Luz e Átomos como ferramentas para Informação Quântica Emaranhamento Multicor



Marcelo Martinelli



Lab. de Manipulação Coerente de Átomos e Luz







Few words about entanglement characterization

• "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]

$$\Delta^{2} \hat{p}_{\inf} = \Delta^{2} \hat{p}_{1} \left(1 - \frac{\langle \delta \hat{p}_{1} \delta \hat{p}_{2} \rangle^{2}}{\Delta^{2} \hat{p}_{1} \Delta^{2} \hat{p}_{2}} \right)$$
$$\delta \hat{p}_{i} = \hat{p}_{i} - \langle \hat{p}_{i} \rangle$$
$$\Delta^{2} \hat{p}_{\inf} \Delta^{2} \hat{q}_{\inf} \geq 1$$

Entanglement Test - DGCZ

•DGCZ separability criterion:

$$\rho = \sum_{i} p_i \ \rho_i = \sum_{i} p_i \ \rho_i^1 \otimes \rho_i^2 \qquad [\hat{q}_i, \hat{p}_j] = 2i\delta_{ij}$$

$$\hat{u} = a\hat{q}_1 + \frac{1}{a}\hat{q}_2,$$
$$\hat{v} = a\hat{p}_1 - \frac{1}{a}\hat{p}_2,$$

Separability
$$\Rightarrow \langle (\Delta \hat{u})^2 \rangle_{\rho} + \langle (\Delta \hat{v})^2 \rangle_{\rho} \ge 2 \ (a^2 + \frac{1}{a^2})$$

Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller, Inseparability criterion for continuous variable systems, Phys. Rev. Lett. 84, 2722 (2000).

•After some (simple) algebra:

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \ge 0;$$

Entanglement Test - DGCZ

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

 $C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \ge 0$$

Entanglement Test - DGCZ



 $p_1 + p_2, q_1 + q_2$

 $p_1 - p_2$, $q_1 - q_2$





 $p_1 - p_2$, $q_1 - q_2$

Entanglement Test - Peres & Horodecki

• Positivity under Partial Transposition (discrete variables) Separability Criterion for Density Matrices

Asher Peres*

PRL 77, 1413 (1996)

$$\rho = \sum_{A} w_{A} \rho_{A}' \otimes \rho_{A}'' \qquad \Longrightarrow \quad \sigma = \sum_{A} w_{A} (\rho_{A}')^{T} \otimes \rho_{A}''$$

non-negative eigenvalues -> Separability



• Continuous variables: PT: $W(q_1, p_1, q_2, p_2) \rightarrow W(q_1, p_1, q_2, -p_2)$ $V + \frac{i}{2} \Omega \ge 0$ $\Omega = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}$ $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ Peres-Horodecki Separability Criterion for Continuous Variable Systems PRL 84, 2726 (2000) $\tilde{V} + \frac{i}{2} \Omega \ge 0$ $\tilde{V} + \frac{i}{2} \Omega \ge 0$ $\tilde{V} = \Lambda V\Lambda$ $\Lambda = \text{diag}(1, 1, 1, -1)$

Simplectic Eigenvalues >1

Diagonalize:
$$-(\Omega \tilde{V})^2$$

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle$$
$$S_{xj} = C_{xjxj}$$

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & -C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & -C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & -C_{p2q2} \\ -C_{p1q2} & -C_{q1q2} & -C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle$$
$$S_{xj} = C_{xjxj}$$



 $p_1 - p_2, q_1 + q_2$

 $p_1 + p_2$, $q_1 - q_2$

Tripartite Entanglement

• Extend DGCZ criterion to three variables

Detecting genuine multipartite continuous-variable entanglement

PHYSICAL REVIEW A 67, 052315 (2003) Peter van Loock¹ and Akira Furusawa²

$$\hat{u} \equiv h_1 \hat{x}_1 + h_2 \hat{x}_2 + h_3 \hat{x}_3, \quad \hat{v} \equiv g_1 \hat{p}_1 + g_2 \hat{p}_2 + g_3 \hat{p}_3,$$

$$\langle (\Delta \hat{u})^2 \rangle_{\rho} + \langle (\Delta \hat{v})^2 \rangle_{\rho} \geq f(h_1, h_2, h_3, g_1, g_2, g_3),$$

• Apply PPT to multiple partitions

Bound Entangled Gaussian States

R. F. Werner* and M. M. Wolf^{\dagger}

Gaussian states of $1 \times N$ systems

ppt implies separability.

PHYSICAL REVIEW LETTERS VOLUME 86, NUMBER 16 DOI: 10.1103/PhysRevLett.86.3658 Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$$



Generation of Squeezed States by Parametric Down Conversion



Generation of Squeezed States by Parametric Down Conversion



Observation of Quantum Noise Reduction on Twin Laser Beams

A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, and C. Fabre

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and

G. Camy Laboratoire de Physique des Lasers, Université de Paris Nord, 93430 Villetaneuse, France (Received 3 August 1987)



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Europhys. Lett., 40 (1), pp. 25-30 (1997) Observation of squeezing using cascaded nonlinearity

K. KASAI(*), GAO JIANGRUI(**) and C. FABRE

Laboratoire Kastler Brossel (***) UPMC - Case 74 75252 Paris Cedex 05, France

(received 20 January 1997; accepted in final form 2 September 1997)

Abstract. – We have observed that the pump beam reflected by a triply resonant optical parametric oscillator, after a cascaded second-order nonlinear interaction in the crystal, is significantly squeezed. The maximum measured squeezing in our device is 30% (output beam squeezing inferred: 48%). The direction of the noise ellipse depends on the cavity detuning and can be adjusted from intensity squeezing to phase squeezing.



Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng^(a)

Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125 (Received 20 February 1992)

The Einstein-Podolsky-Rosen paradox is demonstrated experimentally for dynamical variables having a continuous spectrum. As opposed to previous work with discrete spin or polarization variables, the continuous optical amplitudes of a signal beam are inferred in turn from those of a spatially separated but strongly correlated idler beam generated by nondegenerate parametric amplification. The uncertainty product for the variances of these inferences is observed to be 0.70 ± 0.01 , which is below the limit of unity required for the demonstration of the paradox.

Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables



Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng^(a)

How can we measure the phase?





And if we look for a complete characterization of the OPO, we have to measure three fields of diferent colors!

Is it possible to perform a homodyne measurement without a local oscillator?

Measurement of the Field in the time domain



Measurement of the Field in the frequency domain





Measurement of the Field in the frequency domain

$$\hat{a}(t) = \int_{-\infty}^{\infty} \hat{a}(\Omega) \exp(-i\Omega t) \ d\Omega$$

$$\hat{a}(\Omega) = \hat{x}(\Omega) + i\hat{y}(\Omega)$$





 $E(t) = A \operatorname{Re}\{i \kappa exp[i(\omega - \Omega)t] + exp(i\omega t) + i \kappa exp[i(\omega + \Omega)t]\}$

Phase Rotation of Noise Ellipse



Alessandro S. Villar, The conversion of phase to amplitude fluctuations of a light beam by an optical cavity American Journal of Physics **76**, pp. 922-929 (2008).



Generation of Bright Two-Color Continuous Variable Entanglement

A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig*

Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil

We present the first measurement of squeezed-state entanglement between the twin beams produced in an optical parametric oscillator operating above threshold. In addition to the usual squeezing in the intensity difference between the twin beams, we have measured squeezing in the sum of phase quadratures. Our scheme enables us to measure such phase anticorrelations between fields of different frequencies. In the present measurements, wavelengths differ by ≈ 1 nm. Entanglement is demonstrated according to the Duan *et al.* criterion [Phys. Rev. Lett. 84, 2722 (2000)] $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$. This experiment opens the way for new potential applications such as the transfer of quantum information between different parts of the electromagnetic spectrum.



Generation of Bright Two-Color Continuous Variable Entanglement

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INSIGHT REVIEW

The quantum internet

H. J. Kimble¹



Direct Production of Tripartite Pump-Signal-Idler Entanglement in the Above-Threshold Optical Parametric Oscillator





Experimental Creation of a Fully Inseparable Tripartite Continuous-Variable State

Takao Aoki,* Nobuyuki Takei, Hidehiro Yonezawa, Kentaro Wakui, Takuji Hiraoka, and Akira Furusawa Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan

Peter van Loock

Quantum Information Theory Group, Zentrum für Moderne Optik, Universität Erlangen-Nürnberg, 91058 Erlangen, Germany



Demonstration of a quantum teleportation network for continuous variables

Hidehiro Yonezawa^{1,2}, Takao Aoki^{1,2} & Akira Furusawa^{1,2}

¹Department of Applied Physics, School of Engineering, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan ²CREST, Japan Science and Technology Agency, 1-1-9 Yaesu, Chuo-ku, Tokyo 103-0028, Japan




Quantum error correction beyond qubits

Takao Aoki¹*, Go Takahashi^{1,2}, Tadashi Kajiya^{1,2}, Jun-ichi Yoshikawa^{1,2}, Samuel L. Braunstein³, Peter van Loock⁴ and Akira Furusawa^{1,2}†



Tripartite entanglement?







Quantum Correlations between pump, signal and idler for $P_0 > P_{th}$

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q2} & 0 & S_{q2} & 0 & C_{q2q0} \\ \hline C_{p1p0} & 0 & C_{p2p0} & 0 & \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

K. N. Cassemiro *et al.* Opt. Lett. **32**, 695 (2007)
K. N. Cassemiro *et al.* Opt. Exp. **15**, 18326 (2007)
N. B. Grosse *et al.* PRL **100**, 243601 (2008) (degenerate case)

Three-color quantum correlations

K. N. Cassemiro *et al.* Opt. Lett. **32**, 695 (2007)



Analysis cavities detuning, Δ (relative to bandwidth)

$$\Delta^2 \hat{p}_- + \Delta^2 (\hat{q}_+ - \alpha_0 \, \hat{p}_0) \ge 2$$
, with $\alpha_0 = \frac{C_{\hat{p}_0 \, \hat{q}_+}}{\Delta^2 \hat{p}_0}$

$$\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ - \beta_0 \ge 2$$
, with $\beta_0 = \frac{C_{\hat{p}_0 \hat{q}_+}^2}{\Delta^2 \hat{p}_0}$



The quest for three-color entanglement...

K. N. Cassemiro et al. Opt. Exp. 15, 18326 (2007)





Observation of Entanglement between Two Light Beams Spanning an Octave in Optical Frequency

Nicolai B. Grosse,¹ Syed Assad,^{1,2} Moritz Mehmet,³ Roman Schnabel,³ Thomas Symul,¹ and Ping Koy Lam¹





An open question: What is the source of the noise in the crystal?



Entanglement in the above-threshold optical parametric oscillator

Alessandro S. Villar and Katiúscia N. Cassemiro Kaled Dechoum and Antonio Z. Khoury Marcelo Martinelli and Paulo Nussenzveig Vol. 24, No. 2/February 2007/J. Opt. Soc. Am. B 249

Noise is everywhere!





Extra phase noise from thermal fluctuations in nonlinear optical crystals

J. E. S. César,¹ A. S. Coelho,¹ K. N. Cassemiro,² A. S. Villar,³ M. Lassen,⁴ P. Nussenzveig,¹ and M. Martinelli¹

$$\delta Q_j(\Omega) = \frac{n_j k_j}{2\varepsilon_j} \alpha_j \int |u_j^{(h)}(\vec{r})|^2 \delta \varepsilon_j(\vec{r}, \Omega) \, dx \, dy \, dz$$

$$\begin{aligned} \langle \delta Q_j(\Omega) \delta Q_k(-\Omega) \rangle &= \\ k_j k_k \frac{n_j^3 n_k^3}{4hc} \, l_c^3 \, c_{jk}(\Omega) \, \left(\frac{\ell \sqrt{\lambda_j \lambda_k}}{\pi w_{jk}^2} \right) \sqrt{P_j P_k} \\ &= \eta_{jk} \sqrt{P_j P_k} \,. \end{aligned}$$

$$\begin{aligned} \eta_{00} &= 0.53/W, \ \eta_{11} &= 0.15/W, \\ \text{and} \ \eta_{22} &= 0.14/W. \quad \eta_{01} &= 0.14/W, \ \eta_{02} &= 0.15/W, \\ \text{and} \ \eta_{12} &= 0.087/W. \end{aligned}$$

 $\eta_{00}(T) = [5.92(46)10^{-3} \times T/K - 1.38(13)]/W$



 $\vec{X} = [\delta \hat{p}_0, \delta \hat{q}_0, \delta \hat{p}_1, \delta \hat{q}_1, \delta \hat{p}_2, \delta \hat{q}_2]^T$

$$\begin{split} \tau \frac{\partial}{\partial t} \vec{X} = M_A \vec{X} + M_\gamma \vec{X}_1^{in} + M_\mu \vec{X}_2^{in} + \vec{Q} \\ \vec{X}^{out}(\Omega) = M_\gamma \vec{X}(\Omega) - \vec{X}_1^{in} \end{split}$$

$$V = I + V_{pure} + V_{loss} + V_{phase}$$

Way to go











Three-Color Entanglement

A. S. Coelho,¹ F. A. S. Barbosa,¹ K. N. Cassemiro,² A. S. Villar,^{2,3} M. Martinelli,¹ P. Nussenzveig¹*

Entanglement is an essential quantum resource for the acceleration of information processing as well as for sophisticated quantum communication protocols. Quantum information networks are expected to convey information from one place to another by using entangled light beams. We demonstrated the generation of entanglement among three bright beams of light, all of different wavelengths (532.251, 1062.102, and 1066.915 nanometers). We also observed disentanglement for finite channel losses, the continuous variable counterpart to entanglement sudden death.



Three-Color Entanglement

A. S. Coelho, *et al. Science* **326**, 823 (2009); DOI: 10.1126/science.1178683

Three-Color Entanglement

A. S. Coelho,¹ F. A. S. Barbosa,¹ K. N. Cassemiro,² A. S. Villar,^{2,3} M. Martinel

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The effect of losses



 $\Delta \hat{X}_{b,\varphi}^2 = \eta \Delta \hat{X}_{a,\varphi}^2 + (1-\eta) \Delta \hat{X}_{v,\varphi}^2$

$$\Delta \hat{X}_{b,\varphi}^2 - 1 = \eta \left(\Delta \hat{X}_{a,\varphi}^2 - 1 \right)$$

The problem of decoherence

Is the main problem for an eventual quantum computer, operating over many entangled qubits.

What is the limit for this entanglement?

Interaction with the environment!

Why producing and keeping them is a hard task?

Decoherence: as if the environment where continuously measuring the system!

Famous example: Schrödinger Cat Paradox (1935). Also an entangled state





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REVIEW Sudden Death of Entanglement

Ting Yu^{1*} and J. H. Eberly^{2*}



REPORTS Environment-Induced Sudden Death of Entanglement

M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, L. Davidovich*

SCIENCE VOL 316 27 APRIL 2007





Disentanglement for a Bipartite & Gaussian state



Scenario (1): robust entanglement



Scenario (2): disentanglement



Robustness of bipartite Gaussian entangled beams propagating in lossy channels

F. A. S. Barbosa¹, A. S. Coelho¹, A. J. de Faria¹, K. N. Cassemiro², A. S. Villar^{2,3}, P. Nussenzveig¹ and M. Martinelli^{1*}





Disentanglement for a simpler model: Attenuation on a single beam



Tighter conditions for transmission of quantum entanglement!

Early Stage Disentanglement in Bipartite Continuous-Variable Systems

F. A. S. Barbosa¹, A. S. Coelho¹, A. J. de Faria¹, K. N. Cassemiro², A. S. Villar^{2,3}, and M. Martinelli¹

Duan (optimized) $(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \ge 0;$





arXiv:1009.4255v1

What's next?

The sideband problem: non-unitary purity for unitary operations? Are we missing something in our $S(\Omega)$ measurement?

Quantum state of an injected TROPO above threshold: purity, Glauber function and photon number distribution

T. Golubeva¹, Yu. Golubev¹, C. Fabre², and N. Treps^{2, a}

Eur. Phys. J. D 46, 179–193 (2008) DOI: 10.1140/epjd/e2007-00290-6

YES WE ARE!

 $S(\Omega)$ includes information in the complex part

(ignored up to the moment).

And represents a pair of sidebands!

V is not 6 x 6, it is 12 x 12 (2 quadratures of 2

sidebands of 3 modes)

And we are measuring it right now!



More to follow: use the OPO as a colored entangling tool



Teleportation

Unconditional Quantum Teleportation A. Furusawa, *et al. Science* **282**, 706 (1998);







Light - Matter Interaction



Raiders of the Lost Entanglement

Felippe Barbosa Antônio Sales Jonatas César Alencar Faria Luciano Cruz Paulo Valente Mikael Lassen (MPI) Alessandro Villar Katiúscia Cassemiro Kaled Dechoum A. Zelaquett Khoury Claude Fabre (LKB) Marcelo Martinelli Paulo Nussenzveig


Laboratório de Manipulação Coerente de Átomos e Luz

Paulo Nussenzveig (1996) Marcelo Martinelli (2004) Alessandro Villar (Post-doc) Márcio Heráclyto (Post-doc) Antônio Sales (PhD – MSc 2008) Felippe Barbosa (PhD – MSc 2008) Hans Marin Torres (MSc) Flávio Moraes (MSc) Paula Meirelles (PhD)

















