My Fair Light: Bright Multicolor Entanglement

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Seminário "dublê" – JAS 2008

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FAPESP CNPq CAPES CT BRASIL PADCT

Entanglement and quantum information with macroscopic light fields

• Entanglement is one the most striking features of quantum theory. It can be found in discrete systems, such as collections of two-level systems (qubits), and in continuous variable systems, such as macroscopic light fields. Besides its intrinsic interest, as a counterintuitive quantum property, entanglement is viewed now as a potential resource in the field of quantum information.

• Solving an old problem: entanglement of the bright (non-degenerate) optical beams emitted by an above-threshold OPO was first predicted by Reid and Drummond in 1988. It was only measured for the first time in 2005. We also showed that higher orders of entanglement are expected in this system.

• The entanglement we measured can be used for the implementation of a quantum key distribution protocol and other tasks in quantum information.

• BUT: noise, which at present is still unaccounted for.

Advantages of continuous variables

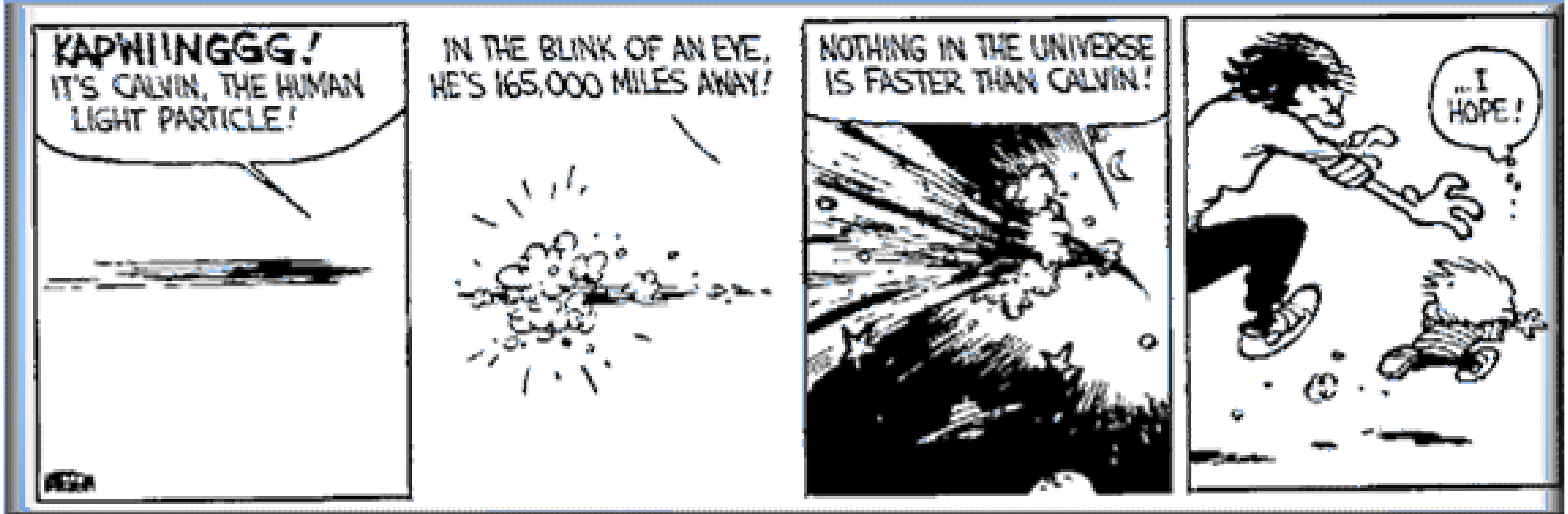
• "Unconditional" state preparation (every inverse bandwidth).

• High detection efficiencies (> 95%).

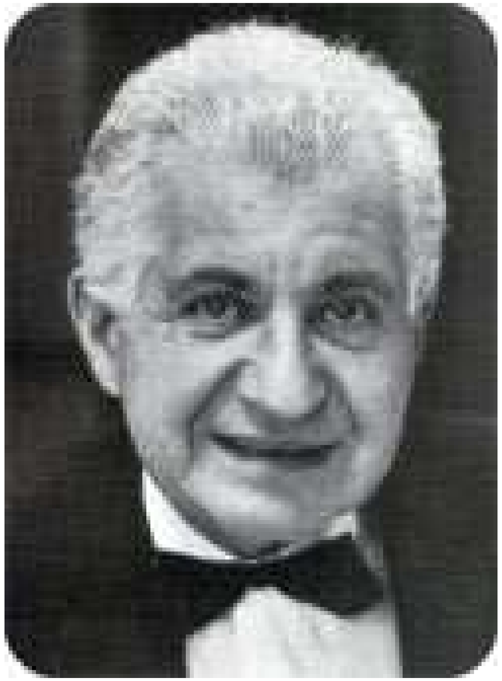
• "Complete Bell detection" with homodyne detection and beamsplitters.

• "Drawbacks": states are not perfect, they depend on the degree of squeezing; most experiments involve Gaussian states, with a Wigner function ≥ 0.

Nothing goes faster than light



Einstein, Podolsky & Rosen's paper



MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality.

Einstein, Podolsky & Rosen's paper

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*

EPR's example

$|\psi\rangle \cong \delta(x_1 - x_2 - \mathrm{L})\delta(p_1 + p_2)$ (localized in $x_1 - x_2$ and $p_1 + p_2$)

> We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

A measurement of x_1 yields x_2, just as a measurement of p_1 gives p_2. But x_2 and p_2 *don't commute*! $\leftrightarrow$ $[x, p] = i\,\hbar$

It's the first example of an entangled state

• The first time the word *entanglement* appeared was in Schrödinger's "cat" paper.

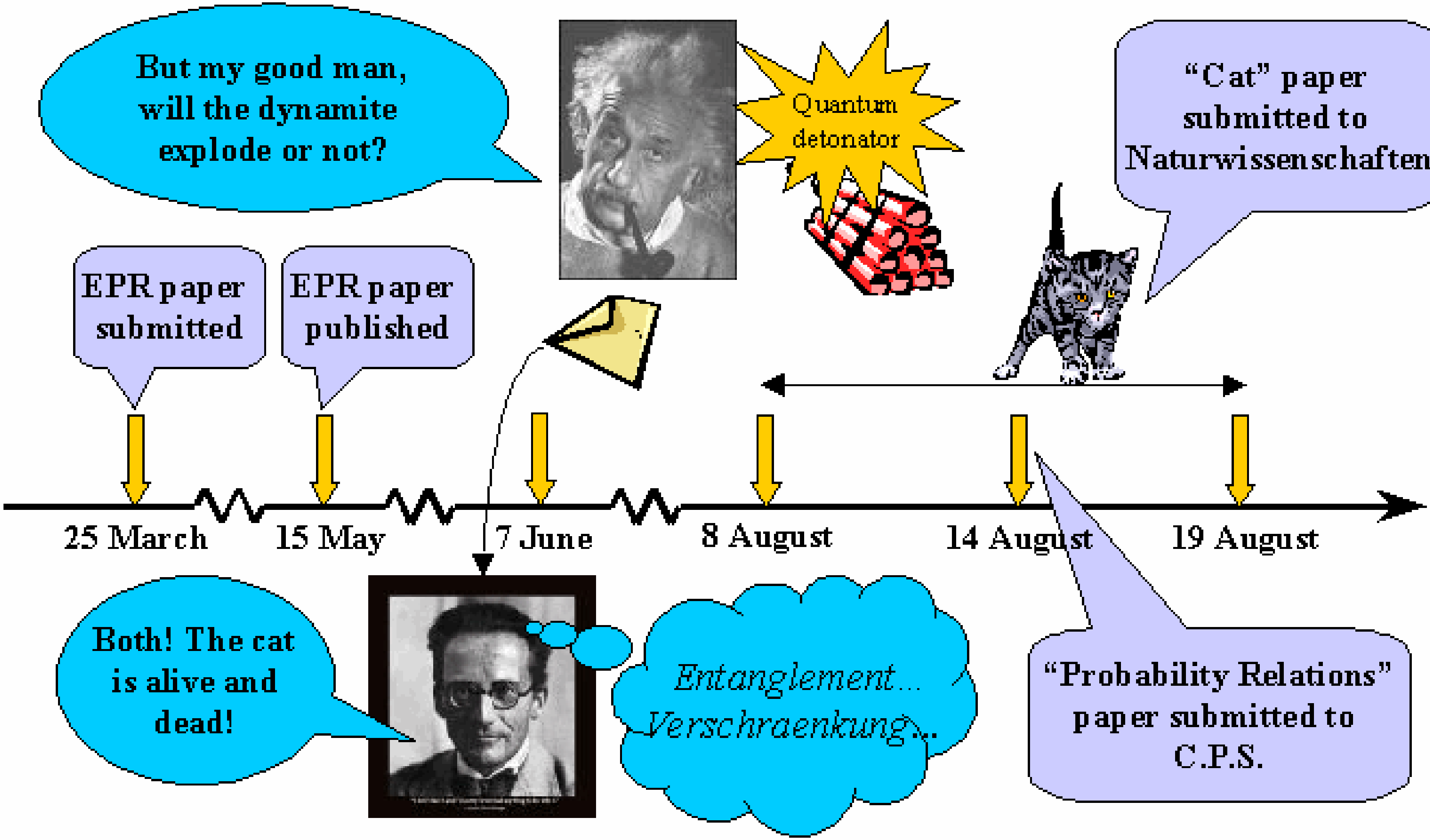
I.11 THE PRESENT SITUATION IN QUANTUM MECHANICS: A TRANSLATION OF SCHRÖDINGER'S "CAT PARADOX" PAPER

ERWIN SCHRÖDINGER (TRANS. JOHN D. TRIMMER*)

[1] E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik," *Naturwissenschaften* **23**: pp. 807–812; 823–

1] E. Schrödinger, "Discussion of probability relations between separated systems", Proceedings of the Cambridge Philosophical Society, 31, (1935), 555-563 [2]

The 1935 Entanglement Timeline



http://qubit.damtp.cam.ac.uk/users/matthias/Entanglement/timeline_sugi.gif

EPR's conclusion

either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*

If (1) is false, then (2) is also false! Hence, (1) should be true: quantum theory, although it allows for correct predictions, must be *incomplete*. Measurements should just reveal pre-existing states, which are not described by this incomplete theory.

Bohr's reply

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

$$[q_1p_1]=[q_2p_2]=ih/2\pi,$$
$$[q_1q_2]=[p_1p_2]=[q_1p_2]=[q_2p_1]=0,$$

$$[Q_1P_1]=ih/2\pi, \qquad [Q_1P_2]=0,$$

$$q_1=Q_1\cos\theta-Q_2\sin\theta \qquad p_1=P_1\cos\theta-P_2\sin\theta$$
$$q_2=Q_1\sin\theta+Q_2\cos\theta \qquad p_2=P_1\sin\theta+P_2\cos\theta.$$

$$Q_1=q_1\cos\theta+q_2\sin\theta,$$
$$P_2=-p_1\sin\theta+p_2\cos\theta,$$

Bohr introduces *complementarity*, but his paper does not give sufficient arguments to *rule out* the EPR program. (This story goes on with the theorems by John Bell and experiments to violate Bell's inequalities, and GHZ-states etc.)

How can we measure continuous variable entanglement?

- "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]

$$\Delta^2 \hat{p}_{\mathrm{inf}} = \Delta^2 \hat{p}_1 \left(1 - \frac{\langle \delta \hat{p}_1 \delta \hat{p}_2 \rangle^2}{\Delta^2 \hat{p}_1 \Delta^2 \hat{p}_2} \right)$$

$$\delta \hat{p}_i = \hat{p}_i - \langle \hat{p}_i \rangle$$

$$\Delta^2 \hat{p}_{\mathrm{inf}} \, \Delta^2 \hat{q}_{\mathrm{inf}} \geq 1$$

How can we measure continuous variable entanglement?

•DGCZ separability criterion:

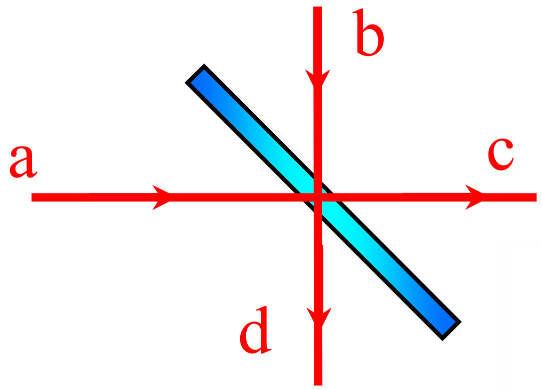
$$\rho = \sum_i p_i \, \rho_i = \sum_i p_i \, \rho_i^1 \otimes \rho_i^2 \qquad [\hat{x}_j, \hat{p}_{j'}] = 2\, i\, \delta_{jj'}$$

$$\hat{u} = |a|\hat{x}_1 + \frac{1}{a}\hat{x}_2$$
$$\hat{v} = |a|\hat{p}_1 - \frac{1}{a}\hat{p}_2$$

Separability $\Rightarrow$ $\langle(\Delta\hat{u})^2\rangle_\rho + \langle(\Delta\hat{v})^2\rangle_\rho \geq 2\,(a^2 + \frac{1}{a^2})$

Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller,
Inseparability criterion for continuous variable systems, Phys. Rev. Lett. **84**, 2722 (2000).

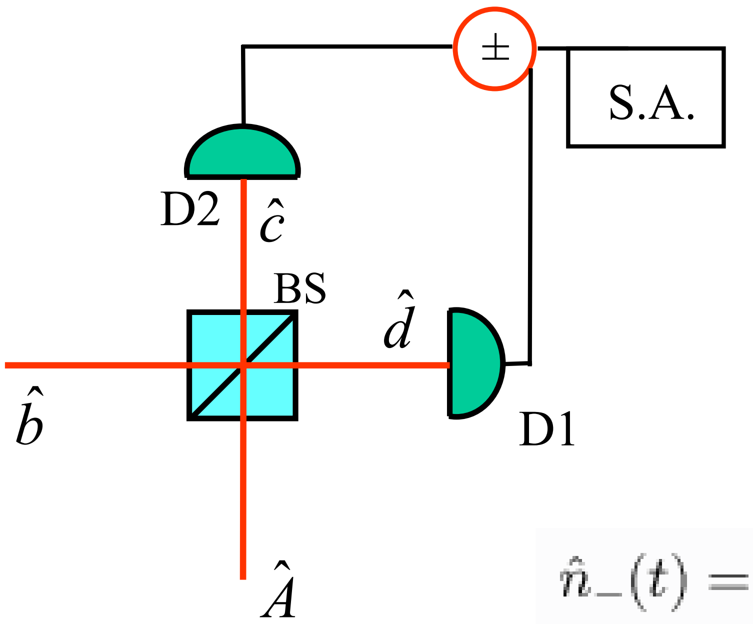
• Two squeezed fields combined in a BS result in entangled output fields.



$$\hat{c} = \frac{1}{\sqrt{2}}\left(\hat{a} + \hat{b}\right) \qquad \hat{X}(\theta) = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}$$
$$\hat{d} = \frac{1}{\sqrt{2}}\left(\hat{b} - \hat{a}\right) \qquad \left[\hat{X}(\theta), \hat{X}\left(\theta + \frac{\pi}{2}\right)\right] = 2i$$

$$\hat{u} = \sqrt{2}\left(\hat{b} + \hat{b}^\dagger\right)$$
$$\hat{v} = i\sqrt{2}\left(\hat{a}^\dagger - \hat{a}\right)$$

Noise Measurements



(Balanced) Homodyne Detection

$$\hat{c}(t) = \frac{1}{\sqrt{2}}\left(\hat{A}(t) + \hat{b}\right)$$

$$\hat{d}(t) = \frac{1}{\sqrt{2}}\left(\hat{b} - \hat{A}(t)\right)$$

$$\hat{n}_-(t) = \hat{c}^\dagger(t)\hat{c}(t) - \hat{d}^\dagger(t)\hat{d}(t) = \hat{b}^\dagger\hat{A}(t) + \hat{A}^\dagger(t)\hat{b}$$

If field *b* is strong, we can replace the operator by its mean value

$$\hat{n}_-(t) = |\beta|\left(\hat{A}(t)e^{-i\theta} + \hat{A}^\dagger(t)e^{i\theta}\right)$$

If field *b* is the vacuum, we can obtain *A*'s intensity noise by measuring n_+

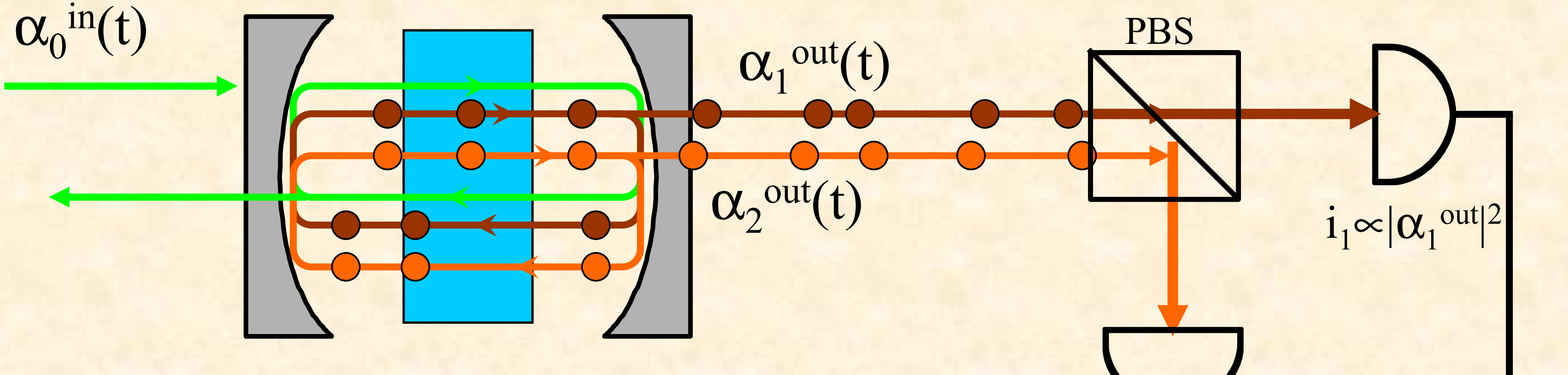
$$\hat{n}_+(t) = \hat{A}^\dagger(t)\hat{A}(t) + \hat{b}^\dagger\hat{b}$$

The Optical Parametric Oscillator (OPO)

Generation of Twin Beams

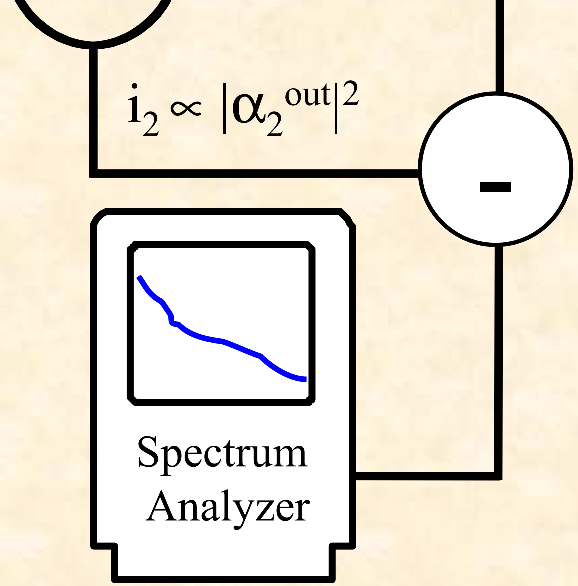
Pump generates twin photons (signal and idler) inside a cavity.

Strong intensity correlations in the output beams.



Correlation measured by the subtraction of detector photocurrents.

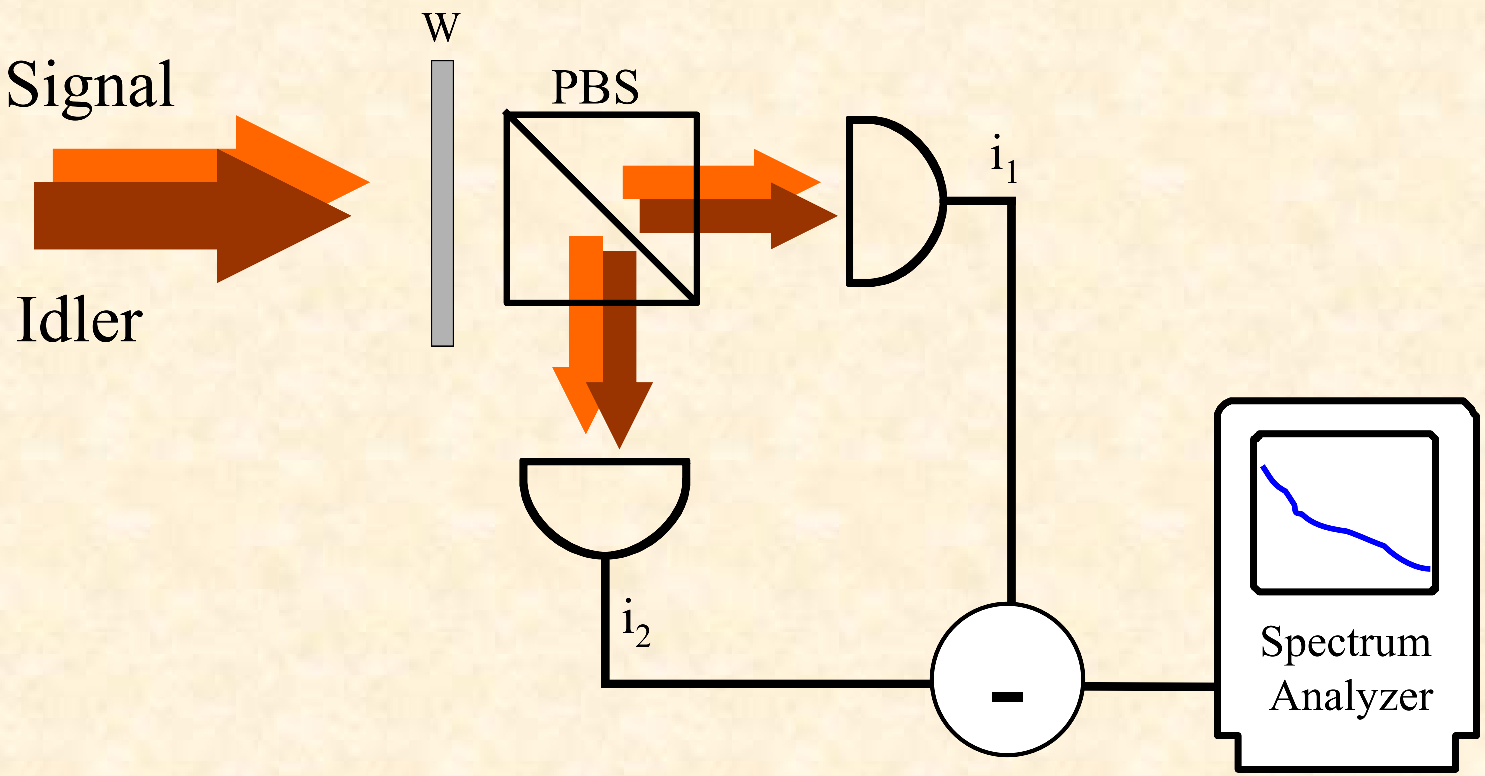
- For long integration times, strong correlations.
- Quantum correlations are not observed if the integration time is shorter than the lifetime of a photon inside the cavity.

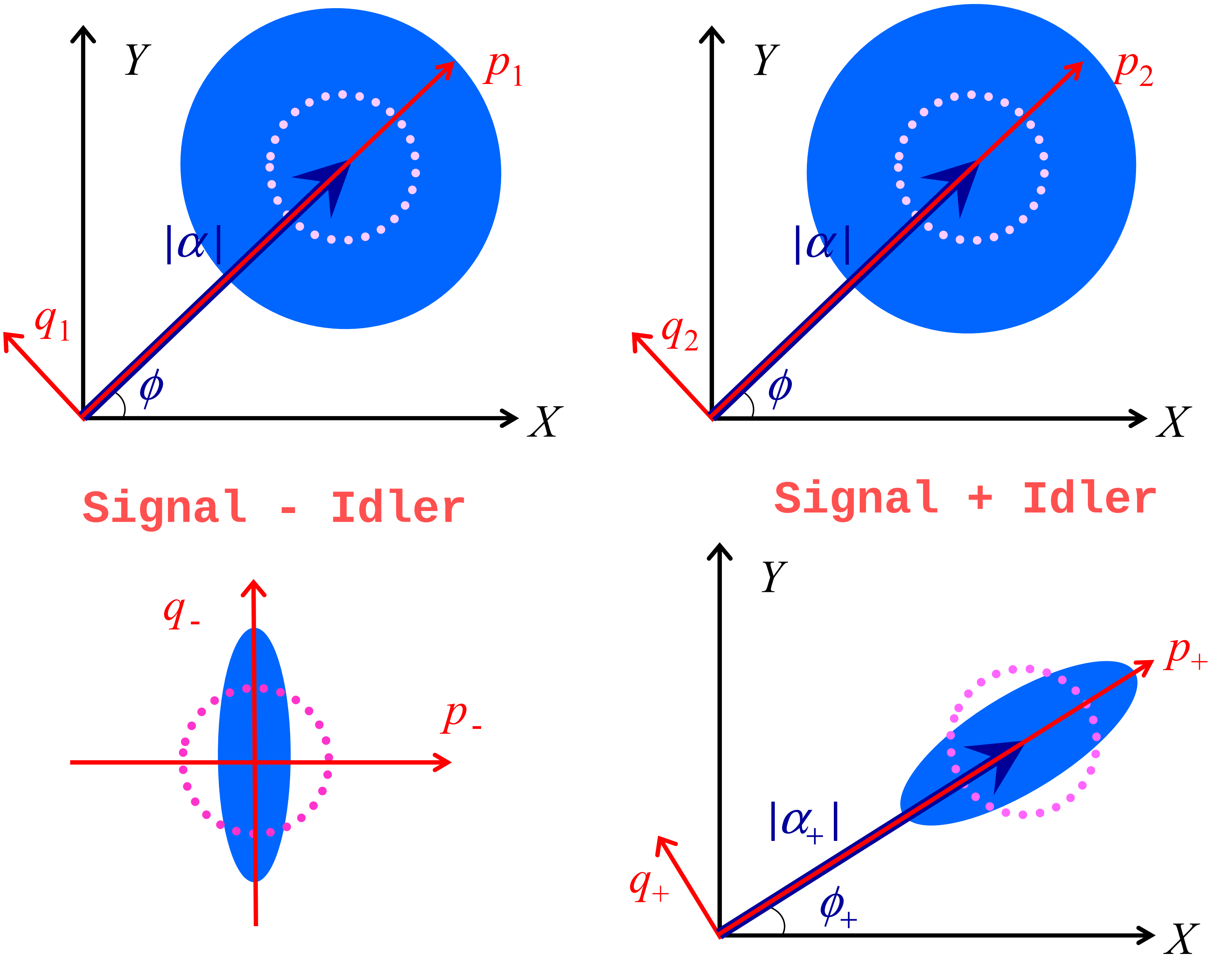


Generation of Twin Beams

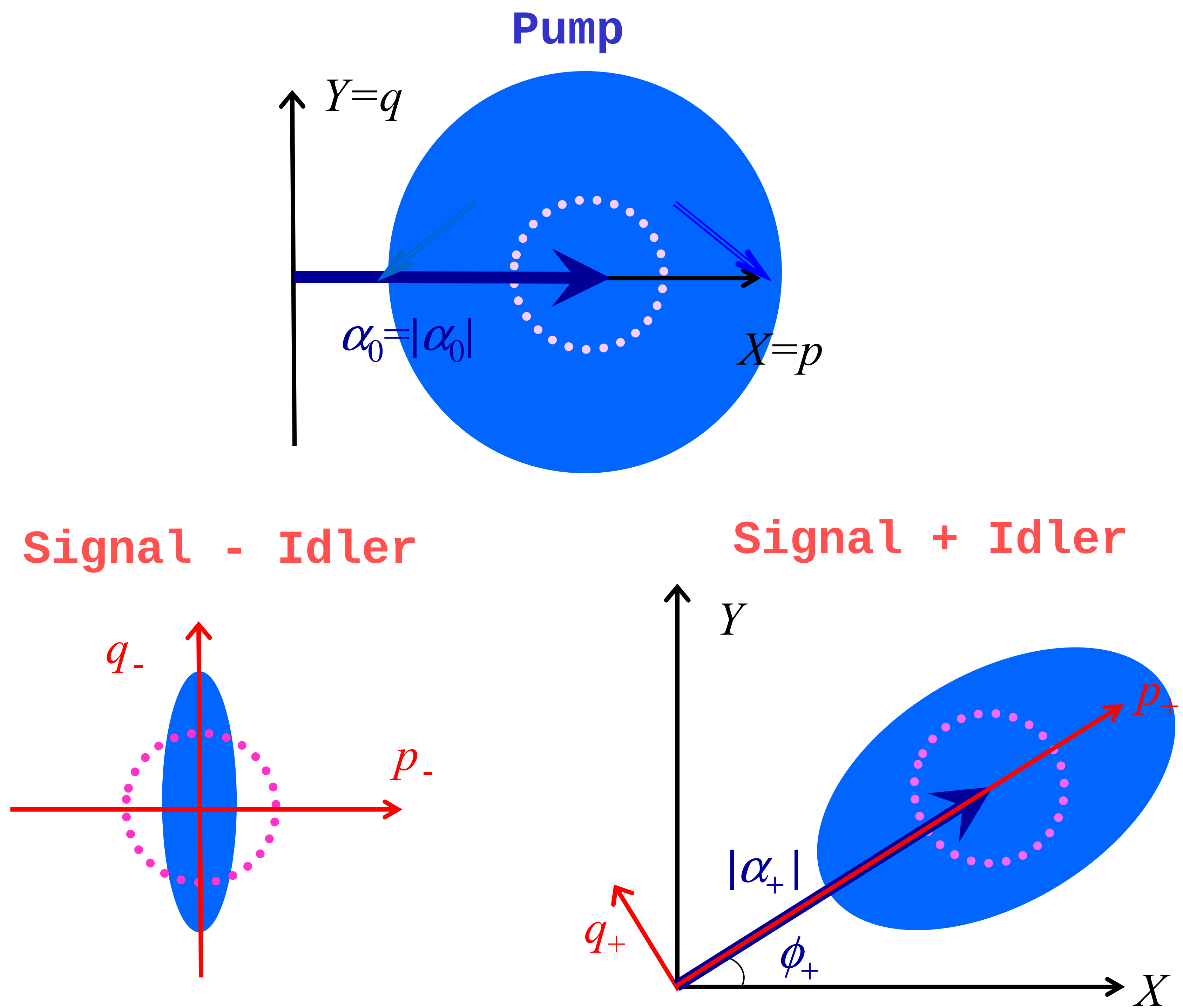
How do we check that we have produced twin beams?

We measure the “shot noise” by mixing signal and idler in the beamsplitter.

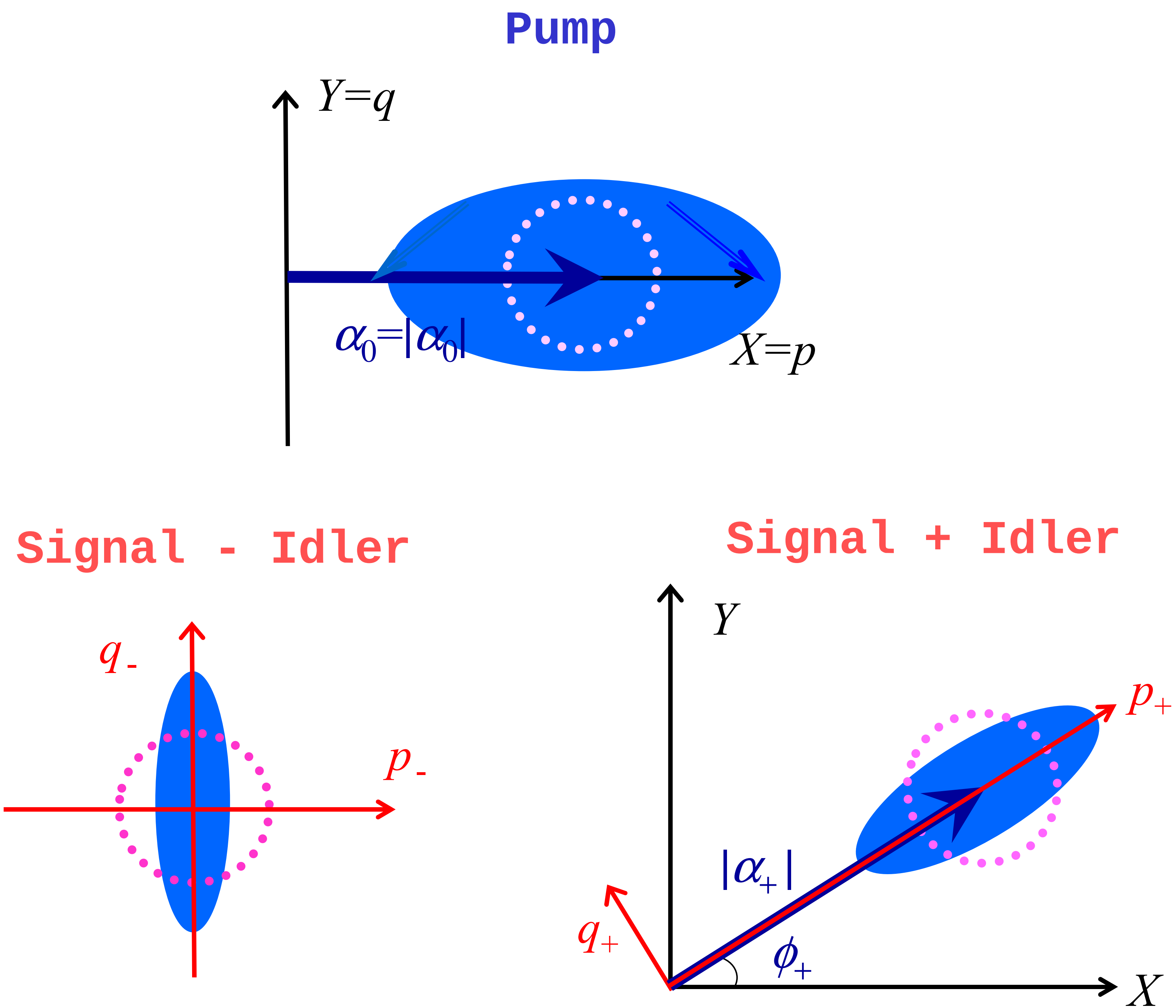




Y
X
p_1
q_1
$|\alpha|$
ϕ
Y
X
p_2
q_2
$|\alpha|$
ϕ
Signal - Idler
Signal + Idler
q_-
p_-
Y
X
p_+
q_+
$|\alpha_+|$
ϕ_+



Pump
Y=q
X=p
α0=|α0|
Signal - Idler
q-
p-
Signal + Idler
Y
X
p+
q+
|α+|
ϕ+



Pump
$Y = q$
$\alpha_0 = |\alpha_0|$
$X = p$
Signal - Idler
q_-
p_-
Signal + Idler
Y
p_+
$|\alpha_+|$
q_+
ϕ_+
X

Energy Conservation

$$\omega_1 + \omega_2 = \omega_0$$

$$\delta I_1 - \delta I_2 = 0$$

Intensity Correlation

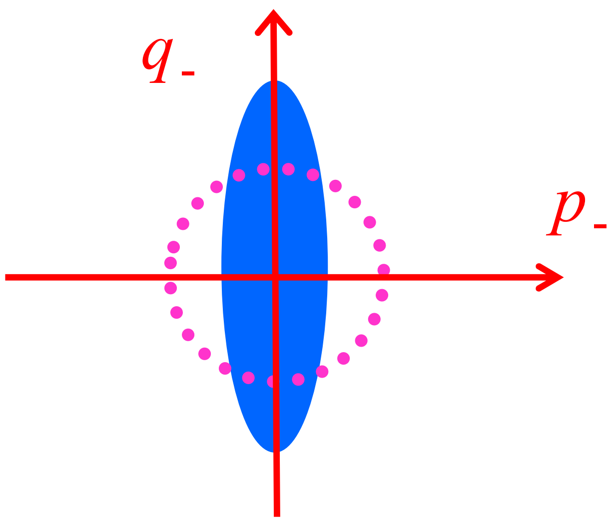
Experiment: A. Heidmann *et al.*, *Phys. Rev. Lett.* **59**, 2555 (1987)

$$\delta\varphi_1 + \delta\varphi_2 = \delta\varphi_0$$

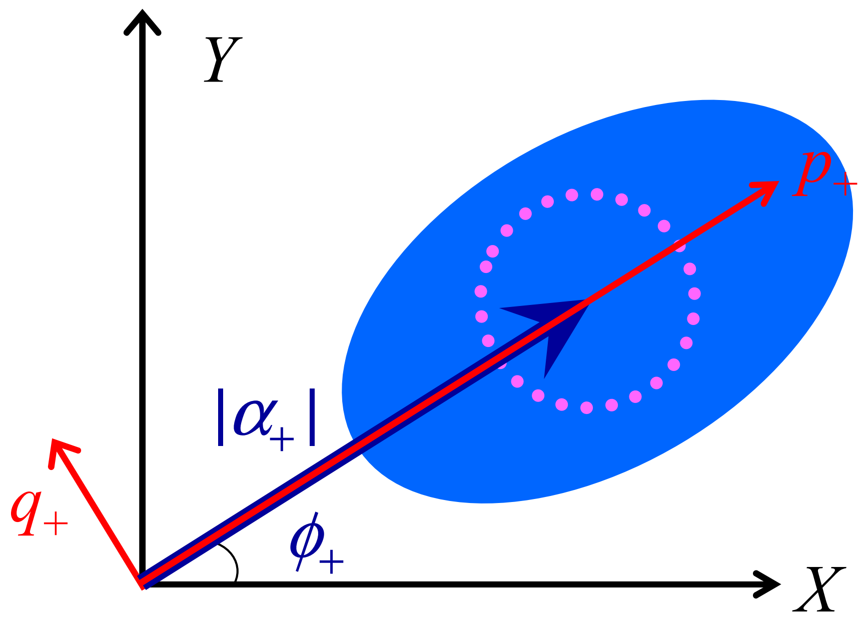
Phase Anti-correlation

Theory: M. D. Reid and P. D. Drummond, *Phys. Rev. Lett.* **60**, 2731 (1988)

Signal - Idler

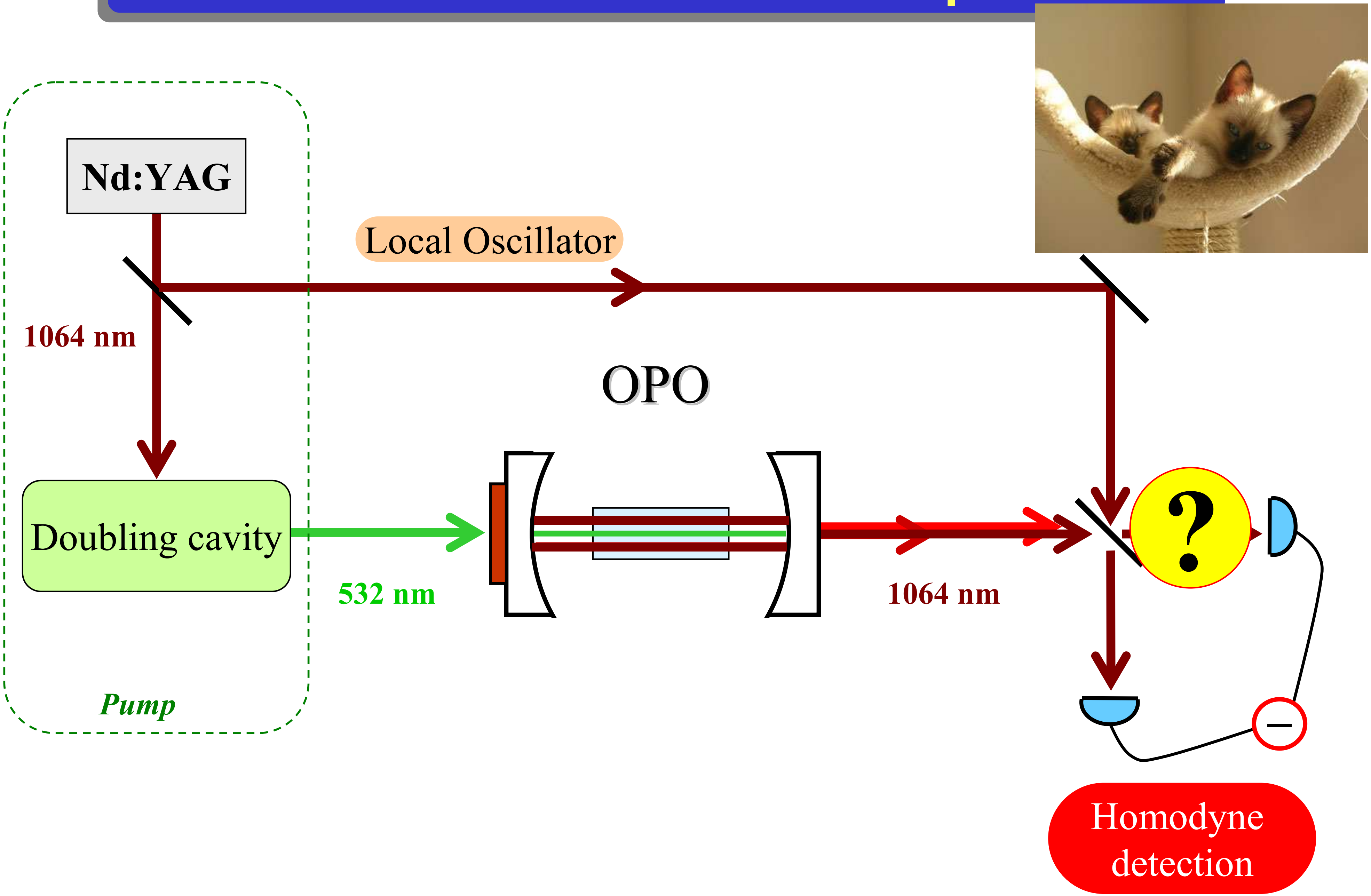


Signal + Idler



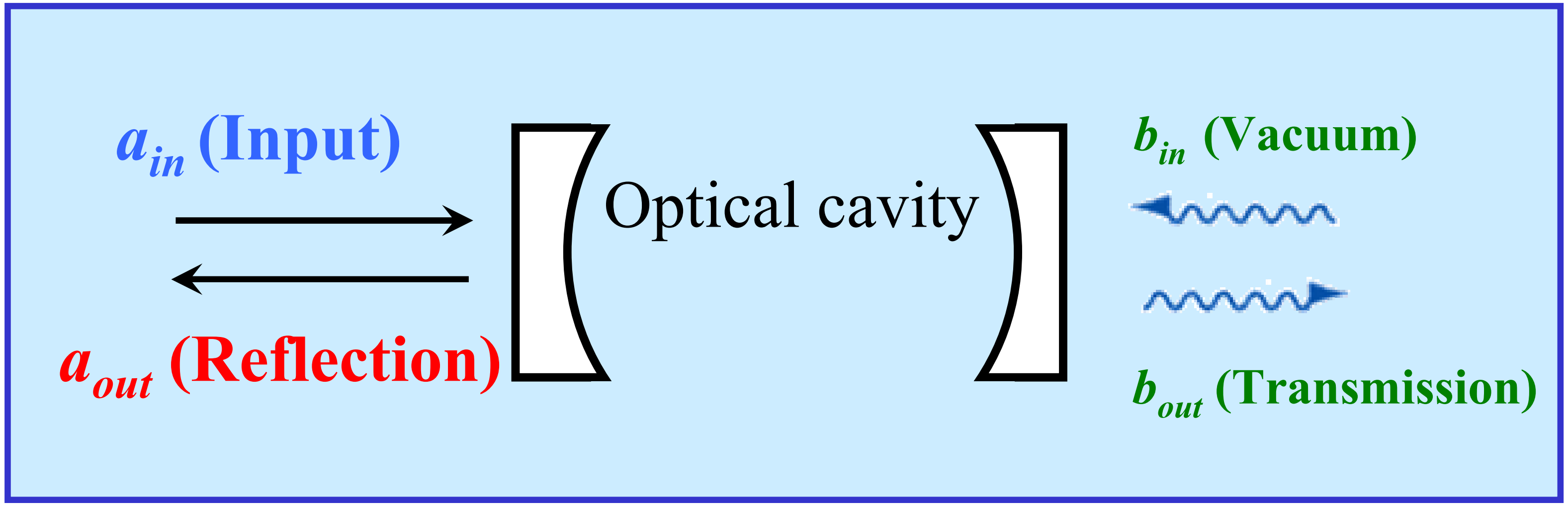
To test for entanglement, we need to measure the phase quadrature. How do we do that without a local oscillator for each beam?

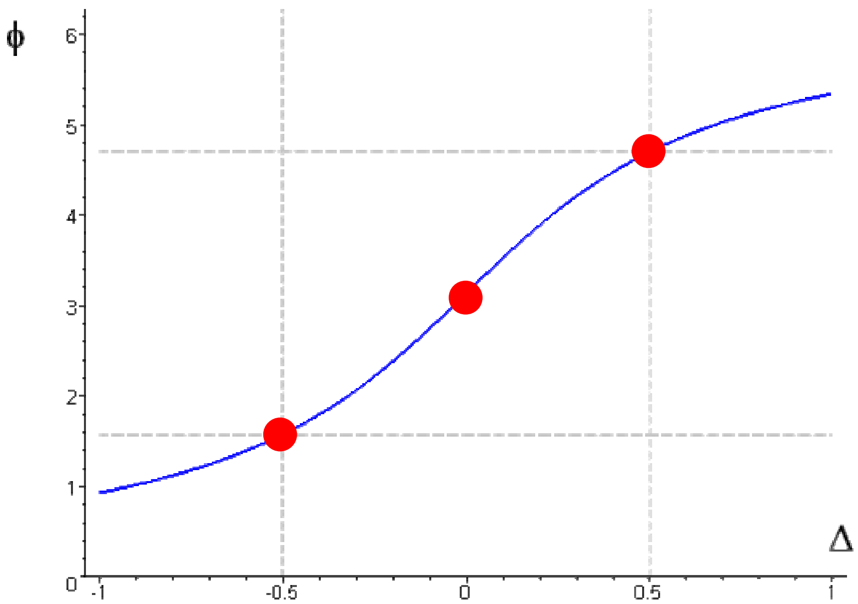
How can we measure the phase?

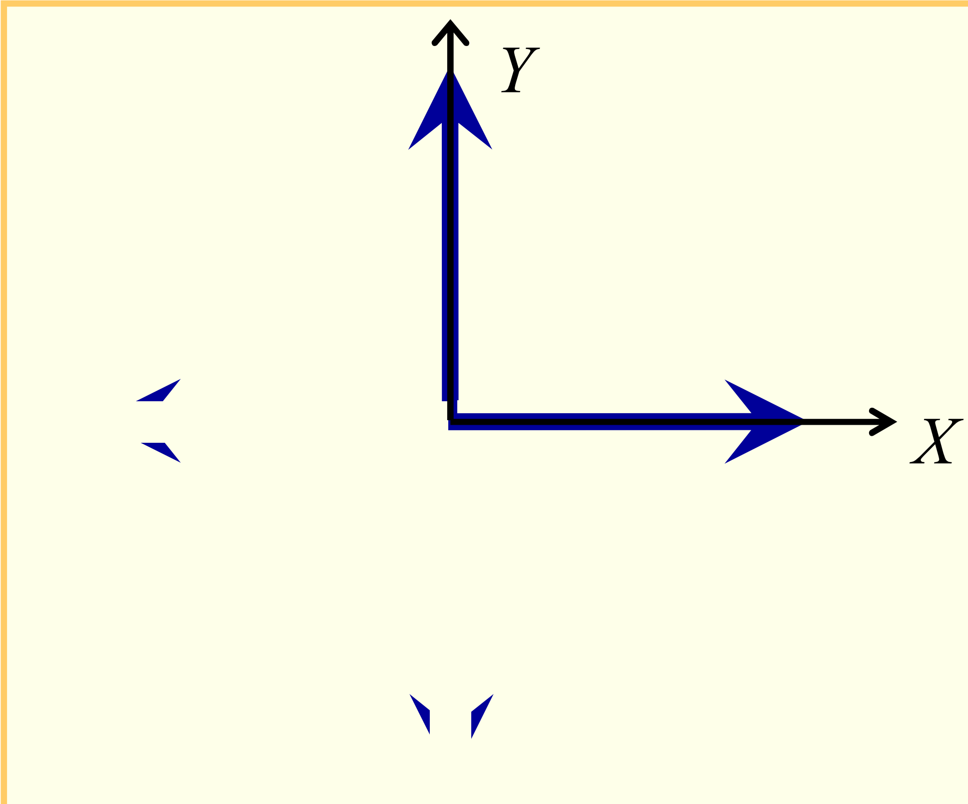


ONLY THEIR MOTHER CAN TELL THEM APART.
TWINS
AN IVAN REITMAN FILM
ARNOLD SCHWARZENEGGER DANNY DEVITO "TWINS" KELLY PRESTON CHLOE WEBB BONNIE BARTLETT WRITTEN BY WILLIAM DAVIES & WILLIAM OSBORNE AND TIMOTHY HARRIS & HERSCHEL WEINGROD MUSIC BY GEORGES DELERUE AND RANDY EDELMAN
PRODUCTION DESIGNER JAMES BISSELL DIRECTOR OF PHOTOGRAPHY ANDRZEJ BARTKOWIAK EXECUTIVE PRODUCERS JOE MEDJUCK AND MICHAEL C. GROSS PRODUCED AND DIRECTED BY IVAN REITMAN
PG PARENTAL GUIDANCE SUGGESTED
OPENS DECEMBER 9TH
A UNIVERSAL PICTURE
PNMP Original Scan

Phase measurement with an Optical Cavity

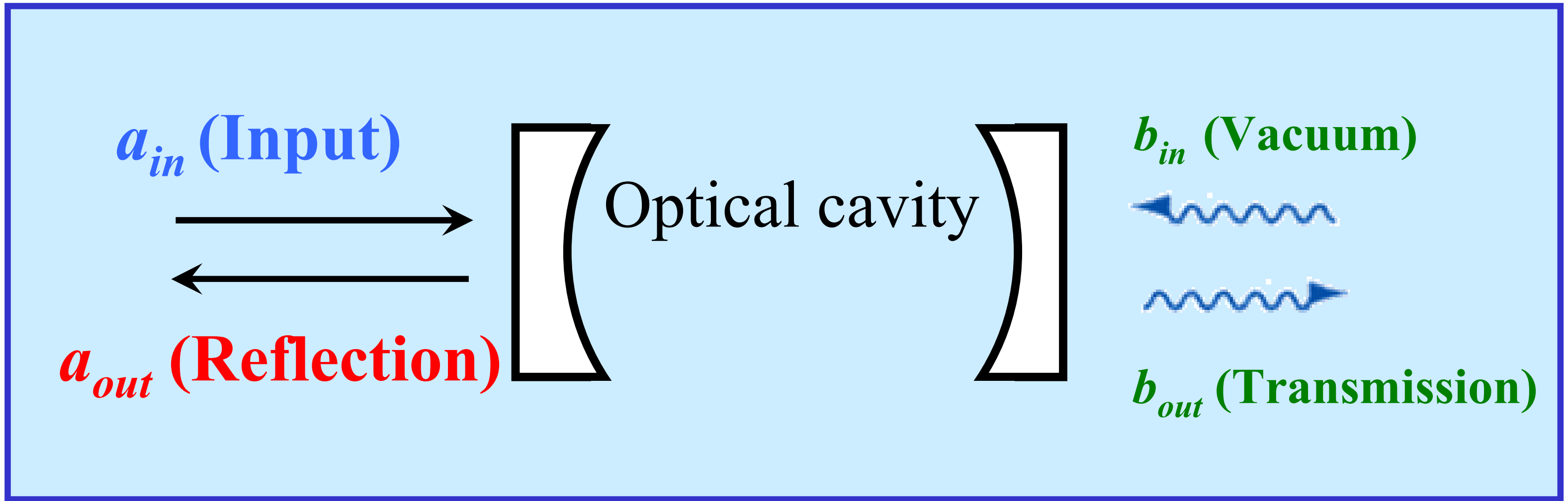


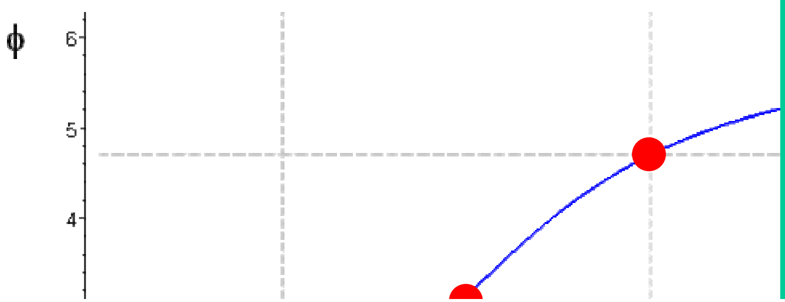




P. Galatola, L. A. Lugiato, M. G. Porreca, P. Tombesi, and G. Leuchs, *Opt. Commun.* **85**, 95 (1991).

Phase measurement with an Optical Cavity





$$\delta I = \alpha^{*}\delta a(\Omega) + \alpha\ \delta a^{\dagger}(-\Omega)$$

$$\delta p = \frac{\delta I}{|\alpha|} = e^{-i\theta_0}\,\delta a(\Omega) + e^{i\theta_0}\,\delta a^{\dagger}(-\Omega)$$

Testing the entanglement of intense beams produced by a non-degenerate optical parametric oscillator

A.S. Villar, M. Martinelli, P. Nussenzveig *

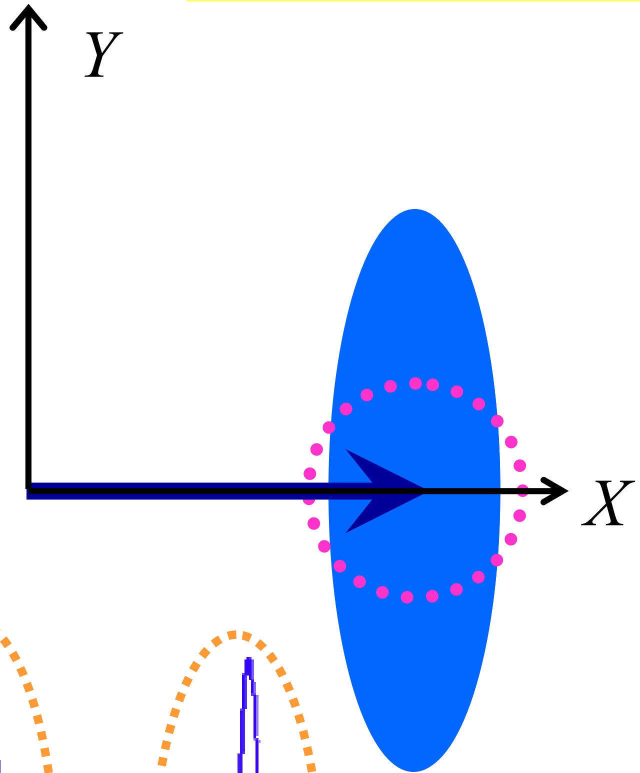
Optics Communications 242 (2004) 551–563

P. Galatola, L. A. Lugiato, M.
Opt. Commun. **85**, 95 (1991).

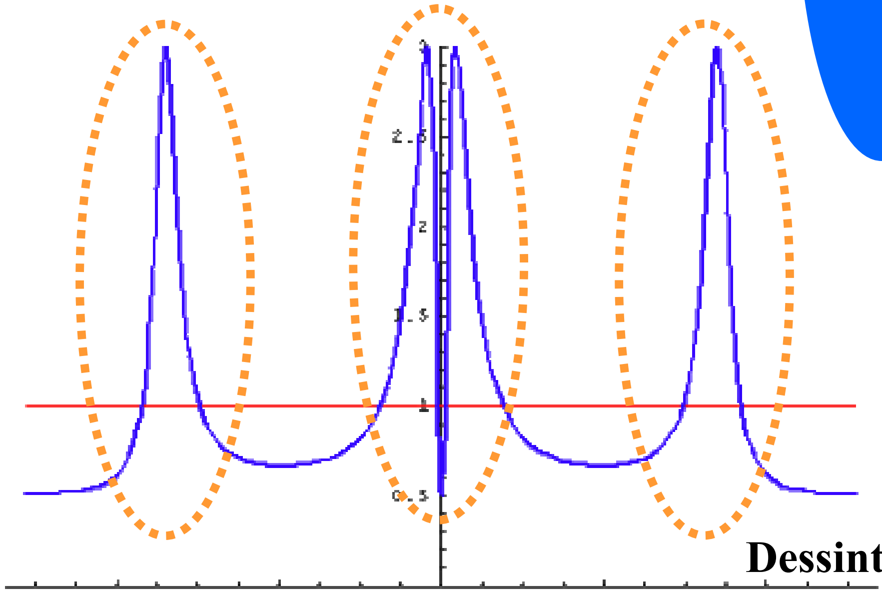
Rotation of the Noise Ellipse

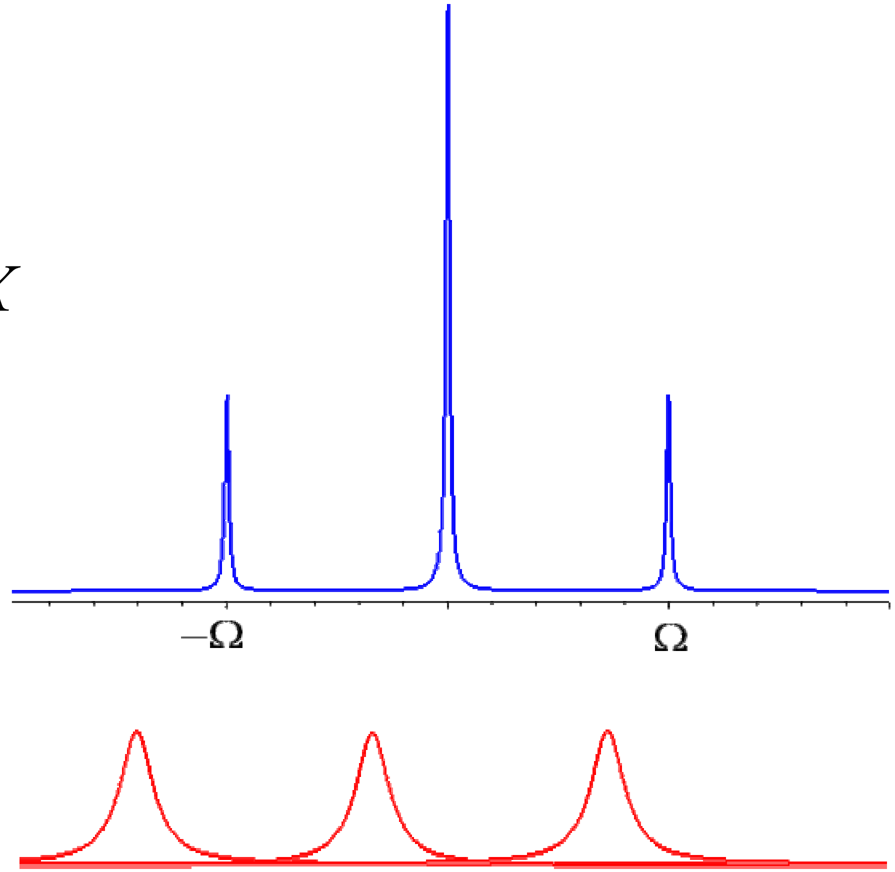
$$\delta p_{in} = \delta a_{in}(\Omega) + \delta a^{\dagger}_{in}(-\Omega)$$

$$\delta p_{out} = e^{-i\theta}\, e^{-i\varphi_{\Omega}}\, \delta a_{in}(\Omega) + e^{i\theta}\, e^{-i\varphi_{-\Omega}}\, \delta a^{\dagger}_{in}(-\Omega)$$



Amplitude Noise of the reflected beam

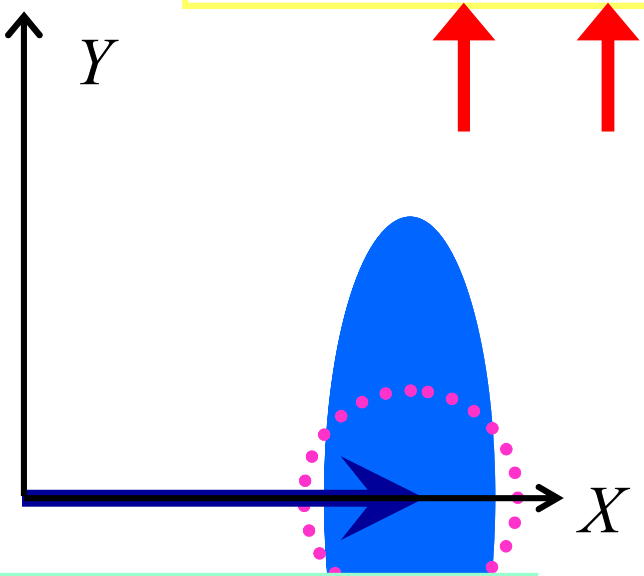




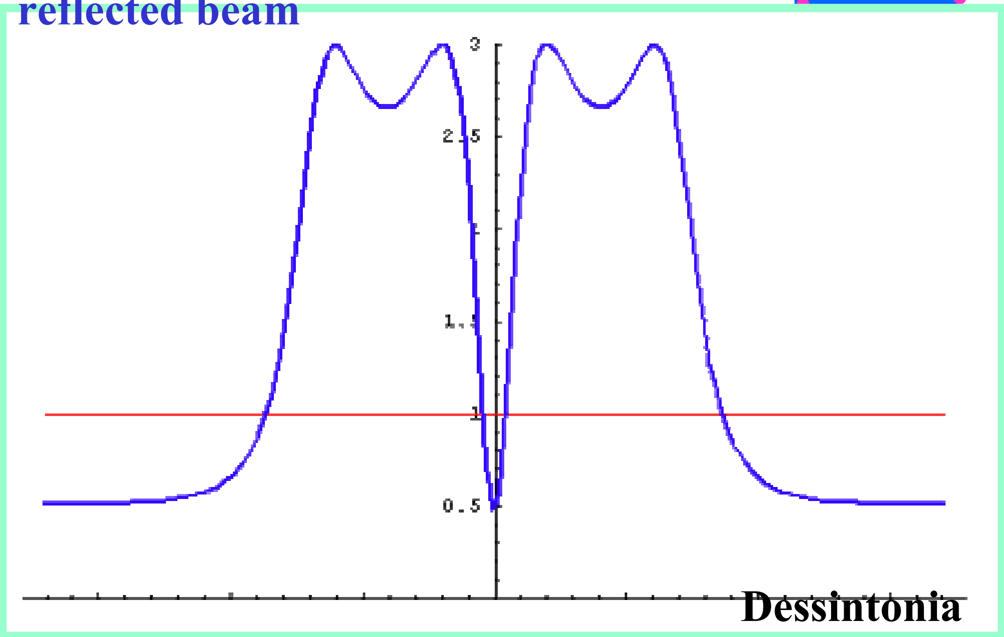
Rotation of the Noise Ellipse

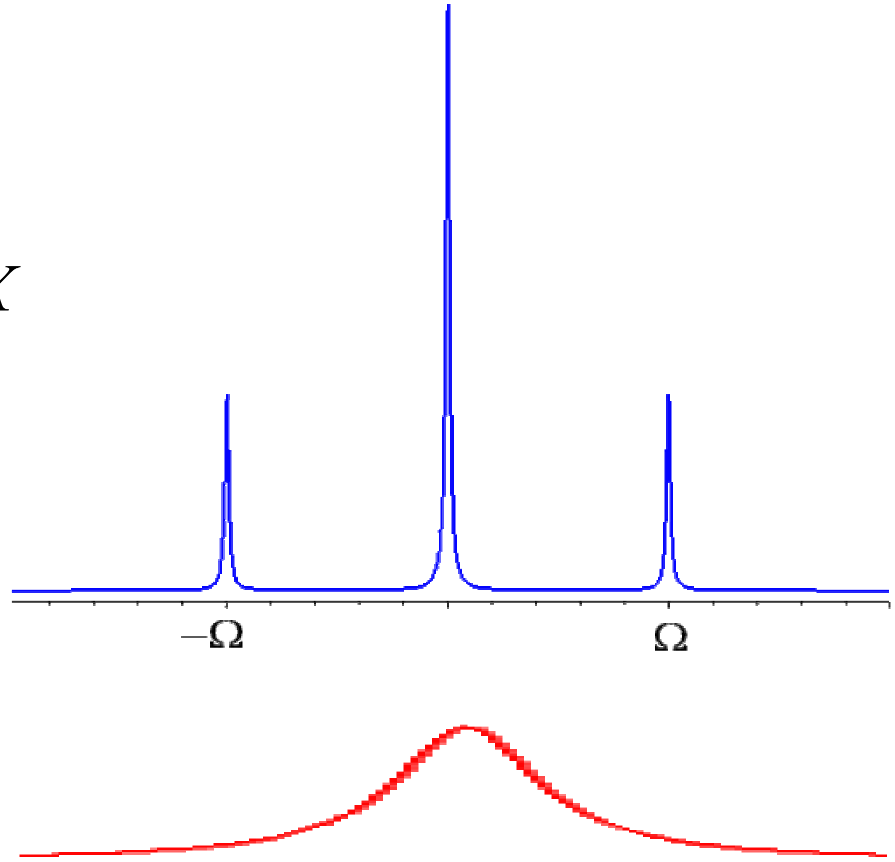
$$\delta p_{in} = \delta a_{in}(\Omega) + \delta a^{\dagger}_{in}(-\Omega)$$

$$\delta p_{out} = \dots\dots\dots\; \delta a_{in}(\Omega) \; + \; \dots\dots\dots\; \delta a^{\dagger}_{in}(-\Omega)$$

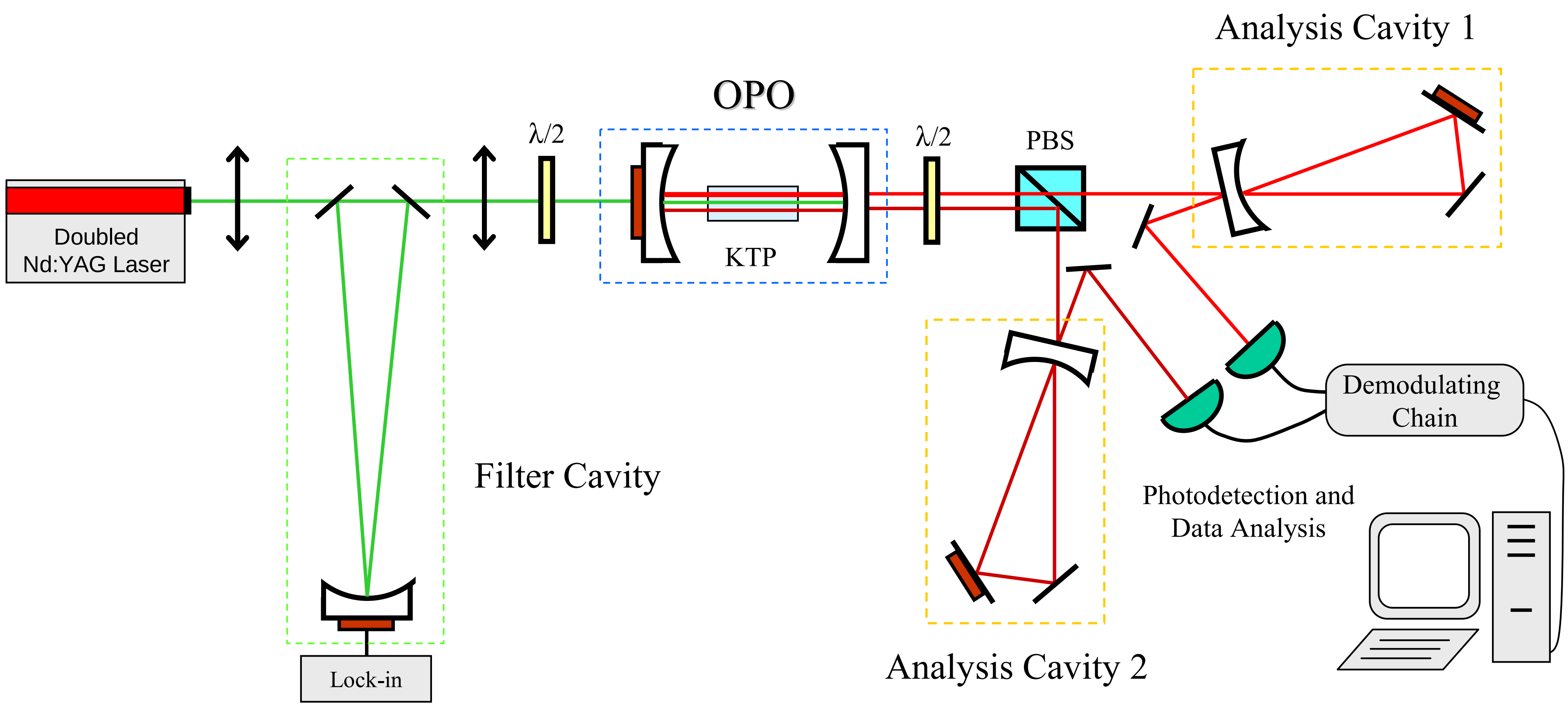


Amplitude Noise of the reflected beam

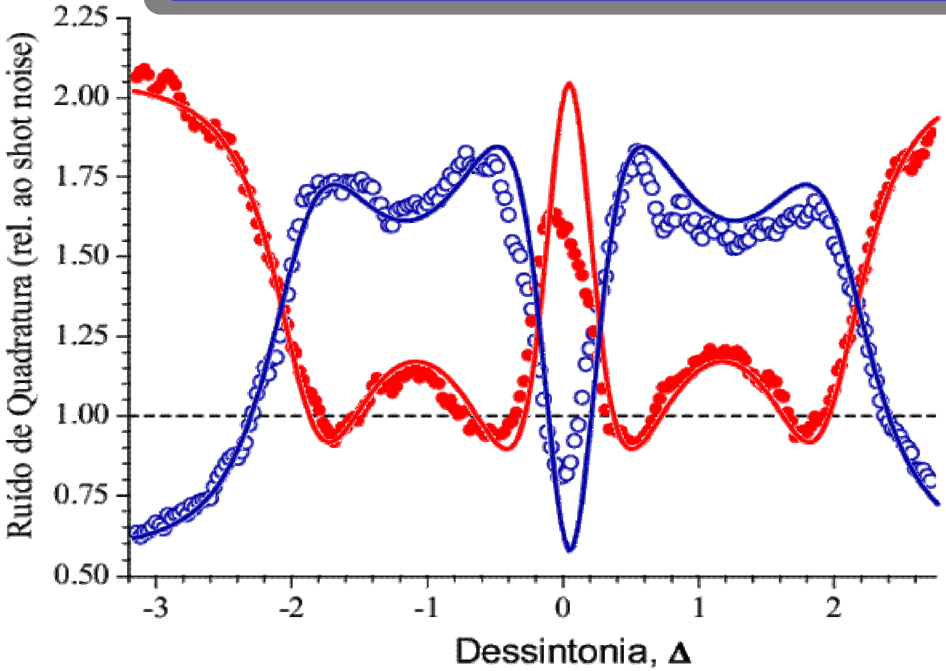


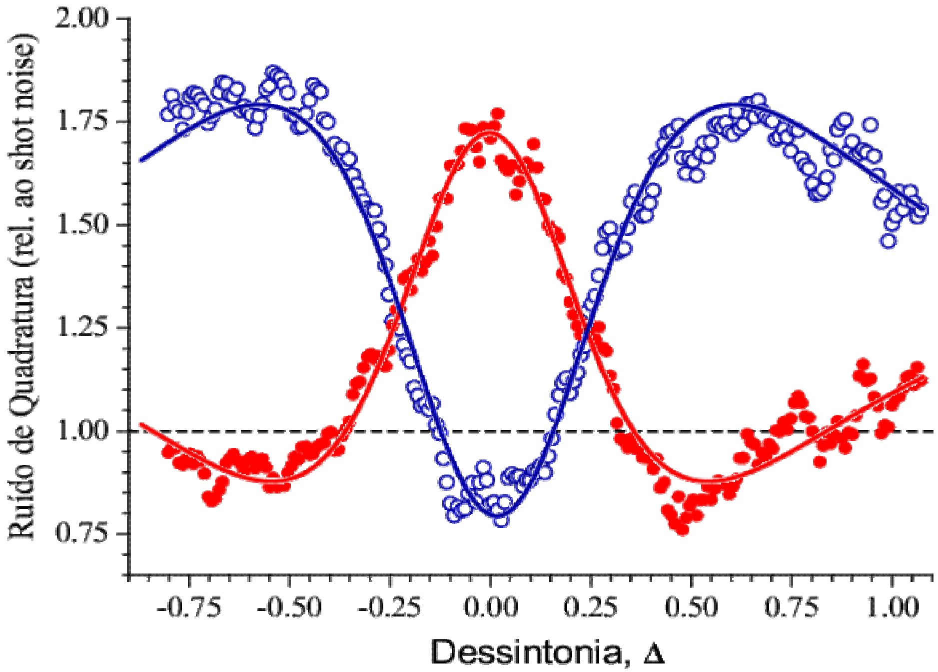


Experimental setup



Results: ν = 21 MHz





$$\Delta^2 p_- + \Delta^2 q_+ = 1.41\ (2) < 2$$

$$\Delta^2 p_- = 0.59\ (1)$$

$$\Delta^2 q_+ = 0.82\ (2)$$

A.S. Villar, L.S. Cruz, K.N. Cassemiro, M. Martinelli, and P. Nussenzveig, *Phys. Rev. Lett.* **95**, 243603 (2005)

$\Delta\lambda = 0.8$ nm
(possible ~10's to 100's nm)

X. Su *et al.*, Opt. Lett. **31**, 1133 (2006)
[Universidade de Shanxi, China]
J. Jing *et al.*, Phys. Rev. A **74**, 041804 (2006)
[Universidade de Virgínia, EUA]

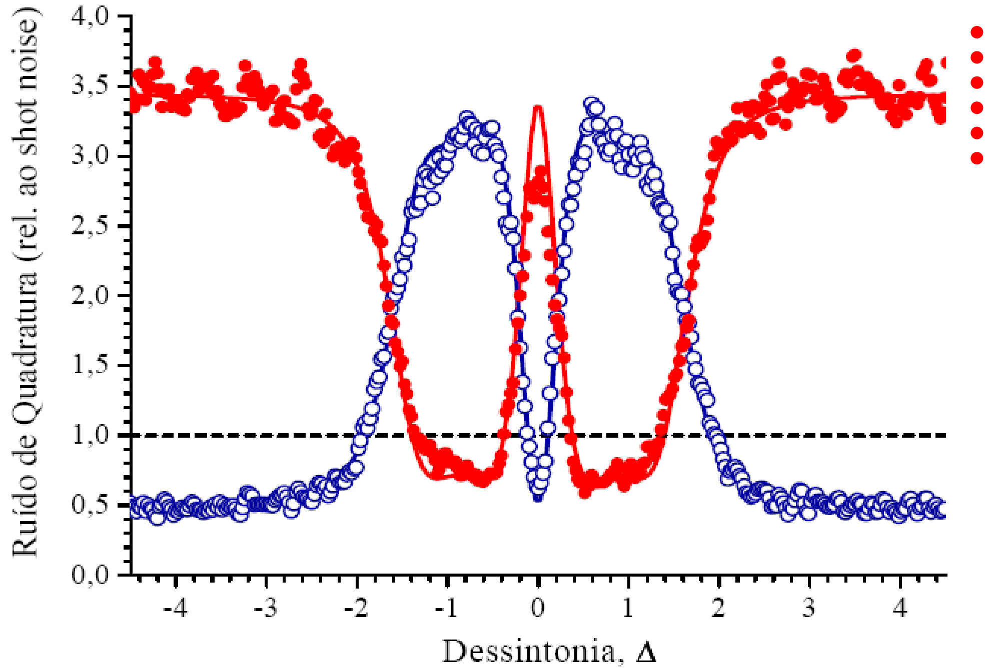
Results: ν = 21 MHz

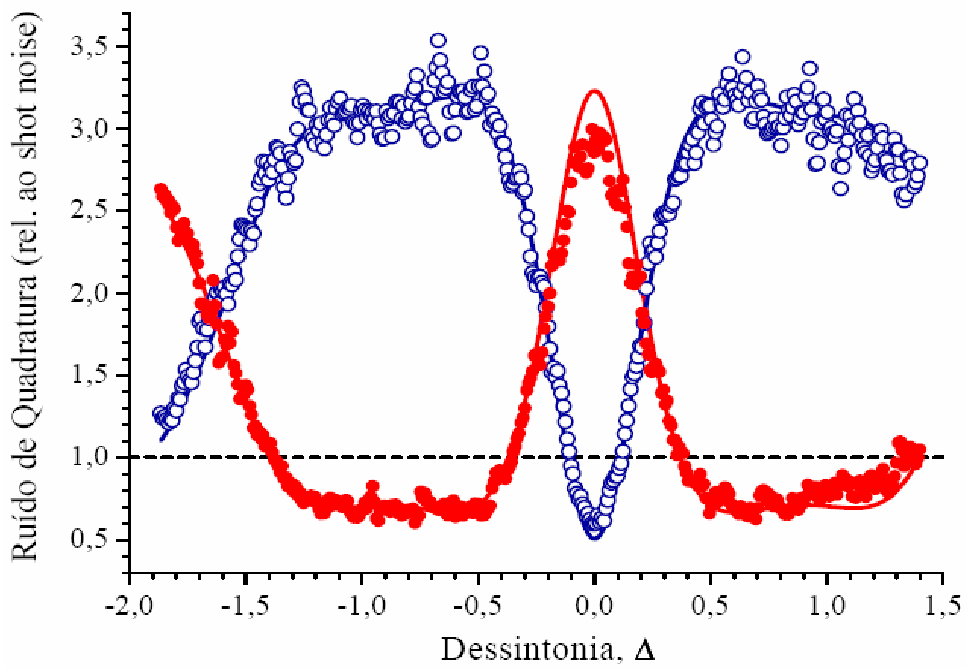
$\Delta^2 p_- + \Delta^2 q_+ = 1.14\ (2) < 2$

$\Delta^2 p_- = 0.49(1)$ [-3. 01dB]

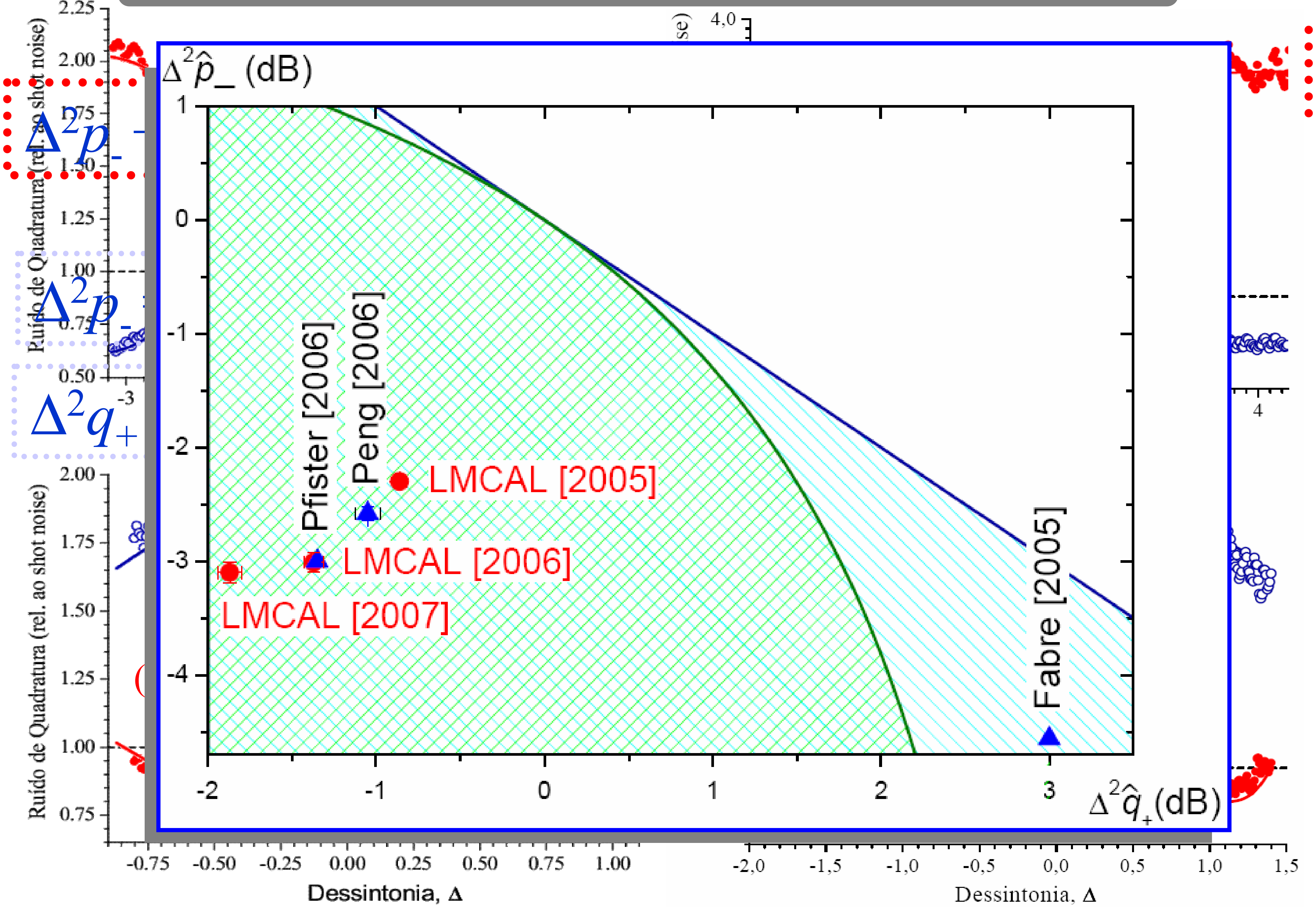
$\Delta^2 q_+ = 0.65(1)$ [-1.87dB]

Δλ = 2.5 nm
(possible ~10's to 100's nm)





Results: ν = 21 MHz

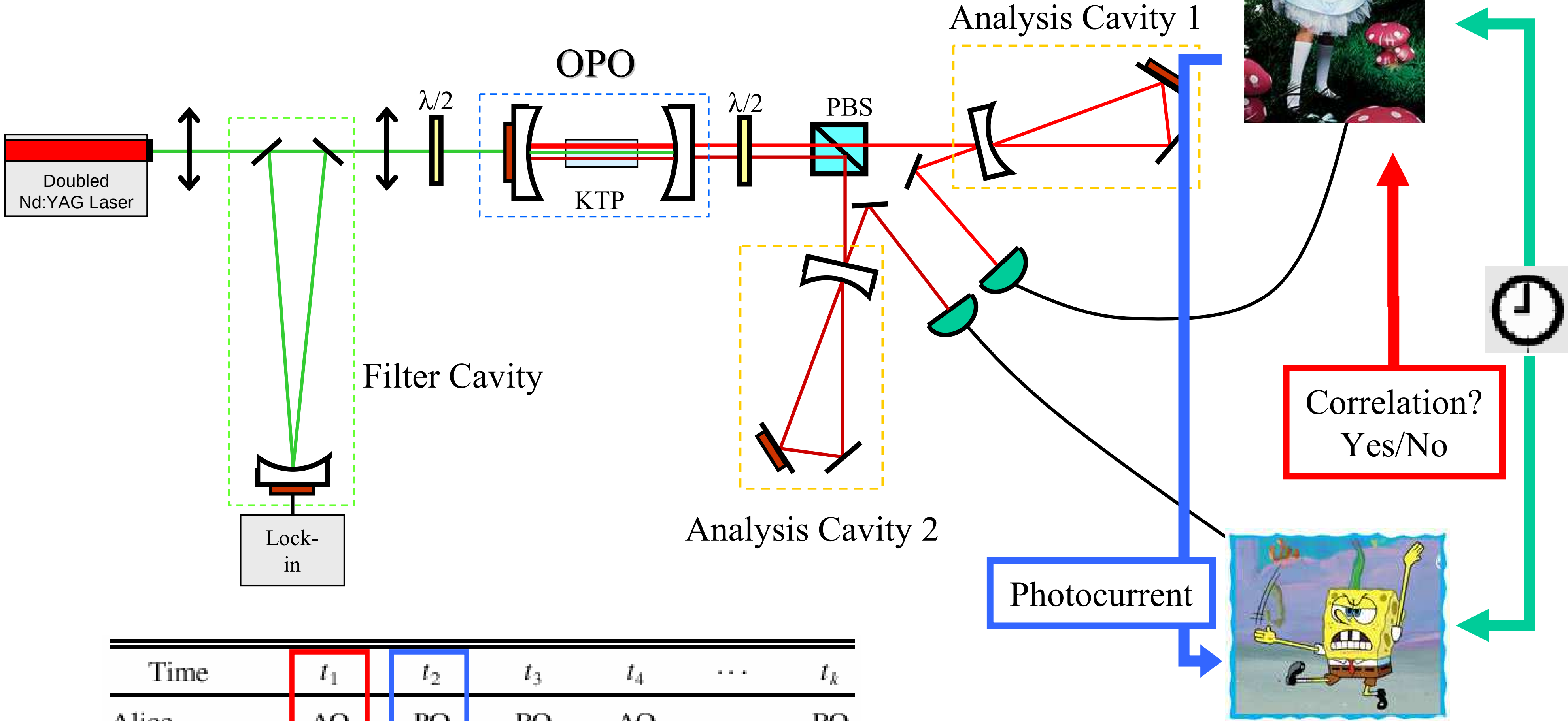


Quantum Key Distribution

VOLUME 88, NUMBER 16 PHYSICAL REVIEW LETTERS 22 APRIL 2002

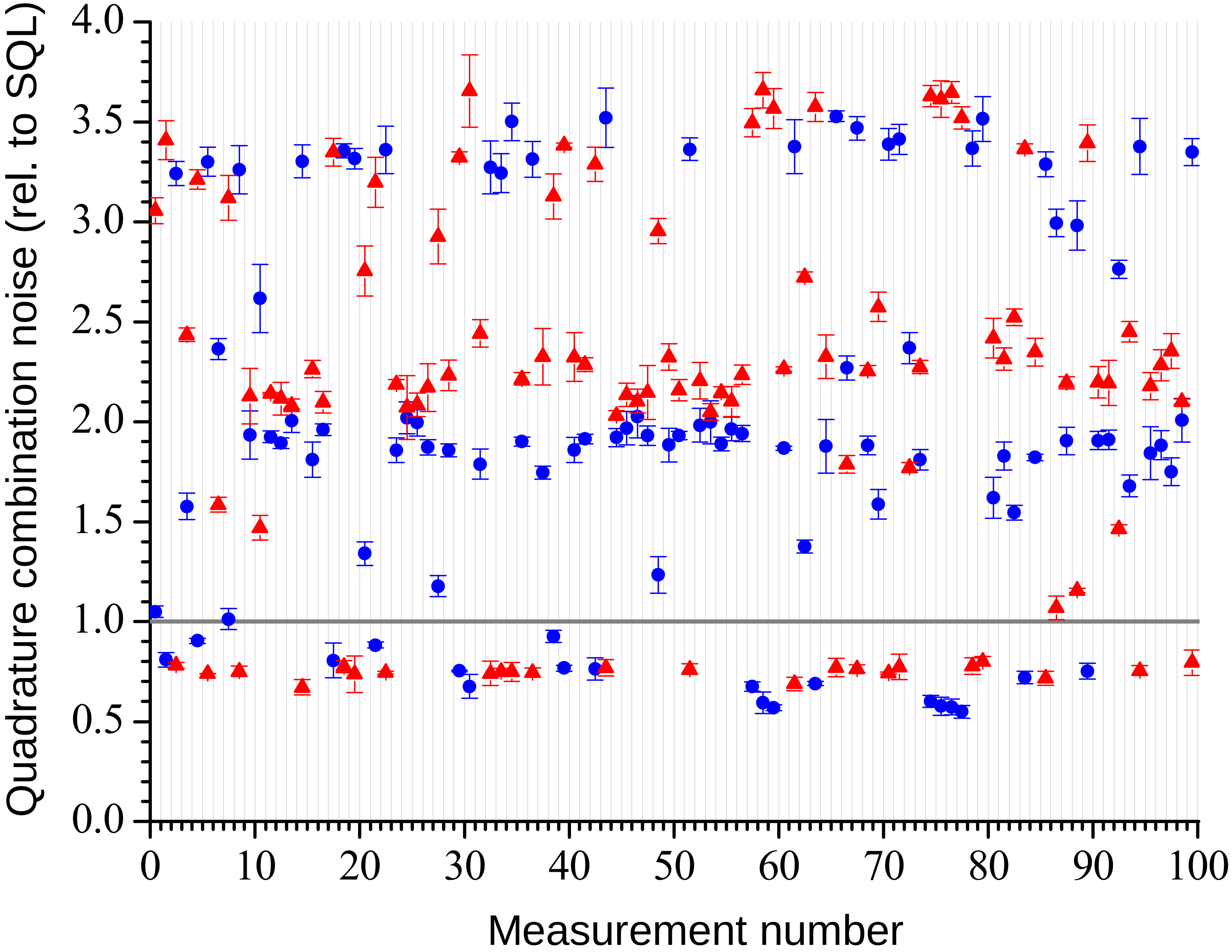
Quantum Key Distribution with Bright Entangled Beams

Ch. Silberhorn, N. Korolkova, and G. Leuchs
Zentrum für Moderne Optik an der Universität Erlangen-Nürnberg, Staudtstraße 7/B2, D-91058 Erlangen, Germany
(Received 14 November 2001; published 8 April 2002)

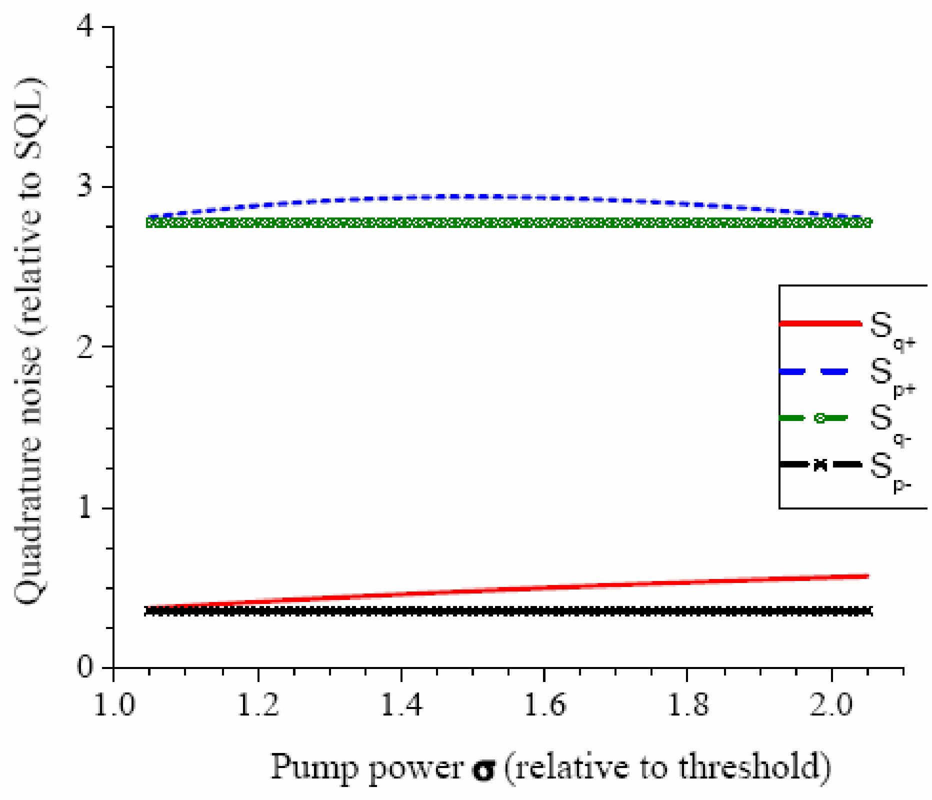


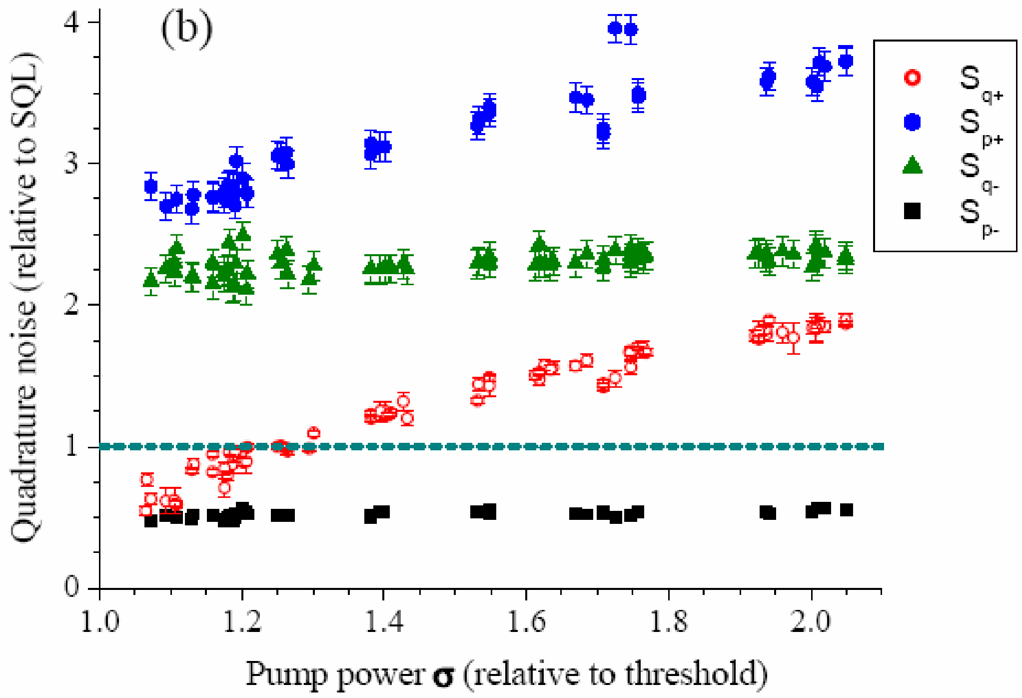
Time	t_1	t_2	t_3	t_4	···	t_k
Alice	AQ	PQ	PQ	AQ	···	PQ
Bob	AQ	AQ	PQ	PQ	···	PQ
Correlation	Yes	No	Yes	No	···	Yes
Key	1	···	0	···	···	0

Quantum Key Distribution

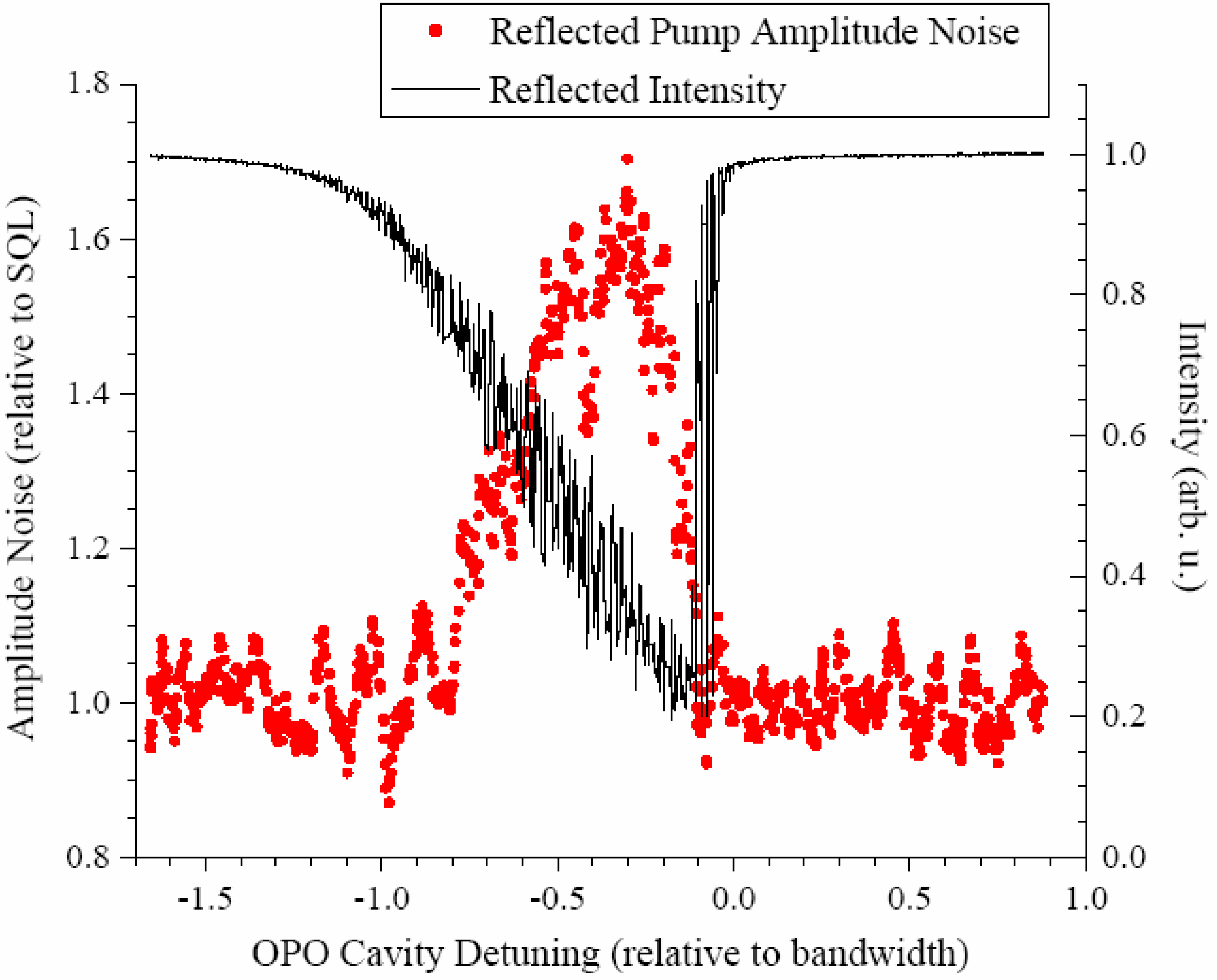


Theory x Experiment





Reflected pump noise



Cavity parameters:

Input

R_{532} = 97 %

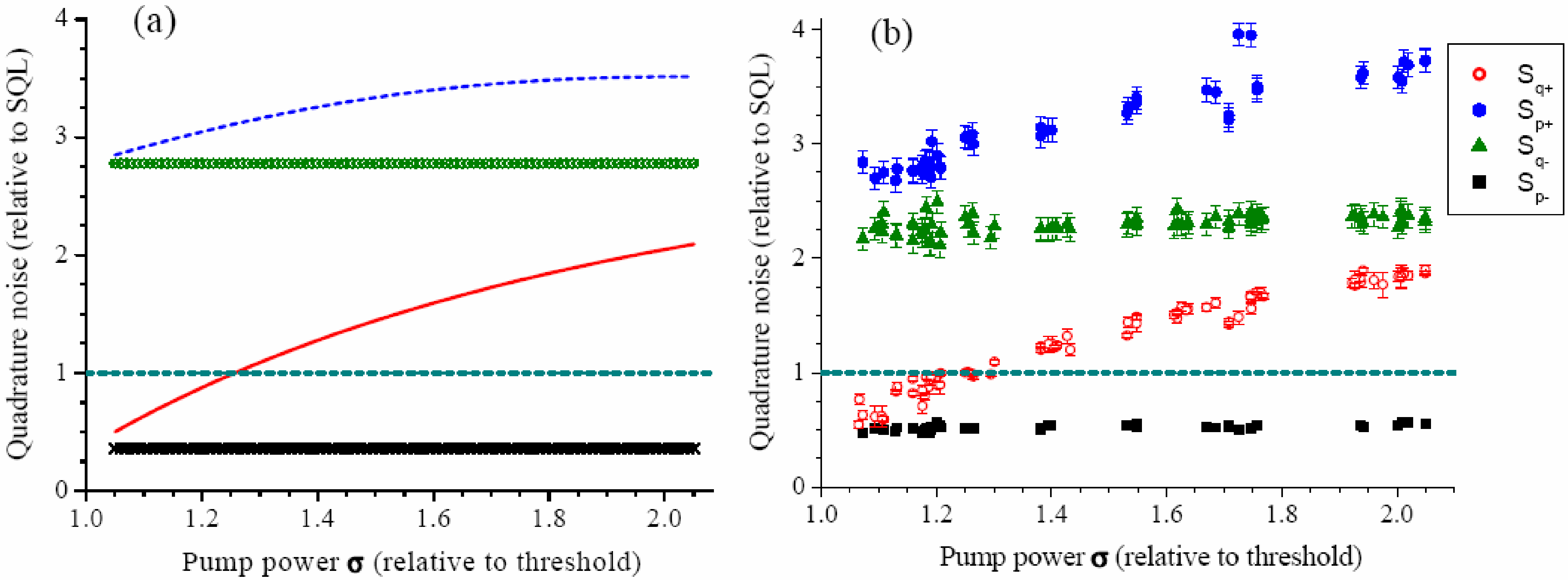
R_{1064} > 99.8 %

Output

R_{532} ~ 99.8 %

R_{1064} = 96 %

Theory x Experiment



Theory x Experiment

Entanglement in the above-threshold optical parametric oscillator

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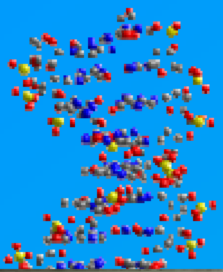
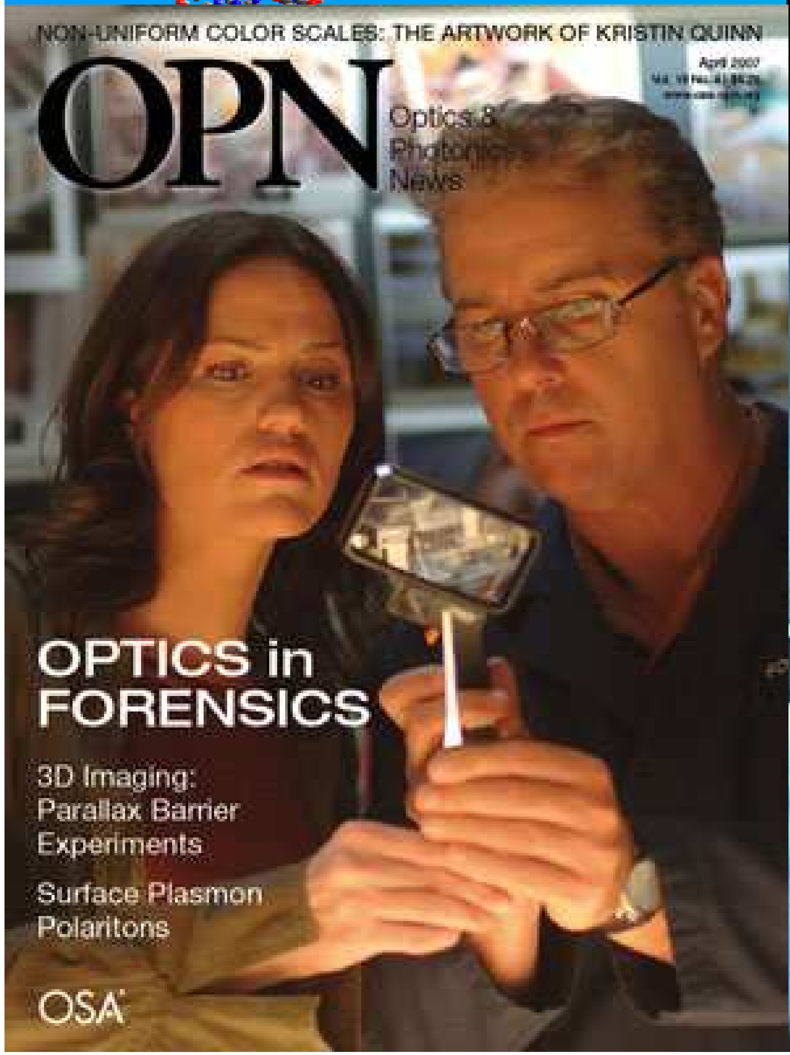
ONLY THEIR MOTHER CAN TELL THEM APART.
TWINS
AN IVAN REITMAN FILM
ARNOLD SCHWARZENEGGER DANNY DEVITO "TWINS" KELLY PRESTON CHLOE WEBB BONNIE BARTLETT WRITTEN BY WILLIAM DAVIES & WILLIAM OSBORNE AND TIMOTHY HARRIS & HERSCHEL WEINGROD MUSIC BY GEORGES DELERUE AND RANDY EDELMAN
PRODUCTION DESIGNER JAMES BISSELL DIRECTOR OF PHOTOGRAPHY ANDRZEJ BARTKOWIAK EXECUTIVE PRODUCERS JOE MEDJUCK AND MICHAEL C. GROSS PRODUCED AND DIRECTED BY IVAN REITMAN
PG PARENTAL GUIDANCE SUGGESTED
OPENS DECEMBER 9TH
A UNIVERSAL PICTURE
PNMP Original Scan

Quantum “DNA test”





Energy Conservation

$$\omega_1 + \omega_2 = \omega_0$$

$$\delta I_1 - \delta I_2 = 0$$

Intensity Correlation

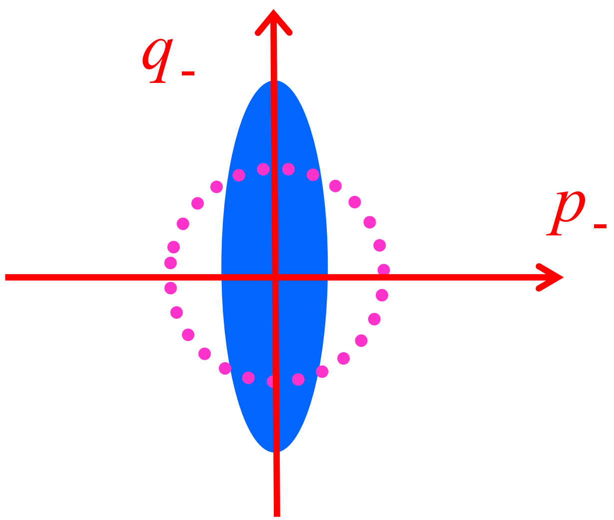
Experiment: A. Heidmann *et al.*, *Phys. Rev. Lett.* **59**, 2555 (1987)

$$\delta\varphi_1 + \delta\varphi_2 = \delta\varphi_0$$

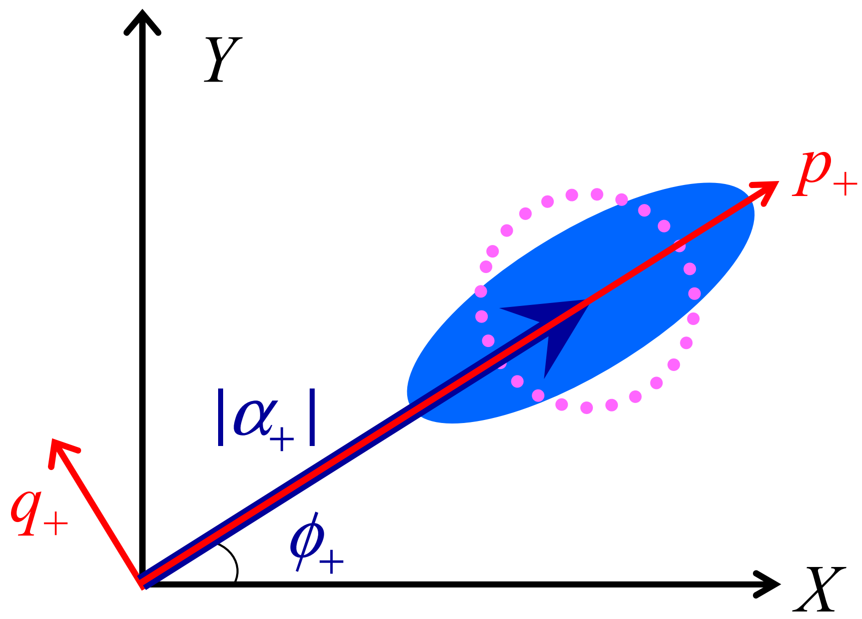
Phase Anti-correlation

Theory: M. D. Reid and P. D. Drummond, *Phys. Rev. Lett.* **60**, 2731 (1988)

Signal - Idler

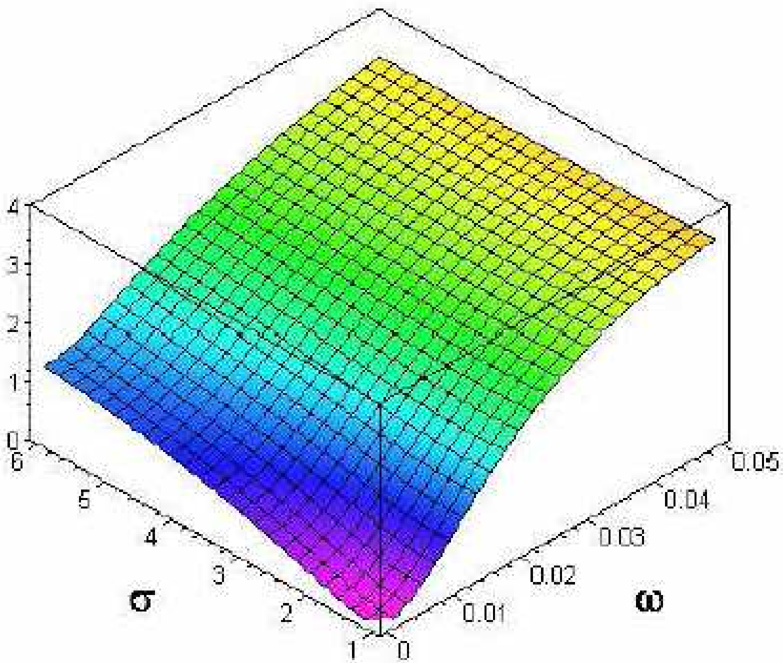


Signal + Idler



Direct Production of Tripartite Pump-Signal-Idler Entanglement in the Above-Threshold Optical Parametric Oscillator

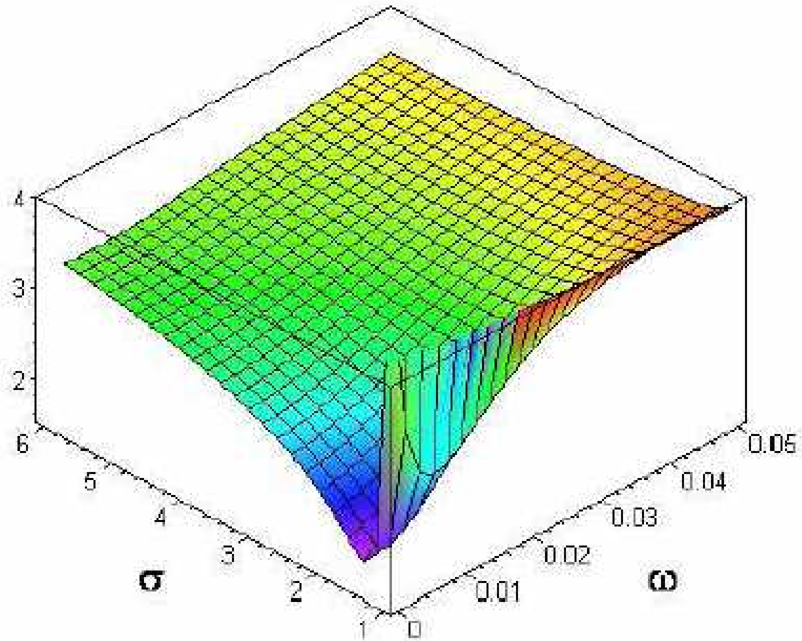
A. S. Villar,[1] M. Martinelli,[1] C. Fabre,[2] and P. Nussenzveig[1,*]



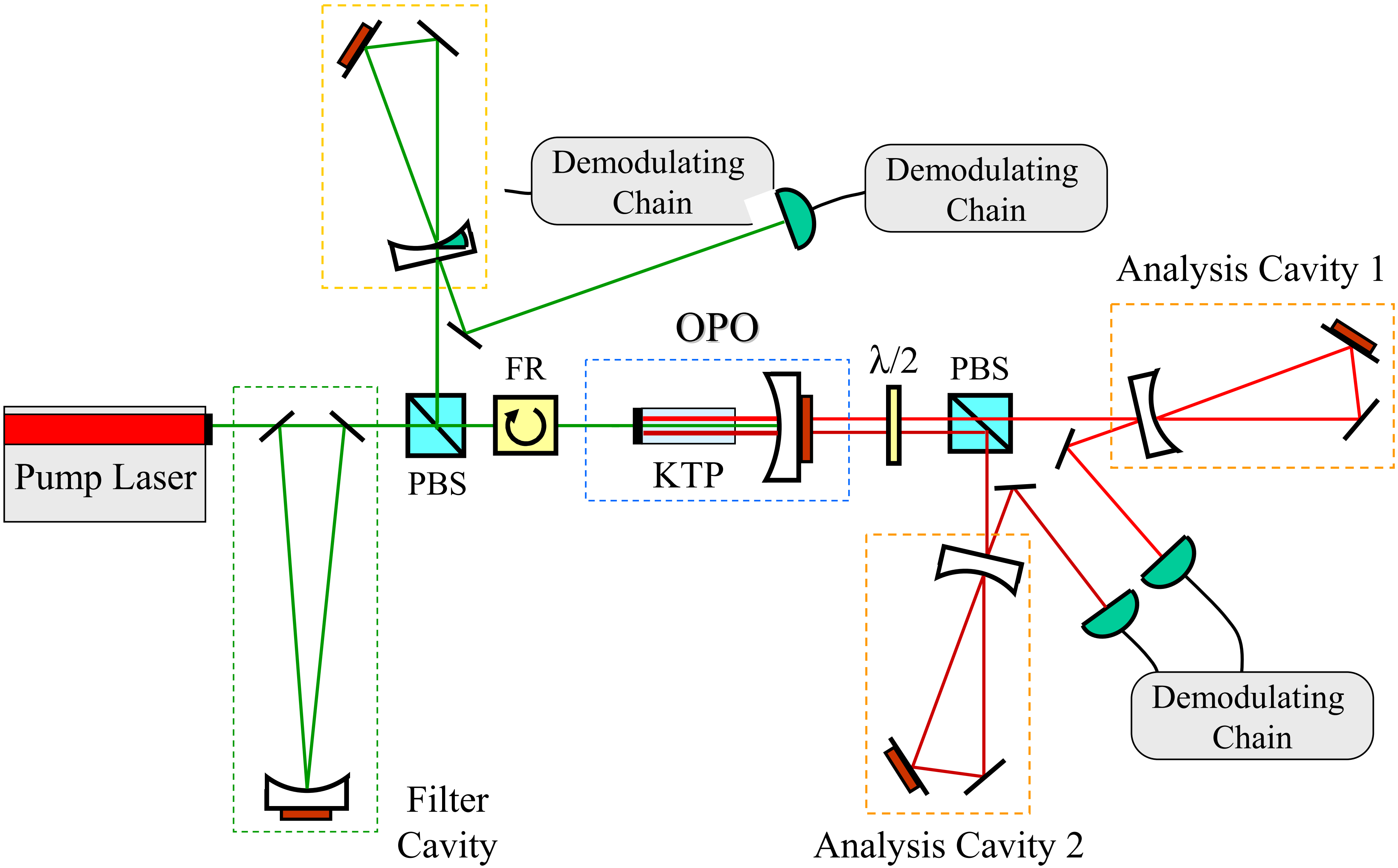
$$V_0 = \Delta^2\left(\frac{\hat{p}_1 - \hat{p}_2}{\sqrt{2}}\right) + \Delta^2\left(\frac{\hat{q}_1 + \hat{q}_2 - \alpha_0\,\hat{q}_0}{\sqrt{2}}\right) \geq 2$$

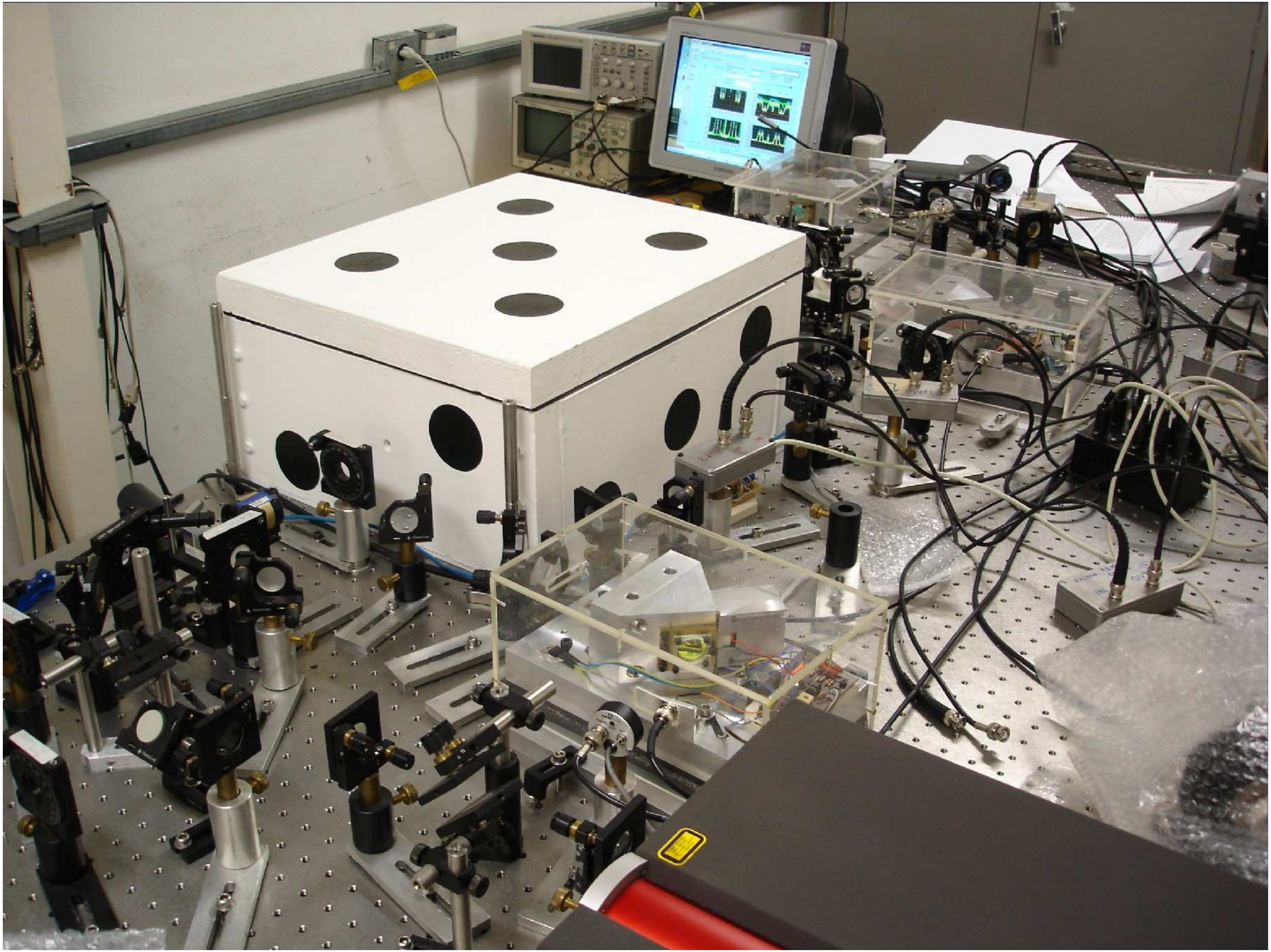
$$V_1 = \Delta^2\left(\frac{\hat{p}_0 + \hat{p}_2}{\sqrt{2}}\right) + \Delta^2\left(\frac{\alpha_1\,\hat{q}_1 + \hat{q}_2 - \hat{q}_0}{\sqrt{2}}\right) \geq 2$$

$$V_2 = \Delta^2\left(\frac{\hat{p}_0 + \hat{p}_1}{\sqrt{2}}\right) + \Delta^2\left(\frac{\hat{q}_1 + \alpha_2\,\hat{q}_2 - \hat{q}_0}{\sqrt{2}}\right) \geq 2$$



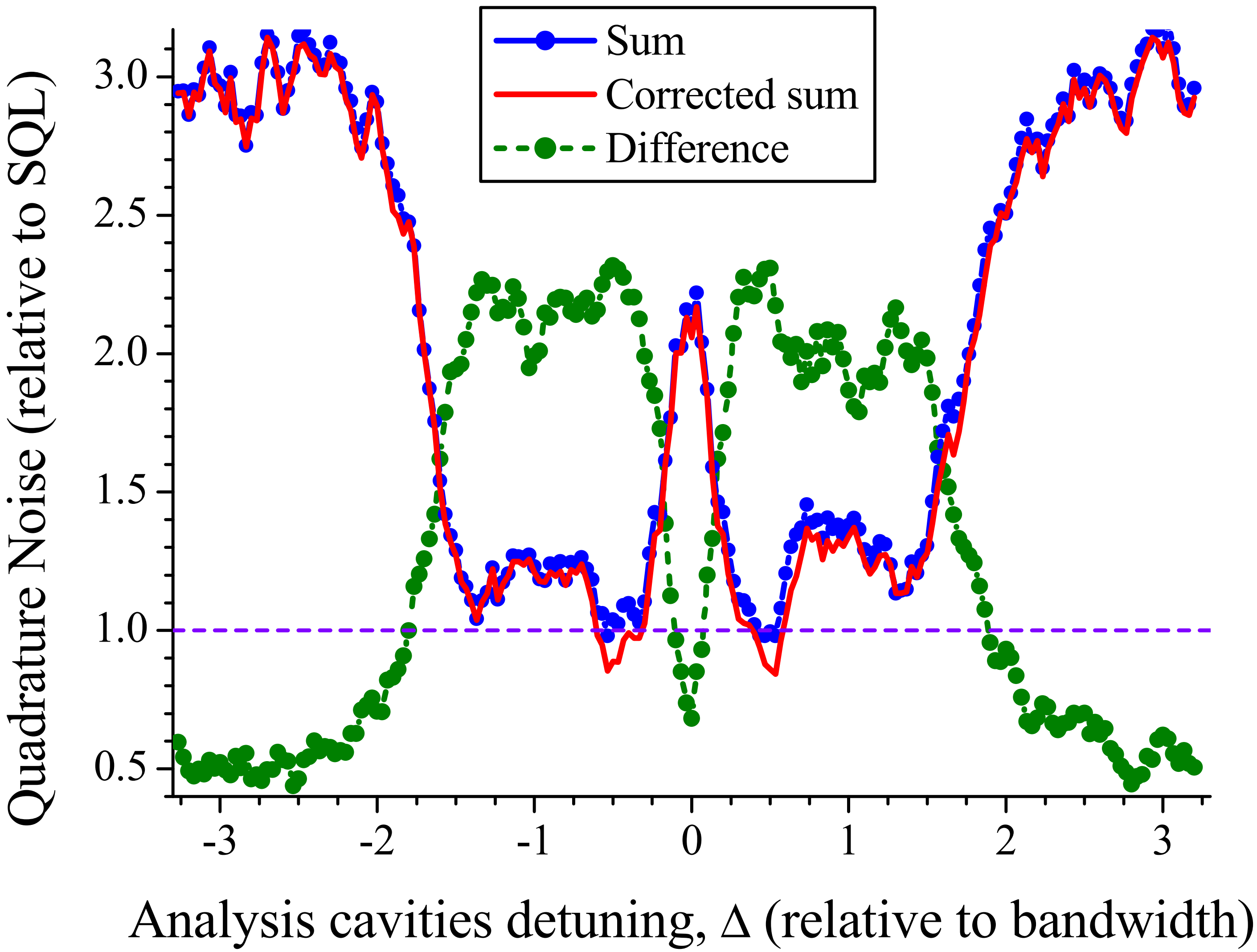
Experimental setup





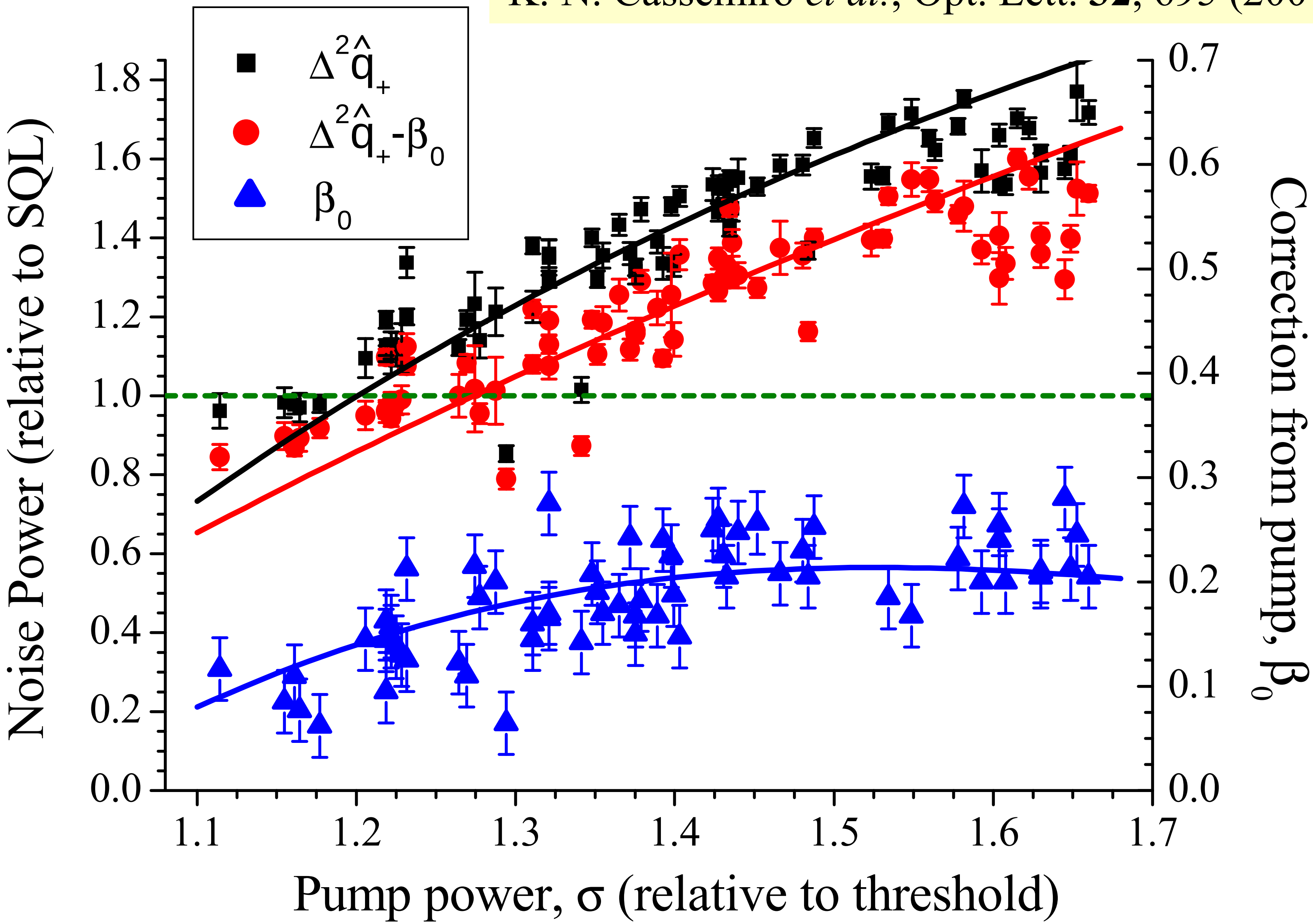
Three-color correlations (27 MHz)

$\Delta^2\hat{q}'_+ \equiv \Delta^2\hat{q}_+ - \beta_0 \,, \quad \text{with} \quad \beta_0 = C^2_{\hat{p}_0\hat{q}_+}/\Delta^2\hat{p}_0 \qquad \Delta^2\hat{q}'_+ < 1 \text{ and } \beta_0 \neq 0$



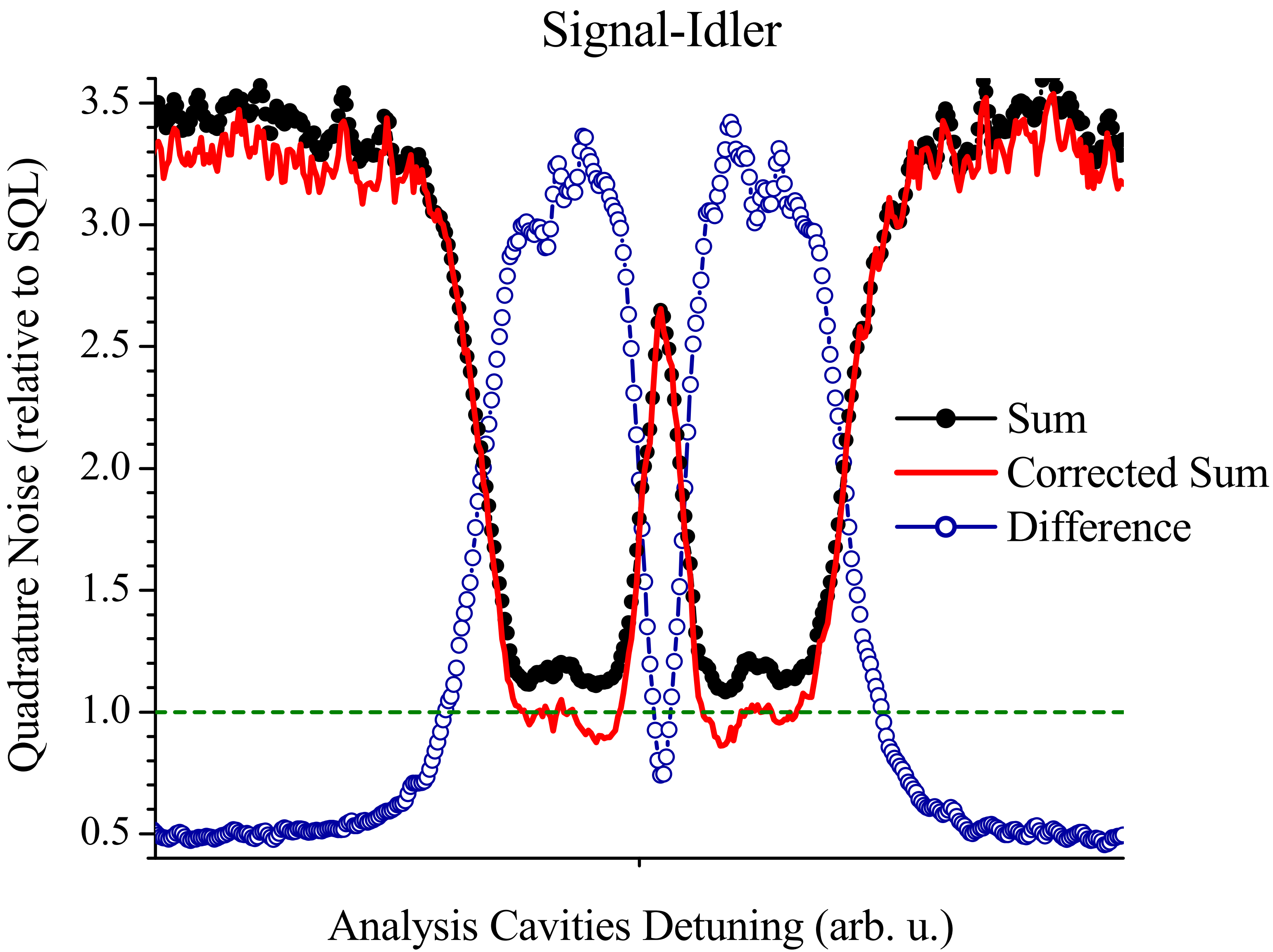
Three-color correlations (27 MHz)

K. N. Cassemiro *et al.*, Opt. Lett. **32**, 695 (2007)

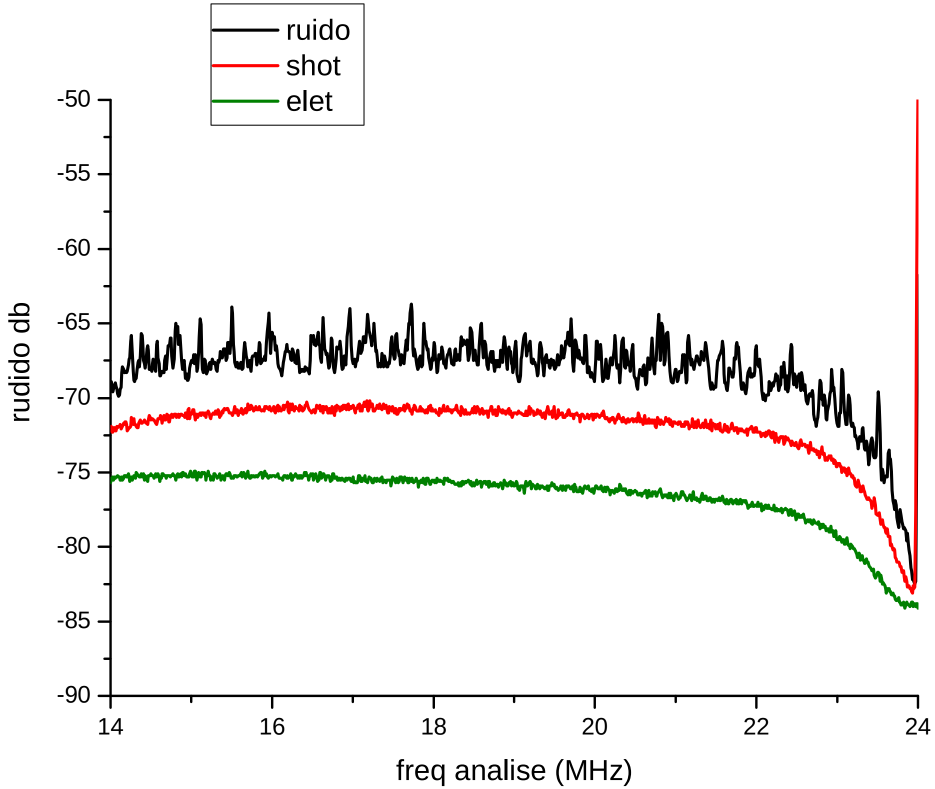


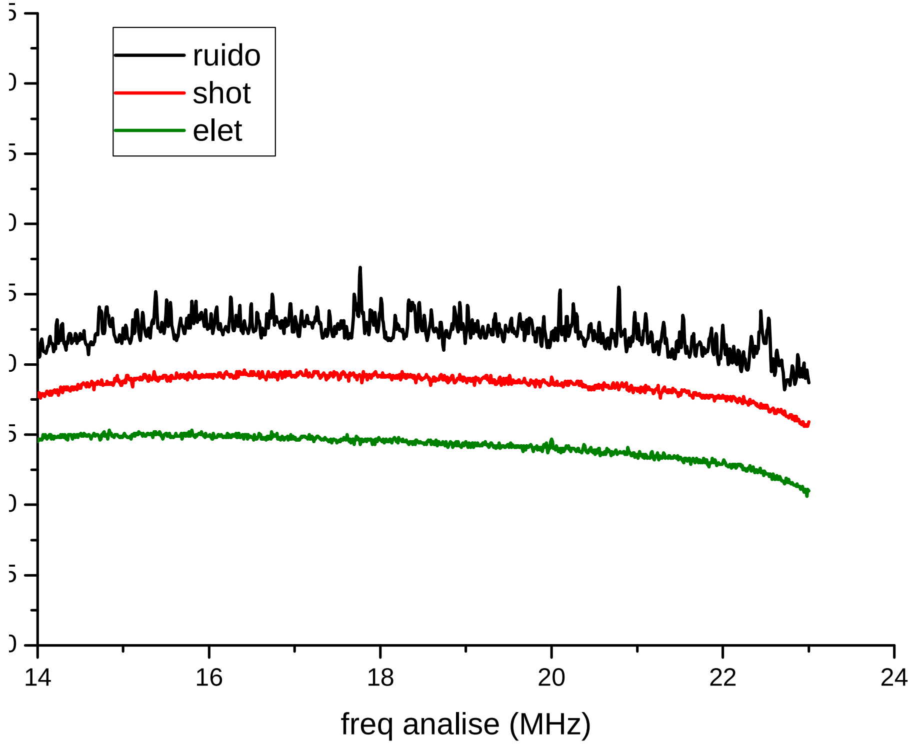
Three-color correlations (21 MHz)

Phase-phase correlations (q_+ and q_0)



Noise!...





Cavity parameters:

<u>Input</u>

$R_{532} = 69.4\ \%$

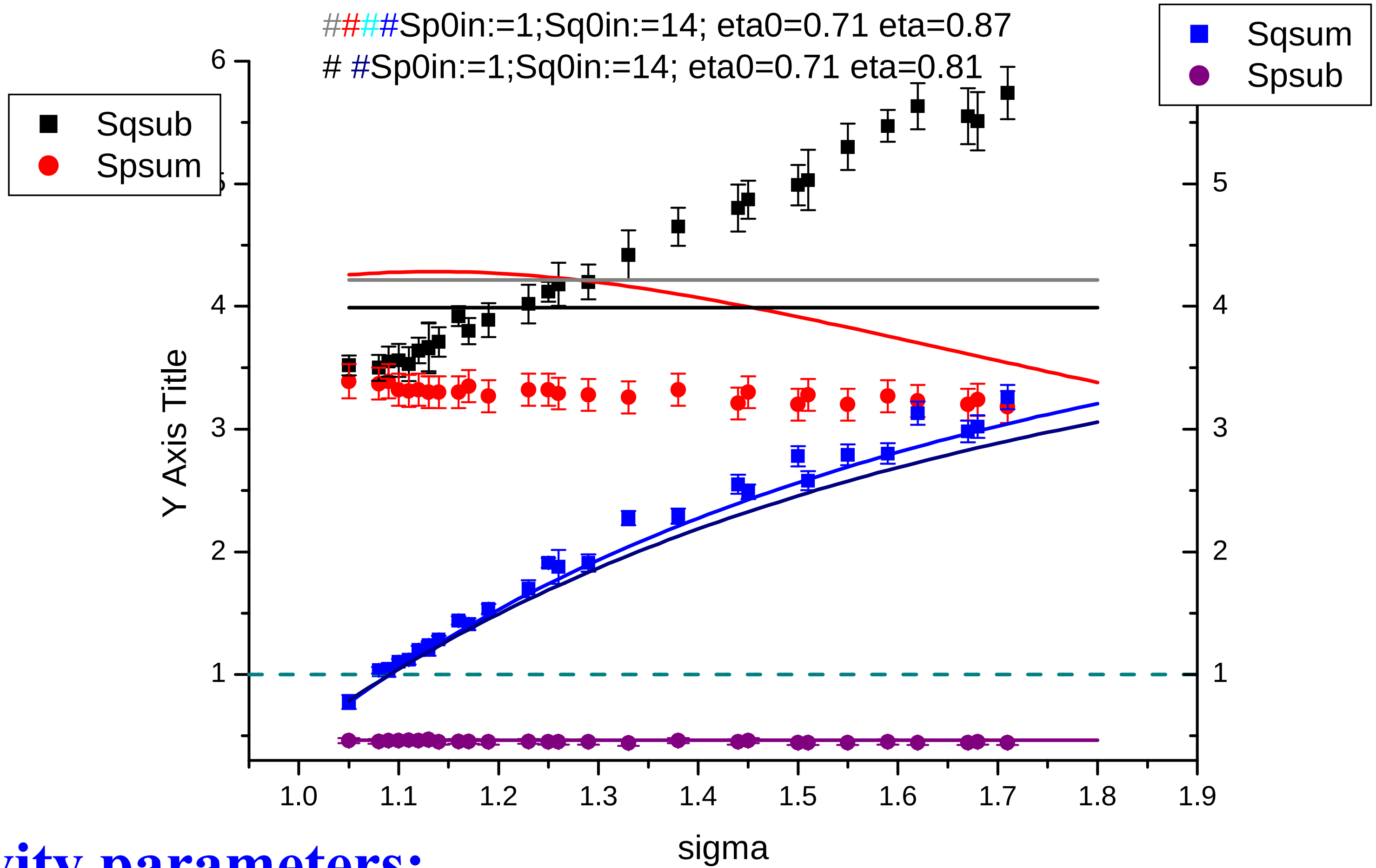
$R_{1064} > 99.8\ \%$

<u>Output</u>

$R_{532} > 99.8\ \%$

$R_{1064} = 96\ \%$

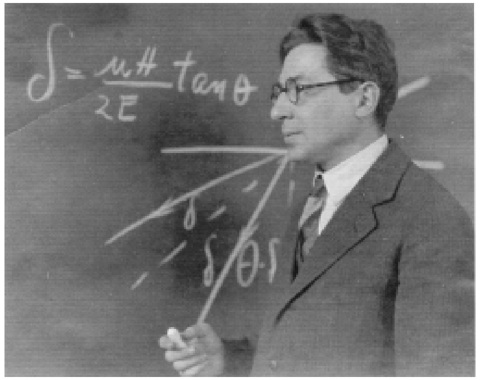
Noise!...



Cavity parameters:

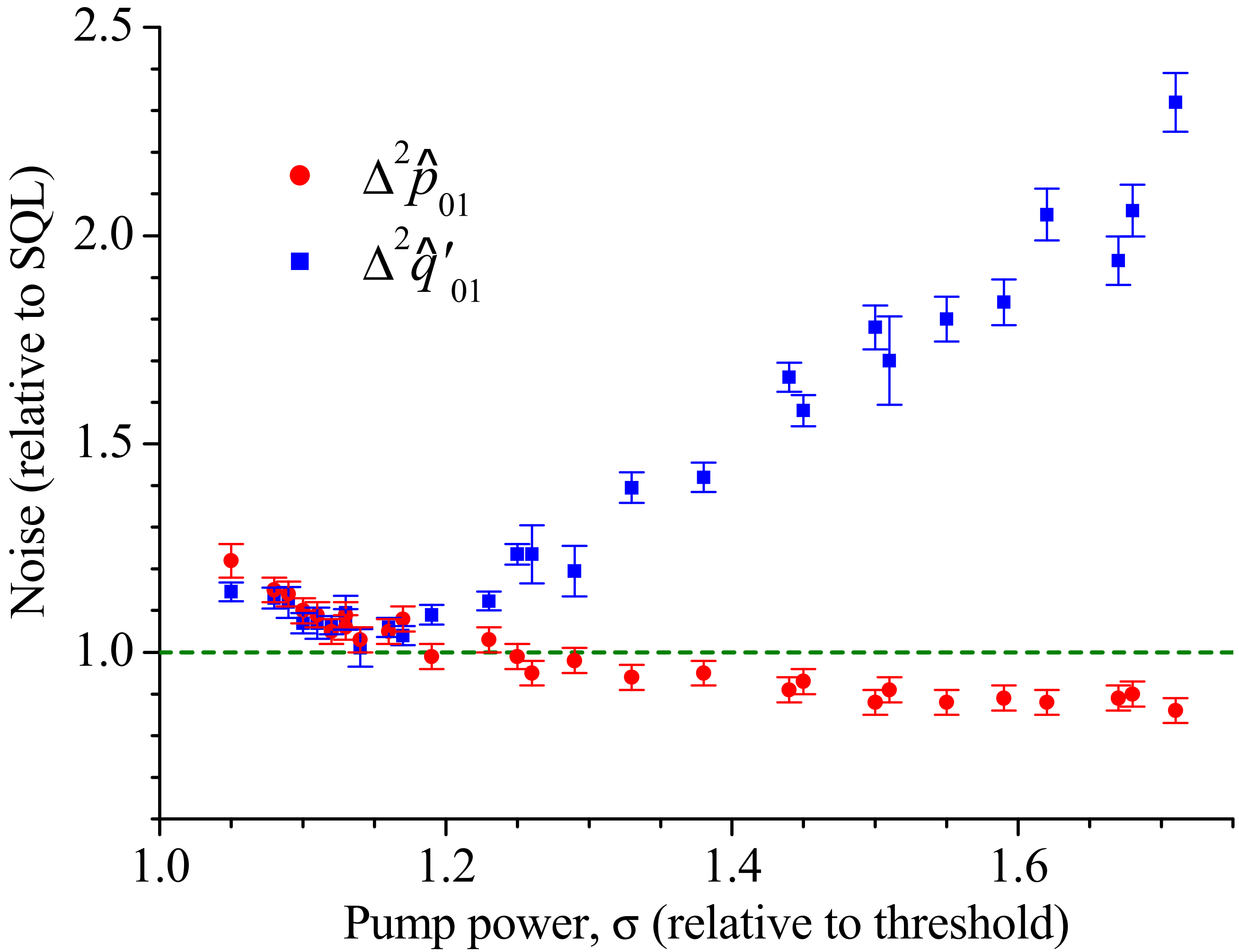
Input	Output
$R_{532} = 69.4$ %	$R_{532} > 99.8$ %
$R_{1064} > 99.8$ %	$R_{1064} = 96$ %

Noise!...



Tripartite entanglement?

1st inequality: OK, consequence of bipartite entanglement (improved).



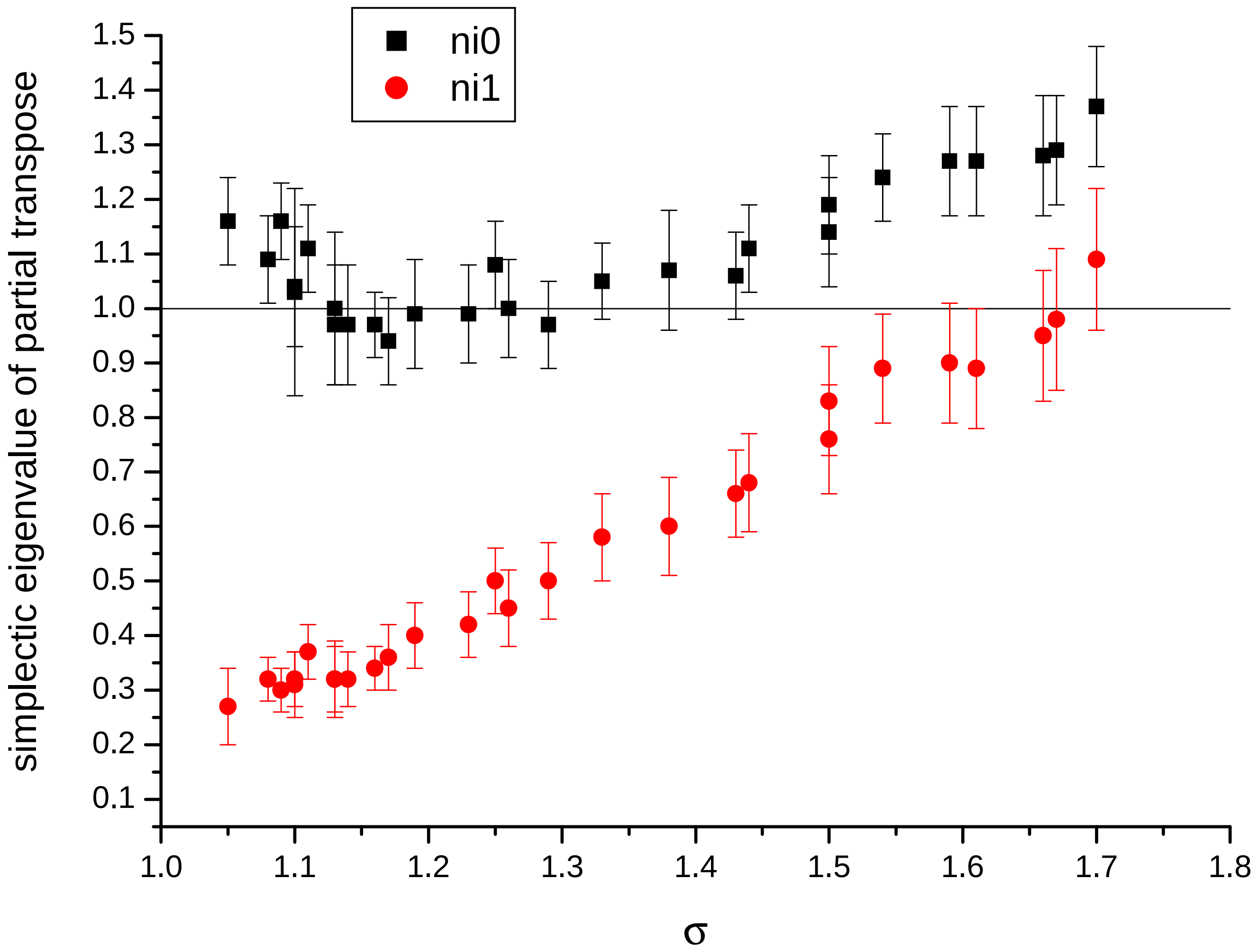
Not yet... ☹

The trouble with the noise…

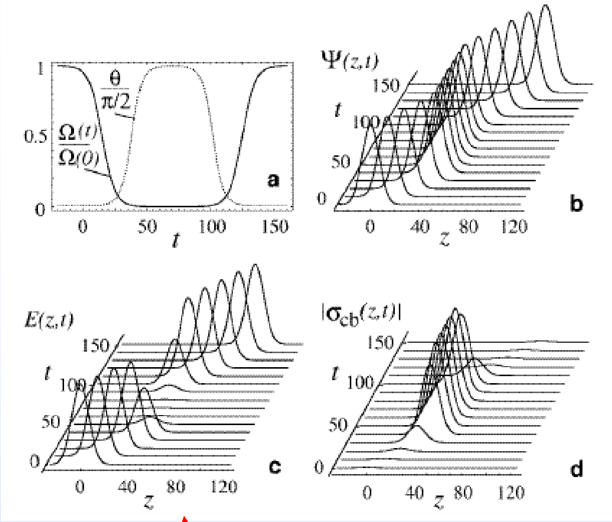
… is not over!

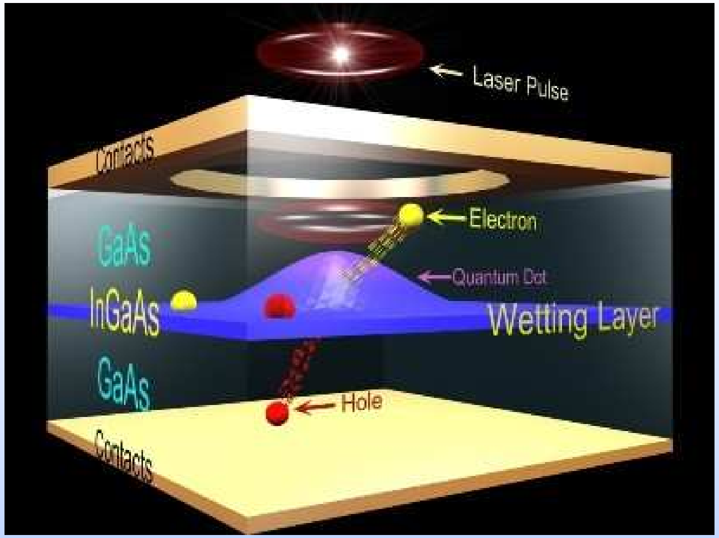
Tripartite entanglement?

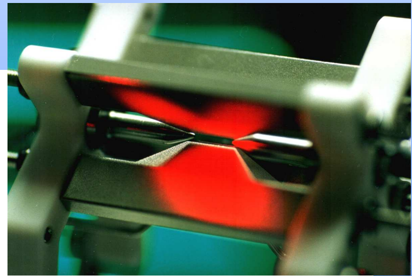
Positivity under partial transpose (preliminary!!!).

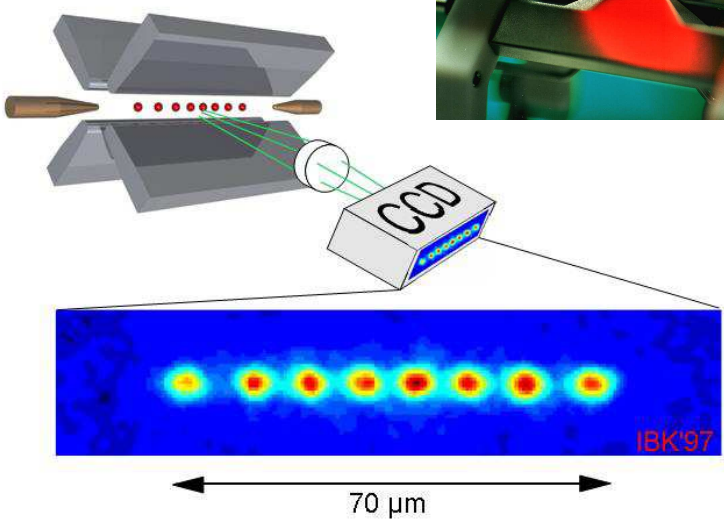


Quantum Networks









Final remarks

❖ Entanglement in the above-threshold OPO: 17 year-old problem solved.

❖ Quantum information color conversion is now possible.