Physics

Students' book Unit 3

Field and potential

Nuffield Advanced Science

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Physics Students' book **Unit 3**

Field and potential

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Field and potential

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Foreword

It is almost a decade since the Trustees of the Nuffield Foundation decided to sponsor curriculum development programmes in science. Over the past few years a succession of materials and aids appropriate to teaching and learning over a wide variety of age and ability ranges has been published. We hope that they may have made a small contribution to the renewal of the science curriculum which is currently so evident in the schools.

The strength of the development has unquestionably lain in the most valuable part that has been played in the work by practising teachers and the guidance and help that have been received from the consultative committees to each Project.

The stage has now been reached for the publication of materials suitable for Advanced courses in the sciences. In many ways the task has been a more difficult one to accomplish. The sixth form has received more than its fair share of study in recent years and there is now an increasing acceptance that an attempt should be made to preserve breadth in studies in the 16–19 year age range. This is no easy task in a system which by virtue of its pattern of tertiary education requires standards for the sixth form which in many other countries might well be found in first year university courses.

Advanced courses are therefore at once both a difficult and an interesting venture. They have been designed to be of value to teacher and student, be they in sixth forms or other forms of education in a similar age range. Furthermore, it is expected that teachers in universities, polytechnics, and colleges of education may find some of the ideas of value in their own work.

If the Advanced Physics course meets with the success and appreciation I believe it deserves, it will be in no small measure due to a very large number of people, in the team so ably led by Jon Ogborn and Dr Paul Black, in the consultative committee, and in the schools in which trials have been held. The programme could not have been brought to a successful conclusion without their help and that of the examination boards, local authorities, the universities, and the professional associations of science teachers.

Finally, the Project materials could not have reached successful publication without the expert assistance that has been received from William Anderson and his editorial staff in the Nuffield Science Publications Unit and from the editorial and production teams of Penguin Education.

K. W. Keohane
Co-ordinator
of the Nuffield Foundation Science Teaching Project

To the student

This book contains some of the things you need to help you to understand the work of this Unit, and some reading which we hope will help you to see how the work is relevant to the practical, everyday world. It does not contain all you need: you will have to consult textbooks and other more general books as well, working through theoretical arguments, reading about experiments, and finding out more about how the ideas can be put to practical use.

This book contains many questions; more than you will be able to do while working on this Unit. Later on, you may wish to use some of them for revision. You will find questions which take you step by step through the theoretical arguments in the course; students who took part in the trials have said that these questions are a good way to understand a piece of theory. You will have to pick and choose, according to your needs and tastes, amongst the other questions. A few give you simple practice in calculation. More invite you to argue about or discuss a problem, and some of these – usually marked *'For discussion'* – are not suited to formal written answers. They are meant to start off a discussion, which may then wander far from the question.

There are a few harder questions to challenge the clever, and you should not expect to be able to tackle every question easily. But most are meant for ordinary human beings, not for budding geniuses. If in doubt, try the obvious answer: usually there is no catch! Most questions are given some kind of answer in the section headed 'Answers', though some of these suggest where you might find the needed information, instead of giving it. We have tried hard not to give wrong answers, but, being fallible like yourselves, may not have succeeded.

Some questions ask you to guess, speculate, or give your private opinion: obviously they have no one right answer.

What you are being asked to learn to do

This course aims to help you to become more like a physicist. Most of you will not become physicists, but will use physics or learn more of it in one of a variety of scientific jobs or in further education. Physics, and the world with it, are changing so fast that no one can tell what bits of physics you will use in, say, ten years' time; however, one can be pretty sure that there are some basic ideas that will be relevant to the new problems of tomorrow. We have tried to build the course around what we believe to be these basic ideas.

So one thing the course aims at is to help you to become able to learn, in the future, the new ideas in physics you may meet, and to help you to become able to use the physics you have learned. It does this because these are the tasks that will face you.

In the future, you will need to be able to learn from books and articles; that is why the course contains a good deal of reading (in a list at the end, you will find details of books referred to in the text). To use the physics you have met, you need to understand it – that is, to be able to use it in new kinds of problems. That is why so many questions in this book ask you to make up arguments about new problems, using what you know.

What is 'understanding'? That is, how does one recognize that someone understands a piece of physics? We think it is something like this. Suppose a group of people are talking about a problem in physics. Very rarely, even among research workers, will anyone immediately see an answer. More often, they each have some ideas which they try out in discussion with colleagues. Those who 'understand' their physics are the ones who can offer sensible, relevant ideas that would help towards clearing up the problem. A reasonably competent physicist expects himself and others to be able to draw on their knowledge and use it to make sensible contributions to the discussion of problems.

So to test whether you understand a piece of physics, it is asking too much to expect you to solve a new problem completely and correctly; few – if any – experts can do that. The test should be that of physicists talking together: can you produce sensible ideas that are relevant and would help a bit towards clearing up a problem? This is the test that will be used in the examination, and is the way to decide how well you have managed a question or problem in the work of the course.

The course also aims to show you what doing physics is like, and this is another reason for encouraging plenty of discussion of problems, for that is the way physicists work. It tries to show what kinds of questions physicists ask themselves and what sorts of ways they use to tackle them. We think this is important because to use physics successfully and to judge its claims and achievements you need to understand what it can, and what it cannot do. That is why several questions ask you about such things as how theories, models, experiments, and facts fit together. Physicists also guess, estimate, and speculate, so other questions ask you to do these things too, to find out what doing them is like and to become better at doing them.

There are a lot of misunderstandings about what physics is like. Some say it is all facts; others that it is all theory, having little to do with what happens in practice. Many are puzzled; asking whether what physics says is true or not, or how physicists arrive at their ideas. We hope you will find chances in this course to think about such matters, and that you will form your own views.

Some of the questions ask about how physics can be used in engineering and technology, and the articles in this book are also about that, because we think that you will rightly want to know when what you learn is of practical value.

Finally, one of the main reasons we want to offer you some physics is that we like the subject and get excited about it. So we hope you enjoy it too.

Summary of Unit 3
Field and potential

This Unit is mostly about electric fields, but the gravitational field is also introduced, to illustrate how the same ideas work for more than one kind of problem.

The ideas in this Unit will be used again and again in the course, especially in Unit 5, *Atomic structure*, Unit 8, *Electromagnetic waves*, and Unit 10, *Waves, particles, and atoms.* Part Four tries to show you another way in which the ideas are used, and will be good practice for the later uses.

Part One
The uniform electric field

The concept of electric field
Forces on charges between charged plates. The uniformity of the field. $E = V/d$. Shapes of other fields.

Experiments with capacitors
Relations between field, p.d., area, spacing, and charge.

A new tool
Use of reed switch, capacitors in parallel and in series.

A fundamental constant
Charge density $= \varepsilon_0 \times E$.

Potential difference and field
Relation between V and E again, using the flame probe.

Part Two

Gravitational field and potential

'Field' as an important idea in physics

Fields, action at a distance.

Newton's basic ideas

$F = GM_1M_2/r^2$.

A fundamental constant

Measuring G.

Seeing how field, force, and energy are linked

Problems using space flight data.

Theoretical links between ideas

$1/r^2$ variation of force; $1/r$ variation of potential energy. Checks using space flight data.

A theoretical concept

Potential at a point. Escape velocity.

Part Three

The electrical inverse square law

Use of analogy

Potential variations near a charged sphere, using flame probe and ideas from Part Two.

A fundamental law

Coulomb's Law, and experimental tests of it.

More theoretical links

Relation between uniform field and inverse square field. Coulomb's Law constant $= 1/4\pi\varepsilon_0$.

Pictures of fields

Different sorts of maps of fields, for different purposes.

Part Four
Ionic crystals

How one kind of theoretical physics works: practice in thinking about force and energy relationships

The energy and structure of sodium chloride, and the forces between its ions. Theoretical discussion of energy and forces using ideas of electric field and potential.

Questions

Part One

The electric field

Questions 1 to 3

These are about circuits containing more than one capacitor and are intended to show you how their capacitances combine.

1 *Capacitors in parallel*

Total capacitance $C = C_1 + C_2$ (figure 1 *a*).

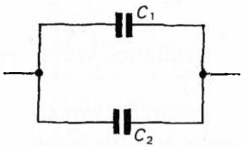


Figure 1a

The following questions show why the capacitances of capacitors in parallel add up. Capacitors of one sort or another will be used in parallel in several experiments in Unit 3.

a What charge is stored on a 10 μF capacitor connected to a 10 V supply (figure 1 *b*)?

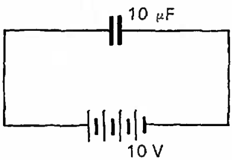


Figure 1b

b What charge is stored on a 20 μF capacitor connected to a 10 V supply?

c In figure 1 *c*, will both capacitors charge up until there are 10 V across each?

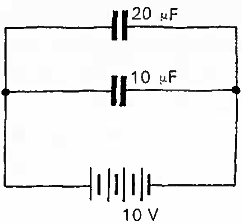


Figure 1c

d What is the total charge stored in the arrangement of figure 1 *c*?

e If you wanted to replace the two capacitors in figure 1 *c* by a single capacitor which would store the same charge as the two together, what capacitance would you have to use? (Use the answers to **c** and **d**.)

f Would you say that the following statement is true?
'Capacitances in parallel add up because their charges add up and the p.d. across each capacitor is the same.'

2 *Sharing charge between capacitors*

This question is included because there are experiments in Unit 3 which use charge-sharing as part of the measurement of charge with an electrometer.

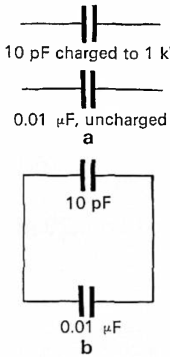
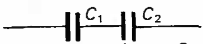


Figure 2

a 1 pF = 10^{-12} F; 1 μF = 10^{-6} F. What is the ratio of the capacitances of the capacitors shown in figure 2*a*?
b What is the charge on the 10 pF capacitor?
c Figure 2*b* shows the capacitors connected. The high voltage across the charged capacitor will drive charge onto the uncharged one. What can you say about the p.d. across each capacitor when this flow of charge stops?
d $C = Q/V$. What will be the ratio of the charges on the capacitors in figure 2*b* when flow stops? Which has the larger charge?
e Write down, without further calculation, a good approximation to the charge on the 0.01 μF capacitor. (Use the result from **b**.)
f Is the value in **e** too large or too small? How big is the error as a fraction of the estimate in **e**?
g In another trial, the 10 pF capacitor was charged to a different voltage, and connected as above to the uncharged 0.01 μF capacitor. An electrometer shows that the p.d. across the 0.01 μF capacitor became 0.48 V. What is a good estimate of the charge originally on the 10 pF capacitor?

3 *Capacitors in series*

Figure 3a



Total capacitance C

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

The problem below shows how it comes about that when two capacitors are in series, the above equation gives the total capacitance. This result, particularly the consequence that the combined capacitance is less than that of either component (prove it), will be useful in Unit 6, *Electronics and reactive circuits*.

a Suppose 10^{-4} C of charge flows from the battery (figure 3 *b*) to the plate A of a 10 μF capacitor. What charge flows from the plate B to the negative terminal of the battery?

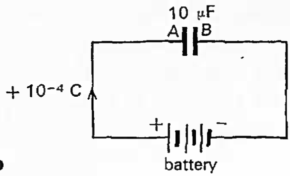


Figure 3b

b What is the p.d. across the capacitor?
c What would be the p.d. across the capacitor had the capacitance been 20 μF and had the same charge flowed?
d In figure 3 *c*, a 10 μF and a 20 μF capacitor are in series. A charge of 10^{-4} C flows onto plate A, and a charge of 10^{-4} C leaves plate B. Instead of going to the battery this charge arrives at plate X. What charge leaves plate Y?

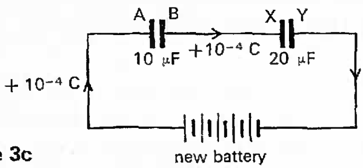


Figure 3c

e If the two capacitors were hidden in a box, and 10^{-4} C went in at one end and out at the other, what charge would you say had been stored on the 'capacitor' in the box?

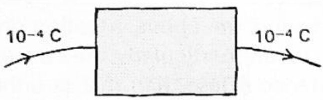


Figure 3d

f Using the answers to **b** and **c**, what must be the p.d. of the new battery in figure 3 *c*?

g Using the answers to **e** and **f**, what is the capacitance of the series combination? ($C = Q/V$.)

h If the charge on the series combination is Q, the charges on each capacitor being Q also, write down formulae in terms of Q, C_1, C_2 for:

the p.d. across C_1

the p.d. across C_2

the p.d. across the total capacitance C.

You should see that it follows that $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Questions 4 to 12

These are a variety of problems about detecting, measuring, and using electric fields.

4 A conducting ball hangs on an insulating thread between two plates connected to a battery. It rattles continually backwards and forwards from plate to plate. Why does it keep moving? As it rattles it must lose energy every time it collides with the plate – where is it getting energy from to keep going? What forces act on the ball?

If during this experiment two galvanometers are included in the circuit as shown, they indicate that a current is flowing. Say in which direction you expect the current to be flowing in each galvanometer.

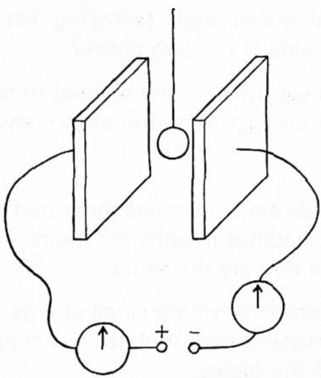


Figure 4

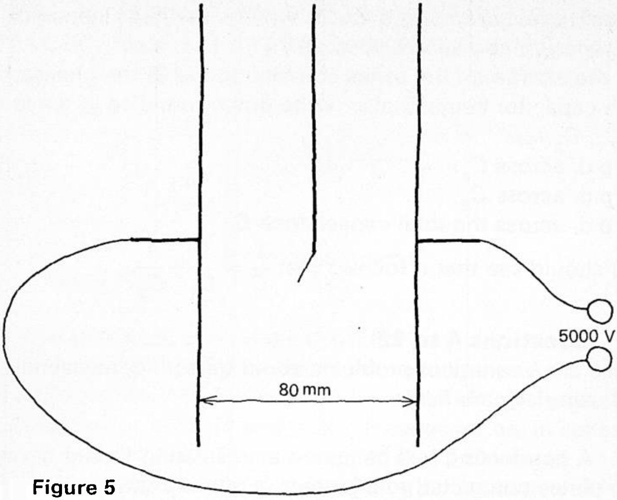


Figure 5

5 A small charged strip of foil on an insulating handle is held between two large charged plates which are connected to a 5000 V supply. The plates are 80 mm apart; the angle at which the strip hangs is noted.

The plates are now moved until they are only 40 mm apart, leaving them connected to the supply at the same p.d. Will the strip hang at a larger angle (showing *more* force on its test charge) or a smaller angle (showing *less* force)? How big are the electric fields in the two cases?

To what value must the p.d. be changed to get the same force as at first on the charged test strip at the new, smaller spacing?

6 Electric fields are sometimes measured in newtons per coulomb and sometimes in volts per metre. This question is about why these two are the same.

Suppose an oil drop carrying a small charge, say 10^{-17} coulomb, is between a pair of plates 5 mm apart with a p.d. of 500 volts across the plates.

a If the drop were pulled by electrical forces from one plate to the other, how much energy would be transformed by the electrical forces?
b In pulling the drop across, an electrical force moves the drop 5 mm (5×10^{-3} metre). How big is the electrical force? (Assume that the field is uniform, so that the force is steady all the way across.)
c How big is the electric field, calculated from the force per *coulomb of charge it acts upon?*
d Divide the p.d. V (500 volts) by the plate spacing d (5×10^{-3} metre). The result should be the same as the answer to **c**. Write down some lines of algebra to explain why. (Try writing F = force, E = field, q = charge, and following the questions through.)
e One unit for electric field is V m^{-1}.
Express the unit V *in terms of joules* J *and coulombs* C.
Express the unit J in terms of newtons N and metres m.
Show that V m^{-1} can be written N C^{-1}.

7 **a** In the sparking plug of a motor car engine there are two electrodes separated by a spacing of about 0.67 mm. If air begins to ionize when the electric field is about 3×10^6 volts per metre (at atmospheric pressure), *roughly what* p.d. must be applied across the electrodes to cause a spark in air?

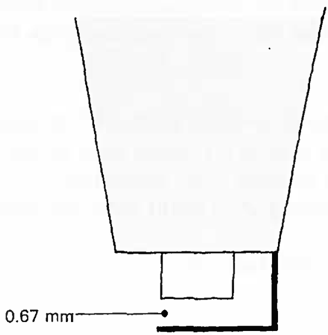


Figure 6

b The p.d. needed to cause a spark will depend on the gas pressure. Explain what effect you think a change of pressure will have on this p.d. (Look back at the *Students' book*, Unit 2, *Electricity, electrons, and energy levels*, Part Five, question 78, and also at 'The evidence for the existence of energy levels in atoms' in the same Unit.)
Is the gas pressure in a motor car engine greater or less than atmospheric pressure when the spark is needed?
You might like to read about induction coils to find out how the high voltages required are produced when a 12 V car battery is the source of electrical energy.

8 When two new smooth sheets of aluminium cooking foil are laid loosely on the bench, with an insulating polythene sheet between them, and are connected to a p.d. of, say, 5000 volts, the foils can be seen (and heard) flattening. Why?

If the p.d. is disconnected, leaving the foils charged, would any electrical force have to be overcome if they were pulled apart with an insulating handle? (**Don't try it** – you could receive a lethal shock.) What would happen to the energy of the arrangement as they were pulled apart, if no charge leaked away? What would happen to the p.d. across them?

(Hard) If the experiment is tried, charge is very likely to leak away, and confuse the predicted results. Remembering the sharp edges of the foil, why should charge leak away so easily?

9 This question is meant to be real guesswork. Physicists value the ability to make a reasonable guess at an answer they are unable to work out completely.
a The electric field at A is 5000 volts per metre. *Guess* what it is at B.
b *Guess* the electric field at C.

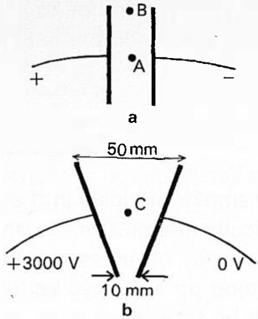


Figure 7

10 **a** Make a very rough estimate of the electric field in the air (say at P, figure 8*a*) between the terminals of a 1.5 volt dry cell. You may start by guessing at the length of the path you would have to travel to go from one terminal to the other through P. In travelling that distance the potential changes by 1.5 volts, so what is the mean field? (Or, if you took one coulomb along the path, the energy change would be so many joules, and so many joules for a path of so many metres gives how many newtons per coulomb acting along the path?)

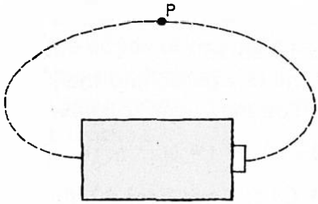
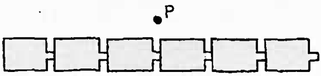


Figure 8a

b Six such cells are clamped in line to give a 9 volt battery (figure 8*b*). What do you think the field at P is now?

Figure 8b



11 Many of the smoke particles in the smoke from a furnace are electrically charged, some positively, some negatively. Devise a way of removing such particles electrically from the smoke going up a flue, so as to reduce the pollution of the air by smoke particles. Would your method offer any advantages over more obvious methods, such as fine gauze filters in the flue?

12 **a** Check the accuracy of the following statement. 'If a reed switch is used to discharge a 1 μF capacitor through an ammeter 50 times a second, a 1 mA meter will be adequate if the capacitor is charged to 10 V.'
b What order of magnitude of capacitance can be used in a similar experiment, again using 10 V, if the reed switch discharges it 100 times a second through a meter which gives a measurable deflection for a steady current of 1 μA?

Questions 13 to 18

These are about the charge on and the p.d. across parallel plate capacitors. You may need to know:

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}$$

$$\approx 10^{-11}\ \mathrm{C^2\ N^{-1}\ m^{-2}} \text{ for rough calculations.}$$

For parallel plates,

$$C = \varepsilon_0 A/d$$

or $$Q/A = \varepsilon_0 V/d$$

$$E = V/d$$

and $C = Q/V$ as for any capacitor.

13 A plate is charged to 4 kV and held by a very well insulated support 50 mm above the bench, which conducts quite well. Its p.d. drops from 4 kV to 3 kV in 10 minutes. If its area is 2×10^{-3} m^2, make a rough guess at the conductivity of the air, assuming the leakage is due to air conduction and that this is uniform over the plate area. (You may be able to show that the question has given you two data you do not have to use.)

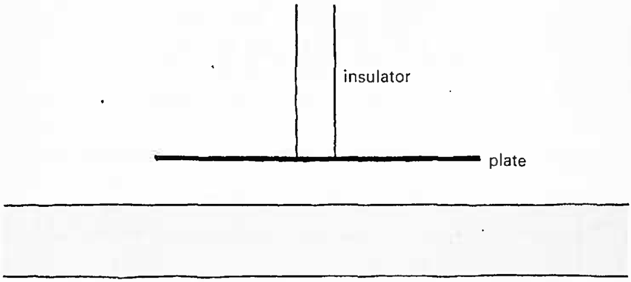


Figure 9

(Harder) If you obtained a value for the conductivity of air from an experimental measurement of this type, your value would not be accurate because the current through the air is not the same near the edges of the plate as in the middle. Would your value for the conductivity be too low or too high? Why do you think the assumption of uniform flow from the plate surface might be wrong?

14 Consider the equation $Q/A = \varepsilon_0 E$.

a To measure ε_0 experimentally, what quantities would you have to measure?

b How might an electric field be measured in practice, using a voltmeter and a ruler?

c Question 6 shows that the uniform electric field E is equal to V/d, where V is the p.d. between the plates and d is their spacing.

$$E = V/d.$$

Why is there no constant, like ε_0, in this equation?

15 How much electric charge would have to be spread out on each square metre of table top to produce an electric field of 100 volts per metre at a height **a** 20 mm, **b** 40 mm, above the table top?
(Watch out – this question isn't playing quite fair.)

Data: see page 16. Use a suitably approximate value of ε_0.

16 A 1 μF capacitor is to be made as follows. Long, 50 mm wide strips of metal foil, B and D, and of insulating paper 0.1 mm thick, A and C, are arranged in a sandwich as shown in figure 10.

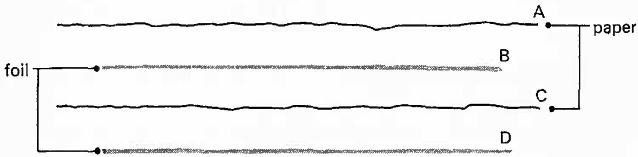


Figure 10

Then the long sandwich is rolled up to make a cylinder. It is known that the paper, when it fills a space between the plates of a capacitor, leads to a doubling of the charge stored at a given voltage (as compared to the charge when there is air in the space).
a At what point in the manufacture does the need for the top sheet of paper, A, become obvious?
b About how long a sandwich is needed to get a final capacitance of 1 μF?
c Estimate roughly how thick the rolled-up cylinder will be. You might compare its size with that of a commercial 'paper' capacitor of about the same value.

Use a suitably approximate value of ε_0.

17 There is an electric field at the surface of the Earth which in normal weather is about 100 volts per metre. The field is normal to the surface of the Earth's surface and is directed down towards the surface.

a What is the charge density (in coulombs per square metre) on the surface of the Earth? Is this charge positive or negative?

b Suppose a thundercloud with a large flat base comes overhead, and the field is observed to reverse its previous direction and reach a size of 90 000 volts per metre. This would be *mainly* due to a layer of charge carried on the base of the thundercloud. Make a rough estimate of the charge per unit area on the base of the cloud. Is the charge positive or negative? If the charge on an area about 3000 metres × 3000 metres is discharged in a single lightning stroke lasting 10^{-3} second, what is the mean current flowing down the lightning stroke?

(Regard the Earth and the cloud as if they were conductors.)

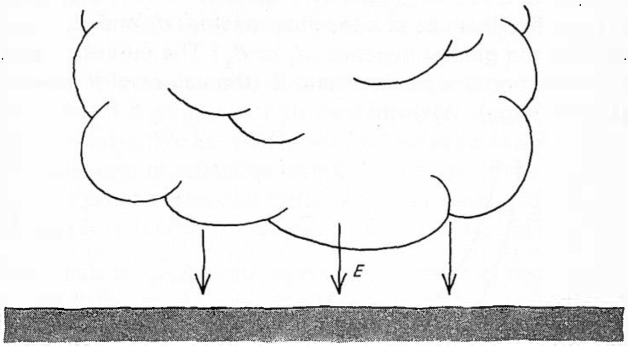


Figure 11

Use a suitably approximate value of ε_0.

18 This question is meant to give practice in thinking about the shapes of graphs, and the links between graphs and equations. Translating from one to the other is a skill often needed in science.

A parallel plate capacitor carries charge Q, has a p.d. V across the plates, and a plate separation d. The field in the space is equal to V/d.

Also $Q/A = \varepsilon_0 V/d$, where A is the plate area.

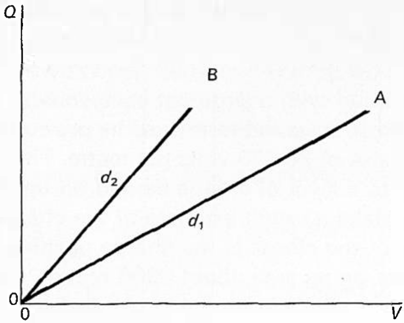


Figure 12a

a Figure 12*a* shows graphs of Q against V, drawn for two different fixed values of capacitor spacing, d_1 and d_2. Which is the greater distance, d_1 or d_2? The curves cannot be drawn beyond the points A and B (the values of V at A and B are quite large). Why not?

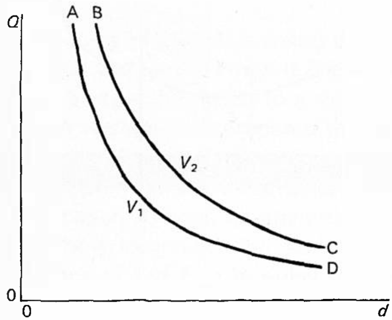


Figure 12b

b Figure 12*b* shows graphs of Q against d for two different, fixed values of V, V_1 and V_2. Why is the shape so different from figure 12*a*? Which is the greater voltage, V_1 or V_2? Is the field at A, B, where the curves stop, very large or very small? (Beyond C and D the curves don't follow the equation, given above, for values of d comparable with the size of the plates – *you might like to suggest why this happens.*)

c Can you sketch your own graph of *V* against *d* at fixed *Q*? Try and draw two, one for a large value of *Q*, one for a small value of *Q*.

Part Two

Gravitational field and potential

Questions 19 and 20

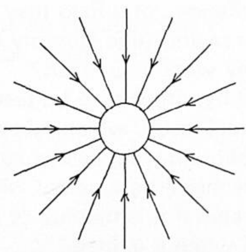
These are of a general, rather philosophical, kind. They may give you practice in putting down ideas coherently at some length.

19 *For discussion*

The following extract is taken from a book by Albert Einstein and Leopold Infeld. They have this to say about fields.

The new concepts originated in connection with the phenomena of electricity, but it is simpler to introduce them, for the first time, through mechanics. We know that two particles attract each other and that this force of attraction decreases with the square of the distance. We can represent this fact in a new way, and shall do so even though it is difficult to understand the advantage of this.

The small circle in our drawing represents an attracting body, say, the sun. Actually, our diagram should be imagined as a model in space and not as a drawing on a plane. Our small circle, then, stands for a sphere in space, say, the sun. A body, the so-called *test body*, brought somewhere within the vicinity of the sun will be attracted along the line connecting the centres of the two bodies. Thus the lines in our drawing indicate the direction of the attracting



force of the sun for different positions of the test body. The arrow on each line shows that the force is directed towards the sun; this means the force is an attraction. These are the *lines of force of the gravitational field.* For the moment, this is merely a name and there is no reason for stressing it further. There is one characteristic feature of our drawing which will be emphasized later. The lines of force are constructed in space, where no matter is present.

For the moment, all the lines of force, or briefly speaking, the *field,* indicate only how a test body would behave if brought into the vicinity of the sphere for which the field is constructed.

(Einstein, A., and Infeld, L. (1938) The evolution of physics, *Simon and Schuster; extract reproduced with permission.)*

Mary is worried about it. She says, 'I don't see what it means to say there is a field in empty space – how can there be an effect if there is nothing there?'
a How might Einstein and Infeld answer her? Look at what they say is actually observed.
Jean says, 'They seem to me to say that a field is just a way of saying what you would observe if you put a test object anywhere. But you can't put a test object everywhere, especially not all at once, so how can they have this picture of the field being everywhere all at once?'
Ann is less bothered: 'Do you really think you could find somewhere on the Earth where gravity won't pull you?'
Mary remembers a philosophical problem she has heard about. She says, 'I don't see how you can know that anything is really there when you are not actually observing it.'
b Do you think that when physicists like Einstein and Infeld are trying to explain the idea of a field they are:
1 Making definitions; saying (like Humpty Dumpty) that the word means what they want it to mean?
2 Suggesting a useful hypothesis, to be tested experimentally?
3 Giving a mathematical result, which can be proved by reasoning alone, to follow from previous suppositions?
4 Suggesting that this might be a useful way of looking at what happens, to be kept if it is helpful, or rejected if it seems clumsy and awkward when it is tried?

20 Write an essay about 'Fields'.
A good essay will give one or more definite examples to illustrate each point made. You should try to say clearly what the view is that you are putting forward, and give at least one argument in its favour, or you may wish to say that you cannot decide, and give arguments on either side.

There is no need for the views you express to conform to some 'correct' position; you may, for example, say, if you wish, that the idea of a field is dangerous nonsense, as long as you give arguments for and illustrations of your views.

Here are some lines of thought you might follow up:

Is a field a 'real thing' like a football, or is it an abstract idea, like friendship, or is it perhaps something in between?

Can fields be measured? What does measuring a field mean?

Do fields obey laws?

Is the repeated appearance of the inverse square law remarkable, or not?

Is it absurd, as Newton thought it was, to think of one object affecting another across space with nothing whatever between them?

Faraday said that it was the fact that magnetic field lines could curve that made him think the field lines were something 'real' like taut strings of elastic.

Some say that fields are an unnecessary mental extravagance, and that all one needs to get any answer to a problem is an equation like $F = GM_1M_2/r^2$, which doesn't mention a field at all.

Questions 21 to 24

These all use data from the Apollo 11 space flight, which was the first manned Moon landing. They show how the changing forces on the spacecraft may be calculated as it moves in the Earth's gravitational field.

21 This question is about the laws of dynamics applied to spacecraft.

About 26 hours 45 minutes after the launch of Apollo 11, the spacecraft was travelling on its way to the Moon with the rocket motors shut down. At this time, the motors were turned on for 3.5 seconds to make a course correction, changing both the speed and the direction of travel. The rocket gave a thrust of 96000 N. The mass of the spacecraft and rocket was 44000 kg.

a What acceleration did the rocket thrust produce?
b What was the force on each kilogramme of spacecraft?
c What change of velocity occurred in 3.5 s?
d The spacecraft was travelling at 1530 m s^{-1} at this time. Neglecting the change in velocity, how far did it travel while the rocket was turned on?
e If the thrust were directed along the direction of travel, how much energy would have been transformed as work, increasing the kinetic energy of the spacecraft?
f What is the change in energy per kilogramme of spacecraft?
g If the thrust were directed at right angles to the direction of travel, through what angle would the flight of the spacecraft be turned?
h In **g**, why would the change in energy of the spacecraft be very small?

22 After the rocket thrust mentioned in question 21, the Apollo 11 spacecraft was 209×10^6 m from the centre of the Earth, travelling at 1527 m s^{-1} on a path almost but not quite pointing directly away from the Earth, as shown in figure 13 (point 7). The motor was not used between this time, 26 hours 45 minutes after launch, and a later time, 32 hours 50

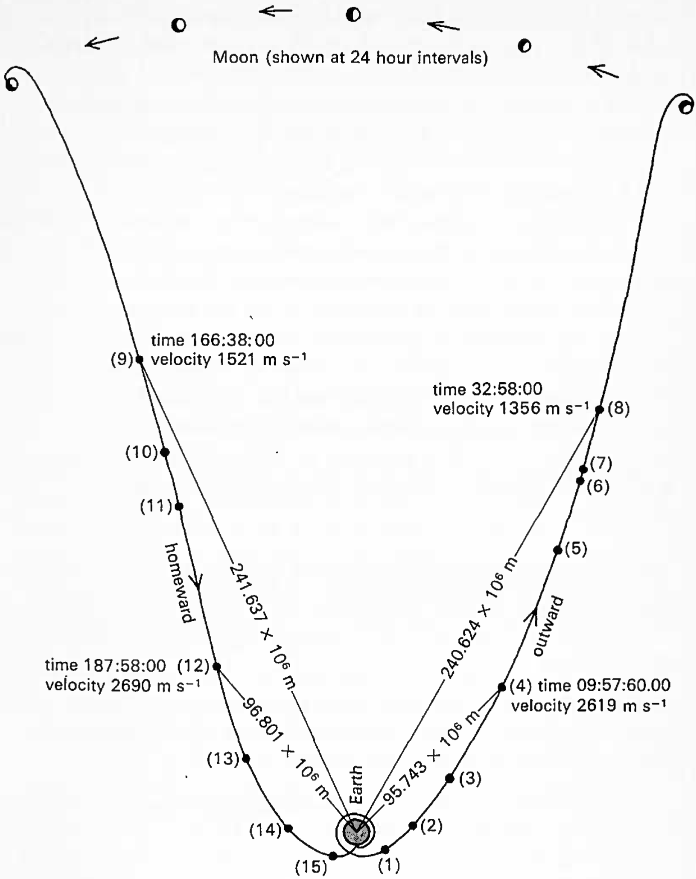


Figure 13
Sketch of typical trajectory of manned Moon flight (from plan of Apollo 12; data from Apollo 11).

minutes after launch (point 8). But over the interval between these times the velocity of the spacecraft fell to 1356 m s^{-1}, and at the later time it was 241×10^6 m from the centre of the Earth.

a How could the velocity decrease, even though the rocket motors were not in use for forward or reverse thrust?

b At what mean rate did the spacecraft decelerate? The interval between times 26 hours 45 minutes and 32 hours 58 minutes is 22 380 s.

c Over this interval, what must have been the mean gravitational pull of the Earth on each kilogramme of spacecraft? Use the answer to **b**. (You need make no further calculation to write this down.)

d In answering **c**, it may have occurred to you that the Moon might be pulling on the spacecraft as well as the Earth. It does, but the force is too small to matter. At distance r from the Earth, if the mass of the Earth is M, the pull of the Earth on one kilogramme of spacecraft is GM/r^2, towards the centre of the Earth.

For the Earth, the product GM is 4.0×10^{14} N m² kg^{-1}. Calculate the gravitational force per kilogramme (the gravitational field) at a distance of 225×10^6 m from the Earth (the average of the two distances given above). Comment.

23 This question uses the rate of change of velocity of the freely coasting Apollo 11 spacecraft to find whether the Earth's gravitational pull on it varied as the inverse square of the distance.

The data in *table 1* cover a period when the spacecraft was travelling more or less directly away from the centre of the Earth, with the motors shut down.

Time from launch/ hours : minutes : seconds		Distance from Earth's centre $r/10^6$ m	Velocity v/m s^{-1}
03:58:00	(2)	26.3	5374
04:08:00	(2*A*)	29.0	5102
05:58:00	(3)	54.4	3633
06:08:00	(3*A*)	56.4	3560
09:58:00	(4)	95.7	2619
10:08:00	(4*A*)	97.2	2594
19:58:00	(5)	169.9	1796
20:08:00	(5*A*)	170.9	1788

Table 1

a The pair of values, (2) and (2*A*), are only ten minutes apart in time. In this time the velocity of the spacecraft decreased by 272 m s^{-1}. At what mean rate was the velocity changing?
b Write down (without further calculation) an estimate for the gravitational pull of the Earth on each kilogramme of the spacecraft at a distance of 27.7×10^6 m from the Earth, this being the average of the distance at points (2) and (2*A*).
c Use the other pairs of data points to plot a graph which tests whether the gravitational force on one kilogramme – the gravitational field – varies as the inverse square of the distance.

24 This question discusses the changes of energy of the Apollo 11 spacecraft. It introduces the idea of the potential energy of a mass in a gravitational field. See figure 13.

When Apollo 11 was 11.0×10^6 m from the centre of the Earth (point 1), its velocity was 8406 m s^{-1}. As it coasted away from the Earth, its velocity fell to 1532 m s^{-1} when it was 209×10^6 m from the Earth (point 6).
a What was the change in the kinetic energy of each kilogramme of spacecraft? Account for it, remembering that the engines were not used.
b On the return journey, the engines were again not used between a distance 210×10^6 m from the Earth (point 10), when the velocity was 1676 m s^{-1} and shortly before re-entry (point 15*A*), when the craft was travelling at 8854 m s^{-1} and was 10.0×10^6 m from the centre of the Earth. (The Earth's radius is 6.38×10^6 m.)
What was the change in kinetic energy of each kilogramme of the spacecraft? Account for it.
c Compare the answers to **a** and **b**. How do you think they would have compared if the pairs of distances had been identical?
d Is there any reason to think that in **a** energy vanished, only to reappear in **b**?

e Suggest a reason why a spacecraft a long way from the Earth is said to have 'potential' energy, on the basis of the data in this question. In ordinary usage, 'potential' means something like 'hidden, but capable of being produced'. The change in potential energy per kilogramme has a special name: it is called the *gravitational potential difference.*

Questions 25 to 27

These are rather more general questions about gravitational fields.

25 A spacecraft needs an energy of about 60×10^6 joules for each kilogramme of its mass to reach a distance of some 200×10^6 m from the Earth, starting near the Earth.

a Where is this energy stored when the spacecraft is sitting on the launching pad before takeoff?

b The burning of one kilogramme of petrol yields about 50×10^6 J kg^{-1}. Suggest a reason why petrol is not used in rockets.

c Why is something like 90 per cent of the mass of a rocket on the launching pad simply fuel?

d Imagine that a rocket engineer has drawn a map of the space near the Earth, with rings (like contour lines) surrounding the Earth, each labelled with the minimum energy needed to get there from the Earth. Why are the rings centred on the Earth?

e If the Earth were half the size but just as massive as it really is, would the numbering of the rings change?

26 The force acting on each of two one-kilogramme masses one metre apart is 6.67×10^{-11} newton.

a Suggest a pair of common objects that have masses of about one kilogramme.

b Suggest an object whose weight, on the Earth, is of the order of 10^{-10} newton (which is the order of magnitude of the force above). (g = 10 N kg^{-1} approximately.)

c Think now of a one-kilogramme mass and of another large body, imagined to be made up of many (say N) one-kilogramme masses, a long way from the first kilogramme mass. Suppose that a pair of one-kilogramme masses as far away as this attract each other with a force F on each. Argue that the force on the large body is $N \times F$.
d Argue that the force on the small body is also $N \times F$.

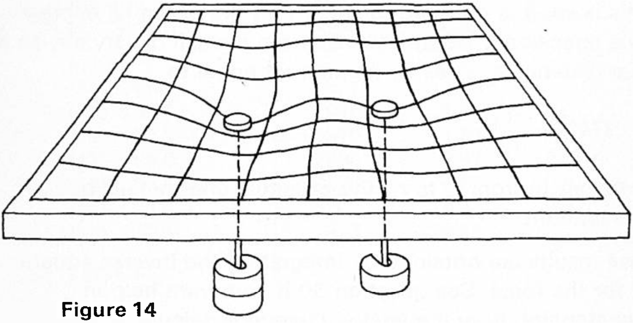


Figure 14

27 Figure 14 shows a thin rubber sheet held in a large frame. The sheet is pulled downwards at two places, one depression being deeper than the other.
a What analogy could there be between such an arrangement and the gravitational potential energy in the vicinity of two masses, such as the Earth and the Moon?
b Could any feature of the rubber sheet arrangement be analogous to the gravitational field? (Look for a place where the field must be zero.)

Questions 28 to 31

These are about $1/r$ variations of potential energy.

28 In question 23, data for the Apollo 11 flight were used to check whether the gravitational force on the spacecraft varied as the inverse square of the distance.

That is, field (force per kilogramme) = GM/r^2.

If this is so, it is not hard to show that the potential energy of the spacecraft, when it travels from a small distance r_1 to a larger distance r_2, rises by an amount equal to

$$GM\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

If it travels in from r_2 to r_1, the potential energy falls by the same amount.

These results are obtained by integrating the inverse square law for the force. See question 30 if you want help in understanding how the energy change is calculated from the variation of the force.

If the rocket motors are not used, any rise in potential energy is matched by an equal fall in kinetic energy, and any fall in potential energy is matched by a rise in kinetic energy. So the changes in potential energy can be calculated from the changes in kinetic energy, which can be got from the speed of the spacecraft. This question uses the data for the Apollo 11 space flight to test the $1/r$ variation of potential energy.

The data in table 2 cover two periods in the flight, one on the outward journey to the Moon and one on the return to Earth. Within both periods, the motors were not in use, though they were used between these periods to land on the Moon and take off again.

Distance from centre of Earth $r/10^6$ m	Velocity v/m s^{-1}	Kinetic energy per kilogramme $\frac{1}{2}v^2/10^6$ J kg^{-1}
Outward		
(1) 11.0	8406	35.33
(2) 26.3	5374	14.44
(3) 54.4	3653	6.60
(4) 95.7	2619	3.43
(5) 169.9	1796	1.61
(6) 209.2	1532	1.17
(8) 240.6	1356	0.92
Return		
(9) 241.6	1521	1.16
(10) 209.7	1676	1.41
(11) 170.9	1915	1.83
(12) 96.8	2690	3.62
(13) 56.4	3626	6.57
(14) 28.4	5201	13.52
(15) 13.3	7673	29.44

Table 2

a At $r = 11.0 \times 10^6$ m on the outward journey, the kinetic energy of each kilogramme of the spacecraft is 35.33×10^6 J kg^{-1}. At $r = 240.6 \times 10^6$ m it is only 0.92×10^6 J kg^{-1}. By how much has the potential energy per kilogramme changed?
b Is it possible that at some distance r_0 the kinetic energy would become zero?
c What would be the change in potential energy per kilogramme between $r = 11.0 \times 10^6$ m and r_0?
d What would be the change in potential energy per kilogramme between $r = 26.3 \times 10^6$ m and r_0?
e Write an equation for the change in potential energy per kilogramme between r and r_0.
f The magnitude (ignoring the sign) of the change in potential energy per kilogramme between distance r and distance r_0 is equal to the kinetic energy per kilogramme given against r in the table. The change in potential energy per kilogramme is also called the *gravitational potential difference.* (The change in energy per coulomb is called the electrical potential difference.)

Plot graphs of the magnitude of the potential energy difference between distance r and the (unknown) distance r_0, for both the outward and homeward sets of data, against $1/r$. r_0 is not the same for these two sets of data, but is constant for all the data in one set.

g What should the shape of each of the two graphs be, if the equation given in the answer to **e** is correct?

h What should the slopes of the graphs be? Should they have the same slope?

i What intercept should each graph have with the axis of $1/r$? Will this be the same for both graphs?

Like questions 23 and 29, this question uses information about the motion of a particle to infer something about the forces on it or the way its energy varies. This is a kind of calculation physicists often make.

29 Figure 15 shows a ball projected onto a curved hill, which it rolls up and then down. The light for the picture was flashed on at regular intervals. What happens to the speed of the ball as it comes closer to the centre of the hill (the white spot)? Measure the speed v of the ball at various distances r from the centre. Remembering that the kinetic energy of the ball is $\frac{1}{2}mv^2$, obtain a list of numbers, from your values v, proportional to the kinetic energy at various distances r. (This ignores any spinning kinetic energy the ball may have; also, the photograph yields only the horizontal component of velocity.)

Plot a graph of the number proportional to kinetic energy against r. Why does the kinetic energy decrease as r decreases? How would the potential energy of the ball vary as r decreases, if little or no energy were transformed by friction? Can you tell anything about the shape of the hill from your graph?

Figure 15

30 *Optional*

This question is about how, if gravitational force varies as $1/r^2$, gravitational potential energy varies as $1/r$. A graphical check is made on the result. The gravitational field due to the Earth is $g = GM/r^2$ N kg^{-1}, and, for the Earth, $GM = 4 \times 10^{14}$ N m^2 kg^{-1}. The graph, figure 16, shows the gravitational field from r = 10000 km (10^7 m) to r = 50000 km.

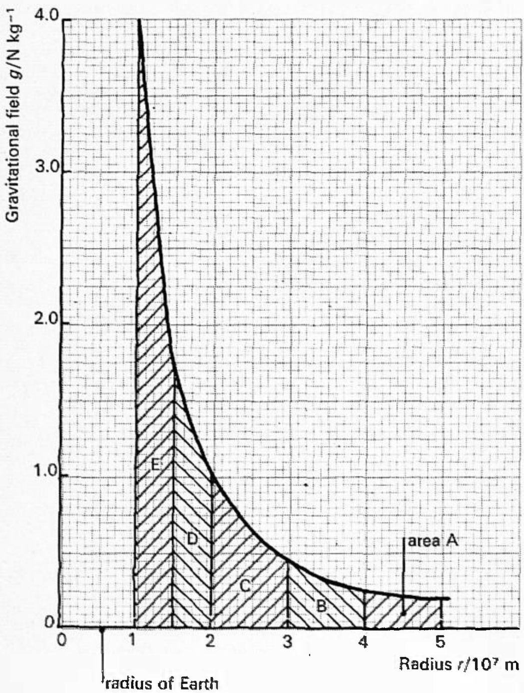


Figure 16
Graph of gravitational field of the Earth against distance from the centre.

a Estimate the radius of the Earth, if g = 10 N kg^{-1} at the surface.

b Explain why the area A below the graph, between $r = 4 \times 10^7$ and 5×10^7 m, represents the change in energy

per kilogramme for an object moving between these two distances.
c Find the area A, by counting squares, or by drawing a line across so that the two small 'triangular' areas look equal (see figure 17) and treating the area as a rectangle.

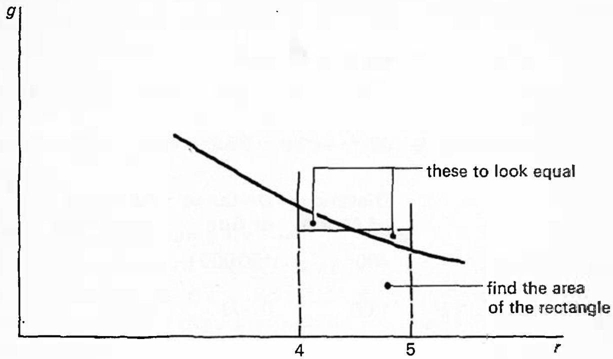


Figure 17

Put the value of this area in table 3.
d Find the area B, and put the area $A+B$ in table 3 to represent the work done in going from $r = 5\times10^7$ m to $r = 3\times10^7$ m.
e Find the areas C, D, E and complete table 3.

$r/10^7$ m	Area = work done in going from 5×10^7 m to r
4.0	A
3.0	$A+B$
2.0	$A+B+C$
1.5	$A+B+C+D$
1.0	$A+B+C+D+E$

Table 3

f Plot a graph of the areas (work done) in the table against $1/r$, to see if the algebraic prediction that the potential varies as $1/r$ seems to fit your data.
g What value has r where the graph crosses the $1/r$ axis? Explain.

31 This question is about the energy needed to escape from the Earth, and introduces the idea of the *gravitational potential* at a point, and the minus sign it is conventionally given.

Table 4 gives values of the quantity GM/r for the Earth, at different distances r from the centre.

	Earth's surface						
$r/10^6$ m	6.38	6.39	6.75	10.0	20.0	40.0	80.0
$\frac{GM}{r}\Big/10^6$ J kg^{-1}	62.7	62.6	59.3	40.0	20.0	10.0	5.0

	Distance of Moon	Distance of Sun	As far as you please
$r/10^6$ m	400	150000	∞
$\frac{GM}{r}\Big/10^6$ J kg^{-1}	1.00	0.003	0.000

Table 4

a How much energy is needed for one kilogramme to go from the Earth's surface, $r = 6.38 \times 10^6$ m, to a height of 10 km, $r = 6.39 \times 10^6$ m, using the data in the table?
b What force acting on one kilogramme would transform this amount of energy, if it were uniform over the distance 10 km? Comment.
c How much energy is needed for one kilogramme to go from the Earth's surface to a distance equal to that of the Moon?
d Why is the energy in **c** very much less than the product of the distance to the Moon and g, 10 N kg^{-1}?
e How much more energy per kilogramme is needed to go on from the Moon's distance to the Sun's distance?
f How much energy per kilogramme is needed to go from the Earth's surface as far away as you please?
g What velocity will be needed for a mass of one kilogramme to have the energy in **f**? Why will a mass of any size need just the same velocity to escape?

h If a one-kilogramme mass very far from the Earth has a certain potential energy, how much more or less potential energy will a one-kilogramme mass have at the surface of the Earth?

i If the 'certain potential energy' mentioned in **h** is given the value zero, what is the potential energy of a one-kilogramme mass at the Earth's surface? (Give a value, with a plus or minus sign.) Write an algebraic expression for it as well.

j The quantity $-GM/r$ is called the *gravitational potential* at distance r from mass M, and can be symbolized by V_{gravity}.

Either **1** If $V_{\text{gravity}} = -GM/r$, write down an expression for $-\dfrac{dV_{\text{gravity}}}{dr}$.

What is the usual name for $-dV_{\text{gravity}}/dr$?

Or **2** Draw a graph of $-GM/r$ against r, as in figure 18, from, say, $r = 10 \times 10^6$ m to $r = 80 \times 10^6$ m. Calculate values not given in the table if they are needed for drawing a smooth curve.

Draw tangents at a pair of radii r and $2r$, and compare their slopes. How should the slopes compare? (Look back at **a** and **b**.)

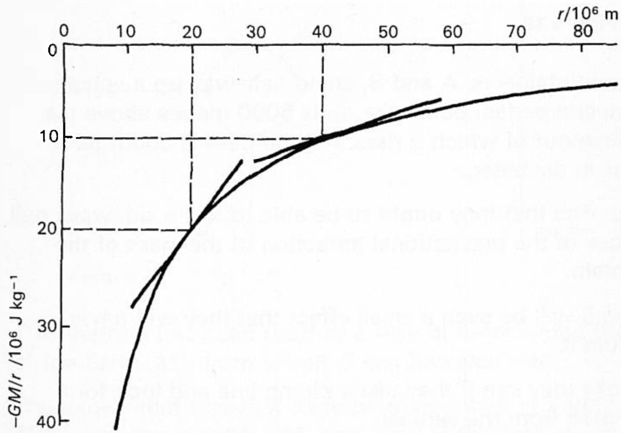


Figure 18

Questions 32 and 33
Further inverse square law problems.

32 If you understand gravitation, you should be able to manage this problem.

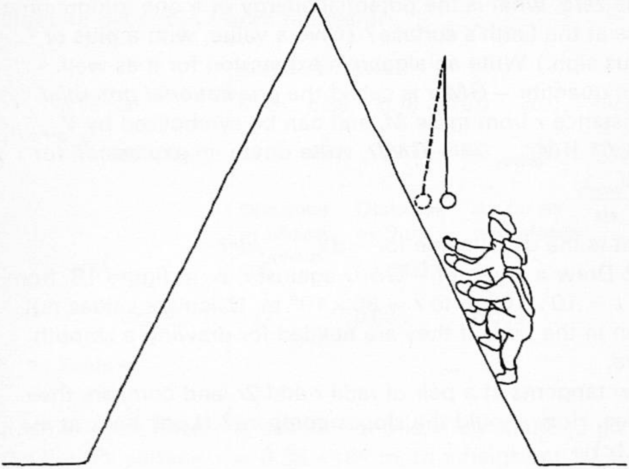


Figure 19

Two mountaineers, A and B, stand half-way up a volcano. It is almost a perfect cone: the tip is 5000 metres above the flat plain out of which it rises and the base is about 5000 metres in diameter.

A suggests that they ought to be able to feel a sideways pull because of the gravitational attraction of the mass of the mountain.

B says it will be such a small effect that they will never measure it.

A thinks they can if they use a plumb line and look for a deflection from the vertical.

B asks how A will know where 'vertical' is and A proposes to use the direction of a fixed star, seen in a telescope at a definite time, as a fixed direction. This does work: can you explain how they might set about looking for a shift in the plumb line angle? A is trying to work out the angle to the vertical at which the plumb line will hang. A piece of A's note-book is shown in figure 20. Can you complete the (very rough) estimate of θ he was trying to make?

Mass of the Earth $\approx 6 \times 10^{24}$ kg.

Radius of the Earth $\approx$ 6000 km.

Density of surface rocks $\approx 3.0 \times 10^{3}$ kg m^{-3}.

If you don't know how to work out the volume of a cone, guess what fraction it is of the volume of a cylinder.

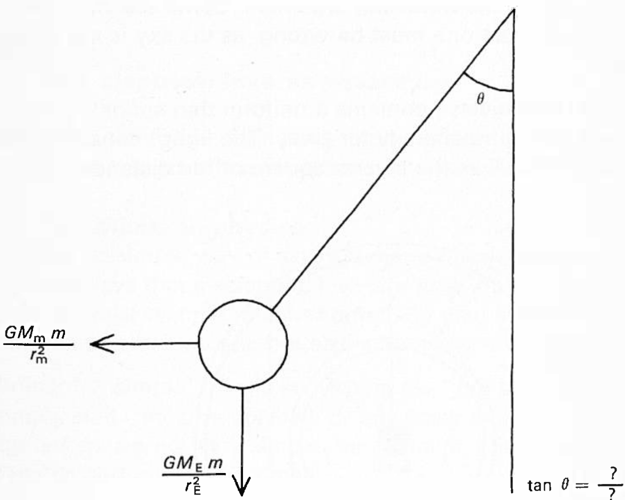


Figure 20

This method has been used as a way of determining the mass of the Earth, M_E, from which G can be calculated.

The experiment was first done on a mountain in Peru in 1740, and the result for M_E gave one of the first proofs that the Earth was not a hollow shell or globe filled with water (as many had believed until then). The result for M_E was so big that it showed that the mean density of the Earth had to be higher than that of the surface rocks (the mean density of the Earth as a whole is about 5.5×10^3 kg m^{-3}). The English physicist Cavendish thought about repeating the experiment on Skiddaw in the Lake District, but it was eventually repeated by Maskeleyne in 1774 on Schiehallion, a mountain in Perthshire.

33 *Optional*

This question illustrates an argument based on an inverse square law, but relating to intensities of light.

The sky is dark at night (of course!). You may not have realized that it is quite easy to show that it ought to be bright, not dark. Study the following argument. Some assumptions are made. At least one must be wrong, as the sky is *not* bright at night.

Suppose the Universe contains a uniform thin sprinkling of stars, extending indefinitely far away. The light intensity from each star falls off as the inverse square of the distance.

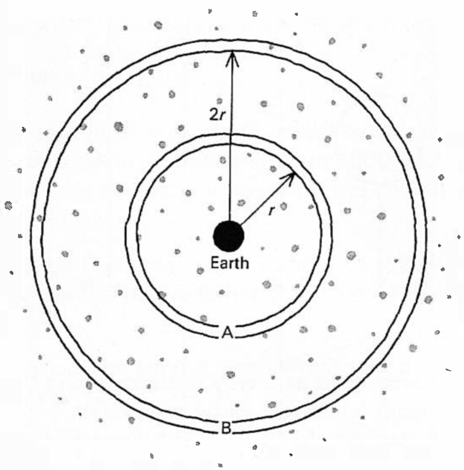


Figure 21

Think of two spherical shells of space, A and B, of the same thickness, with radii r and $2r$. How does the number of stars in the thickness of shell B compare with the number in the thickness of shell A? If the average amount of light given out by a star is the same everywhere, how does the light received at the Earth from A compare with that from B? What should the sky look like at night, if a whole series of shells extending to infinity contributed light? (There are two good answers to this. One ignores the size of the stars; the other does not.) Each star has an inverse square gravity field too. Would there be a force on a mass at the Earth as a result of the rest of the Universe acting on it? (Which direction would it be in?)

Part Three
The electrical inverse square law

34 Read the following passage and answer the questions that follow it or use them as a basis for discussion.

Simplicity in physics

There is no simple way of saying what a physicist means when he says that a scientific law or a new idea is 'simple'. To a physicist 'simple' means something very like what it means to other people, but also something rather different.

Ordinarily, 'simple' means something like 'not too complicated'. Income tax laws or university entrance regulations are not very simple, for example. They have too many ifs and buts to be simple.

Rules saying that something may not happen are often simpler than those which allow something to happen. The law, 'You cannot alter the total amount of energy in a system' is an absolute prohibition. The trick of simplicity lies in the absoluteness of the prohibition. In no circumstances is an energy change allowed. Indeed, any rule which applies to everything without exception can be cast as a prohibition. 'All men must die' becomes 'No man may live for ever', or 'Total momentum is always constant' becomes 'Total momentum is not allowed to change'. This is one kind of simplicity that physicists talk about; the simplicity of allowing no exceptions. Newton's Laws (until Einstein) and the inverse square law of electricity are fundamental, general, all-embracing laws of this kind, whereas the laws of Ohm, Hooke, or Boyle are not. (Ohm's Law says that for some things, under some circumstances, the current is proportional to the potential difference. Try making that into a prohibition.)

But this is not the whole of the story, as may be illustrated by the inverse square law. A number of unrelated things obey inverse square laws: electrical forces, gravitational forces, and the brightness of light from a small lamp, for example, besides the magnetic field from a little bit of an electric circuit and the sound from a whistle in the open air. The mere fact that the inverse square form of law turns up in several places might make a physicist call it a 'simple' pattern, for when one has solved a problem in, say, gravitation, there is a corresponding problem in, say, electricity that one has also solved without trying to. For example, if one has proved, as Newton did, that the gravitational force outside a large sphere is just as if all the mass were at the centre, one knows at once that a similar thing is true for the electrical force outside a spherical charged ion.

Inverse square laws have a yet deeper, more mathematical kind of simplicity. If sound or light or radio energy spreads out evenly in three dimensions, when the energy spreads out to a sphere of twice the radius it has to cover four times the area. So the loudness, brightness, or intensity, which is the energy crossing a small fixed area, simply must obey an inverse square law because energy is conserved.

The interesting point about this emerges when one goes back to, say, the electrical inverse square law. Because the field happens to obey an inverse square law, there is some sort of parallel with energy flow in three dimensions. The field crossing an area multiplied by that area must share with an energy flow the property of not 'getting lost', though it is certainly *not* an energy flow. But field times area behaves *enough like* a flow to be called 'flux', though nothing flows so far as we know. It must behave so because the field is *inverse square*: *contrast this with light or sound*, where the logic is the other way round. There the intensity is inverse square because there is a flow of energy which doesn't get lost.

If there is simplicity here, it might be compared with that of a formation dance. Watch one pair of dancers and you will see a complicated set of moves. Watch them all from the right point of view, and you see something quite new, a set of simple patterns.

One last note: physicists and mathematicians grew so fond of this simple 'flux' aspect of field behaviour that it annoyed them that the mathematics was cumbersome, as it turned out to be. So what did they do? They invented a new, simple way of writing the mathematics. The equation, $\mathrm{div}\, E = 0$, was invented to say that flux (field times area) behaves (in empty space – no materials around) as if it doesn't get lost. ($\mathrm{div}\, E$ is how much is created or lost; the zero says that none is.) This is the best simplicity of all: making things as easy as possible.

a Think of another example of a law of physics usually cast as a prohibition. (Look up the Second Law of Thermodynamics.)
b Put a general, fundamental law of physics in prohibition form. (Try Newton's First Law of Motion.)
c What goes wrong if you try to put, say, Ohm's Law in the form of a prohibition?
d The force between charges Q_1 and Q_2 is proportional to $Q_1 Q_2$. How does the product $Q_1 Q_2$ represent with simplicity the rule, 'Like charges repel, unlike charges attract'?
e Give another example of a problem in gravitation which, when solved, deals with a similar problem in electricity. (Such parallels are called 'formal analogies'.)
f 'A single steady tone from an oscillator is not the simplest sort of music. It is too simple to be called music at all. But the noise from traffic is too complicated and disorganized to be music either. Physics is much the same as music; to be interesting the relationships must have a kind of complex simplicity.' Do you agree?

35 Photographs in figure 24 show the position in which a small charged polystyrene ball, suspended like a pendulum as in figure 24*a*, hangs when a second charged ball on a rod, seen in figures 24*b* to *g*, is pushed up close to it. The fine nylon thread used to suspend the first ball is not visible.

The pictures were taken with the apparatus in figure 22.

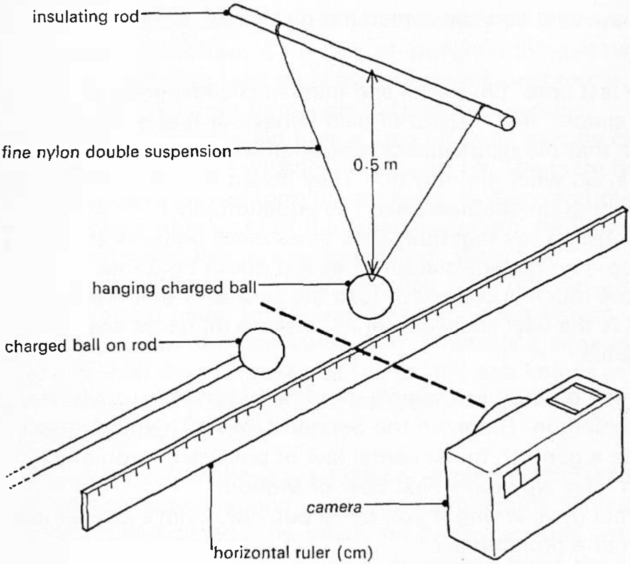


Figure 22

When the charged ball on the rod is a long way away, the suspended ball hangs vertically, in the position shown in figure 24*a*. As the second ball is brought closer, the suspended ball is pushed to the right until there is a big enough sideways force on it to balance the repulsive force due to the second ball (figure 23).

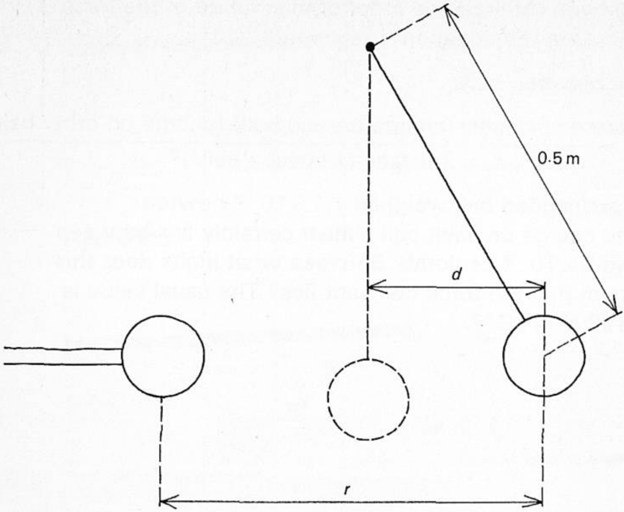


Figure 23

a If the sideways force on the suspended ball is doubled, how (approximately) will its deflection d change? How are d and the force F related?

b Measure the deflection d, and the separation r between the centres of the balls, for photographs b to g. Plot a graph to test whether the sideways force F on the suspended balls varies as $1/r^2$.

c The balls were given equal charges, each about 5×10^{-9} coulomb, from a high voltage source. The charging was repeated for each picture and the charge was measured by sharing the charge on a ball with a 0.01 μF capacitor, which was then found, by means of an electrometer, to have a voltage of 0.5 volt across it. Check that the charge stated above is correct. The measurements of charge indicated that the charge varied by a few per cent on different occasions. Would such fluctuations explain any feature of your graph?

d *Can you estimate the order of magnitude of the force constant in the equation:*

force between balls

$$= \frac{\text{(force constant) (charge on one ball) (charge on other ball)}}{\text{(distance between balls)}^2}$$

The suspended ball weighed 1.1×10^{-3} newton.
e The charge on each ball almost certainly lies between 4 and 6×10^{-9} coulomb. Between what limits does this suggest that the force constant lies? *The usual value is* 9×10^9 N m^2 C^{-2}.

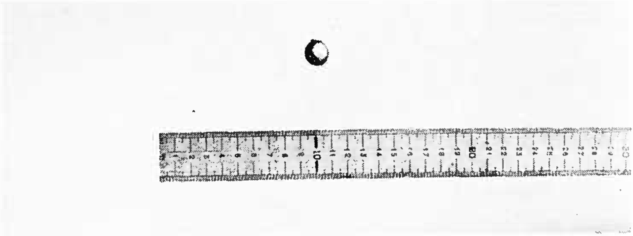


Figure 24a

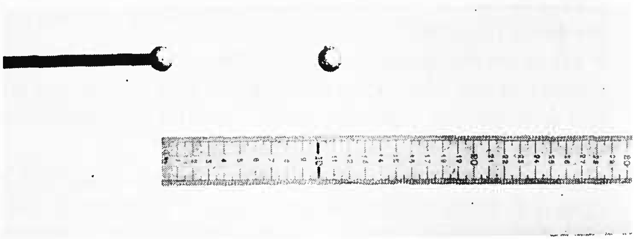


Figure 24b

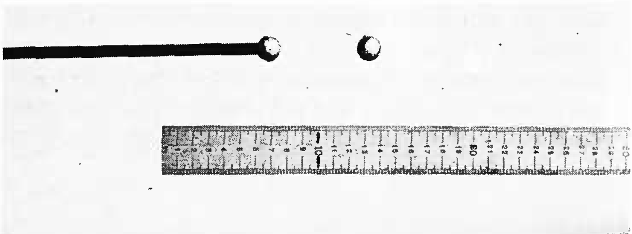


Figure 24c

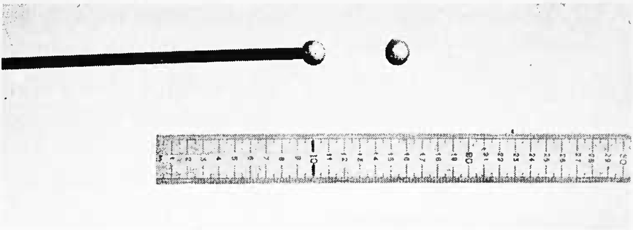


Figure 24d

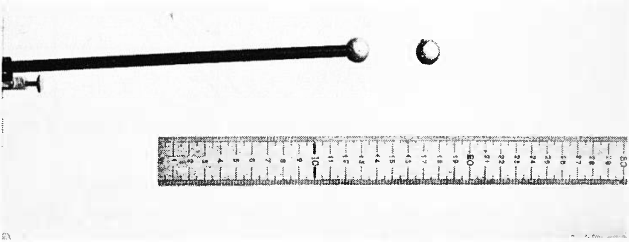


Figure 24e

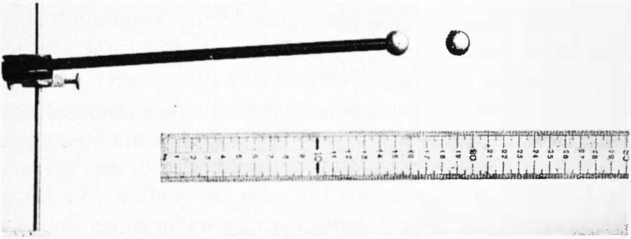


Figure 24f

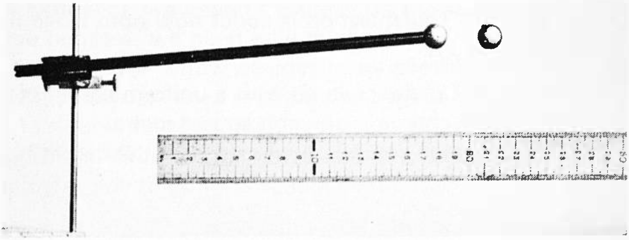


Figure 24g

36 This question suggests why it is often hard to make tests of Coulomb's Law work well.

In a test of Coulomb's Law, equal charges were placed on two small suspended expanded polystyrene spheres each weighing about 10^{-3} N, which pushed each other aside at an angle of the order of 0.1 radian (say 5 to 10 degrees).

Coulomb's Law says that:

$$\text{Force} = 9 \times 10^9 \frac{Q_1 Q_2}{r^2}$$

and $$V = 9 \times 10^9 \frac{Q}{r}.$$

All the following questions require order of magnitude answers only.

a What was the electrical force on each sphere?

b If the spheres were of the order 10^{-2} m apart, what charge did they each carry?

c What current would carry this charge away in 10^2 seconds?

d What was the potential at a place near one sphere (say 10^{-2} m from the centre)?

e What resistance would carry the current from **c** with the p.d. from **d** across it?

It follows that if the suspension has a resistance of not more than 10^{14} ohms, which may well be so if the suspension is at all damp, the experiment can easily fail.

37 Point charges have an inverse square law electric field. Yet the same charges, spread out like a carpet of charge on a flat plate (or on a pair of parallel flat plates), produce a uniform field. This question is about how both these things can come about.

Think of a flat plate, covered with a uniform carpet of charges, with charge σ on each square metre.

a What charge is carried on an area A of the carpet?

Suppose there is a surveyor at S (figure 25) investigating the field, and he starts by looking at the effect of a small patch of the carpet which is marked out by a bundle of four lines reaching down from S to touch the carpet at ABCD. ABCD is a rectangle with sides of length L and width w.

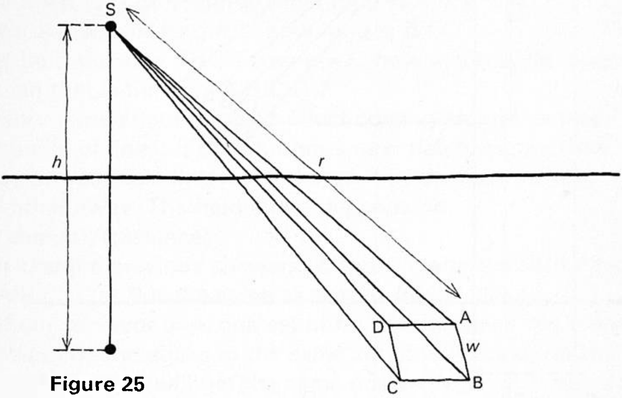


Figure 25

b What is the charge on the patch ABCD?
The distance from S to the patch ABCD is r, and the field due to a charge q depends on charge/(distance)^2.
c Write an algebraic expression for (charge)/(distance)^2 relating to the field at S due to the patch ABCD.
The surveyor has to find the effect at S of *all* the carpet, which can be done by adding the effect of all the patches like ABCD which go to make up the carpet. So the total field is the total effect of all the little bits of field coming along the different bundles which mark out the patches on the surface. For this argument, we needn't actually do the sum of adding up all the bundles; we need only think about one of them in detail. The surveyor S now decides to try another distance from the plane, so he rises up from the original height (which was h) to double this height ($2h$). See figures 26 and 27. As he goes up he carries up his bundle of lines with him – keeping them pointing in the same direction.

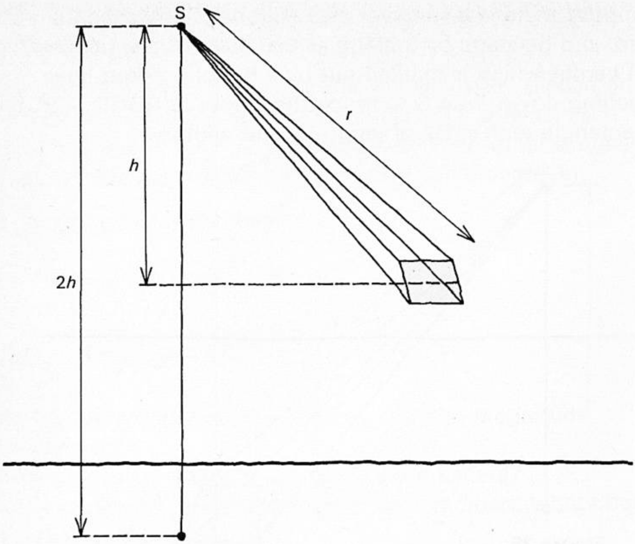


Figure 26

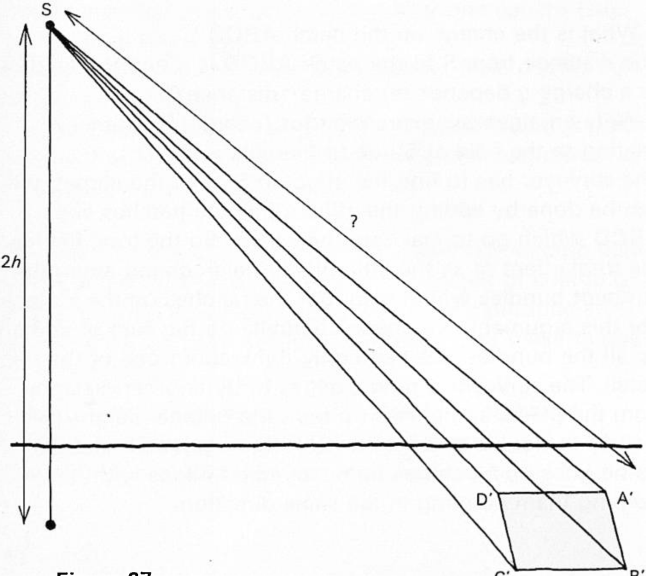


Figure 27

Now they don't reach the plane, so S lengthens each line until they all hit the surface again at A′B′C′D′.

d Because the height has gone up from *h* to 2*h*, the length SA′ is bigger than SA was. How many times bigger? The new patch A′B′C′D′ will be bigger than ABCD was.

e If AB was of length *w*, how long is A′B′?

f If BC was of length *L*, how long is B′C′?

g So if the area ABCD was $w \times L$, how many times bigger than that is the area A′B′C′D′?

Now think about the field effect coming along this new bundle of lines; it comes from a new patch, bigger than before, so including more charge. But the patch is also further away. The field effect depends on (charge)/(distance)^2.

h Use the previous answers to explain why the field effect of A′B′C′D′ at S is the same as that of ABCD was.

If our surveyor uses one set of bundles to cover the plane at height *h,* and sticks to the *same set* of bundles to do the job at 2*h,* then he will get the same answer from each bundle, and so he'll get the same answer when he adds up the whole lot.

If our surveyor is very close to a large sheet, then only a few of his bundles will be near the edge (figure 28).

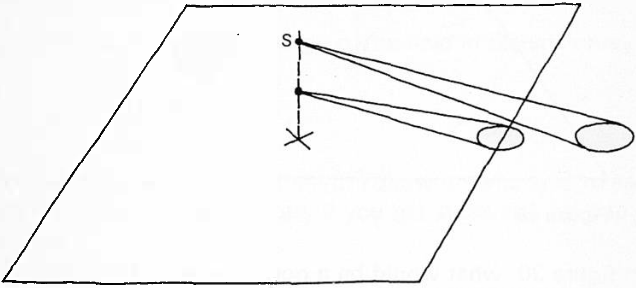


Figure 28

i Since the distance away for these edge bundles is large, will the field effect for them be small or large?

For these edge bundles, when he goes up higher there will be a difference in the effect of the bundles, since they then come off the plane and there is no charge at the end of them.

But if the plane is big and h is small, the effect of the edge bundles is small so that the field does not change appreciably with height.

j In figure 29 the field (will, or will not?) change with height because now the effect of the edge bundles is (large or small?).

In figure 30 the surveyor has gone up so high that the plane has seemingly shrunk down and appears to him like a small point of charge.

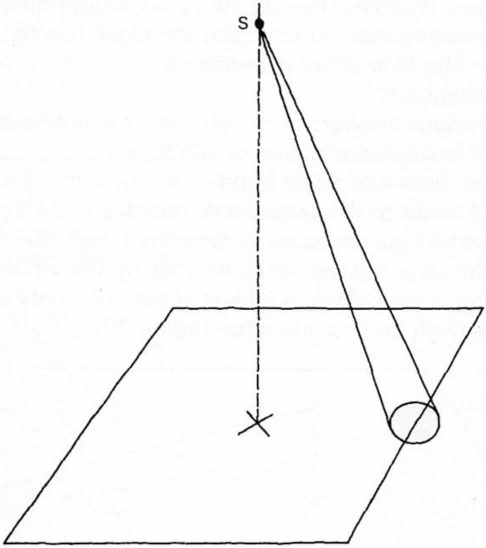


Figure 29

k In figure 30, what would be a good guess for the way the field at S varies with height h?

You can find this argument summarized in PSSC *College physics*, page 446, or PSSC *Physics* (2nd edition), page 491.

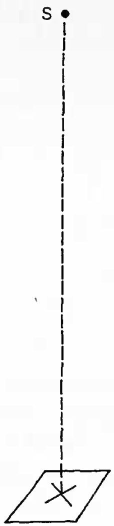


Figure 30

38 This question shows how to perform the adding up of field from a carpet of charge, as outlined in question 37. It shows that if $E = \varepsilon_0 \times$ (charge density) for parallel plates, then the constant in the inverse square law should be written $\frac{1}{4\pi\varepsilon_0}$; that is, for a point charge q, the field at distance r is

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$$

You need not learn the argument. Follow it some way to see what it is like, but don't worry if you get stuck and have to give up.

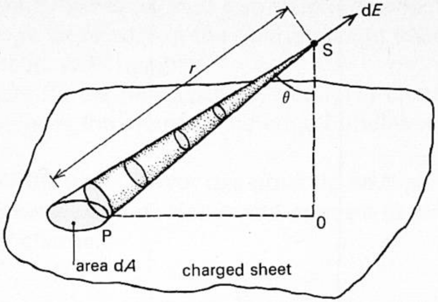


Figure 31

a *Figure 31 shows a flat sheet carrying a charge* σ on each square metre. The patch P produces a field d*E* at the place S. What charge is on the patch P, *area dA?*

b From Coulomb's Law, a charge q occupying a small region gives a field $9 \times 10^9\ q/r^2$. What is the field dE at S due to the charged patch P?

c Why does the field dE at S point in the direction shown in figure 31?

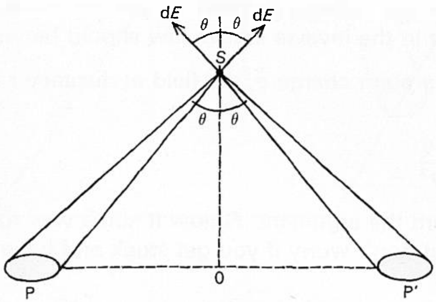


Figure 32

d Figure 32 shows another patch, P′, with the same area as P. P′ and P are the same distance from S, and P′ is on the line from P passing through O, the point directly below the place S. In what direction will a charge at S be pushed by the combined fields of P and P′?

e Why is $\mathrm{d}E \cos\theta$, the component of $\mathrm{d}E$ along OS, the only component of the field $\mathrm{d}E$ from P at S which will contribute to the total field of the whole sheet at S? What about the component $\mathrm{d}E \sin\theta$ parallel to the sheet?

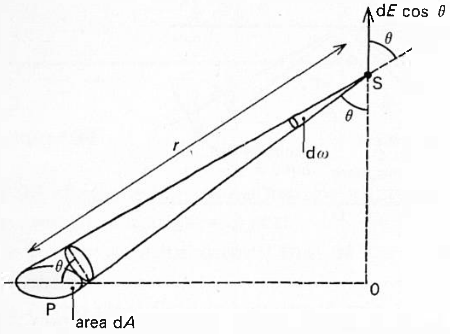


Figure 33

f Figure 33 shows the patch P again, and the component $\mathrm{d}E \cos\theta$ along OS of its field at S. What is $\mathrm{d}E \cos\theta$ in terms of σ, $\mathrm{d}A$, and r?

g In the expression $(9\times 10^9\ \sigma\ \mathrm{d}A \cos\theta)/r^2$, the product $\mathrm{d}A \cos\theta$ can be re-interpreted. It turns out that re-interpreting it is a useful trick to help in adding up the field components from all patches. What region in figure 33 has area $\mathrm{d}A \cos\theta$?

h If you know about solid angles, try **1**. If not, try **2**.

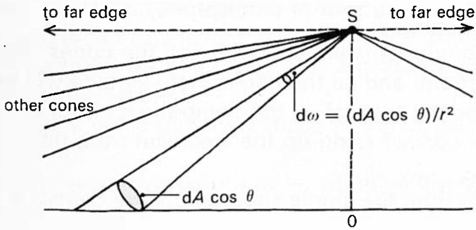


Figure 34

1. See figure 34. The area $\mathrm{d}A \cos\theta$ cuts off the end of a cone, radius r, from S. The cone has solid angle $\mathrm{d}\omega = (\mathrm{d}A \cos\theta)/r^2$. Thus the field component from P at S can be written

$9 \times 10^9\ \sigma\ d\omega$. Patches over other parts of the sheet will be seen from S along cones going out at angles θ up to $\pi/2$ on either side of OS. What is the *total* of $9 \times 10^9\ \sigma\ d\omega$ for all such cones?

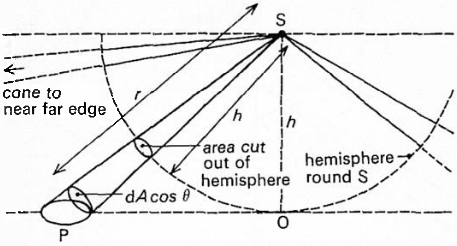


Figure 35

2 All the cones from P which reach the charged plate come below a horizontal plane through S, so each cone cuts through a hemisphere drawn below S. Any hemisphere will do: one of radius h is shown in figure 35.
The *area* cut out of the hemisphere by the cone to patch P is smaller than the area $dA \cos\theta$ at its end by the ratio h^2/r^2; for spheres or sections of spheres *have area proportional* to the square of their radius.
So the area cut out of the hemisphere is $h^2/r^2 \times dA \cos\theta$.
But the electric field component of patch P was $(9 \times 10^9\ \sigma\ dA \cos\theta)/r^2$, so it can be written as:

$$9 \times 10^9 \times \sigma \times (\text{area cut out of hemisphere})/h^2$$

When *all these components are added up*, all the cones will be taken into account, and all the hemisphere surface will be used. What will be the total of all the components from *all* patches and their cones? (Add up the areas cut from the hemisphere.)
i The field E_1 at S from the single sheet of charge density σ comes to:

$$E_1 = 2\pi \times 9 \times 10^9 \times \sigma$$

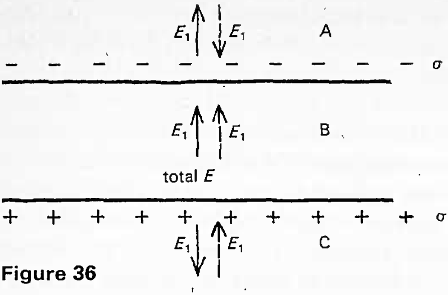


Figure 36

Figure 36 shows a pair of oppositely charged plates, with similar charge densities $+\sigma$ and $-\sigma$. The field E_1 of the positive plate is shown as a full arrow; that of the negative plate as a broken arrow.

1 What is the total field in regions A and C?

2 What is the total field E in region B, between the plates, expressed in terms of σ?

j The field between capacitor plates is written as:

$$E = \sigma/\varepsilon_0$$

as well as being, from **i** above

$$E = 4\pi \times 9 \times 10^9 \times \sigma$$

What is the Coulomb force constant, 9×10^9 N m^2 C^{-2}, in terms of ε_0?

39 In a television set, the position of the picture on the screen is not noticeably affected by the gravitational pull of the Earth on the electron beam. Discuss the plausibility of the following reasons:

a The TV set is too short (back to front) for any effect to show up.

b Electrons are too light to be significantly affected.

c The electrons travel too fast for an effect to be seen.

d The number of electrons in the beam is too great for an effect to be seen.

e The charge on an electron is too small for the effect to be observable.

f Gravity forces do not affect electrons.
g The electrons are converted into light, and light is not affected by gravity.

40 (Do a only, if time is short.)
a What is the electrical potential at 0.5×10^{-10} m from a proton? What is the potential energy of an electron at this distance from a proton? This arrangement (electron 0.5×10^{-10} m from a proton) is a simple and not altogether realistic model of a hydrogen atom. The energy needed to move an electron from the first energy level in hydrogen to a long way away from the proton is 13.6 electronvolts. How does this compare with your answer?

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$e = 1.6 \times 10^{-19} \text{ C.}$$

b The ionized hydrogen molecule (H_2^+) consists of two protons, 1.1×10^{-10} m apart, and a single electron.

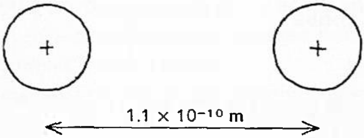


Figure 37

If you wanted to make the potential energy of this electron as low as possible (that is, bind it as strongly as possible to the two protons) where would you put it? The total potential energy of the H_2^+ ion is the sum of three potential energies: write down values for each, putting the electron in the favourable position chosen above. What is the *net* potential energy (terms corresponding to attraction will be of opposite sign to those corresponding to repulsion)? The energy for the first energy level of H_2^+ is about 2.5 eV – compare this with your answer.
The answers you get should be about the right order of magnitude, but won't be much more accurate than that.

This is because you are assuming that the electrons are at rest – if they have to have kinetic energy as well (which they do), then there is another energy contribution to be reckoned with, and, if they are not at rest at one point, then your potential energy calculation won't be accurate either. But the simple calculations do show one very important point – that the Coulomb Law for electrical attraction gives a very *rough order of magnitude for the binding energy of atoms and* molecules. You would be right to suppose that electrical energies are important here, but are not the whole story.

Charge on electron $= -1.6\times10^{-19}$ C

Charge on proton $= +1.6\times10^{-19}$ C

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \text{N m}^2\ \text{C}^{-2}$$

41 *Optional – good practice in calculating and adding up fields*

The electrical properties of many insulators are controlled by the fact that they contain an array of dipoles. A dipole is a pair of charges equal in size and of opposite sign, separated by a short distance, and the distribution of the electrons in many

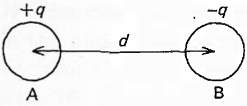


Figure 38a

molecules often leads to the existence of small dipoles: for example, A and B could be a pair of ions, one of which had given an electron to the other. This question is about the difference between the electric field effect of a dipole and that of a single charge. Say that A is a charge of -1.6×10^{-19} C (the charge of one electron) and B one of $+1.6\times10^{-19}$ C, and that *d* is 2×10^{-10} m (see figure 38*b*). What is the *resultant* field at C, 10^{-10} m from B? What is the field at D, 20×10^{-10} m from B? Work out the ratio of these answers, then do the same calculations for a single charge at B (A taken away) and work out a C to D ratio for this case. Does

the field due to a dipole fall off more or less rapidly with distance than that of a single charge? Do you think your answer can easily be explained? (*An explanation might start 'A long way away the separation* of the two equal and opposite charges does not matter, so . . .'.)
Ionic crystals have enormous electric fields inside them whilst giving rise to little or no field effects outside. Does your calculation give you a clue as to why this might be so?

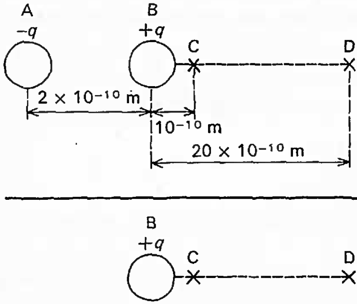


Figure 38b

42 Alpha particles (with a positive charge equal to 2 electron charges) are shot out from radioactive nuclei. It is common to find them travelling with kinetic energy of about 5 million electronvolts.

How much energy is that in joules?

If the alpha particle started at rest near the nucleus, how much potential energy would it then have?

If the potential energy was all due to the electrical forces between nucleus and alpha particle, what would be the electrical potential at the point near the nucleus where the particle was at rest? (*Not* 5 million volts.)

Electron charge $= 1.6 \times 10^{-19}$ coulomb

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}.$$

43 Here are some different methods of 'mapping out' the electric field near to a hydrogen nucleus (a proton with charge $+1.6 \times 10^{-19}$ coulomb).

The same values would be obtained for, say, an Na^{+} ion, although there would be difficulties close to it where one starts to 'go into' the electron cloud.

(Assume $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ N m² C⁻².)

1 Values plotted on a regular grid
Field = number plotted $\times 10^{10}$ volts per metre.
The grid lines are spaced at 10^{-10} metre apart.

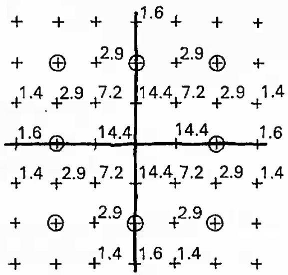


Figure 39

The numerical values have been put in on some of the points in figure 39. Check these values and then see if you can fill in the missing values for the points marked thus ⊕

2 Lines show field direction

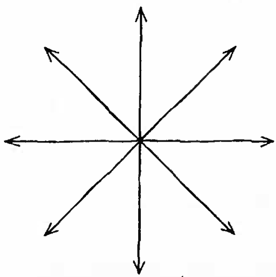


Figure 40

3 Shading of varying density

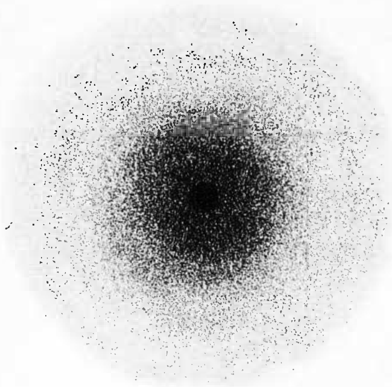


Figure 41

4 Contour lines of equal intensity
Lines are drawn through points of the same field strength. Can you plot approximate numbers on these contour lines? (Use the values given in *1*.)

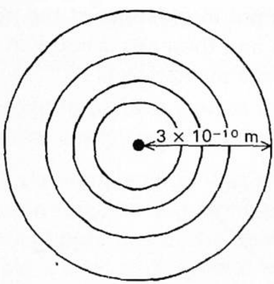


Figure 42

5 Arrows in direction of field
Arrow length is proportional to field strength (at mid-point of arrow). See figure 43.

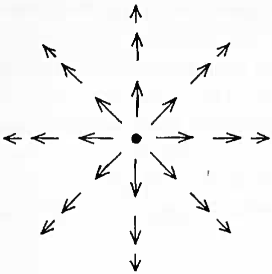


Figure 43

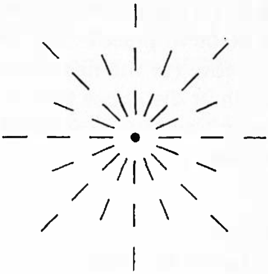


Figure 44

6 Lines in direction of field
The number of lines per centimetre on the paper is proportional to the field strength. See figure 44.

7 A graph of field against distance
Can you draw in the curve?

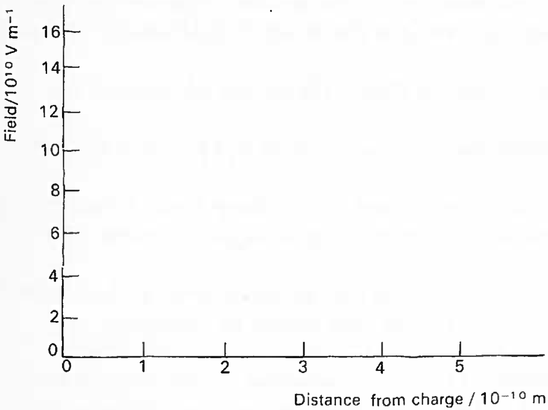


Figure 45

Which methods of mapping might be useful, and which might *not* if you wanted:

a To get a quick impression of the direction of the field at any point?

b To get a rough idea of the variation in strength of the field from place to place?

c To know (to the nearest whole number $\times 10^{10}$) the field strength at distances from 1 to 4×10^{-10} metre?

Suggest any other virtues or defects of the various 'maps' that strike you.

Part Four

Ionic crystals

Questions 44 to 47

The main argument in Part Four of this Unit is developed in a separate chapter (page 85). Questions 44 to 47 below are supplementary and rather hard. You may think that the questions in the chapter are quite enough for you.

44 *Hard*

Suppose an Na^+ ion and a Cl^- ion are initially 8×10^{-10} m apart and the distance between them is then reduced to 4×10^{-10} m, but that even at the smaller distance they are still not 'touching'. What is the effect of this halving of their distance:

a On the force between them? (Work out the ratio of the forces.)

b On the energy they possess? (Work out the ratio of the energies.)

c As they come together, has energy been passed *from* the ions *to* whatever holds them in position, or the other way round?

d If the distance were to be halved again, then at the shorter distance (2×10^{-10} m) the ions would be 'touching' and extra repulsion forces would have an appreciable effect. Would the ratios of forces and energies for the change from 4 to 2×10^{-10} m be smaller than, the same as, or larger than the *ratios* you worked out for the change from 8 to 4×10^{-10} m?

45 *Hard*

Here is a graph showing changes in energy of the atoms or ions in three different (imaginary) solids as the atoms or ions are moved closer together or further apart.

a Compare solids A and B. Do you expect the distances at which the atoms settle down in equilibrium in solid A to be bigger than, the same as, or smaller than the distances in solid B?

b Would solid A need more, the same, or less energy to vaporize it than solid B?

c Repeat **a**, comparing B with C.

d Repeat **b**, comparing B with C.

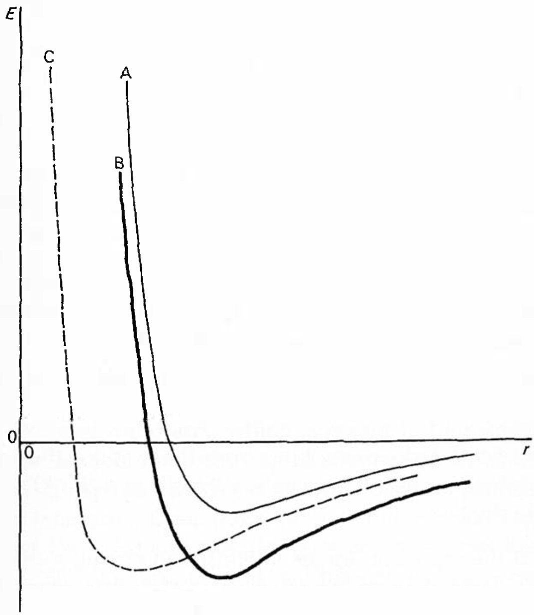


Figure 46

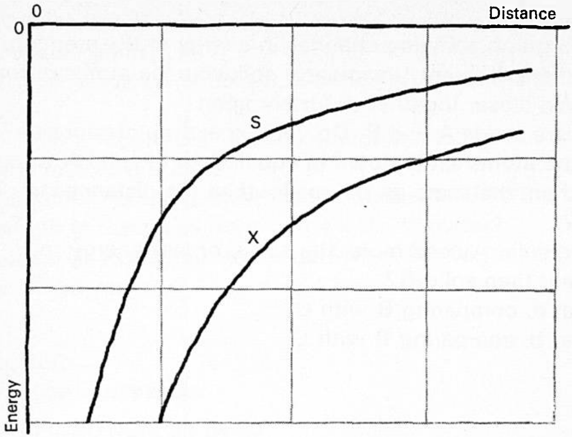


Figure 47

46 *Hard*

In figure 47, curve S shows how the attractive electrical energy for NaCl changes as the distance between the ions changes. Curve X shows the same thing, but for another ionic crystal X.

A says that the curves show that in substance X the electric forces between the ions are greater than in NaCl.

B says that they show the opposite, that in X the electrical attraction forces are smaller.

C says you can't deduce anything from them about the electrical forces in the two crystals. Who do you think is right, and why?

(Harder) If the repulsion forces in NaCl and crystal X were exactly the same (had the same value at the same distance), would you expect the distance at which the ions are in equilibrium in X to be smaller than or greater than the distance for NaCl?

47 *Hard*

Figure 48 is a small part of a graph of energy against average distance-apart-of-atoms in an (imaginary) solid. If the solid is very, very cold the atoms have very little vibrational energy and are more or less fixed at a distance r_0 from each other, with energy E_0. When the solid is warmed, the atoms vibrate. On average, their distance apart can vary between *a* and *b*, because at these distances all the energy (on average) *available to an atom has been transformed into* potential energy. (Compare a pendulum swinging – the more the energy the wider the swing.)

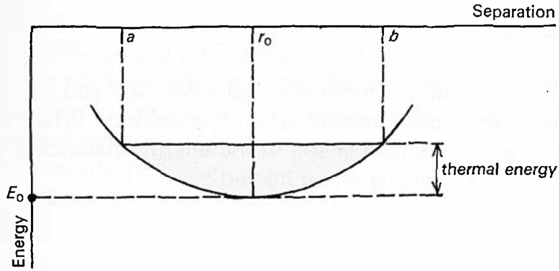


Figure 48

So we have a picture of all the *atoms vibrating, coming as* close as distance *a* together and going as far apart as distance *b*.

a Looking at the graph, which is meant to be symmetrical, what is the *average* distance apart of the vibrating atoms?

b A solid expands when the average distance apart of its atoms increases. Does the simple picture above predict that on heating the solid will expand, contract, or neither expand nor contract? Think about what will happen to the average distance apart of the ions.

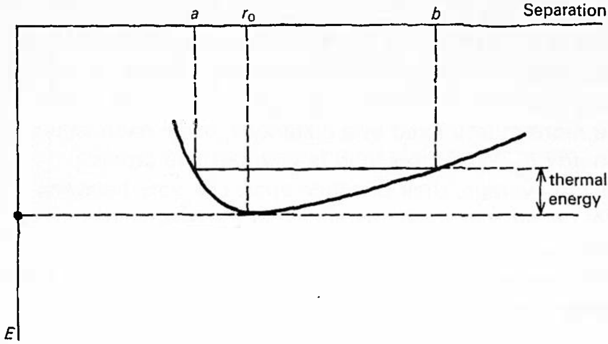


Figure 49

c For NaCl, the graph is more like figure 49. Can you explain why NaCl *does* expand when it is heated? What can you say about the general shape of the energy-distance graph of *any* solid that expands when heated?

Answers

1 a 10^{-4} C.
b 2×10^{-4} C.
c Yes.
d 3×10^{-4} C.
e 3 μF.
f We think this is true.

2 a 0.01 μF $= 10^{-8}$ F; 10 pF $= 10^{-11}$ F; ratio 1000 to 1.
b 10^{-8} C.
c Charge will flow from the higher p.d. to the lower; it will stop when the p.d.s are equal.
d Since the p.d.s are the same, the charges will be in proportion to the capacitances, ratio 1000. The 0.01 μF capacitor has the larger charge.
e Very nearly 10^{-8} C.
f Too large; 1/1000 of this amount is on the other capacitor.
g 0.48×10^{-8} C, with error about one in a thousand.

3 a 10^{-4} C.
b 10 V.
c 5 V.
d 10^{-4} C.
e 10^{-4} C.
f 15 V.
g 20/3 μF.
h

$$V_1 = Q/C_1$$
$$V_2 = Q/C_2$$
$$V = Q/C = V_1 + V_2 = Q/C_1 + Q/C_2$$
$$1/C = 1/C_1 + 1/C_2$$

4 *Your answer could include these points.*
The ball is in an electric field directed from left to right. When the ball is positively charged, it goes to the right, when negatively, to the left.
Or you could discuss this in terms of the pushes and pulls of the charges on the plates on the ball. The ultimate energy source must be the power supply. Avoid the temptation of

thinking that there is only a force on the ball when it touches a charged plate. There is a force on it when it is between the plates. Since the ball always moves to the plate of opposite charge, on arrival it delivers up the charge it ferried across. As it goes back again, it takes the opposite kind of charge in the opposite direction; the current in the rest of the circuit is, however, in the same direction whichever way the ball is going.

5 At the smaller spacing with the same p.d. the field (volts per metre) is larger, and the foil hangs at a larger angle. The fields are 5000/0.08 V m^{-1} and 5000/0.04 V m^{-1}. To get the same field at 40 mm, reduce the p.d. to 2500 V.

6 **a** The energy transformed (charge × p.d.) is 5×10^{-15} J.
b 10^{-12} N.
c 10^5 N C^{-1}.
d 10^5 V m^{-1}.
Energy $= Vq$; $F = Vq/d$; $E = F/q = V/d$.
e Unit V = J C^{-1}.
Unit J = N m.
Unit V m^{-1} = J C^{-1} m^{-1} = N m C^{-1} m^{-1} = N C^{-1}.

7 **a** Use $E = V/d$. Approximately, $V = 2000$ V.
b Briefly, the higher the pressure the shorter the distance a particle moves before a collision with another. If in that distance the occasional ion or electron can acquire enough energy (about 30 electronvolts) from the p.d. it moves through, it may ionize the molecule it hits, and the ion or electron so freed may make another in the same way, leading to an 'avalanche' of ions, and a spark. At higher pressures, then, a greater p.d. is needed to start a spark, for the ions have to acquire a fixed energy in a shorter distance. The question suggests that the sparking p.d. in a car engine exceeds 2 kV, for the pressure in the engine is several times atmospheric pressure.

8 The foils have equal and opposite charges, and such charges attract each other. If they were pulled apart, having been disconnected, there would be a force to be overcome, work would be done, and the energy would rise. So would the p.d. For the leakage problem look up 'action at points' or 'lightning conductors' in a textbook.

9 a We think it might be about half that at A.
b 10^3 V m^{-1}?

10 a 10 V m^{-1}?
b About the same.

11 There isn't one right answer, though obviously you use charged plates. But what shape would collect charges well without interfering with the gas flow? Would the charged particles get carried past plates that were too short? How would the dust be got off the plates; and how much might there be, say, every day in a power station flue?

12 a We think the statement is correct.
b 10^{-9} F.

13 Between 10^{-14} and 10^{-15} Ω^{-1} m^{-1} (10^{14} to 10^{15} Ω m resistivity). There would be extra conduction spreading out beyond the edges of the plate, giving an overestimate of the conductivity. The flow will not be uniform because as the p.d. drops, so does the current, as in the ordinary decay of charge on a capacitor.

The question can be done without knowing the area of the plate or its distance above the bench.

14 a Charge, area, and electric field.
b p.d. divided by spacing, using parallel plates.
c There is no more a constant here than in area = length × width. It follows from the meaning of a uniform electric field that $E = V/d$. But Q/A and E have different units, and there is a constant, with units, relating them.

15 a and **b** are the same, about 10^{-9} C m^{-2}.

16 a When it is rolled up, for B and D must not touch.
b About 100 m.
c This is not easy to do if you try to work out the thickness of a spiral. A zigzag of width L would have $100/L$ layers. If a layer were 0.1 mm thick, then $100/L$ layers would make a stack of height L, if L were about 0.1 m.

17 a About 10^{-9} C m^{-2}, negative.
b About 10^{-6} C m^{-2}, negative. Current about 10^4 A.

18 a d_1 is the larger. At some high electric field, value V/d, a spark passes between the plates. This happens at B at a lower voltage than at A because the spacing at B is smaller.
b V_2 is the larger. The shape is different because Q is proportional to $1/d$ (fixed V), while Q is proportional to V (fixed d). At A and B the field is large (Q is big, d is small). Beyond C and D, the spacing is as big as the width of the plates, the field is nowhere very uniform, and Q is no longer proportional to $1/d$.
c See figure 50.

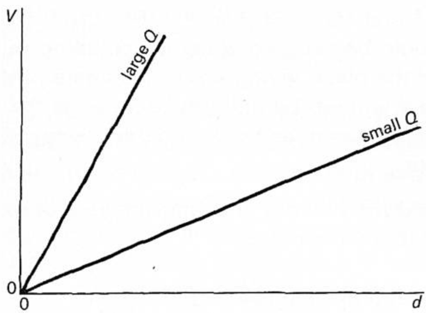


Figure 50

19 This is a question meant to raise problems more than to solve them. In considering **a**, notice that Einstein and Infeld say that the lines show how a test body *would* behave *if it were* brought into the vicinity. . . .

In **b**, answer **2** only makes sense if someone is saying what the size or direction of the field actually is. But if they are saying how they would measure a field, and so explaining what they mean by 'a field', **2** makes no sense. Similar remarks apply to **3**, though it must be doubtful whether anything in science can be established by reason alone. **1** and **4** make most sense, we think. Of course, a defined term doesn't have to go on proving useful. 'Caloric' had a clear meaning, but it isn't much in vogue nowadays as something that conveys a fruitful idea.

20 The question itself tries to explain what a good essay would be like. You may say what you please, but remember: give arguments and illustrations. Try imagining your essay being marked by two people. One gives half the marks for clear views clearly expressed. The other gives marks for the physics that he can see you understand from the variety and relevance of your examples.

21 **a** 2.18 m s^{-2}.
b 2.18 N kg^{-1}.
c 7.64 m s^{-1}.
d 5350 m.
e 5.14×10^8 J.
f 1.17×10^4 J kg^{-1}.
g 2.5×10^{-3} radian.
h The speed remains almost the same. If the force does not remain exactly at right angles to the path, some energy change occurs, but is small. The distance moved under the thrust is small (about 13 m, from $s = \frac{1}{2}at^2$).

22 **a** The Earth's gravitational attraction, directed almost opposite to the motion of the spacecraft, slows it down. Its kinetic energy falls, but its potential energy increases.
b 7.65×10^{-3} m s^{-2}.
c 7.65×10^{-3} N kg^{-1}.
d Gravitational pull towards the Earth's centre $= 7.9 \times 10^{-3}$ N kg^{-1}.

The answers to **c** and to **d** are nearly, but not exactly, the same. They are nearly the same because it is the inverse square gravitational force which slows the spacecraft down. One possible source of error makes the agreement worse, if it is allowed for. This is the fact that the average force is not equal to the force at the average distance, but is larger than this because the force decreases as the inverse square of the distance and is disproportionately larger at the smaller distances.
Another source of error probably explains the discrepancy. The flight plan, figure 13, shows that the path of the spacecraft at a distance of some 225×10^6 m from the Earth makes an angle of a few degrees with the line joining it to the Earth's centre. So the component of the gravitational pull along the flight path is less than the pull along the line to the Earth's centre.

23 **a** 0.453 m s^{-2}.
b 0.453 N kg^{-1}.
c See figure 51 and table 5.

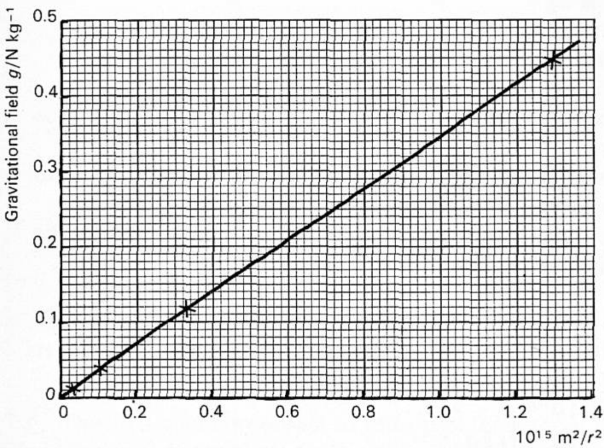


Figure 51

Distance from centre of Earth /10^6 m	**Force per kilogramme /N kg^{-1}**
(*2*), (*2A*) 27.7	0.453
(*3*), (*3A*) 55.4	0.122
(*4*), (*4A*) 96.5	0.042
(*5*), (*5A*) 170.4	0.013

Table 5

The gravitational field varies as the reciprocal of the square of the distance from the centrè of the Earth.

24 **a** The kinetic energy per kg at velocity v is given by $\frac{1}{2}v^2$. (Why?) The change in energy is 57.2×10^6 J kg^{-1}. The spacecraft was flying away from the Earth, so that the attraction of the Earth slowed it down even though the engines weren't used.
b 59.2×10^6 J kg^{-1}. This time the spacecraft was approaching the Earth, being accelerated all the time by the Earth's gravitational pull.
c The two values are similar. It seems likely that if the pairs of distances had been identical, the changes in energy would have been the same. Certainly one would expect that result if the paths were identical, for there is no friction to transform energy and the motors weren't used. Actually, a gravitational field has the interesting property that the changes in energy attributable to its action do not depend at all on the path taken. It is results like those quoted for Apollo 11 that show that the gravitational field has this property. (It is said to be a *conservative* field.)
d Yes and no, we think. Energy isn't something you see or touch, but something you calculate. Certainly some calculated kinetic energy, $\frac{1}{2}mv^2$, does vanish and reappear. Because it does reappear, it seems sensible to think of it 'really' being there all the time, somehow. If you put a pound in the Post Office Savings Bank, you can get it back. You say you had it all the time, even though it had vanished from your pocket, just because you could get it back. Actually, money in

savings is only roughly conserved; *you could argue that it* increases because of interest earned, or decreases because of inflation – that is, its buying power drops.
If you believe very strongly that energy is conserved you may say 'But of course the energy is really there all the time; it just changed its form'. But just why does anyone believe that energy is conserved? Because when some seems to get lost . . . and the argument starts again at the beginning.
e See the answer to **d**. 'Potential' energy is a good name for energy that seems to be really there, even though kinetic energy has gone, because the energy can appear again as kinetic energy, on a return path.

25 **a** *it is available from the fuels, when they react.*
b It looks as if a rocket made entirely of petrol, not even allowing for the mass of oxygen needed to burn it, would not have enough energy to get the petrol itself far away from the Earth. Not all the fuel travels all the way, which complicates things, however.
c See the answer to **b**. Chemical bonds do not differ too much in the energy they release, so the best imaginable fuel of a chemical kind will not have much in hand when one remembers that at least part of the fuel has to travel away from the Earth, and that the rocket motor will convert much less than all the fuel energy into energy of motion and, ultimately, gravitational potential energy.
d *Because the gravitational field is directed towards the* Earth's centre.
e Yes, because the journey would start nearer to the same mass, in effect. On the Earth's surface, you would weigh four times as much as you do now.

26 **a** A lot of things have. A textbook? A bottle of wine? A bag of sugar? A pair of boots?
b Weight 10^{-10} N; mass 10^{-11} kg; volume 10^{-14} m^3 if the density is that of water; size 2×10^{-5} m, or 0.02 mm. This is *some hundreds of wavelengths of visible light, bigger than* most bacteria. Maybe a pencil dot would do, or a marine micro-organism.
c, d See figure 52.

Figure 52

27 a The depth of the sheet below its original surface represents the fall in potential energy near a mass. Notice how a marble would need energy to get out of both 'wells'.
b The steepness of the slope corresponds to the field, which is zero on the flat top of the 'saddle' between 'Earth' and 'Moon'. Notice how on either side of the saddle on the Earth-Moon line the field pulls a mass away from the saddle, but on either side along a line at right angles to the Earth-Moon line, a mass is pulled towards the saddle.

28 a $(35.33-0.92)\times 10^6 = 34.41\times 10^6$ J kg^{-1}.
b Yes, at a distance r_0 where all the kinetic energy is transformed into potential energy. Possibly, the spacecraft has enough kinetic energy to go as far as one might please, and still have some left. Actually, Apollo 11 does not have enough kinetic energy to go as far as one might please, and its kinetic energy would all be transformed into potential energy at some large distance r_0. *You will be able to estimate* r_0 later in this question.
c 35.33×10^6 J kg^{-1}.
d 14.44×10^6 J kg^{-1}.
e Change in potential energy per kilogramme $= GM\left[\frac{1}{r}-\frac{1}{r_0}\right]$.

f See figure 53.

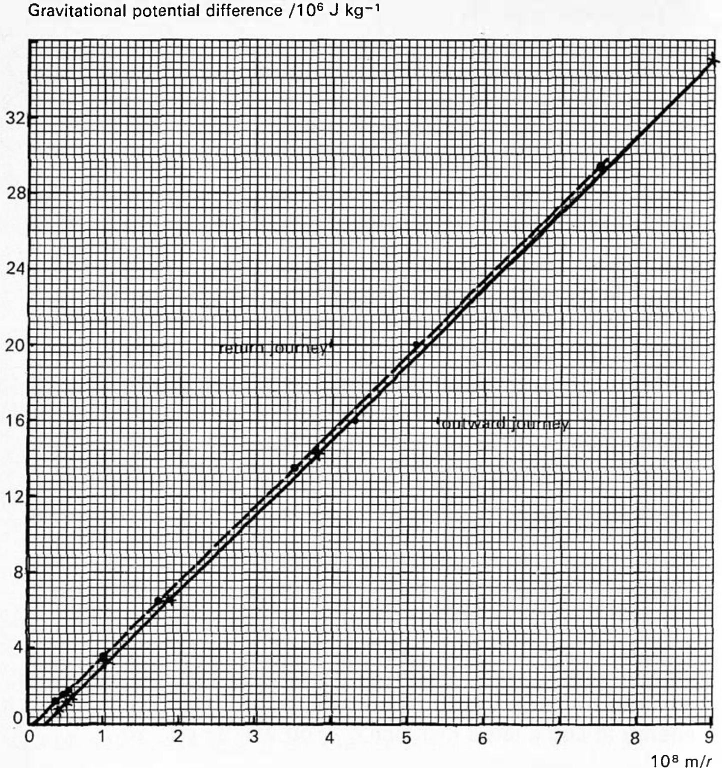


Figure 53

g Both graphs should be straight lines, as the potential difference is proportional to $1/r$, and r_0 is constant for any one graph.

h Both should have the same slope, equal to GM. The slope agrees well with $GM = 4.0 \times 10^{14}$ N m^2 kg^{-1}, calculated from the gravitational force constant G and the mass M of the Earth.

i The intercept is $1/r_0$; actually 10^8 m/r_0 on the graph in figure 53. r_0 is not the same for the two journeys, being a little larger for the return journey, indicating that the return journey has the larger total energy.

29 The hill used actually has a $1/r$ shape. It is possible but not easy to check this using data from the photograph. You can certainly assert that the hill rises towards the centre, and that the slope increases as r decreases.

30 **a** $r^2 = GM/g$, giving $r = 6 \times 10^6$ m approximately.
b You need an argument which splits the distance up into small slices Δr, over each of which the force is more or less constant. Then $F\Delta r$, the work done, is equal to the area of a strip F high and Δr wide. The area below the graph is the total of many such strips.
c We got an area A of 1.8×10^6 J kg^{-1} when we tried.
d We got $B = 3.3 \times 10^6$ J kg^{-1}, so that $A+B = 5.1 \times 10^6$ J kg^{-1}.
e Table 6 gives our estimates from a graph. Your estimates may differ from them a little.

$r/10^7$ m	Area/10^6 J kg^{-1}	
4.0	1.8	A
3.0	5.1	$A+B$
2.0	11.8	$A+B+C$
1.5	18.4	$A+B+C+D$
1.0	31.6	$A+B+C+D+E$

Table 6

f, g The graph of area against $1/r$ is a straight line. The area is zero at a value of $1/r$ corresponding to $r = 5 \times 10^7$ m. This is natural enough, because this is where you started counting areas from. All potential energies have an arbitrary zero. You will find that it is usual to adopt a zero value for large values of r, at which $1/r$ itself falls to zero.

31 **a** 0.1×10^6 J kg^{-1}.
b 10 N. This is about the value of the gravitational field at the surface, as it should be.
c 61.7×10^6 J kg^{-1}.
d The force is much less than 10 N kg^{-1} most of the way.
e 0.997×10^6 J kg^{-1}.

f 62.7×10^6 J kg^{-1}.
g $\frac{1}{2}v^2 = 62.7 \times 10^6$; $v = 11.2 \times 10^3$ m s^{-1}.
For mass m, $\frac{1}{2}mv^2 = 62.7 \times 10^6 m$, so m cancels.
h 62.7×10^6 J kg^{-1} *less*.
i *Minus* 62.7×10^6 J kg^{-1}; $-GM/r$.
j1 $-\frac{dV_{\text{gravity}}}{dr} = -\frac{GM}{r^2}$. The usual name is the *gravitational field*. The sign is the conventional one for an attractive field.
2 The slopes give the gravitational fields, in the ratio 4:1.

32 Shift in plumb line angle: one way would be to go round to the other side of the mountain, where the plumb line should be pulled the other way.

$$\tan\theta = \frac{M_m}{r_m^2} \Big/ \frac{M_E}{r_E^2}.$$

$$M_E = 6 \times 10^{24} \text{ kg}.$$

$$r_E = 6 \times 10^6 \text{ m}.$$

$$M_m = \tfrac{1}{3} \times (\text{base area}) \times (\text{height}) \times (\text{density})$$

$$\approx 10^{14} \text{ kg}.$$

$$r_m \approx 10^3 \text{ m}.$$

$$\tan\theta \approx 6 \times 10^{-4}.$$

33 Number in B is four times number in A.

Light arriving at the Earth from one star in B is one quarter that from one star in A; light from all B is the same as light from all A. If there were infinitely many shells, the total light at the Earth would be infinite. Actually, the nearer stars intercept light from further ones, with the result that it should be as bright at the Earth as at the surface of a star.

If you are interested in how the resolution of this puzzle is connected with the expansion of the Universe, look at Bondi, *The Universe at large,* Chapter II.

34 a An example is, 'It is impossible for heat to flow unaided from a colder place to a hotter one'.
b 'No object may change its motion unless acted on by an outside force.'
c It reads 'No materials disobey Ohm's Law except those which do.'
d By using + and − signs, and the rules for products. Even the 'commutation rule' $Q_1Q_2 = Q_2Q_1$ represents the fact that each exerts the same force on the other. Very neat!
e If field $= GM/r^2$, potential $= -GM/r$.
In electricity, if field $\propto 1/r^2$, potential $\propto 1/r$.
f This can only be your opinion.

35 a Approximately, F is proportional to d for small angles. The deflection is doubled.
b The points on a graph of d against $1/r^2$ lie near to a straight line.
c 0.01 μF $= 10^{-8}$ F. At 0.5 V the charge on it is 0.5×10^{-8} C. This is nearly the charge that was on the ball. See question 2. The fluctuations in charge could explain why the points plotted in **b** are scattered on either side of a straight line.
d, e The force constant is of the order 10^{10} N m^2 C^{-2}, and certainly lies between 5×10^9 and 5×10^{10} N m^2 C^{-2}. You may think the limits are closer than this.

36 a 10^{-4} N.
b 10^{-9} C.
c 10^{-11} A.
d 10^3 V.
e 10^{14} Ω.

37 a σA
b σwL.
c $\sigma wL/r^2$.
d Twice as big.
e $2w$.
f $2L$.
g $4wL$.

h The charge on A′B′C′D′ is four times that on ABCD, because the area is quadrupled. But A′B′C′D′ is twice as far from S as was ABCD; on this account the field is reduced by a factor four, by the inverse square law. Overall, the fields of A′B′C′D′ and ABCD at S are the same.
i Small.
j The field in figure 29 will change with height, for the effect of the missing edge bundles is large.
k Varies inversely as the square of the height.

38 a σdA.
b $dE = 9 \times 10^9\ \sigma dA/r^2$.
c The field of a point charge is directed radially from that charge.
d Along OS, perpendicular to the sheet, for the components of the fields parallel to the sheet are equal and opposite.
e For every patch P, another can be found whose field component $dE \sin\theta$ parallel to the sheet cancels that of P. The components $dE \cos\theta$ perpendicular to the sheet are all that remain, in effect.
f $dE \cos\theta = (9 \times 10^9\ \sigma\ dA \cos\theta)/r^2$.
g $dA \cos\theta$ is the area of the end of the cone, cut at right angles to the cone. See figure 33.
h 1 Over the region below a plane, solid angles $d\omega$ add up to 2π. So the total field is: $2\pi \times 9 \times 10^9\ \sigma$.
2 The area of the hemisphere is $2\pi h^2$, so that the total field comes to $2\pi \times 9 \times 10^9\ \sigma$.
i 1 Zero in both.
2 $E = 2E_1 = 4\pi \times 9 \times 10^9\ \sigma$.
j The Coulomb force constant 9×10^9 N m^2 C^{-2} $= 1/4\pi\varepsilon_0$.

39 Here is what we think, very briefly. You should be able to give reasons.
a Possibly.
b Rubbish.
c Possibly.
d Rubbish.

e Maybe, though actually most people would think that the charge on an electron was very 'big' compared to its mass, for the electrical force between two electrons far outweighs the gravitational force between them.
f Not true so far as anyone knows. But gravity forces on electrons are so hard to observe that we haven't actually heard of a direct observation ever being made.
g Nonsense.

40 **a** 28.8 V; 28.8 electronvolts or 46.1×10^{-19} J.
b Put the electron in the middle.

$$-26.2-26.2+13.1 = 39.3 \text{ electronvolts or}$$

$$-62.8 \times 10^{-19} \text{ J.}$$

The comparison with the measured ionization energy of H_2^+ is not very favourable, indicating that, if the electrical part of the calculation is right, there must be other energies there as well. In fact, the kinetic energy of the electron accounts for the discrepancy, together with proper allowance in the electrical energy for times when it is not between the protons.

41 Dipole field at C = 2.05×10^{-10} V m^{-1}.
Dipole field at D = 1.0×10^{-13} V m^{-1}.
Single charge field at C = 2.3×10^{-10} V m^{-1}.
Single charge field at D = 5.75×10^{-13} V m^{-1}.
Ratio of dipole fields = 2×10^3.
Ratio of single charge fields = 0.4×10^3.

The field from a pair of opposite charges falls off faster than the field from a single charge. The field outside an array of mixed positives and negatives could be very small indeed.

42 8×10^{-13} J.
Potential energy 8×10^{-13} J.
2.5 million volts (an alpha particle carries two electron charges).

43 **a** Methods *2, 5*, or *6*.
b Perhaps methods *3, 5*, or *6*, and maybe others.
c Methods *1, 4*, or *7*.

44 **a** Force rises by a factor 4.
b Energy rises by a factor 2.
c Energy passed *from* ions.
d Both ratios not so large.

45 **a** Same equilibrium distances.
b A needs less energy than B.
c C has a smaller equilibrium distance than B.
d B and C need the same energy.

46 A is correct.
The ions of X would be in equilibrium at a smaller distance than those in NaCl.

47 **a** r_0.
b The prediction is that the solid will neither expand nor contract! If the ions oscillate equal distances on either side of their average position, spending as long at smaller than average distances as at larger than average distances, the average separation will be unchanged.
Of course, nearly all solids do expand, which means that the simple, symmetrical energy curve is not applicable to them.
c The average separation is now more than r_0, since the minimum *a* is closer to r_0 than the maximum *b*. A solid that expands on heating must have an energy 'well' that is less curved at distances larger than the mean spacing than it is at distances smaller than the mean spacing.

Ionic crystals

The purpose of this chapter

This chapter deals with Part Four of this Unit and contains a long, tough argument. It takes you into one of the kinds of problem that face working physicists trying to understand why materials behave as they do. In it you will see how many different kinds of ideas can be put together to make new knowledge.

But the argument *is* long. So it has been broken up into stages, and those who feel they have had enough can stop at any stage. Something valuable is gained by going through to any point, even the earliest, and the argument is not so vital to understanding physics that you ought each to suffer it to the end. Yet we hope that some will see just how far they can go; and we think that they will find value and pleasure in using their knowledge and skill to tackle a problem in which the ideas of electricity and dynamics, together with some mathematics, will all help to reveal something of the reasons for the observable properties of ionic crystals.

The argument is presented mainly in the form of questions. Each has an answer at the bottom of the righthand page, and you may wish to cover up the answers at first. If you find difficulty, it may be helpful to look at the answer, and then consider why it is correct. (We hope it is.)

The way ahead

Unit 1, *Materials and structure*, was about the structure of solids: the idea was that crystals are made by stacking atoms together in a regular pattern. What holds the atoms together? Solids do not fall apart; indeed many are very strong and can only be pulled apart by large forces.

The forces that hold them together are electrical forces. Unit 3 has been about electrical forces between charges, the inverse square law of force, and the potential energy

$$\frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r}$$

of two point charges q_1, q_2, placed a distance r apart. This section tries to show how these electrical ideas can be used to understand the forces that hold atoms together in solids and the energy needed to pull them apart, when, for instance, a solid turns into a gas.

How can the distance between atoms be found? Unit 1 showed that X-ray evidence helps with this. What charges may the atoms carry? This workbook will consider only one simple problem, an ionic crystal, and one in which pairs of atoms have exchanged one electron so that each atom becomes an ion with charge $e = 1.6 \times 10^{-19}$ coulomb, positive or negative.

What part do electrical forces between charged ions play in holding the ions together? Are these forces enough to explain the strength of the crystal, or the energy needed to tear the ions apart? These are the problems with which this section is concerned. The sodium chloride crystal is the ionic crystal chosen for detailed discussion.

Stage one

The energy needed to tear the ions apart

Experimental evidence

Figure 54 suggests the process of tearing apart a piece of NaCl crystal into separate ions. The energy needed to tear 1 mole of NaCl crystal apart into one mole of ions in a vapour can be calculated by putting together results of measurements on vaporizing NaCl, turning the atoms into ions, and so on.

It is: 77×10^4 joules per mole.

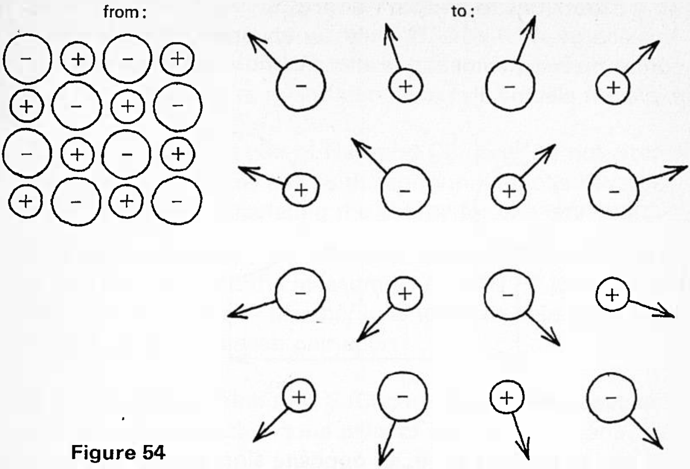


Figure 54

The tear-apart energy of a pair of ions

One mole of NaCl contains 6×10^{23} NaCl units (the Avogadro number).

1 How many Na^+ ions are there in 1 mole of NaCl?

2 How many Cl^- ions are there in 1 mole of NaCl?

3 How many *pairs* of Na^+ and Cl^- ions are there in 1 mole of NaCl?

4 How much energy is needed, on average, to pull apart *one pair* of Na^+ and Cl^- ions?

Answers

1 6×10^{23}.
2 6×10^{23}.
3 6×10^{23}.
4 12.7×10^{-19} joule per pair of ions.

Electrical tear-apart energy

This energy, 12.7×10^{-19} joule, to tear apart each pair of ions comes from experimental results. We now see if we can *explain* it electrically.

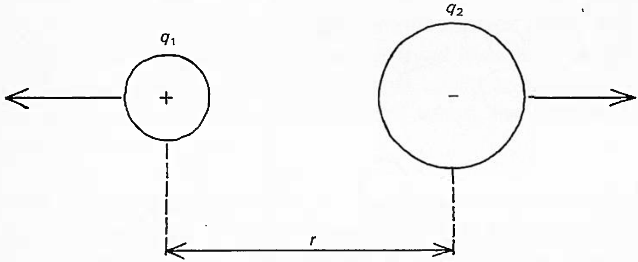


Figure 55

If a pair of charges q_1, q_2 of opposite sign, distant r from each other, are then pulled apart to a big distance, the energy transferred *from* whatever is pulling them *to* electrical energy is:

$$\text{Energy} = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

where $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^-$, with q in coulomb, r in metre.

5 Given that the Na^+ ion has charge $+1.6 \times 10^{-19}$ coulomb and the Cl^- ion has charge -1.6×10^{-19} coulomb, what else do you need to know to calculate this energy for pairs of ions in an NaCl crystal?

6 The Na^+ and Cl^- ions are 2.8×10^{-10} metre apart in NaCl. What is the electrical energy stored when a pair of Na^+ and Cl^- ions are pulled well apart from a distance of 2.8×10^{-10} metre?

7 The calculated value, 8.2×10^{-19} joule, is *less* than the experimental value, 12.7×10^{-19} joule. Have these two values the same order of magnitude or not?

8 If the calculated value were to be raised, by thinking of more forces that could act on the ions, would we have to think of more forces tending to hold the ions together, or of more forces tending to push them apart?

9 In the crystal, a pair of Na^+ and Cl^- ions are not alone (figure 54). The forces due to the surrounding ions have been ignored so far, by calculating the energy for one lone pair.

If this helps to explain the difference between the calculated 8.2×10^{-19} joule and the measured 12.7×10^{-19} joule, do you think that the presence of other ions tends to hold each pair together, or to push each pair apart?

10 Why do you think that NaCl consists of a vast regular array of ions, rather than lone pairs of Na^+ and Cl^- ions?

Summary

The electrical forces between pairs of ions will produce approximately enough energy to explain the tear-apart energy of an ionic crystal. The agreement is not perfect, indicating a need to think further, particularly about the effect of all the other ions in the crystal. As a later stage shows, it is necessary also to think about repulsions between ions as well as attractions.

You could stop here.

Answers

5 The distance *r* at which the ions are spaced in NaCl.
6 8.2×10^{-19} joule per pair of ions.
7 Yes.
8 More forces holding the ions *together.*
9 Tends to hold each pair together.
10 Perhaps because the presence of the other ions produces more holding-together energy than the charges on each pair can do on their own.

Stage two

The existence of repulsive forces between ions

The tear-apart energy of a regular lattice of ions

From here to question 11, the text attempts no more than to show you roughly how the presence of all the other ions in the crystal affects the tear-apart energy of a pair of ions. It need not be followed in any great detail. See figure 56.

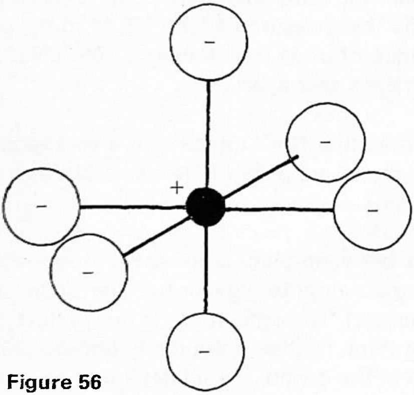


Figure 56

Round each Na^+ ion there are *six* nearby Cl^- ions, each 2.8×10^{-10} metre away. The energy needed to tear these from the Na^+ ion can be found, as was done for the single pair.

See figure 57. Nestling among the six Cl^- ions, are *twelve* Na^+ ions, each of which is repelled by the central Na^+ ion. The energy that can be released as these repelling ions come apart can go towards reducing the total tear-apart energy.

See figure 58. Next, arranged between the twelve Na^+ ions are *eight* Cl^- ions, and energy is required to tear these away from the central Na^+ ion.

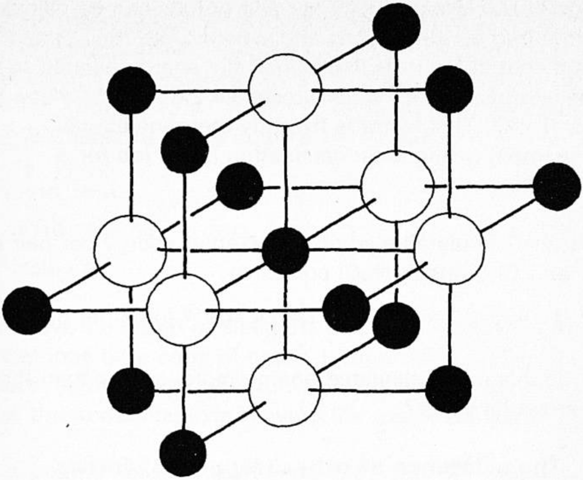


Figure 57

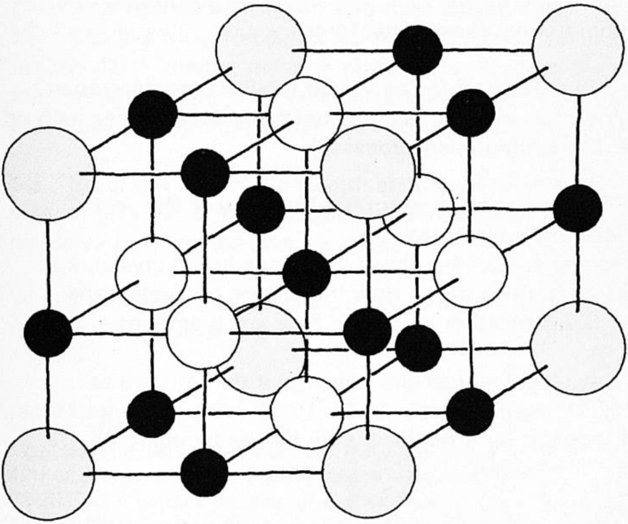


Figure 58

The total tear-apart energy per pair of ions can be calculated by counting up all the ions and working out their distances from a central ion. It is larger than the energy needed to tear apart one pair of ions from a distance 2.8×10^{-10} metre by a factor 1.747. This factor is fixed by the geometrical arrangement of the ions: calculating it is a job for a computer.

Thus, the calculated electrical tear-apart energy per pair of Na^+ and Cl^- ions in NaCl comes to:

$$1.747 \times 8.2 \times 10^{-19} = 14.3 \times 10^{-19} \text{ joule per pair of ions.}$$

11 Is this new calculated energy more, or less than the experimental value?

The existence of other (repulsive) forces

If the experimental value is trustworthy, and if we have correctly added up all the electrical energy, some other forces must be acting on the ions. We shall now try to find out something about these other forces.

12 If the electrical forces would require more tear-apart energy than is actually needed, must any other forces help or hinder the tearing apart process?

The changes in electrical energy if the ion-distance changes

To discover something about the forces in the crystal, it is necessary to think about how the energy changes if the crystal is squeezed or expanded by a small amount.

13 The electrical tear-apart energy if the ions are at 2.8×10^{-10} metre apart is 14.3×10^{-19} joule per pair of ions. What would it be if the ions were half as far apart (1.4×10^{-10} metre)?

14 What would the electrical tear-apart energy be if the ions were 2.0×10^{-10} metre apart?

Graph of variation of electrical energy with distance apart of ions

Values of the electrical energy E for distances from 2.0×10^{-10} metre to 4.0×10^{-10} metre are tabulated in table 7. Check the value at 4.0×10^{-10} metre.

$r/10^{-10}$ m	2.0	2.8	3.0	3.5	4.0
$E/10^{-19}$ joule	20.0	14.3	13.3	11.4	10.0

Table 7

Now draw the graph of electrical energy E against distance apart of ions on a copy of graph 1 (figure 59), using the scales suggested. For later convenience, this graph is drawn *below* the horizontal axis, leaving the top of the paper blank.

The experimental tear-apart energy, 12.7×10^{-19} joule per pair of ions, has already been marked on the graph.

The forces acting on the ions at 2.8×10^{-10} metre

15 The electrical forces on the ions, at their normal distance of 2.8×10^{-10} metre, must be pulling the ions together. If the ions are in balance at this distance, they must be pushed apart as well. How big must this repulsive force be?

16 So, if the size of the electrical attractive force between ions at 2.8×10^{-10} metre could be found, the size of the repulsive force would also be known.

Would anything then be known about the way in which the repulsive force arises, or its nature?

Answers

11 More. The discrepancy has been reduced a little.
12 Help. Such forces must be repulsions between ions.
13 28.6×10^{-19} joule per pair. (Twice as much, energy varies as $1/r$.)
14 20×10^{-19} joule per pair.
($2.8/2.0 = 1.4$ times as much as that at 2.8×10^{-10} metre.)
15 The same size as the electrical force of attraction.
16 No. Only its size would be known.

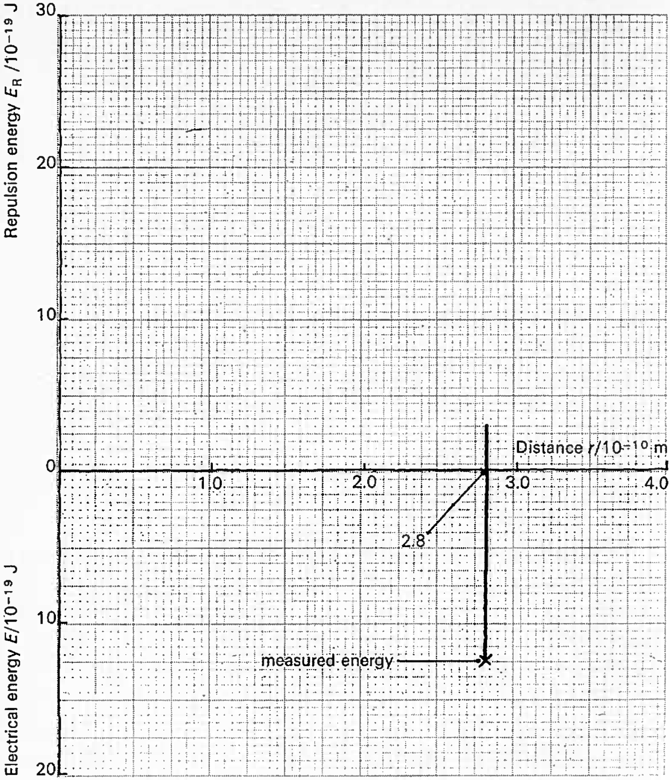


Figure 59

Finding the electrical force

Suppose the electrical force were F newton. If the ions moved a small distance Δr, energy $F\Delta r$ would be transformed, equal to a change ΔE in the electrical energy stored. See figure 60, which represents part of the graph in figure 59, which shows a small change Δr in r and the change ΔE in energy E.

Since $F\Delta r$ is equal to ΔE, F is equal to $\Delta E/\Delta r$.

But $\Delta E/\Delta r$ is the slope, or steepness of the graph at $r = 2.8 \times 10^{-10}$ metre.

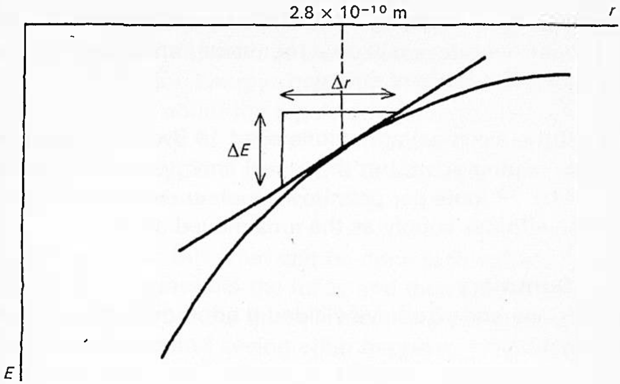


Figure 60

17 Draw a tangent to the graph of E against r at 2.8×10^{-10} metre. Measure the slope, finding

$$F = \frac{\text{change of } E}{\text{change of } r}.$$

Optional calculation

The force $F = \dfrac{dE}{dr}$ and $E = -1.747 \times \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{e^2}{r}$

so that $F = 1.747 \times \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{e^2}{r^2}$.

Check the value 5.1×10^{-9} newton by substitution.

18 How big is the force of repulsion between ions at 2.8×10^{-10} metre?

Answers

17 The force is 5.1×10^{-9} newton.
18 5.1×10^{-9} newton.

The energy transformed by the repulsive forces when the crystal is torn apart

The ions have to be pulled apart against the electrical attractive force, needing 14.3×10^{-19} joule per pair of ions. The repulsive forces will *help* the tearing apart, and can supply some of the necessary energy.

19 If the electrical attractions need 14.3×10^{-19} joule per pair for tearing apart, but the actual energy needed is only 12.7×10^{-19} joule per pair, how much energy did the repulsive forces supply as the ions moved apart?

Summary

Stages one and two have yielded a good deal of numerical information.

20 Fill in values in a copy of table 8, using the answers to previous questions.

The ions are normally in balance at a distance of	2.8×10^{-10} metre
The electrical energy needed to tear apart *one isolated* pair of ions is	8.2×10^{-19} joule
The *measured* energy needed to tear apart each pair of ions is	joule
The electrical tear apart energy per pair of ions, *allowing for other nearby ions,* is	joule
The electrical force on an ion, at 2.8×10^{-10} m separation, is	newton
The repulsive force on an ion, at 2.8×10^{-10} m separation, is	newton
The energy available from repulsions to push ions apart is (per pair)	joule

Table 8

You could stop here.

Stage three

The variation of repulsion with distance

For the electrical attraction energy of the charged ions the $1/r$ rule is known, and from this the attraction *force* was found from the attraction *energy* (question 17). The next step is to try and find out about the repulsion effect. For the repulsion we know the *force* at 2.8×10^{-10} metre and also the *energy* (see question 19). The problem is to work out the *rule* for the variation of repulsion with distance.

It can't be done. But what can be done is to make a guess and see if the guess fits the facts, and then see what else can be worked out from the guess. This is like doing an experiment, trying something and seeing what happens, only, instead of apparatus, there are numbers and algebra to connect together and get working.

A guess

The electrical energy varies as $1/r$. The electrical force varies as $1/r^2$. The way the repulsion energy varies is unknown, but it might be sensible to suppose that it varies as $1/r^n$, where n is some number not yet known. A value of n will be found in this stage.

Guess: energy due to repulsion $E_R = \dfrac{k}{r^n}$ where k and n are not known yet.

Answers

19 1.6×10^{-19} joule per ion pair (the difference).

Some mathematics

Arguments are given below to link the way the energy E_R and the force F_R due to repulsion vary with distance. Look at each, and try to follow the one that seems to you easiest. The first uses calculus, the second does not. For all arguments, the starting point is that if there is a force F, then if distance r changes by a small amount Δr, the energy transformed, ΔE, will be $F\Delta r$ in size.

Argument A

If $$E_R = \frac{k}{r^n}$$

then $$F_R = -\frac{dE_R}{dr} = \frac{nk}{r^{n+1}}.$$

Thus $$\frac{F_R}{E_R} = \frac{n}{r}.$$

Now go on to question 21.

Argument B

Suppose r is increased by one-thousandth of its original value, or 0.1 per cent.

Then new r = old $r \times 1.001$

or $$\frac{\text{change in } r}{\text{original } r} = 0.001.$$

If E varies as $1/r$, E will be multiplied by $\dfrac{1}{1.001}$.

If E varies as $1/r^2$, E will be multiplied by $\dfrac{1}{(1.001)^2}$.

If E varies as $1/r^n$, E will be multiplied by $\dfrac{1}{(1.001)^n}$.

The values in table 9, calculated to nine decimal places, show what happens, as n is altered.

n	$\frac{1}{(1.001)^n}$	**Approximately**
1	0.999 000 999	$1-0.001$
2	0.998 002 996	$1-0.002$
3	0.997 005 990	$1-0.003$
4	0.996 009 980	$1-0.004$
5	0.995 014 965	$1-0.005$
6	0.994 020 944	$1-0.006$
7	0.993 027 916	$1-0.007$
8	0.992 035 880	$1-0.008$
9	0.991 044 835	$1-0.009$

Table 9

By inspection, if r is raised by 0.001 of its value, $1/r^n$ is reduced by very nearly 0.001 n of its value. (If a factor 0.0001 were used, the approximation would be still better.)

As a general rule,

$$\frac{\text{change in } E_R}{E_R} = n\left(\frac{\text{change in } r}{r}\right) \text{ if } E_R \propto \frac{1}{r^n}$$

or, writing Δ for 'change in'

$$\frac{\Delta E_R}{E_R} = n\frac{\Delta r}{r}.$$

The force F_R also is connected with the change in energy ΔE_R. In fact (question 17), the force is the slope of the energy variation with distance.

$$\Delta E_R = F_R \, \Delta r.$$

Substituting for ΔE_R and cancelling Δr gives

$$\frac{F_R}{E_R} = \frac{n}{r}.$$

Now go on to question 21.

21 At $r = 2.8 \times 10^{-10}$ metre, what is E_R for NaCl?

22 At this distance, what is F_R for NaCl?

23 Use $\frac{F_R}{E_R} = \frac{n}{r}$, from the preceding mathematics, to find n to the nearest whole number.

Repulsion energy $1/r^9$ – new knowledge or not?

A guess that the repulsion energy varied as $1/r^n$ was made. If this guess is to fit the known facts, then n must be about 9. There is no proof that the energy *must* vary as $1/r^9$, only that if it is like $1/r^n$, then $n = 9$. (Other guesses might have been tried. A good one is that E_R varies like $\exp(-r/\rho)$, where ρ is a distance that can be found like n was. If you can differentiate an exponential, you can find ρ using the same data.)

From the equation $\frac{F_R}{E_R} = \frac{n}{r}$, it is clear that the less the repulsion energy E_R the bigger n will be. That is right because if the repulsive forces drop off sharply, they will contribute very little energy as the ions move appreciably apart, while the electrical forces are still quite big. Thus the electrical calculation of energy will be nearer and nearer to the observed energy as n gets bigger. The value $n = 9$ reflects the quite small discrepancy 1.6×10^{-19} joule in 12.7×10^{-19} joule.

Drawing the graph of repulsion energy and distance

So far, you know three things about the repulsion energy E_R.

24 Its value at 2.8×10^{-10} metre. What is it?

25 The *slope* of the curve of E_R with distance at 2.8×10^{-10} metre. What is the slope?

26 Thirdly, you know that E_R varies roughly as $1/r^9$ as r varies.
If r decreases from 2.8×10^{-10} metre, by a factor of 1.1 (to 2.54×10^{-10} metre), by what factor does E_R change?

27 If r decreases, does E_R increase or decrease?

28 $(1.1)^9$ is equal to 2.36. Thus when r drops by a factor 1.1 (from 2.8 to 2.54×10^{-10} metre), E_R increases by a factor 2.36.

$$E_R \text{ (at 2.54)} = 2.36 \times 1.6 \times 10^{-19} \text{ joule}$$
$$= 3.78 \times 10^{-19} \text{ joule.}$$

If r *increases* by a factor 1.1 from 2.8×10^{-10} metre to 3.08×10^{-10} metre, what does E_R become?

Table 10 shows values of E_R, varying as $1/r^9$, found in the way shown in questions 26 to 28.

Distance r /10^{-10} metre	3.36	3.08	2.8	2.54	2.34	2.16	2.0
Repulsion energy E_R/10^{-19} joule	0.31	0.68	1.6	3.78	8.25	17	33

Table 10

r was increased by factors 1.1, 1.2, and decreased by factors 1.1, 1.2, 1.3, 1.4. E_R was multiplied or divided by the factor to the ninth power.

Answers

21 $E_R = 1.6 \times 10^{-19}$ joule per pair of ions.
22 $F_R = 5.1 \times 10^{-9}$ newton.
23 $n = 9$.
24 1.6×10^{-19} joule per pair of ions.
25 Slope $= F_R = 5.1 \times 10^{-9}$ newton.
26 $(1.1)^9$.
27 Increases, $E_R \propto \dfrac{1}{r^9}$.
28 E_R becomes $\dfrac{1.6 \times 10^{-19}}{(1.1)^9} = 0.68 \times 10^{-19}$ joule per pair of ions.

29 On the graph in figure 59, plot out E_R against *r* in the space *above* the axis, using the same scales. How must the slope of this curve compare, at 2.8×10^{-10} metre, with the slope of the electrical energy curve?

30 The slope of the repulsion energy curve is a repulsive force, $+5.1 \times 10^{-9}$ newton at 2.8×10^{-10} metre. What is the slope of the electrical energy curve at this distance (the attractive electrical force)?

31 What is the net force on an ion at 2.8×10^{-10} metre?

The total energy needed to tear the crystal apart

If we add together the repulsion energy and the electrical energy, we shall have the total energy. For instance, at 2.8×10^{-10} metre,

$$\text{total energy} = 14.3 \times 10^{-19} - 1.6 \times 10^{-19}$$
$$= 12.7 \times 10^{-19} \text{ joule per pair.}$$

This is the measured energy, of course. The value 1.6×10^{-19} was obtained *from* the measured energy. But now other values can be put in on the graph. Where E_R is very small, the total energy graph follows the electrical energy graph closely.

32 Plot out the total energy curve on figure 59, passing through the 12.7×10^{-19} joule point marked. What shape has this curve?

33 Why is the slope of the total energy curve zero at 2.8×10^{-10} metre?

Answers

29 Same in size but opposite in direction.
30 -5.1×10^{-9} newton.
31 Zero. The two forces balance.
32 Bowl-shaped, with a minimum at 2.8×10^{-10} metre.
33 Because the total force on an ion is zero. Two opposite slopes of energy curves – repulsive and attractive forces – have been arranged to cancel.

Summary

The argument has come a long way. What has been done? To begin with, the idea was that electrical tear-apart energy, varying as $1/r$, might explain the measured tear-apart energy of NaCl. It nearly did, but not quite, and in any case some repulsive forces must be there as well to hold the ions at their known distance apart in the crystal.

The energy E_R transformed by these repulsive forces was found, as was the force itself. Guessing that if r were to change, E_R would vary as $1/r^n$, n was found to be nearly 9. The total picture now fits the facts about NaCl, with zero total force at $r = 2.8 \times 10^{-10}$ metre and minimum total energy equal to the measured value.

The question 'Why bother with all this?' is rather more penetrating than it may seem. All that has been done is to *make calculated and experimental results fit together*; no new information has emerged, except that they *can* be made to fit together.

The ninth power variation may seem new, but it too is the result of seeking to make a guess fit the facts. *If* the variation is anything like $1/r^n$ then n must be around 9. No one goes around claiming a new force law in physics, or a new kind of force. In fact, physicists have been able to think of several ways in which repulsive forces might arise (and of a few extra attractions too!) and we have shown that, all told, these forces vary in roughly the way we have found. The forces are basically electrical in origin, but arise from the way in which the electrons in an ion must be changed in distribution as the ions come together.

But something new *can* be made to come out of this kind of thinking, and can test the adequacy of the picture of NaCl developed so far. The forces between atoms help to decide many of the properties of the material they make up: its strength; elasticity; how much the atoms move if they vibrate, and how much the material expands if the atoms vibrate when it is hotter.

In stage four, just one of these properties will be predicted. It is this: how much force is needed to squeeze the crystal by a given amount?

You could stop here.

Stage four

A prediction of crystal strength

Electrical forces

To make the prediction of strength, it will be necessary to consider how the forces vary, rather than the energies, when r is altered by squeezing the crystal.

34 How does the electrical force F vary with distance?

35 The electrical force at 2.8×10^{-10} metre is 5.1×10^{-9} newton. What is the force at 1.0×10^{-10} metre?

36 What is the force at 2.0×10^{-10} metre?

Table 11 shows the electrical forces calculated at distances from 2.0 to 4.0×10^{-10} metre.

Distance $r/10^{-10}$ metre	2.0	2.5	2.8	3.0	4.0
Force F (electrical) $/10^{-9}$ newton	10	6.4	5.1	4.4	2.5

Table 11

The electrical force has already been plotted out on the lower part of the graph in figure 61.

Repulsive forces

Next the variation of repulsive forces as the crystal is squeezed needs consideration.

37 The energy stored by repulsion was found to vary as $1/r^9$. E_R is the energy; what is $-\dfrac{dE_R}{dr}$?

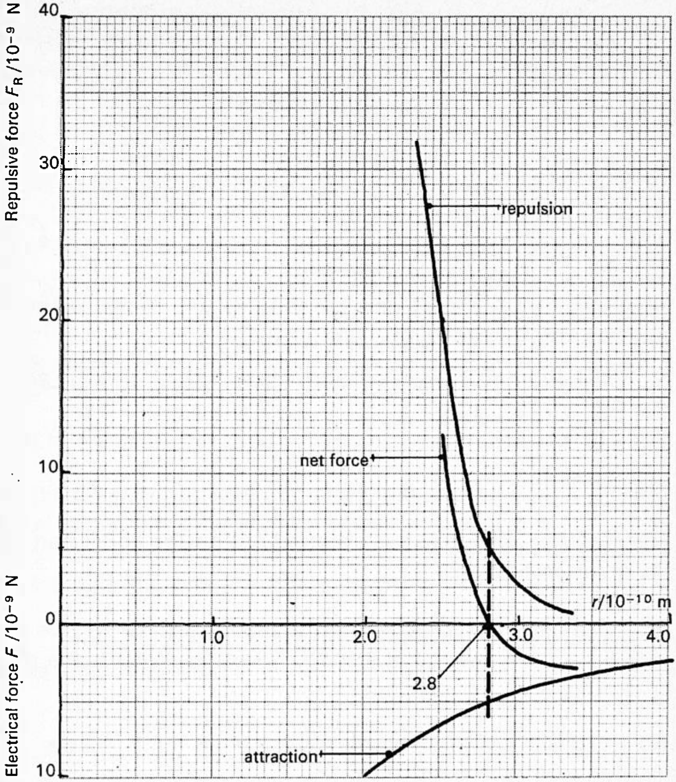


Figure 61

38 $\frac{d}{dr}(1/r^9) = -9/r^{10}$, so the force varies with distance as $1/r^{10}$. If r decreases by a factor 1.1, by what factor will F_R increase?

Answers

34 $F \propto 1/r^2$.
35 40×10^{-9} newton (multiply by 2.8^2).
36 10×10^{-9} newton.
37 The repulsive force F_R.
38 $(1.1)^{10}$.

Calculation of repulsive forces at various distances

$(1.1)^{10}$ is 2.59, so that

at $r = 2.54 \times 10^{-10}$ metre (that is $\dfrac{2.8 \times 10^{-10}}{1.1}$ metre),

$$F_R \text{ will be } (1.1)^{10} \times 5.1 \times 10^{-9} = 2.59 \times 5.1 \times 10^{-9}$$
$$= 13.2 \times 10^{-9} \text{ newton.}$$

Table 12 gives forces calculated from the value 5.1×10^{-9} newton at 2.8×10^{-10} metres, when r is increased and decreased by factors 1.1 and 1.2 so that F_R is decreased and increased by factors $(1.1)^{10}$ and $(1.2)^{10}$.

Distance $r/10^{-10}$ metre	3.36	3.08	2.8	2.54	2.34
Force (repulsive) $/10^{-9}$ newton	0.82	1.97	5.1	13.2	31.6

Table 12

The repulsive force has been plotted out on the upper part of the graph in figure 61.

39 The electrical force F, and the repulsive force F_R, are both 5.1×10^{-9} newton at 2.8×10^{-10} metre. What is the net or total force at this distance?

40 You will see that a third curve, the net force, passes through zero at 2.8×10^{-10} metre.

At 3.0×10^{-10} metre, the electrical force was plotted at 4.4×10^{-9} newton (table 11). Read off the value of the repulsive force at this distance. It should be about 2.5×10^{-9} newton.

What should the net force be, at 3.0×10^{-10} metre? Check that the curve is correct at this point.

41 Why does the net force curve rise above the axis when r is less than 2.8×10^{-10} metre?

Compressing the NaCl crystal

Suppose that a piece of crystal were squeezed in all directions, so that the spacing became, say, 1 per cent less than 2.8×10^{-10} m. See figure 62.

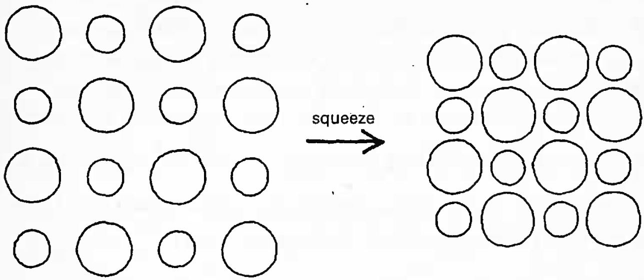


Figure 62

As the ions are squeezed together, the net force per pair rises from zero. Draw a tangent to the net force curve at 2.8×10^{-10} metre on figure 61. It will be possible then to find how much the force rises for a 1 per cent decrease in r.

42 Draw the tangent right back to the force axis. Where does it cut the axis?

43 The tangent, rising at the rate the force rises at 2.8×10^{-10} metre, increases by 42×10^{-9} newton when r decreases by 100 per cent (to zero). How much does the force rise if r decreases by 1 per cent?

44 As r decreases by 1 per cent the net force rises from zero to 0.42×10^{-9} newton. What is the average force over this change of distance?

Answers

39 Zero.
40 The difference between 4.4 and 2.5×10^{-9} newton.
41 The repulsive force is now larger than the attractive force.
42 At about 42×10^{-9} newton.
43 One per cent of $42 \times 10^{-9} = 0.42 \times 10^{-9}$ newton.
44 0.21×10^{-9} newton.

45 An average force, per pair of ions, of 0.21×10^{-9} newton is exerted as the distance is reduced by $\frac{2.8}{100} \times 10^{-10}$ metre.

How much energy, per pair of ions, is transformed?

46 Imagine a cubical block of crystal, 1 metre each way, being squeezed so that each side changes length by 1 per cent, as we assumed. If the ions are distance r apart, each ion occupies a volume of about r^3 (Na^+ ions less, Cl^- ions more). Thus a metre cube of crystal contains $\frac{1}{r^3}$ ions, which with $r = 2.8 \times 10^{-10}$ metre, comes to 4.56×10^{28} ions. How many *pairs* of ions are there in a metre cube?

47 After a 1 per cent squeeze, each pair has stored energy equal to 0.59×10^{-21} joule (see question 45).

How much energy is stored in a metre cube by a 1 per cent squeeze?

48 This energy came from the forces squeezing the block from outside. See figure 63. If the block is squeezed on all sides by a pressure that rises from 0 to p newtons per square metre, the *force* exerted on each face also rises from 0 to p newtons, for a metre cube.

What is the average force acting on each face of the cube, if the force on each face rises from 0 to p?

49 How much energy does each force transform, as it squeezes the block face a distance of 1/100 metre (1 per cent)?

There are three forces, in each of three perpendicular directions, each transforming energy $\frac{p}{2} \times \frac{1}{100}$. Thus the total energy transformed by the external forces is $3 \times \frac{p}{2} \times \frac{1}{100}$.

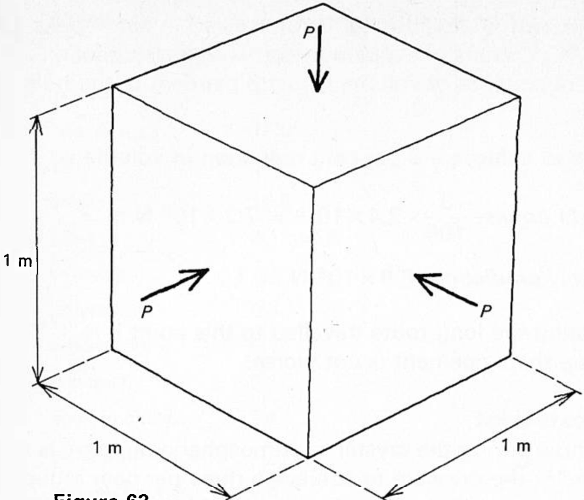


Figure 63

50 This energy is the same as that stored in the bonds between ions, if the crystal does not get hotter. But that (question 47) was 1.35×10^7 joules. What is p?

51 If this pressure squeezes the sides of the cube by 1 per cent, by what percentage does it change the volume?

Thus a pressure of 8.8×10^8 newtons per square metre should compress the volume of a block of NaCl by 3 per cent.

Answers

45 $(0.21 \times 10^{-9}) \times (\frac{2.8}{100} \times 10^{-10}) = 0.59 \times 10^{-21}$ joule per pair of ions.
46 2.28×10^{28} pairs.
47 1.35×10^7 joules.
48 $p/2$.
49 $\frac{p}{2} \times \frac{1}{100}$ joule.
50 $p = 8.8 \times 10^8$ newtons on each square metre.

$$p = \left(\frac{2 \times 100}{3}\right)(1.35 \times 10^7).$$

51 Three per cent. Recall the discussion of small changes in stage three. Volume $= d^3$ and the percentage volume change is 3 times the percentage change in d.

The '*bulk elastic modulus*' of NaCl is listed in data books as 2.4×10^{10} newtons per square metre. The pressure for a 3 per cent squeeze of volume is just 3 per cent of the bulk modulus.

Pressure to achieve a 3 per cent reduction in volume is:

from data books: $\frac{3}{100} \times 2.4 \times 10^{10} = 7.2 \times 10^{8}\ \mathrm{N\ m^{-2}}$;

theoretical prediction: $8.8 \times 10^{8}\ \mathrm{N\ m^{-2}}$.

Considering the long route travelled to this point it is surprising the agreement is not worse.

Postscript

Notice how strong the crystal is. Atmospheric pressure is only $10^{5}\ \mathrm{N\ m^{-2}}$; the pressure to achieve a three per cent reduction in volume of a sodium chloride crystal is 10^{4} times greater than atmospheric pressure.

The calculations, tough and long though they are, really only apply to one very special kind of solid: an ionic crystal. However, the energies (of the order 10^{-18} J) and the force variations (of the order $10^{2}\ \mathrm{N\ m^{-1}}$) are about right for many other sorts of interatomic bonds, though not for all. Table 13 gives values of the bulk modulus for a selection of materials. The pressure for a one per cent reduction in volume is one per cent of the value given.

Material	Bulk modulus/10^{10} N m^{-2}
aluminium	7.5
copper	13.8
cast iron	11.0
tin	5.8
steel	16.8
glass	4.1 to 5.8
Perspex	0.6
polystyrene	0.4
rubber (pure)	0.1
concrete	2
sodium chloride	2.4
potassium chloride	1.7
sodium fluoride	4.6

Table 13

Leaving out rubber and plastics, which have long chain molecules, the values for the metals, ceramics, and ionic crystals given fall within one order of magnitude. The range for solids whose strength comes from balanced atoms or ions in a rigid structure is quite small. The same kind of calculation as for ionic crystals, if it can be done, and if it can it is usually much harder, seems as if it would give broadly the same kind of answers. Physicists now have a glimmering of an idea why when you pick up one end of a ruler the other end follows; a problem Rutherford said was a first class mystery.

This is where we stop.

Physicists go on, if they can, studying other materials, trying to base the guesses about forces more firmly on what is known about how atoms behave when they are squashed, and trying to understand other behaviour, like the expansion of solids when they are heated and the atoms vibrate more. There are plenty of such problems where no one has yet got very far.

Thunderstorms

A large thunderstorm is an impressive and frightening affair, and it is no wonder that men have thought of thunderstorms as expressing the anger of the gods.

This article is about how large-scale electrical activity is generated in the atmosphere, and about some of the consequences of thunderstorm activity, including the fact that the Earth as a whole has an electric charge.

Figure 64
Photograph, R. F. Addie.

Formation of thunderclouds

The formation of thunderclouds, or indeed any cloud, is due to warm moist air rising into colder regions of the atmosphere. Air near the ground is warm because the ground is heated by the sun. This expanded, low density, warm air rises, and cools. If the air is dry the atmosphere settles down to a steady condition, with a definite temperature drop as one goes higher. But if the rising air is moist it cools less than rising dry air and remains warmer and so less dense than the surrounding air. It thus keeps on rising. This rising warm air is replaced by a descending column of cold air.

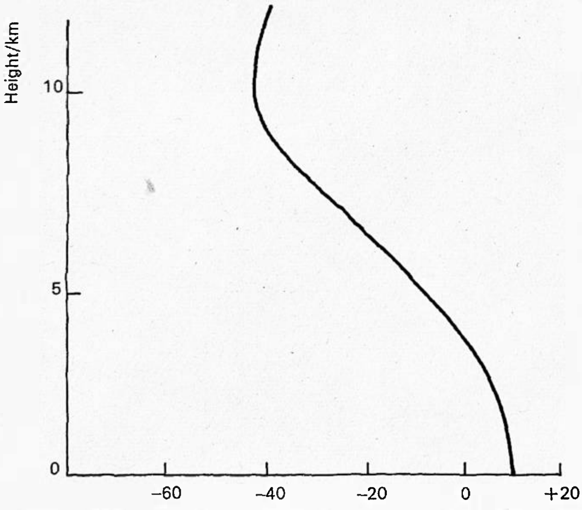


Figure 65
Steady conditions with dry air.

The rising moist air cools and vapour begins to condense. This appears as clouds, seen at this stage as a patchwork pattern of small cumulus clouds. A cloud appears above each rising air column. If the amount of water vapour is high enough the clouds grow and spread into each other. This

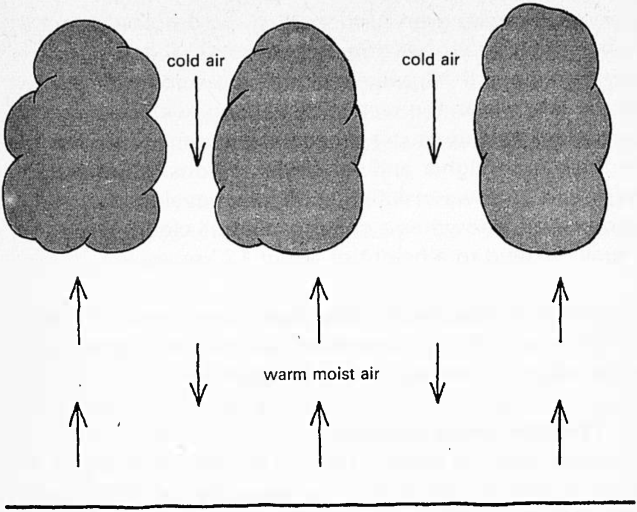


Figure 66
Convection currents in the atmosphere.

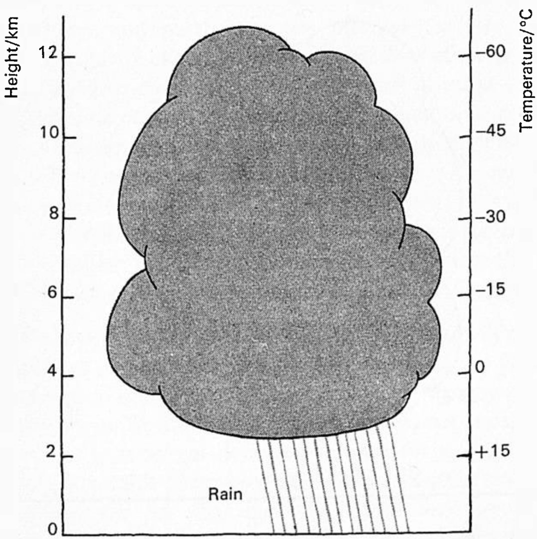


Figure 67
Typical thundercloud.

larger cloud grows even further when the droplets of water in its upper layers start freezing. If water at 0° C is to freeze into ice at 0 °C, quite a significant amount of energy must be taken from the water. This energy warms the surrounding air and causes a decrease in density so that the cloud rises into higher and yet colder regions. More water freezes and it grows even larger. A fully developed thundercloud, known as a cumulo-nimbus cloud, will generally extend to a height of about 12 km.

Thunderclouds can also be produced as the result of a slab of cold air (a 'cold front') meeting a slab of warm moist air. The encounter forces the warm air upwards.

Thundercloud charges

A thundercloud has regions that are electrically charged. The charge is generally positive in the upper part of the cloud and negative in the lower part. In addition there is often a smaller positive charge region near the base of the cloud. Some

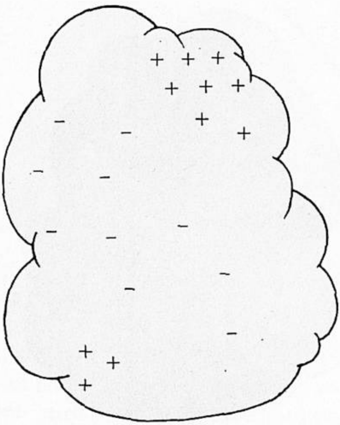


Figure 68
Charge distribution in thundercloud.

evidence for this charge distribution can be got from a study of the lightning flashes within and between clouds, as the flashes always occur between oppositely charged parts of clouds. Other evidence is based on the results of experiments with aircraft or balloons in thunderstorm clouds.

How are these charges produced? No one is too sure, as is shown in the following extract from 'Thunderstorm theories' by J. A. Chalmers.

(The author has explained before this extract starts that many theories account for the low-lying negative charge by supposing that it is large heavy objects that become negatively charged in a cloud, while light ones become positive. What then are these objects?)

Thunderstorm theories

One theory, put forward by Workman and Reynolds, is based on observations of charging when small super-cooled water droplets impinged on a suspended hailstone; it was found that the droplet partly freezes and partly splashes off and that the hailstone acquires a negative charge, while the fragments of water splashing off carry positive charges. It is very difficult to make laboratory conditions close enough to conditions in a cloud to be certain that the same phenomena would occur. A serious difficulty in this theory is that the process would occur only when the temperature is sufficiently close to 0 °C for the droplet to freeze only partially; at lower temperatures, there would be complete freezing. And, recently, Latham and Mason have found that, when this process occurs, the water splashing off carries a *negative* charge.

When it was first discovered that the main separation of charge in a thundercloud occurs where the precipitation must be in the form of ice, Simpson and Scrase suggested that the agency generating the charge might be friction of ice particles against one another. It was very difficult to get definite evidence for this process, but eventually Reynolds discovered that when two pieces of ice come into contact, the one that had the higher conductivity gets a negative charge and the other a positive; the mechanism suggested is a transfer of protons, which are the particles responsible for the larger part of the flow of current in ice. Reynolds found that the higher conductivity could be produced either by higher temperature or by the presence of impurities. Reynolds then suggested that, in the region of the thundercloud where there is a separation of charge, there exist together solid precipitation particles, solid cloud particles, and super-cooled water droplets. When the super-cooled water droplets impinge on the precipitation particles, latent heat is released on freezing, and the precipitation particles are now warmer than the surroundings; thus when the solid cloud particles bounce off the precipitation particles the separation of charge gives a negative charge to the precipitation particles and a positive charge to the smaller cloud particles. Latham and Mason have been able to measure this charging and find that, though it exists and has a magnitude corresponding to theoretical predictions, it is not enough, by a factor of 10^4, to account for the amount of charge in a thundercloud.

But Latham and Mason have found another effect which is important. When a super-cooled water droplet freezes suddenly by impact on a precipitation particle, it freezes first on the outside and then, when the inner liquid freezes, it bursts the skin (as water freezing bursts a pipe) and sends out splinters. Latham and Mason found that these splinters carry away positive charge, leaving the residue, on the precipitation particle, carrying a negative charge; as the ice freezes, the outer part becomes colder than the inner and so gets a positive charge, which is thrown off in the bursting. Measurements have been made of the amount of charge separated for each super-cooled droplet and the calculations show that the effect is just about enough to account for the quantities of charge produced in a thundercloud.

(Chalmers, J. A., 1961, 'Thunderstorm theories', Physics Bulletin **12**.

Figure 69
Cumulo-nimbus clouds.
Photograph, R. K. Pilsbury.

Figure 70
Cloud to cloud lightning.
Photograph, Dr D. Winstanley.

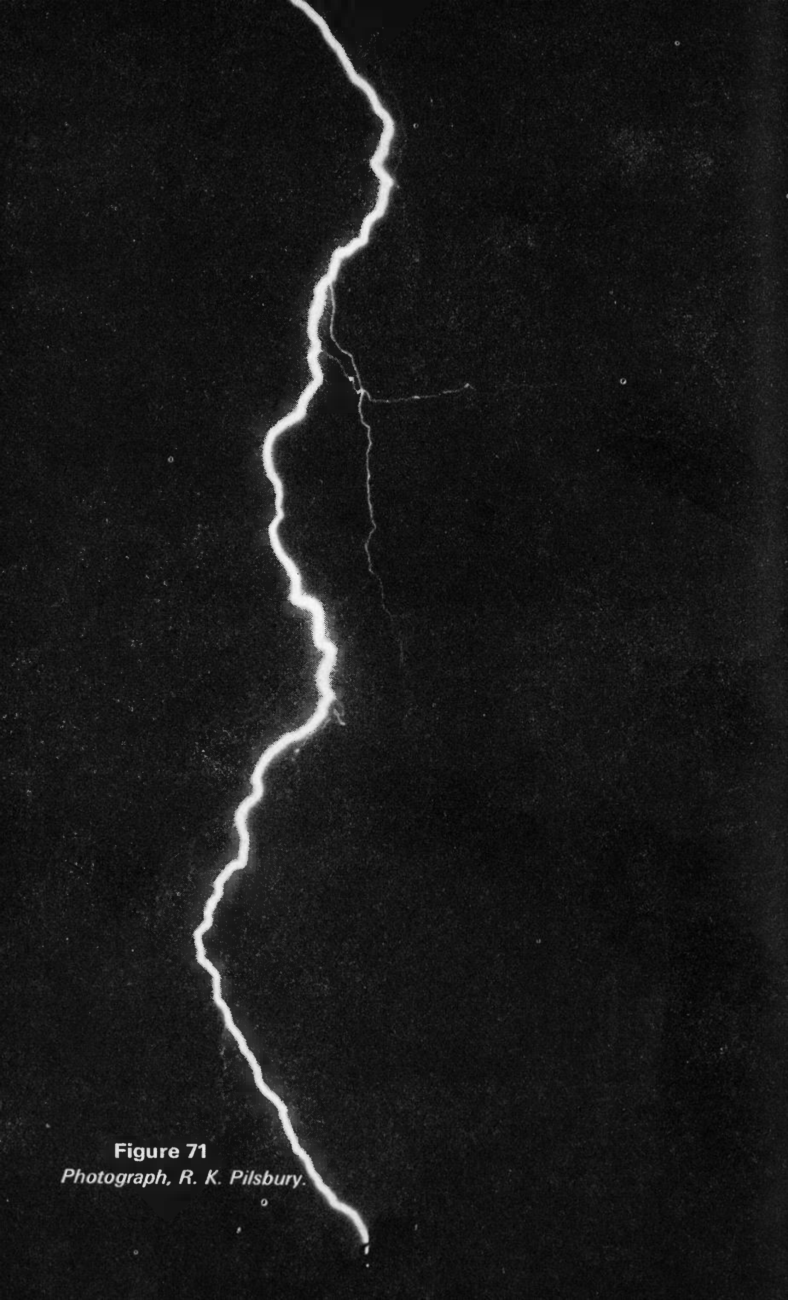


Figure 71
Photograph, R. K. Pilsbury.

Lightning

Most lightning flashes never touch the ground. Most of them occur inside a thundercloud or between thunderclouds. Lightning, either between clouds or to the ground, generally follows a tortuous path with one main channel and several branches. The flash is not usually a simple one, but generally consists of many strokes following each other in rapid succession, every few hundredths of a second.

The first stroke to the ground is followed by a return stroke from the ground to the cloud along nearly the same path. The sequence is generally repeated a number of times and as many as 42 strokes have been recorded for a single lightning flash.

The air near the path of the lightning discharge becomes heated to quite a high temperature, of the order of 15000 to 20000 °C. The rapid expansion of this hot air is the explosion we call a thunderclap. The lightning flashes occupy only a few hundredths of a second. Why then does the thunder rumble on for many seconds? The sound is produced all the way along the flash path and some of the sound has further to travel from more distant parts of the flash path to the observer. The sound is still further prolonged by reflections from regions in the air of varying temperature or moisture content.

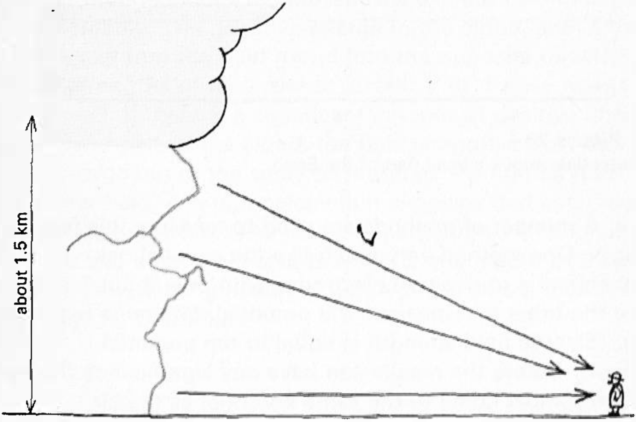


Figure 72

Different sound paths (speed of sound about 340 m s^{-1}; speed of light 3×10^8 m s^{-1}).

The electric field in the atmosphere

In fine weather there is an electric field near the Earth's surface of about 130 V m^{-1}, the Earth being negative and the upper atmosphère being positive. Can this be true? Is the air near your head 200–300 V above the air near your feet? Because your body is a relatively good conductor both you and the ground will tend to be at the same potential, and the equipotentials in the air around you will be distorted; your head will therefore not be at a potential of 200–300 V higher than your feet. On rare occasions people have touched disconnected, insulated radio aerials on roofs and received a

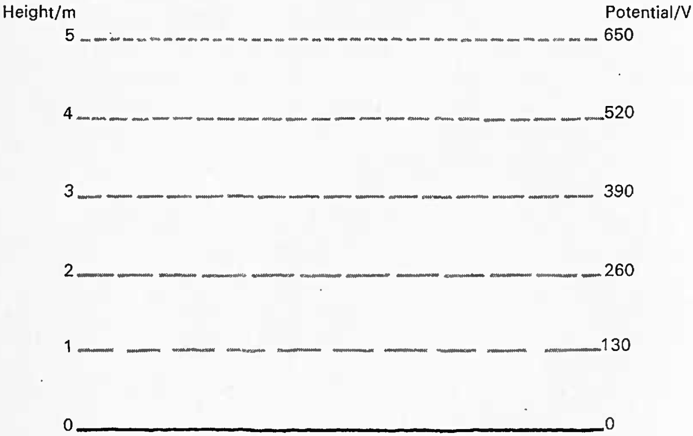


Figure 73
Equipotentials above a level part of the Earth.

shock. A number of methods are used to measure this field strength. One method very much like the aerial shock arrangement is to use two electrodes with one about 1 metre above the other and measure the potential difference between them. (Electric field strength is equal to the potential gradient.) Before the results can have any significance the two electrodes must come to the same potential as the air.

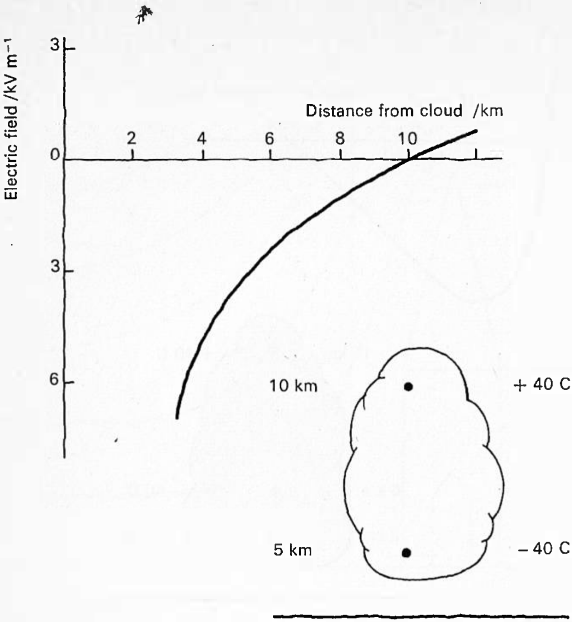


Figure 74

When thunderclouds are present the electric field changes considerably. The negative charge in the cloud, above the Earth, can turn the field round into the opposite direction. Field strengths of the order of 20 000 V m^{-1} have been observed. If there is a significant amount of positive charge in the lower part of the cloud, the field strength below this can be as large but in the same direction as the normal fine weather field. A rough calculation suggests that such fields could be produced by a positive charge of 40 C at a height of 10 km and a negative charge of 40 C at a height of 5 km. If there is a low lying positive charge in the cloud, then it might be, say, 10 C at 2 km.

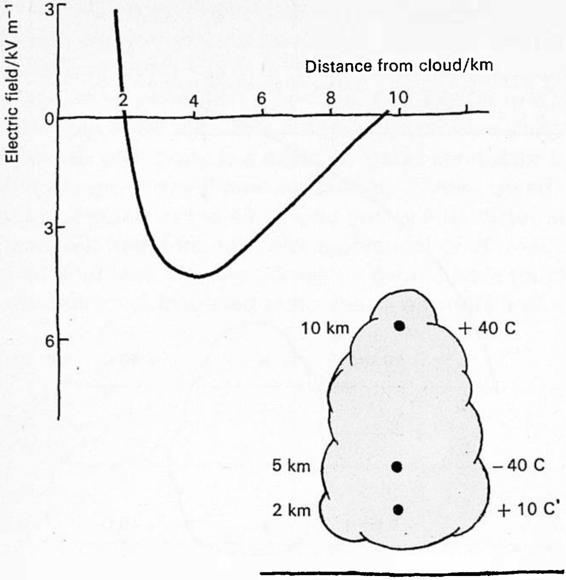


Figure 75

Why does lightning tend to strike tall trees and buildings? They are good conductors, compared with air, and the top of a tree is at nearly the same potential as the ground it is connected to. But nearby air is at a much higher potential, especially in the strong field below a cloud. So the p.d. between tree top and nearby air can be very big. If it is big enough, the field will tear electrons off gas atoms, *and ionize* the air, which then conducts better and provides an easy path for the lightning flash. It is not possible to calculate just what effect a complicated object like a tree will have, and so experiments have been done to find out.

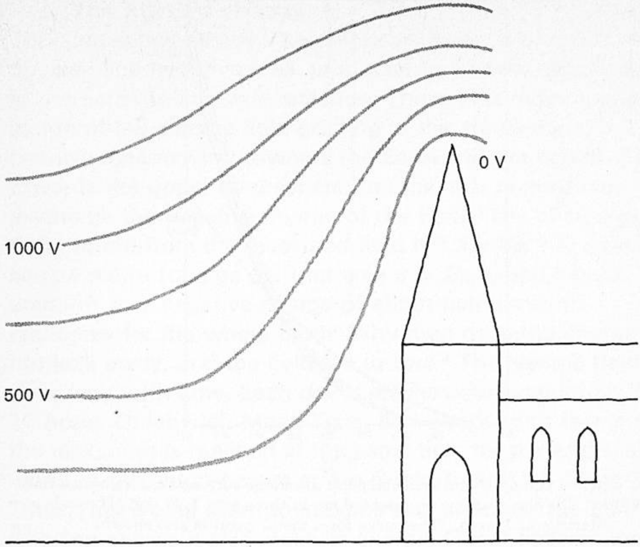


Figure 76
Equipotentials.

The following account is taken from 'Point-discharge currents through small trees in artificial fields', by D. S. Jhawar and J. A. Chalmers.

Measurements were made with a small spruce tree *(Picea pungens)* of height about 60 cm, grown in a plant pot mounted on insulators and arranged to have its tip midway between two metal plates across which a voltage up to 50 kV could be applied. The current through the tree could be measured by a galvanometer connected between the pot and earth.

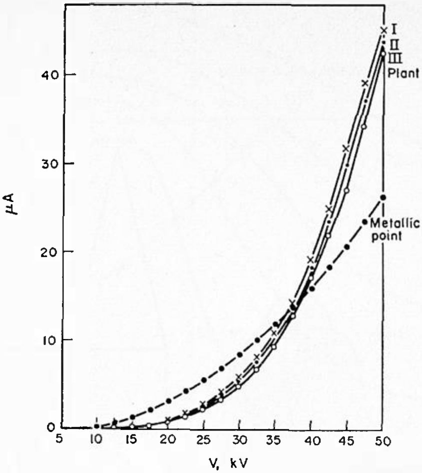


Fig. 2. Applied voltage V and artificial point discharge current I through the small spruce tree. The observations represented are in three sets, I, II and III, made at intervals of 1 week. The curve for a metal point is also shown.

Figure 77

3 Results of measurements

Figure 2 [77] shows the results of measurements of point-discharge currents through the spruce tree for different applied voltages on the plate. After changing the applied voltage, it was found that the current took several minutes to reach a new steady value and this was allowed for. The three curves represent readings on three different days, when the tree was under somewhat different conditions of moisture, temperature etc.

For comparison, the results are also given for the current through a single steel needle of diameter 0.005 cm, with its point replacing the top of the spruce tree.

(Jhawar, D. S., and Chalmers, J. A., 1967, 'Point-discharge currents through small trees in artificial fields.' J. Atmospheric and Terrestrial Physics, **29**, 1459.)

The Earth's charge

The atmosphere is not a complete insulator. It contains positive and negative ions, produced by natural radioactivity in the Earth and cosmic radiation. These ions move under the action of the electric field existing in the atmosphere, the positive ions moving towards the Earth and the negative ions towards the upper atmosphere. In time, this current must discharge the negative charge of the Earth. The charge on the Earth found from the measured field is 1.1×10^{-9} C on each square metre (charge per unit area $\sigma = E\varepsilon_0$), and hence amounts to a negative charge of about half a million coulombs for the whole Earth. Why then does the charge not leak away, and the field fall to zero? The electric field does vary with time. Each day it reaches a maximum at 19 hours Greenwich Mean Time, and a surprising fact is that the maximum is reached at the same time for the entire world. A minimum occurs at 4 o'clock Greenwich Mean Time. This would seem to indicate that although the Earth is losing charge it also receives charge. Direct observations and satellite observations indicate that the thunderstorm

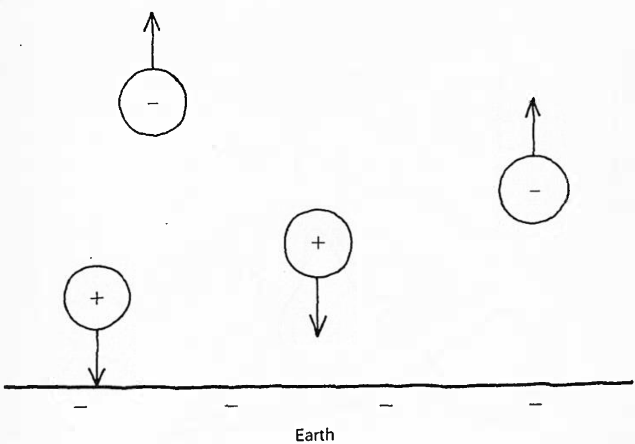


Figure 78
Fine weather movement of ions in the atmosphere.

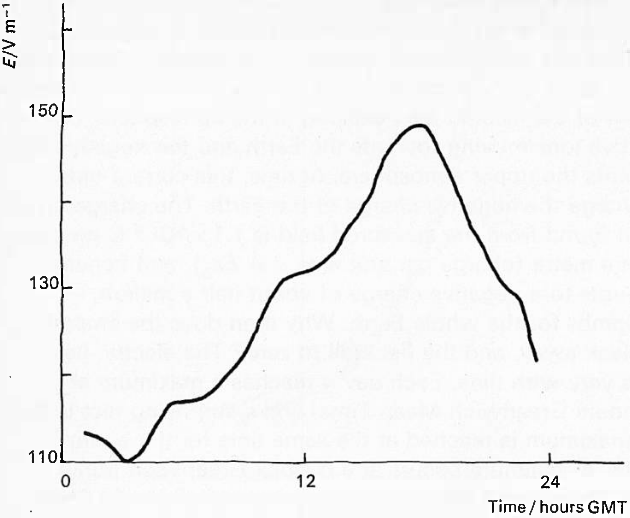


Figure 79
Field strength over oceans.

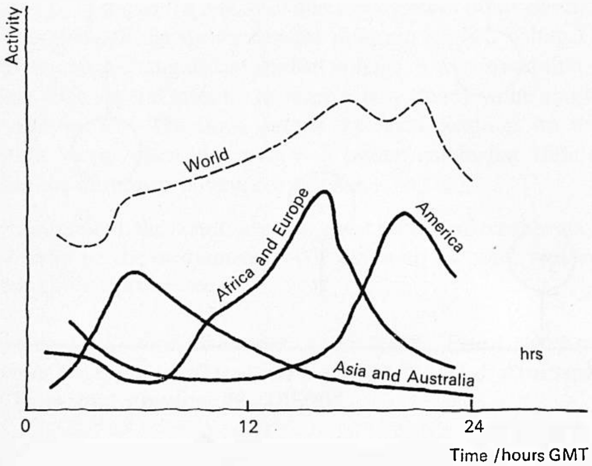


Figure 80
Thunderstorm activity.

activity over the Earth reaches a maximum at 19 hours Greenwich Mean Time. Thunderstorms would seem to be the mechanism by which the earth gains negative charge to replace the loss by discharge.

The current flowing through the atmosphere can be determined from a measurement of the conductivity and the electric field. Consider a one metre cube of air. Such a cube has a resistance of about $43 \times 10^{12}\ \Omega$ between two opposite faces. The p.d. from top to bottom is equal to 130 V (if the field is 130 V m^{-1}). The current crossing a square metre is thus given by:

$$\text{current density} = \frac{V}{R} = \frac{130}{43 \times 10^{12}} \approx 3 \times 10^{-12}\ \mathrm{A\ m^{-2}}.$$

The area of the Earth is 5×10^{14} square metres so the total discharge current is $3 \times 10^{-12} \times 5 \times 10^{14} \approx 1500$ A.

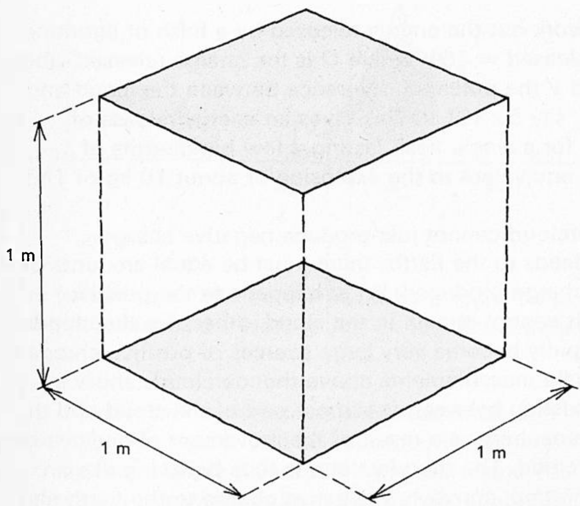


Figure 81

This would discharge the half a million coulomb charge of the Earth rather soon, in about 5 minutes.

So something has to replace the charge continually, and we have seen some evidence that thunderclouds are the electrical machine that does it.

We can work out how much thunder and lightning are needed. A typical lightning flash transfers about 20 coulombs to the Earth. To replace the half a million coulombs every 5 minutes would require $\frac{0.5 \times 10^6}{20}$ flashes. This is about 25 000 flashes every 5 minutes or 85 flashes per second. This is for the entire world.

How often do you see lightning flashes? If we assume you can see an area of about 2 km × 2 km and the flashes are equally spread over the entire world, then you should see one flash about every 20 days. This would be equivalent to a ten-flash storm once or twice a year.

We can work out the energy released by a flash of lightning. Energy released $= \frac{1}{2}QV$ where Q is the charge released, about 20 C, and V the potential difference between the cloud and the Earth, say 5×10^7 V. This gives an energy release of 5×10^8 J for a single flash lasting a few hundredths of a second – equivalent to the explosion of about 10 kg of TNT.

A thundercloud cannot just produce negative charge which it feeds to the Earth; there must be equal amounts of positive charge produced. What happens to the positive charge? It cannot remain in the cloud, otherwise the clouds would rapidly become very large sources of positive charge. Electric field measurements above thunderclouds show a current existing between the upper part of the cloud and the upper atmosphere as a result of the movement of positive ions from the cloud. The thundercloud is thus behaving like an electrical pump, conveying negative charge to the Earth and positive charge to the upper atmosphere. The effect balances out the negative charge lost from the Earth to the upper atmosphere in fine weather.

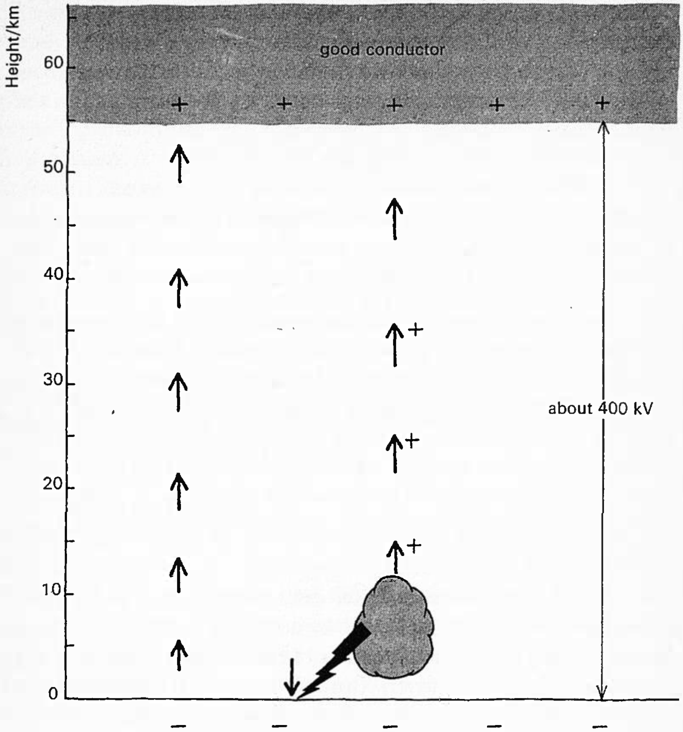


Figure 82

The following extract from a paper by Stergis, Rein, and Kangas indicates the work going on to substantiate this theory.

Electric field measurements above thunderstorms

Abstract – The electric field and conductivity of the atmosphere were measured above thunderstorms in the central Florida area in a series of twenty-five balloon flights made at altitudes ranging from 70000 to 90000 ft above sea level. These tests indicate that at these altitudes and for the storms investigated there is a positive current of approximately 1.3 A, on the average, flowing upward from the top of the thunderstorm towards the ionosphere and confirm Wilson's hypothesis that thunderstorms are the generators which supply the electric current necessary for maintaining the Earth's negative charge.

2. *Experimental method*

The electric current flowing above thunderstorms was determined by measuring the conductivity and the electric field, since the product of these two quantities gives the conduction current.

The conductivity chamber consists essentially of two concentric cylinders across which a given voltage is applied. A fan attached at one end of this chamber draws air through the chamber at a rate of about 3000 ft/min. As the air passes through the chamber, it gives up a fraction of its electric charge to the inner electrode which is connected to a d.c. electrometer-amplifier. This in turn drives a standard radiosonde transmitter operating at 1680 Mc/s. The signals from this transmitter are received at the ground and automatically recorded.

The electric field meter consists essentially of two radioactive probes (polonium 210) separated by a distance of 1 metre. These probes quickly attain the potentials of their surroundings, and the potential difference between them is measured by a cathode-follower type of electrometer circuit. The electrometer drives a standard radiosonde transmitter which sends the desired information to a ground receiver.

The conductivity chamber or the electric field meter is transported into the stratosphere by means of a polyethylene balloon at an average rate of 1000 ft/min. To minimize the electrostatic effects of the balloon on the measurements, the instrument is suspended 200–300 ft below the balloon.

The balloon is launched on the east coast of Florida in advance of forecasted thunderstorm formation. The balloon ascends to a height of 70000 to 90000 ft, depending on the size of the balloon and on the load. It then floats toward the west coast of Florida, since the upper air winds in Florida during the summer months are invariably from east to west. The magnitude of the wind in this region of the stratosphere is from 35 to 55 m.p.h. In its passage across Florida, the balloon usually encounters one or more thunderstorms (assuming a correct forecast was made in the first place). The electrical measurements above thunderstorms are made by the instrument suspended below the balloon and the information is telemetered to the ground. When the balloon arrives at the west coast of Florida, it is cut down automatically and the instrument descends by parachute.

[*Note:* 70000 to 90000 ft is about 21 to 27 km, 3000 ft min^{-1} is about 15 m s^{-1}, Mc/s is MHz, 1000 ft min^{-1} is about 5 m s^{-1}, 200–300 ft is about 60 to 90 m, 35 to 55 mph is about 56 to 88 km h^{-1}.]

(Stergis, C. G., Rein, G. C., and Kangas, T., 1957, 'Electric field measurements above thunderstorms', J. Atmospheric and Terrestrial Physics, **11**, *83.)*

What next?

A great deal needs to be done before the processes involved in the production of a thundercloud, the effects produced by the charges in the cloud, and the lightning can be fully explained. What is the mechanism of charge production in a cloud? What happens during the lightning discharge? One of the difficulties of research in this field is the size and complexity of a thundercloud. Aircraft and balloons can only take readings for a small section of a cloud and laboratory simulation is difficult. Thunderclouds are more than just a means of producing a flash of light; their production and effects are problems spread over a large area of science.

Feynman on gravity

The following extract is taken from *The Feynman lectures on physics.*

What is gravity?

But is this such a simple law? What about the machinery of it? All we have done is to describe *how* the Earth moves around the Sun, but we have not said *what makes it go.* Newton made no hypotheses about this; he was satisfied to find *what* it did without going into the machinery of it. *No one has since given any machinery.* It is characteristic of the physical laws that they have this abstract character. The law of conservation of energy is a theorem concerning quantities that have to be calculated and added together, with no mention of the machinery, and likewise the great laws of mechanics are quantitative mathematical laws for which no machinery is available. Why can we use mathematics to describe nature without a mechanism behind it? No one knows. We have to keep going because we find out more that way.

Many mechanisms for gravitation have been suggested. It is interesting to consider one of these, which many people have thought of from time to time. At first, one is quite excited and happy when he 'discovers' it, but he soon finds that it is not correct. It was first discovered about 1750. Suppose there were many particles moving in space at a very high speed in all directions and being only slightly absorbed in going through matter. When they *are* absorbed, they give an impulse to the Earth. However, since there are as many going one way as another, the impulses all balance. But when the Sun is nearby, the particles coming toward the Earth through the Sun are partially absorbed, so fewer of them are coming from the Sun than are coming from the other side. Therefore, the Earth feels a net impulse toward the Sun and it does not take one long to see that it is inversely as the square of the distance – because of the variation of the solid angle that the sun subtends as we vary the distance. What is wrong with that machinery? It involves some new consequences which are *not true.* This particular idea has the following trouble: the Earth, in moving around the Sun, would impinge on more particles which are coming from its forward side than from its hind side (when you run in the rain, the rain in your face is stronger than that on the back of your head!). Therefore there would be more impulse given the Earth from the front, and the Earth would feel a *resistance to motion* and would be slowing up in its orbit. One can calculate how long it would take for the Earth to stop as a result of the resistance, and it would not take long enough for the Earth to still be in its orbit, so this mechanism does not work. No machinery has ever been invented that 'explains' gravity without also predicting some other phenomenon that does *not* exist.

(Feynman, R. P., Leighton, R. B., and Sands, M., 1963,
The Feynman lectures on physics, *Volume I, Addison-Wesley; Chapter 7.)*

Books for Unit 3

Textbooks

Arons, A. B. (1965) *Development of concepts of physics.* Addison-Wesley. (Chapters 7, 10–15.)

Bennet, G. A. G. (1968) *Electricity and modern physics.* MKS version. Arnold. (Chapters 10, 11.)

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Further reading

Bondi, H. (1961) *The Universe at large.* Heinemann.

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Project Physics (1971) Text, Unit 2 *Motion in the heavens.* Holt, Rinehat, and Winston Inc. (Chapter 8.)

Project Physics (1971) Reader, Unit 2 *Motion in the heavens.* Holt, Rinehat, and Winston Inc. (Chapter 12, 23, 24.)

Project Physics (1971) Reader, Unit 4 *Light and electromagnetism.* Holt, Rinehart, and Winston Inc. (Chapters 5, 6, 8, 10, 18.)

Rogers, E. M. (to be published in 1972) Nuffield O-level Physics Pupils' Guide. *Astronomy.* Longman/Penguin.

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Ryan, P. (1969) *The invasion of the Moon 1969.* Penguin.

Articles

Felici, N. J. (1965) 'Electrostatic engineering.' (From *Science journal*; out of print, but due to be re-published as part of the Nuffield Advanced Physics publications in 1972 in *Physics and the engineer*, a collection of *Science journal* reprints.)

Gamow, G. (1961) 'Gravity.' *Scientific American* Offprint No. 273. W. H. Freeman.

Formulae, data, and circuit symbols

Formulae	Formula	Unit
Capacitors in parallel	$C = C_1 + C_2 + \ldots$	F
Capacitors in series	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$	F
Capacitance of large parallel plate capacitor, area A, spacing d	$C = \frac{\varepsilon_0 A}{d}$	F
Force on charge q in field E	$F = qE$ (defines E)	N
Electric field between parallel plates with charge density σ	$E = \frac{\sigma}{\varepsilon_0}$	$V\ m^{-1}$ or $N\ C^{-1}$
Electric field at distance r from a point charge q	$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$	$V\ m^{-1}$ or $N\ C^{-1}$
Force between two point charges q_1, q_2, distance r apart	$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r^2}$	N
Field E where potential changes by ΔV over distance Δx	$E = -\frac{\Delta V}{\Delta x}$ along Δx	$V\ m^{-1}$ or $N\ C^{-1}$
Electric potential due to point charge q at distance r	$V = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r}$	V
Electric potential energy of point charges q_1, q_2, distance r apart	$\text{energy} = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$	J
Gravitational force between point masses m_1, m_2, distance r apart	$F = \frac{G m_1 m_2}{r}$	N
Gravitational field of point mass M at distance r	$g = \frac{GM}{r^2}$	$N\ kg^{-1}$ or $m\ s^{-2}$
Gravitational field of large sphere total mass M, at distance r	$g = \frac{GM}{r^2}$ if $r >$ radius of sphere	$N\ kg^{-1}$ or $m\ s^{-2}$
Gravitational potential of point mass M at distance r	$\text{gravitational potential} = -\frac{GM}{r}$	$J\ kg^{-1}$
Gravitational field where potential changes over distance Δx	$g \text{ along } \Delta x = -\frac{\text{(change of gravitational potential)}}{\Delta x}$	$N\ kg^{-1}$ or $m\ s^{-2}$

Data

Name	Symbol	Value	Unit
Charge on electron	e	1.6×10^{-19}	C
One electronvolt	1 eV	1.6×10^{-19}	C
Electric field constant (Permittivity of a vacuum)	ε_0	8.854×10^{-12}	$C^2\ N^{-1}\ m^{-2}$ or $F\ m^{-1}$
Coulomb force constant	$\frac{1}{4\pi\varepsilon_0}$	nearly 9×10^9	$N\ m^2\ C^{-2}$
Gravitational force constant	G	6.67×10^{-11}	$N\ m^2\ kg^{-2}$
Gravitational field at surface of Earth (acceleration of free fall)	g	9.8	$N\ kg^{-1}$ or $m\ s^{-2}$
GM for Earth		4.0×10^{14}	$N\ m^2\ kg^{-1}$
Mass of Earth		5.98×10^{24}	kg
Mass of Moon		7.34×10^{22}	kg
Mass of Sun		1.98×10^{30}	kg
Radius of Earth		6.38×10^6	m
Radius of Moon		1.74×10^6	m
Radius of Moon's orbit round Earth		3.8×10^8	m
Radius of Earth's orbit round Sun		1.5×10^{11}	m
Sodium chloride			
Energy needed to turn 1 mole of crystalline NaCl into a gas of ions		77×10^4	$J\ mol^{-1}$
Energy as above, per pair of ions		12.7×10^{-19}	*J per ion pair*
Equilibrium distance of nearest neighbour ions in crystalline NaCl		2.8×10^{-10}	m

Symbols for circuit diagrams

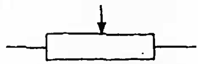
Some of the symbols for circuit diagrams used in this book are shown below. They follow British Standard 3939, *Graphical symbols for electric power, telecommunications and electronics diagrams* (1966–70).

Resistor general symbol

variable resistor

resistor with preset adjustment

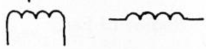
resistor with moving contact



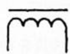
Capacitor general symbol

polarized electrolytic capacitor

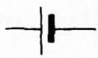
Inductor general symbol



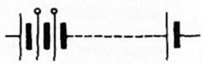
inductor with core



Battery primary or secondary cell



battery with tappings



pn diode

Transistor (npn)

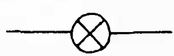
Measuring instruments

voltmeter

ammeter

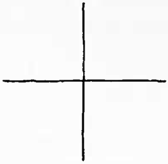
galvanometer

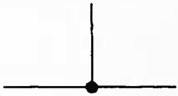
Signal lamp

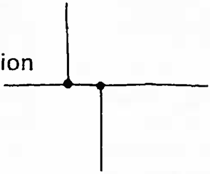
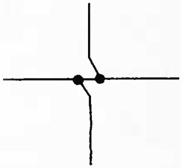


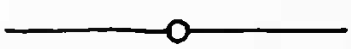
Lamp for illumination 

Wires, junctions, terminals

crossing of wires,
no electrical contact 

junction 

double junction  

terminal 

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Where significant information is contained in an illustration or diagram, the page reference is italicized.

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This *Students' book* contains a summary of Unit 3, *Field and potential*, and questions on its main work. The Unit is divided into four Parts: 'The uniform electric field', 'Gravitational field and potential', 'The electrical inverse square law', and 'Ionic crystals'. The book also includes answers to the questions, chapters on 'Thunderstorms' and 'Feynman on gravity', a list of background reading, and lists of formulae, data, and circuit symbols.

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