Luz e Átomos como ferramentas para Informação Quântica
Oscilador Paramétrico Ótico

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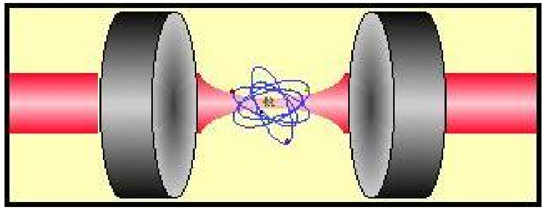

Lab. de Manipulação Coerente de Átomos e Luz

FAPESP
CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico
CAPES
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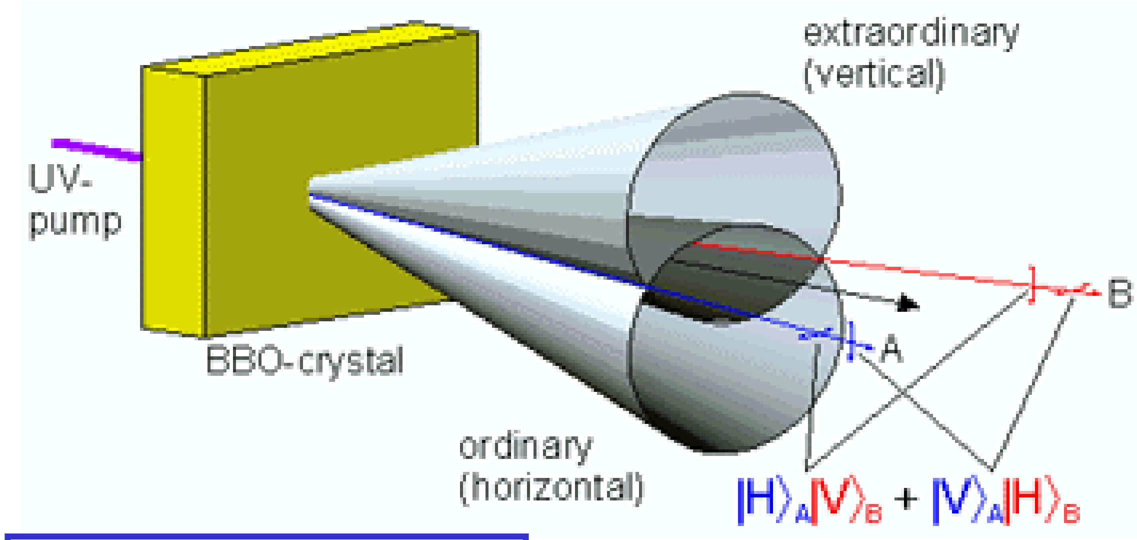
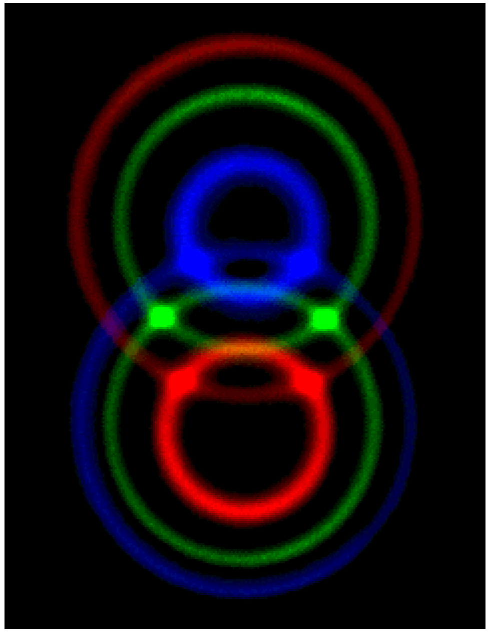
Parametric Down Conversion





$$\boxed{P = \chi^{(1)} E + \chi^{(2)} E^2}$$

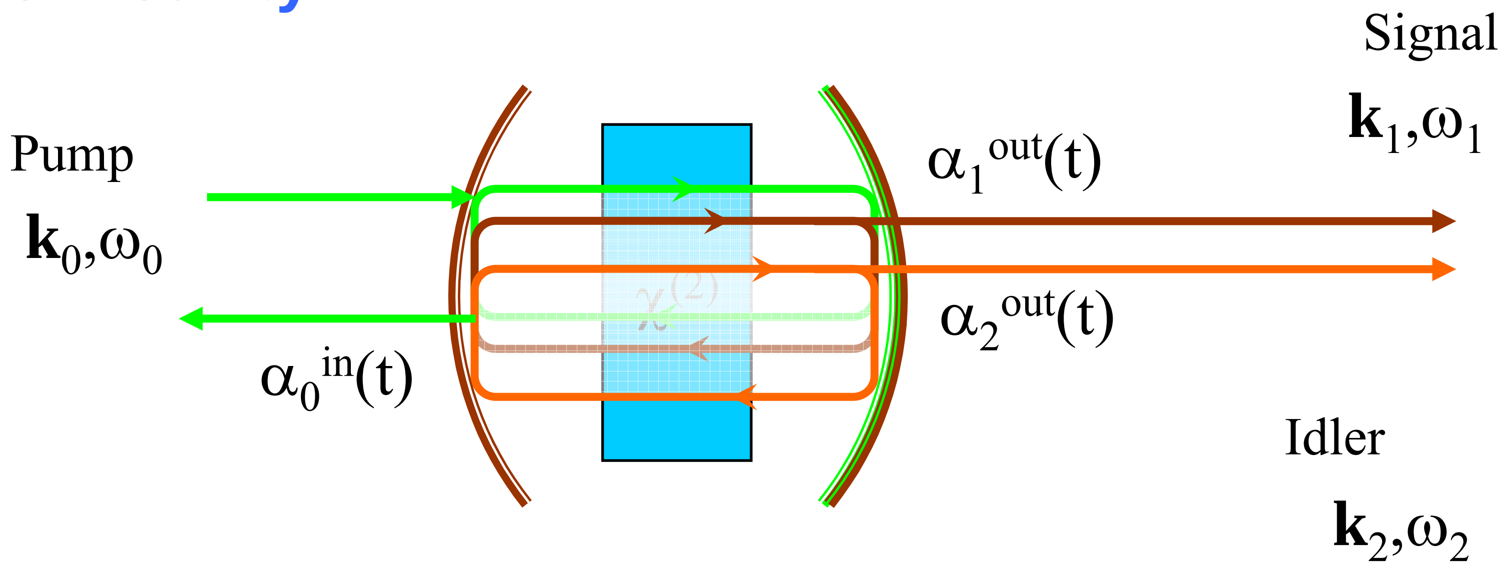
Energy and momentum conservation

$$\boxed{\omega_0 = \omega_1 + \omega_2} \qquad \boxed{\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2}$$

Polarization and transverse momentum correlations

Optical Parametric Oscillator

PDC + Cavity



PHYSICAL REVIEW LETTERS

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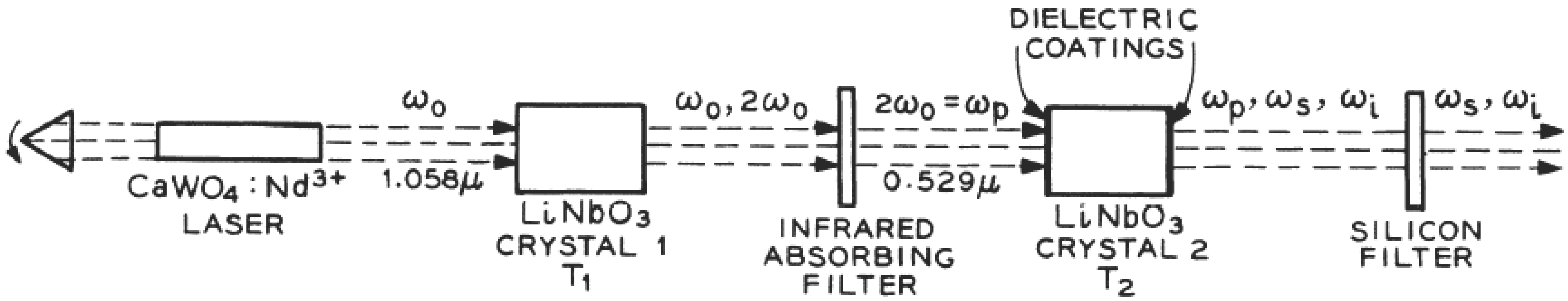
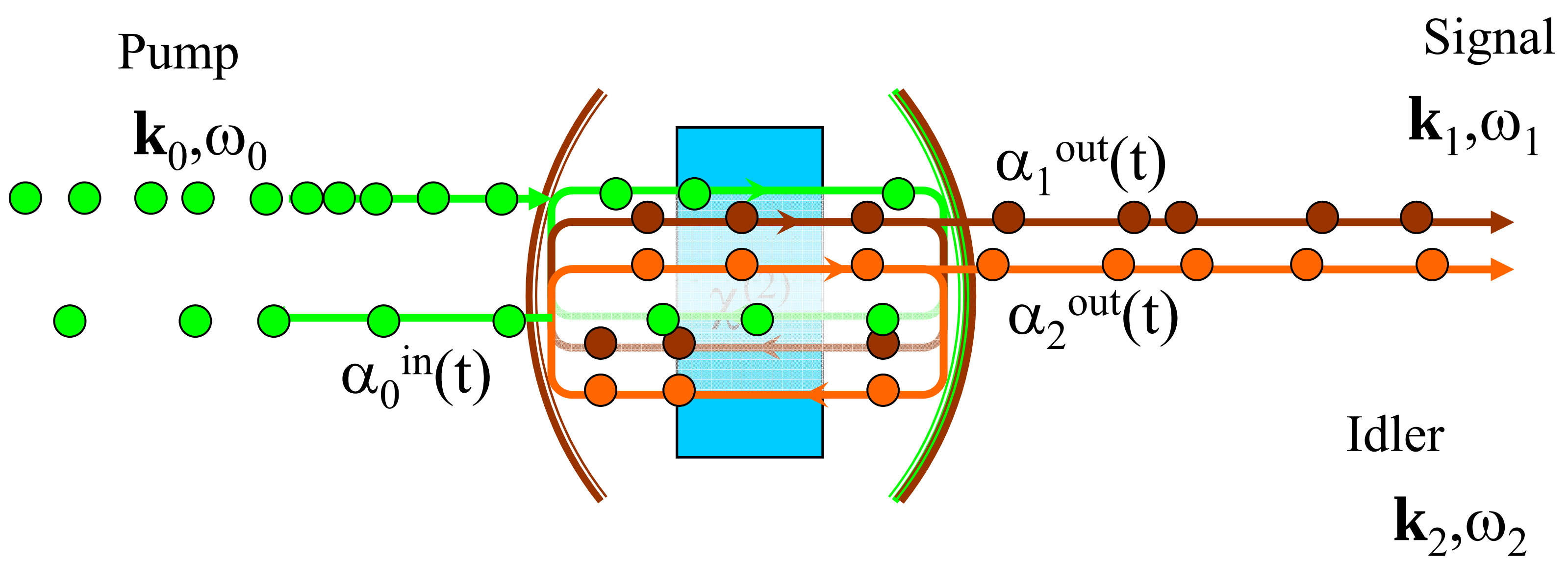


Optical Parametric Oscillator

PDC + Cavity



Twin photons + phase correlation

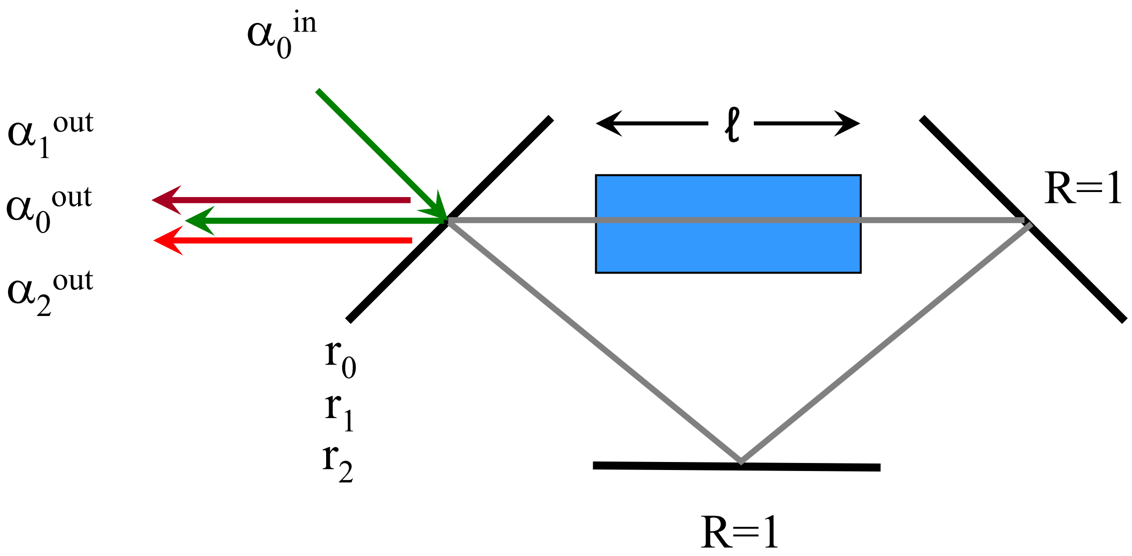
- Sub-threshold
  - squeezed vacuum (degenerate case) - OPA
  - entangled fields (non-degenerate case)

- Above threshold: intense entangled fields

Optical Parametric Oscillator (OPO) - Classical

Let us describe classical properties of the system before we analyze quantum properties. We'll consider a Triply Resonant OPO (TR-OPO) in a ring cavity (for simplicity).

Debuisschert *et al.* J. Opt. Soc. Am. B/Vol. 10, 1668 1993

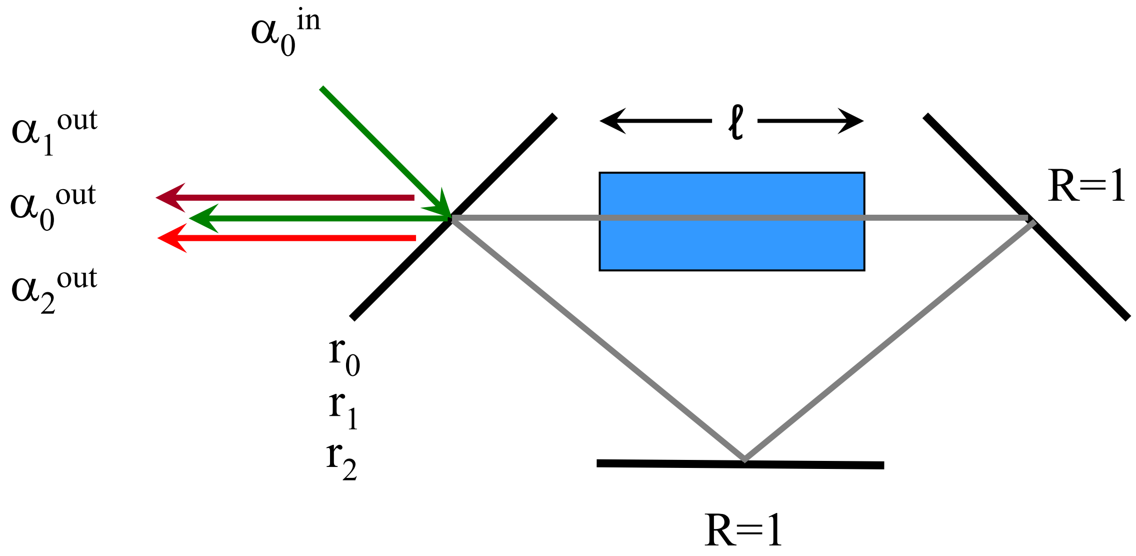


$$r_j = e^{(-\gamma_j)} \simeq 1 - \gamma_j;$$
$$t_j = (2\gamma_j)^{1/2};$$

If we consider that the single pass gain is small, we can approximate the equations for the amplification inside the crystal

$$\alpha_0(\ell) = \alpha_0(0) - 2\chi^* \alpha_1(0)\alpha_2(0)$$
$$\alpha_1(\ell) = \alpha_1(0) + 2\chi \alpha_0(0)\alpha_2^*(0)$$
$$\alpha_2(\ell) = \alpha_2(0) + 2\chi \alpha_0(0)\alpha_1^*(0)$$

Optical Parametric Oscillator (OPO) - Classical



Consistency of the field for a round trip gives us

$$\begin{aligned}\alpha_0 &= r_0 e^{i\varphi_0}(\alpha_0 - 2\chi\alpha_2\alpha_1) - \mu_0\alpha_0 + t_0\alpha_0^{in} \ ,\\ \alpha_1 &= r_1 e^{i\varphi_1}(\alpha_1 + 2\chi\alpha_0\alpha_2^*) - \mu_1\alpha_1 \ ,\\ \alpha_2 &= r_2 e^{i\varphi_2}(\alpha_2 + 2\chi\alpha_0\alpha_1^*) - \mu_2\alpha_2 \ ,\end{aligned}$$

$$\varphi_j = 2p_j\pi + \delta\varphi_j$$

Optical Parametric Oscillator (OPO) - Classical

If $\delta\varphi_j$ is small, we can write:

$$\begin{aligned}
\alpha_0(\gamma_0' - i\delta\varphi_0) &= -2\chi\alpha_1\alpha_2 + \sqrt{2\gamma_0}\alpha_0^{in} \\
\alpha_1(\gamma_1' - i\delta\varphi_1) &= 2\chi\alpha_0\alpha_2^* , \\
\alpha_2(\gamma_2' - i\delta\varphi_2) &= 2\chi\alpha_0\alpha_1^* ,
\end{aligned}$$

where the total loss for each mode is defined

$$\gamma_j' = \gamma_j + \mu_j$$

Normalizing the detuning, we have

$$\Delta_j = \delta\varphi_j / \gamma_j'$$

$$\boxed{\begin{gathered}
\alpha_0\gamma_0'(1 - i\Delta_0) = -2\chi\alpha_1\alpha_2 + \sqrt{2\gamma_0}\alpha_0^{in} , \\
\alpha_1\gamma_1'(1 - i\Delta_1) = 2\chi\alpha_0\alpha_2^* , \\
\alpha_2\gamma_2'(1 - i\Delta_2) = 2\chi\alpha_0\alpha_1^* .
\end{gathered}}$$

Optical Parametric Oscillator (OPO) - Classical

A first solution of these equations is $\alpha_1 = \alpha_2 = 0$, corresponding to operation below threshold. We are more interested in above-threshold operation. Multiplying the complex conjugate of the third equation by the second, we have: $\gamma_1'\gamma_2'(1 - i\Delta_1)(1 + i\Delta_2) = 4|\chi|^2|\alpha_0|^2$ $\rightarrow$ $\Delta_1 = \Delta_2 = \Delta$

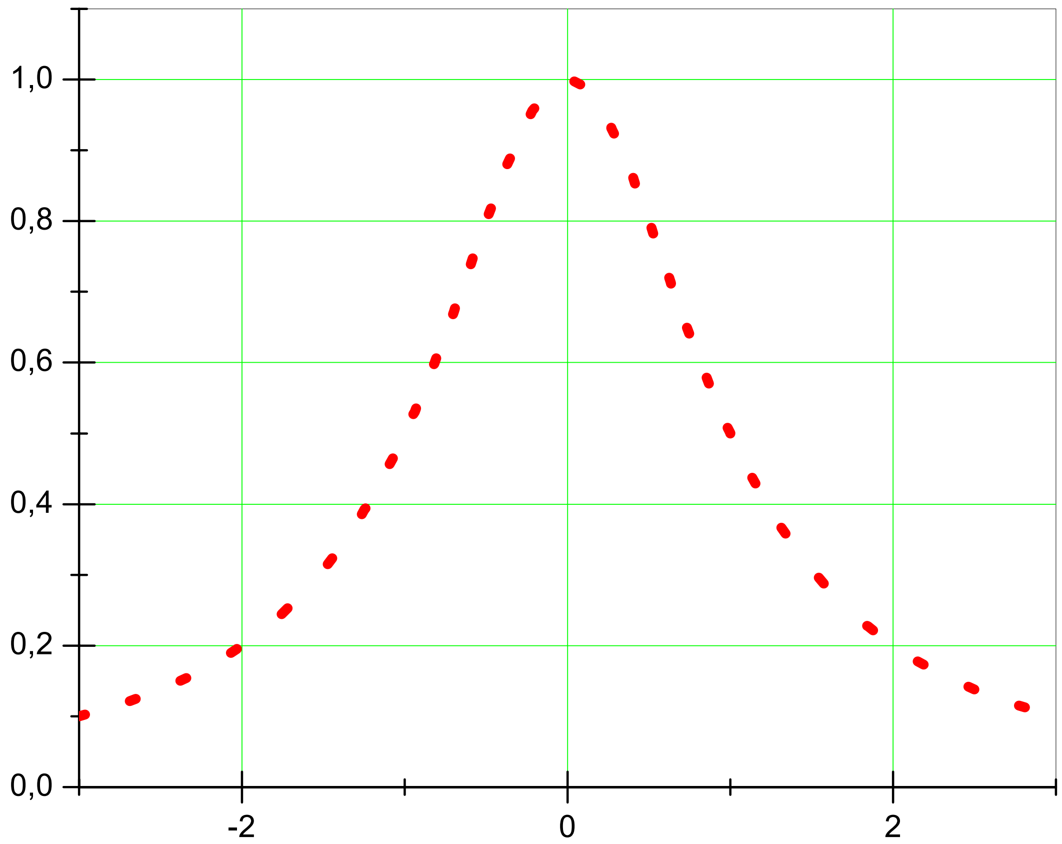
The intracavity pump power is easily obtained and we see it is "clipped": above-threshold it is always the same

$$|\alpha_0|^2 = \frac{\gamma_1'\gamma_2'(1 + \Delta^2)}{4|\chi|^2}$$

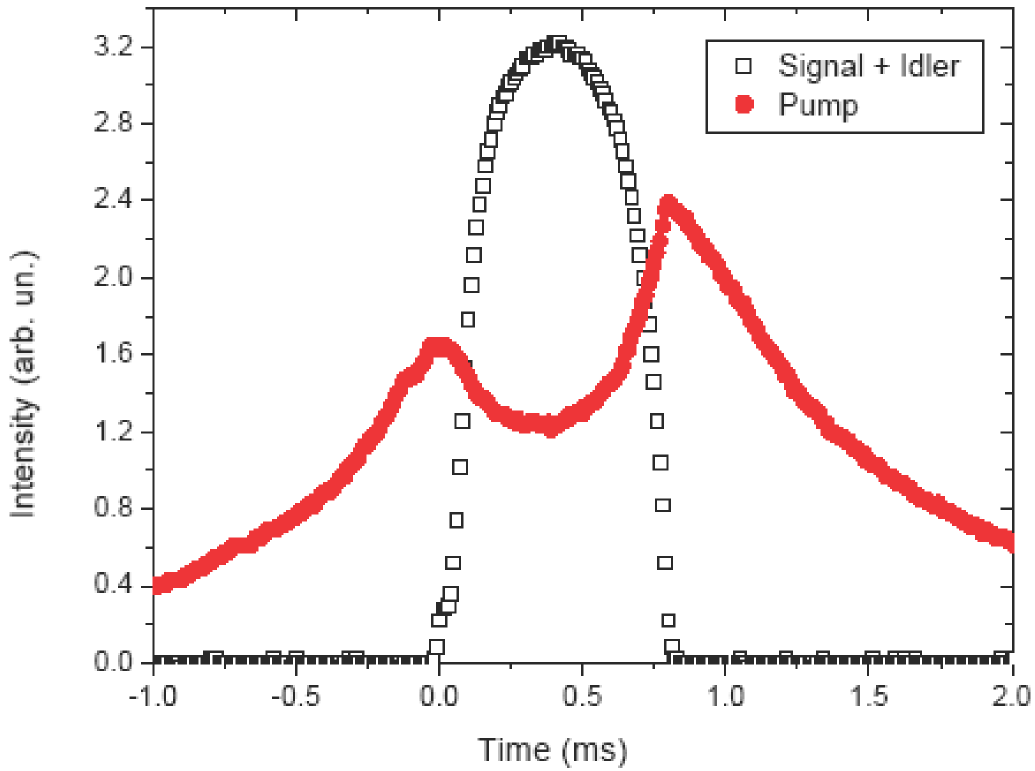
Besides, for $\Delta_1 = \Delta_2 = \Delta$, we also have $\gamma_1'|\alpha_1|^2 = \gamma_2'|\alpha_2|^2$

The classical equations are already signaling that the intensities of signal and idler beams should be strongly correlated and that the pump must be depleted.

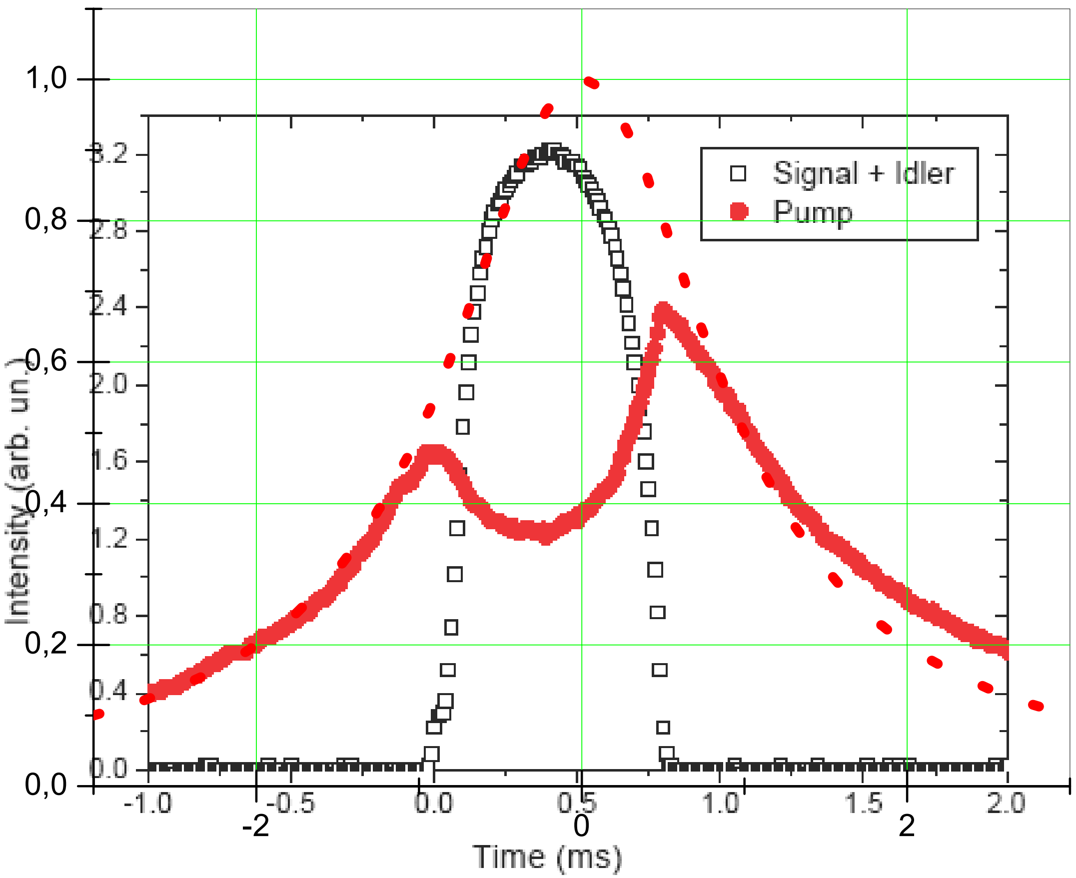
Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical

From the first equation we can derive the threshold power, given the intracavity pump field ($\alpha_1 = \alpha_2 = 0$)

$$|\alpha_0^{in}|^2_{th} = \frac{\gamma_0'^2 \gamma_1' \gamma_2' (1+\Delta^2)(1+\Delta_0^2)}{8|\chi|^2 \gamma_0}$$

An important parameter will be the ratio of incident power to threshold power on resonance:

$$\sigma = \frac{|\alpha_0^{in}|^2}{|\alpha_0^{in}|^2_{res}} = \frac{P_{in}}{P_{th}}$$

Substituting α_2 in the first equation, we have

$$\alpha_0 \gamma_0' (1 - i\Delta_0) = -\frac{4|\chi|^2 \alpha_0 |\alpha_1|^2}{\gamma_2'(1 - i\Delta)} + \sqrt{2\gamma_0}\alpha_0^{in}$$

Optical Parametric Oscillator (OPO) - Classical

$$\frac{2\gamma_0}{\gamma_0'^2}(1+\Delta)^2\frac{|\alpha_0^{in}|^2}{|\alpha_0|^2} = \left[1-\Delta\Delta_0+\frac{4|\chi|^2|\alpha_1|^2}{\gamma_0'\gamma_2'}\right]^2 + (\Delta+\Delta_0)^2$$

Since $|\alpha_0|^2 = \frac{\gamma_1'\gamma_2'(1+\Delta^2)}{4|\chi|^2}$ and $|\alpha_0^{in}|^2_{th} = \frac{\gamma_0'^2\gamma_1'\gamma_2'(1+\Delta^2)(1+\Delta_0^2)}{8|\chi|^2\gamma_0}$

$$|\alpha_0^{in}(\Delta=\Delta_0=0)|^2_{th} = \frac{\gamma_0'\gamma_1'\gamma_2'}{8|\chi|^2\gamma_0}$$

We get $\sigma = \left(1-\Delta\Delta_0+\frac{4|\chi|^2|\alpha_1|^2}{\gamma_2'\gamma_0'}\right)^2 + (\Delta+\Delta_0)^2$

Solving for α_j $|\alpha_j|^2 = \frac{\gamma_k'\gamma_0'}{4|\chi|^2}(\sqrt{\sigma}-1) \rightarrow$ $|\alpha_j^{out}|^2 = \frac{\gamma_j\gamma_k'\gamma_0'}{2|\chi|^2}(\sqrt{\sigma}-1)$

$$|\alpha_0|^2 = \frac{\gamma_1'\gamma_2'(1+\Delta^2)}{4|\chi|^2}$$

Optical Parametric Oscillator (OPO) - Classical

This gives the photon flux. Considering, for the sake of the argument, the frequency-degenerate case ($\omega_1=\omega_2=\omega_0/2$), we can obtain the total output power and the efficiency

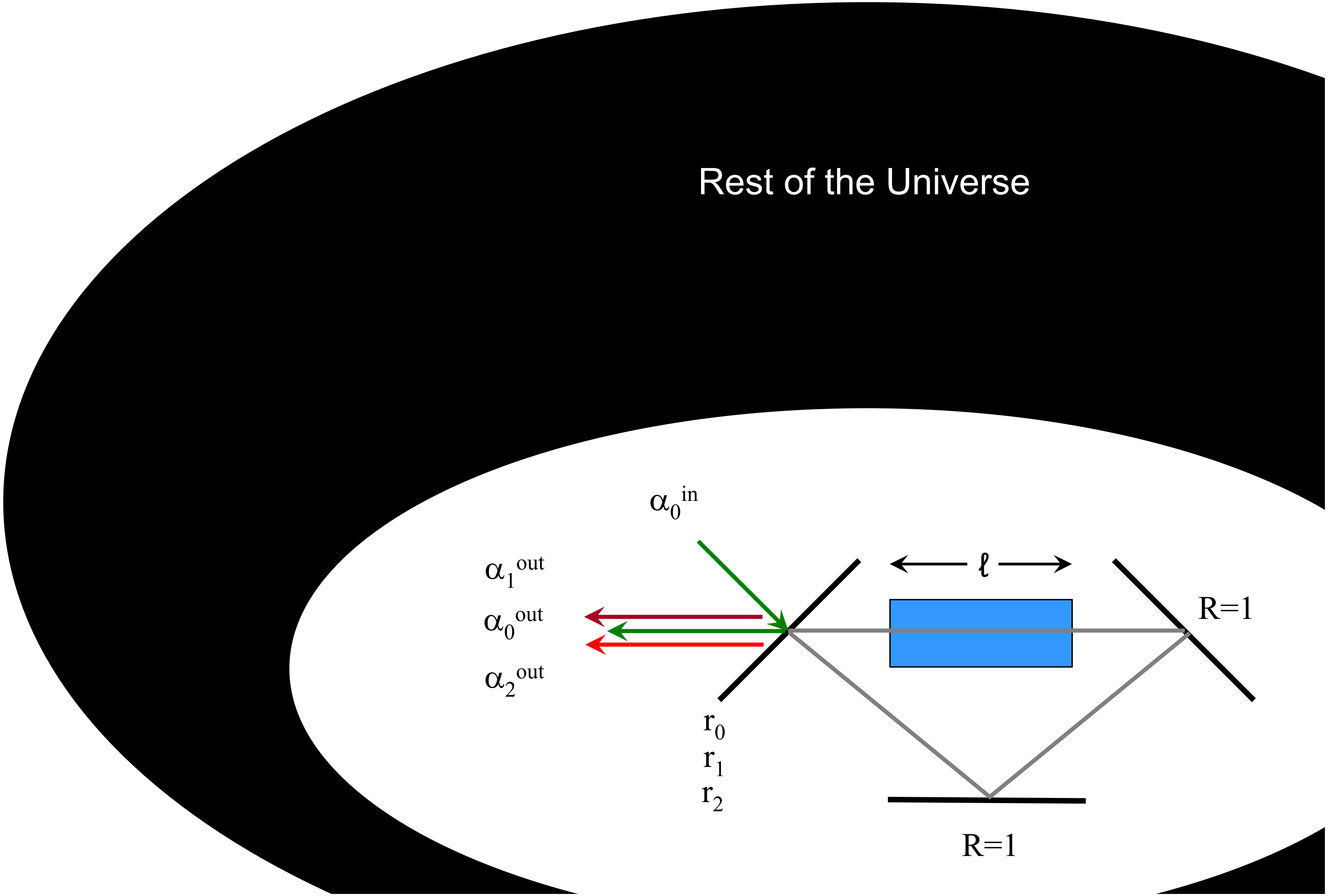
$$P_{out} = \hbar\omega_0 \left[\frac{\gamma\gamma'\gamma_0'}{2|\chi|^2}\left(\sqrt{\sigma}-1\right)\right] = 4\eta_{max}\left(\sqrt{P\cdot P_{th}} - P_{th}\right)$$

Where η_{max} is the maximum efficiency leading to

$$\eta_{max} = \frac{\gamma}{\gamma'}\frac{\gamma_0}{\gamma_0'} = \xi\xi_0 \qquad\qquad \xi_j = \gamma_j/\gamma_j'$$

We will see that the parameter ξ determines the maximum squeezing in the above-threshold OPO.

Optical Parametric Oscillator (OPO) - Quantum



Optical Parametric Oscillator (OPO) – Master Equation

Evolution of the density operator $$\frac{d}{dt}\hat{\rho}_{sr} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{sr}]$$

$$\boxed{\hat{H} = \hat{H}_s + \hat{H}_r + \hat{V}}$$

System + Reservoir + Interaction

$$\hat{H}_r = \sum_j \hbar\omega_j \hat{b}_j^\dagger \hat{b}_j \qquad \hat{V} = \hbar \sum_j (g_j \hat{a}_j^\dagger \hat{b}_j + g_j^* \hat{b}_j^\dagger \hat{a}_j)$$

Evolution of an operator acting only on the system:

$$\langle \hat{O}(t) \rangle = tr_s\{\hat{O}\, tr_r \hat{\rho}_{sr}(t)\} = tr_s\{\hat{O}\hat{\rho}_s(t)\}$$

Master Equation: Evolution of ρ_s

Quantum Properties of the OPO

Hamiltonian and the master equation:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}_f + \hat{H}_i + \hat{H}_{ext}, \hat{\rho}\right] + \left(\Lambda_0 + \Lambda_1 + \Lambda_2\right)\hat{\rho}$$

$$\boxed{\hat{H}_f = -\hbar\Delta_0\frac{\gamma_0'}{\tau}\hat{a}_0^\dagger\hat{a}_0 - \hbar\Delta_1\frac{\gamma_1'}{\tau}\hat{a}_1^\dagger\hat{a}_1 - \hbar\Delta_2\frac{\gamma_2'}{\tau}\hat{a}_2^\dagger\hat{a}_2}$$

$$\boxed{\hat{H}_{ext} = i\hbar\frac{\gamma_0}{\tau}\varepsilon\left(\hat{a}_0^\dagger - \hat{a}_0\right)}$$

$$\boxed{\hat{H}_i = i\hbar\frac{2\chi}{\tau}\left(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_0 - \hat{a}_1\hat{a}_2\hat{a}_0^\dagger\right)}$$

$$\boxed{\Lambda_j\hat{\rho} = \frac{\gamma_j'}{\tau}\left(2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\hat{\rho} - \hat{\rho}\hat{a}_j^\dagger\hat{a}_j\right)}$$

OK, simpler now?

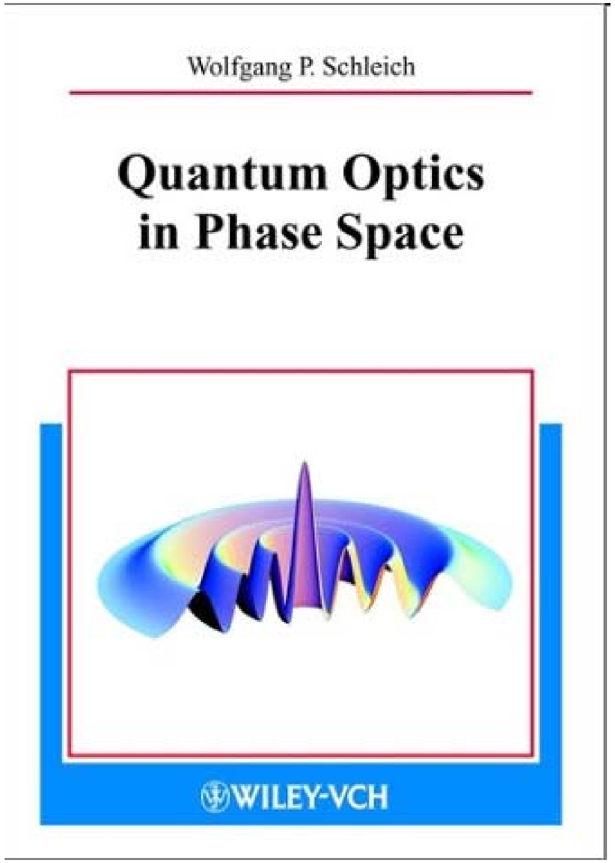
We can improve this if we change from the density matrix into an equivalent representation: it will replace (ordering sensitive) operators by c-numbers.

But the nonclassicallity makes P representation a tricky choice...

Quasi-Probability Representations

P- Glauber - Sudarshan $\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\,\mathrm{d}^2\alpha$

Wigner $W(\alpha) = \dfrac{1}{\pi^2}\displaystyle\int e^{\eta^*\alpha-\eta\alpha^*}\chi(\eta)d^2\eta$ $\qquad \chi(\eta) = Tr[\hat{\rho}e^{\eta\hat{a}^\dagger-\eta^*\hat{a}}]$



$$\bar{W}(\bar{x},\bar{p}) = \frac{1}{\pi\hbar}\int dy\langle\bar{x}+y|\rho|\bar{x}-y\rangle\exp(-2iy\bar{p}/\hbar)$$

$$\langle\{a^r(a^\dagger)^s\}_{\mathrm{sym}}\rangle = \int d^2\alpha\,\alpha^r(\alpha^*)^s W(\alpha,\alpha^*).$$

C.W. Gardiner P. Zoller

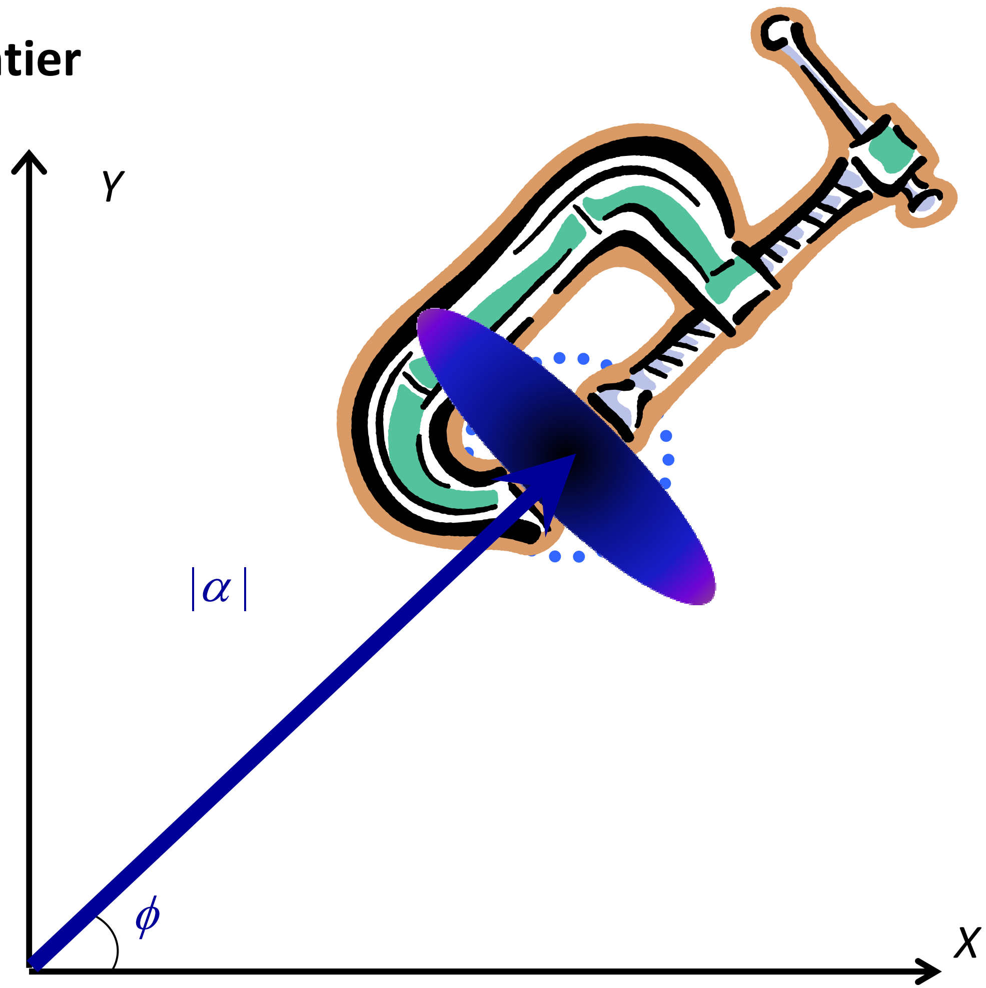
Quantum Noise

A Handbook
of Markovian and Non-Markovian
Quantum Stochastic Methods
with Applications to Quantum Optics

Wigner Representation

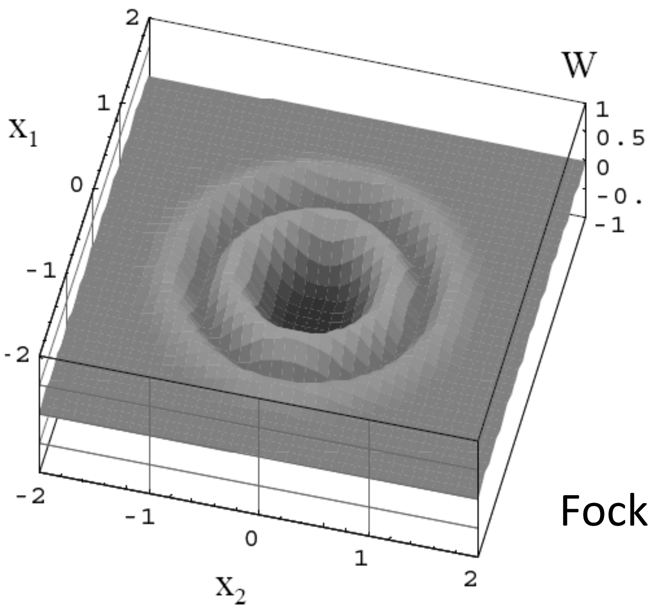
Evident quantum/ classical frontier

Squeezed states



States with W<0

n=3



Fock states $|n\rangle$

Quantum Properties of the OPO

The operators $(\hat{a}^\dagger, \hat{a})$ are replaced by amplitudes (α^*, α)

and the density operator is replaced by $W(\boldsymbol{\alpha})$

$$\boldsymbol{\alpha} = (\alpha_0, \alpha_0^*, \alpha_1, \alpha_1^*, \alpha_2, \alpha_2^*)$$

Using the rules

$$\hat{a}\hat{\rho}_s \Longleftrightarrow \left(\alpha + \frac{1}{2}\frac{\partial}{\partial\alpha^*}\right) W \qquad \hat{a}^\dagger\hat{\rho}_s \Longleftrightarrow \left(\alpha^* - \frac{1}{2}\frac{\partial}{\partial\alpha}\right) W$$

$$\hat{\rho}_s\hat{a} \Longleftrightarrow \left(\alpha - \frac{1}{2}\frac{\partial}{\partial\alpha^*}\right) W \qquad \hat{\rho}_s\hat{a}^\dagger \Longleftrightarrow \left(\alpha^* + \frac{1}{2}\frac{\partial}{\partial\alpha}\right) W.$$

Quantum Properties of the OPO

We obtain

$$\frac{\partial}{\partial t}W(\boldsymbol{\alpha}) = \sum_{j=1}^{3}\frac{\gamma_j'}{\tau}\left[i\Delta_j\left(\frac{\partial}{\partial\alpha_j^*}\alpha_j^* - \frac{\partial}{\partial\alpha_j}\alpha_j\right) + \left(\frac{\partial}{\partial\alpha_j^*}\alpha_j^* + \frac{\partial}{\partial\alpha_j}\alpha_j\right)\right]W(\boldsymbol{\alpha})$$
$$+\frac{2\chi}{\tau}\left(\alpha_1\alpha_2\frac{\partial}{\partial\alpha_0^*} + \alpha_1^*\alpha_2^*\frac{\partial}{\partial\alpha_0}\right)W(\boldsymbol{\alpha}) - \frac{2\chi}{\tau}\left(\alpha_0\alpha_1^*\frac{\partial}{\partial\alpha_2} + \alpha_0^*\alpha_1\frac{\partial}{\partial\alpha_2^*}\right)W(\boldsymbol{\alpha})$$
$$-\frac{2\chi}{\tau}\left(\alpha_0\alpha_2^*\frac{\partial}{\partial\alpha_1} + \alpha_0^*\alpha_2\frac{\partial}{\partial\alpha_1^*}\right)W(\boldsymbol{\alpha}) - \frac{\gamma_0}{\tau}\varepsilon\left(\frac{\partial}{\partial\alpha_0^*} + \frac{\partial}{\partial\alpha_0}\right)W(\boldsymbol{\alpha})$$
$$+\sum_{j=1}^{3}\frac{\gamma_j'}{\tau}\frac{\partial^2}{\partial\alpha_j\partial\alpha_j^*}W(\boldsymbol{\alpha}) - \frac{\chi}{2\tau}\frac{\partial^3}{\partial\alpha_0^*\partial\alpha_1^*\partial\alpha_2^*}W(\boldsymbol{\alpha}).$$

Fokker-Planck equation

$$\frac{\partial}{\partial t}W(\boldsymbol{\alpha}) = -\sum_j\frac{\partial}{\partial\alpha_j}A_jW(\boldsymbol{\alpha}) + \frac{1}{2}\sum_{j,k}\frac{\partial}{\partial\alpha_j}\frac{\partial}{\partial\alpha_k}\left[\mathbf{B}\mathbf{B}^T\right]_{jk}W(\boldsymbol{\alpha})$$

Quantum Properties of the OPO

Which is equivalent to a set of Langevin equations
(Do you remember the Brownian Motion ?)

$$\boxed{\frac{d}{dt}\alpha_j = A_j + [\mathbf{B}\boldsymbol{\sigma}(t)]_j}$$

$$\boxed{\langle \sigma_i(t)\sigma_j(t')\rangle = \delta_{ij}\delta(t-t')}$$

The mean values in steady state are the same as in the classical treatment.

$$\langle \vec{A}(\vec{X},t)\rangle \simeq \vec{A}(\langle \vec{X}\rangle, t) = 0$$

Since we will (typically) deal with intense fields, we proceed by linearizing the fluctuations, neglecting products of fluctuating terms:

$$\delta\vec{X} = \vec{X} - \langle \vec{X}\rangle \qquad \alpha_j(t) = \bar{\alpha}_j + \delta\alpha_j(t)$$

$$\alpha_j = p_j + iq_j$$

Quantum Properties of the OPO

$$\frac{d}{dt}\delta\alpha_0 = -\frac{\gamma_0'}{\tau}(1 - i\Delta_0)\delta\alpha_0 - \frac{2\chi}{\tau}\alpha_+\delta\alpha_+ + \frac{\sqrt{2\gamma_0'}}{\tau}\sigma_1(t)$$

$$\frac{d}{dt}\delta\alpha_0^* = -\frac{\gamma_0'}{\tau}(1 + i\Delta_0)\delta\alpha_0^* - \frac{2\chi}{\tau}\alpha_+^*\delta\alpha_+^* + \frac{\sqrt{2\gamma_0'}}{\tau}\sigma_2(t)$$

$$\frac{d}{dt}\delta\alpha_+ = \frac{2\chi}{\tau}\alpha_0\delta\alpha_+^* - \frac{2\chi}{\tau}\alpha_+^*\delta\alpha_0 - \frac{\gamma'}{\tau}(1 - i\Delta)\delta\alpha_+ + \frac{\sqrt{2\gamma'}}{\tau}\sigma_3(t)$$

$$\frac{d}{dt}\delta\alpha_+^* = \frac{2\chi}{\tau}\alpha_0^*\delta\alpha_+ - \frac{2\chi}{\tau}\alpha_+\delta\alpha_0^* - \frac{\gamma'}{\tau}(1 + i\Delta)\delta\alpha_+^* + \frac{\sqrt{2\gamma'}}{\tau}\sigma_4(t)$$

$$\frac{d}{dt}\delta\alpha_- = -\frac{2\chi}{\tau}\alpha_0\delta\alpha_-^* - \frac{\gamma'}{\tau}(1 - i\Delta)\delta\alpha_- + \frac{\sqrt{2\gamma'}}{\tau}\sigma_5(t)$$

$$\frac{d}{dt}\delta\alpha_-^* = -\frac{2\chi}{\tau}\alpha_0^*\delta\alpha_- - \frac{\gamma'}{\tau}(1 + i\Delta)\delta\alpha_-^* + \frac{\sqrt{2\gamma'}}{\tau}\sigma_6(t).$$

$$\alpha_+ = \frac{\alpha_1 + \alpha_2}{\sqrt{2}} \qquad \alpha_- = \frac{\alpha_1 - \alpha_2}{\sqrt{2}}$$

Quantum Properties of the OPO

Defining

$\alpha_j = p_j \, \mathrm{e}^{i\varphi_j}$ with $p_1 = p_2 \equiv p$ $\delta\alpha_j(t) = \frac{e^{i\varphi_j}}{2} \left[\, \delta p_j(t) + i \, \delta q_j(t) \,\right]$

We get

Quantum Properties of the OPO

$$
\begin{aligned}
\tau \frac{d}{dt}\,\delta p_- &= -2\gamma'\,\delta p_- + \sqrt{2\gamma}\,\delta u_{p_-} + \sqrt{2\mu}\,\delta v_{p_-}\ , \\
\tau \frac{d}{dt}\,\delta q_- &= 2\Delta\gamma'\,\delta p_- + \sqrt{2\gamma}\,\delta u_{q_-} + \sqrt{2\mu}\,\delta v_{q_-}\ , \\
\tau \frac{d}{dt}\,\delta p_+ &= -2\Delta\gamma'\,\delta q_+ + \sqrt{2}\gamma'\beta\,\delta p_0 + \sqrt{2}\Delta\gamma'\beta\,\delta q_0 + \sqrt{2\gamma}\,\delta u_{p_+} + \sqrt{2\mu}\,\delta v_{p_+} \\
\tau \frac{d}{dt}\,\delta q_+ &= -2\gamma'\,\delta q_+ - \sqrt{2}\Delta\gamma'\beta\,\delta p_0 + \sqrt{2}\gamma'\beta\,\delta q_0 + \sqrt{2\gamma}\,\delta u_{q_+} + \sqrt{2\mu}\,\delta v_{q_+} \\
\tau \frac{d}{dt}\,\delta p_0 &= -\sqrt{2}\gamma'\beta\,\delta p_+ + \sqrt{2}\Delta\gamma'\beta\,\delta q_+ - \gamma_0'\,\delta p_0 - \Delta_0\gamma_0'\,\delta q_0 + \\
&\quad + \sqrt{2\gamma_0}\,\cos\varphi_0\,\delta p_0^{\mathrm{in}} + \sqrt{2\gamma_0}\,\mathrm{sen}\,\varphi_0\,\delta q_0^{\mathrm{in}} + \sqrt{2\mu_0}\,\delta v_{p_0}\ , \\
\tau \frac{d}{dt}\,\delta q_0 &= -\sqrt{2}\Delta\gamma'\beta\,\delta p_+ - \sqrt{2}\gamma'\beta\,\delta q_+ + \Delta_0\gamma_0'\,\delta p_0 - \gamma_0'\,\delta q_0 - \\
&\quad - \sqrt{2\gamma_0}\,\mathrm{sen}\,\varphi_0\,\delta p_0^{\mathrm{in}} + \sqrt{2\gamma_0}\,\cos\varphi_0\,\delta q_0^{\mathrm{in}} + \sqrt{2\mu_0}\,\delta v_{q_0}\ .
\end{aligned}
$$

$$\delta p_\pm = (\delta p_1 \pm \delta p_2)/\sqrt{2}\ , \qquad \delta q_\pm = (\delta q_1 \pm \delta q_2)/\sqrt{2} \qquad \beta = p/p_0$$

Quantum Properties of the OPO

$$\begin{aligned}
\tau \frac{d}{dt}\,\delta p_- &= -2\gamma'\,\delta p_- + \sqrt{2\mu}\,\delta v_{p_-} \\
\tau \frac{d}{dt}\,\delta q_- &= \sqrt{2\mu}\,\delta v_{q_-}
\end{aligned}$$

The subspace related to the subtraction of the fields decouples from the sum and the pump fluctuations. However, q_- does not have any decay term, thus the solutions are not strictly stable. As a matter of fact, there is *phase diffusion* and the subtraction of the phases is unbounded. Nevertheless, this is a *slow* process and we will be interested in measuring phases with respect to the phase of the mean field (in other words, we will follow “adiabatically” the diffusion).

Instead of solving these equations in the time domain, we look in the frequency domain.

Usual treatment of the OPO: Master Equation

$$\frac{d\hat{\rho}}{dt} = \frac{i}{\hbar}\left[\hat{H}_0 + \hat{H}_1, \hat{\rho}\right] + \frac{\gamma}{2}\left[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right]$$

Quasi-probability representation

$$\frac{\partial P(\vec{X},t)}{\partial t} = \left[-\sum_i \frac{\partial}{\partial x_i} A_i(\vec{X},t) + \frac{1}{2}\sum_{ij}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} D_{ij}(\vec{X},t)\right] P(\vec{X},t)$$

$$\mathbb{D}(\vec{X},t) = \mathbb{B}(\vec{X},t)\mathbb{B}^T(\vec{X},t)$$

Langevin Equation

$$\frac{d\vec{X}}{dt} = \mathbb{A}(\vec{X},t) + \mathbb{B}(\vec{X},t)\vec{X}^{in}(t)$$

Usual treatment of the OPO: Langevin Equation

Linearization

$$\frac{d\delta\vec{X}(t)}{dt} = \mathbb{A}\delta\vec{X}(t) + \mathbb{B}\vec{X}^{in}(t)$$

Input – Output Formalism

$$\delta\vec{X}^{out}(t) = \mathbb{B}\delta\vec{X}(t) - \mathbb{I}\vec{X}^{in}(t)$$

Frequency Domain

$$\vec{X}(\Omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\delta\vec{X}(t)exp(-i\Omega t)dt$$

$$\vec{X}(\Omega) = \left[-(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}\right]\vec{X}^{in}(\Omega)$$

$$\vec{X}^{out}(\Omega) = -\left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B} + \mathbb{I}\right]\vec{X}^{in}(\Omega)$$

$$\vec{X}(\Omega) = -\mathbb{M}_I(\Omega)\vec{X}^{in}(\Omega)$$

$$\vec{X}^{out}(\Omega) = -\mathbb{M}_O(\Omega)\vec{X}^{in}(\Omega).$$

$$\mathbb{M}_I(\Omega) = (\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}$$

$$\mathbb{M}_O(\Omega) = \mathbb{I} + \left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}\right]$$

Covariance Matrix X Spectral Matrix

$$\mathbb{V}(t,t+\tau) = \mathbb{V}(\tau) = \langle \delta\vec{X}^{out}(t)[\delta\vec{X}^{out}(t+\tau)]^T\rangle \qquad \mathbb{S}(\Omega) = \langle \vec{X}^{out}(\Omega)[\vec{X}^{out}(-\Omega)]^T\rangle$$

$$\mathbb{V}(\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} S(\Omega)\exp(i\Omega\tau)d\Omega$$

Complete description of the state: Wigner function (for a Gaussian State)

$$W(\vec{X}) = \frac{1}{4\pi^2\sqrt{\det \mathbb{V}_i}}\exp\left(-\frac{1}{2}\vec{X}^T\mathbb{V}_i^{-1}\vec{X}\right)$$

Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} & C_{p1p0} & C_{p1q0} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} & C_{q1p0} & C_{q1q0} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} & C_{p2p0} & C_{p2q0} \\ C_{p1q2} & C_{q1q0} & C_{p2q2} & S_{q2} & C_{q2p0} & C_{q2q0} \\ C_{p1p0} & C_{q1p0} & C_{p2p0} & C_{q2p0} & S_{p0} & C_{p0q0} \\ C_{p1q0} & C_{q1q0} & C_{p2q0} & C_{q2q0} & C_{p0q0} & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad S_{xj} = C_{xjxj}$$

36 independent terms !

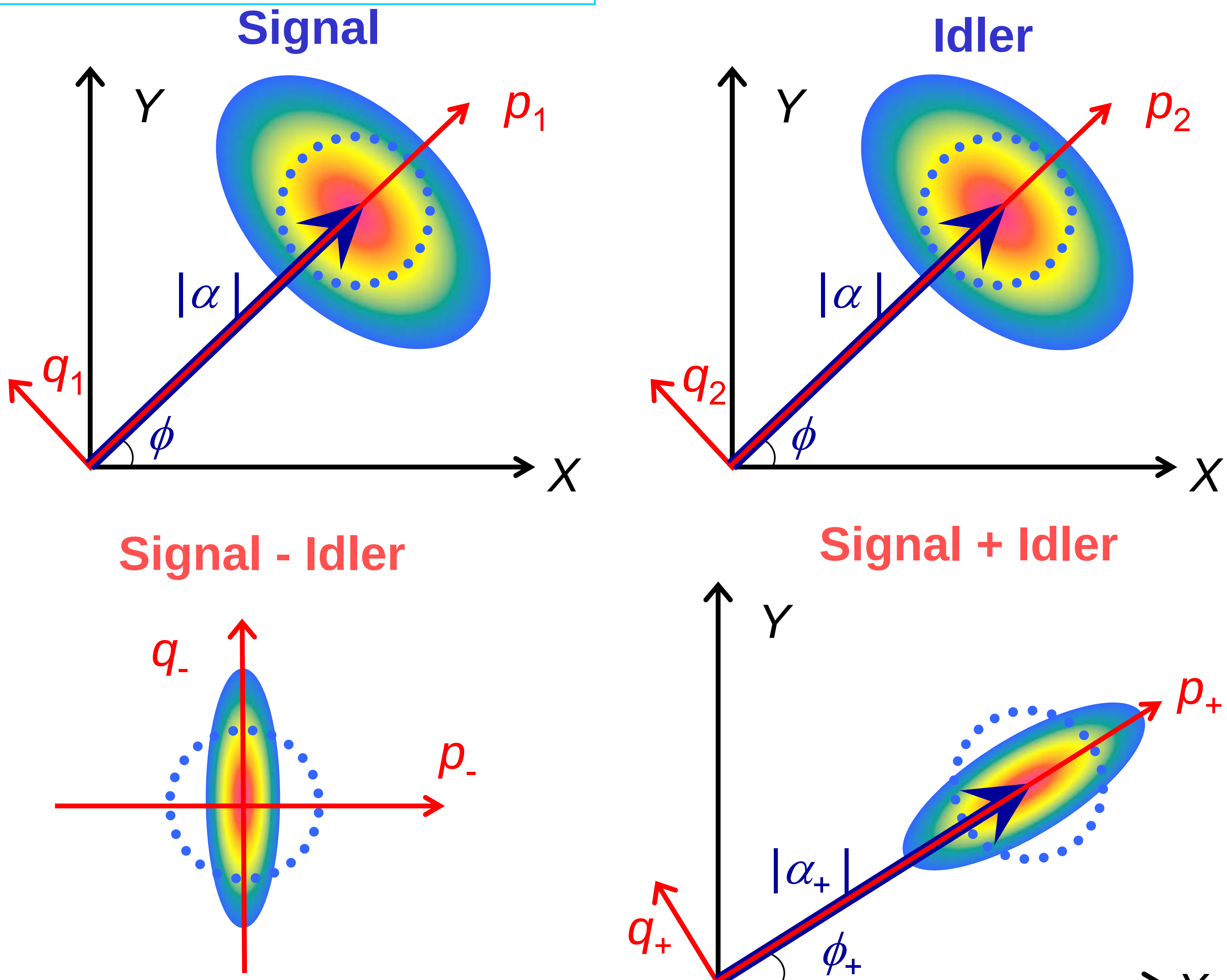
Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2}\langle\{x_i, x_j\}\rangle - \langle x_i\rangle\langle x_j\rangle \qquad\qquad S_{xj} = C_{xjxj}$$

18 independent terms !

Noise correlations



Energy Conservation

$$\omega_1 + \omega_2 = \omega_0$$

$$\delta I_1 - \delta I_2 = 0$$

Intensity Correlation

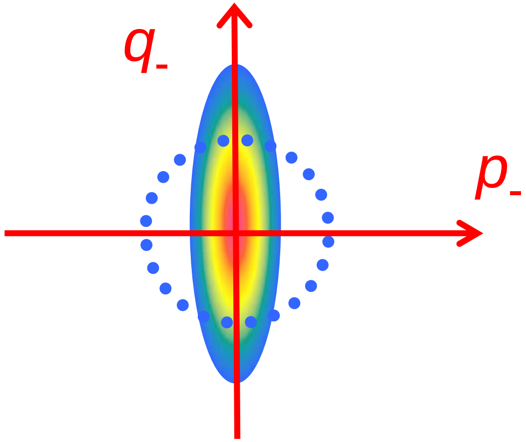
A. Heidmann *et al.*, PRL. **59**, 2555 (1987)

$$\delta\varphi_1 + \delta\varphi_2 = \delta\varphi_0$$

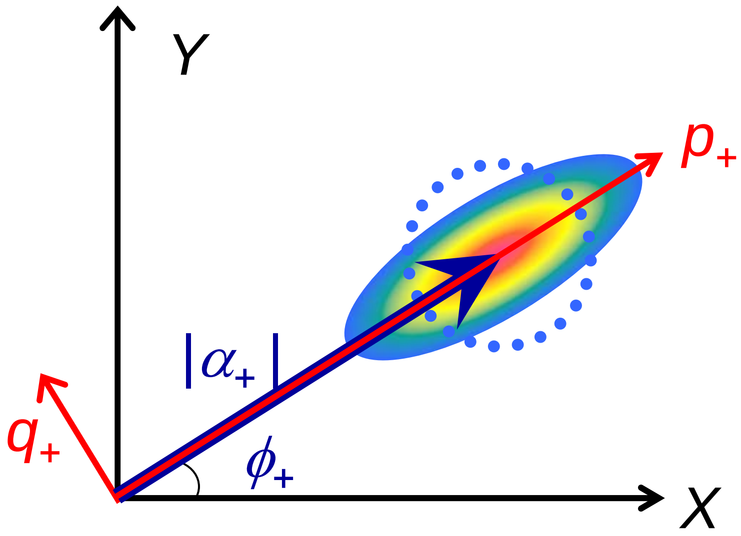
Phase Anti-correlation

A. S. Villar *et al.*, PRL **95**, 243603 (2005)

Signal - Idler



Signal + Idler



EPR's example

$|\psi\rangle \cong \delta(x_1 - x_2 - L)\delta(p_1 + p_2)$ (localized in $x_1 - x_2$ e $p_1 + p_2$)

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

A measurement of x_1 yields x_2, as well as a measurement of p_1 gives p_2. But x_2 and p_2 *don't commute*! $\leftrightarrow [x, p] = i\hbar$

Bohr's reply

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Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

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(Received July 13, 1935)

$$[q_1p_1]=[q_2p_2]=ih/2\pi,$$
$$[q_1q_2]=[p_1p_2]=[q_1p_2]=[q_2p_1]=0,$$

$$q_1=Q_1\cos\theta-Q_2\sin\theta \qquad p_1=P_1\cos\theta-P_2\sin\theta$$
$$q_2=Q_1\sin\theta+Q_2\cos\theta \qquad p_2=P_1\sin\theta+P_2\cos\theta.$$

$$[Q_1P_1]=ih/2\pi, \qquad [Q_1P_2]=0,$$

$$Q_1=q_1\cos\theta+q_2\sin\theta,$$

$$P_2=-p_1\sin\theta+p_2\cos\theta,$$