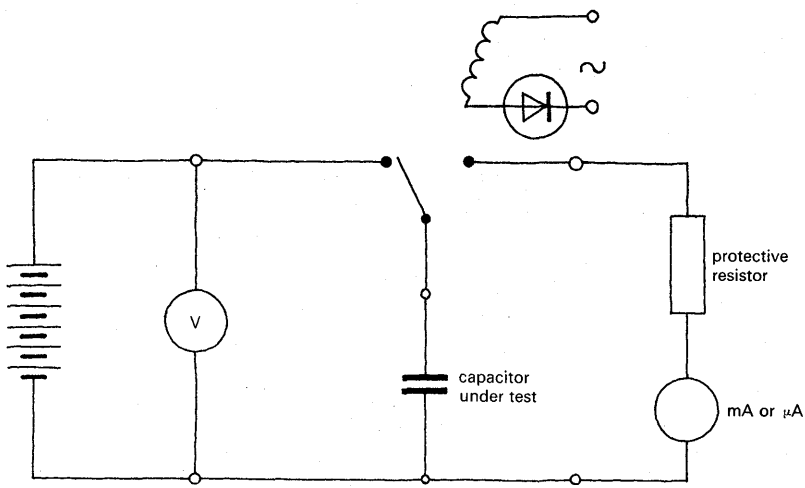


Figure 7
Basic reed switch circuit.

Experiment
3.4 Reed switch and a parallel plate capacitor

The proportionality of Q to V may be tested by plotting the reed switch current against the potential difference.

The spacing d can be varied in equal steps by stacking small insulating spacers at the corners of the plates. The spacers are small, so that their own influence on the charge stored is negligible. If desired, a sheet of insulating material can be sandwiched between the plates to show that the charge stored is larger than for the same air gap. (The ratio is, of course, the relative permittivity of the insulating material, neglecting any edge effects.)

The area can be varied by altering the area of overlap of the plates, it being undesirable to cut them up.

Voltages of 0–500 V are suitable, with plate spacings of 5–25 mm. It is helpful in interpreting the graph of Q against $1/d$ to have a measurement of charge stored when the plates are a long way apart, say half a metre.

A plate spacing of 10 mm is suitable for an experiment to test whether Q is proportional to V.

It is desirable to test whether nearly all the charge has passed to the 0.01 μF capacitor by trying the charge measurement a second time, having discharged the 0.01 μF capacitor but without having charged or discharged the plate. See Unit 2, page 60, for a note on this aspect of the use of the electrometer. It can be discussed, if need be, when the results of experiment 3.6 on the parallel connection of capacitors are reported.

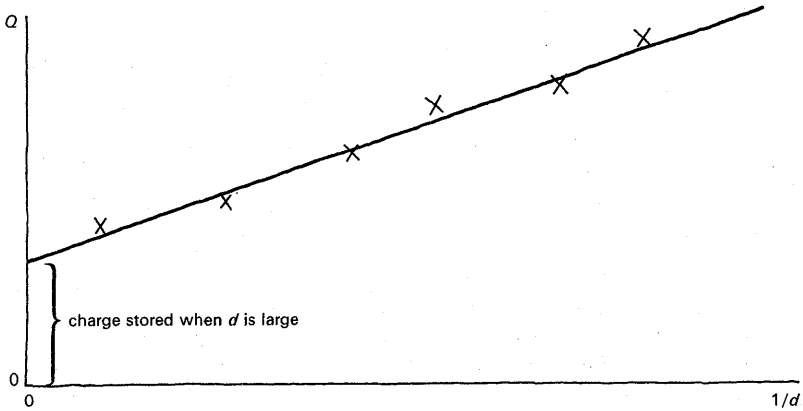


Figure 9
Graph of Q against $1/d$.

$Q \propto V$ using the electrometer: circular argument?

A student may notice that the charges are being measured by supposing that the p.d. across the input capacitor of the electrometer is proportional to the charge on it, and thus, that a test of whether the p.d. across parallel plates is proportional to the charge on them seems to assume what it sets out to test.

In Unit 2, experiment 2.16, charges were spooned onto the electrometer and it emerged that the p.d. across its input capacitor rose in proportion to the charge. It is then fair to use the device to test the point for other capacitors.

Students' book

Question 2 deals with the sharing of charge, used in experiment 3.5.

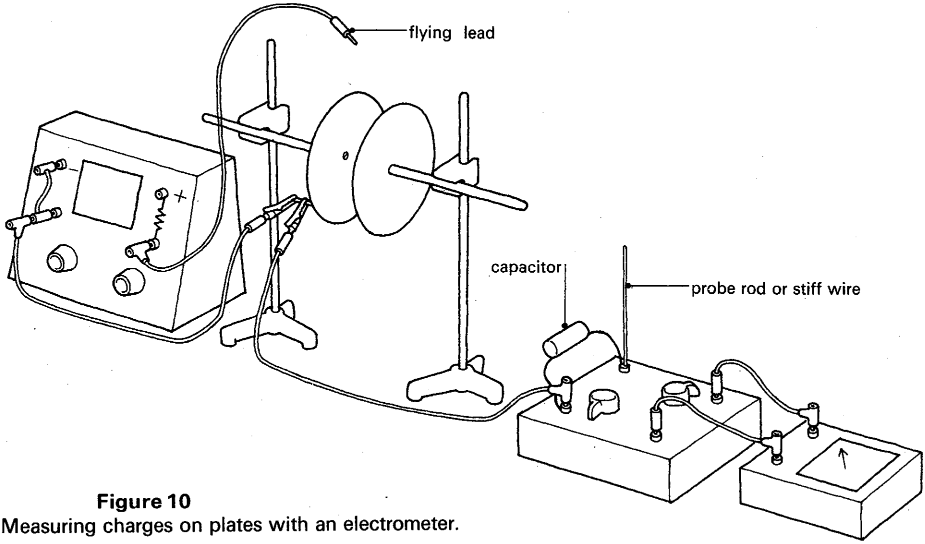


Figure 10
Measuring charges on plates with an electrometer.

Even so, the electrometer readings will show noticeable variations, and its sensitivity may drift. Students should be encouraged to make several observations of each value, so as to estimate the range within which the value may lie. This is the more important because the charge stored does not fall to zero as the spacing is made very large, so that a graph of Q against $1/d$ (figure 9) does not pass through the origin.

The intercept should agree with the value of the charge stored when d is large, due to the capacitance between the plate and the bench or other nearby objects.

Experiment
3.6 Test of reed switch; capacitors in series and in parallel

1010 reed switch

1009 signal generator
or
27 transformer (if fixed frequency will serve)

1002 microammeter (100 μA)

1003/1 milliammeter (1 mA)

1018 capacitance substitution box *2*

1017 resistance substitution box
or
1051 resistor 220 Ω *and*
resistor 2 kΩ

1051 capacitor, paper, 2 μF

1040 clip component holder *2*

1004/2 voltmeter (10 V)

1033 cell holder (four U2 cells each) *2*

158 class oscilloscope

1000 leads

The circuit is shown in figure 7, page 23, where the reed switch is described. For capacitances of a few microfarads, use the milliammeter and a 220 Ω protective resistor. For capacitances of a few tenths of a microfarad, use the 100 μA meter and a 2 kΩ protective resistor.

These protective resistors are suited to a reed switch frequency of 50 Hz. At higher frequencies they may need to be reduced, so that the capacitor is effectively discharged within the time available between switching operations. The completeness of discharge can be investigated with an oscilloscope connected across the capacitor.

Possible experiments are to show that Q is proportional to V; to show that the reed switch current is proportional to the switching frequency; to measure a capacitance; to investigate parallel and series combinations.

Students' book

Questions 1 and 3 develop the parallel and series relationships for capacitors. The series connection is only of use in this course in Unit 6, *Electronics and reactive circuits*. The parallel connection is needed on more occasions, for instance in understanding experiment 3.5, and in any case illustrates the important idea of charge conservation.

The constant ε_0

ε_0 is here presented as a measurable quantity, which relates charge density to electric field. Later, it, or rather $1/4\pi\varepsilon_0$, will also relate the field and the force between two charges to the magnitude of those charges.

Appendix C gives a fuller discussion of the role of the constant ε_0 in electricity and, in particular in the SI. It advises against using words which imply that ε_0 is an 'electrical property' of a vacuum whose value depends on some supposed behaviour of a vacuum, especially if that supposed behaviour is compared with the behaviour of an insulator of relative permittivity ε_r which might fill the space between parallel plates.

Experiment

3.6 Test of reed switch; capacitors in series and in parallel

The story told about the reed switch deserves some investigation, and the action of the reed switch can be discussed by the class as a whole when this investigation is reported.

a A capacitor of known nominal value can be used in the circuit of figure 7, and a check made on whether the current has the expected value. Unless the capacitance is known accurately, a more precise estimate can now be made by working backwards from the measured current.

b The frequency can be varied, to see if the current is proportional to the frequency. Students who work quickly can use an oscilloscope to investigate the effect of using a very large protective resistor, large enough for the discharge to be incomplete at high switching frequencies.

c Capacitors can be connected in parallel and in series. In parallel, the charges add up; in series the charge stored is less than that of either capacitor on its own.

Summary and the value of ε_0

As reports of experiments 3.4, 3.5, and 3.6 are given, the work so far can be summarized. The constant ε_0, its value, and its meaning can be introduced.

The results of the experiments can be collected together in two equivalent ways.

a Remembering that V/d is the electric field, the field produced by charge on the plates is given by

$$Q/A = \varepsilon_0 \, V/d$$

or charge density $= \varepsilon_0 \times$ electric field, ignoring any difficulties about variations in charge density at the edges of the plates.

ε_0 is a constant, making the proportionality into an equality. Its value can be found very roughly from the results of experiment 3.5 with the electrometer. A more accurate measurement can then be planned.

Electric field and volts per metre

An experiment with a flame probe, experiment 3.8, which looks at the variation of potential across a parallel plate capacitor, could be introduced into the discussion during this summary. It appears later in the *Guide* only so that the summary can be concise and unbroken.

Students' book

Questions 13 to 18 deal with several aspects of the charge on and p.d. across a parallel plate capacitor.

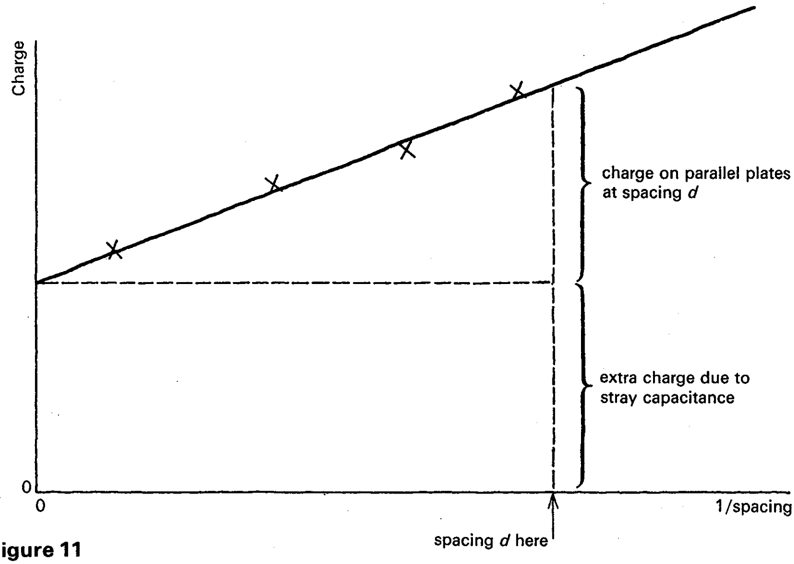


Figure 11
Finding the charge stored on the parallel plates.

The charge on the parallel plates can be found, as in figure 11, for some particular plate spacing, and the charge density written down, after the area of one plate has been measured. The electric field can also be written down. ε_0 is the ratio of the charge density to the electric field, and an estimate of the order of

$$10^{-11}\,\frac{\mathrm{C\ m^{-2}}}{\mathrm{V\ m^{-1}}}$$

should be obtained.

b Since $C = Q/V$, the results may also be summarized as

$$C = \varepsilon_0 A/d$$

giving a way of calculating the capacitance of a parallel plate capacitor (approximately, neglecting edge effects which raise the capacitance somewhat).

Units for ε_0

The last equation, $C = \varepsilon_0 A/d$, gives the simplest-looking unit for ε_0, farads per metre, symbol $\mathrm{F\ m^{-1}}$.

Taking ε_0 as the ratio of charge density to electric field gives units of either

$$\frac{\mathrm{C\ m^{-2}}}{\mathrm{V\ m^{-1}}} \text{ or } \frac{\mathrm{C\ m^{-2}}}{\mathrm{N\ C^{-1}}}$$

These may be reduced in several ways, a useful one being to $\mathrm{C^2\ N^{-1}\ m^{-2}}$. The quoted value is close to $8.854 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$.

Long experiment

3.7 Measurement of ε_0

1010 reed switch

1009 signal generator

1025 capacitor plates with spacers *1 pair*

15 h.t. power supply (25 V)
or
59 l.t. variable voltage supply
and
1064 low voltage smoothing unit

1004/3 voltmeter (100 V)

1001 galvanometer (internal light beam)

1017 resistance substitution box

501 metre rule

1055 micrometer or Vernier callipers

1000 leads

Full details are given in the *Students' laboratory book* in the section on long experiments. This one is essentially the same as experiment 3.4, but the measurements now include the frequency of the signal generator, the current sensitivity of the galvanometer (which can be taken from the maker's data), the p.d. across the plates, the area of one plate, and the spacing.

Demonstration

3.8 Flame probe investigation of voltages in a parallel plate capacitor

flame probe (see page 34)

51A, gold leaf electroscope with hook

14 e.h.t. power supply

94A lamp, holder and stand

27 transformer

52K crocodile clip

1025 capacitor plates *1 pair*

51M square polythene tile *2*

30 slotted base *3*

51G polythene strip

1054 copper wire, bare, 22 s.w.g., 0.5 m

503–6 retort stand base, rod, boss, clamp

1000 leads

Long experiment

3.7 Measurement of ε_0

One group of students can now, or later, attempt a reasonably accurate measurement of ε_0, using the reed switch and the large, square capacitor plates. They should be able to improve considerably on the rough estimate suggested above.

Demonstration

3.8 Flame probe investigation of voltages in a parallel plate capacitor

This experiment, which may form part of the summary above, has two aims. Firstly, to consider again the relation between field, potential difference, and distance, and secondly, to suggest that the strange talk so far of effects 'in' the space between the plates may be a fruitful way of thinking.

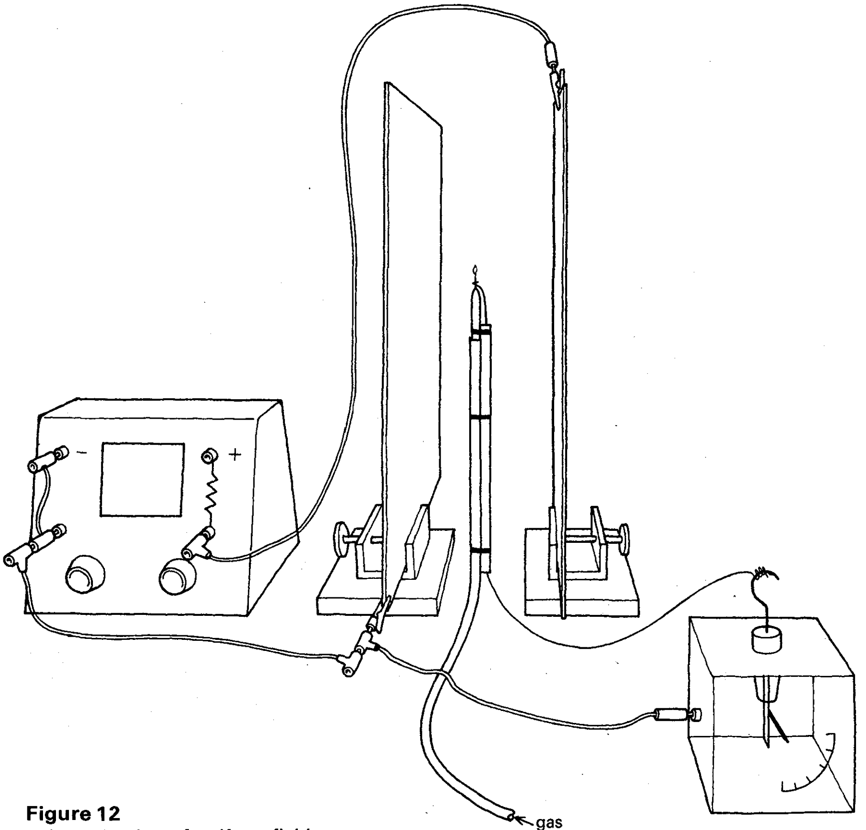


Figure 12
Flame probe investigation of uniform field.

Construction of a flame probe

92X PVC covered copper wire

1055 hypodermic syringe (1 cm³)

1055 25 gauge hypodermic needle

1055 PVC tubing (2 m, 6.5 mm bore)

1055 Perspex rod (0.5 m, 10 mm diameter)

1053 adhesive tape

1053 razor blade

The probe can be made from a plastic disposable hypodermic syringe. The plunger is removed and the open end of the body is cut off so that a length of PVC tubing can be pushed over the syringe body. This junction should be gas-tight.

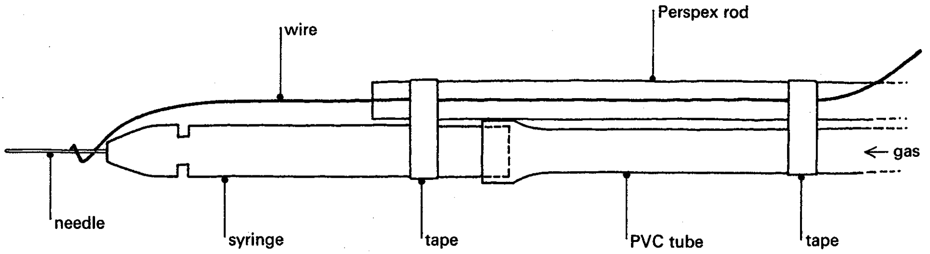


Figure 13

The syringe and PVC tube are fastened with Sellotape, at about 0.2 m intervals, to the Perspex rod. A piece of wire 2 m long is hooked to the base of the needle (as above) and taped to the rod as well.

Using the probe

The PVC tube is connected to the gas supply and the tube filled with gas by removing the needle and turning the supply on for a moment or two. The needle should then be replaced, the gas turned full on, and the jet lit. The probe is used with as low a flame as possible. To obtain this, reduce the gas supply slowly until the jet goes out (the red-hot end of the needle can be seen to 'disappear'), increase the supply a little, and re-light.

It is a little more difficult to light the jet with natural gas. If the gas pressure is too high, the jet will not burn at the needle and it is necessary to reduce the supply very slowly to find the right adjustment at which it will. A Hoffmann clip across the supply tubing makes the adjustment a little easier.

The free end of the wire is taken, clear of the bench or other objects, to a hook on a gold leaf electroscope, the terminal on the case being earthed to an e.h.t. power supply. A lamp and transformer should be used to throw a shadow of the leaf onto the ground glass side of the electroscope case. The electroscope should be calibrated beforehand by connecting the e.h.t. supply directly across it. It may prove possible to use the electrometer (item 1006), suitably adapted, in place of the electroscope.

A wire connected to an electroscope, previously calibrated as a voltmeter, is moved about in the space between the plates. When the tip of the wire carries a flame (any source of ions would serve) the probe reaches the potential of its surroundings, and if it is moved, it appears that the difference in potential between one (earthed) plate and a point between them varies linearly with distance along a line joining the plates.

A student may suggest moving the probe along a line parallel to the plates, when the voltage indicated by the electroscope should stay constant.

The action of the probe needs some explanation, which should be as brief as possible. Suppose the probe, with no flame on it, is moved from outside until it is close to the positive plate. Some of the electrons in the wire connected to the probe, which is not electrically connected to anything, for it ends at the electroscope leaf, will be pulled to the tip. This negative charge at the tip helps to counterbalance any effect of the positive charge on the plate, and indeed the electroscope gives only a small, maybe zero, indication.

The behaviour is much the same if the probe is put into the space inside a coil of wire which is connected to the e.h.t. supply. (Figure 14.)

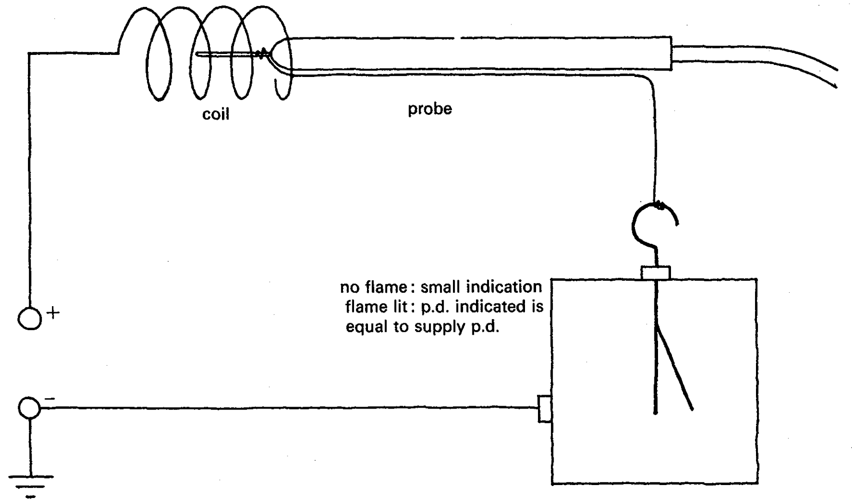


Figure 14
Flame probe tested inside a coil at a high voltage.

In the flame probe experiment, a p.d. of 1.5 kV across the plates is suitable. The plates can be about 0.15 m apart, standing in slotted bases on the polythene tiles.

For the test with the coil (figure 13), a coil of 5 to 10 turns, about 10 mm diameter and 10 mm long, is suitable. The coil can be taped to the end of a polythene strip which stands in a slotted base. The coil is connected to the positive e.h.t. through 50 MΩ, the earthed negative supply being joined to the electroscope case. The probe should not touch the coil.

Without the flame, if the coil is at 2 kV, the electroscope may indicate about 0.1 kV. When the flame is lit, the probe reaches the same voltage as that supplied to the coil.

What p.d. does the electroscope indicate?

The electroscope indicates the p.d. between the leaf and the earthed case. When the flame probe has settled down, no current flows along the conductor between probe and leaf, so there is zero p.d. across this conductor. If one plate is earthed, the p.d. between that plate and the probe is the same as that between the electroscope leaf and its case.

$$E = -\frac{dV}{dx}$$

Some classes will be able to move on now to this more general relationship. Most will do better to develop it gradually, starting with the similar gravitational case in Part Two and then that of space-varying electric fields in Part Three. But those who can take it should meet it now.

Later use of the flame probe

The probe will be used in Part Three to investigate the variation of potential near a charged sphere, giving indirect evidence of the inverse square law of force for charges which is rather hard to obtain directly in the conditions of a school laboratory.

Students' book

See 'Thunderstorms' for a use of a similar potential probe.

But when the flame is lit, there are ions of both sorts near the tip. The flame is not necessary: a little man sitting on the tip throwing pairs of ions into the air would serve as well. The negative ions are pushed away from the tip if there is negative charge on it, but the positive ions tend to stay near the tip, being attracted by its charge. So, if it is negatively charged, it loses negative charge. If it were positively charged, the story would be the same, but it would lose positive charge. Sooner or later, depending on how fast ions are produced, the tip will end up with no charge.

With the probe inside the coil, when the flame is lit the electroscope soon shows that the probe has reached the voltage of the supply (the p.d. between probe and earth is the same as the p.d. between the coil and earth).

With the probe placed between the plates, when the flame is lit the electroscope indication also rises to a steady value. This value depends on where the probe tip is placed, and is less than the p.d. across the plates.

There being no charge on the probe tip, the p.d. between the earthed plate and the probe is the same as the p.d. would be between the earthed plate and the place where the probe is, whether the probe is there or not. This is also the p.d. indicated by the electroscope.

Variation of voltage with distance

Arguing, as earlier, that a charge moving a distance, x, along the direction of a constant field, E, will transform energy such that $V = Ex$, the potential difference between the probe and the earthed plate ought to rise linearly with distance x, if the field is truly constant.

If the field were not constant, V would not rise in equal steps with equal distances. It is interesting that one does seem to be able to measure the p.d. not just between the plates, but between one plate and a point in the space in the middle. Such a result might incline one the more to think it sensible to say that there is an electric field 'in' the space.

Alternative: empirical test of probe

If the discussion of why the probe works would not go well (perhaps because it leaves out too much), the experiment can be treated as an empirical test of the probe itself. Accepting that the field is constant, the probe is put into such a constant field to see if it does measure potential differences between plates. As it seems to do so, it can be used later on to investigate other situations. No more need be said about the flame than that the electroscope has capacitance, and that a current, provided by the ions in the flame, is needed to charge it up.

Part Two

Gravitational field and potential

Time: rather less than a week

This Part tries to generalize the notion of a field by switching attention to the gravitational field. The gravitational field is also used to discuss the idea of potential, and to develop the description of inverse square law situations. It seems the natural way to approach both these problems, since firsthand experience and evidence from reports of space flight and astronomy can be called upon. In particular, the variation of potential as $1/r$ when the field varies as $1/r^2$ is developed here, for later use in electric field problems.

Talking about fields

Part Two begins by raising some general questions about fields, in a philosophical spirit. Sixth form students are often ready for a little philosophy, though the teacher will need to play it gently.

Teachers will have to decide how much, or how little, of this abstract discussion their classes can take. Students often have worries about what a field 'really is' and it may help them to know that others have shared this concern.

The idea of a field has proved very useful to physicists, and many are content with that, brushing aside philosophical problems, while some still wish to emphasize the strangeness of this idea. But students should realize that the concept is a human invention, which has value in so far as it enables people to think more easily or more clearly about some kinds of problem.

Students' book

Questions 19 and 20 raise some of these general issues. Question 19 is more suited to class discussion than to individual work.

Sources

The extracts from Newton's letters appear in Newton, *Principia*, University of California edition, pages 633–4, and are reprinted by permission of the Regents of the University of California. They are discussed in Hesse, *Forces and fields*, Chapter VI, which is a book teachers would find helpful.

Reading

Gamow, 'Gravity' (*Scientific American* Offprint).

Gamow, *Gravity* (book).

Feynman, *The character of physical law.*

Rothman, *The laws of physics* (Chapter 7).

A particularly useful specific piece of reading comes from *The Feynman lectures on physics*, Volume I, Chapter 7.7. This is reprinted in the *Students' book*. Feynman discusses here whether one could ask why gravity obeys a simple law. He describes a model in which space is filled with randomly moving particles, out of which an inverse square law of attraction emerges, because bodies shield each other from impacts with the particles. He then shows what goes wrong with this model, and says that no one has ever found any other which has not a similar flaw. It could be used as the basis for a short piece of teaching about laws and explanations. It has value as a sample of the way physicists talk.

The idea of a 'field'

Pairs of charges, masses, and magnets all seem to affect each other across space. Students may be helped by being invited once again to feel two strong magnets repelling each other.

Some people have found it strange that a thing can act where it is not; indeed some have said that the idea is absurd, thinking it 'obvious' that things can only push or pull each other if they touch. But physicists now often use pictures of 'influences across space'; indeed, on an atomic scale, 'touching' is understood as forces between atoms close together, and, as will appear later in the course, these are looked at in the same action-at-a-distance way.

Some physicists are inclined to say that because there are effects in space, the space is 'occupied' by a field. This view was adopted in Part One. Again, physicists like to talk this way for other action-at-a-distance forces. Over there is the Moon, getting on for half a million kilometres away, and over here are the tides going up and down: it is said that there is a gravity field and that that is why there are tides. What has this explained? Why bother to say it at all? Over there is a magnet, and here a compass – the same idea is used; it is said that there is a 'magnetic field'.

But this way of thinking has grown up only gradually, as people found the field idea useful. Newton thought it obvious that action across empty space was an absurdity, as the following extract from a letter to Richard Bentley (25 February 1692) shows:

> 'That gravity should be innate, inherent, and essential to matter, so that one body may act upon another without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking, can ever fall into it.'

(Note that for Newton, the word 'philosophical' meant almost the same as the word 'scientific' does nowadays.)

In an earlier letter, also to Bentley, Newton shows that he knows that the idea of a gravitational field is just a useful idea, and that to say 'there is a field' is not the same as explaining why things attract each other.

> 'You sometimes speak of gravity as essential and inherent to matter. Pray, do not ascribe that notion to me; for the cause of gravity is what I do not pretend to know, and therefore would take more time to consider of it.'

O-Level astronomy

Students who have taken the fifth year of Nuffield O-level Physics, with its survey of planetary astronomy, will need little extra here. Others may need more, and the teacher will have to judge how much.

Textbooks

Students can be referred to:

Arons, *Development of concepts of physics*. (Chapter 15.)
Holton and Roller, *Foundations of modern physical science*. (Chapters 6–12.)
Nuffield O-level Physics Pupils' Guide, *Astronomy*.
Project Physics, Reader, Unit 2, *Motion in the heavens*. (Chapter 12.)
Project Physics, Text, Unit 2, *Motion in the heavens*. (Chapter 8.)
PSSC, *College physics*. (Chapter 15.)
PSSC, *Physics*, 2nd edition. (Chapter 21.)
Rogers, *Physics for the inquiring mind*. (Chapters 1, 22–24.)

Long experiment

3.9 Measurement of the gravitational force constant *G*

1026	kit to make gravitational constant apparatus	
535	bottle of mercury (100 cm^3 needed)	
94A, 94B	lamp holder and stand with shield	
524	mercury tray	
1055	soft container (e.g. polythene beakers) to hold flask	*2*
501	metre rule	
50/1	cylindrical magnet	
507	stopwatch or stopclock	
1054	graph paper (mm squares)	
1053	aluminium foil	
1053	polythene bag (about 0.25 m square)	
1053	rubber band	
1053	Durafix glue	
1053	adhesive tape	
1053	razor blade	

Assembling the apparatus, particularly the suspension, and then handling the apparatus require considerable patience and manipulative skill, and are not within the capacity of all students.

Full instructions appear in the *Students' laboratory book.*

Space flight problems

The first problems suggested illustrate gravitational forces and the inverse square law force field. Later problems help to develop the idea of gravitational potential. The problems suggested take students step by step from gravitational field to gravitational potential difference, and then to gravitational potential. At each step, a problem illustrates the meaning of algebraic relationships by means of a practical example.

The inverse square law for gravitational fields

Assuming that students are already aware that the gravitational field, or the force due to gravity, obeys an inverse square law, there can be some discussion of the evidence for the law. Newton guessed at an inverse square law to explain the motions of the Moon around the Earth and of the planets around the Sun, and it worked. Nowadays, satellites and lunar probes test the inverse square law whenever they fly: it has never been found wanting in predicting or explaining their paths.

The gravitational force F between point masses m_1 and m_2 a distance r apart is proportional to m_1, to m_2, and inversely proportional to r^2.

$$F \propto \frac{m_1 m_2}{r^2}$$

The size of the force for given masses and distances can be measured, giving

$$F = G\frac{m_1 m_2}{r^2}$$

where G is an experimentally measured constant, 6.7×10^{-11} newton for 1 kilogramme masses 1 metre apart. That is, $G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.

Long experiment

3.9 Measurement of the gravitational force constant *G*

The experiment is very difficult to do, though not impossible. It will be beyond the abilities of many students.

Students can be shown the apparatus, and should find it easy to see what the experiment would be.

The best stratagem may be to say that none of them will succeed in measuring G, and allow the occasional student who wishes to prove the teacher wrong to try. He or she may succeed; it has been done. The experience of trying to do such a difficult task may be valuable for the right student, whether or not a value of G emerges in the end.

Gravity and space flight

A series of calculations using data from real space flights can illustrate how such data support the inverse square law, give practice in calculation, and show what it means to say that there is a field varying from place to place. See table 1.

Film

Short, inexpensive 8 mm films of the *Apollo* flights can be obtained from the *Daily Express*. See page 114.

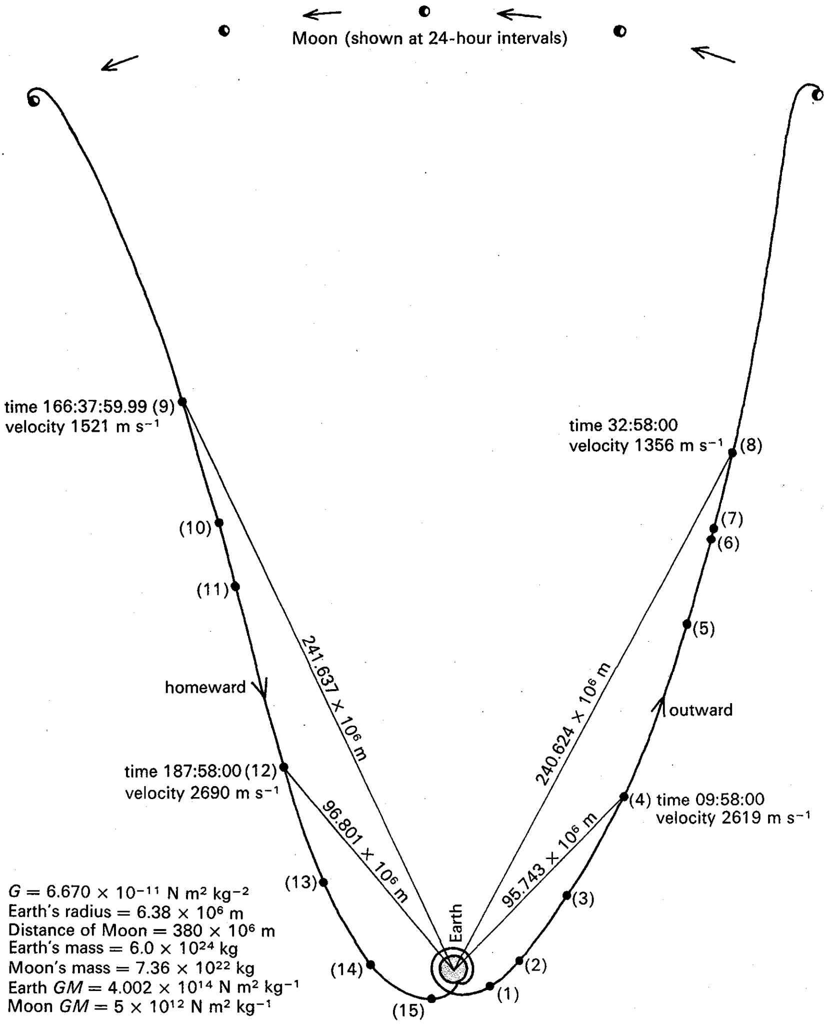


Figure 15
Sketch of typical trajectory for manned Moon flight (from plan for *Apollo* 12; data from *Apollo* 11).

Table 1 (opposite)
Data from *Apollo* 11.

Events	Ground elapsed time/hours: minutes: seconds		Distance r from centre of Earth/10^6 m	Velocity v/m s^{-1}
launch from surface of Earth	00:00:00			
ignition to inject into coasting orbit to Moon (5 m 20 s burn)	02:44			
rocket not burning: coast begins	03:08:00	(*1*)	11.054	8406
	03:58:00	(*2*)	26.306	5374
	04:08:00	(*2A*)	29.030	5102
	05:58:00	(*3*)	54.356	3633
	06:08:00	(*3A*)	56.368	3560
	09:58:00	(*4*)	95.743	2619
	10:08:00	(*4A*)	97.242	2594
	19:58:00	(*5*)	169.900	1796
	20:08:00	(*5A*)	170.945	1788
no rocket burn until this time	26:44:57.92	(*6*)	209.228	1531.56
3-second burn, changing speed and direction	26:45:01.47	(*7*)	209.232	1527.16
	32:58:00	(*8*)	240.624	1356
	33:08:00	(*8A*)	241.417	1352
after landing and setting back				
rocket burn	150:28			
coasting	166:38:00	(*9*)	241.637	1521
	166:48:00	(*9A*)	240.740	1524
	172:18:00	(*10*)	209.722	1676
	172:28:00	(*10A*)	208.737	1681
	178:28:00	(*11*)	170.891	1915
	178:38:00	(*11A*)	169.766	1923
	187:58:00	(*12*)	96.801	2690
	188:08:00	(*12A*)	95.241	2715
	191:48:00	(*13*)	56.368	3626
	191:58:00	(*13A*)	54.310	3699
	193:48:00	(*14*)	28.427	5201
	193:58:00	(*14A*)	25.640	5486
	194:38:00	(*15*)	13.311	7673
	194:48:00	(*15A*)	10.036	8854
rocket burn on re-entry	195:03			

Sources of data

The data used all come from the first manned Moon landing space flight, *Apollo* 11. It is taken from Ryan, *The invasion of the Moon*, and from computer print-out supplied by NASA.

The data have been converted from a variety of units, which may themselves have been converted from other units, so they contain rounding off errors.

Teachers will naturally use more recent data if they can be obtained.

Students' book

Questions 21 to 23 pose the problems about gravitational forces and fields. It is suggested that students try them first and discuss them afterwards. For the convenience of teachers the calculations are summarized in this *Guide*.

Calculations suggested in the *Students' book*

a Question 21 concerns simple calculations of acceleration, velocity change, and energy change.

The calculation concerns points (*6*) and (*7*) in table 1. The thrust from the rocket was 96 000 N on a mass of 44 000 kg, giving a rate of change of velocity of 2.18 m s^{-2}. The force per kilogramme is also 2.18 N kg^{-1}, of course. In a 3.5 s burn, the velocity change was therefore 7.64 m s^{-1}.

Had the thrust been directed along the direction of motion, the spacecraft would have travelled about 5350 m while the thrust was acting (travelling at about 1530 m s^{-1} for 3.5 s). The energy transformed would have been 5.14×10^8 J, or 1.17×10^4 J kg^{-1}, from the thrust and the distance travelled.

Had the thrust been at right angles to the path, the path would have been turned through an angle of 5.0×10^{-3} radian, very little energy being transformed.

b Question 22 asks for the calculation of the gravitational field, from the mean rate of change of velocity, between points (*7*) and (*8*). The velocity decreases by 171 m s^{-1} over 22 380 s, at 7.65×10^{-3} m s^{-2}, so the estimate of the mean gravitational field is 7.65×10^{-3} N kg^{-1}.

The mean distance is 225×10^6 m; at this distance GM/r^2 comes to 7.9×10^{-3} N kg^{-1}. Allowing for the disproportionately large field at the smaller distances makes the agreement worse. The discrepancy is probably the result of the small angle between the trajectory and the line to the centre of the Earth, so that the estimated field is the component of the field along the trajectory.

c Question 23 invites the plotting of a graph of gravitational field against $1/r^2$. The field is calculated for the pairs of points (*2*) (*2A*), and (*3*) (*3A*), and so on, by working out the rate of change of velocity between each pair of points.

	Distance from centre of Earth $/10^6$ m	Force per kilogramme /N kg^{-1}
(*2*) (*2A*)	27.668	0.453
(*3*) (*3A*)	55.362	0.122
(*4*) (*4A*)	96.493	0.042
(*5*) (*5A*)	170.423	0.013

Table 2

Force and field calculations

The following are examples of possible calculations.

a Simple calculations of acceleration, velocity change, and energy change when the rocket exerts a thrust. These can use the data at points (*6*) and (*7*) in table 1.

Such calculations can bring out the difference between force and energy calculations, and emphasize that force, acceleration, and velocity change are vector quantities.

This is also a good place to remind students, as part of a calculation, that the force per kilogramme exerted on the rocket is identical with the acceleration, as this point will be needed in later questions which use the data to find the gravitational field strength at different distances from the Earth from the rate of change of velocity. It is also convenient here to introduce the idea of calculating the kinetic energy per kilogramme. This will be needed in later questions about the gravitational potential difference, which is also the energy change per kilogramme.

b As an introduction to using the data to calculate the gravitational field, the velocity change between a pair of points such as (*7*) and (*8*) can be used. The rocket motor was not used between these points, so the mean rate of change of velocity can be compared with the gravitational force per kilogramme calculated from GM/r^2 for the mean distance r. The vector nature of the changes involved can again be pointed out.

c The data can be used to test the inverse square law for gravity. The data points are given in pairs, (*2*) and (*2A*), (*3*) and (*3A*), and so on, the two points of a pair being close together in time and distance, so that the velocity change between the two points divided by the time between points (actually 10 minutes) gives a good estimate of the rate of change of velocity at the mean distance. This gives the force per kilogramme at once.

The various pairs are well separated in time and distance, so that the test of the inverse square law can cover a wide range of distances. See figure 16.

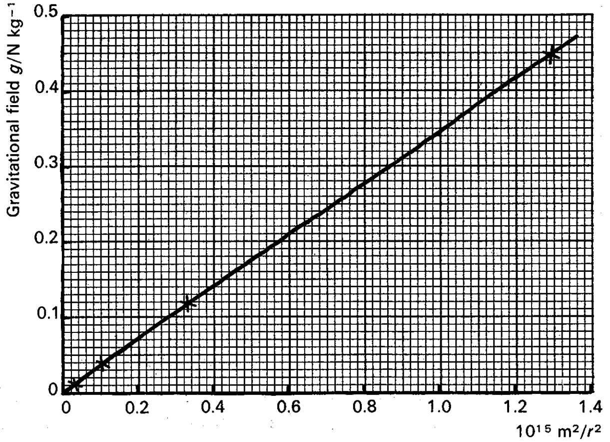


Figure 16
Gravitational field against $1/r^2$.

Students' book

Question 24 goes through the calculation of the kinetic energy changes of the spacecraft.

Rockets and fuel

Petrol is not, of course, used as a rocket fuel. A full discussion of the amount of fuel needed to lift a rocket is very complicated, and is not suggested. Partly because of the inevitable inefficiency of the rocket motor, the fuel situation is very critical, and the payload is only a tiny fraction of the total rocket mass. Although 'the fuel needed to get there' is necessarily an imprecise idea, it may help a student for whom gravitational potential difference is at first too abstract.

It may also help to say that the idea of a potential is useful in electricity too, and will be used in working out the energies of electrons in atoms and the energy of ions in a crystal.

Gravitational potential difference

Further problems using the space flight data can illustrate how, without any use of the rocket motor, the kinetic energy of the spacecraft falls as it moves away from the Earth, and rises as it approaches the Earth.

Kinetic energy changes

As a further problem, the kinetic energy changes can be calculated for values (*1*) and (*6*) and for values (*10*) and (*15A*). Again, it will be useful later, and easier now, to calculate the energy change for each kilogramme of the spacecraft. (Table 3.)

	Distance/m	Velocity v/m s^{-1}	Kinetic energy per kilogramme $\frac{1}{2}v^2$/J kg^{-1}
(*1*)	11.054×10^6	8406	35.33×10^6
			difference 34.16×10^6
(*6*)	209.183×10^6	1532	1.17×10^6
(*10*)	209.722×10^6	1676	1.41×10^6
			difference 37.78×10^6
(*15A*)	10.036×10^6	8854	39.19×10^6

Table 3
Changes of kinetic energy of *Apollo* 11.

Potential energy

The kinetic energy that vanishes on the outward journey can be got back again on the inward journey. It has been stored up meanwhile. It is also an experimental fact about gravity that the energy stored, and so the changes in kinetic energy, are exactly the same between pairs of identical distances. The data suggest as much, but because the distances are not exactly equal, they do not prove the point (and most of the difference is due to the small difference between the distances close to the Earth).

Question 24 looks at this matter more carefully.

The size of the energy changes

The change of kinetic energy per kilogramme calculated above was around 35×10^6 J kg^{-1}. From near the Earth's surface, the energy is nearer 60×10^6 J kg^{-1}.

The burning of one kilogramme of petrol yields around 50×10^6 J kg^{-1}. In view of the size of the energy changes per kilogramme of spacecraft, it is no wonder that a moon rocket is almost entirely fuel. The petrol would use all of its energy simply in getting itself there.

g as gravitational field

The quantity g, often called the acceleration due to gravity, is treated as the gravitational field in Nuffield O-level Physics. We here continue to use that helpful viewpoint.

The minus signs in field $= -\mathrm{d}V/\mathrm{d}x$ and in $V = -GM/r$

There is always a minus sign in the equation, field $= -\mathrm{d}V/\mathrm{d}x$, whatever the nature of the field, for movement in the same direction as the field results in a decrease in potential energy. Distance x is taken to increase along the field direction. The minus sign in the equation $V = -GM/r$ results from the forces being attractive, together with the convention that the potential energy of bodies a long way apart shall be called zero.

In general, common sense will serve for solving problems if one asks oneself whether the potential or kinetic energy will rise or fall as the result of a change. If in doubt about the formal rules, one goes back to physical understanding (not the other way around). But students who can cope with the formal rules should certainly be shown them.

Students' book

Question 28 presents the data in table 4, and uses them to check the integral both by direct calculation, and by plotting a graph to test the $1/r$ variation, as shown in this *Guide* in figure 17.

Integrals and area measurement

The integral of $1/r^2$ is simple, and delightfully compact, elegant, general, and powerful. It deserves emphasis with all those who can handle it.

But there will be students who do not understand what is going on. Some may not be able to integrate $1/r^2$ at all; others may be able to do it as a mechanical task without having much idea of what it means.

Appendix D suggests a means of helping these students to grasp what is being said when they are told the result of the integration. It involves plotting the $1/r^2$ field of the Earth, and adding up areas beneath the curve. The areas are then tested for agreement with the stated result of integrating by plotting them against $1/r$. The argument is laid out, step by step, in the optional question, 30, in the *Students' book.*

What decides the size of the energy change from place to place? This interests rocket engineers, who need to know the energy per kilogramme necessary to move from one place to another. This quantity is called the *gravitational potential difference*. Using ideas practised in the problems, it should be easy to write, for a small change in distance Δr and field g,

gravitational potential difference $= g\ \Delta r$

For the uniform electric field, the similar relationship $V = Ed$ was developed. Is there a uniform gravitational field? (Near the surface of the Earth.) If the field g does not decrease too much with change in height h, then

gravitational potential difference $= gh$.

Integration of $1/r^2$ field variation to obtain gravitational potential difference

For the spacecraft, the force varies with distance, so a simple force times distance calculation will not serve to compute the energy changes.

As long as the rocket motor is not used, any increase in kinetic energy of the spacecraft corresponds to an equal fall in potential energy.

Table 4 gives the kinetic energy per kilogramme of the spacecraft over two series of distances from the Earth, one on the outward journey and one on the return journey. The difference in kinetic energy per kilogramme between two points in any one series gives the gravitational potential difference between those points.

	Distance from centre of Earth $r/10^6$ m	**Kinetic energy per kilogramme $\frac{1}{2}v^2/10^6$ J kg^{-1}**	**10^8 m/r**
outward			
(*1*)	11.054	35.33	9.05
(*2*)	26.306	14.44	3.79
(*3*)	54.356	6.60	1.84
(*4*)	95.743	3.43	1.05
(*5*)	169.900	1.61	0.59
(*6*)	209.187	1.17	0.48
return			
(*9*)	241.637	1.16	0.41
(*10*)	209.722	1.41	0.48
(*11*)	170.891	1.83	0.58
(*12*)	96.801	3.62	1.03
(*13*)	56.368	6.57	1.77
(*14*)	28.427	13.52	3.52
(*15*)	13.311	29.44	7.52

Table 4
Kinetic energy variation with distance for *Apollo 11*.

Effect of the Moon

The distances used in figure 17 extend to some 200×10^6 m from the Earth, at which place the gravitational potential of the Moon is at most 0.03×10^6 J kg^{-1}, compared with 2×10^6 J kg^{-1} for that of the Earth. So the presence of the Moon does not appreciably affect the $1/r$ plot.

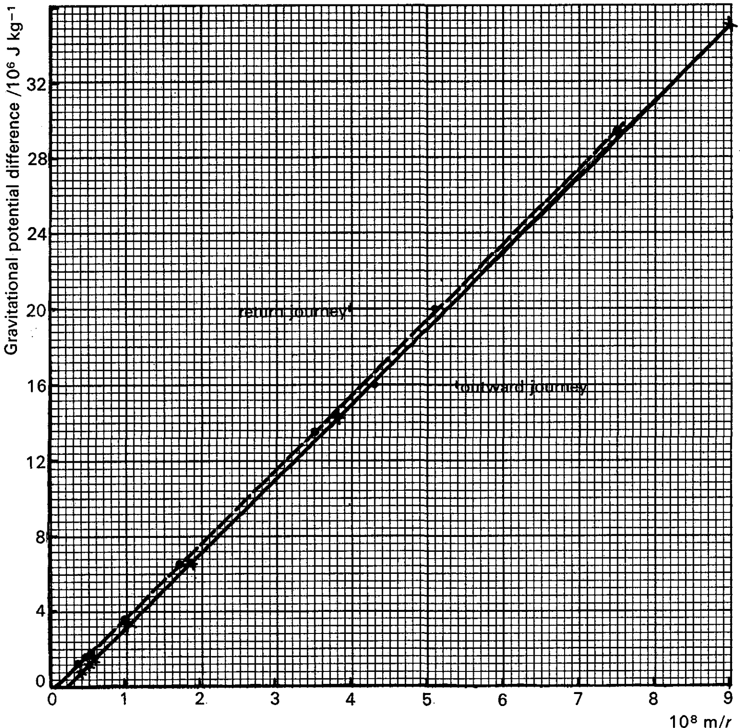


Figure 17

Sign of GM/r

In the integration and the argument leading to the graph shown in figure 17, the values of GM/r have not been given the minus signs proper to them on the convention that gravitational potential is zero at large distances. Teachers may prefer to introduce the sign now. We have deferred it until later, thinking it better to treat the potential energy changes as differences between terms GM/r_1 and GM/r_2, with the sign of the change – increase or decrease of energy – being argued out physically.

For example, at point (*1*) the kinetic energy per kilogramme is 35.33×10^6 J kg^{-1}; at point (*2*) it is 14.44×10^6 J kg^{-1}. The gravitational potential difference between points (*1*) and (*2*) is 20.89×10^6 J kg^{-1}.

Can this difference be calculated from the known way the force varies between these two points?

At distance r, the field, the force on a kilogramme, is GM/r^2. If the spacecraft moves a small distance Δr, the energy transformed is $GM\,\Delta r/r^2$, this being stored up as a gain in potential energy if the craft is moving away from the Earth.

Adding up many such changes as r varies is a matter for integration. The class may know the integral.

Total energy transformed per kilogramme (gravitational potential difference) $= \displaystyle\int_{r_1}^{r_2} \frac{GM}{r^2}\,dr = GM\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

For the Earth, $GM = 4 \times 10^{14}$ N m^2 kg^{-1}, and for the points (*1*) and (*2*), $r_1 = 11.054 \times 10^6$ m and $r_2 = 26.306 \times 10^6$ m. Using these values, the equation gives 21.0×10^6 J kg^{-1} for the gravitational potential difference, in good agreement with the value obtained from the motion of the spacecraft.

Graphical test of the $1/r$ variation

In table 4, as r increases, the kinetic energy per kilogramme falls. At some large distance r_0, it will become zero, though not at the same distance for the two series of data, because the rockets were fired between them (indeed, the craft landed on the Moon). At this distance r_0, the energy would all be potential energy. The kinetic energy per kilogramme given for each distance thus represents the gravitational potential difference between each distance and the large distance r_0 where the kinetic energy is zero.

A plot can be made of the kinetic energy per kilogramme, and so of the gravitational potential difference, at each distance r, against $1/r$. Since the gravitational potential difference is expected to be $GM\,(1/r - 1/r_0)$, and r_0 is a constant quantity for each of the two outward and homeward series of data, the graph should give two straight lines, one for each journey. Having different constants r_0, the intercepts will differ, but the slopes should be the same, both being equal to GM.

Figure 17 shows such a graph.

Proof that $1/r^2$ law holds close to a spherical distribution

It is suggested that the result be quoted without proof. However, a few pupils might like to see a formal proof, and teachers may wish to have one to hand.

Consider a spherical shell, mass ρ per unit area (figure 18 *a*).

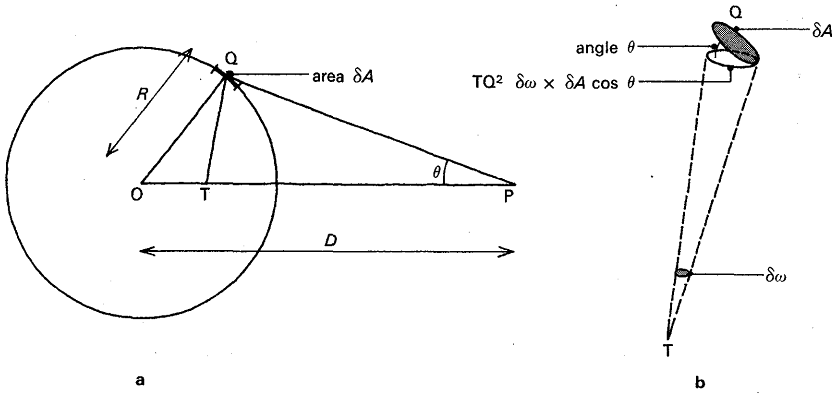


Figure 18
Field of a spherical shell.

Consider any small area δA at Q. The component of the field along OP at P is,

$$\frac{G\rho\delta A}{\mathrm{PQ}^2}\cos\theta \qquad (1)$$

Choose a point T so that Δ OQT is similar to Δ OQP, and call the solid angle subtended by δA at T, $\delta\omega$ (figure 18*b*).

Then $\delta A\cos\theta = \mathrm{QT}^2 \times \delta\omega \qquad (2)$

Because the triangles are similar,

$$\frac{\mathrm{PQ}}{\mathrm{PO}} = \frac{\mathrm{QT}}{\mathrm{OQ}} \text{ or } \mathrm{PQ} = \frac{D}{R} \times \mathrm{QT}$$

Thus $\mathrm{PQ}^2 = \dfrac{D^2}{R^2} \times \mathrm{QT}^2 \qquad (3)$

Putting (*3*) and (*2*) into (*1*) we have

$$\text{field} = \frac{G\rho R^2\delta\omega}{D^2}$$

Adding up over all such pieces $\delta\omega$ gives

$$\text{total field of shell} = \frac{G\rho 4\pi R^2}{D^2} = \frac{G(\text{mass of shell})}{D^2}$$

$1/r^2$ and $1/r$ close to a sphere

At point (*1*) the spacecraft was less than two Earth radii from the Earth's centre. The fact that this point lies on the same line as the others in figure 17 indicates that the $1/r$ rule, and so the $1/r^2$ rule, still work even though the Earth is nothing like a point mass at such a distance.

It is by no means clear that the inverse square law will go on working, for some of the Earth is much nearer to the craft than is the centre of the Earth, and some is further away.

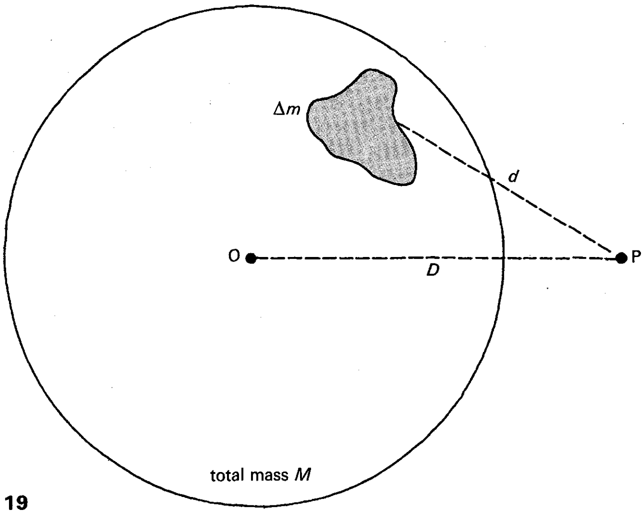


Figure 19
Forces on a mass near the Earth.

In figure 19, a test mass at P is pulled on by many small masses Δm, at varying distances d from P. The total pull is as if the whole mass M of the sphere were at the centre O, distance D from P.

The problem of adding up the effect of all of a sphere on a point close to it was first solved by Newton. It should be clear from symmetry why the total pull is towards the centre. It is not so easy to show that for an inverse square law force, the size of this pull is just the same as if all the mass of the sphere were at its centre.

This is an important result, because it works for electric forces as well. In Part Four, the forces between sodium and chlorine ions can be written down easily even though these more or less spherical ions are pretty well touching each other.

Demonstration

3.10 Ball rolling on a $1/r$ shaped hill

171	photographic accessories kit
133	camera (with cable release)
134/2	xenon flasher
1028	alpha scattering analogue ($1/r$ hill)
7B	vegetable black
503–5	retort stand base, rod, and boss
113/3	lens 2.5D
1054	developer, fixer, printing paper
	slide projector and screen

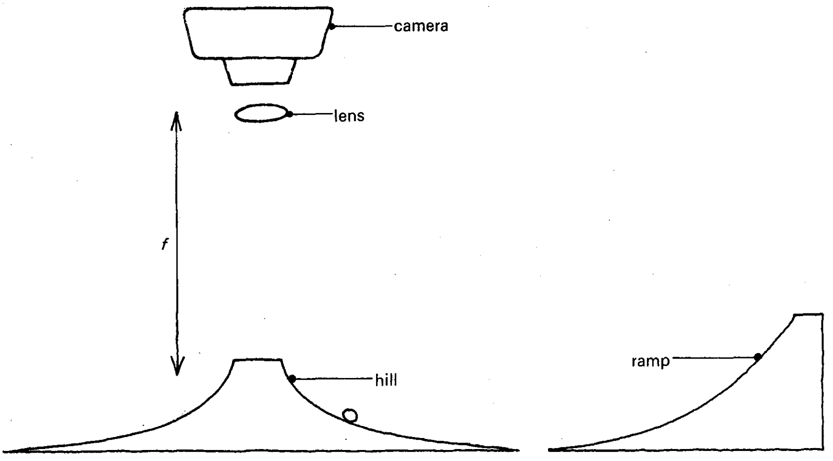


Figure 21

Unless the camera will focus down sufficiently for the hill to fill the field of view, an auxiliary lens should be used, as shown in figure 21, with the camera focused at infinity.

The xenon flasher is placed so that as little light as possible falls on the part of the hill where the ball will roll, but so that the polished steel ball shows a strong highlight visible to the camera. The hill needs to be painted matt black. A flash rate of 3000 per minute is suitable. The ball may be released by hand from some point on the hill, and it may also be projected at the hill using the ramp provided.

A number proportional to the kinetic energy of the ball may be found by squaring the distance between adjacent images of the ball in the photograph. The distance from the centre of the hill to the point midway between the pair of images is a fair measure of the distance from the centre at which the ball has this energy. It is convenient to project the negative onto a large screen, and to make measurements on the screen, or to make a copy of the ball positions on a large piece of paper over the screen.

Demonstration

3.10 Ball rolling on a $1/r$ shaped hill

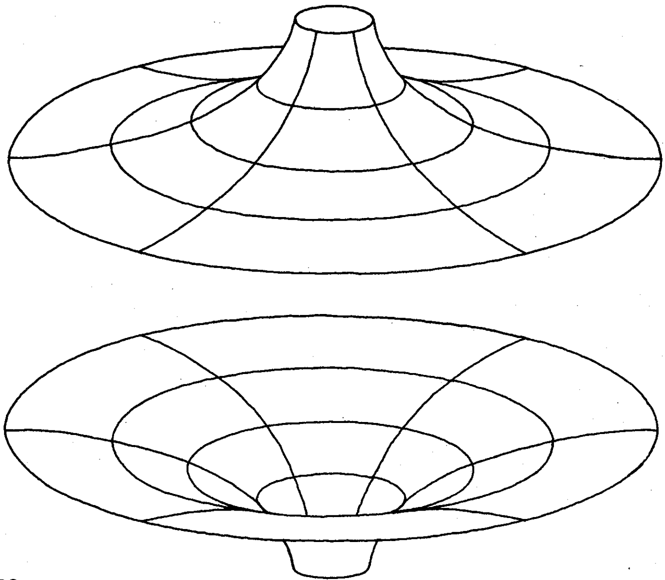


Figure 22
$1/r$ shaped hill.

A ball may be rolled on a hill (which can be inverted to be a bowl) whose sides rise a distance above the bench proportional to the reciprocal of the distance from the centre. Ask if the hill or the bowl is correct for gravitational attraction (the bowl), and what the hill might represent (repulsion between charges for instance).

Optional analysis

Photographs of a ball rolling on the hill can be taken or prepared in advance. Ask about the energy changes. The ball slows down as it climbs the hill, gaining potential energy and losing kinetic energy. A photograph may be analysed, giving velocities v at various distances r from the centre.

The values of v^2 indicate changes in kinetic energy, and the decreases in v^2 from the value near the edge represent increases in potential energy. A graph may be drawn of the variation of potential energy with r.

Another possibility is to allow the ball to roll down the hill from various heights, and to measure the velocity as it comes off the hill.

Stroboscopic photography

See Nuffield O-level Physics *Guide to apparatus*, page 150, and *Guide to experiments IV*, Appendix I and Appendix II. The photograph shows only the horizontal components of the velocity, so the energy measured from the photograph is not the total kinetic energy. In practice, the error is small. See also *Students' laboratory book.*

Students' book

A stroboscopic photograph of a ball rolling on a $1/r$ shaped hill is printed in question 29, and could be used if time is short.

Potential at 'infinity'

It is advisable not to speak as if 'infinity' were a place. To 'go to infinity' means to go as far as one pleases, without limit. So the important point is that on a journey in steps of, say, 10^6 metres the first few steps contribute nearly all the energy. On a journey away from the Earth, the first million 10^6 metre steps contribute over 60 million J kg^{-1}, while the next million 10^6 metre steps contribute only about 200 J kg^{-1}.

Students' book

Question 31 deals with gravitational potential and escape velocity, and with the conventional sign attached to gravitational potential.

The potential at a place and its sign

In the discussion of gravitational potential energy, the changes of potential

$GM\left[\dfrac{1}{r_1}-\dfrac{1}{r_2}\right]$ are conveniently calculated as the differences

between terms $\dfrac{GM}{r_1}$ and $\dfrac{GM}{r_2}$. These terms, with the correct sign, are called the potential at r_1 and the potential at r_2. The following discussion suggests a meaning that can be given to the potential at a place, and decides upon a sign.

How much energy is needed to escape from the Earth?

If the hill in experiment 3.10 extended a very long way, what shape would it be? (Nearly flat.) The energy would then change very little with distance.

Consider calculations of $\dfrac{GM}{r}$ for the Earth, as in table 5.

$r/10^6$ m	6.38 (Earth's surface)	6.75	202	380 (distance to Moon)	150 000 (distance to Sun)	'infinity'
$\dfrac{GM}{r}/10^6$ J kg^{-1}	62.7	59.3	1.98	1.05	0.003	0.000

Table 5
GM/r for the Earth.

An energy $(62.7 - 1.05) \times 10^6$ J kg^{-1} will get a spacecraft to the orbit of the Moon. A little more $(62.7 - 0.003) \times 10^6$ J kg^{-1}, will take it to the distance of the Sun. How much is needed to travel just as far as one likes? (62.7×10^6 J kg^{-1}.)

From $\frac{1}{2}v^2 = GM/r$, the escape velocity v from the surface of the Earth is 11.2×10^3 m s^{-1} (40 300 kilometres per hour, 25 000 miles per hour).

Sign of the potential

Energy is *needed* to get away from the Earth, because a mass on the surface has *less* potential energy than it has far away. It is a convention to call the energy far away zero, when the energy GM/r near the surface will be a negative quantity, written $-GM/r$. Then each value in the second line of the table has a minus sign, and the potential rises, from large negative values to smaller ones and then to zero, as r increases to very big values.

Graph of gravitational potential against r

It is convenient to draw the curve 'like a well'. Recall that a bowl, not a hill, represents attraction for a rolling ball. Students will be drawing such curves, and drawing slopes to them, several times during the work on ionic crystals and this practice now should be helpful later on.

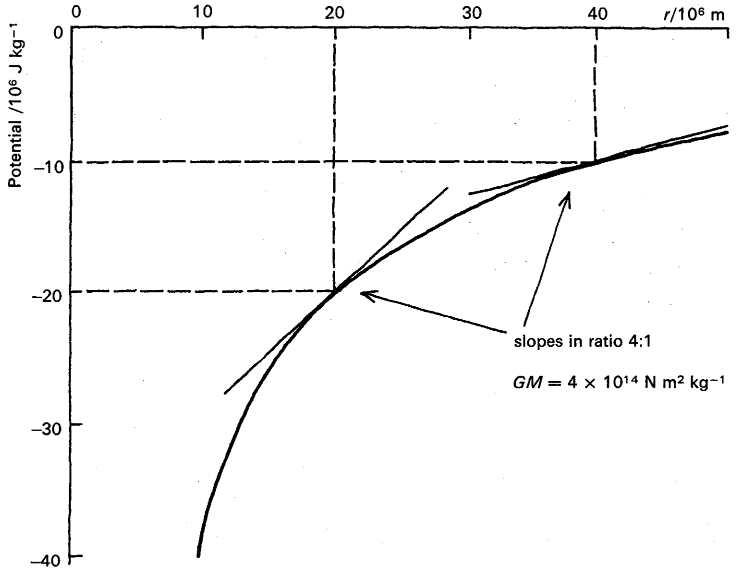


Figure 23
Graph of gravitational potential against r.

Summary

a The gravitational field of a mass M at distance r is GM/r^2.

b The field is the force on one kilogramme, unit N kg^{-1}.

c In moving from r_1 to r_2, the potential energy of one kilogramme changes by $GM\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$.

d The potential energy of mass m changes by m times the change in potential.

e The potential at a place, distance r from mass M, is $-GM/r$, the minus sign showing that the energy is less than that at very big distances, which is arbitrarily given the value zero.

f The unit of gravitational potential is J kg^{-1}.

g The field g at a place where the potential V_{gravity} changes by $\Delta V_{\text{gravity}}$ over a distance Δr is $g = \Delta V_{\text{gravity}}/\Delta r$ or, in the limit, dV_{gravity}/dr. The field is the slope of the graph of potential against distance.

The last point can be reinforced by drawing a curve of $-GM/r$ and drawing slopes at distances r and $2r$ as in figure 23. The slopes differ by a factor 4. A $1/r$ variation of potential is a direct consequence of a $1/r^2$ force law, and vice versa. This will enable a simple test of the $1/r^2$ force law for electric fields to be made at the start of Part Three, by measuring the electrical potential variations near a charge.

Part Three

The electrical inverse square law

Time: about a week

Having introduced the electric field by way of the uniform field between parallel plates in Part One, and considered the $1/r^2$ law for gravitational force and the $1/r$ law for gravitational potential in Part Two, this part of the Unit describes the inverse square law for the forces between electric charges. The $1/r^2$ law for force, and the corresponding $1/r$ law for electrical potential are needed for later work, especially Unit 5, *Atomic structure* and Unit 10, *Waves, particles, and atoms.*

We hope that students will see that the electrical inverse square law is of great use in understanding the nature of atoms; indeed, were the course not concerned with charged electrons and nuclei, there would be little point in thinking about the inverse square law for charges at all.

Part Four, 'Ionic crystals', attempts to illustrate this message by using the $1/r^2$ and $1/r$ laws at once on a physical problem, so that they are quickly seen to be useful, and so that students are not tempted to suppose that these laws mainly describe the behaviour of charged balls.

Introduction of $1/r^2$ law from $1/r$ variation of potential

Experiments to investigate directly the force between electric charges are all too often spoiled by leakage, whilst the flame probe enables quite reliable experimenting to be done on the $1/r$ variation of potential. Other reasons for suggesting that a start be made with the variation of potential are that it follows smoothly from the previous Part and again illustrates that if a (conservative) field obeys an inverse square law its potential necessarily obeys a $1/r$ law, and vice versa.

Photographs of electric field patterns

See:
PSSC *Physics* (2nd edition), page 489.
PSSC *College physics*, page 444.

'Simple'

When a physicist says something is 'simple' he doesn't necessarily mean that it is easy, but means roughly that the relationships have not too many terms, that they have a certain elegance and that they cannot readily be broken down into something deeper. 'Simple' and 'fundamental' are quite close in meaning, though to suppose that what is fundamental must be simple and elegant can be no more than an article of faith. Some physicists say that when they don't know what to try next in the search for fundamental laws, they use simplicity and elegance as a guide.

Question 34 makes an opportunity for a discussion of simplicity in physics.

The importance of forces between charges

The work of Part One found its practical application in the design of capacitors, and, within physics, in the ability to produce an electric field of known magnitude for experiments such as the Millikan experiment.

This Part concerns the forces between charges which are concentrated near one place. Why should such forces interest a physicist or chemist? Some further questions about what particles go to make up atoms, the existence of ions, the suggestion that a crystal of, say, sodium chloride is made up of positively and negatively charged ions, and the evidence from Unit 2 (*Electricity, electrons, and energy levels*) that energy is needed to remove an electron from an atom, leaving it ionized, can bring out that an understanding of forces between such charges and the energy involved ought to be of wide-ranging value in delving further into the atomic structure of matter and into the structure of atoms themselves.

It is because atoms are made of charged particles that this work appears in the course at all, and will find frequent uses within the course.

Reminder of the appearance of the field of a point charge

Demonstration 3.3 showed what the field around the charged tip of a wire looked like. It may be worth showing this again; certainly the class should be reminded of its shape and the suggestion that the field is directed radially.

The uniform field was simple in that it was the same over a very considerable region. A physicist would also call the radial field 'simple', as compared with that between, say, two oppositely charged points. The class may or may not think that the radial field looks the simpler of the last pair: the physicist's reasons for saying that it is are mainly that the equations that describe it are simpler, though it also has more symmetry.

The variation of potential near a charged sphere

The teaching may begin by exploiting the previous development of gravitational potential, noting that there would be an electric potential around a charged object.

Suppose the force from a charged sphere on another charge outside it is in fact radial in direction. If the outside charge moves and also changes its electrical energy, how must it have moved? (Radially, or in a direction with a radial component.) Can it move and not change its energy? (Yes, if it moves on a circle round the centre of the sphere.)

Demonstration

3.11 Flame probe investigation of the potential around a charged sphere

1053	plastic football (see below for means of support)
97B	Aquadag
14	e.h.t. power supply
51A, J	gold leaf electroscope with hook
94A	lamp, holder, and stand
27	transformer
52K	crocodile clip
	flame probe (see demonstration 3.8)
1000	leads

The construction of the flame probe is described in demonstration 3.8, page 34.

In this experiment the probe measures the difference in potential between places near the sphere and the distant floor and walls of the room, to which the electroscope and power supply are connected through the earth connection.

A plastic football at least 0.2 m in diameter, given a good coating of Aquadag, makes a suitable conducting sphere. The football can alternatively be wrapped as smoothly as possible with aluminium foil, but the inevitable wrinkles make this a less satisfactory expedient.

The football can be hung on nylon threads, using a triple suspension to hold it in one place, or it may be supported from below on a plastic beaker (item 164) or other insulating collar such as that from an old vacuum flask, itself fixed to a Perspex rod at least 0.5 m long supported by a retort stand base (item 503).

The sphere must be as far as possible from the floor, walls, benches, students, and other earthed conductors. The wire joining the probe to the electroscope (figure 24) must not touch the bench or any earthed conductor.

A potential of 1500 V on the sphere is suitable. The electroscope needs to have been calibrated beforehand to read potential differences, as in experiment 3.8. It is best to have the probe and flame vertical, placed as in figure 24 so that the flame is in the same horizontal plane as the centre of the sphere. The width of the flame, if small, then introduces relatively little uncertainty into the distance r.

Because of this uncertainty about the effective position of the probe, there is an advantage in plotting $1/V$ against r, rather than V against $1/r$.

It may be possible to use an electrometer (item 1006), suitably adapted, to indicate the potential of the probe.

Demonstration

3.11 Flame probe investigation of the potential around a charged sphere

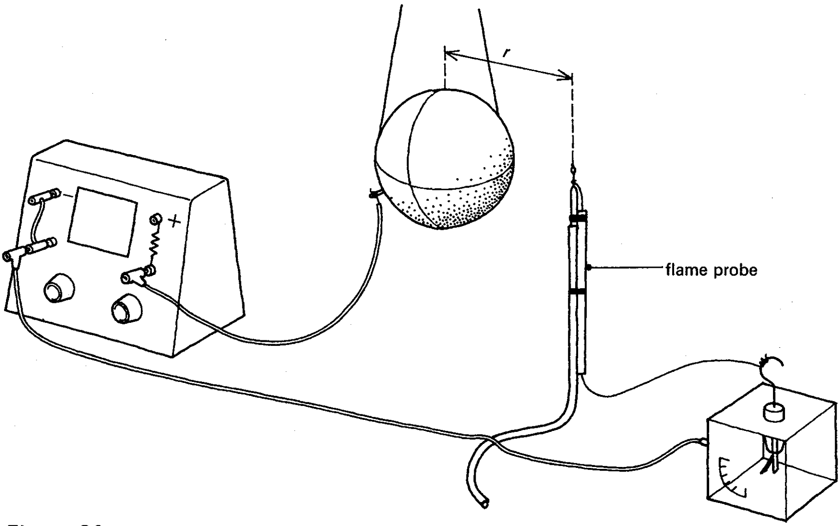


Figure 24
Flame probe used to investigate the potential near a charged sphere.

If the field is radial, why will a charge moving so as to keep at a fixed distance from the sphere not change its electrical potential energy? (No force component in that direction; compare the gravitational problems discussed in Part Two.) This can be checked by moving the probe about while keeping the distance fixed. It will be necessary to make it clear that the probe and electroscope indicate the difference in potential between a place near the sphere and any place a long way off; again, the gravitational analogy should be helpful.

Why does the electroscope indicate a potential smaller than that on the sphere? Why is the potential higher nearer the sphere and lower further away? (More energy is needed to bring a charge from a long way off to a point nearer the sphere.)

If the electric field of a small charge obeys an inverse square law, two things follow as in the gravitational problem:

a A sphere of charge will produce the same field outside itself as if all the charge were at the centre (as long as the charge is evenly spread over the sphere).

Demonstration

3.12 Measurement of the constant of proportionality in $V \propto Q/r$

Add to apparatus for 3.11:

1006	electrometer with probe rod or wire
1003/1	milliammeter (1 mA) if not built into the electrometer
1051	capacitor, 0.01 μF, low leakage polystyrene type, unless the electrometer has a switched charge range 10^{-8} C
1041	potentiometer holder *with*
1051	preset potentiometer 5 kΩ
1033	cell holder with one U2 cell
1005	multirange meter
1000	leads

The electrometer is used to measure the charge on the sphere when it is at a known potential. A potential rather less than 1000 V is suitable, giving a charge of nearly 10^{-8} C for a sphere 0.2 m in diameter.

Before use, the electrometer must be zeroed and its sensitivity checked by applying 1.0 V across its input and observing the output meter reading or, better, adjusting the sensitivity until the output is 1.0 mA (or whatever current is appropriate to the electrometer being used). Output readings can then be converted into input p.d., and these into charge if the charge is on a capacitor of known value placed across the input.

It is important when charging the sphere to touch it with the lead from the e.h.t. supply held on an insulating rod, so that an earthed hand is not nearby. Then the lead is removed, and the sphere is allowed to touch a stiff probe rod or wire inserted into the electrometer input, the electrometer being moved up to the sphere from a position rather far from the sphere.

If the sphere were at about 1000 V, and if, when its charge is shared with the input capacitor, the p.d. across this capacitor is about 1 V, only 0.1 per cent of the charge on the sphere would not have passed to the capacitor.

b The potential at a distance r from a concentrated charge will vary as $1/r$, and using **a**, so will the potential around a sphere of charge.

Values of potential V and distance r can now be taken, noting particularly that just at the surface of the sphere, the potential is that indicated by the e.h.t. supply connected to it, and that it is half as big at a distance from the centre of the sphere equal to twice its radius.

Ask what graph or graphs could be used to test whether the potential varies as $1/r$ (V against $1/r$, $1/V$ against r, $\lg V$ against $\lg r$).

Demonstration

3.12 Measurement of the constant of proportionality in $V \propto Q/r$

There is a variation of gravitational potential around a mass because masses attract each other. Why is there a variation of potential around a charged sphere? (Because charges attract or repel each other.) Earlier experiments, or a test now, indicate that Q is proportional to V. A measurement can now be made of the magnitude of the charge Q needed to produce a potential V at a distance r from a concentrated charge, by measuring the charge on the sphere with an electrometer. The potential V was shown in 3.11 to be the same as that of the e.h.t. supply, if the distance r is equal to the sphere's radius. Alternatively, the slope of the graph obtained previously can be used.

The measurement, made accurately, yields very nearly:

$$V = 9 \times 10^9 \, Q/r.$$

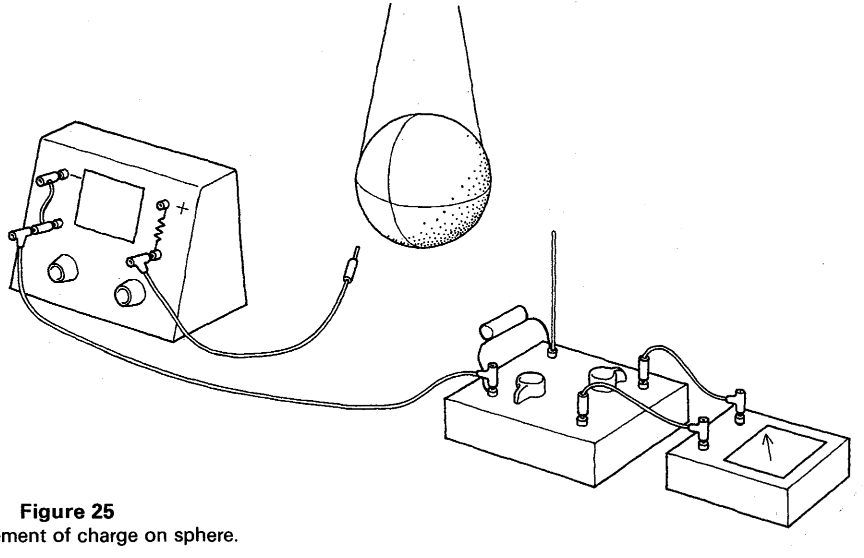


Figure 25
Measurement of charge on sphere.

$1/4\pi\varepsilon_0$

A better value for the force constant is 8.98×10^9 N m² C⁻². This constant is equal to $1/4\pi\varepsilon_0$. Some teachers will want to say so, anticipating the arguments that will follow which make the connection between the radial field and the uniform field, and which show that the constant here is in fact $1/4\pi\varepsilon_0$. Others will simply prefer to carry the numerical value along for the time being. The value is used often enough to be worth posting up in the laboratory.

Use of formal analogy

Students may wrongly suppose that this use of analogy supposes that gravity and electricity are connected. Compare the formal analogy between the energy of a stretched spring, $\frac{1}{2}Fx$, and that of a charged capacitor, $\frac{1}{2}VQ$, or between the traffic carried by vehicles on a road and the current carried by electrons in a wire. Physicists are fond of these cases where the different quantities in different relationships fit together in the same way, for, having done the mathematics for one, they need not do it again for the other. Teachers will know of other examples which are widely used in practice. For instance, the analogy between electric field in space and electric current density in a tank of conducting fluid. Or again, the height of a stretched light membrane is analogous to the variation of potential in two dimensions if the boundary conditions are the same. Perhaps most fruitful of all are the analogies between flows of various sorts, real and imaginary, such as heat flow, current flow, water flow, and the 'flow' of magnetic flux round a 'magnetic circuit'.

Use of calculus

Students who cannot handle the differentiation with fluency should be encouraged to write down the answers by analogy. But they should also be reminded that $\Delta V = -E\Delta r$, as in the gravitational case (from the meanings of potential difference and field) and also of the deductive (not experimental) link between a $1/r^2$ field and a $1/r$ potential.

Recalling units

In an examination, the value 9×10^9 N m² C⁻² would be given, with its units. There could, however, be a question asking why it has these units.

Textbooks

PSSC, *Physics* (2nd edition), Chapter 27, and
PSSC, *College physics*, Chapter 24, show Coulomb's original apparatus.
Rogers, *Physics for the inquiring mind*, describes the principles behind experiments to test Coulomb's Law.

Fundamental laws

A student should realize at least that there is some difference worth noting between laws of general and laws of restricted application, and between those which we think can be explained in terms of deeper laws and those which, so far, cannot be further unravelled. This is a part of understanding what physics is like, and how it works, and it is one of the general aims of the course to achieve this understanding.

It is not intended that students be expected to conform to one view in such debatable matters, but that they see that such debate can arise and become more willing to take part in it.

The constant for forces between charges

The constant measured above has some analogy with G, the gravitational force constant.

For gravity	Force $= GM_1 M_2/r^2$	Field $= GM/r^2$	Potential $= -GM/r$.
For electricity			Potential $V = +9 \times 10^9\, Q/r$.

The plus sign appears because like charges repel, not attract. When like charges come closer together, their potential energy rises, whereas it falls when masses come closer to each other.

Can one write down the electric field and force expressions by analogy? Using $E = -\mathrm{d}V/\mathrm{d}r$, the field E can be written down, and since force $= EQ$, the analogous results are:

$$\text{Force} = 9 \times 10^9\, Q_1\, Q_2/r^2 \qquad E = 9 \times 10^9\, Q/r^2 \qquad V = +9 \times 10^9\, Q/r.$$

This can be done, not because gravity is the same as electricity (nobody is too sure if they are related or not), but because the different quantities fit together in the same way. From the expression for the force, the constant 9×10^9 is a force divided by the product of two charges and multiplied by the square of a distance. Its units are N m^2 C^{-2}. Those of G are, similarly, N m^2 kg^{-2}. The expression for the potential gives the same unit for this constant, since the unit of potential is J C^{-1}, which may be written N m C^{-1}. Thus Vr/Q has units N m^2 C^{-2}.

Coulomb's Law

Historically, the expression:

$$\text{Force} = 9 \times 10^9\, Q_1\, Q_2/r^2$$

was where quantitative electricity began, when in 1875, the French scientist Coulomb measured the forces between electrically charged balls.

Like Newton's law of gravitation it is regarded as a fundamental law, while Ohm's Law and Hooke's Law are not. What reasons could one have for calling a law fundamental? The class might suggest some of the following:

The law is quite general, with no known exceptions.

The law can be used to explain many other things.

No one has found something deeper which, in turn, would explain the law.

The law doesn't seem to be too complicated.

Does Coulomb's Law explain *why* charges obey an inverse square law? (No, it only says that they do.)

Practical difficulties of Coulomb's Law tests

These experiments are easy enough for a single experimenter on his own on a dry day with dry apparatus. The humidity in a classroom full of students (all necessarily breathing) rises rapidly, and is likely to produce serious leakage difficulties.

Teachers will often have difficulties in arranging for these experiments to be done, especially if all the class must remain in one room. Means of drying apparatus, available to all, is a great help.

In the event of failure, the *Students' book*, question 35, contains photographs of some Coulomb's Law tests, from which measurements can be made. They may also be useful for revision.

Question 36 is about why the experiment can easily fail in the classroom.

Experiment
3.13 Tests of Coulomb's Law

51D	metallized polystyrene ball *2*
51E	reel of nylon suspension
51L	proof plane *2*
94A	lamp, holder, and stand
27	transformer
	Means of charging the polystyrene balls:
14	e.h.t. power supply *or*
60/1	Van de Graaff generator *or*
51I, 51K, 51M	electrophorus plate, rubber and polythene tile
503–6	retort stand base, rod, boss, and clamp
1054	graph paper
1053	glue (Durafix or Evo-stik 863)
1053	adhesive tape
1000	leads

One of the polystyrene balls is glued to an insulating rod, for which the small proof plane from the Malvern electrostatics kit serves well. The other ball is hung from the other insulating rod by a double suspension of nylon thread. This double suspension is necessary because with a single nylon thread suspension, the ball would swing sideways as well as away from the fixed ball.

To make the double suspension, one end of the nylon thread is fixed to the insulating rod with a small piece of adhesive tape. About 1 m of nylon thread should be unrolled along the bench and the ball fixed to it with a dab of glue at a point about 0.5 m from the fixed end. The free end of the thread is then taped to the insulating rod some 0.1 m from the other suspension point and after adjusting the length so that the ball hangs evenly on two threads of equal length. Care must be taken not to handle the thread more than is absolutely necessary when doing this, or leakage may well result. The insulating rod to which a ball is glued should be cleaned to remove finger grease before starting the experiments. In damp conditions, a hair drier can be used to dry the rods and possibly stop further condensation and consequent leakage of charge.

Experiment
3.13 **Tests of Coulomb's Law**

The class may be shown a simple arrangement with which it is possible to investigate, at least roughly, the forces between charged spheres. As shown in figure 26, one light charged ball is suspended on a pair of fine nylon threads ('trapeze') near another ball which may be moved about. Shadows of the balls are cast on a screen so that measurements may be made without touching the balls.

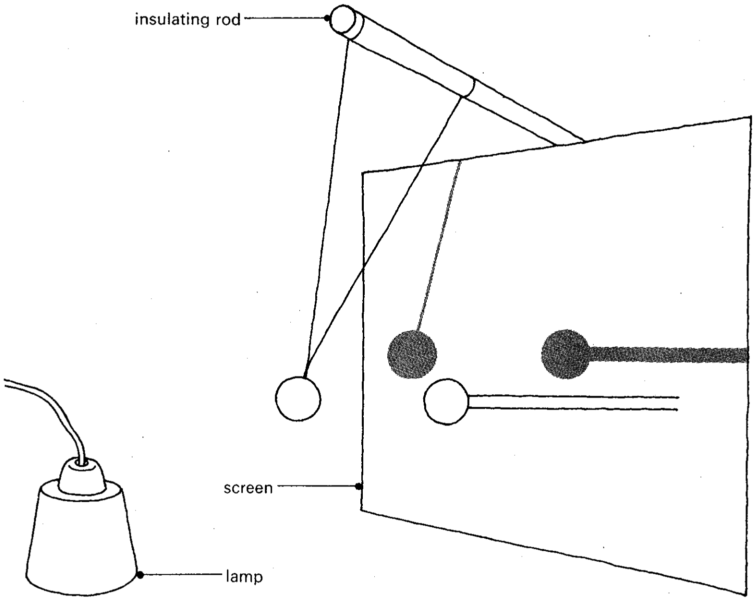


Figure 26
Inverse square law test.

The balls should be positioned in front of and at the same height as the lamp so that their shadows are cast on a sheet of graph paper pinned onto a screen.

The spheres may be charged by contact using the plate of an electrophorus, or the dome of a Van de Graaff machine, or by using an e.h.t. power supply. In 'leaky' conditions, there is something to be gained by using an e.h.t. unit because the spheres can be recharged before each measurement. Of course, in these circumstances, the output voltage should be kept at a constant value and charging carried out, under the same conditions each time, with the negative terminal of the unit earthed.

Some improvement is sometimes gained by suspending both balls on nylon threads, each on a double suspension, one of which passes through the other.

3.13a $F \propto 1/r^2$

The rest position of the shadow of the suspended ball should be marked. After bringing up the other charged ball, the distance between the centres of the shadows, *r*, and the sideways deflection of the suspended ball need to be recorded.

3.13b $F \propto Q_1 Q_2$

Additional apparatus required:

1006	electrometer (with probe)
1051	capacitor, polystyrene, 0.01 μF (see below)
1003/1	milliammeter (if not provided with electrometer)
51D	metallized polystyrene ball (on a nylon thread)
51L	insulating rod for the above

The distance between the balls (and between the shadows) is kept fixed and the force between them is measured by the sideways deflection of the suspended one. Touching one ball with a similar uncharged ball will halve the charge. Alternatively, the balls may be charged to different potentials with the e.h.t. unit or Van de Graaff machine. In this case, the charge on each ball must be measured by allowing it to touch the probe of the electrometer with an input capacitance of 0.01 μF.

A 0.01 μF capacitor should be connected directly across the electrometer input, so that a p.d. of 1 V represents a charge of 10^{-8} C. This will not be necessary if the electrometer has a suitable switched charge range.

3.13c The constant 9×10^9 N m^2 C^{-2}

Access to a balance to weigh 1 mg is required. The charges are measured as in **b**. The angle is found from the length of the suspension and the sideways deflection. The true deflection and true separation of the balls must also be calculated from the measurements of the positions of the shadows of the balls.

3.13d Alternative demonstration $F \propto 1/r^2$

Additional apparatus:

1053	nylon fishing line
1053	large insulated conducting sphere (see below)
97B	Aquadag

3.13a $F \propto 1/r^2$

The distance between the charged balls is varied and the force on the suspended one is estimated in arbitrary units from its sideways deflection.

3.13b $F \propto q_1, F \propto q_2$

The charge on either ball may be halved by touching it with a similar uncharged ball. This was Coulomb's original and elegant method. The forces become small and it is hard to show that when both charges are halved, the force is reduced by a factor of four. If charging the balls from an e.h.t. supply gives adequate deflections, it may be possible to vary the p.d. used to charge the balls. Alternatively, the balls might be charged from a Van de Graaff machine several times and an attempt be made to measure the charge on each ball with the electrometer on each occasion. The product of the charges should be proportional to the force on the suspended ball.

3.13c The constant 9×10^9 N m^2 C^{-2}

It is possible, but not easy, to measure the constant in the expression $F = kQ_1Q_2/r^2$ by measuring the charge on each ball. The force is measured by noting the angle at which the suspended ball hangs, and weighing the ball. An accurate value is not to be expected, but the right order of magnitude can be obtained.

In view of the earlier measurement of this constant (experiment 3.12), it may not be necessary to press students to attempt **c**.

Alternative demonstration

3.13d $F \propto 1/r^2$

If it can be agreed that, should the inverse square law hold, the force due to a large charged sphere will still follow an inverse square law as in the gravitational case, a slightly easier experiment is available.

A large insulated conducting sphere is charged from a Van de Graaff machine (or the cap of the machine is used if the charge does not leak away when the machine is stopped) and used to repel a ball suspended as in the previous experiments. The deflections can now be larger and the experiment is easier to perform. Because of the use of a large sphere, the experiment has an air about it of assuming what one wishes to demonstrate. It might, alternatively, be seen as a test of the inverse square law near a large charged sphere.

The large insulated sphere may be a cheap plastic football painted with Aquadag to make it conducting. It should be suspended by means of long lengths of fine nylon fishing line from a screw inserted in the valve aperture. A triple suspension will stop the ball swinging. A Van de Graaff generator should be used for charging the football. It may be possible to use the dome of the Van de Graaff generator as the large sphere if the machine is clean and dry so that leakage is small. The small polystyrene sphere is suspended as before.

Difficulties with Coulomb's Law experiments

Students are, reasonably enough, annoyed by experiments that 'don't work', though the calculation on page 79 may comfort them. Although the difficulties of leakage that beset traditional 'electrostatic' experiments have been mitigated by modern insulating materials and the existence of suitable high voltage supplies, they have not been removed altogether.

This experiment seems to us important enough to be worth the attempt, even if the attempt fails. It was the start of the serious study of electricity; it remains one of the foundation stones of physics; it is involved in almost all theories or arguments about atoms, molecules, or the structure of materials. Moreover, no one has ever found a theory to explain why charges exert forces on each other. They do, and that is all that is known.

Optional experiment

Force on a hanging ball pulled sideways

Classes with little knowledge of vectors will need some help in seeing that the sideways force is proportional to the sideways deflection, for small deflections. It may help to pull a suspended five-kilogramme mass sideways, using a spring balance reading to 10 N, as in figure 27.

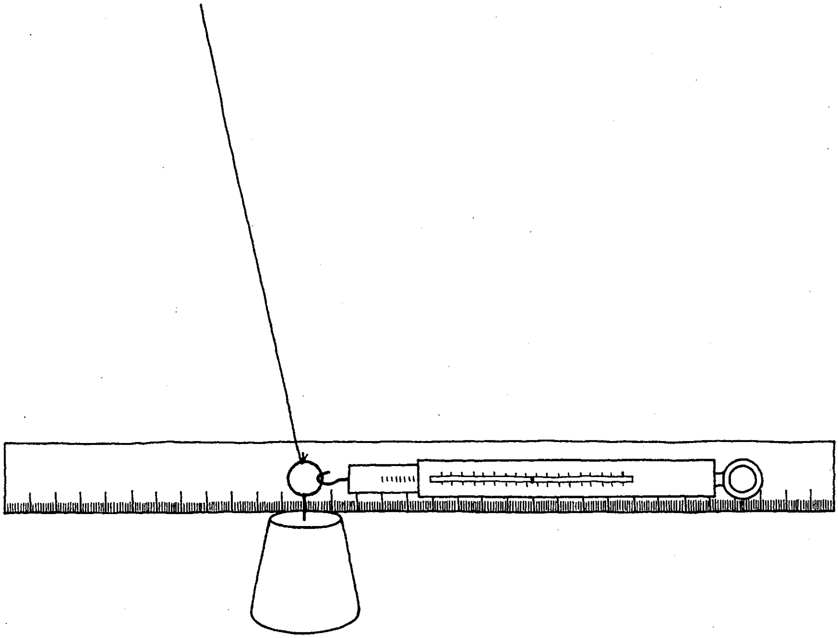


Figure 27
Force on a mass pulled sideways.

'Why these experiments don't work well'

A useful order of magnitude calculation can be done. The expanded polystyrene balls weigh about 10^{-3} N (mass 0.1 g) and so the force between a pair is of the order 10^{-4} N, since they hang at a rather small angle. If they are 10^{-2} m apart, with the same charge on each, what charge would they have? (About 10^{-9} C.)

A current of 10^{-11} A will carry that charge off in a minute or two. The voltage used exceeds 1000 V, so even if the 'insulators' have resistance exceeding 10^{14} Ω, the experiment may fail through leakage.

The radial and uniform field argument ('cones' argument)

The argument laid out in this *Guide* is borrowed from PSSC (see 'Textbooks' below). It shows the nature of the adding up process, for an electric field, by taking a result known for one geometry, and using it to 'predict' the result for another.

The argument shows that if the field from a point charge follows an inverse square law, that from a large plane sheet of charge is uniform. The necessary integration is not completed, so the argument fails to show that if $\sigma = \varepsilon_0 E$ for the uniform field, the constant in the inverse square law must be $1/4\pi\varepsilon_0$, that is $E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q_1 Q_2}{r^2}$. This integration appears in the *Students' book*, in the form of a series of questions.

We think that all students should try to understand the character of the argument, and that the 'cones' argument is a good way of doing so. Those who can perform the integration may be encouraged to do so, but it is not essential for all to see the details. Those who do not will have to be told the result, and be satisfied that the proof is available for inspection.

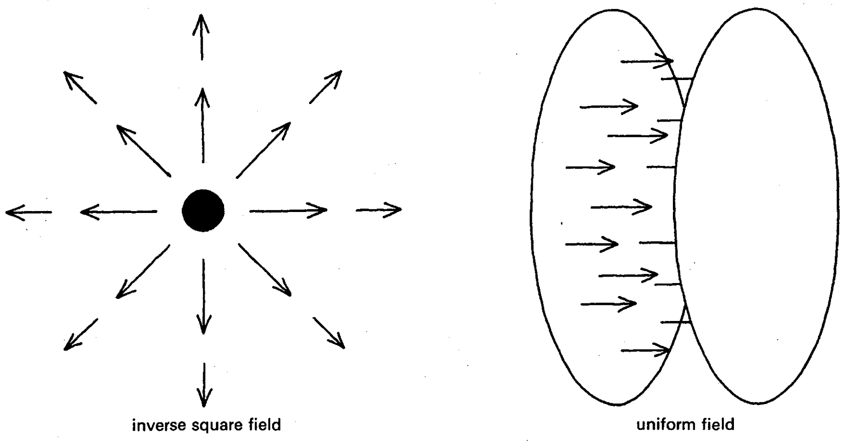


Figure 28
Two field shapes.

Students' book

Question 37 goes through the 'cones' argument step by step, and it would be useful for students to try it before the class discussion, which can then concentrate more on the character of what is going on and less on the detailed machinery.

Question 38 lays out the complete integration, for those who wish to follow it through.

Textbooks

PSSC, *Physics* (2nd edition), page 491.
PSSC, *College physics*, page 446.

The radial field and the uniform field

A single small charge, perhaps an electron, gives a field which dies away far from it (figure 28), but the electrons spread out on a pair of plates give a field which has the same value at most points between the plates. How can a carpet of point charges and their inverse square law give a uniform field? A geometrical argument gives the clue to the solution of this puzzle.

Consider an observer at P, height h above a flat carpet of charges, and concentrate attention on the bit of field coming at him in the direction E from the patch of charge marked out by lines of length w and L. See figure 29.

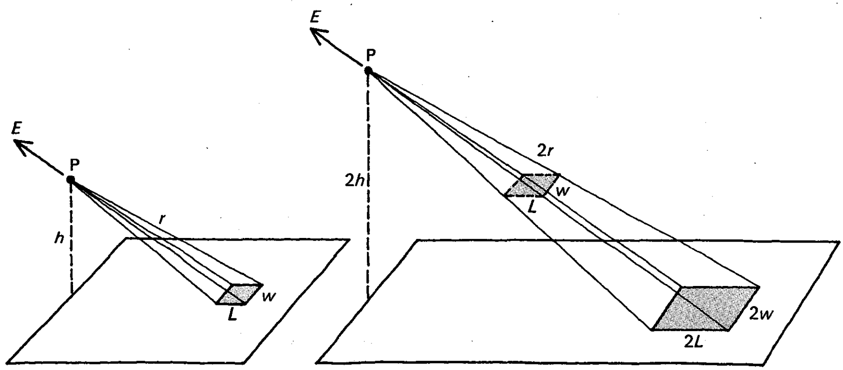


Figure 29
Field 'seen' along cones.

If the edges of the patch are joined to P, they mark out a small cone. The problem of adding the effects of all the patches on the plane can be looked at as a problem of adding all the cones, each having its own contribution to E. The size of the contribution depends on the charge in the patch marked out by the cone, and on $1/r^2$.

Say the observer now moves up to a height $2h$, and decides to calculate his new field by using exactly similar cones to mark off patches on the plane. The cone originally used at height h now marks off an area $2w \times 2L$ on the plane. So the charge in this cone's patch is now four times bigger, but its distance is also doubled and the net contribution, which depends on

$$2w \times 2L \times \frac{1}{(2r)^2} = w \times L \times \frac{1}{(r)^2} \text{ is unchanged.}$$

In fact, if the observer sticks to the same set of cones as the height alters, each cone gives the same answer at all heights. Is the total field bound to be the same at all heights?

Yes, if the plane is very big, so that there is no need to worry about the cones pointing beyond the edge of the plane as h increases.

Laws, constants, theoretical arguments

This is a good place to bring out, in comments alongside the teaching, the roles of laws, constants, and theoretical deductions in physics.

Laws

The $1/r^2$ and $1/r$ laws are excellent examples of simple law-like relationships; they are perhaps the best examples in the course and, because of their wide validity and exactness, among the best in physics. The power 2 in the $1/r^2$ law for electric charges is known, experimentally, to be good to within $\pm 10^{-9}$. The power 2 in the law for gravity needs a tiny correction if general relativity is a correct description of gravity.

Interested students might consult Feynman, *The character of physical law*, for a simple and readable discussion of the essential simplicity of the inverse square law.

Constants

The constant $1/4\pi\varepsilon_0$ is one of the few *universal* constants (it is related, through μ_0, to the value of the velocity of light). Unlike a spring constant, which describes a particular spring, $1/4\pi\varepsilon_0$ describes the size of the forces between all pairs of charges everywhere, so far as is known.

Students may ask why it is supposed that $1/4\pi\varepsilon_0$ is a universal constant. Consider g, which is not. The pull of the Earth on a mass at the surface is sure to depend on how big the Earth is, whether it is round, and so on. As expected for such theoretical reasons, when g is measured at different places, it varies, and the variations can easily be explained.

Contrast G. There are no theoretical reasons for supposing that G *will* vary from place to place or time to time (though some have tentatively proposed that G should depend on the age of the Universe). Measurements of G, using earthbound data or estimates from astronomical data, vary, but in no regular manner, the variations being explicable bearing in mind the inevitable uncertainties of the measurements.

The charge on the electron, e, presents an interesting case history. There are no theoretical reasons for supposing that e will vary, but for some years in the late 1930s there was a discrepancy between Millikan's experimental value and another which was obtained from X-ray diffraction in crystals. Until an error in Millikan's data was discovered, there was speculation as to whether the charge on an electron tied to an atom in a crystal was different from the charge on a free electron.

More recently it has been suggested that the charge on a proton might not be exactly numerically equal to that on an electron, in order to explain the expansion of the Universe as due to electrostatic repulsion. But an experimental trial revealed no difference. Until the suggestion was made, no one tried the necessary accurate test of the equality of the two charges.

In general, if there is no reason to think that a constant is not universal, and measurements of it are consistent, it is simplest to assume that it is universal. Nobody usually bothers to investigate the matter until there is a suspicion, usually theoretical, that such an investigation may be worth while.

Deduction

The 'cones' argument is a good example of a piece of theoretical deduction. The inverse square field and the uniform field appear to have little in common. Then it turns out that the second is an inevitable consequence of the first.

This kind of connection is common in physics – 'If that idea is true, this other one must also be true.'

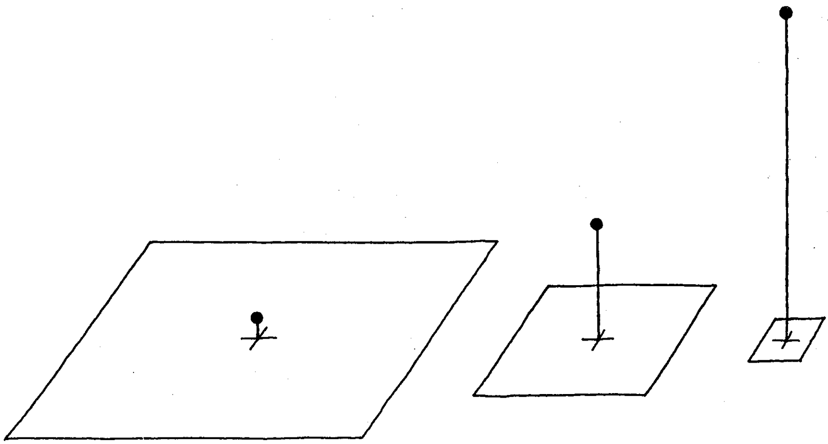


Figure 30
Fields at different distances from flat sheets of charge.

In figure 30, the observer in *a* doesn't have to worry about this, for only his very outermost cones go beyond the edge; in *b*, many of his cones are over the edge and the argument is no good – there wouldn't be a uniform field; in *c*, the plane of charge has receded so far that it begins to look like a point and there would be something close to inverse square law behaviour.

ε_0 and the Coulomb's Law constant $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2 C^{-2}

In Part One, the constant $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 N^{-1} m^{-2} was introduced into the expression

$$Q/A = \varepsilon_0 E$$

for the field between parallel plates.

If this field is regarded as the total field due to carpets of point charges, each point charge obeying Coulomb's Law,

$$E = 9 \times 10^9 \frac{Q}{r^2},$$

then the theoretical adding up exercise shows that the Coulomb's Law constant is equal to $1/4\pi\varepsilon_0$.

The units agree (C^2 N^{-1} m^{-2} for ε_0, N m^2 C^{-2} for $1/4\pi\varepsilon_0$).

The value calculated for $1/4\pi\varepsilon_0$, from the accurate value of ε_0 is 8.98×10^9 N m^2 C^{-2}, agreeing with the rougher value 9×10^9 N m^2 C^{-2}. Alternatively, the measured value of the Coulomb constant is about 12 times (4π times) smaller than the reciprocal of ε_0.

Arguments like this, which are just powerful logic, can only be found where one can say something exact and definite to start with. The inverse square law is a fine example of such a definite starting point for logic to work on.

Useful constants

It is convenient to post up in the laboratory values of useful constants such as

$$\frac{1}{4\pi\varepsilon_0} = 8.98 \times 10^9 \text{ N m}^2 \text{ C}^{-2} \text{ } (9 \times 10^9 \text{ approximately})$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$$

and others such as G, μ_0, c, e, h as they appear in the course.

Recalling formulae

Students should certainly be expected to recognize these formulae when they see them, and to recall that $E \propto Q$; $E \propto 1/r^2$; $V \propto Q$; $V \propto 1/r$. They will not, however, be asked in examinations to write down from memory the full, explicit formulae such as

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{Q_1 Q_2}{r^2}$$

But a question might ask for a proposal to test some aspect of such relationships, or a discussion as to the value of a proposed experiment as such a test.

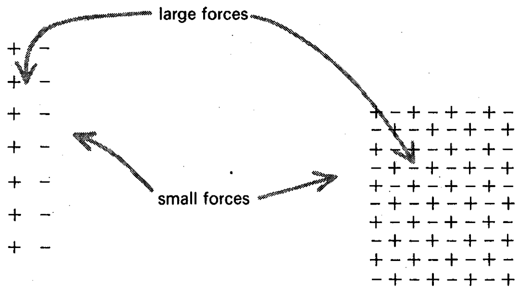


Figure 31
Forces produced by arrays of charges.

Another viewpoint

Teachers may like to make use of the following quotation from Feynman, R. P., Leighton, R. B., and Sands, M. (1964) *The Feynman lectures on physics,* Volume II, Chapter 1, Addison-Wesley.

> 'Consider a force like gravitation which varies predominantly inversely as the square of the distance, but which is about a *billion-billion-billion-billion* times stronger. And with another difference. There are two kinds of "matter", which we can call positive and negative. Like kinds repel and unlike kinds attract – unlike gravity where there is only attraction. What would happen?
>
> 'A bunch of positives would repel with an enormous force and spread out in all directions. A bunch of negatives would do the same. But an evenly mixed bunch of positives and negatives would do something completely different. The opposite pieces would be pulled together by the enormous attractions. The net result would be that the terrific forces would balance themselves out almost perfectly, by forming tight, fine mixtures of the positive and the negative, and between two separate bunches of such mixtures there would be practically no attraction or repulsion at all.

Coulomb's Law now reads $E = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$

The potential V is given by $V = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r}$

The size of electrical forces

The force between two charges is:

$$F = 9 \times 10^9 \times \frac{Q_1\, Q_2}{r^2}$$

Compare the role of the factor 9×10^9 with that of G in $F = G\,\frac{m_1\, m_2}{r^2}$.

How big is this force? Take 10 g of water: this contains about 3×10^{23} water molecules. Say that one molecule in 100 lost an electron, and that the 10 g mass was placed 0.1 m from a similar electron-deficient mass. The force would be

$$F = 9 \times 10^9 \times \frac{(3 \times 10^{21} \times 1.6 \times 10^{-19})^2}{(10^{-1})^2} = 2.1 \times 10^{17}\ \mathrm{N}$$

a force sufficient to support the weight of 10^{13} cubic metres of surface rock against gravity (say the whole of the British Isles to a depth of 20 metres). The gravitational attraction between such masses is only 6×10^{-13} N.

All the electric force experiments showed quite small forces – how can that be reconciled with this enormous result? It is, first of all, clear that a collection of only positive or only negative charges would repel with enormous force: if matter contains such charges the fact that it doesn't explode spontaneously must be due to an almost exact balance; there must be a close-knit collection of positives and negatives so that attractions balance out repulsions. But why aren't there enormous electric fields near pieces of matter? The parallel plates give a clue. Between the sheets of charge, there is a field; outside the two sheets the net effect is nothing. If a solid had alternating sheets of charge, then between the sheets there would be enormous electric fields, but outside the solid – nothing. Not all solids are like this, but the general idea that far away from equal quantities of two signs of charge, the net effect is nothing, must be true.

Any two identical grammes of (say) water must not have lost even as many as one electron in 10^{10} molecules if enormous forces are to be avoided. In all of the experiments commonly performed with currents or stationary charges, only minute fractions of the charges in the atoms involved are being moved or exchanged to give the effects observed.

Within the arrays of charged objects which make up matter there are very large forces. In fact, it is these forces which hold atoms together in solids and which account for their strength. How are the ions in sodium chloride arranged? Would there be a big field outside such a crystal?

'There is such a force: the electrical force. And all matter is a mixture of positive protons and negative electrons which are attracting and repelling with this great force. So perfect is the balance, however, that when you stand near someone else you don't feel any force at all. If there were even a little bit of unbalance you would know it. If you were standing arm's length from someone and each of you had *one per cent* more electrons than protons the repelling force would be incredible. How great? Enough to lift the Empire State Building? No! To lift Mount Everest? No! The repulsion would be enough to lift a "weight" equal to that of the entire Earth!

'With such enormous forces so perfectly balanced in this intimate mixture, it is not hard to understand that matter, trying to keep its positive and negative charges in the finest balance, can have a great stiffness and strength. The Empire State Building, for example, swings only eight feet in the wind because the electrical forces hold every electron and proton more or less in its proper place. On the other hand, if we look at matter on a scale small enough that we see only a few atoms, any small piece will not, usually, have an equal number of positive and negative charges, and so there will be strong residual electrical forces. Even when there are equal numbers of both charges in two neighbouring small pieces, there may still be large net electrical forces because the forces between individual charges vary inversely as the square of the distance. A net force can arise if a negative charge of one piece is closer to the positive than to the negative charges of the other piece. The attractive forces can then be larger than the repulsive ones and there can be a net attraction between two small pieces with no excess charges. The force that holds the atoms together, and the chemical forces that hold molecules together, are really electrical forces acting in regions where the balance of charge is not perfect, or where the distances are very small.'

Reading

See Feynman *et al., The Feynman lectures on physics,* Volume 1, Chapter 7.7.
See Project Physics, Reader, Unit 2, *Motion in the heavens,* Chapter 24 on *'Negative mass'*, by Banesh Hoffmann. The same reader contains (in Chapter 12) *'Universal gravitation'* by Feynman.

The 'weak' and the 'strong' interactions

The two types of nuclear force are known as the 'weak' interaction and the 'strong' interaction – it is not suggested that there should be any discussion of these. Magnetic forces may come up: these are not reckoned as separate because they can be seen to arise from viewing 'electrostatic' effects from moving frames of reference; given Coulomb's Law and the transformation laws of special relativity, the magnetic force laws can be deduced.

Looking ahead

This discussion of electrical and other forces looks ahead to later work in the course, especially Unit 10, *Waves, particles, and atoms*. We hope it will also serve to indicate something of the wide sweep of ideas one is engaged in when one studies fields.

The particular point about the close mixture of positive and negative charge in matter looks forward directly to the last part of this Unit, on 'Ionic crystals' (pages 85 and 111 in the *Students' book*). Students should embark on the last part expecting to see a sample of how the strength of matter is studied theoretically, not anticipating information which is vital for its own sake. So we start here from a very general point of view.

Electricity, gravity, and other forces

The law for attraction due to gravity looks similar to Coulomb's Law. The constant G plays a similar role to $1/4\pi\varepsilon_0$. The outstanding difference is that gravity forces are always attractive; there are not, as far as is known, two types of gravity matter corresponding to positive and negative charge. Electric fields hold matter together, gravity fields hold solar systems, stars, and galaxies, together. To compare electric and gravity forces, consider two electrons a distance r apart. Then,

$$\frac{\text{electric force}}{\text{gravity force}} = \frac{e^2}{4\pi\varepsilon_0 r^2} \times \frac{r^2}{Gm^2} = \frac{e^2}{m^2} \times \frac{1}{4\pi\varepsilon_0 G} \approx 4 \times 10^{42}.$$

For two protons, the charges would have been the same, but m^2 would have been about 3×10^6 times larger and the ratio smaller. Physicists have puzzled about the theoretical significance (if any?) of these enormous numbers.

The four known interactions

There are four kinds of force known in physics. Two are forces which hold nuclear matter together and are not experienced unless one studies interactions between particles which are as close together (10^{-14} m) as they are in nuclei. The other two are electrical and gravitational forces.

The electrical one dominates inside matter: the forces here are responsible for holding atoms together and for bonding together different atoms in solids. By external means we can only make tiny distortions to atoms and only add or remove minute fractions of the total charge. But all stable matter has well-balanced charge-systems, so that the net external electric effects are small, and for man-sized lumps it is just possible to measure the gravity effect. On an astronomical scale, the gravity effect dominates.

There are many problems connected with the electric force which are left unsolved. Since it is so large, why don't all the unlike charges in atoms collapse together and coalesce? What about the smallest charge yet found – that of the electron – what kind of 'stuff' is it made of? If this is a sensible question, then one bit of this 'stuff' must repel another bit in the same electron with an enormous force – so after asking why atoms don't collapse to nothing one is led to ask why electrons don't fly apart.

The first of these questions will be looked at later in the course. It is the first question to ask if one wishes to understand how atoms exist and how they interact. For the second question, it can only be said that it has never been answered, although theoretical physicists have considered that it is a sufficiently sensible question to be worth considerable efforts in thinking about possible answers.

Later uses of graphs of electric field and potential

It may well be worth asking students to preserve these graphs. They will be useful for reference in Part Four, 'Ionic crystals', in Unit 5, *Atomic structure*, and ultimately in the final Unit, on *Waves, particles, and atoms*, as well as being valuable for answering questions.

Students' book

See question 43 which would allow the mapping exercises to be done at home.

Lines of force

See Appendix B, 'lines of force', for a discussion of reasons for not stressing the line of force representation too heavily.

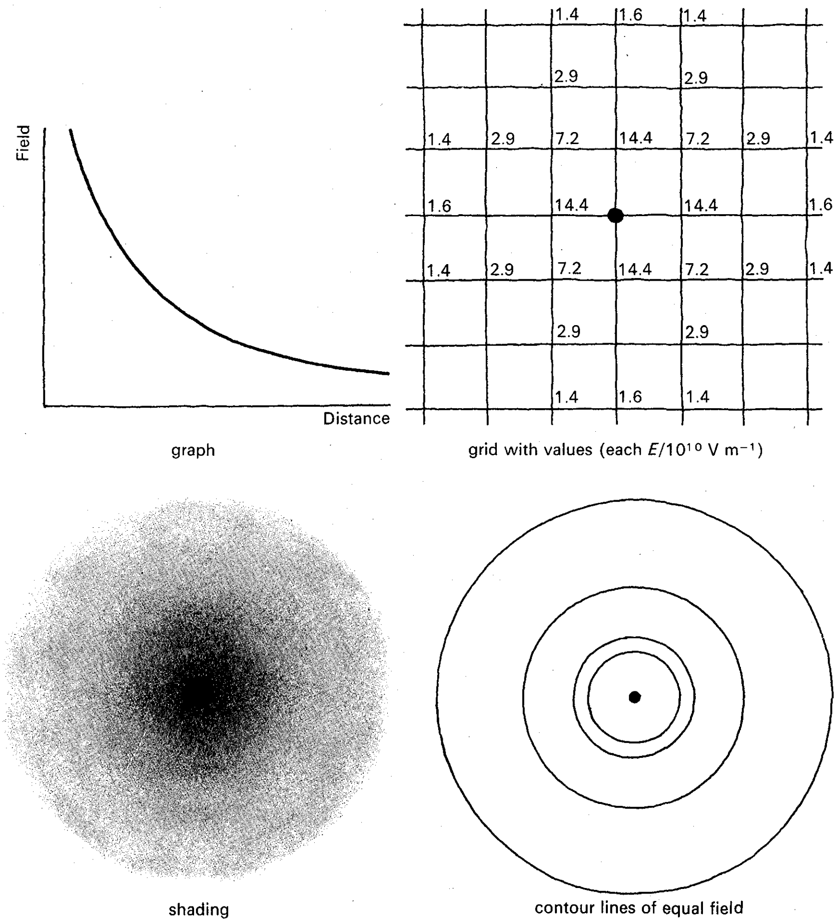


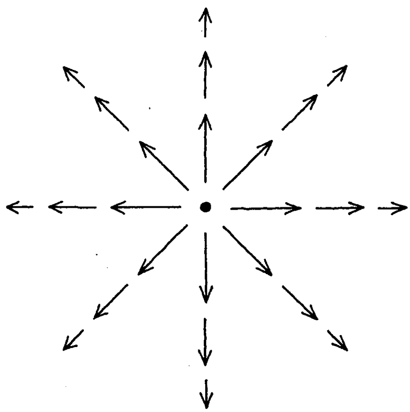
Figure 32
Various electric field maps. (Values refer to $Q = 1.6 \times 10^{-19}$ C, grid spacing 10^{-10}m.)

Mapping and graphing fields

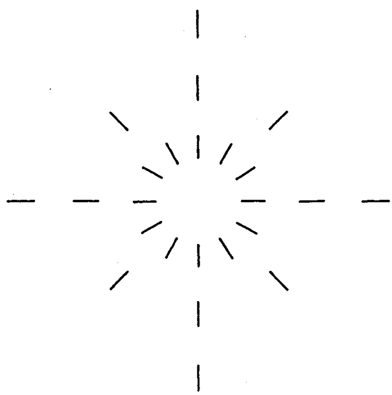
Ask for values of the electric field and the electric potential near to a single electronic charge, 1.6×10^{-19} C. Distances of up to 4×10^{-10} m would be suitable (how big are atoms?).

All should draw graphs of E and V against r.

The problem of drawing a picture of the field is worth raising. Students may be asked to invent or choose some way of making a two-dimensional map of the field round an electron. See figure 32. There is no 'right' way to represent a vector field, and the object of the exercise is to give acquaintance with various ways of doing it – shading, a numbered grid, arrows, lines, contour lines, etc. – and at the same time to give an acquaintance with orders of magnitude involved. The virtues and defects of some methods would bear brief discussion.



arrows, length proportional to field



lines, number per centimetre proportional to field

Figure 32 (*Continued.*)

Part Four

Ionic crystals

Time:
It would be best not to prolong this Part beyond, say, two double periods and an intervening homework, or the equivalent amount of time. Really interested students can always pursue the work on their own, using the detailed treatment in the *Students' book.*

Kinetic theory of gases.
A short treatment of the kinetic theory of gases ought to be considered as a substitute for Part Four, if students missed the theory at O-level. See Nuffield O-level Physics, *Teachers' guide IV,* Chapter 2; and the O-level Pupils' Guide *Molecules and motion* which is shortly to be published.

Why study ionic crystals?

The work provides a use for, and therefore an opportunity to gain more confidence with, the ideas of electric force, and potential. It should help students to use these ideas and to use the general idea of a pair potential, more readily in future.

The ideas are used in arguments in which we take seriously the idea that matter is made of regular arrays of atoms (Unit 1, *Materials and structure*), and use data from macroscopic experiments (results for electron charge, $1/4\pi\varepsilon_0$) to begin to show that electric forces are the key to the understanding of matter on the atomic scale. The use of a $1/r$ potential for charges of order e and for atomic distances will be required later in the course in the study of Rutherford scattering (Unit 5, *Atomic structure*) and in work on fitting electron waves in the potential well of an atom in Unit 10, *Waves, particles, and atoms.* The result for the force required to squeeze a crystal is used in Unit 4, *Waves and oscillations*, in a calculation on the frequency of oscillations in NaCl, which accounts for its absorption of electromagnetic waves. The work therefore plays a useful role in the course as a whole, but it is not essential to any of the main ideas; if it were omitted, other topics might need more careful attention ($1/r$ potential in Unit 5) or might be omitted (NaCl oscillations in Unit 4).

The work does give an excellent example of the development of a piece of theory in physics – having a mixture of abstract ideas, empirical facts, judicious guesses, and finally a predictive test. So there are good opportunities for talking about the nature of inquiry in physics.

Use of the step by step treatment in the *Students' book*

The whole argument is developed step by step in the *Students' book*, and we envisage that students will learn by working through this material, with the teacher acting as guide and consultant.

At the end of each of the stages one to four described here, the *Students' book* invites the student to stop if the going is too tough. Something of value is gained even by a student who stops at an early stage. Those who go on to the end have real cause for pride. We have found in trials that students have welcomed the challenge even of so tough a piece of work, and have taken pleasure in meeting that challenge.

At the end, the teacher could usefully summarize the sweep of the argument for those who have not pursued it right through.

Use of the *Teachers' guide*

The righthand text pages summarize the argument that appears in greater detail in the *Students' book*. The lefthand pages give further background information for teachers. None of the latter is meant to be taught.

This Part tries to show how ideas about the structure of a crystal (from Unit 1, *Materials and structure*) and ideas about electrical forces and energies (from Unit 3) can be used to explain how a solid is held together. The particular type of solid studied is an ionic crystal, and sodium chloride is chosen as a particular example.

Overall view of the argument

Stage one compares the experimental value of the energy needed to tear solid sodium chloride apart into a gas of separated ions, with the calculated energy needed to pull apart pairs of ions, starting from the measured equilibrium separation in the solid. The calculated value is about right, but is too low.

Stage two considers the effect on the electrical energy calculation of the presence of the other ions arranged in a lattice around any one ion. The new calculated value is a little too large. This could be explained if there were a contribution to the energy from the repulsive forces which are in any case needed to keep the ions at an equilibrium distance.

Stage three makes a guess that the repulsion energy follows a $1/r^n$ law, and n is shown to be nearly 9. The combined electrical ($1/r$) and repulsion terms yield an energy curve with a minimum at the equilibrium distance.

Stage four discusses the forces involved, and calculates the net force needed to reduce the spacing between ions by a small amount. From this, the bulk compressibility of the model of sodium chloride can be calculated, and compared with the tabulated value as a test of the model.

Introductory discussion

Recall Unit 1 and the evidence that the atoms of solids are held together rather strongly. One solid whose structure was mentioned in Unit 1 was salt; because it is made of charged ions, the forces holding solid salt together can be thought about in the light of the ideas developed in Unit 3.

For example, the energy of two charged ions a distance r apart is given by

$$\frac{1}{4\pi\varepsilon_0} \times \frac{Q_1\,Q_2}{r}$$

What experiments give information about r? (X-ray diffraction.) What are the charges Q? (Both equal to the charge on an electron.) With these data, it is possible to begin to ask questions like how much energy is needed to tear apart a salt crystal into separate ions.

Stage one

Experimental values of energy can be found from calorimetric measurements on the heat of formation of NaCl from Na metal and Cl_2 gas. The 'tear-apart' energy used here is the energy to get to separated ions, and so the heat of formation measurements have to be corrected for the energy to tear apart Na metal, the energy to tear apart Cl_2 molecules, the energy to ionize Na atoms, and the energy released when Cl^- is formed. Another way is to measure the heat of sublimation of NaCl, which produces NaCl molecules in a vapour phase; the tear-apart energy of such vapour molecules can also be measured.

Stage two

The argument shows how energy depends on the geometry of a structure. A different structure would give a factor different from 1.747. For CsCl with Cs ions at cube corners and Cl ions at cube centres, it is 1.763. This difference is one of the factors which explains why solids choose the structures they do – but it is only one of the factors. NaCl would have bigger attraction energy in a CsCl structural arrangement, but the repulsion energy would go up even more steeply because the relative size of Na and Cl ions is such that they would overlap and interpenetrate a lot in a CsCl arrangement.

The result tells us why an NaCl crystal is more stable than isolated NaCl molecules; but such isolated molecules are found in the vapour above a hot crystal – they are more stable than isolated Na^+ and Cl^- ions, as shown in stage one.

Throughout the calculation, we fix on one ion and reckon the interaction energies with every neighbour (summed up in the factor 1.747). If we then multiplied by the total number of ions, then the single energy (say of attraction) between ion X and ion Y would be counted twice, once as part of the energy sum for ion X and once again as part of the energy sum of ion Y; to avoid this error, we should multiply only by the number of ion pairs – so that it is correct to say that the energy is the energy per ion pair, not the energy per ion.

The actual drawing of the curves, here and in stage three, is an important part of the work, in giving confidence in reducing the abstract ideas to one's own calculated numbers and in getting the feel of the shape of the effects. It would be better to stop at the end of stage two or stage three than to save time by showing curves already drawn.

Stage three

The *Students' book* offers two arguments for the fact that if $E \propto 1/r^n$, the fractional change in E will be n times the fractional change in r. The first uses calculus, and will be easy for some students. The second uses arithmetic, and may be needed for part of the class. All students should be encouraged to think of this as a useful technical fact, not something to puzzle long over at this point.

The repulsion term arises because of the interaction effects of the electron charge clouds on one another. The $1/r$ attraction law assumes point charges, which is the same as assuming uniform spheres of charge for points lying outside the spheres. When charge clouds interpenetrate, the simple electric theory goes wrong; there are more important effects arising because the actual energy levels of electrons in an atom will be altered if the charges of a neighbouring atom or ion get very close. All of the effects can be calculated on the basis of Coulomb's Law if we know the right rules for the motion of electrons (wave mechanics) but it is clear that the atomic charge clouds will no longer behave as point charge clouds when they overlap. So the repulsion force is not a new type of force in physics – it's just a guess at representing a very complicated interaction of electron systems: it probably works quite well because we don't expect it to be right except in a very small region around 2.8×10^{-10} m where it is adjusted to be right.

Stage one

The experimental value of tear-apart energy for NaCl is compared with the value of

$$\frac{1}{4\pi\varepsilon_0} \times \frac{Q_1 Q_2}{r}$$

at $r = 2.8 \times 10^{-10}$ m (from the diffraction measurements), taking Q_1, Q_2 as 1.6×10^{-19} coulomb each (because we believe that each Na atom loses an electron and each Cl atom gains one). The energy given is about the right size – showing that we are using the right sort of charges and distances in the right sort of energy law. But the predicted tear-apart energy is too small.

But this calculation gives the energy that might be needed to tear apart sodium chloride if it were made up of isolated pairs of ions, each pair not influenced by any other pair, which is not too realistic.

Stage two

We have to look at the actual structure of NaCl because each ion has a lot of neighbours rather than just one; on balance, this puts up the tear-apart energy predicted by 1.747 times the energy for a single pair of ions. Now the predicted value turns out to be too big, so there must be some other, repulsive, effect. From the $1/r$ law, we can predict how the attraction effect varies with distance and draw an energy-distance curve. The slope of this curve gives the force of attraction: the force at 2.8×10^{-10} m can be measured from a tangent to the curve. The repulsion effect must give (*a*) a force equal and opposite to this to keep the crystal in equilibrium, (*b*) a contribution to the energy to explain the small difference between the measured and predicted tear-apart energies.

Stage three

We guess that the repulsion energy follows a $1/r^n$ law.

Then,

$$\text{repulsive force} = \frac{-\Delta E_R}{\Delta r} = \frac{nE_R}{r} \qquad \text{(numerical value)}$$

and substitution of values for NaCl from Stage two gives $n = 9$. A graph of repulsion energy against distance can be drawn. This repulsion energy can be combined with the attraction energy graph to give a graph of total energy against distance showing a minimum at 2.8×10^{-10} m, the slope being zero because the forces were made to balance at that point.

We guess that $E \propto 1/r^9$, and we might write this formally as $E = K/r^9$. K would include both factors governing the size of the repulsion effect between any two neighbours, plus a term giving the net effect of summing over all neighbours in a crystal (like the factor 1.747 used in stage two, but not the same, since that depended on the $1/r$ law). In all the subsequent calculations we don't need to know K: either the equations need $\frac{\Delta E_R}{E_R}$ or they are of the form $\frac{E_{r_1}}{E_{r_2}} = \left(\frac{r_2}{r_1}\right)^9$, so K always cancels.

The pair potential derived here is a particular example of a general idea: any stable condensed system, a molecule or a liquid or any solid, must follow the general pattern of attraction ($r>r_0$), zero force ($r=r_0$), repulsion ($r<r_0$), with an energy minimum at $r=r_0$. So this idea turns up frequently in the physics of matter. In ionic crystals the laws of attractive and repulsive potentials are $1/r$ and $1/r^9$ respectively; in solids held by the van der Waals forces the laws are $1/r^6$ and $1/r^{12}$; for covalent bonds there are no simple laws but the same general shape must be found.

Stage four

Compressibility is measured simply by placing specimens in a fluid, say in a piston, and measuring the volume change as pressure is applied hydrostatically. We have chosen to deal with uniform compression because it is the only distortion which can be dealt with by simple means. Other deformations, e.g. a tensile strain, are far more complicated: the whole three-dimensional array of the crystal is altered, the 1.747 constant (and a corresponding constant in the repulsion curve) will alter, and our energy distance and force distance curves don't tell us what will happen for such deformations.

There is a certain looseness in the use of the word 'force' in the argument. Once the calculation of the energy has moved from one pair of ions to the whole crystal, there are not one but many real forces acting on any one ion, surrounded as it is by many others. While for one pair, the force between them is dE/dr, when E becomes the energy per pair in the crystal as a whole, dE/dr cannot be one single real force. The argument used can be justified rigorously at every stage, the only change needed being a rather more cautious use of words.

Although the energy has been expressed as an energy per pair of ions, neither the energy nor the 'forces' dE/dr derivable from it relate to any one particular pair of ions. The whole crystal has now been included in the calculation. A change in r in the expression dE/dr relates only to a change in which every ion is moved closer to or further from its neighbours by the same amount, or in other words, a change in which the crystal is uniformly compressed or expanded.

Unlike the multitude of real forces acting on each ion as the crystal is compressed, dE/dr has no definite direction, and cannot be a real, single force. If it is thought of as a 'force', it can only mean the size of a force that would, moving through the distance dr, transform as much energy as is in fact transformed per pair of ions by the many real forces between ions as the crystal is squeezed down, achieving a uniform change dr in all the ion spacings r.

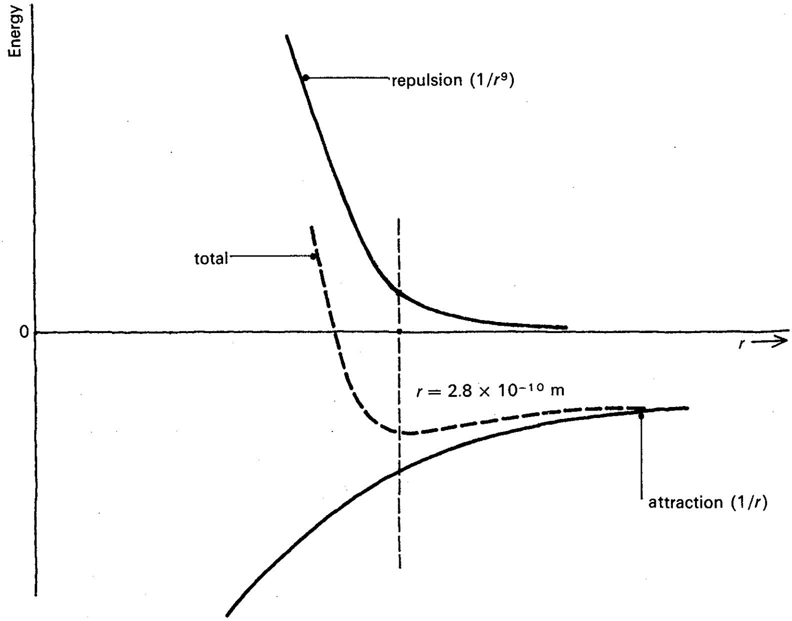


Figure 33

Up to this point, rules about a repulsion law have been invented and adjusted to fit the facts. A self-consistent picture or model has been made – but it fits because it was invented to do just that.

Stage four

The model is tested by seeing whether it can predict something new; something which it wasn't invented especially to fit.

The rates of change with distance of the laws for potential energy variations ($1/r$ and $1/r^9$) give the force laws for attraction and repulsion as $1/r^2$ (already known) and $1/r^{10}$ (new).

Knowing the value of the attractive force and that of the repulsive force at the equilibrium distance, 2.8×10^{-10} m, the actual force-distance curves and their resultant sum can be drawn. The slope of the resultant curve at $r = 2.8 \times 10^{-10}$ m can be used to find the extra force needed to squeeze the crystal, so from the curve we can predict the energy needed to compress the crystal by (say) 1 per cent in distance (3 per cent in volume), and, by energy arguments, use this to predict the pressure needed for a 3 per cent volume change. This can be checked against values of the compressibility modulus obtained experimentally and reported in tables of data.

Concluding discussion

This is not covered in the *Students' book*, and will only be worth pursuing if the interest of the class in this sort of question can be aroused. If such discussion can be encouraged, however, it will help to gain full advantage from the work put in on ionic crystals. Too long should not be spent on it – there will be many opportunities later in the course for arguments about models, theories, guesses, predictions, and so on. The aim is to encourage students to question and think about what they are doing in physics; clear tidy answers will not be found.

A task of this type can be carried through for ionic crystals because the stability of the closed shell ions (Na^+ and Cl^-) means that all we need to know about the attraction force is the electric force law for two uniform spheres; we can get away with knowing nothing about wave mechanics of electrons in atoms because the repulsion energy contribution is small and we deal with it empirically. For, say, covalent or metallic crystals, the main attraction energy is due to the rearrangement of electron energy levels under the effect of the electric forces of the neighbours and it is not possible to begin without knowing about wave mechanics of electrons.

Further details about calculations on ionic crystals may be found in Kittel, *Introduction to solid state physics* (3rd edition), and details about pair potentials in general in Tabor, *Gases, liquids and solids.*

Students' book

Questions 44 to 47 concern the ionic crystals argument. All are rather hard, and not all students should attempt them.

Question 44 discusses forces between charged ions. Questions 45 and 46 ask for interpretation of potential wells drawn as graphs.

Question 47 goes beyond the work so far, and suggests a new use for similar calculations, to discover something about what happens when the ions vibrate with larger amplitude, as they will do if the solid is heated.

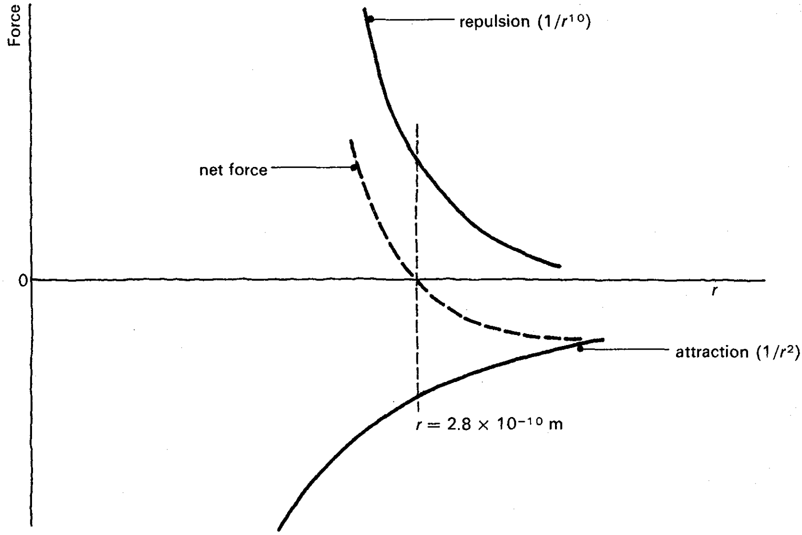


Figure 34

Concluding discussion

After work with the *Students' book* has finished, there could be some class discussion about the sort of job the class has done. Have we proved that solids are held together by the same forces that push and pull charged pith balls? (The same ε_0 was used.) Where did we stop cooking the theory to fit the facts and really show that it could stand on its own feet and do something? It can do more than compute a compressibility: the expansion of the solid, when it is heated and the ions vibrate more, can be analysed, for example. Some might try to see if they think the crystal would get harder to squeeze or easier to squeeze as it expanded (force-distance slope becomes less; it would be easier).

Students might try to list the things that went into the argument. Which are experimental facts? (The spacing, the energy, charge on the electron, etc.) Which are abstract ideas? (Potential; force $= -\Delta E/\Delta r$; energy changes on squeezing, etc.) Which were the guesses? (Electrical forces are responsible in the first place, $1/r$ law.) All have to come together and be stirred up with the right sort of spoon to produce results. Why do physicists think it is worth all this trouble? Are the ideas of interest only to physicists?

A better theory would use fewer guesses. There are theories which will enable the repulsion law to be predicted just as we could predict the $1/r$ law. They are very complicated because they depend on knowing how electrons in ions start to rearrange themselves when the ions start to touch. But they work quite well, and nowadays it is possible to *predict* the spacing and the energy, from atomic properties alone.

Appendices

Appendix A
Applications

The following are samples of the kind of information about applications which we hope teachers may collect, and use from time to time to illustrate the teaching.

1 The capacitor microphone

The flexible diaphragm of the microphone constitutes one of the plates of the capacitor, the rigid back plate forming the other. Sound incident on the diaphragm causes it to flex and so to alter the separation of the capacitor plates. The resulting changes in capacitance produce changes in the potential difference across the capacitor (the capacitor is in series with a d.c. supply and a resistor), which can then be passed on to the tape recorder, etc. The smallness of the changes means that a high degree of amplification is necessary. The frequency response of the capacitor microphone can be very good.

2 Tiltmeter/seismograph

(This information is taken from Jones, R. V., 1967, Physics bulletin, *Vol.* 18, *p.* 329.*)*

Basically, the apparatus consists of two capacitors formed between a plate, P_a, and an electrode, P_c, and between a plate, P_b, and P_c. P_a and P_b are vertical parallel plates and P_c is the bob of a short, 30 mm, plumb line which is free to move between the plates P_a and P_b. Tilting of the instrument affects the capacitance of the two capacitors. (Figure 36.)

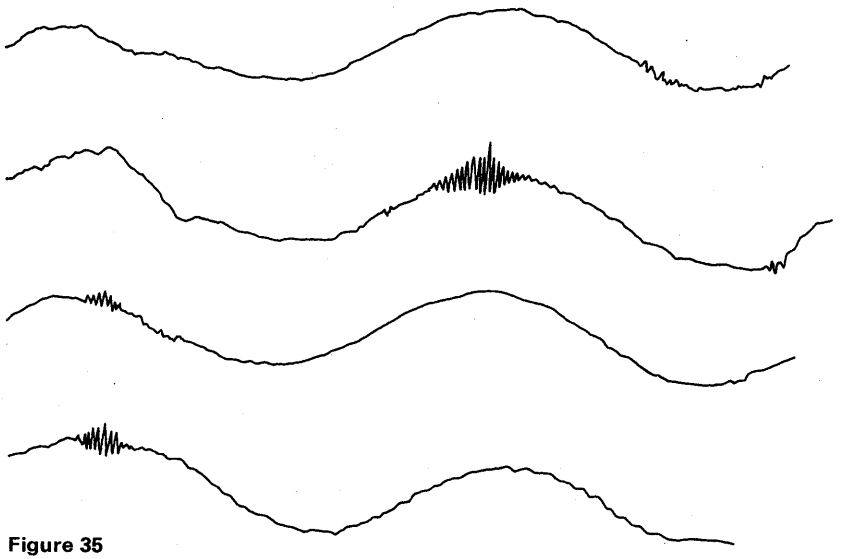


Figure 35
Typical tiltmeter record. This shows the effect of the tide in the North Sea on King's College in Aberdeen on four successive days.

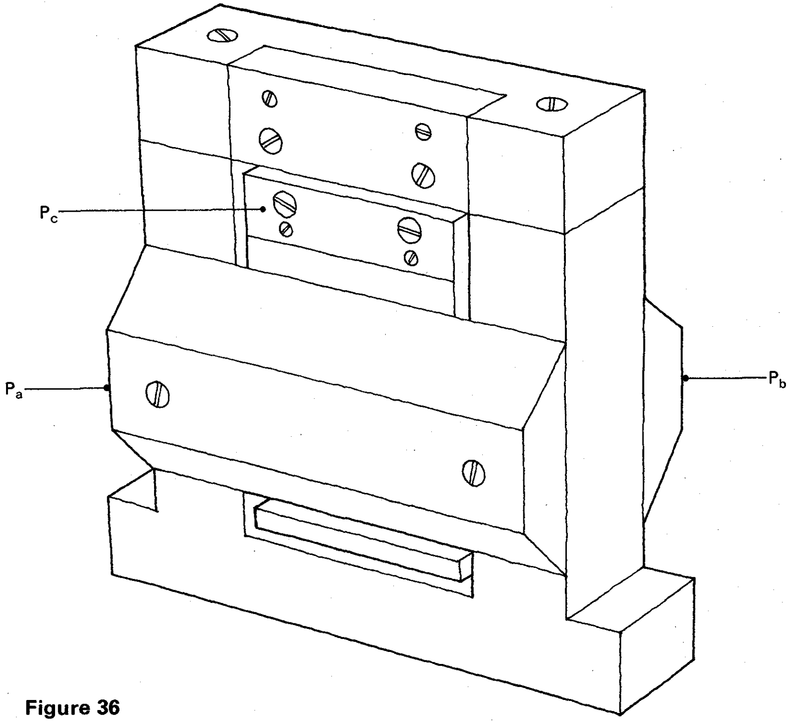


Figure 36

Figures 35 *and* 36 *are based on Jones, R. V.* (1967), Physics bulletin, **18**.

3 Electrostatic spraying and powder coating

Essentially, this method of coating an object depends on the coating medium being broken down into fine particles which are then charged and drawn onto the required surface by an electric field. In one form, the particles are directed by an air blast through a nozzle. The particles are charged at the nozzle and attracted to the required surface by the electric field between the spray gun and the object, the object generally being earthed.

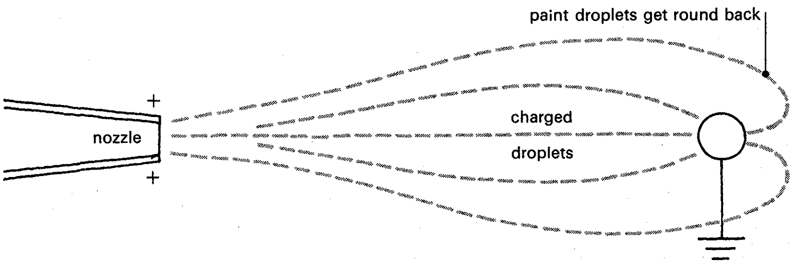


Figure 37

4 Static charges

When stepping out of a car you may have received an electrical shock. Sometimes it is possible to produce a spark over 10 mm long between a knuckle and the car body. Static charge built up as you slide across the plastic-covered seats is usually responsible. Static build-up on plastic sheeting is a nuisance in many situations: rolls of plastic cannot be unrolled owing to the sticking together of adjacent charged layers; spark hazards can give explosions if plastic film is being coated with adhesive in an inflammable solvent; dust spots adhere to charged ciné film during processing.

Static eliminators consist of devices to ionize the air near charged articles, so allowing the charge to leak away.

5 The electrostatic loudspeaker

A loudspeaker operates by converting electrical signals into mechanical movements of a diaphragm. The simplest form consists of two parallel mesh plates with a flexible metal-coated film stretched over a frame and placed midway between the two plates. A high steady potential difference is applied between the two mesh plates and the alternating potential is applied to the central flexible film. This causes the film to vibrate and emit sound.

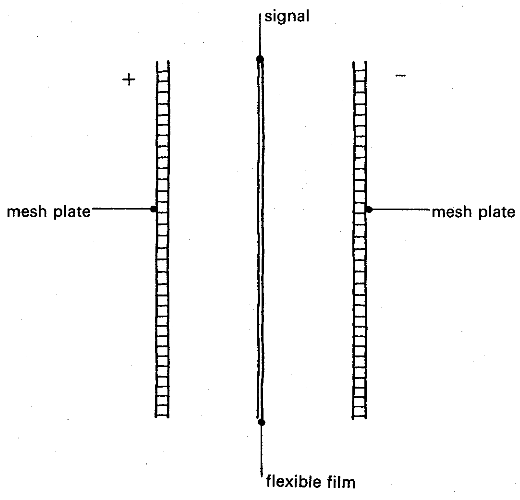


Figure 38

6 Electrical potential prospecting

Electrical prospecting methods are generally used in searching for metals and minerals and are effective only for shallow exploration down to about 500 m. They are also used by engineers studying the ground structure at the proposed sites of large structures such as dams, and by archaeologists.

If two electrodes are inserted in the ground some distance apart and a voltage applied across them, current will flow through the earth from one electrode to another. If the earth between the electrodes is homogeneous, the current flow lines will be symmetrical about the line joining the centres of the electrodes. Suppose, however, that there are clusters of buried material between the electrodes, perhaps a cluster with a lower resistance than the soil.

In this case, the current lines will look something like those in figure 39.
It is not possible to measure the direction of the current lines in the earth directly. Instead, use is made of the principle that the lines of current flow always pass at right angles across the lines along which the potential is equal. The distortion of the equipotential lines shows the presence of high or low resistance regions of material. In equipotential prospecting, two electrodes are buried about 500 m apart and a potential of 2000 V applied.

The actual equipotential lines are plotted by using search electrodes inserted into the ground. One is kept fixed and the other is moved until no current flows through a galvanometer in a lead joining them. By finding no-current-flow-points, it is possible to map out the equipotential lines.

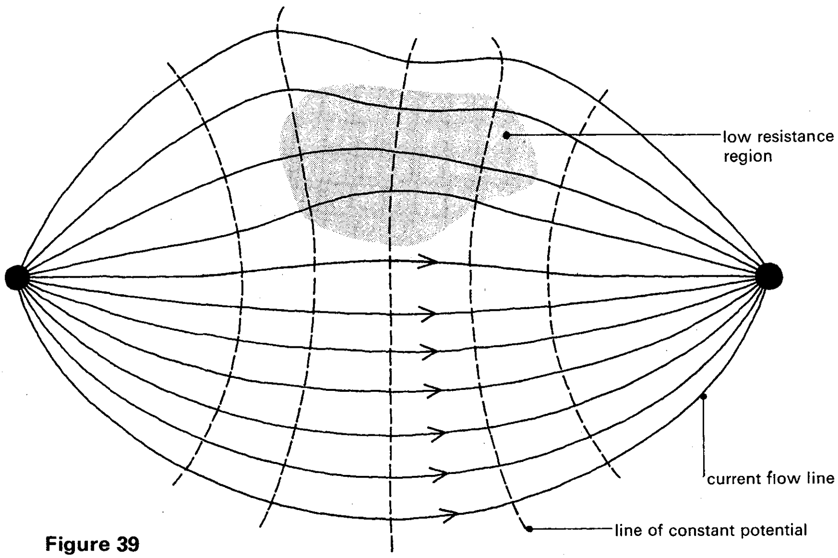


Figure 39

Appendix B
Lines of force

The suggested teaching in Unit 3, and elsewhere in the course, makes no use of the concept of an electric line of force as a continuous unbroken line through space, with line density representing field strength and perhaps with the lines endowed with some of the properties of elastic threads.

We do not think it can be argued that such a concept of a line of force contains any actual error, and the decision to avoid it is based on a matter of judgment, not on any matter of fact or principle.

The concept of a line of force can lead teachers and students into error or confusion, though it need not. One error 'derives' Gauss's theorem by sleight of hand. The lines are introduced and are said both to map the field by their density and to be continuous. Then Gauss's theorem is said to follow by an appeal to the continuity of the lines. This discussion reverses the proper dependence of ideas on one another. If lines are drawn to map the field by their density, it is Gauss's theorem which says that they may be drawn as continuous lines.

Simpler confusions can arise. Students sometimes suppose that there is zero field between the lines, especially when told that one, or that 4π lines emerge from a unit charge. The lines easily acquire a spurious reality of this kind. Too strong an insistence on lines suggests the error of supposing that a charged particle will always travel along a line of force.

At a later stage, there are some subtler difficulties. The electric field of a moving charge is not the same as that of a stationary charge, and whether an observer detects effects of moving charges as magnetic field or electric field depends on how that observer is moving. So the pattern of lines one observer draws is not the same as that drawn by another. The existence of a line of force does not survive a change of the frame of reference.

This is not to deny the value of the line of force representation for some purposes. We have talked with electrical engineers and plasma physicists who use the idea daily and value it highly. They are, however, careful to avoid the traps set by an over-literal use of the idea. Others use the idea little, and are conscious of its dangers. We recommend teachers who wish to develop the idea to do so with caution.

Appendix C
The constant ε_0

There are two kinds of difficulty about the status of ε_0, one relating to units and definitions, the other to debates about models of the aether or vacuum, and the relationship, if any, of such models to the electrical behaviour of materials like polythene.

Definitions and units

In the SI, the ampere is a fundamental quantity, so that charge is measured with an ammeter and a clock, in ampere seconds. At this point in this course, the position is similar, except that ammeter readings are temporarily taken on trust prior to discussing the unit of current in Unit 7, *Magnetic fields*.

Thus ε_0 in equations such as

$\sigma = \varepsilon_0 E$ (σ = charge surface density)

or $F = Q_1 Q_2 / 4\pi\varepsilon_0 r^2$

is a measurable quantity related to the size of forces between charges. (The disposition of the 4π in these equations is a consequence of 'rationalization'.)

It is not necessary, in a different unit system, to proceed in this way. The constant k in $F = kQ_1 Q_2/r^2$ can be chosen to have any arbitrary value, so fixing the unit of charge in another way. If $k = 1$, the charge unit is the 'electrostatic' unit, if F is measured in dynes and r in centimetres, for example.

It is not, however, quite enough to say that ε_0 specifies the size of the force between charges a certain distance apart, and as a consequence the p.d. across and the field inside a capacitor with a given charge density. The size of the charge unit was specified in terms of the force between moving charges or currents. The value of ε_0 ultimately compares the electrical force between charges at rest, with respect to the measuring instruments, with the magnetic force between these charges when moving, with respect to the measuring instruments.

This situation arises because there are two force measurements – electrical and magnetic – that can be made, but one of them must be used to choose one of the units, either of current or of charge. This may be brought out by writing down the forces between moving and between stationary charges.

Fixed charges (electrical) $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$

Moving charges (magnetic) $F = \dfrac{\mu_0 Q_1 Q_2 v^2}{4\pi r^2}$

(The charges Q_1, Q_2 move parallel to each other, both with velocity v.)

The value of μ_0 in the second equation is fixed at $4\pi \times 10^{-7}$ N A^{-2} in the SI by the choice of the unit of current in terms of the force, arbitrarily fixed at 2×10^{-7} N m^{-1} on a long straight wire parallel to another, one metre away, when both are said to carry one ampere.

The ratio of these two forces is $F_E/F_m = v^2 \varepsilon_0 \mu_0 = v^2/c^2$ because $(\varepsilon_0 \mu_0)^{-\frac{1}{2}}$ is, on Maxwell's theory, the velocity of light. Ultimately, then, a measurement of ε_0 is a comparison between forces on stationary and moving charges, which, with the velocity of the moving charges, amounts to an electrical measurement of the velocity of electromagnetic waves.

In more advanced teaching, when the electric and magnetic fields can both be discussed from the outset, there may be a case for treating $\varepsilon_0 \mu_0$, equal to $1/c^2$, as the only fundamental measurable quantity. Then if μ_0 is a defined quantity, as in the SI, ε_0 is to be found by calculation from the values of μ_0 and $1/c^2$. Indeed, one of the quantities ε_0 or μ_0 can then be disposed of altogether, with $1/\varepsilon_0 c^2$ replacing μ_0, or vice versa.

The vacuum as a dielectric 'substance'

If the plates of a capacitor are filled with, say, polythene, the charge stored for a given potential difference is larger and the field E is smaller for a given charge density, both by a factor ε_r, the relative permittivity of polythene.

$$C = \varepsilon_r \varepsilon_0 A/d$$

$$\text{or } \sigma = \varepsilon_r \varepsilon_0 E$$

The value of the relative permittivity ε_r of a material substance can be explained, in principle, in terms of the effects of an electric field upon its molecules.

If one had some model to represent empty space such as physicists of the nineteenth century strove to invent, it might make sense to interpret ε_0 in terms of the model, and to treat ε_0 and $\varepsilon = \varepsilon_r \varepsilon_0$ as similar kinds of quantity, both being permittivities in need of microscopic explanation.

But, as the previous considerations show, the present view among physicists is otherwise. A vacuum is simply 'nothing there', no model could 'explain' the value of ε_0, and arguments about ε_0 are arguments at first about forces between charges across empty space, and finally about the constant velocity c of propagation of electromagnetic fields in empty space. Of course, the nineteenth century physicist could transform a question about interpreting why ε_0 was a permittivity of a certain size into a question about why the fields propagate at this velocity, both being related to some model of the vacuum. The present position is simpler, if less agreeable: no one knows why c has its particular value, nor is there any other fundamental velocity with which to compare it.

Appendix D
1/r variation of potential

For the spacecraft, the force varies with distance, so a simple force times distance calculation will not serve to compute the energy change.

The gravitational field of the Earth can be plotted for distances of, say, 10×10^6 m to 50×10^6 m. See figure 40.

Distance $r/10^6$ m	10.0	14.14	20.0	30.0	40.0	50.0
Field g/N kg^{-1}	4.00	2.00	1.00	0.44	0.25	0.15

Table 6
Gravitational field of the Earth.

The energy needed for a 1 kg mass to go from 19×10^6 m to 21×10^6 m will be close to 1 N kg^{-1} multiplied by 2×10^6 m, or 2×10^6 J kg^{-1}, illustrating how the energy is given by the area below a section of the graph.

Areas between distances 10×10^6 m apart can be estimated, and totalled, giving values such as those in table 7.

$r/10^6$ m from	to	Area (energy per kilogramme)/10^6 J kg^{-1}	Running total energy/10^6 J kg^{-1}	r/10^6 m
			0	50
50	40	1.8	1.8	40
40	30	3.3	5.1	30
30	20	6.7	11.8	20
20	15	6.6	18.4	15
15	10	13.2	31.6	10

Table 7
Changes in potential near the Earth.

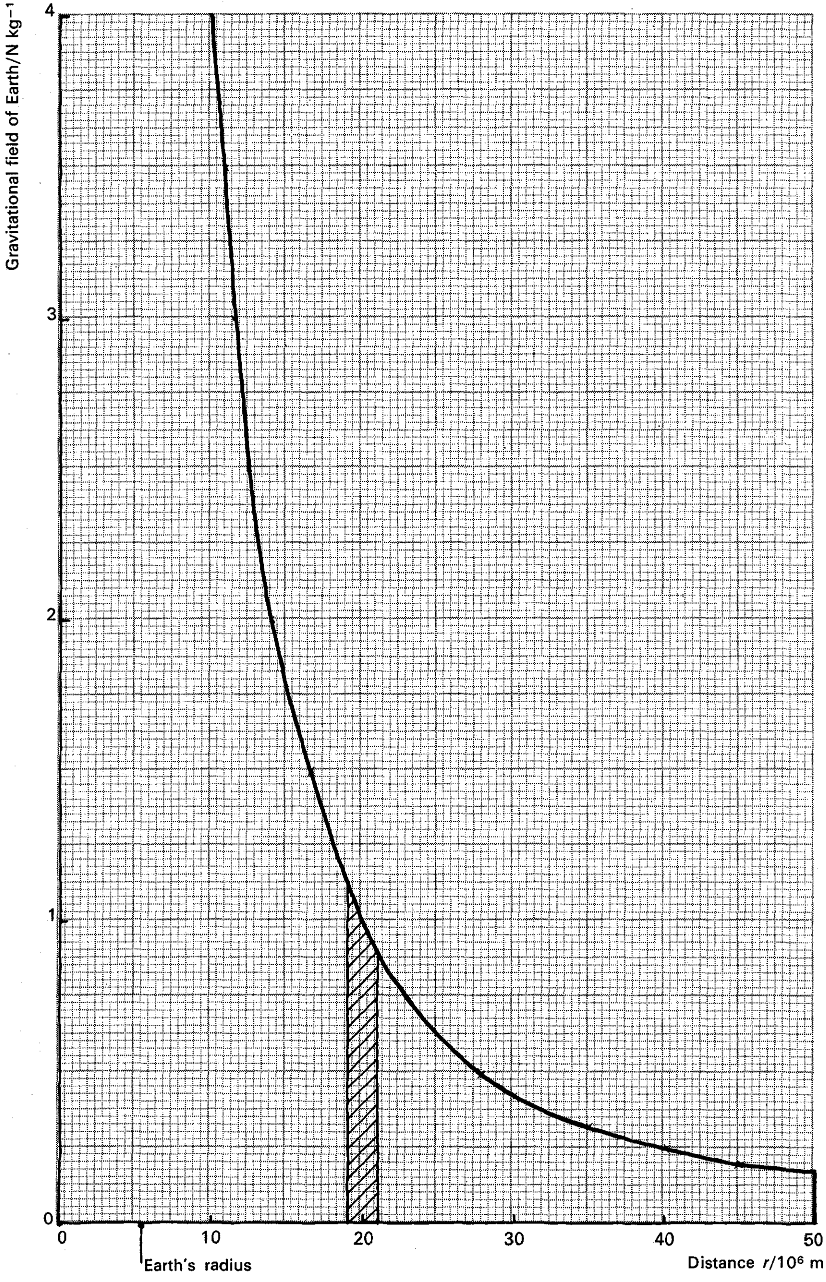
The energy per kilogramme transformed in going from 50×10^6 m to 10×10^6 m is thus about 31.6×10^6 J kg^{-1}.

The integral gives

$$\text{Energy transformed per kilogramme} = \int_{r_1}^{r_2} \frac{GM}{r^2}\,\mathrm{d}r = GM\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$
$$= -32 \times 10^6 \text{ J kg}^{-1}$$

The potential energy has fallen. The agreement is good.

Further, the energy transformed in going from a distance 50×10^6 m to distance r, (the running total above) plotted against $1/r$ gives a good straight line. See figure 41.



field

Figure 40
Gravitational field of the Earth.

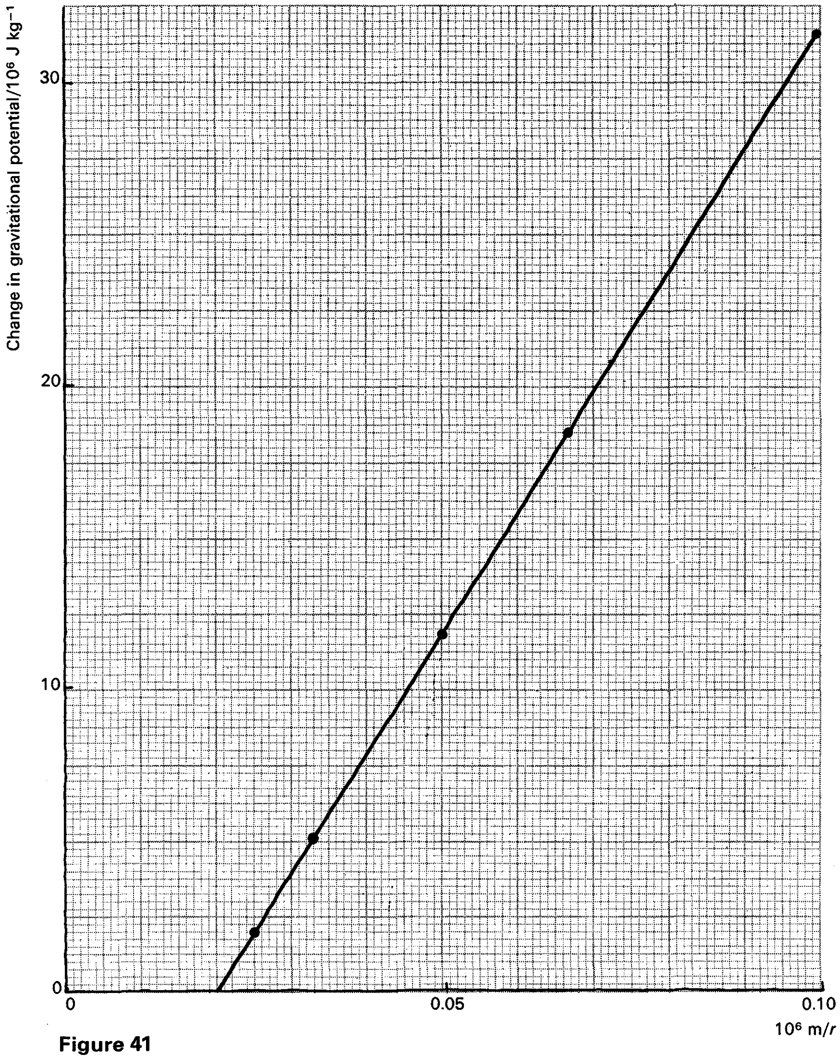


Figure 41
Changes in potential energy against $1/r$.

Books, articles, films, and apparatus

Books

Page numbers of references in this *Guide* appear in bold type.

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Articles

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8 mm films

A brochure, 'Camera on the Moon', available from Moon Productions, *Daily Express,* 116 Fleet Street, London E.C.4, gives details of 8 mm films taken on several of the Apollo flights.

	Apparatus	Experiment
7B	vegetable black	3.10
14	e.h.t. power supply	3.1, 3.2, 3.3, 3.5, 3.8, 3.11, 3.13
15	h.t. power supply	3.4, 3.7
27	transformer	3.6, 3.8, 3.11, 3.13
30	slotted base	3.2, 3.8
32	1 kg weight	3.4
50/1	cylindrical magnet	3.9
51A	gold leaf electroscope	3.8, 3.11
51D	metallized polystyrene balls	3.13
51E	reel of nylon suspension	3.13
51G	polythene strip	3.1, 3.2, 3.8
51I	'rubber'	3.13
51K	electrophorus plate	3.13
51L	proof planes	3.13
51M	square polythene tiles	3.2, 3.8, 3.13
52K	crocodile clip	3.8, 3.11
57K	reel nylon sewing thread	3.1
57L	table tennis ball coated with Aquadag	3.1
59	l.t. variable voltage supply	3.4, 3.7
60/1	Van de Graaff generator	3.3, 3.13
65	metal plates with insulating handles	3.1, 3.5
92X	PVC-covered copper wire	3.8, 3.11
94A	lamp, holder, and stand	3.8, 3.9, 3.11, 3.13
97B	bottle of Aquadag	3.11, 3.13
113/3	lens + 2.5D	3.10
133	camera	3.10
134/2	xenon flasher	3.10
149	electric field apparatus	3.3
158	class oscilloscope	3.4, 3.6
171	photographic accessories kit	3.10
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A *Teachers' guide* has been produced for each of the ten Units forming the Advanced Physics course. This is the *Guide* for Unit 3, *Field and potential.* It is intended to provide whatever information and ideas are required for the day-to-day teaching of the Unit. The book begins with an Introduction setting out the purpose of the Unit, a summary of the Unit, and a list of suggested experiments. Following this, the main text consists of four Parts, 'The uniform electric field', 'Gravitational field and potential', 'The electrical inverse square law', and 'Ionic crystals'. It contains teaching suggestions, details of experiments, and a commentary giving background information and other guidance. There are also Appendices on 'Applications', 'Lines of force', 'The constant ϵ_0', and '1/r variation of potential', and lists of relevant books, articles, films, and apparatus.

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