Multipartite entanglement and sudden death in Quantum Optics: continuous variables domain Part III - The OPO - Entangled

Marcelo Martinelli

The New York Times

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually. PRINCETON, N. J., May 3.—Professor Albert Einstein will attack science's important theory of quantum mechanics, a theory of which he was a sort of grandfather. He concludes that while it is "correct" it is not "complete."

With two colleagues at the Institute for Advanced Study here, the noted scientist is about to report to the American Physical Society what is wrong with the theory of quantum mechanics, it has been learned exclusively by Science Service.

The quantum theory, with which science predicts with some success inter-atomic happenings, does not meet the requirements for a satisfactory physical theory, Professor Einstein will report in a joint paper with Dr. Boris Podolsky and Dr. N. Rosen.

EPR and Entanglement

Anybody who is not shocked by quantum theory has not understood it.





Niels Bohr



MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality.

Einstein, Podolsky & Rosen's paper

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

either (1) the quantummechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.





 $|\psi\rangle \cong \delta(x_1 - x_2 - L)\delta(p_1 + p_2)$ (localized in $x_1 - x_2 e p_1 + p_2$)

We see therefore that, as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions. On the other hand, since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system.

A measurement of x_1 yields x_2 , as well as a measurement of p_1 gives p_2 . But x_2 and p_2 don't commute! $\leftrightarrow [x, p] = i\hbar$

EPR's conclusion

either (1) the quantummechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality.

If (1) is false, then (2) is also false! Hence, (1) should be true: quantum theory, although it allows for correct predictions, must be *incomplete*. Measurements should just reveal pre-existing states, which are not described by this incomplete theory.

Bohr's reply

OCTOBER 15, 1935

PHYSICAL REVIEW

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, Institute for Theoretical Physics, University, Copenhagen (Received July 13, 1935)

$$\begin{bmatrix} q_1 p_1 \end{bmatrix} = \begin{bmatrix} q_2 p_2 \end{bmatrix} = ih/2\pi, \\ \begin{bmatrix} q_1 q_2 \end{bmatrix} = \begin{bmatrix} p_1 p_2 \end{bmatrix} = \begin{bmatrix} q_1 p_2 \end{bmatrix} = \begin{bmatrix} q_2 p_1 \end{bmatrix} = 0,$$

$$\begin{array}{ll} q_1 = Q_1 \cos \theta - Q_2 \sin \theta & p_1 = P_1 \cos \theta - P_2 \sin \theta \\ q_2 = Q_1 \sin \theta + Q_2 \cos \theta & p_2 = P_1 \sin \theta + P_2 \cos \theta. \end{array}$$

 $[Q_1P_1] = ih/2\pi, \qquad [Q_1P_2] = 0,$

$$Q_1 = q_1 \cos \theta + q_2 \sin \theta,$$
$$P_2 = -p_1 \sin \theta + p_2 \cos \theta,$$

Few words about entanglement characterization

• "EPR" criterion [M. D. Reid, PRA **40**, 913 (1989), M. D. Reid and P. D. Drummond, PRL **60**, 2731 (1988) & PRA **40**, 4493 (1989)]

$$\Delta^{2} \hat{p}_{\text{inf}} = \Delta^{2} \hat{p}_{1} \left(1 - \frac{\langle \delta \hat{p}_{1} \delta \hat{p}_{2} \rangle^{2}}{\Delta^{2} \hat{p}_{1} \Delta^{2} \hat{p}_{2}} \right)$$
$$\delta \hat{p}_{i} = \hat{p}_{i} - \langle \hat{p}_{i} \rangle$$
$$\Delta^{2} \hat{p}_{\text{inf}} \Delta^{2} \hat{q}_{\text{inf}} \ge 1$$

Entanglement Test - DGCZ

•DGCZ separability criterion:

$$\rho = \sum_{i} p_i \ \rho_i = \sum_{i} p_i \ \rho_i^1 \otimes \rho_i^2 \qquad [\hat{q}_i, \hat{p}_j] = 2i\delta_{ij}$$

$$\hat{u} = a\hat{q}_1 + \frac{1}{a}\hat{q}_2,$$
$$\hat{v} = a\hat{p}_1 - \frac{1}{a}\hat{p}_2,$$

Separability
$$\Rightarrow \langle (\Delta \hat{u})^2 \rangle_{\rho} + \langle (\Delta \hat{v})^2 \rangle_{\rho} \ge 2 \ (a^2 + \frac{1}{a^2})$$

Lu-Ming Duan, G. Giedke, J.I. Cirac, P. Zoller, Inseparability criterion for continuous variable systems, Phys. Rev. Lett. 84, 2722 (2000).

•After some (simple) algebra:

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \ge 0;$$

Entanglement Test - DGCZ

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

 $C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$

$$(\Delta^2 p_1 + \Delta^2 q_1 - 2)(\Delta^2 p_2 + \Delta^2 q_2 - 2) - (|c_p| + |c_q|)^2 \ge 0$$

Entanglement Test - DGCZ



 $p_1 + p_2, q_1 + q_2$

 $p_1 - p_2$, $q_1 - q_2$





 $p_1 - p_2$, $q_1 - q_2$

Entanglement Test - Peres & Horodecki

• Positivity under Partial Transposition (discrete variables) Separability Criterion for Density Matrices

Asher Peres*

PRL 77, 1413 (1996)

$$\rho = \sum_{A} w_{A} \rho_{A}' \otimes \rho_{A}'' \qquad \Longrightarrow \quad \sigma = \sum_{A} w_{A} (\rho_{A}')^{T} \otimes \rho_{A}''$$

non-negative eigenvalues -> Separability



• Continuous variables: PT: $W(q_1, p_1, q_2, p_2) \rightarrow W(q_1, p_1, q_2, -p_2)$ $V + \frac{i}{2} \Omega \ge 0$ $\Omega = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}$ $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ Peres-Horodecki Separability Criterion for Continuous Variable Systems PRL 84, 2726 (2000) $\tilde{V} + \frac{i}{2} \Omega \ge 0$ $\tilde{V} + \frac{i}{2} \Omega \ge 0$ $\tilde{V} = \Lambda V\Lambda$ $\Lambda = \text{diag}(1, 1, 1, -1)$

Simplectic Eigenvalues >1

Diagonalize:
$$-(\Omega \tilde{V})^2$$

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} \\ C_{p1q2} & C_{q1q2} & C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle$$
$$S_{xj} = C_{xjxj}$$

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & -C_{p1q2} \\ C_{p1q1} & S_{q1} & C_{q1p2} & -C_{q1q2} \\ C_{p1p2} & C_{q1p2} & S_{p2} & -C_{p2q2} \\ -C_{p1q2} & -C_{q1q2} & -C_{p2q2} & S_{q2} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle$$
$$S_{xj} = C_{xjxj}$$



 $p_1 - p_2, q_1 + q_2$

 $p_1 + p_2$, $q_1 - q_2$

Tripartite Entanglement

• Extend DGCZ criterion to three variables

Detecting genuine multipartite continuous-variable entanglement

PHYSICAL REVIEW A 67, 052315 (2003) Peter van Loock¹ and Akira Furusawa²

$$\hat{u} \equiv h_1 \hat{x}_1 + h_2 \hat{x}_2 + h_3 \hat{x}_3, \quad \hat{v} \equiv g_1 \hat{p}_1 + g_2 \hat{p}_2 + g_3 \hat{p}_3,$$

$$\langle (\Delta \hat{u})^2 \rangle_{\rho} + \langle (\Delta \hat{v})^2 \rangle_{\rho} \geq f(h_1, h_2, h_3, g_1, g_2, g_3),$$

• Apply PPT to multiple partitions

Bound Entangled Gaussian States

R. F. Werner* and M. M. Wolf^{\dagger}

Gaussian states of $1 \times N$ systems

ppt implies separability.

PHYSICAL REVIEW LETTERS VOLUME 86, NUMBER 16 DOI: 10.1103/PhysRevLett.86.3658

OPO and Entanglement



Twin photons + phase correlation

- Sub-threshold

squeezed vacuum (degenerate case) - OPA entangled fields (non-degenerate case) -Above threshold: Intense entangled fields Squeezing of the pump Usual treatment of the OPO: Master Equation

$$\frac{d\hat{\rho}}{dt} = \frac{i}{\hbar} \left[\hat{H}_0 + \hat{H}_1, \hat{\rho} \right] + \frac{\gamma}{2} \left[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger} \right]$$

Quasi-probability representation

$$\frac{\partial P(\vec{X},t)}{\partial t} = \left[-\sum_{i} \frac{\partial}{\partial x_{i}} A_{i}(\vec{X},t) + \frac{1}{2} \sum_{ij} \frac{\partial}{\partial x_{i}} \frac{\partial}{\partial x_{j}} D_{ij}(\vec{X},t) \right] P(\vec{X},t)$$

$$\mathbb{D}(\vec{X},t) = \mathbb{B}(\vec{X},t)\mathbb{B}^T(\vec{X},t)$$

Langevin Equation

$$\frac{d\vec{X}}{dt} = \mathbb{A}(\vec{X}, t) + \mathbb{B}(\vec{X}, t)\vec{X}^{in}(t)$$

Usual treatment of the OPO: Langevin Equation

Linearization

$$\frac{d\delta \vec{X}(t)}{dt} = \mathbb{A}\delta \vec{X}(t) + \mathbb{B}\vec{X}^{in}(t)$$

Input – Output Formalism $\delta \vec{X}^{out}(t) = \mathbb{B}\delta \vec{X}(t) - \mathbb{I}\vec{X}^{in}(t)$

Frequency Domain
$$\vec{X}(\Omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \delta \vec{X}(t) exp(-i\Omega t) dt$$

$$\vec{X}(\Omega) = \left[-(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B} \right] \vec{X}^{in}(\Omega) \qquad \qquad \vec{X}^{out}(\Omega) = -\left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B} + \mathbb{I} \right] \vec{X}^{in}(\Omega)$$

 $\vec{X}(\Omega) = -\mathbb{M}_{I}(\Omega)\vec{X}^{in}(\Omega) \qquad \qquad \vec{X}^{out}(\Omega) = -\mathbb{M}_{O}(\Omega)\vec{X}^{in}(\Omega).$ $\mathbb{M}_{I}(\Omega) = (\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B} \qquad \qquad \mathbb{M}_{O}(\Omega) = \mathbb{I} + \left[\mathbb{B}(\mathbb{A} + i\Omega\mathbb{I})^{-1}\mathbb{B}\right]$

 $\begin{array}{ll} \underline{Covariance\ Matrix} & X & \underline{Spectral\ Matrix} \\ \mathbb{V}(t,t+\tau) = \mathbb{V}(\tau) = \langle \delta \vec{X}^{out}(t) [\delta \vec{X}^{out}(t+\tau)]^T \rangle & \mathbb{S}(\Omega) = \langle \vec{X}^{out}(\Omega) [\vec{X}^{out}(-\Omega)]^T \rangle \end{array}$

$$\mathbb{V}(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\Omega) \exp(i\Omega\tau) d\Omega$$

Complete description of the state: Wigner function (for a Gaussian State)

$$W(\vec{X}) = \frac{1}{4\pi^2 \sqrt{\det \mathbb{V}_i}} \exp\left(-\frac{1}{2}\vec{X}^T \mathbb{V}_i^{-1} \vec{X}\right)$$

Where is the complete information about the OPO state?

Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & C_{p1q1} & C_{p1p2} & C_{p1q2} & C_{p1p0} & C_{p1q0} \\ C_{p1q1} & S_{q1} & C_{q1p2} & C_{q1q2} & C_{q1p0} & C_{q1q0} \\ C_{p1p2} & C_{q1p2} & S_{p2} & C_{p2q2} & C_{p2p0} & C_{p2q0} \\ C_{p1q2} & C_{q1q0} & C_{p2q2} & S_{q2} & C_{q2p0} & C_{q2q0} \\ C_{p1p0} & C_{q1p0} & C_{p2p0} & C_{q2p0} & S_{p0} & C_{p0q0} \\ C_{p1q0} & C_{q1q0} & C_{p2q0} & C_{q2q0} & C_{p0q0} & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$$

36 independent terms !

Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$$

18 independent terms !



Energy Conservation

 $\omega_1 + \omega_2 = \omega_0$

 $\delta I_1 - \delta I_2 = 0$

Intensity Correlation A. Heidmann *et al.*, PRL. **59**, 2555 (1987)

 $\delta \phi_1 + \delta \phi_2 = \delta \phi_0$

Phase Anti-correlation A. S. Villar *et al.*, PRL **95**, 243603 (2005)





But if we look for a complete characterization of the OPO, we have to measure three fields of diferent colors!

Is it possible to perform a homodyne measurement without a local oscillator?

Measurement of the Field in the time domain



Measurement of the Field in the frequency domain





Measurement of the Field in the frequency domain

$$\hat{a}(t) = \int_{-\infty}^{\infty} \hat{a}(\Omega) \exp(-i\Omega t) \ d\Omega$$

$$\hat{a}(\Omega) = \hat{x}(\Omega) + i\hat{y}(\Omega)$$





Phase Rotation of Noise Ellipse



Alessandro S. Villar, The conversion of phase to amplitude fluctuations of a light beam by an optical cavity American Journal of Physics **76**, pp. 922-929 (2008).



Covariance Matrix

$$V = \begin{bmatrix} S_{p1} & 0 & C_{p1p2} & 0 & C_{p1p0} & 0 \\ 0 & S_{q1} & 0 & C_{q1q2} & 0 & C_{q1q0} \\ C_{p1p2} & 0 & S_{p2} & 0 & C_{p2p0} & 0 \\ 0 & C_{q1q0} & 0 & S_{q2} & 0 & C_{q2q0} \\ C_{p1p0} & 0 & C_{p2p0} & 0 & S_{p0} & 0 \\ 0 & C_{q1q0} & 0 & C_{q2q0} & 0 & S_{q0} \end{bmatrix}$$

$$C_{xixj} = \frac{1}{2} \langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle \qquad \qquad S_{xj} = C_{xjxj}$$



Generation of Squeezed States by Parametric Down Conversion



Generation of Squeezed States by Parametric Down Conversion



Observation of Quantum Noise Reduction on Twin Laser Beams

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Experimental investigation of amplitude and phase quantum correlations in a type II optical parametric oscillator above threshold: from nondegenerate to degenerate operation

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Quantum Correlations of Phase in Nondegenerate Parametric Oscillation

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The squeezing spectrum for nondegenerate parametric oscillation above threshold is calculated, including phase diffusion. A *nonclassical* correlation in phase *and* intensity occurs which is an example of the Einstein-Podolsky-Rosen paradox, even in fields of large photon number. Р

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FIG. 1. Plot of $V(\theta, -\theta, \omega)$, the spectrum of fluctuation in the signal and idler quadrature amplitude difference $X_1^{\theta} - X_2^{-\theta}$: Solid line, near threshold. $E/E_T = 1.01$, $\kappa_3/\kappa = 0.01$, I = 10. Dash-dotted line, well above threshold with a good pump. $E/E_T = 50$, $\kappa_3/\kappa = 0.1$ ($I^0 > 10$). Dashed line, well above threshold with excellent pump. $E/E_T = 20$, $\kappa_3/\kappa = 0.01$ ($I^0 > 10$).

P. D. Drummond

Europhys. Lett., 40 (1), pp. 25-30 (1997) Observation of squeezing using cascaded nonlinearity

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(received 20 January 1997; accepted in final form 2 September 1997)

Abstract. – We have observed that the pump beam reflected by a triply resonant optical parametric oscillator, after a cascaded second-order nonlinear interaction in the crystal, is significantly squeezed. The maximum measured squeezing in our device is 30% (output beam squeezing inferred: 48%). The direction of the noise ellipse depends on the cavity detuning and can be adjusted from intensity squeezing to phase squeezing.



Generation of bright squeezed light at 1.06 µm using cascaded nonlinearities in a triply resonant cw periodically-poled lithium niobate optical parametric oscillator

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We have used an ultralow threshold continuous-wave optical parametric oscillator (OPO) to reduce the quantum fluctuations of the reflected pump beam below the shot noise limit. The OPO consisted of a triply resonant cavity containing a periodically poled lithium niobate crystal pumped by a Nd:YAG (yttrium aluminum garnets) laser and giving signal and idler wavelengths close to 2.12 μ m and a threshold as low as 300 μ W. We detected the quantum fluctuations of the pump beam reflected by the OPO using a slightly modified homodyne detection technique. The measured noise reduction was 30% (inferred noise reduction at the output of the OPO 38%).



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Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables

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The Einstein-Podolsky-Rosen paradox is demonstrated experimentally for dynamical variables having a continuous spectrum. As opposed to previous work with discrete spin or polarization variables, the continuous optical amplitudes of a signal beam are inferred in turn from those of a spatially separated but strongly correlated idler beam generated by nondegenerate parametric amplification. The uncertainty product for the variances of these inferences is observed to be 0.70 ± 0.01 , which is below the limit of unity required for the demonstration of the paradox.

Realization of the Einstein-Podolsky-Rosen Paradox for Continuous Variables



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How can we measure the phase?





Generation of Bright Two-Color Continuous Variable Entanglement

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We present the first measurement of squeezed-state entanglement between the twin beams produced in an optical parametric oscillator operating above threshold. In addition to the usual squeezing in the intensity difference between the twin beams, we have measured squeezing in the sum of phase quadratures. Our scheme enables us to measure such phase anticorrelations between fields of different frequencies. In the present measurements, wavelengths differ by ≈ 1 nm. Entanglement is demonstrated according to the Duan *et al.* criterion [Phys. Rev. Lett. 84, 2722 (2000)] $\Delta^2 \hat{p}_- + \Delta^2 \hat{q}_+ = 1.41(2) < 2$. This experiment opens the way for new potential applications such as the transfer of quantum information between different parts of the electromagnetic spectrum.



Generation of Bright Two-Color Continuous Variable Entanglement

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Braz. J. Phys. **31**, 597 (2001)

























