PROJECT PHYSICS

Text and Handbook 3

The Triumph of Mechanics

PROJECT PHYSICS

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PROJECT PHYSICS

Unit 3 Text and Handbook

The Triumph of Mechanics

A partial list of staff and consultants to Harvard Project Physics appears in the Text and Handbook, Unit 1.

This Text and Handbook, Unit 3 is one of the many instructional materials developed for the Project Physics Course. These materials include Text, Handbook, Resource Book, Readers, Programmed Instruction booklets, Film Loops, Transparencies, 16mm films, and laboratory equipment.

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Science is an adventure of the whole human race to learn to live in and perhaps to love the universe in which they are. To be a part of it is to understand, to understand oneself, to begin to feel that there is a capacity within man far beyond what he felt he had, of an infinite extension of human possibilities . . . .

I propose that science be taught at whatever level, from the lowest to the highest, in the humanistic way. It should be taught with a certain historical understanding, with a certain philosophical understanding, with a social understanding and a human understanding in the sense of the biography, the nature of the people who made this construction, the triumphs, the trials, the tribulations.

I. I. RABI
Nobel Laureate in Physics

Preface

Background The Project Physics Course is based on the ideas and research of a national curriculum development project that worked in three phases. First, the authors—a high school physics instructor, a university physicist, and a professor of science education—collaborated to lay out the main goals and topics of a new introductory physics course. They worked together from 1962 to 1964 with financial support from the Carnegie Corporation of New York, and the first version of the text was tried out in two schools with encouraging results.

These preliminary results led to the second phase of the Project when a series of major grants were obtained from the U.S. Office of Education and the National Science Foundation, starting in 1964. Invaluable additional financial support was also provided by the Ford Foundation, the Alfred P. Sloan Foundation, the Carnegie Corporation, and Harvard University. A large number of collaborators were brought together from all parts of the nation, and the group worked together for over four years under the title *Harvard Project Physics*. At the Project's center, located at Harvard University, Cambridge, Massachusetts, the staff and consultants included college and high school physics instructors, astronomers, chemists, historians and philosophers of science, science educators, psychologists, evaluation specialists, engineers, film makers, artists and graphic designers. The instructors serving as field consultants and the students in the trial classes were also of vital importance to the success of Harvard Project Physics. As each successive experimental version of the course was developed, it was tried out in schools throughout the United States and Canada. The instructors and students in those schools reported their criticisms and suggestions to the staff in Cambridge, and these reports became the basis for the subsequent revisions of the course materials. In the Preface to the *Text* you will find a list of the major aims of the course.

We wish it were possible to list in detail the contributions of each person who participated in some part of Harvard Project Physics. Unhappily it is not feasible, since most staff members worked on a variety of materials and had multiple responsibilities. Furthermore, every text chapter, experiment, piece of apparatus, film or other item in the experimental program benefitted from the contributions of a great many people. Beginning on page A21 of the *Text* Appendix is a partial list of contributors to Harvard Project Physics. There were, in fact, many other contributors too numerous to mention. These include school administrators in participating schools, directors and staff members of training institutes for teachers, instructors who tried the course after the evaluation year, and most of all the thousands of students who not only agreed to take the experimental version of the course, but who were also willing to appraise it critically and contribute their opinions and suggestions.

The Project Physics Course Today. Using the last of the experimental versions of the course developed by Harvard Project Physics in 1964–68 as a starting point, and taking into account the evaluation results from the tryouts, the three original collaborators set out to develop the version suitable for large-scale publication. We take particular pleasure in acknowledging the assistance of Dr. Andrew Ahlgren of the University of Minnesota. Dr. Ahlgren was invaluable because of his skill as a physics instructor, his editorial talent, his versatility and energy, and above all, his commitment to the goals of Harvard Project Physics.

We would also especially like to thank Ms. Joan Laws whose administrative skills, dependability, and thoughtfulness contributed so much to our work. The publisher, Holt, Rinehart and Winston, Inc. of New York, provided the coordination, editorial support, and general backing necessary to the large undertaking of preparing the final version of all components of the Project Physics Course, including texts, laboratory apparatus, films, etc. Damon-Educational Division, a company located in Westwood, Massachusetts, worked closely with us to improve the engineering design of the laboratory apparatus and to see that it was properly integrated into the program.

In the years ahead, the learning materials of the Project Physics Course will be revised as often as is necessary to remove remaining ambiguities, clarify instructions, and to continue to make the materials more interesting and relevant to students. We therefore urge all students and instructors who use this course to send to us (in care of Holt, Rinehart and Winston, Inc., 383 Madison Avenue, New York, New York 10017) any criticism or suggestions they may have.

F. James Rutherford
Gerald Holton
Fletcher G. Watson

Answers to End-of-Section Questions

Chapter 9

Q1 False
Q2 No. Don't confuse mass with volume or mass with weight.
Q3 Answer C
Q4 No. Change speed to velocity and perform additions by vector techniques.
Q5 (a), (c) and (d) (Their momenta before collision are equal in magnitude and opposite in direction.)
Q6 Least momentum: a pitched baseball (small mass and fairly small speed)
Greatest momentum: a jet plane in flight (very large mass and high speed)
Q7 (a) about 4 cm/sec. Faster ball delivers more momentum to girl.
(b) about 4 cm/sec. More massive ball delivers more momentum to girl.
(c) about 1 cm/sec. With same gain in momentum more massive girl gains less speed.
(d) about 4 cm/sec. Momentum change of ball is greater if its direction reverses.
(These answers assume the mass of the ball is much less than the mass of the girl.)
Q8 It can be applied to situations where only masses and speeds can be determined.
Q9 Conservation of mass: No substances are added or allowed to escape.
Conservation of momentum: No net force from outside the system acts upon any body considered to be part of the system.
Q10 None of these is an isolated system. In cases (a) and (b) the earth exerts a net force on the system. In case (c) the sun exerts a net force on the system.
Q11 Answer (c) (Perfectly elastic collisions can only occur between atoms or subatomic particles.)
Q12 Answer (d) (This assumes mass is always positive.)
Q13 Answer (c)
Q14 (a) It becomes stored as the object rises.
(b) It becomes "dissipated among the small parts" which form the earth and the object.

Chapter 10

Q1 Answer (b)
Q2 Answer (b)
Q3 Answer (c)
Q4 Answer (c) The increase in potential energy equals the work done on the spring.
Q5 Answer (e) You must do work on the objects to push them closer together.
Q6 Answer (e) Kinetic energy increases as gravitational potential energy decreases. Their sum remains the same (if air resistance is negligible).
Q7 Potential energy is greatest at extreme position where the speed of the string is zero. Kinetic energy is greatest at midpoint where the string is unstretched.
Q8 The less massive treble string will gain more speed although both gain the same amount of kinetic energy (equal to elastic potential energy given by guitarist).
Q9 Multiply the weight of the boulder (estimated from density and volume) by the distance above ground level that it seems to be. (For further discussion see SG 10.15.)
Q10 None. Centripetal force is directed inward along the radius which is always perpendicular to the direction of motion for a circular orbit.
Q11 Same, if initial and final positions are identical.
Q12 Same, if frictional forces are negligible. Less if frictional forces between skis and snow are taken into account.
Q13 Answer (c)
Q14 Answer (c)
Q15 False. It was the other way around.
Q16 Chemical, heat, kinetic or mechanical
Q17 Answer (b)
Q18 Answer (d)
Q19 It is a unit of power, or rate of doing work, equal to 746 watts.
Q20 Answer (d)
Q21 Answer (b)
Q22 Nearly all. A small amount was transformed into kinetic energy of the slowly descending weights and the water container would also have been warmed.
Q23 Answer (a)
Q24 Answer (e)
Q25 The statement means that the energy which the lion obtains from eating comes ultimately from sunlight. He eats animals, which eat plants which grow by absorbed sunlight.
Q26 Answer (c)
Q27 Answer (a)
Q28 Answer (c)
Q29 Answer (c)
Q30 ΔE is the change in the total energy of the system
ΔW is the net work (the work done on the system — the work done by the system)
ΔH is the net heat exchange (heat added to the system — heat lost by the system)
Q31 1. heating (or cooling) it
2. doing work on it (or allowing it to do work)

Chapter 11

Q1 Answer (c)
Q2 True
Q3 False
Q4 Answer (b)
Q5 In gases the molecules are far enough apart that the rather complicated intermolecular forces can safely be neglected.
Q6 Answer (b)
Q7 Answer (b)
Q8 Answer (d)
Q9 Answer (c)

(continued on p. A1)

Project Physics

Text 3

The Triumph of Mechanics

Things to Do and Use

Experiments

3-1 Collisions in One Dimension I
3-2 Collisions in One Dimension II
3-3 Collisions in Two Dimensions I
3-4 Collisons in Two Dimensions II
3-5 Conservation of Energy I
3-6 Conservation of Energy II
3-7 Measuring the Speed of a Bullet
3-8 Energy Analysis of a Pendulum Swing
3-9 Least Energy
3-10 Temperature and Thermometers
3-11 Calorimetry
3-12 Ice Calorimetry
3-13 Monte Carlo Experiment on Molecular Collisions
3-14 Behavior of Gases
3-15 Wave Properties
3-16 Waves in a Ripple Tank
3-17 Measuring Wavelength
3-18 Sound
3-19 Ultrasound

Activities

Is Mass Conserved?
Exchange of Momentum Devices
Student Horsepower
Steam-powered Boat
Problems of Scientific and Technological Growth
Predicting the Range of an Arrow
Drinking Duck
Mechanical Equivalent of Heat
A Diver in a Bottle
Rockets
How to Weigh a Car with a Tire Pressure Gauge
Perpetual Motion Machines?
Standing Waves on a Drum and Violin
Moire Patterns
Music and Speech Activities
Measurement of the Speed of Sound
Mechanical Wave Machines
Resource Letter

Film Loops

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L19 One-Dimensional Collisions II
L20 Inelastic One-Dimensional Collisions
L21 Two-Dimensional Collisions I
L22 Two-Dimensional Collisions II
L23 Inelastic Two-Dimensional Collisions
L24 Scattering of a Cluster of Objects
L25 Explosion of a Cluster of Objects
L26 Finding the Speed of a Rifle Bullet I
L27 Finding the Speed of a Rifle Bullet II
L28 Recoil
L29 Colliding Freight Cars
L30 Dynamics of a Billiard Ball
L31 A Method of Measuring Enegy—Nails Driven into Wood
L32 Gravitational Potential Energy
L33 Kinetic Energy
L34 Conservation of Energy—Pole Vault
L35 Conversation of Energy—Aircraft Take-off
L36 Reversibility of Time
L37 Superposition
L38 Standing Waves on a String
L39 Standing Waves in a Gas
L40 Vibrations of a Wire
L41 Vibrations of a Rubber Hose
L42 Vibrations of a Drum
L43 Vibrations of a Metal Plate

Programmed Instruction Booklets

The Kinetic-Molecular Theory of Gases
Waves 1 The Superposition Principle
Waves 2 Periodic Waves

Reader Articles

1 *Silence, Please*
by Arthur C. Clarke
2 *The Steam Engine Comes of Age*
by R. J. Forbes and E. J. Dijksterhuis
3 *The Great Conservation Principles*
by Richard P. Feynman
4 *The Barometer Story*
by Alexander Calandra
5 *The Great Molecular Theory of Gases*
by Eric M. Rogers
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by Kenneth W. Ford
7 *The Law of Disorder*
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9 *The Arrow of Time*
by Jacob Bronowski
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by Leo L. Beranek
12 *Randomness and the Twentieth Century*
by Alfred M. Bork
13 *Waves*
by Richard Stevenson and R. B. Moore
14 *What is a Wave?*
by Albert Einstein and Leopold Infeld
15 *Musical Instruments and Scales*
by Harvey E. White
16 *Founding a Family of Fiddles*
by Carleen M. Hutchins
17 *The Seven Images of Science*
by Gerald Holton
18 *Scientific Cranks*
by Martin Gardner
19 *Physics and the Vertical Jump*
by Elmer L. Offenbacher

Transparencies

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T21 Unequal Mass Two-Dimensional Collisions
T22 Inelastic Two-Dimensional Collisions
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T26 Square Wave Analysis
T27 Standing Waves
T28 Two-Slit Interference
T29 Interference Pattern Analysis

Contents TEXT, UNIT 3


MUNDI
SYSTEMATE
LIBER TERTIUS

UNIT 3

The Triumph of Mechanics

CHAPTERS

PROLOGUE The success of Isaac Newton in uniting the studies of astronomy and of terrestrial motion is one of the glories of the human mind. It was a turning point in the development of science and humanity. Never before had a scientific theory been so successful in finding simple order in observable events. Never before had the possibilities for using one's rational faculties for solving any kind of problem seemed so promising. So it is not surprising that after his death in 1727 Newton was looked upon almost as a god, especially in England. Many poems like this one appeared:

Newton the unparalled'd, whose Name
No Time will wear out of the Book of Fame,
Celestial Science has promoted more,
Than all the Sages that have shone before.
Nature compell'd his piercing Mind obeys,
And gladly shows him all her secret Ways;
'Gainst Mathematics she has no defence,
And yields t' experimental Consequence;
His tow'ring Genius, from its certain Cause
Ev'ry Appearance *a priori* draws
And shews th' Almighty Architect's unalter'd Laws.

(From J. T. Desaguliers, *The Newtonian System of the World the Best Model of Government, an Allegorical Poem.*)

Newton's success in mechanics altered profoundly the way in which scientists viewed the universe. Physicists after Newton explained the motion of the planets around the sun by treating the solar system as a huge machine. Its "parts" were held together by gravitational forces rather than by nuts and bolts. But the motions of these parts relative to each other, according to Newton's theory, were determined once and for all after the system had first been put together.

We call this model of the solar system the *Newtonian world-machine.* As is true for any model, certain things are left out. The mathematical equations which govern the motions of the model cover only the main properties of the

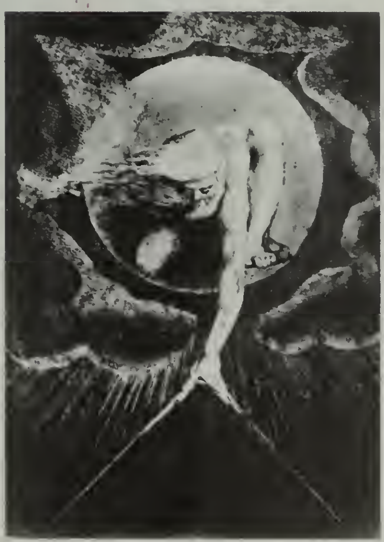
Beginning of Book 3, "The Systems of the World," in the 1713 edition of Newton's *Principia.*

real solar system. The masses, positions and velocities of the parts of the system, and the gravitational forces among them are well described. But the Newtonian model neglects the internal structure and chemical composition of the planets, heat, light, and electric and magnetic forces. Nevertheless, it serves splendidly to deal with observed motions. Moreover, it turned out that Newton's approach to science and many of his concepts became useful later in the study of those aspects he had to leave aside.

The idea of a world machine does not trace back only to Newton's work. In his *Principles of Philosophy* (1644), Rene Descartes, the most influential French philosopher of the seventeenth century, had written:

> I do not recognize any difference between the machines that artisans make and the different bodies that nature alone composes, unless it be that the effects of the machines depend only upon the adjustment of certain tubes or springs, or other instruments, that, having necessarily some proportion with the hands of those who make them, are always so large that their shapes and motions can be seen, while the tubes and springs that cause the effects of natural bodies are ordinarily too small to be perceived by our senses. And it is certain that all the laws of Mechanics belong to Physics, so that all the things that are artificial, are at the same time natural.

Robert Boyle (1627-1691), a British scientist, is known particularly for his studies of the properties of air. (See Chapter 11.) Boyle, a pious man, expressed the "mechanistic" viewpoint even in his religious writings. He argued that a God who could design a universe that ran by itself like a machine was more wonderful than a God who simply created several different kinds of matter and gave each a natural tendency to behave as it does. Boyle also thought it was insulting to God to believe that the world machine would be so badly designed as to require any further divine adjustment once it had been created. He suggested that an engineer's skill in designing "an elaborate engine" is more deserving of praise if the engine never needs supervision or repair. "Just so," he continued,



"The Ancient of Days" by William Blake, an English poet who had little sympathy with the Newtonian style of "natural philosophy."

> . . . it more sets off the wisdom of God in the fabric of the universe, that he can make so vast a machine perform all those many things, which he designed it should, by the meer contrivance of brute matter managed by certain laws of local motion, and upheld by his ordinary and general concourse, than if he employed from time to time an intelligent overseer, such as nature is fancied to be, to regulate, assist, and controul the motions of the parts. . . .

Boyle and many other scientists in the seventeenth and eighteenth centuries tended to think of God as a supreme engineer and physicist. God had set down the laws of matter and motion. Human scientists could best glorify the Creator by discovering and proclaiming these laws.

Our main concern in this unit is with physics as it developed after Newton. In mechanics, Newton's theory was extended to cover a wide range of phenomena, and new concepts were introduced. The conservation laws to be discussed in Chapters 9 and 10 became increasingly important. These powerful principles offered a new way of thinking about mechanics. They opened up new areas to the study of physics—for example, heat and wave motion.

Newtonian mechanics treated directly only a small range of experiences. It dealt with the motion of simple bodies, or those largely isolated from others as are planets, projectiles, or sliding discs. Do the same laws work when applied to complex phenomena? Do real solids, liquids, and gases behave like machines or mechanical systems? Can their behavior be explained by using the same ideas about matter and motion that Newton used to explain the solar system?

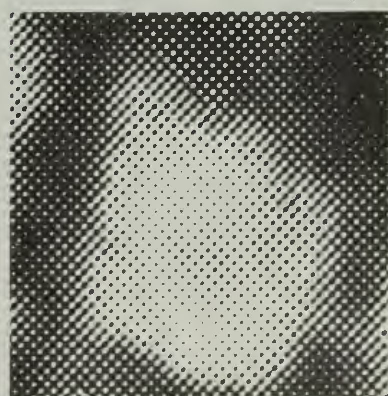
At first, it might seem unlikely that everything can be reduced to matter and motion, the principles of mechanics. What about temperature, colors, sounds, odors, hardness, and so forth? Newton himself believed that the mechanical view would essentially show how to investigate these and all other properties. In the preface to the *Principia* he wrote:

> I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are mutually impelled towards one another, and cohere according to regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of Philosophy.

Scientists after Newton strove to understand nature in many different areas, "by the same kind of reasoning from mechanical principles." We will see in this unit how wide was the success of Newtonian mechanics—but you will see also some evidence of limits to its applicability.

Ironically, Newton himself explicitly rejected the deterministic aspects of the "World-Machine" which his followers had popularized.

A small area from the center of the picture has been enlarged to show what the picture is "really" like. Is the picture only a collection of dots? Knowing the underlying structure doesn't spoil our other reactions to the picture, but rather gives us another dimension of understanding it.



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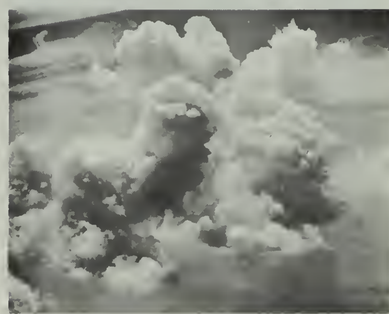
CHAPTER NINE

Conservation of Mass and Momentum

9.1 Conservation of mass

The idea that despite ever-present, obvious change all around us the total amount of material in the universe does not change is really very old. The Roman poet Lucretius restated (in the first century B.C.) a belief held in Greece as early as the fifth century B.C.:

> . . . and no force can change the sum of things; for there is no thing outside, either into which any kind of matter can emerge out of the universe or out of which a new supply can arise and burst into the universe and change all the nature of things and alter their motions. [*On the Nature of Things*]



SG 9.1

Just twenty-four years before Newton's birth, the English philosopher Francis Bacon included the following among his basic principles of modern science in *Novum Organum* (1620):

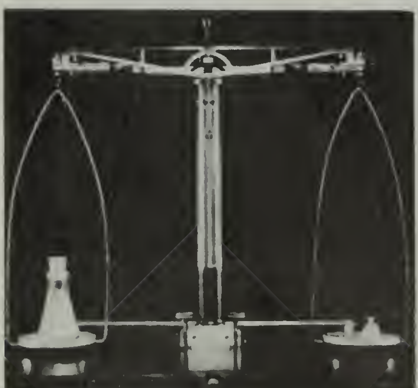
> There is nothing more true in nature than the twin propositions that "nothing is produced from nothing" and "nothing is reduced to nothing" . . . the sum total of matter remains unchanged, without increase or diminution.

This view agrees with everyday observation to some extent. While the form in which matter exists may change, in much of our ordinary experience matter appears somehow indestructible. For example, we may see a large boulder crushed to pebbles, and not feel that the amount of matter in the universe has diminished or increased. But what if an object is burned to ashes or dissolved in acid? Does the amount of matter remain unchanged even in such chemical reactions? Or what of large-scale changes such as the forming of rain clouds or of seasonal variations?

In some open-air chemical reactions, the mass of objects seems to decrease, while in others it seems to increase.

Note the closed flask shown in the portrait on p. 7.

SG 9.2



Conservation of mass was demonstrated in experiments on chemical reactions in closed flasks.

The meaning of the phrase "closed system" will be discussed in more detail in Sec. 9.5.

To test whether the total quantity of matter actually remains constant, we must know how to measure that quantity. Clearly it cannot simply be measured by its volume. For example, we might put water in a container, mark the water level, and then freeze the water. If so, we find that the volume of the ice is larger than the volume of the water we started with. This is true even if we carefully seal the container so that no water can possibly come in from the outside. Similarly, suppose we compress some gas in a closed container. The volume of the gas decreases even though no gas escapes from the container.

Following Newton, we regard the *mass* of an object as the proper measure of the amount of matter it contains. In all our examples in Units 1 and 2, we assumed that the mass of a given object does not change. But a burnt match has a smaller mass than an unburnt one; and an iron nail increases in mass as it rusts. Scientists had long assumed that something escapes from the match into the atmosphere, and that something is added from the surroundings to the iron of the nail. Therefore nothing is really "lost" or "created" in these changes. But not until the end of the eighteenth century was sound experimental evidence for this assumption provided. The French chemist Antoine Lavoisier produced this evidence.

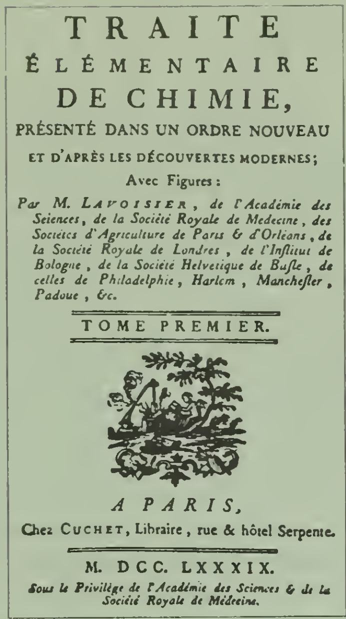
Lavoisier caused chemical reactions to occur in *closed* flasks. He carefully weighed the flasks and their contents before and after the reaction. For example, he burned iron in a closed flask. The mass of the iron oxide produced equalled the sum of the masses of the iron and oxygen used in the reaction. With experimental evidence like this at hand, he could announce with confidence in *Traité Élémentaire de Chimie* (1789):

> We may lay it down as an incontestable axiom that in all the operations of art and nature, nothing is created; an equal quantity of matter exists both before and after the experiment, . . . and nothing takes place beyond changes and modifications in the combinations of these elements. Upon this principle, the whole art of performing chemical experiments depends.

Lavoisier knew that if he put some material in a well-sealed bottle and measured its mass, he could return at any later time and find the same mass. It would not matter what had happened to the material inside the bottle. It might change from solid to liquid or liquid to gas, change color or consistency, or even undergo violent chemical reactions. But at least one thing would remain unchanged—the *total* mass of all the different materials in the bottle.

In the years after Lavoisier's pioneering work, a vast number of similar experiments were performed with ever increasing accuracy. The result was always the same. As far as we now can measure with sensitive balances (having a precision of better than 0.000001%), mass is *conserved*—that is, it remains constant—in chemical reactions.

To sum up: despite changes in location, shape, chemical composition and so forth, *the mass of any closed system remains constant.* This is the statement of what we will call the *law of conservation of mass.* This law is basic to both physics and chemistry.



TRAITE
ÉLÉMENTAIRE
DE CHIMIE,
PRÉSENTÉ DANS UN ORDRE NOUVEAU
ET D'APRÈS LES DÉCOUVERTES MODERNES;
Avec Figures :

Par M. LAVOISIER, de l'Académie des Sciences, de la Société Royale de Medecine, des Sociétés d'Agriculture de Paris & d'Orléans, de la Société Royale de Londres, de l'Institut de Bologne, de la Société Helvetique de Basle, de celles de Philadelphie, Harlem, Manchester, Padoue, &c.

TOME PREMIER.

A PARIS,
Chez CUCHET, Libraire, rue & hôtel Serpente.

M. DCC. LXXXIX.

Sous le Privilège de l'Académie des Sciences & de la Société Royale de Médecine.

Antoine Laurent Lavoisier (1743-1794) is known as the "father of modern chemistry" because he showed the decisive importance of quantitative measurements, confirmed the principle of conservation of mass in chemical reactions, and helped develop the present system of nomenclature for the chemical elements. He also showed that organic processes such as digestion and respiration are similar to burning.

To earn money for his scientific research, Lavoisier invested in a private company which collected taxes for the French government. Because the tax collectors were allowed to keep any extra tax which they could collect from the public, they became one of the most hated groups in France. Lavoisier was not directly engaged in tax collecting, but he had married the daughter of an important executive of the company, and his association with the company was one of the reasons why Lavoisier was guillotined during the French Revolution.

Also shown in the elegant portrait by David is Madame Lavoisier. She had been only fourteen at the time of her marriage. Intelligent as well as beautiful, she assisted her husband by taking data, translating scientific works from English into French, and making illustrations. About ten years after her husband's execution, she married another scientist, Count Rumford, who is remembered for his experiments which cast doubt on the caloric theory of heat.

"The change in the total mass is zero" can be expressed symbolically as $\Delta \Sigma m = 0$ where Σ represents the sum of the masses of m in all parts of the system.

Obviously, one must know whether a given system is closed or not before applying this law to it. For example, it is perhaps surprising that the earth itself is not exactly a closed system within which all mass would be conserved. Rather, the earth, including its atmosphere gains and loses matter constantly. The most important addition occurs in the form of dust particles. These particles are detected by their impacts on satellites that are outside most of the atmosphere. Also, they create light and ionization when they pass through the atmosphere and are slowed down by it. The number of such particles is larger for those particles which are of smaller size. The great majority are very thin particles on the order of 10^{-4} cm diameter. Such small particles cannot be individually detected from the ground when they enter the atmosphere. They are far too small to appear as meteorites, which result when particles at least several millimeters in diameter vaporize. The total estimated inflow of mass of all these particles, large and small, is about 10^5g/sec over the whole surface of the earth. (Note: the mass of the earth is about 6×10^{27}g.) This gain is not balanced by any loss of dust or larger particles, not counting an occasional spacecraft and its debris. The earth also collects some of the hot gas evaporating from the sun, but this amount is comparatively small.

The earth does lose mass by evaporation of molecules from the top of the atmosphere. The rate of this evaporation depends on how many molecules are near enough to the top of the atmosphere to escape without colliding with other molecules. Also, such molecules must have velocities high enough to escape the earth's gravitational pull. The velocities of the molecules are determined by the temperature of the upper atmosphere. Therefore the rate of evaporation depends greatly on this temperature. At present the rate is probably less than 5×10^3g/sec over the whole earth. This loss is very small compared with the addition of dust. (No water molecules are likely to be lost directly by atmospheric "evaporation;" they would first have to be dissociated into hydrogen and oxygen molecules.)

Try these end-of-section questions before going on.

Q1 *True or false:* Mass is conserved in a closed system only if there is no chemical reaction in the system.

Q2 If 50 cm³ of alcohol is mixed with 50 cm³ of water, the mixture amounts to only 98 cm³. An instrument pack on the moon weighs much less than on earth. Are these examples of contradictions with the law of conservation of mass?

SG 9.3–9.7

Q3 Which one of the following statements is true?

(a) Lavoisier was the first person to believe that the amount of material stuff in the universe did not change.

(b) Mass is measurably increased when heat enters a system.

(c) A closed system was used to establish the law of conservation of mass experimentally.

9.2 Collisions

Looking at moving things in the world around us easily leads to the conclusion that everything set in motion eventually stops. Every clock, every machine eventually runs down. It appears that the amount of motion in the universe must be decreasing. The universe, like any machine, must be running down.

Many philosophers of the 1600's could not accept the idea of a universe that was running down. The concept clashed with their idea of the perfection of God, who surely would not construct such an imperfect mechanism. Some definition of "motion" was needed which would permit one to make the statement that "the quantity of motion in the universe is constant."

Is there such a constant factor in motion that keeps the world machine going? To answer these questions most directly, we can do some simple laboratory experiments. We will use a pair of identical carts with nearly frictionless wheels, or better, two dry-ice discs or two air-track gliders. In the first experiment, a lump of putty is attached so that the carts will stick together when they collide. The carts are each given a push so that they approach each other with equal speeds and collide head-on. As you will see when you do the experiment, both carts stop in the collision: their motion ceases. But is there anything related to their motions which does not change?

Yes, there is. If we add the velocity $\vec{v}_A$ of one cart to the velocity $\vec{v}_B$ of the other cart, we find that the *vector sum* does not change. The vector sum of the velocities of these oppositely moving carts is zero *before* the collision. It is also zero for the carts at rest *after* the collision.

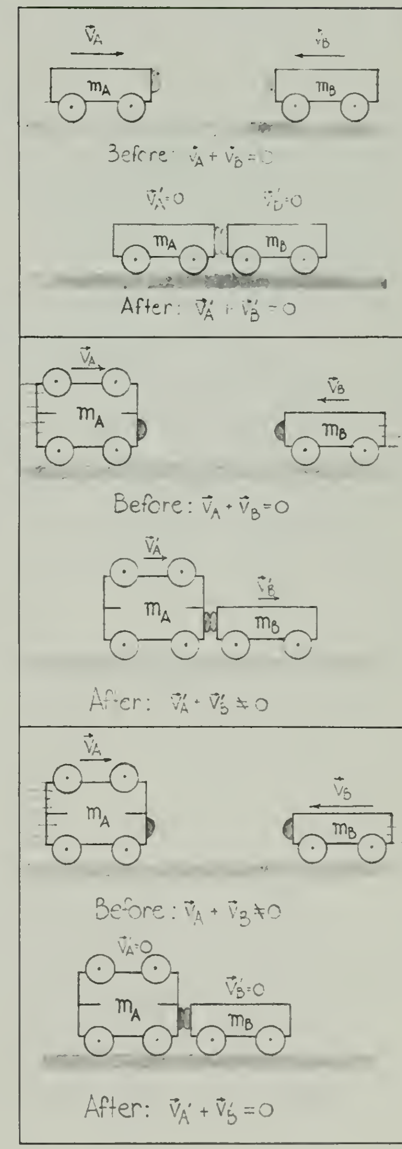
We might wonder whether this finding holds for all collisions. In other words, is there a "law of conservation of velocity"? The example above was a very special circumstance. Carts with equal masses approach each other with equal speeds. Suppose we make the mass of one of the carts twice the mass of the other cart. (We can conveniently double the mass of one cart by putting another cart on top of it.) Now let the carts approach each other with equal speeds and collide, as before. This time the carts do *not* come to rest. There is some motion remaining. Both objects move together in the direction of the initial velocity of the more massive object. Our guess that the vector sum of the velocities might be conserved in all collisions is wrong.

Another example of a collision will confirm this conclusion. This time let the first cart have twice the mass of the second, but only half the speed. When the carts collide head-on and stick together, they stop. The vector sum of the velocities is equal to zero *after* the collision. But it was not equal to zero *before* the collision. Again, there is no conservation of velocity.

We have been trying to show that the "quantity of motion" is always the same before and after the collision. But our results indicate that the proper definition of "quantity of motion" may involve the *mass* of a body

Note that in Units 1 and 2 we dealt mostly with phenomena in which this fact did not seem to be faced.

In symbols $\vec{v}_A + \vec{v}_B = 0$ in *this* particular case



as well as its speed. Descartes had suggested that the proper measure of a body's quantity of motion was the product of its mass and its speed. Speed does not involve direction and is considered always to have a positive value. The examples above, however, show that this product (a scalar and always positive) is not a conserved quantity. In the first and third collisions, for example, the products of mass and speed are zero for the stopped carts *after* the collision. But they obviously are not equal to zero *before* the collision.

But if we make one very important change in Descartes' definition, we do obtain a conserved quantity. Instead of defining "quantity of motion" as the product of mass and *speed, mv,* we can define it (as Newton did) as the product of the mass and *velocity,* $m\vec{v}$. In this way we include the idea of the *direction* of motion as well as the speed. On the next page the quantities $m\vec{v}$ are analyzed for the three collisions we have considered. In all three head-on collisions, the motion of both carts before and after collision is described by the equation:

In general symbols, [illegible]

$$\underbrace{m_A\vec{v}_A + m_B\vec{v}_B}_{\text{before collision}} = \underbrace{m_A\vec{v}_A{}' + m_B\vec{v}_B{}'}_{\text{after collision}}$$

In Unit 1, initial and final velocities were represented as v and v'. Here they are represented by v and v' because we now need to add subscripts such as A and B.

Where m_A and m_B represents the masses of the carts, $\vec{v}_A$ and $\vec{v}_B$ represent their velocities before the collision and $\vec{v}_A{}'$ and $\vec{v}_B{}'$ represent their velocities after the collision.

SG 9.8, 9.9

In words: *the vector sum of the quantities mass × velocity is constant, or conserved, in all these collisions.* This is a very important and useful equation, leading directly to a powerful law.

Q4 Descartes defined the quantity of motion of an object as the product of its mass and its speed. Is his quantity of motion conserved as he believed it was? If not, how would you modify his definition so the quantity of motion would be conserved?

Q5 Two carts collide head-on and stick together. In which of the following cases will the carts be at rest immediately after the collision?

	Cart A		*Cart B*	
	mass	speed before	mass	speed before
(a)	2 kg	3 m/sec	2 kg	3 m/sec
(b)	2	2	3	3
(c)	2	3	3	2
(d)	2	3	1	6

9.3 Conservation of momentum

Since the momentum of a system is the vector sum of the momentum of its parts, it is sometimes called the "total momentum" of the system. We will assume that "total" is understood.

The product of mass and velocity often plays an interesting role in mechanics. It therefore has been given a special name. Instead of being called "quantity of motion," as in Newton's time, it is now called *momentum.* The total momentum of a system of objects (for example, the two carts) is the vector sum of the momenta of all objects in the system.

Analyses of Three Collisions

In Section 9.2 we discuss three examples of collisions between two carts. In each case the carts approached each other head-on, collided, and stuck together. We will show here that in each collision the motion of the carts before and after the collision is described by the *general* equation

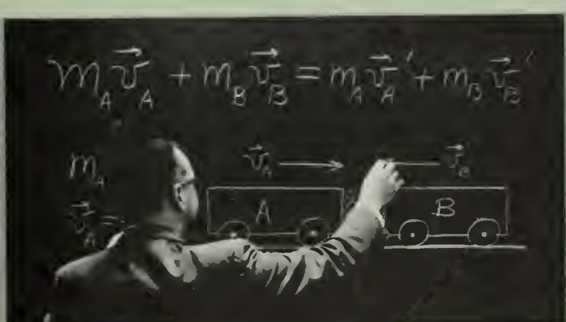
$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

where m_A and m_B represent the masses of the carts, $\vec{v}_A$ and $\vec{v}_B$ their velocities before collision, and $\vec{v}_A'$ and $\vec{v}_B'$ their velocities after the collision.

Example 1: Two carts with equal masses move with equal speeds—but in opposite directions—before the collision. The speed of the stuck-together carts after the collision is zero. Before collision, the product of mass and velocity has the same magnitude for each cart, but opposite direction. So their vector sum is obviously zero. After collision, each velocity is zero, so the product of mass and velocity is also zero.

This simple case could be described in a few sentences. More complicated cases are much easier to handle by using an equation and substituting values in the equation. To show how this works, we will go back to the simple case above, even though it will seem like a lot of trouble for such an obvious result. We substitute specific values into the general equation given in the first paragraph above for two colliding bodies. In this specific case $m_A = m_B$, $\vec{v}_B = -\vec{v}_A$, and $\vec{v}_A' = \vec{v}_B' = 0$. Just before collision, the vector sum of the separate momenta is given by $m_A\vec{v}_A + m_B\vec{v}_B$, which in this case is equal to $m_A\vec{v}_A + m_A(-\vec{v}_A)$ or $m_A\vec{v}_A - m_A\vec{v}_A$ which equals zero.

After the collision, the vector sum of the momenta is given by $m_A\vec{v}_A' + m_B\vec{v}_B'$. Since both velocities after collision is zero, then



$$m_A(0) + m_B(0) = 0$$

Thus, before the collision, the vector sum of the products of mass and velocity is zero, and the same is true for the vector sum after the collision. The general equation is therefore "obeyed" in this case.

Example 2: The carts move with equal speeds toward each other before the collision. The mass of one cart is twice that of the other. After the collision, the velocity of the stuck-together carts is found to be $\frac{1}{3}$ the original velocity of the more massive cart. In symbols: $m_A = 2m_B$, $\vec{v}_B = -\vec{v}_A$, and $\vec{v}_A' = \vec{v}_B' = \frac{1}{3}\vec{v}_A$. Before the collision:

$$\begin{aligned} m_A\vec{v}_A = m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B(-\vec{v}_A) \\ &= 2m_B\vec{v}_A - m_B\vec{v}_A \\ &= m_B\vec{v}_A \end{aligned}$$

After the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\tfrac{1}{3}\vec{v}_A + m_B\tfrac{1}{3}\vec{v}_A \\ &= \tfrac{2}{3}m_B\vec{v}_A + \tfrac{1}{3}m_B\vec{v}_A \\ &= m_B\vec{v}_A \end{aligned}$$

Again, the sum of $m\vec{v}$'s is the same before and after the collision. Therefore, the general equation describes the collision correctly.

Example 3: Two carts approach each other; the mass of one cart is twice that of the other. Before the collision, the speed of the less massive cart is twice that of the more massive cart. The speed of the stuck-together carts after the collision is found to be zero. In symbols: $m_A = 2m_B$, $\vec{v}_B = -2\vec{v}_A$ and $\vec{v}_A' = \vec{v}_B' = 0$. Before the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B(-2\,\vec{v}_A) \\ &= 2m_B\vec{v}_A - 2m_B\vec{v}_A \\ &= 0 \end{aligned}$$

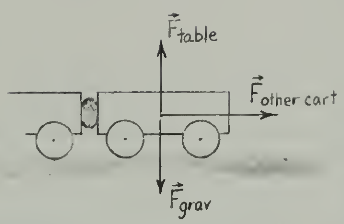
After the collision:

$$m_A(0) + m_B(0) = 0$$

Again, the principle holds. Indeed, *it holds for all collisions of this kind* on which no external pushes or pulls are exerted, regardless of their masses and their initial velocities.

In these examples all motion has been along a straight line. However, the principle is most useful for collisions that are not directly head-on and where the bodies go off at different angles. An example of such a collision is on page 23.

SG 9.10, 9.11



Forces on cart *B* during collision.

Consider each of the collisions that we examined. The momentum of the system as a whole—the vector sum of the individual parts—is the same before and after collision. Moreover, the total momentum doesn't change *during* the collision, as the results of a typical experiment on page 10 show. Thus, we can summarize the results of the experiments briefly: *the momentum of the system is conserved.*

We arrived at this rule (or law, or principle) by observing the special case of collisions between two carts that stuck together after colliding. But in fact this *law of conservation of momentum* is a completely general, universal law. The momentum of *any* system is conserved *if one condition is met:* that no net force is acting on the system.

To see just what this condition means, let us examine the forces acting on one of the carts. Each cart experiences three main forces. There is of course a downward pull $\vec{F}_{grav}$ exerted by the earth and an upward push $\vec{F}_{table}$ exerted by the table. During the collision, there is also a push $\vec{F}_{from\ other\ cart}$ exerted by the other cart. The first two forces evidently cancel, since the cart is not accelerating up or down. Thus the net force on each cart is just the force exerted on it by the other cart as they collide. (To simplify, we assume that frictional forces exerted by the table and the air are small enough to neglect. That was the reason for using dry-ice disks, air-track gliders, or carts with "frictionless" wheels. This assumption makes it easier to discuss the law of conservation of momentum. But we will see that the law holds whether friction exists or not.)

The two carts form a *system* of bodies, each cart being a part of the system. The force exerted by one cart on the other cart is a force exerted by one part of the system on another part. But it is *not* a force on the system as a whole. The outside forces acting on the carts (by the earth and by the table) exactly cancel. Thus, there is no *net* outside force. We can say that the system is "isolated." This condition must be met in order for the momentum of a system to stay constant, to be conserved.

In general, for *n* objects the law can be written

$$\sum (m v) = \sum (m v)$$

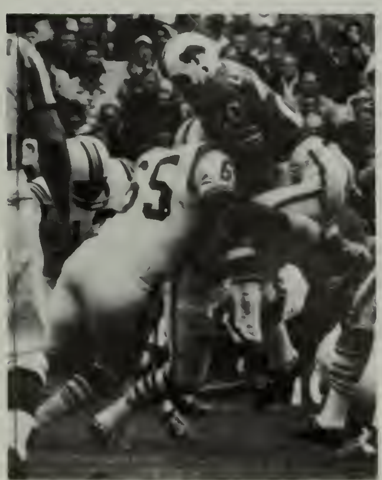
If the net force on a system of bodies is zero, the momentum of the system will not change. This is the *law of conservation of momentum* for systems of bodies that are moving with linear velocity $\vec{v}$.

So far we have considered only cases in which two bodies collide directly and stick together. But the remarkable thing about the law of conservation of momentum is how universally it applies. For example:

(a) It holds true no matter what *kind* of forces the bodies exert on each other. They may be gravitational forces, electric or magnetic forces, tension in strings, compression in springs, attraction or repulsion. The sum of the $m\vec{v}$'s before is equal to the sum of $m\vec{v}$'s after any interaction.

(b) It doesn't matter whether the bodies stick together or scrape against each other or bounce apart. They don't even have to touch. When two strong magnets repel or when an alpha particle is repelled by a nucleus, conservation of momentum still holds.

(c) The law is not restricted to systems of only two objects; there can be any number of objects in the system. In those cases, the basic conservation equation is made more general simply by adding a term for each object to both sides of the equation.



Example of the Use of the Conservation of Momentum

Here is an example that illustrates how one can use the law of conservation of momentum.

(a) A space capsule at rest in space, far from the sun or planets, has a mass of 1,000 kg. A meteorite with a mass of 0.1 kg moves towards it with a speed of 1,000 m/sec. How fast does the capsule (with the meteorite stuck in it) move after being hit?

m_A mass of the meteorite = 0.1 Kg
m_B mass of the capsule = 1,000 Kg
$\vec{v}_A$ initial velocity of meteorite = 1,000 m/sec
$\vec{v}_B$ initial velocity of capsule = 0
$\vec{v}_A'$ final velocity of meteorite } = ?
$\vec{v}_B'$ final velocity of capsule }

The law of conservation of momentum states that

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

Inserting the values given,

$$(0.1 \text{ kg})(1000 \text{ m/sec}) + (1000 \text{ kg})(0) = (0.1 \text{ kg})\vec{v}_A' + (1000 \text{ kg})\vec{v}_B'$$

$$100 \text{ kg}\cdot\text{m/sec} = (0.1 \text{ kg})\vec{v}_A' + (1000 \text{ kg})\vec{v}_A'$$

Since the meteorite sticks to the capsule, $\vec{v}_B' = \vec{v}_A'$ so we can write

$$100 \text{ kg}\cdot\text{m/sec} = (0.1 \text{ kg})\vec{v}_A' + (1000 \text{ kg})\vec{v}_A'$$
$$100 \text{ kg}\cdot\text{m/sec} = (1000.1 \text{ kg})\vec{v}_A'$$

Therefore

$$\vec{v}_A' = \frac{100 \text{ kg}\cdot\text{m/sec}}{1000.1 \text{ kg}} = 0.1 \text{ m/sec}$$

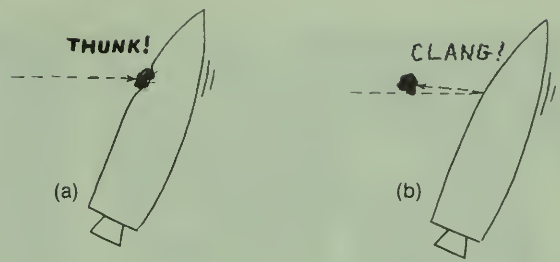
(in the original direction of the motion of the meteorite). Thus, the capsule (with the stuck meteorite) moves on with a speed of 0.1 m/sec.

Another approach to the solution is to handle the symbols first, and substitute in the values only as a final step. Substituting $\vec{v}_A'$ for $\vec{v}_B'$ and letting $\vec{v}_B' = 0$ would leave the equation $m_A\vec{v}_A = m_A\vec{v}_A' + m_B\vec{v}_A' = (m_A + m_B)\vec{v}_A'$. Solving for v_A'

$$\vec{v}_A' = \frac{m_A\vec{v}_A}{(m_A + m_B)}$$

This equation holds true for any projectile hitting (and staying with) a body initially at rest that moves on in a straight line after collision.

(b) An identical capsule at rest nearby is hit by a meteorite of the same mass as the other.



But this meteorite, hitting another part of the capsule, does not penetrate. Instead it bounces straight back with almost no change of speed. (Some support for the reasonableness of this claim is given in SG 9.24.) How fast does the capsule move on after being hit? Since all these motions are along a straight line, we can drop the vector notation from the symbols and indicate the reversal in direction of the meteorite with a minus sign.

The same symbols are appropriate as in (a):

m_A = 0.1 kg	v_B = 0
m_B = 1000 kg	v_A' = −1000 m/sec
v_A = 1000 m/sec	v_B' = ?

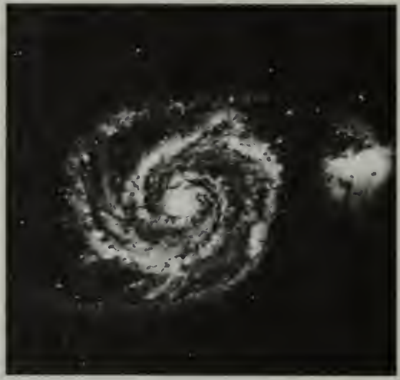
The law of conservation of momentum stated that $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$. Here

$$(0.1 \text{ kg})(1000 \text{ m/sec}) + (1000 \text{ kg})(0) = (0.1 \text{ kg})(-1000 \text{ m/sec}) + (1000 \text{ kg})v_B'$$
$$100 \text{ kg}\cdot\text{m/sec} = -100 \text{ kg}\cdot\text{m/sec} + (1000 \text{ kg})v_B'$$

$$v_B' = \frac{200 \text{ kg}\cdot\text{m/sec}}{1000 \text{ kg}} = 0.2 \text{ m/sec}$$

Thus, the struck capsule moves on with about twice the speed of the capsule in (a). (A general symbolic approach can be taken to this solution, too. But the result is valid only for the special case of a projectile rebounding perfectly elastically from a body of much greater mass.)

There is a general lesson here. It follows from the law of conservation of momentum that a struck object is given less momentum if it *absorbs* the projectile than if it *reflects* it. (A goalie who catches the soccer ball is pushed back less than one who lets the ball bounce off his chest.) Some thought will help you to understand this idea: an interaction that merely stops the projectile is not as great as an interaction that first stops it and then propels it back again.



(d) The size of the system is not important. The law applies to a galaxy as well as to an atom.

(e) The angle of the collision does not matter. All of our examples so far have involved collisions between two bodies moving along the same straight line. They were "one-dimensional collisions." But if two bodies make a glancing collision rather than a head-on collision, each will move off at an angle to the line of approach. The law of conservation of momentum applies to such two-dimensional collisions also. (Remember that momentum is a vector quantity.) The law of conservation of momentum also applies in *three* dimensions. The vector sum of the momenta is still the same before and after the collision.

SG 9.12–9.15

One of the stroboscopic photographs that appears in the *Handbook*.

On page 13 is a worked-out example that will help you become familiar with the law of conservation of momentum. At the end of the chapter is a special page on the analysis of a two-dimensional collision. There are also stroboscopic photographs in the Project Physics *Handbook* and film loops of colliding bodies and exploding objects. These include collisions and explosions in two dimensions. The more of them you analyze, the more convinced you will be that the law of conservation of momentum applies to *any* isolated system.

The worked-out example of page 13 displays a characteristic feature of physics. Again and again, physics problems are solved by applying the expression of a *general* law to a specific situation. Both the beginning student and the veteran research physicist find it helpful, but also somewhat mysterious, that one can *do* this. It seems strange that a few general laws enable one to solve an almost infinite number of specific individual problems. Everyday life seems different. There you usually cannot calculate answers from general laws. Rather, you have to make quick decisions, some based on rational analysis, others based on "intuition." But the use of general laws to solve scientific problems becomes, with practice, quite natural also.

Q6 Which of the following has the least momentum? Which has the greatest momentum?

(a) a pitched baseball

(b) a jet plane in flight

(c) a jet plane taxiing toward the terminal

Q7 A girl on ice skates is at rest on a horizontal sheet of smooth ice. As a result of catching a rubber ball moving horizontally toward her, she moves at 2 cm/sec. Give a rough estimate of what her speed would have been

(a) if the rubber ball were thrown twice as fast
(b) if the rubber ball had twice the mass
(c) if the girl had twice the mass
(d) if the rubber ball were not caught by the girl, but bounced off and went straight back with no change of speed.

9.4 Momentum and Newton's laws of motion

In Section 9.2 we developed the concept of momentum and the law of conservation of momentum by considering experiments with colliding carts. The law was an "empirical" law. That is, we arrived at it as a summary of experimental results, not from theory. The law was discovered—perhaps "invented" or "induced" are better terms—as a generalization from experiment. SG 9.16

We can show, however, that the law of conservation of momentum follows directly from Newton's laws of motion. It takes only a little algebra. We will first put Newton's second law into a somewhat different form than we used before.

Newton's second law expresses a relation between the net force $\vec{F}_{net}$ acting on a body, the mass m of the body, and its acceleration a. We wrote this as $\vec{F}_{net} = m\vec{a}$. But we can also write this law in terms of *change of momentum* of the body. Recalling that acceleration is the rate-of-change of velocity, $\vec{a} = \Delta\vec{v}/\Delta t$, we can write:

$$\vec{F}_{net} = m\frac{\Delta\vec{v}}{\Delta t}$$

or

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

If m is a constant,
$\Delta(m\vec{v}) = m\vec{v}' - m\vec{v}$
$= m(\vec{v}' - \vec{v})$
$= m\Delta\vec{v}$

If the mass of the body is constant, the change in its momentum, $\Delta(m\vec{v})$, is the same as its mass times its change in velocity, $m(\Delta\vec{v})$. So then we can write:

$$\vec{F}_{net}\ \Delta t = \Delta(m\vec{v})$$

$\vec{F}\Delta t$ is called the "impulse."

SG 9.17–9.20

That is, *the product of the net force on a body and the time interval during which this force acts equals the change in momentum of the body.*

In Newton's second law, "change of motion" meant change of momentum - see Definition II at the beginning of the *Principia*.

This statement of Newton's second law is more nearly what Newton used in the *Principia*. Together with Newton's third law, it enables us to derive the law of conservation of momentum for the cases we have studied. The details of the derivation are given on page 16. Thus Newton's laws and the law of conservation of momentum are not separate, independent laws of nature.

Deriving Conservation of Momentum from Newton's Laws

Suppose two bodies with masses m_A and m_B exert forces on each other (by gravitation or by mutual friction, etc.). $\vec{F}_{AB}$ is the force exerted on a body *A* by body *B*, and $\vec{F}_{BA}$ is the force exerted on body *B* by body *A*. No other unbalanced force acts on either body; they form an isolated system. By Newton's third law, the forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are at every instant equal in magnitude and opposite in direction. Each body acts on the other for exactly the same time Δt. Newton's second law, applied to each of the bodies, says

$$\vec{F}_{AB}\,\Delta t=\Delta(m_A\vec{v}_A)$$

and

$$\vec{F}_{BA}\,\Delta t=\Delta(m_B\vec{v}_B)$$

By Newton's third law

$$\vec{F}_{AB}\,\Delta t=-\vec{F}_{BA}\Delta t$$

therefore

$$\Delta(m_A\vec{v}_A)=-\Delta(m_B\vec{v}_B)$$

Suppose that each of the masses m_A and m_B are constant. Let $\vec{v}_A$ and $\vec{v}_B$ stand for the velocities of the two bodies at some instant and let $\vec{v}_A'$ and $\vec{v}_B'$ stand for their velocities at some later instant. Then we can write the last equation as

$$m_A\vec{v}_A'-m_A\vec{v}_A = -(m_B\vec{v}_B'-m_B\vec{v}_B)$$

A little rearrangement of terms leads to

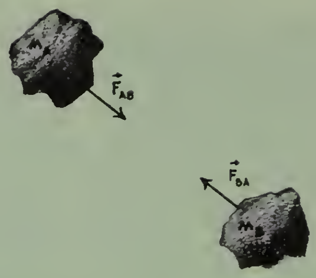
$$m_A\vec{v}_A' - m_A\vec{v}_A=-m_B\vec{v}_B' + m_B\vec{v}_B$$

and

$$m_A\vec{v}_A' + m_B v_B'=m_A\vec{v}_A + m_B\vec{v}_B$$

You will recognize this as our original expression of the law of conservation of momentum.

Here we are dealing with a system consisting of two bodies. But this method works equally well for a system consisting of any number of bodies. For example, SG 9.21 shows you how to derive the law of conservation of momentum for a system of three bodies.



Globular clusters of stars like this one contain tens of thousands of suns held together by gravitational attraction.

In all examples we considered each body to have constant mass. But a change of momentum can arise from a change of mass as well as from (or in addition to) a change of velocity. For example, as a rocket spews out exhaust gases, its mass decreases. The mass of a train of coal cars increases as it moves under a hopper which drops coal into the cars. In Unit 5 you will find that *any* body's mass increases as it moves faster and faster. (However, this effect is great enough to notice only at extremely high speeds.) The equation $F_{net} = ma$ is a form of Newton's second law that works in special cases where the mass is constant. But this form is not appropriate for situations where mass changes. Nor do the forms of the law of conservation of momentum that are based on $F_{net} = ma$ work in such cases. But other forms of the law can be derived for systems where mass is not constant. See, for example, the first pages of the article "Space Travel" in *Reader 5*.

SG 9.21–9.24

In one form or another, the law of conservation of momentum can be derived from Newton's second and third laws. Nevertheless, the law of conservation of momentum is often the preferred tool because it enables us to solve many problems which would be difficult to solve using Newton's laws directly. For example, suppose a cannon that is free to move fires a shell horizontally. Although it was initially at rest, the cannon is forced to move while firing the shell; it *recoils*. The expanding gases in the cannon barrel push the cannon backward just as hard as they push the shell forward. Suppose we had a continuous record of the magnitude of the force. We could then apply Newton's second law separately to the cannon and to the shell to find their respective accelerations. After a few more steps (involving calculus) we could find the speed of the shell and the recoil speed of the cannon. But in practice it is very difficult to get a continuous record of the magnitude of the force. For one thing, the force almost certainly decreases as the shell moves toward the end of the barrel. So it would be very difficult to use Newton's laws to find the final speeds.

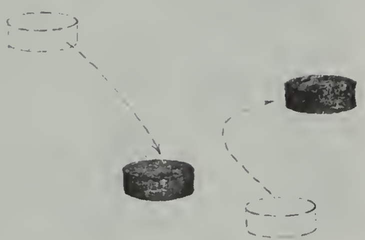
However, we can use the law of conservation of momentum even if we know nothing about the force. The law of conservation of momentum is a law of the kind that says "before = after." Thus, it works in cases where we do not have enough information to apply Newton's laws during the whole interval between "before" and "after." In the case of the cannon and shell the momentum of the system (cannon plus shell) is zero initially. Therefore, by the law of conservation of momentum, the momentum will also be zero after the shell is fired. If we know the masses and the speed of one, after firing we can calculate the speed of the other. (The film loop titled "Recoil" provides just such an event for you to analyze.) On the other hand, if both speeds can be measured afterwards, then the ratio of the masses can be calculated. In the Supplemental Unit entitled *The Nucleus* you will see how just such an approach was used to find the mass of the neutron when it was originally discovered.

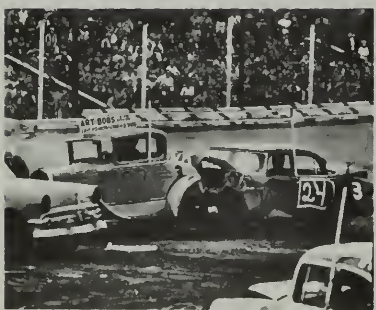
SG 9.25

SG 9.26
SG 9.27

Q8 Since the law of conservation of momentum can be derived from Newton's laws, what good is it?

9.5 Isolated systems





There are important similarities between the conservation law of mass and that of momentum. We test both laws by observing systems that are in some sense isolated from the rest of the universe. When testing or using the law of conservation of *mass,* we arrange an isolated system such as a sealed flask. Matter can neither enter or leave this system. When testing or using the law of conservation of *momentum,* we arrange another kind of isolated system. Such a system is closed in the sense that each body in it experiences no net force from outside the system.

Consider for example two dry-ice pucks colliding on a smooth horizontal table. The very low friction pucks form a very nearly closed or isolated system. We need not include in it the table and the earth, for their individual effects on each puck cancel. Each puck experiences a downward gravitational force exerted by the earth. But the table exerts an equally strong upward push.

Even in this artificial example, the system is not entirely closed. There is always a little friction with the outside world. The layer of gas under the puck and air currents, for example, exert friction. All outside forces are not *completely* balanced, and so the two pucks do not form a truly isolated system. Whenever this is unacceptable, one can expand or extend the system so that it *includes* the bodies that are responsible for the external forces. The result is a new system on which the unbalanced forces are small enough to ignore.

For example, picture two cars skidding toward a collision on an icy road. The frictional forces exerted by the road on each car may be several hundred pounds. These forces are very small compared to the immense force (many tons) exerted by each car on the other when they collide. Thus, for many purposes, we can forget about the action of the road. For such purposes, the two skidding cars *during the collision* are nearly enough an isolated system. However, if friction with the road (or the table on which the pucks move) is too great to ignore, the law of conservation of momentum still holds, but we must apply it to a larger system, one which includes the road or table. In the case of the skidding cars or the pucks, the table or road is attached to the earth. So we would have to include the entire earth in a "closed system."

SG 9.28–9.33

Q9 Define what is meant by "closed" or "isolated" system for the purpose of the law of conservation of mass; for the purpose of the law of conservation of momentum.

Q10 Explain whether or not each of the following can be considered as an isolated system.

(a) a baseball thrown horizontally

(b) an artificial earth satellite
(c) the earth and the moon

9.6 Elastic collisions

In 1666, members of the recently-formed Royal Society of London witnessed a demonstration. Two hardwood balls of equal size were suspended at the ends of two strings to form two pendula. One ball was released from rest at a certain height. It swung down and struck the other, which had been hanging at rest.

After impact, the first ball stopped at the point of impact while the second ball swung from this point to the same height as that from which the first ball had been released. When the second ball returned and struck the first, it now was the second ball which stopped at the point of impact as the first swung up to almost the same height from which it had started. This motion repeated itself for several swings.

This demonstration aroused great interest among members of the Society. In the next few years, it also caused heated and often confusing arguments. Why did the balls rise each time to nearly the same height after each collision? Why was the motion "transferred" from one ball to the other when they collided? Why didn't the first ball bounce back from the point of collision, or continue moving forward after the second ball moved away from the collision point?

Our law of momentum conservation explains what is observed. But it would also allow quite different results. It says only that the momentum of ball A just before it strikes ball B is equal to the total momentum of A and B just after collision. It does not say how A and B share the momentum. The actual result is just one of infinitely many different outcomes that would all agree with conservation of momentum. For example, suppose (though it is never observed to happen) that ball A bounced back with ten times its initial speed. Momentum would still be conserved *if* ball B went ahead at eleven times A's initial speed.

In 1668 three men reported to the Royal Society on the whole matter of impact. The three men were the mathematician John Wallis, the architect and scientist Christopher Wren, and the physicist Christian Huygens. Wallis and Wren offered partial answers for some of the features of collisions; Huygens analyzed the problem in complete detail.

Huygens explained that in such collisions *another conservation law* also holds, in addition to the law of conservation of momentum. Not only was the vector sum of $m\vec{v}$'s conserved, but so was the ordinary arithmetic sum of mv^2's! In modern algebraic form, the relationship he discovered can be expressed as

In general symbols, [illegible] 0

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2$$

Compare this equation with the conservation of momentum equation on page 10.

SG 9.34–9.37

The scalar quantity $\frac{1}{2}mv^2$ has come to be called *kinetic energy.* (The reason for the $\frac{1}{2}$, which doesn't really affect the rule here, will become clear in the next chapter.) The equation stated above, then, is the mathematical expression of the conservation of kinetic energy. This relationship holds for the collision of two "perfectly hard" objects similar to those observed at the Royal Society meeting. There, ball A stopped and ball B went on at A's initial speed. A little algebra will show that this is the *only* result that agrees with *both* conservation of momentum and conservation of kinetic energy. (See SG 9.33.)

But is the conservation of kinetic energy as general as the law of conservation of momentum? Is the total kinetic energy present conserved in *any* interaction occurring in *any* isolated system?

It is easy to see that it is not. Consider the first example of Section 9.2. Two carts of equal mass (and with putty between the bumping surfaces) approach each other with equal speeds. They meet, stick together, and stop. The kinetic energy of the system after the collision is 0, since the speeds of both carts are zero. Before the collision the kinetic energy of the system was $\frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2$. But both $\frac{1}{2}m_Av_A^2$ and $\frac{1}{2}m_Bv_B^2$ are always positive numbers. Their sum cannot possibly equal zero (unless both v_A and v_B are zero, in which case there would be no collision—and not much of a problem). Kinetic energy is *not* conserved in this collision in which the bodies stick together. In fact, *no* collision in which the bodies stick together will show conservation of kinetic energy. It applies only to the collision of "perfectly hard" bodies that bounce back from each other.

Christiaan Huygens (1629-1695) was a Dutch physicist. He devised an improved telescope with which he discovered a satellite of Saturn and saw Saturn's rings clearly. He was the first to obtain the expression for centripetal acceleration (v^2/R), he worked out a wave theory of light, and he invented a pendulum-controlled clock. His scientific contributions were major, and his reputation would undoubtedly have been greater were he not overshadowed by his contemporary, Newton.

Huygens, and others after him for about a century, did not use the factor . The quantity *mv* was called *vis viva*, Latin for "living force." Seventeenth- and eighteenth-century scientists were greatly interested in distinguishing and naming various "forces." They used the word loosely: it meant sometimes a push or a pull (as in the colloquial modern use of the word force), sometimes what we now call "momentum," and sometimes what we now call "energy." The term *vis viva* is no longer used.

The law of conservation of kinetic energy, then, is *not* as general as the law of conservation of momentum. If two bodies collide, the kinetic energy may or may not be conserved, depending on the type of collision. It *is* conserved if the colliding bodies do not crumple or smash or dent or stick together or heat up or change physically in some other way. We call bodies that rebound without any such change "perfectly elastic." We describe collisions between them as "perfectly elastic collisions." In perfectly elastic collisions, *both* momentum and kinetic energy are conserved.

Most collisions that we witness, are not perfectly elastic and kinetic energy is not conserved. Thus, the sum of the $\frac{1}{2}mv^2$'s after the collision is *less* than before the collision. Depending on how much kinetic energy is "lost," such collisions might be called "partially elastic," or "perfectly inelastic." The loss of kinetic energy is greatest in perfectly inelastic collisions, when the colliding bodies remain together.

Collisions between steel ball-bearings, glass marbles, hardwood balls, billiard balls, or some rubber balls (silicone rubber) are almost perfectly elastic, if the colliding bodies are not damaged in the collision. The total kinetic energy after the collision might be as much as, say, 96% of this value before the collision. Examples of true perfectly elastic collisions are found only in collisions between atoms or sub-atomic particles.

SG 9.38-9.40

Q11 Which phrases correctly complete the statement? Kinetic energy is conserved

(a) in all collisions

(b) whenever momentum is conserved
(c) in some collisions
(d) when the colliding objects are not too hard

Q12 Kinetic energy is never negative because
(a) scalar quantities are always positve
(b) it is impossible to draw vectors with negative length
(c) speed is always greater than zero
(d) it is proportional to the square of the speed

9.7 Leibniz and the conservation law

Rene Descartes believed that the total quantity of motion in the universe did not change. He wrote in his *Principles of Philosophy:*

> It is wholly rational to assume that God, since in the creation of matter He imparted different motions to its parts, and preserves all matter in the same way and conditions in which he created it, so He similarly preserves in it the same quantity of motion.

Descartes proposed to define the quantity of motion of an object as the product of its mass and its speed. But as we saw in Section 1.1 this product is a conserved quantity only in very special cases.

Gottfried Wilhelm Leibniz was aware of the error in Descartes' ideas on motion. In a letter in 1680 he wrote:

> M. Descartes' physics has a great defect; it is that his rules of motion or laws of nature, which are to serve as the basis, are for the most part false. This is demonstrated. And his great principle, that the same quantity of motion is conserved in the world, is an error.

Leibniz, however was as sure as Descartes had been that *something* involving motion was conserved. Leibniz called this something he identified as "force" the quantity mv^2 (which he called *vis viva*). We notice that this is just twice the quantity we now call kinetic energy. (Of course, whatever applies to mv^2 applies equally to $\frac{1}{2}mv^2$.)

As Huygens had pointed out, the quantity $(\frac{1}{2})mv^2$ is conserved only in perfectly elastic collisions. In other collisions the total quantity of $(\frac{1}{2})mv^2$ after collision is always *less* than before the collision. Still, Leibniz was convinced that $(\frac{1}{2})mv^2$ is *always* conserved. In order to save his conservation law, he invented an explanation for the apparent loss of *vis viva*. He maintained that the *vis viva* is *not* lost or destroyed. Rather, it is merely "dissipated among the small parts" of which the colliding bodies are made. This was pure speculation and Leibniz offered no supporting evidence. Nonetheless, his explanation anticipated modern ideas about the connection between energy and the motion of molecules. We will study some of these ideas in Chapter 11.

Descartes (1596-1650) was the most important French scientist of the seventeenth century. In addition to his early contribution to the idea of momentum conservation, he is remembered by scientists as the inventor of coordinate systems and the graphical representation of algebraic equations. His system of philosophy, which used the deductive structure of geometry as its model, is still influential.

Leibniz (1646-1716), a contemporary of Newton, was a German philosopher and diplomat, an advisor to Louis XIV of France and Peter the Great of Russia. Independently of Newton he invented the method of mathematical analysis called calculus. A long public dispute resulted between the two great men concerning charges of plagiarism of ideas.

Leibniz extended conservation ideas to phenomena other than collisions. For example, when a stone is thrown straight upward, its quantity of $(\frac{1}{2})mv^2$ decreases as it rises, even without any collision. At the top of the trajectory, $(\frac{1}{2})mv^2$ is zero for an instant. Then it reappears as the stone falls. Leibniz wondered whether something applied or given to a stone at the start is somehow *stored* as the stone rises, instead of being lost. His idea would mean that $(\frac{1}{2})mv^2$ is just one part of a more general, and really conserved quantity. In Chapter 10, this idea will lead us directly to the most powerful of all laws of science—the law of conservation of energy.

Q13 According to Leibniz, Descartes' principle of conservation of mv was

(a) correct, but trivial.

(b) another way of expressing the conservation of *vis viva*.

(c) incorrect.

(d) correct only in elastic collisions.

Q14 How did Leibniz explain the apparent disappearance of the quantity $(\frac{1}{2})mv^2$

(a) during the upward motion of a thrown object?

(b) when the object strikes the ground?

A Collision in Two Dimensions

The stroboscopic photograph shows a collision between two wooden discs on a "frictionless horizontal table" photographed from straight above the table. The discs are riding on tiny plastic spheres which make their motion nearly frictionless. Body *B* (marked **x**) is at rest before the collision. After the collision it moves to the left and Body *A* (marked –) moves to the right. The mass of Body *B* is known to be twice the mass of Body *A:* $m_B = 2m_A$. We will analyze the photograph to see whether momentum was conserved. (Note: The size reduction factor of the photograph and the [constant] stroboscopic flash rate are not given here. So long as all velocities for this test are measured in the same units, it does not matter what those units are.)

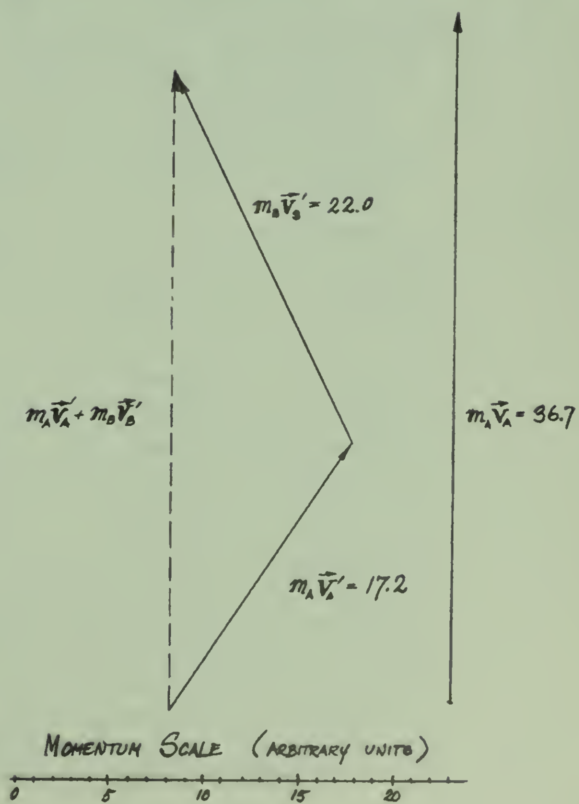
In this analysis we will measure in centimeters the distance the discs moved on the photograph. We will use the time between flashes as the unit of time. Before the collision, Body *A* (coming from the lower part of the photograph) traveled 36.7 mm in the time between flashes: $\vec{v}_A = 36.7$ speed-units. Similarly we find that $\vec{v}_A' = 17.2$ speed-units, and $\vec{v}_B' = 11.0$ speed-units.

The total momentum before the collision is just $m_A\vec{v}_A'$. It is represented by an arrow 36.7 momentum-units long, drawn at right.

The vector diagram shows the momenta $m_A\vec{v}_A'$ and $m_B\vec{v}_B'$ after the collision; $m_A\vec{v}_A'$ is represented by an arrow 17.2 momentum-units long. Since $m_B = 2m_A$, the $m_B\vec{v}_B'$ arrow is 22.0 momentum-units long.

The dotted line represents the vector sum of $m_A\vec{v}_A'$ and $m_B\vec{v}_B'$; that is, the total momentum after the collision. Measurement shows it to be 34.0 momentum-units long. Thus, our measured values of the total momentum before and after the collision differ by 2.7 momentum-units. This is a difference of about –7%. We can also verify that the *direction* of the total is the same before and after the collision to within a small uncertainty.

Have we now demonstrated that momentum was conserved in the collision? Is the 7% difference likely to be due entirely to measurement inaccuracies? Or is there reason to expect that the total momentum of the two discs after the collision is really a bit less than before the collision?



9.1 The Project Physics learning materials particularly appropriate for Chapter 9 include:

Experiments
Collisions in One Dimension
Collisions in Two Dimensions

Film Loops
One-dimensional Collisions I
One-dimensional Collisions II
Inelastic One-dimensional Collisions
Two-dimensional Collisions I
Two-dimensional Collisions II
Inelastic Two-dimensional Collisions
Scattering of a Cluster of Objects
Explosion of a Cluster of Objects

Transparencies
One-dimensional Collisions
Equal Mass Two-dimensional Collisions
Unequal Mass Two-dimensional Collisions
Inelastic Two-dimensional Collisions

In addition, the *Reader 3* articles "The Seven Images of Science" and "Scientific Cranks" are of general interest in the course.

9.2 Certainly Lavoisier did not investigate every possible interaction. What justification did he have for claiming mass was conserved "in all the operations of art and nature"?

9.3 It is estimated that every year at least 2000 tons of meteoric dust fall on to the earth. The dust is mostly debris that was moving in orbits around the sun.

(a) Is the earth (whose mass is about 6×10^{21} tons) reasonably considered to be a closed system with respect to the law of conservation of mass?
(b) How large would the system, including the earth, have to be in order to be completely closed?

9.4 Would you expect that in your lifetime, when more accurate balances are built, you will see experiments which show that the law of conservation of mass does not entirely hold for chemical reactions in closed systems?

9.5 Dayton C. Miller, a renowned experimenter at Case Institute of Technology, was able to show that two objects placed side by side on an equal-arm pan balance did not exactly balance two otherwise identical objects placed one on top of the other. (The reason is that the pull of gravity decreases with distance from the center of the earth.) Does this experiment contradict the law of conservation of mass?

9.6 A children's toy known as a Snake consists of a tiny pill of mercuric thiocyanate. When the pill is ignited, a large, serpent-like foam curls out almost from nothingness. Devise and describe an experiment by which you would test the law of conservation of mass for this demonstration.

9.7 Consider the following chemical reaction, which was studied by Landolt in his tests of the law of conservation of mass. In a closed container, a solution of 19.4 g of potassium chromate in 100.0 g of water is mixed with a solution of 33.1 g of lead nitrate in 100.0 g of water. A bright yellow solid precipitate forms and settles to the bottom of the container. When removed from the liquid, this solid is found to have a mass of 32.3 g and is found to have properties different from either of the reactants.

(a) What is the mass of the remaining liquid? (Assume the combined mass of all substances in the system is conserved.)
(b) If the remaining liquid (after removal of the yellow precipitate) is then heated to 95°C, the water it contains will evaporate, leaving a white solid. What is the mass of this solid? (Assume that the water does not react with anything, either in (a) or in (b).)

9.8 If a stationary cart is struck head-on by a cart with twice the mass, and the two carts stick together, they will move together with a speed $\frac{2}{3}$ as great as the moving cart had before collision. Show that this is consistent with the conservation of momentum equation.

9.9 A freight car of mass 10^5 kg travels at 2.0 m/sec and collides with a motionless freight car of mass 1.5×10^5 kg on a horizontal track. The two cars lock, and roll together after impact. Find the velocity of the two cars after collision.
HINTS:
The general equation for conservation of momentum for a two-body system is:

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

(a) What quantities does the problem give for the equation?
(b) Rearrange terms to get an expression for $\vec{v}_A'$.
(c) Find the value of $\vec{v}_A'$. (Note $\vec{v}_A' = \vec{v}_B'$.)

9.10 You have been given a precise technical definition for the word *momentum*. Look it up in a large dictionary and record its various uses. Can you find anything similar to our definition in these more general meanings? How many of the uses seem to be consistent with the technical definition here given?

9.11 Benjamin Franklin, in correspondence with his friend James Bowdoin (founder and first president of the American Academy of Arts and Sciences), objected to the corpuscular theory of light by saying that a particle traveling with such immense speed (3×10^8 m/sec) would have the impact of a 10-kg ball fired from a cannon at 100 m/sec. What mass did Franklin assign to the "light particle"?

9.12 If powerful magnets are placed on top of each of two carts, and the magnets are so arranged that like poles face each other when one cart is pushed toward the other, the carts bounce away from each other without actually making contact.

(a) In what sense can this be called a collision?
(b) Will the law of conservation of momentum apply?
(c) Describe an arrangement for testing your answer to (b).

9.13 From the equation

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'$$

show that the change in momentum of object *A* is equal and opposite to the change of momentum of object *B*. Using the symbol $\Delta\vec{p}$ for change of momentum, rewrite the law of conservation of momentum for two bodies. What might it be for 3 bodies? for n bodies?

9.14 A person fires a fast ball vertically. Clearly, the momentum of the ball is not conserved; it first loses momentum as it rises, then gains it as it falls. How large is the "closed system" within which the ball's momentum, *together* with that of other bodies (tell which), *is* conserved. What happens to the rest of the system as the ball rises? as it falls?

9.15 If everyone in the world were to stand together in one field and jump up with an initial speed of 1 m/sec,

(a) For how long would they be off the ground?
(b) How high would they go?
(c) What would be the earth's speed downward?
(d) How far would it move?
(e) How big would the field have to be?

9.16 Did Newton arrive at the law of conservation of momentum in the *Principia*? If a copy of the *Principia* is available, read Corollary III and Definition II (just before and just after the three laws).

9.17 If mass remains constant, then $\Delta(mv) = m(\Delta v)$. Verify this relation by substituting some numerical values, for example for the case where m is 3 units and v changes from 4 units to 6 units.

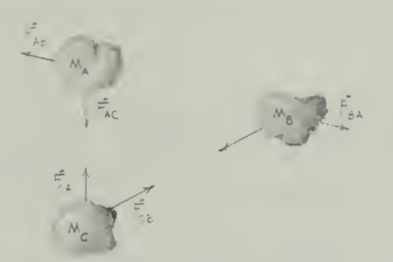
9.18 (a) Why can ocean liners or planes not turn corners sharply?
(b) In the light of your knowledge of the relationship between momentum and force, comment on reports about unidentified flying objects (UFO) turning sharp corners in full flight.

9.19 A girl on skis (mass of 60 kg including skis) reaches the bottom of a hill going 20 m/sec. What is her momentum? She strikes a snowdrift and stops within 3 seconds. What force does the snow exert on the girl? How far does she penetrate the drift? What happened to her momentum?

9.20 During sports, the forces exerted on parts of the body and on the ball, etc., can be astonishingly large. To illustrate this, consider the forces in hitting a golf ball. Assume the ball's mass is .046 kg. From the strobe photo on p. 27 of Unit 1, in which the time interval between strobe flashes was 0.01 sec, estimate:

(a) the speed of the ball after impact
(b) the magnitude of the ball's momentum after impact
(c) how long the impact lasted
(d) the average force exerted on the ball during impact.

9.21 The *Text* derives the law of conservation of momentum for two bodies from Newton's third and second laws. Is the principle of the conservation of mass essential to this derivation? If so, where does it enter?



9.22 Consider an isolated system of three bodies, *A*, *B*, and *C*. The forces acting among the bodies can be indicated by subscript: for example, the force exerted on body *A* by body *B* can be given the symbol $\vec{F}_{AB}$. By Newton's third law of motion, $\vec{F}_{BA} = -\vec{F}_{AB}$. Since the system is isolated, the only force on each body is the sum of the forces exerted on it by the other two; for example, $\vec{F}_A = \vec{F}_{AB} + \vec{F}_{AC}$. Using these principles, show that the total momentum change of the system will be zero.

9.23 In Chapter 4, SG 4.24 was about putting an Apollo capsule into an orbit around the moon.

The question was: "Given the speed v_o necessary for orbit and the present speed v, how long should the rocket engine with thrust F fire to give the capsule of mass m the right speed?" There you solved the problem by considering the acceleration.

(a) Answer the question more directly by considering change in momentum.
(b) What would be the total momentum of all the exhaust from the rocket?
(c) If the "exhaust velocity" were v_e, about what mass of fuel would be required?

9.24 (a) Show that when two bodies collide their changes in velocity are inversely proportional to their masses. That is, if m_A and m_B are the masses and $\Delta\vec{v}_A$ and $\Delta\vec{v}_B$ the velocity changes, show that numerically,

$$\frac{\Delta v_A}{\Delta v_B} = \frac{m_B}{m_A}$$

(b) Show how it follows from conservation of momentum that if a light particle (like a B.B. pellet) bounces off a massive object (like a bowling ball), the velocity of the light particle is changed much more than the velocity of the massive object.

(c) For a head-on elastic collision between a body of mass m_A moving with velocity v_A and a body of mass m_B at rest, combining the equations for conservation of momentum and conservation of kinetic energy leads to the relationship $v_A' = v_A(m_A - m_B) / (m_A + m_B)$. Show that if body *B* has a much greater mass than body *A*, then v_A' is almost exactly the same as v_A—that is, body *A* bounces back with virtually no loss in speed.

9.25 The equation $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'$ is a general equation applicable to countless separate situations. For example, consider a 10-kg shell fired from a 1000-kg cannon. If the shell is given a speed of 1000 m/sec, what would be the recoil speed of the cannon? (Assume the cannon is on an almost frictionless mount.) Hint: your answer could include the following steps:

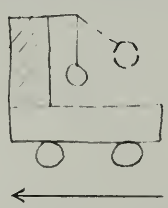
(a) If A refers to the cannon and B to the shell, what are $\vec{v}_A$ and $\vec{v}_B$ (before firing)?
(b) What is the total momentum before firing?
(c) What is the total momentum after firing?
(d) Compare the magnitudes of the momenta of the cannon and of the shell after firing.
(e) Compare the ratios of the speeds and of the masses of the shell and cannon after firing.

9.26 The engines of the first stage of the Apollo/Saturn rocket develop an average thrust of 35 million newtons for 150 seconds. (The entire rocket weighs 28 million newtons near the earth's surface.)

(a) How much momentum will be given to the rocket during that interval?
(b) The final speed of the vehicle is 6100 miles/hour. What would one have to know to compute its mass?

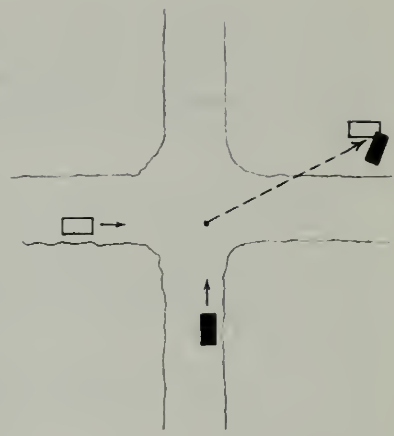
9.27 Newton's second law can be written $\vec{F}\Delta t = \Delta(m\vec{v})$. Use the second law to explain the following:

(a) It is safer to jump into a fire net or a load of hay than onto the hard ground.
(b) When jumping down from some height, you should bend your knees as you come to rest, instead of keeping your legs stiff.
(c) Hammer heads are generally made of steel rather than rubber.
(d) Some cars have plastic bumpers which, temporarily deformed under impact, slowly return to their original shape. Others are designed to have a somewhat pointed front-end bumper.



9.28 A student in a physics class, having learned about elastic collisions and conservation laws, decides that he can make a self-propelled car. He proposes to fix a pendulum on a cart, using a "super-ball" as a pendulum bob. He fixes a block to the cart so that when the ball reaches the bottom of the arc, it strikes the block and rebounds elastically. It is supposed to give the cart a series of bumps that propel it along.

(a) Will his scheme work? (Assume the "super-ball" is perfectly elastic.) Give reasons for your answer.
(b) What would happen if the cart had an initial velocity in the forward direction?
(c) What would happen if the cart had an initial velocity in the backward direction?



9.29 A police report of an accident describes two vehicles colliding (inelastically) at an icy intersection of country roads. The cars slid to a stop in a field as shown in the diagram. Suppose the masses of the cars are approximately the same.

(a) How did the speeds of the two cars compare just before collision?
(b) What information would you need in order to calculate the actual speeds of the automobiles?
(c) What simplifying assumptions have you made in answering (b)?

9.30 Two pucks on a frictionless horizontal surface are joined by a spring.

(a) Can they be considered an isolated system?
(b) How do gravitational forces exerted by the earth affect your answer?
(c) What about forces exerted by the pucks *on* the earth?
(d) How big would the system have to be in order to be considered completely isolated?

9.31 A hunter fires a gun horizontally at a target fixed to a hillside. Describe the changes of momentum to the hunter, the bullet, the target and the earth. Is momentum conserved

(a) when the gun is fired?
(b) when the bullet hits?
(c) during the bullet's flight?

9.32 A billiard ball moving 0.8 m/sec collides with the cushion along the side of the table. The collision is head-on and can here be regarded as perfectly elastic. What is the momentum of the ball

(a) before impact?
(b) after impact?

(Pool sharks will recognize that it depends upon the spin or "English" that the ball has, but to make the problem simpler, neglect this condition.

(c) What is the change in momentum of the bal'
(d) Is momentum conserved?

9.33 Discuss conservation of momentum for the system shown in this sketch from *Le Petit Prince*. What happens
(a) if he leaps in the air?
(b) if he runs around?

Le petit prince sur l'astéroïde B 612.

9.34 When one ball collides with a stationary ball of the same mass, the first ball stops and the second goes on with the speed the first ball had. The claim is made on p. 20 that this result is the only possible result that will be consistent with conservation of both momentum and kinetic energy. (That is, if $m_A = m_B$ and $v_B = 0$, then the result must be $v_A' = 0$ and $v_B' = v_A$.) Combine the equations that express the two conservation laws and show that this is actually the case. (Hint: rewrite the equations with m for m_A and m_B' and $v_B = 0$; solve the simplified momentum equation for v_A'; substitute in the simplified kinetic energy equation; solve for v_B'.)

9.35 Fill in the blanks for the following motions:

Object	m (kg)	v m/sec	mv kg·m/sec	$\frac{1}{2}mv^2$ kg·m^2/sec^2
baseball	0.14	30.0	–	–
hockey puck	–	50.0	8.55	–
superball	0.050	1.5	–	–
light car	1460	–	–	1.79×10^6
mosquito	–	–	2.0×10^5	4.0×10^{-6}
football player	–	–	–	–

9.36 Two balls, one of which has three times the mass of the other, collide head-on, each moving with the same speed. The more massive ball stops, the other rebounds with twice its original speed. Show that both momentum and kinetic energy are conserved.

9.37 If both momentum and kinetic energy are conserved, say that a ball of mass m moving at speed v strikes, elastically, head-on, a second ball of mass $3m$ which is at rest. Using the principle of conservation of momentum and kinetic energy, find the speeds of the two balls after collision.

9.38 Devise a way of giving a numerical estimate just how far from "perfectly elastic" a collision is—for example, the collision between a ball and the ground from which it bounces.

9.39 Apply the law of conservation of momentum to discuss qualitatively a man swimming; a ship changing course; a man walking; a rocket taking off; a rifle being fired; a propeller plane in straight line motion, and while circling; a jet plane ascending; an apple dropping to earth; a comet being captured by the sun; a spaceship leaving earth; an atomic nucleus emitting a small particle.

9.40 Describe the changes of kinetic energy involved in pole vaulting from the start of the vaulter's run to his landing.

CHAPTER TEN

Energy

10.1 Work and kinetic energy

In everyday language we say that pitching, catching, and running on the baseball field is "playing," while sitting at a desk, reading, writing, and thinking is "working." But, in the language of physics, studying involves very little work, while playing baseball involves a great deal of work. The term "doing work" means something very definite in physics. It means "exerting a force on an object while the object moves in the direction of the force." When you throw a baseball, you exert a large force on it while it moves forward for about one meter. In doing so, you do a large amount of work. By contrast, in writing or in turning the pages of a book you exert only a small force over a short distance. This does not require much work, as the term work is understood in physics.

Suppose you are employed in a factory to lift boxes from the floor straight upward to a conveyor belt at waist height. Here the language of common usage and the physics both agree that you are doing work. If you lift two boxes at once you do twice as much work as you do if you lift one box. And if the conveyor belt were twice as high above the floor, you would do twice as much work to lift a box to it. The work you do depends on both the *magnitude* of the force you must exert on the box and the *distance* through which the box moves in the direction of the force.

SG 10.1

Note that work you do on a box does not depend on how fast you do your job.

With this example in mind, we can define work in a way that allows us to give a numerical value to the concept. The work W done on an object by a force $\vec{F}$ is defined as the product of the magnitude F of the force and the distance d that the object moves *in the direction of* $\vec{F}$ while the force is being exerted:

$$W = Fd$$

The way d is defined here, the $W = Fd$ is correct. It does not, however, explicity tell how to compute W if the motion is not in exactly the same direction as the force. The definition of d implies that it would be the component of the displacement along the direction of F, and this is entirely correct.

To lift a box weighing 100 newtons upward through 0.8 meters requires you to apply an upward force of 100 newtons. The work you do on the box is 100 newtons × 0.8 meters = 80 newton-meters.

From our definition of work it follows that no work is done if there is no displacement. No matter how hard you push on a wall, no work is done if the wall does not move. Also, no work is done if the only motion is perpendicular to the direction of the force. For example, suppose you are carrying a book bag. You must pull up against the downward pull of gravity to keep the bag at a constant height. But as long as you are standing still you do no work on the bag. Even if you walk along with it steadily in a horizontal line, the only work you do is in moving it forward against the small resisting force of the air.

Note that work is a scalar quantity. A more general definition of work will be given in Sec. 10.4.

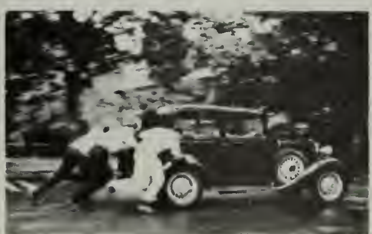
The equation $W = Fd$ implies that work is always a positive quantity. However, by convention, when the force on a body and its displacement are in opposite directions, the work is negative. This implies that the body's energy would be decreased. The sign convention follows naturally from the more rigorous definition of mechanical work as $W = Fl \cos \theta$ where θ is the angle between $\vec{F}$ and l.

Work is a useful concept in itself. But the concept is most useful in understanding the concept of *energy*. There are a great many forms of energy. A few of them will be discussed in this chapter. We will define them, in the sense of describing how they can be measured and how they can be expressed algebraically. We will also discuss how energy changes from one form to another. The *general* concept of energy is very difficult to define. But to define some *particular* forms of energy is easy enough. The concept of work helps greatly in making such definitions.

The chief importance of the concept of work is that work represents an amount of energy transformed from one form to another. For example, when you throw a ball you do work on it. You also transform chemical energy, which your body obtains from food and oxygen, into energy of motion of the ball. When you lift a stone (doing work on it), you transform chemical energy into gravitational potential energy. If you release the stone, the earth pulls it downward (does work on it); gravitational potential energy is transformed into energy of motion. When the stone strikes the ground, it compresses the ground below it (does work on it); energy of motion is transformed into heat. These are some of the forms energy takes; and work is a measure of how much energy is transferred.

SG 10.2

The form of energy we have been calling "energy of motion" is perhaps the simplest to deal with. We can use the definition of work $W = Fd$, together with Newton's laws of motion, to get an expression of this form of energy. Remember that a moving body has many attributes which are related by separate ideas. For example, we have studied speed v (Chapter 1), velocity $\vec{v}$ (Chapter 3), momentum $m\vec{v}$ (Chapter 9). We also saw how the seventeenth century thinkers groped for a clear idea of some *conserved* quantity in all motion. Now let us imagine that we exert a constant net force F on an object of mass m. This force accelerates the object over a distance d from rest to a speed v. Using Newton's second law of motion, we can show in a few steps of algebra that



$$Fd = \tfrac{1}{2}mv^2$$

The details of this derivation are given on the first half of page 32 "Doing Work on a Sled."

We recognize Fd as the expression for the work done on the object by whatever exerted the force F. The work done on the object equals the amount of energy transformed from some form into energy of motion of the object. So $\frac{1}{2}mv^2$ is the expression for the energy of motion of the object. The energy of motion of an object at any instant is given by the quantity $\frac{1}{2}mv^2$ at that instant, and is called *kinetic energy*. We will use the symbol KE to represent kinetic energy. By definition then,

$$KE = \frac{1}{2}mv^2$$

The Greek word kinetos means "moving."

Now it is clearer why we wrote $\frac{1}{2}mv^2$ instead of just mv^2 in Chapter 9. If one is conserved, so must be the other—and conservation was all that we were concerned with there. But $\frac{1}{2}mv^2$ also relates directly to the concept of work, and so provides a more useful expression for energy of motion.

The speed of an object must be measured relative to some reference frame, so kinetic energy is a relative quantity also. See SG 10.3.

The equation $Fd = \frac{1}{2}mv^2$ was obtained by considering the case of an object initially at rest. In other words, the object had an initial kinetic energy of zero. But the relation also holds for an object already in motion when the net force is applied. In that case the work done on the object still equals the change in its kinetic energy:

SG 10.3–10.8

$$Fd = \Delta(KE)$$

The quantity $\Delta(KE)$ is by definition equal to $(\frac{1}{2}mv^2)_{\text{final}} - (\frac{1}{2}mv^2)_{\text{initial}}$. The proof of this general equation appears on the second half of page 32.

Work is defined as the product of a force and a distance. Therefore, its units in the mks system are *newtons* × *meters* or newton·meters. A newton·meter is also called a *joule* (abbreviated *J*). The joule is the unit of work or of energy.

The name of the unit of energy and work commemorates J. P. Joule, a nineteenth-century English physicist famous for his experiments showing that heat is a form of energy (see Sec. 10.7). There is no general agreement today whether the name should be pronounced like "jool" or like "jowl." The majority of physicists favor the former.

Q1 If a force F is exerted on an object while the object moves a distance d in the direction of the force, the work done on the object is (a) F (b) Fd (c) F/d (d) $\frac{1}{2}Fd^2$

Q2 The kinetic energy of a body of mass m moving at a speed v is (a) $\frac{1}{2}mv$ (b) $\frac{1}{2}mv^2$ (c) mv^2 (d) $2mv^2$ (e) m^2v^2

10.2 Potential energy

As we have seen in the previous section, doing work on an object can increase its kinetic energy. But work can be done on an object *without* increasing its kinetic energy. For example, you might lift a book straight up at a small, constant speed, so that its kinetic energy stays the same. But you are still doing work on the book. And by doing work you are using your body's store of chemical energy. Into what form of energy is it being transformed?

The answer, as Leibniz suggested, is that there is "energy" associated with height above the earth. This energy is called *gravitational potential energy*. Lifting the book higher and higher increases the gravitational potential energy. You can see clear evidence of this effect when you drop the book. The gravitational potential energy is transformed rapidly into kinetic energy of fall. In general terms, suppose a force $\vec{F}$ is used to displace an object upwards a distance d, without changing its KE. Then

Doing Work on a Sled

Suppose a loaded sled of mass m is initially at rest on low-friction ice. You, wearing spiked shoes, exert a constant horizontal force F on the sled. The weight of the sled is balancced by the upward push exerted by the ice, so F is the net force on the sled. You keep pushing, running faster and faster as the sled accelerates, until the sled has moved a total distance d.

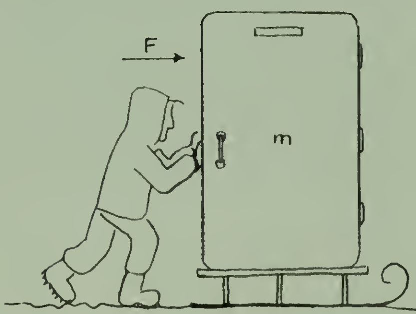
Since the net force F is constant, the acceleration of the sled is constant. Two equations that apply to motion starting from rest with constant acceleration are

$$v = at$$

and

$$d = \tfrac{1}{2}at^2$$

where a is the acceleration of the body, t is the time interval during which it accelerates



(that is, the time interval during which a net force acts on the body), v is the final speed of the body and d is the distance it moves in the time interval t.

According to the first equation $t = v/a$. If we substitute this expression for t into the second equation, we obtain

$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2}a\frac{v^2}{a^2} = \tfrac{1}{2}\frac{v^2}{a}$$

The work done on the sled is $W=Fd$. From Newton's second law, $F=ma$, so

$$W = Fd$$
$$= ma \times \tfrac{1}{2}\frac{v^2}{a}$$

The acceleration cancels out, giving

$$W = \tfrac{1}{2}mv^2$$

So the work done in this case can be found from just the mass of the body and its final speed. With more advanced mathematics, it can be shown that the result is the same whether the force is constant or not.

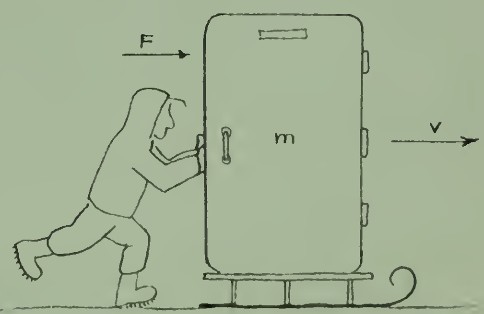
More generally, we can show that the change in kinetic energy of a body already moving is equal to the work done on the body. By the definition of average speed,

$$d = v_{av}t$$

If we consider a uniformly accelerated body whose speed changes from v_0 to v, the average speed during t is $\tfrac{1}{2}(v + v_0)$. Thus

$$d = \frac{v + v_0}{2} \times t$$

By the definition of acceleration, $a = \Delta v/t$;



therefore $t = \Delta v/a = (v - v_0)/a$
Substituting $(v - v_0)/a$ for t gives

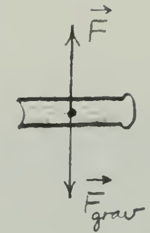
$$d = \frac{v + v_0}{2} \times \frac{v - v_0}{a}$$
$$= \frac{(v + v_0)(v - v_0)}{2a}$$
$$= \frac{v^2 - v_0^2}{2a}$$

The work W done is $W=Fd$, or, since $F=ma$,

$$W = ma \times d$$
$$= ma \times \frac{v^2 - v_0^2}{2a}$$
$$= \frac{m}{2}(v^2 - v_0^2)$$
$$= \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

the increase in gravitational potential energy, $\Delta(PE)_{grav}$, is

$$\Delta(PE)_{grav} = Fd$$



To lift the book at constant speed you must exert an upward force F equal in magnitude to the weight F_{grav} of the book. The work you do in lifting the book through distance d is Fd, which is numerically equal to $F_{grav}d$. See SG 10.9 and 10.10.

Potential energy can be thought of as *stored* energy. As the book falls, its gravitational potential energy decreases while its kinetic energy increases correspondingly. When the book reaches its original height, all of the gravitational potential energy stored during the lift will have been transformed into kinetic energy.

Many useful applications follow from this idea of potential or stored energy. For example, the steam hammer used by construction crews is driven up by high-pressure steam ("pumping in" energy). When the hammer drops, the gravitational potential energy is converted to kinetic energy. Another example is the proposal to use extra available energy from electric power plants during low demand periods to pump water into a high reservoir. When there is a large demand for electricity later, the water is allowed to run down and drive the electric generators.

A set mouse-trap contains elastic potential energy.

There are forms of potential energy other than gravitational. For example, if you stretch a rubber band or a spring, you increase its *elastic potential energy*. When you release the rubber band, it can deliver the stored energy to a projectile in the form of kinetic energy. Some of the work done in blowing up an elastic balloon is also stored as potential energy.

Other forms of potential energy are associated with other kinds of forces. In an atom, the negatively charged electrons are attracted by the positively charged nucleus. If an externally applied force pulls an electron *away* from the nucleus, the *electric potential energy* increases. If the electron is pulled back and moves *toward* the nucleus, the potential energy decreases as the electron's kinetic energy increases.

If two magnets are pushed together with north poles facing, the *magnetic potential energy* increases. When released, the magnets will move apart, gaining kinetic energy as they lose potential energy. SG 10.11

Where is the potential energy located in all these cases? It might seem at first that it "belongs" to the body that has been moved. But this is not the most useful way of thinking about it. For without the other object—the earth, the nucleus, the other magnet—the work would not increase any potential form of energy. Rather, it would increase only the kinetic energy of the object on which work was done. The potential energy belongs not to *one* body, but to the whole system of interacting bodies! This is evident in the fact that the potential energy is available to any one or to all of these interacting bodies. For example, you could give either magnet all the kinetic energy, just by releasing it and holding the other in place. Or suppose you could fix the book somehow to a hook that would hold it at one point in space. The earth would then "fall" up toward the book. Eventually the earth would gain just as much kinetic energy at the expense of stored potential energy as the book would if it were free to fall. SG 10.12 SG 10.13

The increase in gravitational potential energy "belongs" to the earth-book *system*, not to the book alone. The work is done by an "outside"

The work you have done on the earth-book system is equal to the energy you have given up from your store of chemical energy

agent (you), increasing the total energy of the earth-book system. When the book falls, it is responding to forces exerted by one part of the system on another. The *total energy* of the system does not change—it just is converted from *PE* to *KE*. This is discussed in more detail in the next section.

Q3 If a stone of mass m falls a vertical distance d, pulled by its weight $F_{grav} = ma_g$, the decrease in gravitational potential energy is (a) md (b) ma_g (c) ma_gd (d) $\frac{1}{2}md^2$ (e) d.

Q4 When you compress a coil spring you do work on it. The elastic potential energy (a) disappears (b) breaks the spring (c) increases (d) decreases.

Q5 Two electrically charged objects repel one another. To increase the electric potential energy, you must

(a) make the objects move faster
(b) move one object in a circle around the other object
(c) attach a rubber band to the objects
(d) pull the objects farther apart
(e) push the objects closer together.

10.3 Conservation of mechanical energy

In Section 10.1 we stated that the amount of work done on an object *equals* the amount of energy transformed from one form to another. For example, the chemical energy of a muscle is transformed into the kinetic energy of a thrown ball. Our statement implied that the *amount* of energy involved does not change—only its *form* changes. This is particularly obvious in motions where no "outside" force is applied to a mechanical system.

While a stone falls freely, for example, the gravitational potential energy of the stone-earth system is continually transformed into kinetic energy. Neglecting air friction, the *decrease* in gravitational potential energy is, for any portion of the path, equal to the *increase* in kinetic energy. Or consider a stone thrown upward. Between any two points in its path, the *increase* in gravitational potential energy equals the *decrease* in kinetic energy. For a stone falling or rising (without external forces such as friction),

$$\Delta(PE)_{grav} = -\Delta(KE)$$

The equations in this section are true only if friction is negligible. We shall extend the range later to include friction, which can cause the conversion of mechanical energy into heat energy.

This relationship can be rewritten as

$$\Delta(KE) + \Delta(PE)_{grav} = 0$$

or still more concisely as

$$\Delta(KE + PE_{grav}) = 0$$

If $(KE + PE_{grav})$ represents the *total mechanical energy* of the system, then the *change* in the system's total mechanical energy is *zero*. In other words, the total mechanical energy, $\Delta(KE + PE_{grav})$ remains constant; it is *conserved*.

SG 10.14

A similar statement can be made for a vibrating guitar string. While the string is being pulled away from its unstretched position, the string-guitar system gains elastic potential energy. When the string is released, the elastic potential energy decreases while the kinetic energy of the string increases. The string coasts through its unstretched position and becomes stretched in the other direction. Its kinetic energy then decreases as the elastic potential energy increases. As it vibrates, there is a repeated transformation of elastic potential energy into kinetic energy and back again. The string loses some mechanical energy—for example, sound waves radiate away. Otherwise, the decrease in elastic potential energy over any part of the string's motion would be accompanied by an equal increase in kinetic energy, and *vice versa:*

$$\Delta(PE)_{\text{elastic}} = -\Delta(KE)$$

In such an ideal case, the total mechanical energy $(KE + PE_{\text{elastic}})$ remains constant; it is conserved.

Up to here we have always considered only *changes* in PE. There is some subtlety in defining an actual value of PE. See SG 10.15

We have seen that the potential energy of a system can be transformed into the kinetic energy of some part of the system, and *vice versa*. Potential energy also can be transformed into another form of potential energy without change in the *total energy* $(KE + PE)$. We can write this rule in several equivalent ways:

$$\Delta KE = -\Delta PE$$

or

$$\Delta KE > \Delta PE = 0$$

or

$$\Delta(KE + PE) = 0$$

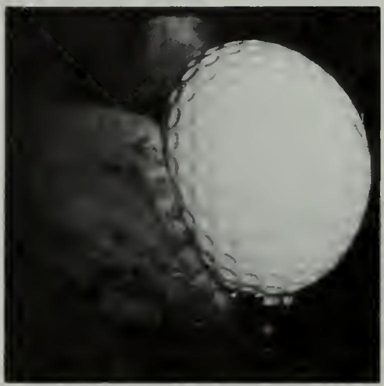
or

$$KE + PE = \text{constant}$$

These equations are different ways of expressing the *law of conservation of mechanical energy* when there is no "external" force. But suppose that an amount of work W is done on part of the system by some external force. Then the energy of the system is increased by an amount equal to W. Consider, for example, a suitcase-earth system. You must do work on the suitcase to pull it away from the earth up to the second floor. This work increases the total mechanical energy of the earth + suitcase system. If you yourself are included in the system, then your internal chemical energy decreases in proportion to the work you do. Therefore, the *total* energy of the lifter + suitcase + earth system does not change.

The law of conservation of energy can be derived from Newton's laws of motion. Therefore, it tells us nothing that we could not, in principle, compute directly from Newton's laws of motion. However, there are situations where there is simply not enough information about the forces involved to apply Newton's laws. It is in these cases that the law of conservation of mechanical energy demonstrates its usefulness. Before long you will see how the law came to be very useful in understanding a huge variety of natural phenomena.

A perfectly elastic collision is a good example of a situation where we often cannot apply Newton's laws of motion. In such collisions we do not know and cannot easily measure the force that one object exerts on the other. We do know that during the actual collision, the objects distort one another. (See the photograph of the golf ball in the margin.) The distortions are produced against elastic forces. Thus, some of the combined kinetic energy of the objects is transformed into elastic potential energy as they distort one another. Then elastic potential energy is transformed back into kinetic energy as the objects separate. In an ideal case, both the objects and their surroundings are exactly the same after colliding as they were before. They have the same shape, same temperature, etc. In such a case, all of the elastic potential energy is converted back into kinetic energy.



During its contact with a golf club, a golf ball is distorted, as is shown in the high-speed photograph. As the ball moves away from the club, the ball recovers its normal spherical shape, and elastic potential energy is transformed into kinetic energy.

This is useful but incomplete knowledge. The law of conservation of mechanical energy gives only the *total* kinetic energy of the objects after the collision. It does not give the kinetic energy of each object separately. (If enough information were available, we could apply Newton's laws to get more detailed results: namely, the speed of *each* object.) You may recall that the law of conservation of momentum also left us with useful but incomplete knowledge. We can use it to find the *total* momentum, but not the *individual* momentum vectors, of elastic objects in collision. In Chapter 9 we saw how conservation of momentum and conservation of mechanical energy together limit the possible outcomes of perfectly elastic collisions. For two colliding objects, these two restrictions are enough to give an exact solution for the two velocities after collision. For more complicated systems, conservation of energy remains important. We usually are not interested in the detailed motion of each of every part of a complex system. We are not likely to care, for example, about the motion of every molecule in a rocket exhaust. Rather, we probably want to know only about the overall thrust and temperature. The principle of conservation of energy applies to total, defined systems, and such systems usually interest us most.

Q6 As a stone falls frictionlessly

(a) its kinetic energy is conserved

(b) its gravitational potential energy is conserved

(c) its kinetic energy changes into gravitational potential energy

(d) no work is done on the stone

(e) there is no change in the total energy

Q7 In what position is the elastic potential energy of the vibrating guitar string greatest? At which position is its kinetic energy greatest?

Q8 If a guitarist gives the same amount of elastic potential energy to a bass string and to a treble string, which one will gain more speed when released? (The mass of a meter of bass string is greater than that of a meter of treble string.)

Q9 How would you compute the potential energy stored in the system shown in the margin made up of the top boulder and the earth?

10.4 Forces that do no work

In Section 10.1 we defined the *work* done on an object. It is the product of the magnitude of the force $\vec{F}$ applied to the object and the magnitude of the distance $\vec{d}$ in the direction of $\vec{F}$ through which the object moves while the force is being applied. In all our examples so far, the object moved in the same direction as that of the force vector.

But usually the direction of motion and the direction of the force are *not* the same. For example, suppose you carry a book at constant speed and horizontally, so that its kinetic energy does not change. Since there is no change in the book's energy, you are doing no work on the book (by our definition of work). You do apply a force on the book, and the book does move through a distance. But here the applied force and the distance are at right angles. You exert a vertical force on the book—upwards to balance its weight. But the book moves horizontally. Here, an applied force $\vec{F}$ is exerted on an object while the object moves at right angles to the direction of the force. Therefore $\vec{F}$ has no component in the direction of $\vec{d}$, and so the force *does no work*. This statement agrees entirely with the idea of work as *energy being transformed from one form to another*. Since the book's speed is constant, its kinetic energy is constant. And since its distance from the earth is constant, its gravitational potential energy is constant. So there is no transfer of mechanical energy.

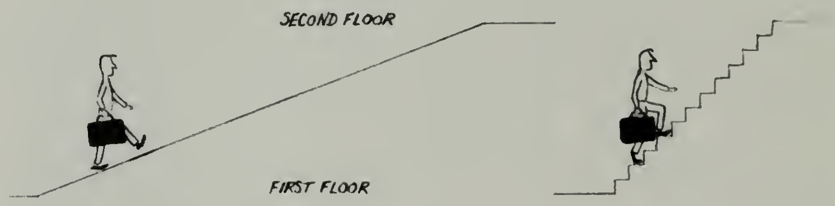
A similar reasoning, but not so obvious, applies to a satellite in a circular orbit. The speed and the distance from the earth are both constant. Therefore, the kinetic energy and the gravitational potential energy are both constant, and there is no energy transformation. For a circular orbit the centripetal force vector is perpendicular to the tangential direction of motion at any instant. So no work is being done. To put an artificial satellite into a circular orbit requires work. But once it is in orbit, the *KE* and *PE* stay constant and no further work is done on the satellite.

When the orbit is eccentric, the force vector is usually not perpendicular to the direction of motion. In such cases energy is continually transformed between kinetic and gravitational potential forms. The total energy of the system remains constant, of course.

Situations where the net force is exactly perpendicular to the motion are as rare as situations where the force and motion are in exactly the same direction. What about the more usual case, involving some angle between the force and the motion?

SG 10.16

In general, the work done on an object depends on how far the body moves *in the direction of the force*. As stated before, the equation $W = Fd$ properly defines work only if d is the distance moved in the direction of the force. Consider the example of a child sliding down a playground slide. The gravitational force $\vec{F}_{grav}$ is directed *down*. So only the distance *down* determines the amount of work done by $\vec{F}_{grav}$. It does not matter how long the slide is, or what its shape is. Change in gravitational potential energy depends *only* on change in height—near the earth's surface, at least. For example, consider raising a suitcase from the first floor to the second floor. The same increase in PE_{grav} of the suitcase-earth system occurs regardless of the path by which the suitcase is raised. Also, each path requires the same amount of work.



More generally, change in PE_{grav} depends only on change of position. The details of the path followed in making the change make no difference at all. The same is true for changes in elastic potential energy and electric potential energy. The changes depend only on the initial and final positions, and not on the path taken between these positions.

If frictional forces also have to be overcome, additional work will be needed, and that additional work may depend on the path chosen—for example, whether it is long or short.

An interesting conclusion follows from the fact that change in PE_{grav} depends only on change in height. For the example of the child on the slide, the gravitational potential energy decreases as his altitude decreases. If frictional forces are vanishingly small, all the work goes into transforming PE_{grav} into KE. Therefore, the increases in KE depend only on the decrease in altitude. In other words, the child's speed when he reaches the ground will be the same whether he slides down or jumps off the top. A similar principle holds for satellites in orbit and for electrons in TV tubes: in the absence of friction, the change in kinetic energy depends only on the initial and final positions, and not on the path taken between them. This principle gives great simplicity to some physical laws, as we will see when we consider gravitational and electric fields in Chapter 14.

SG 10.17

Q10 How much work is done on a satellite during each revolution if its mass is m, its period is T, its speed is v, and its orbit is a circle of radius R?

Q11 Two skiers were together at the top of a hill. While one skier skied down the slope and went off the jump, the other rode the ski-lift down. Compare their changes in gravitational potential energy.

Q12 A third skier went directly down a straight slope. How would his speed at the bottom compare with that of the skier who went off the jump?

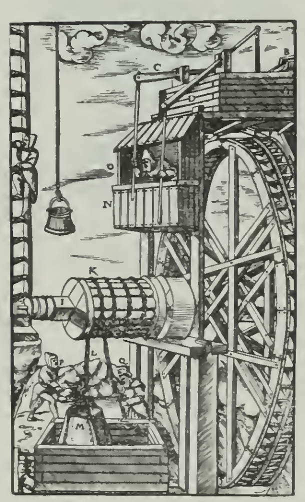
Q13 No work is done when

(a) a heavy box is pushed at constant speed along a rough horizontal floor
(b) a nail is hammered into a board
(c) there is no component of force parallel to the direction of motion
(d) there is no component of force perpendicular to the direction of motion.

10.5 Heat energy and the steam engine

So far we have assumed that our equations for work and energy hold only if friction is absent or very small. Why? Suppose that frictional forces *do* affect a suitcase or other object as it is being lifted. The object must do work against these forces as it moves. (This work in fact serves to warm up the stairs, the air, etc.). Consequently, that much less work is available to increase *PE* or *KE* or both. How can we modify our expression of the law of conservation of mechanical energy to include these effects?

Suppose that a book on a table has been given a shove and is sliding across the table top. If the surface is rough, it will exert a fairly large frictional force and the book will stop quickly. Its kinetic energy will rapidly disappear. But no corresponding increase in potential energy will occur, since there is no change in height. It appears that, in this example, mechanical energy is not conserved.



However, close examination of the book and the tabletop show that they are warmer than before. The disappearance of kinetic energy of the book is accompanied by the appearance of *heat*. This suggests—but by no means proves—that the kinetic energy of the book was transformed into heat. If so, then heat must be one form of energy. This section deals with how the idea of heat as a form of energy gained acceptance during the nineteenth century. You will see how theory was aided by practical knowledge of the relation of heat and work. This knowledge was gained in developing, for very practical reasons, the steam engine.

Until about 200 years ago, most work was done by people or animals. Work was obtained from wind and water also, but these were generally unreliable as sources of energy. For one thing, they were not always available when and where they were needed. In the eighteenth century, miners began to dig deeper and deeper in search of greater coal supply. But water tended to seep in and flood these deeper mines. The need arose for an economical method for pumping water out of mines. The steam engine was developed initially to meet this very practical need.

The steam engine is a device for converting the energy of some kind of fuel into heat energy. For example, the chemical energy of coal or oil, or the nuclear energy of uranium is converted to heat. The heat energy in turn is converted into mechanical energy. This mechanical energy can be used directly to do work, as in a steam locomotive, or can be transformed into electrical energy. In typical twentieth-century industrial societies, most of the energy used in factories and homes comes from electrical energy. Falling water is used to generate electricity in some parts of the country. But steam engines still generate most of the electrical energy used in the United States today. There are other heat engines, too—internal combustion engines and turbines for example. But the steam engine remains a good model for the basic operation of this whole family of engines.

The generation and transmission of electrical energy, and its conversion into mechanical energy, will be discussed in Chapter 15. Here we will focus on the central and historic link in the chain of energy conversion, the steam engine.

A model of Heron's aeolipile. Steam produced in the boiler escapes through the nozzles on the sphere, causing it to rotate.

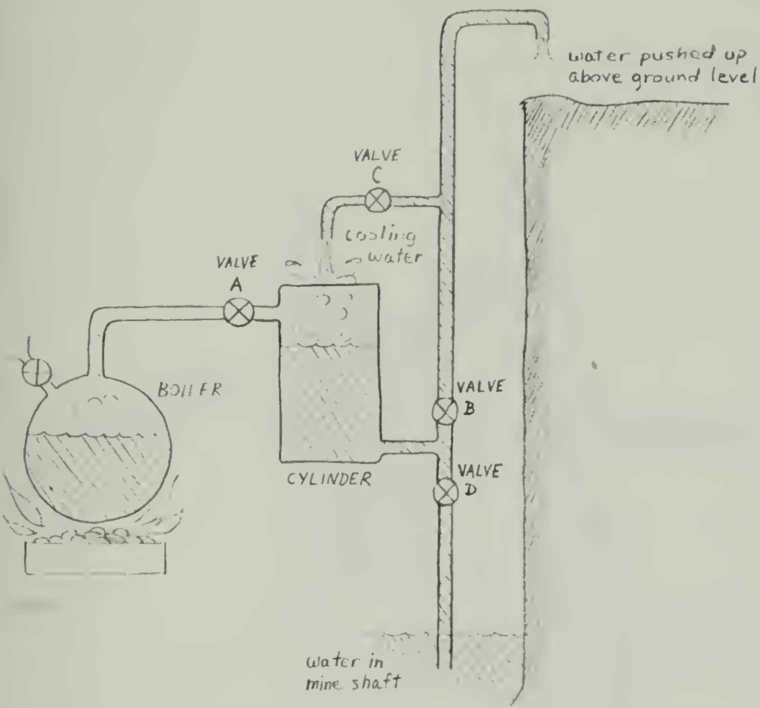
Since ancient times it had been known that heat could be used to produce steam, which could then do mechanical work. The "aeolipile," invented by Heron of Alexandria about 100 A.D., worked on the principle of Newton's third law. (See margin.) The rotating lawn sprinkler works the same way except that the driving force is water pressure instead of steam pressure.

Heron's aeolipile was a toy, meant to entertain rather than to do any useful work. Perhaps the most "useful" application of steam to do work in the ancient world was another of Heron's inventions. This steam-driven device astonished worshippers in a temple by causing a door to open when a fire was built on the altar. Not until late in the eighteenth century, however, were commercially successful steam engines invented.

Today we would say that a steam engine uses a supply of heat energy to do mechanical work. That is, it converts heat energy into mechanical energy. But many inventors in the eighteenth and nineteenth centuries did not think of heat in this way. They regarded heat as a thin, invisible substance that could be used over and over again to do work without being used up itself. But they did not need to learn all the presently known laws of physics in order to become successful engineers. In fact, the sequence of events was just the opposite. Steam engines were developed first by inventors who knew relatively little about science. Their main interest lay in making money, or in improving the effectiveness and safety of mining. Later, scientists with both a practical knowledge of *what* would work and a curiosity about *how* it worked made new discoveries in physics.

The first commercially successful steam engine was invented by Thomas Savery (1650-1715), an English military engineer. Follow the explanation of it one sentence at a time, referring to the diagram on page 41. In the Savery engine the water in the mine shaft is connected by a pipe and a valve **D** to a chamber called the cylinder. With valve **D** closed and valve **B** open, high-pressure steam from the boiler is admitted to the cylinder through valve **A.** This forces the water out of the cylinder and up the pipe. The water empties at the top and runs off at ground level. Valve **A** and valve **B** are closed. Valve **D** is opened, allowing an open connection between the cylinder and the water in the mine shaft.

When valve **C** is opened, cold water pours over the cylinder. The



Schematic diagram of Savery engine.

steam left in the cylinder cools and condenses to water. Since water occupies a much smaller volume than the same mass of steam, a partial vacuum forms in the cylinder. This vacuum allows the air pressure in the mine to force water from the mine shaft up the pipe into the cylinder.

The same process, started by closing valve **D** and opening valves **A** and **B**, is repeated over and over. The engine is in effect a pump. It moves water from the mine shaft to the cylinder, then from the cylinder to the ground above.

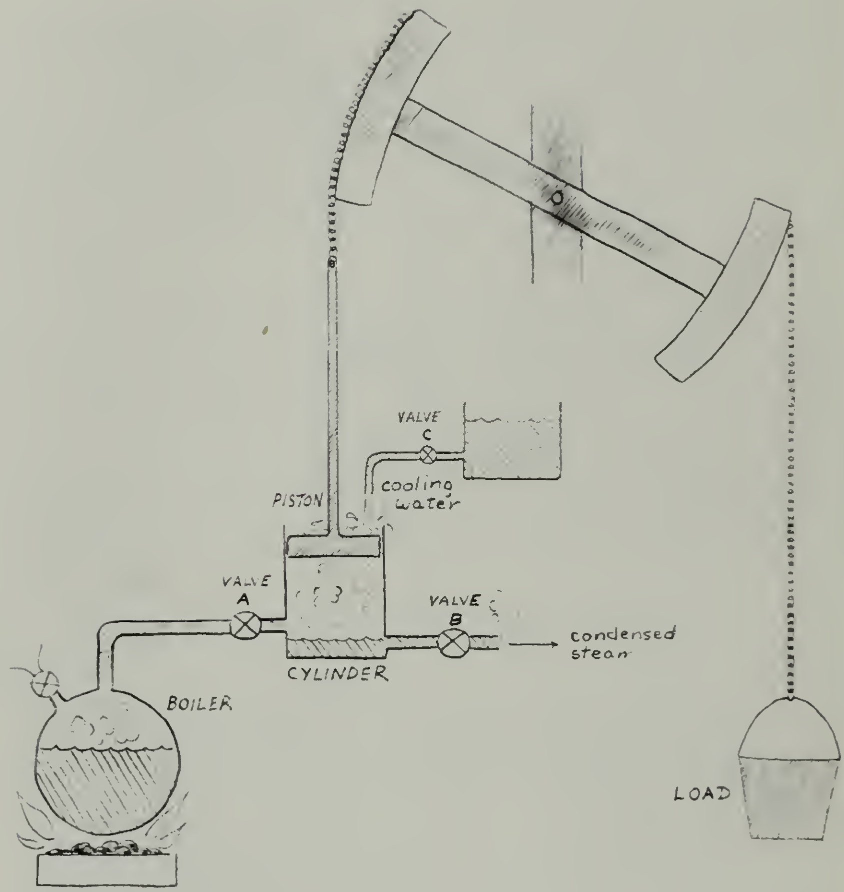
However, the Savery engine's use of high-pressure steam produced a serious risk of boiler or cylinder explosions. This defect was remedied by Thomas Newcomen (1663-1729), another Englishman. Newcomen invented an engine that used steam at lower pressure. His engine was superior in other ways also. For example, it could raise loads other than water.

The Newcomen engine features a rocking beam. This beam connects to the load on one side and to a piston in a cylinder on the other side. When valve **A** is open, the cylinder is filled with steam at normal atmospheric pressure. The beam is balanced so that the weight of the load raises the piston to the upper end of the cylinder. While the piston is coming toward this position, valve **A** is still open and valve **B** is still closed.

But when the piston reaches its highest position, valve **A** is closed and valve **C** is opened. Cooling water flows over the cylinder and the steam condenses, making a partial vacuum in the cylinder. This allows the pressure of the atmosphere to push the piston down. As the piston reaches the bottom of the cylinder, valve **C** is closed and valve **B** is opened briefly.

In the words of Erasmus Darwin the engine:

Bade with cold streams the quick expansion stop,
And sunk the immense of vapour to a drop.
Press'd by the ponderous air the Piston falls
Resistless, sliding through its iron walls;
Quick moves the balanced beam, of giant-birth,
Wields his large limbs, and nodding shakes the earth.



Schematic diagram of Newcomen engine. In the original Newcomen engine the load was water being lifted from a mine shaft.

The cooled and condensed steam runs off. The valve **A** is opened, and the cycle begins all over again.

Originally someone had to open and close the valves by hand at the proper times in the cycle. But later models did this automatically. The automatic method used the rhythm and some of the energy of the moving parts of the engine itself to control the sequence of operation. This idea, of using part of the output of the process to regulate the process itself, is called *feedback*. It is an essential part of the design of many modern mechanical and electronic systems. (See the article "Systems, Feedback, Cybernetics" in Unit 3 *Reader*.

SG 10.18

The Newcomen engine was widely used in Britain and other European countries throughout the eighteenth century. By modern standards it was not a very good engine. It burned a large amount of coal but did only a small amount of work at a slow, jerky rate. But the great demand for machines to pump water from mines produced a good market even for Newcomen's uneconomical engine.

Q14 When a book slides to a stop on the horizontal rough surface of a table

At the left, a contemporary engraving of a working Newcomen steam engine. In July, 1698 Savery was granted a patent for "A new invention for raising of water and occasioning motion to all sorts of mill work by the impellent force of fire, which will be of great use and advantage for drayning mines, serving townes with water, and for the working of all sorts of mills where they have not the benefitt of water nor constant windes." The patent was good for 35 years and prevented Newcomen from making much money from his superior engine during this period.

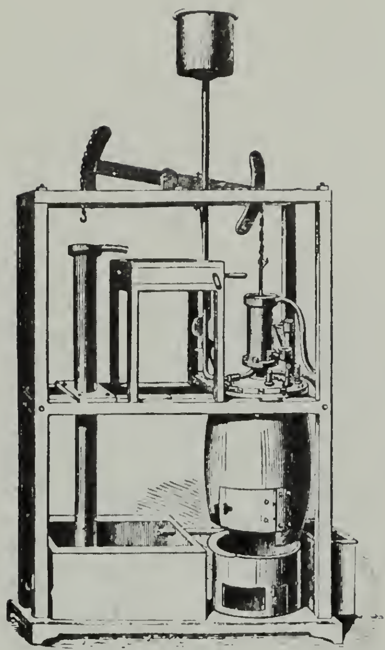
(a) the kinetic energy of the book is transformed into potential energy.
(b) heat is transformed into mechanical energy.
(c) the kinetic energy of the book is transformed into heat energy.
(d) the momentum of the book itself is conserved.

Q15 True or false: The invention of the steam engine depended strongly on theoretical developments in the physics of heat.

Q16 In Savery's steam engine, the ______________ energy of coal was changed (by burning) into ______________ energy which in turn was converted into the ______________ energy of the pump.

10.6 James Watt and the Industrial Revolution

A greatly improved steam engine originated in the work of a Scotsman, James Watt. Watt's father was a carpenter who had a successful business selling equipment to ship owners. Watt was in poor health much of his life and gained most of his early education at home. In



The actual model of the Newcomen engine that inspired Watt to conceive of the separation of condenser and piston.

his father's attic workshop, he developed considerable skill in using tools. He wanted to become an instrument-maker and went to London to learn the trade. Upon his return to Scotland in 1757, he obtained a position as instrument maker at the University of Glasgow.

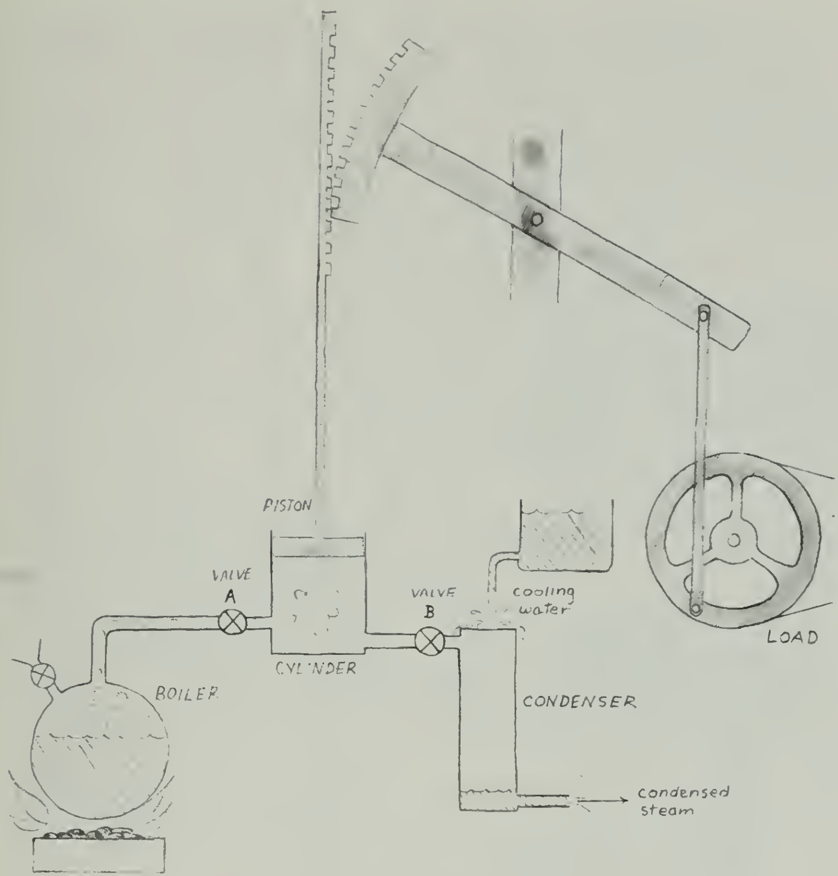
In the winter of 1763-1764, Watt was asked to repair a model of Newcomen's engine that was used for demonstration lectures at the university. As it turned out, this assignment had immense worldwide consequences. In acquainting himself with the model, Watt was impressed by how much steam was required to run the engine. He undertook a series of experiments on the behavior of steam and found that a major problem was the temperature of the cylinder walls. Newcomen's engine wasted most of its heat in warming up the walls of its cylinders. The walls were then cooled again every time cold water was injected to condense the steam.

Early in 1765, Watt remedied this wasteful defect by devising a modified type of steam engine. In retrospect, it sounds like a simple idea. The steam in its cylinder, after pushing the piston up, was admitted to a *separate* container to be condensed. With this system, the cylinder could be kept hot all the time and the condenser could be kept cool all the time.

The diagram opposite represents Watt's engine. With valve **A** open and valve **B** closed, steam under pressure enters the cylinder and pushes the piston upward. When the piston nears the top of the cylinder, valve **A** is closed to shut off the steam supply. Then valve **B** is opened, so that steam leaves the cylinder and enters the condenser. The condenser is kept cool by water flowing over it, so the steam condenses. As steam leaves the cylinder, the pressure there decreases. Atmospheric pressure (helped by the inertia of the flywheel) pushes the piston down. When the piston reaches the bottom of the cylinder, valve **B** is closed and valve **A** is opened, starting the cycle again.

Watt's invention of the separate condenser might seem only a small step in the development of steam engines. But in fact it was a decisive one. Not having to reheat the cylinder again and again allowed huge fuel

Watt in his workshop contemplating a model of a Newcomen engine. (A romanticized engraving from a nineteenth-century volume on technology.)



Schematic diagram of Watt engine.

savings. Watt's engine could do more than twice as much work as Newcomen's with the same amount of fuel. This improvement enabled Watt to make a fortune by selling or renting his engines to mine owners.

The fee that Watt charged for the use of his engines depended on their *power*. Power is defined as the *rate* of doing work (or the rate at which energy is transformed from one form to another). The mks unit of power is the joule-per-second, which is now fittingly called one *watt:*

Matthew Boulton (Watt's business partner) proclaimed to Boswell (the biographer of Samuel Johnson), "I sell here, Sir, what all the world desires to have: POWER!"

$$1 \text{ watt} = 1 \text{ joule/sec}$$

James Watt expressed the power of his engines in different units.

One "foot-pound" is defined as the work done when a force of one pound is exerted on an object while the object moves a distance of one foot. (In mks units, this corresponds roughly to a force of 4 newtons while the object moves $\frac{1}{3}$ meter. Thus, 1 foot-pound is approximately $\frac{4}{3}$ newton-meters.) Watt found that a strong workhorse, working steadily, could lift a 150-pound weight at the rate of almost four feet per second. In other words, it could do about 550 foot-pounds of work per second. Watt used this as a definition of a convenient unit for expressing the power of his engines: the *horsepower*. To this day the "horsepower" unit is used in engineering—although it is now defined as precisely 746 watts.

Typical power ratings (in horsepower)

SG 10.19–10.26

Man turning a crank	0.06 h.p.
Overshot waterwheel	3
Turret windmill	10
Savery steam engine (1702)	1
Newcomen engine (1732)	12
Smeaton's Long Benton engine (1772)	40
Watt engine (of 1778)	14
Cornish engine for London waterworks (1837)	135
Electric power station engines (1900)	1000
Nuclear power station turbine (1970)	300,000

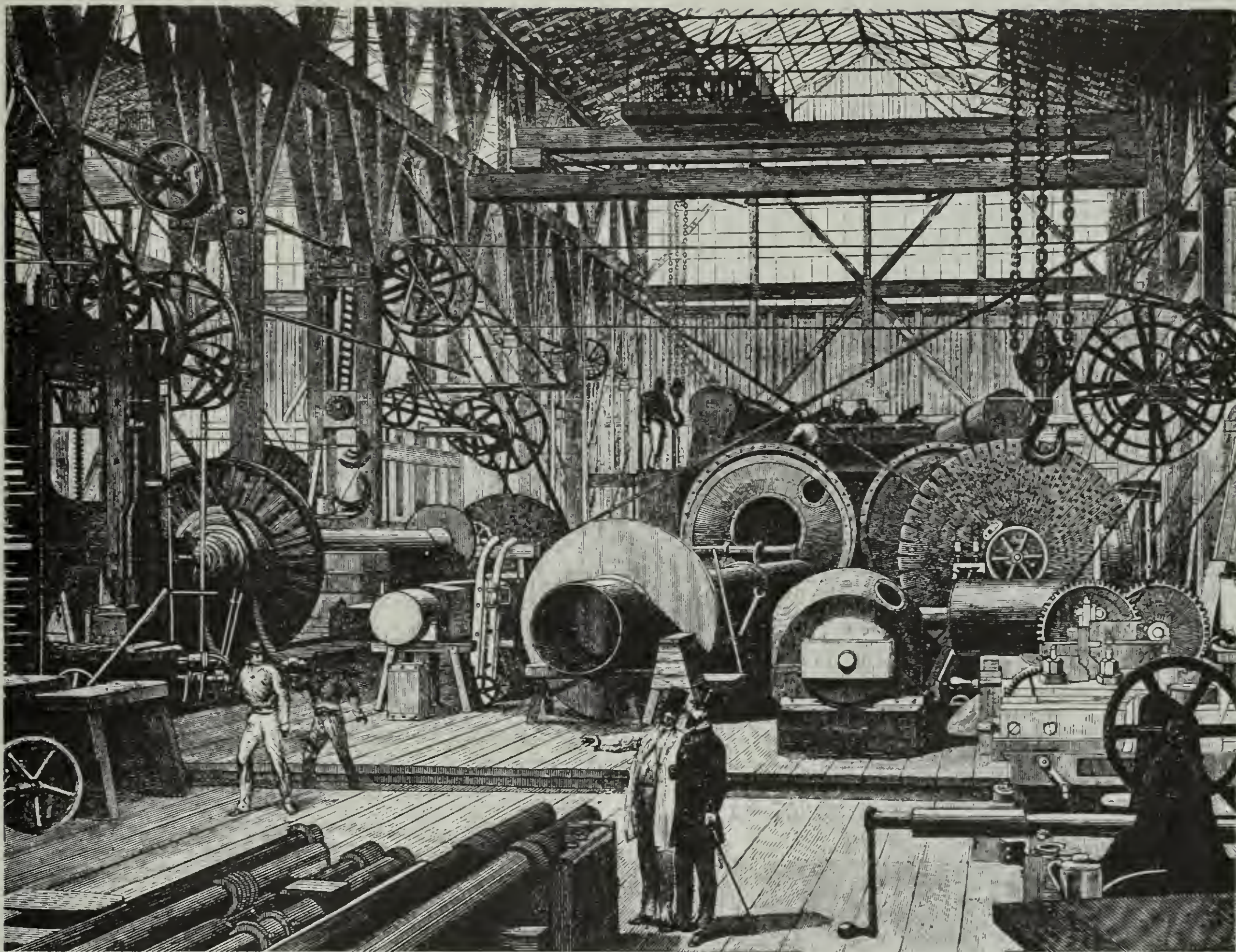
[Adapted from R. J. Forbes, in C. Singer et al, *History of Technology.*]

A steam locomotive from the early part of the 20th century.

Watt's invention, so superior to Newcomen's engine, stimulated the development of engines that could do many other jobs. Steam drove machines in factories, railway locomotives, steamboats, and so forth. Watt's engine gave an enormous stimulus to the growth of industry in Europe and America. It thereby helped transform the economic and social structure of Western civilization.

The widespread development of engines and machines revolutionized mass production of consumer goods, construction, and transportation. The average standard of living in Western Europe and the United States rose sharply. Nowadays it is difficult for most people in the industrially "developed" countries to imagine what life was like before the Industrial Revolution. But not all the effects of industrialization have been beneficial. The nineteenth-century factory system provided an opportunity for some greedy and cruel employers to treat workers almost like slaves. These employers made huge profits, while keeping employees and their families on the edge of starvation. This situation, which was especially serious in England early in the nineteenth century, led to demands for reform. New laws eventually eliminated the worst excesses.

More and more people left the farms—voluntarily or forced by poverty and new land laws—to work in factories. Conflict grew intense between the working class, made up of employees, and the middle class, made up of employers and professional men. At the same time, some artists and intellectuals began to attack the new tendencies of their society. They saw this society becoming increasingly dominated by commerce, machinery, and an emphasis on material goods. In some cases they confused research science itself with its technical applications (as is still done today). In some cases scientists were accused of explaining away all the awesome mysteries of nature. They denounced both science and technology, while often refusing to learn anything about them. In a poem by William Blake we find the questions:

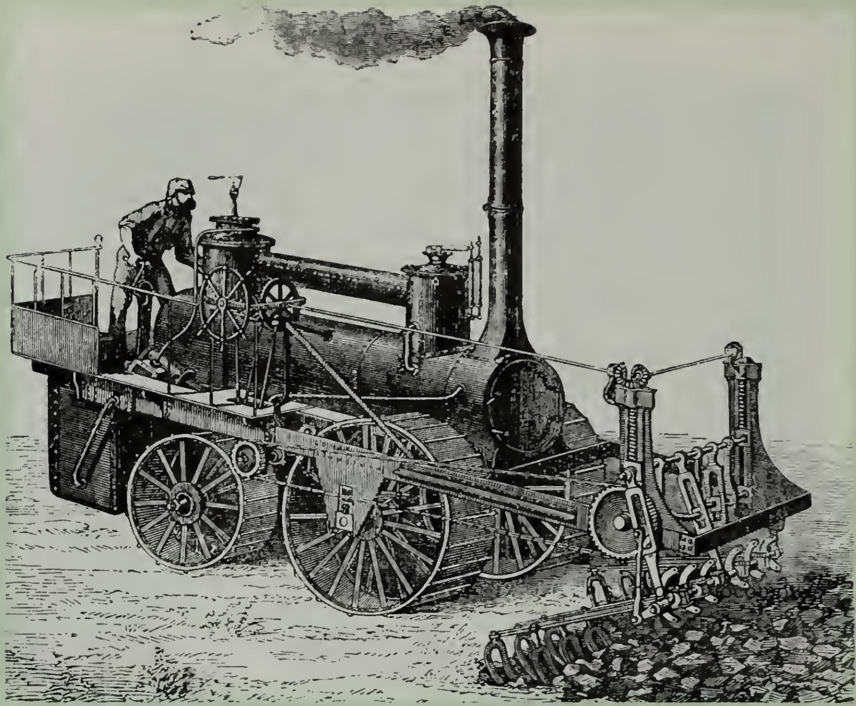


And did the Countenance Divine
  Shine forth upon our clouded hills?
And was Jerusalem builded here
  Among these dark Satanic mills?

Elsewhere, Blake advised his readers "To cast off Bacon, Locke, and Newton." John Keats was complaining about science when he included in a poem the line: "Do not all charms fly/At the mere touch of cold philosophy?" These attitudes are part of an old tradition, going back to the ancient Greek opponents of Democritus' atomism. We saw that Galilean and Newtonian physics also was attacked for distorting values. The same type of accusation can still be heard today.

Steam engines are no longer widely used as direct sources of power in industry and transportation. But steam is indirectly still the major source of power. The steam turbine, invented by the English engineer Charles Parsons in 1884, has now largely replaced older kinds of steam engines. At present, steam turbines drive the electric generators in most electric-

Richard Trevithick's railroad at Euston Square, London, 1809.



A nineteenth-century French steam cultivator.

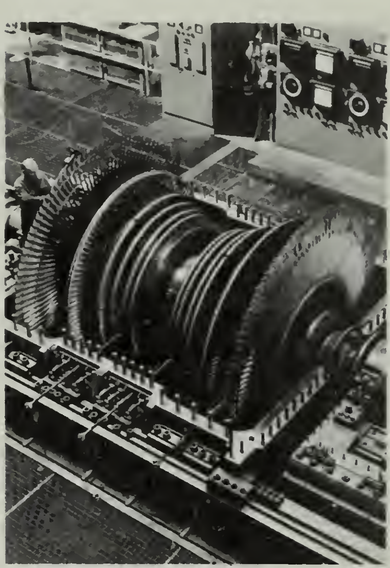
The "Charlotte Dundas," the first practical steamboat, built by William Symington, an engineer who had patented his own improved steam engine. It was tried out on the Forth and Clyde Canal in 1801.

power stations. These steam-run generators supply most of the power for the machinery of modern civilization. Even in nuclear power stations, the nuclear energy is generally used to produce steam, which then drives turbines and electric generators.

The basic principle of the Parsons turbine is simpler than that of the Newcomen and Watt engines. A jet of high-pressure steam strikes the blades of a rotor, driving the rotor around at high speed. A description of the type of steam turbine used in modern power stations shows the change of scale since Heron's toy:

> The boiler at this station [in Brooklyn, New York] is as tall as a 14-story building. It weighs about 3,000 tons, more than a U.S. Navy destroyer. It heats steam to a temperature of 1,050° F and to a pressure of 1,500 pounds per square inch. It generates more than 1,300,000 pounds of steam an hour. This steam runs a turbine to make 150,000 kilowatts of electricity, enough to supply all the homes in a city the size of Houston, Texas. The boiler burns 60 tons (about one carload) of coal an hour.
>
> The 14-story boiler does not rest on the ground. It hangs—all 3,000 tons of it—from a steel framework. Some boilers are even bigger—as tall as the Statue of Liberty—and will make over 3,000,000 pounds of steam in one hour. This steam spins a turbine that will make 450,000 kilowatts of electricity—all of the residential needs for a city of over 4,000,000 people!

Below, a 200 thousand kilowatt turbine being assembled. Notice the thousands of blades on the rotor.



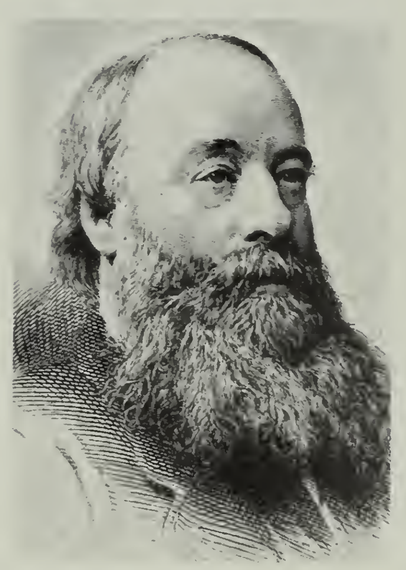
Q17 The purpose of the separate condenser in Watt's steam engine is to

(a) save the water so it can be used again
(b) save fuel by not having to cool and reheat the cylinder
(c) keep the steam pressure as low as possible
(d) make the engine more compact

Q18 The history of the steam engine suggests that the social and economic effects of technology are

(a) always beneficial to everyone
(b) mostly undesirable
(c) unimportant one way or another
(d) very different for different levels of society

Q19 What is horsepower?



James Prescott Joule (1818-1889) Joule was the son of a wealthy Manchester brewer. He is said to have become first interested in his arduous experiments by the desire to develop more efficient engines for the family brewery.

10.7 The experiments of Joule

In the steam engine a certain amount of heat does a certain amount of work. What happens to the heat in doing the work?

Early in the nineteenth century, most scientists and engineers thought that the amount of heat remained constant; and that heat could do work as it passed from a region at one temperature to a region at a lower temperature. For example, early steam engines condensed steam at high temperatures to water at low temperature. Heat was considered to be a substance called "caloric." The total amount of caloric in the universe was thought to be conserved.

According to the caloric theory, heat could do work in much the same way that water can do work. Water falling from a high level to a low level can do work, with the total amount of water used remaining the same. The caloric explanation seemed reasonable. And most scientists accepted it, even though no one measured the amount of heat before and after it did work.

A few scientists, however, disagreed. Some favored the view that heat is a form of energy. One who held this view was the English physicist James Prescott Joule. During the 1840's Joule conducted a long series of experiments designed to show that heat is a form of energy. He hoped to demonstrate in a variety of different experiments that the same decrease in mechanical energy always produced the same amount of heat. This, Joule reasoned, would mean that heat is a form of energy.

The idea of heat as a conserved substance is consistent with many phenomena. An experiment showing this is "Calorimetry" in the *Handbook*.

For one of his early experiments he constructed a simple electric generator, which was driven by a falling weight. The electric current that was generated heated a wire. The wire was immersed in a container of water which it heated. From the distance that the weight descended he calculated the work done (the decrease in gravitational potential energy). The product of the mass of the water and its temperature rise gave him a measure of the amount of heat produced. In another experiment he compressed gas in a bottle immersed in water, measuring the amount of work done to compress the gas. He then measured the amount of heat given to the water as the gas got hotter on compression.

But his most famous experiments involved an apparatus in which slowly descending weights turned a paddle-wheel in a container of water. Owing to the friction between the wheel and the liquid, work was done on the liquid, raising its temperature.

Joule repeated this experiment many times, constantly improving the apparatus and refining his analysis of the data. He learned to take very great care to insulate the container so that heat was not lost to the room. He measured the temperature rise with a precision of a small fraction of a degree. And he allowed for the small amount of kinetic energy the descending weights had when they reached the floor.

Joule published his results in 1849. He reported:

> 1st. That the quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of [energy] expended. And 2nd. That the quantity of heat capable of increasing the temperature of a pound of water . . . by 1° Fahr. requires for its evolution the expenditure of a mechanical energy represented by the fall of 772 lb through the distance of one foot.

Joule used the word "force" instead of "energy." The current scientific vocabulary was still being formed.

The first statement is the evidence that heat is a form of energy, contrary to the caloric theory. The second statement gives the numerical magnitude of the ratio he had found. This ratio related a unit of mechanical energy (the foot-pound) and a unit of heat (the heat required to raise the temperature of one pound of water by one degree on the Fahrenheit scale).

This unit is called a *British Thermal Unit* (BTU)

In the mks system, the unit of heat is the kilocalorie and the unit of mechanical energy is the joule. Joule's results are equivalent to the statement that 1 kilocalorie equals 4,150 joules. Joule's paddle-wheel experiment and other basically similar ones have since been performed with great accuracy. The currently accepted value for the "mechanical equivalent of heat" is

SG 10.27, 10.28

A kilocalorie is what some dictionaries call "large calorie." It is the amount of heat required to raise the temperature of 1 kilogram of water by 1° Celsius ("centigrade"). This unit is identical to the "Calorie" (with a capital C) used to express the energy content of foods in dietetics.

$$1 \text{ kilocalorie} = 4{,}184 \text{ joules}$$

We might, therefore, consider heat to be a form of energy. We will consider the nature of the "internal" energy associated with temperature further in Chapter 11.

Joule's finding a value for the "mechanical equivalent of heat" made it possible to describe engines in a new way. The concept of *efficiency* applies to an engine or any device that transforms energy from one form to another. Efficiency is defined as the percentage of the input energy that appears as useful output. Since energy is conserved, the greatest possible efficiency is 100%—when *all* of the input energy appears as useful output. Obviously, efficiency must be considered as seriously as power output in designing engines. However, there are theoretical limits on efficiency. Thus, even a perfectly designed machine could never do work at 100% efficiency. We will hear more about this in Chapter 11.

The efficiency of a steam engine is roughly 15–20%; for an automobile it is about 22%, and for a diesel engine it is as high as 40%.

In Sec. 10.10 we mention some qualifications that must be placed on the simple idea of heat as a form of energy.

Q20 According to the caloric theory of heat, caloric
(a) can do work when it passes between two objects at the same temperature
(b) is another name for temperature
(c) is produced by steam engines
(d) is a substance that is conserved

Q21 The kilocalorie is
(a) a unit of temperature
(b) a unit of energy
(c) 1 kilogram of water at 1°C
(d) one pound of water at 1°F

Q22 In Joule's paddle-wheel experiment, was all the change of gravitational potential energy used to heat the water?

10.8 Energy in biological systems

All living things need a supply of energy to maintain life and to carry on their normal activities. Human beings are no exception; like all animals, we depend on food to supply us with energy.

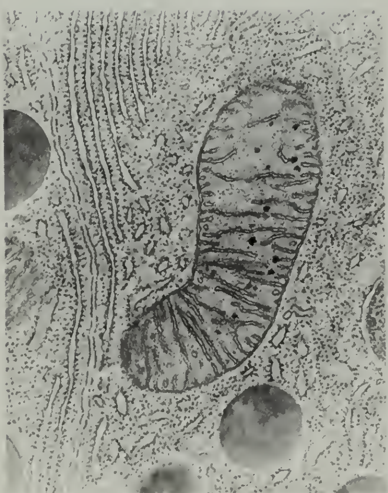
Most human beings are omnivores; that is, they eat both animal and plant materials. Some animals are herbivores, eating only plants, while others are carnivores, eating only animal flesh. But all animals, even carnivores, ultimately obtain their food energy from plant material. The animal eaten by the lion has previously dined on plant material, or on another animal which had eaten plants.

Green plants obtain energy from sunlight. Some of that energy is used by the plant to perform the functions of life. Much of the energy is used to make carbohydrates out of water (H_2O) and carbon dioxide (CO_2). The energy used to synthesize carbohydrates is not lost; it is stored in the carbohydrate molecules as chemical energy.

Carbohydrates are molecules made of carbon, hydrogen, and oxygen. A simple example is the sugar glucose, the chemical formula for which is $C_6H_{12}O_6$.

The process by which plants synthesize carbohydrates is called photosynthesis. It is still not completely understood and research in this field is lively. We know that the synthesis takes place in many small steps, and many of the steps are well understood. It is conceivable that we may learn how to photosynthesize carbohydrates without plants thus producing food economically for the rapidly increasing world population. The overall process of producing carbohydrates (the sugar glucose, for example) by photosynthesis can be represented as follows:

$$\text{carbon dioxide} + \text{water} + \text{energy} \longrightarrow \text{glucose} + \text{oxygen}$$



Electron micrograph of an energy-converting mitochondrion in a bat cell (200,000 times actual size).

The energy stored in the glucose molecules is used by the animal that eats the plant. This energy maintains the body temperature, keeps its heart, lungs, and other organs operating, and enables various chemical reactions to occur in the body. The animal also uses it to do work on external objects. The process by which the energy stored in sugar molecules is made available to the cell is very complex. It takes place mostly in tiny bodies called mitochondria, which are found in all cells. Each mitochondrion contains enzymes which, in a series of about ten steps, split glucose molecules into simpler molecules. In another sequence of reactions these molecules are oxidized (combined with oxygen), thereby releasing most of the stored energy and forming carbon dioxide and water.

$$\text{glucose} + \text{oxygen} \longrightarrow \text{carbon dioxide} + \text{water} + \text{energy}$$

Proteins and fats are used to build and restore tissue and enzymes, and to pad delicate organs. They also can be used to provide energy. Both proteins and fats can enter into chemical reactions which produce the same molecules as the split carbohydrates. From that point, the energy-releasing process is the same as in the case of carbohydrates.

The released energy is used to change a molecule called adenosine diphosphate (ADP) into adenosin triphosphate (ATP). In short, chemical energy originally stored in glucose molecules in plants is eventually stored as chemical energy in ATP molecules in animals. The ATP molecules pass out of the mitochondrion into the body of the cell. Wherever energy is needed in the cell, it can be supplied by an ATP molecule. As it releases its stored energy, the ATP changes back to ADP. Later, back in a mitochondrion, the ADP is reconverted to energy-rich ATP.

The overall process in the mitochondrion involves breaking glucose, in the presence of oxygen, into carbon dioxide and water. The energy released is transferred to ATP and stored there until needed by the animal's body.

The chemical and physical operations of the living body are in some ways like those in an engine. Just as a steam engine uses chemical energy stored in coal or oil, the body uses chemical energy stored in food. In both cases the fuel is oxidized to release its stored energy. The oxidation is vigorous in the steam engine, and gentle, in small steps, in the body. In both the steam engine and the body, some of the input energy is used to do work; the rest is used up internally and eventually "lost" as heat to the surroundings.

Some foods supply more energy per unit mass than others. The energy stored in food is usually measured in kilocalories. (1 kilocalorie = 10^3 calories). However it could just as well be measured in joules or foot-pounds or British Thermal Units. The table in the margin gives the energy content of some foods. (The "calorie" or "large calorie" used by dieticians, is identical to what we have defined as the kilocalorie.)

Much of the energy you obtain from food keeps your body's internal "machinery" running and keeps your body warm. Even when asleep your body uses about one kilocalorie every minute. This amount of energy is needed just to keep alive.

To do work, you need more energy. Yet only a fraction of this energy can be used to do work; the rest is wasted as heat. Like any engine, the body of humans or other animals is not 100% efficient. Its efficiency when it does work varies with the job and the physical condition and skill of the worker. But efficiency probably never exceeds 25%, and usually is less. Studies of this sort are carried out in *bioenergetics,* one of the many fascinating and useful fields where physics and biology overlap.

The table in the margin gives the results of experiments done in the United States of the rate at which a healthy young person of average build and metabolism uses energy in various activities. The estimates were made by measuring the amount of carbon dioxide exhaled. Thus, they show the total amount of food energy used, including the amount necessary just to keep the body functioning.

According to this table, if the subject did nothing but sleep for eight hours a day and lie quietly the rest of the time, he would still need at least 1,700 kilocalories of energy each day. There are countries where large numbers of working people exist on less than 1,700 kilocalories a day. The U.N. Yearbook of National Accounts Statistics for 1964 shows that in India the average food intake was about 1,600 kilocalories per day. The United States average was 3,100 kilocalories per day. About half the population of Southeast Asia is at or below the starvation line. Vast numbers of people elsewhere in the world, including some parts of the United States, are also close to that line. It is estimated that if the available food were equally distributed among all the earth's inhabitants, each would have about 2,400 kilocalories a day on the average. This is only a little more than the minimum required by a working person.

It is now estimated that at the current rate of increase, the population of the world may double in 30 years. Thus by the year 2000 it would be 7 billion or more. Furthermore, the *rate* at which the population is increasing is itself increasing! Meanwhile, the production of food supply per person has not increased markedly on a global scale. For example, in

Approximate Energy Content of Various Foods (In Calories per kilogram)

Butter	7000
Chocolate (sweetened)	5000
Beef (hamburger)	4000
Bread	2600
Milk (whole)	700
Apples (raw)	500
Lettuce	150

Adapted from U.S. Department of Agriculture, Agriculture Handbook No. 8, June 1950.

Approximate Rates of Using Energy During Various Activities (In Calories per hour)

Sleeping	70
Lying down (awake)	80
Sitting still	100
Standing	120
Typewriting rapidly	140
Walking (3 mph)	220
Digging a ditch	400
Running fast	600
Rowing in a race	1200

Adapted from a handbook of the U.S. Department of Agriculture.

SG 10.29-10.31

SG 10.32

the last ten years the increase in crop yield per acre in the poorer countries has averaged less than one percent per year, far less than the increase in population. The problem of supplying food energy for the world's hungry is one of the most difficult problems facing humanity today.

The physics of energy transformations in biological processes is one example of a lively interdisciplinary field, namely biophysics (where physics, biology, chemistry, and nutrition all enter). Another connection to physics is provided by the problem of inadequate world food supply. Here, too, many physicists, with others, are presently trying to provide solutions through work using their special competence.

In this problem of life-and-death importance, what are the roles science and technology can play? Obviously, better agricultural practice should help, both by opening up new land for farming and by increasing production per acre on existing land. The application of fertilizers can increase crop yields, and factories that make fertilizers are not too difficult to build. But right here we meet a general law on the use of applications of science through technology: Before applying technology, study all the consequences that may be expected; otherwise you may create two new problems for every old one that you wish to "fix."

In any particular country, the questions to ask include these: How will fertilizers interact with the plant being grown and with the soil? Will some of the fertilizer run off and spoil rivers and lakes and the fishing industry in that locality? How much water will be required? What variety of the desired plant is the best to use within the local ecological framework? How will the ordinary farmer be able to learn the new techniques? How will he be able to pay for using them?

Upon study of this sort it may turn out that in addition to fertilizer, a country may need just as urgently a better system of bank loans to small farmers, and better agricultural education to help the farmer. Such training has played key roles in the rapid rise of productivity in the richer countries. Japan, for example, produces 7,000 college graduate agriculturalists each year. All of Latin America produces only 1,100 per year. In Japan there is one farm advisor for each 600 farms. Compare this with perhaps one advisor for 10,000 farms in Colombia, and one advisor per 100,000 farms in Indonesia.

But for long-run solutions, the problem of increasing food production in the poorer countries goes far beyond changing agricultural practices. Virtually all facets of the economies and cultures of the affected countries are involved. Important factors range from international economic aid and internal food pricing policies to urbanization, industrial growth, public health, and family planning practice.

Where, in all this, can the research scientist's contribution come in to help? It is usually true that one of the causes of some of the worse social problems is ignorance, including the absence of specific scientific knowledge. For example, knowledge of how food plants can grow efficiently in the tropics is lamentably sparse. Better ways of removing salt from sea water or brackish ground water are needed to allow irrigating fields with water from these plentiful sources. But before this will be economically possible, more basic knowledge will be needed on just how the molecules in liquids are structured, and how molecules move through membranes of the sort usable in de-salting equipment. Answers to such questions, and many like them, can only come through research in "pure" science, from trained research workers having access to adequate research facilities.

"The Repast of the Lion"
by Henri Rousseau
The Metropolitan Museum of Art

Q23 Animals obtain the energy they need from food, but plants

(a) obtain energy from sunlight
(b) obtain energy from water and carbon dioxide
(c) obtain energy from seeds
(d) do not need a supply of energy

Q24 The human body has an efficiency of about 20%. This means that

(a) only one-fifth of the food you eat is digested
(b) four-fifths of the energy you obtain from food is destroyed
(c) one-fifth of the energy you obtain from food is used to run the "machinery" of the body
(d) you should spend 80% of each day lying quietly without working
(e) only one-fifth of the energy you obtain from food can be used to enable your body to do work on external objects

Q25 Explain this statement: "The repast of the lion is sunlight."

10.9 Arriving at a general law

In Section 10.3 we introduced the law of conservation of *mechanical* energy. This law applies only in situations where no mechanical energy is transformed into heat energy or *vice versa*. But early in the nineteenth century, developments in science, engineering and philosophy suggested new ideas about energy. It appeared that all forms of energy (including heat) could be transformed into one another with no loss. Therefore the total amount of energy in the universe must be constant.

Volta's invention of the electric battery in 1800 showed that chemical reactions could produce electricity. It was soon found that electric currents could produce heat and light. In 1820, Hans Christian Oersted, a Danish physicist, discovered that an electric current produces magnetic effects. And in 1831, Michael Faraday, the great English scientist, discovered electromagnetic induction: the effect that when a magnet moves near a coil or a wire, an electric current is produced in the coil or wire. To some thinkers, these discoveries suggested that all the phenomena of nature were somehow united. Perhaps all natural events resulted from the same basic "force." This idea, though vague and imprecise, later bore fruit in the form of the law of conservation of energy. All natural events involve a transformation of energy from one form to another. But the total *quantity* of energy does not change during the transformation.

Joule began his long series of experiments by investigating the "duty" of electric motors. In this case duty was measured by the work the motor could do when a certain amount of zinc was used up in the battery that ran the motor. Joule's interest was to see whether motors could be made economically competitive with steam engines.

The invention and use of steam engines helped to establish the law of conservation of energy by showing how to measure energy changes. Almost from the beginning, steam engines were rated according to a quantity termed their "duty." This term referred to how heavy a load an engine could lift using a given supply of fuel. In other words, the test was how much *work* an engine could do for the price of a ton of coal. This very practical approach is typical of the engineering tradition in which the steam engine was developed.

The concept of work began to develop about this time as a measure of the amount of energy transformed from one form to another. (The actual words "work" and "energy" were not used until later.) This made possible quantitative statements about the transformation of energy. For example, Joule used the work done by descending weights as a measure of the amount of gravitational potential energy transformed into heat energy.

In 1843, Joule had stated that whenever a certain amount of mechanical energy seemed to disappear, a definite amount of heat always appeared. To him, this was an indication of the conservation of what we now call energy. Joule said that he was

> . . . satisfied that the grand agents of nature are by the Creator's fiat *indestructible;* and that, wherever mechanical [energy] is expended, an exact equivalent of heat is *always* obtained.

Having said this, Joule got back to his work in the laboratory. He was basically a practical man who had little time to speculate about a deeper philosophical meaning of his findings. But others, though using speculative arguments, were also concluding that the total amount of energy in the universe is constant.

A year before Joule's remark, for example, Julius Robert Mayer, a German physician, had proposed a general law of conservation of energy. Unlike Joule, Mayer had done no quantitative experiments. But he had observed body processes involving heat and respiration. And he had used other scientists' published data on the thermal properties of air to calculate the mechanical equivalent of heat. (Mayer obtained about the same value that Joule did.)

Johann Wolfgang von Goethe (1749-1832)
Goethe thought that his color theory (which most modern scientists consider useless) exceeded in importance all his literary works.

Mayer had been influenced strongly by the German philosophical school now known as *Naturphilosophie* or "nature-philosophy." This movement flourished in Germany during the late eighteenth and early nineteenth centuries. (See also the Epilogue to Unit 2.) Its most influential leaders were Johann Wolfgang von Goethe and Friedrich von Schelling. Neither of these men is known today as a scientist. Goethe is generally considered Germany's greatest poet and dramatist, while Schelling is remembered as a minor philosopher. But both men had great influence on the generation of German scientists educated at the beginning of the nineteenth century. The nature-philosophers were closely associated with the Romantic movement in literature, art, and music. The Romantics protested against the idea of the universe as a great machine. This idea, which had arisen after Newton's success in the seventeenth century, seemed morally empty and artistically worthless to them. The nature-philosophers also detested the mechanical world view. They refused to believe that the richness of natural phenomena—including human intellect, emotions, and hopes—could be understood as the result of the motions of particles.

Friedrich von Schelling (1775-1854)

At first glance, nature-philosophy would seem to have little to do with the law of conservation of energy. That law is practical and quantitative, whereas nature-philosophers tended to be speculative and qualitative. But nature-philosophy did insist on the value of searching for the underlying reality of nature. And this attitude did influence the discovery of the law of conservation of energy. Also, the nature-philosophers believed that the various phenomena of nature—gravity, electricity, magnetism, etc.—are not really separate from one another. Rather, they are simply different forms of one basic "force." This philosophy encouraged scientists to look for connections between different "forces" (or, in modern terms, between different forms of energy). It is perhaps ironic that in this way, it stimulated the experiments and theories that led to the law of conservation of energy.

The nature-philosophers claimed that nature could be understood as it "really" is only by direct observation. But no complicated "artificial" apparatus must be used—only feelings and intuitions. Goethe and Schelling were both very much interested in science and thought that their philosophy could uncover the hidden, inner meaning of nature. For Goethe the goal was "That I may detect the inmost force which binds the world, and guides its course."

By the time conservation of energy was established and generally accepted, however, nature-philosophy was no longer popular. Scientists who had previously been influenced by it, including Mayer, now strongly opposed it. In fact, some hard-headed scientists at first doubted the law of conservation of energy simply because of their distrust of nature-philosophy. For example, William Barton Rogers, founder of the Massachusetts Institute of Technology, wrote in 1858:

> To me it seems as if many of those who are discussing this question of the conservation of force are plunging into the fog of mysticism.

However, the law was so quickly and successfully put to use in physics that its philosophical origins were soon forgotten.

This episode is a reminder of a lesson we learned before: In the ordinary day-to-day work of scientists, experiment and mathematical theory are the usual guides. But in making a truly major advance in science, philosophical speculation often also plays an important role.

Mayer and Joule were only two of at least a dozen people who, between 1832 and 1854, proposed in some form the idea that energy is conserved. Some expressed the idea vaguely; others expressed it quite clearly. Some arrived at their belief mainly through philosophy: others from a practical concern with engines and machines, or from laboratory investigations; still others from a combination of factors. Many, including Mayer and Joule, worked quite independently of one another. The idea of energy conservation was somehow "in the air," leading to essentially simultaneous, separate discovery.

Hermann von Helmholtz (1821-1894)

Helmholtz's paper "Zur Erhaltung der Kraft" was tightly reasoned and mathematically sophisticated. It related the law of conservation of energy to the established principles of Newtonian mechanics and thereby helped make the law scientifically respectable.

The wide acceptance of the law of conservation of energy owes much to the influence of a paper published in 1847. This was two years before Joule published the results of his most precise experiments. The author, a young German physician and physicist named Hermann von Helmholtz, entitled his work "On the Conservation of Force." Helmholtz boldly asserted the idea that others were only vaguely expressing; namely, "that it is impossible to create a lasting motive force out of nothing." He restated this theme even more clearly many years later in one of his popular lectures:

> We arrive at the conclusion that Nature as a whole possesses a store of force which cannot in any way be either increased or diminished, and that, therefore, the quantity of force in Nature is just as eternal and unalterable as the quantity of matter. Expressed in this form, I have named the general law 'The Principle of the Conservation of Force.'

Any machine or engine that does work (provides energy) can do so only by drawing from some source of energy. The machine cannot supply more energy than it obtains from the source. When the source runs out, the machine will stop working. Machines and engines can only *transform* energy; they cannot create it or destroy it.

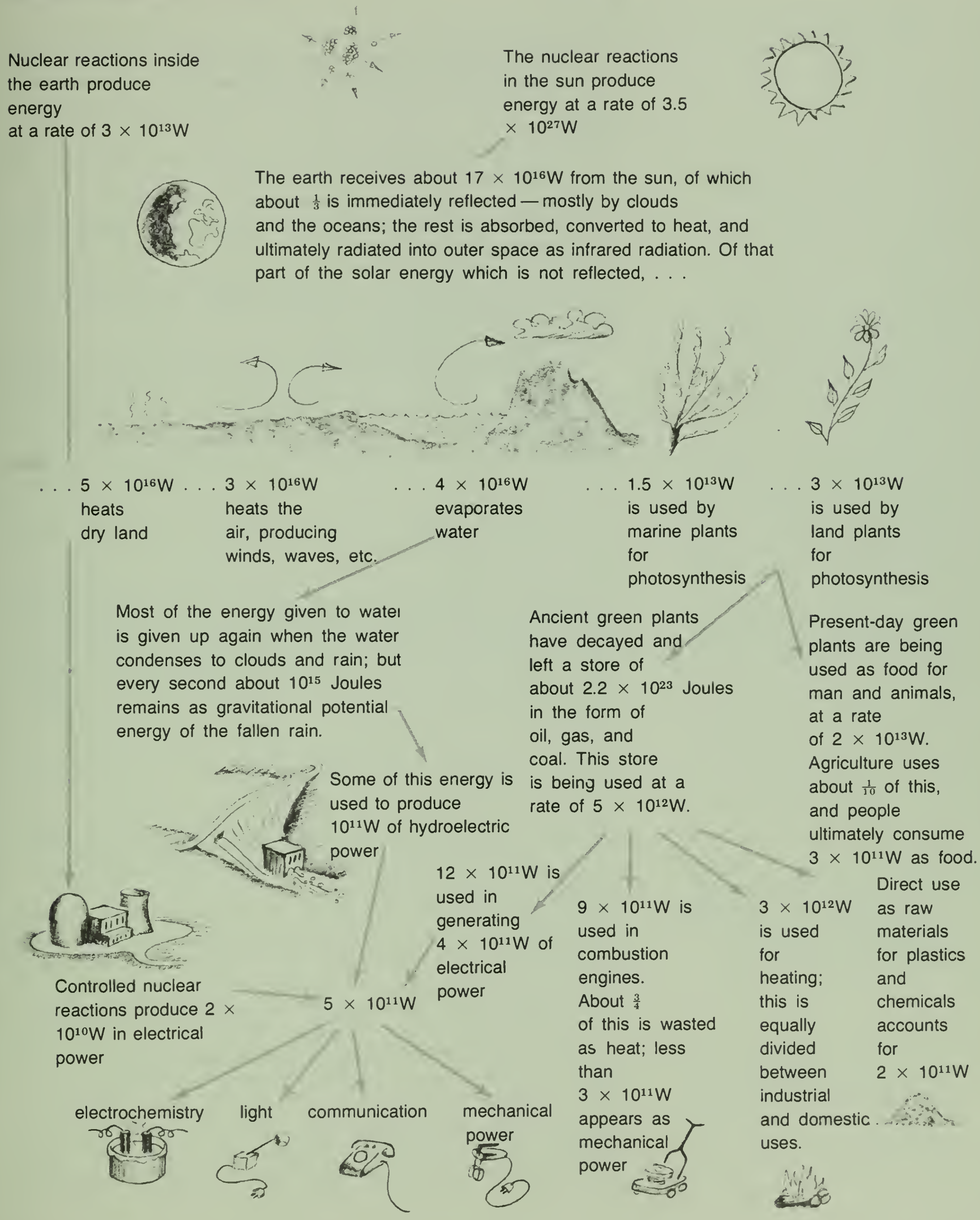
Q26 The significance of German nature philosophy in the history of science is that it

SG 10.33

(a) was the most extreme form of the mechanistic viewpoint

(b) was a reaction against excessive speculation

Energy Conservation on Earth



(c) stimulated speculation about the unity of natural phenomena

(d) delayed progress in science by opposing Newtonian mechanics

Q27 Discoveries in electricity and magnetism early in the nineteenth century contributed to the discovery of the law of conservation of energy because

(a) they attracted attention to the transformation of energy from one form to another

(b) they made it possible to produce more energy at less cost

(c) they revealed what happened to the energy that was apparently lost in steam engines

(d) they made it possible to transmit energy over long distances

Q28 The development of steam engines helped the discovery of the law of conservation of energy because

(a) steam engines produce a large amount of energy

(b) the caloric theory could not explain how steam engines worked

(c) the precise idea of work was developed to rate steam engines

(d) the internal energy of a steam engine was always found to be conserved

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10.10 A precise and general statement of energy conservation

We can now try to pull many of the ideas in this chapter together into a precise statement of the law of conservation of energy. It would be pleasingly simple to call heat "internal" energy associated with temperature. We could then add heat to the potential and kinetic energy of a system, and call this sum the total energy that is conserved. In fact this works well for a great variety of phenomena, including the experiments of Joule. But difficulties arise with the idea of the heat "content" of a system. For example, when a solid is heated to its melting point, further heat input causes melting *without increasing the temperature*. (You may have seen this in the experiment on Calorimetry.) So simply adding the idea of heat as one form of a system's energy will not give us a complete general law. To get *that*, we must invent some additional terms with which to think.

If you do not want to know what the detailed difficulties are, you can skip to the conclusion in the last paragraph on the next page.

Instead of "heat," let us use the idea of an *internal energy*, an energy in the system that may take forms not directly related to temperature. We can then use the word "heat" to refer only to a *transfer* of energy between a system and its surroundings. (In a similar way, the term *work* is not used to describe something contained in the system. Rather, it describes the transfer of energy from one system to another.)

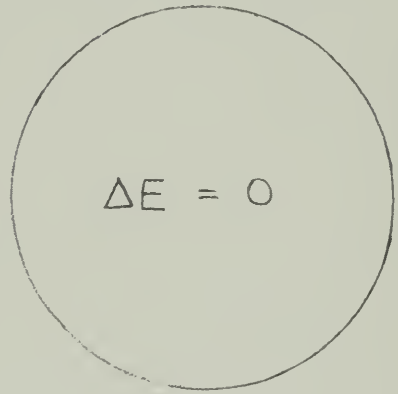
The word "heat" is used rather loosely, even by physicists. This restriction on its meaning is not necessary in most contexts, but it is important for the discussion in this section.

Yet even these definitions do not permit a simple statement like "heat input to a system increases its internal energy, and work done on a system increases its mechanical energy." For heat input to a system can have effects other than increasing internal energy. In a steam engine, for example, heat input increases the mechanical energy of the piston. Similarly, *work* done on a system can have effects other than increasing mechanical energy. In rubbing your hands together, for example, the work you do increases the internal energy of the skin of your hands.

Therefore, a general conservation law of energy must include *both* work and heat transfer. Further, it must deal with change in the *total energy* of a system, not with a "mechanical" part and an "internal" part.

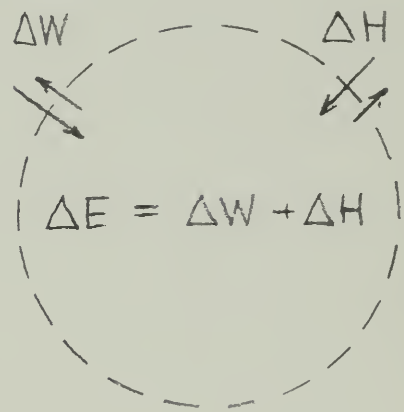
As we mentioned before in discussing conservation laws, such laws can be expressed in two ways: (a) in terms of an isolated system, in which the total quantity of something does not change, or (b) in terms of how to measure the increases and decreases of the total quantity in an open (or non-isolated) system. The two ways of expressing the law are logically related by the definition of "isolated." For example, conservation of momentum can be expressed either: (a) If no net outside force acts on a system, then the total $m\vec{v}$ of the system is constant; or (b) If a net outside force $\vec{F}$ acts on a system for a time Δt, the change in the total $m\vec{v}$ of the system is $\vec{F} \times \Delta t$. In (a), the absence of the net force is a condition of isolation. In (b), one describes how the presence of a net force affects momentum. Form (b) is obviously more generally useful.

Special case of an isolated system:



A similar situation exists for the law of conservation of energy. We can say that the total energy of a system remains constant if the system is isolated. (By isolated we mean that no work is done on or by the system, and no heat passes between the system and its surroundings.) Or we can say that the *change* in energy of a *non*-isolated system is equal to the net work done on the system plus the net heat added to it. More precisely, we can let ΔW stand for the net work on the system, which is all the work done *on* the system minus all the work done *by* the system. We can let ΔH represent the net heat transfer to the system, or the heat added to the system minus the heat lost by the system. Then the change in *total* energy of the system, ΔE, is given by

In general:



$$\Delta E = \Delta W + \Delta H$$

This is a simple and useful form of the law of conservation of energy, and is sometimes called *the first law of thermodynamics*.

Thermodynamics is the study of the relation between heat and mechanical energy.

This general expression includes as special cases the preliminary versions of the conservation law given earlier in the chapter. If there is no heat transfer at all, then $\Delta H = 0$, and so $\Delta E = \Delta W$. In this case, the change in energy of a system equals the net work done on it. On the other hand, if work is done neither on nor by a system, then $\Delta W = 0$, and $\Delta E = \Delta H$. Here the change in energy of a system is equal to the net heat transfer.

We still lack a description of the part of the total energy of a system that we have called heat (or better, "internal" energy). So far we have seen only that an increase in internal energy is sometimes associated with an increase in temperature. We also mentioned the long-held suspicion that internal energy involves the motion of the "small parts" of bodies. We will take up this problem in detail in Chapter 11.

SG 10.34–10.38

Q29 The first law of thermodynamics is
(a) true only for steam engines
(b) true only when there is no friction
(c) a completely general statement of conservation of energy
(d) the only way to express conservation of energy

Q30 Define ΔE, ΔW, and ΔH for a system.

Q31 What two ways are there for changing the total energy of a system?

10.11 Faith in the conservation of energy

For over a century, the law of conservation of energy has stood as one of the most fundamental laws of science. We encounter it again and again in this course, in studying electricity and magnetism, the structure of the atom, and nuclear physics. Throughout the other sciences, from chemistry to biology, and throughout engineering studies, the same law applies. Indeed, no other law so clearly brings together the various scientific fields, giving all scientists a common set of concepts.

The principle of conservation of energy has been immensely successsful. It is so firmly believed that it seems almost impossible that any new discovery could disprove it. Sometimes energy seems to appear or disappear in a system, without being accounted for by changes in known forms of energy. In such cases, physicists prefer to assume that some hitherto unknown kind of energy is involved, rather than consider seriously the possibility that energy is not conserved. We have already pointed out Leibniz's proposal that energy could be dissipated among "the small parts" of bodies. He advanced this idea specifically in order to maintain the principle of conservation of energy in inelastic collisions and frictional processes. His faith in energy conservation was justified. Other evidence showed that "internal energy" changed by just the right amount to explain observed changes in external energy.

SG 10.39, 10.40

Another recent example is the "invention" of the neutrino by the physicist Wolfgang Pauli in 1933. Experiments had suggested that energy disappeared in certain nuclear reactions. But Pauli proposed that a tiny particle, named the "neutrino" by Enrico Fermi, was produced in these reactions. Unnoticed, the neutrino carried off some of the energy. Physicists accepted the neutrino theory for more than twenty years even though neutrinos could not be detected by any method. Finally, in 1956, neutrinos were detected in experiments using the radiation from a nuclear reactor. (The experiment could not have been done in 1933, since no nuclear reactor existed until nearly a decade later.) Again, faith in the law of conservation of energy turned out to be justified.

The theme of "conservation" is so powerful in science that we believe it will always be justified. We believe that any apparent exceptions to the law will sooner or later be understood in a way which does not require us to give up the law. At most, they may lead us to discover new forms of energy making the law even more general and powerful.

The French mathematician and philosopher Henri Poincaré expressed this idea back in 1903 in his book *Science and Hypothesis:*

> . . . the principle of conservation of energy signifies simply that there is *something* which remains constant. Indeed, no matter what new notions future experiences will give us of the world, we are sure in advance that there will be something which will remain constant, and which we shall be able to call *energy*.

Today it is agreed that the discovery of conservation laws was one of the most important achievements of science. They are powerful and valuable tools of analysis. All of them basically affirm that, whatever happens within a system of interacting bodies, certain measurable quantities will remain constant as long as the system remains isolated.

The list of known conservation laws has grown in recent years. The area of fundamental (or "elementary") particles has yielded much of this new knowledge. Some of the newer laws are imperfectly and incompletely understood. Others are on uncertain ground and are still being argued.

Below is a list of conservation laws as it now stands. One cannot say that the list is complete or eternal. But it does include the conservation laws that make up the working tool-kit of physicists today. Those which are starred are discussed in the basic text portions of this course. The others are treated in supplemental (optional) units, for example, the Supplemental Unit entitled *Elementary Particles*.

1. Linear momentum*
2. Energy (including mass)*
3. Angular momentum (including spin)
4. Charge*
5. Electron-family number
6. Muon-family number
7. Baryon-family number
8. Strangeness number
9. Isotopic spin

Numbers 5 through 9 result from work in nuclear physics, high energy physics, or elementary or fundamental particle physics. If this aspect interests you, you will find the essay "Conservation Laws" (in the Reader entitled *The Nucleus*) worth reading at this stage. The first seven of the laws in the above listing are discussed in this selection.

10.1 The Project Physics materials particularly appropriate for Chapter 10 include:

Experiments
- Conservation of Energy
- Measuring the Speed of a Bullet
- Temperature and Thermometers
- Calorimetry
- Ice Calorimetry

Activities
- Student Horsepower
- Steam Powered Boat
- Predicting the Range of an Arrow

Film Loops
- Finding the Speed of a Rifle Bullet – I
- Finding the Speed of a Rifle Bullet – II
- Recoil
- Colliding Freight Cars
- Dynamics of a Billiard Ball
- A Method of Measuring Energy – Nail Driven into Wood
- Gravitational Potential Energy
- Kinetic Energy
- Conservation of Energy – Pole Vault
- Conservation of Energy – Aircraft Takeoff

Reader Articles
- The Steam Engine Comes of Age
- The Great Conservation Principles

Transparencies
- Slow Collisions
- The Watt Engine

10.2 A man carries a heavy load across the level floor of a building. Draw an arrow to represent the force he applies to the load, and one to represent the direction of his motion. By the definition of work given, how much work does he do on the load? Do you feel uncomfortable about this result? Why?

10.3 The speed of an object is always *relative* – that is, it will be different when measured from different reference frames. Since kinetic energy depends on speed, it too is only a relative quantity. If you are interested in the idea of the relativity of kinetic energy, consider this problem: An object of mass m is accelerated uniformly by a force F through a distance d, changing its speed from v_1 to v_2. The work done, Fd, is equal to the change in kinetic energy $\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. (For simplicity, assume the case of motion in only one direction along a straight line.) Now: describe this event from a reference frame which is itself moving with speed u along the same direction.

(a) What are the speeds as observed in the new reference frame?

(b) Are the kinetic energies observed to have the same value in both reference frames?

(c) Does the *change* in kinetic energy have the same value?

(d) Is the calculated amount of work the same? Hint: by the Galilean relativity principle, the magnitude of the acceleration – and therefore force – will be the same when viewed from frames of reference moving uniformly relative to each other.)

(e) Is the change in kinetic energy still equal to the work done?

(f) Which of the following are "invariant" for changes in reference frame (moving uniformly relative to one another)?
  i. the quantity $\frac{1}{2}mv^2$
  ii. the quantity Fd
  iii. the relationship $Fd = \Delta(\frac{1}{2}mv^2)$

(g) Explain why it is misleading to consider kinetic energy as something a body *has*, instead of only a quantity calculated from measurements.

10.4 An electron of mass about 9.1×10^{-31} kg is traveling at a speed of about 2×10^8 m/sec toward the screen of a television set. What is its kinetic energy? How many electrons like this one would be needed for a total kinetic energy of one joule?

10.5 Estimate the kinetic energy of each of the following: (a) a pitched baseball (b) a jet plane (c) a sprinter in a 100-yard dash (d) the earth in its motion around the sun.

10.6 A 200-kilogram iceboat is supported by a smooth surface of a frozen lake. The wind exerts on the boat a constant force of 400 newtons while the boat moves 900 meters. Assume that frictional forces are negligible, and that the boat starts from rest. Find the speed attained at the end of a 900 meter run by each of the following methods:

(a) Use Newton's second law to find the acceleration of the boat. How long does it take to move 900 meters? How fast will it be moving then?

(b) Find the final speed of the boat by equating the work done on it by the wind and the increase in its kinetic energy. Compare your result with your answer in (a).

10.7 A 2-gram bullet is shot into a tree stump. It enters at a speed of 300 m/sec and comes to rest after having penetrated 5 cm in a straight line.

(a) What was the change in the bullet's kinetic energy?

(b) How much work did the tree do on the bullet?

(c) What was the average force during impact?

10.8 Refer back to SG 9.20. How much work does the golf club do on the golf ball? How much work does the golf ball do on the golf club?

10.9 A penny has a mass of about 3.0 grams and is about 1.5 millimeters thick. You have 50 pennies which you pile one above the other.

(a) How much more gravitational potential energy has the top penny than the bottom one?

(b) How much more have all 50 pennies together than the bottom one alone?

10.10 (a) How high can you raise a book weighing 5 newtons if you have available one joule of energy?

(b) How many joules of energy are needed just to lift a 727 jet airliner weighing 7×10^5 newtons (fully loaded) to its cruising altitude of 10,000 meters?

10.11 As a home experiment, hang weights on a rubber band and measure its elongation. Plot the force vs. stretch on graph paper. How could you measure the stored energy?

10.12 For length, time and mass there are standards (for example, a standard meter). But energy is a "derived quantity," for which no standards need be kept. Nevertheless, assume someone asks you to supply him one joule of energy. Describe in as much detail as you can how you would do it.

10.13 (a) Estimate how long it would take for the earth to fall up 1 meter to a 1-kg stone if this stone were somehow rigidly fixed in space.
(b) Estimate how far the earth will actually move up while a 1-kg stone falls 1 meter from rest.

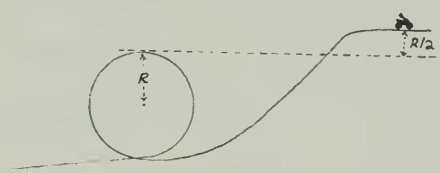
10.14 The photograph below shows a massive lead wrecking ball being used to demolish a wall. Discuss the transformations of energy involved.

10.15 This discussion will show that the *PE* of an object is relative to the frame of reference in which it is measured. The boulder in the photograph on page 37 was not lifted to its perch–rather the rest of the land has eroded away, leaving it where it may have been almost since the formation of the earth. Consider the question "What is the gravitational potential energy of the system boulder + earth?" You can easily calculate what the change in potential energy would be if the rock fell—it would be the product of the rock's weight and the distance it fell. But would that be the actual value of the gravitational energy that had been stored in the boulder-earth system? Imagine that there happened to be a deep mine shaft nearby and the boulder fell into the shaft. It would then fall much farther reducing the gravitational potential energy much more. Apparently the amount of energy stored depends on how far you imagine the boulder can fall.

(a) What is the greatest possible decrease in gravitational potential energy the isolated system boulder + earth could have?
(b) Is the system earth + boulder really isolated?
(c) Is there a true absolute bottom of gravitational potential energy for any system that includes the boulder and the earth?

These questions suggest that potential energy, like kinetic energy, is a relative quantity. The value of *PE* depends on the location of the (resting) frame of reference from which it is measured. This is not a serious problem, because we are concerned only with *changes* in energy. In any given problem, physicists will choose some convenient reference for the "zero-level" of potential energy, usually one that simplifies calculations. What would be a convenient zero-level for the gravitational potential energy of
(a) a pendulum?
(b) a roller coaster?
(c) a weight oscillating up and down a spring?
(d) a planet in orbit around the sun?

10.16 The figure below (not drawn to scale) shows a model of a carnival "loop-the-loop." A car starting from a platform above the top of the loop coasts down and around the loop without falling off the track. Show that to traverse the



loop successfully, the car must start from a height at least one-half a radius above the top of the loop. Hint: The car's weight must not be greater than the centripetal force required to keep it on the circular path at the top of the loop.

10.17 Discuss the conversion between kinetic and potential forms of energy in the system of a comet orbiting around the sun.

10.18 Sketch an addition to one of the steam engine diagrams of a mechanical linkage that would open and close the valves automatically.

10.19 Show that if a constant propelling force F keeps a vehicle moving at a constant speed v (against the friction of the surrounding) the power required is equal to Fv.

10.20 The Queen Mary, one of Britain's largest steamships, has been retired to a marine museum on our west coast after completing 1,000 crossings of the Atlantic. Her mass is 81,000 tons (75 million kilograms) and her maximum engine power of 234,000 horsepower (174 million watts) allows her to reach a maximum speed of 30.63 knots (16 meters per second).

(a) What is her kinetic energy at full speed?
(b) Assume that at maximum speed all the power output of her engines goes into overcoming water drag. If the engines are suddenly stopped, how far will the ship coast before stopping? (Assume water drag is constant.)
(c) What constant force would be required to bring her to a stop from full speed within 1 nautical mile (2000 meters)?
(d) The assumptions made in (b) are not valid for the following reasons:
1. Only about 60% of the power delivered to the propellor shafts results in a forward thrust to the ship; the rest results in friction and turbulence, eventually warming the water.
2. Water drag is less for lower speed than for high speed.
3. If the propellors are not free-wheeling, they add an increased drag.

Which of the above factors tend to increase, which to decrease the coasting distance?
(e) Explain why tugboats are important for docking big ships.

10.21 Devise an experiment to measure the power output of
(a) a man riding a bicycle
(b) a motorcycle
(c) an electric motor.

10.22 Refer to the table of "Typical Power Ratings" on p. 46.
(a) What advantages would Newcomen's engine have over a "turret windmill"?
(b) What advantage would you expect Watt's engine (1778) to have over Smeaton's engine (1772)?

10.23 Besides horsepower, another term used in Watt's day to describe the performance of steam engines was *duty*. The duty of a steam engine was defined as the distance in feet that an engine can lift a load of one million pounds, using one bushel of coal as fuel. For example, Newcomen's engine had a duty of 4.3: it could perform 4.3 million foot-pounds of work by burning a bushel of coal. Which do you think would have been more important to the engineers building steam engines—increasing the horsepower or increasing the duty?

10.24 A modern term that is related to the "duty" of an engine is ***efficiency***. The efficiency of an engine (or any device that transforms energy from one form to another) is defined as the percentage of the energy input that appears as useful output.
(a) Why would it have been impossible to find a value for the efficiency of an engine before Joule?
(b) The efficiency of "internal combustion" engines is seldom greater than 10%. For example, only about 10% of the chemical energy released in burning gasoline in an automobile engine goes into moving the automobile. What becomes of the other 90%?

10.25 Engine A operates at a greater power than engine B does, but its efficiency is less. This means that engine A does (a) more work with the same amount of fuel, but more slowly (b) less work with the same amount of fuel, but more quickly (c) more work with the same amount of fuel and does it faster (d) less work with the same amount of fuel and does it more slowly.

10.26 A table of rates for truck transportation is given below. How does the charge depend on the amount of work done?

Truck Transportation (1965)

Weight	Moving rates (including pickup and delivery) from Boston to:		
	Chicago (967 miles)	Denver (1969 miles)	Los Angeles (2994 miles)
100 lbs	$ 18.40	$ 24.00	$ 27.25
500	92.00	120.00	136.25
1000	128.50	185.50	220.50
2000	225.00	336.00	406.00
4000	383.00	606.00	748.00
6000	576.00	909.00	1122.00

10.27 Consider the following hypothetical values for a paddle-wheel experiment like Joule's: a 1-kilogram weight descends through a distance of 1 meter, turning a paddle-wheel immersed in 5 kilograms of water.
(a) About how many times must the weight be allowed to fall in order that the temperature of the water will be increased by $\frac{1}{2}$ Celsius degree?
(b) List ways you could modify the experiment so that the same temperature rise would be produced with fewer falls of the weight? (There are at least three possible ways.)

10.28 While traveling in Switzerland, Joule attempted to measure the difference in temperature of the water at the top and at the bottom of a waterfall. Assuming that the amount of heat produced at the bottom is equal to the decrease in gravitational potential energy, calculate roughly the temperature difference you would expect to observe between the top and bottom of a waterfall about 50 meters high, such as Niagra Falls. Does it matter how much water goes down the fall?

10.29 Find the power equivalent in watts or in horsepower of one of the activities listed in the table on p. 53.

10.30 About how many kilograms of hamburgers would you have to eat to supply the energy for a half-hour of digging? Assume that your body is 20% efficient.

10.31 When a person's food intake supplies less energy than he uses, he starts "burning" his own stored fat for energy. The oxidation of a pound of animal fat provides about 4,300 kilocalories of energy. Suppose that on your present diet of 4,000 kilocalories a day you neither gain nor lose weight. If you cut your diet to 3,000 kilocalories and maintain your present physical activity, how long would it take to reduce your mass by 5 pounds?

10.32 In order to engage in normal light work. a person in India has been found to need on the average about 1,950 kilocalories of food energy a day, whereas an average West European needs about 3,000 kilocalories a day. Explain how each of the following statements makes the difference in energy need understandable.

(a) The average adult Indian weighs about 110 pounds; the average adult West European weighs about 150 pounds.

(b) India has a warm climate.

(c) The age distribution of the population for which these averages have been obtained is different in the two areas.

10.33 No other concept in physics has the economic significance that "energy" does. Discuss the statement: "We could express energy in dollars just as well as in joules or calories."

10.34 Show how the conservation laws for energy and for momentum can be applied to a rocket during the period of its lift off.

10.35 Discuss the following statement: "During a typical trip, all the chemical energy of the gasoline used in an automobile is used to heat up the car, the road and the air."

10.36 Show how all the equations we have given in Chapter 10 to express conservation of energy are special cases of the general statement $\Delta E = \Delta W + \Delta H$. Hint: let one or more of the terms equal zero.)

10.37 (a) Describe the procedure by which a space capsule can be changed from a high circular orbit to a lower circular orbit.
(b) How does the kinetic energy in the lower orbit compare with that in the higher orbit?
(c) How does the gravitational potential energy for the lower orbit compare with that of the higher orbit?
(d) It can be shown (by using calculus) that the change in gravitational potential energy in going from one circular orbit to another will be *twice* the change in kinetic energy. How, then, will the total energy for the lower circular orbit compare with that for the higher orbit?
(e) How do you account for the change in total energy?

10.38 Any of the terms in the equation $\Delta E = \Delta H + \Delta W$ can have negative values.
(a) What would be true for a system for which
i. ΔE is negative?
ii. ΔH is negative?
iii. ΔW is negative?
(b) Which terms would be negative for the following systems?
i. a man digging a ditch
ii. a car battery while starting a car
iii. an electric light bulb just after it is turned on
iv. an electric light bulb an hour after it is turned on
v. a running refrigerator
vi. an exploding firecracker

10.39 In each of the following, trace the chain of energy transformations from the sun to the energy in its final form:
(a) A pot of water is boiled on an electric stove.
(b) An automobile accelerates from rest on a level road, climbs a hill at constant speed, and comes to stop at a traffic light.
(c) A windmill pumps water out of a flooded field.

10.40 Show how the law of conservation of energy applies to the motion of each of the situations listed in SG 9.39 and 9.40, p. 27.

Bubbles of gas from high-pressure tanks expand as the pressure decreases on the way to the surface.

CHAPTER ELEVEN

The Kinetic Theory of Gases

11.1 An overview of the chapter

SG 11.1

During the 1840's, many scientists recognized that heat is not a substance, but a form of energy which can be converted into other forms. Two of these scientists, James Prescott Joule and Rudolf Clausius, went a step further. They based this advance on the fact that heat can produce mechanical energy and mechanical energy can produce heat. Therefore, they reasoned, the "heat energy" of a substance is simply the kinetic energy of its atoms and molecules. In this chapter we will see that this idea is largely correct. It forms the basis of the *kinetic-molecular theory of heat*.

Molecules are the smallest pieces of a substance—they may be combinations of atoms cf simpler substances.

However, even the idea of atoms and molecules was not completely accepted in the nineteenth century. If such small bits of matter really existed, they would be too small to observe even in the most powerful microscopes. Since scientists could not observe molecules, they could not check directly the hypothesis that heat is molecular kinetic energy. Instead, they had to derive from this hypothesis predictions about the behavior of measurably large samples of matter. Then they could test these predictions by experiment. For reasons which we will explain, it is easiest to test such hypotheses by observing the properties of gases. Therefore, this chapter deals mainly with the kinetic theory as applied to gases.

The development of the kinetic theory of gases in the nineteenth century led to the last major triumph of Newtonian mechanics. The method involved using a simple theoretical model of a gas. Newton's laws of motion were applied to the gas molecules assumed in this model as if they were tiny billiard balls. This method produced equations that related the easily observable properties of gases—such as pressure, density, and temperature—to properties not directly observable—such as the sizes and speeds of molecules. For example, the kinetic theory:

(1) explained rules that had been found previously by trial-and-error methods. (An example is "Boyle's law," which relates the pressure and the volume of a gas.)

(2) predicted new relations. (One surprising result was that the friction between layers of gas moving at different speeds increases with temperature, but is independent of the density of the gas.)

(3) led to values for the sizes and speeds of gas molecules.

Thus the successes of kinetic theory showed that Newtonian mechanics provided a way for understanding the effects and behavior of invisible molecules.

But applying Newtonian mechanics to a mechanical model of gases resulted in some predictions that did *not* agree with the facts. That is, the model is not valid for all phenomena. According to kinetic theory, for example, the energy of a group of molecules should be shared equally among all the different motions of the molecules and their atoms. But the properties of gases predicted from this "equal sharing" principle clearly disagreed with experimental evidence. Newtonian mechanics could be applied successfully to a wide range of motions and collisions of molecules in a gas. But it did not work for the motions of atoms inside molecules. It was not until the twentieth century that an adequate theory of the behavior of atoms— "quantum mechanics"—was developed. (Some ideas from quantum mechanics are discussed in Unit 5.)

Kinetic theory based on Newtonian mechanics also had trouble dealing with the fact that most phenomena are not reversible. An inelastic collision is an irreversible process. Other examples are the mixing of two gases, or scrambling an egg. In Newtonian theory, however, the reverse of any event is just as reasonable as the event itself. Can irreversible processes be described by a theory based on Newtonian theory? Or do they involve some new fundamental law of nature? In discussing this problem from the viewpoint of kinetic theory, we will see how the concept of "randomness" entered physics.

Modern physicists do not take too seriously the "billiard ball" idea of gas molecules—nor did most nineteenth century physicists. All models oversimplify the actual facts. Therefore, the simple assumptions of a model often need adjustment in order to get a theory that agrees well with experimental data. Nevertheless the kinetic theory is still very useful. Physicists are fond of it, and often present it as an example of how a physical theory should be developed. Section 11.5 gives one of the mathematical derivations from the model used in kinetic theory. This derivation is not given to be memorized in detail; it simply illustrates mathematical reasoning based on models. Physicists have found this method very useful in understanding many natural phenomena.

Q1 Early forms of the kinetic molecular theory were based on the assumption that heat energy is

(a) a liquid

(b) a gas

(c) the kinetic energy of molecules

(d) made of molecules

Q2 True or false: In the kinetic theory of gases, as developed in the nineteenth century, it was assumed that Newton's laws of motion apply to the motion and collisions of molecules.

Q3 True or false: In the twentieth century, Newtonian mechanics was found to be applicable not only to molecules but also to the atoms inside molecules.

Balloon for carrying apparatus used for weather forecasting.

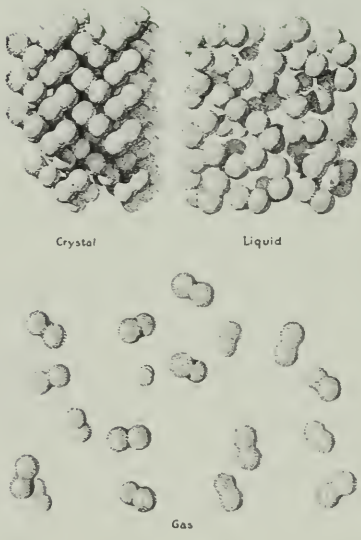
Gases can be confined without a container. A star, for example, is a mass of gas confined by gravitational force. Another example is the earth's atmosphere

11.2 A model for the gaseous state

What are the differences between a gas and a liquid or solid? We know by observation that liquids and solids have definite volume. Even if their shapes change, they still take up the same amount of space. A gas, on the other hand, will expand to fill any container (such as a room). If not confined, it will leak out and spread in all directions. Gases have low densities compared to liquids and solids—typically about 1,000 times smaller. Gas molecules are usually relatively far apart from one another, and they only occasionally collide. In the kinetic theory model, forces between molecules act only over very short distances. Therefore, gas molecules are considered to be moving freely most of the time. In liquids, the molecules are closer together; forces act among them continually and keep them from flying apart. In solids the molecules are usually even closer together, and the forces between them keep them in a definite orderly arrangement.

The initial model of a gas is a very simple model. The molecules are considered to behave like miniature billiard balls—that is, tiny spheres or clumps of spheres which exert no force at all on each other except when they make contact. Moreover, all the collisions of these spheres are assumed to be perfectly elastic. Thus, the total kinetic energy of two spheres is the same before and after they collide.

Note that the word "model" is used in two different senses in science. In Chapter 10, we mentioned the model of Newcomen's engine which James Watt was given to repair. That was a *working model.* It actually did function, although it was much smaller than the original engine, and contained some parts made of different materials. But now we are discussing a *theoretical model* of a gas. This model exists only in our imagination. Like the points, lines, triangles, and spheres studied in geometry, this theoretical model can be discussed mathematically. The results of such a discussion may help us to understand the real world of experience.



A very simplified "model" of the three states of matter:
(From *General Chemistry*, second edition, by Linus Pauling, W. H. Freeman and Company, © 1953.)

In order to emphasize that our model is a theoretical one, we will use the word "particle" instead of "atom" or "molecule." There is now no doubt that atoms and molecules exist and have their own definite properties. The particles in the kinetic theory model, on the other hand, are idealized and imaginary. We imagine such objects as perfectly elastic spheres, whose supposed properties are hopefully similar to those of actual atoms and molecules.

Our model represents the gas as consisting of *a large number of very*

The word "gas" was originally derived from the Greek word *chaos*; it was first used by the Belgian chemist Jan Baptista van Helmont (1580-1644)

small particles in rapid, disordered motion. Let us define some of these terms. "A large number" means something like a billion billion (10^{18}) or more particles in a sample as small as a bubble in a soft drink. "Very small" means a diameter about a hundred-millionth of a centimeter (10^{-10} meter). "Rapid motion" means an average speed of a few hundred miles per hour. What is meant by "disordered" motion? Nineteenth-century kinetic theorists assumed that each individual molecule moved in a definite way, determined by Newton's laws of motion. Of course, in practice it is impossible to follow a billion billion particles at the same time. They move in all directions, and each particle changes its direction and speed during collision with another particle. Therefore, we cannot make a definite prediction of the motion of any one *individual* particle. Instead, we must be content with describing the *average* behavior of large collections of particles. We still believe that from moment to moment each individual molecule behaves according to the laws of motion. But it turns out to be easier to describe the *average* behavior if we assume complete ignorance about any *individual* motions. To see why this is so, consider the results of flipping a large number of coins all at once. It would be very hard to predict how a single coin would behave. But if you assume they behave randomly, you can confidently predict that flipping a million coins will give approximately 50% heads and 50% tails. The same principle applies to molecules bouncing around in a container. You can safely bet that about as many are moving in one direction as in another. Further, the particles are equally likely to be found in any cubic centimeter of space inside the container. This is true no matter where such a region is located, and even though we do not know where a given particle is at any given time. "Disordered," then, means that velocities and positions are distributed randomly. Each molecule is just as likely to be moving to the right as to the left (or in any other direction). And it is just as likely to be near the center as near the edge (or any other position).

On the opposite page you will find a more detailed discussion of the idea of random fluctuations

SG 11.2

The idea of disorder is elaborated in the *Reader 3* articles "The Law of Disorder," "The Law," "The Arrow of Time," and "Randomness in the Twentieth Century."

In summary, we are going to discuss the properties of a model of a gas. The model is imagined to consist of a large number of very small particles in rapid, disordered motion. The particles move freely most of the time, exerting forces on one another only when they collide. The model is designed to represent the structure of real gases in many ways. However, it is simplified in order to make calculations manageable. By comparing the results of these calculations with the observed properties of gases, we can estimate the speeds and sizes of molecules.

Q4 In the kinetic theory, particles are thought to exert significant forces on one another

(a) only when they are far apart
(b) only when they are close together
(c) all the time
(d) never

Q5 Why was the kinetic theory first applied to gases rather than to liquids or solids?

Averages and Fluctuations

Molecules are too small, too numerous, and too fast for us to measure the speed of any one molecule, or its kinetic energy, or how far it moves before colliding with another molecule. For this reason the kinetic theory of gases concerns itself with making predictions about *average* values. The theory enables us to predict quite precisely the *average* speed of the molecules in a sample of gas, or the *average* kinetic energy, or the *average* distance the molecules move between collisions.

Any measurement made on a sample of gas reflects the combined effect of billions of molecules, averaged over some interval of time. Such average values measured at different times, or in different parts of the sample, will be slightly different. We assume that the molecules are moving randomly. Thus we can use the mathematical rules of statistics to estimate just how different the averages are likely to be. We will call on two basic rules of statistics for random samples:

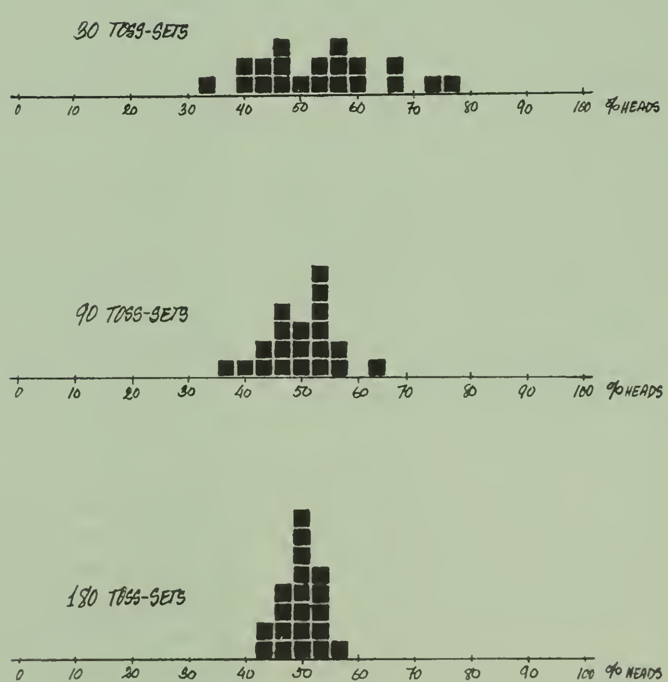
1. Large variations away from the average are less likely than small variations. (For example, if you toss 10 coins you are less likely to get 9 heads and 1 tail than to get 6 heads and 4 tails.)
2. Percentage variations are likely to be smaller for large samples. (For example, you are likely to get nearer to 50% heads by flipping 1,000 coins than by flipping just 10 coins.)

A simple statistical prediction is the statement that if a coin is tossed many times, it will land "heads" 50 percent of the time and "tails" 50 percent of the time. For small sets of tosses there will be many "fluctuations" (variations) to either side of the predicted average of 50% heads. Both statistical rules are evident in the charts at the right. The top chart shows the percentage of heads in sets of 30 tosses each. Each of the 10 black squares represents a set of 30 tosses. Its position along the horizontal scale indicates the percent of heads. As we would expect from rule 1, there are more values near the theoretical 50% than far from it. The second chart is similar to the first, but here each square represents a set of *90* tosses. As before, there are more values near 50% than far from it. And, as we would expect from rule 2, there are fewer values far from 50% than in the first chart.

The third chart is similar to the first two, but now each square represents a set of 180 tosses. Large fluctuations from 50% are less common still than for the smaller sets.

Statistical theory shows that the *average* fluctuation from 50% shrinks in proportion to the square root of the number of tosses. We can use this rule to compare the average fluctuation for sets of, say, 30,000,000 tosses with the average fluctuation for sets of 30 tosses. The 30,000,000-toss sets have 1,000,000 times as many tosses as the 30-toss sets. Thus, their average fluctuation in percent of heads should be 1,000 times smaller!

These same principles hold for fluctuations from average values of any randomly-distributed quantities, such as molecular speed or distance between collisions. Since even a small bubble of air contains about a quintillion (10^{18}) molecules, fluctuations in the average value for any isolated sample of gas are not likely to be large enough to be measurable. A measurably large fluctuation is not *impossible*, but extremely unlikely.



11.3 The speeds of molecules

The basic idea of the kinetic theory is that heat is related to the kinetic energy of molecular motion. This idea had been frequently suggested in the past. However, many difficulties stood in the way of its general acceptance. Some of these difficulties are well worth mentioning. They show that not all good ideas in science (any more than outside of science) are immediately successful.

SG 11.3

In 1738, the Swiss mathematician Daniel Bernoulli showed how a kinetic model could explain a well-known property of gases. This property is described by Boyle's law: as long as the temperature does not change, the pressure of a gas is proportional to its density. Bernoulli assumed that the pressure of a gas is simply a result of the impacts of individual molecules striking the wall of the container. If the density of the gas were twice as great there would be twice as many molecules per cubic centimeter. Thus, Bernoulli said, there would be twice as many molecules striking the wall per second, and hence twice the pressure. Bernoulli's proposal seems to have been the first step toward the modern kinetic theory of gases. Yet it was generally ignored by other scientists in the eighteenth century. One reason for this was that Newton had proposed a different theory in his *Principia* (1687). Newton showed that Boyle's law *could* be explained by a model in which particles at rest exert forces that repel neighboring particles. Newton did not claim that he had proved that gases really *are* composed of such repelling particles. But most scientists, impressed by Newton's discoveries, simply assumed that his treatment of gas pressure was also right. (As it turned out, it was not.)

Pressure is defined as the perpendicular force on a surface divided by the area of the surface

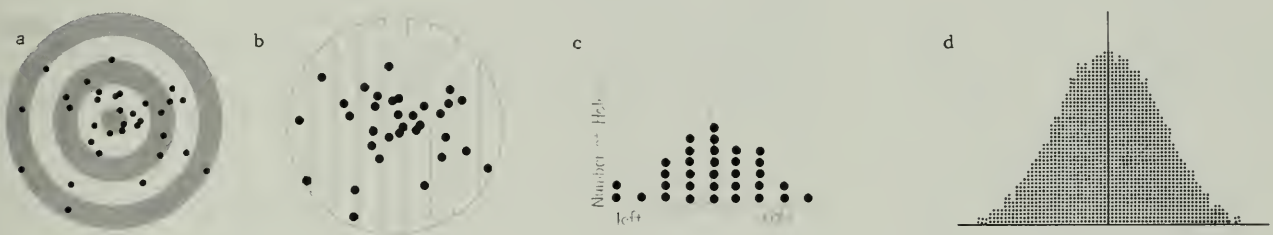
The kinetic theory of gases was proposed again in 1820 by an English physicist, John Herapath. Herapath rediscovered Bernoulli's results on the relations between pressure and density of a gas and the speeds of the particles. But Herapath's work also was ignored by most other scientists. His earlier writings on the kinetic theory had been rejected for publication by the Royal Society of London. Despite a long and bitter battle Herapath did not succeed in getting recognition for his theory.

James Prescott Joule, however, did see the value of Herapath's work. In 1848 he read a paper to the Manchester Literary and Philosophical Society in which he tried to revive the kinetic theory. Joule showed how the speed of a hydrogen molecule could be computed (as Herapath had done). He reported a value of 2,000 meters per second at 0°C, the freezing temperature of water. This paper, too, was ignored by other scientists. For one thing, physicists do not generally look in the publications of a "literary and philosophical society" for scientifically important papers. But evidence for the equivalence of heat and mechanical energy continued to mount. Several other physicists independently worked out the consequences of the hypothesis that heat energy in a gas is the kinetic energy of molecules. Rudolf Clausius in Germany published a paper in 1856 on "The Nature of the Motion we call Heat." This paper established the basic principles of kinetic theory essentially in the form we accept today. Soon afterward, James Clerk Maxwell in Britain and Ludwig Bolzmann in Austria set forth the full mathematical details of the theory.

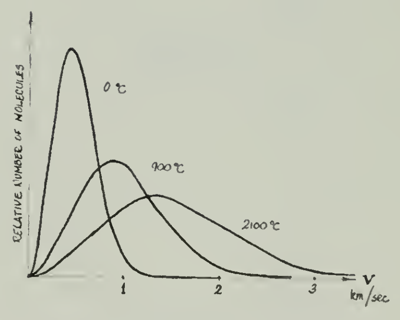
The Maxwell velocity distribution. It did not seem likely that all molecules in a gas would have the same speed. In 1859 Maxwell applied the mathematics of probability to this problem. He suggested that the speeds of molecules in a gas are distributed over all possible values. Most molecules have speeds not very far from the average speed. But some have much lower speeds and some much higher speeds.

A simple example will help you to understand Maxwell's distribution of molecular speeds. Suppose a marksman shoots a gun at a practice target many times. Some bullets will probably hit the bullseye. Others will miss by smaller or larger amounts, as shown in (a) in the sketch below. We count the number of bullets scattered at various distances to the left and right of the bullseye in (b). Then we can make a graph showing the number of bullets at these distances as shown in (c).

TARGET PRACTICE EXPERIMENT
(a) Scatter of holes in target; (b) target marked off in distance intervals left and right of center; (c) graph of number of holes per strip to left and right of center; (d) For a very large number of bullets and narrow strips, the envelope of the graph often closely approximates the mathematical curve called the "normal distribution" curve.



This graph showing the distribution of hits illustrates a general principle of statistics, namely, if any quantity varies randomly about an average value, the graph showing the distribution of variations will resemble the one shown in (d) above in the margin. There will be a peak at the average value and a smooth decline on either side. A similar "bell-shaped curve," as it is called, describes the distribution of many kinds of physical measurements. The *normal distribution law* applies even to large groups of people. For example, consider the distribution of heights in a large crowd. Such a distribution results from the combined effect of a great many *independent* factors. A person's height, for example, depends upon many independent genes as well as environmental factors. Thus the distribution of heights will closely follow a normal distribution. The velocity of a gas molecule is determined by a very large number of independent collisions. So the distribution of velocities is also smoothly "bell-shaped."



Maxwell's distribution of speeds in gases at different temperatures.

Maxwell's distribution law for molecular velocities in a gas is shown in the margin in graphical form for three different temperatures. The curve is not symmetrical since no molecule can have less than zero speed, but some have very large speeds. For a gas at any given temperature, the "tail" of each curve is much longer on the right (high speeds) than on the left (low speeds). As the temperature increases, the peak of the curve shifts to higher speeds. Then the speed distribution becomes more broadly spread out.

What evidence do we have that Maxwell's distribution law really applies to molecular speeds? Several successful predictions based on this law gave indirect support to it. But not until the 1920's was a direct experimental check possible. Otto Stern in Germany, and later Zartmann in the United States, devised a method for measuring the speeds in a beam of molecules. (See the illustration of Zartmann's method on the next page.) Stern, Zartmann, and others found that molecular speeds are indeed distributed according to Maxwell's law. Virtually all of the individual molecules in a gas change speed as they collide again and again. Yet if a confined sample of gas is isolated, the *distribution* of speeds remains very much the same. For the tremendous number of
SG 11.4 molecules in almost any sample of gas, the *average* speed has an
SG 11.5 extremely stable value.

Q6 In the kinetic theory of gases, it is assumed that the pressure of a gas on the walls of the container is due to

(a) gas molecules colliding with one another
(b) gas molecules colliding against the walls of the container
(c) repelling forces exerted by molecules on one another

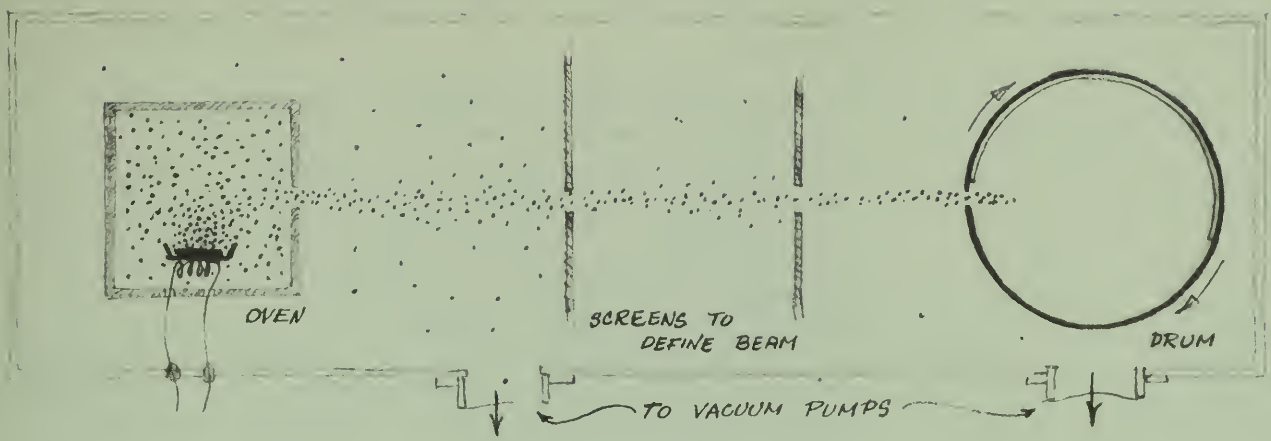
Q7 The idea of speed distribution for gas molecules means that

(a) each molecule always has the same speed
(b) there is a wide range at speeds of gas molecules
(c) molecules are moving fastest near the center of the gas

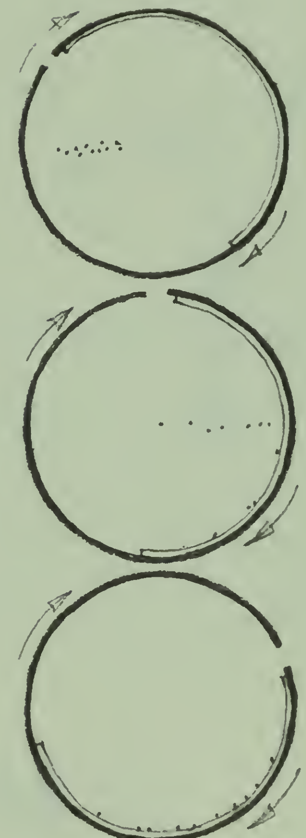
11.4 The sizes of molecules

Is it reasonable to suppose that gases consist of molecules moving at speeds up to several hundred meters per second? If this model were correct, gases should mix with each other very rapidly. But anyone who has studied chemistry knows that they do not. Suppose hydrogen sulfide or chlorine is generated at the end of a large room. Several minutes may pass before the odor is noticed at the other end. But according to our kinetic-theory calculations, each of the gas molecules should have crossed the room hundreds of times by then. Something must be wrong with our kinetic-theory model.

Rudolf Clausius recognized this as a valid objection to his own version of the kinetic theory. His 1856 paper had assumed that the particles are so small that they can be treated like mathematical points. If this were true, particles would almost never collide with one another. However, the observed *slowness* of diffusion and mixing convinced Clausius to change his model. He thought it likely that the molecules of a gas are not vanishingly small, but of a finite size. Particles of finite size moving very rapidly would often collide with one another. An individual molecule might have an instantaneous speed of several hundred meters per second. But it changes its direction of motion every time it collides with another molecule. The more often it collides with other molecules, the less likely it



Direct Measurement of Molecular Speeds



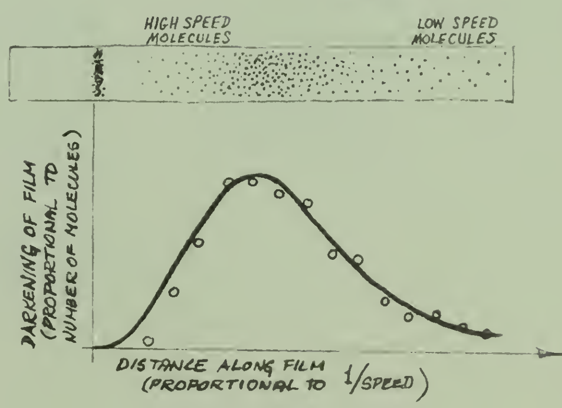
A narrow beam of molecules is formed by letting molecules of a hot gas pass through a series of slits. In order to keep the beam from spreading out, collisions with randomly moving molecules must be avoided. Therefore, the source of gas and the slits are housed in a highly evacuated chamber. The molecules are then allowed to pass through a slit in the side of a cylindrical drum which can be spun very rapidly. The general scheme is shown in the drawing above.

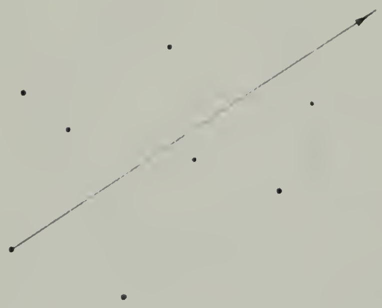
As the drum rotates, the slit moves out of the beam of molecules. No more molecules can enter until the drum has rotated through a whole revolution. Meanwhile the molecules in the drum continue moving to the right, some moving fast and some moving slowly.

Fastened to the inside of the drum is a sensitive film which acts as a detector. Any molecule striking the film leaves a mark. The faster molecules strike the film first, before the drum has rotated very far.

The slower molecules hit the film later, after the drum has rotated farther, in general, molecules of different speeds strike different parts of the film. The darkness of the film at any point is proportional to the number of molecules which hit it there. Measurement of the darkening of the film shows the relative distribution of molecular speeds. The speckled strip at the right represents the unrolled film, showing the impact position of molecules over many revolutions of the drum. The heavy band indicates where the beam struck the film before the drum started rotating. (It also marks the place to which infinitely fast molecules would get once the drum was rotating.)

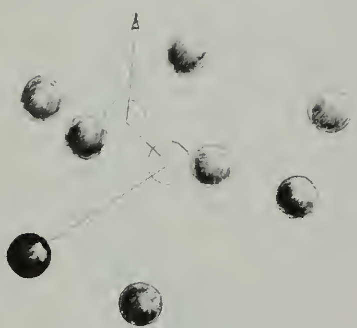
A comparison of some experimental results with those predicted from theory is shown in the graph. The dots show the experimental results and the solid line represents the predictions from the kinetic theory.





is to move very far in any one direction. How often collisions occur depends on how crowded the molecules are and on their size. For most purposes one can think of molecules as being relatively far apart and of very small size. But they are just large enough and crowded enough to get in one another's way. Realizing this, Clausius could modify his model to explain why gases mix so slowly. Further, he derived a precise quantitative relationship between the molecule's size and the average distance they moved between collisions.

Clausius now was faced with a problem that plagues every theoretical physicist. If a simple model is modified to explain better the observed properties, it becomes more complicated. Some plausible adjustment or approximation may be necessary in order to make any predictions from the model. If the predictions disagree with experimental data, one doesn't know whether to blame a flaw in the model or calculation error introduced by the approximations. The development of a theory often involves a compromise between adequate explanation of the data, and mathematical convenience.



The larger the molecules are, the more likely they are to collide with each other.

Nonetheless, it soon became clear that the new model was a great improvement over the old one. It turned out that certain other properties of gases also depend on the size of the molecules. By combining data on several such properties it was possible to work backwards and find fairly reliable values for molecular sizes. Here we can only report the result of these calculations. Typically, the diameter of gas molecules came out to be of the order of 10^{-10} meters to 10^{-9} meters. This is not far from the modern values—an amazingly good result. After all, no one previously had known whether a molecule was 1,000 times smaller or bigger than that.

SG 11.6

In fact, as Lord Kelvin remarked:

> The idea of an atom has been so constantly associated with incredible assumptions of infinite strength, absolute rigidity, mystical actions at a distance and indivisibility, that chemists and many other reasonable naturalists of modern times, losing all patience with it, have dismissed it to the realms of metaphysics, and made it smaller than 'anything we can conceive.'

SG 11.7
SG 11.8

Kelvin showed that other methods could also be used to estimate the size of atoms. None of these methods gave results as reliable as did the kinetic theory. But it was encouraging that they all led to the same order of magnitude (within about 50%).

Q8 In his revised kinetic-theory model Clausius assumed that the particles have a finite size, instead of being mathematical points, because

(a) obviously everything must have some size

(b) it was necessary to assume a finite size in order to calculate the speed of molecules.

(c) the size of a molecule was already well known before Clausius' time

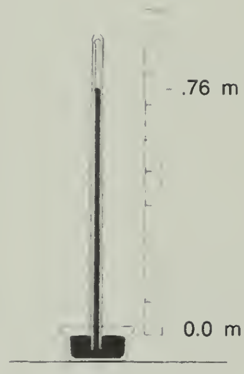
(d) a finite size of molecules could account for the slowness of diffusion.

11.5 Predicting the behavior of gases from the kinetic theory

One of the most easily measured characteristics of a confined gas is pressure. Our experience with balloons and tires makes the idea of air pressure seem obvious; but it was not always so.

Galileo, in his book on mechanics, *Two New Sciences* (1638), noted that a lift-type pump cannot raise water more than 34 feet. This fact was well known. Such pumps were widely used to obtain drinking water from wells and to remove water from mines. We already have seen one important consequence of this limited ability of pumps to lift water out of deep mines. This need provided the initial stimulus for the development of steam engines. Another consequence was that physicists became curious about why the lift pump worked at all. Also, why should there be a limit to its ability to raise water?

Air Pressure. The puzzle was solved as a result of experiments by Torricelli (a student of Galileo), Guericke, Pascal, and Boyle. By 1660, it was fairly clear that the operation of a "lift" pump depends on the pressure of the air. The pump merely reduces the pressure at the top of the pipe. It is the pressure exerted by the atmosphere on the pool of water below which forces water up the pipe. A good pump can reduce the pressure at the top of the pipe to nearly zero. Then the atmospheric pressure can force water up to about 34 feet above the pool—but no higher. Atmospheric pressure at sea level is not great enough to support a column of water any higher. Mercury is almost 14 times as dense as water. Thus, ordinary pressure on a pool of mercury can support a column only $\frac{1}{14}$ as high, about $2\frac{1}{2}$ feet (0.76 meter). This is a more convenient height for laboratory experiments. Therefore, much of the seventeenth-century research on air pressure was done with a column of mercury, or mercury "barometer." The first of these was designed by Torricelli.

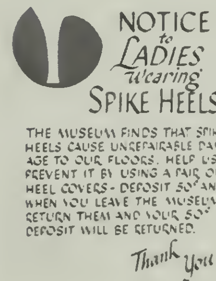


SG 11.9

SG 11.10

Torricelli's barometer is a glass tube standing in a pool of mercury. The top most part of the tube is empty of air. The air pressure on the pool supports the weight of the column of mercury in the tube up to a height of about $2\frac{1}{2}$ feet (0.76 meter).

The height of the mercury column which can be supported by air pressure does not depend on the diameter of the tube. That is, it depends not on the total amount of mercury, but only on its height. This may seem strange at first. To understand it, we must understand the difference between *pressure* and *force*. Pressure is defined as the magnitude of the force acting perpendicularly on a surface divided by the area of that surface: $P = F_{\perp}/A$. Thus a large force may produce only a small pressure if it is spread over a large area. For example, you can walk on snow without sinking in it if you wear snowshoes. On the other hand, a small force can produce a very large pressure if it is concentrated on a small area. Women's spike heel shoes have ruined many a wooden floor or carpet. The pressure at the place where the heel touched the floor was greater than that under an elephant's foot.



In 1661 two English scientists, Richard Towneley and Henry Power, discovered an important basic relation. They found that *the pressure exerted by a gas is directly proportional to the density of that gas.* Using P for pressure and D for density, we can write this relationship as $P \propto D$ or $P = kD$ where k is some constant. For example, if the density of a given quantity of air is doubled (say by compressing it), its pressure also doubles. Robert Boyle confirmed this relation by extensive experiments. It is an empirical rule, now generally known as *Boyle's Law*. But the law

SG 11.11

SG 11.12

holds true only under special conditions.

The effect of temperature on gas pressure. Boyle recognized that if the temperature of a gas changes during an experiment, the relation $P = kD$ no longer applies. For example, the pressure exerted by a gas in a closed container increases if the gas is heated, even though its density stays constant.

Many scientists throughout the eighteenth century investigated the expansion of gases by heat. The experimental results were not consistent enough to establish a quantitative relation between density (or volume) and temperature. But eventually, evidence for a surprisingly simple general law appeared. The French chemist Joseph-Louis Gay-Lussac (1778-1850) found that all the gases he studied—air, oxygen, hydrogen, nitrogen, nitrous oxide, ammonia, hydrogen chloride, sulfur dioxide, and carbon dioxide—changed their volume in the same way. If the pressure remained constant, then the change in volume was proportional to the change in temperature. On the other hand, if the volume remained constant, the change in pressure was proportional to the change in temperature.

A single equation summarizes all the experimental data obtained by Boyle, Gay-Lussac, and many other scientists. It is known as the *ideal gas law:*

$$P = kD(t + 273°)$$

On the Celsius scale, water freezes at 0° and boils at 100°, when the pressure is equal to normal atmospheric pressure. On the Fahrenheit scale, water freezes at 32° and boils at 212°. Some of the details involved in defining temperature scales are part of the experiment Hotness and Temperature in the *Handbook.*

Here t is the temperature on the Celsius scale. The proportionality constant k depends only on the kind of gas (and on the units used for P, D and t).

We call this equation the *ideal* gas law because it is not completely accurate for real gases except at very low pressures. Thus, it is not a law of physics in the same sense as the law of conservation of momentum. Rather, it simply gives an experimental and approximate summary of the observed properties of real gases. It does not apply when pressure is so high, or temperature so low, that the gas is nearly changing to a liquid.

Why does the number 273 appear in the ideal gas law? Simply because we are measuring temperature on the Celsius scale. If we had chosen to use the Fahrenheit scale, the equation for the ideal gas law would be

$$P = k'D(t + 460°)$$

where t is the temperature measured on the Fahrenheit scale. In other words, the fact that the number is 273 or 460 has no great importance. It just depends on our choice of a particular scale for measuring temperature. However, it *is* important to note what would happen if t were decreased to −273°C or −460°F. Then the entire factor involving temperature would be zero. And, according to the ideal gas law, the pressure of any gas would also fall to zero at this temperature. The chemical properties of the gas no longer makes sense. Real gases become liquid long before a temperature of −273°C is reached. Both experiment and thermodynamic theory indicate that it is impossible actually to cool anything—gas, liquid, or solid—down to precisely

If the pressure were kept constant then according to the ideal gas law the *volume* of a sample of gas would shrink to zero at −273°C.

this temperature. However, a series of cooling operations has produced temperatures less than 0.0001 degree above this limit.

In view of the unique meaning of this lowest temperature, Lord Kelvin proposed a new temperature scale. He called it the *absolute temperature scale,* and put its zero at −273°C. Sometimes it is called the Kelvin scale. The temperature of −273°C is now referred to as 0°K on the absolute scale, and is called the *absolute zero* of temperature.

This absolute zero point on the temperature scale has been found to be 273.16 Celsius (459 69 F).

The ideal gas law may now be written in simpler form:

$$P = kDT$$

For our purposes it is sufficiently accurate to say the absolute temperature of any sample (symbolized by the letter *T* and measured in degrees Kelvin, or K) is equal to the Celsius temperature *t* plus 273

T *t* 273

The boiling point of water for example is 373 K on the absolute scale.

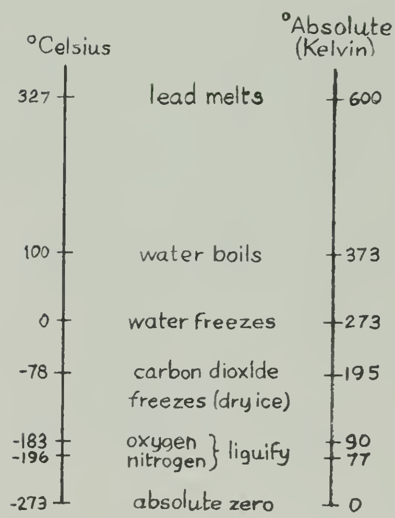
SG 11.13, 11.14

T is the temperature in degrees Kelvin and k is the proportionality constant.

The equation $P = kDT$ summarizes *experimental facts* about gases. Now we can see whether the kinetic-theory model offers a *theoretical* explanation for these facts.

Kinetic explanation of gas pressure. According to the kinetic theory, the pressure of a gas results from the continual impacts of gas particles against the container wall. This explains why pressure is proportional to density: the greater the density, the greater the number of particles colliding with the wall. But pressure also depends on the *speed* of the individual particles. This speed determines the force exerted on the wall during each impact and the frequency of the impacts. If the collisions with the wall are perfectly elastic, the law of conservation of momentum will describe the results of the impact. The detailed reasoning for this procedure is worked out on pages 82 and 83. This is a beautifully simple application of Newtonian mechanics. The result is clear: applying Newtonian mechanics to the kinetic molecular model of gases leads to the conclusion that $P = \frac{1}{3}D(v^2)_{av}$ where $(v^2)_{av}$ is the average of the squared speed of the molecules.

So we have two expressions for the pressure of a gas. One summarizes the experimental facts, $P = kDT$. The other is derived by Newton's laws from a theoretical model, $P = \frac{1}{3}D(v^2)_{av}$. The *theoretical* expression will agree with the *experimental* expression only if $kT = \frac{1}{3}(v^2)_{av}$. This would mean that *the temperature of a gas is proportional to* $(v^2)_{av}$. The mass m of each molecule is a constant, so we can also say that the temperature is proportional to $\frac{1}{2}m(v^2)_{av}$. In equation form, $T \propto \frac{1}{2}m(v^2)_{av}$. You should recall that $\frac{1}{2}m(v^2)$ is our expression for kinetic energy. Thus, the kinetic theory leads to the conclusion that the temperature of a gas is proportional to the average kinetic energy of its molecules! We already had some idea that raising the temperature of a material somehow affected the motion of its "small parts." We were aware that the higher the temperature of a gas, the more rapidly its molecules are moving. But the conclusion $T \propto \frac{1}{2}m(v^2)_{av}$ is a precise quantitative relationship derived from the kinetic model and empirical laws.



Comparison of the Celsius and absolute temperature scales.

Many different kinds of experimental evidence support this conclusion, and therefore also support the kinetic-theory model. Perhaps the best evidence is the motion of microscopic particles suspended in a gas or liquid,

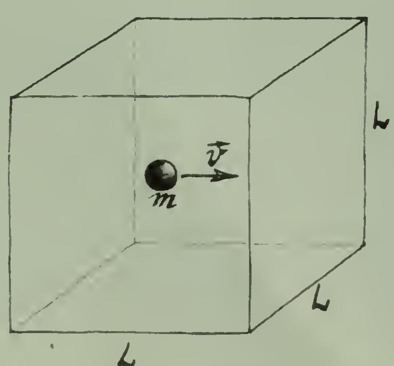
Deriving an Expression For Pressure From the Kinetic Theory

We begin with the model of a gas described in Section 11.2: "a large number of very small particles in rapid, disordered motion." We can assume here that the particles are points with vanishingly small size, so that collisions between them can be ignored. If the particles did have finite size, the results of the calculation would be slightly different. But the approximation used here is accurate enough for most purposes.

The motions of particles moving in all directions with many different velocities are too complex as a starting point for a model. So we fix our attention first on one particle that is simply bouncing back and forth between two opposite walls of a box. Hardly any molecules in a real gas would actually move like this. But we will begin here in this simple way, and later in this chapter extend the argument to include other motions. This later part of the argument will require that one of the walls be movable. So let us arrange for that wall to be movable, but to fit snugly into the box.

In SG 9.24 we saw how the laws of conservation of momentum and energy apply to cases like this. When a very light particle hits a more massive object, like our wall, very little kinetic energy is transferred. If the collision is elastic, the particle will reverse its direction with very little change in speed. In fact, if a force on the outside of the wall keeps it stationary against the impact from inside, the wall will not move during the collisions. Thus *no work* is done on it, and the particles rebound without any change in speed.

How large a force will these particles exert on the wall when they hit it? By Newton's third law the

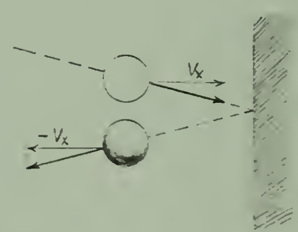


average force acting on the wall is equal and opposite to the average force with which the wall acts on the particles. The force on each particle is equal to the product of its mass times its acceleration ($\vec{F} = m\vec{a}$), by Newton's second law. As shown in Section 9.4, the force can also be written as

$$\vec{F} = \frac{\Delta(m\vec{v})}{\Delta t}$$

where $\Delta(m\vec{v})$ is the change in momentum. Thus, to find the average force acting on the wall we need to find the change in momentum per second due to molecule-wall collisions.

Imagine that a particle, moving with speed v_x (the component of $\vec{v}$ in the x direction) is about to collide with the wall at the right. The component of the particle's momentum in the x direction is $m\vec{v}_x$. Since the particle collides elastically with the wall, it rebounds with the same speed. Therefore, the momentum in the x direction after the collision is $m(-\vec{v}_x) = -m\vec{v}_x$. The change in the

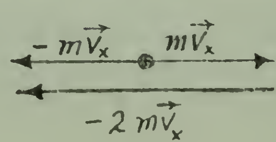


momentum of the particle as a result of this collision is

$$\frac{\text{final}}{\text{momentum}} - \frac{\text{initial}}{\text{momentum}} = \frac{\text{change in}}{\text{momentum}}$$

$$(-mv_x) \quad - \quad (mv_x) \quad = \quad (-2mv_x)$$

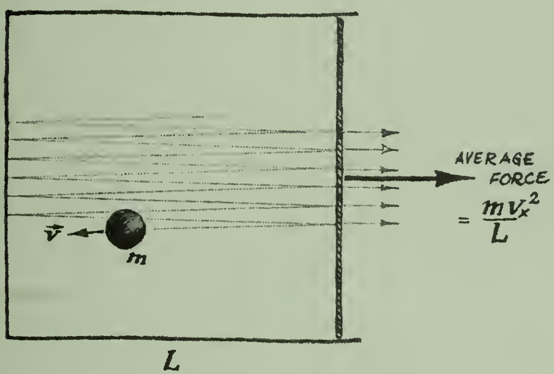
Note that all the vector quantities considered in this derivation have only two possible directions: to the right or to the left. We can therefore indicate direction by using a + or a −



sign respectively.

Now think of a single particle of mass m moving in a cubical container of volume L^3 as shown in the figure.

The time between collisions of one particle with the right-hand wall is the time required to cover a distance $2L$ at a speed of v_x; that is, $2L/v_x$. If $2L/v_x$ = the time between collisions,



then $v_x/2L$ = the number of collisions per second. Thus, the change in momentum per second is given by

(change in momentum in one collision)	×	(number of collisions per second)	=	(change in momentum per second)

$$(-2mv_x) \times (v_x/2L) = \frac{-mv_x^2}{L}$$

The net force equals the rate of change of momentum. Thus, the average force acting on the molecule (due to the wall) is equal to $-mv_x^2/L$; and by Newton's third law, the average force acting on the wall (due to the molecule) is equal to $+mv_x^2/L$. So the average pressure on the wall due to the collisions made by one molecule moving with speed v_x is

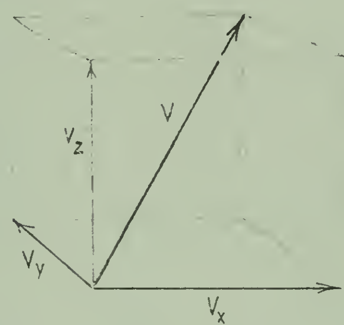
$$P = \frac{F}{A} = \frac{F}{L^2} = \frac{mv_x^2}{L^3} = \frac{mv_x^2}{V}$$

where V (here L^3) is the volume of the cubical container.

Actually there are not one but N molecules in the container. They do not all have the same speed, but we need only the average speed in order to find the pressure they exert. More precisely, we need the average of the square of their speeds in the x direction. We call this quantity $(v_x^2)_{av}$. The pressure on the wall due to N molecules will be N times the pressure due to one molecule, or

$$P = \frac{nm(v_x^2)_{av}}{V}$$

In a real gas, the molecules will be moving in all directions, not just in the x direction. That



is, a molecule moving with speed v will have three components: v_x, v_y, and v_z. If the motion is random, then there is no preferred direction of motion for a large collection of molecules, and $(v_x^2)_{av} = (v_y^2)_{av} = (v_z^2)_{av}$. It can be shown from a theorem in geometry that $v^2 = v_x^2 + v_y^2 + v_z^2$. These last two expressions can be combined to give

$$(v^2)_{av} = 3(v_x^2)_{av}$$

or

$$(v_x^2)_{av} = \tfrac{1}{3}(v^2)_{av}$$

By substituting this expression for $(v_x^2)_{av}$ in the pressure formula, we get

$$P = \frac{Nm \times 1/3(v^2)_{av}}{V}$$

$$= \frac{1}{3}\frac{Nm}{V}(v^2)_{av}$$

Notice now that Nm is the total mass of the gas and therefore Nm/V is just the density D. So

$$P = \tfrac{1}{3}D(v^2)_{av}$$

This is our theoretical expression for the pressure P exerted on a wall by a gas in terms of its density D and the molecular speed v.

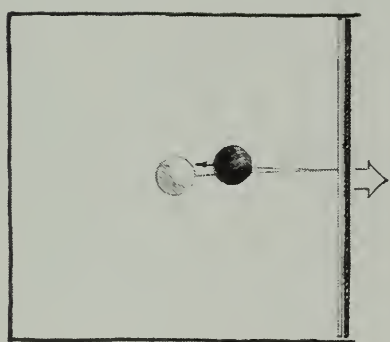
SG 11.15

Brownian motion was named after the English botanist Robert Brown who in 1827 observed the phenomenon while looking at a suspension of the microscopic grains of plant pollen. The same kind of motion of particles ("thermal motion") exists also in liquids and solids, but there the particles are far more constrained than in gases.

called *Brownian Movement*. The gas or liquid molecules themselves are too small to be seen directly. But their effects on a larger particle (for example, a particle of smoke) can be observed through the microscope. At any instant, molecules moving at very different speeds are striking the larger particle from all sides. Nevertheless, so many molecules are taking part that their total effect *nearly* cancels. Any remaining effect changes in magnitude and direction from moment to moment. Hence the impact of the invisible molecules makes the visible particle "dance" in the viewfield of the microscope. The hotter the gas, the more lively the motion, as the equation $T \propto \frac{1}{2}m(v^2)_{av}$ predicts.

This experiment is simple to set up and fascinating to watch. You should do it as soon as you can in the laboratory. It gives visible evidence that the smallest parts of all matter in the universe are in a perpetual state of lively, random motion. In the words of the twentieth-century physicist Max Born, we live in a "restless universe."

But we need a more extensive argument in order to make confident quantitative predictions from kinetic theory. We know by experience that when a gas is compressed or expanded very slowly, its temperature changes hardly at all. Thus Boyle's simple law ($P = kD$) applies. But when a gas is compressed or condensed rapidly, the temperature does change. Then, only the more general gas law ($P = kDT$) applies. Can our model explain this?



In the model used on the special pages, particles were bouncing back and forth between the walls of a box. Every collision with the wall was perfectly elastic, so the particles rebounded with no loss in speed. Suppose we suddenly reduce the outside force that holds one wall in place. What will happen to the wall? The force exerted on the wall by the collisions of the particles will now be greater than the outside force. Therefore, the wall will move outward.

As long as the wall was stationary, the particles did no work on it, and the wall did no work on the particles. But now the wall moves in the same direction as the force exerted on it by the particles. Thus, the particles must be doing work on the wall. The energy needed to do this work must come from somewhere. But the only available source of energy here is the kinetic energy ($\frac{1}{2}mv^2$) of the particles. In fact, we can show that molecules colliding perfectly elastically with a receding wall rebound with slightly less speed (see SG 11.16). Therefore the kinetic energy of the particles must decrease. But the relationship $T \propto \frac{1}{2}m(v^2)_{av}$ implies that the temperature of the gas will then drop. And this is exactly what happens!

This phenomenon can be demonstrated by means of the expansion cloud chamber, cooling of CO_2 fire extinguisher, etc. The "wall" is here the air mass being pushed away.

If we increase the outside force on the wall instead of decreasing it, just the opposite happens. The gas is suddenly compressed as the wall moves inward, doing work on the particles and increasing their kinetic energy. As $\frac{1}{2}mv^2$ goes up, we expect the temperature of the gas to rise—which is just what happens when we compress a gas quickly.

Diesel engines have no spark plugs; ignition is produced by temperature rise during the high compression of the air-fuel vapor mixture.

The model also predicts that, for *slow* motion of the wall, Boyle's law applies. However, the gas must not be insulated from its surroundings. Suppose we keep the surroundings of the gas at a constant temperature—for example, by immersing the gas container in a large water bath. Small changes in the temperature of the gas will then be cancelled by exchange of heat with the surroundings. Whenever the kinetic energy of the molecules momentarily decreases (as during expansion), the temperature of the gas will drop below that of its surroundings. Unless the walls of the container are heat insulators, heat will then flow into the gas until its temperature rises to that of the surroundings. Whenever the kinetic energy momentarily increases (as during compression), the temperature of the gas will rise above that of its surroundings. Heat will then flow out of the gas until its temperature falls to the temperature of the surroundings. This natural tendency of heat to flow from hot bodies to cold bodies explains why the average kinetic energy of the particles remains nearly constant when gas is slowly compressed or expanded.

SG 11.17 - 11.22

Q9 The relationship between the density and pressure of a gas expressed by Boyle's law, $P = kD$, holds true

(a) for any gas under any conditions
(b) for some gases under any conditions
(c) only if the temperature is kept constant
(d) only if the density is constant

Q10 If a piston is pushed rapidly into a container of gas, what will happen to the kinetic energy of the molecules of gas? What will happen to the temperature of the gas?

Q11 Which of the following conclusions result only when the ideal gas law and the kinetic theory model are *both* considered to apply?

(a) P is proportional to T.
(b) P is proportional to $(v^2)_{av}$.
(c) $(v^2)_{av}$ is proportional to T.

11.6 The second law of thermodynamics and the dissipation of energy

We have seen that the kinetic-theory model can explain the way a gas behaves when it is compressed or expanded, warmed or cooled. In the late nineteenth century, the model was refined to take into account many effects we have not discussed. There proved to be limits beyond which the model breaks down. For example, radiated heat comes to us from the sun through the vacuum of space. This is not explainable in terms of the thermal motion of particles. But in most cases the model worked splendidly, explaining the phenomena of heat in terms of the ordinary

SG 11.23

motions of particles. This was indeed a triumph of Newtonian mechanics. It fulfilled much of the hope Newton had expressed in the *Principia:* that all phenomena of nature could be explained in terms of the motion of the small parts of matter. In the rest of this chapter we will touch briefly on the further development of thermodynamic theory. (Additional discussion appears in several articles in *Reader 3.)*

Our life runs down in sending up
the clock
The brook runs down in sending
up our life.
The sun runs down in sending up
the brook
And there is something sending
up the sun
It is this backward motion toward
the source.
Against the stream, that most we
see ourselves in
It is from this in nature we are
from
It is most us
[Robert Frost *West-Running Brook*]

The first additional concept arose out of a basic philosophical theme of the Newtonian cosmology: the idea that the world is like a machine whose parts never wear out, and which never runs down. This idea inspired the search for conservation laws applying to matter and motion. So far in this text, it might seem that this search has been successful. We can measure "matter" by mass, and "motion" by momentum or by kinetic energy. By 1850 the law of conservation of mass had been firmly established in chemistry. In physics, the laws of conservation of momentum and of energy had been equally well established.

Yet these successful conservation laws could not banish the suspicion that somehow the world *is* running down, the parts of the machine *are* wearing out. Energy may be conserved in burning fuel, but it loses its *usefulness* as the heat goes off into the atmosphere. Mass may be conserved in scrambling an egg, but its organized *structure* is lost. In these transformations, something is conserved, but something is also lost. Some processes are irreversible—they will not run backwards. There is no way to *un*scramble an egg, although such a change would not violate mass conservation. There is no way to draw smoke and hot fumes back onto a blackened stick, forming a new, unburned match.

The first attempts to find quantitative laws for such irreversible processes were stimulated by the development of steam engines. During the eighteenth and nineteenth centuries, engineers steadily increased the efficiency of steam engines. Recall that *efficiency* refers to the amount of mechanical work obtainable from a given amount of fuel energy. (See Section 10.6.) In 1824 a young French engineer, Sadi Carnot, published a short book entitled *Reflections on the Motive Power of Fire.* Carnot raised the question: Is there a maximum possible efficiency of an engine? Conservation of energy, of course, requires a limit of 100%, since energy output can never be greater than energy input. But, by analyzing the flow of heat in engines, Carnot proved that the maximum efficiency actually is always *less* than 100%. That is, the useful energy output can never even be as much as the input energy. There is a fixed limit on the amount of mechanical energy obtainable from a given amount of heat by using an engine. This limit can never be exceeded regardless of what substance—steam, air, or anything else—is used in the engine.


Sadi Carnot (1796–1832)

Modern steam engines have a theoretical limit of about 35% efficiency, but in practice they seldom have better than 20%.

In addition to this limit on efficiency even for ideal engines, real engines operate at still lower efficiency in practice. For example, heat leaks from the hot parts of the engine to the cooler parts. Usually, this heat bypasses the part of the engine where it could be used to generate mechanical energy.

Carnot's analysis of steam engines shows that there is an *unavoidable* waste of mechanical energy, even under ideal circumstances. The total amount of energy in the high-temperature steam is *conserved* as it passes through the engine. But while part of it is transformed into useful mechanical energy, the rest is discharged in the exhaust. It then joins the relatively low temperature pool of the surrounding world. Carnot reasoned that there always must be some such "rejection" of heat from any kind of engine. This rejected heat goes off into the surroundings and becomes unavailable for useful work.

These conclusions about heat engines became the basis for the *Second Law of Thermodynamics*. This law has been stated in various ways, all of which are roughly equivalent. It expresses the idea that it is impossible to convert a given amount of heat fully into work.

The first law of thermodynamics, or the general law of conservation of energy, does not forbid the full conversion of heat into mechanical energy. The second law is an additional constraint on what can happen in nature.

Carnot's analysis implies more than this purely negative statement, however. In 1852, Lord Kelvin asserted that the second law of thermodynamics applies even more generally. There is, he said, a universal tendency in nature toward the "degradation" or "dissipation" of energy. Another way of stating this principle was suggested by Rudolph Clausius, in 1865. Clausius introduced a new concept, *entropy* (from the Greek word for transformation). In thermodynamics, entropy is defined quantitatively in terms of temperature and heat transfer. But here we will find it more useful to associate entropy with *disorder*. Increases in entropy occur with increasing disorder of motion in the parts of a system.

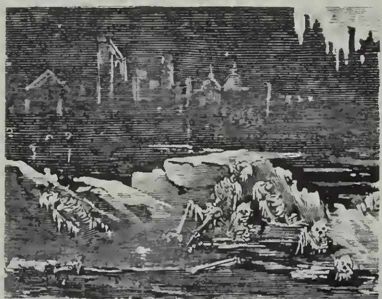
SG 11.24-11.26

For example, think of a falling ball. If its temperature is very low, the random motion of its parts is very low too. Thus, the motion of all particles during the falling is mainly downward (and hence "ordered"). The ball strikes the floor and bounces several times. During each bounce, the mechanical energy of the ball decreases and the ball warms up. Now the random thermal motion of the parts of the heated ball is far more vigorous. Finally, the ball as a whole lies still (no "ordered" motion). The disordered motion of its molecules (and of the molecules of the floor where it bounced) is all the motion left. According to the entropy concept, *all* motion of whole bodies will run down like this. In other words, as with the bouncing ball, all motions tend from ordered to disordered. In fact, entropy can be defined mathematically as a measure of the disorder of a system (though we will not go into the mathematics here). The general version of the second law of thermodynamics, as stated by Clausius, is therefore quite simple: *the entropy of an isolated system always tends to increase.*

Irreversible processes are processes for which entropy increases. For example, heat will not flow by itself from cold bodies to hot bodies A ball lying on the floor will not somehow gather the kinetic energy of its randomly moving parts and suddenly leap up. An egg will not unscramble itself. An ocean liner cannot be powered by an engine that takes heat from the ocean water and exhausts ice cubes. All these and many other events could occur without violating any principles of Newtonian mechanics, including the law of conservation of energy. But they do not happen; they are "forbidden" by the second law of thermodynamics. (We say "forbidden" in the sense that Nature does not show that such things happen. Hence, the second law, formulated by the human mind, describes

Two illustrations from Flammarion's novel, *Le Fin du Monde*.

"La miserable race humaine pêrira par le froid."



"Ce sera la fin."

well what Nature does or does not do.)

We haven't pointed it out yet, but *all* familiar processes are to some degree irreversible. Thus, Lord Kelvin predicted that all bodies in the universe would eventually reach the same temperature by exchanging heat with each other. When this happened, it would be impossible to produce any useful work from heat. After all, work can only be done by means of heat engines when heat flows from a hot body to a cold body. Finally, the sun and other stars would cool, all life on earth would cease, and the universe would be dead.

This general "heat-death" idea, based on predictions from thermodynamics, aroused some popular interest at the end of the nineteenth century. It appeared in several books of that time, such as H. G. Wells' *The Time Machine*. The French astronomer Camille Flammarion wrote a book describing ways in which the world would end. The American historian Henry Adams had learned about thermodynamics through the writings of one of America's greatest scientists, J. Willard Gibbs. Adams attempted to extend the application of the second law from physics to human history in a series of essays entitled *The Degradation of the Democratic Dogma*.

SG 11.27

Q12 The presumed "heat death of the universe" refers to a state
(a) in which all mechanical energy has been transformed into heat energy
(b) in which all heat energy has been transformed into other forms of energy
(c) in which the temperature of the universe decreases to absolute zero
(d) in which the supply of coal and oil has been used up.

Q13 Which of the following statements agrees with the second law of thermodynamics?
(a) Heat does not naturally flow from cold bodies to hot bodies.
(b) Energy tends to transform itself into less useful forms.
(c) No engine can transform all its heat input into mechanical energy.
(d) Most processes in nature are reversible.

11.7 Maxwell's demon and the statistical view of the second law of thermodynamics

Is there any way of avoiding the "heat death?" Is irreversibility a basic law of physics, or only an approximation based on our limited experience of natural processes?

The Austrian physicist Ludwig Boltzmann investigated the theory of irreversibility. He concluded that the tendency toward dissipation of energy is not an *absolute* law of physics that holds rigidly always. Rather, it is only a *statistical* law. Think of a can of air containing about 10^{22} molecules. Boltzmann argued that, of all conceivable arrangements of the gas molecules at a given instant, nearly all would be almost completely "disordered." Only a relatively few arrangements would have most of the molecules moving in the same direction. And even if a momentarily ordered arrangement of molecules occurred by chance, it would soon become less ordered by collisions, etc. Fluctuations from complete disorder will of course occur. But the greater the fluctuations, the less likely it is to occur. For collections of particles as large as 10^{22}, the chance of a fluctuation large enough to be measurable is vanishingly small. It is *conceivable* that a cold kettle of water will heat up on its own after being struck by only the most energetic molecules in the surrounding air. It is also *conceivable* that air molecules will "gang up" and strike only one side of a rock, pushing it uphill. But such events, while conceivable, are *utterly improbable*.

Consider also a pool table—the ordered motion of a cue ball moving into a stack of resting ones gets soon "randomized."

To illustrate Boltzmann's argument consider a pack of cards when it is shuffled. Most possible arrangements of the cards after shuffling are fairly disordered. If we start with an ordered arrangement—for example, the cards sorted by suit and rank—then shuffling would almost certainly lead to a more disordered arrangement. (Nevertheless it does occasionally happen that a player is dealt 13 spades—even if no one has stacked the deck.)

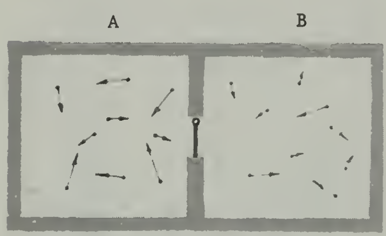
For *small* collections of particles, however, it is a different story. For example, it is quite probable that the average height of people on a bus will be considerably greater or less than the national average. In the same way, it is probable that more molecules will hit one side of a microscopic particle than the other side. Thus we can observe the "Brownian movement" of microscopic particles. Fluctuations are an important aspect of the world of very small particles. But they are virtually undetectable for any large collection of molecules familiar to us in the everyday world.

Still, the second law is different in character from all the other fundamental laws of physics we have studied so far. The difference is that it deals with probabilities, not uncertainties.

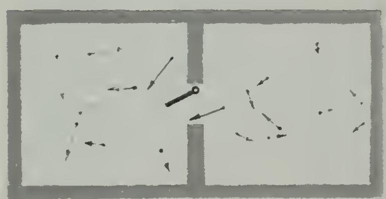
Maxwell proposed an interesting "thought experiment" to show how the second law of themodynamics could be violated or disobeyed. It involved an imaginary being who could observe individual molecules and sort them out in such a way that heat would flow from cold to hot. Suppose a container of gas is divided by a diaphragm into two parts, **A** and **B**. Initially the gas in **A** is hotter than the gas in **B**. This means that the molecules in **A** have greater average *KE* and therefore greater average speeds than those in **B**. However, the speeds are distributed according to

Drawing by Steinberg; © 1963, The New Yorker Magazine, Inc.

How Maxwell's "demon" could use a small, massless door to *increase* the order of a system and make heat flow from a cold gas to a hot gas.



Initially the average *KE* of molecules is greater in *A*.



Only fast molecules are allowed to go from *B* to *A*.

Only slow molecules are allowed to go from *A* to *B*.

As this continues, the average *KE* in *A* increases and the average *KE* in *B* decreases.

Maxwell's distribution (Section 11.3). Therefore many molecules in **A** have speeds less than the average in **A.**

"Now conceive a finite being," Maxwell suggested, "who knows the paths and velocities of all the molecules by simple inspection but who can do no work except open and close a hole in the diaphragm by means of a slide without mass." (If the slide or door has no mass, no work will be needed to move it.) This "finite being" observes the molecules in **A**. When he sees one coming whose speed is less than the average speed of the molecules in **B,** he opens the hole and lets it go into **B.** Now the average speed of the molecules of **B** is even lower than it was before. Next, the "being" watches for a molecule of **B** with a speed greater than the average speed in **A.** When he sees one, he opens the hole to let the molecule go into **A.** Now the average speed in **A** is even higher than it was before. Maxwell concludes:

> Then the number of molecules in **A** and **B** are the same as at first, but the energy in **A** is increased and that in **B** diminished, that is, the hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat-fingered being has been employed.

In the same way, a group of such beings could keep watch over a swarm of randomly moving molecules. By allowing passage to those molecules moving only in some given direction, they could establish a region of orderly molecular motion.

The imaginary "being who knows the paths and velocities of all the molecules" has come to be known as "Maxwell's demon." Maxwell's thought experiment shows that if there were any way to sort out individual molecules, the tendency to increasing entropy could be reversed. Some biologists have suggested that certain large molecules, such as enzymes, may function in just this way. They may influence the motions of smaller molecules, building up ordered molecular systems in living beings. This would help to explain why growing plants or animals do not tend toward higher disorder, while lifeless objects do.

As interesting as this suggestion is, it shows a misunderstanding of the second law of thermodynamics. The second law doesn't say that the order can never (or is extremely unlikely to) increase in *any* system. It makes that claim only for an isolated, or closed system. The order of *part* of a closed system may increase, but only if the order of the *other parts* decreases by as much or more. This point is made nicely in the following passage from a UNESCO document on environmental pollution.

SG 11.28

> Some scientists used to feel that the occurrence, reproduction, and growth of order in living systems presented an exception to the second law. This is no longer believed to be so. True, the living system may increase in order, but only by diffusing energy to the surroundings and by converting complicated molecules (carbohydrates, fats) called food into simple

molecules (CO_2, H_2O). For example, to maintain a healthy human being at constant weight for one year requires the degradation of about 500 kilograms (one half ton) of food, and the diffusion into the surroundings (from the human and the food) of about 500,000 kilocalories (two million kilojoules) of energy. The "order" in the human may stay constant or even increase, but the order in the surroundings decreases much, much more. Maintenance of life is an expensive process in terms of generation of disorder, and no one can understand the full implications of human ecology and environmental pollution without understanding that first.

James Clerk Maxwell (1831–1879)

Q14 In each of the following pairs, which situation is more ordered?

(a) an unbroken egg; a scrambled egg.

(b) a glass of ice and warm water; a glass of water at uniform temperature.

Q15 True or false?

(a) Maxwell's demon was able to get around the second law of thermodynamics.

(b) Scientists have made a Maxwell demon.

(c) Maxwell believed that his demon actually existed.

11.8 Time's arrow and the recurrence paradox

Late in the nineteenth century, a small but influential group of scientists began to question the basic philosophical assumptions of Newtonian mechanics. They even questioned the very idea of atoms. The Austrian physicist Ernst Mach argued that scientific theories should not depend on assuming the existence of things (such as atoms) which could not be directly observed. Typical of the attacks on atomic theory was the argument used by the mathematician Ernst Zermelo and others against kinetic theory. Zermelo believed that: (1) The second law of thermodynamics is an absolutely valid law of physics because it agrees with all the experimental data. However, (2) kinetic theory allows the possibility of exceptions to this law (due to large fluctuations). Therefore, (3) kinetic theory must be wrong. It is an interesting historical episode on a point that is still not quite settled.

The critics of kinetic theory pointed to two apparent contradictions between kinetic theory and the principle of dissipation of energy. These were the *reversibility paradox* and the *recurrence paradox*. Both paradoxes are based on possible exceptions to the second law; both could be thought to cast doubt on the kinetic theory.

The reversibility paradox: Can a model based on reversible events explain a world in which so many events are irreversible? (Also see photographs on next page.)

The *reversibility* paradox was discovered in the 1870's by Lord Kelvin and Josef Loschmidt, both of whom supported atomic theory. It was not regarded as a serious objection to the kinetic theory until the 1890's. The paradox is based on the simple fact that Newton's laws of motion are reversible in time. For example, if we watch a motion picture of a

bouncing ball, it is easy to tell whether the film is being run forward or backward. We know that the collisions of the ball with the floor are inelastic, and that the ball rises less high after each bounce. If, however, the ball made perfectly elastic bounces, it would rise to the same height after each bounce. Then we could not tell whether the film was being run forward or backward. In the kinetic theory, molecules *are* assumed to make perfectly elastic collisions. Imagine that we could take a motion picture of gas molecules colliding elastically according to this assumption. When showing this motion picture, there would be no way to tell whether it was being run forward or backward. Either way would show valid sequences of collisions. But—and this is the paradox—consider motion pictures of interactions involving large objects, containing many molecules. One can immediately tell the difference between forward (true) and backward (impossible) time direction. For example, a smashed lightbulb does not reassemble itself in real life, though a movie run backward can make it appear to do so.

The kinetic theory is based on laws of motion which are reversible for each individual molecular interaction. How, then, can it explain the existence of *irreversible* processes on a large scale? The existence of such processes seems to indicate that time flows in a definite direction—from past to future. This contradicts the possibility implied in Newton's laws of motion: that it does not matter whether we think of time as flowing forward or backward. As Lord Kelvin expressed the paradox,

> If . . . the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy, and throw the mass up the fall in drops reforming into a close column of ascending water. Heat which had been generated by the friction of solids and dissipated by conduction, and radiation with absorption, would come again to the place of contact, and throw the moving body back against the force to which it had previously yielded. Boulders would recover from the mud the materials required to rebuild them into their previous jagged forms, and would become reunited to the mountain peak from which they had formerly broken away. And if also the materialistic hypothesis of life were true, living creatures would grow backwards, with conscious knowledge of the future, but no memory of the past, and would become again unborn. But the real phenomena of life infinitely transcend human science; and speculation regarding consequences of their imagined reversal is utterly unprofitable.

Kelvin himself, and later Boltzmann, used statistical probability to explain why we do not observe such large-scale reversals. There are almost infinitely many possible disordered arrangements of water molecules at the bottom of a waterfall. Only an extremely small number of these arrangements would lead to the process described above. Reversals of this kind are possible *in principle,* but for all practical purposes they are out of the question.

SG 11.29

The answer to Zermelo's argument is that his first claim is incorrect. The second law of thermodynamics is not an absolute law, but a statistical law. It assigns a very low probability to ever detecting any overall increase in order, but does not declare it impossible.

However, another small possibility allowed in kinetic theory leads to a situation that seems unavoidably to contradict the dissipation of energy. The *recurrence paradox* revived an idea that appeared frequently in ancient philosophies and present also in Hindu philosophy to this day: the myth of the "eternal return." According to this myth, the long-range history of the world is cyclic. All historical events eventually repeat themselves, perhaps many times. Given enough time, even the matter that people were made of will eventually reassemble by chance. Then people who have died may be born again and go through the same life. The German philosopher Friedrich Nietzsche was convinced of the truth of this idea. He even tried to prove it by appealing to the principle of conservation of energy. He wrote:

The World's great age begins anew
  The golden years return
The earth doth like a snake renew
  His winter weeds outworn
Another Athens shall arise
  And to remoter time
Bequeath like sunset to the skies,
  The splendour of its prime
[Percy Bysshe Shelley, Hellas (1822)]

Lord Kelvin (1824–1907)

> If the universe may be conceived as a definite quantity of energy, as a definite number of centres of energy—and every other concept remains indefinite and therefore useless—it follows therefrom that the universe must go through a calculable number of combinations in the great game of chance which constitutes its existence. In infinity [of time], at some moment or other, every possible combination must once have been realized; not only this, but it must have been realized an infinite number of times.

If the number of molecules is finite, there is only a finite number of possible arrangements of molecules. Hence, somewhere in infinite time the same combination of molecules is bound to come up again. At the same point, all the molecules in the universe would reach exactly the same arrangement they had at some previous time. All events following this point would have to be exactly the same as the events that followed it before. That is, if any single instant in the history of the universe is ever exactly repeated, then the entire history of the universe will be repeated. And, as a little thought shows, it would then be repeated over and over again to infinity. Thus, energy would *not* endlessly become dissipated. Nietzsche claimed that this view of the eternal return disproved the "heat death" theory. At about the same time, in 1889, the French mathematician Henri Poincaré published a theorem on the possibility of recurrence in mechanical systems. According to Poincaré, even though the universe might undergo a heat death, it would ultimately come alive again:

SG 11.30-11.32

> A bounded world, governed only by the laws of mechanics, will always pass through a state very close to its initial state. On the other hand, according to accepted experimental laws (if one

> attributes absolute validity to them, and if one is willing to press their consequences to the extreme), the universe tends toward a certain final state, from which it will never depart. In this final state, from which will be a kind of death, all bodies will be at rest at the same temperature.
>
> . . . the kinetic theories can extricate themselves from this contradiction. The world, according to them, tends at first toward a state where it remains for a long time without apparent change; and this is consistent with experience; but it does not remain that way forever; . . . it merely stays there for an enormously long time, a time which is longer the more numerous are the molecules. This state will not be the final death of the universe, but a sort of slumber, from which it will awake after millions of centuries.
>
> According to this theory, to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and the dexterity of Maxwell's demon; it will suffice to have a little patience.

SG 11.33

Poincaré was willing to accept the possibility of a violation of the second law after a very long time. But others refused to admit even this possibility. In 1896, Zermelo published a paper attacking not only the kinetic theory but the mechanistic world view in general. This view, he asserted, contradicted the second law. Boltzmann replied, repeating his earlier explanations of the statistical nature of irreversibility.

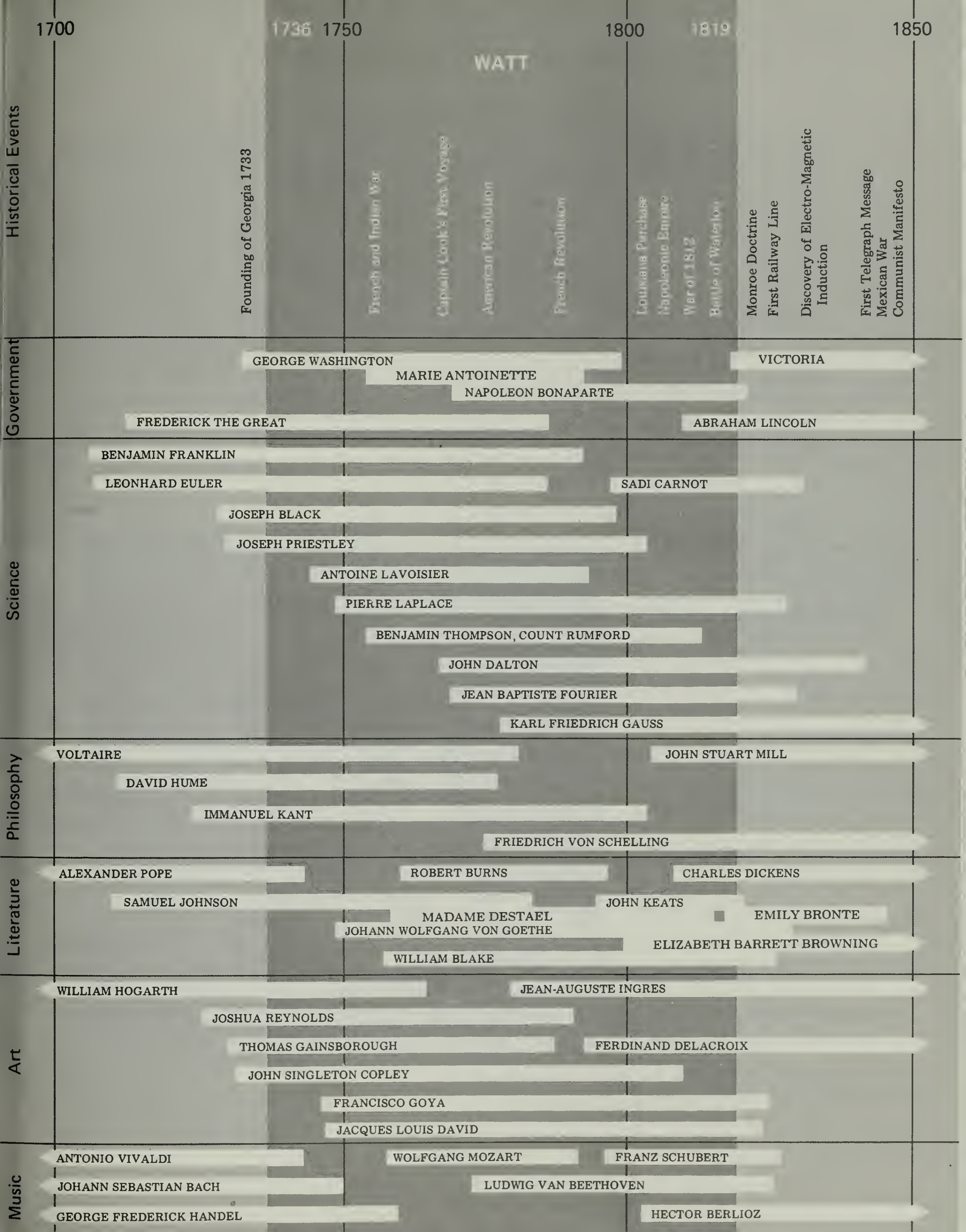
The final outcome of the dispute between Boltzmann and his critics was that both sides were partly right and partly wrong. Mach and Zermelo were correct in believing that Newton's laws of mechanics cannot fully describe molecular and atomic processes. (We will come back to this subject in Unit 5.) For example, it is only approximately valid to describe gases in terms of collections of frantic little billiard balls. But Boltzmann was right in defending the usefulness of the molecular model. The kinetic theory is very nearly correct except for those properties of matter which involve the structure of molecules themselves.

Record of a particle in Brownian motion. Successive positions, recorded every 20 seconds, are connected by straight lines. The actual paths between recorded positions would be as erratic as the overall path.

In 1905, Albert Einstein pointed out that the fluctuations predicted by kinetic theory could be used to calculate the rate of displacement for particles in "Brownian" movement. Precise quantitative studies of Brownian movement confirmed Einstein's theoretical calculations. This new success of kinetic theory—along with discoveries in radioactivity and atomic physics—persuaded almost all the critics that atoms and molecules do exist. But the problems of irreversibility and of whether the laws of physics must distinguish between past and future survived. In new form, these issues still interest physicists today.

This chapter concludes the application of Newtonian mechanics to individual particles. The story was mainly one of triumphant success. Toward the end, however, we have hinted that, like all theories, Newtonian mechanics has serious limitations. These will be explored later.

The last chapter in this unit covers the successful use of Newtonian mechanics in the case of mechanical wave motion. This will complete the list of possibilities of particle motion. In Unit 1 we treated the motion of single particles or isolated objects. The motion of a system of objects bound by a



1700
1736
1750
1800
1819
1850
WATT
Historical Events
Founding of Georgia 1733
French and Indian War
Captain Cook's First Voyage
American Revolution
French Revolution
Louisiana Purchase
Napoleonic Empire
War of 1812
Battle of Waterloo
Monroe Doctrine
First Railway Line
Discovery of Electro-Magnetic Induction
First Telegraph Message
Mexican War
Communist Manifesto
Government
GEORGE WASHINGTON
VICTORIA
MARIE ANTOINETTE
NAPOLEON BONAPARTE
FREDERICK THE GREAT
ABRAHAM LINCOLN
Science
BENJAMIN FRANKLIN
LEONHARD EULER
SADI CARNOT
JOSEPH BLACK
JOSEPH PRIESTLEY
ANTOINE LAVOISIER
PIERRE LAPLACE
BENJAMIN THOMPSON, COUNT RUMFORD
JOHN DALTON
JEAN BAPTISTE FOURIER
KARL FRIEDRICH GAUSS
Philosophy
VOLTAIRE
JOHN STUART MILL
DAVID HUME
IMMANUEL KANT
FRIEDRICH VON SCHELLING
Literature
ALEXANDER POPE
ROBERT BURNS
CHARLES DICKENS
SAMUEL JOHNSON
JOHN KEATS
MADAME DESTAEL
EMILY BRONTE
JOHANN WOLFGANG VON GOETHE
ELIZABETH BARRETT BROWNING
WILLIAM BLAKE
Art
WILLIAM HOGARTH
JEAN-AUGUSTE INGRES
JOSHUA REYNOLDS
THOMAS GAINSBOROUGH
FERDINAND DELACROIX
JOHN SINGLETON COPLEY
FRANCISCO GOYA
JACQUES LOUIS DAVID
Music
ANTONIO VIVALDI
WOLFGANG MOZART
FRANZ SCHUBERT
JOHANN SEBASTIAN BACH
LUDWIG VAN BEETHOVEN
GEORGE FREDERICK HANDEL
HECTOR BERLIOZ

force of interaction, such as the Earth and Sun, was treated in Unit 2 and in Chapters 9 and 10 of this unit. In this chapter we discussed the motions of a system of a very large number of separate objects. Finally, in Chapter 12 we will study the action of many particles going back and forth together as a wave passes.

Q16 The kinetic energy of a falling stone is transformed into heat when the stone strikes the ground. Obviously this is an irreversible process; we never see the heat transform into kinetic energy of the stone, so that the stone rises off the ground. We believe that the process is irreversible because

(a) Newton's laws of motion prohibit the reversed process.

SG 11.34
SG 11.35

(b) the probability of such a sudden ordering of molecular motion is extremely small.

(c) the reversed process would not conserve energy.

(d) the reverse process would violate the second law of thermodynamics.

The ruins of a Greek temple at Delphi are as elegant a testimony to the continual encroachment of disorder, as is the tree to the persistent development of islands of order by living organisms.

11.1 The Project Physics materials particularly appropriate for Chapter 11 include:

Experiments

Monte Carlo Experiment on Molecular Collisions

Behavior of Gases

Activities

Drinking Duck

Mechanical Equivalent of Heat

A Diver in a Bottle

Rockets

How to Weigh a Car with a Tire Pressure Gauge

Perpetual-Motion Machines

Film Loop

Reversibility of Time

Reader Articles

The Barometer Story

The Great Molecular Theory of Gases

Entropy and the Second Law of Thermodynamics

The Law of Disorder

The Law

The Arrow of Time

James Clerk Maxwell

Randomness and the Twentieth Century

11.2 The idea of randomness can be used in predicting the results of flipping a large number of coins. Give some other examples where randomness is useful.

11.3 The examples of early kinetic theories given in Sec. 11.3 include only *quantitative* models. Some of the underlying ideas are thousands of years old. Compare the kinetic molecular theory of gases to these Greek ideas expressed by the Roman poet Lucretius in about 60 B.C.:

> If you think that the atoms can stop and by their stopping generate new motions in things, you are wandering far from the path of truth. Since the atoms are moving freely through the void, they must all be kept in motion either by their own weight or on occasion by the impact of another atom. For it must often happen that two of them in their course knock together and immediately bounce apart in opposite directions, a natural consequence of their hardness and solidity and the absence of anything behind to stop them . . . .
>
> It clearly follows that no rest is given to the atoms in their course through the depths of space. Driven along in an incessant but variable movement, some of them bounce far apart after a collision while others recoil only a short distance from the impact. From those that do not recoil far, being driven into a closer union and held there by the entanglement of their own interlocking shapes, are composed firmly rooted rock, the stubborn strength of steel and the like. Those others that move freely through larger tracts of space, springing far apart and carried far by the rebound—these provide for us thin air and blazing sunlight. Besides these, there are many other atoms at large in empty space which have been thrown out of compound bodies and have nowhere even been granted admittance so as to bring their motions into harmony.

11.4 Consider these aspects of the curves showing Maxwell's distribution of molecular speeds:

(a) All show a peak.

(b) The peaks move toward higher speed at higher temperatures.

(c) They are not symmetrical, like normal distribution curves.

Explain these characteristics on the basis of the kinetic model.

11.5 The measured speed of sound in a gas turns out to be nearly the same as the average speed of the gas molecules. Is this a coincidence? Discuss.

11.6 How did Clausius modify the simple kinetic model for a gas? What was he able to explain with this new model?

11.7 Benjamin Franklin observed in 1765 that a teaspoonful of oil would spread out to cover half an acre of a pond. This helps to give an estimate of the upper limit of the size of a molecule. Suppose that one cubic centimeter of oil forms a continuous layer one molecule thick that just covers an area on water of 1000 square meters.

(a) How thick is the layer?

(b) What is the size of a single molecule of the oil (considered to be a cube for simplicity)?

11.8 Knowing the size of molecules allows us to compute the number of molecules in a sample of material. If we assume that molecules in a solid or liquid are packed close together, something like apples in a bin, then the total volume of a material is approximately equal to the volume of one molecule times the number of molecules in the material.

(a) Roughly how many molecules are there in 1 cubic centimeter of water? (For this approximation, you can take the volume of a molecule to be d^3 if its diameter is d.)

(b) The density of a gas (at 1 atmosphere pressure and 0°C) is about 1/1000 the density of a liquid. Roughly how many molecules are there in 1 cc. of gas? Does this estimate support the kinetic model of a gas as described on p. 82?

11.9 How high could water be raised with a lift pump on the moon?

11.10 At sea level, the atmospheric pressure of air ordinarily can balance a barometer column of mercury of height 0.76 meters or 10.5 meters of water. Air is approximately a thousand times less dense than liquid water. What can you say about the minimum height to which the atmosphere goes above the Earth?

11.11 How many atmospheres of pressure do you exert on the ground when you stand on flat-heeled shoes? skis? skates? (1 atmosphere is about 15 lbs/in.)

11.12 From the definition of density, $D = M/V$ (where M is the mass of a sample and V is its volume), write an expression relating pressure P and volume V of a gas.

11.13 Show how all the proportionalities describing gas behavior on p. 79 are included in the ideal gas law: $P = kD\,(t + 273°)$

11.14 The following information appeared in a pamphlet published by an oil company:

> HOW'S YOUR TIRE PRESSURE?
>
> If you last checked the pressure in your tires on a warm day, one cold morning you may find your tires seriously underinflated.
>
> The Rubber Manufacturers Association warns that tire pressures drop approximately one pound for every 10-degree dip in outside air. If your tires register 24 pounds pressure on an 80-degree day, for example, they'll have only 19 pounds pressure when the outside air plunges to 30° Fahrenheit.
>
> If you keep your car in a heated garage at 60°, and drive out into a 20 degrees-below-zero morning, your tire pressure drops from 24 pounds to 18 pounds.

Are these statements consistent with the ideal gas law? (Note: The pressure registered on a tire gauge is the pressure *above* normal atmospheric pressure of about 15 pounds/sq. in.)

11.15 Distinguish between two uses of the word "model" in science.

11.16 If a light particle rebounds from a massive, stationary wall with almost no loss of speed, then, according to the principle of Galilean relativity, it would still rebound from a *moving* wall without changing speed as seen in *the frame of reference of the moving piston.* Show that the rebound speed as measured *in the laboratory* would be less from a retreating wall (as is claimed at the bottom of p. 84).
(Hint: First write an expression relating the particle's speed relative-to-the-wall to its speed relative-to-the-laboratory.)

11.17 What would you expect to happen to the temperature of a gas that was released from a container in empty space (that is, with nothing to push back)?

11.18 List some of the directly observable properties of gases.

11.19 What aspects of the behavior of gases can the kinetic molecular theory be used to explain successfully?

11.20 Many products are now sold in spray cans. Explain in terms of the kinetic theory of gases why it is dangerous to expose the cans to high temperatures.

11.21 When a gas in an enclosure is compressed by pushing in a piston, its temperature increases.

Explain this fact in two ways:

(a) by using the first law of thermodynamics.

(b) by using the kinetic theory of gases.

The compressed air eventually cools down to the same temperature as the surroundings. Describe this heat transfer in terms of molecular collisions.

11.22 From the point of view of the kinetic theory, how can one explain (a) that a hot gas would not cool itself down while in a perfectly insulating container? (b) how a kettle of cold water, when put on the stove, reaches a boiling temperature. (Hint: At a given temperature the molecules in and on the walls of the solid container are also in motion, although, being part of a solid, they do not often get far away.)

11.23 In the *Principia* Newton expressed the hope that all phenomena could be explained in terms of the motion of atoms. How does Newton's view compare with this Greek view expressed by Lucretius in about 60 B.C.?

> I will now set out in order *the stages by which the initial concentration of matter laid the foundations of earth and sky,* of the ocean depths and the orbits of sun and moon. Certainly the atoms did not post themselves purposefully in due order by an act of intelligence, nor did they stipulate what movements each should perform. But multitudinous atoms, swept along in multitudinous courses through infinite time by mutual clashes and their own weight, have come together in every possible way and realized everything that could be formed by their combinations. So it comes about that a voyage of immense duration, in which they have experienced every variety of movement and conjunction, has at length brought together those whose sudden encounter normally forms the starting-point of substantial fabrics—earth and sea and sky and the races of living creatures.

11.24 Clausius' statement of the second law of thermodynamics is: "Heat will not of its own accord pass from a cooler to a hotter body." Give examples of the operation of this law. Describe how a refrigerator can operate, and show that it does not contradict the Clausius statement.

11.25 There is a tremendous amount of internal energy in the oceans and in the atmosphere. What would you think of an invention that purported to draw on this source of energy to do mechanical work? (For example, a ship that sucked in sea water and exhausted blocks of ice, using the heat from the water to run the ship.)

11.26 Imagine a room that is perfectly insulated so that no heat can enter or leave. In the room is a refrigerator that is plugged into an electric outlet in the wall. If the door of the refrigerator is left open, what happens to the temperature of the room?

11.27 Since there is a tendency for heat to flow from hot to cold, will the universe eventually reach absolute zero?

11.28 Does Maxwell's demon get around the second law of thermodynamics? List the assumptions in Maxwell's argument. Which of them do you believe are likely to be true?

11.29 Since all the evidence is that molecular motions are random, one might expect that any given arrangement of molecules will recur if one just waited long enough. Explain how a paradox arises when this prediction is compared with the second law of thermodynamics.

11.30 (a) Explain what is meant by the statement that Newton's laws of motion are time-reversible.
(b) Describe how a paradox arises when the time-reversibility of Newton's laws of motion is compared with the second law of thermodynamics.

11.31 If there is a finite probability of an exact repetition of a state of the universe, there is also a finite probability of its exact opposite—that is, a state where molecules are in the same position but with reversed velocities. What would this imply about the subsequent history of the universe?

11.32 List the assumptions in the "recurrence" theory. Which of them do you believe to be true?

11.33 Some philosophical and religious systems of the Far East and the Middle East include the ideas of the eternal return. If you have read about some of these philosophies, discuss what analogies exist with some of the ideas in the last part of this chapter. Is it appropriate to take the existence of such analogies to mean there is some direct connection between these philosophical and physical ideas?

11.34 Where did Newtonian mechanics run into difficulties in explaining the behavior of molecules?

11.35 What are some advantages and disadvantages of theoretical models?

CHAPTER TWELVE

Waves

12.1 Introduction

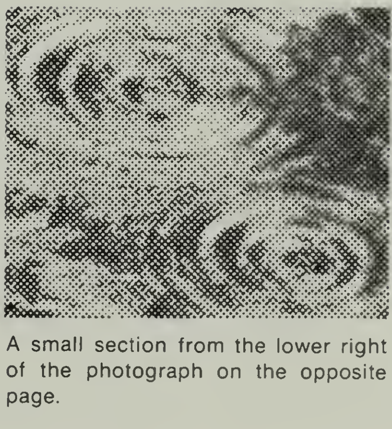
The world is continually criss-crossed by waves of all sorts. Water waves, whether giant rollers in the middle of the ocean or gently-formed rain ripples on a still pond, are sources of wonder or pleasure. If the earth's crust shifts, violent waves in the solid earth cause tremors thousands of miles away. A musician plucks a guitar string and sound waves pulse against our ears. Wave disturbances may come in a concentrated bundle like the shock front from an airplane flying at supersonic speeds. Or the disturbances may come in succession like the train of waves sent out from a steadily vibrating source, such as a bell or a string.

SG 12.1

All of these are *mechanical* waves, in which bodies or particles physically move back and forth. But there are also wave disturbances in electric and magnetic fields. In Unit 4, you will learn that such waves are responsible for what our senses experience as light. In all cases involving waves, however, the effects produced depend on the flow of energy as the wave moves forward.

So far in this text we have considered motion in terms of individual particles. In this chapter we begin to study the cooperative motion of collections of particles in "continuous media" moving in the form of mechanical waves. We will see how closely related are the ideas of particles and waves which we use to describe events in nature.

A comparison will help us here. Look at a black and white photograph in a newspaper or magazine with a magnifying glass. You will see that the picture is made up of many little black dots printed on a white page (up to 20,000 dots per square inch). Without the magnifier, you do not see the individual dots. Rather, you see a pattern with all possible shadings between completely black and completely white. These two views emphasize different aspects of the same thing. In much the same way, the physicist can sometimes choose between two (or more) ways of viewing events. For the most part, a particle view has been emphasized in



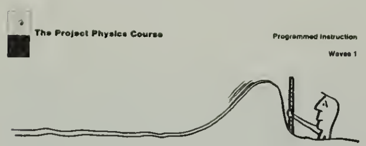
A small section from the lower right of the photograph on the opposite page.

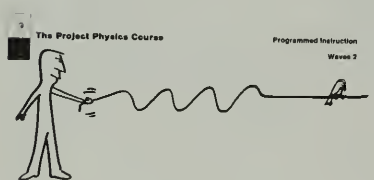
the first three units of the *Text*. In Unit 2 for example, we treated each planet as a particle undergoing the sun's gravitational attraction. We described the behavior of the solar system in terms of the positions, velocities, and accelerations of point-like objects. For someone interested only in planetary motions, this is fine. But for someone interested in, say, the chemistry of materials on Mars, it is not very helpful.

In the last chapter we saw two different descriptions of a gas. One was in terms of the behavior of the individual particles making up the gas. We used Newton's laws of motion to describe what each *individual* particle does. Then we used average values of speed or energy to describe the behavior of the gas. But we also discussed concepts such as pressure, temperature, heat, and entropy. These refer directly to a sample of gas *as a whole*. This is the viewpoint of thermodynamics, which does not depend on assuming Newton's laws or even the existence of particles. Each of these viewpoints served a useful purpose and helped us to understand what we cannot directly see.

Now we are about to study waves, and once again we find the possibility of using different points of view. Most of the waves discussed in this chapter can be described in terms of the behavior of particles. But we also want to understand waves as disturbances traveling in a continuous medium. We want, in other words, to see both the forest and the trees—the picture as a whole, not only individual dots.

Waves should be studied in the laboratory. Most of this chapter is only a summary of some of what you will learn there. The articles "Waves" and "What is a Wave" in *Reader 3* give additional discussion of wave behavior. Transparencies and film loops on waves are listed in SG 12.1. The Programmed Instruction booklets *Waves 1* and *Waves 2* may help you with the mathematics of periodic waves (see Sec. 12.4) and wave superposition (see Sec 12.5).





12.2 Properties of waves

Suppose that two people are holding opposite ends of a rope. Suddenly one person snaps the rope up and down quickly once. That "disturbs" the rope and puts a hump in it which travels along the rope toward the other person. We can call the traveling hump one kind of a wave, a *pulse*.

Originally, the rope was motionless. The height of each point on the rope depended only upon its position along the rope, and did not change in time. But when one person snaps the rope, he creates a rapid change in the height of one end. This disturbance then moves away from its source. The height of each point on the rope depends upon time as well as position along the rope.

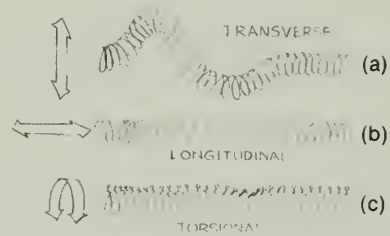
The disturbance is a pattern of *displacement* along the rope. The motion of the displacement pattern from one end of the rope toward the other is an example of a *wave*. The hand snapping one end is the *source* of the wave. The rope is the *medium* in which the wave moves. These four terms are common to all mechanical wave situations.

Consider another example. When a pebble falls into a pool of still liquid, a series of circular crests and troughs spreads over the surface. This moving displacement pattern of the liquid surface is a wave. The pebble is the source, the moving pattern of crests and troughs is the wave, and the liquid surface is the medium. Leaves, sticks, or other objects floating on the surface of the liquid bob up and down as each wave passes. But they do not experience any net displacement on the average.

No *material* has moved from the wave source, either on the surface or among the particles of the liquid. The same holds for rope waves, sound waves in air, etc.

As any one of these waves moves through a medium, the wave produces a changing displacement of the successive parts of the medium. Thus we can refer to these waves as *waves of displacement*. If we can see the medium and recognize the displacements, then we can see waves. But waves also may exist in media we cannot see (such as air). Or they may form as disturbances of a state we cannot detect with our eyes (such as pressure, or an electric field).

You can use a loose spring coil to demonstrate three different kinds of motion in the medium through which a wave passes. First, move the end of the spring from side to side, or up and down as in sketch (a) in the margin. A wave of side-to-side or up-and-down displacement will travel along the spring. Now push the end of the spring back and forth, along the direction of the spring itself, as in sketch (b). A wave of back-and-forth displacement will travel along the spring. Finally, twist the end of the spring clockwise and counterclockwise, as in sketch (c). A wave of angular displacement will travel along the spring. Waves like those in (a), in which the displacements are perpendicular to the direction the wave travels, are called *transverse* waves. Waves like those in (b), in which the displacements are in the direction the wave travels, are called *longitudinal* waves. And waves like those in (c), in which the displacements are twisting in a plane perpendicular to the direction the wave travels, are called *torsional* waves.



"Snapshots" of three types of waves. In (c), small markers have been put on the top of each coil in the spring.

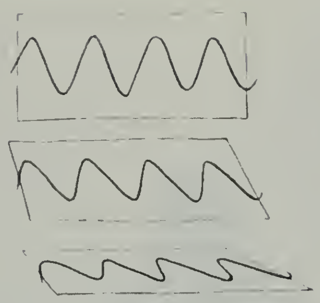
All three types of wave motion can be set up in solids. In fluids, however, transverse and torsional waves die out very quickly, and usually cannot be produced at all. So sound waves in air and water are longitudinal. The molecules of the medium are displaced back and forth along the direction that the sound travels.

It is often useful to make a graph of wave patterns in a medium. However, a graph on paper always has a transverse appearance, even if it represents a longitudinal or torsional wave. For example, the graph at the right represents the pattern of compressions in a sound wave in air. The sound waves are longitudinal, but the graph line goes up and down. This is because the graph represents the increasing and decreasing density of the air. It does *not* represent an up-and-down motion of the air.

(a) "Snapshot" representation of a sound wave progressing to the right. The dots represent the density of air molecules. (b) Graph of air pressure P vs. position x at the instant of the snapshot.

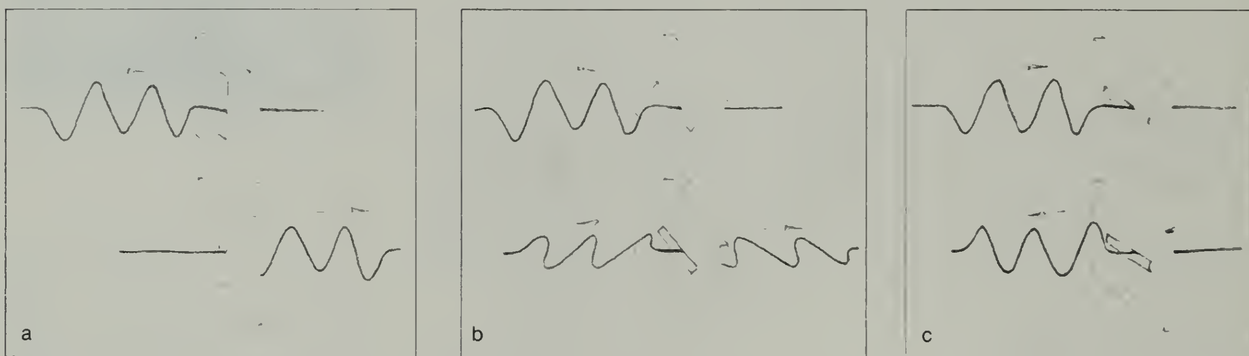
To describe completely transverse waves, such as those in ropes, you must specify the *direction* of displacement. Longitudinal and torsional waves do not require this specification. The displacement of a longitudinal wave can be in only one direction—along the direction of travel of the wave. Similarly, the angular displacements of a torsional wave can be around only one axis—the direction of travel of the wave. But the displacements of a transverse wave can be in any and all of an infinite number of directions. The only requirement is that they be at right angles to the direction of travel of the wave. You can see this by shaking one end of a rope randomly instead of straight up and down or straight left and right. For simplicity, our diagrams of rope and spring waves here show transverse displacements consistently in only one of all the possible planes.

When the displacement pattern of a transverse wave does lie in a



Three of the infinitely many different polarization planes of a transverse wave.

single plane, we say the wave is *polarized*. For waves on ropes and springs, we can observe the polarization directly. Thus, in the photograph on the previous page, the waves the person makes are in the horizontal plane. However, whether we can see the wave directly or not, there is a general test for polarization. The test involves finding some effect of the wave which depends on the angular position of a medium or obstacle through which it travels. An example of the principle is illustrated in the diagram below. Here, the transmission of a rope wave depends on the angle at which a slotted board is held. Each of the three sketches begins with the same wave approaching the obstacle (top line). Whether the wave passes through (bottom line) depends on the angle the slot makes with the plane of the rope's mechanical motion.



The same short wave train on the rope approaches the slotted board in each of the three sketches (top). Depending on the orientation of the slot, the train of waves (a) goes entirely through the slot; (b) is partly reflected and partly transmitted with changed angles of rope vibration; or (c) is completely reflected.

In general, if some effect of a wave depends similarly on the angular position of an obstacle or medium, the wave must be polarized. Further, we can conclude that the wave is transverse rather than longitudinal or torsional. Some interesting and important examples of this principle will be presented in Chapter 13.

All three kinds of wave—longitudinal, transverse, and torsional—have an important characteristic in common. The disturbances move away from their sources through the media and *continue on their own*. We stress this particular characteristic by saying that these waves *propagate*. This means more than just that they "travel" or "move." An example will clarify the difference between waves which propagate and those which do not. You probably have read some description of the great wheat plains of our Middle West, Canada, or Central Europe. Such descriptions usually mention the "beautiful, wind-formed waves that roll for miles across the fields." The medium for such a wave is the wheat, and the disturbance is the swaying motion of the wheat. This disturbance does indeed travel, but it does *not* propagate. That is, the disturbance does not originate at a source and then go on by *itself*. Rather, it must be continually fanned by the wind. When the wind stops, the disturbance does not roll on, but stops, too. The traveling "waves" of swaying wheat are not at all the same as our rope and water waves. We will concentrate on waves that do originate at sources and propagate themselves. For the purposes of this chapter, *waves are disturbances which propagate in a medium.*

Q1 What kinds of mechanical waves can propagate in a solid?

Q2 What kinds of mechanical waves can propagate in a fluid?

Q3 What kinds of mechanical waves can be polarized?

Q4 Suppose that a mouse runs along under a rug, causing a bump in the rug that travels with the mouse across the room. Is this moving disturbance a propagating wave?

12.3 Wave propagation

Waves and their behavior are perhaps best studied by beginning with large mechanical models and focusing our attention on pulses. Consider for example a freight train, with many cars attached to a powerful locomotive, but standing still. If the locomotive now starts abruptly, it sends a displacement wave running down the line of cars. The shock of the starting displacement proceeds from locomotive to caboose, clacking through the couplings one by one. In this example, the locomotive is the source of the disturbance, while the freight cars and their couplings are the medium. The "bump" traveling along the line of cars is the wave. The disturbance proceeds all the way from end to end, and with it goes *energy* of displacement and motion. Yet no particles of matter are transferred that far; each car only jerks ahead a bit.

A very important point: energy transfer can occur without matter transfer.

How long does it take for the effect of a disturbance created at one point to reach a distant point? The time interval depends upon the speed with which the disturbance or wave propagates. That, in turn, depends upon the type of wave and the characteristics of the medium. In any case, the effect of a disturbance is never transmitted instantly over any distance. Each part of the medium has inertia, and each portion of the medium is compressible. So time is needed to transfer energy from one part to the next.

An engine starting abruptly can start a displacement wave along a line of cars.

The same comments apply also to transverse waves. The series of sketches in the margin represents a wave on a rope. Think of the sketches as frames of a motion picture film, taken at equal time intervals. The material of the rope does *not* travel along with the wave. But each bit of the rope goes through an up-and-down motion as the wave passes. Each bit goes through exactly the same motion as the bit to its left, except a little later.

A rough representation of the forces at the ends of a small section of rope as a transverse pulse moves past.

Consider the small section of rope labeled X in the diagrams. When the pulse traveling on the rope first reaches X, the section of rope just to the left of X exerts an upward force on X. As X is moved upward, a restoring downward force is exerted by the next section. The further upward X moves, the greater the restoring forces become. Eventually X stops moving upward and starts down again. The section of rope to the left of X now exerts a restoring (downward) force, while the section to the right exerts an upward force. Thus, the trip down is similar, but opposite, to the trip upward. Finally, X returns to the equilibrium position and both forces vanish.

The time required for X to go up and down—the time required for the pulse to pass by that portion of the rope—depends on two factors. These factors are the *magnitude of the forces* on X, and the *mass* of X. To put it more generally: the speed with which a wave propagates depends on the *stiffness* and on the *density* of the medium. The stiffer the medium, the greater will be the force each section exerts on neighboring sections. Thus, the greater will be the propagation speed. On the other hand, the greater the density of the medium, the less it will respond to forces. Thus, the slower will be the propagation. In fact, the speed of propagation depends on the *ratio* of the stiffness factor and the density factor.

SG 12.2

The exact meaning of stiffness and density factors is different for different kinds of waves and different media. For tight strings, for example, the stiffness factor is the tension T in the string, and the density factor is the mass per unit length, m/l. The propagation speed v is given by

$$v = \sqrt{\frac{T}{m/l}}$$

Q5 What is transferred along the direction of wave motion?

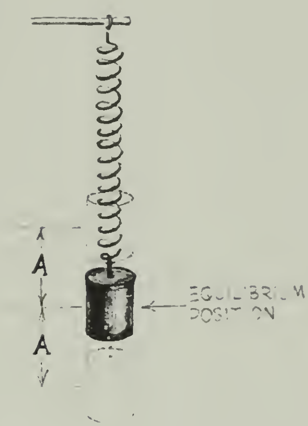
Q6 On what two properites of a medium does wave speed depend?

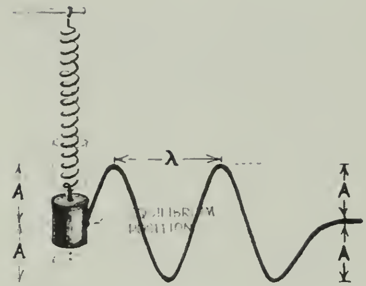
12.4 Periodic waves

Many of the disturbances we have considered up to now have been sudden and short-lived. They were set up by a single disturbance like snapping one end of a rope or suddenly bumping one end of a train. In each case, we see a single wave running along the medium with a certain speed. We call this kind of wave a *pulse*.

Now let us consider *periodic waves*—continuous regular rhythmic disturbances in a medium, resulting from *periodic vibrations* of a source. A good example of a periodic vibration is a swinging pendulum. Each swing is virtually identical to every other swing, and the swing repeats over and over again in time. Another example is the up-and-down motion of a weight at the end of a spring. The maximum displacement from the position of equilibrium is called the *amplitude* A, as shown on page 107. The time taken to complete one vibration is called the *period* T. The number of vibrations per second is called the *frequency* f.

What happens when such a vibration is applied to the end of a rope? Suppose that one end of a rope is fastened to the oscillating (vibrating) weight. As the weight vibrates up and down, we observe a wave propagating along the rope. The wave takes the form of a series of moving crests and troughs along the length of the rope. The source executes "simple harmonic motion" up and down. Ideally, every point along the length of the rope executes simple harmonic motion in turn. The wave travels to the right as crests and troughs follow one another. But each point along the rope simply oscillates up and down at the same frequency as the source. The amplitude of the wave is represented by A. The distance between any two consecutive crests or any two consecutive troughs is the same all along the length of the rope. This distance, called the *wavelength* of the periodic wave, is represented by the Greek letter λ (lambda).





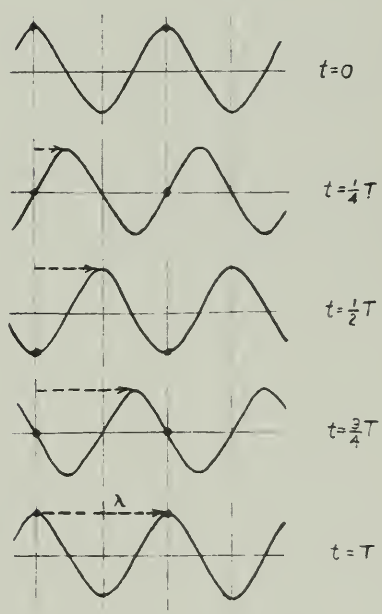
If a single pulse or a wave crest moves fairly slowly through the medium, we can easily find its *speed*. In principle all we need is a clock and a meter stick. By timing the pulse or crest over a measured distance, we get the speed. But it is not always simple to observe the motion of a pulse or a wave crest. As is shown below, however, the speed of a periodic wave can be found indirectly from its frequency and wavelength.

As a wave progresses, each point in the medium oscillates with the frequency and period of the source. The diagram in the margin illustrates a periodic wave moving to the right, as it might look in snapshots taken every $\frac{1}{4}$ period. Follow the progress of the crest that started out from the extreme left at $t = 0$. The time it takes this crest to move a distance of one wavelength is *equal* to the time required for one complete oscillation. That is, the crest moves one wavelength λ in one period of oscillation T. The speed v of the crest is therefore

$$v = \frac{\text{distance moved}}{\text{corresponding time interval}} = \frac{\lambda}{T}$$

All parts of the wave pattern propagate with the same speed. Thus the speed of any one crest is just the speed of the wave. We can say, therefore, that the speed v of the wave is

$$v = \frac{\text{wavelength}}{\text{period of oscillation}} = \frac{\lambda}{T}$$

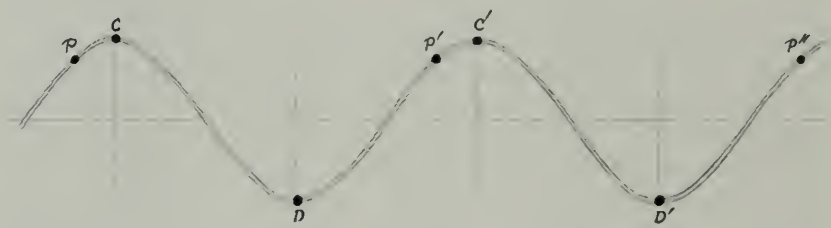


The wave generated by a simple harmonic vibration is a *sine* wave. A "snapshot" of the displacement of the medium would show it has the same shape as a graph of the sine function familiar in trigonometry. This shape is frequently referred to as "sinusoidal."

But $T = 1/f$, where f = frequency (see *Text*, Chapter 4, page 108). Therefore $v = f\lambda$, or wave speed = frequency × wavelength.

We can also write this relationship as $\lambda = v/f$ or $f = v/\lambda$. These expressions imply that, for waves of the same speed, the frequency and wavelength are inversely proportional. That is, a wave of twice the frequency would have only half the wavelength, and so on. This inverse relation of frequency and wavelength will be useful in other parts of this course.

The diagram below represents a periodic wave passing through a medium. Sets of points are marked which are moving "in step" as the periodic wave passes. The crest points C and C' have reached maximum displacement positions in the upward direction. The trough points D and D' have reached maximum displacement positions in the downward direction. The points C and C' have identical displacements and velocities at any instant of time. Their vibrations are identical, and in unison. The same is true for the points D and D'. Indeed there are infinitely many such points along the medium which are vibrating identically when this wave passes. Note that C and C' are a distance λ apart, and so are D and D'.



A "snapshot" of a periodic wave moving to the right. Letters indicate sets of points with the same phase.

Points that move "in step" such as C and C', are said to be *in phase* with one another. Points D and D' also move in phase. Points separated from one another by distances of λ, 2λ, 3λ, . . . and $n\lambda$ (where n is any whole number) are all in phase with one another. These points can be anywhere along the length of the wave. They need not correspond with only the highest or lowest points. For example, points such as P, P', P'', are all in phase with one another. Each is separated from the next by a distance λ.

Some of the points are exactly *out* of step. For example, point C reaches its maximum upward displacement at the same time that D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to go up. Points such as these are called one-half period *out of phase* with respect to one another; C and D' also are one-half period out of phase. Any two points separated from one another by distances of $\lambda/2$, $3\lambda/2$, $5\lambda/2$, . . . are one-half period out of phase.

Q7 Of the wave variables—frequency, wavelength, period, amplitude and polarization—which ones describe

(1) *space* properties of waves?

(2) *time* properties of waves?

Q8 A wave with the displacement as smoothly and simply varying from point to point as that shown in the last illustration above is called a sine wave. How might the "wavelength" be defined for a periodic wave that isn't a sine wave?

Q9 A vibration of 100 cycles per second produces a wave.

(1) What is the wave frequency?

(2) What is the period of the wave?

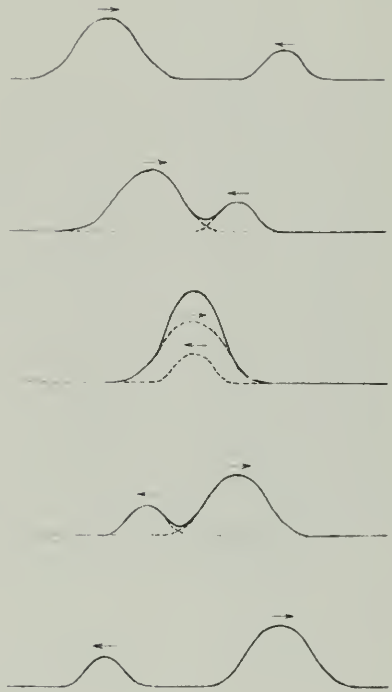
(3) If the wave speed is 10 meters per second what is the wavelength? (If necessary, look back to find the relationship you need to answer this.)

Q10 If points X and Y on a periodic wave are one-half period "out of phase" with each other, which of the following *must* be true?

(a) X oscillates at half the frequency at which Y oscillates.
(b) X and Y always move in opposite directions.
(c) X is a distance of one-half wavelength from Y.

12.5 When waves meet: the superposition principle

So far, we have considered single waves. What happens when two waves encounter each other in the same medium? Suppose two waves approach each other on a rope, one traveling to the right and one traveling to the left. The series of sketches in the margin shows what would happen if you made this experiment. The waves pass through each other without being modified. After the encounter, each wave looks just as it did before and is traveling just as it was before. This phenomenon of passing through each other unchanged can be observed with all types of waves. You can easily see that it is true for surface ripples on water. (Look back, for example, to the opening photograph for the chapter.) You could reason that it must be true for sound waves also, since two conversations can take place across a table without distorting each other. (Note that when *particles* encounter each other, they collide. Waves can pass through each other.)



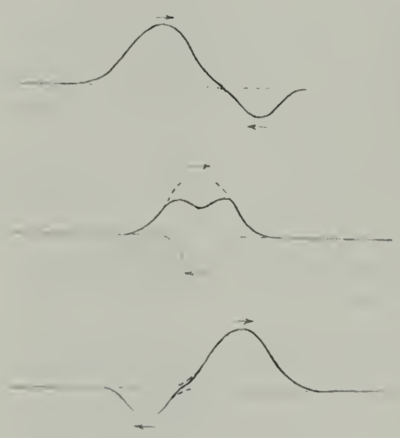
The superposition of two rope waves at a point. The dashed curves are the contributions of the individual waves.

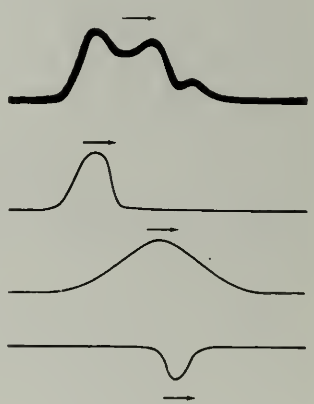
But what happens during the time when the two waves overlap? The displacements they provide add up. At each instant, the displacement of each point in the overlap region is just the *sum* of the displacements that would be caused by each of the two waves separately. An example is shown in the margin. Two waves travel toward each other on a rope. One has a maximum displacement of 0.4 cm upward and the other a maximum displacement of 0.8 cm upward. The total maximum upward displacement of the rope at a point where these two waves pass each other is 1.2 cm.

What a wonderfully simple behavior, and how easy it makes everything! Each wave proceeds along the rope making its own contribution to the rope's displacement no matter what any other wave is doing. We can easily determine what the rope looks like at any given instant. All we need to do is add up the displacements caused by each wave at each point along the rope at that instant. This property of waves is called the *superposition principle.* Another illustration of wave superposition is shown on page 110. Notice that when the displacements are in opposite directions, they tend to cancel each other. One of the two directions of displacement may always be considered negative. Check the diagrams with a ruler. You will find that the net displacement (solid line) is just the sum of the individual displacements (broken lines).

The superposition principle applies no matter how many separate waves or disturbances are present in the medium. In the examples just given, only two waves are present. But we would find by experiment that the superposition principle works equally well for three, ten, or any number of waves. Each wave makes its own contribution, and the net result is simply the sum of all the individual contributions.

We can turn the superposition principle around. If waves add as we have described, then we can think of a complex wave as the sum of a set of simpler waves. In the diagram below (right), a complex pulse has been analyzed into a set of three simpler pulses. In 1807 the French mathematician Jean-Baptiste Fourier advanced a very useful theorem. Fourier stated that any continuing periodic oscillation, however complex, could be analyzed as the sum of simpler, regular wave motions. This, too, can be demonstrated by experiment. The sounds of musical instruments have been analyzed in this way also. Such analysis makes it possible to "imitate" instruments electronically by combining just the right proportions of simple vibrations.





SG 12.4-12.8

Q11 Two periodic waves of amplitudes A_1 and A_2 pass through a point P. What is the greatest displacement of P?

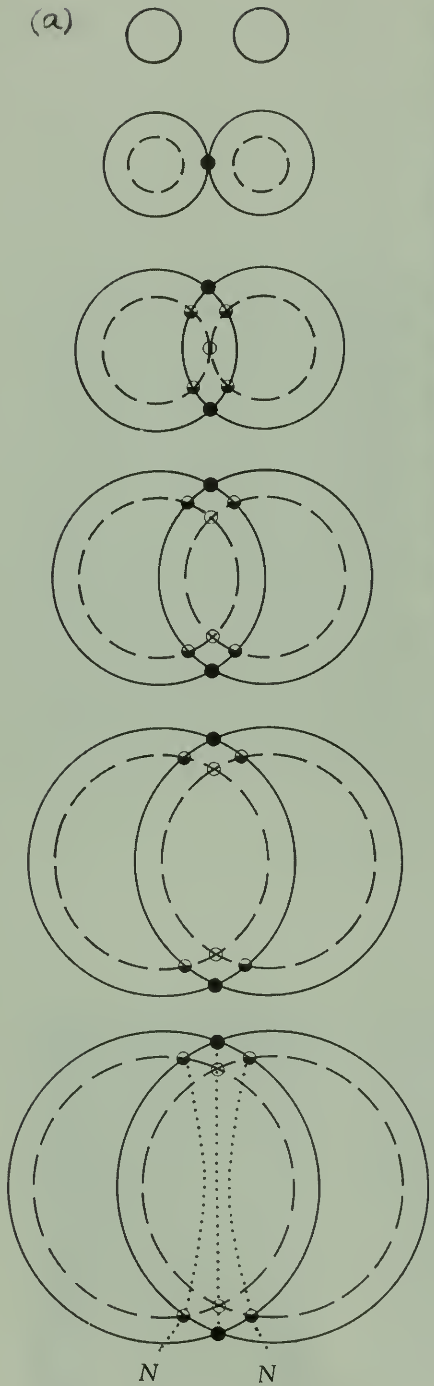
Q12 What is the displacement of a point produced by two waves together if the displacements produced by the waves separately at that instant are +5 cm and −6 cm?

12.6 A two-source interference pattern

The photograph at the right (center) shows ripples spreading from a vibrating source touching the water surface in a "ripple tank." The drawing to the left of it shows a "cut-away" view of the water level pattern at a given instant.

The third photograph (far right) introduces a phenomenon which will play an important role in later parts of the course. It shows the pattern of ripples on a water surface which is disturbed by *two* vibrating sources. The two small sources go through their up and down motions together. That is, they are in phase. Each creates its own set of circular, spreading ripples. The photograph catches the pattern made by the overlapping sets of waves at one instant. It is called an *interference pattern.*

The ripple tank shown in the photograph at the left is being used by students to observe a circular pulse spreading over a thin layer of water. When a vibrating point source is immersed at the edge of the tank, it produces periodic wave trains of crests and troughs, somewhat as shown in the "cut-away" drawing at the left below. The center figure below is an instantaneous photograph of the shadows of ripples produced by a vibrating point source. The crests and troughs on the water surface show up in the image as bright and dark circular bands. Below right, there were two point sources vibrating in phase. The overlapping waves create an interference pattern.



Pattern produced when two circular pulses, each of a crest and a trough, spread through each other. The small circles indicate the net displacement:

● = double height peak
◒ = average level
○ = double depth trough

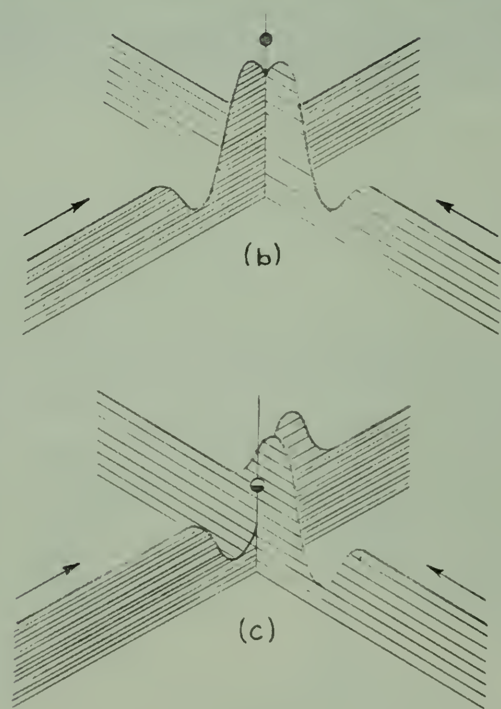


Diagram representing the separate pulses superposing as in the Figure at the left. The top sketch illustrates two crests about to arrive at the vertical line. The bottom sketch illustrates a crest about to arrive together with a trough.

We can interpret what we see in this photograph in terms of what we already know about waves. And we can predict how the pattern will change with time. First, tilt the page so that you are viewing the interference pattern from a glancing direction. You will see more clearly some nearly straight gray bands. This feature can be explained by the superposition principle.

Suppose that two sources produce identical pulses at the same instant. Each pulse contains one crest and one trough. (See page 112 at the left.) In each pulse the height of the crest above the undisturbed or average level is equal to the depth of the trough below. The sketches show the patterns of the water surface after equal time intervals. As the pulses spread out, the points at which they overlap move too. In the figure we have placed a completely darkened circle wherever a crest overlaps another crest. A half-darkened circle marks each point where a crest overlaps a trough. A blank circle indicates the meeting of two troughs. According to the superposition principle, the water level should be highest at the completely darkened circles (where the crests overlap). It should be lowest at the blank circles, and at average height at the half-darkened circles. Each of the sketches on page 112 represents the spatial pattern of the water level at a given instant.

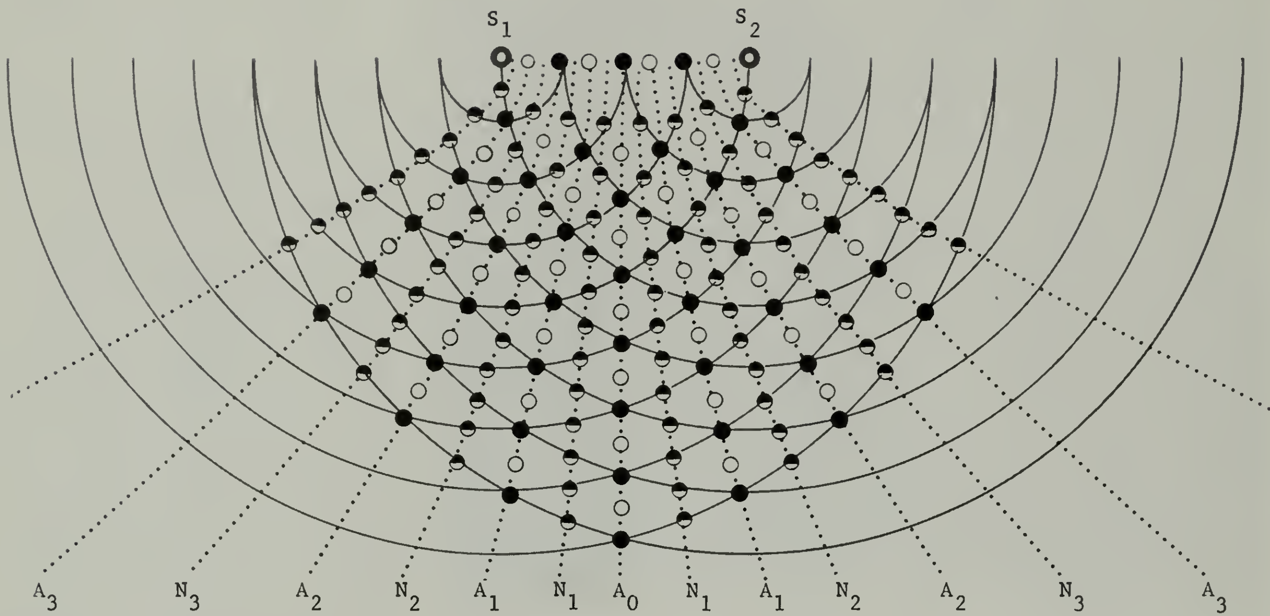
At the points marked with darkened circles in (a), the two pulses arrive in phase, as indicated in (b). At points indicated by blank circles, the pulses also arrive in phase. In either case, the waves reinforce each other, causing a *greater* amplitude of either the crest or the trough. Thus, they are said to *interfere constructively*. In this case, all such points are at the same distance from each source. As the ripples spread, the region of maximum disturbance moves along the central dotted line in (a).

At the points in (a) marked with half-darkened circles, the two pulses arrive completely out of phase, as shown in (c). Here the waves cancel and so are said to interfere *destructively*, leaving the water surface undisturbed. The lines N in (a) show the path along which the overlapping pulses meet when they are just out of phase. All along these lines there is no change or displacement of the water level. Note that all points on these lines are one-crest-trough distance ($\frac{1}{2}\lambda$) further from one source than from the other.

When periodic waves of equal amplitude are sent out instead of single pulses, overlap occurs all over the surface. All along the central dotted line there is a doubled disturbance amplitude. All along the side lines the water height remains undisturbed. Depending on the wavelength and the distance between the sources, there can be many such lines of constructive and destructive interference.

Now we can interpret the ripple tank interference pattern on page 111. The "gray bands" are areas where waves cancel each other, called *nodal* lines. These bands correspond to lines N in the simple case of pulses instead of periodic waves. Between these bands are other bands where crest and trough follow one another, where the waves reinforce. These are called *antinodal* lines.

Look closely at the diagram on page 114. It explains what is happening in the lower right hand photograph on page 111. Notice its symmetry. The central band labeled A_0 is an antinode where



Analysis of interference pattern similar to that of the lower right photograph on p. 111 set up by two in-phase periodic sources. (Here S_1 and S_2 are separated by four wavelengths.) The letters A and N designate antinodal and nodal lines. The dark circles indicate where crest is meeting crest, the blank circles where trough is meeting trough, and the half-dark circles where crest is meeting trough.

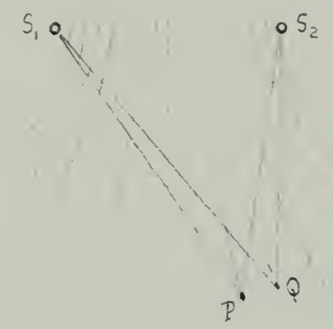
reinforcement is complete. The other lines of maximum constructive interference are labeled A_1, A_2, A_3, etc. Points on these lines move up and down *much more* than they would because of waves from either source alone. The lines labeled N_1, N_2, etc. represent bands along which there is maximum destructive interference. Points on these lines move up and down *much less* than they would because of waves from either source alone. Compare the diagram with the photograph and identify antinodal lines and nodal lines.

Whenever we find such an interference pattern, we know that it is set up by overlapping waves from two sources. For water waves, the interference pattern can be seen directly. But whether visible or not, all waves can set up interference patterns—including earthquake waves, sound waves, or x rays. For example, suppose two loudspeakers are working at the same frequency. By moving about and listening in front of the loudspeakers, you can find the nodal regions where destructive interference causes only little sound to be heard. You also can find the antinodal regions where a strong signal comes through.

The beautiful symmetry of these interference patterns is not accidental. Rather, the whole pattern is determined by the wavelength λ and the source separation S_1S_2. From these we could calculate the angles at which the nodal and antinodal lines radiate out to either side of A_0. Conversely, we might know S_1S_2 and might have found these angles by probing around in the two-source interference pattern. If so, we can calculate the wavelength even if we can't see the crests and troughs of the waves directly. This is very useful, for most waves in nature can't be directly seen. So their wavelength has to be found in just this way: letting waves set up an interference pattern, probing for the nodal and antinodal lines, and calculating λ from the geometry.

SG 12.9

The figure at the right shows part of the pattern of the diagram on the opposite page. At any point P on an *antinodal* line, the waves from the two sources arrive *in phase*. This can happen only if P is equally far from S_1 and S_2, or if P is some whole number of wavelengths farther from one source than from the other. In other words, the difference in distances $(S_1P - S_2P)$ must equal $n\lambda$, λ being the wavelength and n being zero or any whole number. At any point Q on a *nodal* line, the waves from the two sources arrive exactly *out of phase*. This occurs because Q is an odd number of half-wavelengths ($\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.) farther from one source than from the other. This condition can be written $S_1Q - S_2Q = (n + \frac{1}{2})\lambda$.



$$S_1P - S_2P = n\lambda$$

$$S_1Q - S_2Q = (n + \tfrac{1}{2})\lambda$$

Since the sound wave patterns in space are three-dimensional, the nodal or antinodal regions in this case are two-dimensional surfaces. For example, they are planes, not lines.

The distance from the sources to a detection point may be much larger than the source separation d. In that case, there is a simple relationship between the node position, the wavelength λ, and the separation d. The wavelength can be calculated from measurements of the positions of nodal lines. The details of the relationship and the calculation of wavelength are described on the next page.

This analysis allows us to calculate from simple measurements made on an interference pattern the wavelength of any wave. It applies to water ripples, sound, light, etc. You will find this method very useful in later units. One important thing you can do now is find λ for a real case of interference of waves in the laboratory. This practice will help you later in finding the wavelengths of other kinds of waves.

Q13 Are nodal lines in interference patterns regions of cancellation or reinforcement?

Q14 What are antinodal lines? antinodal points?

Q15 Nodal points in an interference pattern are places where
(a) the waves arrive "out of phase"
(b) the waves arrive "in phase"
(c) the point is equidistant from the wave sources
(d) the point is one-half wavelength from both sources.

Q16 Under what circumstances do waves from two in-phase sources arrive at a point out of phase?

12.7 Standing waves

If both ends of a rope are shaken with the same frequency and same amplitude, an interesting thing happens. The interference of the identical waves coming from opposite ends results in certain points on the rope not moving at all! In between these nodal points, the rope oscillates up and down. But there is no apparent propagation of wave patterns in either direction along the rope. This phenomenon is called a *standing wave* or *stationary wave*. (With the aid of Transparency T-27, using the superposition principle, you can see that this effect is just what would be expected from the addition of the two oppositely traveling waves.) The important thing to remember is that the standing oscillation we observe is

Calculating λ from an Interference Pattern

$d = (S_1S_2)$ = separation between S_1 and S_2. (S_1 and S_2 may be actual sources that are in phase, or two slits through which a previously prepared wave front passes.)

$\ell = OQ$ = distance from sources to a far-off line or screen placed parallel to the two sources.

x = distance from center axis to point P along the detection line.

$L = OP$ = distance to point P on detection line measured from sources.

Waves reaching P from S_1 have traveled farther than waves reaching P from S_2. If the extra distance is λ (or 2λ, 3λ, etc.), the waves will arrive at P in phase. Then P will be a point of strong wave disturbance. If the extra distance is $\frac{1}{2}\lambda$ (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.), the waves will arrive out of phase. Then P will be a point of weak or no wave disturbance.

With P as center we draw an arc of a circle of radius PS_2; it is indicated on the figure by the dotted line S_2M. Then line segment PS_2 = line segment PM. Therefore the extra distance that the wave from S travels to reach P is the length of the segment SM.

Now if d is very small compared to ℓ, as we can easily arrange in practice, the circular arc S_2M will then be a very small piece of a large diameter circle—or nearly a straight line. Also, the angle S_1MS_2 is very nearly 90°. Thus, the triangle S_1S_2M can be regarded as a right triangle. Furthermore, angle S_1S_2M is equal to angle POQ. Then right triangle S_1S_2M is a similar triangle POQ.

$$\frac{S_1M}{S_1S_2} = \frac{x}{OP} \text{ or } \frac{S_1M}{d} = \frac{x}{L}$$

If the distance ℓ is large compared to x, the distances ℓ and L are nearly equal, and we can write

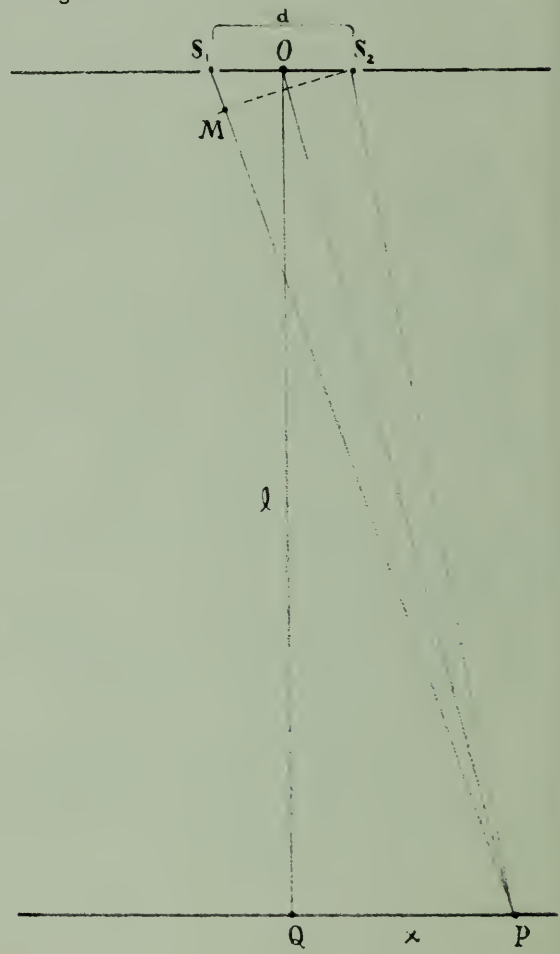
$$\frac{S_1M}{d} = \frac{x}{L}$$

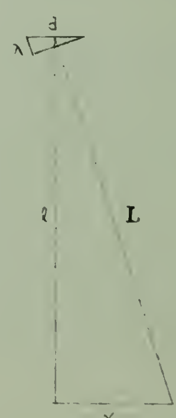
But S_1M is the extra distance traveled by the wave from source S_1. For P to be a point of maximum wave disturbance, S_1M must be equal to $n\lambda$ (where $n = 0$ if P is at Q, and $n = 1$ if P is at the first maximum of wave disturbance found to one side of Q, etc.). So the equation becomes

$$\frac{n\lambda}{d} = \frac{x}{\ell}$$

and $$\lambda = \frac{dx}{n\ell}$$

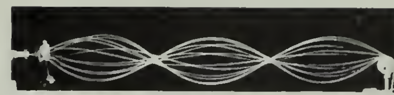
This important result says that if we measure the source separation d, the distance ℓ, and the distance x from the central line to a wave disturbance maximum, we can calculate the wavelength λ.





really the effect of two *traveling* waves.

To make standing waves on a rope, there do not have to be two people shaking the opposite ends. One end can be tied to a hook on a wall. The train of waves sent down the rope by shaking one end will reflect back from the fixed hook. These reflected waves interfere with the new, oncoming waves and can produce a standing pattern of nodes and oscillation. In fact, you can go further and tie both ends of a string to hooks and pluck (or bow) the string. From the plucked point a pair of waves go out in opposite directions and then reflect back from the ends. The interference of these reflected waves traveling in opposite directions can produce a standing pattern just as before. The strings of guitars, violins, pianos, and all other stringed instruments act in just this fashion. The energy given to the strings sets up standing waves. Some of the energy is then transmitted from the vibrating string to the body of the instrument. The sound waves sent forth from there are at essentially the same frequency as the standing waves on the string.



A vibrator at the left produces a wave train that runs along the rope and reflects from the fixed end at the right. The sum of the oncoming and the reflected waves is a standing wave pattern.

Lyre player painted on a Greek vase in the 5th century B.C.

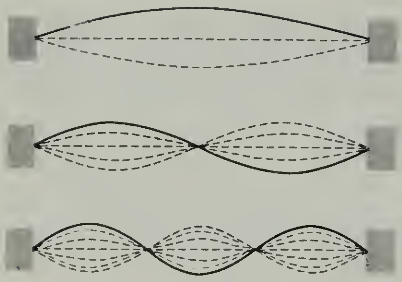
The vibration frequencies at which standing waves can exist depend on two factors. One is the speed of wave propagation along the string. The other is the length of the string. The connection between length of string and musical tone was recognized over two thousand years ago. This relationship contributed greatly to the idea that nature is built on mathematical principles. Early in the development of musical instruments, people learned how to produce certain pleasing harmonies by plucking strings. These harmonies result if the strings are of equal tautness and diameter and if their lengths are in the ratios of small whole numbers. Thus the length ratio 2:1 gives the octave, 3:2 the musical fifth, and 4:3 the musical fourth. This striking connection between music and numbers encouraged the Pythagoreans to search for other numerical ratios or harmonies in the universe. The Pythagorean ideal strongly affected Greek science and many centuries later inspired much of Kepler's work. In a general form, the ideal flourishes to this day in many beautiful applications of mathematics to physical experience.

Using the superposition principle, we can now define the harmonic relationship much more precisely. First, we must stress an important fact about standing wave patterns produced by reflecting waves from the boundaries of a medium. We can imagine an unlimited variety of waves traveling back and forth. But, in fact, *only certain wavelengths (or frequencies) can produce standing waves* in a given medium. In the example of a stringed instrument, the two ends are fixed and so must be nodal points. This fact puts an upper limit on the length of standing waves possible on a fixed rope of length L. Such waves must be those for which one-half wavelength just fits on the rope ($L = \lambda/2$). Shorter waves also can produce standing patterns having more nodes. But *always*, some whole number of one-half wavelengths must just fit on the rope ($L = \lambda/2$).

SG 12.13

We can turn this relationship around to give an expression for all possible wavelengths of standing waves on a fixed rope:

$$\lambda_n = \frac{2L}{n}$$



Or simply, $\lambda_n \propto 1/n$. That is, if λ_1 is the longest wavelength possible, the other possible wavelengths will be $\frac{1}{2}\lambda_1, \frac{1}{3}\lambda_1, \ldots 1/n\lambda_1$. Shorter wavelengths correspond to higher frequencies. Thus, *on any bounded medium, only certain frequencies of standing waves can be set up.* Since frequency f is inversely proportional to wavelength, $f \propto 1/\lambda$, we can rewrite the expression for all possible standing waves as

$$f_n \propto n$$

The lowest possible frequency of a standing wave is usually the one most strongly present when the string vibrates after being plucked or bowed. If f_1 represents this lowest possible frequency, then the other possible standing waves would have frequencies $2f_1, 3f_1, \ldots nf_1$. These higher frequencies are called "overtones" of the "fundamental" frequency f_1. On an "ideal" string, there are in principle an unlimited number of such frequencies, all simple multiples of the lowest frequency.

Film Loops 34—43 show a variety of standing waves, including waves on a string, a drum, and in a tube of air.

Mathematically inclined students are encouraged to pursue the topic of waves and standing waves, for example, Science Study Series paperbacks *Waves and the Ear* and *Horns, Strings and Harmony*

See the *Reader 3* articles "Musical Instruments and Scales" and "Founding a Family of Fiddles."

SG 12.16

In real media, there are practical upper limits to the possible frequencies. Also, the overtones are not exactly simple multiples of the fundamental frequency. That is, the overtones are not strictly "harmonic." This effect is still greater in more complicated systems than stretched strings. In a saxophone or other wind instrument, an *air column* is put into standing wave motion. The overtones produced may not be even approximately harmonic.

As you might guess from the superposition principle, standing waves of different frequencies can exist in the same medium at the same time. A plucked guitar string, for example, oscillates in a pattern which is the superposition of the standing waves of many overtones. The relative oscillation energies of the different instruments determine the "quality" of the sound they produce. Each type of instrument has its own balance of overtones. This is why a violin sounds different from a trumpet, and both sound different from a soprano voice—even if all are sounding at the same fundamental frequency.

Q17 When two identical waves of same frequency travel in opposite directions and interfere to produce a standing wave, what is the motion of the medium at

(1) the nodes of the standing wave?

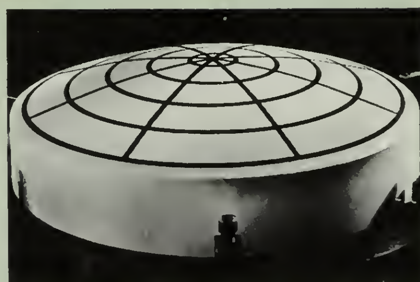
(2) the places between nodes, called "antinodes" or loops, of the standing wave?

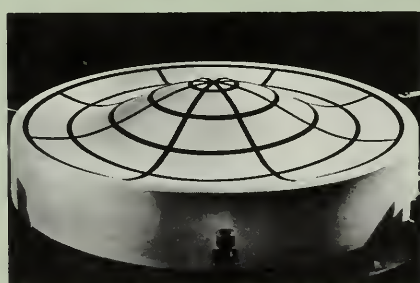
Q18 If the two interfering waves have wavelength λ, what is the distance between the nodal points of the standing wave?

Q19 What is the wavelength of the longest traveling waves which can produce a standing wave on a string of length L?

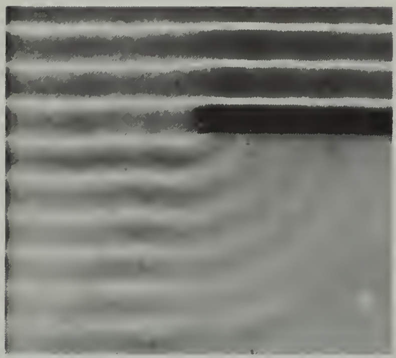
Q20 Can standing waves of *any* frequency, as long as it is higher than the fundamental, be set up in a bounded medium?

In the Film Loop *Vibration of a Drum*, a marked rubber "drumhead" is seen vibrating in several of its possible modes. Below are pairs of still photographs from three of the symmetrical modes and from an antisymmetrical mode.

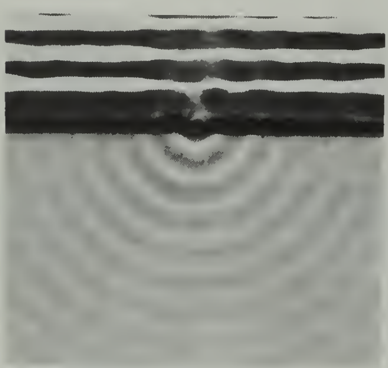




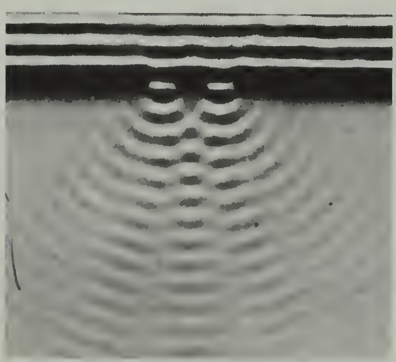


Diffraction of ripples around the edge of a barrier.



Diffraction of ripples through a narrow opening.



Diffraction of ripples through two narrow openings.

12.8 Wave fronts and diffraction

Waves can go around corners. For example, you can hear a voice coming from the other side of a hill, even though there is nothing to reflect the sound to you. We are so used to the fact that sound waves do this that we scarcely notice it. This spreading of the energy of waves into what we would expect to be "shadow" regions is called *diffraction*.

Once again, water waves will illustrate this behavior most clearly. From among all the arrangements that can result in diffraction, we will concentrate on two. The first is shown in the second photograph in the margin at the left. Straight water waves (coming from the top of the picture) are diffracted as they pass through a narrow slit in a straight barrier. Notice that the slit is less than one wavelength wide. The wave emerges and spreads in all directions. Also notice the *pattern* of the diffracted wave. It is basically the same pattern a vibrating point source would set up if it were placed where the slit is.

The bottom photograph shows the second barrier arrangement we want to investigate. Now there are two narrow slits in the barrier. The pattern resulting from superposition of the diffracted waves from both slits is the same as that produced by two point sources vibrating in phase. The same kind of result is obtained when many narrow slits are put in the barrier. That is, the final pattern just matches that which would appear if a point source were put at the center of each slit, with all sources in phase.

We can describe these and all other effects of diffraction if we understand a basic characteristic of waves. It was first stated by Christian Huygens in 1678 and is now known as *Huygens' principle*. But in order to state the principle, we first need the definition of a *wave front*.

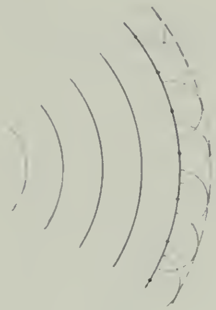
For a water wave, a wave front is an imaginary line along the water's surface. Every point along this line is in exactly the same stage of vibration. That is, all points on the line are *in phase*. Crest lines are wave fronts, since all points on the water's surface along a crest line are in phase. Each has just reached its maximum displacement upward, is momentarily at rest, and will start downward an instant later.

The simplest wave fronts are straight lines parallel to each other, as in the top part of the center photograph at the left. Or they may be circular, as in the bottom part of the same photograph. Sound waves are somewhat different. Since a sound wave spreads not over a surface but in three dimensions, its wave fronts become very nearly spherical surfaces. At large distances from the source, however, the radius of a spherical wave front is also large. Thus, any small section of the wave front is nearly flat. All circular and spherical wave fronts become virtually straight-line or flat-plane fronts at great distances from their sources.

Now Huygens' principle, as it is generally stated today, is that *every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation.* As Huygens said:

> There is the further consideration in the emanation of these waves, that each particle of matter in which a wave spreads, ought not to communicate its motion only to the next

> particle which is in the straight line drawn from the (source), but that it also imparts some of it necessarily to all others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the center.



Each point on a wave front can be thought of as a point source of waves. The waves from all the point sources interfere constructively only along their envelope, which becomes the new wave front.

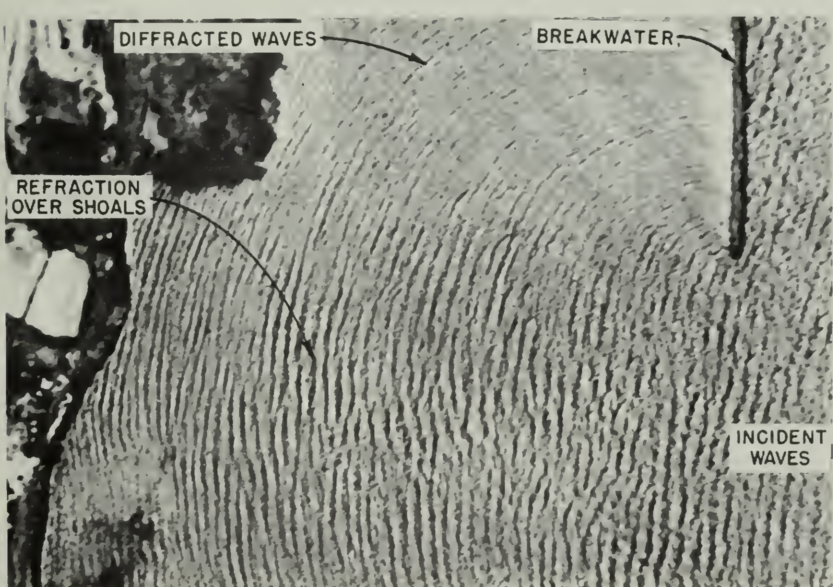
The diffraction patterns seen at slits in a barrier are certainly consistent with Huygens' principle. The wave arriving at the barrier causes the water in the slit to oscillate. The oscillation of the water in the slit acts as a source for waves traveling out from it in all directions. When there are two slits and the wave reaches both slits in phase, the oscillating water in each slit acts like a point source. The resulting interference pattern is similar to the pattern produced by waves from two point sources oscillating in phase.

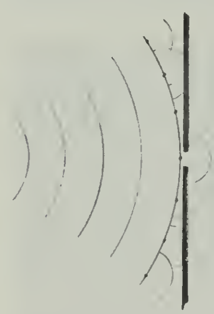
Or consider what happens behind a breakwater wall as in the aerial photograph of the harbor below. By Huygens' principle, water oscillation near the end of the breakwater sends circular waves propagating into the "shadow" region.

We can understand all diffraction patterns if we keep both Huygens' principle and the superposition principle in mind. For example, consider a slit wider than one wavelength. In this case the pattern of diffracted waves contains nodal lines (see the series of four photographs in the margin).

The figure on p. 122 helps to explain why nodal lines appear. There must be points like P that are just λ farther from side A of the slit than from side B. That is, there must be points P for which AP differs from BP

When part of the wave front is blocked, the constructive interference of waves from points on the wave front extends into the "shadow" region.

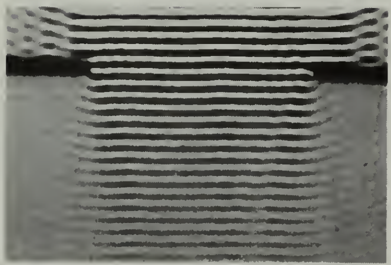




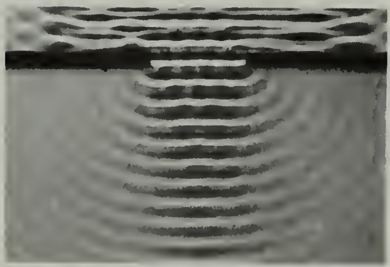
When all but a very small portion of a wave front is blocked, the wave propagating away from that small portion is nearly the same as from a point source.

by exactly λ. For such a point, *AP* and *OP* differ by one-half wavelength, λ/2. By Huygens' principle, we may think of points *A* and *O* as in-phase point sources of circular waves. But since *AP* and *OP* differ by λ/2, the two waves will arrive at *P* completely out of phase. So, according to the superposition principle, the waves from *A* and *O* will cancel at point *P*.

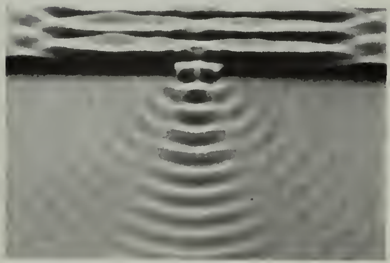
But this argument also holds true for the pair of points consisting of the first point to the right of *A* and the first to the right of *O*. In fact, it holds true for *each* such matched pair of points, all the way across the slit. The waves originating at each such pair of points all cancel at point *P*. Thus, *P* is a nodal point, located on a nodal line. On the other hand, if the slit width is less than λ, then there can be *no* nodal point. This is obvious, since no point can be a distance λ farther from one side of the slit than from the other. Slits of widths less than λ behave nearly as point sources. The narrower they are, the more nearly their behavior resembles that of point sources.



We can easily compute the wavelength of a wave from the interference pattern set up where diffracted waves overlap. For example, we can analyze the two-slit pattern on page 120 exactly as we analyzed the two-source pattern in Section 12.6. This is one of the main reasons for our interest in the interference of diffracted waves. By locating nodal lines formed beyond a set of slits, we can calculate λ even for waves that we cannot see.



For two-slit interference, the larger the wavelength compared to the distance between slits, the more the interference pattern spreads out. That is, as λ increases or *d* decreases, the nodal and antinodal lines make increasingly large angles with the straight-ahead direction. Similarly, for single-slit diffraction, the pattern spreads when the ratio of wavelength to the slit width increases. In general, diffraction of longer wavelengths is more easily detected. Thus, when you hear a band playing around a corner, you hear the bass drums and tubas better than the piccolos and cornets—even though they actually are playing equally loud.



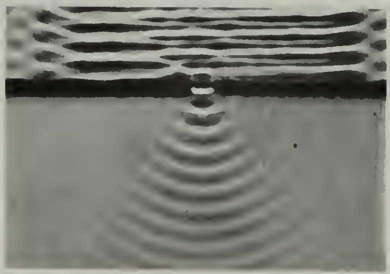
Q21 What characteristic do all points on a wave front have in common?

Q22 State Huygens' principle.

Q23 Why can't there be nodal lines in a diffraction pattern from an opening less than one wavelength wide?

Q24 What happens to the diffraction pattern from an opening as the wavelength of the wave increases?

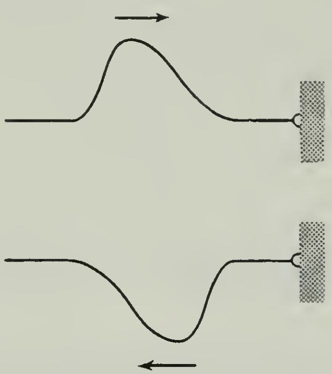
Q25 Can there be diffraction without interference? Interference without diffraction?



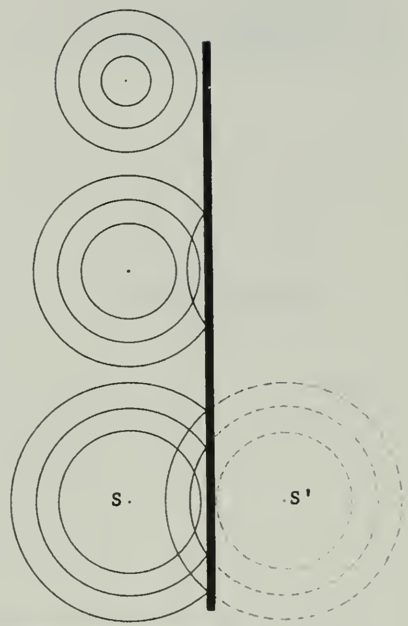
12.9 Reflection

We have seen that waves can pass through one another and spread around obstacles in their paths. Waves also are reflected, at least to some degree, whenever they reach any boundary of the medium in which they travel. Echoes are familiar examples of the reflection of sound waves. All waves share the property of reflection. Again, the superposition principle will help us understand what happens when reflection occurs.

Suppose that one end of a rope is tied tightly to a hook securely fastened to a massive wall. From the other end, we send a pulse wave down the rope toward the hook. Since the hook cannot move, the force exerted by the rope wave can do no work on the hook. Therefore, the energy carried in the wave cannot leave the rope at this fixed end. Instead, the wave bounces back—is *reflected*—ideally with the same energy.



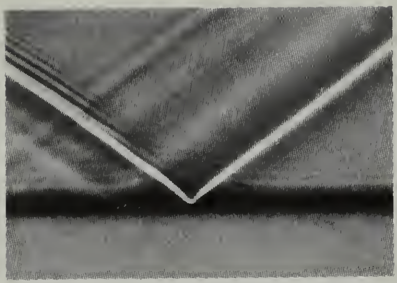
What does the wave look like after it is reflected? The striking result is that the wave seems to flip upside down on reflection. As the wave comes in from left to right and encounters the fixed hook, it pulls up on it. By Newton's third law, the hook must exert a force on the rope in the opposite direction while reflection is taking place. The details of how this force varies in time are complicated. The net effect is that an inverted wave of the same form is sent back down the rope.



SG 12.21

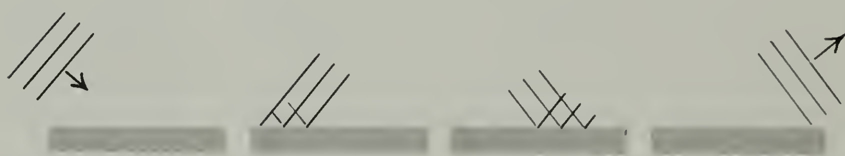
Two-dimensional water-surface waves exhibit a fascinating variety of reflection phenomena. There may be variously shaped crest lines, variously shaped barriers, and various directions from which the waves approach the barrier. If you have never watched closely as water waves are reflected from a fixed barrier, you should do so. Any still pool or water-filled wash basin or tub will do. Watch the circular waves speed outward, reflect from rocks or walls, run through each other, and finally die out. Dip your fingertip into and out of the water quickly, or let a drop of water fall from your finger into the water. Now watch the circular wave approach and then bounce off a straight wall or a board. The long side of a tub is a good straight barrier.

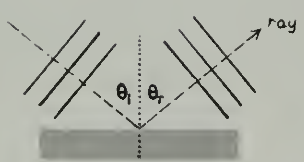
The sketches in the margin picture the results of reflection from a straight wall. Three crests are shown. You may see more or fewer than three clear crests in your observations, but that does not matter. In the upper sketch, the outer crest is approaching the barrier at the right. The next two sketches show the positions of the crests after first one and then



two of them have been reflected. Notice the dashed curves in the last sketch. They attempt to show that the reflected wave appears to originate from a point S' that is as far behind the barrier as S is in front of it. The imaginary source at point S' is called the *image* of the source S.

We mention the reflection of circular waves first, because that is what one usually notices first when studying water waves. But it is easier to see a general principle for explaining reflection by observing a straight wave front, reflected from a straight barrier. The ripple-tank photograph at the left shows one instant during such a reflection. (The wave came in from the upper left at an angle of about 45°.) The sketches below show in more detail what happens as the wave crests reflect from the straight barrier. The first sketch shows three crests approaching the barrier. The last sketch shows the same crests as they move away from the barrier after the encounter. The two sketches between show the reflection process at two different instants during reflection.



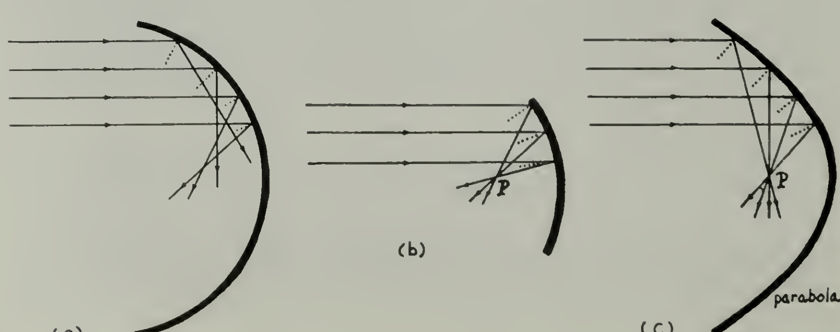


The description of wave behavior is often made easier by drawing lines perpendicular to the wave fronts. Such lines, called *rays*, indicate the direction of propagation of the wave. Notice the drawing at the left, for example. Rays have been drawn for a set of wave crests just before reflection and just after reflection from a barrier. The straight-on direction, perpendicular to the reflecting surface, is shown by a dotted line. The ray for the *incident* crests makes an angle θ_i with the straight-on direction. The ray for the *reflected* crests makes an angle θ_r with it. The *angle of reflection* θ_r is equal to the *angle of incidence* θ_i : that is, $\theta_r = \theta_i$. This is an experimental fact, which you can verify for yourself.

SG 12.22

Many kinds of wave reflectors are in use today, from radar antennae to infrared heaters. Figures (a) and (b) below show how straight-line waves reflect from two circular reflectors. A few incident and reflected rays are shown. (The dotted lines are perpendicular to the barrier surface.) Rays reflected from the half circle (a) head off in all directions. However, rays reflected from a small segment of the circle (b) come close to

SG 12.23–12.25



meeting at a single point. And a barrier with the shape of a parabola (c) focuses straight-line waves precisely at a point. Similarly, a parabolic surface reflects plane waves to a sharp focus. An impressive example is a radio telescope. Its huge parabolic surface reflects faint radio waves from space to focus on a detector.

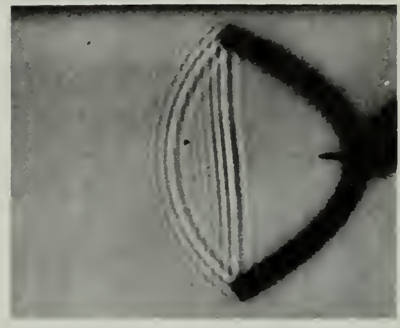
The wave paths indicated in the sketches could just as well be reversed. For example, spherical waves produced at the focus become plane waves when reflected from a parabolic surface. The flashlight and automobile headlamp are familiar applications of this principle. In them, white-hot wires placed at the focus of parabolic reflectors produce almost parallel beams of light.

Q26 What is a "ray"?

Q27 What is the relationship between the angle at which a wave front strikes a barrier and the angle at which it leaves?

Q28 What shape of reflector can reflect parallel wave fronts to a sharp focus?

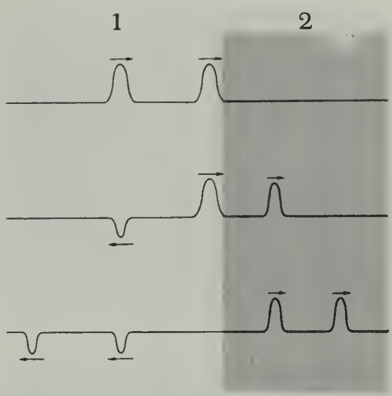
Q29 What happens to wave fronts originating at the focus of such a reflecting surface?



Above: A ripple tank shadow showing how circular waves produced at the focus of a parabolic wall are reflected from the wall into straight waves.

Left: the parabolic surface of a radio telescope reflects radio waves from space to a detector supported at the focus.

Below: the filament of a flashlight bulb is at the focus of a parabolic mirror, so the reflected light forms a nearly parallel beam.

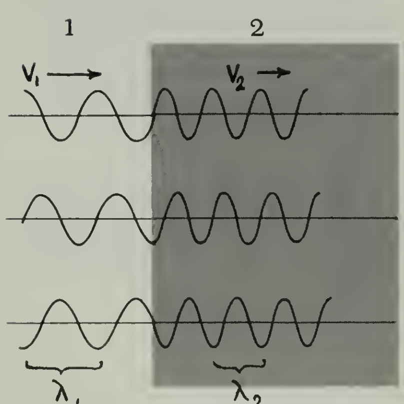


Pulses encountering a boundary between two different media. The speed of propagation is less in medium 2.

12.10 Refraction

What happens when a wave propagates from one medium to another medium in which its speed of propagation is different? We begin with the simple situation pictured in the margin. Two one-dimensional pulses approach a boundary separating two media. The speed of the propagation in medium 1 is greater than it is in medium 2. We might imagine the pulses to be in a light rope (medium 1) tied to a relatively heavy rope (medium 2). Part of each pulse is reflected at the boundary. This reflected component is flipped upside down relative to the original pulse. You will recall the inverted reflection at a hook in a wall discussed earlier. The heavier rope here tends to hold the boundary point fixed in just the same way. But we are not particularly interested here in the reflected wave. We want to see what happens to that part of the wave which continues into the second medium.

As shown in the figure, the transmitted pulses are closer together in medium 2 than they are in medium 1. Is it clear why this is so? The speed of the pulses is less in the heavier rope. So the second pulse is catching up with the first while the second pulse is still in the light rope and the first is already in the heavy rope. In the same way, each separate pulse is itself squeezed into a narrower form. That is, while the front of the pulse is entering the region of less speed, the back part is still moving with greater speed.



Continuous wave train crossing the boundary between two different media. The speed of propagation is less in medium 2.

Something of the same sort happens to a periodic wave at such a boundary. The figure at the left pictures this situation. For the sake of simplicity, we have assumed that all of the wave is transmitted, and none of it reflected. Just as the two pulses were brought closer and each pulse was squeezed narrower, the periodic wave pattern is squeezed together too. Thus, the wavelength λ_2 of the transmitted wave is shorter than the wavelength λ_1 of the incoming, or incident, wave.

Although the wavelength changes when the wave passes across the boundary, the frequency of the wave cannot change. If the rope is unbroken, the pieces immediately on either side of the boundary must go up and down together. The frequencies of the incident and transmitted waves must, then, be equal. So we can simply label both of them f.

We can write our wavelength, frequency, and speed relationship for both the incident and transmitted waves separately:

$$\lambda_1 f = v_1, \text{ and } \lambda_2 f = v_2$$

If we divide one of these equations by the other, eliminating the f's, we get

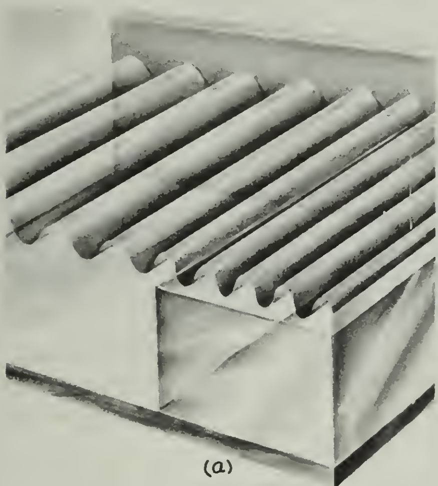
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

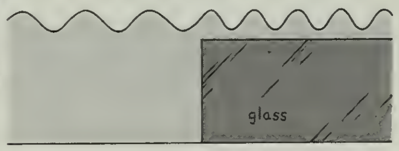
SG 12.26

This equation tells that the ratio of the wavelengths in the two media equals the ratio of the speeds.

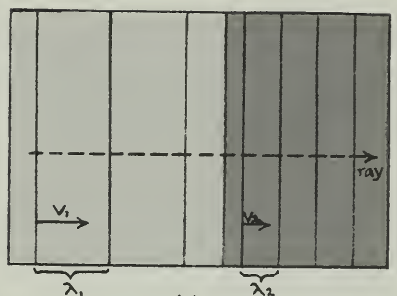
The same sort of thing happens when water ripples cross a boundary.

Experiments show that the ripples move more slowly in shallower water. A piece of plate glass is placed on the bottom of a ripple tank to make the water shallower there. This creates a boundary between the deeper and shallower part (medium 1 and medium 2). Figure (a) below shows the case where this boundary is parallel to the crest lines of the incident wave. As with rope waves, the wavelength of water waves in a medium is proportional to the speed in that medium.


(a)

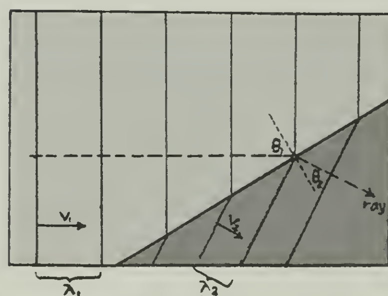


(b)



(c)

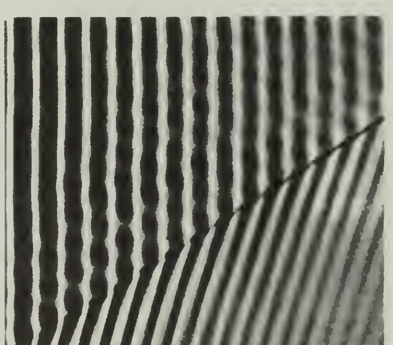
Water waves offer a possibility not present for rope waves. We can arrange to have the crest lines approach the boundary at any angle, not only head-on. The photograph below right shows such an event. A ripple tank wave approaches the boundary at the angle of incidence θ_i. The wavelength and speed of course change as the wave passes across the boundary. But the *direction* of the wave propagation changes too. Figure (d) in the margin shows how this comes about. As each part of a crest line in medium 1 enters medium 2, its speed decreases and it starts to lag behind. In time, the directions of the whole set of crest lines in medium 2 are changed from their directions in medium 1.



(d)

This phenomenon is called *refraction*. It occurs whenever a wave passes into a medium in which the wave velocity is reduced. The wave fronts are turned (refracted) so that they are more nearly parallel to the

SG 12.27–12.31



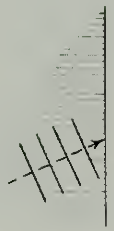
Left: ripples on water (coming from the left) encounter the shallow region over the corner of a submerged glass plate.

Right: ripples on water (coming from the left) encounter a shallow region over a glass plate placed at an angle to the wavefronts.

Aerial photograph of the refraction of ocean waves approaching shore.

boundary. (See the photographs at the bottom of the previous page.) This accounts for something that you may have noticed if you have been at an ocean beach. No matter in what direction the waves are moving far from the shore, when they come near the beach their crest-lines are nearly parallel to the shoreline. A wave's speed is steadily reduced as it moves into water that gets gradually more shallow. So the wave is refracted continuously as if it were always crossing a boundary between different media, as indeed it is. The refraction of sea waves is so great that wave crests can curl around a small island with an all-beach shoreline and provide surf on all sides. (See the photograph on page 139.)

The slowing of star light by increasingly dense layers of the atmosphere produces refraction that changes the apparent position of the star.



Q30 If a periodic wave slows down on entering a new medium, what happens to (1) its frequency? (2) its wavelength? (3) its direction?

Q31 Complete the sketch in the margin to show roughly what happens to a wave train that enters a new medium where its speed is greater.

12.11 Sound waves

Sound waves are mechanical disturbances that propagate through a medium, such as the air. Typically, sound waves are longitudinal waves, producing changes of density and pressure in the medium through which they travel. The medium may be a solid, liquid, or gas. If the waves strike the ear, they can produce the sensation of hearing. The biology and psychology of hearing, as well as the physics of sound, are important to the science of acoustics. But here, of course, we will concentrate on sound as an example of wave motion. Sound has all the properties of wave motion that we have considered so far. It exhibits refraction, diffraction,

Look again at the bottom figure in the margin of p. 103.

and the same relations among frequency, wavelength, and propagation speed and interference. Only the property of polarization is missing, because sound waves are longitudinal, not transverse.

Vibrating sources for sound waves may be as simple as a tuning fork or as complex as the human larynx with its vocal cords. Tuning forks and some special electronic devices produce a steady "pure tone." Most of the energy in such a tone is in simple harmonic motion at a single frequency. The "pitch" of a sound we hear goes up as the frequency of the wave increases.

People can hear sound waves with frequencies between about 20 and 20,000 cycles per second. Dogs can hear over a much wider range (15-50,000 cps). Bats, porpoises, and whales generate and respond to frequencies up to about 120,000 cps.

Loudness (or "volume") of sound is, like pitch, a psychological variable. Loudness is strongly related to the *intensity* of the sound. Sound intensity is a physical quantity. It is defined in terms of *power flow*, such as the number of watts per square centimeter transmitted through a surface perpendicular to the direction of motion of a wave front. The human ear can perceive a vast range of intensities of sound. The table below illustrates this range. It begins at a level of 10^{-16} watts per square centimeter (relative intensity = 1). Below this "threshold" level, the normal ear does not perceive sound.

RELATIVE INTENSITY	SOUND
1	Threshold of hearing
10^1	Normal breathing
10^2	Leaves in a breeze
10^3	
10^4	Library
10^5	Quiet restaurant
10^6	Two-person conversation
10^7	Busy traffic
10^8	Vacuum cleaner
10^9	Roar of Niagara Falls
10^{10}	Subway train
10^{11}	
10^{12}	Propeller plane at takeoff
10^{13}	Machine-gun fire
10^{14}	Small jet plane at takeoff
10^{15}	
10^{16}	Wind tunnel
10^{17}	Space rocket at lift-off

Levels of noise intensity about 10^{12} times threshold intensity can be felt as a tickling sensation in the ear. Beyond 10^{13} times threshold intensity, the sensation changes to pain and may damage the unprotected ear.

SG 12.32

It has always been fairly obvious that sound takes time to travel from source to receiver. Light and sound are often closely associated in the same event—lightning and thunder, for instance. In all such cases, we perceive the sound later. By timing echoes over a known distance, the French mathematician Marin Mersenne in 1640 first computed the speed

Noise and the Sonic Boom

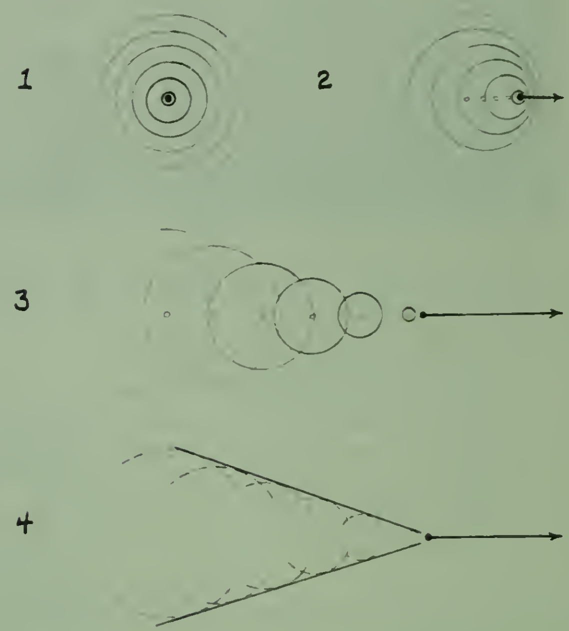
The world seems to be increasingly loud with unpleasant, manmade noise. At worst it is a major nuisance and may be tiring, painful, and sometimes even physically harmful. Loud, prolonged noise can produce temporary deafness. Very loud noise, kept up for a long time, can produce some degree of permanent deafness, especially deafness with respect to high-frequency sounds.

Often the simplest way of reducing noise is by *absorbing* it after it is produced but before it reaches your ears. Like all sound, noise is the energy of back and forth motion of the medium through which the noise goes. Noisy machinery can be muffled by padded enclosures in which the energy of noise is changed to heat energy, which then dissipates. In a house, a thick rug on the floor can absorb 90% of room noise. (A foot of fresh snow is an almost perfect absorber of noise outdoors. Cities and countrysides are remarkably hushed after a snowfall.)

In the last few years a new kind of noise has appeared: the sonic boom. An explosion-like sonic boom is produced whenever an object travels through air at a speed greater than the speed of sound (supersonic speed). Sound travels in air at about 700 miles per hour. Many types of military airplanes can travel at two or three times this speed. Flying at such speeds, the planes unavoidably and continually produce sonic booms. SST (Supersonic Transport) planes are now in civilian use in some countries. The unavoidable boom raises important questions. What is the price of technological "progress"? Who gains, and what fraction of the population? Who and how many pay the price? *Must* we pay it—must SST's be used? How much say has the citizen in decisions that affect his environment so violently?

The formation of a sonic boom is similar to the formation of a wake by a boat. Consider a simple point source of waves. If it remains in the same position in a medium, the wave it produces spreads out symmetrically around it, as in diagram 1. But if the source of the disturbance is *moving* through the medium, each new crest starts from a different point, as in diagram 2.

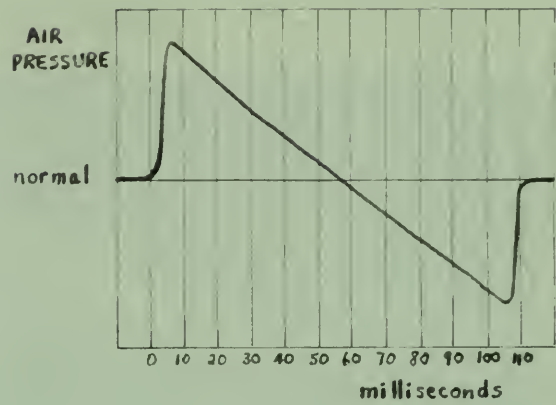
Notice that the wavelength has become shorter in front of the object and longer behind it. (This is called the *Doppler effect.*) In diagram 3, the source is moving through the medium *faster than the wave speed.* Thus the crests and the corresponding troughs overlap and interfere with one another. The interference is mostly destructive everywhere except on the line tangent to the wave fronts, indicated in diagram 4. The result is a wake that spreads like a wedge away from the moving source, as in the photograph below.



All these concepts apply not only to water waves but also to sound waves, including those disturbances set up in air by a moving plane as the wind and body push the air out of the way. If the source of sound is moving faster than the speed of sound wave, then there is a cone-shaped wake (in 3-dimensions) that spreads away from the source.

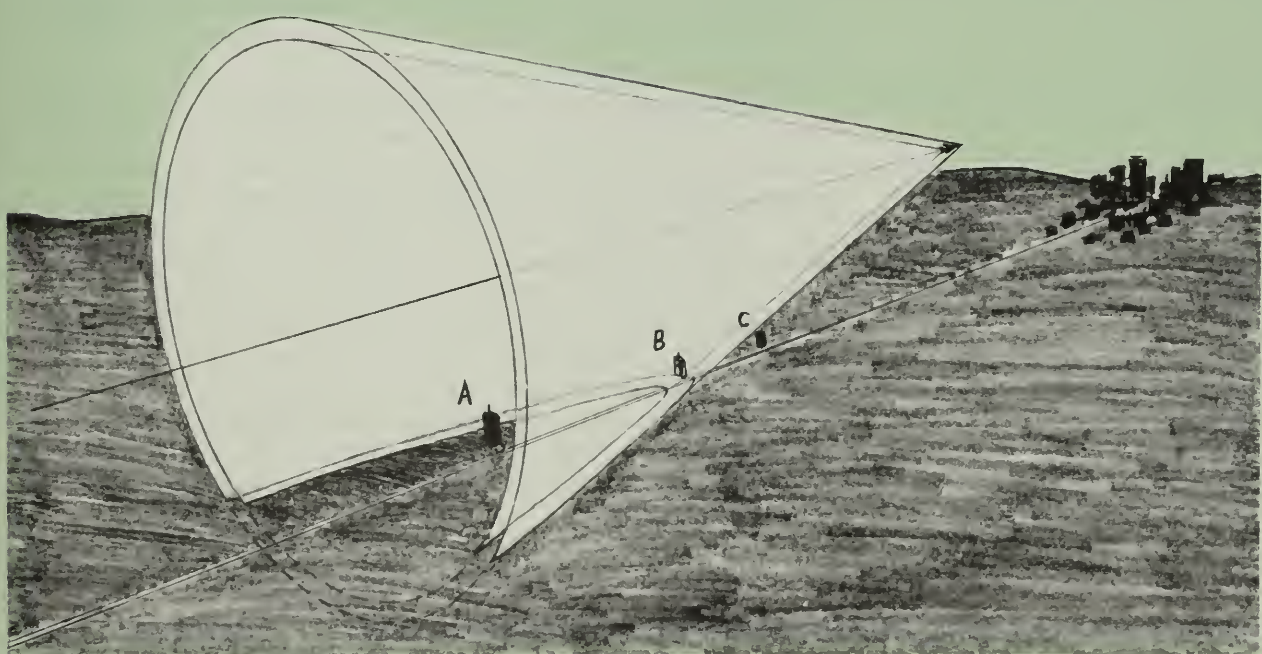
Actually two cones of sharp pressure change are formed: one originating at the front of the airplane and one at the rear, as indicated in the graph at the right.

Because the double shock wave follows along behind the airplane, the region on the ground where people and houses may be struck by the boom (the "sonic-boom carpet," or "bang-zone"), is as long as the supersonic flight path itself. In such an area, typically thousands of miles long and 50 miles wide, there may be millions of people. Tests made with airplanes flying at supersonic speed have shown that a single such cross-country flight by a 350-ton supersonic transport plane would break many thousands of dollars worth of windows, plaster walls, etc., and cause fright and annoyance to millions of people. Thus the supersonic flight of such planes may have to be confined to over-ocean use – though it may even turn out that the annoyance to people on shipboard, on islands, etc., is so great that over-ocean flights, too, will have to be restricted.



This curve represents the typical sonic boom from an airplane flying at supersonic speed (speed greater than about 700 mph). The pressure rises almost instantly, then falls relatively slowly to below-normal pressure, then rises again almost instantaneously. The second pressure rise occurs about 0.1 second after the first one, making the boom sound "double."

Double-cone shock wave, or sonic boom, produced by an airplane that is travelling (at 13-mile altitude) at three times the speed of sound. Building B is just being hit by shock wave, building A was struck a few seconds ago, and building C will be hit a few seconds later.



of sound in air. But it took another seventy years before William Derham in England, comparing the flash and noise from cannons across 12 miles, came close to the modern measurements.

Sound in air at 68°F moves at 1,125 feet per second (about 344 meters per second or 770 mph). As for all waves, the speed of sound waves depends on the properties of the medium—the temperature, density, and elasticity. Sound waves generally travel faster in liquids than in gases, and faster still in solids. In sea water, their speed is about 4,890 ft/sec; in steel, about 16,000 ft/sec; in quartz, about 18,000 ft/sec.

SG 12.33–12.35

Interference of sound waves can be shown in a variety of ways. In a large hall with hard, sound-reflecting surfaces, there will be "dead" spots. At these spots, sound waves coming together after reflection cancel each other. Acoustic engineers must consider this in designing the shape, position, and materials of an auditorium. Another interesting and rather different example of sound interference is the phenomenon known as *beats*. When two notes of slightly different frequency are heard together, they interfere. This interference produces beats, a rhythmic pulsing of the sound. Piano tuners and string players use this fact to tune two strings to the same pitch. They simply adjust one string or the other until the beats disappear.

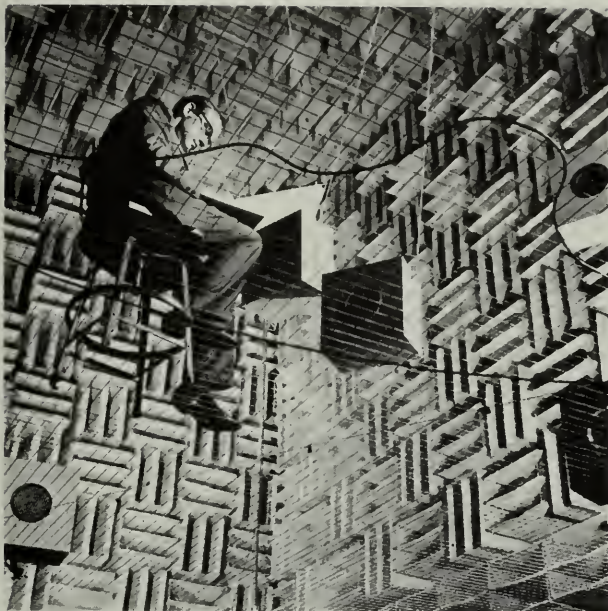
The article "Silence, Please" in *Reader 3* is an amusing fantasy about wave superposition.

Refraction of sound by different layers of air explains why we sometimes see lightning without hearing thunder. Similar refraction of sound occurs in layers of water of different temperatures. .This sometimes causes problems in using *sonar* (*so*und *n*avigation *a*nd *r*anging) devices at sea. Sonic refraction is used for a variety of purposes today. Geologists use them to study the earth's deep structure and to locate fossil fuels and minerals. Very intense sound waves are produced in the ground (as by dynamite blasts). The sound waves travel through the earth and are received by detection devices at different locations. The path of the waves, as refracted by layers in the earth, can be calculated from the relative intensities of sound received. From knowledge of the paths, estimates can be made of the composition of the layers.

We have already mentioned diffraction as a property of sound waves. Sound waves readily bend around corners and barriers to reach the listener within range. Sound waves reflect, as do rope or water waves, wherever they encounter a boundary between different media. Echo chamber effects (which can be artificially produced by electronics) have become familiar to listeners who enjoy popular music. The "live" sound of a bare room results from multiple reflections of waves which normally would be absorbed by furniture, rugs, and curtains. The architechtural accidents called "whispering galleries" show vividly how sound can be focused by reflection from curved surfaces. Laboratory rooms which greatly reduce reflections are called *anechoic* chambers. All these effects are of interest in the study of acoustics. Moreover, the proper acoustical design of public buildings is now recognized as an important function by most good architects.

The acoustic properties of a hall filled with people are very different from those of the empty hall. Acoustical engineers sometimes fill the seats with felt-covered sandbags while making tests.

In this chapter we have explained the basic phenomena of mechanical waves, ending with the theory of sound propagation. These explanations were considered the final triumph of Newtonian mechanics as applied to the transfer of energy of particles in motion. Most of the general principles



An anechoic chamber being used for research in acoustics. Sound is almost completely absorbed during multiple reflections among the wedges of soft material that cover the walls.

The concert hall of the University of Illinois Krannert Center for the Performing Arts was accoustically designed for unamplified performances.

of acoustics were discovered in the 1870's. Since then the study of acoustics has become involved with such fields as quantum physics. But perhaps its most important influence on modern physics has been its effect on the imagination of scientists. The successes of acoustics encouraged them to take seriously the power of the wave viewpoint—even in fields far from the original one, the mechanical motion of particles that move back and forth or up and down in a medium.

Q32 List five wave behaviors that can be demonstrated with sound waves.

Q33 Why can't sound waves be polarized?

EPILOGUE Seventeenth-century scientists thought they could eventually explain all physical phenomena by reducing them to matter and motion. This mechanistic viewpoint became known as the Newtonian worldview or Newtonian cosmology, since its most impressive success was Newton's theory of planetary motion. Newton and other scientists of his time proposed to apply similar methods to other problems, as we mentioned in the Prologue to this unit.

The early enthusiasm for this new approach to science is vividly expressed by Henry Power in his book *Experimental Philosophy* (1664). Addressing his fellow natural philosophers (or scientists, as we would now call them), he wrote:

> You are the enlarged and elastical Souls of the world, who, removing all former rubbish, and prejudicial resistances, do make way for the Springy Intellect to flye out into its desired Expansion . . .
>
> . . . This is the Age wherein (me-thinks) Philosophy comes in with a Spring-tide . . . I see how all the old Rubbish must be thrown away, and carried away with so powerful an Inundation. These are the days that must lay a new Foundation of a more magnificent Philosophy, never to be overthrown: that will Empirically and Sensibly canvass the *Phaenomena* of Nature, deducing the causes of things from such Originals in Nature, as we observe are producible by Art, and the infallible demonstration of Mechanicks; and certainly, this is the way, and no other, to build a true and permanent Philolophy.

In Power's day there were many people who did not regard the old Aristotelian cosmology as rubbish. For them, it provided a comforting sense of unity and interrelation among natural phenomena. They feared that this unity would be lost if everything was reduced simply to atoms moving randomly through space. The poet John Donne, in 1611, complained bitterly of the change already taking place in cosmology:

> And new Philosophy calls all in doubt,
> The Element of fire is quite put out;
> The Sun is lost, and th' earth, and no man's wit
> Can well direct him where to looke for it.
> And freely men confesse that this world's spent,
> When in the Planets, and the Firmament
> They seeke so many new; then see that this
> Is crumbled out againe to his Atomies.
> 'Tis all in peeces, all coherence gone;
> All just supply, and all Relation. . .

Newtonian physics provided powerful methods for analyzing the world and uncovering the basic principles of motion for individual pieces of matter. But the richness and complexity of processes in the real world seemed infinite. Could Newtonian physics deal as successfully with these real events as with ideal processes in a hypothetical vacuum? Could the perceptions of colors, sounds, and smells really be reduced to "nothing but" matter and motion? In the seventeenth century, and even in the eighteenth century, it was too soon to expect Newtonian physics to answer these questions. There was still too much work to do in establishing the basic principles of mechanics and applying them to astronomical problems. A full-scale attack on the properties of matter and energy had to wait until the nineteenth century.

This unit covered several successful applications and extensions of Newtonian mechanics which were accomplished by the end of the nineteenth century. For example, we discussed the conservation laws, new explanations of the properties of heat and gases, and estimates of some properties of molecules. We introduced the concept of energy, linking mechanics to heat and to sound. In Unit 4 we will show similar links to light, electricity, and magnetism. We also noted that applying mechanics on a molecular level requires statistical ideas and presents questions about the direction of time.

Throughout most of this unit we have emphasized the application of mechanics to separate pieces or molecules of matter. But scientists found that the molecular model was not the only way to understand the behavior of matter. Without departing from basic Newtonian cosmology, scientists could also interpret many phenomena (such as sound and light) in terms of wave motions in continuous matter. By the middle of the nineteenth century it was generally believed that all physical phenomena could be explained by a theory that was built on the use of either particles or waves. In the next unit, we will discover how much or how little validity there was in this belief. We will begin to see the rise of a new viewpoint in physics, based on the field concept. Then, in Unit 5, particles, waves, and fields will come together in the context of twentieth-century physics.

12.1 The Project Physics materials particularly appropriate for Chapter 12 include:

Experiments
Sound

Activities
Standing Waves on a Drum and a Violin
Moiré Patterns
Music and Speech Activities
Measurement of the Speed of Sound
Mechanical Wave Machines

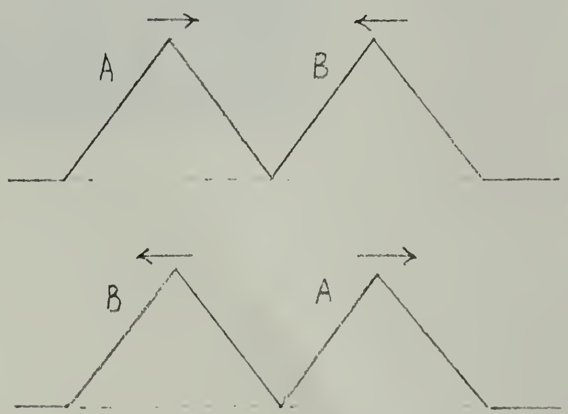
Film Loops
Superposition
Standing Waves in a String
Standing Waves in a Gas
Four loops on vibrations

Reader Articles
Silence, Please
Frontiers of Physics Today: Acoustics
Waves
What is a Wave
Musical Instruments and Scales
Founding a Family of Fiddles

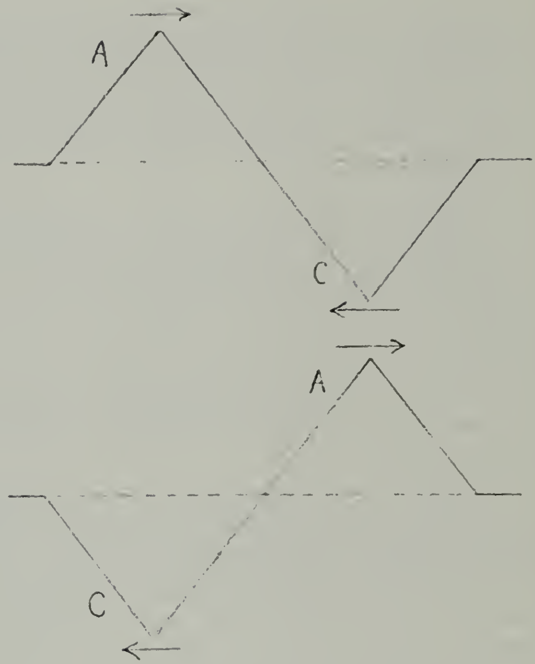
12.2 Some waves propagate at such a high speed that we are usually not aware of any delay in energy transfer. For example, the delay between the flash and the "bang" in watching lightning or fireworks seems peculiar, because the propagation time for sounds produced near us is not noticeable. Give an example of a compression wave in a solid, started by an action at one end, that propagates so quickly that we are not aware of any delay before an effect at the other end.

12.3 Describe the differences in phase of oscillation of various parts of your body as you walk. What points are exactly in phase? Which points are exactly $\frac{1}{2}$ cycle out of phase? Are there any points $\frac{1}{4}$ cycle out of phase?

12.4 Pictured are two pulse waves (A and B) on a rope at the instants before and after they



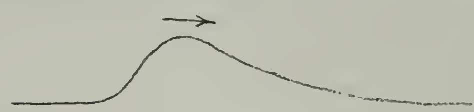
overlap (t_1 and t_2). Divide the elapsed time between t_1 and t_2 into four equal intervals and



plot the shape of the rope at the end of each interval.

12.5 Repeat Exercise 12.3 for the two pulses (*A* and C) pictured at the top.

12.6 The wave below propagates to the right along a rope. What is the shape of the wave propagating to the left that could for an instant cancel this one completely?



12.7 The velocity of a portion of rope at some instant as transverse waves are passing through it is the superposition of the velocities of waves passing through that portion. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that region? Justify your answer.

12.8 Graphically superpose the last three curves of the figure on p. 110 to find their sum (which should be the original curve).

12.9 What shape would the nodal regions have for sound waves from two loudspeakers?

12.10 Imagine a detection device for waves is moved slowly to either the right or left of the point labeled A_0 in the figure on p. 114. Describe what the detection device would register.

12.11 What kind of interference pattern would you expect to see if the separation between two in-phase sources were less than the wavelength λ? Where would the nodal and antinodal lines be if the two in-phase sources were separated by the distance λ? By $\lambda/2$? Convince yourself that one additional nodal line appears on each side of the central antinodal line whenever the separation between the two in-phase sources is increased by one wavelength.

12.12 Derive an equation, similar to $n\lambda l = dx_n$, for *nodal* points in a two-source interference pattern (where d is the separation of the sources, l the distance from the sources, and x_n the distance of the n^{th} node from the center line).

12.13 If you suddenly disturbed a stretched rubber hose or slinky with a frequency that precisely matched a standing wave frequency, would standing waves appear immediately? If not, what factors would determine the time delay?

12.14 Different notes are sounded with the same guitar string by changing its vibrating length (that is, pressing the string against a brass ridge). If the full length of the string is L, what lengths must it be shortened to in order to sound (a) a "musical fourth," (b) a "musical fifth," (c) an "octave"?

12.15 Standing sound waves can be set up in the air in an enclosure (like a bottle or an organ pipe). In a pipe that is closed at one end, the air molecules at the closed end are not free to be displaced, so the standing wave must have a displacement node at the closed end. At the open end, however, the molecules are almost completely free to be displaced, so the standing waves must have an antinode near the open end.
(a) What will be the wavelength of the fundamental standing wave in a pipe of length L closed at one end? (Hint: What is the longest wave that has a node and an antinode a distance L apart?)
(b) What is a general expression for possible wavelengths of standing waves in a pipe closed at one end?
(c) Answer (a) and (b) for the case of a pipe open at *both* ends.

12.16 Imagine a spherical blob of jello in which you can set up standing vibrations. What would be some of the possible modes of vibration? (Hint: what possible symmetrical nodal surfaces could there be?)

12.17 Suppose that straight-line ripple waves approach a thin straight barrier which is a few wavelengths long and which is oriented with its length parallel to the wavefronts. What do you predict about the nature of the diffraction pattern along a straight line behind the barrier which is perpendicular to the barrier and passes through the center of the barrier? Why do people who design breakwaters need to concern themselves with diffraction effects?

12.18 A megaphone directs sound along the megaphone axis if the wavelength of the sound is small compared to the diameter of the opening. Estimate the upper limit of frequencies which are diffracted at a cheerleader's megaphone opening. Can you hear what a cheerleader shouts even though you are far off the axis of the megaphone?

12.19 Explain why it is that the narrower a slit in a barrier is, the more nearly it can act like a point source of waves.

12.20 If light is also a wave, then why have you not seen light being diffracted by the slits, say those of a picket fence, or diffracted around the corner of houses?

12.21 By actual construction with a ruler and compass on a tracing of the photograph on p. 127, show that rays for the reflected wave front appear to come from S'. Show also that this is consistent with $\theta_r = \theta_i$.

12.22 A straight-line wave approaches a right-angle reflecting barrier as shown in the figure. Find the shape, size, and direction of propagation of the wave after it has been completely reflected by the barrier.

12.23 With ruler and compass reproduce part (b) of the figure at the bottom of p. 124 and find the distance from the circle's center to the point P in terms of the radius of the circle r. Make the radius of your circle much larger than the one in the figure. (Hint: the dotted lines are along radii.)

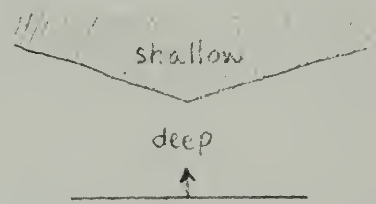
12.24 Convince yourself that a parabolic reflector will actually bring parallel wave-fronts to a sharp focus. Draw a parabola $y = kx^2$ (choosing any convenient value for k) and some parallel rays along the axis as in part (c) of the Figure at the bottom of p. 124. Construct line segments perpendicular to the parabola where the rays hit it, and draw the reflected rays at equal angles on the other side of these lines.

12.25 The *focal length* of a curved reflector is the distance from the reflector to the point where parallel rays are focused. Use the drawing in SG 12.24 to find the focal length of a parabola in terms of k.

12.26 Recalling that water surface waves travel slower in shallow water, what would you expect to happen to the shape of the following wave as it

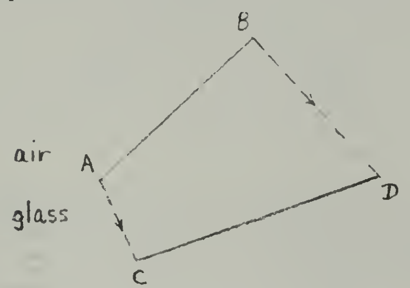
continues to the right? Pay particular attention to the region of varying depth. Can you use the line of reasoning above to give at least a partial explanation of the cause of breakers near a beach?

12.27 A straight-line wave in a ripple tank approaches a boundary between deep and shallow water as shown. Describe the shape of the wave as it passes through the boundary and then as it continues in the shallow water.



12.28 On the opposite page is an aerial photograph of ocean waves entering from the upper right and encountering a small island. Describe the wave phenomena demonstrated by this encounter.

12.29 The diagram below shows two successive positions, *AB* and *CD*, of a wave train of sound or light, before and after crossing an air-glass boundary. The time taken to go from *AB* to *DC* is one period of the wave.



(a) Indicate and label an angle equal to angle of incidence θ_A.
(b) Indicate and label an angle equal to angle of refraction θ_B.
(c) Label the wavelength in air λ_A.
(d) Label the wavelength in glass λ_B.
(e) Show that $v_A/v_B = \lambda_A/\lambda_B$.
(f) If you are familiar with trigonometry, show that $\sin \theta_A/\sin \theta_B = \lambda_A/\lambda_B$.

12.30 A periodic ripple-tank wave passes through a straight boundary between deep and shallow water. The angle of incidence at the boundary is 45° and the angle of refraction is 30°. The propagation speed in the deep water is 0.35 m/sec, and the frequency of the wave is 10 cycles per sec. Find the wavelengths in the deep and shallow water.

12.31 Look at Figure (d) on p. 127. Convince yourself that if a wave were to approach the boundary between medium 1 and medium 2 from below, along the same direction as the refracted ray in the figure, it would be refracted along the direction of the incident ray in the figure. This is another example of a general rule: if a wave follows a set of rays in one direction, then a wave can follow the same set of rays in the opposite direction. In other words, wave paths are reversible.

12.32 Suppose that in an extremely quiet room you can barely hear a buzzing mosquito at a distance of one meter.
(a) What is the sound power output of the mosquito?
(b) How many mosquitoes would it take to supply the power for one 100-watt reading lamp?
(c) If the swarm were at ten meters' distance, what would the sound be like? (Sound intensity diminishes in proportion to the square of the distance from a point source.)

12.33 How can sound waves be used to map the floors of oceans?

12.34 Estimate the wavelength of a 1000 cycles per second sound wave in air; in water; in steel (refer to data in text). Do the same if $f = 10.000$ cps. Design the dimensions of an experiment to show two-source interference for 1000 cps sound waves.

12.35 Waves reflect from an object in a definite direction only when the wavelength is small compared to the dimensions of the object. This is true for sound waves as well as for any other. What does this tell you about the sound frequencies a bat must generate if it is to catch a moth or a fly? Actually some bats can detect the presence of a wire about 0.12 mm in diameter. Approximately what frequency does that require?

Refraction, reflection, and diffraction of waves around Farallon Island, California. There are breakers all around the coast. The swell coming from top right rounds both sides of the island, producing a crossed pattern below. The small islet 'radiates' the waves away in all directions. (U.S. Navy photograph.)

meeting at a single point. And a barrier with the shape of a parabola (c) focuses straight-line waves precisely at a point. Similarly, a parabolic surface reflects plane waves to a sharp focus. An impressive example is a radio telescope. Its huge parabolic surface reflects faint radio waves from space to focus on a detector.

The wave paths indicated in the sketches could just as well be reversed. For example, spherical waves produced at the focus become plane waves when reflected from a parabolic surface. The flashlight and automobile headlamp are familiar applications of this principle. In them, white-hot wires placed at the focus of parabolic reflectors produce almost parallel beams of light.

Q26 What is a "ray"?

Q27 What is the relationship between the angle at which a wave front strikes a barrier and the angle at which it leaves?

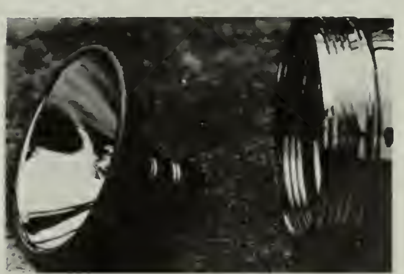
Q28 What shape of reflector can reflect parallel wave fronts to a sharp focus?

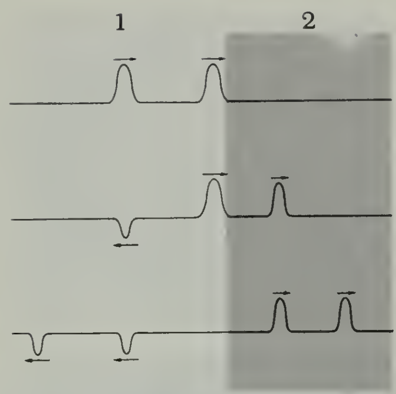
Q29 What happens to wave fronts originating at the focus of such a reflecting surface?

Above: A ripple tank shadow showing how circular waves produced at the focus of a parabolic wall are reflected from the wall into straight waves.

Left: the parabolic surface of a radio telescope reflects radio waves from space to a detector supported at the focus.

Below: the filament of a flashlight bulb is at the focus of a parabolic mirror, so the reflected light forms a nearly parallel beam.



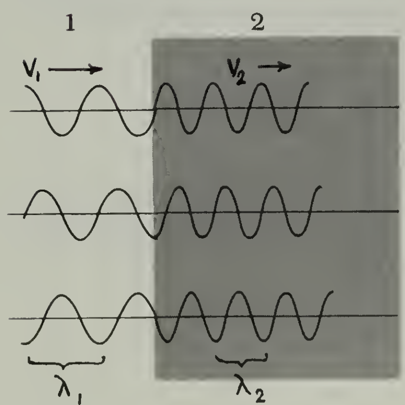


Pulses encountering a boundary between two different media. The speed of propagation is less in medium 2.

12.10 Refraction

What happens when a wave propagates from one medium to another medium in which its speed of propagation is different? We begin with the simple situation pictured in the margin. Two one-dimensional pulses approach a boundary separating two media. The speed of the propagation in medium 1 is greater than it is in medium 2. We might imagine the pulses to be in a light rope (medium 1) tied to a relatively heavy rope (medium 2). Part of each pulse is reflected at the boundary. This reflected component is flipped upside down relative to the original pulse. You will recall the inverted reflection at a hook in a wall discussed earlier. The heavier rope here tends to hold the boundary point fixed in just the same way. But we are not particularly interested here in the reflected wave. We want to see what happens to that part of the wave which continues into the second medium.

As shown in the figure, the transmitted pulses are closer together in medium 2 than they are in medium 1. Is it clear why this is so? The speed of the pulses is less in the heavier rope. So the second pulse is catching up with the first while the second pulse is still in the light rope and the first is already in the heavy rope. In the same way, each separate pulse is itself squeezed into a narrower form. That is, while the front of the pulse is entering the region of less speed, the back part is still moving with greater speed.



Continuous wave train crossing the boundary between two different media. The speed of propagation is less in medium 2.

Something of the same sort happens to a periodic wave at such a boundary. The figure at the left pictures this situation. For the sake of simplicity, we have assumed that all of the wave is transmitted, and none of it reflected. Just as the two pulses were brought closer and each pulse was squeezed narrower, the periodic wave pattern is squeezed together too. Thus, the wavelength λ_2 of the transmitted wave is shorter than the wavelength λ_1 of the incoming, or incident, wave.

Although the wavelength changes when the wave passes across the boundary, the frequency of the wave cannot change. If the rope is unbroken, the pieces immediately on either side of the boundary must go up and down together. The frequencies of the incident and transmitted waves must, then, be equal. So we can simply label both of them f.

We can write our wavelength, frequency, and speed relationship for both the incident and transmitted waves separately:

$$\lambda_1 f = v_1, \text{ and } \lambda_2 f = v_2$$

If we divide one of these equations by the other, eliminating the f's, we get

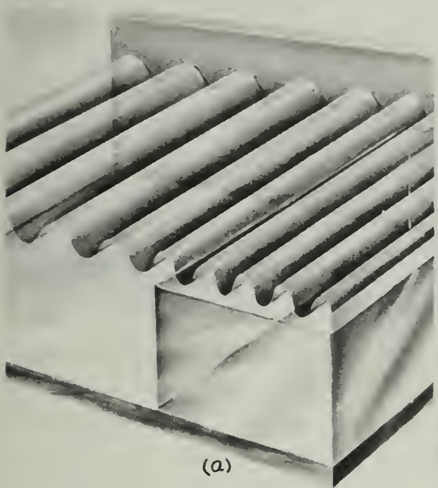
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

SG 12.26

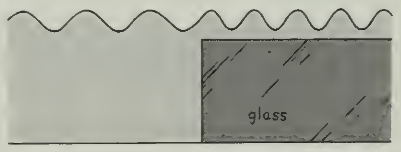
This equation tells that the ratio of the wavelengths in the two media equals the ratio of the speeds.

The same sort of thing happens when water ripples cross a boundary.

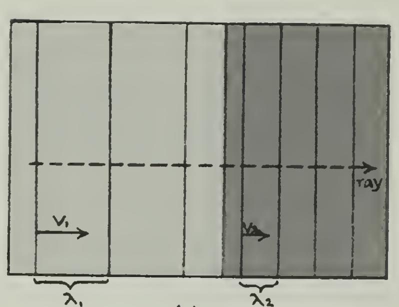
Experiments show that the ripples move more slowly in shallower water. A piece of plate glass is placed on the bottom of a ripple tank to make the water shallower there. This creates a boundary between the deeper and shallower part (medium 1 and medium 2). Figure (a) below shows the case where this boundary is parallel to the crest lines of the incident wave. As with rope waves, the wavelength of water waves in a medium is proportional to the speed in that medium.



(a)

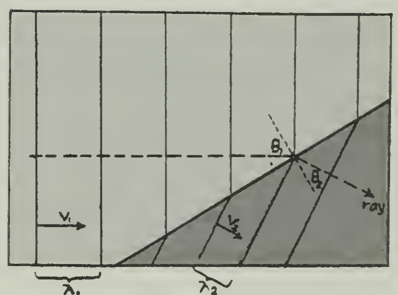


(b)



(c)

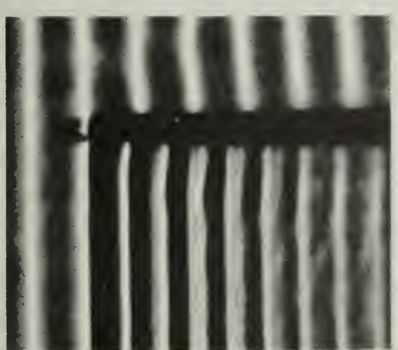
Water waves offer a possibility not present for rope waves. We can arrange to have the crest lines approach the boundary at any angle, not only head-on. The photograph below right shows such an event. A ripple tank wave approaches the boundary at the angle of incidence θ_i. The wavelength and speed of course change as the wave passes across the boundary. But the *direction* of the wave propagation changes too. Figure (d) in the margin shows how this comes about. As each part of a crest line in medium 1 enters medium 2, its speed decreases and it starts to lag behind. In time, the directions of the whole set of crest lines in medium 2 are changed from their directions in medium 1.

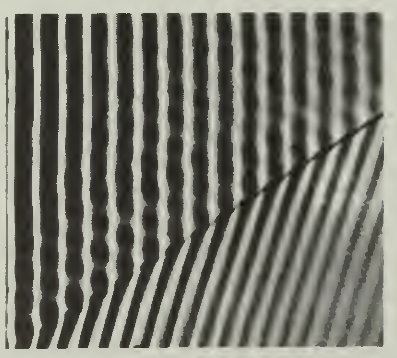


(d)

This phenomenon is called *refraction*. It occurs whenever a wave passes into a medium in which the wave velocity is reduced. The wave fronts are turned (refracted) so that they are more nearly parallel to the

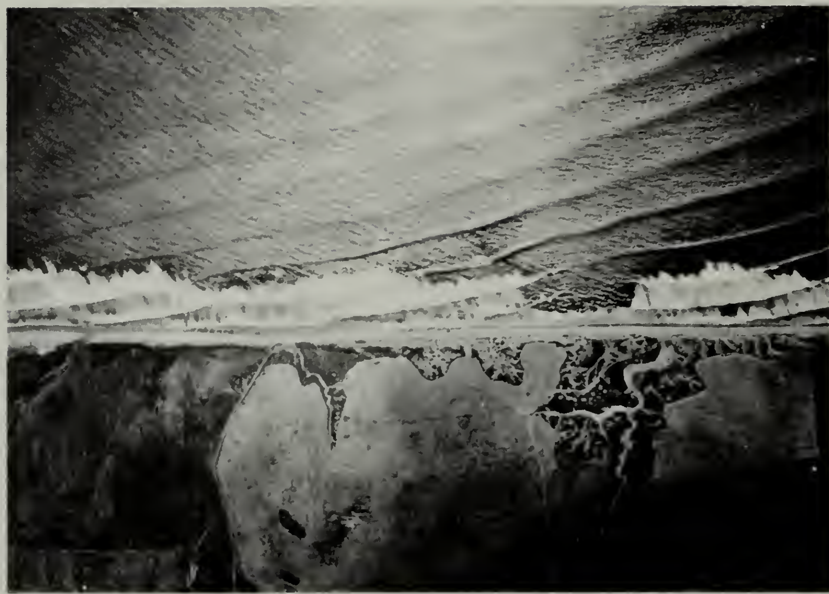
SG 12.27–12.31





Left: ripples on water (coming from the left) encounter the shallow region over the corner of a submerged glass plate.

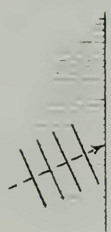
Right: ripples on water (coming from the left) encounter a shallow region over a glass plate placed at an angle to the wavefronts.



Aerial photograph of the refraction of ocean waves approaching shore.

boundary. (See the photographs at the bottom of the previous page.) This accounts for something that you may have noticed if you have been at an ocean beach. No matter in what direction the waves are moving far from the shore, when they come near the beach their crest-lines are nearly parallel to the shoreline. A wave's speed is steadily reduced as it moves into water that gets gradually more shallow. So the wave is refracted continuously as if it were always crossing a boundary between different media, as indeed it is. The refraction of sea waves is so great that wave crests can curl around a small island with an all-beach shoreline and provide surf on all sides. (See the photograph on page 139.)

The slowing of star light by increasingly dense layers of the atmosphere produces refraction that changes the apparent position of the star.



Q30 If a periodic wave slows down on entering a new medium, what happens to (1) its frequency? (2) its wavelength? (3) its direction?

Q31 Complete the sketch in the margin to show roughly what happens to a wave train that enters a new medium where its speed is greater.

12.11 Sound waves

Look again at the bottom figure in the margin of p. 103.

Sound waves are mechanical disturbances that propagate through a medium, such as the air. Typically, sound waves are longitudinal waves, producing changes of density and pressure in the medium through which they travel. The medium may be a solid, liquid, or gas. If the waves strike the ear, they can produce the sensation of hearing. The biology and psychology of hearing, as well as the physics of sound, are important to the science of acoustics. But here, of course, we will concentrate on sound as an example of wave motion. Sound has all the properties of wave motion that we have considered so far. It exhibits refraction, diffraction, and the same relations among frequency, wavelength, and propagation speed and interference. Only the property of polarization is missing, because sound waves are longitudinal, not transverse.

Vibrating sources for sound waves may be as simple as a tuning fork or as complex as the human larynx with its vocal cords. Tuning forks and some special electronic devices produce a steady "pure tone." Most of the energy in such a tone is in simple harmonic motion at a single frequency. The "pitch" of a sound we hear goes up as the frequency of the wave increases.

People can hear sound waves with frequencies between about 20 and 20,000 cycles per second. Dogs can hear over a much wider range (15-50,000 cps). Bats, porpoises, and whales generate and respond to frequencies up to about 120,000 cps.

Loudness (or "volume") of sound is, like pitch, a psychological variable. Loudness is strongly related to the *intensity* of the sound. Sound intensity is a physical quantity. It is defined in terms of *power flow*, such as the number of watts per square centimeter transmitted through a surface perpendicular to the direction of motion of a wave front. The human ear can perceive a vast range of intensities of sound. The table below illustrates this range. It begins at a level of 10^{-16} watts per square centimeter (relative intensity = 1). Below this "threshold" level, the normal ear does not perceive sound.

RELATIVE INTENSITY	SOUND
1	Threshold of hearing
10^1	Normal breathing
10^2	Leaves in a breeze
10^3	
10^4	Library
10^5	Quiet restaurant
10^6	Two-person conversation
10^7	Busy traffic
10^8	Vacuum cleaner
10^9	Roar of Niagara Falls
10^{10}	Subway train
10^{11}	
10^{12}	Propeller plane at takeoff
10^{13}	Machine-gun fire
10^{14}	Small jet plane at takeoff
10^{15}	
10^{16}	Wind tunnel
10^{17}	Space rocket at lift-off

Levels of noise intensity about 10^{12} times threshold intensity can be felt as a tickling sensation in the ear. Beyond 10^{13} times threshold intensity, the sensation changes to pain and may damage the unprotected ear.

SG 12.32

It has always been fairly obvious that sound takes time to travel from source to receiver. Light and sound are often closely associated in the same event—lightning and thunder, for instance. In all such cases, we perceive the sound later. By timing echoes over a known distance, the French mathematician Marin Mersenne in 1640 first computed the speed

Noise and the Sonic Boom

The world seems to be increasingly loud with unpleasant, manmade noise. At worst it is a major nuisance and may be tiring, painful, and sometimes even physically harmful. Loud, prolonged noise can produce temporary deafness. Very loud noise, kept up for a long time, can produce some degree of permanent deafness, especially deafness with respect to high-frequency sounds.

Often the simplest way of reducing noise is by *absorbing* it after it is produced but before it reaches your ears. Like all sound, noise is the energy of back and forth motion of the medium through which the noise goes. Noisy machinery can be muffled by padded enclosures in which the energy of noise is changed to heat energy, which then dissipates. In a house, a thick rug on the floor can absorb 90% of room noise. (A foot of fresh snow is an almost perfect absorber of noise outdoors. Cities and countrysides are remarkably hushed after a snowfall.)

In the last few years a new kind of noise has appeared: the sonic boom. An explosion-like sonic boom is produced whenever an object travels through air at a speed greater than the speed of sound (supersonic speed). Sound travels in air at about 700 miles per hour. Many types of military airplanes can travel at two or three times this speed. Flying at such speeds, the planes unavoidably and continually produce sonic booms. SST (Supersonic Transport) planes are now in civilian use in some countries. The unavoidable boom raises important questions. What is the price of technological "progress"? Who gains, and what fraction of the population? Who and how many pay the price? *Must* we pay it—must SST's be used? How much say has the citizen in decisions that affect his environment so violently?

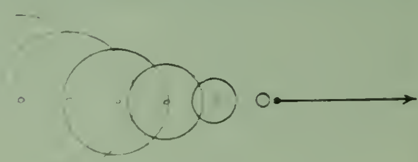
The formation of a sonic boom is similar to the formation of a wake by a boat. Consider a simple point source of waves. If it remains in the same position in a medium, the wave it produces spreads out symmetrically around it, as in diagram 1. But if the source of the disturbance is *moving* through the medium, each new crest starts from a different point, as in diagram 2.

Notice that the wavelength has become shorter in front of the object and longer behind it. (This is called the *Doppler effect.*) In diagram 3, the source is moving through the medium *faster than the wave speed.* Thus the crests and the corresponding troughs overlap and interfere with one another. The interference is mostly destructive everywhere except on the line tangent to the wave fronts, indicated in diagram 4. The result is a wake that spreads like a wedge away from the moving source, as in the photograph below.

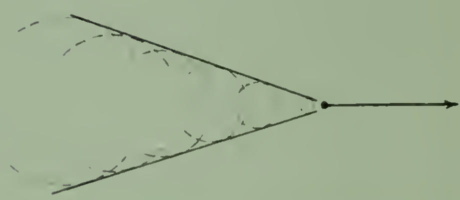
1

2

3



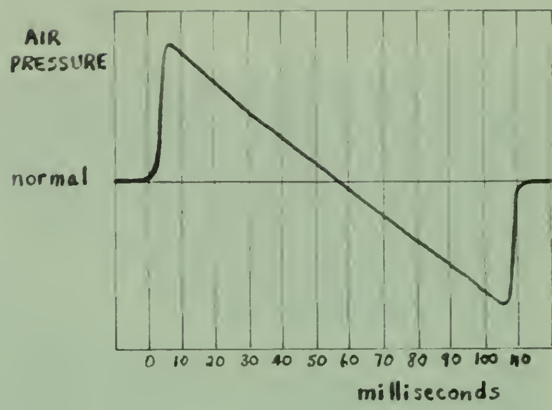
4



All these concepts apply not only to water waves but also to sound waves, including those disturbances set up in air by a moving plane as the wind and body push the air out of the way. If the source of sound is moving faster than the speed of sound wave, then there is a cone-shaped wake (in 3-dimensions) that spreads away from the source.

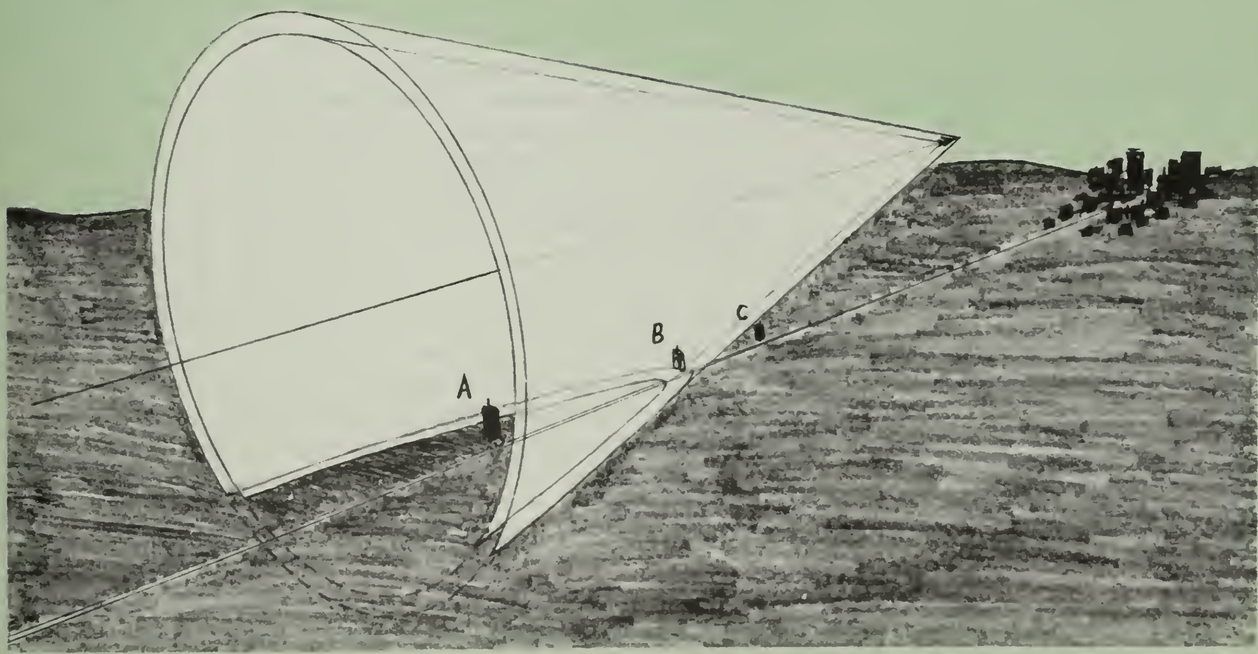
Actually two cones of sharp pressure change are formed: one originating at the front of the airplane and one at the rear, as indicated in the graph at the right.

Because the double shock wave follows along behind the airplane, the region on the ground where people and houses may be struck by the boom (the "sonic-boom carpet," or "bang-zone"), is as long as the supersonic flight path itself. In such an area, typically thousands of miles long and 50 miles wide, there may be millions of people. Tests made with airplanes flying at supersonic speed have shown that a single such cross-country flight by a 350-ton supersonic transport plane would break many thousands of dollars worth of windows, plaster walls, etc., and cause fright and annoyance to millions of people. Thus the supersonic flight of such planes may have to be confined to over-ocean use – though it may even turn out that the annoyance to people on shipboard, on islands, etc., is so great that over-ocean flights, too, will have to be restricted.



This curve represents the typical sonic boom from an airplane flying at supersonic speed (speed greater than about 700 mph). The pressure rises almost instantly, then falls relatively slowly to below-normal pressure, then rises again almost instantaneously. The second pressure rise occurs about 0.1 second after the first one, making the boom sound "double."

Double-cone shock wave, or sonic boom, produced by an airplane that is travelling (at 13-mile altitude) at three times the speed of sound. Building B is just being hit by shock wave, building A was struck a few seconds ago, and building C will be hit a few seconds later.



of sound in air. But it took another seventy years before William Derham in England, comparing the flash and noise from cannons across 12 miles, came close to the modern measurements.

Sound in air at 68°F moves at 1,125 feet per second (about 344 meters per second or 770 mph). As for all waves, the speed of sound waves depends on the properties of the medium—the temperature, density, and elasticity. Sound waves generally travel faster in liquids than in gases, and faster still in solids. In sea water, their speed is about 4,890 ft/sec; in steel, about 16,000 ft/sec; in quartz, about 18,000 ft/sec.

SG 12.33–12.35

The article "Silence, Please" in *Reader 3* is an amusing fantasy about wave superposition.

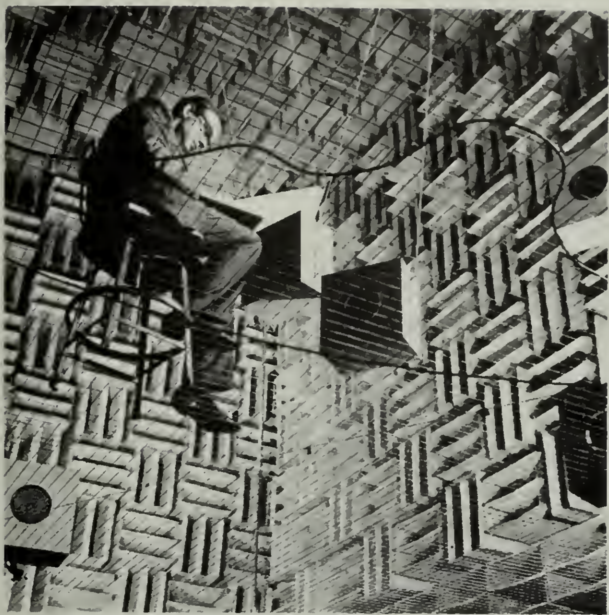
Interference of sound waves can be shown in a variety of ways. In a large hall with hard, sound-reflecting surfaces, there will be "dead" spots. At these spots, sound waves coming together after reflection cancel each other. Acoustic engineers must consider this in designing the shape, position, and materials of an auditorium. Another interesting and rather different example of sound interference is the phenomenon known as *beats*. When two notes of slightly different frequency are heard together, they interfere. This interference produces beats, a rhythmic pulsing of the sound. Piano tuners and string players use this fact to tune two strings to the same pitch. They simply adjust one string or the other until the beats disappear.

Refraction of sound by different layers of air explains why we sometimes see lightning without hearing thunder. Similar refraction of sound occurs in layers of water of different temperatures. .This sometimes causes problems in using *sonar* (*so*und *n*avigation *a*nd *r*anging) devices at sea. Sonic refraction is used for a variety of purposes today. Geologists use them to study the earth's deep structure and to locate fossil fuels and minerals. Very intense sound waves are produced in the ground (as by dynamite blasts). The sound waves travel through the earth and are received by detection devices at different locations. The path of the waves, as refracted by layers in the earth, can be calculated from the relative intensities of sound received. From knowledge of the paths, estimates can be made of the composition of the layers.

We have already mentioned diffraction as a property of sound waves. Sound waves readily bend around corners and barriers to reach the listener within range. Sound waves reflect, as do rope or water waves, wherever they encounter a boundary between different media. Echo chamber effects (which can be artificially produced by electronics) have become familiar to listeners who enjoy popular music. The "live" sound of a bare room results from multiple reflections of waves which normally would be absorbed by furniture, rugs, and curtains. The architechtural accidents called "whispering galleries" show vividly how sound can be focused by reflection from curved surfaces. Laboratory rooms which greatly reduce reflections are called *anechoic* chambers. All these effects are of interest in the study of acoustics. Moreover, the proper acoustical design of public buildings is now recognized as an important function by most good architects.

The acoustic properties of a hall filled with people are very different from those of the empty hall. Acoustical engineers sometimes fill the seats with felt-covered sandbags while making tests.

In this chapter we have explained the basic phenomena of mechanical waves, ending with the theory of sound propagation. These explanations were considered the final triumph of Newtonian mechanics as applied to the transfer of energy of particles in motion. Most of the general principles



An anechoic chamber being used for research in acoustics. Sound is almost completely absorbed during multiple reflections among the wedges of soft material that cover the walls.

The concert hall of the University of Illinois Krannert Center for the Performing Arts was accoustically designed for unamplified performances.

of acoustics were discovered in the 1870's. Since then the study of acoustics has become involved with such fields as quantum physics. But perhaps its most important influence on modern physics has been its effect on the imagination of scientists. The successes of acoustics encouraged them to take seriously the power of the wave viewpoint—even in fields far from the original one, the mechanical motion of particles that move back and forth or up and down in a medium.

Q32 List five wave behaviors that can be demonstrated with sound waves.

Q33 Why can't sound waves be polarized?

EPILOGUE Seventeenth-century scientists thought they could eventually explain all physical phenomena by reducing them to matter and motion. This mechanistic viewpoint became known as the Newtonian worldview or Newtonian cosmology, since its most impressive success was Newton's theory of planetary motion. Newton and other scientists of his time proposed to apply similar methods to other problems, as we mentioned in the Prologue to this unit.

The early enthusiasm for this new approach to science is vividly expressed by Henry Power in his book *Experimental Philosophy* (1664). Addressing his fellow natural philosophers (or scientists, as we would now call them), he wrote:

> You are the enlarged and elastical Souls of the world, who, removing all former rubbish, and prejudicial resistances, do make way for the Springy Intellect to flye out into its desired Expansion . . .
>
> . . . This is the Age wherein (me-thinks) Philosophy comes in with a Spring-tide . . . I see how all the old Rubbish must be thrown away, and carried away with so powerful an Inundation. These are the days that must lay a new Foundation of a more magnificent Philosophy, never to be overthrown: that will Empirically and Sensibly canvass the *Phaenomena* of Nature, deducing the causes of things from such Originals in Nature, as we observe are producible by Art, and the infallible demonstration of Mechanicks; and certainly, this is the way, and no other, to build a true and permanent Philolophy.

In Power's day there were many people who did not regard the old Aristotelian cosmology as rubbish. For them, it provided a comforting sense of unity and interrelation among natural phenomena. They feared that this unity would be lost if everything was reduced simply to atoms moving randomly through space. The poet John Donne, in 1611, complained bitterly of the change already taking place in cosmology:

> And new Philosophy calls all in doubt,
> The Element of fire is quite put out;
> The Sun is lost, and th' earth, and no man's wit
> Can well direct him where to looke for it.
> And freely men confesse that this world's spent,
> When in the Planets, and the Firmament
> They seeke so many new; then see that this
> Is crumbled out againe to his Atomies.
> 'Tis all in peeces, all coherence gone;
> All just supply, and all Relation. . .

Newtonian physics provided powerful methods for analyzing the world and uncovering the basic principles of motion for individual pieces of matter. But the richness and complexity of processes in the real world seemed infinite. Could Newtonian physics deal as successfully with these real events as with ideal processes in a hypothetical vacuum? Could the perceptions of colors, sounds, and smells really be reduced to "nothing but" matter and motion? In the seventeenth century, and even in the eighteenth century, it was too soon to expect Newtonian physics to answer these questions. There was still too much work to do in establishing the basic principles of mechanics and applying them to astronomical problems. A full-scale attack on the properties of matter and energy had to wait until the nineteenth century.

This unit covered several successful applications and extensions of Newtonian mechanics which were accomplished by the end of the nineteenth century. For example, we discussed the conservation laws, new explanations of the properties of heat and gases, and estimates of some properties of molecules. We introduced the concept of energy, linking mechanics to heat and to sound. In Unit 4 we will show similar links to light, electricity, and magnetism. We also noted that applying mechanics on a molecular level requires statistical ideas and presents questions about the direction of time.

Throughout most of this unit we have emphasized the application of mechanics to separate pieces or molecules of matter. But scientists found that the molecular model was not the only way to understand the behavior of matter. Without departing from basic Newtonian cosmology, scientists could also interpret many phenomena (such as sound and light) in terms of wave motions in continuous matter. By the middle of the nineteenth century it was generally believed that all physical phenomena could be explained by a theory that was built on the use of either particles or waves. In the next unit, we will discover how much or how little validity there was in this belief. We will begin to see the rise of a new viewpoint in physics, based on the field concept. Then, in Unit 5, particles, waves, and fields will come together in the context of twentieth-century physics.

12.1 The Project Physics materials particularly appropriate for Chapter 12 include:

Experiments
- Sound

Activities
- Standing Waves on a Drum and a Violin
- Moiré Patterns
- Music and Speech Activities
- Measurement of the Speed of Sound
- Mechanical Wave Machines

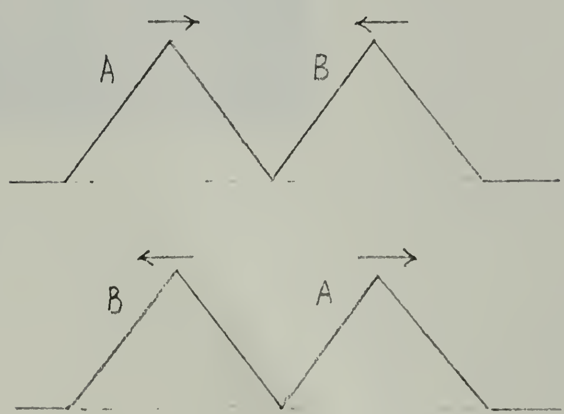
Film Loops
- Superposition
- Standing Waves in a String
- Standing Waves in a Gas
- Four loops on vibrations

Reader Articles
- Silence, Please
- Frontiers of Physics Today: Acoustics
- Waves
- What is a Wave
- Musical Instruments and Scales
- Founding a Family of Fiddles

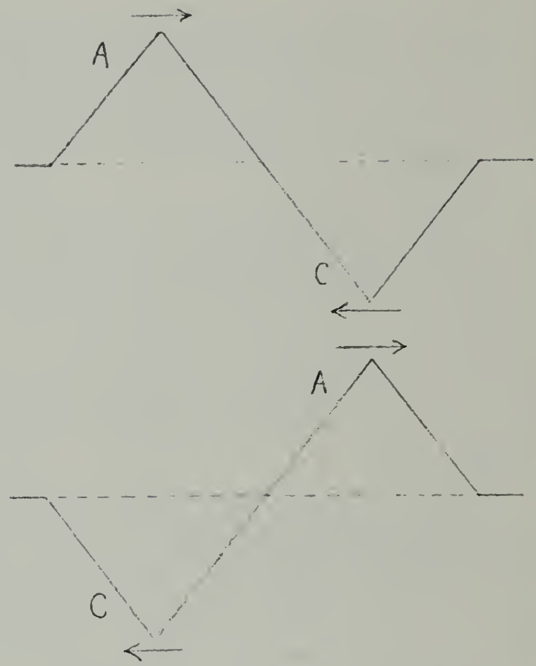
12.2 Some waves propagate at such a high speed that we are usually not aware of any delay in energy transfer. For example, the delay between the flash and the "bang" in watching lightning or fireworks seems peculiar, because the propagation time for sounds produced near us is not noticeable. Give an example of a compression wave in a solid, started by an action at one end, that propagates so quickly that we are not aware of any delay before an effect at the other end.

12.3 Describe the differences in phase of oscillation of various parts of your body as you walk. What points are exactly in phase? Which points are exactly $\frac{1}{2}$ cycle out of phase? Are there any points $\frac{1}{4}$ cycle out of phase?

12.4 Pictured are two pulse waves (A and B) on a rope at the instants before and after they



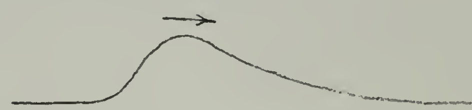
overlap (t_1 and t_2). Divide the elapsed time between t_1 and t_2 into four equal intervals and



plot the shape of the rope at the end of each interval.

12.5 Repeat Exercise 12.3 for the two pulses (*A* and *C*) pictured at the top.

12.6 The wave below propagates to the right along a rope. What is the shape of the wave propagating to the left that could for an instant cancel this one completely?



12.7 The velocity of a portion of rope at some instant as transverse waves are passing through it is the superposition of the velocities of waves passing through that portion. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that region? Justify your answer.

12.8 Graphically superpose the last three curves of the figure on p. 110 to find their sum (which should be the original curve).

12.9 What shape would the nodal regions have for sound waves from two loudspeakers?

12.10 Imagine a detection device for waves is moved slowly to either the right or left of the point labeled A_0 in the figure on p. 114. Describe what the detection device would register.

12.11 What kind of interference pattern would you expect to see if the separation between two in-phase sources were less than the wavelength λ? Where would the nodal and antinodal lines be if the two in-phase sources were separated by the distance λ? By $\lambda/2$? Convince yourself that one additional nodal line appears on each side of the central antinodal line whenever the separation between the two in-phase sources is increased by one wavelength.

12.12 Derive an equation, similar to $n\lambda l = dx_n$, for *nodal* points in a two-source interference pattern (where d is the separation of the sources, l the distance from the sources, and x_n the distance of the n^{th} node from the center line).

12.13 If you suddenly disturbed a stretched rubber hose or slinky with a frequency that precisely matched a standing wave frequency, would standing waves appear immediately? If not, what factors would determine the time delay?

12.14 Different notes are sounded with the same guitar string by changing its vibrating length (that is, pressing the string against a brass ridge). If the full length of the string is L, what lengths must it be shortened to in order to sound (a) a "musical fourth," (b) a "musical fifth," (c) an "octave"?

12.15 Standing sound waves can be set up in the air in an enclosure (like a bottle or an organ pipe). In a pipe that is closed at one end, the air molecules at the closed end are not free to be displaced, so the standing wave must have a displacement node at the closed end. At the open end, however, the molecules are almost completely free to be displaced, so the standing waves must have an antinode near the open end.
(a) What will be the wavelength of the fundamental standing wave in a pipe of length L closed at one end? (Hint: What is the longest wave that has a node and an antinode a distance L apart?)
(b) What is a general expression for possible wavelengths of standing waves in a pipe closed at one end?
(c) Answer (a) and (b) for the case of a pipe open at *both* ends.

12.16 Imagine a spherical blob of jello in which you can set up standing vibrations. What would be some of the possible modes of vibration? (Hint: what possible symmetrical nodal surfaces could there be?)

12.17 Suppose that straight-line ripple waves approach a thin straight barrier which is a few wavelengths long and which is oriented with its length parallel to the wavefronts. What do you predict about the nature of the diffraction pattern along a straight line behind the barrier which is perpendicular to the barrier and passes through the center of the barrier? Why do people who design breakwaters need to concern themselves with diffraction effects?

12.18 A megaphone directs sound along the megaphone axis if the wavelength of the sound is small compared to the diameter of the opening. Estimate the upper limit of frequencies which are diffracted at a cheerleader's megaphone opening. Can you hear what a cheerleader shouts even though you are far off the axis of the megaphone?

12.19 Explain why it is that the narrower a slit in a barrier is, the more nearly it can act like a point source of waves.

12.20 If light is also a wave, then why have you not seen light being diffracted by the slits, say those of a picket fence, or diffracted around the corner of houses?

12.21 By actual construction with a ruler and compass on a tracing of the photograph on p. 127, show that rays for the reflected wave front appear to come from S'. Show also that this is consistent with $\theta_r = \theta_i$.

12.22 A straight-line wave approaches a right-angle reflecting barrier as shown in the figure. Find the shape, size, and direction of propagation of the wave after it has been completely reflected by the barrier.

12.23 With ruler and compass reproduce part (b) of the figure at the bottom of p. 124 and find the distance from the circle's center to the point P in terms of the radius of the circle r. Make the radius of your circle much larger than the one in the figure. (Hint: the dotted lines are along radii.)

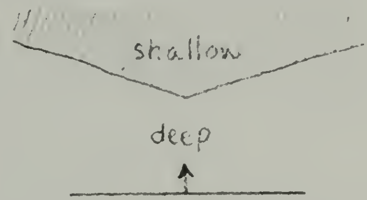
12.24 Convince yourself that a parabolic reflector will actually bring parallel wave-fronts to a sharp focus. Draw a parabola $y = kx^2$ (choosing any convenient value for k) and some parallel rays along the axis as in part (c) of the Figure at the bottom of p. 124. Construct line segments perpendicular to the parabola where the rays hit it, and draw the reflected rays at equal angles on the other side of these lines.

12.25 The *focal length* of a curved reflector is the distance from the reflector to the point where parallel rays are focused. Use the drawing in SG 12.24 to find the focal length of a parabola in terms of k.

12.26 Recalling that water surface waves travel slower in shallow water, what would you expect to happen to the shape of the following wave as it

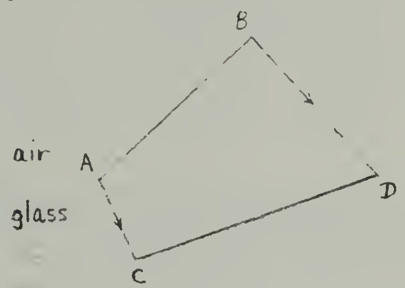
continues to the right? Pay particular attention to the region of varying depth. Can you use the line of reasoning above to give at least a partial explanation of the cause of breakers near a beach?

12.27 A straight-line wave in a ripple tank approaches a boundary between deep and shallow water as shown. Describe the shape of the wave as it passes through the boundary and then as it continues in the shallow water.



12.28 On the opposite page is an aerial photograph of ocean waves entering from the upper right and encountering a small island. Describe the wave phenomena demonstrated by this encounter.

12.29 The diagram below shows two successive positions, *AB* and *CD*, of a wave train of sound or light, before and after crossing an air-glass boundary. The time taken to go from *AB* to *DC* is one period of the wave.



(a) Indicate and label an angle equal to angle of incidence θ_A.
(b) Indicate and label an angle equal to angle of refraction θ_B.
(c) Label the wavelength in air λ_A.
(d) Label the wavelength in glass λ_B.
(e) Show that $v_A/v_B = \lambda_A/\lambda_B$.
(f) If you are familiar with trigonometry, show that $\sin\theta_A/\sin\theta_B = \lambda_A/\lambda_B$.

12.30 A periodic ripple-tank wave passes through a straight boundary between deep and shallow water. The angle of incidence at the boundary is 45° and the angle of refraction is 30°. The propagation speed in the deep water is 0.35 m/sec, and the frequency of the wave is 10 cycles per sec. Find the wavelengths in the deep and shallow water.

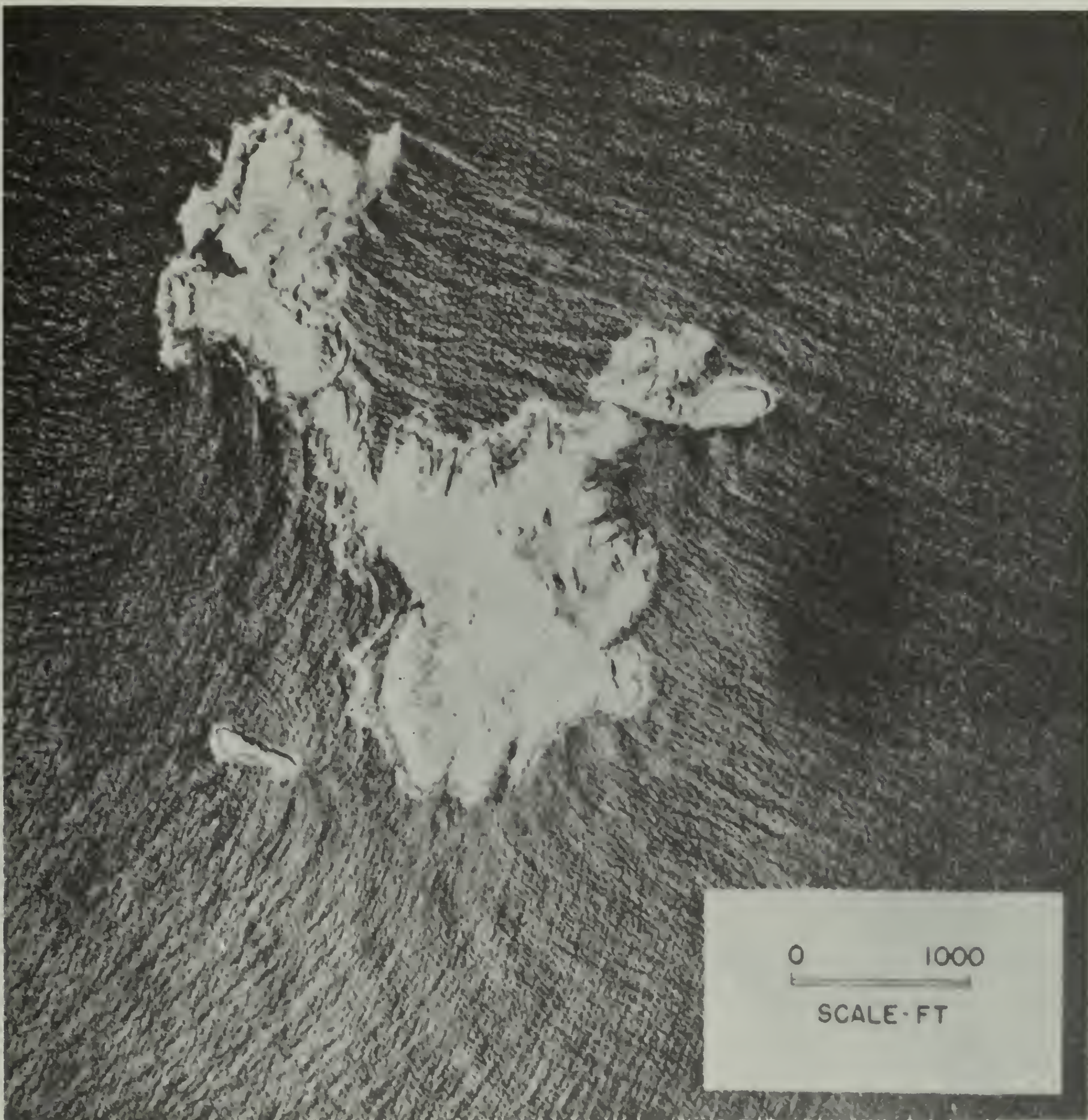
12.31 Look at Figure (d) on p. 127. Convince yourself that if a wave were to approach the boundary between medium 1 and medium 2 from below, along the same direction as the refracted ray in the figure, it would be refracted along the direction of the incident ray in the figure. This is another example of a general rule: if a wave follows a set of rays in one direction, then a wave can follow the same set of rays in the opposite direction. In other words, wave paths are reversible.

12.32 Suppose that in an extremely quiet room you can barely hear a buzzing mosquito at a distance of one meter.
(a) What is the sound power output of the mosquito?
(b) How many mosquitoes would it take to supply the power for one 100-watt reading lamp?
(c) If the swarm were at ten meters' distance, what would the sound be like? (Sound intensity diminishes in proportion to the square of the distance from a point source.)

12.33 How can sound waves be used to map the floors of oceans?

12.34 Estimate the wavelength of a 1000 cycles per second sound wave in air; in water; in steel (refer to data in text). Do the same if $f = 10{,}000$ cps. Design the dimensions of an experiment to show two-source interference for 1000 cps sound waves.

12.35 Waves reflect from an object in a definite direction only when the wavelength is small compared to the dimensions of the object. This is true for sound waves as well as for any other. What does this tell you about the sound frequencies a bat must generate if it is to catch a moth or a fly? Actually some bats can detect the presence of a wire about 0.12 mm in diameter. Approximately what frequency does that require?

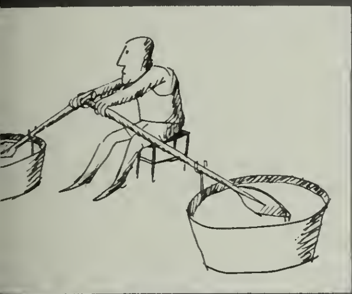


Refraction, reflection, and diffraction of waves around Farallon Island, California. There are breakers all around the coast. The swell coming from top right rounds both sides of the island, producing a crossed pattern below. The small islet 'radiates' the waves away in all directions. (U.S. Navy photograph.)

Project Physics

Handbook **3**

The Triumph of Mechanics



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Page 65 "Physics and Music," *Scientific American*, July 1948.

Page 89 Linda Cronquist, Wheat Ridge High School, Wheat Ridge, Colorado.

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Photographs of laboratory equipment and of students using laboratory equipment were supplied with the cooperation of the Project Physics staff and Damon Corporation.

Contents Project Physics Handbook Unit 3

EXPERIMENTS

EXPERIMENT 3-1 COLLISIONS IN ONE DIMENSION—I

In this experiment you will investigate the motion of two objects interacting in one dimension. The interactions (explosions and collisions in the cases treated here) are called one-dimensional because the objects move along a single straight line. Your purpose is to look for quantities or combinations of quantities that remain unchanged before and after the interaction—that is, quantities that are conserved.

Your experimental explosions and collisions may seem not only tame but also artificial and unlike the ones you see around you in everyday life. But this is typical of many scientific experiments, which simplify the situation so as to make it easier to make meaningful measurements and to discover patterns in the observed behavior. The underlying laws are the same for all phenomena, whether or not they are in a laboratory.

Two different ways of observing interactions are described here (and two others in Experiment 3-2). You will probably use only one of them. In each method, the friction between the interacting objects and their surroundings is kept as small as possible, so that the objects are a nearly isolated system. Whichever method you do follow, you should handle your results in the way described in the final section: *Analysis of data*.

Method A—Dynamics Carts

"Explosions" are easily studied using the low-friction dynamics carts. Squeeze the loop of spring steel flat and slip a loop of thread over it, to hold it compressed. Put the compressed loop between two carts on the floor or on a smooth table (Fig. 1). When you release the spring by burning the thread, the carts fly apart with velocities that you can measure from a strobe photograph or by any of the techniques you learned in earlier experiments.

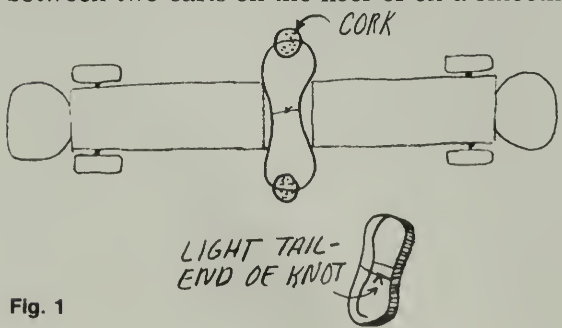


Fig. 1

Load the carts with a variety of weights to create simple ratios of masses, say 2 to 1 or 3 to 2. Take data for as great a variety of mass ratios as time permits. Because friction will gradually slow the carts down, you should make measurements on the speeds immediately after the explosion is over (that is, when the spring is through pushing).

Since you are interested only in comparing the speeds of the two carts, you can express those speeds in any units you wish, without worrying about the exact scale of the photograph and the exact strobe rate. For example, you can use distance units measured directly from the photograph (in millimeters) and use time units equal to the time interval between strobe images. If you follow that procedure, the speeds recorded in your notes will be in mm/interval.

Remember that you can get data from the negative of a Polaroid picture as well as from the positive print.

Method B—Air Track

The air track allows you to observe collisions between objects—"gliders"—that move with almost no friction. You can take stroboscopic photographs of the gliders either with the xenon strobe or by using a rotating slotted disk in front of the camera.

The air track has three gliders: two small ones with the same mass, and a larger one which has just twice the mass of a small one. A small and a large glider can be coupled together to make one glider so that you can have collisions between gliders whose masses are in the ratio of 1:1, 2:1, and 3:1. (If you add light sources to the gliders, their masses will no longer be in the same simple ratios. You can find the masses from the measured weights of the glider and light source.)

You can arrange to have the gliders bounce apart after they collide (elastic collision) or stick together (inelastic collision). Good tech-

nique is important if you are to get consistent results. Before taking any pictures, try both elastic and inelastic collisions with a variety of mass ratios. Then, when you have chosen one to analyze, rehearse each step of your procedure with your partners before you go ahead.

You can use a good photograph to find the speeds of both carts, before and after they collide. Since you are interested only in comparing the speeds before and after each collision, you can express speeds in any unit you wish, without worrying about the exact scale of the photograph or the exact strobe rate. For example, you use distance units measured directly from the photograph (in millimeters) and use time units equal to the time interval between strobe images. If you follow that procedure, the speeds recorded in your notes will be in mm/interval.

Remember that you can get data from the negative of your Polaroid picture as well as from your positive print.

Analysis of Data

Assemble all your data in a table having column headings for the mass of each object, m_A and m_B, the speeds before the interaction, v_A and v_B (for explosions, $v_A = v_B = 0$), and the speeds after the collision, v_A' and v_B'.

Examine your table carefully. Search for quantities or combinations of quantities that remain unchanged before and after the interaction.

1. Is *speed* a conserved quantity? That is, does the quantity $(v_A + v_B)$ equal the quantity $(v_A' + v_B')$?

2. Consider the direction as well as the speed. Define velocity to the right as positive and velocity to the left as negative. Is *velocity* a conserved quantity?

3. If neither speed nor velocity is conserved, try a quantity that combines the mass and velocity of each cart. Compare $(m_A v_A + m_B v_B)$ with $(m_A v_A' + m_B v_B')$ for each interaction. In the same way compare m/v, $m\vec{v}$, $m^2 v$, or any other likely combinations you can think of, before and after interaction. What conclusions do you reach?

EXPERIMENT 3-2 COLLISIONS IN ONE DIMENSION—II

Method A—Film Loops

Film Loops 3-1, 3-2, and 3-3 show one-dimensional collisions that you cannot easily perform in your own laboratory, for they were filmed with a very high speed camera, producing the effect of slow motion when projected at the standard rate. You can make measurements directly from the pictures projected onto graph paper. Since you are interested only in comparing speeds before and after a collision, you can express speeds in any unit you wish—that is, you can make measurements in any convenient distance and time units.

Notes for these film loops are located on pages 76 to 77. If you use these loops, read the notes carefully before taking your data.

Method B—Stroboscopic Photographs

Stroboscopic photographs showing seven different examples of one-dimensional collisions appear on the following pages.* They are useful here for studying momentum and again later for studying kinetic energy.

*Reproduced by permission of National Film Board of Canada

For each event you should find the speeds of the balls before and after collision. From the values for mass and speed of each ball, you should calculate the total momentum before and after collision. You will use the same values to calculate the total kinetic energy before and after collision.

You should read Section I, before analyzing any of the events, in order to find out what measurements to make and how the collisions were produced. After you have made your measurements, turn to Section II for questions to answer about each event.

I. The Measurements You Will Make

To make the necessary measurements you will need a metric ruler marked in millimeters, preferably of transparent plastic with sharp scale markings. Before starting your work, consult Fig. 1 for suggestions on improving your measuring technique.

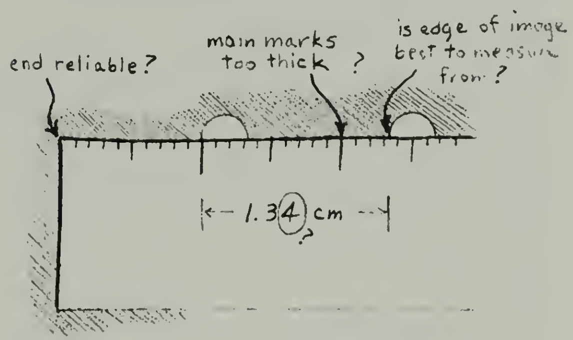


Fig. 1

Fig. 2 shows schematically that the colliding balls were hung from very long wires. The balls were released from rest, and their double-wire (bifilar) suspensions guided them to a squarely head-on collision. Stroboscopes illuminated the 3 × 4 ft rectangle that was the field of view of the camera. The stroboscopes are not shown in Fig. 2.

Notice the two rods whose tops reach into the field of view. These rods were 1 meter (± 2 millimeters) apart, measured from top center of one rod to top center of the other. The tops of these rods are visible in the photographs on

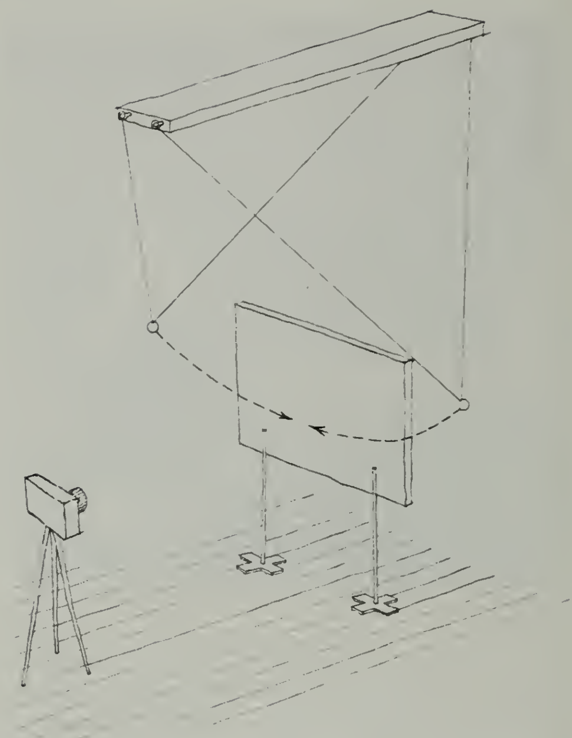


Fig. 2 Set-up for photographing one-dimensional collisions.

which you will make your measurements. This enables you to convert your measurements to actual distances if you wish. However, it is easier to use the lengths in millimeters measured directly off the photograph if you are merely going to compare momenta.

The balls speed up as they move into the field of view. Likewise, as they leave the field of view, they slow down. Therefore successive displacements on the stroboscopic photograph, each of which took exactly the same time, will not necessarily be equal in length. Check this with your ruler.

As you measure a photograph, number the position of each ball at successive flashes of the stroboscope. Note the interval during which the collision occurred. Identify the clearest time interval for finding the velocity of each ball (a) before the collision and (b) after the collision. Then mark this information close on each side of the interval.

II. Questions to be Answered about Each Event

After you have recorded the masses (or relative masses) given for each ball and have recorded the necessary measurements of velocities, answer the following questions.

1. What is the total momentum of the system of two balls before the collision? Keep in mind here that velocity, and therefore momentum, are vector quantities.
2. What is the total momentum of the system of two balls after the collision?
3. Was momentum conserved within the limits of precision of your measurements?

Event 1

The photographs of this Event 1 and all the following events appear below as Figs. 10 to 16. This event is also shown as the first example in *Film Loop* L18, "One-Dimensional Collisions I."

Figure 3 shows that ball B was initially at rest. After the collision both balls moved off to the left. The balls are made of steel.

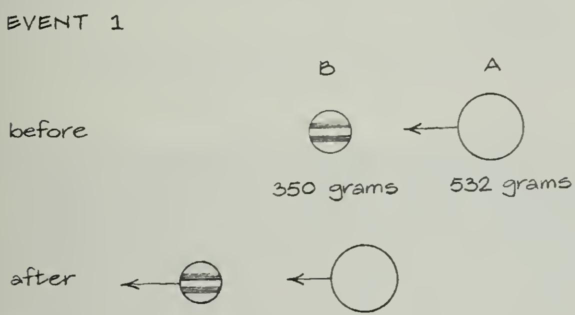


Fig. 3

Event 2

This event, the reverse of Event 1, is shown as the second example in *Film Loop* L18, "One-Dimensional Collisions I."

Fig. 3 shows that ball B came in from the left and that ball A was initially at rest. The collision reversed the direction of motion of ball B and sent ball A off to the right. (The balls are of hardened steel.)

As you can tell by inspection, ball B moved slowly after collision, and thus you may have trouble getting a precise value for its speed. This means that your value for this speed is the

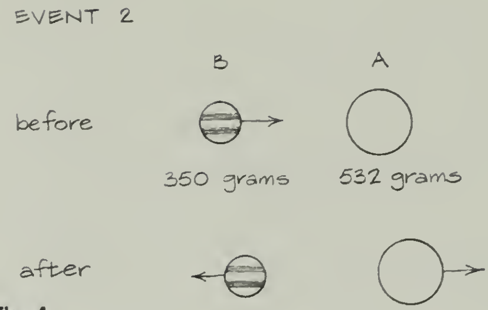


Fig. 4

least reliable of your four speed measurements. Nevertheless, this fact has only a small influence on the reliability of your value for the total momentum after collision. Can you explain why this should be so?

Why was the direction of motion of ball B reversed by the collision?

If you have already studied Event 1, you will notice that the same balls were used in Events 1 and 2. Check your velocity data, and you will find that the *initial* speeds were nearly equal. Thus, Event 2 was truly the reverse of Event 1. Why, then, was the direction of motion of ball A in Event 1 not reversed although the direction of ball B in Event 2 was reversed?

Event 3

This event is shown as the first example in *Film Loop* L19, "One-Dimensional Collisions II." Event 3 is not recommended unless you also study one of the other events. Event 3 is especially recommended as a companion to Event 4.

Fig. 5 shows that a massive ball (A) entered from the left. A less massive ball B came in from the right. The directions of motion of both balls were reversed by the collision. (The balls were made of hardened steel.)

When you compare the momenta before and after the collision you will probably find that they differed by more than any other event so far in this series. Explain why this is so.

Event 4

This event is also shown as the second ex-

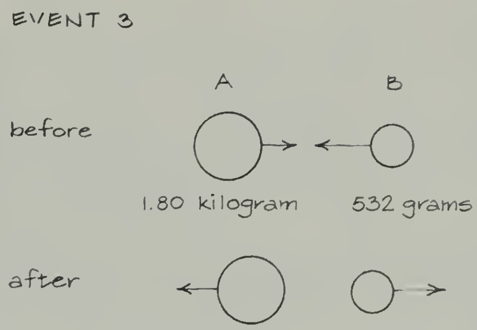


Fig. 5

ample in *Film Loop* L19, "One-Dimensional Collisions II."

Fig. 6 shows that two balls came in from the left, that ball A was far more massive than ball B, and that ball A was moving faster than ball B before collision. The collision occurred when A caught up with B, increasing B's speed at some expense to its own speed. (The balls were made of hardened steel.)

Each ball moved across the camera's field from left to right on the same line. In order to be able to tell successive positions apart on a stroboscopic photograph, the picture was taken

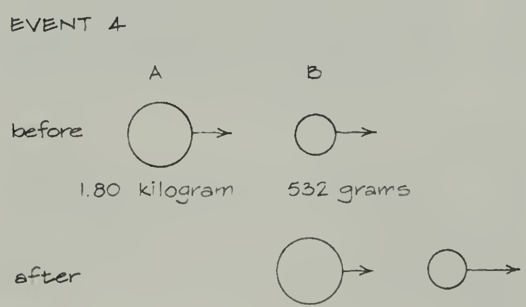


Fig. 6

twice. The first photograph shows only the progress of the large ball A because ball B had been given a thin coat of black paint (of negligible mass). Ball A was painted black when the second picture was taken. It will help you to analyze the collision if you actually number white-ball positions at successive stroboscope flashes in each picture.

Event 5

This event is also shown as the first example in *Film Loop* L20, "Inelastic One-Dimensional Collisions." You should find it interesting to analyze this event or Event 6 or Event 7, but it is not necessary to do more than one.

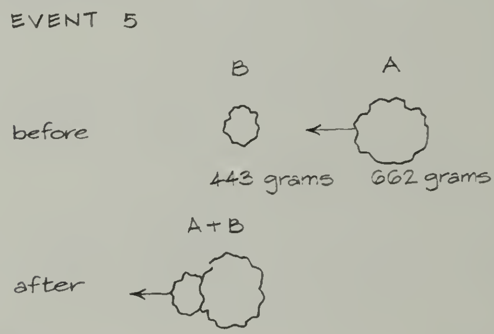


Fig. 7

Fig. 7 shows that ball A came in from the right, striking ball B which was initially at rest. The balls were made of a soft material (plasticene). They remained stuck together after the collision and moved off to the left as one. A collision of this type is called *"perfectly inelastic."*

Event 6

This event is shown as the second example in *Film Loop* L20, "Inelastic One-Dimensional Collisions."

Fig. 8 shows that balls A and B moved in from the right and left, respectively, before collision. The balls were made of a soft material (plasticene). They remained stuck together after the collision and moved off together to the left. This is another "perfectly inelastic" collision, like that in Event 5.

This event was photographed in two parts. The first print shows the conditions before collision, the second print, after collision. Had the picture been taken with the camera shutter open throughout the motion, it would be difficult to take measurements because the combined balls (A + B)—after collision—retraced the path which ball B followed before collision. You can number the positions of each ball before collision at successive flashes of the stroboscope (in the first photo); and you can do likewise for the combined balls (A + B) after the collision in the second photo.

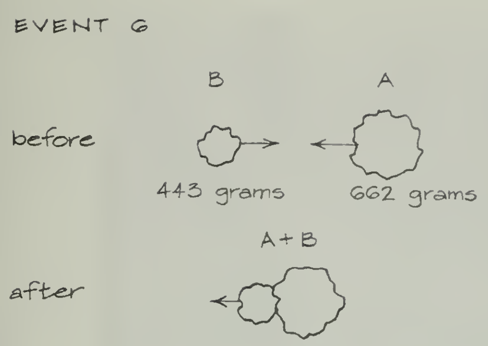


Fig. 8

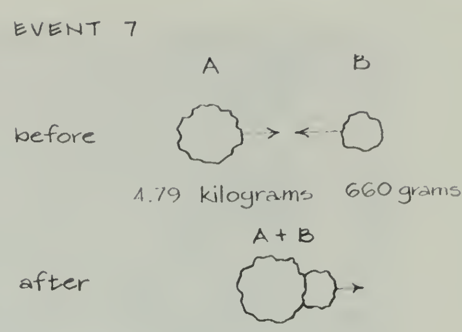


Fig. 9

Event 7

Fig. 9 shows that balls A and B moved in from opposite directions before collision. The balls are made of a soft material (plasticene). They remain stuck together after collision and move off together to the right. This is another so-called "*perfectly inelastic*" collision.

This event was photographed in two parts. The first print shows the conditions before collision, the second print, after collision. Had the picture been made with the camera shutter open throughout the motion, it would be difficult to take measurements because the combined balls (A + B) trace out the same path as incoming ball B. You can number the positions of each ball before collision at successive flashes of the stroboscope (in the first photograph), and you can do likewise for the combined balls (A + B) after collision in the second photograph.

Photographs of the Events

The photographs of the events are shown in Fig. 10 through 16.

Fig. 10 Event 1, 10 flashes/sec

before

after

Fig. 11 Event 2, 10 flashes/sec

before

after

Fig. 12 Event 3, 10 flashes/sec

ball A

ball B

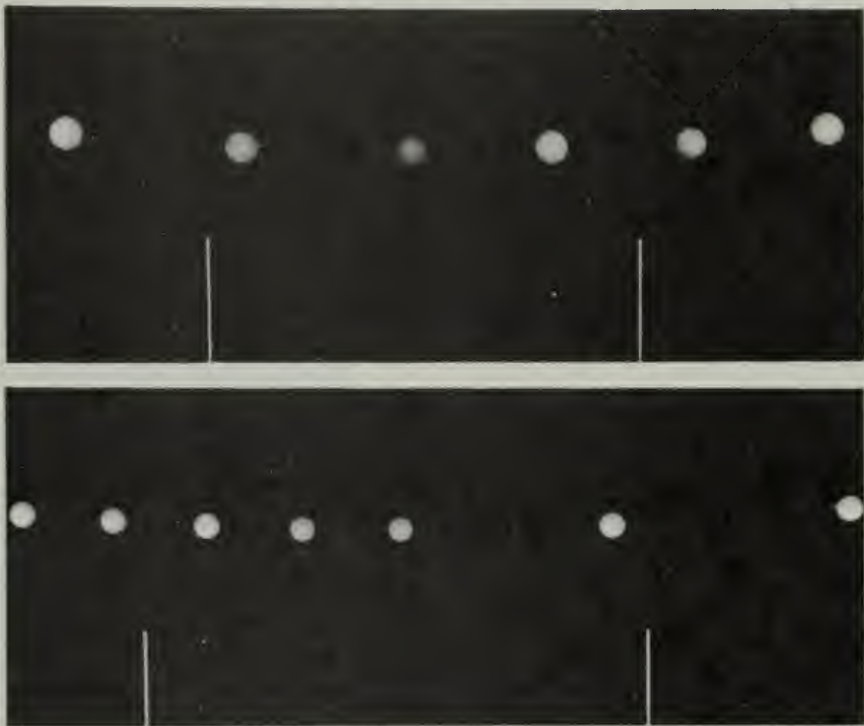


Fig. 13 Event 4, 10 flashes/sec

Fig. 14 Event 5, 10 flashes/sec

before

after

Fig. 15 Event 6, 10 flashes/sec

before

after

Fig. 16 Event 7, 10 flashes/sec

EXPERIMENT 3-3 COLLISIONS IN TWO DIMENSIONS—I

Collisions rarely occur in only one dimension, that is, along a straight line. In billiards, basketball, and tennis, the ball usually rebounds at an angle to its original direction; and ordinary explosions (which can be thought of as collisions in which initial velocities are all zero) send pieces flying off in all directions.

This experiment deals with collisions that occur in two dimensions—that is, in a single plane—instead of along a single straight line. It assumes that you know what momentum is and understand what is meant by "conservation of momentum" in one dimension. In this experiment you will discover a general form of the rule for one dimension that applies also to the conservation of momentum in cases where the parts of the system move in two (or three) dimensions.

Two methods of getting data on two-dimensional collisions are described below (and two others in Experiment 3-4), but you will probably want to follow only one method. Whichever method you use, handle your results in the way described in the last section.

Method A—Colliding Pucks

On a carefully leveled glass tray covered with a sprinkling of Dylite spheres, you can make pucks coast with almost uniform speed in any direction. Set one puck motionless in the center of the table and push a second similar one toward it, a little off-center. You can make excellent pictures of the resulting two-dimensional glancing collision with a camera mounted directly above the surface.

To reduce reflection from the glass tray, the photograph should be taken using the xenon stroboscope with the light on one side and almost level with the glass tray. To make each puck's location clearly visible in the photograph, attach a steel ball or a small white Styrofoam hemisphere to its center.

The large puck has twice the mass of the small puck. You can get a greater variety of masses by stacking pucks one on top of the other and fastening them together with tape (but avoid having the collisions cushioned by the tape).

Two people are needed to do the experiment. One experimenter, after some preliminary practice shots, launches the projectile puck while the other experimenter operates the camera. The resulting picture should consist of a series of white dots in a rough "Y" pattern.

Using your picture, measure and record all the speeds before and after collision. Record the masses in each case too. Since you are interested only in comparing speeds, you can use any convenient speed units. You can simplify your work if you record speeds in mm/dot instead of trying to work them out in cm/sec. Because friction does slow the pucks down, find speeds as close to the impact as you can. You can also use the "puck" instead of the kilogram as your unit of mass.

Method B—Colliding Disk Magnets

Disk magnets will also slide freely on Dylite spheres as described in Method A.

The difference here is that the magnets need never touch during the "collision." Since the interaction forces are not really instantaneous as they are for the pucks, the magnets follow *curving* paths during the interaction. Consequently the "before" velocity should be determined as early as possible and the "after" velocities should be measured as late as possible.

Following the procedure described above for pucks, photograph one of these "collisions." Again, small Styrofoam hemispheres or steel balls attached to the magnets should show up in the strobe picture as a series of white dots. Be sure the paths you photograph are long enough so that the dots near the ends form straight lines rather than curves.

Using your photograph, measure and record the speeds and record the masses. You can simplify your work if you record speeds in mm/dot instead of working them out in cm/sec. You can use the disk instead of the kilogram as your unit of mass.

Analysis of Data

Whichever procedure you used, you should analyze your results in the following way. Multiply the mass of each object by its before-the-collision speed, and add the products.

> 1. Do the same thing for each of the objects in the system after the collision, and add the after-the-collision products together. Does the sum before the collision equal the sum after the collision?

Imagine the collision you observed was an explosion of a cluster of objects at rest; the total quantity mass-times-speed before the explosion will be zero. But surely, the mass-times-speed of each of the flying fragments after the explosion is more than zero! "Mass-times-speed" is obviously *not* conserved in an explosion. You probably found it wasn't conserved in the experiments with pucks and magnets, either. You may already have suspected that you ought to be taking into account the *directions* of motion.

To see what *is* conserved, proceed as follows.

Use your measurements to construct a drawing like Fig. 1, in which you show the directions of motion of all the objects both before and after the collision.

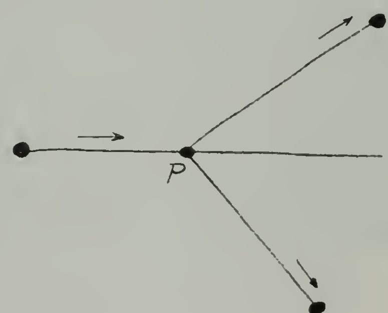


Fig. 1

Have all the direction lines *meet at a single point* in your diagram. The actual paths in your photographs will not do so, because the pucks and magnets are large objects instead of points, but you can still draw the *directions* of motion as lines through the single point P.

On this diagram draw a vector arrow whose magnitude (length) is proportional to the mass times the speed of the projectile *before* the collision. (You can use any convenient scale.) In Fig. 2, this vector is marked $m_A v_A$. Before an explosion there is no motion at all, and hence, no diagram to draw.

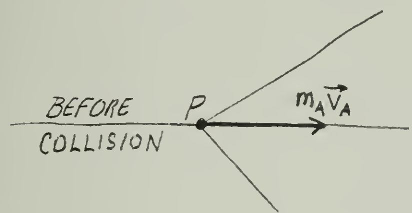


Fig. 2

Below your first diagram draw a second one in which you once more draw the directions of motion of all the objects exactly as before. On this second diagram construct the vectors for mass-times-speed for each of the objects leaving P *after* the collision. For the collisions of pucks and magnets your diagram will resemble Fig. 3. Now construct the "after the collision" vector sum.

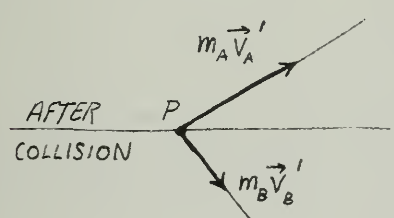


Fig. 3

The length of each of your arrows is given by the product of mass and speed. Since each arrow is drawn in the *direction* of the speed, the arrows represent the product of mass and velocity $\overrightarrow{mv}$ which is called *momentum*. The vector sums "before" and "after" collision therefore represent the total momentum of the system of objects before and after the collision. If the "before" and "after" arrows are equal, then the total momentum of the system of interacting objects is conserved.

2. How does this vector sum compare with the vector sum on your before-the-collision figure? Are they equal within the uncertainty?

3. Is the principle of conservation of momentum for one dimension different from that for two, or merely a special case of it? How can the principle of conservation of momentum be extended to three dimensions? Sketch at least one example.

4. Write an equation that would express the principle of conservation of momentum for collisions of (a) 3 objects in two dimensions, (b) 2 objects in three dimensions, (c) 3 objects in three dimensions.

A 3,000-pound steel ball swung by a crane against the walls of a condemned building. What happens to the momentum of the ball?

EXPERIMENT 3-4 COLLISIONS IN TWO DIMENSIONS—II

Method A—Film Loops

Several *Film Loops* (L21, L22, L23, L24, and L25) show two-dimensional collisions that you cannot conveniently reproduce in the laboratory. Notes on these films appear on pages 77-79. Project one of the loops on the chalkboard or on a sheet of graph paper. Trace the paths of the moving objects and record their masses and measure their speeds. Then go on to the analysis described in the notes for Film Loop L21 on p. 77.

Method B–Stroboscopic Photographs

Stroboscopic photographs* of seven different two-dimensional collisions in a plane are used in this experiment. The photographs (Figs. 5 to 12) are shown on the pages immediately following the description of these events. They were photographed during the making of Film Loops L21 through L25.

I. Material Needed

1. A transparent plastic ruler, marked in millimeters.
2. A large sheet of paper for making vector diagrams. Graph paper is especially convenient.
3. A protractor and two large drawing triangles are useful for transferring direction vectors from the photographs to the vector diagrams.

II. How the Collisions were Produced

Balls were hung on 10-meter wires, as shown schematically in Fig. 1. They were released so as to collide directly above the camera, which was facing upward. Electronic strobe lights (shown in Fig. 4) illuminated the rectangle shown in each picture.

Two white bars are visible at the bottom of each photograph. These are rods that had their tips 1 meter (± 2 millimeters) apart in the actual situation. The rods make it possible for you to convert your measurements to the actual distance. It is not necessary to do so, if you choose instead to use actual on-the-photograph distances in millimeters as you may have done in your study of one-dimensional collisions.

*Reproduced by permission of National Film Board of Canada

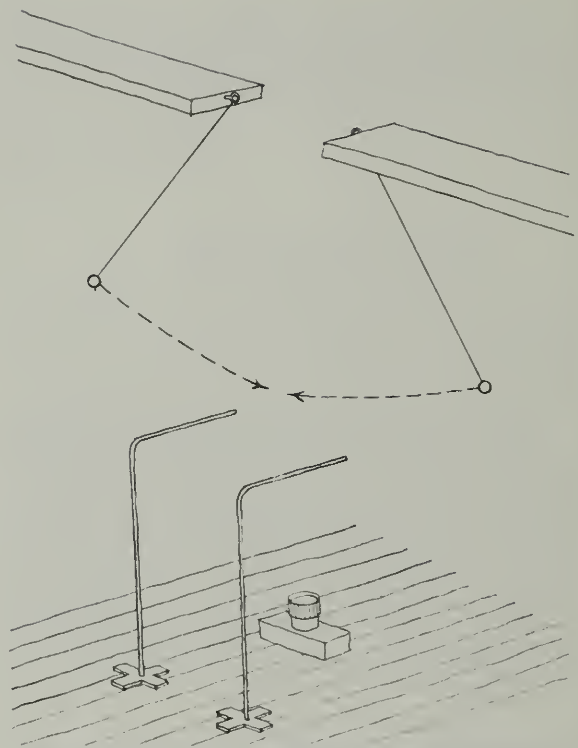


Fig. 1 Set-up for photographing two-dimensional collisions.

Since the balls are pendulum bobs, they move faster near the center of the photographs than near the edge. Your measurements, therefore, should be made near the center.

III. A Sample Procedure

The purpose of your study is to see to what extent momentum seems to be conserved in two-dimensional collisions. For this purpose you need to construct vector diagrams.

Consider an example: in Fig. 2, a 450 g and a 500 g ball are moving toward each other. Ball A has a momentum of 1.8 kg-m/sec, in the direction of the ball's motion. Using the scale shown, you draw a vector 1.8 units long, parallel to the direction of motion of A. Similarly, for ball B you draw a momentum vector of 2.4 units long, parallel to the direction of motion of B.

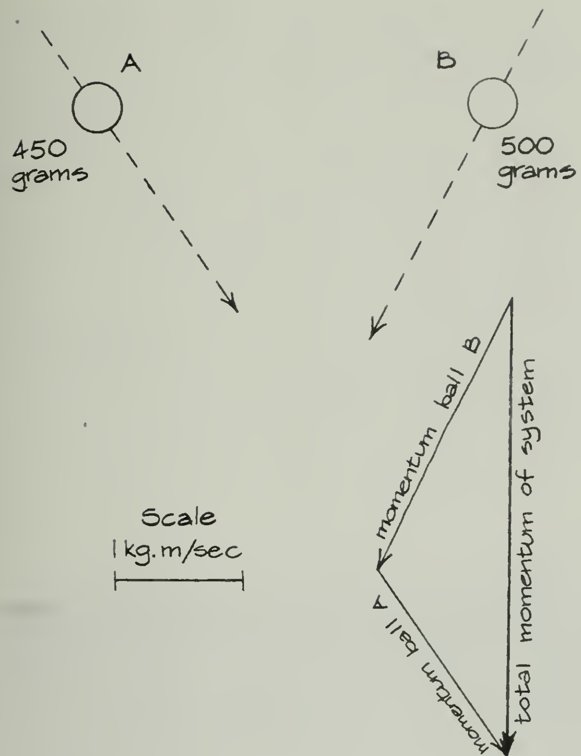


Fig. 2 Two balls moving in a plane. Their individual momenta, which are vectors, are added together vectorially in the diagram on the lower right. The vector sum is the total momentum of the system of two balls. (Your own vector drawings should be at least twice this size.)

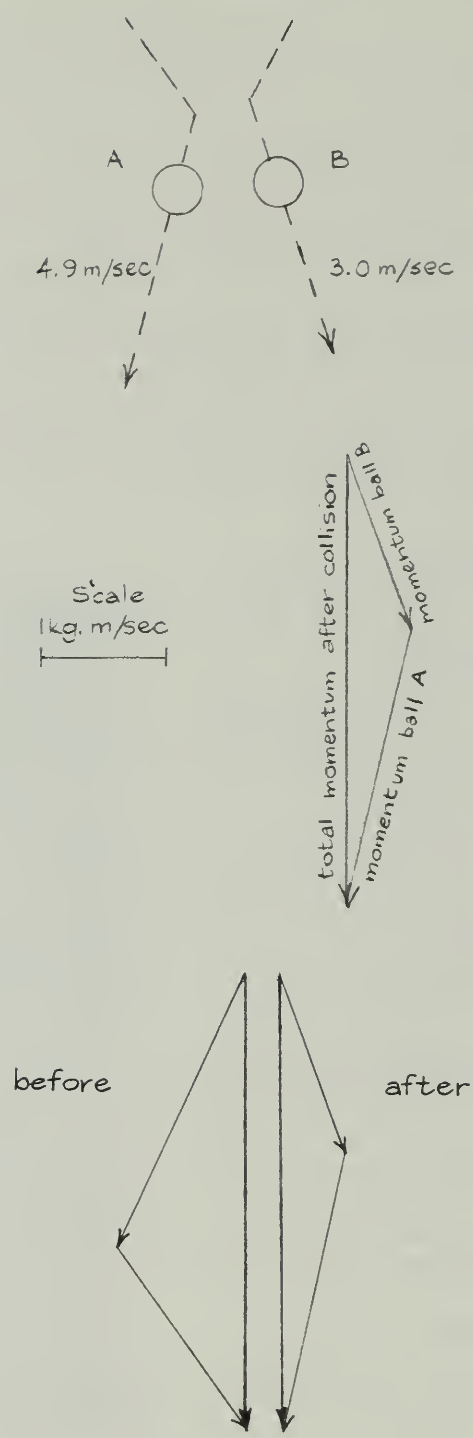


Fig. 3 The two balls collide and move away. Their individual momenta after collision are added vectorially. The resultant vector is the total momentum of the system after collision.

The system of two balls has a total momentum before the collision equal to the vector sum of the two momentum vectors for A and B.

The total momentum after the collision is also found the same way, by adding the momentum vector for A after the collision to that for B after the collision (see Fig. 3).

This same procedure is used for any event you analyze. Determine the momentum (magnitude *and* direction) for each object in the system before the collision, graphically add them, and then do the same thing for each object after the collision.

For each event that you analyze, consider whether momentum is conserved.

Events 8, 9, 10, and 11

Event 8 is also shown as the first example in

Film Loop L22, "Two-Dimensional Collisions: Part II," as well as on Project Physics *Transparency T-20.*

Event 10 is also shown as the second example in *Film Loop* L22.

Event 11 is also shown in *Film Loop* L21, "Two-Dimensional Collisions: Part I," and on Project Physics *Transparency T-21.*

These are all elastic collisions. Events 8 and 10, are simplest to analyze because each shows a collision of equal masses. In Events 8 and 9, one ball is initially at rest.

A small sketch next to each photograph indicates the direction of motion of each ball. The mass of each ball and the strobe rate are also given.

Events 12 and 13

Event 12 is also shown as the first example in *Film Loop* L23, "Inelastic Two-Dimensional Collisions."

Event 13 is also shown as the second example in *Film Loop* L23. A similar event is shown and analyzed in Project Physics *Transparency T-22.*

Since Events 12 and 13 are similar, there is no need to do both.

Events 12 and 13 show inelastic collisions between two plasticene balls that stick together and move off as one compound object after the collision. In 13 the masses are equal; in 12 they are unequal.

Caution: You may find that the two objects rotate slightly about a common center after the collision. For each image after the collision, you should make marks halfway between the centers of the two objects. Then determine the velocity of this "center of mass," and multiply it by the combined mass to get the total momentum after the collision.

Event 14

Do *not* try to analyze Event 14 unless you have done at least one of the simpler events 8 through 13.

Event 14 is also shown on *Film Loop* L24 "Scattering of a Cluster of Objects."

Figure 4 shows the setup used in photographing the scattering of a cluster of balls. The photographer and camera are on the floor, and four electronic stroboscope lights are on tripods in the lower center of the picture.

You are to use the same graphical methods as you used for Events 8 through 13 to see if the conservation of momentum holds for more than two objects. Event 14 is much more complex because you must add seven vectors, rather than two, to get the total momentum after the collision.

In Event 14, one ball comes in and strikes a cluster of six balls of various masses. The balls were initially at rest. Two photographs are included: Print 1 shows only the motion of ball A before the event. Print 2 shows the positions of all seven balls just before the collision and the motion of each of the seven balls after the collision.

You can analyze this event in two different ways. One way is to determine the initial momentum of ball A from measurements taken on Print 1 and then compare it to the total final momentum of the system of seven balls from measurements taken on Print 2. The second method is to determine the total final momentum of the system of seven balls

Fig. 4 Catching the seven scattered balls to avoid tangling in the wires from which they hang. The photographer and the camera are on the floor. The four stroboscopes are seen on tripods in the lower center of the picture.

on Print 2, predict the momentum of ball A, and then take measurements of Print 1 to see whether ball A had the predicted momentum. Choose one method.

The tops of prints 1 and 2 lie in identical positions. To relate measurements on one print to measurements on the other, measure a ball's distance relative to the top of one picture with a rule; the ball would lie in precisely the same position in the other picture if the two pictures could be superimposed.

There are two other matters you must consider. First, the time scales are different on the two prints. Print 1 was taken at a rate of 5 flashes/second, and Print 2 was taken at a rate of 20 flashes/second. Second, the distance scale may not be exactly the same for both prints. Remember that the distance from the center of the tip of one of the white bars to center of the tip of the other is 1 meter (± 2 mm) in real space. Check this scale carefully on both prints to determine the conversion factor.

The stroboscopic photographs for Events 8 to 14 appear in Figs. 5 to 12.

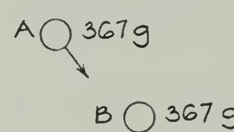


Fig. 5 Event 8, 20 flashes/sec

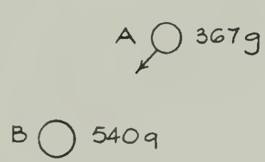
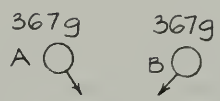


Fig. 6 Event 9, 20 flashes/sec



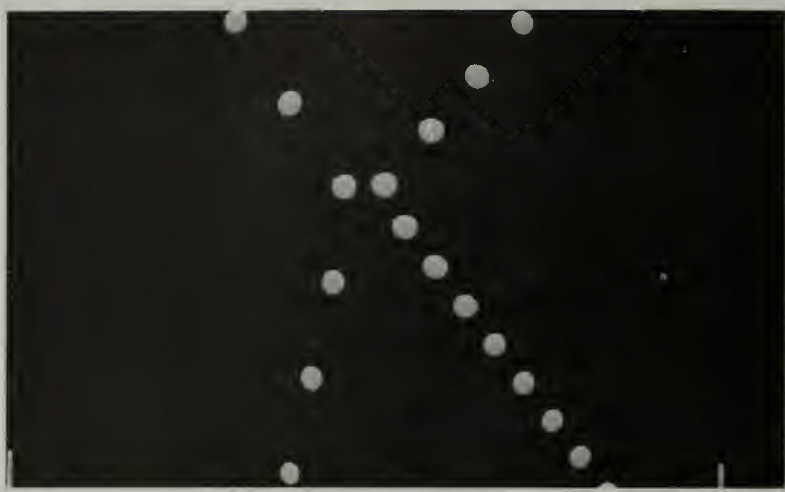


Fig. 7 Event 10, 20 flashes/sec

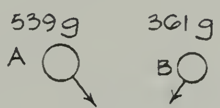


Fig. 8 Event 11, 20 flashes/sec

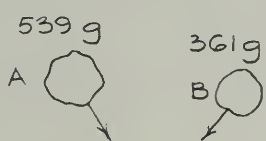


Fig. 9 Event 12, 20 flashes/sec

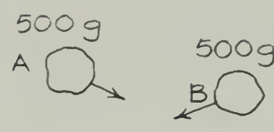
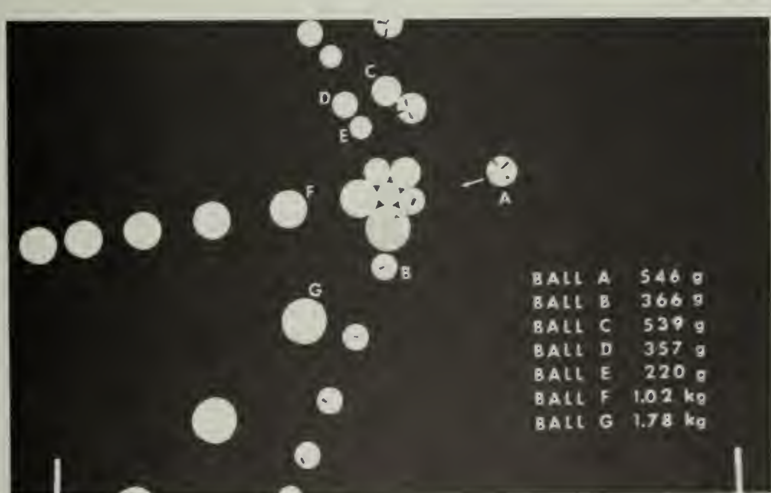


Fig. 10 Event 13, 10 flashes/sec

Fig. 11 Event 14, print 1, 5 flashes/sec



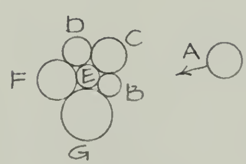


Fig. 12 Event 14, print 2, 20 flashes/sec

EXPERIMENT 3-5 CONSERVATION OF ENERGY—I

In the previous experiments on conservation of momentum, you recorded the results of a number of collisions involving carts and gliders having different initial velocities. You found that within the limits of experimental uncertainty, momentum was conserved in each case. You can now use the results of these collisions to learn about another extremely useful conservation law, the conservation of energy.

Do you have any reason to believe that the product of m and v is the only conserved quantity? In the data obtained from your photographs, look for other combinations of quantities that might be conserved. Find values for m/v, m^2v, and mv^2 for each cart before and after collision, to see if the sum of these quantities for both carts is conserved. Compare the results of the elastic collisions with the inelastic ones. Consider the "explosion" too.

Is there a quantity that is conserved for one type of collision but not for the other?

There are several alternative methods to explore further the answer to this question; you will probably wish to do just one. Check your results against those of classmates who use other methods.

Method A—Dynamics Carts

To take a closer look at the details of an elastic collision, photograph two dynamics carts as you may have done in the previous experiment. Set the carts up as shown in Fig. 1.

The mass of each cart is 1 kg. Extra mass is added to make the total masses 2 kg and 4 kg. Tape a light source on each cart. So that you can distinguish between the images formed by the two lights, make sure that one of the bulbs is slightly higher than the other.

Place the 2-kg cart at the center of the table and push the other cart toward it from the left. If you use the 12-slot disk on the stroboscope, you should get several images during the time that the spring bumpers are touching. You will need to know which image of the right-hand cart was made at the same instant as a given image of the left-hand cart. Matching images will be easier if one of the twelve slots on the stroboscope disk should be slightly more than half-covered with tape. (Fig. 2.) Images formed when that slot is in front of the lens will be fainter than the others.

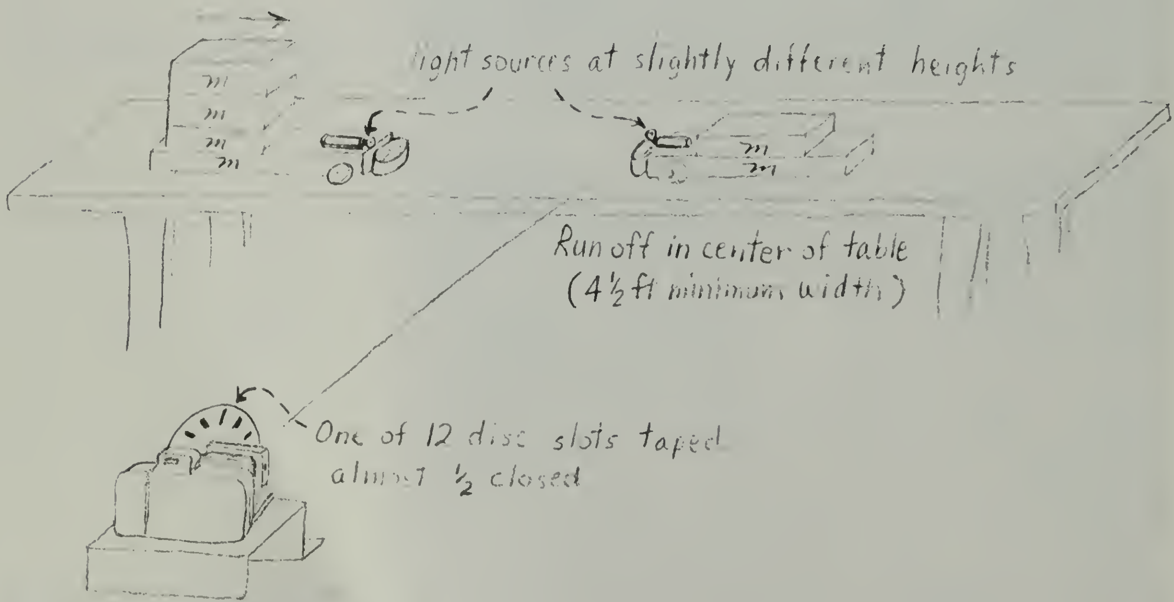


Fig. 1

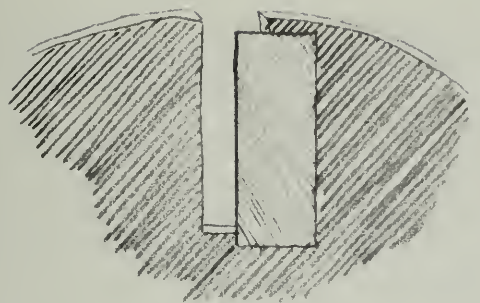


Fig. 2

Compute the values for the momentum, (mv), for each cart for each time interval while the springs were touching, plus at least three intervals before and after the springs touched. List the values in a table, making sure that you pair off the values for the two carts correctly. Remember that the lighter cart was initially at rest while the heavier one moved toward it. This means that the first few values of mv for the lighter cart will be zero.

On a sheet of graph paper, plot the momentum of each cart as a function of time, using the same coordinate axes for both. Connect each set of values with a smooth curve.

Now draw a third curve which shows the sum of the two values of mv, the total momentum of the system for each time interval.

> 1. Compare the final value of mv for the system with the initial value. Was momentum conserved in the collision?
> 2. What happened to the momentum of the system while the springs were touching —was momentum conserved *during* the collision?

Now compute values for the scalar quantity mv^2 for each cart for each time interval, and add them to your table. On another sheet of graph paper, plot the values of mv^2 for each cart for each time interval. Connect each set of values with a smooth curve.

Now draw a third curve which shows the sum of the two values of mv^2 for each time interval.

> 3. Compare the final value of mv^2 for the system with the initial value. Is mv^2 a conserved quantity?

> 4. How would the appearance of your graph change if you multiplied each quantity by $\frac{1}{2}$? (The quantity $\frac{1}{2}mv^2$ is called the *kinetic energy* of the object of mass m and speed v.)

Compute values for the scalar quantity $\frac{1}{2}mv^2$ for each cart for each time interval. On a sheet of graph paper, plot the kinetic energy of each cart as a function of time, using the same coordinate axes for both.

Now draw a third curve which shows the sum of the two values of $\frac{1}{2}mv^2$, for each time interval.

> 5. Does the total amount of kinetic energy vary during the collision? If you found a change in the total kinetic energy, how do you explain it?

Method B—Magnets

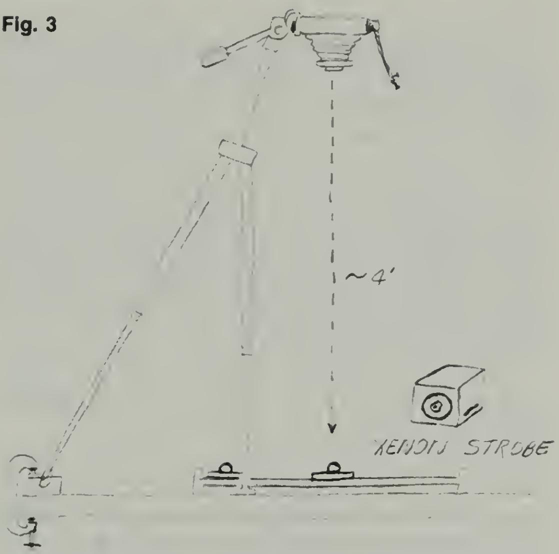
Spread some Dylite spheres (tiny plastic beads) on a glass tray or other hard, flat surface. A disc magnet will slide freely on this low-friction surface. Level the surface carefully.

Put one magnet puck at the center and push a second one toward it, slightly off center. You want the magnets to repel each other without actually touching. Try varying the speed and direction of the pushed magnet until you find conditions that make both magnets move off after the collision with about equal speeds.

To record the interaction, set up a camera directly above the glass tray (using the motor-strobe mount if your camera does not attach directly to the tripod) and a xenon stroboscope to one side as in Fig. 3. Mount a steel ball or Styrofoam hemisphere on the center of each disk with a small piece of clay. The ball will give a sharp reflection of the strobe light.

Take strobe photographs of several interactions. There must be several images before and after the interaction, but you can vary the initial speed and direction of the moving magnet, to get a variety of interactions. Using your photograph, calculate the "before" and "after" speeds of each disk. Since you are interested only in comparing speeds, you can use any convenient units for speed.

Fig. 3



> 1. Is mv^2 a conserved quantity? Is $\frac{1}{2}mv^2$ a conserved quantity?

If you find there has been a decrease in the total kinetic energy of the system of interacting magnets, consider the following: the surface is not perfectly frictionless and a single magnet disk pushed across it will slow down a bit. Make a plot of $\frac{1}{2}mv^2$ against time for a moving puck to estimate the rate at which kinetic energy is lost in this way.

> 2. How much of the loss in $\frac{1}{2}mv^2$ that you observed in the interaction can be due to friction?
>
> 3. What happens to your results if you consider kinetic energy to be a *vector* quantity?

When the two disks are close together (but not touching) there is quite a strong force between them pushing them apart. If you put the two pucks down on the surface close together and release them, they will fly apart: the kinetic energy of the system has increased.

If you have time to go on, you should try to find out what happens to the total quantity $\frac{1}{2}mv^2$ of the disks while they are close together during the interaction. To do this you will need to work at a fairly high strobe rate, and push the projectile magnet at fairly high speed —without letting the two magnets actually touch, of course. Close the camera shutter before the disks are out of the field of view, so that you can match images by counting backward from the last images.

Now, working backward from the last interval, measure v and calculate $\frac{1}{2}mv^2$ for each puck. Make a graph in which you plot $\frac{1}{2}mv^2$ for each puck against time. Draw smooth curves through the two plots.

Now draw a third curve which shows the sum of the two $\frac{1}{2}mv^2$ values for each time interval.

> 4. Is the quantity $\frac{1}{2}mv^2$ conserved *during* the interaction, that is, while the repelling magnets approach very closely?
>
> Try to explain your observations.

Method C—Inclined Air Tracks

Suppose you give the glider a push at the bottom of an inclined air track. As it moves up the slope it slows down, stops momentarily, and then begins to come back down the track.

Clearly the bigger the push you give the glider (the greater its initial velocity v_i), the higher up the track it will climb before stopping. From experience you know that there is some connection between v_i and d, the distance the glider moves along the track.

According to physics texts, when a stone is thrown upward, the kinetic energy that it has initially ($\frac{1}{2}mv_i^2$) is transformed into gravitational potential energy (ma_gh) as the stone moves up. In this experiment, you will test to see whether the same relationship applies to the behavior of the glider on the inclined air track. In particular, your task is to find the initial kinetic energy and the increase in potential energy of the air track glider and to compare them.

The purpose of the first set of measurements is to find the initial kinetic energy $\frac{1}{2}mv_i^2$. You cannot measure v_i directly, but you can find it from your calculation of the *average* velocity v_{av} as follows. In the case of uniform acceleration $v_{av} = \frac{1}{2}(v_i + v_f)$, and since $v_f = 0$ at the top of the track, $v_{av} = \frac{1}{2}v_i$ or $v_i = 2v_{av}$. Remember that $v_{av} = \Delta d/\Delta t$, so $v_i = 2\Delta d/\Delta t$; Δd and Δt are easy to measure with your apparatus.

To measure Δd and Δt three people are needed: one gives the glider the initial push, another marks the highest point on the track that the glider reaches, and the third uses a stopwatch to time the motion from push to rest.

Raise one end of the track a few centimeters above the tabletop. The launcher should practice pushing until he can reproduce a push that will send the glider nearly to the raised end of the track.

Record the distance traveled and time taken for several trials, and weigh the glider. Determine and record the initial kinetic energy.

To calculate the increase in gravitational potential energy, you must measure the vertical height h through which the glider moves for each push. You will probably find that you need to measure from the tabletop to the track

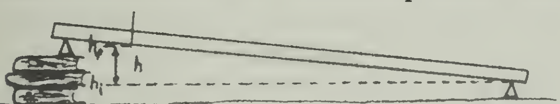


Fig. 4

at the initial and final points of the glider's motion (see Fig. 4), since $h = h_f - h_i$. Calculate the potential energy increase, the quantity ma_gh for each of your trials.

For each trial, compare the kinetic energy loss with the potential energy increase. Be sure that you use consistent units: m in kilograms, v in meters/second, a_g in meters/second^2, h in meters.

> 1. Are the kinetic energy loss and the potential energy increase equal within your experimental uncertainty?
> 2. Explain the significance of your result.

Here are more things to do if you have time to go on:

(a) See if your answer to 1 continues to be true as you make the track steeper and steeper.

(b) When the glider rebounds from the rubber band at the bottom of the track it is momentarily stationary—its kinetic energy is zero. The same is true of its gravitational potential energy, if you use the bottom of the track as the zero level. And yet the glider will rebound from the rubber band (regain its kinetic energy) and go quite a way up the track (gaining gravitational potential energy) before it stops. See if you can explain what happens at the rebound in terms of the conservation of mechanical energy.

(c) The glider does not get quite so far up the track on the second rebound as it did on the first. There is evidently a loss of energy. See if you can measure how much energy is lost each time.

EXPERIMENT 3-6 CONSERVATION OF ENERGY—II

Method A—Film Loops

You may have used one or more of *Film Loops* L18 through L25 in your study of momentum. You will find it helpful to view these slow-motion films of one and two-dimensional collisions again, but this time in the context of the study of energy. The data you collected previously will be sufficient for you to calculate the kinetic energy of each ball before and after the collision. Remember that kinetic energy $\frac{1}{2}mv^2$ is *not* a vector quantity, and hence, you need only use the magnitude of the velocities in your calculations.

On the basis of your analysis you may wish to try to answer such questions as these: Is kinetic energy consumed in such interactions? If not, what happened to it? Is the loss in kinetic energy related to such factors as relative speed, angle of impact, or relative masses of the colliding balls? Is there a difference in the kinetic energy lost in elastic and inelastic collisions?

The film loops were made in a highly controlled laboratory situation. After you have developed the technique of measurement and analysis from film loops, you may want to turn to one or more loops dealing with things outside the laboratory setting. Film Loops L26 through L33 involve freight cars, billiard balls, pole vaulters, and the like. Suggestions for using these loops can be found on pages 81-87.

Method B—Stroboscopic Photographs of Collisions

When studying momentum, you may have

taken measurements on the one-dimensional and two-dimensional collisions shown in stroboscopic photographs on pages 9-12 and 19-21. If so, you can now easily re-examine your data and compute the kinetic energy $\frac{1}{2}mv^2$ for each ball before and after the interaction. Remember that kinetic energy is a scalar quantity, and so you will use the magnitude of the velocity but not the direction in making your computations. You would do well to study one or more of the simpler events (for example, Events 1, 2, 3, 8, 9, or 10) before attempting the more complex ones involving inelastic collisions or several balls. Also you may wish to review the discussions given earlier for each event.

If you find there is a loss of kinetic energy beyond what you would expect from measurement error, try to explain your results. Some questions you might try to answer are these: How does kinetic energy change as a function of the distance from impact? Is it the same before and after impact? How is energy conservation influenced by the relative speed at the time of collision? How is energy conservation influenced by the angle of impact? Is there a difference between elastic and inelastic interactions in the fraction of energy conserved?

EXPERIMENT 3-7 MEASURING THE SPEED OF A BULLET

In this experiment you will use the principle of the conservation of momentum to find the speed of a bullet. Sections 9.2 and 9.3 in the *Text* discuss collisions and define momentum.

You will use the general equation of the principle of conservation of momentum for two-body collisions: $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$.

The experiment consists of firing a projectile into a can packed with cotton or a heavy block that is free to move horizontally. Since all velocities before and after the collision are in the same direction, you may neglect the vector nature of the equation above and work only with speeds. To avoid subscripts, call the mass of the target M and the much smaller mass of the projectile m. Before impact the target is at rest, so you have only the speed v of the projectile to consider. After impact both target and embedded projectile move with a common speed v'. Thus the general equation becomes

$$mv = (M + m)v'$$

or

$$v = \frac{(M + m)v'}{m}$$

Both masses are easy to measure. Therefore, if the comparatively slow speed v' can be found after impact, you can compute the high speed v of the projectile before impact. There are at least two ways to find v'.

Method A—Air Track

The most direct way to find v' is to mount the target on the air track and to time its motion after the impact. (See Fig. 1.) Mount a small can, lightly packed with cotton, on an air-track glider. Make sure that the glider will still ride freely with this extra load. Fire a "bullet" (a pellet from a toy gun that has been checked for safety by your instructor) horizontally, parallel to the length of the air track. If M is large enough, compared to m, the glider's speed will be low enough so that you can use a stopwatch to time it over a meter distance. Repeat the measurement a few times until you get consistent results.

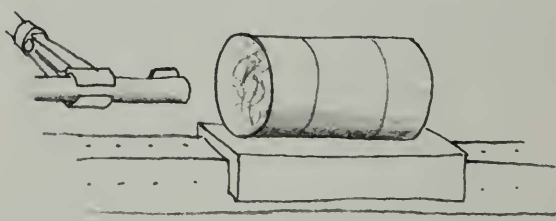


Fig. 1

1. What is your value for the bullet's speed?

2. Suppose the collision between bullet and can was not completely inelastic, so that the bullet bounced back a little after impact. Would this increase or decrease your value for the speed of the bullet?

3. Can you think of an independent way

to measure the speed of the bullet? If you can, go on and make the independent measurement. Then see if you can account for any differences between the two results.

Method B—Ballistic Pendulum

This was the original method of determining the speed of bullets, invented in 1742 and is still used in some ordnance laboratories. A movable block is suspended as a freely swinging pendulum whose motion reveals the bullet's speed.

Obtaining the Speed Equation

The collision is inelastic, so kinetic energy is not conserved in the impact. But during the nearly frictionless swing of the pendulum after the impact, mechanical energy is conserved —that is, the increase in gravitational potential energy of the pendulum at the end of its upward swing is equal to its kinetic energy immediately after impact. Written as an equation, this becomes

$$(M + m)a_g h = \frac{(M + m)v'^2}{2}$$

where h is the increase in height of the pendulum bob.

Solving this equation for v' gives:

$$v' = \sqrt{2a_g h}$$

Substituting this expression for v' in the momentum equation above leads to

$$v' = \left(\frac{M + m}{m}\right)\sqrt{2a_g h}$$

Now you have an equation for the speed v of the bullet in terms of quantities that are known or can be measured.

A Useful Approximation

The change h in vertical height is hard to measure accurately, but the horizontal displacement d may be 10 centimeters or more and can be found easily. Therefore, let's see if h can be replaced by an equivalent expression involving d. The relation between h and d can be found by using a little plane geometry.

In Fig. 2, the center of the circle, O, represents the point from which the pendulum is hung. The length of the cords is l.

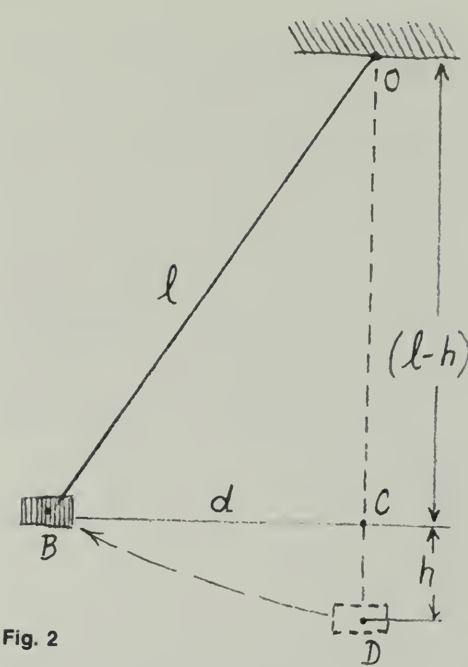


Fig. 2

In the triangle OBC,

$$l^2 = d^2 + (l - h)^2$$

so $$l^2 = d^2 + l^2 - 2lh + h^2$$

and $$2lh = d^2 + h^2$$

For small swings, h is small compared with l and d, so you may neglect h^2 in comparison with d^2, and write the close approximation

$$2lh = d^2$$

or $$h = d^2/2l$$

Putting this value of h into your last equation for v above and simplifying gives:

$$v = \frac{(M + m)d}{m}\sqrt{\frac{a_g}{l}}$$

If the mass of the projectile is small compared with that of the pendulum, this equation can be simplified to another good approximation. How?

Finding the Projectile's Speed

Now you are ready to turn to the experiment. The kind of pendulum you use will depend on the nature and speed of the projectile. If you use pellets from a toy gun, a cylindrical cardboard carton stuffed lightly with cotton and suspended by threads from a laboratory stand will do. If you use a good bow and arrow, stuff

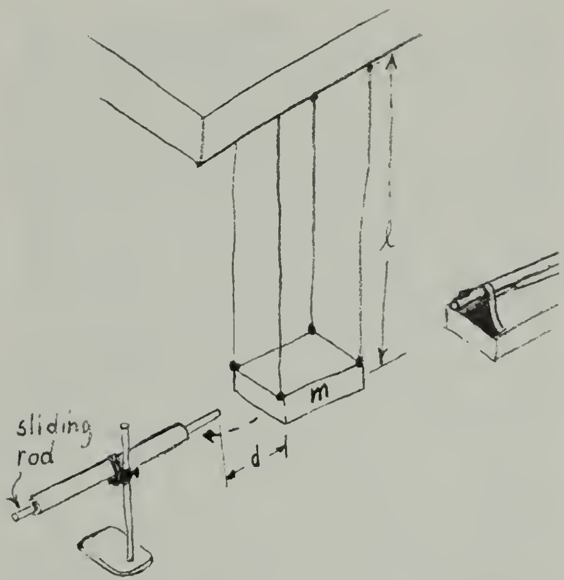


Fig. 3

straw into a fairly stiff corrugated box and hang it from the ceiling. To prevent the target pendulum from twisting, hang it by parallel cords connecting four points on the pendulum to four points directly above them, as in Fig. 3.

To measure d, a light rod (a pencil or a soda straw) is placed in a tube clamped to a stand. The rod extends out of the tube on the side toward the pendulum. As the pendulum swings, it shoves the rod back into the tube so that the rod's final position marks the end of the swing of the pendulum. Of course the pendulum must not hit the tube and there must be sufficient friction between rod and tube so that the rod stops when the pendulum stops. The original rest position of the pendulum is readily found so that the displacement d can be measured.

Repeat the experiment a few times to get an idea of how precise your value for d is. Then substitute your data in the equation for v, the bullet's speed.

1. What is your value for the bullet's speed?

2. From your results, compare the kinetic energy of the bullet before impact with that of the pendulum after impact. Why is there such a large difference in kinetic energy?

3. Can you describe an independent method for finding v? If you have time, try it, and explain any difference between the two v values.

EXPERIMENT 3-8 ENERGY ANALYSIS OF A PENDULUM SWING

According to the law of conservation of energy, the loss in gravitational potential energy of a simple pendulum as it swings from the top of its swing to the bottom is completely transferred into kinetic energy at the bottom of the swing. You can check this with the following photographic method. A one-meter simple pendulum (measured from the support to the *center* of the bob) with a 0.5 kg bob works well. Release the pendulum from a position where it is 10 cm higher than at the bottom of its swing.

To simplify the calculations, set up the camera for 10:1 scale reduction. Two different strobe arrangements have proved successful: (1) tape an AC blinky to the bob, or (2) attach an AA cell and bulb to the bob and use a motor-strobe disk in front of the camera lens. In either case you may need to use a two-string suspension to prevent the pendulum bob from spinning while swinging. Make a time exposure for one swing of the pendulum.

Fig. 1

You can either measure directly from your print (which should look something like the one in Fig. 1), or make pinholes at the center of each image on the photograph and project the hole images onto a larger sheet of paper. Calculate the instantaneous speed v at the bottom of the swing by dividing the distance traveled between the images nearest the bottom of the swing by the time interval between the images. The kinetic energy at the bottom of the swing $\frac{1}{2}mv^2$, should equal the change in potential energy from the top of the swing to the bottom. If Δh is the difference in vertical height between the bottom of the swing and the top, then

$$v = \sqrt{2a_g\Delta h}$$

If you plot both the kinetic and potential energy on the same graph (using the bottommost point as a zero level for gravitational potential energy), and then plot the sum of $KE + PE$, you can check whether total energy is conserved during the entire swing.

EXPERIMENT 3-9 LEAST ENERGY

Concepts such as momentum, kinetic energy, potential energy, and the conservation laws often turn out to be unexpectedly helpful in helping us to understand what at first glance seem to be unrelated phenomena. This experiment offers just one such case in point: How can we explain the observation that if a chain is allowed to hang freely from its two ends, it always assumes the same shape? Hang a three-foot length of beaded chain, the type used on light sockets, from two points as shown in Fig. 1. What shape does the chain assume? At first glance it seems to be a parabola.

Fig. 1

Check whether it is a parabola by finding the equation for the parabola which would go through the vertex and the two fixed points. Determine other points on the parabola by using the equation. Plot them and see whether they match the shape of the chain.

The following example may help you plot the parabola. The vertex in Fig. 1 is at (0,0) and the two fixed points are at (−8, 14.5) and (8, 14.5). All parabolas symmetric to the y axis have the formula $y = kx^2$, where k is a constant. For this example you must have $14.5 = k(8)^2$, or $14.5 = 64k$. Therefore, $k = 0.227$, and the equation for the parabola going through the given vertex and two points is $y = 0.227x^2$. By substituting values for x, we calculated a table of x and y values for our parabola and plotted it.

A more interesting question is why the chain assumes this particular shape, which is called a catenary curve. You will recall that the gravitational potential energy of a body mass m is defined as ma_gh, where a_g is the acceleration due to gravity, and h the height of the body above the reference level chosen. Remember that only a *difference* in energy level is meaningful; a different reference level only adds a constant to each value associated with the original reference level. In theory, you could measure the mass of one bead on the chain, measure the height of each bead above the reference level, and total the potential energies for all the beads to get the total potential energy for the whole chain.

In practice that would be quite tedious, so you will use an approximation that will still allow you to get a reasonably good result. (This would be an excellent computer problem.) Draw vertical parallel lines about 1-inch apart on the paper behind the chain (or use graph paper). In each vertical section, make a mark beside the chain in that section (see Fig. 2 on the following page.)

Fig. 2

The total potential energy for that section of the chain will be approximately $Ma_g h_{av}$, where h_{av} is the average height which you marked, and M is the total mass in that section of chain. Notice that near the ends of the chain there are more beads in one horizontal interval than there are near the center of the chain. To simplify the solution further, assume that M is always an integral number of beads which you can count.

In summary, for each interval multiply the number of beads by the average height for that interval. Total all these products. This total is a good approximation to the gravitational potential energy of the chain.

After doing this for the freely hanging chain, pull the chain with thumbtacks into such different shapes as those shown in Fig. 3. Calculate the total potential energy for each

Fig. 3

shape. Does the catenary curve (the freely-formed shape) or one of these others have the minimum total potential energy?

If you would like to explore other instances of the minimization principle, we suggest the following:

1. When various shapes of wire are dipped into a soap solution, the resulting film always forms so that the total surface area of the film is a minimum. For this minimum surface, the total potential energy due to surface tension is a minimum. In many cases the resulting surface is not at all what you would expect. An excellent source of suggested experiments with soap bubbles, and recipes for good solutions, is the paperback *Soap Bubbles and the Forces that Mould Them*, by C. V. Boys, Doubleday Anchor Books. Also see "The Strange World of Surface Film," *The Physics Teacher*, Sept., 1966.
2. Rivers meander in such a way that the work done by the river is a minimum. For an explanation of this, see "A Meandering River," in the June 1966 issue of *Scientific American*.
3. Suppose that points A and B are placed in a vertical plane as shown in Fig. 4, and you want to build a track between the two points so that a ball will roll from A to B in the least possible

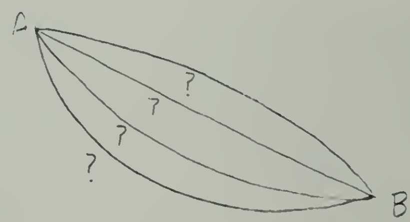


Fig. 4

time. Should the track be straight or in the shape of a circle, parabola, cycloid, catenary, or some other shape? An interesting property of a cycloid is that no matter where on a cycloidal track you release a ball, it will take the same amount of time to reach the bottom of the track. You may want to build a cycloidal track in order to check this. Don't make the track so steep that the ball slips instead of rolling.

A more complete treatment of "The Principle of Least Action" is given in the *Feynman Lectures on Physics, Vol. II, p. 19-1.*

EXPERIMENT 3-10 TEMPERATURE AND THERMOMETERS

You can usually tell just by touch which of two similar bodies is the hotter. But if you want to tell exactly *how* hot something is or to communicate such information to somebody else, you have to find some way of assigning a *number* to "hotness." This number is called temperature, and the instrument used to get this number is called a thermometer.

It's not difficult to think of standard units for measuring intervals of time and distance —the day and the foot are both familiar to us. But try to imagine yourself living in an era before the invention of thermometers and temperature scales, that is, before the time of Galileo. How would you describe, and if possible give a number to, the "degree of hotness" of an object?

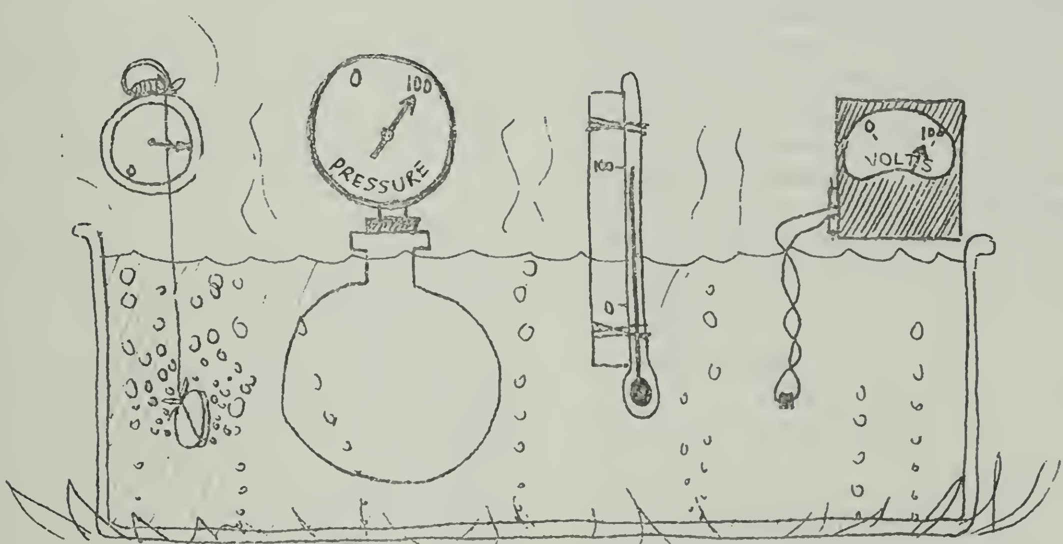
Any property (such as length, volume, density, pressure, or electrical resistance) that changes with hotness and that can be measured could be used as an indication of temperature; and any device that measures this property could be used as a thermometer.

In this experiment you will be using thermometers based on properties of liquid-expansion, gas-expansion, and electrical resistance. (Other common kinds of thermometers are based on electrical voltages, color, or gas-pressure.) Each of these devices has its own particular merits which make it suitable, from a practical point of view, for some applications, and difficult or impossible to use in others.

Of course it's most important that readings given by two different types of thermometers agree. In this experiment you will make your own thermometers, put temperature scales on them, and then compare them to see how well they agree with each other.

Defining a Temperature Scale

How do you make a thermometer? First, you decide what *property* (length, volume, etc.) of what *substance* (mercury, air, etc.) to use in your thermometer. Then you must decide on two fixed points in order to arrive at the size of a degree. A fixed point is based on a physical phenomenon that always occurs at the same degree of hotness. Two convenient fixed points to use are the melting point of ice and the boiling point of water. On the Celsius (centigrade) system they are assigned



Any quantity that varies with hotness can be used to establish a temperature scale (even the time it takes for an alka seltzer tablet to dissolve in water!). Two "fixed points" (such as the freezing and boiling points of water) are needed to define the size of a degree.

the values 0°C and 100°C at ordinary atmospheric pressure.

When you are making a thermometer of any sort, you have to put a scale on it against which you can read the hotness-sensitive quantity. Often a piece of centimeter-marked tape or a short piece of ruler will do. Submit your thermometer to two fixed points of hotness (for example, a bath of boiling water and a bath of ice water) and mark the positions on the indicator.

The length of the column can now be used to define a temperature scale by saying that equal temperature changes cause equal changes along the scale between the two fixed-point positions. Suppose you marked the length of a column of liquid at the freezing point and again at the boiling point of water. You can now divide the total increase in length into equal parts and call each of these parts "one degree" change in temperature.

On the Celsius scale the degree is 1/100 of the temperature range between the boiling and freezing points of water.

To identify temperatures between the fixed points on a thermometer scale, mark off the actual distance between the two fixed points on the vertical axis of a graph and equal intervals for degrees of temperature on the horizontal axis, as in Fig. 1. Then plot the fixed points (x) on the graph and draw a straight line between them.

Now, the temperature on this scale, corresponding to any intermediate position l, can be read off the graph.

Other properties and other substances can

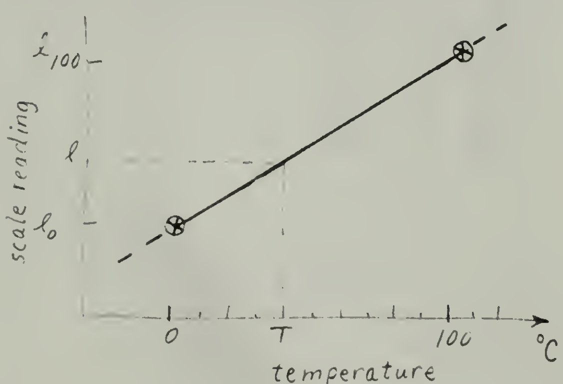


Fig. 1

be used (the volume of different gases, the electrical resistance of different metals, and so on), and the temperature scale defined in the same way. All such thermometers will have to agree at the two fixed points—but do they agree at intermediate temperatures?

If different physical properties do not change in the same way with hotness, then the temperature values you read from thermometers using these properties will not agree. Do similar temperature scales defined by different physical properties agree anywhere besides at the fixed points? That is a question that you can answer from this lab experience.

Comparing Thermometers

You will make or be given two "thermometers" to compare. Take readings of the appropriate quantity—length of liquid column, volume of gas, electrical resistance, thermocouple voltage, or whatever—when the devices are placed in an ice bath, and again when they are placed in a boiling water bath. Record these values. Define these two temperatures as 0° and 100° and draw the straight-line graphs that define intermediate temperatures as described above.

Now put your two thermometers in a series of baths of water at intermediate temperatures, and again measure and record the length, volume, resistance, etc. for each bath. Put both devices in the bath at the same time in case the bath is cooling down. Use your graphs to read off the temperatures of the water baths as indicated by the two devices.

Do the temperatures measured by the two devices agree?

If the two devices do give the same readings at intermediate temperatures, then you could apparently use either as a thermometer. But if they do not agree, you must choose only one of them as a standard thermometer. Give whatever reasons you can for choosing one rather than the other before reading the following discussion. If possible, compare your results with those of classmates using the same or different kinds of thermometers.

There will of course be some uncertainty in your measurements, and you must decide whether the differences you observe between two thermometers might be due only to this uncertainty.

The relationship between the readings from two different thermometers can be displayed on another graph, where one axis is the reading on one thermometer and the other axis is the reading on the other thermometer. Each bath will give a plot point on this graph. If the points fall along a straight line, then the two thermometer properties must change with both in the same way. If, however, a fairly regularly smooth curve can be drawn through the points, then the two thermometer properties probably depend on hotness in different ways. (Figure 2 shows possible results for two thermometers.)

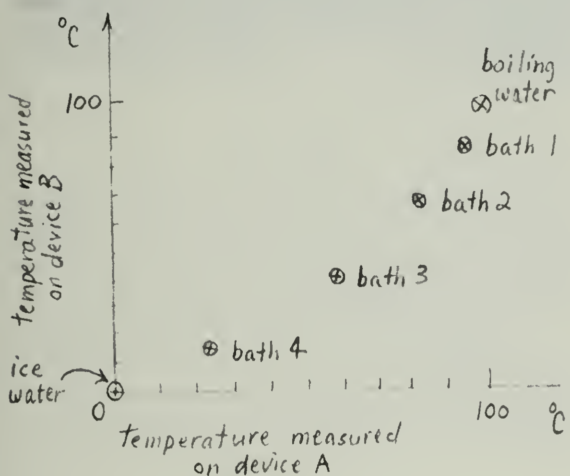


Fig. 2

Discussion

If we compare many gas thermometers—at constant volume as well as pressure, and use different gases, and different initial volumes and pressures—we find that they all behave quantitatively in very much the same way with respect to changes in hotness. If a given hotness change causes a 10% increase in the pressure of gas A, then the same change will also cause a 10% increase in gas B's pressure. Or, if the volume of one gas sample decreases by 20% when transferred to a particular cold bath, then a 20% decrease in volume will also be observed in a sample of any other gas. This means that the temperatures read from different gas thermometers all agree.

This sort of close similarity of behavior between different substances is not found as consistently in the expansion of liquids or solids, or in their other properties—electrical resistance, etc.—and so these thermometers do not agree, as you may have just discovered.

This suggests two things. First, that there is quite a strong case for using the change in pressure (or volume) of a gas to *define* the temperature change. Second, the fact that in such experiments all gases do behave quantitatively in the same way suggests that there may be some underlying simplicity in the behavior of gases not found in liquids and solids, and that if one wants to learn more about the way matter changes with temperature, one would do well to start with gases.

EXPERIMENT 3-11 CALORIMETRY

Speedometers measure speed, voltmeters measure voltage, and accelerometers measure acceleration. In this experiment you will use a device called a calorimeter. As the name suggests, it measures a quantity connected with heat.

Unfortunately heat energy cannot be measured as directly as some of the other quantities mentioned above. In fact, to measure the heat energy absorbed or given off by a substance you must measure the change in temperature of a second substance chosen as a standard. The heat exchange takes place inside a calorimeter, a container in which measured quantities of materials can be mixed together without an appreciable amount of heat being gained from or lost to the outside.

A Preliminary Experiment

The first experiment will give you an idea of how good a calorimeter's insulating ability really is.

Fill a calorimeter cup about half full of ice water. Put the same amount of ice water with one or two ice cubes floating in it in a second cup. In a third cup pour the same amount of water that has been heated to nearly boiling. Measure the temperature of the water in each

cup, and record the temperature and the time of observation.

Repeat the observations at about five-minute intervals throughout the period. Between observations, prepare a sheet of graph paper with coordinate axes so that you can plot temperature as a function of time.

Mixing Hot and Cold Liquids

(You can do this experiment while continuing to take readings of the temperature of the water in your three cups.) You are to make several assumptions about the nature of heat. Then you will use these assumptions to predict what will happen when you mix two samples that are initially at different temperatures. If your prediction is correct, then you can feel some confidence in your assumptions—at least, you can continue to use the assumptions until they lead to a prediction that turns out to be wrong.

First, assume that, in your calorimeter, heat behaves like a fluid that is conserved—that is, it can flow from one substance to another but the total quantity of heat H present in the calorimeter in any given experiment is constant. This implies that heat lost by warm object just equals heat gained by cold object. Or, in symbols

$$-\Delta H_1 = \Delta H_2$$

Next, assume that, if two objects at different temperatures are brought together, heat will flow from the warmer to the cooler object until they reach the same temperature.

Finally, assume that the amount of heat fluid ΔH which enters or leaves an object is proportional to the change in temperature ΔT and to the mass of the object, m. In symbols,

$$\Delta H = cm\Delta T$$

where c is a constant of proportionality that depends on the units—and is different for different substances.

The units in which heat is measured have been defined so that they are convenient for calorimeter experiments. The calorie is defined as the quantity of heat necessary to change the temperature of one gram of water by one Celsius degree. (This definition has to be refined somewhat for very precise work, but it is adequate for your purpose.) In the expression

$$\Delta H = cm\Delta T$$

when m is measured in grams of water and T in Celsius degrees, H will be the number of calories. Because the calorie was defined this way, the proportionality constant c has the value 1 cal/g·C° when water is the only substance in the calorimeter. (The calorie is 1/1000 of the kilocalorie—or Calorie.)

Checking the Assumptions

Measure and record the mass of two empty plastic cups. Then put about $\frac{1}{2}$ cup of cold water in one and about the same amount of hot water in the other, and record the mass and temperature of each. (Don't forget to subtract the mass of the empty cup.) Now mix the two together in one of the cups, stir *gently* with a thermometer, and record the final temperature of the mixture.

Multiply the change in temperature of the cold water by its mass. Do the same for the hot water.

1. What is the product (mass × temperature change) for the cold water?
2. What is this product for the hot water?
3. Are your assumptions confirmed, or is the difference between the two products greater than can be accounted for by uncertainties in your measurement?

Predicting from the Assumptions

Try another mixture using different quantities of water, for example $\frac{1}{4}$ cup of hot water and $\frac{1}{2}$ cup of cold. Before you mix the two, try to predict the final temperature.

4. What do you predict the temperature of the mixture will be?
5. What final temperature do you observe?
6. Estimate the uncertainty of your thermometer readings and your mass measurements. Is this uncertainty enough to account for the difference between your predicted and observed values?
7. Do your results support the assumptions?

Melting

The cups you filled with hot and cold water at the beginning of the period should show a measurable change in temperature by this time. If you are to hold to your assumption of conservation of heat fluid, then it must be that some heat has gone from the hot water into the room and from the room to the cold water.

8. How much has the temperature of the cold water changed?
9. How much has the temperature of the water that had ice in it changed?

The heat that must have gone from the room to the water-ice mixture evidently did not change the temperature of the water as long as the ice was present. But some of the ice melted, so apparently the heat that leaked in was used to melt the ice. Evidently, heat was needed to cause a "change of state" (in this case, to melt ice to water) even if there was no change in temperature. The additional heat required to melt one gram of ice is called "latent heat of melting." Latent means hidden or dormant. The units are cal/g—there is no temperature unit here because no temperature change is involved in latent heat.

Next, you will do an experiment mixing materials other than liquid water in the calorimeter to see if your assumptions about heat as a fluid can still be used. Two such experiments are described below, "Measuring Heat Capacity" and "Measuring Latent Heat." If you have time for only one of them, choose either one. Finally, do "Rate of Cooling" to complete your preliminary experiment.

Measuring Heat Capacity

(While you are doing this experiment, continue to take readings of the temperature of the water in your three test cups.) Measure the mass of a small metal sample. Put just enough cold water in a calorimeter to cover the sample. Tie a thread to the sample and suspend it in a beaker of boiling water. Measure the temperature of the boiling water.

Record the mass and temperature of the water in the calorimeter.

When the sample has been immersed in the boiling water long enough to be heated uniformly (2 or 3 minutes), lift it out and hold it just above the surface for a few seconds to let the water drip off, then transfer it quickly to the calorimeter cup. Stir gently with a thermometer and record the temperature when it reaches a steady value.

> 10. Is the product of mass and temperature change the same for the metal sample and for the water?
> 11. If not, must you modify the assumptions about heat that you made earlier in the experiment?

In the expression $\Delta H = cm\Delta T$, the constant of proportionality c (called the "specific heat capacity") may be different for different materials. For water the constant has the value 1 cal/gC°. You can find a value of c for the metal by using the assumption that heat gained by water equals the heat lost by sample. Or, writing subscripts w and s for water and metal sample, $\Delta H_w = -\Delta H_s$.

Then $$c_w m_w \Delta t_w = -c_s m_s \Delta t_s$$

and $$c_s = \frac{-c_w m_w \Delta t_w}{m_s \Delta t_s}$$

> 12. What is your calculated value for the specific heat capacity c_s for the metal sample you used?

If your assumptions about heat being a fluid are valid, you now ought to be able to predict the final temperature of *any* mixture of water and your material.

Try to verify the usefulness of your value. Predict the final temperature of a mixture of water and a heated piece of your material, using different masses and different initial temperatures.

> 13. Does your result support the fluid model of heat?

Measuring Latent Heat

Use your calorimeter to find the "latent heat of melting" of ice. Start with about ½ cup of water that is a little above room temperature, and record its mass and temperature. Take a small piece of ice from a mixture of ice and water that has been standing for some time; this will assure that the ice is at 0°C and will not have to be warmed up before it can melt. Place the small piece of ice on paper toweling for a moment to dry off water on its surface, and then transfer it quickly to the calorimeter.

Stir gently with a thermometer until the ice is melted and the mixture reaches an equilibrium temperature. Record this temperature and the mass of the water plus melted ice.

> 14. What was the mass of the ice that you added?

The heat given up by the warm water is:

$$\Delta H_w = c_w m_w \Delta t_w$$

The heat gained by the water formed by the melted ice is:

$$H_i = c_w m_i \Delta t_i$$

The specific heat capacity c_w is the same in both cases—the specific heat of water.

The heat given up by the warm water first melts the ice, and then heats the water formed by the melted ice. If we use the symbol ΔH_L for the heat energy required to melt the ice, we can write:

$$-\Delta H_w = \Delta H_L + \Delta H_i$$

So the heat energy needed to melt the ice is

$$\Delta H_L = -\Delta H_w - \Delta H_i$$

The latent heat of melting is the heat energy needed *per gram* of ice, so

$$\text{latent heat of melting} = \frac{\Delta H_L}{m}$$

> 15. What is your value for the latent heat of melting of ice?

When this experiment is done with ice made from distilled water with no inclusions of liquid water, the latent heat is found to be 80 calories per gram of ice. How does your result compare with the accepted value?

Rate of Cooling

If you have been measuring the temperature of the water in your three test cups, you should have enough data by now to plot three curves of temperature against time. Mark the temperature of the air in the room on your graph too.

> 16. How does the rate at which the hot water cools depend on its temperature?
>
> 17. How does the rate at which the cold water heats up depend on its temperature?

Weigh the amount of water in the cups. From the rates of temperature change (degrees/minute) and the masses of water, calculate the rates at which heat leaves or enters the cups at various temperatures. Use this information to estimate the error in your earlier results for latent or specific heat.

EXPERIMENT 3-12 ICE CALORIMETRY

A simple apparatus made up of thermally insulating styrofoam cups can be used for doing some ice calorimetry experiments. Although the apparatus is simple, careful use will give you excellent results. To determine the heat transferred in processes in which heat energy is given off, you will be measuring either the volume of water or the mass of water from a melted sample of ice.

You will need either three cups the same size (8 oz or 6 oz), or two 8 oz and one 6 oz cup. Also have some extra cups ready. One large cup serves as the collector, A, (Fig. 1), the second cup as the ice container, I, and the smaller cup (or one of the same size cut back to fit inside the ice container as shown) as the cover, K.

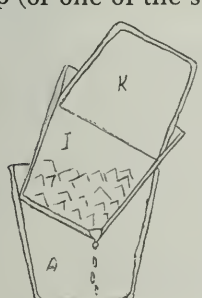


Fig. 1

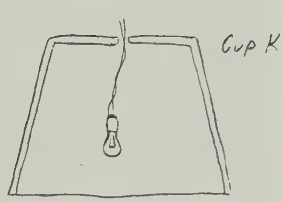


Fig. 2

Cut a hole about ¼-inch in diameter in the bottom of cup I so that melted water can drain out into cup A. To keep the hole from becoming clogged by ice, place a bit of window screening in the bottom of I.

In each experiment, ice is placed in cup I. This ice should be carefully prepared, free of bubbles, and dry, if you want to use the known value of the heat of fusion of ice. However, you can use ordinary crushed ice, and, before doing any of the experiment, determine experimentally the effective heat of melting of this non-ideal ice. (Why should these two values differ?)

In some experiments which require some time to complete (such as Experiment b), you should set up two identical sets of apparatus (same quantity of ice, etc.), except that one does not contain a source of heat. One will serve as a fair measure of the background effect. Measure the amount of water collected in it during the same time, and subtract it from the total amount of water collected in the experimental apparatus, thereby correcting for the amount of ice melted just by the heat leaking in from the room. An efficient method for measuring the amount of water is to place the arrangement on the pan of a balance and lift up cups I and K at regular intervals (about 10 min.) while you weigh A with its contents of melted ice water.

(a) Heat of melting of ice. Fill a cup about ½ to ⅓ full with crushed ice. (Crushed ice has a larger amount of surface area, and so will melt more quickly, thereby minimizing errors due to heat from the room.) Bring a small measured amount of water (say about 20 cc) to a boil in a beaker or large test tube and pour it over the ice in the cup. Stir briefly with a poor heat conductor, such as a glass rod, until equilibrium has been reached. Pour the ice-water mixture through cup I. Collect and measure the final amount of water (m_f) in A. If m_0 is the original mass of hot water at 100°C with which you started, then $m_f - m_0$ is the mass of ice that was melted. The heat energy absorbed by the melting ice is the latent heat of melting for ice, L_i, times the mass of melted ice: $L_i(m_f - m_o)$. This will be equal to the heat energy lost by the boiling water cooling from 100°C to 0°C, so we can write

$$L_i(m_f - m_0) = m_0 \Delta T$$

and $$L_i = \frac{m_0}{m_f - m_0} 100\text{C}^\circ$$

Note: This derivation is correct only if there is still some ice in the cup afterwards. If you start with too little ice, the water will come out at a higher temperature.

For crushed ice which has been standing for some time, the value of L_i will vary between 70 and 75 calories per gram.

(b) Heat exchange and transfer by conduction and radiation. For several possible experiments you will need the following additional apparatus. Make a small hole in the bottom of cup K and thread two wires, soldered to a lightbulb, through the hole. A flashlight bulb which operates with an electric current between 300 and 600 milliamperes is preferable; but even a GE #1130 6-volt automobile headlight bulb (which draws 2.4 amps) has been used with success. (See Fig. 2.) In each experiment, you are to observe how different apparatus affects heat transfer into or out of the system.

1. Place the bulb in the ice and turn it on for 5 minutes. Measure the ice melted.
2. Repeat 1, but place the bulb above the ice for 5 minutes.
3. and 4: Repeat 1 and 2, but cover the inside of cup K with aluminum foil.
5. and 6: Repeat 3 and 4, but in addition cover the inside of cup I with aluminum foil.
7. Prepare "heat absorbing" ice by freezing water to which you have added a small amount of dye, such as India ink. Repeat any or all of experiments 1 through 6 using this "specially prepared" ice.

Some questions to guide your observations: Does any heat escape when the bulb is immersed in the ice? What arrangement keeps in as much heat as possible?

EXPERIMENT 3-13 MONTE CARLO EXPERIMENT ON MOLECULAR COLLISIONS

A model for a gas consisting of a large number of very small particles in rapid random motion has many advantages. One of these is that it makes it possible to estimate the properties of a gas as a whole from the behavior of a comparatively small random sample of its molecules. In this experiment you will not use actual gas particles, but instead employ analogs of molecular collisions. The technique is named the Monte Carlo method after that famous (or infamous) gambling casino in Monaco. The experiment consists of two games, both of which involve the concept of randomness. You will probably have time to play only one.

Game I Collision Probability for a Gas of Marbles

In this part of the experiment, you will try to find the diameter of marbles by rolling a "bombarding marble" into an array of "target marbles" placed at random positions on a level sheet of graph paper. The computation of the marble diameter will be based on the proportion of hits and misses. In order to assure randomness in the motion of the bombarding marble, start at the top of an inclined board studded with nails spaced about an inch apart—a sort of pinball machine (Fig. 1).

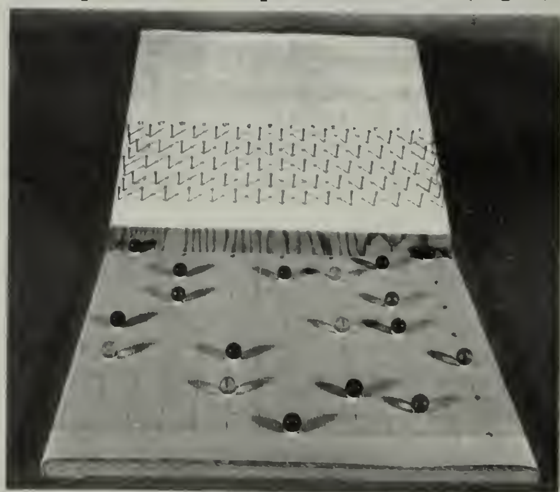


Fig. 1

To get a fairly even, yet random, distribution of the bombarding marble's motion, move its release position over one space for each release in the series.

First you need to place the target marbles at random. Then draw a network of crossed grid lines spaced at least two marble diameters apart on your graph paper. (If you are using marbles whose diameters are half an inch, these grid lines should be spaced 1.5 to 2 inches apart.) Number the grid lines as shown in Fig. 2.

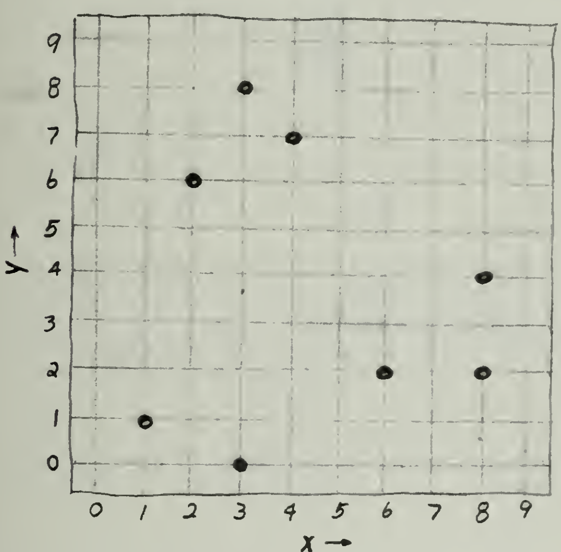


Fig. 2 Eight consecutive two-digit numbers in a table of random numbers were used to place the marbles.

One way of placing the marbles at random is to turn to the table of random numbers at the end of this experiment. Each student should start at a different place in the table and then select the next eight numbers. Use the first two digits of these numbers to locate positions on the grid. The first digit of each number gives the x coordinate, the second gives the y coordinate—or vice versa. Place the target marbles in these positions. Books may be placed along the sides of the graph paper and across the bottom to serve as containing walls.

With your array of marbles in place, make about fifty trials with the bombarding marble. From your record of hits and misses compute R, the ratio between the number of runs in which there are one or more hits to the total number of runs. Remember that you are counting "runs with hits," not hits, and hence, several hits in a single run are still counted as "one."

Inferring the size of the marbles. How does the ratio R lead to the diameter of the target object? The theory applies just as well to determining the size of molecules as it does to marbles, although there would be 10^{20} or so molecules instead of 8 "marble molecules."

If there were no target marbles, the bombarding marble would get a clear view of the full width, say D, of the back wall enclosing the array. There could be no hit. If, however, there were target marbles, the 100% clear view would be cut down. If there were N target marbles, each with diameter d, then the clear path over the width D would be reduced by $N \times d$.

It is assumed that no target marble is hiding behind another. (This corresponds to the assumption that the sizes of molecules are extremely small compared with the distances between them.)

The blocking effect on the bombarding marble is greater than just Nd, however. The bombarding marble will miss a target marble only if its center passes more than a distance of one radius on either side of it. (See Fig. 3 on next page.) This means that a target marble has a blocking effect over twice its diameter (its own diameter plus two radii), so the total blocking effect of N marbles is $2Nd$. Therefore the expected ratio R of hits to total trials is $2Nd/D$ (total blocked width to total width). Thus:

$$R = \frac{2Nd}{D}.$$

which we can rearrange to give an expression for d:

$$d = \frac{RD}{2N}$$

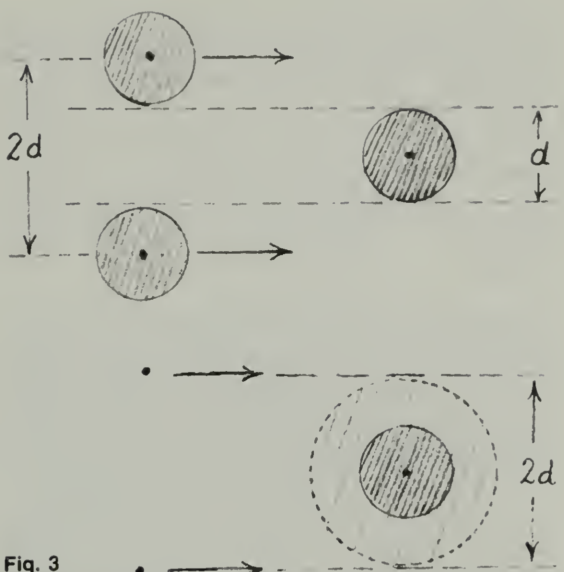


Fig. 3
A projectile will clear a target only if it passes outside a center-to-center distance *d* on either side of it. Therefore, thinking of the projectiles as points, the effective blocking width of the target is 2*d*.

To check the accuracy of the Monte Carlo method compare the value for *d* obtained from the formula above with that obtained by direct measurement of the marbles. (For example, line up your eight marbles against a book. Measure the total length of all of them together and divide by eight to find the diameter *d* of one marble.)

> 1. What value do you calculate for the marble diameter?
> 2. How well does your experimental prediction agree with direct measurement?

Game II Mean Free Path Between Collision Squares

In this part of the experiment you play with blacked-in squares as target molecules in place of marble molecules in a pinball game. On a sheet of graph paper, say 50 units on a side (2,500 squares), you will locate by the Monte Carlo method between 40 and 100 molecules. Each student should choose a different number of molecules.

You will find a table of random numbers (from 0 to 50) at the end of this experiment. Begin anywhere you wish in the table, but then proceed in a regular sequence. Let each pair of numbers be the x and y coordinates of a point on your graph. (If one of the pair is greater than 49, you cannot use it. Ignore it and take the next pair.) Then shade in the squares for which these points are the lower left-hand corners. You now have a random array of square target "molecules."

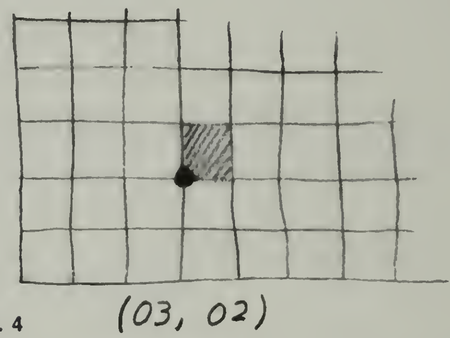


Fig. 4

Rules of the game. The way a bombarding particle passes through this array, it is bound to collide with some of the target particles. There are five rules for this game of collision. All of them are illustrated in Fig. 5.

(a) The particle can travel only along lines of the graph paper, up or down, left or right. They start at some point (chosen at random) on

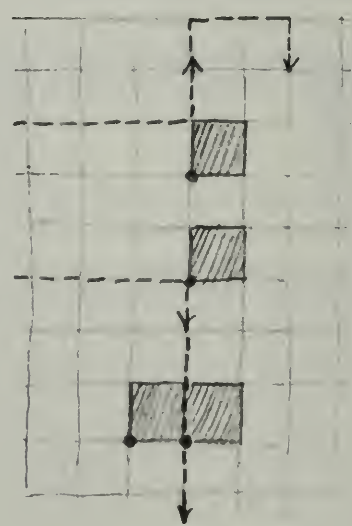


Fig. 5

the left-hand edge of the graph paper. The particle initially moves horizontally from the starting point until it collides with a blackened square or another edge of the graph paper.

(b) If the particle strikes the upper left-hand corner of a target square, it is diverted upward through a right angle. If it should strike a lower left-hand corner it is diverted downward, again through ninety degrees.

(c) When the path of the particle meets an edge or boundary of the graph paper, the particle is *not* reflected directly back. (Such a reversal of path would make the particle retrace its previous paths.) Rather it moves two spaces to its *right* along the boundary edge before reversing its direction.

(d) There is an exception to rule (c). Whenever the particle strikes the edge so near a corner that there isn't room for it to move two spaces to the right without meeting another edge of the graph paper, it moves two spaces to the *left* along the boundary.

(e) Occasionally two target molecules may occupy adjacent squares and the particle may hit touching corners of the two target molecules at the same time. The rule is that this counts as two hits and the particle goes straight through without changing its direction.

Finding the "mean free path." With these collision rules in mind, trace the path of the particle as it bounces about among the random array of target squares. Count the number of collisions with targets. Follow the path of the particle until you get 51 hits with target squares (collisions with the edge do not count). Next, record the 50 lengths of the paths of the particle between collisions. Distances to and from a boundary should be included, but *not* distances *along* a boundary (the two spaces introduced to avoid backtracking). These 50 lengths are the free paths of the particle. Total them and divide by 50 to obtain the mean free path, L, for your random two-dimensional array of square molecules.

In this game your molecule analogs were pure points, i.e., dimensionless. In his investigations Clausius modified this model by giving the particles a definite size. Clausius showed that the average distance L a molecule travels between collisions, the so-called "mean free path," is given by

$$L = \frac{V}{Na}$$

where V is the volume of the gas, N is the number of molecules in that volume, and a is the cross-sectional area of an individual molecule. In this two-dimensional game, the particle was moving over an area A, instead of through a volume V, and was obstructed by targets of width d, instead of cross-sectional area a. A two-dimensional version of Clausius's equation might therefore be:

$$L = \frac{A}{2Nd}$$

where N is the number of blackened square "molecules."

3. What value of L do you get from the data for your runs?

4. Using the two-dimensional version of Clausius's equation, what value do you estimate for d (the width of a square)?

5. How does your calculated value of d compare with the actual value? How do you explain the difference?

TABLE OF 1000 RANDOM TWO-DIGIT NUMBERS
(FROM 0 to 50)

03 47	44 22	30 30	22 00	00 49	22 17	38 30	23 21	20 11	24 33
16 22	36 10	44 39	46 40	24 02	19 36	38 21	45 33	14 23	01 31
33 21	03 29	08 02	20 31	37 07	03 28	47 24	11 29	49 08	10 39
34 29	34 02	43 28	03 43	43 40	26 08	28 06	50 14	21 44	47 21
32 44	11 05	05 05	05 50	23 29	26 00	09 05	27 31	08 43	04 14
18 18	04 02	48 39	48 22	38 18	15 39	48 34	50 28	37 21	15 09
23 42	31 08	19 30	06 00	20 18	30 24	15 33	10 07	14 29	05 24
35 12	11 12	11 04	01 10	25 39	48 50	24 44	03 47	34 04	44 07
12 13	42 10	40 48	45 44	42 35	41 26	41 10	23 05	06 36	08 43
37 35	12 41	02 02	19 11	06 07	42 31	23 47	47 25	10 43	12 38
16 08	18 39	03 31	49 26	07 12	17 31	17 31	35 07	44 38	40 35
31 16	10 47	38 45	28 40	33 34	24 16	42 38	19 09	41 47	50 41
32 43	45 37	30 38	22 01	30 14	02 17	45 18	29 06	13 27	46 24
27 42	03 09	08 32	24 02	05 49	18 05	22 00	23 02	44 43	43 20
00 39	05 03	49 37	23 22	33 42	26 29	00 20	12 03	10 05	02 39
11 27	39 32	13 30	36 45	09 03	46 40	22 07	03 03	05 39	03 46
35 24	22 49	17 33	35 01	01 32	18 09	47 03	39 41	36 23	19 41
16 20	38 36	29 48	07 27	48 14	34 13	07 48	39 12	20 18	19 42
38 23	33 26	15 29	20 02	21 45	04 31	48 13	23 32	37 30	09 24
45 11	27 07	39 43	13 05	47 45	47 45	00 06	41 18	05 02	03 09
18 00	14 21	49 17	30 37	25 15	04 49	24 19	40 23	24 17	17 16
20 46	06 18	45 07	06 28	49 44	10 08	43 00	38 26	34 41	11 16
05 26	50 25	38 47	39 38	42 45	10 08	16 06	43 18	34 48	27 03
21 19	13 42	16 04	00 18	16 46	13 13	16 29	44 10	29 18	22 45
41 23	03 10	35 30	24 36	38 09	25 21	08 40	20 46	39 14	37 31
34 50	20 14	21 46	38 46	12 27	20 44	46 06	01 41	30 49	18 48
39 43	13 04	24 15	08 22	13 29	04 05	42 29	50 47	01 50	01 48
18 14	04 43	27 46	23 07	19 28	07 10	23 19	41 45	25 27	19 10
09 47	34 45	08 45	25 21	49 21	18 46	16 40	35 14	41 28	41 15
44 17	04 33	15 22	12 45	39 07	34 27	14 47	35 33	42 29	47 47
40 33	42 45	07 08	38 15	08 25	22 06	07 26	32 44	03 42	42 34
33 27	10 45	18 40	11 48	48 03	07 16	32 25	20 25	44 22	39 28
06 09	04 26	14 35	36 03	15 22	02 07	46 48	45 12	47 11	30 19
33 32	34 25	45 17	13 26	03 37	33 35	08 13	15 26	09 18	34 25
42 38	40 01	43 31	30 33	39 11	49 41	27 44	11 39	06 19	47 23
15 06	22 08	50 44	50 11	18 16	00 41	07 47	34 25	28 10	50 03
22 35	49 36	44 21	25 12	19 44	31 51	49 18	40 36	00 27	22 12
31 04	32 17	08 23	38 32	01 47	43 53	44 04	10 27	16 00	16 33
39 00	01 50	07 28	35 02	38 00	46 47	33 29	28 41	09 23	47 48
37 32	07 02	07 48	07 41	22 13	37 27	27 12	34 21	07 04	49 34
05 03	36 07	10 15	21 48	14 44	39 39	15 09	23 23	37 31	00 25
17 37	13 41	13 39	40 14	19 48	34 18	08 18	08 06	44 26	12 45
32 24	24 30	29 13	34 39	27 44	11 20	37 40	36 46	35 22	09 09
07 45	29 12	48 35	05 38	43 11	45 18	28 14	04 37	48 38	43 12
14 08	04 04	18 17	10 33	04 32	27 37	33 42	34 41	07 41	49 14
31 38	08 31	38 30	42 10	08 09	17 32	46 15	15 43	15 31	46 45
42 34	46 31	29 03	08 32	11 06	20 21	24 16	13 17	29 34	42 31
16 00	02 48	10 34	32 14	25 39	29 31	18 37	28 50	07 28	08 24
20 15	60 11	21 31	20 49	07 35	41 16	16 17	43 36	20 26	39 38
00 49	14 10	29 01	49 28	21 30	40 15	01 07	16 04	19 09	36 12

EXPERIMENT 3-14 BEHAVIOR OF GASES

Air is elastic or springy. You can feel this when you put your finger over the outlet of a bicycle pump and push down on the pump plunger. You can tell that there is some connection between the volume of the air in the pump and the force you exert in pumping, but the exact relationship is not obvious. About 1660, Robert Boyle performed an experiment that disclosed a very simple relationship between gas pressure and volume, but not until two centuries later was the kinetic theory of gases developed, which accounted for Boyle's law satisfactorily.

The purpose of these experiments is not simply to show that Boyle's Law and Gay Lussac's Law (which relates temperature and volume) are "true." The purpose is also to show some techniques for analyzing data that can lead to such laws.

I. Volume and Pressure

Boyle used a long glass tube in the form of a J to investigate the "spring of the air." The short arm of the J was sealed, and air was trapped in it by pouring mercury into the top of the long arm. (Apparatus for using this method may be available in your school.)

A simpler method requires only a small plastic syringe, calibrated in cc, and mounted so that you can push down the piston by piling weights on it. The volume of the air in the syringe can be read directly from the calibrations on the side. The pressure on the air due to the weights on the piston is equal to the force exerted by the weights divided by the area of the face of the piston:

$$P_w = \frac{F_w}{A}$$

Fig. 1

Because "weights" are usually marked with the value of their *mass*, you will have to compute the force from the relation $F_{grav} = ma_{grav}$. (It will help you to answer this question before going on: What is the weight, in newtons, of a 0.1 kg mass?)

To find the area of the piston, remove it from the syringe. Measure the diameter ($2R$) of the piston face, and compute its area from the familiar formula $A = \pi R^2$.

You will want to both decrease and increase the volume of the air, so insert the piston about halfway down the barrel of the syringe. The piston may tend to stick slightly. Give it a twist to free it and help it come to its equilibrium position. Then record this position.

Add weights to the top of the piston and each time record the equilibrium position, after you have given the piston a twist to help overcome friction.

Record your data in a table with columns for volume, weight, and pressure. Then remove the weights one by one to see if the volumes are the same with the piston coming up as they were going down.

If your apparatus can be turned over so that the weights pull out on the plunger, obtain more readings this way, adding weights to increase the volume. Record these as negative forces. (Stop adding weights before the piston is pulled all the way out of the barrel!) Again remove the weights and record the values on returning.

Interpreting your results. You now have a set of numbers somewhat like the ones Boyle reported for his experiment. One way to look for a relationship between the pressure P_w and the volume V is to plot the data on graph paper, draw a smooth simple curve through the points, and try to find a mathematical expression that would give the same curve when plotted.

Plot volume V (vertical axis) as a function of pressure P_w (horizontal axis). If you are willing to believe that the relationship between P_w and V is fairly simple, then you should try to draw a simple curve. It need not actually go through all the plot points, but should give an overall "best fit."

Since V decreases as P_w increases, you can tell before you plot it that your curve represents an "inverse" relationship. As a first guess at the mathematical description of this curve, try the simplest possibility, that $1/V$ is proportional to P_w. That is, $1/V \propto P_w$. A graph of proportional quantities is a straight line. If $1/V$ *is* proportional to P_w, then a plot of $1/V$ value against P_w will lie on a straight line.

Add another column to your data table for values of $1/V$ and plot this against P_w.

1. Does the curve pass through the origin?

2. If not, at what point does your curve cross the horizontal axis? (In other words, what is the value of P_w for which $1/V$ would be zero?) What is the physical significance of the value of P_w?

In Boyle's time, it was not understood that air is really a mixture of several gases. Do you believe you would find the same relationship between volume and pressure if you tried a variety of pure gases instead of air? If there are other gases available in your laboratory, flush out and refill your apparatus with one of them and try the experiment again.

3. Does the curve you plot have the same shape as the previous one?

II. Volume and Temperature

Boyle suspected that the temperature of his air sample had some influence on its volume, but he did not do a quantitative experiment to find the relationship between volume and temperature. It was not until about 1880, when there were better ways of measuring temperature, that this relationship was established.

You could use several kinds of equipment to investigate the way in which volume changes with temperature. Such a piece of equipment is a glass bulb with a J tube of mercury or the syringe described above. Make sure the gas inside is dry and at atmospheric pressure. Immerse the bulb or syringe in a beaker of cold water and record the volume of gas and temperature of the water (as measured on a suitable thermometer) periodically as you slowly heat the water.

A simpler piece of equipment that will give just as good results can be made from a piece of glass capillary tubing.

Equipment note: assembling a constant-pressure gas thermometer

About 6" of capillary tubing makes a thermometer of convenient size. The dimensions of the tube are not critical, but it is very important that the bore be *dry*. It can be dried by heating, or by rinsing with alcohol and waving it frantically—or better still, by connecting it to a vacuum pump for a few moments.

Filling with air. The dry capillary tube is dipped into a container of mercury, and the end sealed with fingertip as the tube is withdrawn (Fig. 2), so that a pellet of mercury remains in the lower end of the tube.

Fig. 2

Fig. 3

The tube is held at an angle and the end tapped gently on a hard surface until the mercury pellet slides to about the center of the tube (Fig. 3).

One end of the tube is sealed with a dab of silicone sealant; some of the sealant will go up the bore, but this is perfectly all right. The sealant is easily set by immersing it in boiling water for a few moments.

Taking measurements. A scale now must be positioned along the completed tube. The scale will be directly over the bore if a stick is placed as a spacer next to the tube and bound together with rubber bands (Fig. 4). (A long stick makes a convenient handle.) The zero of the scale should be aligned carefully with the end of the gas column, that is, the end of the silicone seal.

Fig. 4

Fig. 5

In use, the thermometer should be completely immersed in whatever one wishes to measure the temperature of, and the end tapped against the side of the container gently to allow the mercury to slide to its final resting place (Fig. 5).

Filling with some other gas. To use some gas other than air, begin by connecting a short length of rubber tubing to a fairly low-pressure supply of gas. As before, trap a pellet of mercury in the end of a capillary tube, but this time do not tap it to the center. Leave it flat so that it will be pushed to the center by the gas pressure (Fig. 6). Open the gas valve slightly for a moment to flush out the rubber tube. With your finger tip closing off the far end of the capillary tube to prevent the mercury being

Fig. 6

Fig. 7

blown out, work the rubber connecting tube over the capillary tube. Open the gas valve slightly, and very cautiously release your finger very slightly for a brief instant until the pellet has been pushed to about the middle of the tube.

Remove from the gas supply, seal off as before (the end that was connected to gas supply), and attach scale. Plot a graph of volume against temperature.

Interpreting your results.

5. With any of the methods mentioned here, the pressure of the gas remains constant. If the curve is a straight line, does this "prove" that the volume of a gas at constant pressure is proportional to its temperature?

6. Remember that the thermometer you used probably depended on the expansion of a liquid such as mercury or alcohol. Would your graph have been a straight line if a different type of thermometer had been used?

7. If you could continue to cool the air, would there be a lower limit to the volume it would occupy?

Draw a straight line as nearly as possible through the points on your *V-T* graph and extend it to the left until it shows the approximate temperature at which the volume would be zero. Of course, you have no reason to assume that gases have this simple linear relationship all the way down to zero volume. (In fact, air would change to a liquid long before it reached the temperature indicated on your graph for zero volume.) However, some gases do show this linear behavior over a wide temperature range, and for these gases the straight line always crosses the *T*-axis at the same point. Since the volume of a sample of gas cannot be less than 0, this point represents the lowest possible temperature of the gases—the "absolute zero" of temperature.

8. What value does your graph give for this temperature?

III. Questions for Discussion

Both the pressure and the temperature of a gas sample affect its volume. In these experiments you were asked to consider each of these factors separately.

9. Were you justified in assuming that the temperature remained constant in the first experiment as you varied the pressure? How could you check this? How would your results be affected if, in fact, the temperature went up each time you added weight to the plunger?

10. In the second experiment the gas was at atmospheric pressure. Would you expect to find the same relationship between volume and temperature if you repeated the experiment with a different pressure acting on the sample?

Gases such as hydrogen, oxygen, nitrogen, and carbon dioxide are very different in their chemical behavior. Yet they all show the same simple relationships between volume, pressure, and temperature that you found in these experiments, over a fairly wide range of pressures and temperatures. This suggests that perhaps there is a simple physical model that will explain the behavior of all gases within these limits of temperature and pressure. Chapter 11 of the *Text* describes just such a simple model and its importance in the development of physics.

EXPERIMENT 3-15 WAVE PROPERTIES

In this laboratory exercise you will become familiar with a variety of wave properties in one- and two-dimensional situations.* Using ropes, springs, Slinkies, or a ripple tank, you can find out what determines the speed of waves, what happens when they collide, and how waves reflect and go around corners.

Waves in a Spring

Many waves move too fast or are too small to watch easily. But in a long "soft" spring you can make big waves that move slowly. With a partner to help you, pull the spring out on a smooth floor to a length of about 20 to 30 feet. Now, with your free hand, grasp the stretched spring two or three feet from the end. Pull the two or three feet of spring together toward the end and then release it, being careful *not* to let go of the fixed end with your other hand! Notice the single wave, called a pulse, that travels along the spring. In such a *longitudinal* pulse the spring coils move back and forth along the same direction as the wave travels. The wave carries energy, and hence, could be used to carry a message from one end of the spring to the other.

You can see a longitudinal wave more easily if you tie pieces of string to several of the loops of the spring and watch their motion when the spring is pulsed.

A *transverse* wave is easier to see. To make one, practice moving your hand very quickly back and forth at right angles to the stretched spring, until you can produce a pulse that travels down only one side of the spring. This pulse is called "transverse" because the individual coils of wire move at right angles to (transverse to) the length of the spring.

Perform experiments to answer the following questions about transverse pulses.

*Adapted from R. F. Brinckerhoff and D. S. Taft, *Modern Laboratory Experiments in Physics*, by permission of Science Electronics, Nashua, N.H.

1. Does the size of the pulse change as it travels along the spring? If so, in what way?
2. Does the pulse reflected from the far end return to you on the same side of the spring as the original pulse, or on the opposite side?
3. Does a change in the tension of the spring have any effect on the speed of the pulses? When you stretch the spring farther, in effect you are changing the nature of the *medium* through which the pulses move.

Next observe what happens when waves go from one material into another—an effect called *refraction*. To one end of your spring attach a length of rope or rubber tubing (or a different kind of spring) and have your partner hold the end of this.

4. What happens to a pulse (size, shape, speed, direction) when it reaches the boundary between the two media? The far end of your spring is now free to move back and forth at the joint which it was unable to do before because your partner was holding it.

Have your partner detach the extra spring and once more grasp the far end of your original spring. Have him send a pulse on the same side, at the same instant you do, so that the two pulses meet. The interaction of the two pulses is called *interference*.

5. What happens (size, shape, speed, direction) when two pulses reach the center of the spring? (It will be easier to see what happens in the interaction if one pulse is larger than the other.)
6. What happens when two pulses on opposite sides of the spring meet?

As the two pulses pass on opposite sides of the spring, can you observe a point on the spring that does not move at all?

7. From these two observations, what can you say about the displacement caused by the addition of two pulses at the same point?

By vibrating your hand steadily back and forth, you can produce a train of pulses, a *periodic wave.* The distance between any two neighboring crests on such a periodic wave is the *wavelength.* The rate at which you vibrate your hand will determine the *frequency* of the periodic wave. Use a long spring and produce short bursts of periodic waves so you can observe them without interference by reflections from the far end.

> 8. How does the wavelength seem to depend on the frequency?

You have now observed the reflection, refraction, and interference of single waves, or pulses, traveling through different materials. These waves, however, moved only along one dimension. So that you can make a more realistic comparison with other forms of traveling energy, in the next experiment you will turn to these same wave properties spread out over a two-dimensional surface.

EXPERIMENT 3-16 WAVES IN A RIPPLE TANK

In the laboratory one or more ripple tanks will have been set up. To the one you and your partner are going to use, add water (if necessary) to a depth of 6 to 8 mm. Check to see that the tank is level so that the water has equal depth at all four corners. Place a large sheet of white paper on the table below the ripple tank, and then switch on the light source. Disturbances on the water surface are projected onto the paper as light and dark patterns, thus allowing you to "see" the shape of the disturbances in the horizontal plane.

To see what a single pulse looks like in a ripple tank, gently touch the water with your fingertip—or, better, let a drop of water fall into it from a medicine dropper held only a few millimeters above the surface.

For certain purposes it is easier to study pulses in water if their crests are straight. To generate single straight pulses, place a three-quarter-inch dowel, or a section of a broom handle, along one edge of the tank and roll it backward a fraction of an inch. A periodic wave, a continuous train of pulses, can be formed by rolling the dowel backward and forward with a uniform frequency.

Use straight pulses in the ripple tank to observe reflection, refraction, and diffraction, and circular pulses from point sources to observe interference.

Reflection

Generate a straight pulse and notice the direction of its motion. Now place a barrier in the water so that it intersects that path. Generate new pulses and observe what happens to the pulses when they strike the barrier. Try different angles between the barrier and the incoming pulse.

> 1. What is the relationship between the *direction* of the incoming pulse and the reflected one?
>
> 2. Replace the straight barrier with a curved one. What is the shape of the reflected pulse?
>
> 3. Find the point where the reflected pulses run together. What happens to the pulse after it converges at this point? At this point—called the *focus*—start a pulse with your finger, or a drop of water. What is the shape of the pulse after reflection from the curved barrier?

Refraction

Lay a sheet of glass in the center of the tank, supported by coins if necessary, to make an area of very shallow water. Try varying the angle at which the pulse strikes the boundary between the deep and shallow water.

> 4. What happens to the wave speed at the boundary?
>
> 5. What happens to the wave direction at the boundary?
>
> 6. How is change in direction related to change in speed?

Interference

Arrange two point sources side by side a few centimeters apart. When tapped gently, they should produce two pulses on the spring. You will see the action of interference better if you vibrate the two point sources continuously with a motor and study the resulting pattern of waves.

7. How does changing the wave frequency affect the original waves?

Find regions in the interference pattern where the waves from the two sources cancel and leave the water undisturbed. Find the regions where the two waves add up to create a doubly great disturbance.

8. Make a sketch of the interference pattern indicating these regions.

9. How does the pattern change as you change the wavelength?

Diffraction

With two-dimensional waves you can observe a new phenomenon—the behavior of a wave when it passes around an obstacle or through an opening. The spreading of the wave into the "shadow" area is called *diffraction*. Generate a steady train of waves by using the motor driven straight-pulse source. Place a small barrier in the path of the waves so that it intercepts part but not all of the wave front. Observe what happens as the waves pass the edge of the barrier. Now vary the wavelength of the incoming wave train by changing the speed of the motor on the source.

10. How does the interaction with the obstacle vary with the wavelength?

Place two long barriers in the tank, leaving a small opening between them.

11. How does the angle by which the wave spreads out beyond the opening depend on the size of the opening?

12. In what way does the spread of the diffraction pattern depend on the length of the waves?

EXPERIMENT 3-17 MEASURING WAVELENGTH

There are three ways you can conveniently measure the wavelength of the waves generated in your ripple tank. You should try them all, if possible, and cross-check the results. If there are differences, indicate which method you believe is most accurate and explain why.

A. *Direct*

Set up a steady train of pulses using either a single point source or a straightline source. Observe the moving waves with a stroboscope, and then adjust the vibrator motor to the lowest frequency that will "freeze" the wave pattern. Place a meter stick across the ripple tank and measure the distance between the crests of a counted number of waves.

B. *Standing Waves*

Place a straight barrier across the center of the tank parallel to the advancing waves. When the distance of the barrier from the generator is properly adjusted, the superposition of the advancing waves and the waves reflected from the barrier will produce *standing waves*. In other words, the reflected waves are at some points reinforcing the original waves, while at other points there is always cancellation. The points of continual cancellation are called *nodes*. The distance between nodes is one-half wavelength.

C. *Interference Pattern*

Set up the ripple tank with two point sources. The two sources should strike the water at the same instant so that the two waves will be exactly in phase and of the same frequency as they leave the sources. Adjust the distance between the two sources

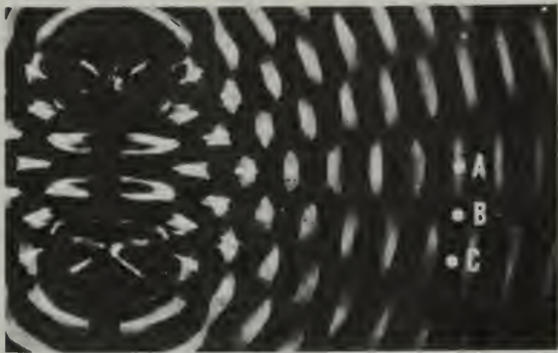


Fig. 1 An interference pattern in water. Two point sources vibrating in phase generate waves in a ripple tank. A and C are points of maximum disturbance (in opposite directions) and B is a point of minimum disturbance.

and the frequency of vibration until a distinct pattern is obtained, such as in Fig. 1.

As you study the pattern of ripples you will notice lines along which the waves cancel almost completely so that the amplitude of the disturbance is almost zero. These lines are called *nodal lines*, or *nodes*. You have already seen nodes in your earlier experiment with standing waves in the ripple tank.

At every point along a node the waves arriving from the two sources are half a wavelength out of step, or "out of phase." This means that for a point (such as B in Fig. 1) to be on a line of nodes it must be ½ or 1½ or 2½ . . . wavelengths farther from one source than from the other.

Between the lines of nodes are regions of maximum disturbance. Points A and C in Fig. 1 are on lines down the center of such regions, called *antinodal lines*. Reinforcement of waves from the two sources is a maximum along these lines.

For reinforcement to occur at a point, the two waves must arrive in step or "in phase." This means that any point on a line of antinodes is a whole number of wavelengths 0, 1, 2, . . . farther from one source than from the other. The relationship between crests, troughs, nodes and antinodes in this situation is summarized schematically in Fig. 2.

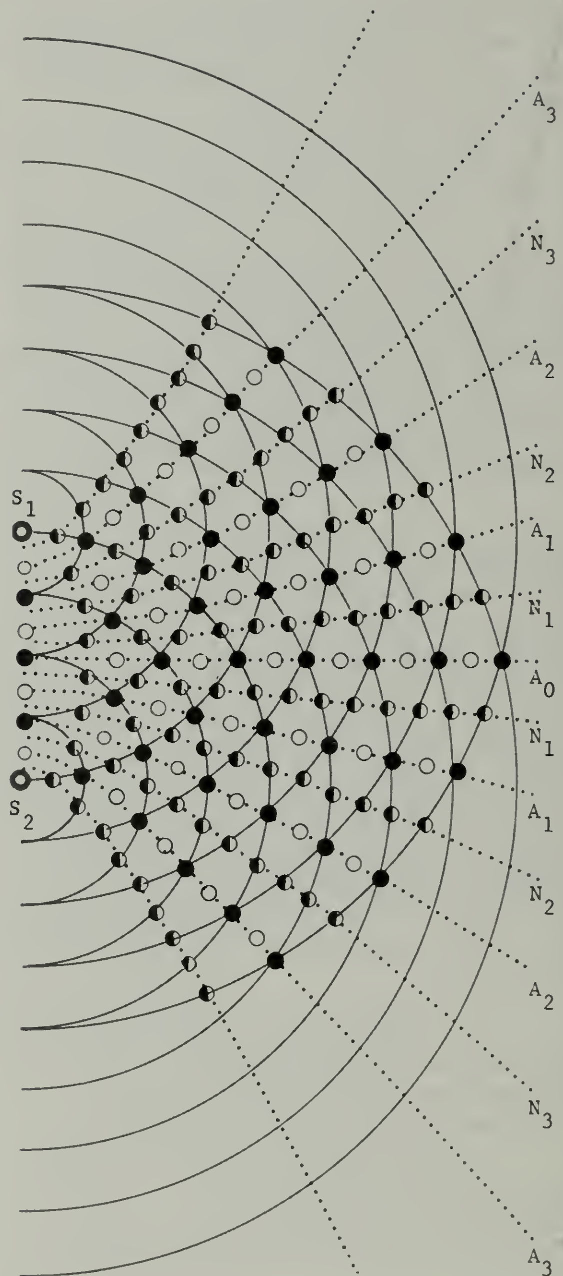


Fig. 2

Analysis of interference pattern similar to that of Fig. 1 at the top of the left column set up by two in-phase periodic sources. (Here S_1 and S_2 are separated by four wavelengths.) The letters A and N designate antinodal and nodal lines. The dark circles indicate where crest is meeting crest, the blank circles where trough is meeting trough, and the half-dark circles where crest is meeting trough.

Most physics textbooks develop the mathematical argument of the relationship of wavelength to the geometry of the interference pattern. (See, for example, p. 119 in Unit 3 of the *Project Physics Text*.) If the distance between the sources is d and the detector is at a comparatively greater distance L from the sources, then d, L, and λ are related by the equations

$$\frac{\lambda}{d} = \frac{x}{L}$$

or

$$\lambda = \frac{xd}{L}$$

where x is the distance between neighboring antinodes (or neighboring nodes).

You now have a method for computing the wavelength λ from the distances that you can measure precisely. Measure x, d, and L in your ripple tank and compute λ.

EXPERIMENT 3-18 SOUND

In previous experiments you observed how waves of relatively low frequency behave in different media. In this experiment you will try to determine to what extent audible sound exhibits similar properties.

At the laboratory station where you work there should be the following: an oscillator, a power supply, two small loudspeakers, and a group of materials to be tested. A loudspeaker is the source of audible sound waves, and your ear is the detector. First connect one of the loudspeakers to the output of the oscillator and adjust the oscillator to a frequency of about 4000 cycles per second. Adjust the loudness so that the signal is just audible one meter away from the speaker. The gain-control setting should be low enough to produce a clear, pure tone. Reflections from the floor, tabletop, and hard-surfaced walls may interfere with your observations so set the sources at the edge of a table, and put soft material over any unavoidably close hard surface that could cause reflective interference.

You may find that you can localize sounds better if you make an "ear trumpet" or stethoscope from a small funnel or thistle tube and a short length of rubber tubing (Fig. 1). Cover

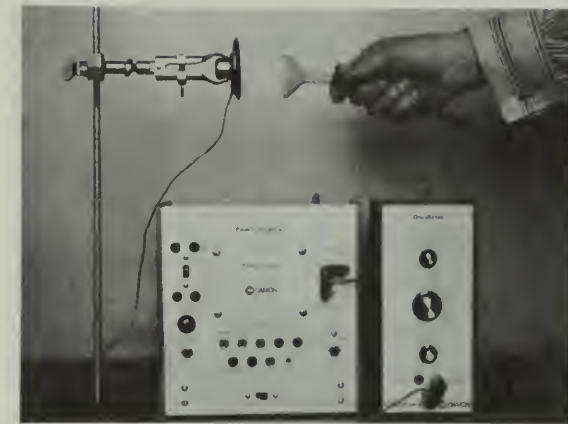


Fig. 1. Sound from the speaker can be detected by using a funnel and rubber hose, the end of which is placed to the ear. The Oscillator's banana plug jacks must be inserted into the −8V, +8V and ground holes of the Power Supply. Insert the speaker's plugs into the sine wave—ground receptacles of the Oscillator. Select the audio range by means of the top knob of the Oscillator and then turn on the Power Supply.

the ear not in use to minimize confusion when you are hunting for nodes and maxima.

Transmission and Reflection

Place samples of various materials at your station between the speaker and the receiver to see how they transmit the sound wave. In a table, record your qualitative judgments as best, good, poor, etc.

Test the same materials for their ability to reflect sound and record your results. Be sure that the sound is really being reflected and is not coming to your detector by some other path. You can check how the intensity varies at the detector when you move the reflector and rotate it about a vertical axis (see Fig. 2).

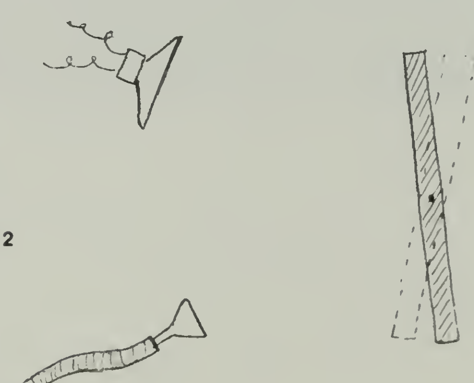


Fig. 2

If suitable materials are available to you, also test the reflection from curved surfaces.

1. On the basis of your findings, what generalizations can you make relating transmission and reflection to the properties of the test materials?

Refraction

You have probably observed the refraction or "bending" of a wave front in a ripple tank as the wave slowed down in passing from water of one depth to shallower water.

You may observe the refraction of sound waves using a "lens" made of gas. Inflate a spherical balloon with carbon dioxide gas to a diameter of about 4 to 6 inches. Explore the area near the balloon on the side away from the source. Locate a point where the sound seems loudest, and then remove the balloon.

2. Do you notice any difference in loudness when the balloon is in place? Explain.

Diffraction

In front of a speaker set up as before place a thick piece of hard material about 25 cm long, mounted vertically about 25 cm directly in front of the speaker. Slowly probe the area about 75 cm beyond the obstacle.

3. Do you hear changes in loudness? Is there sound in the "shadow" area? Are there regions of silence where you would expect to hear sound? Does there seem to be any pattern to the areas of minimum sound?

For another way to test for diffraction, use a large piece of board placed about 25 cm in front of the speaker with one edge aligned with the center of the source. Now explore the area inside the shadow zone and just outside it.

Describe the pattern of sound interference that you detect.

4. Is the pattern analogous to the pattern you observed in the ripple tank?

Wavelength

(a) *Standing wave method* Set your loudspeaker about $\frac{1}{2}$ meter above and facing toward a *hard* tabletop or floor or about that distance from a hard, smooth plaster wall or other good sound reflector (see Fig. 3). Your ear is *most* sensitive to the changes in intensity of faint sounds, so be sure to keep the volume low.

Explore the space between the source and reflector, listening for changes in loudness. Record the positions of minimum loudness, or at least find the approximate distance between two consecutive minima. These minima are located $\frac{1}{2}$ wavelength apart.

5. Does the spacing of the minima depend on the intensity of the wave?

Measure the wavelength of sound at several different frequencies.

6. How does the wavelength change when the frequency is changed?

(b) *Interference Method* Connect the two loudspeakers to the output of the oscillator and mount them at the edge of the table about 25 cm apart. Set the frequency at about 4,000 cycles/sec to produce a high-pitched tone. Keep the gain setting low during the entire experiment to make sure the oscillator is producing a pure tone, and to reduce reflections that would interfere with the experiment.

Move your ear or "stethoscope" along a line parallel to, and about 50 cm from, the line joining the sources. Can you detect distinct maxima and minima? Move farther away from the sources; do you find any change in the pattern spacing?

Fig. 3

7. What effect does a change in the source separation have on the spacing of the nodes?

8. What happens to the spacing of the nodes if you change the frequency of the sound? To make this experiment quantitative, work out for yourself a procedure similar to that used with the ripple tank. (Fig. 2.)

Measure the separation d of the source centers and the distance x between nodes and use this data to calculate the wavelength λ.

9. Does the wavelength change with frequency? If so, does it change directly or inversely?

Calculating the Speed of Sound

The relationship between speed v, wavelength λ, and frequency f is $v = \lambda f$. The oscillator dial gives a rough indication of the frequency (and your teacher can advise you on how to use an oscilloscope to make precise frequency settings). Using your best estimate of λ, calculate the speed of sound. If you have time, extend your data to answering the following questions:

10. Does the speed of the sound waves depend on the intensity of the wave?

11. Does the speed depend on the frequency?

EXPERIMENT 3-19 ULTRASOUND

The equipment needed for this experiment is an oscillator, power supply, and three ultrasonic transducers—crystals that transform electrical impulses into sound waves (or vice versa), and several materials to be tested. The signal from the detecting transducer can be displayed with either an oscilloscope (as in Fig. 1) or an amplifier and meter (Fig. 2). One or two of the transducers, driven by the oscillator are sources of the ultrasound, while the third transducer is a detector. Before you proceed, have the teacher check your setup and help you get a pattern on the oscilloscope screen or a reading on the meter.

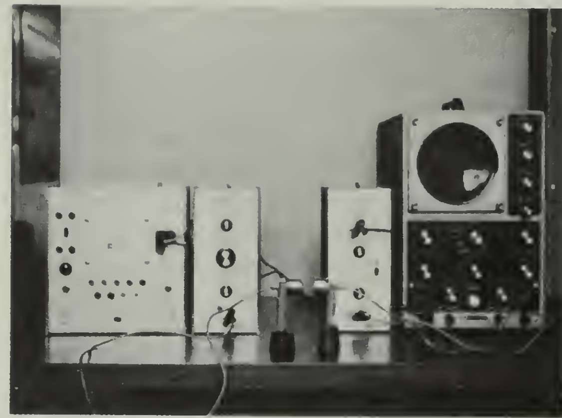


Fig. 1 Complete ultrasound equipment. Plug the +8v, −8v, ground jacks from the Amplifier and Oscillator into the Power Supply. Plug the coaxial cable attached to the transducer to the sine wave output of the Oscillator. Plug the coaxial cable attached to a second transducer into the input terminals of the amplifier. Be sure that the shield of the coaxial cable is attached to ground. Turn the oscillator range switch to the 5K-50K position. Turn the horizontal frequency range switch of the oscilloscope to at least 10kHz. Turn on the Oscillator and Power Supply. Tune the Oscillator for maximum reception, about 40 kilocycles.

The energy output of the transducer is highest at about 40,000 cycles per second, and

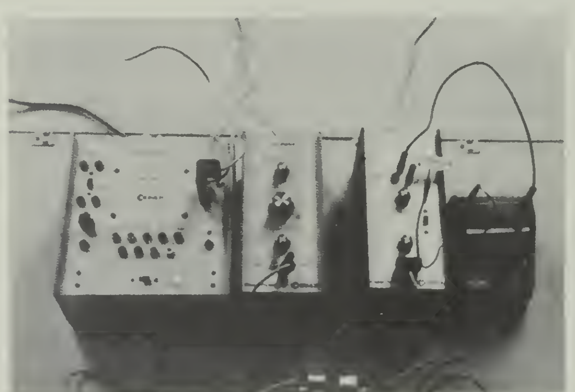


Fig. 2 Above, ultrasound transmitter and receiver. The signal strength is displayed on a microammeter connected to the receiver amplifier. Below, a diode connected between the amplifier and the meter, to rectify the output current. The amplifier selector switch should be turned to *ac*. The *gain* control on the amplifier should be adjusted so that the meter will deflect about full-scale for the loudest signal expected during the experiment. The *offset* control should be adjusted until the meter reads zero when there is no signal.

the oscillator must be carefully "tuned" to that frequency. Place the detector a few centimeters directly in front of the source and set the oscillator range to the 5-50 kilocycle position. Tune the oscillator carefully around 40,000 cycles/second for maximum deflection of the meter or the scope track. If the signal output is too weak to detect beyond 25 cm, plug the detector transducer into an amplifier and connect the output of the amplifier to the oscilloscope or meter input.

Transmission and Reflection

Test the various samples at your station to see how they *transmit* the ultrasound. Record your judgments as best, good, poor, etc. Hold the sample of the material being tested close to the detector.

Test the same materials for their ability to *reflect* ultrasound. Be sure that the ultrasound is really being reflected and is not coming to your detector by some other path. You can check this by seeing how the intensity varies at the detector when you move the reflector.

Make a table of your observations.

> 1. What happens to the energy of ultrasonic waves in a material that neither reflects nor transmits well?

Diffraction

To observe diffraction around an obstacle, put a piece of hard material about 3 cm wide 8 or 10 cm in front of the source (see Fig. 3.) Explore the region 5-10 cm behind the obstacle.

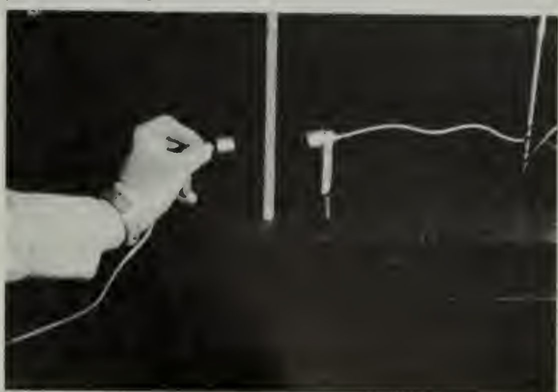


Fig. 3 Detecting diffraction of ultrasound around a barrier.

> 2. Do you find any signal in the "shadow" area? Do you find minima in the regions where you would expect a signal to be? Does there seem to be any pattern relating the areas of minimum and maximum signals?

Put a larger sheet of absorbing material 10 cm in front of the source so that the edge obstructs about one-half of the source.

Again probe the "shadow" area and the area near the edge to see if a pattern of maxima and minima seems to appear.

Measuring Wavelength

(a) *Standing Wave Method*

Investigate the standing waves set up between a source and a reflector, such as a hard tabletop or metal plate. Place the source about 10 to 15 cm from the reflector with the detector.

> 3. Does the spacing of nodes depend on the intensity of the waves?

Find the approximate distance between two consecutive maxima or two consecutive minima. This distance is one half the wavelength.

(b) *Interference Method* For sources, connect two transducers to the output of the oscillator and set them about 5 cm apart. Set the oscillator switch to the 5-50 kilocycle position. For a detector, connect a third transducer to an oscil-

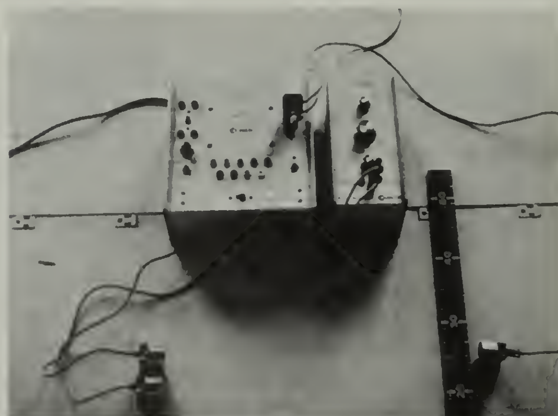


Fig. 4 Set-up for determination of wavelength by the interference method.

loscope or amplifier and meter as described in Part A of the experiment. Then tune the oscillator for maximum signal from the detector when it is held near one of the sources (about 40,000 cycles/sec). Move the detector along a line parallel to and about 25 cm in front of a line connecting the sources. Do you find distinct maxima and minima? Move closer to the sources. Do you find any change in the pattern spacing?

> 4. What effect does a change in the separation of the sources have on the spacing of the nulls?

To make this experiment quantitative, work out a procedure for yourself similar to that used with the ripple tank. Measure the appropriate distances and then calculate the wavelength using the relationship

$$\lambda = \frac{xd}{L}$$

derived earlier for interference patterns in a ripple tank.

> 5. In using that equation, what assumptions are you making?

The Speed of Ultrasound

The relationship between speed v, wavelength λ, and frequency f is $v = \lambda f$. Using your best estimate of λ, calculate the speed of sound.

> 6. Does the speed of the ultrasound waves depend on the intensity of the wave?
>
> 7. How does the speed of sound in the inaudible range compare with the speed of audible sound?

ACTIVITIES

IS MASS CONSERVED?

You have read about some of the difficulties in establishing the law of conservation of mass. You can do several different experiments to check this law.

Alka-Seltzer.

You will need the following equipment: Alka-Seltzer tablets; 2-liter flask, or plastic one-gallon jug (such as is used for bleach, distilled water, or duplicating fluid); stopper for flask or jug; warm water; balance (sensitivity better than 0.1 g) spring scale (sensitivity better than 0.5 g).

Balance a tablet and 2-liter flask containing 200-300 cc of water on a sensitive balance. Drop the tablet in the flask. When the tablet disappears and no more bubbles appear, readjust the balance. Record any change in mass. If there is a change, what caused it?

Repeat the procedure above, but include the rubber stopper in the initial balancing. Immediately after dropping in the tablet, place the stopper tightly in the mouth of the flask. (The pressure in a 2-liter flask goes up by no more than 20 per cent, so it is not necessary to tape or wire the stopper to the flask. Do not use smaller flasks in which proportionately higher pressure would be built up.) Is there a change in mass? Remove the stopper after all reaction has ceased; what happens? Discuss the difference between the two procedures.

Brightly Colored Precipitate.

You will need: 20 g lead nitrate; 11 g potassium iodide; Erlenmeyer flask, 1000 cc with stopper; test tube, 25 × 150 mm; balance.

Place 400 cc of water in the Erlenmeyer flask, add the lead nitrate, and stir until dissolved. Place the potassium iodide in the test tube, add 30 cc of water, and shake until dissolved. Place the test tube, open and upward, carefully inside the flask and seal the flask with the stopper. Place the flask on the balance and bring the balance to equilibrium. Tip the flask to mix the solutions. Replace the flask on the balance. Does the total mass remain conserved? What *does* change in this experiment?

Magnesium Flash Bulb.

On the most sensitive balance you have available, measure the mass of an unflashed magnesium flash bulb. Repeat the measurement several times to make an estimate of the precision of the measurement.

Flash the bulb by connecting it to a battery. Be careful to touch the bulb as little as possible, so as not to wear away any material or leave any fingerprints. Measure the mass of the bulb several times, as before. You can get a feeling for how small a mass change your balance could have detected by seeing how large a piece of tissue paper you have to put on the balance to produce a detectable difference.

EXCHANGE OF MOMENTUM DEVICES

The four situations described below are more complex tests for conservation of momentum, to give you a deeper understanding of the generality of the conservation law and of the importance of your frame of reference. (a) Fasten a section of HO gauge model railroad track to two ring stands as shown in Fig. 1. Set one truck of wheels, removed from a car, on the track and from it suspend an object with mass roughly equal to that of the truck. Hold the truck, pull the object to one side, parallel to the track, and release both at the same instant. What happens?

Fig. 1

Predict what you expect to see happen if you released the truck an instant after releasing the object. Try it.

Try increasing the suspended mass. (b) Fig. 2 shows a similar situation, using an air track supported on ring stands. An object of 20 g mass was suspended by a 50 cm string from one of the small air-track gliders. (One student trial continued for 166 swings.)

Fig. 2

(c) Fasten two dynamics carts together with four hacksaw blades as shown in Fig. 3. Push the top one to the right, the bottom to the left, and release them. Try giving the bottom cart a push across the room at the same instant you release them.

What would happen when you released the two if there were 10 or 20 bearing balls or small wooden balls hung as pendula from the top cart?

Fig. 3

(d) Push two large rubber stoppers onto a short piece of glass tubing or wood (Fig. 4). Let the "dumbbell" roll down a wooden wedge so that the stoppers do not touch the table until the dumbbell is almost to the bottom. When the dumbbell touches the table, it suddenly increases its linear momentum as it moves off along the table. Principles of rotational momentum and energy are involved here that are not covered in the *Text*, but even without

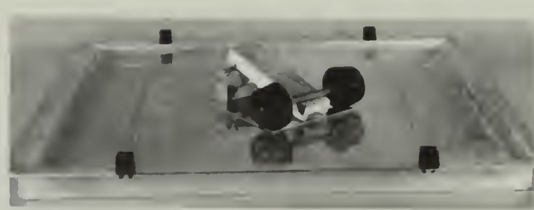


Fig. 4

extending the *Text*, you can deal with the "mysterious" increase in linear momentum when the stoppers touch the table.

Using what you have learned about conservation of momentum, what do you think could account for this increase? (Hint: set the wedge on a piece of cardboard supported on plastic beads and try it.)

STUDENT HORSEPOWER

When you walk up a flight of stairs, the work you do goes into frictional heating and increasing gravitational potential energy. The $\Delta(PE)_{grav}$, in joules, is the product of your weight in newtons and the height of the stairs in meters. (In foot-pounds, it is your weight in pounds times the height of the stairs in feet.)

Your useful power output is the average rate at which you did the lifting work—that is, the total change in $(PE)_{grav}$, divided by the time it took to do the work.

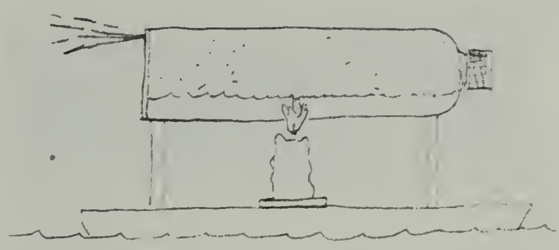
Walk or run up a flight of stairs and have someone time how long it takes. Determine the total vertical height that you lifted yourself by measuring one step and multiplying by the number of steps.

Calculate your useful work output and your power, in both units of watts and in horsepower. (One horsepower is equal to 550 foot-pounds/sec which is equal to 746 watts.)

STEAM-POWERED BOAT

You can make a steam-propelled boat that will demonstrate the principle of Heron's steam engine (*Text* Sec. 10.5) from a small toothpowder or talcum-powder can, a piece of candle, a soap dish, and some wire.

Place the candle in the soap dish. Punch a hole near the edge of the bottom of the can with a needle. Construct wire legs which are long enough to support the can horizontally



over the candle and the soap dish. Rotate the can so that the needle-hole is at the top. Half fill the can with water, replace the cover, and place this "boiler" over the candle and light the candle. If this boat is now placed in a large pan of water, it will be propelled across the pan.

Can you explain the operation of this boat in terms of the conservation of momentum? of the conservation of energy?

PROBLEMS OF SCIENTIFIC AND TECHNOLOGICAL GROWTH

The Industrial Revolution of the eighteenth and nineteenth centuries is rich in examples of man's disquiet and ambivalence in the face of technological change. Instead of living among pastoral waterwheel scenes, men began to live in areas with pollution problems as bad or worse than those we face today, as is shown in the scene at Wolverhampton, England in 1866. As quoted in the *Text*, William Blake lamented in "Stanzas from Milton,"

And did the Countenance Divine
Shine forth upon our clouded hills?
And was Jerusalem builded here
Among these dark Satanic mills?

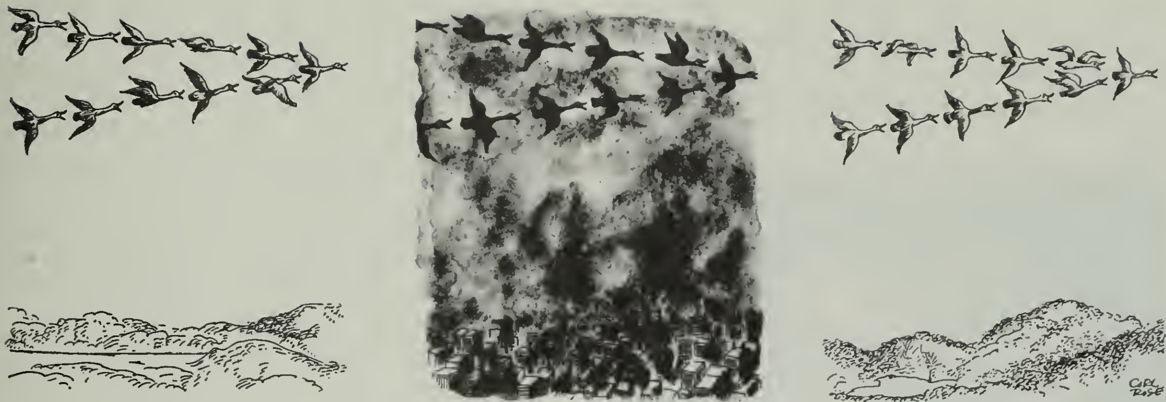
Ever since the revolution began, we have profited from advances in technology. But we also still face problems like those of pollution and of displacement of men by machines.

One of the major problems is a growing lack of communication between people working in science and those working in other fields. When C. P. Snow published his book *The Two Cultures and the Scientific Revolution* in 1959, he initiated a wave of debate which is still going on.

In your own community there are probably some pollution problems of which you are aware. Find out how science and technology

A portrayal of the scene near Wolverhampton, England in 1866, called "Black Country."

ACTIVITIES



"Honk, honk, honk, honk, honk, honk, honk, cough, cough, cough, cough, cough, cough, cough, honk, honk, honk, honk, honk, honk, honk."

may have contributed to these problems—and how they can contribute to solutions!

PREDICTING THE RANGE OF AN ARROW

If you are interested in predicting the range of a projectile from the work you do on a slingshot while drawing it back, ask your teacher about it. Perhaps he will do this with you or tell you how to do it yourself.

Another challenging problem is to estimate the range of an arrow by calculating the work done in drawing the bow. To calculate work, you need to know how the force used in drawing the string changed with the string displacement. A bow behaves even less according to Hooke's law than a slingshot; the force vs. displacement graph is definitely not a straight line.

To find how the force depends on string displacement, fasten the bow *securely* in a vise or some solid mounting. Attach a spring balance to the bow and record values of force (in newtons) as the bowstring is drawn back one centimeter at a time from its rest position (*without* having an arrow notched). Or, have someone stand on a bathroom scale, holding the bow, then pull upwards on the string; the force on the string at each position will equal this apparent *loss* of weight.

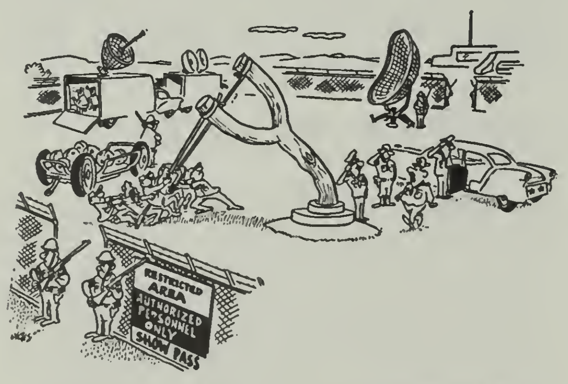
Now to calculate the amount of work done, plot a force vs. displacement graph. Count squares to find the "area" (force units times displacement units) under the graph; this is the work done on the bow—equal to the elastic potential energy of the drawn bow.

Assume that all the elastic potential energy of the bow is converted into the kinetic energy of the arrow and predict the range of the arrow by the same method used in predicting the range of a slingshot projectile.

A recent magazine article stated that an alert deer can leap out of the path of an approaching arrow when he hears the twang of the bowstring. Under what conditions do you think this is possible?

DRINKING DUCK

A toy called a Drinking Duck (No. 60,264 in Catalogue 671, about $1.00, Edmund Scientific Co., Barrington, New Jersey 08007) demonstrates very well the conversion of heat



"What's this scheme of yours for an economical method of launching a satellite?"

energy into energy of gross motion by the processes of evaporation and condensation. The duck will continue to bob up and down as long as there is enough water in the cup to wet his beak.

Rather than dampen your spirit of adventure, we won't tell you how it works. First, see if you can figure out a possible mechanism for yourself. If you can't, George Gamow's book, *The Biography of Physics*, has a very good explanation. Gamow also calculates how far the duck could raise water in order to feed himself. An interesting extension is to replace the water with rubbing alcohol. What do you think will happen?

Lest you think this device useful only as a toy, an article in the June 3, 1967, *Saturday Review* described a practical application being considered by the Rand Corporation. A group of engineers built a 7-foot "bird" using Freon 11 as the working fluid. Their intention was to investigate the possible use of large-size ducks for irrigation purposes in the Nile River Valley.

MECHANICAL EQUIVALENT OF HEAT

By dropping a quantity of lead shot from a measured height and measuring the resulting change in temperature of the lead, you can get a value for the ratio of work units to heat units—the "mechanical equivalent of heat."

You will need the following equipment:

Cardboard tube	Lead shot (1 to 2 kg)
Stoppers	Thermometer

Close one end of the tube with a stopper, and put in 1 to 2 kg lead shot that has been cooled about 5°C below room temperature. Close the other end of the tube with a stopper in which a hole has been drilled and a thermometer inserted. Carefully roll the shot to this end of the tube and record its temperature. Quickly invert the tube, remove the thermometer, and plug the hole in the stopper. Now invert the tube so the lead falls the full length of the tube and repeat this quickly one hundred times. Reinsert the thermometer and measure the temperature. Measure the average distance the shot falls, which is the length of the tube minus the thickness of the layer of shot in the tube.

If the average distance the shot falls is h and the tube is inverted N times, the work you did raising the shot of mass m is:

$$\Delta W = N \times ma_g \times h$$

The heat ΔH needed to raise the temperature of the shot by an amount ΔT is:

$$\Delta H = cm\Delta T$$

where c is the specific heat capacity of lead, 0.031 cal/gC°.

The mechanical equivalent of heat is $\Delta W/\Delta H$. The accepted experimental value is 4.184 newton-meters per kilocalorie.

A DIVER IN A BOTTLE

Descartes is a name well known in physics. When we graphed motion in *Text* Sec. 1.5, we used Cartesian coordinates, which Descartes introduced. Using Snell's law of refraction, Descartes traced a thousand rays through a sphere and came up with an explanation of the rainbow. He and his astronomer friend Gassendi were a bulwark against Aristotelian physics. Descartes belonged to the generation between Galileo and Newton.

On the lighter side, Descartes is known for a toy called the Cartesian diver which was very popular in the eighteenth century when very elaborate ones were made. To make one, first you will need a column of water. You may find a large cylindrical graduate about the laboratory, the taller the better. If not, you can improvise one out of a gallon jug or any other tall glass container. Fill the container almost to the top with water. Attach a piece of glass tubing that has been fire-polished on each end. Lubricate the glass tubing and the hole in the stopper with water and carefully insert the glass tubing. Fit the rubber stopper into the top of the container as shown in Fig. 1.

Next construct the diver. You may limit yourself to pure essentials, namely a small pill bottle or vial, which may be weighted with wire and partially filled with water so it just barely floats *upside down* at the top of the water column. If you are so inclined, you can decorate the bottle so it looks like a real underwater swimmer (or creature, if you prefer). The essential things are that you have a diver that just floats and that the volume of water can be changed.

Now to make the diver perform, just blow momentarily on the rubber tube. According to Boyle's law, the increased pressure (transmitted by the water) decreases the volume of trapped air and water enters the diver. The buoyant force decreases, according to Archimedes' principle, and the diver begins to sink.

If the original pressure is restored, the diver rises again. However, if you are lucky, you will find that as you cautiously make him sink deeper and deeper down into the column of water, he is more and more reluctant to return to the surface as the additional surface pressure is released. Indeed, you may find a depth at which he remains almost stationary. However, this apparent equilibrium, at which his weight just equals the buoyant force, is unstable. A bit above this depth, the diver will freely rise to the surface, and a bit below this depth he will sink to the bottom of the water column from which he can be brought to the surface only by vigorous sucking on the tube.

If you are mathematically inclined, you can compute what this depth would be in terms of the atmospheric pressure at the surface, the volume of the trapped air, and the weight of the diver. If not, you can juggle with the volume of the trapped air so that the point of unstable

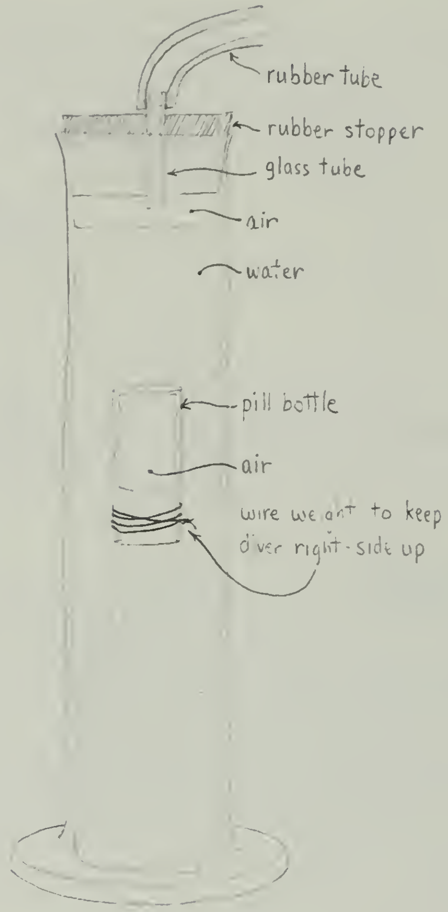
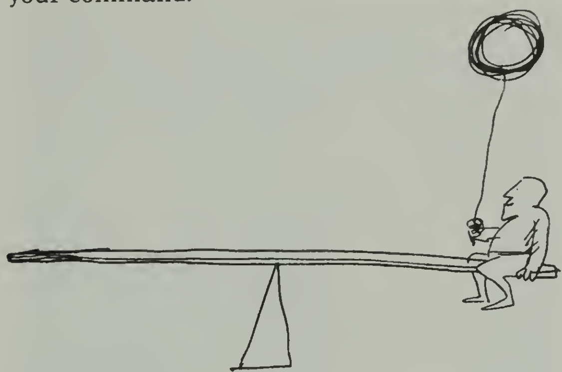


Fig. 1

equilibrium comes about halfway down the water column.

The diver raises interesting questions. Suppose you have a well-behaved diver who "floats" at room temperature just halfway down the water column. Where will he "float" if the atmospheric drops? Where will he "float" if the water is cooled or is heated? Perhaps the ideal gas law is not enough to answer this question, and you may have to do a bit of reading about the "vapor pressure" of water.

After demonstrating the performance of your large-scale model by blowing or sucking in the rubber tube, you can mystify your audience by making a small scale model in a bottle. A plastic bottle with flat sides can act like a diaphragm which increases the pressure within as the sides are pushed together. The bottle and diver are tightly sealed. In this case, add a rubber tube leading to a *holeless* stopper. Your classmates blowing as hard as they will cannot make the diver sink; but you, by secretly squeezing the bottle, can make him perform at your command.



ROCKETS

If it is legal to set off rockets in your area, and their use is supervised, they can provide excellent projects for studying conversion from kinetic to potential energy, thrust, etc.

Ask your teacher for instructions on how to build small test stands for taking thrust data to use in predicting the maximum height, range, etc. of the rockets. (Estes Industries, Box 227, Penrose, Colorado 81240, will send a very complete free catalogue and safety rules on request.)

HOW TO WEIGH A CAR WITH A TIRE PRESSURE GAUGE

Reduce the pressure in all four of your auto tires so that the pressure is the same in each and somewhat below recommended tire pressure.

Drive the car onto four sheets of graph paper placed so that you can outline the area of the tire in contact with each piece of paper. The car should be on a reasonably flat surface (garage floor or smooth driveway). The flattened part of the tire is in equilibrium between the vertical force of the ground upward and the downward force of air pressure within.

Measure the air pressure in the tires, and the area of the flattened areas. If you use inch graph paper, you can determine the area in square inches by counting squares.

Pressure P (in pounds per square inch) is defined as F/A, where F is the downward force (in pounds) acting perpendicularly on the flattened area A (in square inches). Since the tire pressure gauge indicates the pressure *above* the normal atmospheric pressure of 15 lb/in^2 you must add this value to the gauge reading. Compute the four forces as pressure times area. Their sum gives the weight of the car.

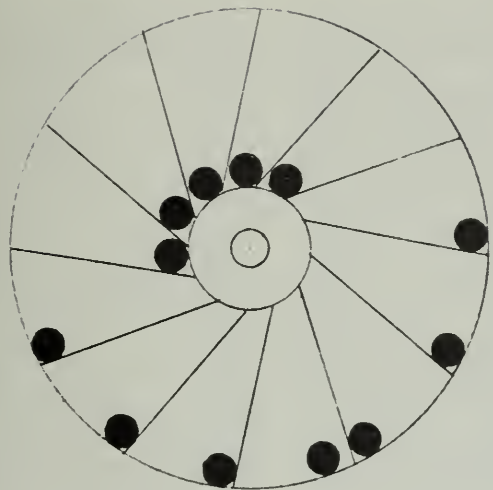
PERPETUAL-MOTION MACHINES?

You must have heard of "perpetual-motion" machines which, once started, will continue running and doing useful work forever. These proposed devices are inconsistent with laws of thermodynamics. (It is tempting to say that they *violate* laws of thermodynamics—but this implies that laws are rules by which Nature must run, instead of descriptions men have thought up.) We now believe that it is in principle impossible to build such a machine.

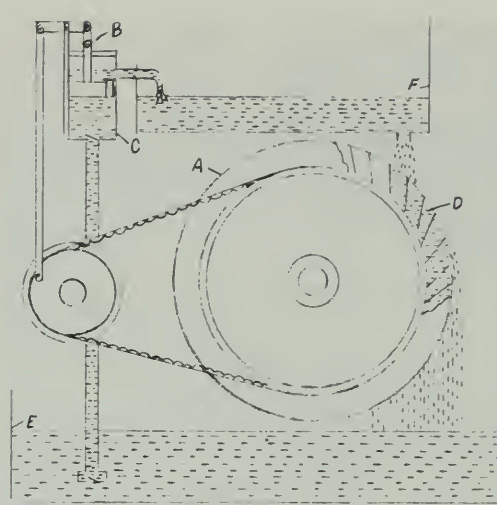
But the dream dies hard! Daily there are new proposals. Thus S. Raymond Smedile, in *Perpetual Motion and Modern Research for Cheap Power* (Science Publications of Boston, 1962), maintains that this attitude of "it can't be done" negatively influences our search for new sources of cheap power. His book gives sixteen examples of proposed machines, of which two are shown here.

Number 5 represents a wheel composed of twelve chambers marked A. Each chamber contains a lead ball B, which is free to roll. As the wheel turns, each ball rolls to the lowest level possible in its chamber. As the balls roll out to the right edge of the wheel, they create a preponderance of turning effects on the right side as against those balls that roll toward the hub on the left side. Thus, it is claimed the wheel is driven clockwise perpetually. If you think this will not work, explain why not.

Number 7 represents a water-driven wheel marked A. D represents the buckets on the perimeter of the waterwheel for receiving water draining from the tank marked F. The waterwheel is connected to pump B by a belt and wheel. As the overshot wheel is operated by water dropping on it, it operates the pump



Number 5



Number 7

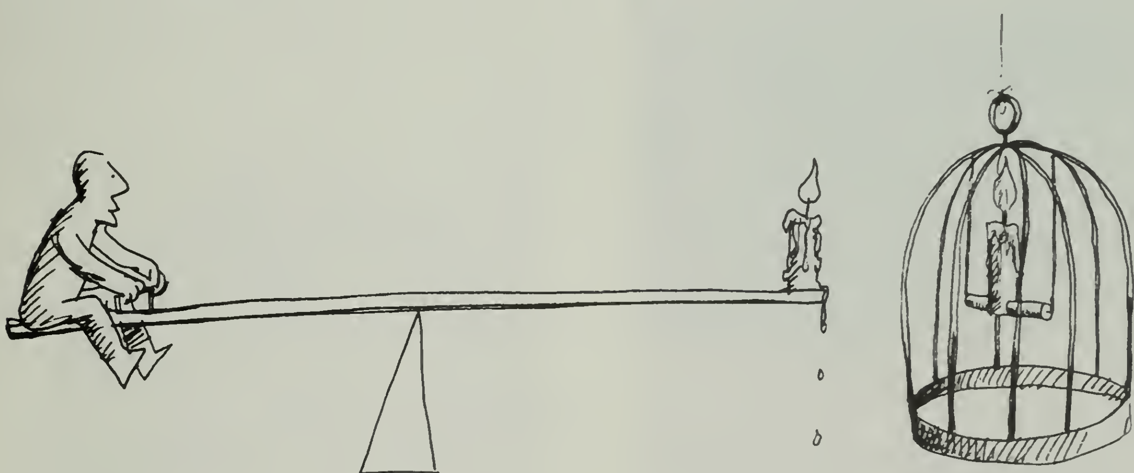
which sucks water into C from which it enters into tank F. This operation is supposed to go on perpetually. If you think otherwise, explain why.

If such machines would operate, would the conservation laws necessarily be wrong?

Is the reason that true perpetual motion machines are not found due to "theoretical" or "practical" deficiencies?

STANDING WAVES ON A DRUM AND A VIOLIN

You can demonstrate many different patterns of standing waves on a rubber membrane using a method very similar to that used in Film Loop 42, "Vibrations of A Drum." If you have not yet seen this loop, view it if possible before setting up the demonstration in your lab.



The cartoons above (and others of the same style which are scattered through the *Handbook*) were drawn in response to some ideas in the Project Physics Course by a cartoonist who was unfamiliar with physics. On being informed that the drawing on the left did not represent conservation because the candle wasn't a closed system, he offered the solution at the right. (Whether a system is "closed" depends, of course, upon what you are trying to conserve.)

Fig. 1 shows the apparatus in action, producing one pattern of standing waves. The

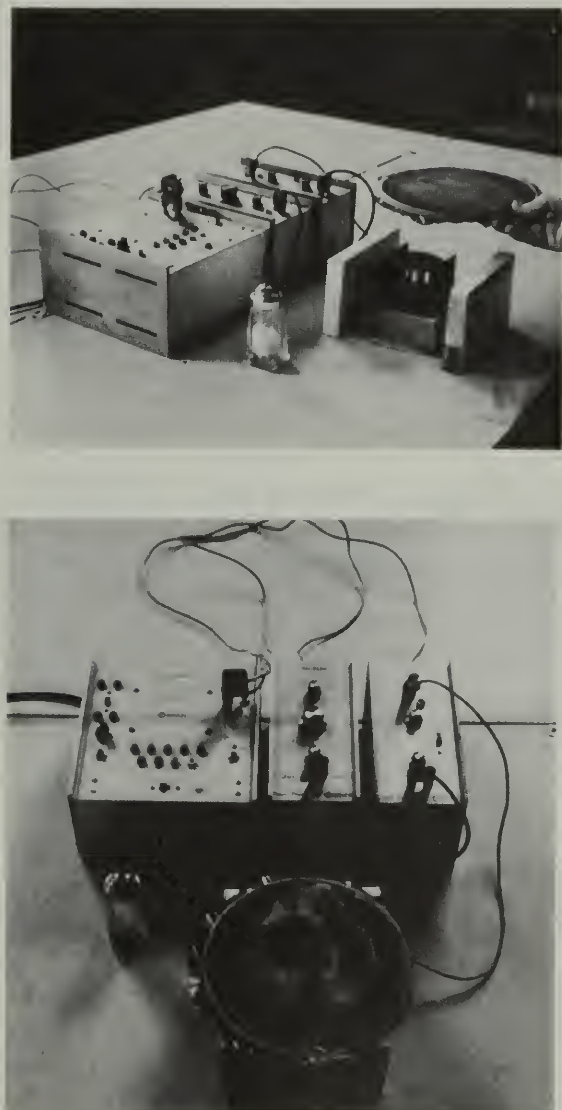


Fig. 1

drumhead in the figure is an ordinary 7-inch embroidery hoop with the end of a large balloon stretched over it. If you make your drumhead in this way, use as large and as strong a balloon as possible, and cut its neck off with scissors. A flat piece of sheet rubber (dental dam) gives better results, since even tension over the entire drumhead is much easier to maintain if the rubber is not curved to begin with. Try other sizes and shapes of hoops, as well as other drumhead materials.

A 4-inch, 45-ohm speaker, lying under the drum and facing upward toward it, drives the vibrations. Connect the speaker to the output of an oscillator. If necessary, amplify the oscillator output.

Turn on the oscillator and sprinkle salt or sand on the drumhead. If the frequency is near one of the resonant frequencies of the surface, standing waves will be produced. The salt will collect along the nodes and be thrown off from the antinodes, thus outlining the pattern of the vibration. Vary the frequency until you get a clear pattern, then photograph or sketch the pattern and move on to the next frequency where you get a pattern.

When the speaker is centered, the vibration pattern is symmetrical around the center of the surface. In order to get antisymmetric modes of vibration, move the speaker toward the edge of the drumhead. Experiment with the spacing between the speaker and the drumhead until you find the position that gives the clearest pattern; this position may be different for different frequencies.

If your patterns are distorted, the tension of the drumhead is probably not uniform. If you have used a balloon, you may not be able to remedy the distortion, since the curvature of the balloon makes the edges tighter than the center. By pulling gently on the rubber, however, you may at least be able to make the tension even all around the edge.

A similar procedure, used 150 years ago and still used in analyzing the performance of violins, is shown in these photos reprinted from *Scientific American*, "Physics and Music."

MOIRE PATTERNS

You are probably noticing a disturbing visual effect from the patterns in Figs. 1 and 2 on the opposite page. Much of "op art" depends on similar effects, many of which are caused by moiré patterns.

If you make a photographic negative of the pattern in Fig. 1 or Fig. 2 and place it on top of the same figure, you can use it to

Chladni Plates indicate the vibration of the body of a violin. These patterns were produced by covering a violin-shaped brass plate with sand and drawing a violin bow across its edge. When the bow caused the plate to vibrate, the sand concentrated along quiet nodes between the vibrating areas. Bowing the plate at various points, indicated by the round white marker, produces different frequencies of vibration and different patterns. Low tones produce a pattern of a few large areas; high tones a pattern of many small areas. Violin bodies have a few such natural modes of vibration which tend to strengthen certain tones sounded by the strings. Poor violin bodies accentuate squeaky top notes. This sand-and-plate method of analysis was devised 150 years ago by the German acoustical physicist Earnst Chladni.

study the interference pattern produced by two point sources. The same thing is done on Transparency 28, Two Slit Interference.

Long before op art, there was an increasing number of scientific applications of moiré patterns. Because of the great visual changes caused by very small shifts in two regular overlapping patterns, they can be used to make measurements to an accuracy of +0.0000001%. Some specific examples of the use of moiré patterns are visualization of two- or multiple-source interference patterns, measurement of

Fig. 1

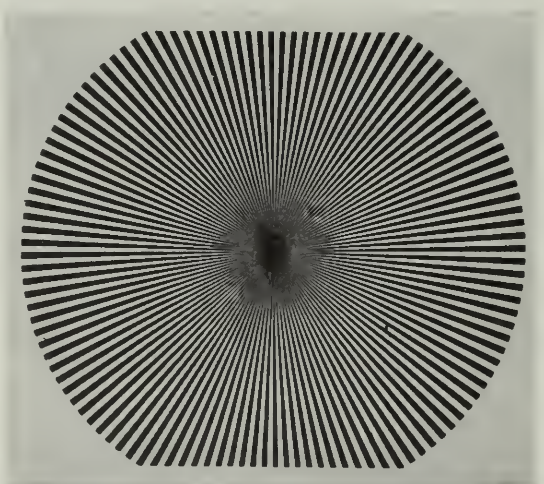


Fig. 2

small angular shifts, measurements of diffusion rates of solids into liquids, and representations of electric, magnetic, and gravitation fields. Some of the patterns created still cannot be expressed mathematically.

Scientific American, May 1963, has an excellent article; "Moiré Patterns" by Gerald Oster and Yasunori Nishijima. *The Science of Moiré Patterns*, a book by G. Oster, is available from Edmund Scientific Co., Barrington, N.J. Edmund also has various inexpensive sets of different patterns, which save much drawing time, and that are much more precise than hand-drawn patterns.

MUSIC AND SPEECH ACTIVITIES

(a) Frequency ranges: Set up a microphone and oscilloscope so you can display the pressure variations in sound waves. Play different instruments and see how "high C" differs on them.

(b) Some beautiful oscilloscope patterns result when you display the sound of the new computermusic records which use sound-synthesizers instead of conventional instruments.

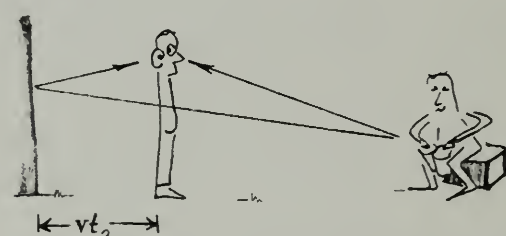
(c) For interesting background, see the following articles in *Scientific American:* "Physics and Music," July 1948; "The Physics of Violins," November 1962; "The Physics of Wood Winds," October 1960; and "Computer Music," December 1959.

(d) The Bell Telephone Company has an interesting educational item, which may be available through your local Bell Telephone office. A 33⅓ LP record, "The Science of Sounds," has ten bands demonstrating different ideas about sound. For instance, racing cars demonstrate the Doppler shift, and a soprano, a piano, and a factory whistle all sound alike when overtones are filtered out electronically. The record is also available on the Folkways label FX 6136.

MEASUREMENT OF THE SPEED OF SOUND

For this experiment you need to work outside in the vicinity of a large flat wall that produces a good echo. You also need some source of loud pulses of sound at regular intervals, about one a second or less. A friend beating on a drum or something with a higher pitch will do. The important thing is that the time between one pulse and the next doesn't vary, so a metronome would help. The sound source should be fairly far away from the wall, say a couple of hundred yards in front of it.

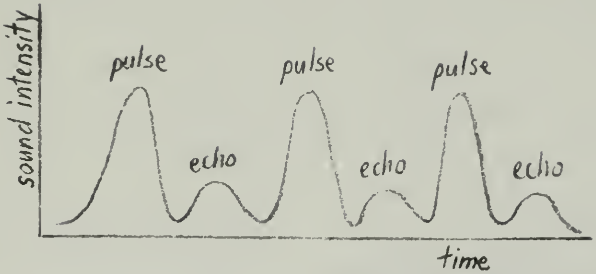
Stand somewhere between the reflecting wall and the source of pulses. You will hear both the direct sound and the sound reflected from the wall. The direct sound will reach you first because the reflected sound must travel the additional distance from you to the wall and back again. As you approach the wall, this additional distance decreases, as does the time interval between the direct sound and the echo. Movement away from the wall increases the interval.



If the distance from the source to the wall is great enough, the added time taken by the echo to reach you can amount to more than the time between drum beats. You will be able to find a position at which you hear the *echo* of one pulse at the same time you hear the *direct* sound of the next pulse. Then you know that the sound took a time equal to the interval between pulses to travel from you to the wall and back to you.

Measure your distance from the source. Find the time interval between pulses by measuring the time for a large number of pulses. Use these two values to calculate the speed of sound.

(If you cannot get far enough away from the wall to get this synchronization, increase the speed of the sound source. If this is impossible, you may be able to find a place where you hear the echoes exactly halfway between the pulses as shown at the top of the opposite page. You will hear a pulse, then an echo, then the next pulse. Adjust your position so that these three sounds seem equally spaced in time. At



this point you know that the time taken for the return trip from you to the wall and back is equal to *half* the time interval between pulses.)

MECHANICAL WAVE MACHINES

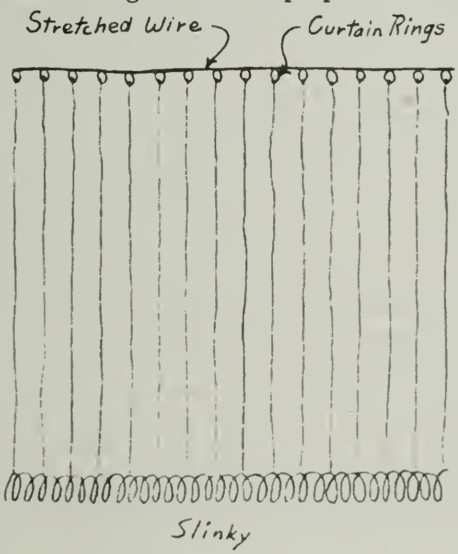
Several types of mechanical wave machines are described below. They help a great deal in understanding the various properties of waves.

(a) Slinky

The spring called a Slinky behaves much better when it is freed of friction with the floor or table. Hang a Slinky horizontally from strings at least three feet long tied to rings on a wire stretched from two solid supports. Tie strings to the Slinky at every fifth spiral for proper support.

Fasten one end of the Slinky *securely* and then stretch it out to about 20 or 30 feet. By holding onto a ten-foot piece of string tied to the end of the Slinky, you can illustrate "open-ended" reflection of waves.

See Experiment 3-15 for more details on demonstrating the various properties of waves.



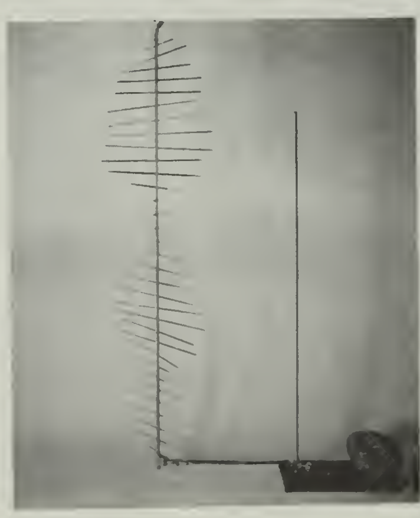
(b) Rubber Tubing and Welding Rod

Clamp both ends of a four-foot piece of rubber tubing to a table so it is under slight tension. Punch holes through the tubing every inch with a hammer and nail. (Put a block of wood under the tubing to protect the table.)

Put enough one-foot lengths of welding rod for all the holes you punched in the tubing. Unclamp the tubing, and insert one rod in each of the holes. Hang the rubber tubing vertically, as shown below, and give its lower end a twist to demonstrate transverse waves. Performance and visibility are improved by adding weights to the ends of the rods or to the lower end of the tubing.

(c) A Better Wave Machine

An inexpensive paperback, *Similarities in Wave Behavior*, by John N. Shive of Bell Telephone Laboratories, has instructions for building a better torsional wave machine than that described in (b) above. The book is available from Garden State-Novo, Inc., 630 9th Avenue, New York, N.Y. 10036.



ACTIVITIES

Resource Letter TLA-1 on Technology, Literature, and Art * since World War II

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(Received 10 December 1969)

I. INTRODUCTION

This resource letter lists materials for collateral reading in classes in physics and other sciences as well as in new cross-discipline courses; it also offers professors and students alike an opportunity to see how modern science and technology appear to artists and writers—in other words, to see themselves as others see them. A sampling of books and articles in the increasingly publicized area of cross relationships between technology and society in general would require a separate resource letter, which may one day materialize. The reader will note that the proper distinctions between science and technology are often blurred, as indeed they are daily by the public. The basic hope in this letter is to promote the mutual communication and understanding between disciplines so necessary for personal growth and so vital in such areas as top-level decision making.

As an earlier letter has said, the following listings are suggestions, not prescriptions. They are samplings guided by personal taste and experience, offered with the notion of tempting readers to go farther and deeper on their own.

By agreement with the editors, the listings contain a minimum of overlap with earlier letters, two of which are cited below; indeed, they are intended to pick up where the former left off. Anyone wishing bibliography on the "Two Cultures" argument, missing the presence of earlier "must" items by Mumford, Giedion, Holton, Bronowski, Barzun *et al.*, or seeking historical material can probably find what he wants in the earlier letters. This bibliography is arranged alphabetically by author or editor, except where no such identification exists; in the latter instances, titles are listed in the appropriate alphabetical sequence.

*Prepared at the request of the Committee on Resource Letters of the American Association of Physics Teachers, supported by a grant from the National Science Foundation and published in the *American Journal of Physics*, Vol. 38, No. 4, April 1970, pp. 407-414.

II. RELEVANT RESOURCE LETTERS

1. **"Science and Literature" (SL-1).** Marjorie Nicolson. Amer. J. Phys. **33**, 175 (1965).
2. **"Resource Letter ColR-1 on Collateral Reading for Physics Courses,"** Alfred M. Bork and Arnold B. Arons. Amer. J. Phys. **35**, 71 (1967).

III. TECHNOLOGY, LITERATURE, AND ART SINCE WORLD WAR II: INTERPLAY AND CROSS RELATIONS

1. **"Two Cultures in Engineering Design,"** Anonymous. Engineering **197**, 373 (13 Mar. 1964). A model essay demonstrating that dams and highway-lighting standards can be and should be both useful and beautiful.
2. **Poetics of Space.** Gaston Bachelard. (Orion Press, New York, 1964). A physicist–philosopher justifies poetry as an answer to technology and formulas. In a provocative discussion of the "spaceness" of cellars, attics, and closets and of their relative effects on us, of which we are generally unaware, the author makes us see the familiar in a new light. He offers stimulating contrasts with common notions of space in physics and in the public mind, as influenced by Apollo missions.
3. **"Science: Tool of Culture,"** Cyril Bibby. Saturday Rev. **48**, No. 23, 51 (6 June 1964). Pits the scientists and creative artists against the purely verbal scholars, asks for more science (better taught) in schools, and appeals to administrators to change their methods of training teachers, so that science will appear not as an ogre but as a fairy godmother.
4. **Voices from the Crowd (Against the H-Bomb).** David Boulton, Ed. (Peter Owen, London, 1964.) This anthology of poetry and prose, stemming from the Campaign for Nuclear Disarmament, is a good example of direct testimony of the effect of the Bomb on thinking and writing. Among the literary people included: Priestley, Comfort, Russell, Read, Osborne, Braine (*Room at the Top*).
5. **Poetry and Politics: 1900–1960.** C. M. Bowra. (University Press, Cambridge, England, 1966.) Discusses in part the effect of Hiroshima on poets, notably Edith Stilwell, whose form and vision were radically affected by the event, and the Russian, Andrei Voznesensky, whose poem on the death of Marilyn Monroe foresaw a universal disaster.

6. **"Science as a Humanistic Discipline,"** J. Bronowski. Bull. Atomic Scientists **24,** No. 8, 33 (1968). The author of *Science and Human Values* here covers the history of humanism, values, choice, and man as a unique creature. It is the duty of science to transmit this sense of uniqueness, to teach the world that man is guided by self-created values and thus comfort it for loss of absolute purpose.
7. **"Artist in a World of Science,"** Pearl Buck. Saturday Rev. **41,** No. 38, 15–16, 42–44 (1958). Asks for artists to be strong, challenges writers to use the findings of science and illuminate them so that "human beings will no longer be afraid."
8. **The Novel Now.** Anthony Burgess. (W. W. Norton and Co.,.New York, 1967.) Prominent British novelist discusses the aftermath of nuclear war as a gloomy aspect of fictional future time and advances the thesis that comparatively few good novels came out of the war that ended with Hiroshima, although a good deal of ordinary fiction has the shadow of the Bomb in it.
9. **Beyond Modern Sculpture: Effects of Science & Technology on the Sculpture of this Century.** Jack Burnham. (George Braziller Inc., New York, 1969.) "Today's sculpture is preparing man for his replacement by information-processing energy" Burnham sees an argument for a mechanistic teleological interpretation of life in which culture, including art, becomes a vehicle for qualitative changes in man's biological status. [See review by Charlotte Willard in Saturday Rev. **52,** No. 2, 19 (1969).]
10. **Cultures in Conflict.** David K. Cornelius and Edwin St. Vincent, Eds. (Scott, Foresman and Company, Glenview, Ill., 1964). A useful anthology of primary and secondary materials on the continuing C. P. Snow debates.
11. **"The Computer and the Poet,"** Norman Cousins. Saturday Rev. **49,** No. 30, 42 (23 July 1966). Suggests editorially (and movingly) that poets and programmers should get together to "see a larger panorama of possibilities than technology alone may inspire" and warns against the "tendency to mistake data for wisdom."
12. **Engineers and Ivory Towers.** Hardy Cross. Robert C. Goodpasture, Ed. (McGraw-Hill Book Co., New York, 1952). A sort of common-sense bible covering the education of an engineer, the full life, and concepts of technological art.
13. **Engineering: Its Role and Function in Human Society.** William H. Davenport and Daniel Rosenthal, Eds. (Pergamon Press, Inc., New York, 1967). An anthology with four sections on the viewpoint of the humanist, the attitudes of the engineer, man and machine, and technology and the future. Many of the writers in this bibliography are represented in an effort to present historical and contemporary perspectives on technology and society.
14. **"Art and Technology—The New Combine,"** Douglas M. Davis. Art in Amer. **56,** 28 (Jan.–Feb. 1968). Notes a new enthusiasm among many modern artists because of the forms, effects, and materials made possible by the new technology. Envisions full partnership between artist and machine in the creative process.
15. **So Human an Animal.** René Dubos. (Charles Scribner's & Sons, New York, 1968). Dubos, a prominent microbiologist, won a Pulitzer Prize for this work, and it deserves wide reading. Motivated by humanistic impulses, writing now like a philosopher and again like a poet, he discusses man's threatened dehumanization under technological advance. Man can adjust, Dubos says—at a price But first he must understand himself as a creature of heredity and environment and then learn the science of life, not merely science.
16. **The Theatre of the Absurd.** Martin Esslin. (Anchor Books–Doubleday and Co., Inc., Garden City, N. J., 1961). The drama director for the British Broadcasting Company explains the work of Beckett, Ionesco, Albee, and others as a reaction to loss of values, reason, and control in an age of totalitarianism and of that technological development, the Bomb.
17. **Engineering and the Liberal Arts.** Samuel C. Florman. (McGraw-Hill Book Co., New York, 1968). The subtitle tells the story: *A Technologist's Guide to History, Literature, Philosophy, Art, and Music.* Explores the relationships between technology and the liberal arts—historical, aesthetic, functional. Useful reading lists are included.
18. **The Creative Process.** Brewster Ghiselin, Ed. (University of California Press, Berkeley, 1952; Mentor Books, The New American Library, Inc., New York, paperback, 1961). Mathematicians, musicians, painters, and poets, in a symposium on the personal experience of creativity. Of use to those interested in the interplay between science and art.
19. **Postwar British Fiction: New Accents and Attitudes.** James Gindin. (University of California Press, Berkeley, 1963). Traces the comic or existentialist view of the world in recent British novels as resulting in part from the threat of the hydrogen bomb.
20. **The Poet and the Machine.** Paul Ginestier. Martin B. Friedman, Transl. (University of North Carolina Press, Chapel Hill, 1961; College and University Press, New Haven, Conn., paperback, 1964). Considers through analysis of generous examples from modern and contemporary poetry the effect of the machine on subject matter, form, and attitude. An original approach to the value, meaning, and influence, as the author puts it, of the poetry of our technology-oriented era.
21. **"Nihilism in Contemporary Literature,"** Charles I. Glicksberg. Nineteenth Century 144, 214 (Oct., 1948). An example of the extreme view that man is lost in a whirlpool of electronic energy, that cosmic doubts, aloneness, and fear of cataclysmic doom have led to a prevailing mood of nihilism in writing.
22. **"Impact of Technological Change on the Humanities,"**

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MAXWELL H. GOLDBERG. Educational Record 146, No. 4, 388–399 (1965). It is up to the humanities to soften the impact of advancing technology upon the pressured individual. One thing they can do is to help us pass the almost unlimited leisure time prophesied for the near future under automation and, thus, to avoid Shaw's definition of hell.

23. **"A Poet's Investigation of Science,"** ROBERT GRAVES. Saturday Rev. 46, 82 (7 Dec. 1963). The dean of English poets in a lecture at the Massachusetts Institute of Technology takes technologists to task good-humoredly but with a sting, too. He is concerned about the upset of nature's balance, the weakening of man's powers through labor-saving devices, synthetic foods, artificial urban life, dulling of imagination by commercialized art, loss of privacy—all products or results of technology. Graves finds no secret mystique among advanced technologists, only a sense of fate that makes them go on, limited to objective views and factual accuracy, forgetting the life of emotions and becoming diminished people.

24. **Social History of Art.** ARNOLD HAUSER. (Vintage Books, Random House, Inc., 4 vols., New York, 1951). In the fourth volume of this paperback edition of a standard work, considerable space is given to the cultural problem of technics and the subject of film and technics.

25. **"Automation and Imagination,"** JACQUETTA HAWKES. Harper's 231, 92 (Oct. 1965). Prominent archaeologist fears loss of man's imaginative roots under years of technical training. While the technological revolution sweeps on toward a total efficiency of means, she says, we must control the ends and not forget the significance of the individual.

26. **The Future as Nightmare.** MARK R. HILLEGAS. (Oxford University Press, New York, 1967). A study that begins with Wells and ends with recent science fiction by Ray Bradbury, Kurt Vonnegut, and Walter Miller, Jr. The latter three are worried about the mindless life of modern man with his radio, TV, and high-speed travel; the need to learn nothing more than how to press buttons; the machine's robbing man of the pleasure of working with his hands, leaving him nothing useful to do, and lately making decisions for him; and, of course, the coming nuclear holocaust.

27. **Science and Culture.** GERALD HOLTON, Ed. (Beacon Press, Boston, 1967). Almost all of the 15 essays in this outstanding collection appeared, several in different form, in the Winter 1965 issue of Daedalus. Of particular relevance to the area of this bibliography are Herbert Marcuse's view of science as ultimately just technology; Gyorgy Kepes' criticism of modern artists for missing vital connections with technological reality; René Dubos' contention that technological applications are becoming increasingly alienated from human needs; and Oscar Handlin's documentation of the ambivalent attitude of modern society toward technology.

28. **"The Fiction of Anti-Utopia,"** IRVING HOWE. New Republic 146, 13 (23 Apr. 1962). An analysis of the effect on modern fiction of the splitting apart of technique and values and the appearance of technical means to alter human nature, both events leading to the American dream's becoming a nightmare.

29. **The Idea of the Modern.** IRVING HOWE, Ed. (Horizon Press, New York, 1967). A perspective on post-Hiroshima literature and its relation to technology calls for a frame of reference on modernism in art and literature in general. A useful set of ideas is contained in this volume, the summing-up of which is that "nihilism lies at the center of all that we mean by modernist literature."

30. **The Machine.** K. G. P. HULTÉN, Ed. (Museum of Modern Art, New York, 1968). A metal-covered book of pictorial reproductions with introduction and running text, actually an exhibition catalogue, offering clear visual evidence of the interplay of modern art and modern technology in forms and materials.

31. **Literature and Science.** ALDOUS HUXLEY. (Harper & Row, Publishers, New York, 1963). A literary and highly literate attempt to show bridges between the two cultures. Technological know-how tempered by human understanding and respect for nature will dominate the scene for some time to come, but only if men of letters and men of science advance together.

32. **The Inland Island.** JOSEPHINE JOHNSON. (Simon & Schuster, Inc., New York, 1969). One way to avoid the evils of a technological society is to spend a year on an abandoned farm, study the good and the cruel aspects of nature, and write a series of sketches about the experience. Escapist, perhaps, but food for thought.

33. **The Sciences and the Humanities.** W. T. JONES. (University of California Press, Berkeley, 1965). A professor of philosophy discusses conflict and reconciliation between the two cultures, largely in terms of the nature of reality and the need to understand each other's language.

34. **"The Literary Mind,"** ALFRED KAZIN. Nation 201, 203 (20 Sept. 1965). Advances the thesis that it is more than fear of the Bomb that produces absurdist and existentialist writing, it is dissatisfaction that comes from easy self-gratifications: "Art has become too easy."

35. **"Imagination and the Age,"** ALFRED KAZIN. Reporter 34, No. 9, 32 (5 May 1966). Analyzes the crisis mentality behind modern fiction, the guilt feelings going back to Auschwitz and Hiroshima. Salvation from the materialism of modern living lies in language and in art.

36. **New Landscape in Science and Art.** GYORGY KEPES. (Paul Theobald, Chicago, 1967). Like the earlier *Vision in Motion* by L. Moholy-Nagy (Paul Theobald, Chicago, 1947), this work will make the reader see more, better, and differently. Essays and comments by Gabo, Giedion, Gropius, Rossi, Wiener, and others

plus lavish illustration assist Kepes, author of the influential *Language of Vision* and head of the program on advanced visual design at the Massachusetts Institute of Technology, to discuss morphology in art and science, form in engineering, esthetic motivation in science—in short, to demonstrate that science and its applications belong to the humanities, that, in Frank Lloyd Wright's words, "we must look to the artist brain . . . to grasp the significance to society of this thing we call the machine."

37. **"If You Don't Mind My Saying So . . .,"** JOSEPH WOOD KRUTCH. Amer. Scholar 37, 572 (Autumn, 1968). Expresses fear over extending of experimentation with ecology and belief that salvation does not lie with manipulation and conditioning but may come from philosophy and art.

38. **The Scientist vs the Humanist.** GEORGE LEVINE AND OWEN THOMAS, Eds. (W. W. Norton, New York, 1963). Among the most relevant items are I. I. Rabi's "Scientist and Humanist"; Oppenheimer's "The Tree of Knowledge"; Howard Mumford Jones's "The Humanities and the Common Reader" (which treats technological jargon); and P. W. Bridgman's "Quo Vadis."

39. **Death in Life: Survivors of Hiroshima.** ROBERT J. LIFTON. (Random House, Inc., New York, 1967). Chapter 10, "Creative Response: A-Bomb Literature," offers samples of diaries, memoirs, and poems by survivors, running the gamut from protest to reconstruction. See, also, Lifton's "On Death and Death Symbolism: The Hiroshima Disaster" in Amer. Scholar 257 (Spring, 1965).

40. **"The Poet and the Press,"** ARCHIBALD MACLEISH. Atlantic **203**, No. 3, 40 (March, 1959). Discusses the "divorce between knowing and feeling" about Hiroshima as part of a social crisis involving the decay of the life of the imagination and the loss of individual freedom. The danger here is growing acquiescence to a managed order and satisfaction with the car, TV, and the material products of our era.

41. **"The Great American Frustration,"** ARCHIBALD MACLEISH. Saturday Rev. **51**, No. 28, 13 (13 July 1968). Prior to Hiroshima, it seemed that technology would serve human needs; after the event, it appeared that technology is bound to do what it can do. We are no longer men, but consumers filled with frustrations that produce the satirical novels of the period. We must try to recover the management of technology and once more produce truly educated men.

42. **"The New Poetry,"** FRANK MACSHANE. Amer. Scholar **37**, 642 (Autumn, 1968). Frequently, the modern poet writes of confrontation of man and machine. He is both attracted and repelled by technological change, which both benefits and blights.

43. **The Machine in the Garden: Technology and the Pastoral Ideal in America.** LEO MARX. (Oxford University Press, New York, 1964; Galaxy, Oxford Univ. Press, New York, paperback, 1967). One of the three most significant contemporary works on the interplay of literature and technology [along with Sussman (69) and Sypher (72)], this study concentrates on 19th-century American authors and their ambivalent reactions to the sudden appearance of the machine on the landscape. Whitman, Emerson, Thoreau, Hawthorne, Melville, and others reveal, under Marx's scrutiny, the meaning inherent in productivity and power. Whitman assimilated the machine, Emerson welcomed it but disliked ugly mills, Thoreau respected tools but hated the noise and smoke, Hawthorne and Melville noted man's growing alienation with the green fields gone, Henry Adams set the theme for the "ancient war between the kingdom of love and the kingdom of power...waged endlessly in American writing ever since." The domination of the machine has divested of meaning the older notions of beauty and order, says Marx, leaving the American hero dead, alienated, or no hero at all. Aptly used quotations, chronological order, and clarity of perspective and statement (with which all may not agree) make this a "must" for basic reading in this special category. Furthermore, there are links to Frost, Hemingway, Faulkner, and other modern writers.

44. **Technology and Culture in Perspective.** ILENE MONTANA, Ed. (The Church Society for College Work, Cambridge, 1967). Includes "Technology and Democracy" by Harvey Cox, "The Spiritual Meaning of Technology and Culture," by Walter Ong, and "The Artist's Response to the Scientific World," by Gyorgy Kepes.

45. **"Science, Art and Technology,"** CHARLES MORRIS. Kenyon Rev. 1, No. 4, 409 (Autumn, 1939). A tight study of three forms of discourse—scientific, aesthetic, and technological and a plea that the respective users acquire vision enough to see that each complements the other and needs the other's support.

46. **"Scientist and Man of Letters,"** HERBERT J. MULLER. Yale Rev. **31**, No. 2, 279 (Dec., 1941). Similarities and differences again. Science has had some bad effects on literature, should be a co-worker; literature can give science perspective on its social function.

47. **The Myth of the Machine.** LEWIS MUMFORD. (Harcourt, Brace & World, Inc., New York, 1967). Important historical study of human cultural development, that shows a major shift of emphasis from human being to machine, questions our commitment to technical progress, and warns against the downplaying of literature and fine arts so vital to complete life experience. See also his earlier *Art and Technics* (Columbia University Press, New York, 1952).

48. **"Utopia, the City, and the Machine,"** LEWIS MUMFORD. Daedalus **94**, No. 2, of the Proceedings of the American Academy of Arts and Sciences, 271 (Spring, 1965). The machine has become a god beyond challenge. The only group to understand the dehumanizing effects and eventual price of technology are the *avant-garde* artists, who have resorted to caricature.

49. **"Utopias for Reformers,"** FRANCOIS BLOCH-LAINE.

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Daedalus **94,** No. 2, 419 (1965). Discusses the aim of two utopias—technological and democratic—to increase man's fulfillment by different approaches, which must be combined.

50. **"Utopia and the Good Life,"** GEORGE KATEB, Daedalus **94,** No. 2, 454 (1965). Describes the thrust of technology in freeing men from routine drudgery and setting up leisure and abundance, with attendant problems, however.

51. **Utopia and Utopian Thought.** FRANK MANUEL, Ed. (Houghton Mifflin Co., Boston, 1966). A gathering of the preceding three, and other materials in substantially the same form.

52. **Aesthetics and Technology in Building.** PIER LUIGI NERVI. (Harvard University Press, Cambridge, Mass., 1965). "Nervi's thesis is that good architecture is a synthesis of technology and art," according to an expert review by Carl W. Condit, in Technol. and Culture **7,** No. 3, 432 (Summer, 1966), which we also recommend.

53. **Liberal Learning for the Engineer.** STERLING P. OLMSTED. (Amer. Soc. Eng. Educ., Washington, D. C., 1968). The most recent and comprehensive report on the state of liberal studies in the engineering and technical colleges and institutes of the U. S. Theory, specific recommendations, bibliography.

54. **Road to Wigan Pier.** GEORGE ORWELL. (Berkley Publishing Corporation, New York, 1967). A paperback reissue of the 1937 work by the author of *1984*. Contains a 20-page digression, outspoken and controversial, on the evils of the machine, which has made a fully human life impossible, led to decay of taste, and acquired the status of a god beyond criticism.

55. **"Art and Technology: 'Cybernetic Serendipity',"** S. K. OVERBECK. The Alicia Patterson Fund, 535 Fifth Ave., New York, N. Y., 10017, SKO-1 (10 June 1968). The first of a dozen illustrated newsletter-articles by Overbeck that are published by the Fund. The series (space limitations forbid separate listings) describes various foreign exhibitions of computer music, electronic sculpture, and sound and light, which, in turn, recall "Nine Evenings: Theater and Engineering" staged in New York, fall 1966, by Experiments in Art and Technology, with outside help, "to familiarize the artist with the realities of technology while indulging the technician's penchant to transcend the mere potentialities of his discipline."

56. **"Myths, Emotions, and the Great Audience,"** JAMES PARSONS. Poetry **77,** 89 (Nov., 1950). Poetry is important to man's survival because it is a myth maker at a time when "it is the developing rationale of assembly-line production that all society be hitched to the machine."

57. **"Public and Private Problems in Modern Drama,"** RONALD PEACOCK. Tulane Drama Rev. **3,** No. 3, 58 (March, 1959). The dehumanizing effects of technocratic society as seen in modern plays going back as far as Georg Kaiser's *Gas* (1918).

58. **"The American Poet in Relation to Science,"** NORMAN HOLMES PEARSON. Amer. Quart. **1,** No. 2, 116 (Summer, 1949). Science and technology have done a service to poets by forcing them into new modes of expression; however, the poet remains the strongest force in the preservation of the freedom of the individual.

59. **Science, Faith and Society.** MICHAEL POLANYI. (University of Chicago Press, Chicago, 1964). Originally published by Oxford University Press, London, in 1946, this work appears in a new format with a new introduction by the author, which fits the present theme, inasmuch as it considers the idea that all great discoveries are beautiful and that scientific discovery is like the creative act in the fine arts.

60. **Avant-Garde: The Experimental Theater in France.** LEONARD C. PRONKO. (University of California Press, Berkeley, 1966). A keen analysis of the work of Beckett, Ionesco, Genêt, and others, which no longer reflects a rational world but the irrational world of the atom bomb.

61. **"Scientist and Humanist: Can the Minds Meet?"** I. I. RABI. Atlantic **197,** 64 (Jan., 1956). Discusses modern antiintellectualism and the urge to keep up with the Russians in technology. Calls for wisdom, which is unobtainable as long as sciences and humanities remain separate disciplines.

62. **"Integral Science and Atomized Art,"** EUGENE RABINOWITCH. Bull. Atomic Scientists **15,** No. 2, 65 (February, 1959). Through its own form and expression, art could help man find the harmony now threatened by the forces of atomism and fear of nuclear catastrophe.

63. **"Art and Life,"** SIR HERBERT READ. Saturday Evening Post **232,** 34 (26 Sept. 1959). Modern violence and restlessness stem in great part from a neurosis in men who have stopped making things by hand. Production, not grace or beauty, is the guiding force of technological civilization. Recommends the activity of art to release creative, rather than destructive, forces.

64. **The New Poets: American and British Poetry Since World War II.** M. L. ROSENTHAL. (Oxford University Press, New York, 1967). Detects a dominant concern among contemporary poets with violence and war and links it to a general alienation of sensibility, due in great part to the fact that human values are being displaced by technology.

65. **"The Vocation of the Poet in the Modern World,"** DELMORE SCHWARTZ. Poetry **78,** 223 (July, 1951). The vocation of the poet today is to maintain faith in and love of poetry, until he is destroyed as a human being by the doom of a civilization from which he has become alienated.

66. **"Science and Literature,"** ELIZABETH SEWELL. Commonweal **73,** No. 2, 218 (13 May 1966). Myth and the simple affirmation of the human mind and body are the only two forms of imagination capable of facing modern enormities. The two terminal points of our technological age were Auschwitz and

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Hiroshima; in literature about them, we may yet see "the affirmation of simple humanity."

67. **"Is Technology Taking Over?"** CHARLES E. SILBERMAN. Fortune **73**, No. 2, 112 (Feb., 1966). A brisk discussion of familiar topics: art as defense; technology as an end; dehumanization and destruction; mass idleness; meaninglessness. Technology may not determine our destiny, but it surely affects it and, in enlarging choice, creates new dangers. As the author points out, however, borrowing from Whitehead, the great ages have been the dangerous and disturbed ones.

68. **"One Way to Spell Man,"** WALLACE STEGNER. Saturday Rev. **41**, No. 21, 8; 43 (24 May 1958). Finds a real quarrel between the arts and technology but not between the arts and science, the latter two being open to exploitation by the technology of mass production. Reminds us that nonscientific experience is valid, and nonverifiable truth important.

69. **Victorians and the Machine: The Literary Response to Technology.** HERBERT L. SUSSMAN. (Harvard University Press, Cambridge, Mass., 1968). Does for English writers of the 19th century what Leo Marx [43] did for the Americans, with substantially similar conclusions. Writers stressed are Carlyle, Butler, Dickens, Wells, Ruskin, Kipling, and Morris, whose thought and art centered on the effects of mechanization on the intellectual and aesthetic life of their day. A major study of the machine as image, symbol, servant, and god—something feared and respected, ugly and beautiful, functional and destructive—as seen by the significant Victorian literary figures, this work also helps explain the thrust of much contemporary writing.

70. **"The Poet as Anti-Specialist,"** MAY SWENSON. Saturday Rev. **48**, No. 5, 16 (30 Jan. 1965). A poet tells how her art can show man how to stay human in a technologized age, compares and contrasts the languages of science and poetry, wonders about the denerving and desensualizing of astronauts "trained to become a piece of equipment."

71. **"The Poem as Defense,"** WYLIE SYPHER. Amer. Scholar **37**, 85 (Winter, 1967). The author is not worried about opposition between science and art, but about opposition between both of them together and technology. Technique can even absorb criticism of itself. Technological mentality kills the magic of surprise, grace, and chance. If Pop art, computer poetry, and obscene novels are insolent, society is even more so in trying to engineer people.

72. **Literature and Technology.** WYLIE SYPHER. (Random House, Inc., New York, 1968). The best, almost the only, general study of its kind, to be required reading along with Leo Marx [43] and Herbert Sussman [69]. Develops the thesis that technology dreads waste and, being concerned with economy and precaution, lives by an ethic of thrift. The humanities, including art, exist on the notion that every full life includes waste—of virtue, intention, thinking, and work. The thesis is illustrated by examples from literature and art. Although, historically, technology minimizes individual participation and resultant pleasure, Sypher concedes that lately "technology has been touched by the joy of finding in its solutions the play of intellect that satisfies man's need to invent."

73. **Dialogue on Technology.** ROBERT THEOBALD, Ed. (The Bobbs-Merrill Co., Inc., Indianapolis, 1967). Contains essays on the admiration of technique, human imagination in the space age, educational technology and value systems, technology and theology, technology and art.

74. **Science, Man and Morals.** W. H. THORPE. (Cornell University Press, Ithaca, N. Y., 1965). Brings out interplay among science, religion, and art, accenting an over-all tendency toward wholeness and unity. Traces modern plight in some degree to the Bomb.

75. **"Modern Literature and Science,"** I. TRASCHEN. College English **25**, 248 (Jan., 1964). Explores the common interests of scientist and poet in their search for truth as well as their differences, which produce alienation and literary reaction.

76. **"The New English Realism,"** OSSIA TRILLING. Tulane Drama Rev. **7**, No. 2, 184 (Winter, 1962). Ever since Osborne's "Look Back in Anger," the modern British theatre has shown a realism based on revolt against class structure and the dilemma of threatening nuclear destruction, although scarcely touching on the new technology itself.

77. **"The Poet in the Machine Age,"** PETER VIERECK. J. History Ideas **10**, No. 1, 88 (Jan., 1949). A classification of antimachine poets, who for esthetic, pious, instinctual, or timid reasons have backed away, and promachine poets, who, as materialists, cultists, or adapters, have used the new gadgets to advantage. We must try to unite the world of machinery and the world of the spirit, or "our road to hell will be paved with good inventions."

78. **The Industrial Muse.** With introduction by JEREMY WARBURG, Ed. (Oxford University Press, New York, 1958). An amusing and informative anthology of verse from 1754 to the 1950's dealing in all moods with engines, factories, steamboats, railways, machines, and airplanes.

79. **"Poetry and Industrialism,"** JEREMY WARBURG. Modern Language Rev. **53**, No. 2, 163 (1958). Treats the problem of imaginative comprehension as the modern poet strives to assimilate the new technology, make statements, and find terms for a new form of expression.

80. **Reflections on Big Science.** ALVIN WEINBERG. (The MIT Press, Cambridge, Mass., 1967). The director of Oak Ridge National Laboratory devotes his first chapter, "The Promise of Scientific Technology; The New Revolutions," to nuclear energy, cheap electricity, technology of information, the Bomb, and dealing with nuclear garbage. He calls upon the humanists to restore meaning and purpose to our lives.

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81. **The Theater of Protest and Paradox.** George Wellwarth. (New York University Press, 1964). A discussion of contemporary playwrights, e.g., Ionesco, who finds a machine-made preplanned city essentially drab; and Dürrenmatt, whose "The Physicists" teaches the lesson that mankind can be saved only through suppression of technical knowledge.
82. **Flesh of Steel: Literature and the Machine in American Culture.** Thomas Reed West. (Vanderbilt University Press, Nashville, Tenn., 1967). A consideration of the writings of Sherwood Anderson, Dos Passos, Sandburg, Sinclair Lewis, Mumford, and Veblen which, while conceding that most of them are antimachine most of the time, preaches the positive virtues of the Machine: law, order, energy, discipline, which, at a price, produce a city like New York, where artists and writers may live and work on their own terms who could not exist if the machine stopped.
83. **"The Discipline of the History of Technology,"** Lynn White, Jr. Eng. Educ. **54**, No. 10, 349 (June, 1964). Technologists have begun to see that they have an intellectual need for the knowledge of the tradition of what they are doing. Engineers, too, must meet the mark of a profession, namely, the knowledge of its history. Even the humanists are realizing what this explosive new discipline can contribute to their personal awareness.
84. **Drama in a World of Science.** Glynne Wickham. (University of Toronto Press, Toronto, Canada, 1962). Treats the renascence of English theatre in the 50's, the Bomb as topic, the individual confused by technology and its tyranny as protagonist, and mass conformity, violence, or apathy as themes.
85. **"The Scientist and Society."** J. Tuzo Wilson. Imperial Oil Rev., 20–22 (Dec., 1963). The humanist who pretends to have no interest in science and the technocrat who relies completely on science are equally deluded. Calls for tolerance and understanding among all intellectual disciplines. The scientist must reconsider his position *vis-à-vis* the humanities and the arts.
86. **"Science is Everybody's Business,"** J. Tuzo Wilson. Amer. Scientist **52**, 266A (1964). Includes new directions in technology.
87. **"On the History of Science,"** J. Tuzo Wilson, Saturday Rev. **47**, No. 18, 50 (2 May 1964). Suggests new university departments to train scientifically literate humanists.
88. **"The Long Battle between Art and the Machine."** Edgar Wind. Harper's 228, 65 (Feb., 1964). Contemplation of fake-modern buildings, dehumanized music, and mass-produced furniture raises once more the old question of whether the artist uses the machine or becomes its slave.

Postscript to Sec. III

Since most of the foregoing material is critical or expository, except for quoted illustration, readers may wish to make a start with firsthand creative literary pieces. Here are some suggestions (unless otherwise indicated, items are available in various paperback editions; see the current issue of *Paperbound Books in Print*, R. R. Bowker Co., New York).

Plays

On the theme of machine replacing man, there are two early modern classics for background:

89. **R.U.R.** Karel Capek.
90. **The Adding Machine.** Elmer Rice.

Three British plays deal directly with the Bomb, and the fourth, the only one available in paper, alludes to it:

91. **The Tiger and the Horse.** Robert Bolt. In *Three Plays* (Mercury Books, London, 1963).
92. **The Offshore Island.** Marghanita Laski. (Cresset Press, London, 1959).
93. **Each His Own Wilderness.** Doris Lessing. In *New English Dramatists*, E. Martin Browne, Ed. (Penguin Plays, London).
94. **Look Back in Anger.** John Osborne.

Two recent plays dealing with physicists:

95. **The Physicists.** Friedrich Durrenmatt.
96. **In the Matter of J. Robert Oppenheimer.** Heinar Kipphardt.

Fiction

A quartet of Utopian or anti-Utopian novels:

97. **Brave New World.** Aldous Huxley.
98. **Nineteen Eighty-Four.** George Orwell.
99. **Walden II.** B. F. Skinner.
100. **We.** E. Zamiatan.

A quartet of science fiction:

101. **Fahrenheit** *451*. Ray Bradbury.
102. **Canticle for Leibowitz.** Walter Miller, Jr.
103. **Player Piano.** Kurt Vonnegut, Jr.
104. **Cat's Cradle.** Kurt Vonnegut, Jr.

A trio of short stories:

105. **"By the Waters of Babylon,"** Stephen V. Benet.
106. **"The Portable Phonograph,"** Walter Van Tilburg Clark. In *The Art of Modern Fiction*, R. West and R. Stallman, Eds., alternate ed. (Holt, Rinehart, & Winston, Inc., New York, 1949).
107. **"The Machine Stops,"** E. M. Forster. In *Modern Short Stories*, L. Brown, Ed. (Harcourt, Brace & World, Inc., New York, 1937).

Poetry

See Ginestier [20], Warburg [78], and Boulton [4] above. Also:

108. The Modern Poets. John M. Brinnin and Bill Read, Eds. (McGraw-Hill Book Co., New York, 1963). Contains poems by Hoffman, Lowell, Moss, and Nemerov pertaining to the Bomb.

109. Weep Before God. John Wain. (The Macmillan Company, London, 1961). Sections VI–VII consider the Machine.

110. Wildtrack. John Wain. (The Macmillan Company, London, 1965). Pages 10–12 satirize Henry Ford and the assembly line.

111. Today's Poets. Chad Walsh, Ed. (Charles Scribner's Sons, New York, 1964). The Introduction mentions the Bomb, and a poem by Gil Orlovitz spoofs the computer.

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None of the above is responsible for errors, omissions, or final choices.

FILM LOOP NOTES

FILM LOOP 18 ONE-DIMENSIONAL COLLISIONS I

Two different head-on collisions of a pair of steel balls are shown. The balls hang from long, thin wires that confine each ball's motion to the same circular arc. The radius is large compared with the part of the arc, so the curvature is hardly noticeable. Since the collisions take place along a straight line, they can be called one-dimensional.

In the first example, ball B, weighing 350 grams, is initially at rest. In the second example, ball A, with a mass of 532 grams, is the one at rest.

With this film, you can make detailed measurements on the total momentum and energy of the balls before and after collision. Momentum is a vector, but in this one-dimensional case you need only worry about its sign. Since momentum is the product of mass and velocity, its sign is determined by the sign of the velocity.

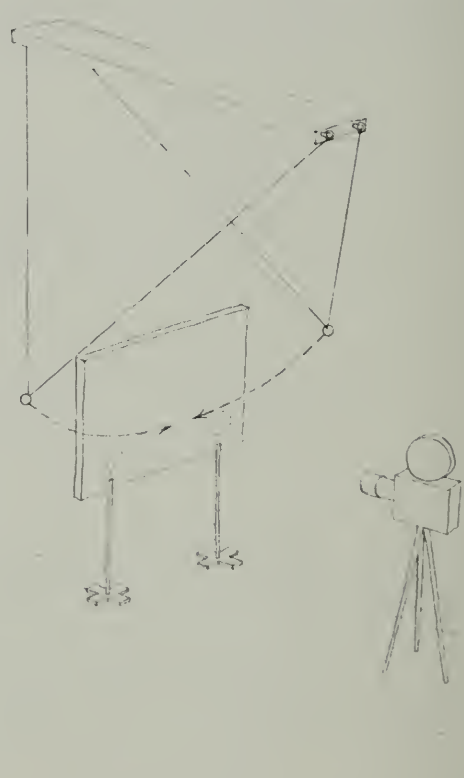
You know the masses of the balls. Velocities can be measured by finding the distance traveled in a known time.

After viewing the film, you can decide on what strategy to use for distance and time measurements. One possibility would be to time the motion through a given distance with a stopwatch, perhaps making two lines on the paper. You need the velocity just before and after the collision. Since the balls are hanging from wires, their velocity is not constant. On the other hand, using a small arc increases the chances of distance-time uncertainties. As with most measuring situations, a number of conflicting factors must be considered.

You will find it useful to mark the crosses on the paper on which you are projecting, since this will allow you to correct for projector movement and film jitter. You might want to give some thought to measuring distances. You may use a ruler with marks in millimeters, so you can estimate to a tenth of a millimeter. Is it wise to try to use the zero end of the ruler, or should you use positions in the middle? Should you use the thicker or the thinner marks on the ruler? Should you rely on one measurement, or should you make a number of measurements and average them?

Estimate the uncertainty in distance and time measurements, and the uncertainty in velocity. What can you learn from this about the uncertainty in momentum?

When you compute the total momentum



before and after collision (the sum of the momentum of each ball), remember that you must consider the direction of the momentum.

Are the differences between the momentum before and after collision significant, or are they within the experimental error already estimated?

Save the data you collect so that later you can make similar calculations on total kinetic energy for both balls just before and just after collision.

FILM LOOP 19 ONE-DIMENSION COLLISIONS II

Two different head-on collisions of a pair of steel balls are shown, with the same setup as that used in *Film Loop 18*, "One-Dimensional Collisions I."

In the first example, ball A with a mass of 1.8 kilograms collides head on with ball B, with a mass of 532 grams. In the second example, ball A catches up with ball B. The instructions for *Film Loop 18*, "One-Dimension Collisions I" may be followed for completing this investigation also.

FILM LOOP 20 INELASTIC ONE-DIMENSIONAL COLLISIONS

In this film, two steel balls covered with plasticene hang from long supports. Two collisions are shown. The two balls stick together after colliding, so the collision is "inelastic." In the first example, ball A, weighing 443 grams, is at rest when ball B, with a mass of 662 grams, hits it. In the second example, the same two balls move toward each other. Two other films, "One-Dimensional Collisions I" and "One-Dimensional Collisions II" show collisions where the two balls bounce off each other. What different results might you expect from measurements of an inelastic one-dimensional collision?

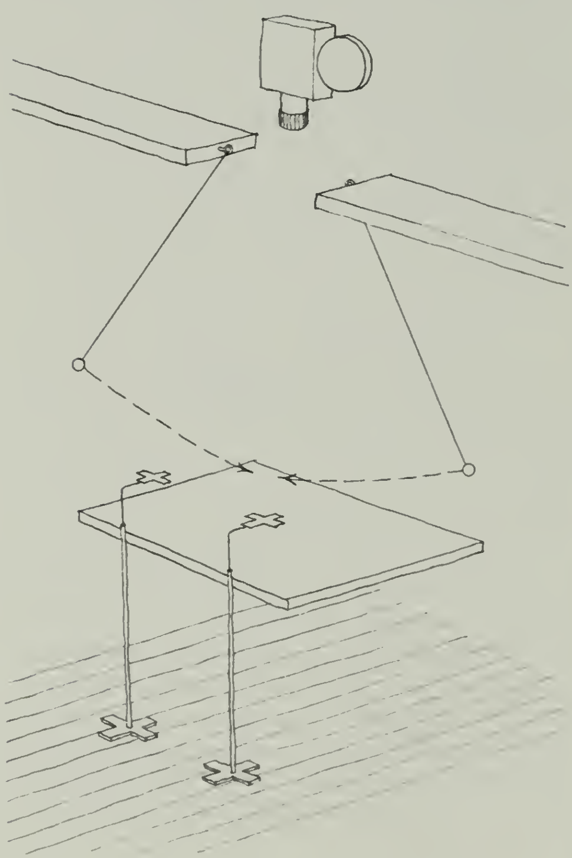
The instructions for *Film Loop 18*, "One-Dimensional Collisions I" may be followed for completing this investigation.

Are the differences between momentum before and after collision significant, or are they within the experimental error already estimated?

Save your data so that later you can make similar calculations on total kinetic energy for both balls just before and just after the collision. Is whatever difference you may have obtained explainable by experimental error? Is there a noticeable difference between elastic and inelastic collisions as far as the conservation of kinetic energy is concerned?

FILM LOOP 21 TWO-DIMENSIONAL COLLISIONS I

Two hard steel balls, hanging from long, thin wires, collide. Unlike the collisions in *Film Loops 18* and *20*, the balls do not move along the *same* straight line before or after the collisions. Although strictly the balls do not all move in a plane, as each motion is an arc of a circle, to a good approximation everything occurs in one plane. Hence, the collisions are two-dimensional. Two collisions are filmed in slow motion, with ball A having a mass of 539



grams, and ball B having a mass of 361 grams. Two more cases are shown in *Film Loop* 22.

Using this film, you can find both the momentum and the kinetic energy of each ball before and after the collision, and thus study total momentum and total kinetic energy conservation in this situation. Thus, you should save your momentum data for later use when studying energy.

Both direction and magnitude of momentum should be taken into account, since the balls do not move on the same line. To find momentum you need velocities. Distance measurements accurate to a fraction of a millimeter and time measurements to about a tenth of a second are suggested, so choose measuring instruments accordingly.

You can project directly onto a large piece of paper. An initial problem is to determine lines on which the balls move. If you make many marks at the centers of the balls, running the film several times, you may find that these do not form a perfect line. This is due both to the inaccuracies in your measurements and to the inherent difficulties of high speed photography. Cameras photographing at a rate of 2,000 to 3,000 frames a second "jitter," because the film moves so rapidly through the camera that accurate frame registration is not possible. Decide which line is the "best" approximation to determine direction for velocities for the balls before and after collision.

You will also need the *magnitude* of the velocity, the speed. One possibility is to measure the time it takes the ball to move across two lines marked on the paper. Accuracy suggests a number of different measurements to determine which values to use for the speeds and how much error is present.

Compare the sum of the momentum before collision for both balls with the total momentum after collision. If you do not know how to add vector diagrams, you should consult your teacher or the Programmed Instruction Booklet *Vectors II*. The momentum of each object is represented by an arrow whose direction is that of the motion and whose length is proportional to the magnitude of the momentum. Then, if the head of one arrow is placed on the tail of the other, moving the line parallel to itself, the vector sum is represented by the arrow which joins the "free" tail to the "free" head.

What can you say about momentum conservation? Remember to consider measurement errors.

FILM LOOP 22 TWO-DIMENSIONAL COLLISION II

Two hard steel balls, hanging from long thin wires, collide. Unlike the collisions in *Film Loops 18* and *20*, the balls do not move along the *same* straight line before or after the collisions. Although the balls do not strictly all move in a plane, as each motion is an arc of a circle, everything occurs in one plane. Hence, the collisions are two-dimensional. Two collisions are filmed in slow motion, with both balls having a mass of 367 grams. Two other cases are shown in *Film Loop 21*.

Using this film you can find both the kinetic energy and the momentum of each ball before and after the collision, and thus study total momentum and total energy conservation in this situation. Follow the instructions given for *Film Loop 21*, "Two-dimensional Collisions I," in completing this investigation.

FILM LOOP 23 INELASTIC TWO-DIMENSIONAL COLLISIONS

Two hard steel balls, hanging from long, thin wires, collide. Unlike the collisions in *Film Loops 18* and *20*, the balls do not move along the *same* straight line before or after the collision. Although the balls do not strictly all move in a plane, as each motion is an arc of a circle, to a good approximation the motion occurs in one plane. Hence, the collisions are two-dimensional. Two collisions are filmed in slow motion. Each ball has a mass of 500 grams. The plasticene balls stick together after collision, moving as a single mass.

Using this film, you can find both the kinetic energy and the momentum of each ball before and after the collision, and thus study total momentum and total energy conservation in this situation. Follow the instructions given for *Film Loop 21*, "Two-dimensional Collisions I," in completing this investigation.

FILM LOOP 24 SCATTERING OF A CLUSTER OF OBJECTS

This film and also Film Loop 3-8 each contain one advanced quantitative problem. We recommend that you do not work on these loops until you have analyzed one of the Events 8 to 13 of the series, *Stroboscopic Still Photographs of Two-Dimensional Collisions*, or one of the examples in the film loops entitled "Two-Dimensional Collisions: Part II," or "Inelastic Two-Dimensional Collisions." All these examples involve two-body collisions, whereas the film here described involves seven objects and *Film Loop* 25, five.

In this film seven balls are suspended from long, thin wires. The camera sees only a small portion of their motion, so the balls all move approximately along straight lines. The slow-motion camera is above the balls. Six balls are initially at rest. A hardened steel ball strikes the cluster of resting objects. The diagram in Fig. 1 shows the masses of each of the balls.

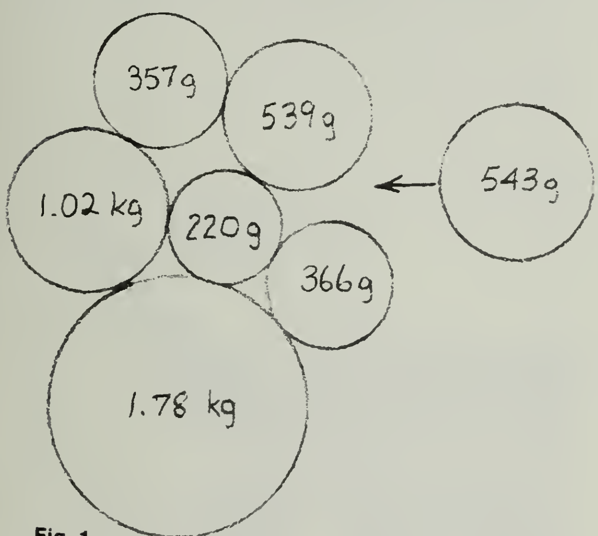
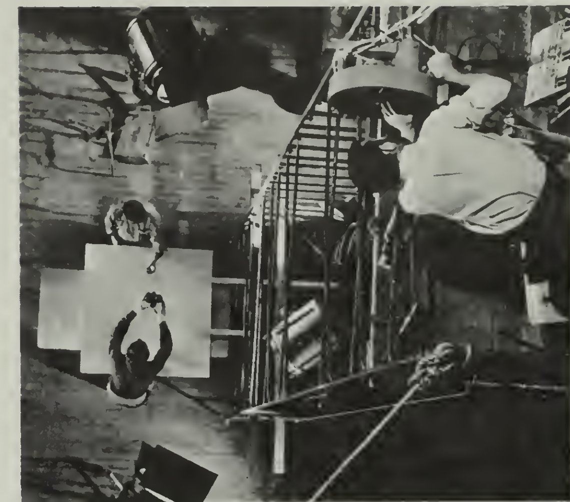


Fig. 1

Part of the film is photographed in slow motion at 2,000 frames per second. By projecting this section of the film on paper several times and making measurements of distances and times, you can determine the directions and magnitudes of the velocities of each of the balls. Distance and time measurements are needed. Discussions of how to make such measurements are contained in the Film Notes for one-dimensional and two-dimensional collisions. (See *Film Loops 18* and *21*.)



Compare the total momentum of the system both before and after the collision. Remember that momentum has both direction and magnitude. You can add momenta after collision by representing the momentum of each ball by an arrow, and "adding" arrows geometrically. What can you say about the accuracy of your calculations and measurements? Is momentum conserved? You might also wish to consider energy conservation.

FILM LOOP 25 EXPLOSION OF A CLUSTER OF OBJECTS

Five balls are suspended independently from long thin wires. The balls are initially at rest, with a small cylinder containing gunpowder in the center of the group of balls. The masses and initial positions of the ball are shown in Fig. 2. The charge is exploded and each of the balls moves off in an independent direction. In the slow-motion sequence the camera is mounted directly above the resting objects. The camera sees only a small part of the motion, so that the paths of the balls are almost straight lines.

In your first viewing, you may be interested

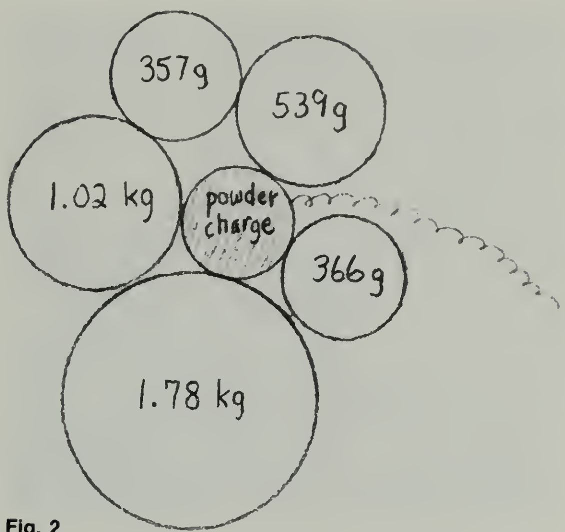


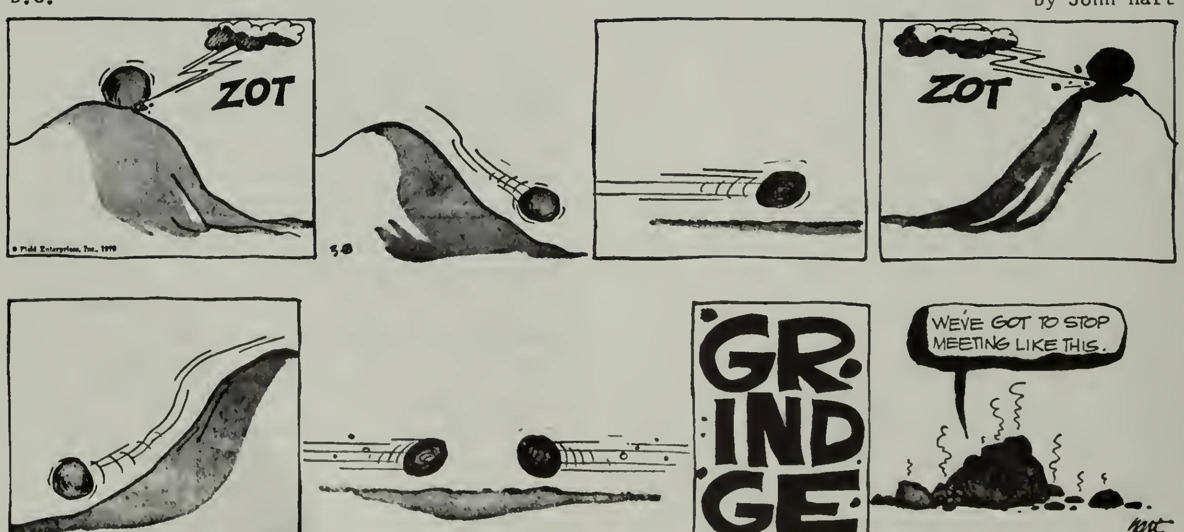
Fig. 2

in trying to predict where the "missing" balls will emerge. Several of the balls are hidden at first by the smoke from the charge of powder. All the balls except one are visible for some time. What information could you use that would help you make a quick decision about where this last ball will appear? What physical quantity is important? How can you use this quantity to make a quick estimate? When you see the ball emerge from the cloud, you can determine whether or not your prediction was correct. The animated elliptical ring identifies this final ball toward the end of the film.

You can also make detailed measurements, similar to the momentum conservation measurements you may have made using other Project Physics Film Loops. During the slow-motion sequence find the magnitude and direction of the velocity of each of the balls after the explosion by projecting the film on paper, measuring distances and times. The notes on previous films in this series, *Film Loops 18* and *21*, will provide you with information about how to make such measurements if you need assistance.

Determine the total momentum of all the balls after the explosion. What was the momentum before the explosion? You may find these results slightly puzzling. Can you account for any discrepancy that you find? Watch the film again and pay close attention to what happens during the explosion.

B.C. by John Hart



KINETIC ENERGY CALCULATIONS

You may have used one or more of *Film Loops 18* through 25 in your study of momentum. You will find it helpful to view these slow-motion films of one and two-dimensional collisions again, but this time in the context of the study of energy. The data you collected previously will be sufficient for you to calculate the kinetic energy of each ball before and after the collision. Remember that kinetic energy $\frac{1}{2}mv^2$ is *not* a vector quantity, and hence, you need only use the magnitude of the velocities in your calculations.

On the basis of your analysis you may wish to try to answer such questions as these: Is kinetic energy consumed in such interactions? If not, what happened to it? Is the loss in kinetic energy related to such factors as relative speed, angle of impact, or relative masses of the colliding balls? Is there a difference in the kinetic energy lost in elastic and inelastic collisions?

FILM LOOP 26 FINDING THE SPEED OF A RIFLE BULLET I

In this film a rifle bullet of 13.9 grams is fired into an 8.44 kg log. The log is initially at rest, and the bullet imbeds itself in the log. The two bodies move together after this violent collision. The height of the log is 15.0 centimeters. You can use this information to convert distances to centimeters. The setup is illustrated in Fig. 1 and 2.

Fig. 1

BALLISTIC PENDULUM

Fig. 2 Schematic diagram of ballistic pendulum (not to scale).

You can make measurements in this film using the extreme slow-motion sequence. The high-speed camera used to film this sequence operated at an average rate of 2850 frames per second; if your projector runs at 18 frames per second, the slow-motion factor is 158. Although there was some variation in the speed of this camera, the average frame rate of 2850 is quite accurate. For velocity measurements in centimeters per second, a convenient unit to use in considering a rifle bullet, convert the apparent time of the film to seconds. Find the exact duration with a timer or a stop-watch by timing the interval from the yellow circle at the beginning to the one at the end of the film. There are 3490 frames in the film, so you can determine the precise speed of the projector.

Project the film onto a piece of white paper or graph paper to make your measurements of distance and time. View the film before making decisions about which measuring instruments to use. As suggested above, you can convert your distance and time measurements to centimeters and seconds.

After measuring the speed of the log after impact, calculate the bullet speed at the moment when it entered the log. What physical laws do you need for the calculation? Calculate the kinetic energy given to the bullet, and also calculate the kinetic energy of the log after the bullet enters it. Compare these two energies and discuss any differences that you might find. Is kinetic energy conserved?

A final sequence in the film allows you to find a *lower limit* for the bullet's speed. Three successive frames are shown, so the time between each is 1/2850 of a second. The frames are each printed many times, so each is held on the screen. How does this lower limit compare with your measured velocity?

FILM LOOP 27 FINDING THE SPEED OF A RIFLE BULLET II

The problem proposed by this film is that of determining the speed of the bullet just before it hits a log. The wooden log with a mass of 4.05 kilograms is initially at rest. A bullet fired from a rifle enters the log. (Fig. 1.) The mass of the bullet is 7.12 grams. The bullet is imbedded in the thick log and the two move together after the impact. The extreme slow-motion sequence is intended for taking measurements.

The log is suspended from thin wires, so that it behaves like a pendulum that is free to swing. As the bullet strikes the log it starts to rise. When the log reaches its highest point, it momentarily stops, and then begins to swing back down. This point of zero velocity is visible in the slow-motion sequence in the film.

Fig. 1

The bullet plus the log *after* impact forms a closed system, so you would expect the total amount of mechanical energy of such a system to be conserved. The total mechanical energy is the sum of kinetic energy plus potential energy. If you conveniently take the potential energy as zero at the moment of impact for the lowest position of the log, then the energy at that time is all kinetic energy. As the log begins to move, the potential energy is proportional to the vertical distance above its lowest point, and it increases while the kinetic energy, depending upon the speed, decreases. The kinetic energy becomes zero at the point where the log reverses its direction, because the log's speed is zero at that point. All the mechanical energy at the reversal point is potential energy. Because energy is conserved, the initial kinetic energy at the lowest point should equal the potential energy at the top of the swing. On the basis of this result, write an equation that relates the initial log speed to the final height of rise. You might check this result with your teacher or with other students in the class.

If you measure the vertical height of the rise of the log, you can calculate the log's initial speed, using the equation just derived. What is the initial speed that you find for the log? If you wish to convert distance measurements to centimeters, it is useful to know that the vertical dimension of the log is 9.0 centimeters.

Find the speed of the rifle bullet at the moment it hits the log, using conservation of momentum.

Calculate the kinetic energy of the rifle bullet before it strikes and the kinetic energy of the log plus bullet after impact. Compare the two kinetic energies, and discuss any difference.

FILM LOOP 28 RECOIL

Conservation laws can be used to determine recoil velocity of a gun, given the experimental information that this film provides.

The preliminary scene shows the recoil of a cannon firing at the fort on Ste. Hélène Island,

Fig. 1

near Montreal, Canada. (Fig. 1.) The small brass laboratory "cannon" in the rest of the film is suspended by long wires. It has a mass of 350 grams. The projectile has a mass of 3.50 grams. When the firing is photographed in slow motion, you can see a time lapse between the time the fuse is lighted and the time when the bullet emerges from the cannon. Why is this delay observed? The camera used here exposes 8000 frames per second.

Project the film on paper. It is convenient to use a horizontal distance scale in centimeters. Find the bullet's velocity by timing the bullet over a large fraction of its motion. (Only relative values are needed, so it is not necessary to convert this velocity into cm/sec.)

Use momentum conservation to predict the gun's recoil velocity. The system (gun plus bullet) is one dimensional; all motion is along one straight line. The momentum before the gun is fired is zero in the coordinate system in which the gun is at rest. So the momentum of the cannon after collision should be equal and opposite to the momentum of the bullet.

Test your prediction of the recoil velocity by running the film again and timing the gun to find its recoil velocity experimentally. What margin of error might you expect? Do the predicted and observed values agree? Give reasons for any difference you observe. Is kinetic energy conserved? Explain your answer.

FILM LOOP 29 COLLIDING FREIGHT CARS

This film shows a test of freight-car coupling. The collisions, in some cases, were violent enough to break the couplings. The "hammer car" coasting down a ramp, reaches a speed of about 6 miles per hour. The momentary force between the cars is about 1,000,000 pounds. The photograph below (Fig. 1) shows cou-

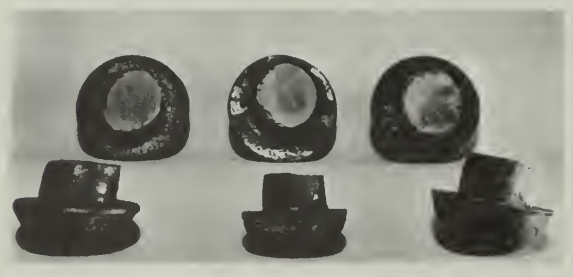


Fig. 1 Broken coupling pins from colliding freight cars.

pling pins that were sheared off by the force of the collision. The slow-motion collision allows you to measure speeds before and after impact, and thus to test conservation of momemtum. The collisions are *partially* elastic, as the cars separate to some extent after collision.

The masses of the cars are:
Hammer car: m_1 = 95,000 kg (210,000 lb)
Target car: m_2 = 120,000 kg (264,000 lb)

To find velocities, measure the film time for the car to move through a given distance. (You may need to run the film several times.) Use any convenient units for velocities.

Simple timing will give v_1 and v_2. The film was made on a cold winter day and friction was appreciable for the hammer car after collision. One way to allow for friction is to make a velocity time graph, assume a uniform negative acceleration, and extrapolate to the instant after impact.

An example might help. Suppose the hammer car coasts 3 squares on graph paper in 5 seconds after collision, and it also coasts 6 squares in 12 seconds after collision. The *average* velocity during the first 5 seconds was v_1 = (3 squares)/(5 sec) = 0.60 squares/sec. The average velocity during any short interval approximately equals the instantaneous velocity at the mid-time of that interval, so the car's velocity was about v_1 = 0.60 squares/sec at t = 2.5 sec. For the interval 0-12 seconds, the velocity was v_1 = 0.50 squares/sec at t = 6.0 sec. Now plot a graph like that shown in Fig. 1. This graph shows by extrapolation that v_1 = 0.67 squares/sec at t = 0, just after the collision.

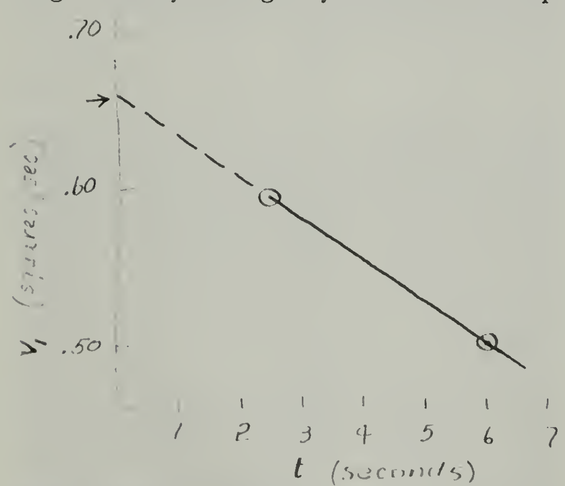


Fig. 1 Extrapolation backwards in time to allow for friction in estimating the value of v_1 immediately after the collision.

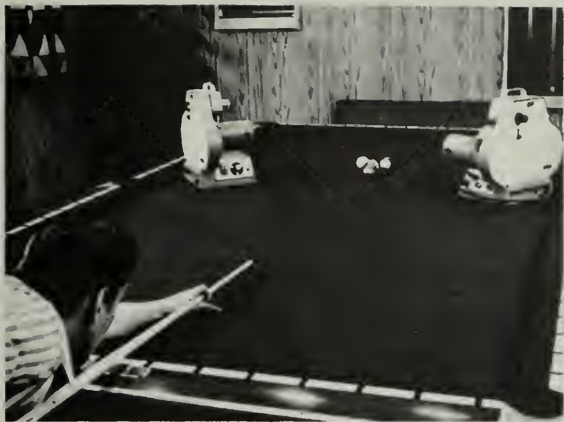
Compare the total momentum of the system before collision with the total momentum after collision. Calculate the kinetic energy of the freight cars before and after collision. What fraction of the hammer car's original kinetic energy has been "lost"? Can you account for this loss?

FILM LOOP 30 DYNAMICS OF A BILLIARD BALL

The event pictured in this film is one you have probably seen many times—the striking of a ball, in this case a billiard ball, by a second ball. Here, the camera is used to "slow down" time so that you can see details in this event which you probably have never observed. The ability of the camera to alter space and time is important in both science and art. The slow-motion scenes were shot at 3000 frames per second.

The "world" of your physics course often has some simplifications in it. Thus, in your textbook, much of the discussion of mechanics of bodies probably assumes that the objects are point objects, with no size. But clearly these massive billiard balls have size, as do all the things you encounter. For a point particle we can speak in a simple, meaningful way of its position, its velocity, and so on.

But the particles photographed here are billiard balls and not points. What information might be needed to describe their positions and velocities? Looking at the film may suggest possibilities. What motions can you see besides simply the linear forward motion? Watch each ball carefully, just before and just after the collision, watching not only the overall motion of the ball, but also "internal" motions. Can any of these motions be appropriately de-



Billiard balls near impact. The two cameras took side views of the collision, which are not shown in this film loop.

scribed by the word "spin"? Can you distinguish the cases where the ball is rolling along the table, so that there is no slippage between the ball and the table, from the situations where the ball is skidding along the table without rolling? Does the first ball move *immediately* after the collision? You can see that even this simple phenomenon is a good bit more complex than you might have expected.

Can you write a careful verbal description of the event? How might you go about giving a more careful mathematical description?

Using the slow-motion sequence you can make a momentum analysis, at least partially, of this collision. Measure the velocity of the cue ball before impact and the velocity of both balls after impact. Remember that there is friction between the ball and the table, so velocity is *not* constant. The balls have the same mass, so conservation of momentum predicts that

velocity of cue ball just before collision	=	sum of velocities of the balls just after collision

How closely do the results of your measurements agree with this principle? What reasons, considering the complexity of the phenomenon, might you suggest to account for any disagreement? What motions are you neglecting in your analysis?

FILM LOOP 31 A METHOD OF MEASURING ENERGY—NAILS DRIVEN INTO WOOD

Some physical quantities, such as distance, can be measured directly in simple ways. Other concepts can be connected with the world of experience only through a long series of measurements and calculations. One quantity that we often would like to measure is *energy*. In certain situations, simple and reliable methods of determining energy are possible. Here, you are concerned with the energy of a moving object.

This film allows you to check the validity of one way of measuring mechanical energy. If a moving object strikes a nail, the object will lose all of its energy. This energy has some effect, in that the nail is driven into the wood. The energy of the object becomes work done on the nail, driving it into the block of wood.

The first scenes in the film show a construction site. A pile driver strikes a pile over and over again, "planting" it in the ground. The laboratory situation duplicates this situation under more controlled circumstances. Each of the blows is the same as any other because the massive object is always raised to the same height above the nail. The nail is hit ten times. Because the conditions are kept the same, you expect the energy by the impact to be the same for each blow. Hence, the work from each blow is the same. Use the film to find if the distance the nail is driven into the wood is proportional to the energy or work. Or, better, you want to know how you can find the energy if you know the depth of penetration of the nail.

The simplest way to display the measurements made with this film may be to plot the depth of nail penetration versus the number of blows. Do the experimental points that you obtain lie approximately along a straight line? If the line is a good approximation, then the energy is about proportional to the depth of penetration of the nail. Thus, depth of penetration can be used in the analysis of other films to measure the energy of the striking object.

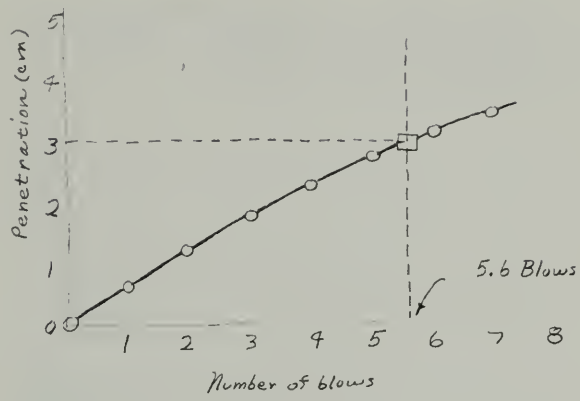


Fig. 1

If the graph is not a straight line, you can still use these results to calibrate your energy-measuring device. By use of penetration versus the number of blows, an observed penetration (in centimeters, as measured on the screen), can be converted into a number of blows, and therefore an amount proportional to the work done on the nail, or the energy transferred to the nail. Thus in Fig. 1, a penetration of 3 cm signifies 5.6 units of energy.

FILM LOOP 32 GRAVITATIONAL POTENTIAL ENERGY

Introductory physics courses usually do not give a complete definition of potential energy, because of the mathematics involved. Only particular kinds of potential energy, such as gravitational potential energy, are considered.

You may know the expression for the gravitational potential energy of an object near the earth—the product of the weight of the object and its height. The height is measured from a location chosen arbitrarily as the zero level for potential energy. It is almost impossible to "test" a formula without other physics concepts. Here we require a method of measuring energy. The previous *Film Loop 31* "A Method of Measuring Energy," demonstrated that the depth of penetration of a nail into wood, due to a blow, is a good measure of the energy at the moment of impact of the object.

Although you are concerned with potential energy you will calculate it by first finding kinetic energy. Where there is no loss of energy through heat, the sum of the kinetic energy and potential energy is constant. If you measure potential energy from the point at which the weight strikes the nail, at the moment of striking all the energy will be kinetic energy. On the other hand, at the moment an object is released, the kinetic energy is zero, and all the energy is potential energy. These two must, by conservation of energy, be equal.

Since energy is conserved, you can figure the initial potential energy that the object had from the depth of penetration of the nail by using the results of the measurement connecting energy and nail penetration.

Two types of measurements are possible with this film. The numbered scenes are all photographed from the same position. In the first scenes (Fig. 1) you can determine how gravitational potential energy depends upon weight. Objects of different mass fall from the same distance. Project the film on paper and measure the positions of the nailheads before and after the impact of the falling objects.

Fig. 1

Make a graph relating the penetration depth and the weight ma_g. Use the results of the previous experiment to convert this relation into a relation between gravitational potential energy and weight. What can you learn from this graph? What factors are you holding constant? What conclusions can you reach from your data?

Later scenes (Fig. 2), provide information for studying the relationship between gravitational potential energy and position. Bodies of equal mass are raised to different heights and allowed to fall. Study the relationship between the distance of fall and the gravitational potential energy. What graphs might be useful? What conclusion can you reach from your measurements?

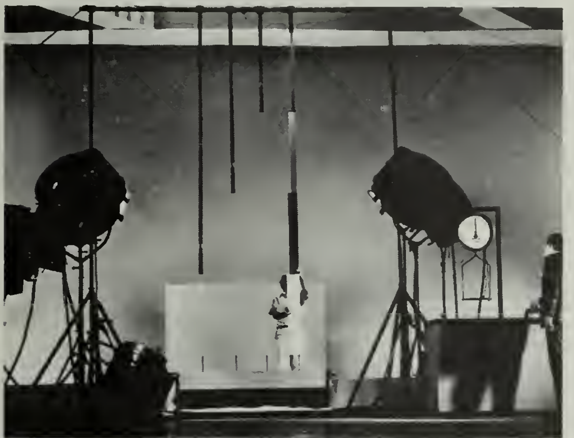


Fig. 2

Can you relate the results of these measurements with statements in your text concerning gravitational potential energy?

FILM LOOP 33 KINETIC ENERGY

In this film you can test how kinetic energy *(KE)* depends on speed *(v)*. You measure both *KE* and *v*, keeping the mass *m* constant.

Penetration of a nail driven into wood is a good measure of the work done on the nail, and hence is a measure of the energy lost by whatever object strikes the nail. The speed of the moving object can be measured in several ways.

The preliminary scenes show that the object falls on the nail. Only the speed just before the object strikes the nail is important. The scenes intended for measurement were photographed with the camera on its side, so the body appears to move horizontally toward the nail.

The speeds can be measured by timing the motion of the leading edge of the object as it moves from one reference mark to the other. The clock in the film (Fig. 1) is a disk that rotates at 3000 revolutions per minute. Project the film on paper and mark the positions of the clock pointer when the body crosses each reference mark. The time is proportional to the angle through which the pointer turns. The speeds are proportional to the reciprocals of the times, since the distance is the same in each case. Since you are testing only the *form* of the kinetic energy dependence on speed, any convenient unit can be used. Measure the speed for each of the five trials.

Fig. 1

The kinetic energy of the moving object is transformed into the work required to drive the nail into the wood. In *Film Loop 31,* "A Method of Measuring Energy," you relate the work to the distance of penetration. Measure the nail penetration for each trial, and use your results from the previous film.

How does *KE* depend on *v*? The conservation law derived from Newton's laws indicates that *KE* is proportional to v^2, the square of the speed, not to *v*. Test this by making two graphs. In one graph, plot *KE* vertically and plot v^2 horizontally. For comparison, plot *KE* versus *v*. What can you conclude? Do you have any assurance that a similar relation will hold, if the speeds or masses are very different from those found here? How might you go about determining this?

FILM LOOP 34 CONSERVATION OF ENERGY—POLE VAULT

This quantitative film can help you study conservation of energy. A pole vaulter (mass 68 kg, height 6 ft) is shown, first at normal speed and then in slow motion, clearing a bar at 11.5 ft. You can measure the total energy of the system at two points in time just before the jumper starts to rise and part way up, when the pole has a distorted shape. The total energy of the system is constant, although it is divided differently at different times. Since it takes work to bend the pole, the pole has elastic potential energy when bent. This elastic energy comes from some of the kinetic energy the vaulter has as he runs horizontally before inserting the pole in the socket. Later, the elastic potential energy of the bent pole is transformed into some of the jumper's gravitational potential energy when he is at the top of the jump.

Position 1 The energy is entirely kinetic energy, $\frac{1}{2}mv^2$. To help you measure the runner's speed, successive frames are held as the runner moves past two markers 1 meter apart. Each "freeze frame" represents a time interval of 1/250 sec, the camera speed. Find the runner's average speed over this meter, and then find the kinetic energy. If m is in kg and v is in m/sec, E will be in joules.

Position 2 The jumper's center of gravity is about 1.02 meters above the soles of his feet. Three types of energy are involved at the intermediate positions. Use the stop-frame sequence to obtain the speed of the jumper. The seat of his pants can be used as a reference. Calculate the kinetic energy and gravitational potential energy as already described.

The work done in deforming the pole is stored as elastic potential energy. In the final scene, a chain windlass bends the pole to a shape similar to that which it assumes during the jump in position 2. When the chain is shortened, work is done on the pole: work = (average force) × (displacement). During the cranking sequence, the force varied. The average force can be approximated by adding the initial and final values, found from the scale and dividing by two. Convert this force to newtons. The displacement can be estimated from the number of times the crank handle is pulled. A close-up shows how far the chain moves during a single stroke. Calculate the work done to crank the pole into its distorted shape.

You now can add and find the total energy. How does this compare with the original kinetic energy?

Position 3 Gravitational potential energy is the work done to raise the jumper's center of gravity. From the given data, estimate the vertical rise of the center of gravity as the jumper moves from position (1) to position (3). (His center of gravity clears the bar by about a foot, or 0.3m.) Multiply this height of rise by the jumper's weight to get potential energy. If weight is in newtons and height is in meters, the potential energy will be in joules. A small additional source of energy is in the jumper's muscles: judge for yourself how far he lifts his body by using his arm muscles as he nears the highest point. This is a small correction, so a relatively crude estimate will suffice. Perhaps he pulls with a force equal to his own weight through a vertical distance of $\frac{2}{3}$ of a meter.

How does the initial kinetic energy, plus the muscular energy expended in the pull-up, compare with the final gravitational potential energy? (An agreement to within about 10 per cent is about as good as you can expect from a measurement of this type.)

As a general reference see "Mechanics of the Pole Vault," 16th ed., by Dr. R. V. Ganslen; John Swift & Co., St. Louis, Mo. (1965).

FILM LOOP 35 CONSERVATION OF ENERGY—AIRCRAFT TAKEOFF

The pilot of a Cessna 150 holds the plane at constant speed in level flight, just above the surface of the runway. Then, keeping the throttle fixed, he pulls back on the stick, and the plane begins to rise. With the same throttle setting, he levels off at several hundred feet. At this altitude the aircraft's speed is less than at ground level. You can use this film to make a crude test of energy conservation. The plane's initial speed was constant, indicating that the net force on it was zero. In terms of an approximation, air resistance remained the same

after lift-off. How good is this approximation? What would you expect air resistance to depend on? When the plane rose, its gravitational potential energy increased, at the expense of the initial kinetic energy of the plane. At the upper level, the plane's kinetic energy is less, but its potential energy is greater. According to the principle of conservation of energy, the total energy $(KE + PE)$ remained constant, assuming that air resistance and any other similar factors are neglected. But are these negligible? Here is the data concerning the film and the airplane:

Length of plane: 7.5 m (23 ft)
Mass of plane: 550 kg
Weight of plane:
550 kg × 9.8 m/sec^2 = 5400 newtons (1200 lb)
Camera speed: 45 frames/sec

Project the film on paper. Mark the length of the plane to calibrate distances.

Stop-frame photography helps you measure the speed of 45 frames per second. In printing the measurement section of the film only every third frame was used. Each of these frames was repeated ("stopped") a number of times, enough to allow time to mark a position on the screen. The effect is one of "holding" time, and then jumping a fifteenth of a second.

Measure the speeds in all three situations, and also the heights above the ground. You have the data needed for calculating kinetic energy ($\frac{1}{2}mv^2$) and gravitational potential energy (ma_gh) at each of the three levels. Calculate the total energy at each of the three levels.

Can you make any comments concerning air resistance? Make a table showing (for each level) KE, PE, and E total. Do your results substantiate the law of conservation of energy within experimental error?

Steve Aacker of Wheat Ridge High School, Wheat Ridge, Colorado, seems a bit skeptical about elastic potential energy.

FILM LOOP 36 REVERSIBILITY OF TIME

It may sound strange to speak of "reversing time." In the world of common experience we have no control over time direction, in contrast to the many aspects of the world that we can modify. Yet physicists are much concerned with the reversibility of time; perhaps no other issue so clearly illustrates the imaginative and speculative nature of modern physics.

The camera gives us a way to manipulate time. If you project film backward, the events pictured happen in reverse time order. This film has sequences in both directions, some shown in their "natural" time order and some in reverse order.

The film concentrates on the motion of objects. Consider each scene from the standpoint of time direction: Is the scene being shown as it was taken, or is it being reversed and shown backward? Many sequences are paired, the same film being used in both time senses. Is it always clear which one is forward in time and which is backward? With what types of events is it difficult to tell the "natural" direction?

The Newtonian laws of motion do *not* depend on time direction. Any filmed motion of particles following strict Newtonian laws should look completely "natural" whether seen forward or backward. Since Newtonian laws are "invariant" under time reversal, changing the direction of time, you could not tell by examining a motion obeying these laws whether the sequence is forward or backward. Any motion which could occur forward in time can also occur, under suitable conditions, with the events in the opposite order.

With more complicated physical systems, with extremely large number of particles, the situation changes. If ink were dropped into water, you would have no difficulty in determining which sequence was photographed forward in time and which backward. So certain physical phenomena at least *appear* to be irreversible, taking place in only one time direction. Are these processes *fundamentally* irreversible, or is this only some limitation on human powers? This is not an easy question to answer. It could still be considered, in spite of a fifty-year history, a frontier problem.

Reversibility of time has been used in many ways in twentieth-century physics. For example, an interesting way of viewing the two kinds of charge in the universe, positive and negative, is to think of some particles as "moving" backward in time. Thus, if the electron is viewed as moving forward in time, the positron can be considered as exactly the same particle moving backward in time. This backward motion is equivalent to the forward-moving particle having the *opposite* charge! This was one of the keys to the development of the space-time view of quantum electrodynamics which R. P. Feynman described in his Nobel Prize lecture.

For a general introduction to time reversibility, see the Martin Gardner article, "Can Time Go Backward?" originally published in *Scientific American* January, 1967.

FILM LOOP 37 SUPERPOSITION

Using this film, you study an important physical idea—superposition. The film was made by photographing patterns displayed on the face of the cathode ray tube (CRT) of an oscilloscope, similar to a television set. You may have such an oscilloscope in your laboratory.

Still photographs of some of these patterns appearing on the CRT screen are shown in Figs. 1 to 2. The two patterns at the top of the screen are called *sinusoidal.* They are not just any wavy lines, but lines generated in a precise fashion. If you are familiar with the sine and cosine functions, you will recognize them here. The sine function is the special case where the origin of the coordinate system is located where the function is zero and starting to rise. No origin is shown, so it is arbitrary as to whether one calls these sine curves, cosine curves, or some other sinusoidal type. What physical situations might lead to curves of this type? (You might want to consult books of someone else about simple harmonic oscillators.) Here the curves are produced by electronic circuits which generate an electrical voltage changing in time so as to cause the curve to be displayed on the cathode ray

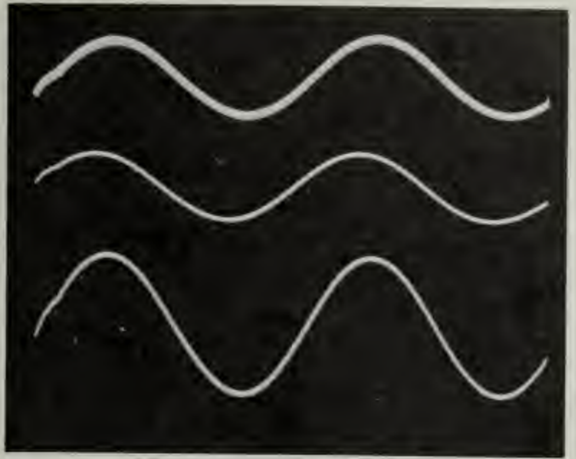


Fig. 1

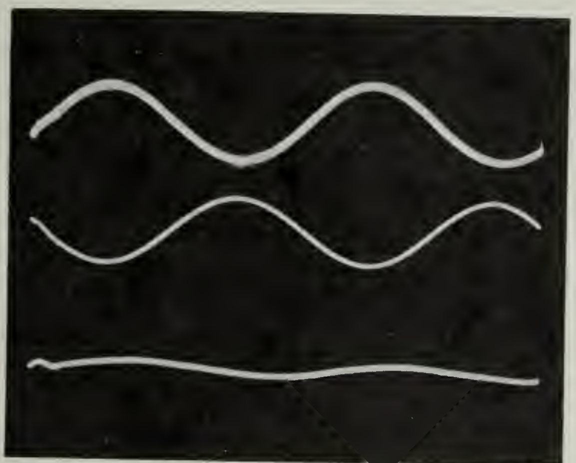


Fig. 2

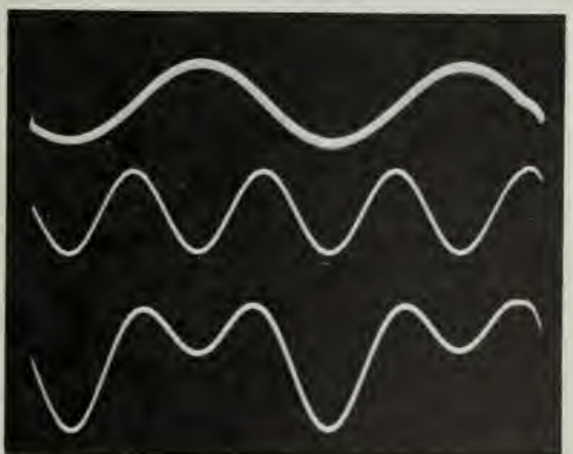


Fig. 3

tube. The oscilloscope operator can adjust the magnitudes and phases of the two top functions.

The bottom curve is obtained by a point-by-point adding of the top curves. Imagine a horizontal axis going through each of the two top curves, and positive and negative distances measured vertically from this axis. The bottom curve is at each point the algebraic sum of the two points above it on the top curves, as measured from their respective axes. This point-by-point algebraic addition, when applied to actual waves, is called superposition.

Two cautions are necessary. First, you are not seeing waves, but *models* of waves. A wave is a disturbance that propagates in time, but, at least in some of the cases shown, there is no propagation. A model always has some limitations. Second, you should not think that all waves are sinusoidal. The form of whatever is propagating can be any shape. Sinusoidal waves constitute only one important class of waves. Another common wave is the pulse, such as a sound wave produced by a sharp blow on a table. The pulse is *not* a sinusoidal wave.

Several examples of superposition are shown in the film. If, as approximated in Fig. 1, two sinusoidal curves of equal period and amplitude are in phase, both having zeroes at the same places, the result is a double-sized function of the same shape. On the other hand, if the curves are combined out of phase, where one has a positive displacement while the other one has a negative displacement, the result is *zero* at each point (Fig. 2). If functions of different periods are combined (Figs. 3, 4, and 5), the result of the superposition is not sinusoidal, but more complex in shape. You are asked to interpret both verbally and quantitatively, the superpositions shown in the film.

FILM LOOP 38 STANDING WAVES ON A STRING

Tension determines the speed of a wave traveling down a string. When a wave reaches a fixed end of a string, it is reflected back again. The reflected wave and the original wave are

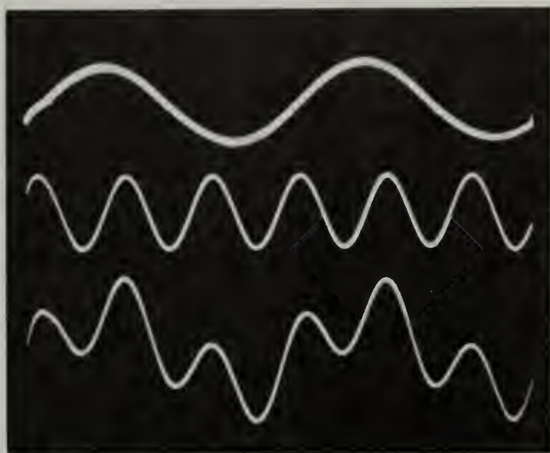


Fig. 4

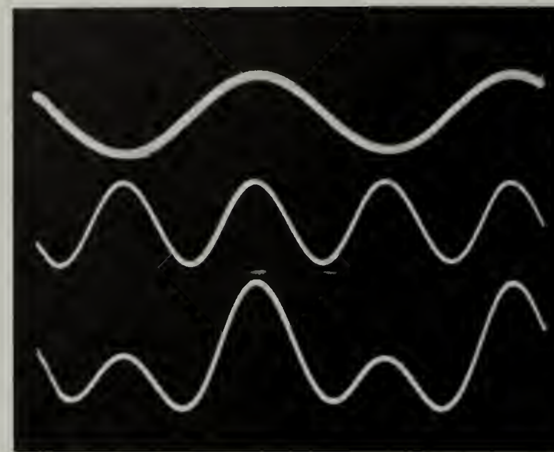


Fig. 5

superimposed or added together. If the tension (and therefore the speed) is just right, the resulting wave will be a "standing wave." Certain nodes will always stand still on the string. Other points on the string will continue to move in accordance with superposition. When the tension in a vibrating string is adjusted, standing waves can be set up when the tension has one of a set of "right" values.

In the film, one end of a string is attached to a tuning fork with a frequency of 72 vibrations per second. The other end is attached to a cylinder. The tension of the string is adjusted by sliding the cylinder back and forth.

Several standing wave patterns are shown. For example, in the third mode the string vibrates in 3 segments with 2 nodes (points of no motion) between the nodes at each end. The nodes are half a wavelength apart. Between the nodes are points of maximum possible vibration called antinodes.

You tune the strings of a violin or guitar by changing the tension on a string of fixed length, higher tension corresponding to higher pitch. Different notes are produced by placing a finger on the string to shorten the vibrating part. In this film the frequency of vibration of a string is fixed, because the string is always driven at 72 vib/sec. When the frequency remains constant, the wavelength changes as the tension is adjusted because velocity depends on tension.

A high-speed snapshot of the string at any time would show its instantaneous shape. Sections of the string move, except at the nodes. The eye sees a blurred or "time exposure" superposition of string shapes because of the frequency of the string. In the film, this blurred effect is reproduced by photographing at a slow rate: Each frame is exposed for about 1/15 sec.

Some of the vibration modes are photographed by a stroboscopic method. If the string vibrates at 72 vib/sec and frames are exposed in the camera at the rate of 70 times per sec, the string seems to go through its complete cycle of vibration at a slower frequency when projected at a normal speed. In this way, a slow-motion effect is obtained.

FILM LOOP 39 STANDING WAVES IN A GAS

Standing waves are set up in air in a large glass tube. (Fig. 1.) The tube is closed at one end by an adjustable piston. A loudspeaker at the other end supplies the sound wave. The speaker is driven by a variable-frequency oscillator and amplifier. About 20 watts of audiopower are used, giving notice to everyone in a large building that filming is in progress! The waves are reflected from the piston.

A standing wave is formed when the frequency of the oscillator is adjusted to one of several discrete values. Most frequencies do *not* give standing waves. Resonance is indi-

Fig. 1

cated in each mode of vibration by nodes and antinodes. There is always a node at the fixed end (where air molecules cannot move) and an antinode at the speaker (where air is set into motion). Between the fixed end and the speaker there may be additional nodes and antinodes.

The patterns can be observed in several ways, two of which are used in the film. One method of making visible the presence of a stationary acoustic wave in the gas in the tube is to place cork dust along the tube. At resonance the dust is blown into a cloud by the movement of air at the antinodes; the dust remains stationary at the nodes where the air is not moving. In the first part of the film, the dust shows standing wave patterns for these frequencies:

Frequency (vib/sec)	Number of half wavelengths
230	1.5
370	2.5
530	3.5
670	4.5
1900	12.5

The pattern for $f = 530$ is shown in Fig. 2. From node to node is $\frac{1}{2}\lambda$, and the length of the pipe is $3\lambda + \frac{1}{2}\lambda$ (the extra $\frac{1}{2}\lambda$ is from the speaker antinode to the first node). There are, generally, $(n + \frac{1}{2})$ half-wavelengths in the fixed length, so $\lambda \propto 1/(n + \frac{1}{2})$. Since $f \propto 1/\lambda$, $f \propto (n + \frac{1}{2})$. Divide each frequency in the table by $(n + \frac{1}{2})$ to find whether the result is reasonably constant.

In all modes the dust remains motionless near the stationary piston which is a node.

In the second part of the film nodes and antinodes are made visible by a different

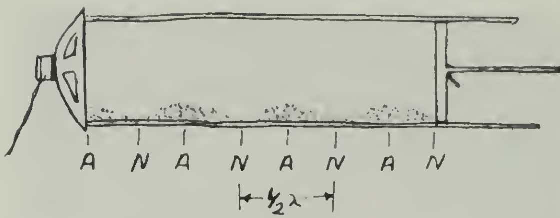


Fig. 2

method. A wire is placed in the tube near the top. This wire is heated electrically to a dull red. When a standing wave is set up, the wire is cooled at the antinodes, because the air carries heat away from the wire when it is in vigorous motion. So the wire is cooled at antinodes and glows less. The bright regions correspond to nodes where there are no air currents. The oscillator frequency is adjusted to give several standing wave patterns with successively smaller wavelengths. How many nodes and antinodes are there in each case? Can you find the frequency used in each case?

FILM LOOP 40 VIBRATIONS OF A WIRE

This film shows standing-wave patterns in thin but stiff wires. The wave speed is determined by the wire's cross section and by the elastic constants of the metal. There is no external tension. Two shapes of wire, straight and circular, are used.

The wire passes between the poles of a strong magnet. When a switch is closed, a steady electric current from a battery is set up in one direction through the wire. The interaction of this current and the magnetic field leads to a downward force on the wire. When the direction of the current is reversed, the force on the wire is upward. Repeated rapid reversal of the current direction can make the wire vibrate up and down.

The battery is replaced by a source of variable frequency alternating current whose frequency can be changed. When the frequency is adjusted to match one of the natural frequencies of the wire, a standing wave builds up. Several modes are shown, each excited by a different frequency.

The first scenes show a straight brass wire, 2.4 mm in diameter (Fig. 1). The "boundary

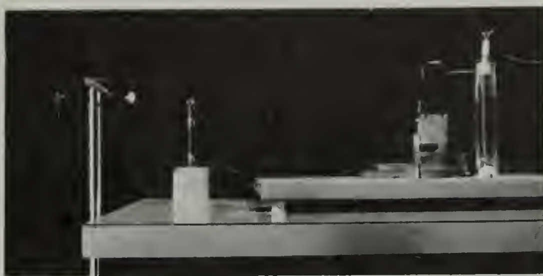


Fig. 1

conditions" for motion require that, in any mode, the fixed end of the wire is a node and the free end is an antinode. (A horizontal plastic rod is used to support the wire at another node.) The wire is photographed in two ways: in a blurred "time exposure," as the eye sees it, and in "slow motion," simulated through stroboscopic photography.

Study the location of the nodes and antinodes in one of the higher modes of vibration. They are not equally spaced along the wire, as for vibrating string (see *Film Loop 38*). This is because the wire is stiff whereas the string is perfectly flexible.

In the second sequence, the wire is bent into a horizontal loop, supported at one point (Fig. 2). The boundary conditions require a node at this point; there can be additional nodes, equally spaced around the ring. Several modes are shown, both in "time exposure" and in "slow motion." To some extent the vibrating circular wire is a helpful model for the wave behavior of an electron orbit in an atom such as hydrogen; the discrete modes correspond to discrete energy states for the atom.

Fig. 2

FILM LOOP 41 VIBRATIONS OF A RUBBER HOSE

You can generate standing waves in many physical systems. When a wave is set up in a medium, it is usually reflected at the boundaries. Characteristic patterns will be formed, depending on the shape of the medium, the frequency of the wave, and the material. At certain points or lines in these patterns there is no vibration, because all the partial waves passing through these points just manage to cancel each other through superposition.

Standing-wave patterns only occur for certain frequencies. The physical process selects a *spectrum* of frequencies from all the possible ones. Often there are an infinite number of such discrete frequencies. Sometimes there are simple mathematical relations between the selected frequencies, but for other bodies the relations are more complex. Several films in this series show vibrating systems with such patterns.

This film uses a rubber hose, clamped at the top. Such a stationary point is called a node. The bottom of the stretched hose is attached to a motor whose speed is increased during the film. An eccentric arm attached to the motor shakes the bottom end of the hose. Thus this end moves slightly, but this motion is so small that the bottom end also is a node.

The motor begins at a frequency below that for the first standing-wave pattern. As the motor is gradually speeded up, the amplitude of the vibrations increase until a well-defined

Fig. 1

steady loop is formed between the nodes. This loop has its maximum motion at the center. The pattern is half a wavelength long. Increasing the speed of the motor leads to other harmonics, each one being a standing-wave pattern with both nodes and antinodes, points of maximum vibration. These resonances can be seen in the film to occur only at certain sharp frequencies. For other motor frequencies, no such simple pattern is seen. You can count as many as eleven loops with the highest frequency case shown.

It would be interesting to have a sound track for this film. The sound of the motor is by no means constant during the process of increasing the frequency. The stationary resonance pattern corresponds to points where the motor is running much more quietly, because the motor does not need to "fight" against the hose. This sound distinction is particularly noticeable in the higher harmonics.

If you play a violin cello, or other stringed instrument, you might ask how the harmonies observed in this film are related to musical properties of vibrating strings. What can be done with a violin string to change the frequency of vibration? What musical relation exists between two notes if one of them is twice the frequency of the other?

What would happen if you kept increasing the frequency of the motor? Would you expect to get arbitrarily high resonances, or would something "give?"

FILM LOOP 42 VIBRATIONS OF A DRUM

The standing-wave patterns in this film are formed in a stretched circular rubber membrane driven by a loudspeaker. The loudspeaker is fed large amounts of power, about 30 watts, more power than you would want to use with your living room television set or phonograph. The frequency of the sound can be changed electronically. The lines drawn on the membrane make it easier for you to see the patterns.

The rim of the drum cannot move, so in all cases it must be a nodal circle, a circle that does not move as the waves bounce back and forth on the drum. By operating the camera at a frequency only slightly different from the resonant frequency, a stroboscopic effect enables you to see the rapid vibrations as if in slow motion.

In the first part of the film, the loudspeaker is directly under the membrane, and the vibratory patterns are symmetrical. In the fundamental harmonic, the membrane rises and falls as a whole. At a higher frequency, a second circular node shows up between the center and the rim.

In the second part of the film, the speaker is placed to one side, so that a different set of modes, asymmetrical modes, are generated in the membrane. You can see an antisymmetrical mode where there is a node along the diameter, with a hill on one side and a valley on the other.

Various symmetric and antisymmetric vibration modes are shown. Describe each mode, identifying the nodal lines and circles.

In contrast to the one-dimensional hose in *Film Loop 41* there is no *simple* relation of the resonant frequencies for this two-dimensional system. The frequencies are *not* integral multiples of any basic frequency. There is a relation between values in the frequency spectrum, but it is more complex than that for the hose.

FILM LOOP 43 VIBRATIONS OF A METAL PLATE

The physical system in this film is a square metal plate. The various vibrational modes are produced by a loudspeaker, as with the vibrating membrane in *Film Loop 42*. The metal plate is clamped at the center, so that point is always a node for each of the standing-wave patterns. Because this is a stiff metal plate, the vibrations are too slight in amplitude to be seen directly. The trick used to make the patterns visible is to sprinkle sand along the plates. This sand is jiggled away from the parts of the plates which are in rapid motion, and tends to fall along the nodal lines, which are not moving. The beautiful patterns of sand are known as Chladni figures. These patterns have often been much admired by

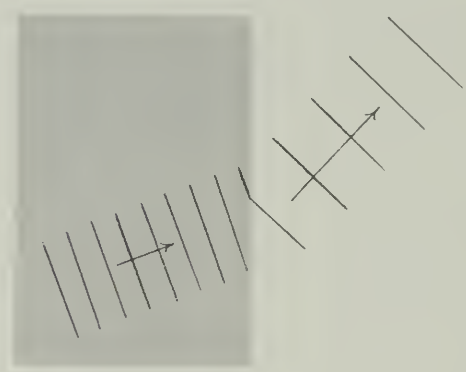
artists. These and similar patterns are also formed when a metal plate is caused to vibrate by means of a violin bow, as seen at the end of this film, and in the Activity, "Standing Waves on a Drum and a Violin."

Not all frequencies will lead to stable patterns. As in the case of the drum, these harmonic frequencies for the metal plate obey complex mathematical relations, rather than the simple arithmetic progression seen in a one-dimensional string. But again they are discrete events. As the frequency spectrum is scanned, only at certain sharp well-defined frequencies are these elegant patterns produced.

Q10 Both will increase.
Q11 Answer (c)
Q12 Answer (a)
Q13 Answers a, b, c are correct
Q14 (a) unbroken egg
(b) a glass of ice and warm water
Q15 (a) True
(b) False
(c) False
Q16 Answer (b)

Chapter 12

Q1 Transverse, longitudinal and torsional
Q2 Longitudinal. Fluids can be compressed but they are not stiff enough to be bent or twisted.
Q3 Transverse
Q4 No. The movement of the bump in the rug depends on the movement of the mouse; it does not go on by itself.
Q5 Energy (Particles of the medium are *not* transferred along the direction of the wave motion.)
Q6 The stiffness and the density
Q7 (1) Wavelength, amplitude, polarization
(2) Frequency, period
Q8 The distance between any two successive points that have identical positions in the wave pattern.
Q9 (1) 100 cps
(2) $T = \frac{1}{f} = \frac{1}{100 \text{ cps}} = 0.01$ sec.
(3) $\lambda = \frac{v}{f} = \frac{10 \text{ m/sec}}{100 \text{ cps}} = 0.1$ meter
Q10 Answer (b)
Q11 $A_1 + A_2$
Q12 Yes. The resulting displacement would be $5 + (-6) = -1$ cm
Q13 Cancellation
Q14 Antinodal lines are formed by a series of antinodal points. Antinodal points are places where waves arrive in phase and maximum reinforcement occurs. (The amplitude there is greatest.)
Q15 Answer (a)
Q16 When the difference in path lengths to the two sources is an odd number of half wavelengths ($\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.).
Q17 (1) No motion at the nodes
(2) Oscillates with maximum amplitude
Q18 $\frac{\lambda}{2}$
Q19 $2L$, so that one-half wavelength just fits on the string.
Q20 No, only frequencies which are whole number multiples of the fundamental frequency are possible.
Q21 All points on a wave front have the same phase; that is, they all correspond to crests or troughs (or any other set of similar parts of the wavelength pattern).
Q22 Every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation.
Q23 If the opening is less than one-half a wavelength wide the difference in distance to a point P from the two edges of the opening cannot be equal to $\lambda/2$.
Q24 As the wavelength increases, the diffraction pattern becomes more spread out and the number of nodal lines decreases until pattern resembles one half of that produced by a point source oscillator.
Q25 Yes to both (final photograph shows diffraction without interference; interference occurs whenever waves pass each other).
Q26 A ray is a line drawn perpendicular to a wave front and indicates the direction of propagation of the wave.
Q27 The angles are equal.
Q28 Parabolic
Q29 The reflected wave fronts are parallel wave fronts.
Q30 (1) Stays the same
(2) Becomes smaller
(3) Changes so that the wave fronts are more nearly parallel to the boundary. (Or its direction of propagation becomes closer to the perpendicular between the media.)
Q31



Q32 (1) $f\lambda = v$ relationship
(2) Reflection
(3) Refraction
(4) Diffraction
(5) Interference
Q33 Sound waves are longitudinal.

Brief Answers to Study Guide Questions

Chapter 9

9.1 Information
9.2 Discussion
9.3 (a) Yes
(b) The solar system
9.4 Discussion
9.5 No
9.6 Discussion
9.7 (a) 220.2 g
(b) 20.2 g
9.8 Derivation
9.9 (a) All except v_A' (which $= v_B'$)
(b) $v_A' = \frac{m_A v_A}{m_A + m_B}$
(c) 0.8 m/sec
9.10 Dictionary comment
9.11 3.3×10^{-6} kg
9.12 Discussion
9.13 Derivation
9.14 Discussion
9.15 (a) 0.2 sec
(b) About 0.05 m
(c) 5×10^{-14} m/sec
(d) 2.5×10^{-15} m
(e) About 15×10^8 m^2 or a square of about 40 km on a side
9.16 Yes
9.17 Derivation
9.18 Discussion
9.19 1.2×10^3 kg m/sec; 4×10^2 newtons; 30 meters
9.20 (a) about 100 m/sec
(b) about 4.6 kg m/sec
(c) less than 0.003 sec
(d) at least 1.5×10^3 newtons
9.21 Yes
9.22 Derivation
9.23 (a) $\Delta t = \frac{m\,(v_o - v)}{F}$
(b) $m\,(v_o - v)$
(c) $\frac{m\,(v_o - v)}{v_e}$
9.24 Derivation
9.25 10 m/sec
9.26 10.5×10^8 kg m/sec
9.27 Discussion
9.28 Discussion
9.29 Discussion
9.30 Discussion
9.31 Discussion
9.32 (a) 0.8 × mass of ball
(b) −0.8 × mass of ball
(c) 1.6 × mass of ball
(d) Depends on system considered
9.33 Discussion
9.34 Derivation
9.35 Table
9.36 Derivation
9.37 Both speeds $= \frac{v}{2}$ but in opposite directions
9.38 Discussion
9.39 Discussion

Chapter 10

10.1 Information
10.2 Discussion
10.3 (a) $v_1 - u$ and $v_2 - u$
(b) No
(c) No
(d) No
(e) Yes
(f) iii
(g) Discussion
10.4 5×10^{-15} joules, 2×10^{14} electrons
10.5 (a) 67.5 joules
(b) 4.5×10^9 joules
(c) 3.75×10^3 joules
(d) 2.7×10^{33} joules
10.6 (a) 2 m/sec^2, 30 sec, 60 m/sec
(b) 60 m/sec
10.7 (a) −90 joules
(b) 90 joules
(c) 18×10^2 newtons
10.8 2.3×10^2 joules
10.9 (a) 2.2×10^{-3} joules
(b) 5.4×10^{-2} joules
10.10 (a) 0.2 meter
(b) 7×10^9 joules
10.11 Discussion
10.12 Discussion
10.13 (a) 1.1×10^{12} seconds (about 3×10^4 years)
(b) 1.6×10^{-25} meters
10.14 Discussion
10.15 Discussion
10.16 Derivation
10.17 Discussion
10.18 Sketch
10.19 Proof
10.20 (a) 96×10^8 joules
(b) 8.8×10^2 meters
(c) 48×10^5 newtons
(d) Discussion
(e) Discussion
10.21 Discussion
10.22 Discussion
10.23 Discussion
10.24 Discussion
10.25 (b)
10.26 No
10.27 (a) >1000
(b) Discussion
10.28 1/8° C; no
10.29 Rowing 1375 watts or 1.8 H.P.
10.30 1/4 kg
10.31 21.5 days

10.32 Discussion
10.33 Discussion
10.34 Discussion
10.35 Discussion
10.36 Discussion
10.37 (a) Discussion
(b) Greater in lower orbit
(c) Less
(d) Less
(e) Discussion
10.38 (a) Discussion
(b) i: all three, ii: all three, iii: ΔH, iv: ΔH, v: all three, vi: ΔH
10.39 Discussion
10.40 Discussion

Chapter 11

11.1 Information
11.2 Discussion
11.3 Discussion
11.4 Discussion
11.5 No
11.6 Discussion
11.7 (a) 10^{-9} m
(b) 10^{-9} m
11.8 (a) 10^{21}
(b) 10^{18}
11.9 *Zero* meters
11.10 10.5 kilometers
11.11 Shoes—about 1/7 atm
Skis—about 1/60 atm
Skates—about 3 atm
11.12 Derivation
11.13 Discussion
11.14 Discussion
11.15 Discussion
11.16 Derivation
11.17 No change
11.18 Pressure, mass, volume, temperature
11.19 Discussion
11.20 Discussion
11.21 Discussion
11.22 Discussion
11.23 Discussion
11.24 Discussion
11.25 Discussion
11.26 Temperature will rise
11.27 No
11.28 Discussion
11.29 Discussion
11.30 Discussion
11.31 Discussion
11.32 Discussion
11.33 Discussion
11.34 Discussion
11.35 Discussion

Chapter 12

12.1 Information
12.2 Discussion
12.3 Discussion
12.4 Construction
12.5 Construction
12.6 Discussion
12.7 Discussion
12.8 Construction
12.9 Discussion
12.10 Discussion
12.11 Construction
12.12 Derivation
12.13 No; discussion
12.14 (a) $3/4\ L$
(b) $2/3\ L$
(c) $1/2\ L$
12.15 (a) $\lambda = 4L$
(b) $\lambda = \dfrac{4L}{2n + l}$
(c) $\lambda = 2L$, $\lambda = \dfrac{4L}{n + l}$ ($n = 0, 1, 2, 3$, etc.)
12.16 Discussion
12.17 Maximum
12.18 100 and 1000 cps; yes
12.19 Discussion
12.20 Discussion
12.21 Construction
12.22 Straight line
12.23 $\dfrac{R}{2}$
12.24 Construction
12.25 $\dfrac{1}{4k}$
12.26 Discussion
12.27 Two straight-line waves inclined toward each other.
12.28 Discussion
12.29 (a) $\theta_A = \angle BAD$
(b) $\theta_B = \angle CDA$
(c) $\lambda_A = BD$
(d) $\lambda_B = AC$
(e) Derivation
(f) Derivation
12.30 $\lambda_P = 0.035$ m
$\lambda_S = 0.025$ m
12.31 Discussion
12.32 (a) 1.27×10^{-11} watts
(b) 8×10^{12} mosquitoes
(c) subway train
12.33 $2d = vt$
12.34 1000 cps: { air 1.125 ft; sea water 4.8 ft; steel 16 ft
One tenth of each of these values for 10,000 cps
Discussion
12.35 3×10^5 cps
2.5×10^7 cps

INDEX/TEXT

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