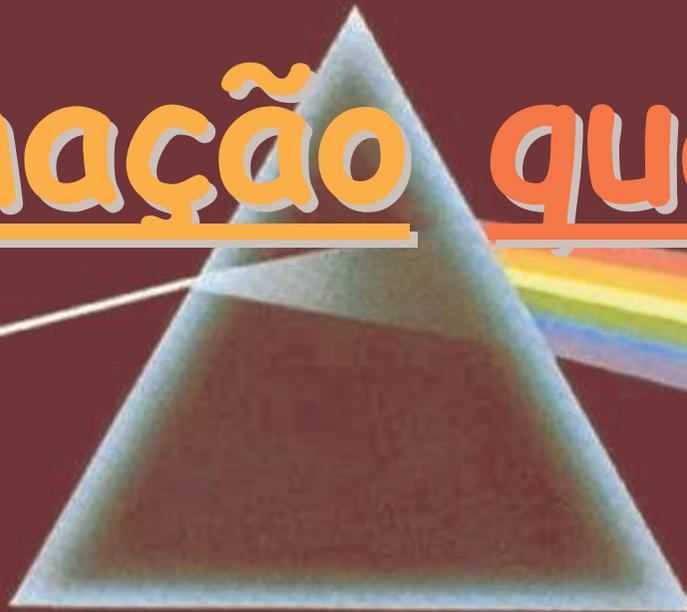
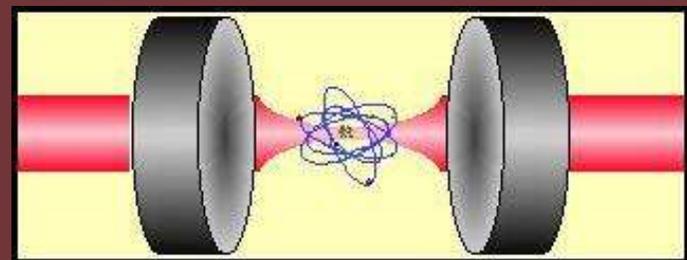


Colocando cores em

informação quântica



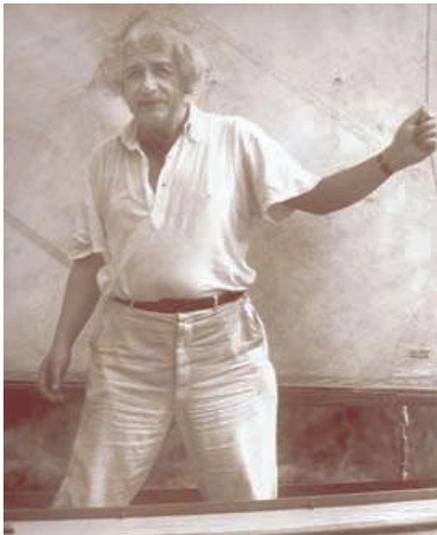
Prof. Marcelo Martinelli
Laboratório de Manipulação
Coerente de Átomos e Luz



EPR e Desigualdade de Bell

Anybody who is not shocked by quantum theory has not understood it.

Niels Bohr



MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality.

O exemplo de EPR



$$|\psi\rangle \cong \delta(x_1 - x_2 - L)\delta(p_1 + p_2) \quad (\text{localizada em } x_1 - x_2 \text{ e } p_1 + p_2)$$

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

Uma medida de x_1 fornece x_2 , assim como uma medida de p_1 fornece p_2 . Mas x_2 e p_2 *não comutam!* $\leftrightarrow [x, p] = i \hbar$

A conclusão de EPR

either (1) *the quantum-mechanical description of reality given by the wave function is not complete* or (2) *when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.*

Se (1) é falso, então (2) também é falso! Portanto, (1) deve ser verdadeiro: a teoria quântica, embora forneça previsões corretas, deve ser *incompleta*. As medidas devem apenas revelar estados já pré-existentes, ainda não descritos pela teoria.

A resposta de Bohr

OCTOBER 15, 1935

PHYSICAL REVIEW

VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

$$\begin{aligned} [q_1 p_1] &= [q_2 p_2] = i\hbar/2\pi, \\ [q_1 q_2] &= [p_1 p_2] = [q_1 p_2] = [q_2 p_1] = 0, \end{aligned}$$

$$\begin{aligned} q_1 &= Q_1 \cos \theta - Q_2 \sin \theta & p_1 &= P_1 \cos \theta - P_2 \sin \theta \\ q_2 &= Q_1 \sin \theta + Q_2 \cos \theta & p_2 &= P_1 \sin \theta + P_2 \cos \theta. \end{aligned}$$

$$[Q_1 P_1] = i\hbar/2\pi, \quad [Q_1 P_2] = 0,$$

$$Q_1 = q_1 \cos \theta + q_2 \sin \theta,$$

$$P_2 = -p_1 \sin \theta + p_2 \cos \theta,$$

Bohr introduz a noção de *complementaridade*, mas sua resposta não contém elementos que permitam *descartar* o programa proposto por EPR.

As desigualdades de Bell



Somente em 1964/1966 o “paradoxo” de EPR se tornou mais interessante para a comunidade de físicos. O irlandês John Bell conseguiu demonstrar uma desigualdade que deveria ser satisfeita por teorias de variáveis ocultas que respeitassem a condição de *localidade*. Seria possível realizar *testes experimentais*.

REVIEWS OF MODERN PHYSICS

VOLUME 38, NUMBER 3

JULY 1966

On the Problem of Hidden Variables in Quantum Mechanics*

JOHN S. BELL†

Mas a proposta original de EPR envolvia variáveis contínuas, posição e momento.

Variáveis contínuas do campo EM

$$E(t) = \text{Re}[\alpha \exp(i\omega t)]$$

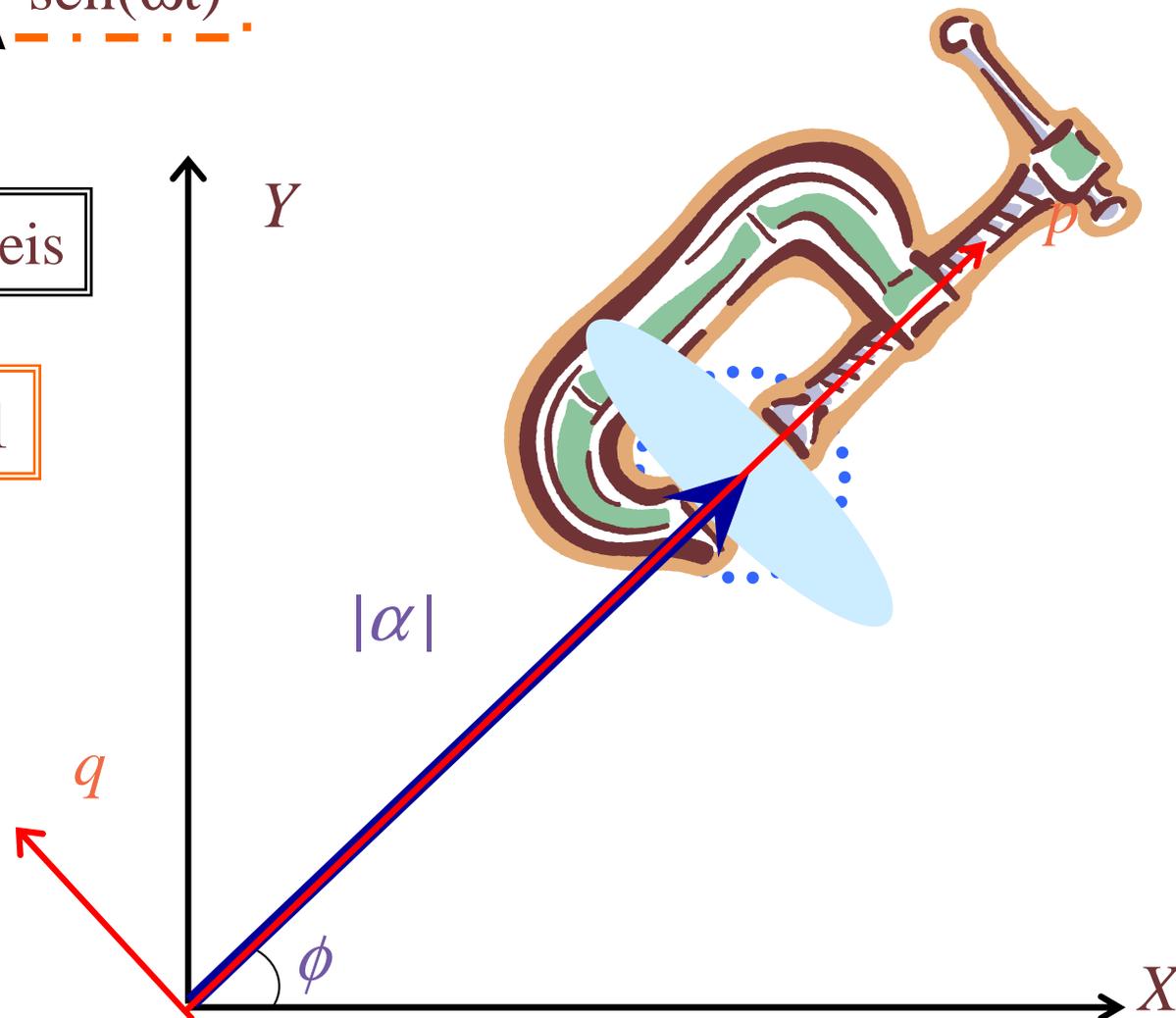
$$E(t) = X \cos(\omega t) + Y \sin(\omega t)$$

$$\alpha = X + i Y$$

observáveis

$$\Delta^2 X \Delta^2 Y = \Delta^2 p \Delta^2 q > 1$$

Heisenberg



Vantagens de variáveis contínuas

- Preparação “incondicional” de estados (a cada inverso de largura de banda).
- Medidas com alta eficiência de detecção (eficiência maior que 95%).
- “Complete Bell detection” com detecção homodina e beamsplitters.
- “Drawbacks”: estados não são perfeitos, dependem do grau de squeezing; maioria dos experimentos envolvem estados gaussianos, com função de Wigner ≥ 0 .

Como medir emaranhamento?

- Critério “EPR” [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]

$$\Delta^2 \hat{p}_{\text{inf}} = \Delta^2 \hat{p}_1 \left(1 - \frac{\langle \delta \hat{p}_1 \delta \hat{p}_2 \rangle^2}{\Delta^2 \hat{p}_1 \Delta^2 \hat{p}_2} \right) \quad \delta \hat{p}_i = \hat{p}_i - \langle \hat{p}_i \rangle$$

$$\Delta^2 \hat{p}_{\text{inf}} \Delta^2 \hat{q}_{\text{inf}} \geq 1$$

- Critério DGCZ [Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller, *Inseparability criterion for continuous variable systems*, Phys. Rev. Lett. **84**, 2722 (2000).

$$\rho = \sum_i p_i \rho_i = \sum_i p_i \rho_i^1 \otimes \rho_i^2$$

$$[\hat{x}_j, \hat{p}_{j'}] = 2i \delta_{jj'}$$

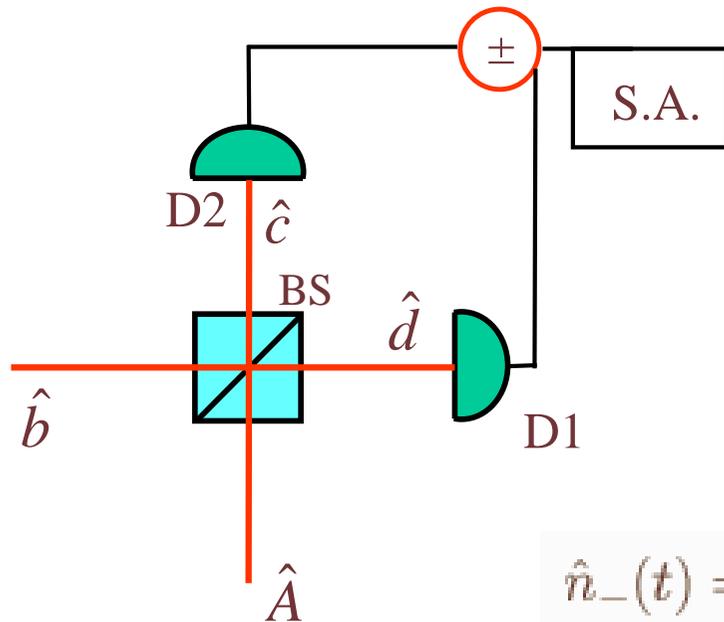
$$\hat{u} = |a| \hat{x}_1 + \frac{1}{a} \hat{x}_2$$

$$\hat{v} = |a| \hat{p}_1 - \frac{1}{a} \hat{p}_2$$

Separability

$$\langle (\Delta \hat{u})^2 \rangle_\rho + \langle (\Delta \hat{v})^2 \rangle_\rho \geq 2 \left(a^2 + \frac{1}{a^2} \right)$$

Medidas de ruído



(Balanced) Homodyne Detection

$$\hat{c}(t) = \frac{1}{\sqrt{2}} (\hat{A}(t) + \hat{b})$$

$$\hat{d}(t) = \frac{1}{\sqrt{2}} (\hat{b} - \hat{A}(t))$$

$$\hat{n}_-(t) = \hat{c}^\dagger(t)\hat{c}(t) - \hat{d}^\dagger(t)\hat{d}(t) = \hat{b}^\dagger\hat{A}(t) + \hat{A}^\dagger(t)\hat{b}$$

Se o campo b é intenso, trocamos o operador por seu valor médio

$$\hat{n}_-(t) = |\beta| \left(\hat{A}(t)e^{-i\theta} + \hat{A}^\dagger(t)e^{i\theta} \right)$$

Se b é o vácuo, obtemos o ruído de intensidade de A medindo n_+

$$\hat{n}_+(t) = \hat{A}^\dagger(t)\hat{A}(t) + \hat{b}^\dagger\hat{b}$$

Como gerar emaranhamento?

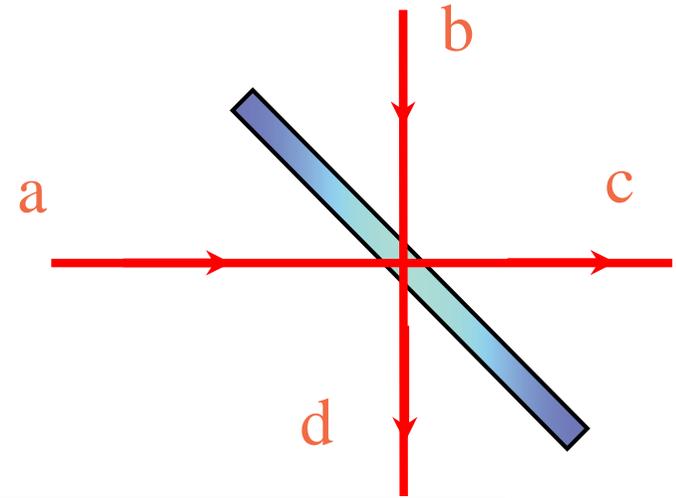
Estados comprimidos em um divisor de feixe

$$\hat{X}(\theta) = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}$$

$$[\hat{X}(\theta), \hat{X}(\theta + \frac{\pi}{2})] = 2i$$

$$\hat{u} = \sqrt{2} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{v} = i\sqrt{2} (\hat{a}^\dagger - \hat{a})$$

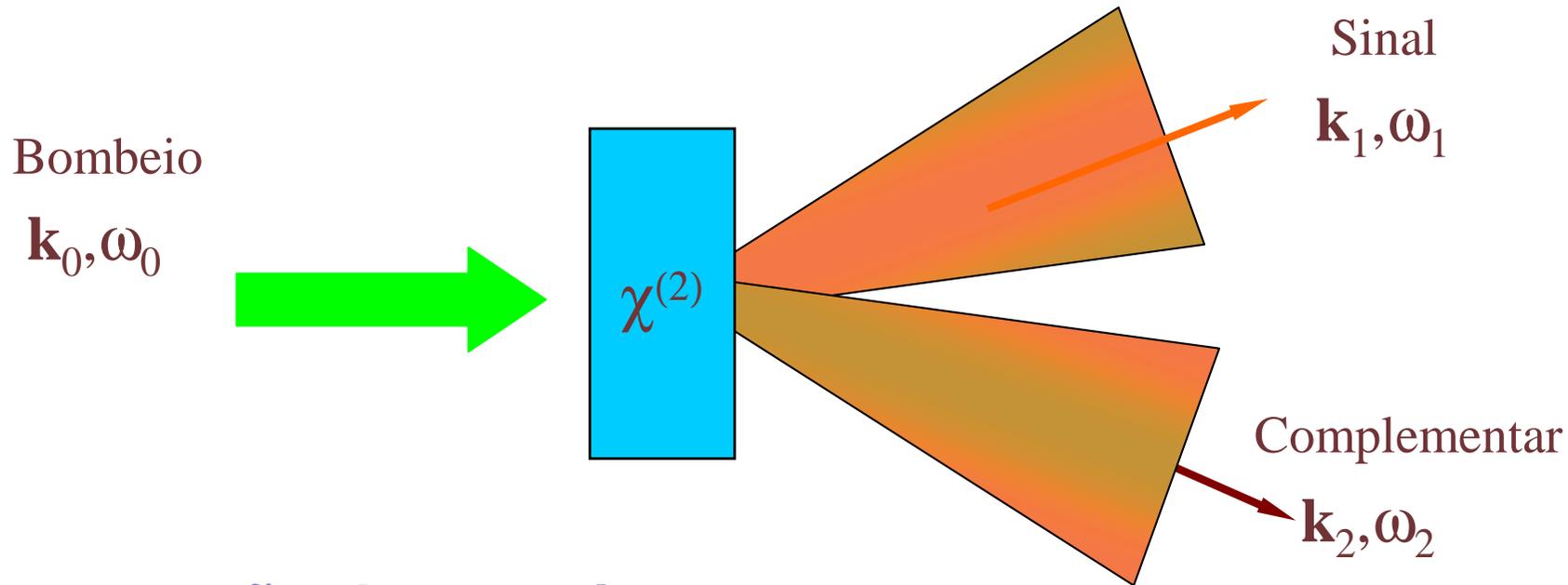


$$\hat{c} = \frac{1}{\sqrt{2}} (\hat{a} + \hat{b})$$

$$\hat{d} = \frac{1}{\sqrt{2}} (\hat{b} - \hat{a})$$

Oscilador Paramétrico Ótico

Conversão Paramétrica Descendente



Conservação de energia e momento

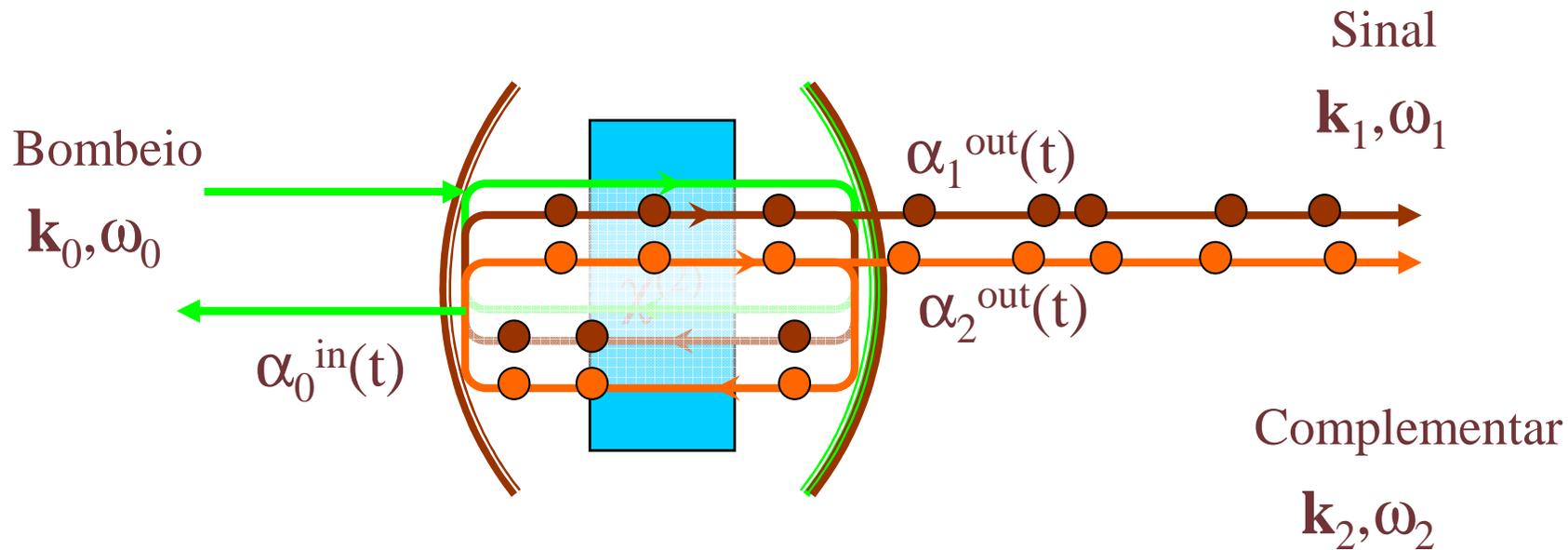
$$\omega_0 = \omega_1 + \omega_2$$

$$\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$$

Emaranhamento de polarização e momento transversal

Oscilador Paramétrico Ótico

CPD + Cavidade



Geração de pares de fótons + correlação de fase

- Abaixo do limiar

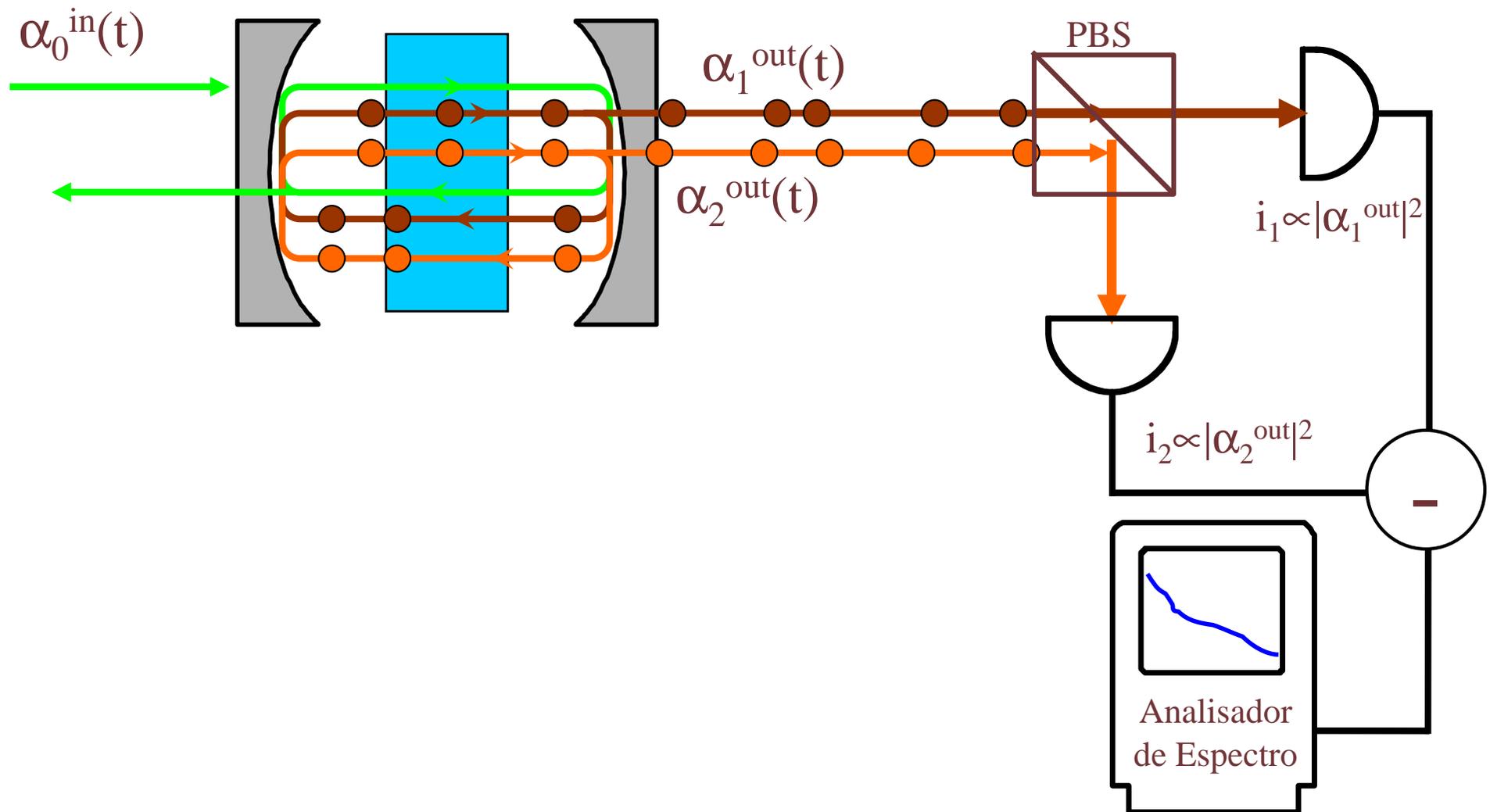
vácuo comprimido (caso degenerado) - OPA

campos emaranhados (caso não degenerado)

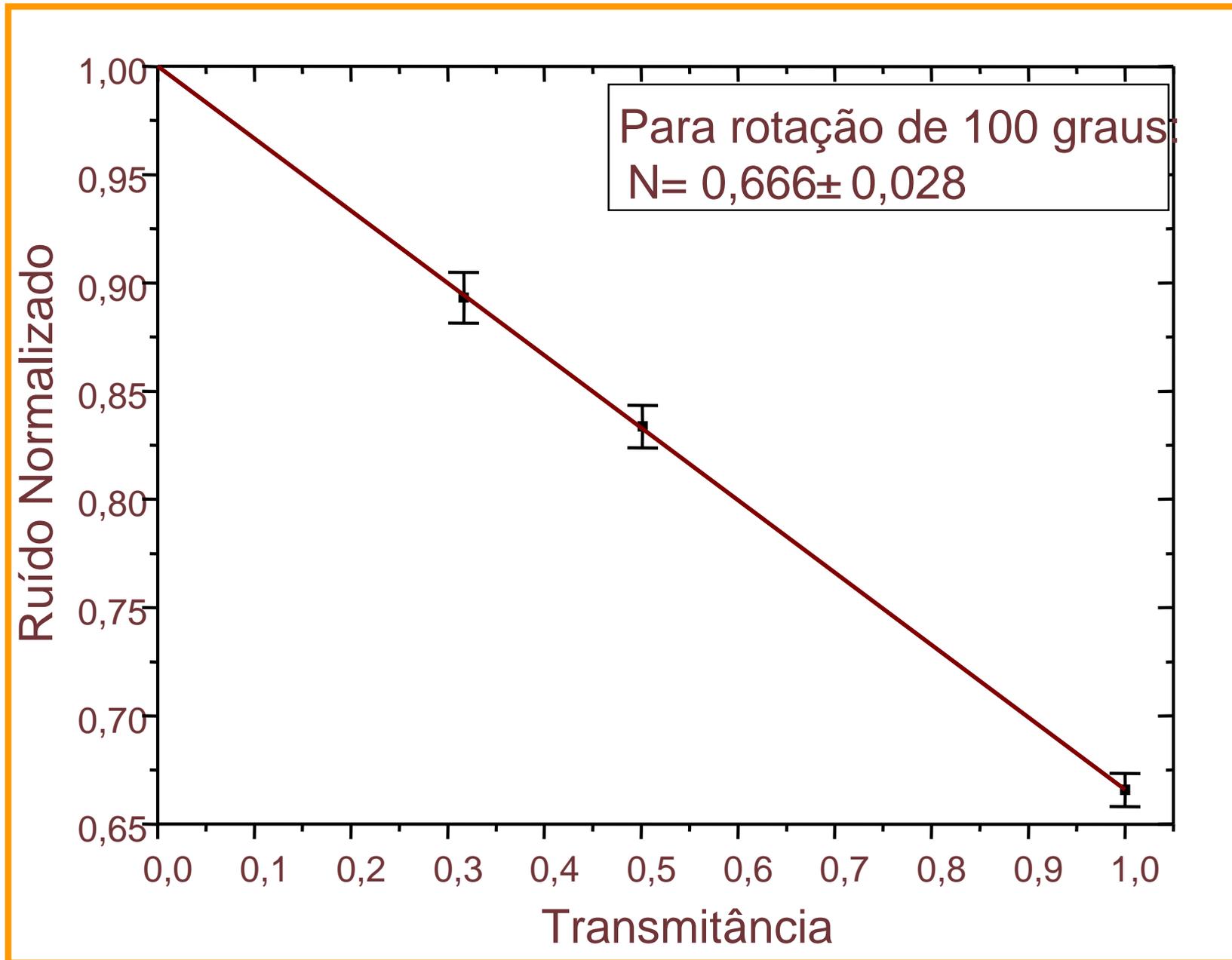
- Acima do limiar: campos intensos de saída emaranhados

Oscilador Paramétrico Ótico

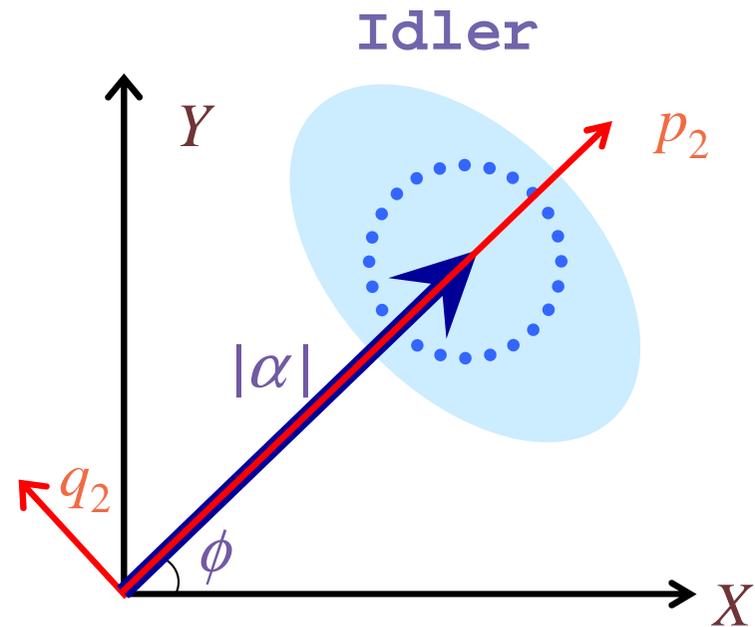
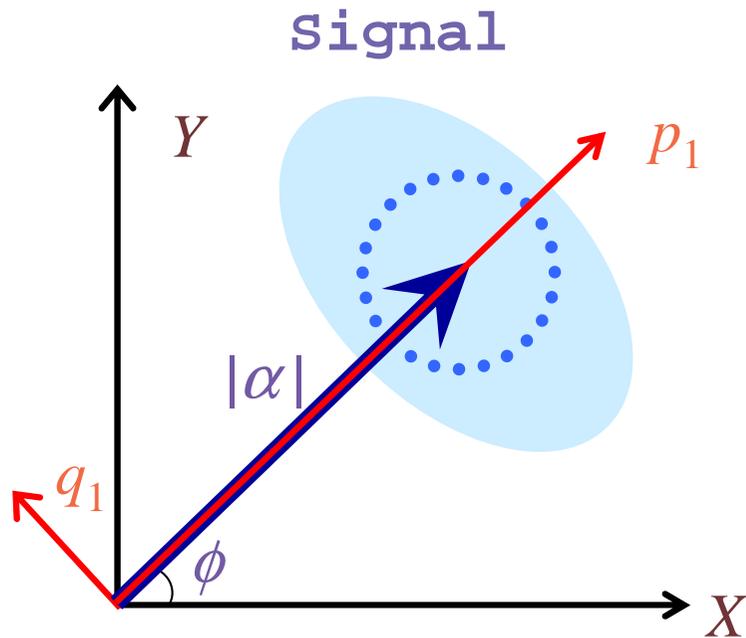
Feixes Gêmeos - correlação quântica de intensidade



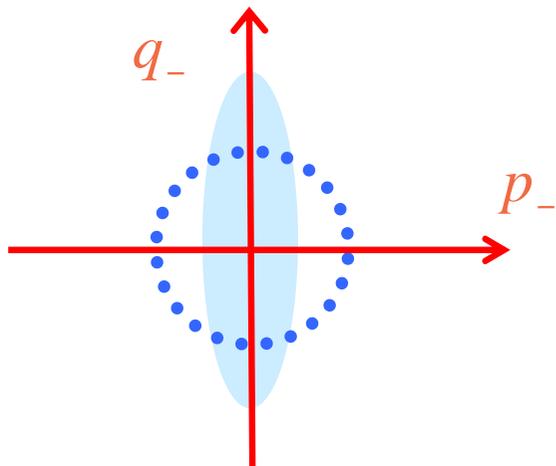
Compressão de Ruído em Feixes Gêmeos



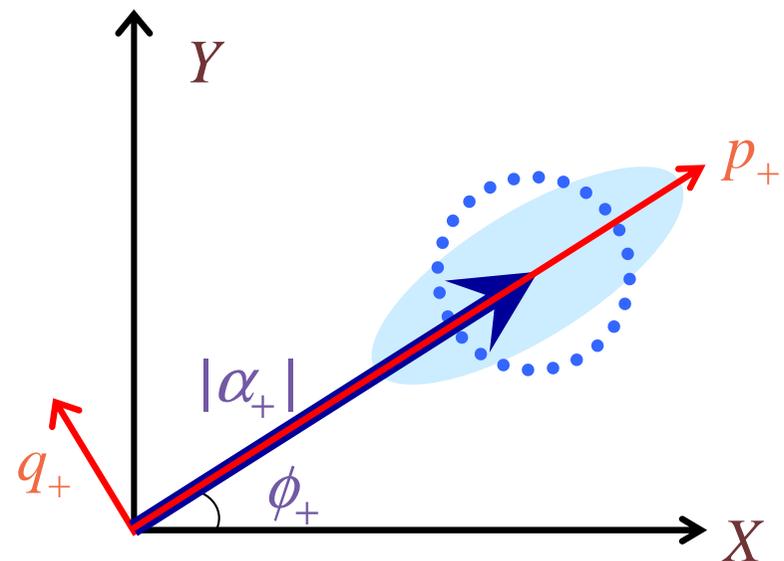
Correlações de Ruído



Signal - Idler



Signal + Idler



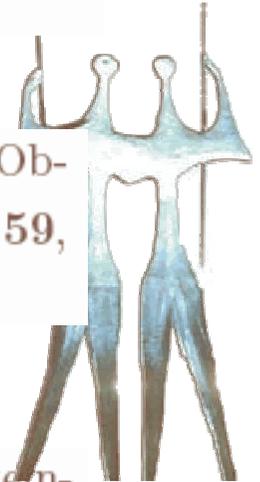


Vácuo comprimido

L.-A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, “Generation of Squeezed States by Parametric Down Conversion”, *Phys. Rev. Lett.* **57**, 2520-2523 (1986).

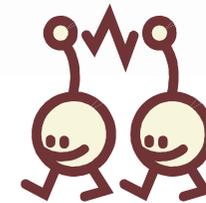
Feixes Gêmeos

A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, “Observation of Quantum Noise Reduction on Twin Laser Beams”, *Phys. Rev. Lett.* **59**, 2555-2557 (1987).



Campos Emaranhados - Vácuo

Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, “Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables”, *Phys. Rev. Lett.* **68**, 3663-3666 (1992).



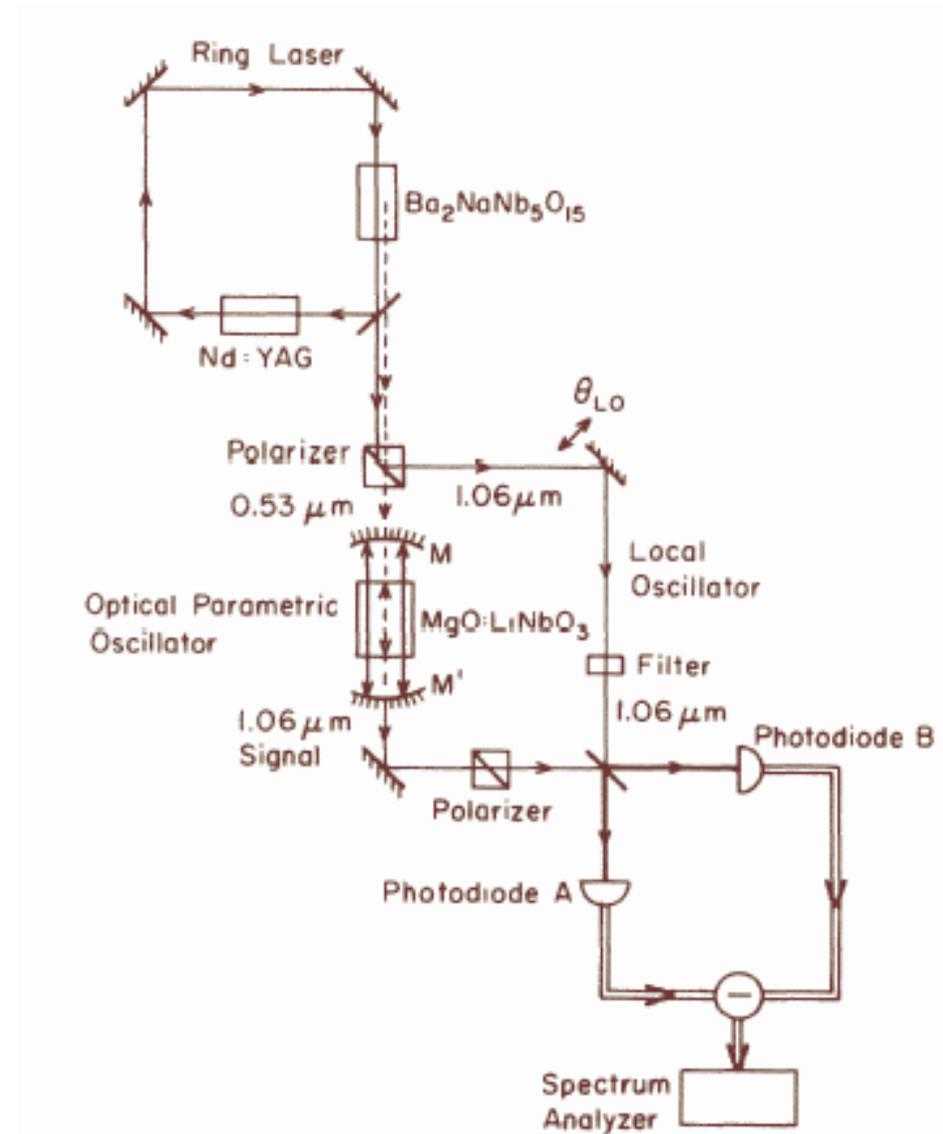
Teletransporte

A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, “Unconditional Quantum Teleportation”, *Science* **282**, 706-709 (1998).

OPO – faltando alguma coisa...

Campos emaranhados (vácuo) são interessantes, mas para criptografia seria muito útil contar com feixes intensos.

Porém para medir feixes intensos no OPO acima do limiar precisamos de um oscilador local, e portanto de um OPO degenerado.



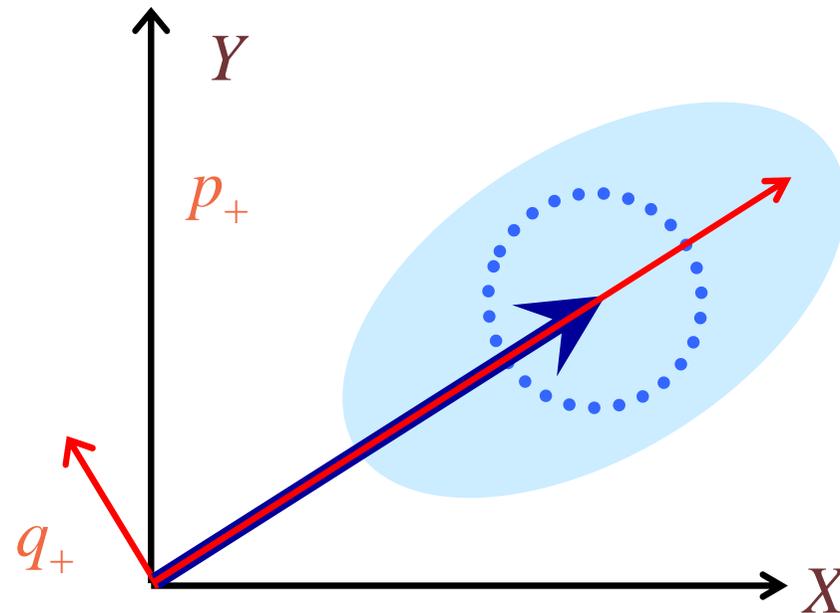
Realmente necessitamos disso?

Medindo a quadratura fase do feixe

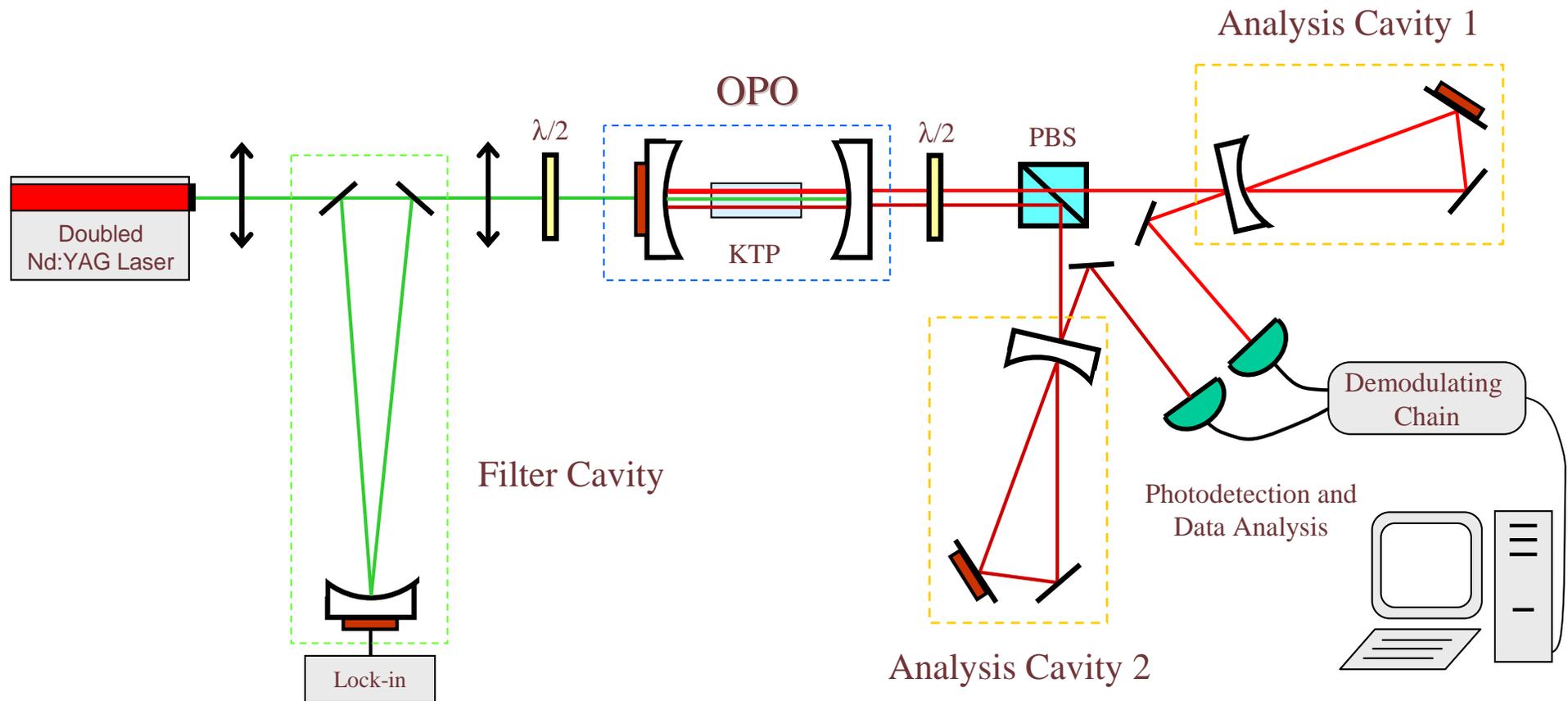


Flutuações de fase
⇒ intensidade

$$\Omega \geq \sqrt{2} \delta\omega$$

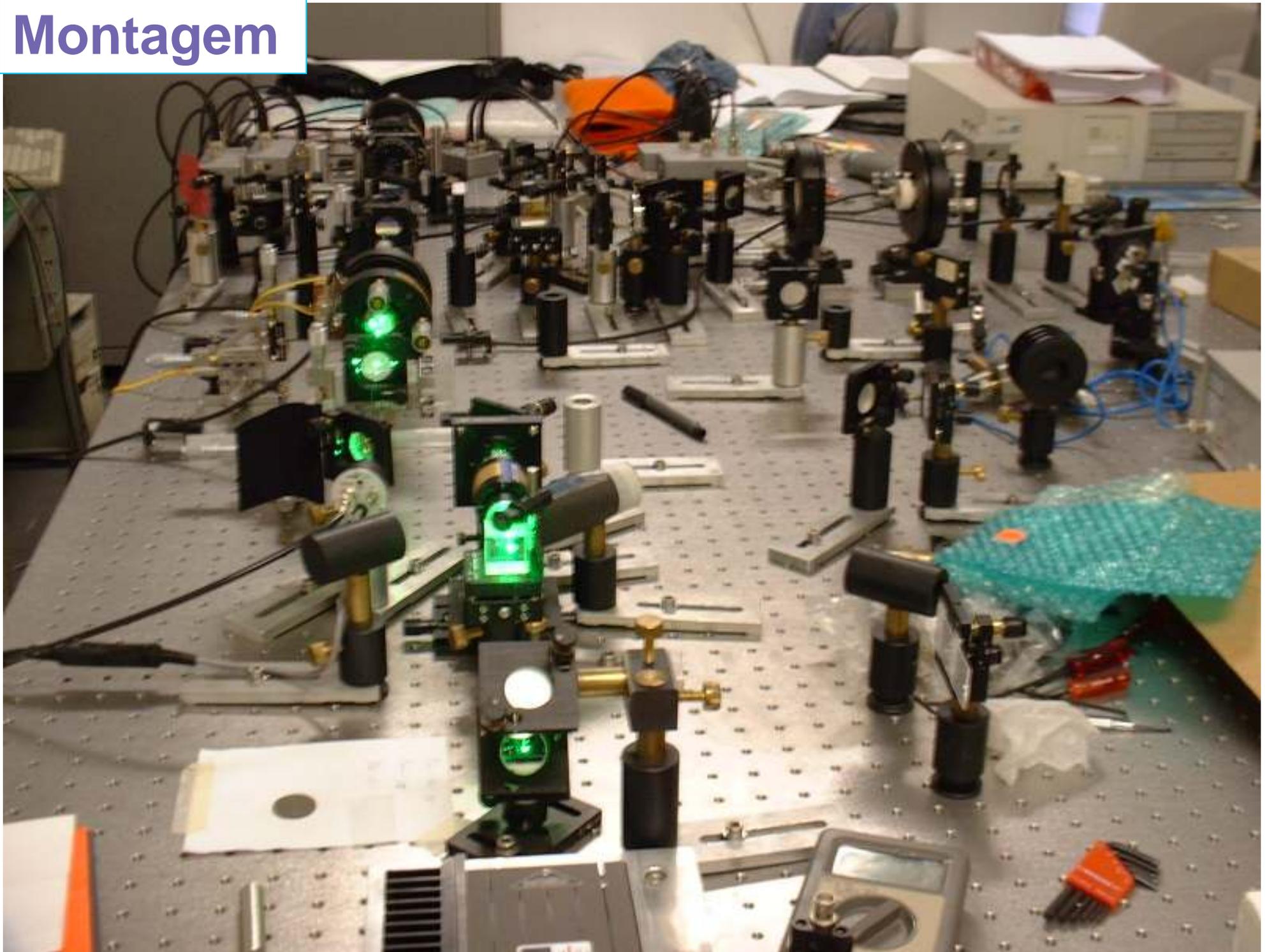


Montagem



A. S. Villar, M. Martinelli, and P. Nussenzveig, "Testing the entanglement of intense beams produced by a non-degenerate Optical Parametric Oscillator", *Opt. Commun.* 242, 551-563 (2004).

Montagem



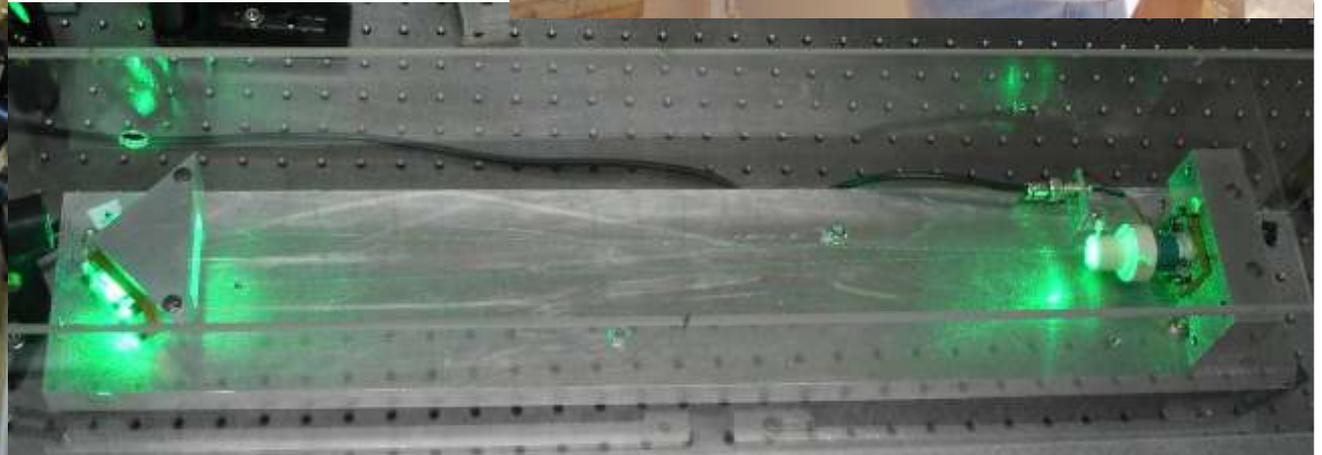
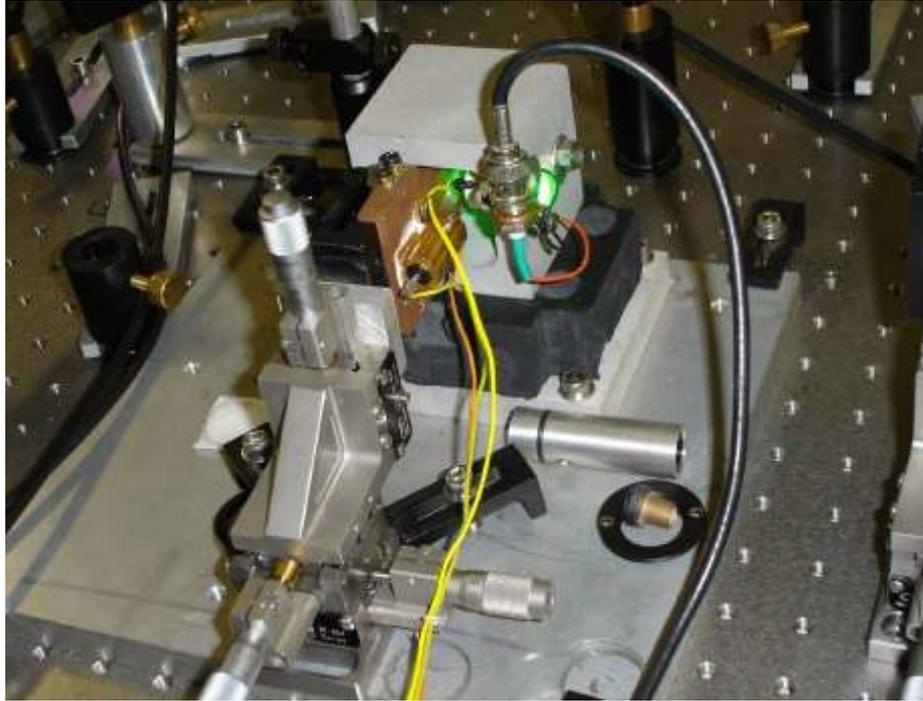
Problemas!!!



Soluções...



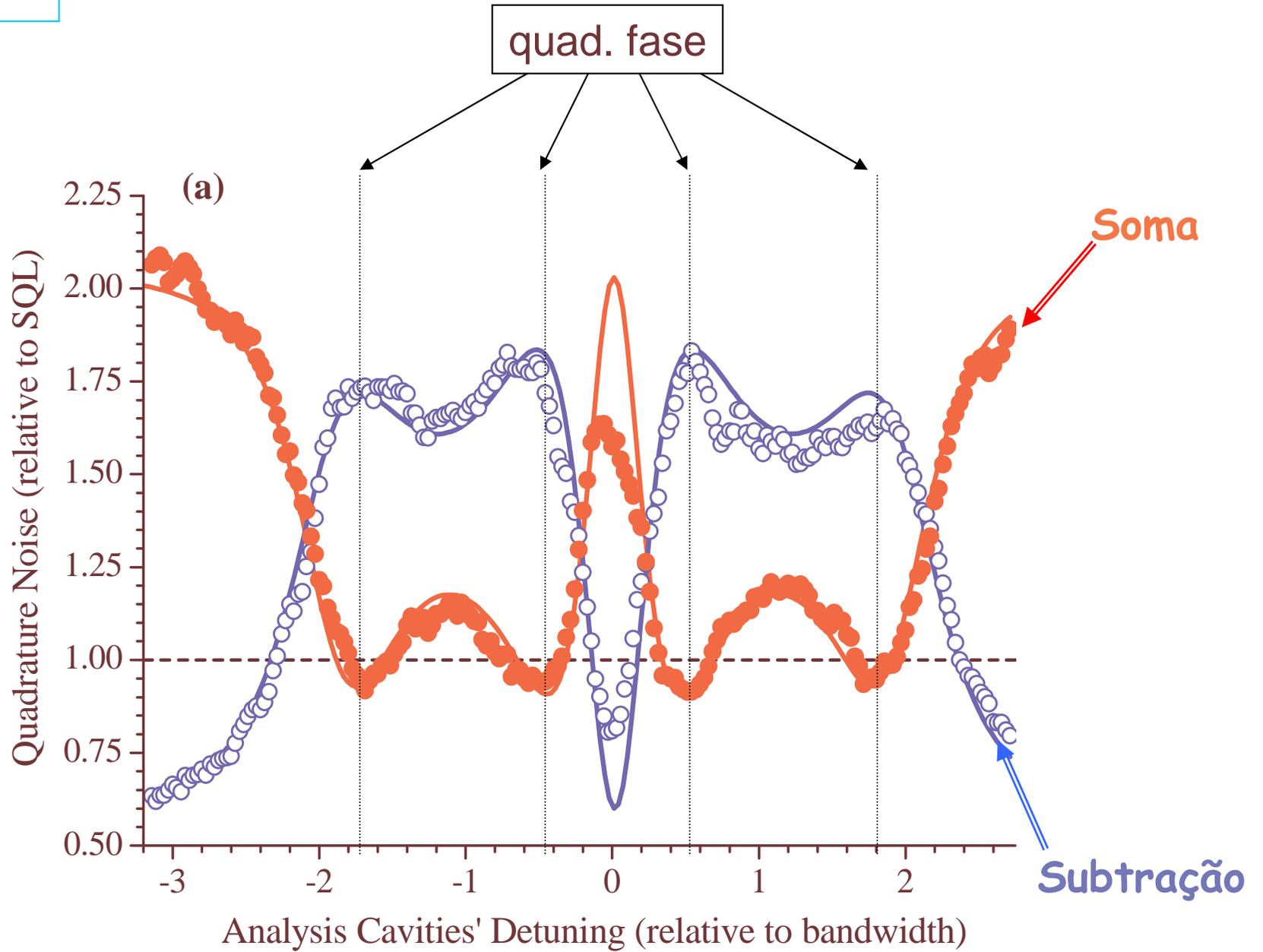
Soluções...

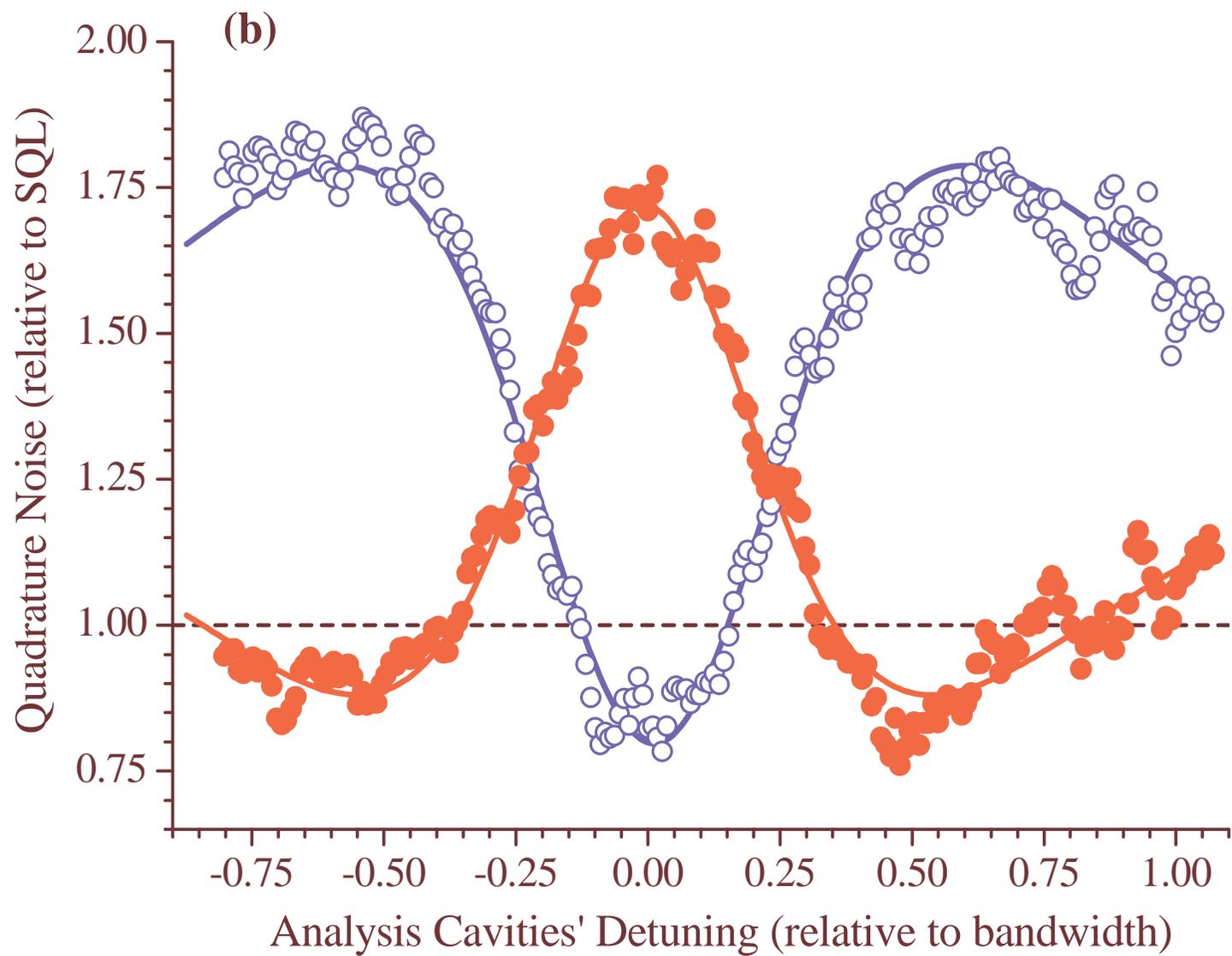


Medida



Medida





Generation of Bright Two-Color Continuous Variable Entanglement

A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig*

Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil

(Received 16 June 2005; revised manuscript received 16 September 2005; published 9 December 2005)

EPR

$$\Delta^2 \hat{p}_{\text{inf}} \Delta^2 \hat{q}_{\text{inf}} = 0.95(-3/ + 6)$$

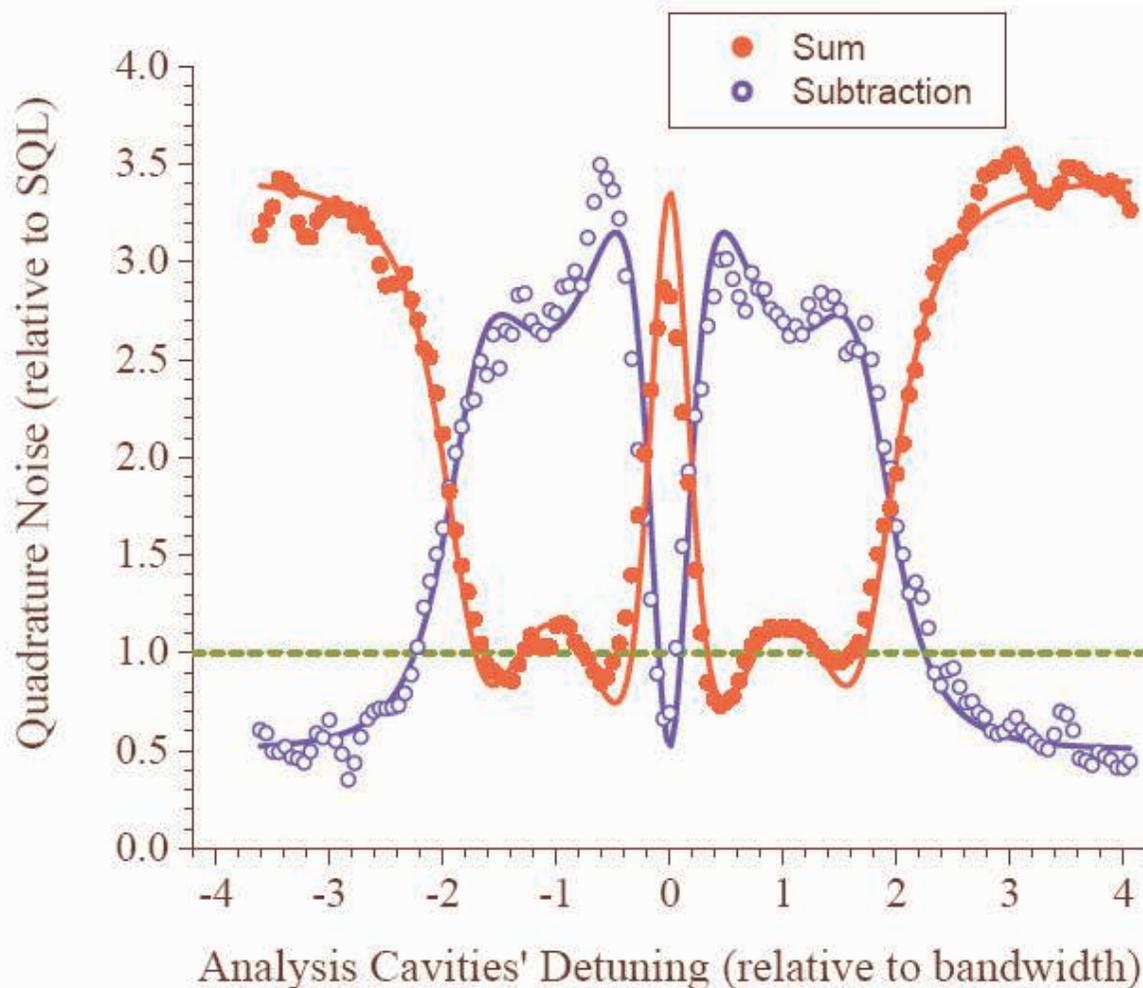
Duan

$$\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$$

Entanglement in the above-threshold optical parametric oscillator

Alessandro S. Villar¹, Katiúscia N. Cassemiro¹, Kaled Dechoum², Antônio Z. Khoury², Marcelo Martinelli¹, and Paulo Nussenzveig¹

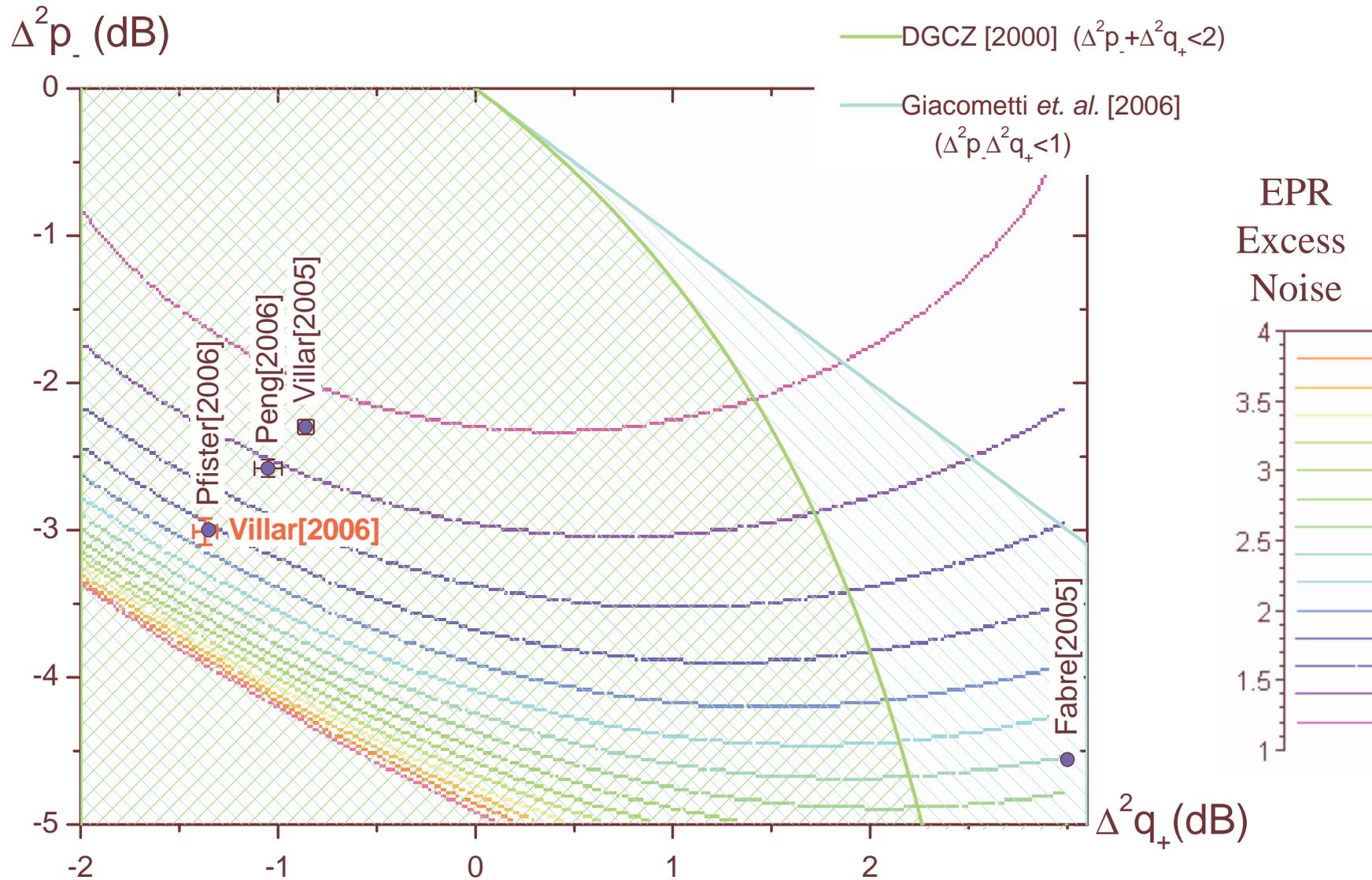
$$\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.23(2) < 2$$



**Aceito - JOSAB
ed. especial (2007)**

Sucesso!

Feixes intensos (> 1 mW), emaranhados, de comprimentos de onda diferentes ($\lambda_1 - \lambda_2 \sim 1$ nm).



Tomografia

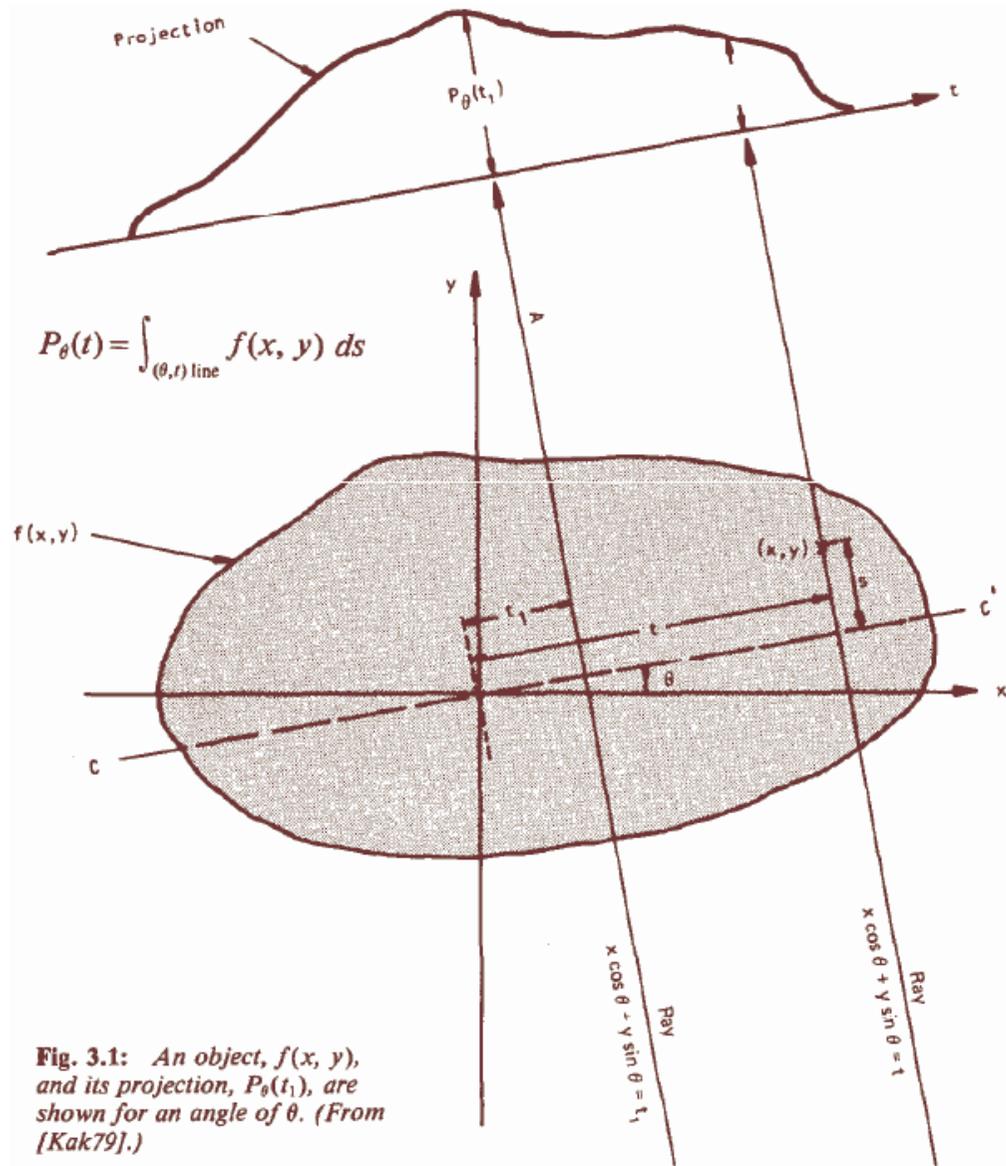
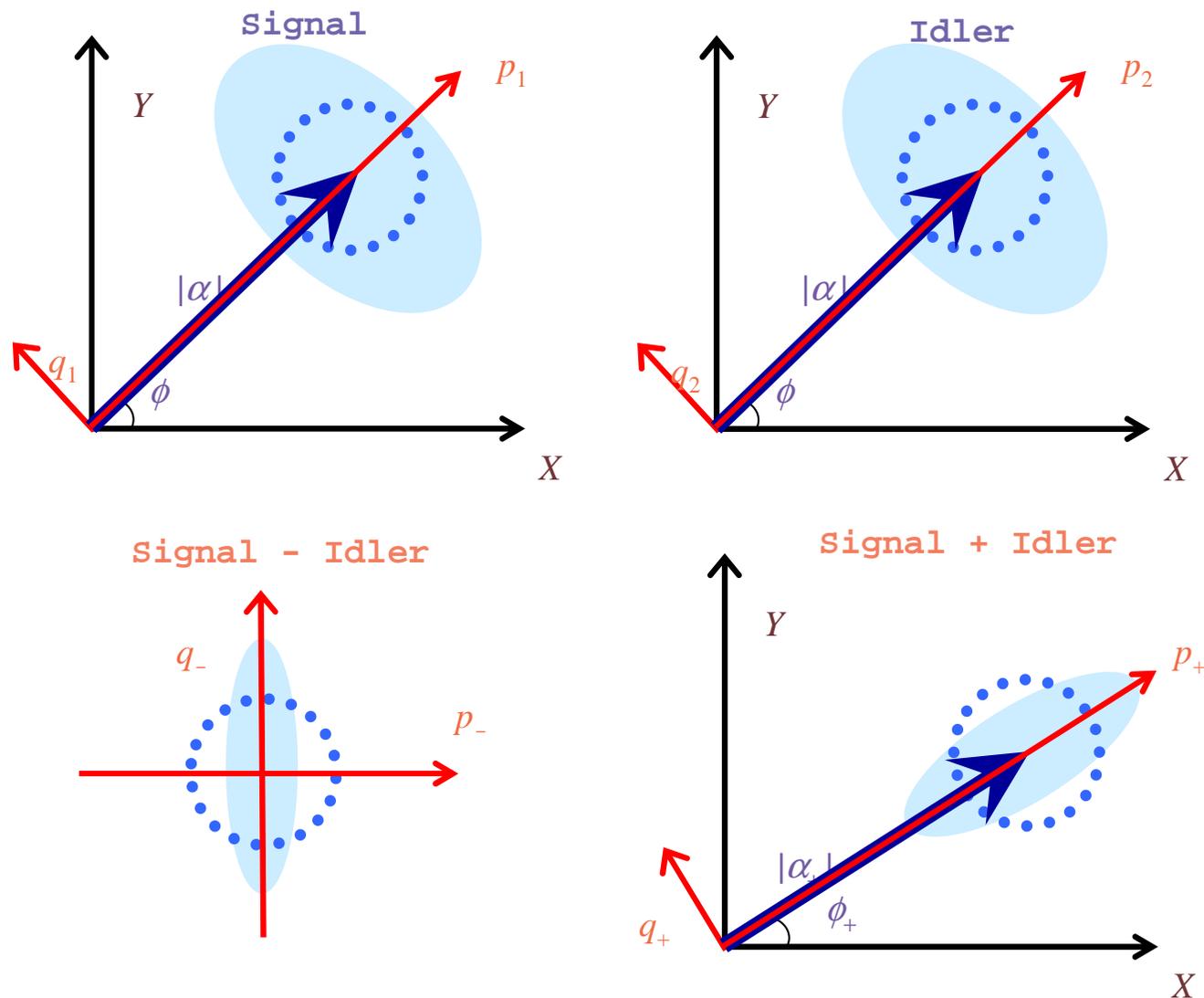


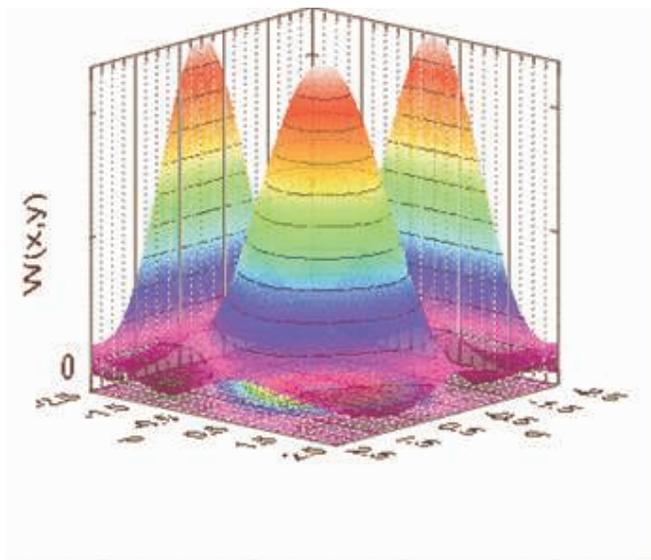
Fig. 3.1: An object, $f(x, y)$, and its projection, $P_\theta(t_1)$, are shown for an angle of θ . (From [Kak79].)

Tomografia

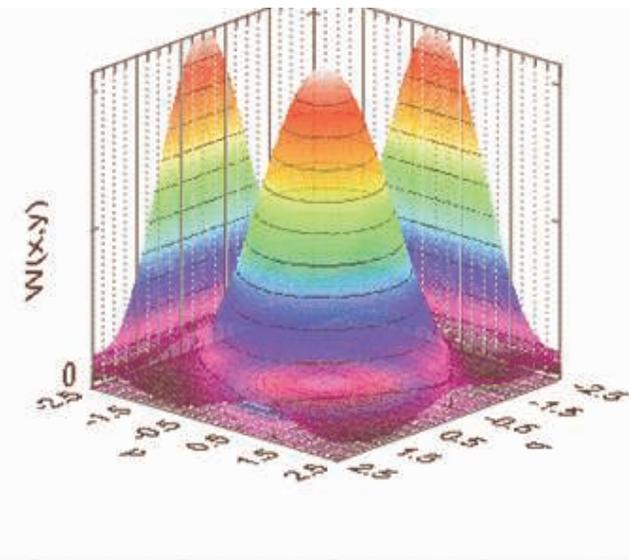


Tomografia

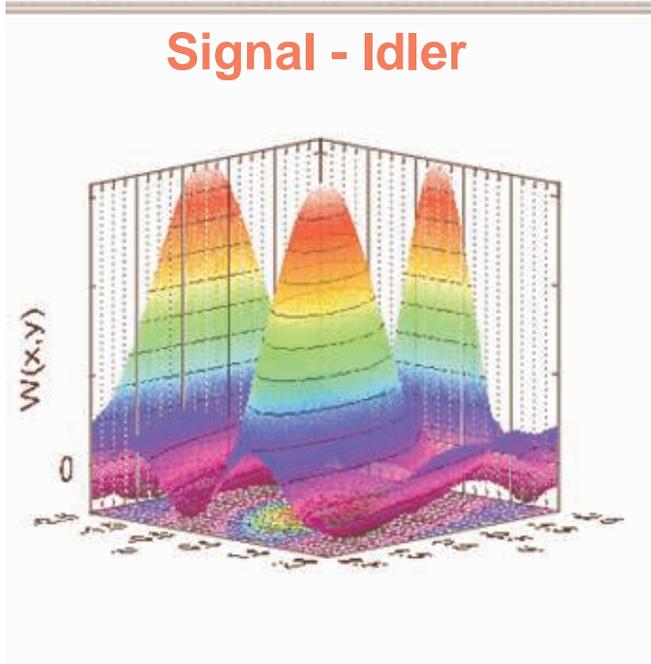
Signal



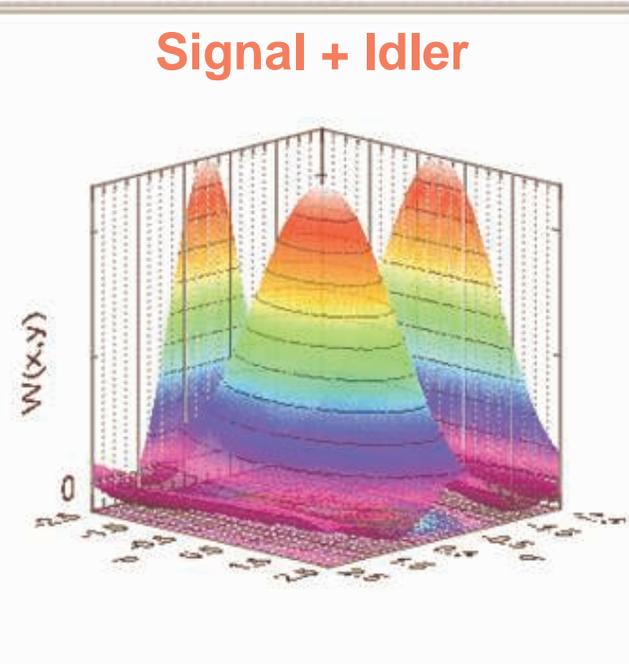
Idler



Signal - Idler

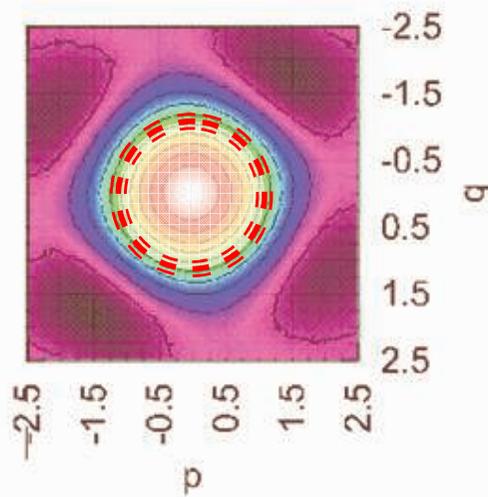


Signal + Idler

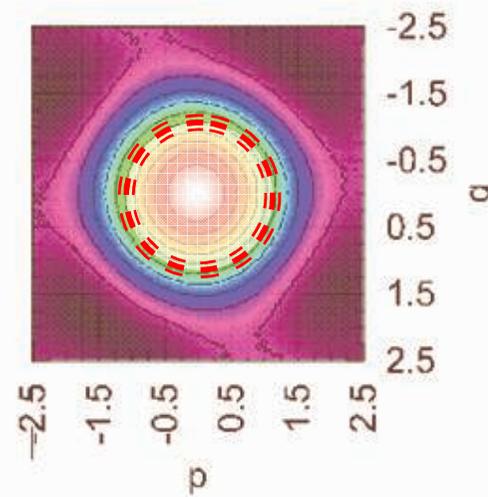


Tomografia

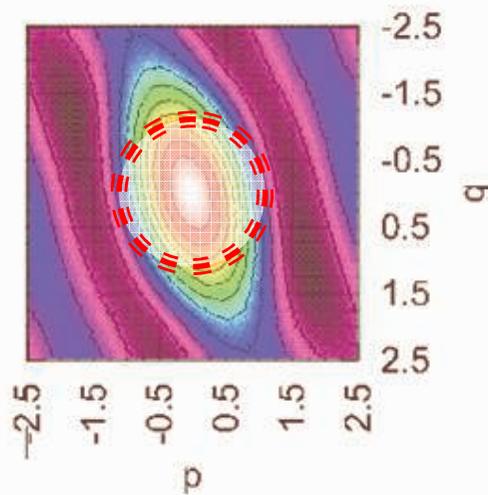
Signal



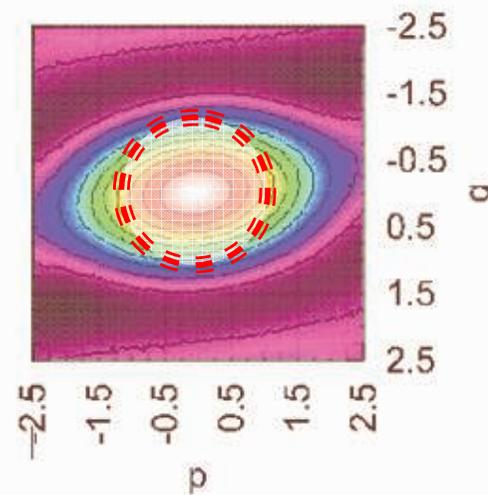
Idler



Signal - Idler

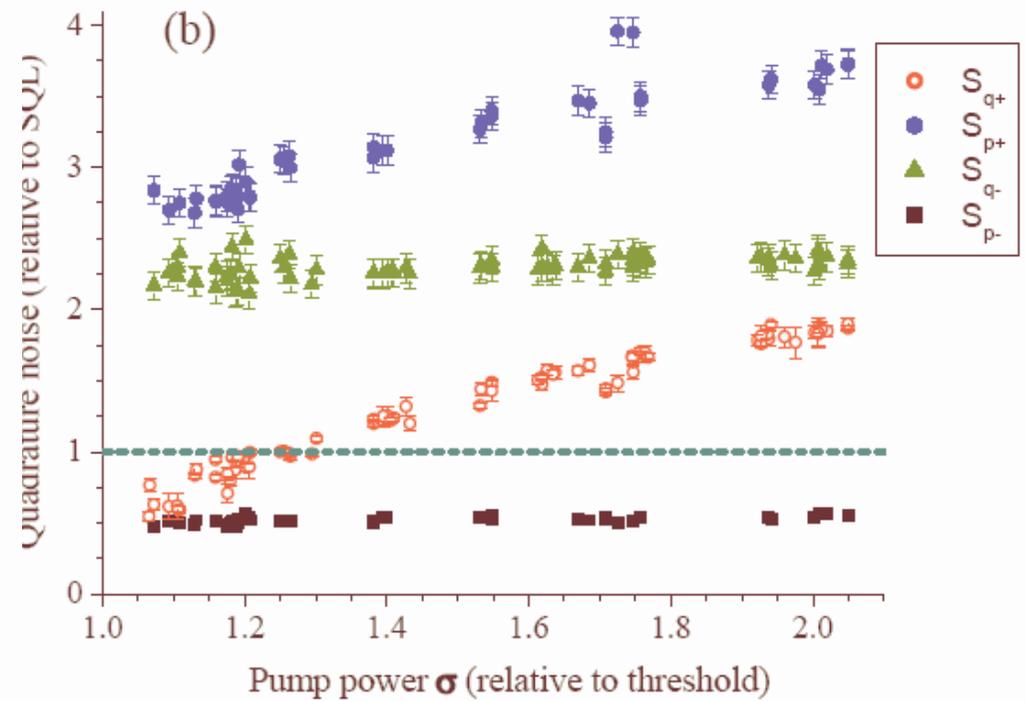
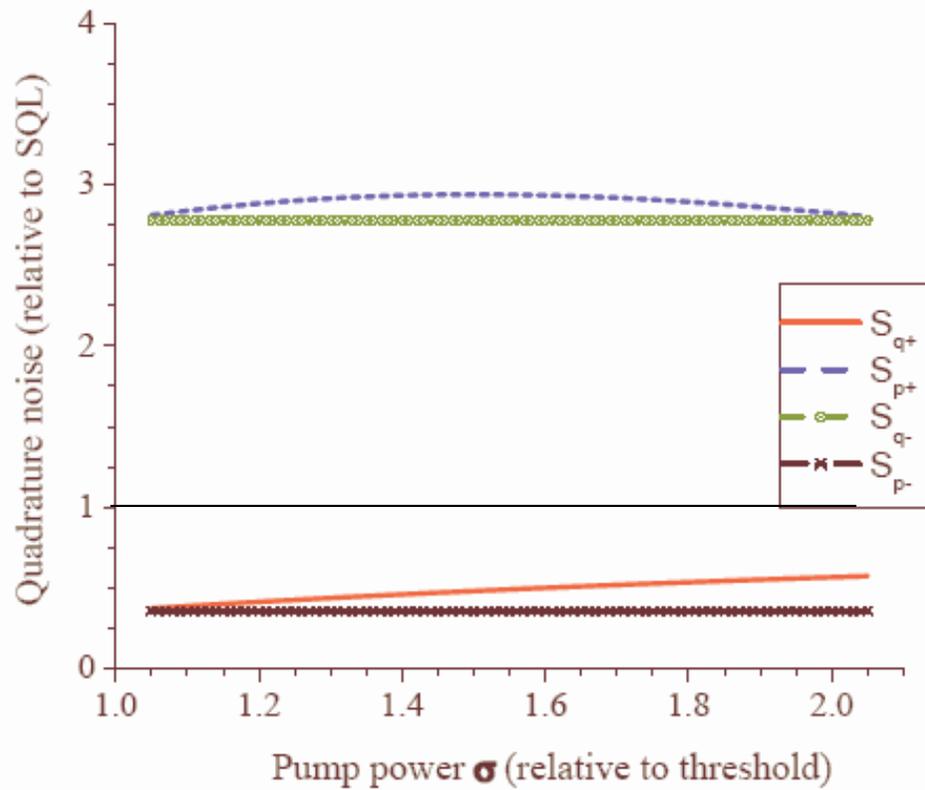


Signal + Idler

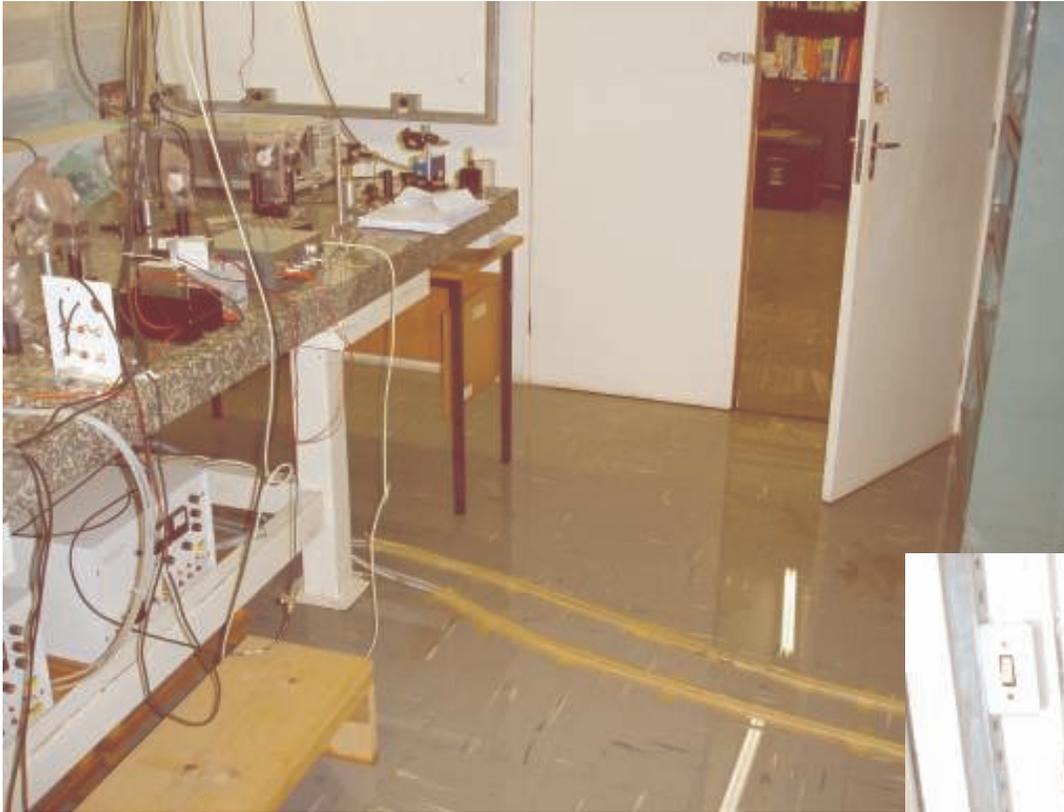


Sucesso?

Teoria x Experiência

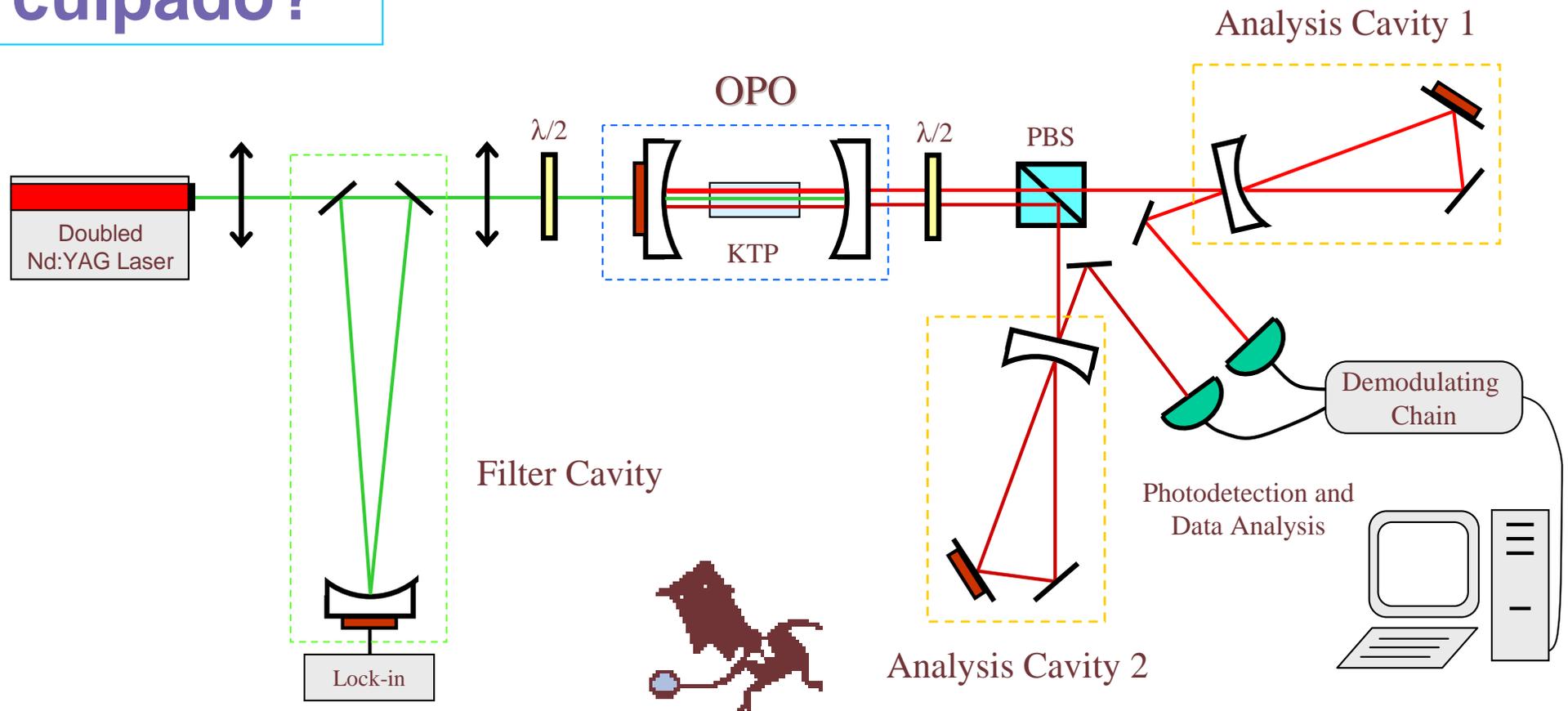


Que outras surpresas nos esperam?



Quem é o culpado?

A montagem ?



Quem é o culpado?

A teoria?

$$\frac{d}{dt}\rho = -\frac{i}{\hbar} [H_f + H_i + H_{ext}, \rho] + (\Lambda_0 + \Lambda_1 + \Lambda_2) \rho.$$

$$H_{ext} = i\hbar \frac{\gamma_0}{\tau} \varepsilon (a_0^\dagger - a_0)$$

$$H_i = i\hbar \frac{2\chi}{\tau} (a_1^\dagger a_2^\dagger a_0 - a_1 a_2 a_0^\dagger)$$

$$H_f = -\hbar\Delta_0 \frac{\gamma'_0}{\tau} a_0^\dagger a_0 - \hbar\Delta_1 \frac{\gamma'_1}{\tau} a_1^\dagger a_1 - \hbar\Delta_2 \frac{\gamma'_2}{\tau} a_2^\dagger a_2$$

$$\Lambda_j \rho = \frac{\gamma'_j}{\tau} (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j)$$

$$\begin{aligned}
\frac{\partial}{\partial t} W(\{\alpha_j\}) = & \sum_{j=1}^3 \frac{\gamma'_j}{\tau} \left[i\Delta_j \left(\frac{\partial}{\partial \alpha_j^*} \alpha_j^* - \frac{\partial}{\partial \alpha_j} \alpha_j \right) \right. \\
& \left. + \left(\frac{\partial}{\partial \alpha_j^*} \alpha_j^* + \frac{\partial}{\partial \alpha_j} \alpha_j \right) \right] W(\{\alpha_j\}) \\
& + \frac{2\chi}{\tau} \left(\alpha_1 \alpha_2 \frac{\partial}{\partial \alpha_0^*} + \alpha_1^* \alpha_2^* \frac{\partial}{\partial \alpha_0} \right) W(\{\alpha_j\}) \\
& - \frac{2\chi}{\tau} \left(\alpha_0 \alpha_1^* \frac{\partial}{\partial \alpha_2} + \alpha_0^* \alpha_1 \frac{\partial}{\partial \alpha_2^*} \right) W(\{\alpha_j\}) \\
& - \frac{2\chi}{\tau} \left(\alpha_0 \alpha_2^* \frac{\partial}{\partial \alpha_1} + \alpha_0^* \alpha_2 \frac{\partial}{\partial \alpha_1^*} \right) W(\{\alpha_j\}) \\
& - \frac{\gamma_0}{\tau} \varepsilon \left(\frac{\partial}{\partial \alpha_0^*} + \frac{\partial}{\partial \alpha_0} \right) W(\{\alpha_j\}) \\
& + \sum_{j=1}^3 \frac{\gamma'_j}{\tau} \frac{\partial^2}{\partial \alpha_j \partial \alpha_j^*} W(\{\alpha_j\}) \\
& - \frac{\chi}{2\tau} \frac{\partial^3}{\partial \alpha_0^* \partial \alpha_1^* \partial \alpha_2^*} W(\{\alpha_j\})
\end{aligned}$$

Linearização ?

Não !

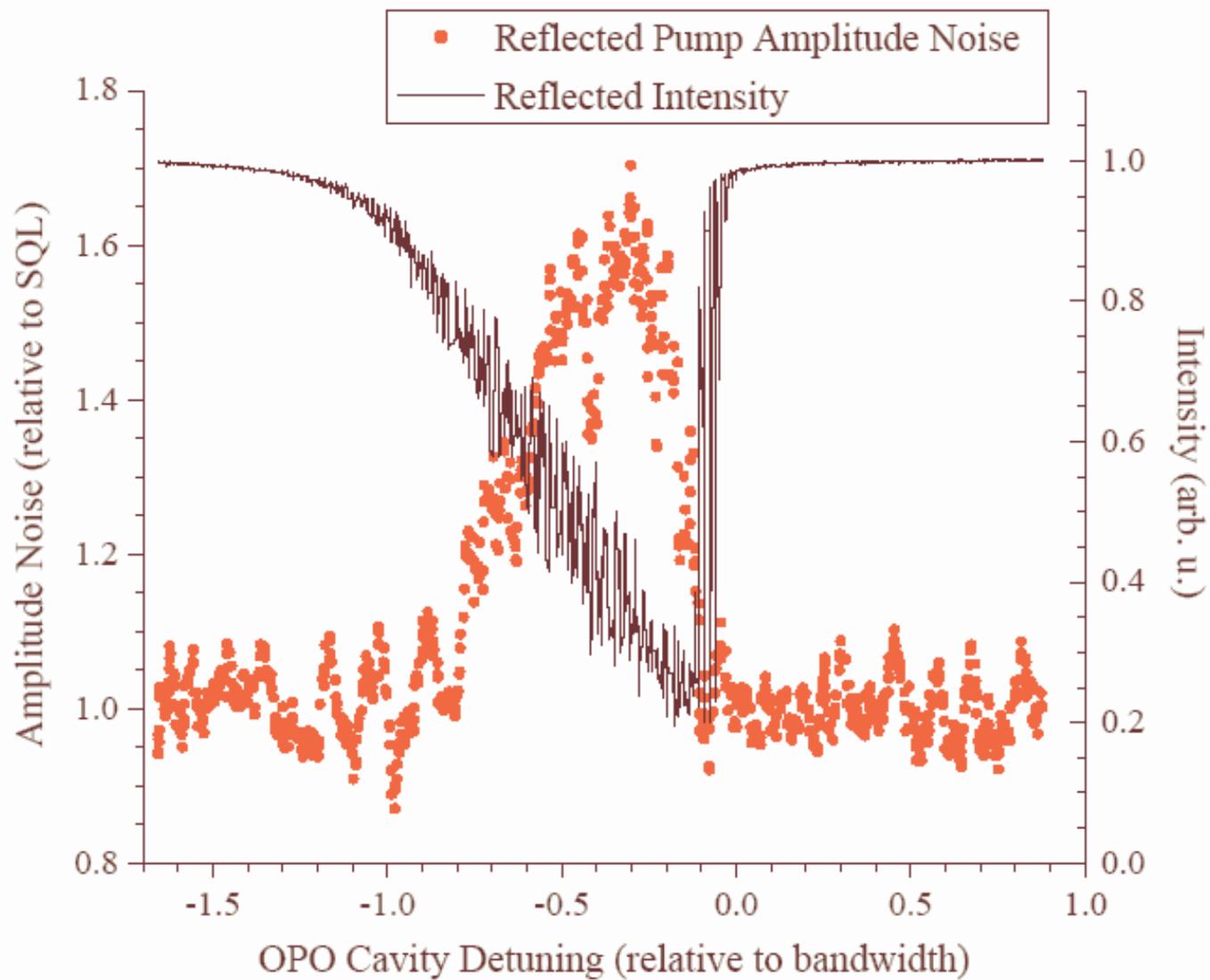
Eq. de Fokker-Planck

$$\begin{aligned}
\frac{\partial}{\partial t} W(\{\alpha_j\}) = & - \sum_j \frac{\partial}{\partial \alpha_j} A_j W(\{\alpha_j\}) \\
& + \frac{1}{2} \sum_{j,k} \frac{\partial}{\partial \alpha_j} \frac{\partial}{\partial \alpha_k} [\mathbf{B}\mathbf{B}^T]_{jk} W(\{\alpha_j\})
\end{aligned}$$

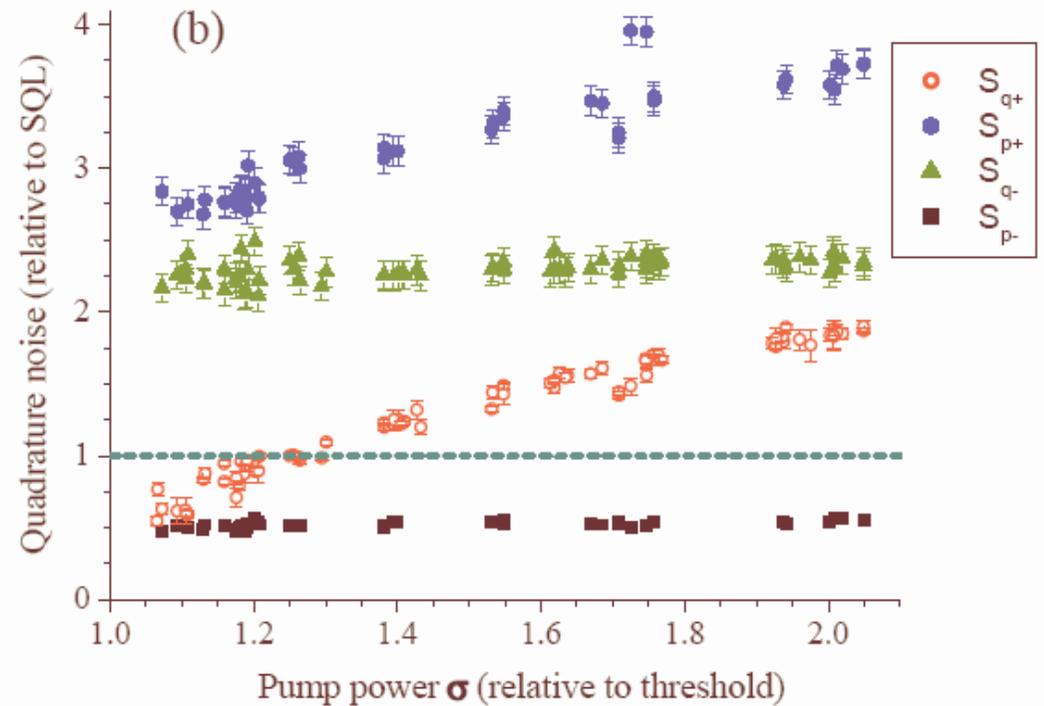
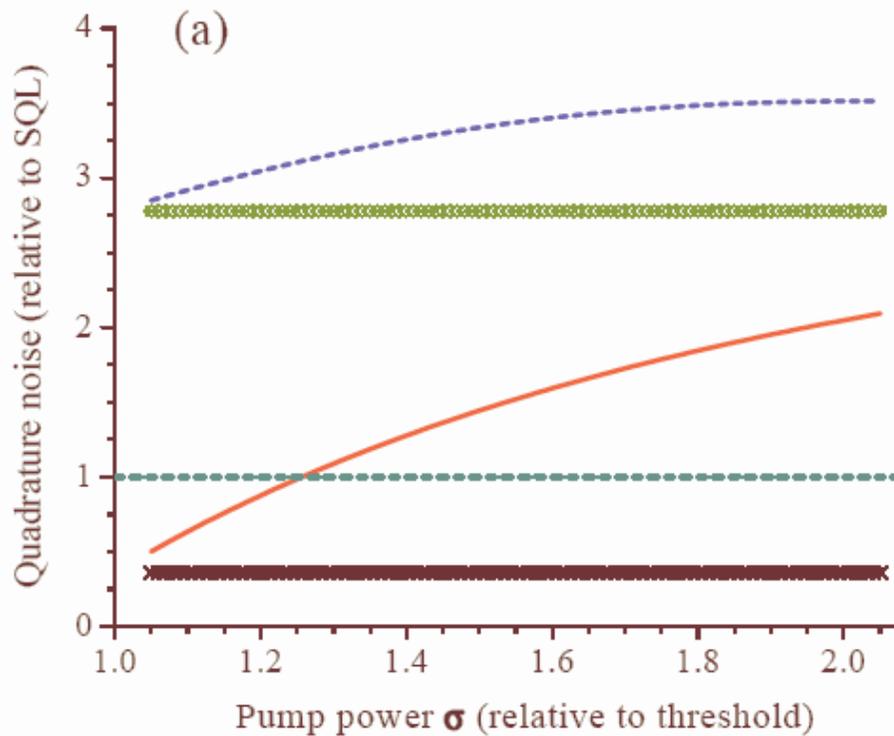
Eqs. de Langevin

$$\frac{d}{dt} \alpha_j = A_j + [\mathbf{B}\boldsymbol{\sigma}(\mathbf{t})]_j$$

Ruído do bombeio refletido !



Teoria x Experiência



$$\Delta^2 p_0 = 1,8 \quad \Delta^2 q_0 = 5,5$$

**Permanece a questão:
qual o mecanismo de geração de ruído no cristal ?**

Ainda falta algo...

$$\tau \frac{d}{dt} P = -AP + BP_{\text{in}},$$

$$P = [\delta p_-(t), \delta q_-(t), \delta p_+(t), \delta q_+(t), \delta p_0(t), \delta q_0(t)]^T$$

$$\eta = \frac{|\alpha|}{|\alpha_0|}$$

$$\mathbf{A} = \begin{bmatrix} 2\gamma & 0 & 0 & 0 & 0 & 0 \\ -2\gamma\Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma\Delta & -\sqrt{2}\gamma\eta & \sqrt{2}\gamma\Delta\eta \\ 0 & 0 & 0 & 2\gamma & -2\gamma\Delta\eta & -2\gamma\eta \\ 0 & 0 & \sqrt{2}\gamma\eta & \sqrt{2}\gamma\Delta\eta & \gamma_0 & -\gamma_0\Delta_0 \\ 0 & 0 & -\sqrt{2}\gamma\Delta\eta & \sqrt{2}\gamma\eta & \gamma_0\Delta_0 & \gamma_0 \end{bmatrix}$$

Matriz de covariância

Feixes Gêmeos ($\eta > 0$)

Vácuo comprimido ($\eta = 0$, caso degenerado)

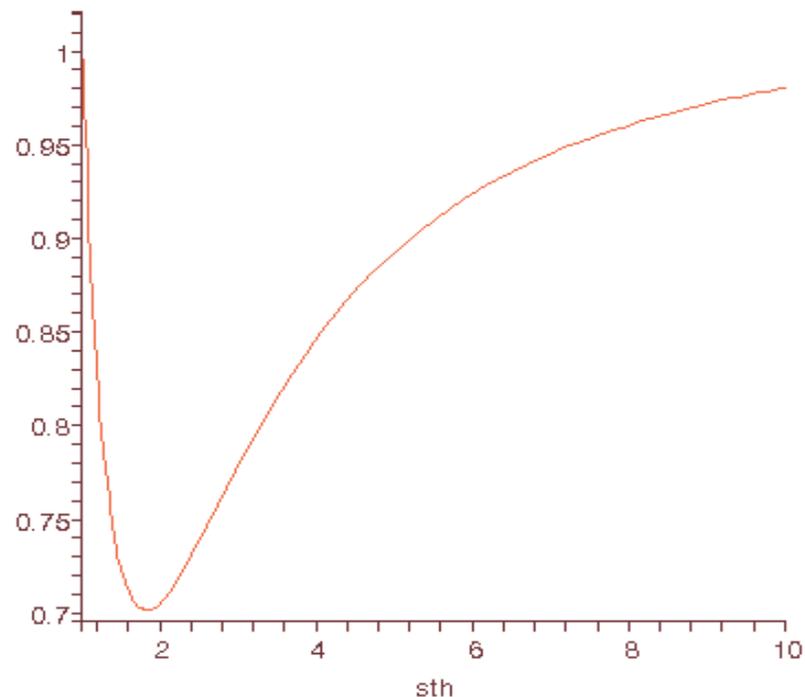
$$V = \begin{bmatrix} S_{p-} & 0 & 0 & 0 & 0 & 0 \\ 0 & S_{q-} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{p+} & 0 & C_{p+p0} & 0 \\ 0 & 0 & 0 & S_{q+} & 0 & C_{q+q0} \\ 0 & 0 & C_{p+p0} & 0 & S_{p0} & 0 \\ 0 & 0 & 0 & C_{q+q0} & 0 & S_{q0} \end{bmatrix}$$

Campos Emaranhados
 Vácuo ($\eta = 0$, emaranhamento máximo)
 Emaranhamento de campos intensos ($\eta > 0$)

Compressão do bombeio ($\eta > 0$)

Pureza de p+ com potência de bombeio indo de P_{thr} a $10P_{thr}$ ($\Omega = \delta\nu_{OPO} / 2$)

Para onde vai a pureza ?



$$V = \begin{bmatrix} S_p & 0 & 0 & 0 & 0 & 0 \\ 0 & S_s & 0 & 0 & 0 & 0 \\ 0 & 0 & S_p & 0 & C_{p-s0} & 0 \\ 0 & 0 & 0 & S_s & 0 & C_{s-p0} \\ 0 & 0 & C_{p-s0} & 0 & S_{p0} & 0 \\ 0 & 0 & 0 & C_{s-p0} & 0 & S_{s0} \end{bmatrix}$$

Correlações entre bombeio, sinal e complementar ($\eta > 0$)

Direct Production of Tripartite Pump-Signal-Idler Entanglement in the Above-Threshold OPO

A. S. Villar¹, M. Martinelli¹, C. Fabre², and P. Nussenzveig^{1*}

¹*Instituto de Física, Universidade de São Paulo,*

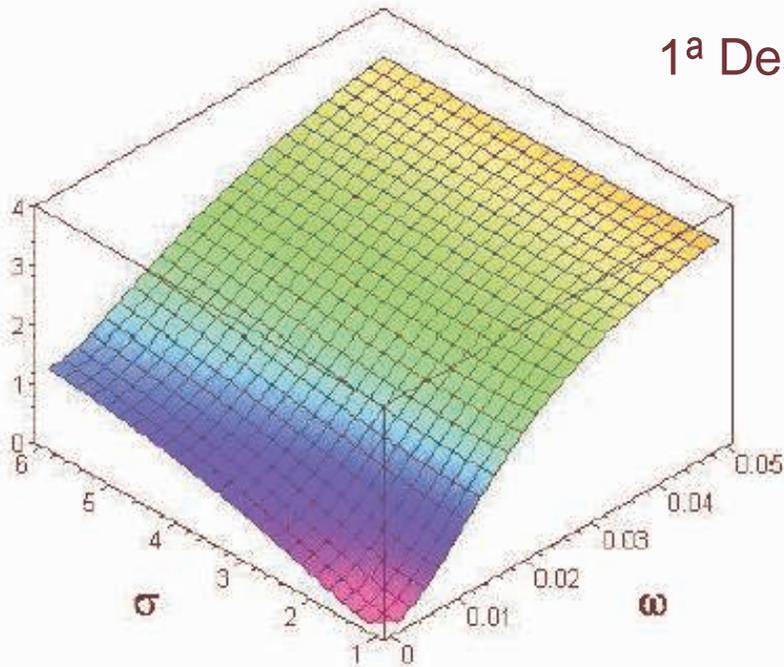
Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil

²*Laboratoire Kastler Brossel, Case 74, Université Pierre et Marie*

Curie - Paris 6, 4 Place Jussieu, 75252 Paris Cedex 05, France

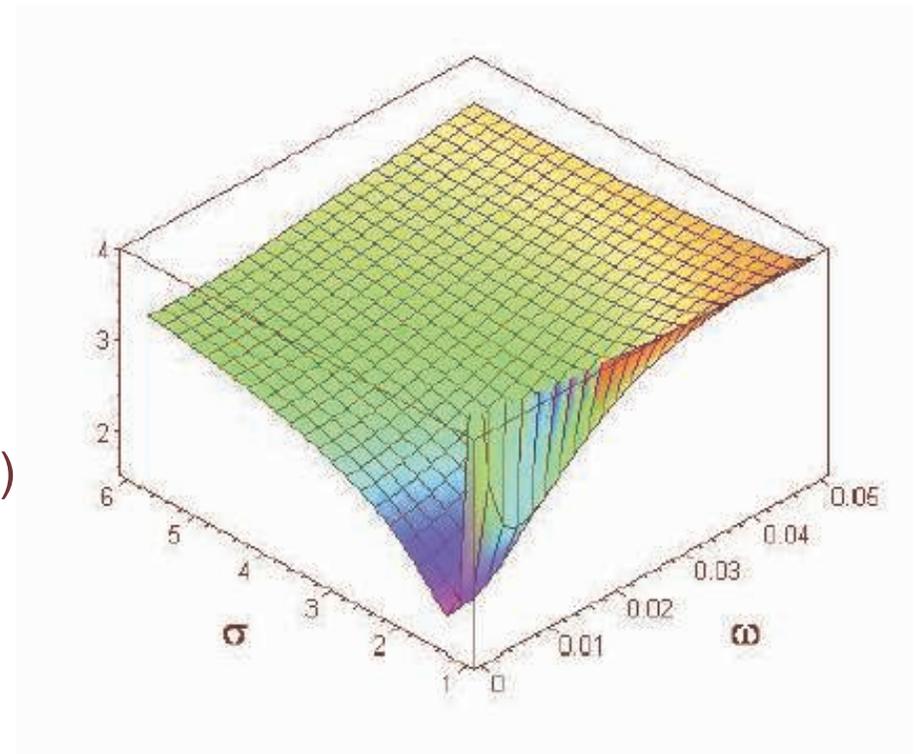
Aceito ! (PRL)

1ª Desigualdade: $\Delta^2(p_1 - p_2) + \Delta^2(q_1 + q_2 - \alpha q_0) \geq 4$

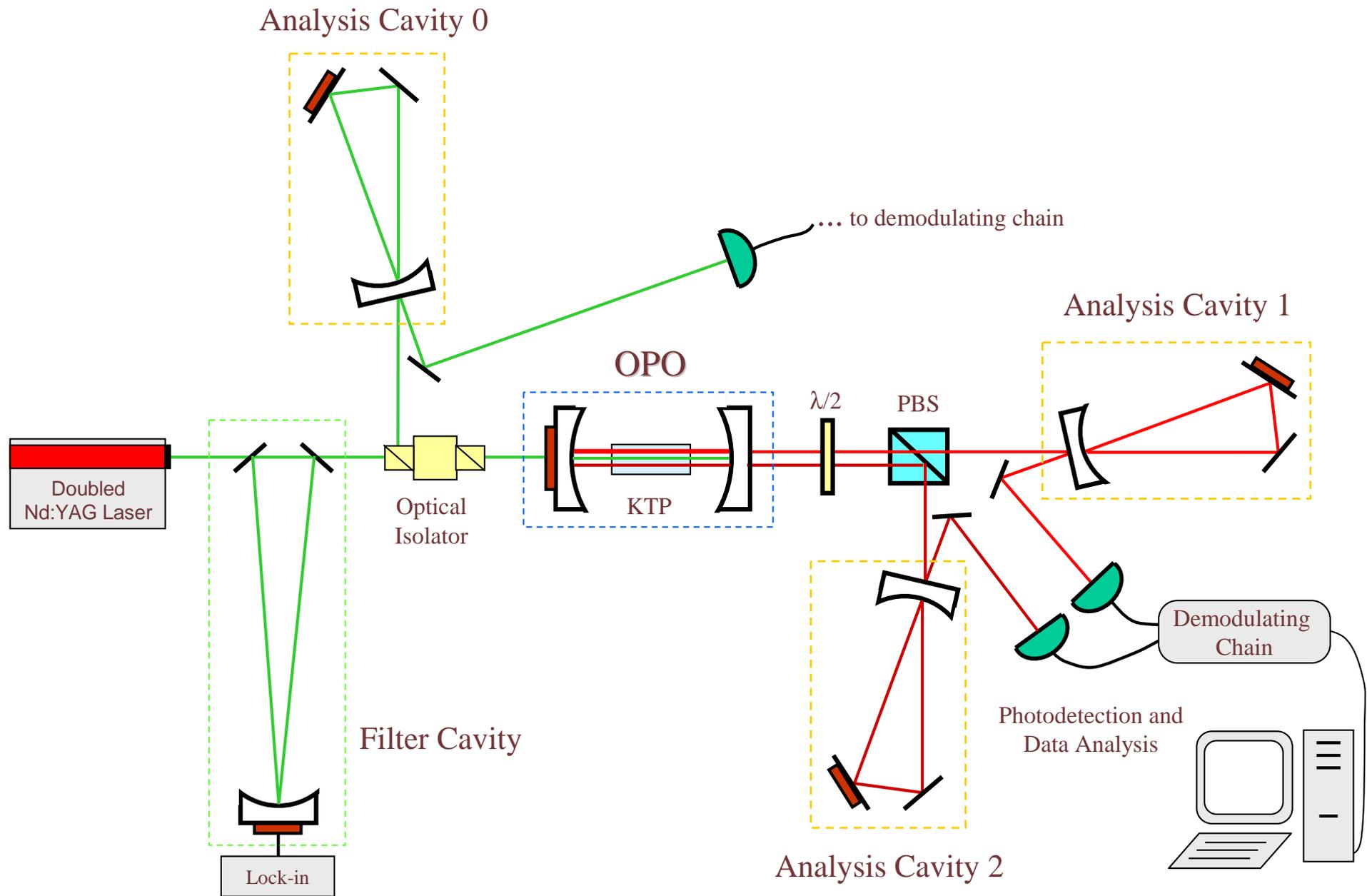


2ª Desigualdade: $\Delta^2(p_1 + p_0) + \Delta^2(q_1 + \beta q_2 - q_0)$

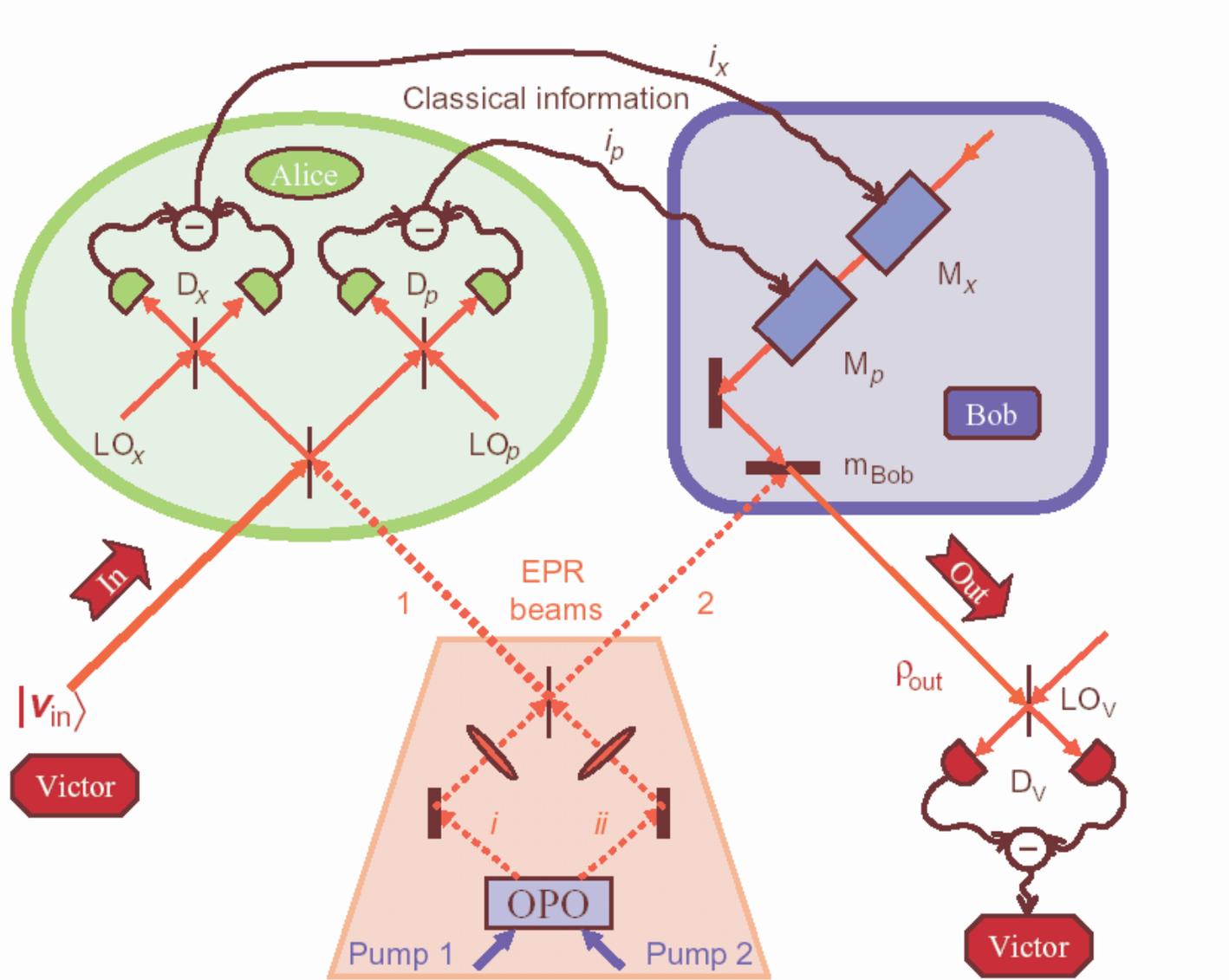
[ou $\Delta^2(p_2 + p_0) + \Delta^2(\gamma q_1 + q_2 - q_0) \geq 4$]



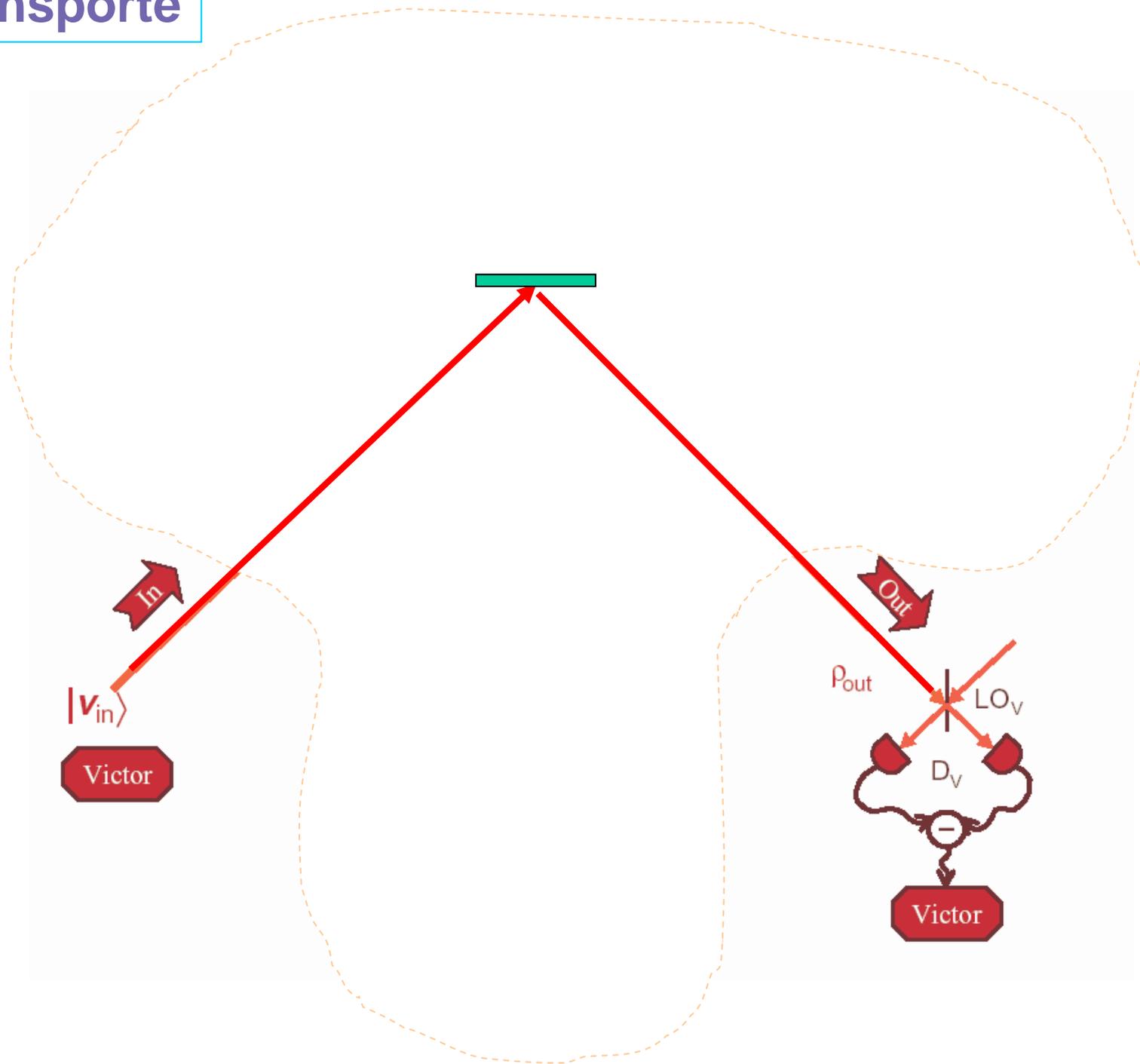
Proposta de medida



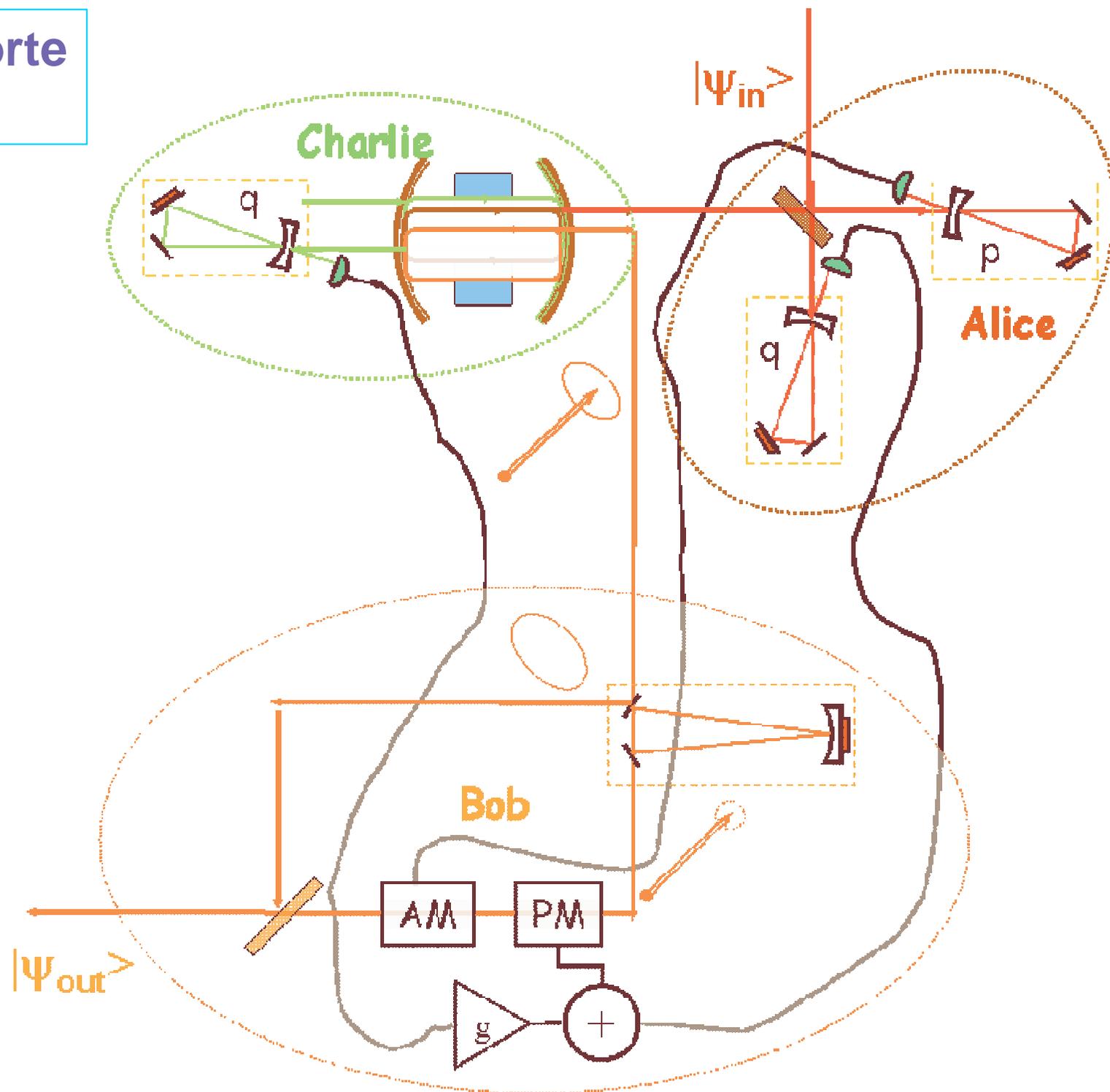
Teletransporte



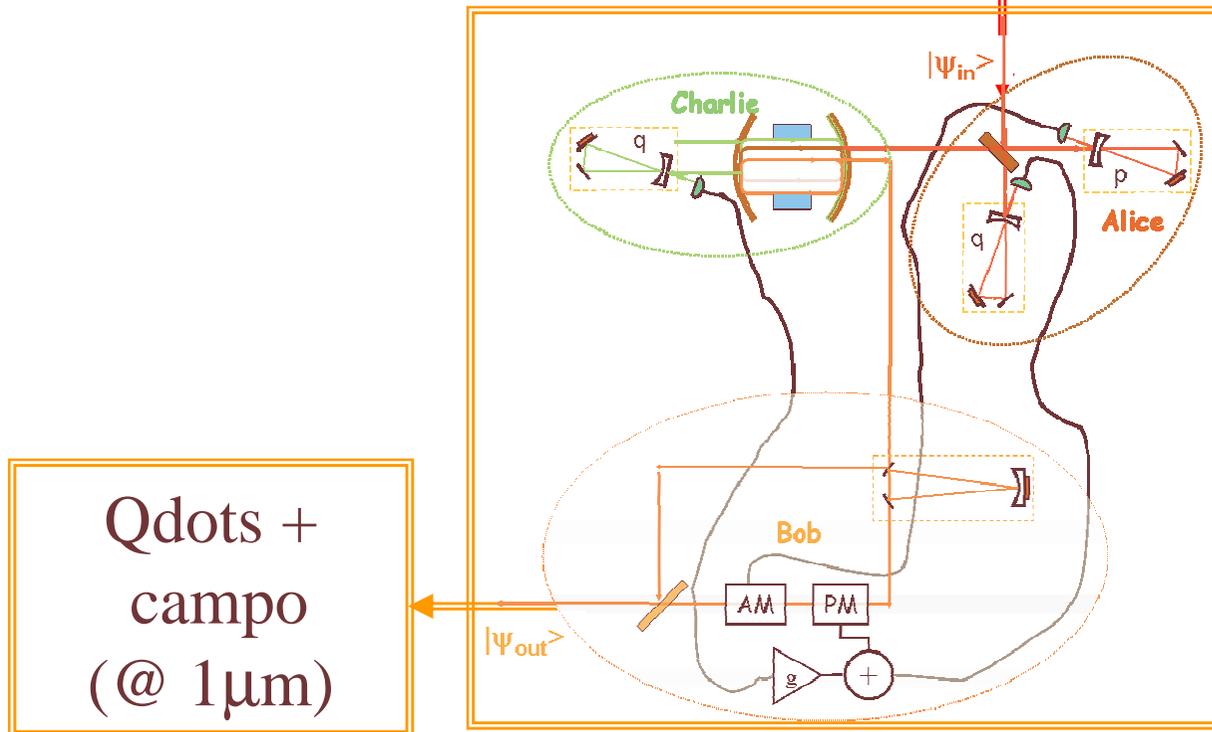
Teletransporte



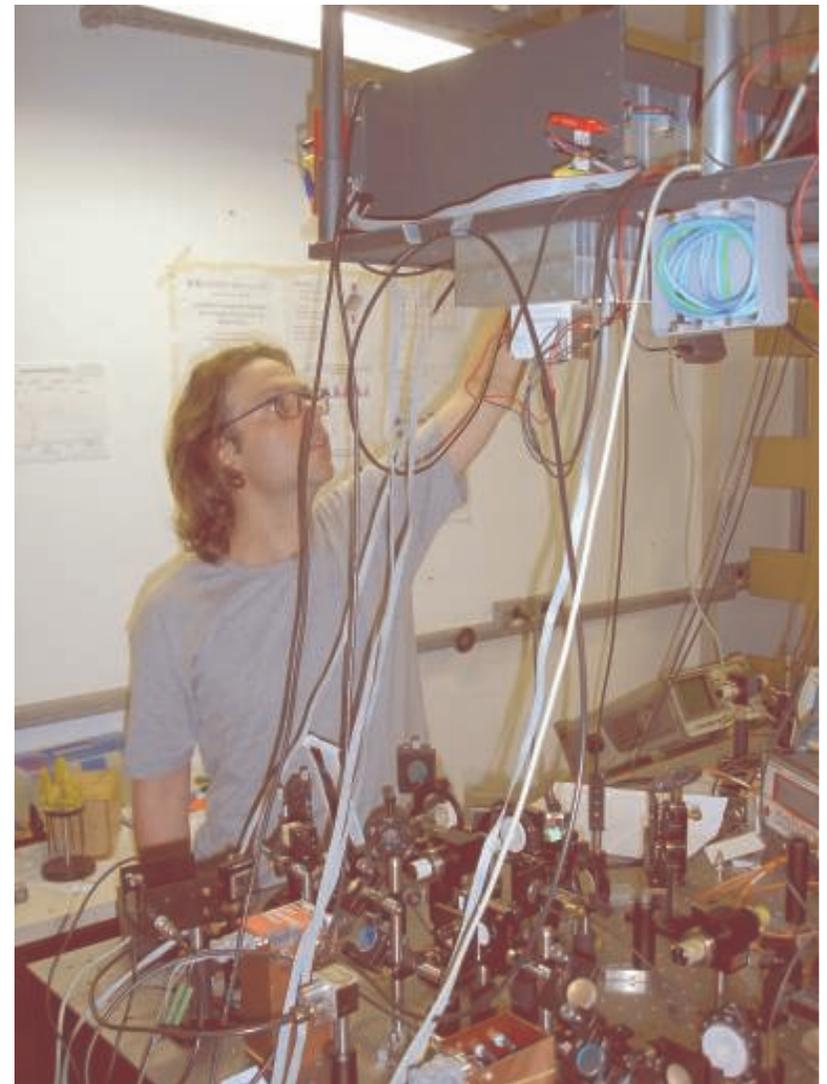
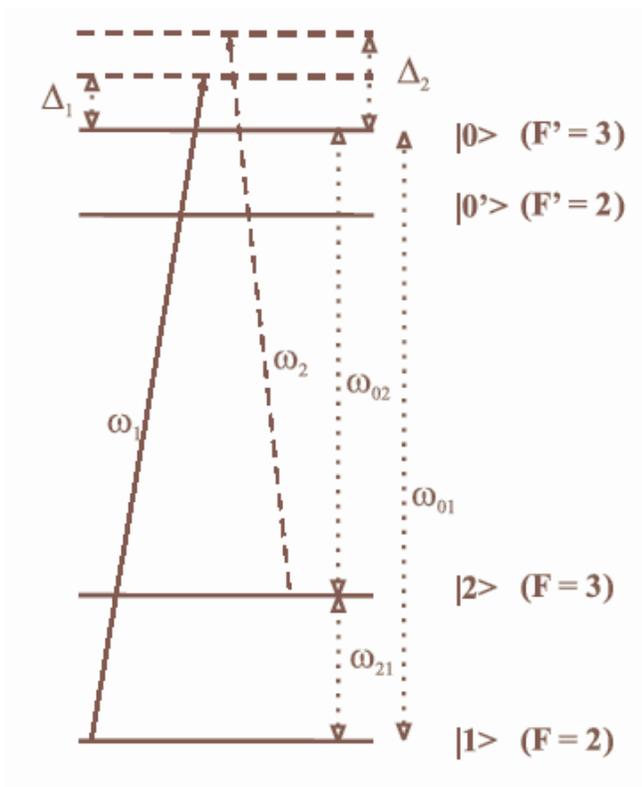
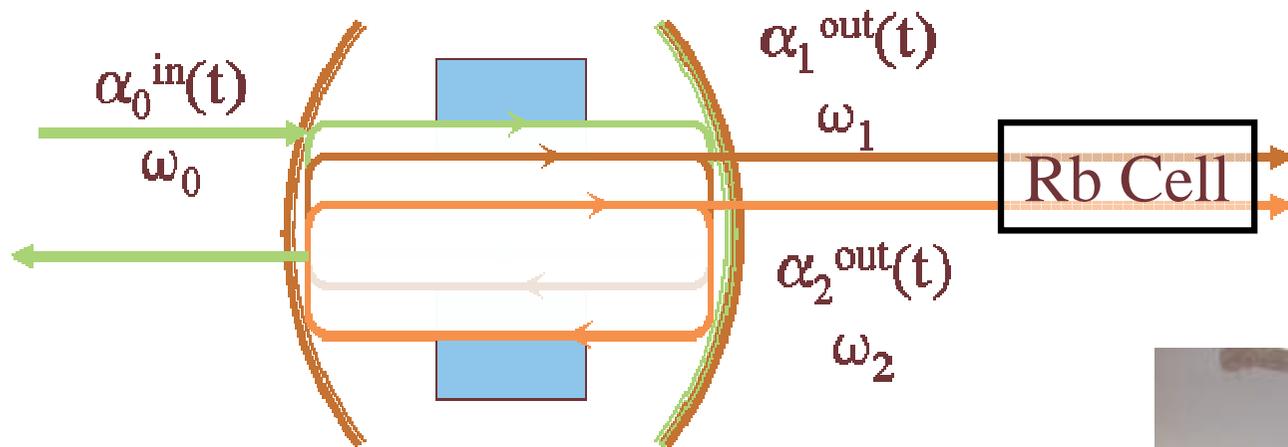
Teletransporte multi-cor !



Átomos +
campo
(@ 780 nm)



Qdots +
campo
(@ 1 μm)

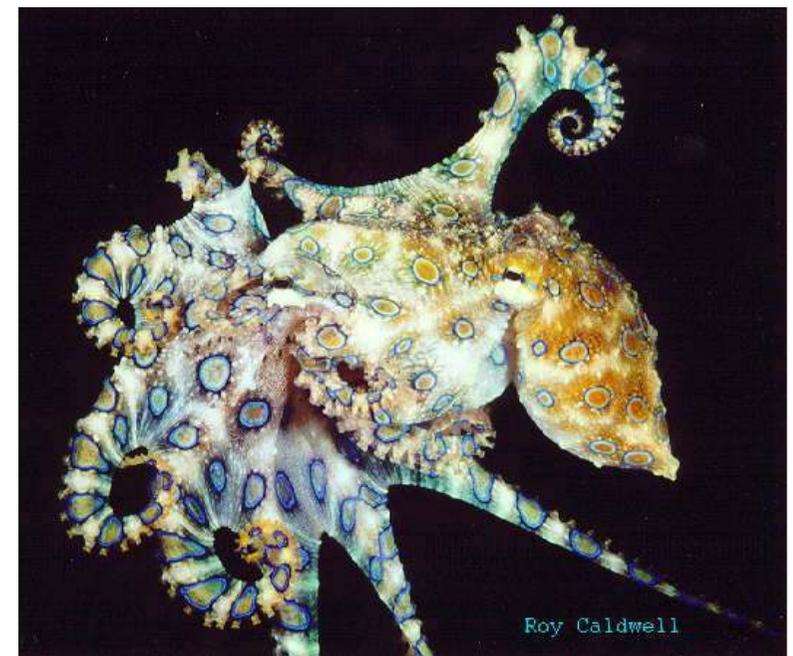


Conclusão

- o Caracterizamos o OPO como uma fonte de feixes *EMARANHADOS, INTENSOS, e de CORES DISTINTAS !*
- o Previsto desde 1988!
- o Perspectivas:
 - Criptografia em redes de fibra ótica.
 - Teleportação de estados quânticos entre diferentes regiões do espectro.
 - Emaranhamento de três campos!
 - Problemas abertos em Ótica Quântica e Não-Linear!



Obrigado!



Paulo A. Nussenzveig – MS5
Marcelo Martinelli – MS3
Paulo Valente – Pos-Doc
Alessandro de Sousa Villar – Dr
Katiúscia Nadyne Casseiro – Dr
Hélio Zhang He – MSc
Jônatas E. Silva César - MSc
Clodoaldo José da Silva - IC
Márcio Lopes - IC
Fábio Moreira da Silva – IC
Vitor Manfrinato - IC
Douglas Canducci – IC

<http://axpfep1.if.usp.br/~lmcal>

