Frequency entangled photons

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What for?

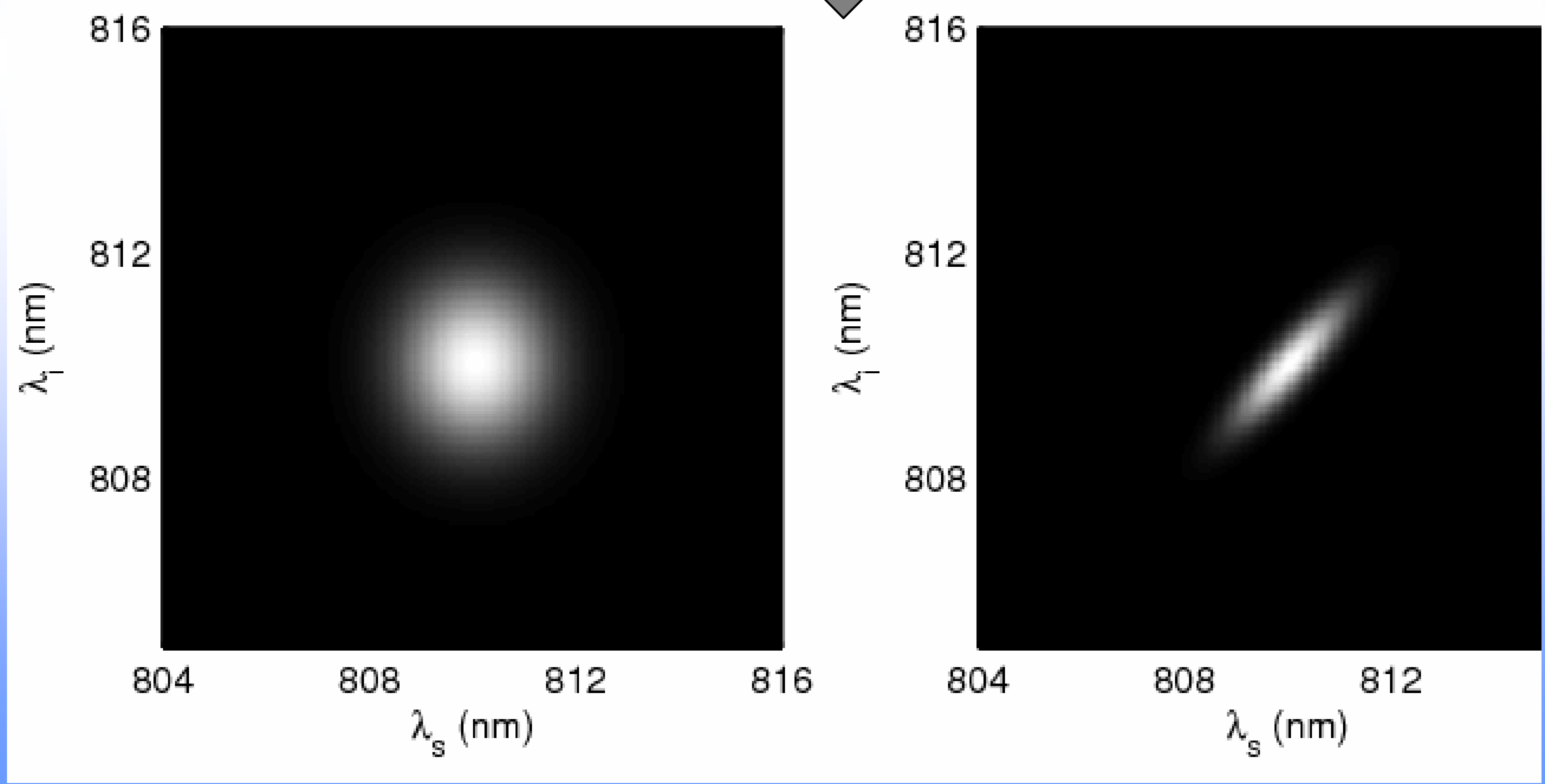
- You have an infinite dimensional Hilbert space that you could use to implement protocols

- You want perfect entanglement in polarization, then you DON'T want entanglement in frequency

- New applications: timing, OCT, etc.

Frequency entanglement

$$|\psi\rangle = \int d\omega_s \int d\omega_i \Phi(\omega_s,\omega_i) a_s^+(\omega_s) a_i^+(\omega_i) |0,0\rangle$$



Frequency entanglement

- It is difficult to measure the frequency entangled state:
  - We don't have the equivalent of the transversal modes

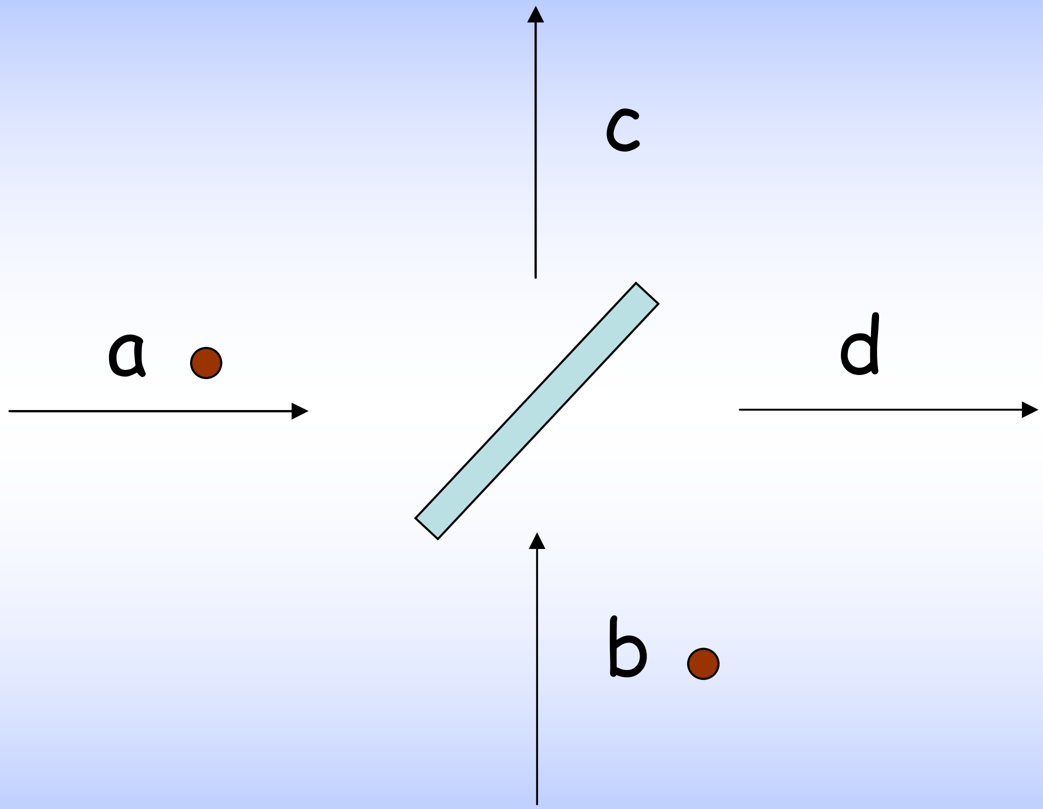
- Realization of quantum information protocols with frequency states is VERY difficult

Measuring frequency states

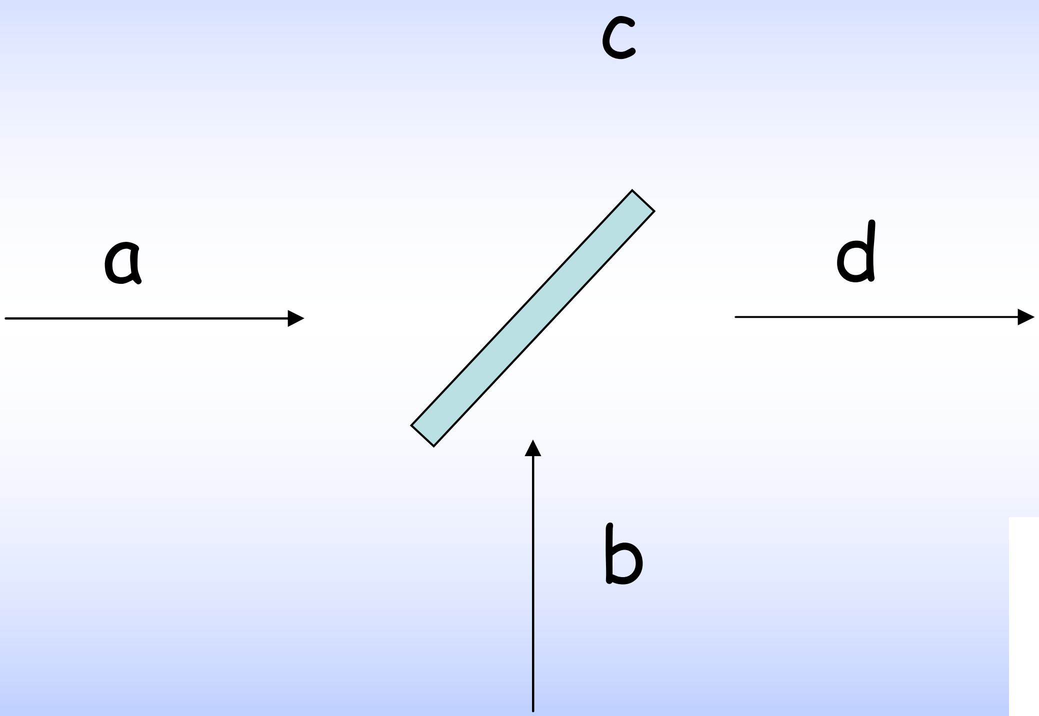
- We can use frequency filters: then, we don't have information on the phases of the states

- We could use time measurements BUT:
  - Present detectors are orders of magnitudes slower than needed
  - Turn to Hong Ou Mandel type measurements

Hong Ou Mandel



Hong Ou Mandel



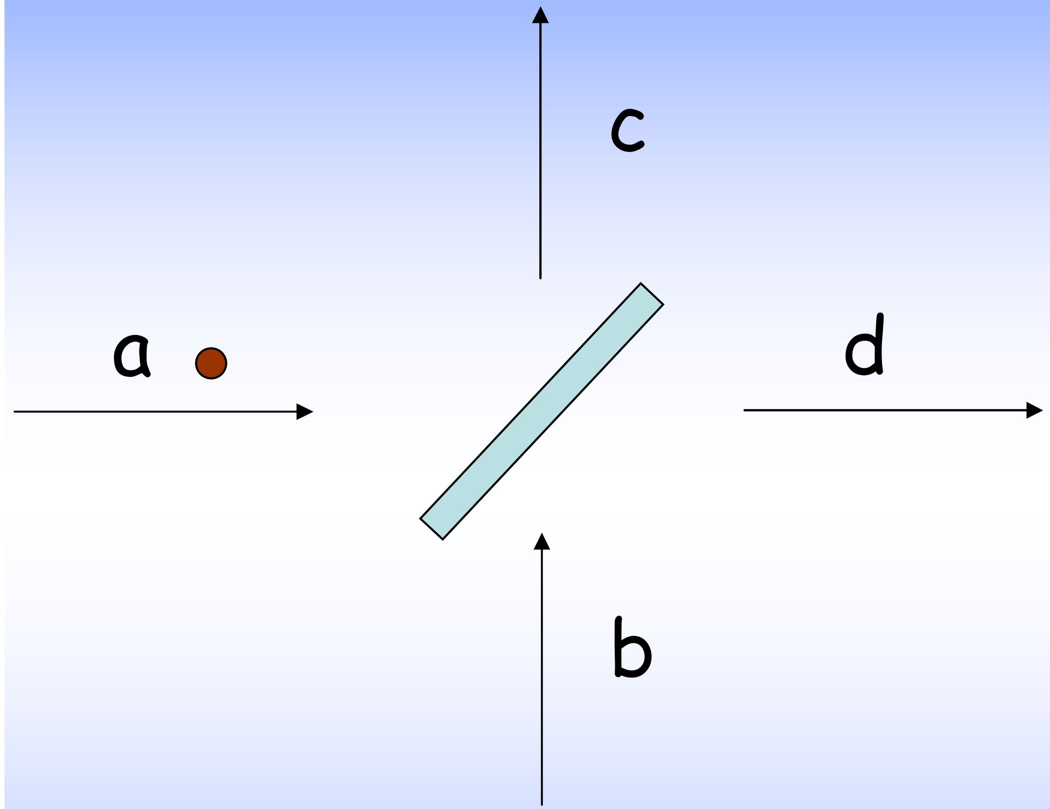
$$c = Ra + Tb$$

$$d = Ta + Rb$$

$$|R|^2 + |T|^2 = 1$$

$$arg(R^*T) = \pi/2$$

Hong Ou Mandel



$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\langle c^\dagger c\rangle$$

Hong Ou Mandel

$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\langle c^\dagger c\rangle = \langle\Psi|(R^* a^\dagger + T^* b^\dagger)(Ra^\dagger + Tb)|\Psi\rangle =$$

$$\langle 0|a(|R|^2 a^\dagger a + |T|^2 b^\dagger b + R^* T a b^\dagger + R T^* a^\dagger b)a^\dagger|0\rangle$$

$$= |R|^2\langle 0|aa^\dagger a a^\dagger|0\rangle$$

$$\langle 0|aa^\dagger a a^\dagger|0\rangle = \langle 0|aa^\dagger[a, a^\dagger]|0\rangle + \langle 0|aa^\dagger a^\dagger a|0\rangle$$

$$= \langle 0|aa^\dagger|0\rangle = 1$$

Hong Ou Mandel

$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\langle c^\dagger c\rangle = \langle\Psi|(R^* a^\dagger + T^* b^\dagger)(R a^\dagger + T b)|\Psi\rangle =$$

$$\langle 0|a(|R|^2 a^\dagger a + |T|^2 b^\dagger b + R^* T a b^\dagger + R T^* a^\dagger b) a^\dagger|0\rangle$$

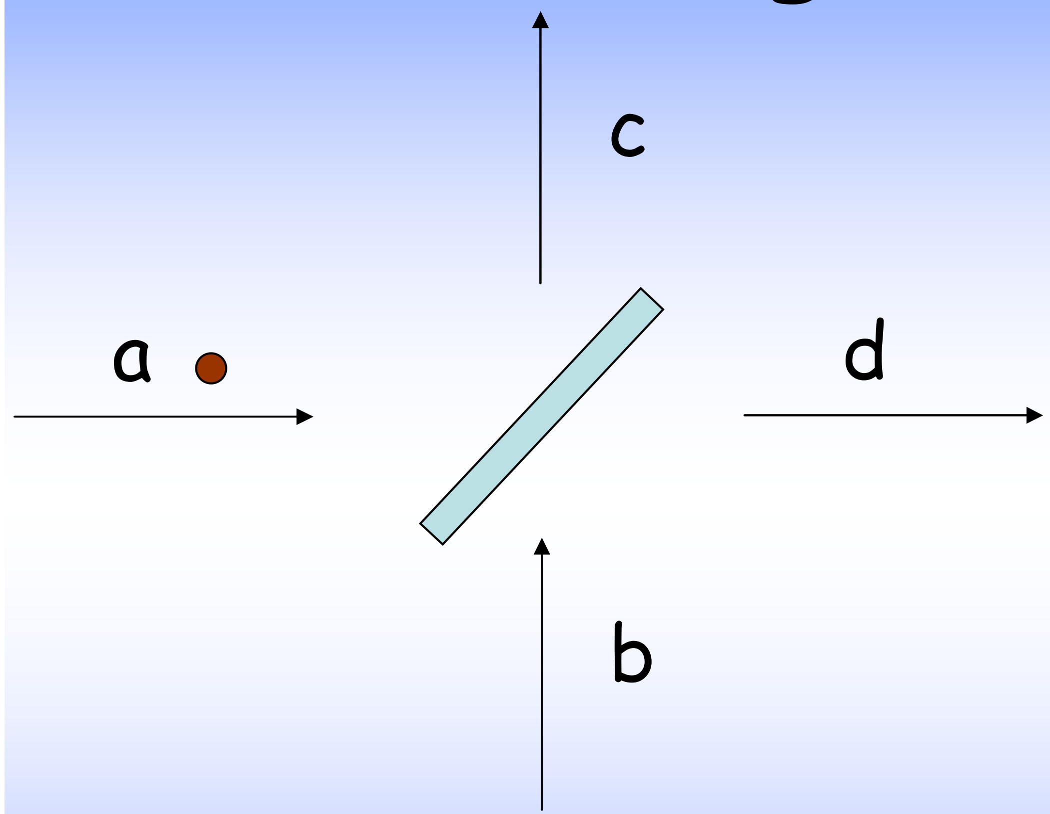
$$= |R| \quad \langle c^\dagger c\rangle = |R|^2$$

$$\langle 0|a a^\dagger a a^\dagger|0\rangle = \langle 0|a a^\dagger[a, a^\dagger]|0\rangle + \langle 0|a a^\dagger a^\dagger a|0\rangle$$

0

$$= \langle 0|a a^\dagger|0\rangle = 1$$

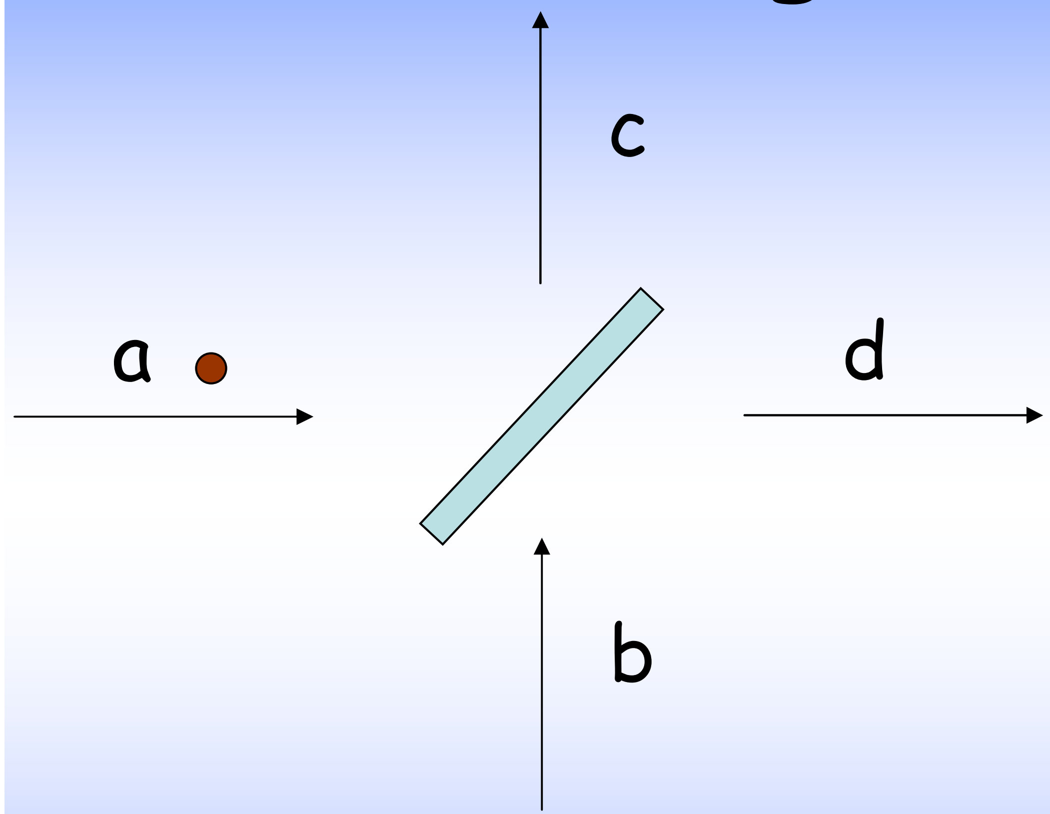
Hong Ou Mandel



$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\langle c^\dagger c\rangle = |R|^2$$

Hong Ou Mandel



$$|\Psi\rangle = a^\dagger|0\rangle$$

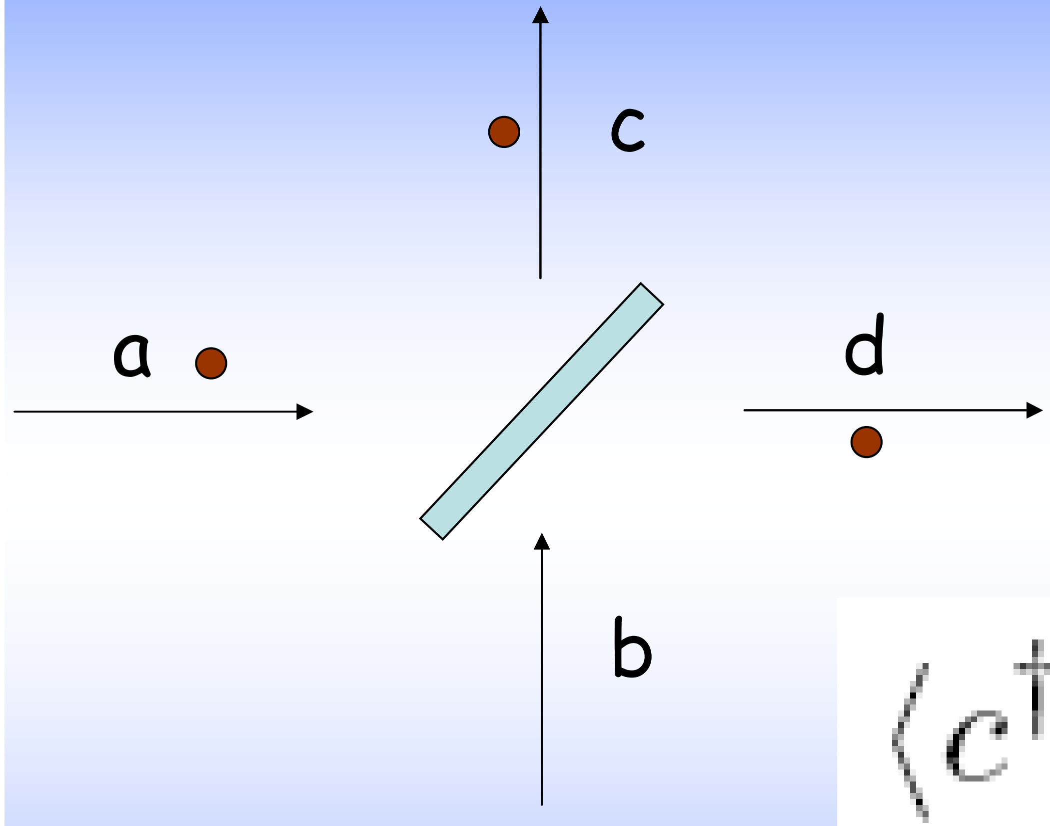
$$\langle c^\dagger d^\dagger d c\rangle$$

Hong Ou Mandel

$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\begin{aligned}
\langle c^\dagger d^\dagger d c\rangle &= \\
&= \langle\Psi|(R^* a^\dagger + T^* b^\dagger)(T^* a^\dagger + R^* b^\dagger)(T a^\dagger + R b)(R a^\dagger + T b)|\Psi\rangle \\
&= |R|^4 \langle 0|a a^\dagger a^\dagger a a a^\dagger|0\rangle = 0
\end{aligned}$$

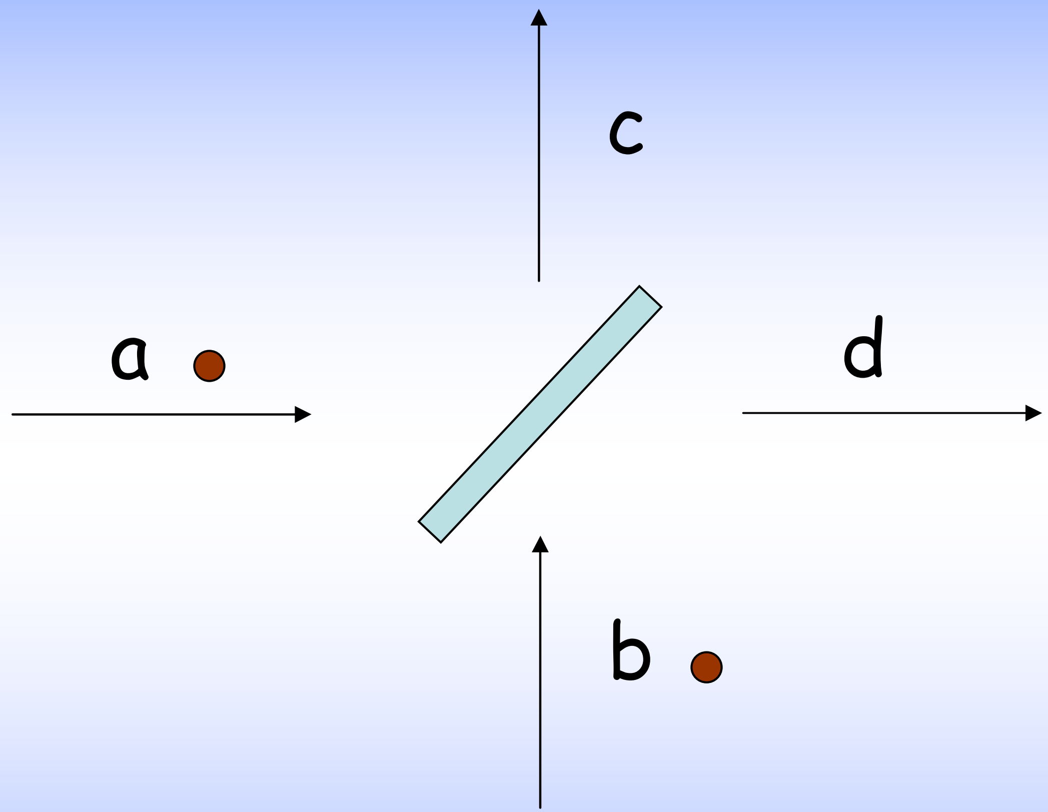
Hong Ou Mandel



$$|\Psi\rangle = a^\dagger|0\rangle$$

$$\langle c^\dagger d^\dagger d c\rangle = 0$$

Hong Ou Mandel



$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

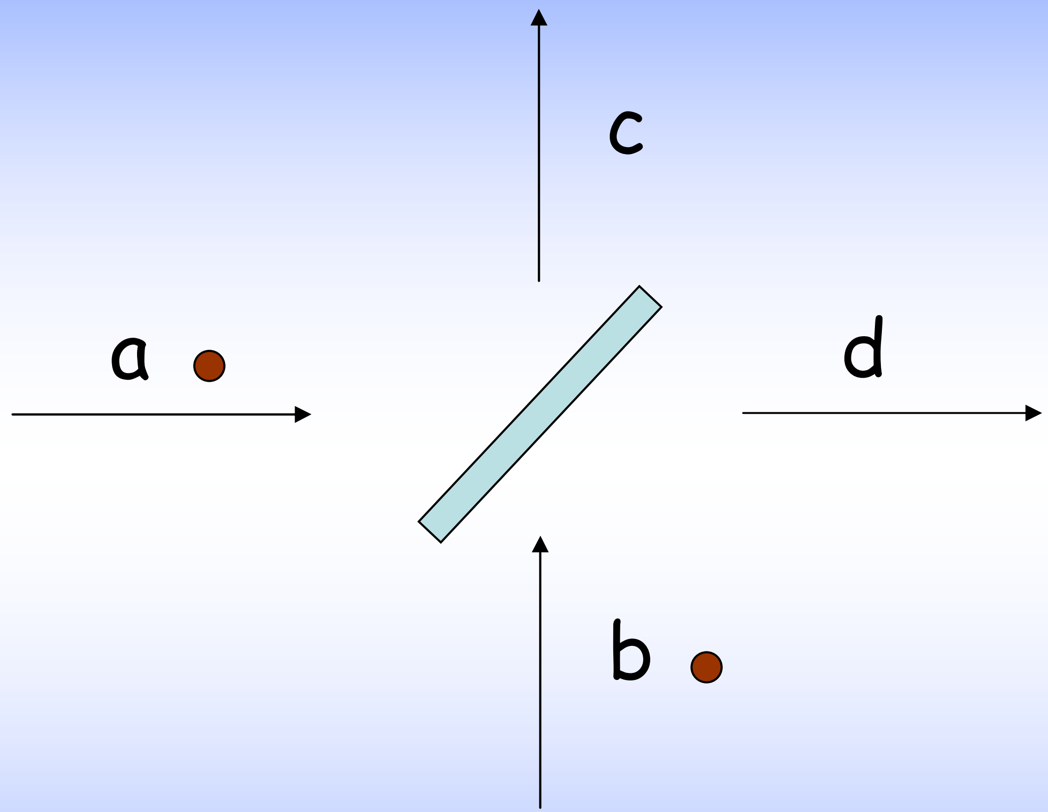
$$\langle c^\dagger c\rangle$$

Hong Ou Mandel

$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

$$\begin{aligned}
\langle c^\dagger c\rangle &= \\
&= \langle 0|ba(|R|^2 a^\dagger a + |T|^2 b^\dagger b + R^* T a b^\dagger + R T^* a^\dagger b) a^\dagger b^\dagger |0\rangle \\
&= |R|^2 \langle 0|a a^\dagger a a^\dagger|0\rangle\langle 0|b b^\dagger|0\rangle + |T|^2\langle 0|b b^\dagger b b^\dagger|0\rangle\langle 0|a a^\dagger|0\rangle = 1
\end{aligned}$$

Hong Ou Mandel



$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

$$\langle c^\dagger d^\dagger d c \rangle$$

Hong Ou Mandel

$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

$$\begin{aligned}
\langle c^\dagger d^\dagger d c\rangle &= \\
&= \langle\Psi|(R^* a^\dagger + T^* b^\dagger)(T^* a^\dagger + R^* b^\dagger)(T a^\dagger + R b)(R a^\dagger + T b)|\Psi\rangle \\
&= \langle 0|ba(R^{*2} a^\dagger b^\dagger + \cancel{R^* T^* a^\dagger a^\dagger} + \cancel{R^* T^* b^\dagger b^\dagger} + T^{*2} b^\dagger a^\dagger) \\
&\quad (R^2 ab + \cancel{RTaa} + \cancel{RTbb} + T^2 ba) a^\dagger b^\dagger |0\rangle \\
&= \langle 0|aa^\dagger aa^\dagger|0\rangle\langle 0|bb^\dagger bb^\dagger|0\rangle(|R|^4 + (RT^*)^2 + (R^*T)^2 + |R|^4)
\end{aligned}$$

1   1   i*i=-1   i*i=-1

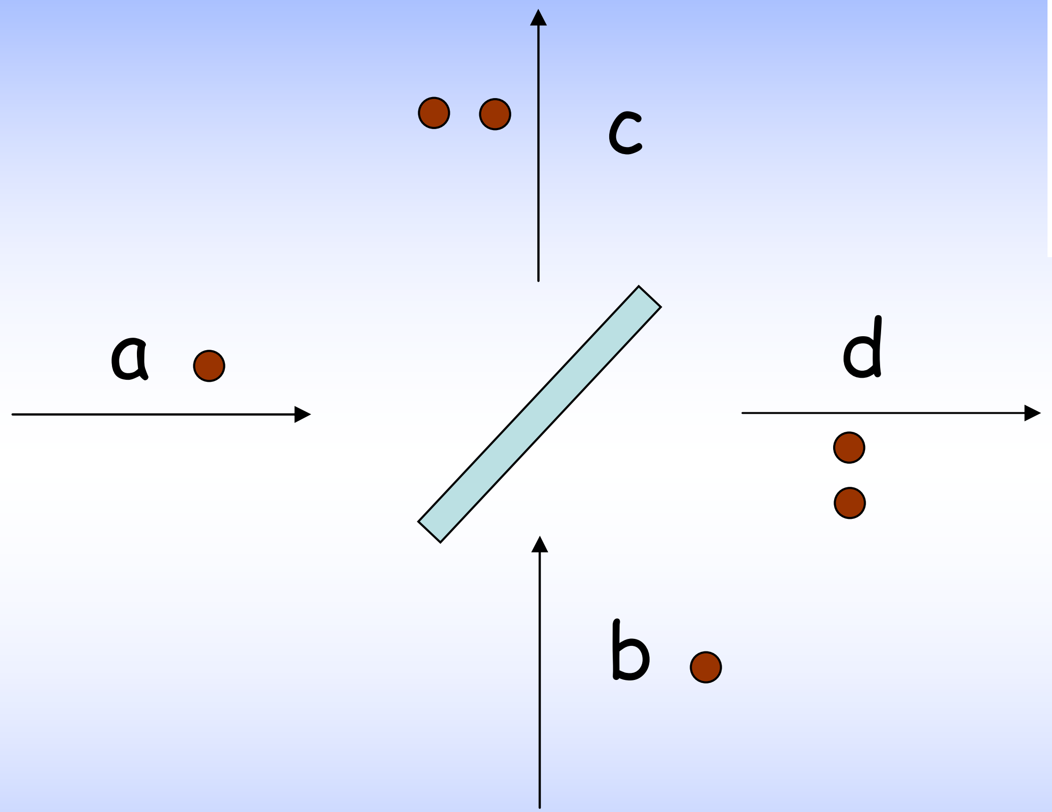
Hong Ou Mandel

$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

If |R|^2=|T|^2=1/2

$$\langle c^\dagger d^\dagger d c\rangle = 0$$

Hong Ou Mandel

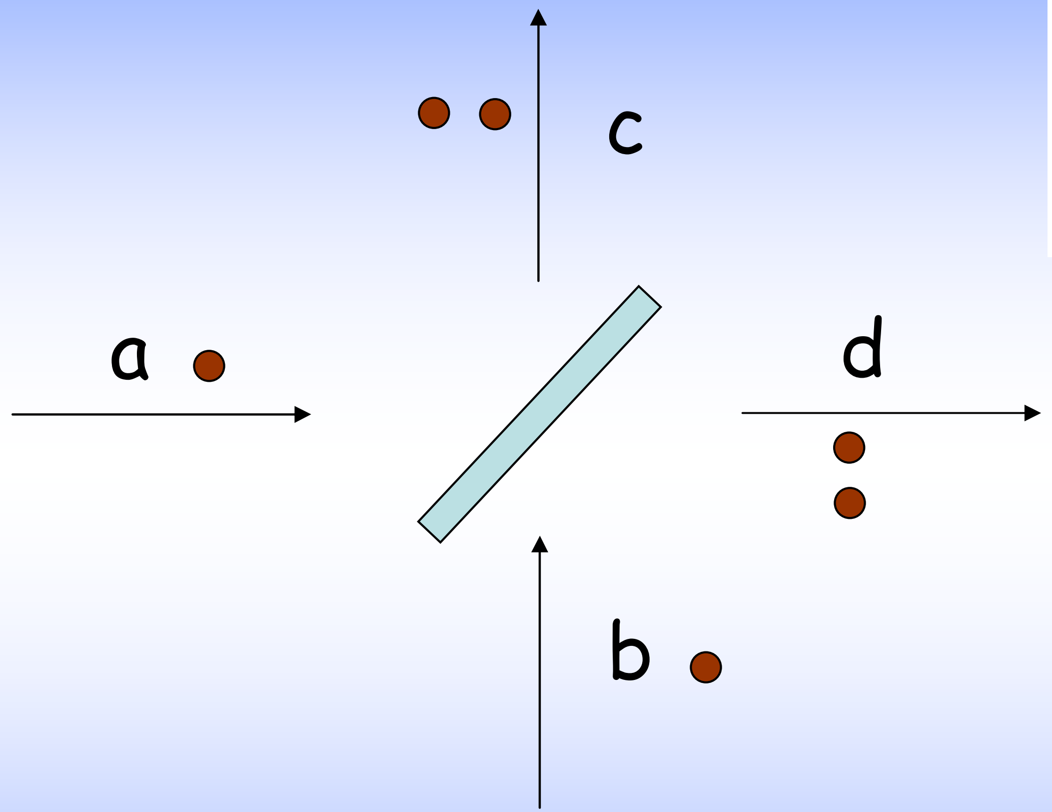


$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

$$\langle c^\dagger c\rangle \;=\; 1$$

$$\langle c^\dagger d^\dagger d c\rangle \;=\; 0$$

Hong Ou Mandel



$$|\Psi\rangle = a^\dagger b^\dagger |0\rangle$$

$$\langle c^\dagger c\rangle \;=\; 1$$

$$\langle c^\dagger d^\dagger d c\rangle \;=\; 0$$

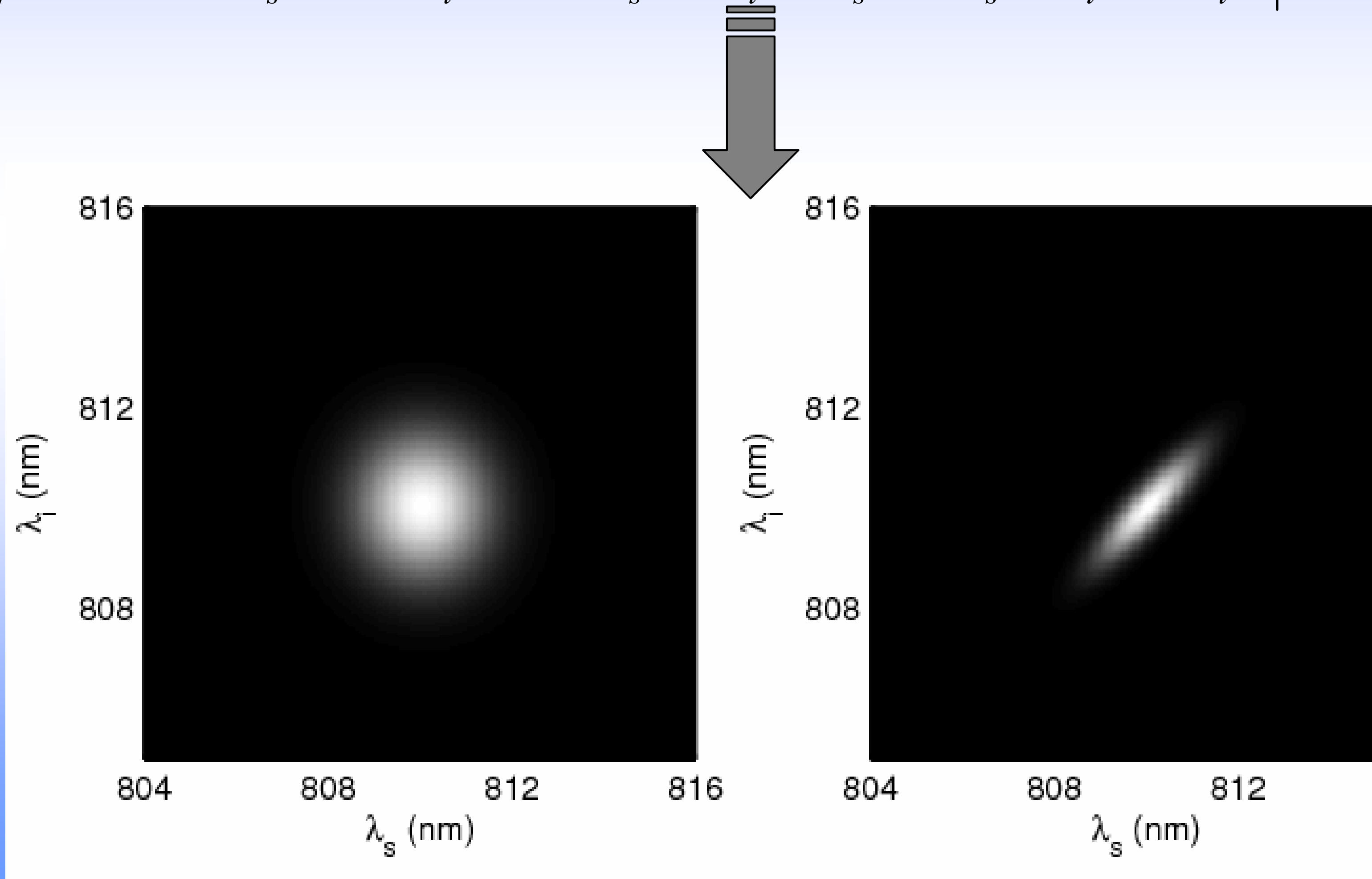
Hong Ou Mandel

Two photon interference effect

You can use it for measuring frequency properties of photons

Generation of frequency entanglement

$$|\psi\rangle = \int d\omega_s \int d\omega_i \Phi(\omega_s, \omega_i) a_s^+(\omega_s) a_i^+(\omega_i) |0,0\rangle$$

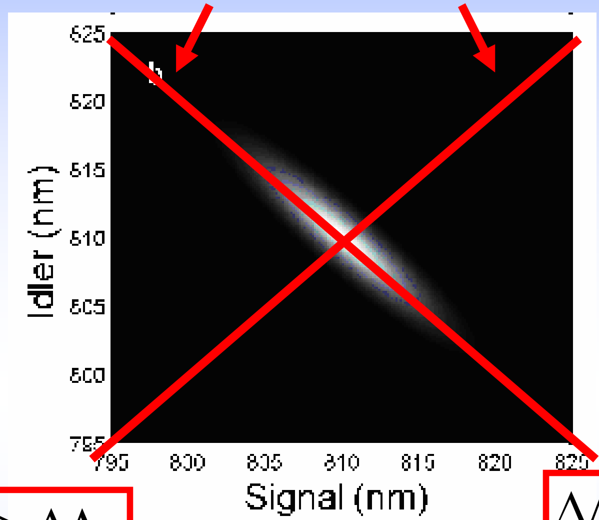


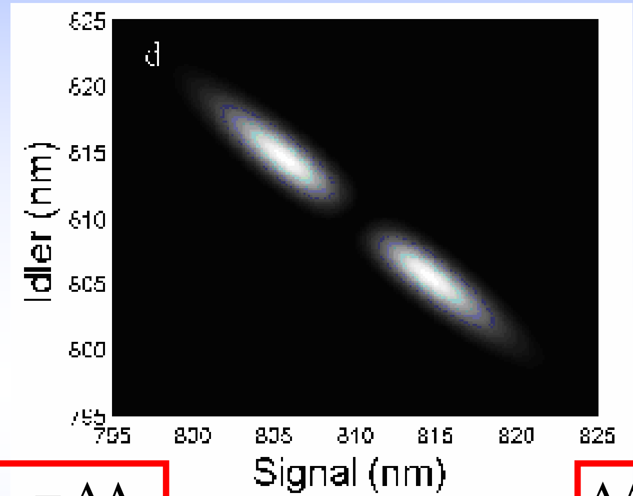
Engineering the joint spectrum

- Waveform
- Bandwidth
- Type of freq. correlations

$\Lambda_{-} = \Lambda_s - \Lambda_i$

$\Lambda_{+} = \Lambda_s + \Lambda_i$

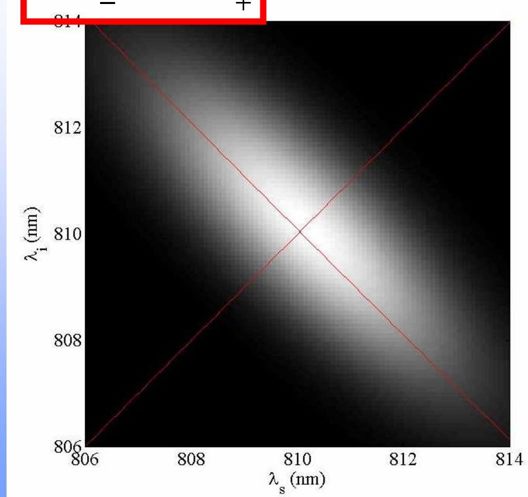




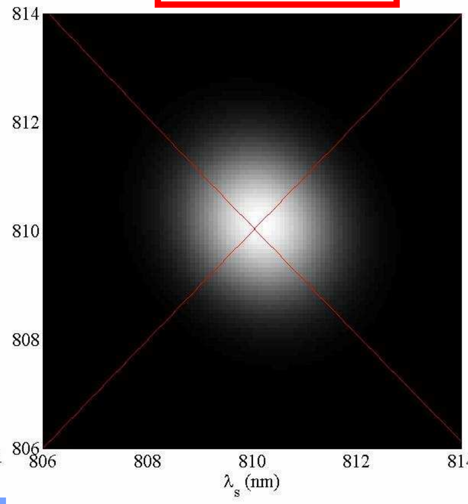
$\Delta\Lambda_{-} > \Delta\Lambda_{+}$

$\Delta\Lambda_{-} = \Delta\Lambda_{+}$

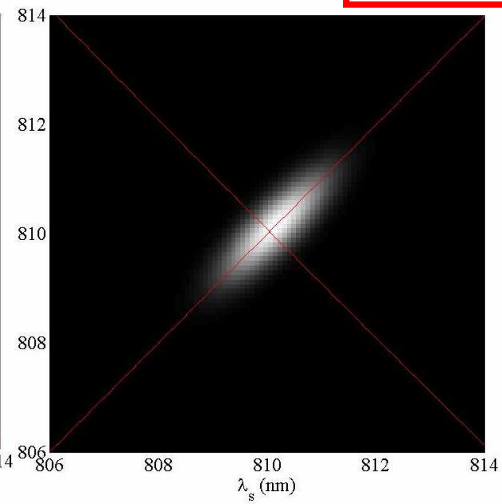
$\Delta\Lambda_{-} < \Delta\Lambda_{+}$



Frequency Anticorrelation



Frequency Uncorrelation

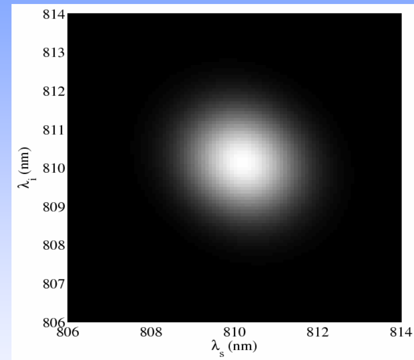


Frequency Correlation

Why to engineer the spectrum of photons??

1) Generation of heralded pure single photons

Completely uncorrelated frequency photons

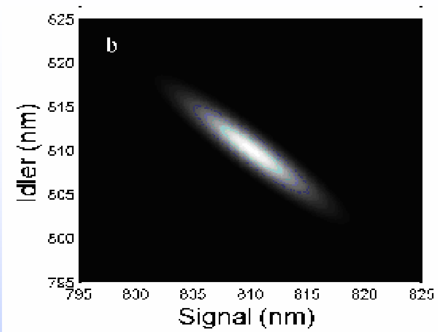


2) Quantum metrology

- **Timing and positioning protocols based on second order correlation measurements**

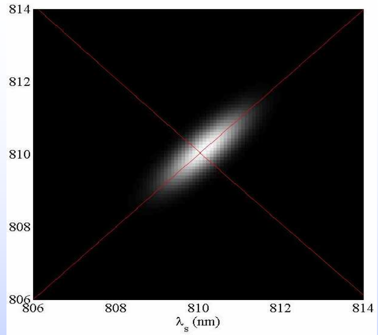
To get narrow

$$\Delta(t_1 - t_2)$$



- **Timing and positioning protocols based on the use of frequency correlated photons**

Correlated frequency photons



3) Quantum optical coherence tomography

Large bandwidth
hundreds of nanometers (THz)

4) Atom-photon interaction

Narow bandwidth
less than a nanometer (MHz)

- Also

  •Optical Coherence Tomography (OCT)
  Largely enhanced bandwidth

  •Quantum Optical Coherence Tomography (OCT)
  Largely enhanced bandwidth + dispersion compensation with frequency correlated photons

Generation of frequency entanglement

- You can generate a given state by choosing a proper crystal and a proper pump

- You can use gratings for the pump and signal and idler to control the output state (pulse-front technique)

- You can use the spatial-frequency correlations to change the state (spatial to spectral mapping)

1) Tilt pulse front technique

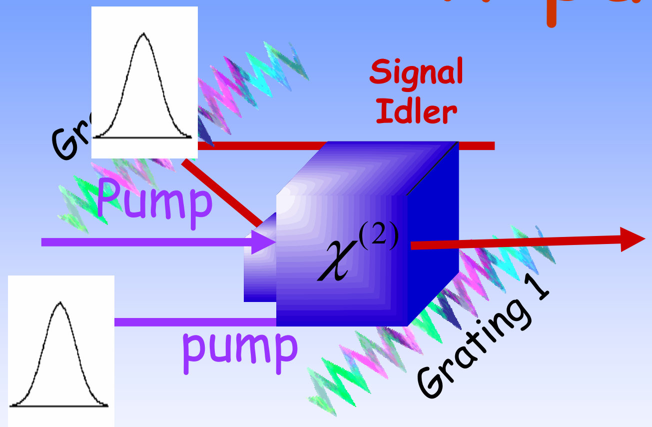
M. Hendrych et. al., Opt. Lett. **32**, 2339 (2007)

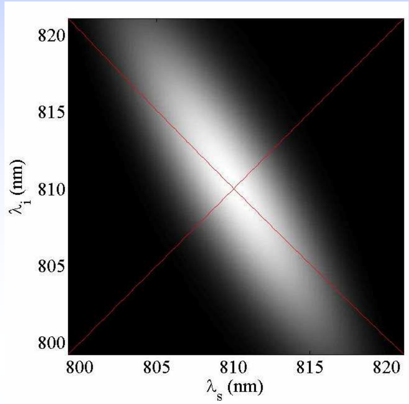
J.P. Torres et. al., Phys. Rev. A. **71**, 022320 (2005)

J.P. Torres et. al., Opt. Lett. **30**, 314 (2005)

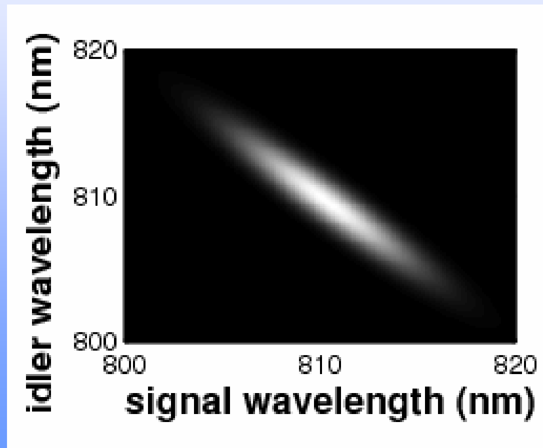
Tilt pulse front technique

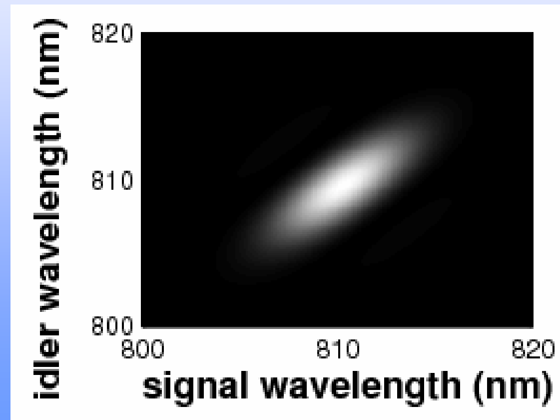
Pulsed SPDC



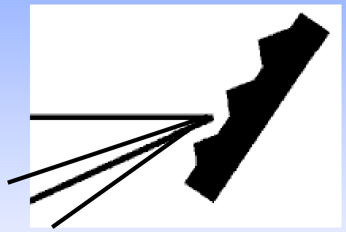


By introducing the appropiate angular dispersion it is possible to modify the type of frequency correlations and the bandwidth of the photons





Pulse-front techniques (angular dispersion)



Induce dispersion in free space,
broadband SHG
temporal solitons in second order nonlinear media

- New inverse group velocity

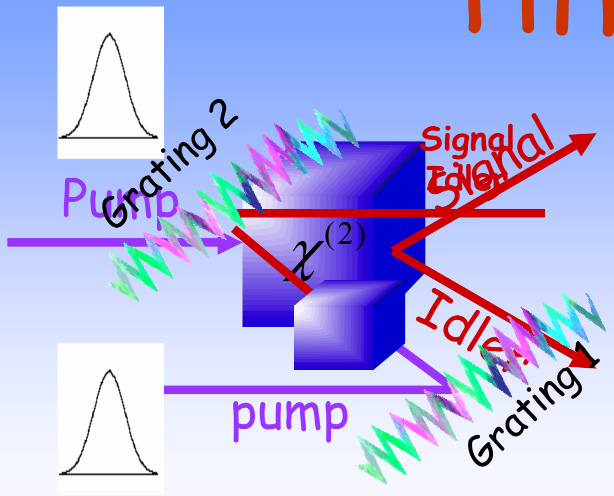
$$u = k' + \frac{\tan\Psi \tan\rho}{c}$$

- New group velocity dispersion

$$g = k'' - \frac{(\tan\Psi)^2}{kc^2}$$

Torres et al., Opt. Lett. 25, 1735 (2000)

Tilt pulse technique



$$\phi(\Omega_s,\Omega_i) \propto E_{\omega p}(\Omega_s+\Omega_i)\, Sinc\,\frac{\Delta k L}{2}\, e^{i\frac{\Delta k L}{2}}$$

$$\omega_j \equiv \omega_j^0 + \Omega_j \qquad N_j \equiv \frac{dk_j}{d\omega} \qquad g_j \equiv \frac{d^2 k_j}{d\omega^2}$$

$$\Delta k \approx (N_p - N_s)\Omega_s + (N_p - N_i)\Omega_i - \frac{1}{2} g_s \Omega_s^2 - \frac{1}{2} g_i \Omega_i^2 - \frac{1}{2} g_p \Omega_p^2$$

- Spatial Walkoff → Walkoff angle ρ
- Angular Dispersion → Tilt angle ξ

$$N_j' \equiv \left(\frac{dk}{d\omega}\right)^{-1} + \tan\rho_j \frac{\tan\xi}{c}$$

$$g_j' \equiv \frac{d^2 k}{d\omega^2} - \frac{1}{|\vec{k}_j|}\left(\frac{\tan\xi}{c}\right)^2$$

Control over group velocities

Control over the bandwidth

Modifying bandwidth and type of frequency correlations via Tilt pulse technique

$$\Delta k \approx (N'_p - N'_s)\Omega_s + (N'_p - N'_i)\Omega_i$$

if $\Delta k \to 0$ ⟹ $$\frac{\Omega_i}{\Omega_s} \approx -\frac{(N'_p - N'_s)}{(N'_p - N'_i)}$$ ⟹

By choosing the appropiate tilt angle it is possible to modify the slope of the joint spectrum

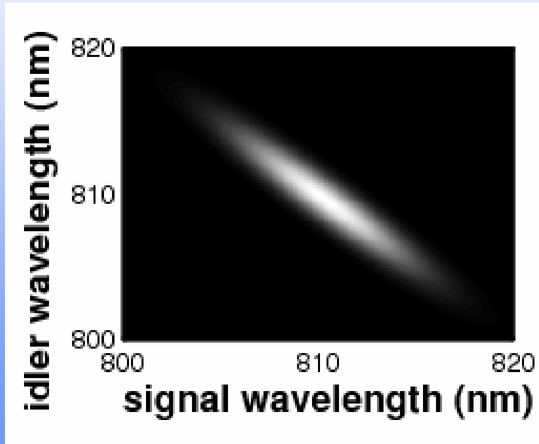
$$\Delta k \approx \left[N'_p - \frac{(N'_s + N'_i)}{2}\right]\Omega_+ - [N'_s - N'_i]\Omega_-$$

$$\Omega_+ \equiv \Omega_s + \Omega_i$$

$$\Omega_- \equiv \Omega_s - \Omega_i$$

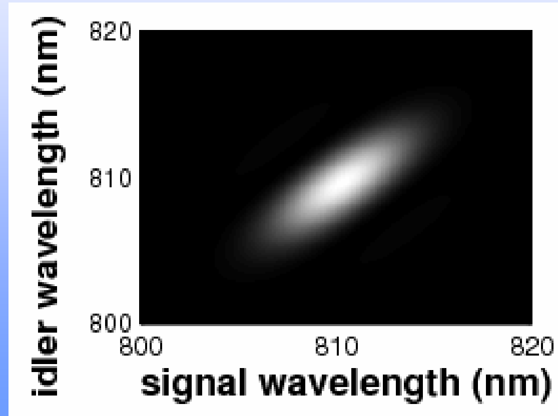
Anticorrelated frequency photons

$\Omega_s = -\Omega_i$ Achieved when $N'_s = N'_i$

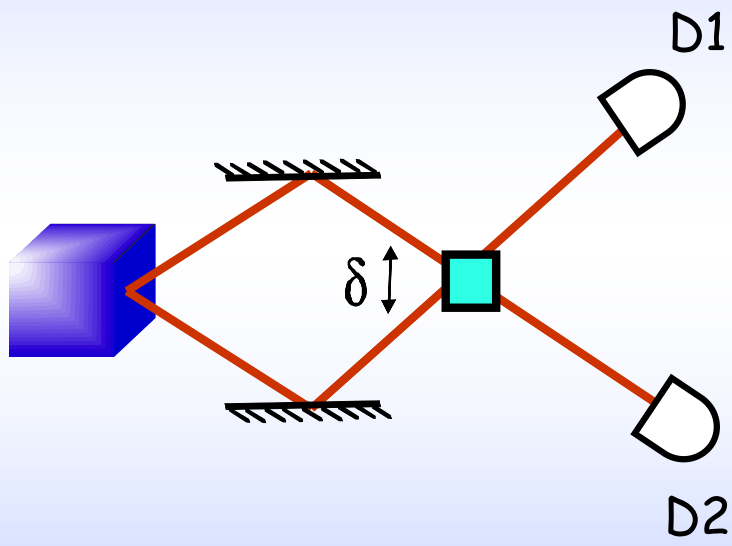


correlated frequency photons

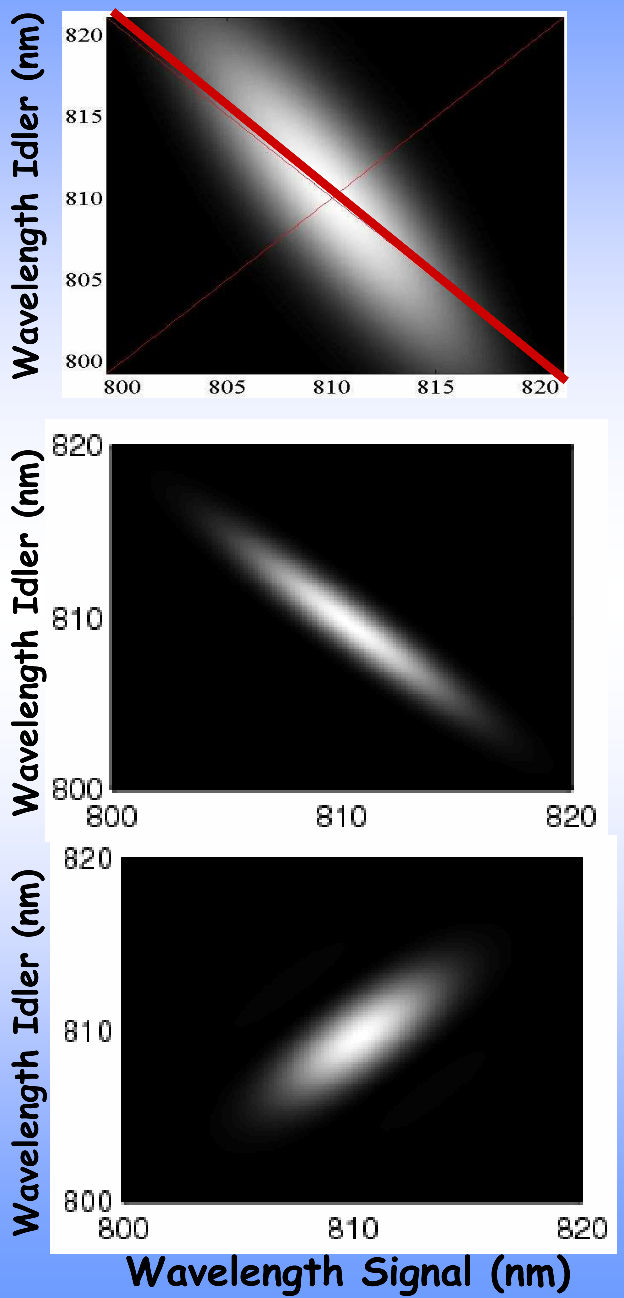
$\Omega_s = \Omega_i$ Achieved when $N'_p = \frac{N'_s + N'_i}{2}$

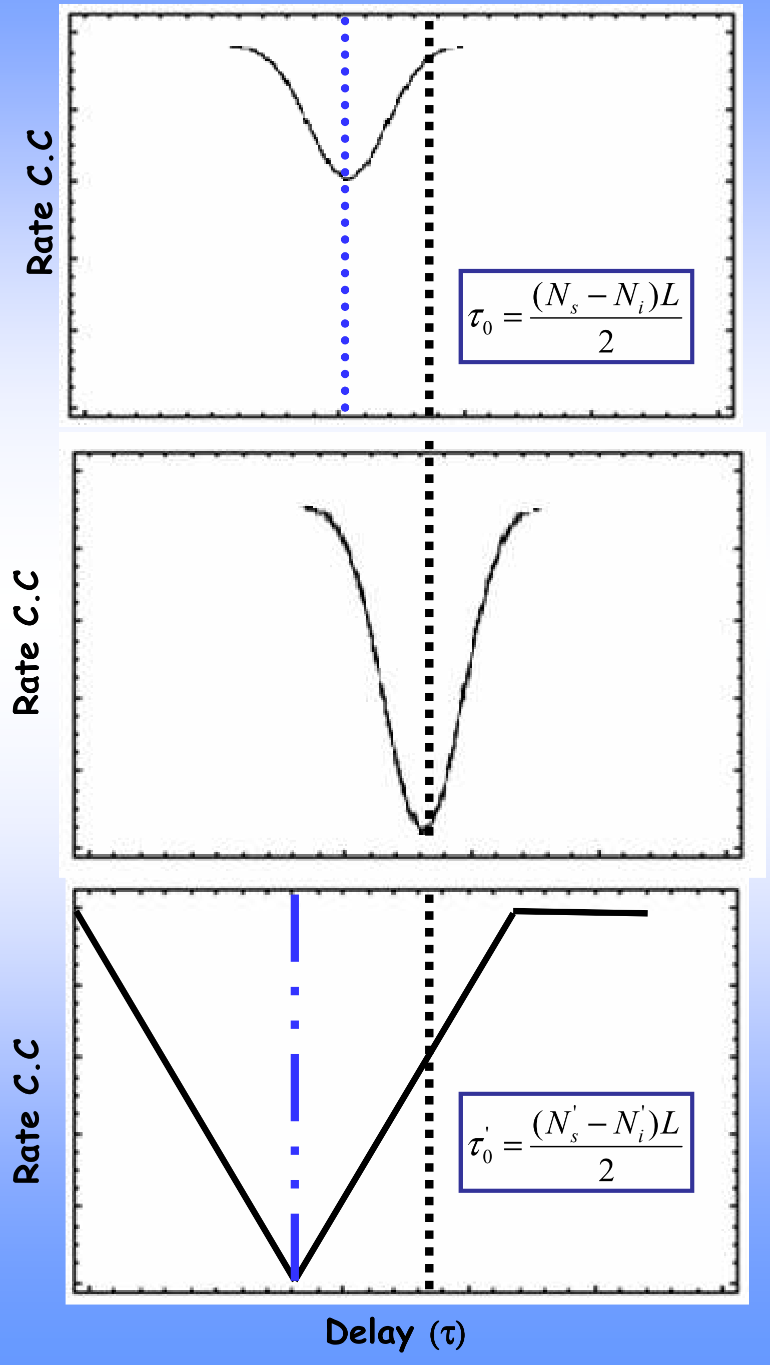


Two-photon Interference

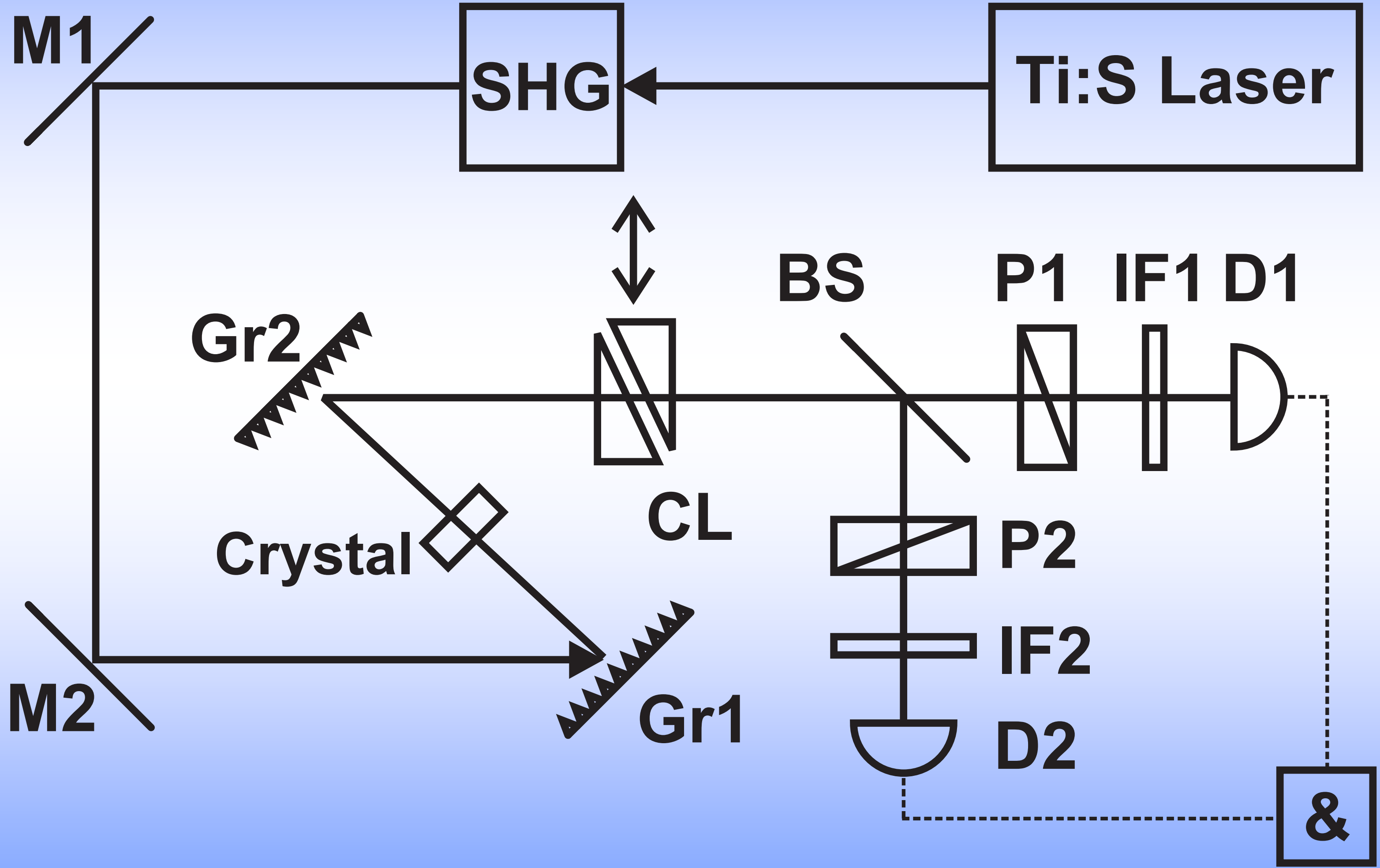


In a two-photon interference experiment:



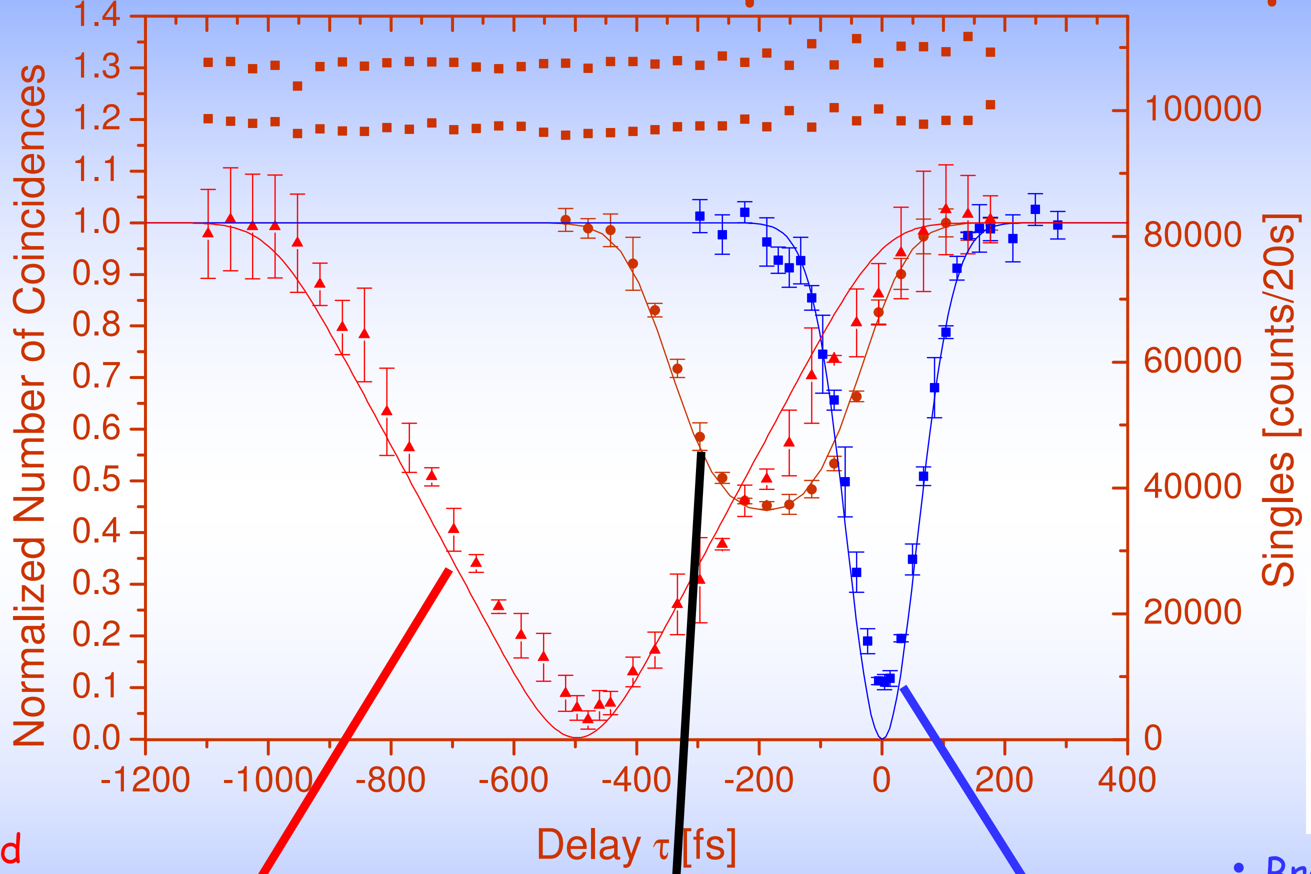


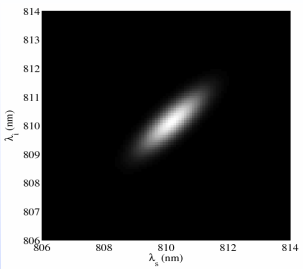
Modifying bandwidth and type of frequency correlations via Tilt pulse technique



M Hendrych, M. Micuda and J.P. Torres Opt. Let. **32** 2339 (2007)

Modifying bandwidth and type of frequency correlations via Tilt pulse technique

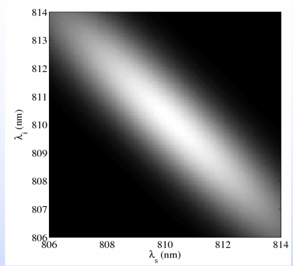




- narrowband and triangular shape like type-II process
- Dip shifted from zero delay

Visibility 93%

Visibility 38%



- Broadband like type-I process
- Dip centered at zero delay

Visibility 83%

Spatial-to-Spectral Mapping

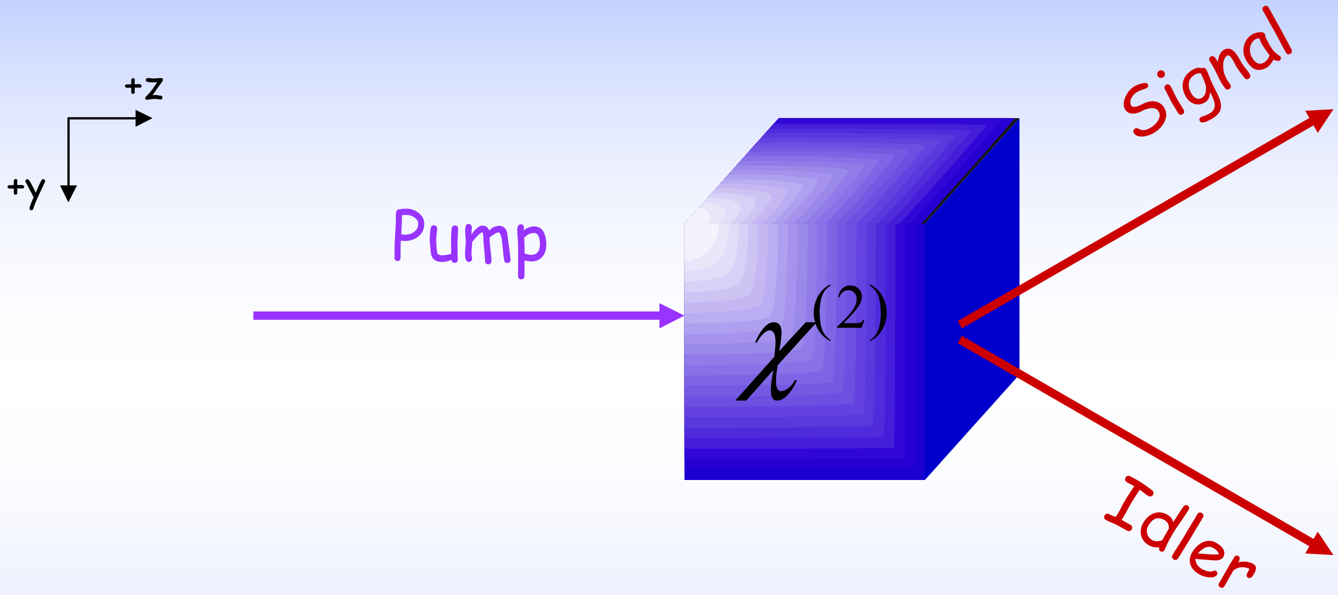
A. Valencia et. al., Phys. Rev. Lett. (2007)

S. Carrasco et. al., Phys. Rev. A **70**, 043817 (2004)

Modifying the type of frequency correlations of paired photons via spatial-to-spectral mapping

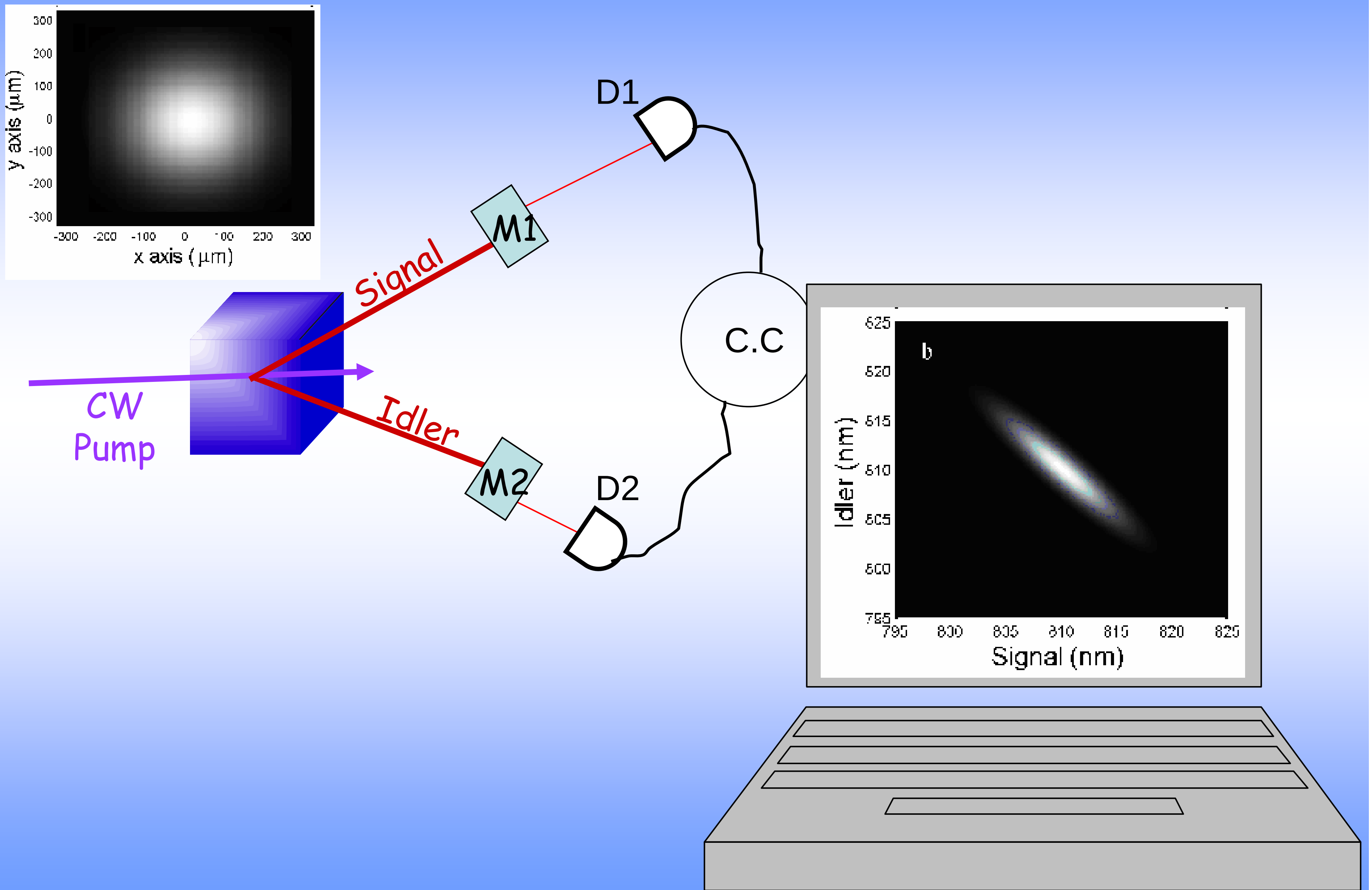
$$\phi(\Omega_s,\Omega_i)\propto E_{\omega p}(\Omega_s+\Omega_i)E_{qp}(0,\Delta_0)Sinc\frac{\Delta kL}{2}e^{i\frac{\Delta kL}{2}}$$

Spatial-to-Spectral Mapping

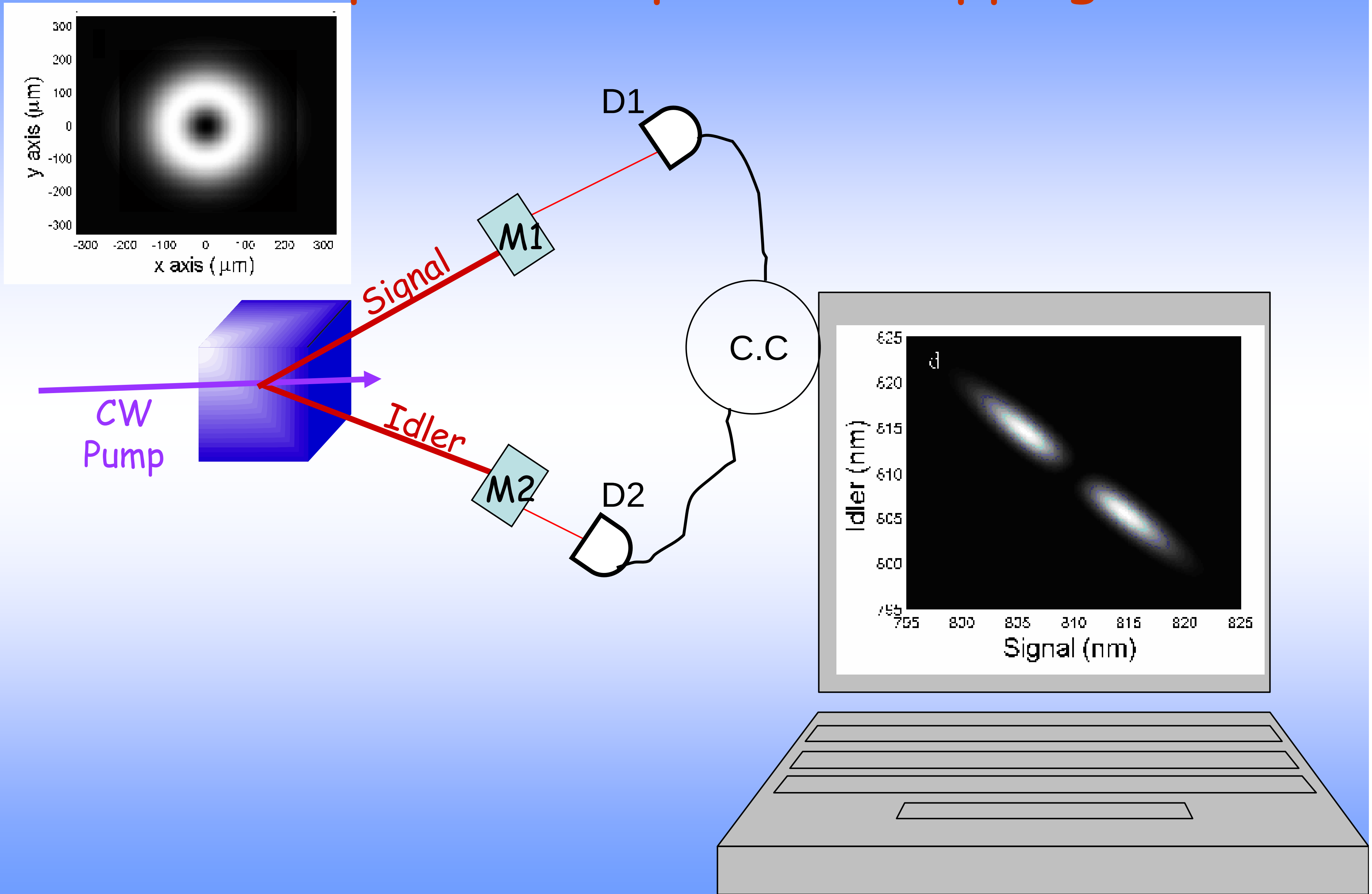


The *noncollinear geometry* madiates the mapping of spatial characteristics of the pump into the joint spectrum of the downconverted photons

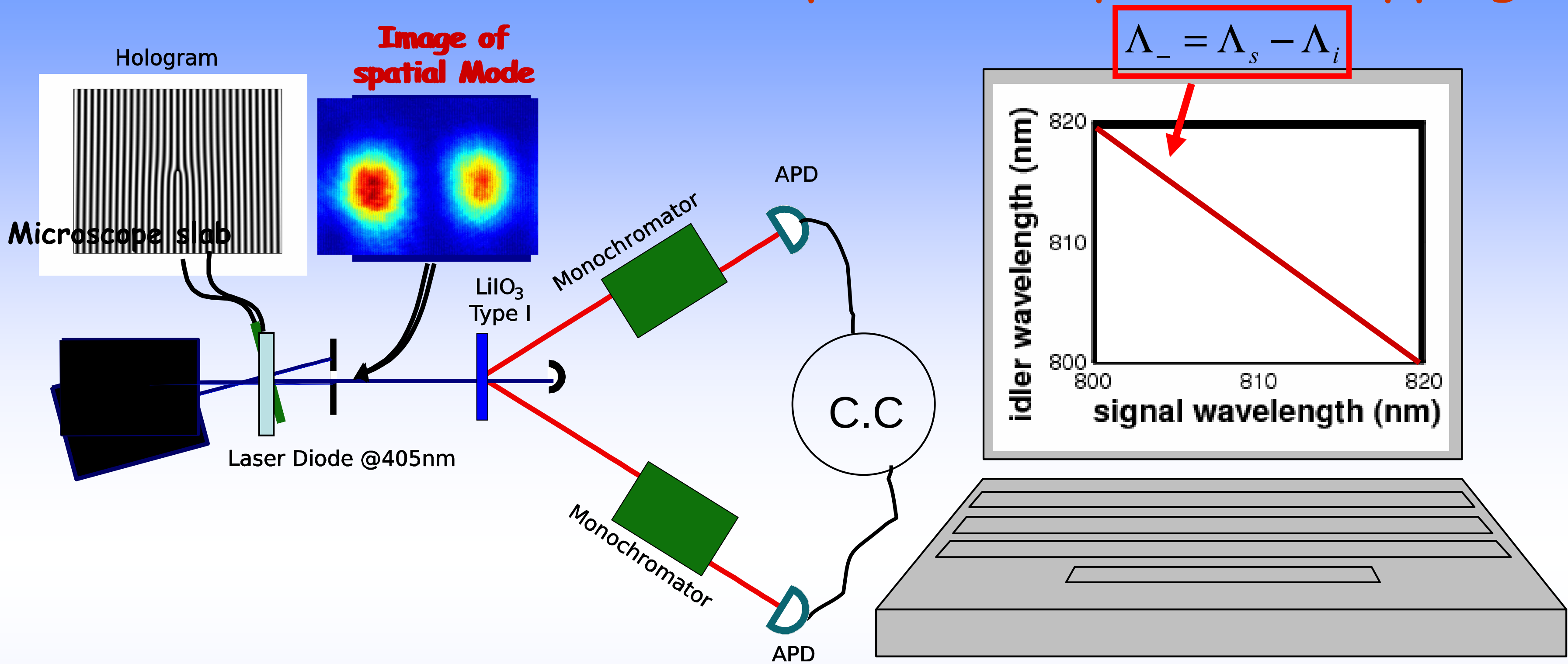
spatial-to-spectral mapping



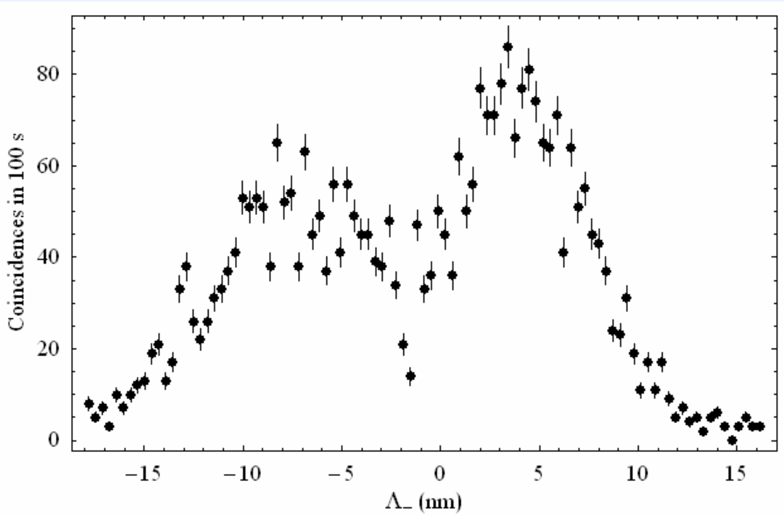
spatial-to-spectral mapping



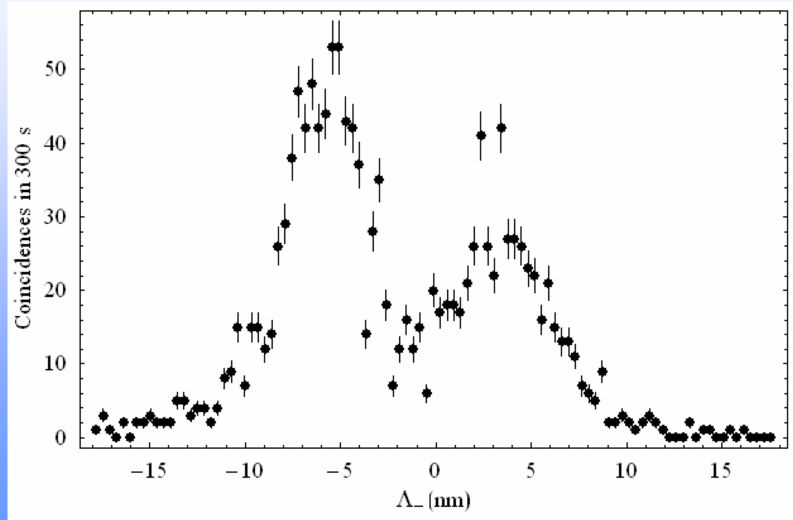
Experimental demonstration of spatial-to-spectral mapping



Pump beam modified by hologram



Pump beam modified by microscope slab



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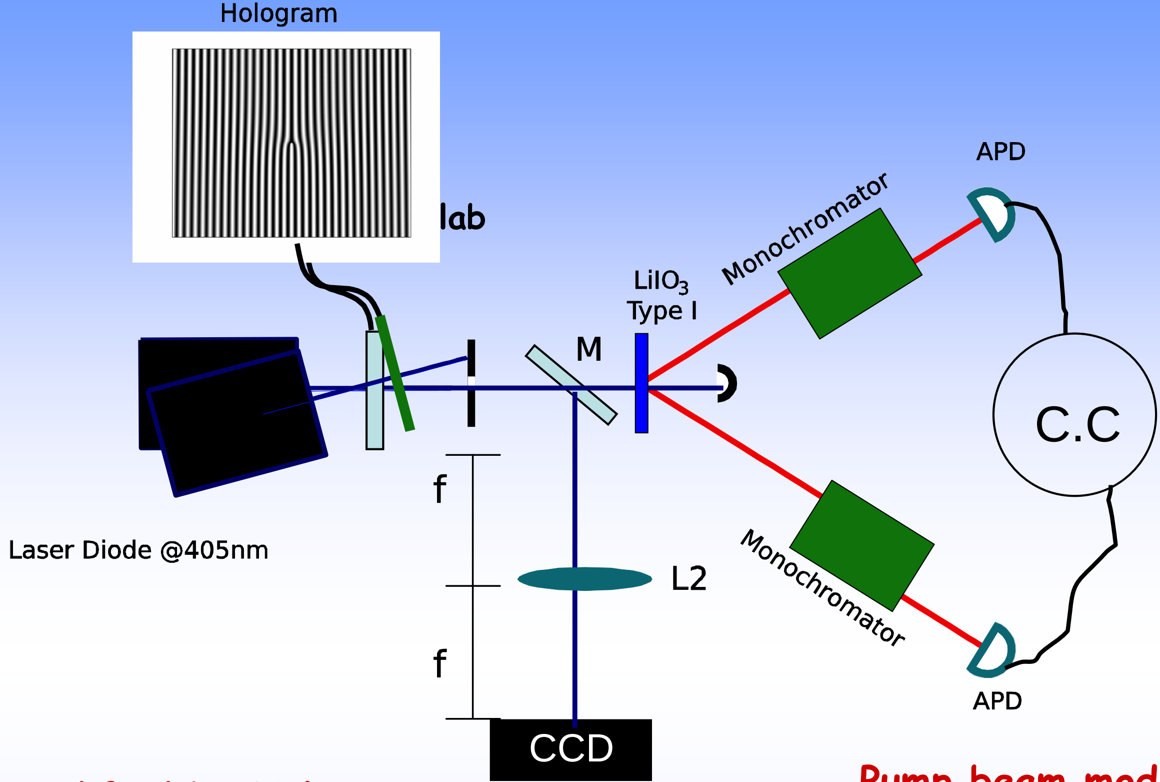
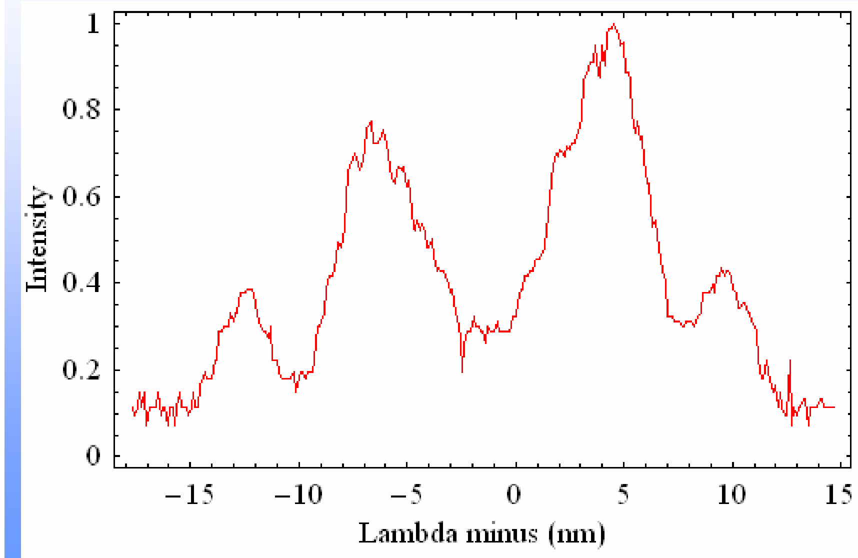


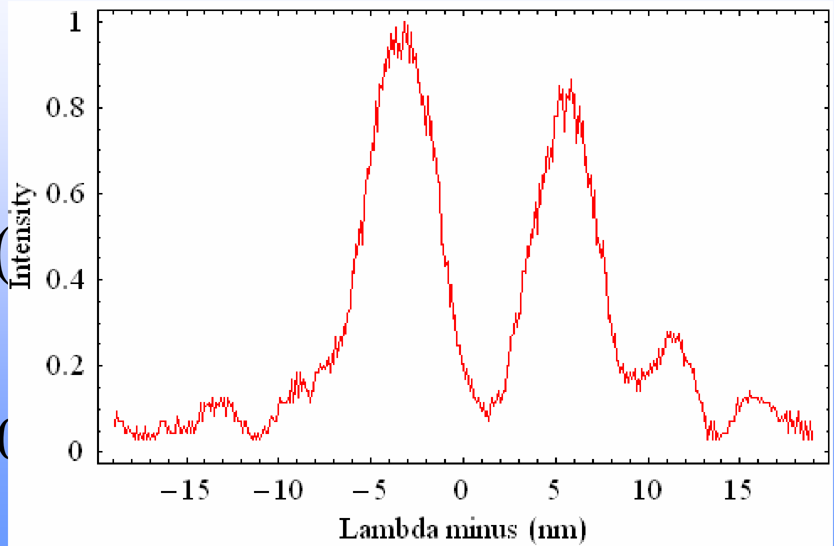
Pump beam modified by Hologram



$y) \propto E_{qp}($

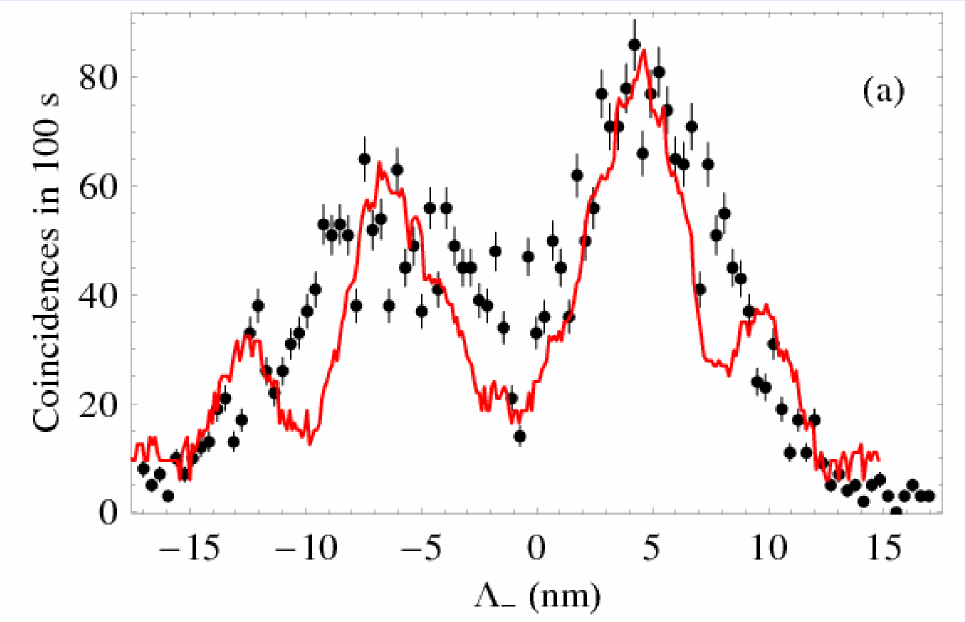
$+\Omega_i)E_{qp}(($

Pump beam modified by microscope slab

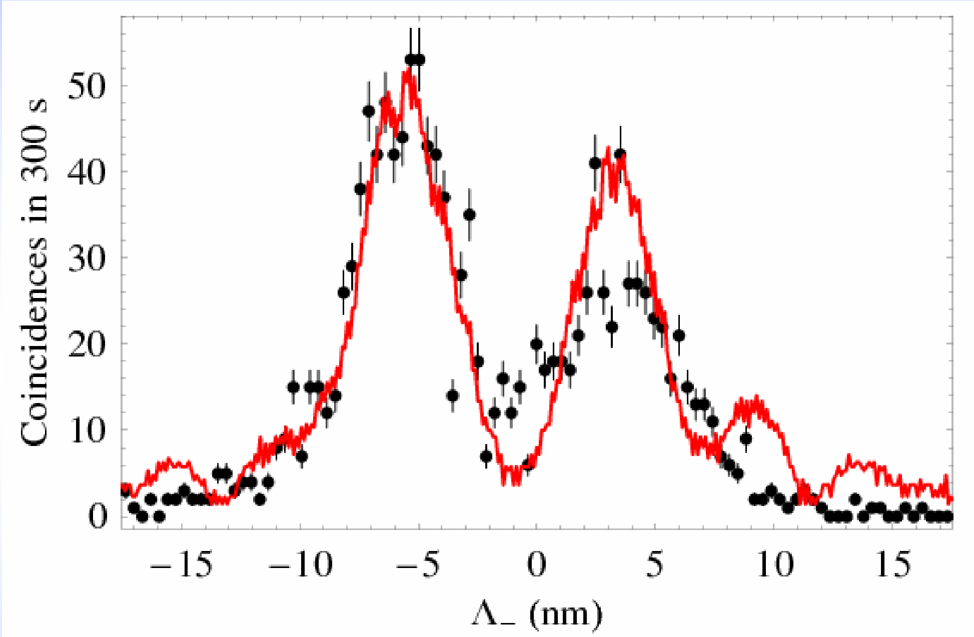


Experimental demonstration of spatial-to-spectral mapping

Pump beam modified by Hologram



Pump beam modified by microscope slab

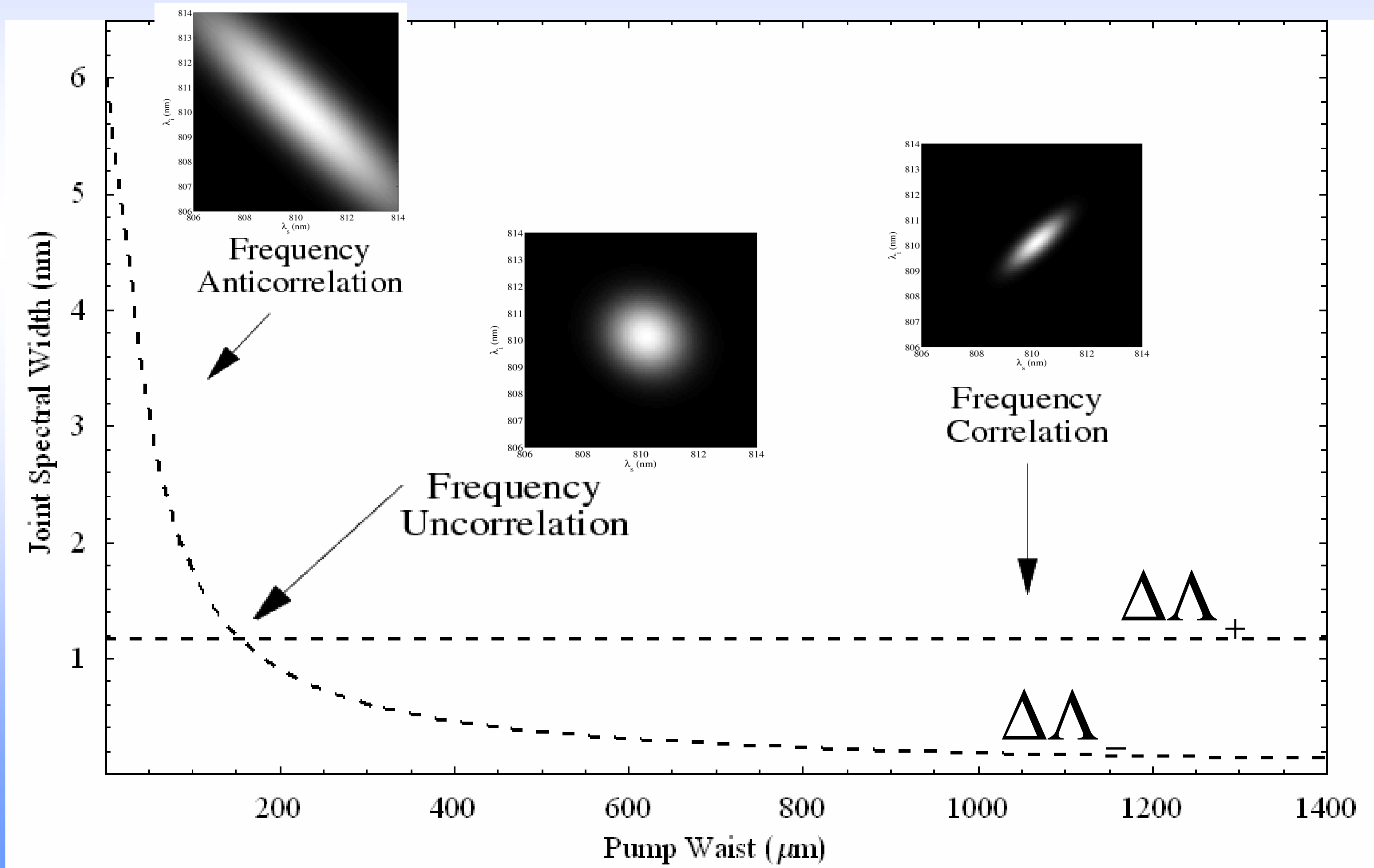


Spatial distribution of the pump

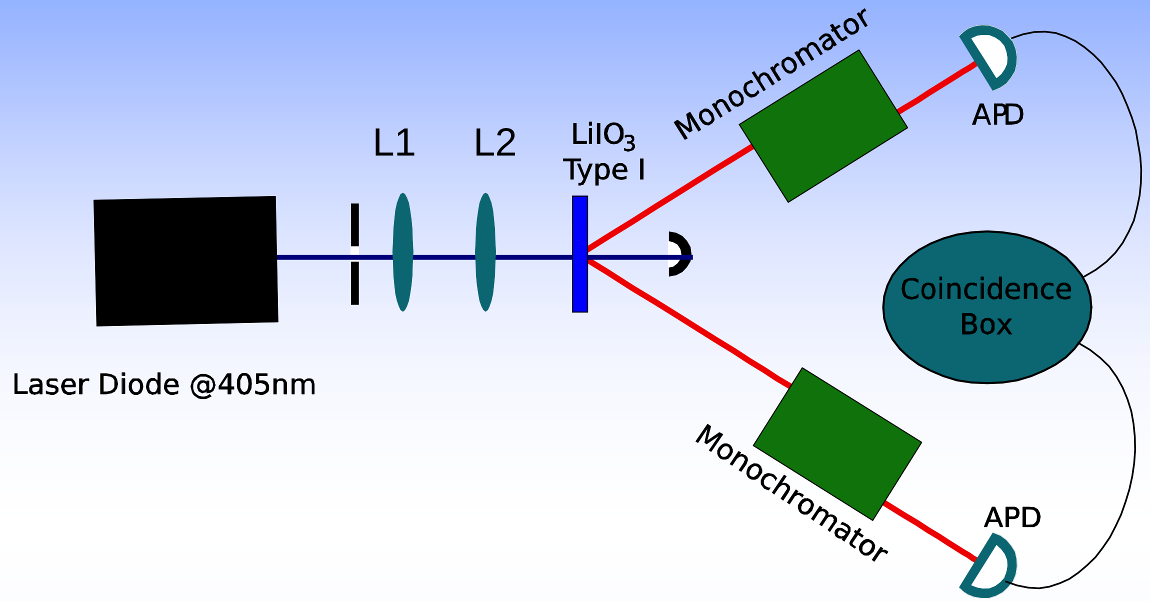
Joint spectrum

Modifying the type of frequency correlations of paired photons via spatial-to-spectral mapping

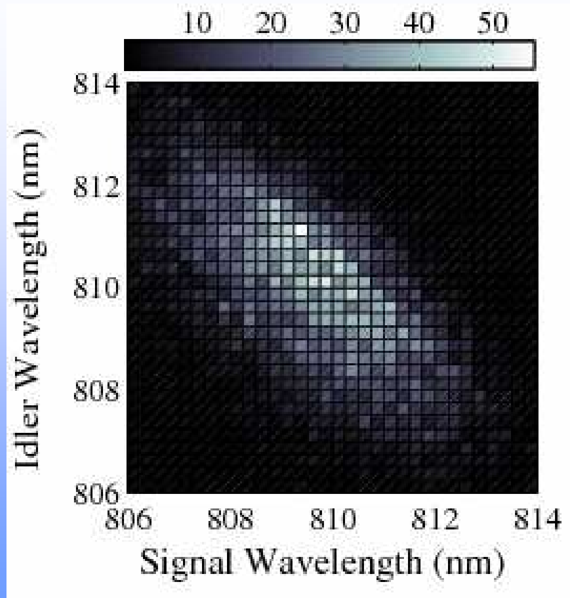
$$\left|\Phi(\lambda_s,\lambda_i)\right|^2 \approx e^{-\frac{\Lambda_+^2}{2\Delta\Lambda_+^2}} e^{-\frac{\Lambda_-^2}{2\Delta\Lambda_-^2}}$$

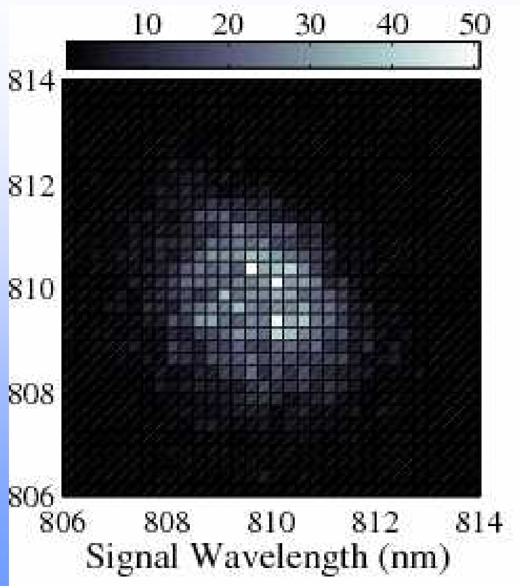


Modifying the type of frequency correlations of paired photons via spatial-to-spectral mapping



$w_0 = 30\mu m$





$w_0 = 462\mu m$

Juan

Alejandra

Martin

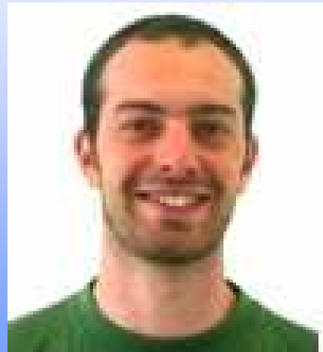
Noelia

Clara

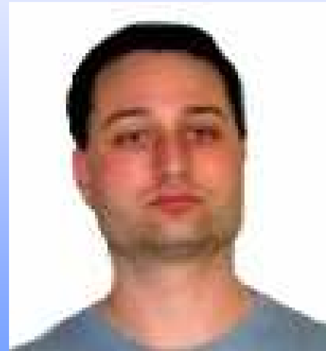
Yana

Xiajuan

Roser



Alessandro



Michal