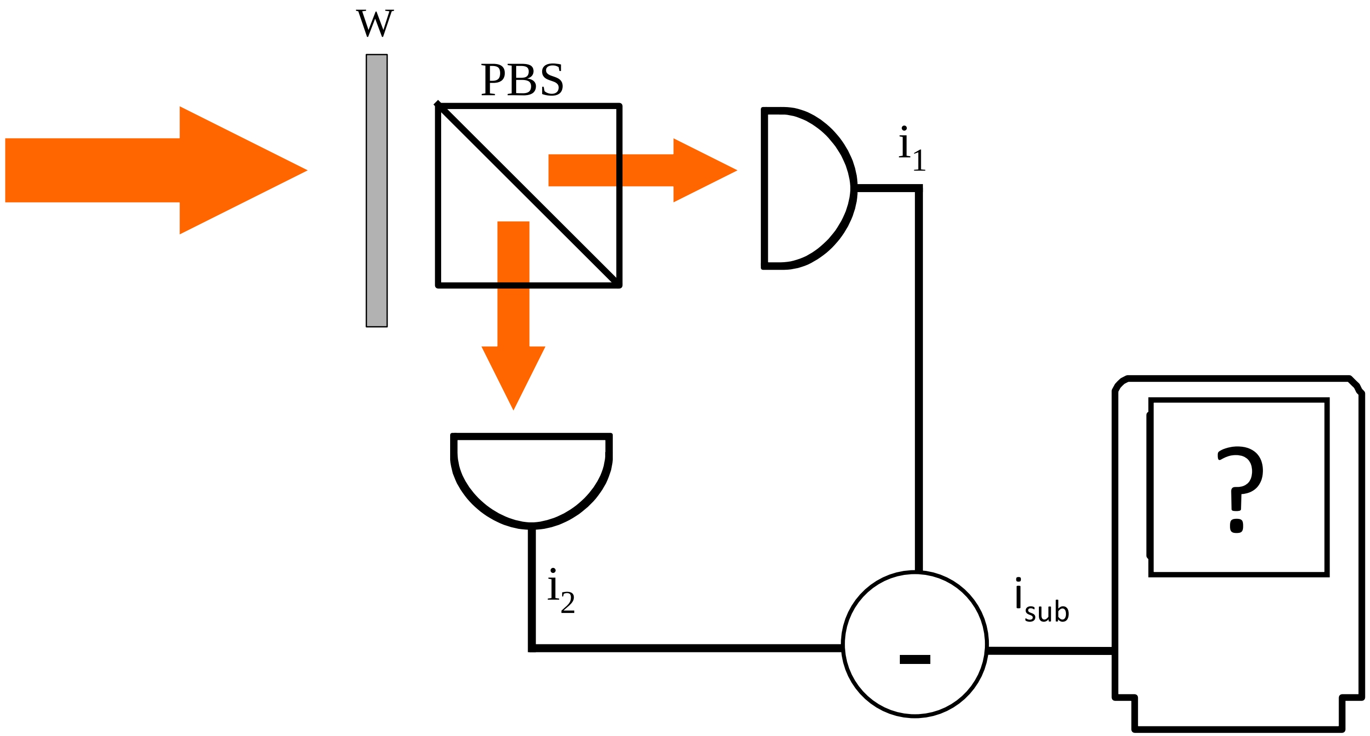
Multipartite entanglement and sudden death in Quantum Optics: continuous variables domain

Part II - The OPO

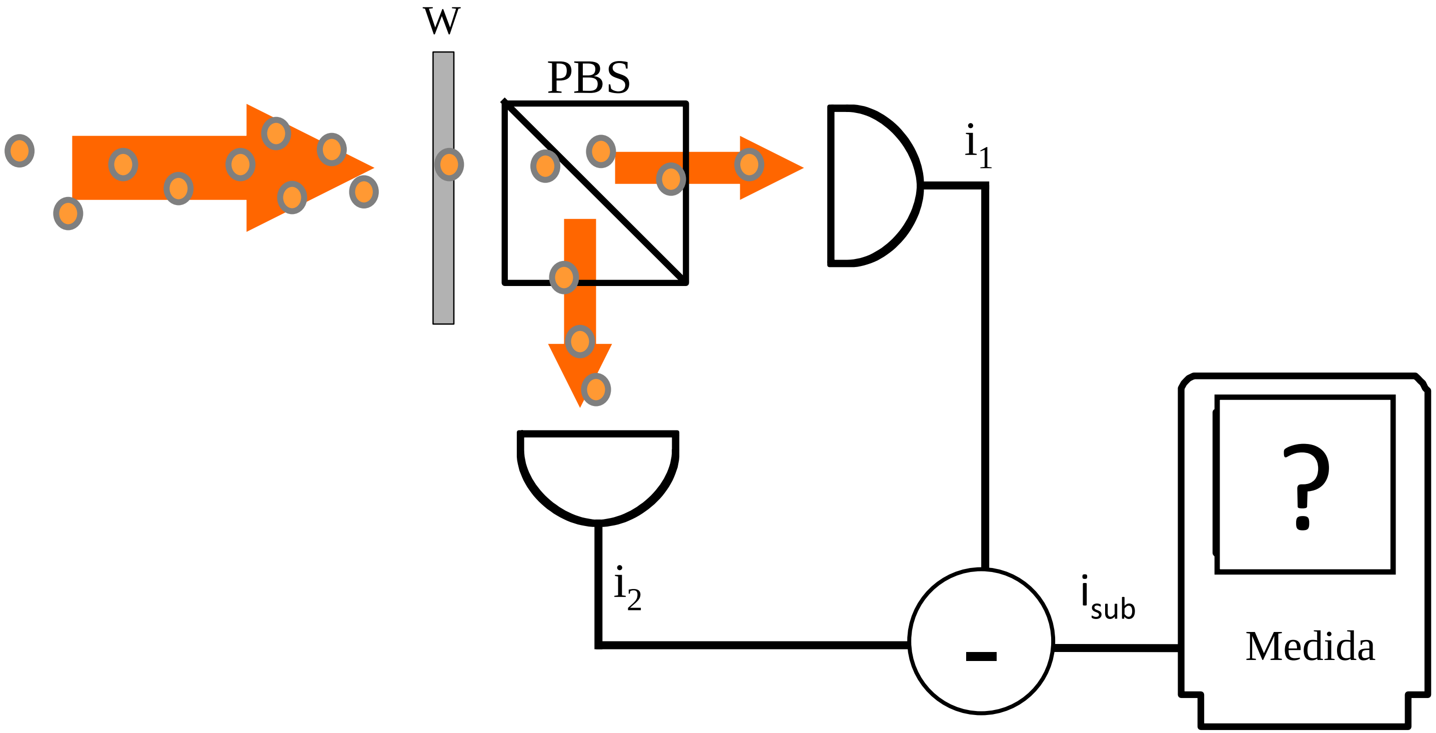
Marcelo Martinelli

Question:

Dividing the incident beam in two “equal” parts, what will be the result?



Answer:



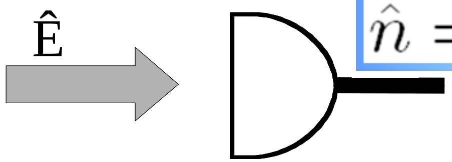
Classically: $i_{sub} = 0$

Quantically: “photons are clicks on photodetectors” (A. Zeilinger)

$$\langle i_{sub} \rangle = 0, \qquad \Delta^2 i_{sub} > 0 \,!$$

Quantum Optics – Measurement of the Intense Field

We can easily measure photon flux: optical power



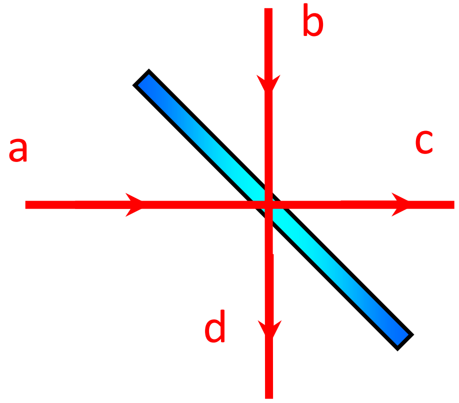
$$\hat{n} = \hat{a}^\dagger \hat{a}$$

$$\hat{a}^\dagger = \alpha + \delta\hat{a}^\dagger$$

$$\alpha = |\alpha| exp(i\varphi)$$

$$\hat{n} = |\alpha|^2 + |\alpha| e^{i\varphi}\delta\hat{a}^\dagger + |\alpha| e^{-i\varphi}\delta\hat{a} + \delta\hat{a}^\dagger\delta\hat{a}$$

$$\hat{n} = |\alpha|^2 + |\alpha|\delta\hat{p} + O(2)$$



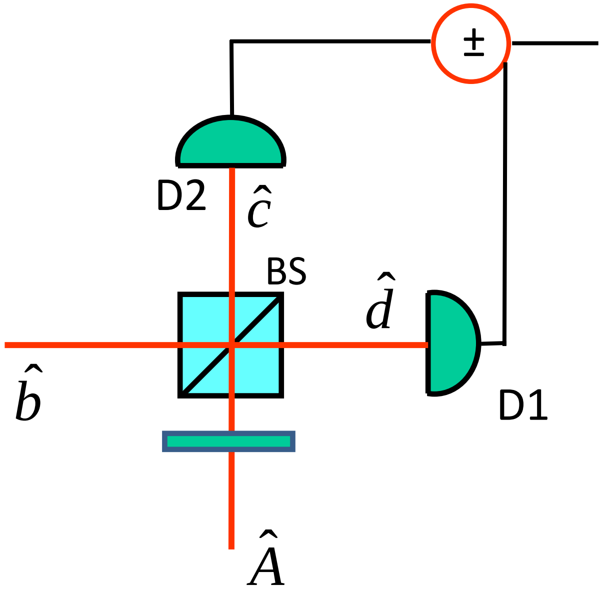
$$\hat{n}_c = \frac{\hat{n}_a + \hat{n}_b + \hat{a}^\dagger\hat{b} + \hat{b}^\dagger\hat{a}}{2}$$

$$\hat{n}_d = \frac{\hat{n}_a + \hat{n}_b - \hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{a}}{2}$$

$$\hat{c} = \frac{1}{\sqrt{2}}\left(\hat{a} + \hat{b}\right)$$

$$\hat{d} = \frac{1}{\sqrt{2}}\left(\hat{b} - \hat{a}\right)$$

Building an Interferometer – The Beam Splitter



$$\hat{n}_+ = \hat{n}_a + \hat{n}_b$$

$$\hat{n}_- = \hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}$$

Homodyning if $<|\hat{a}|> \ll <|\hat{b}|>$

$$\hat{n}_-(t) = |\beta| \left(\hat{A}(t) e^{-i\theta} + \hat{A}^\dagger(t) e^{i\theta} \right)$$

Vacuum Homodyning Calibration of the Standard Quantum Level

$$\hat{n}_+ = \hat{n}_b$$

$$\langle \hat{n}_- \rangle = 0$$

$$\Delta^2 \hat{n}_- = \langle \hat{n}_b \rangle$$

$$\begin{aligned}
\Delta^2 \hat{n} &= \langle (\hat{n} - \langle \hat{n} \rangle)^2 \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2 \\
&= \langle \hat{a}^\dagger \hat{a} \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2 + \langle \hat{a}^\dagger \hat{a} \rangle \\
&= \langle : \hat{n}^2 : \rangle - \langle \hat{n} \rangle^2 + \langle \hat{n} \rangle
\end{aligned}$$

"Classical" Variance

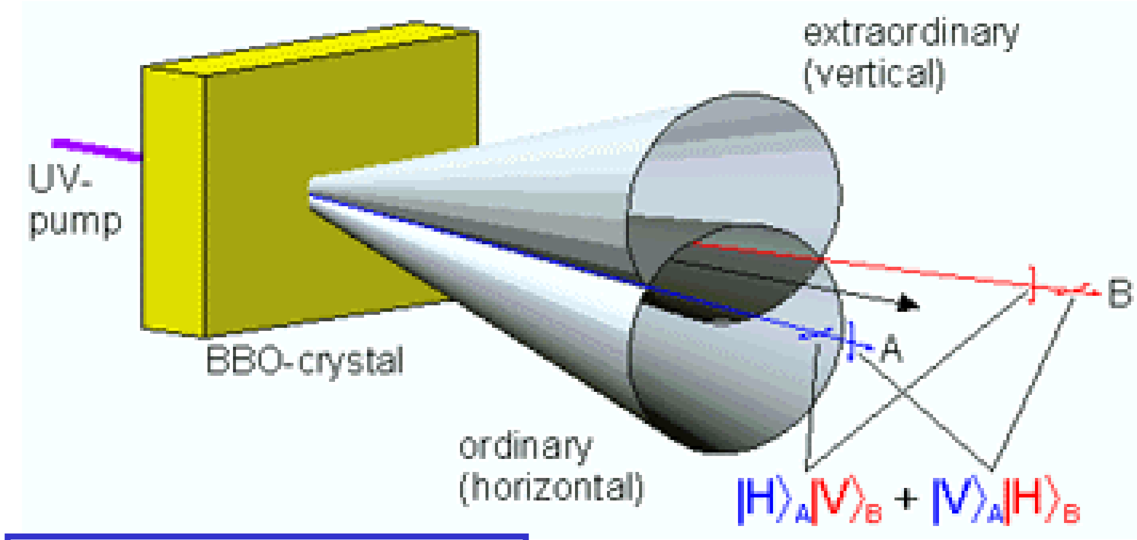
Shot noise !

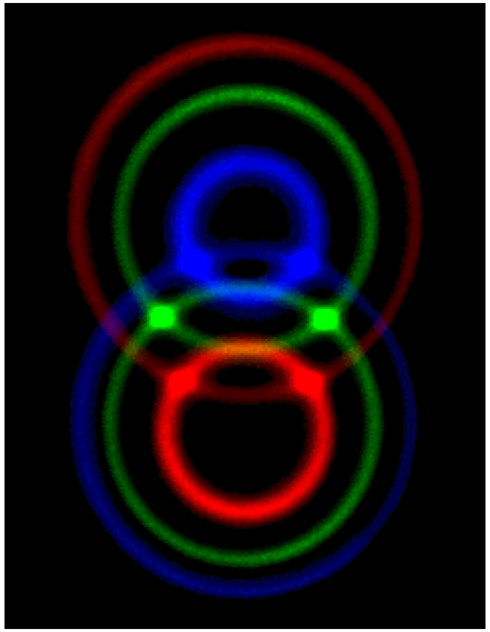
Vacuum Homodyning allows the calibration of the detection, producing a Poissonian distribution in the output (just like a coherent state).

$$|\alpha\rangle = exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$p_n = |\langle \hat{n} | \alpha \rangle|^2 = exp(-|\alpha|^2) \frac{|\alpha|^{2n}}{n!}$$

Parametric Down Conversion





$$P = \chi^{(1)} E + \chi^{(2)} E^2$$

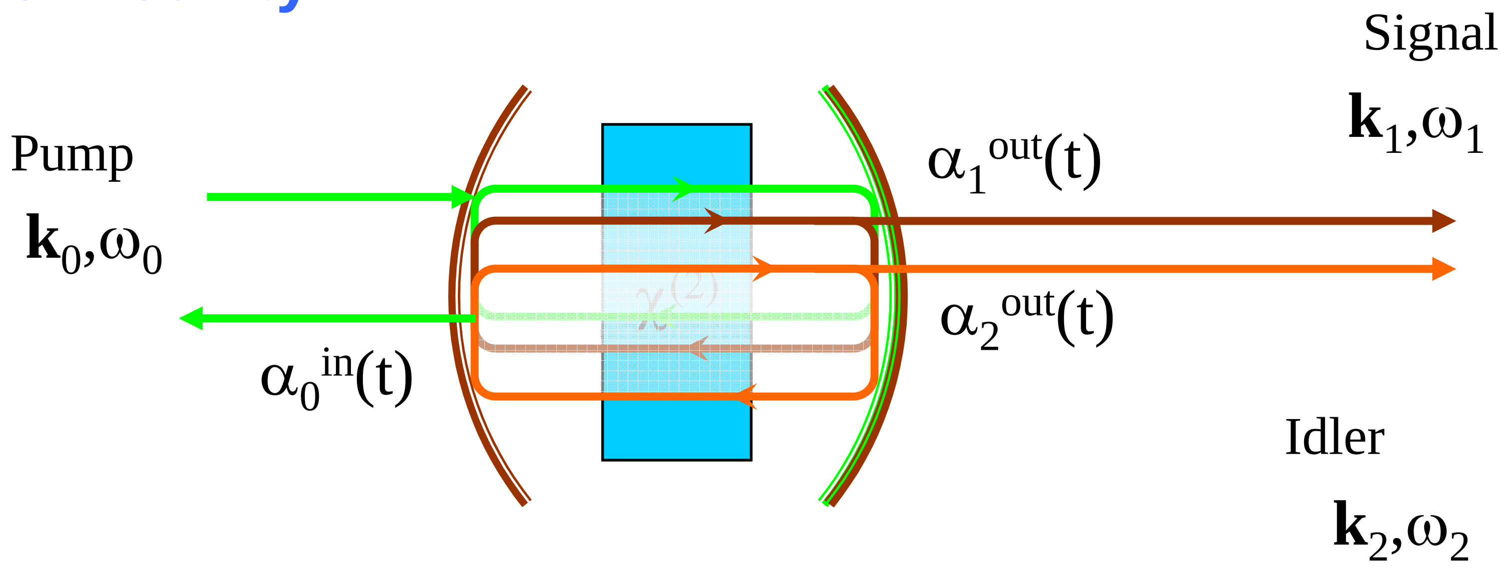
Energy and momentum conservation

$\omega_0 = \omega_1 + \omega_2$ $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$

Polarization and transverse momentum correlations

Optical Parametric Oscillator

PDC + Cavity



PHYSICAL REVIEW LETTERS

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TUNABLE COHERENT PARAMETRIC OSCILLATION IN $LiNbO_3$ AT OPTICAL FREQUENCIES

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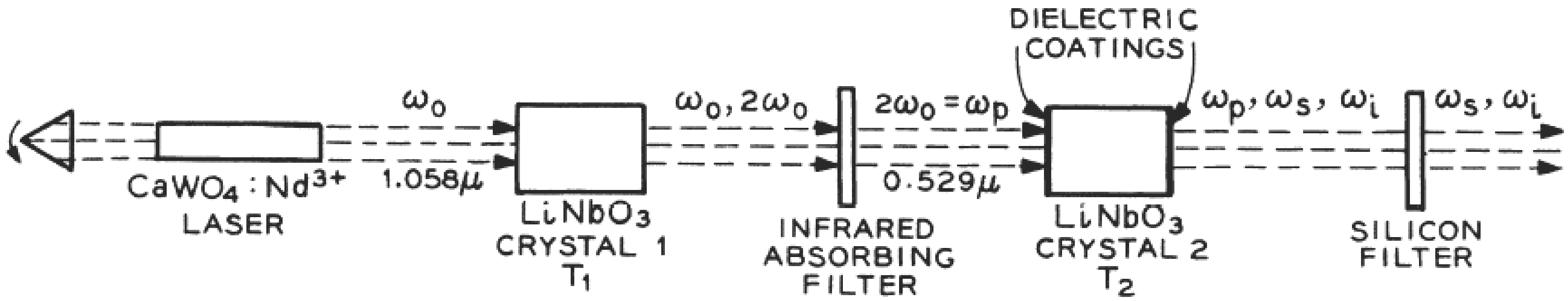
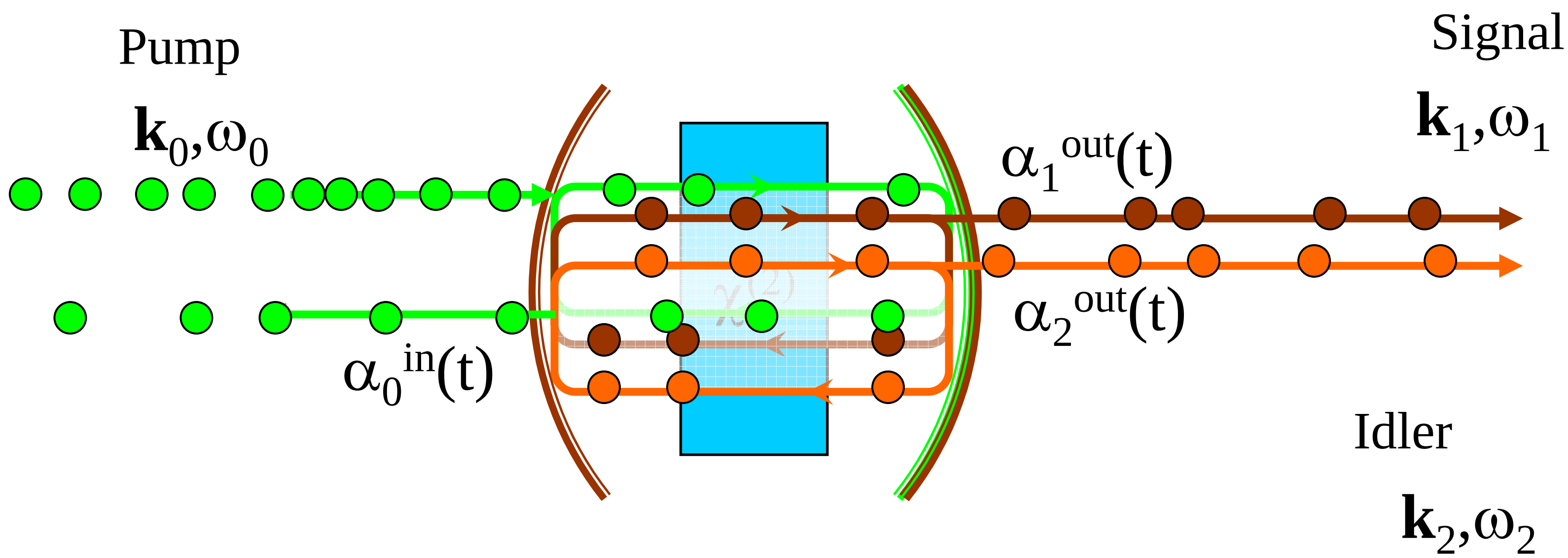


Optical Parametric Oscillator

PDC + Cavity



Twin photons + phase correlation

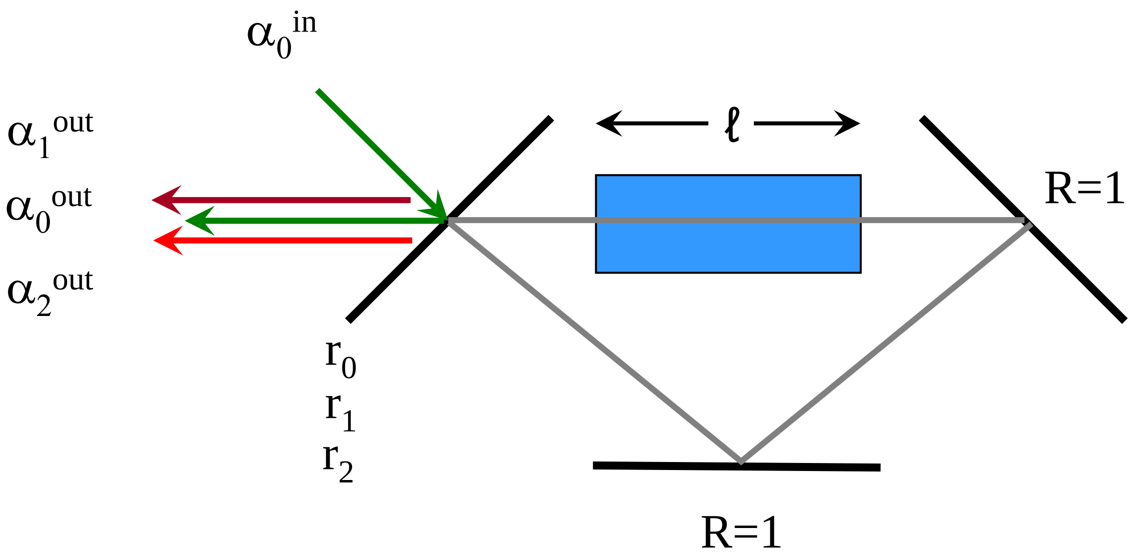
- Sub-threshold
  - squeezed vacuum (degenerate case) - OPA
  - entangled fields (non-degenerate case)

- Above threshold: intense entangled fields

Optical Parametric Oscillator (OPO) - Classical

Let us describe classical properties of the system before we analyze quantum properties. We'll consider a Triply Resonant OPO (TR-OPO) in a ring cavity (for simplicity).

Debuisschert *et al.* J. Opt. Soc. Am. B/Vol. 10, 1668 1993

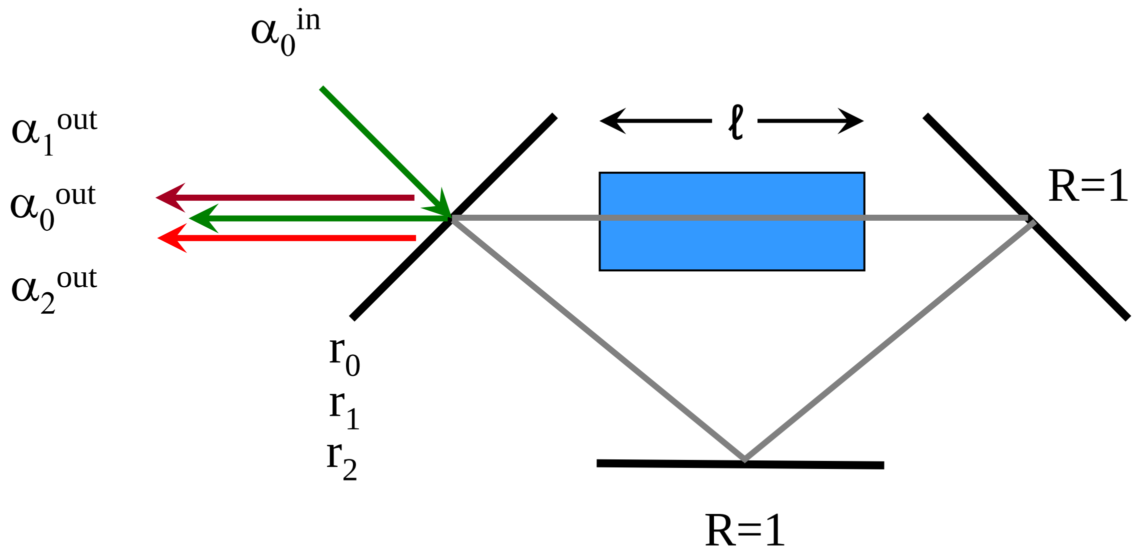


$$r_j = e^{(-\gamma_j)} \simeq 1 - \gamma_j;$$
$$t_j = (2\gamma_j)^{1/2};$$

If we consider that the single pass gain is small, we can approximate the equations for the amplification inside the crystal

$$\alpha_0(\ell) = \alpha_0(0) - 2\chi^*\alpha_1(0)\alpha_2(0)$$
$$\alpha_1(\ell) = \alpha_1(0) + 2\chi\alpha_0(0)\alpha_2^*(0)$$
$$\alpha_2(\ell) = \alpha_2(0) + 2\chi\alpha_0(0)\alpha_1^*(0)$$

Optical Parametric Oscillator (OPO) - Classical



Consistency of the field for a round trip gives us

$$\alpha_0 = r_0 e^{i\varphi_0}(\alpha_0 - 2\chi\alpha_2\alpha_1) - \mu_0\alpha_0 + t_0\alpha_0^{in} ,$$

$$\alpha_1 = r_1 e^{i\varphi_1}(\alpha_1 + 2\chi\alpha_0\alpha_2^*) - \mu_1\alpha_1 ,$$

$$\alpha_2 = r_2 e^{i\varphi_2}(\alpha_2 + 2\chi\alpha_0\alpha_1^*) - \mu_2\alpha_2 ,$$

$$\varphi_j = 2p_j\pi + \delta\varphi_j$$

Optical Parametric Oscillator (OPO) - Classical

If $\delta\varphi_j$ is small, we can write:

$$\begin{aligned} \alpha_0(\gamma_0' - i\delta\varphi_0) &= -2\chi\alpha_1\alpha_2 + \sqrt{2\gamma_0}\alpha_0^{in} \\ \alpha_1(\gamma_1' - i\delta\varphi_1) &= 2\chi\alpha_0\alpha_2^* \, , \\ \alpha_2(\gamma_2' - i\delta\varphi_2) &= 2\chi\alpha_0\alpha_1^* \, , \end{aligned}$$

where the total loss for each mode is defined

$$\gamma_j' = \gamma_j + \mu_j$$

Normalizing the detuning, we have

$$\Delta_j = \delta\varphi_j / \gamma_j'$$

$$\alpha_0\gamma_0'(1 - i\Delta_0) = -2\chi\alpha_1\alpha_2 + \sqrt{2\gamma_0}\alpha_0^{in} \, ,$$

$$\alpha_1\gamma_1'(1 - i\Delta_1) = 2\chi\alpha_0\alpha_2^* \, ,$$

$$\alpha_2\gamma_2'(1 - i\Delta_2) = 2\chi\alpha_0\alpha_1^* \, .$$

Optical Parametric Oscillator (OPO) - Classical

A first solution of these equations is $\alpha_1 = \alpha_2 = 0$, corresponding to operation below threshold. We are more interested in above-threshold operation. Multiplying the complex conjugate of the third equation by the second, we have: $\gamma_1'\gamma_2'(1 - i\Delta_1)(1 + i\Delta_2) = 4|\chi|^2|\alpha_0|^2$ → $\Delta_1 = \Delta_2 = \Delta$

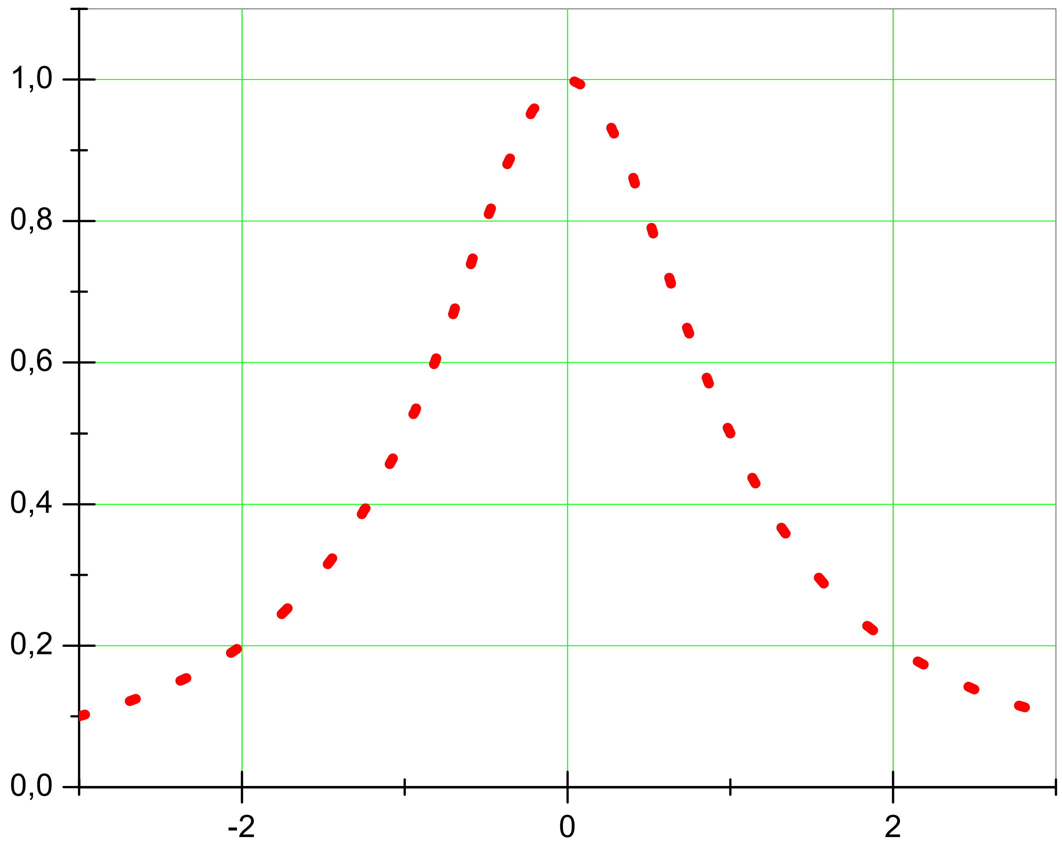
The intracavity pump power is easily obtained and we see it is "clipped": above-threshold it is always the same

$$|\alpha_0|^2 = \frac{\gamma_1'\gamma_2'(1 + \Delta^2)}{4|\chi|^2}$$

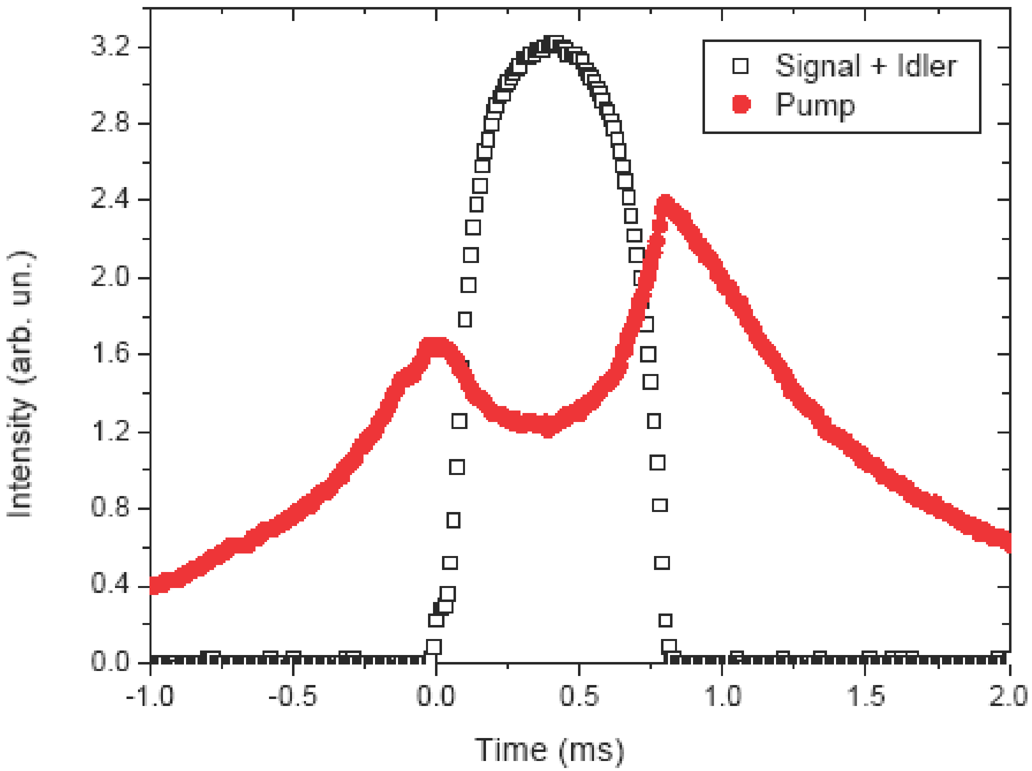
Besides, for $\Delta_1 = \Delta_2 = \Delta$, we also have $\gamma_1'|\alpha_1|^2 = \gamma_2'|\alpha_2|^2$

The classical equations are already signaling that the intensities of signal and idler beams should be strongly correlated and that the pump must be depleted.

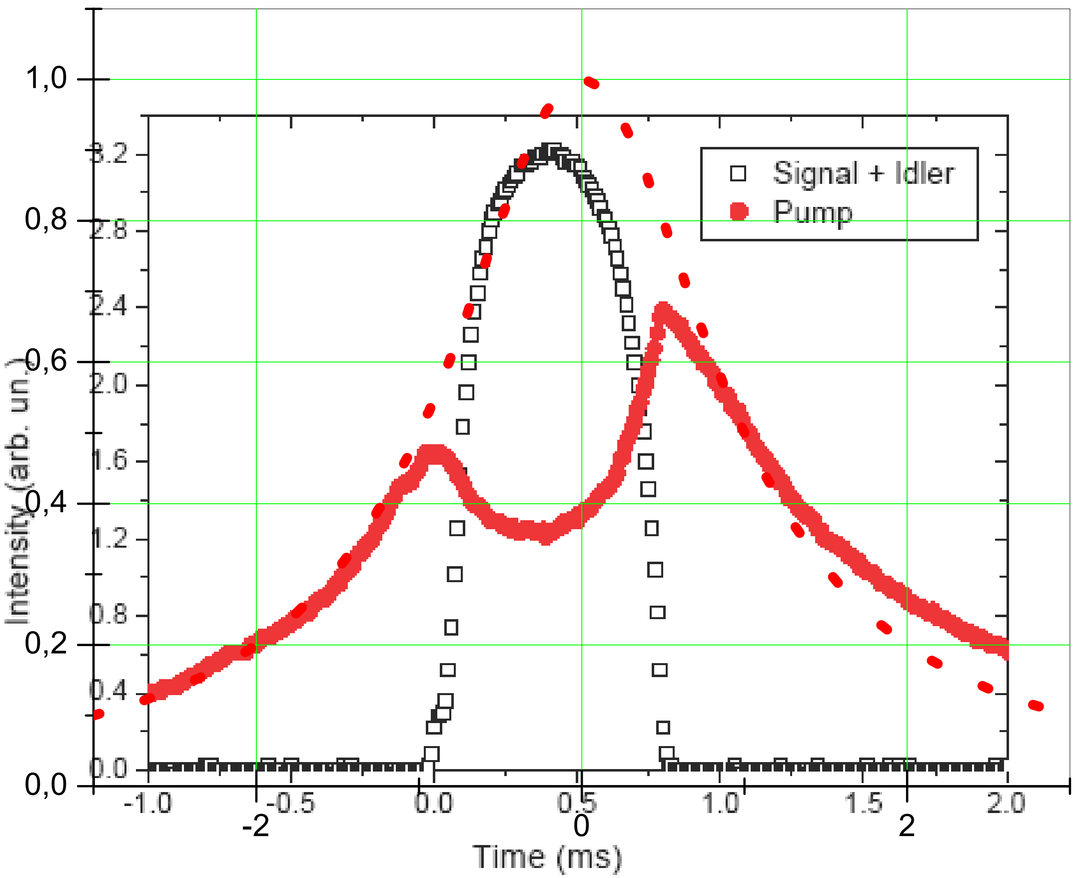
Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical



Optical Parametric Oscillator (OPO) - Classical

From the first equation we can derive the threshold power, given the intracavity pump field ($\alpha_1 = \alpha_2 = 0$)

$$|\alpha_0^{in}|^2_{th} = \frac{\gamma_0'^2 \gamma_1' \gamma_2' (1+\Delta^2)(1+\Delta_0^2)}{8|\chi|^2 \gamma_0}$$

An important parameter will be the ratio of incident power to threshold power on resonance:

$$\sigma = \frac{|\alpha_0^{in}|^2}{|\alpha_0^{in}|^2_{res}} = \frac{P_{in}}{P_{th}}$$

Substituting α_2 in the first equation, we have

$$\alpha_0 \gamma_0'(1 - i\Delta_0) = -\frac{4|\chi|^2 \alpha_0 |\alpha_1|^2}{\gamma_2'(1 - i\Delta)} + \sqrt{2\gamma_0}\,\alpha_0^{in}$$

Optical Parametric Oscillator (OPO) - Classical

$$\frac{2\gamma_0}{\gamma_0'^2}(1+\Delta)^2\frac{|\alpha_0^{in}|^2}{|\alpha_0|^2} = \left[1 - \Delta\Delta_0 + \frac{4|\chi|^2|\alpha_1|^2}{\gamma_0'\gamma_2'}\right]^2 + (\Delta+\Delta_0)^2$$

Since $|\alpha_0|^2 = \dfrac{\gamma_1'\gamma_2'(1+\Delta^2)}{4|\chi|^2}$ and $|\alpha_0^{in}|^2_{th} = \dfrac{\gamma_0'^2\gamma_1'\gamma_2'(1+\Delta^2)(1+\Delta_0^2)}{8|\chi|^2\gamma_0}$

$$|\alpha_0^{in}(\Delta=\Delta_0=0)|^2_{th} = \frac{\gamma_0'\gamma_1'\gamma_2'}{8|\chi|^2\gamma_0}$$

We get $\sigma = \left(1 - \Delta\Delta_0 + \dfrac{4|\chi|^2|\alpha_1|^2}{\gamma_2'\gamma_0'}\right)^2 + (\Delta+\Delta_0)^2$

Solving for α_j $|\alpha_j|^2 = \dfrac{\gamma_k'\gamma_0'}{4|\chi|^2}(\sqrt{\sigma}-1)$ → $|\alpha_j^{out}|^2 = \dfrac{\gamma_j\gamma_k'\gamma_0'}{2|\chi|^2}(\sqrt{\sigma}-1)$

$$|\alpha_0|^2 = \frac{\gamma_1'\gamma_2'(1+\Delta^2)}{4|\chi|^2}$$

Optical Parametric Oscillator (OPO) - Classical

This gives the photon flux. Considering, for the sake of the argument, the frequency-degenerate case ($\omega_1=\omega_2=\omega_0/2$), we can obtain the total output power and the efficiency

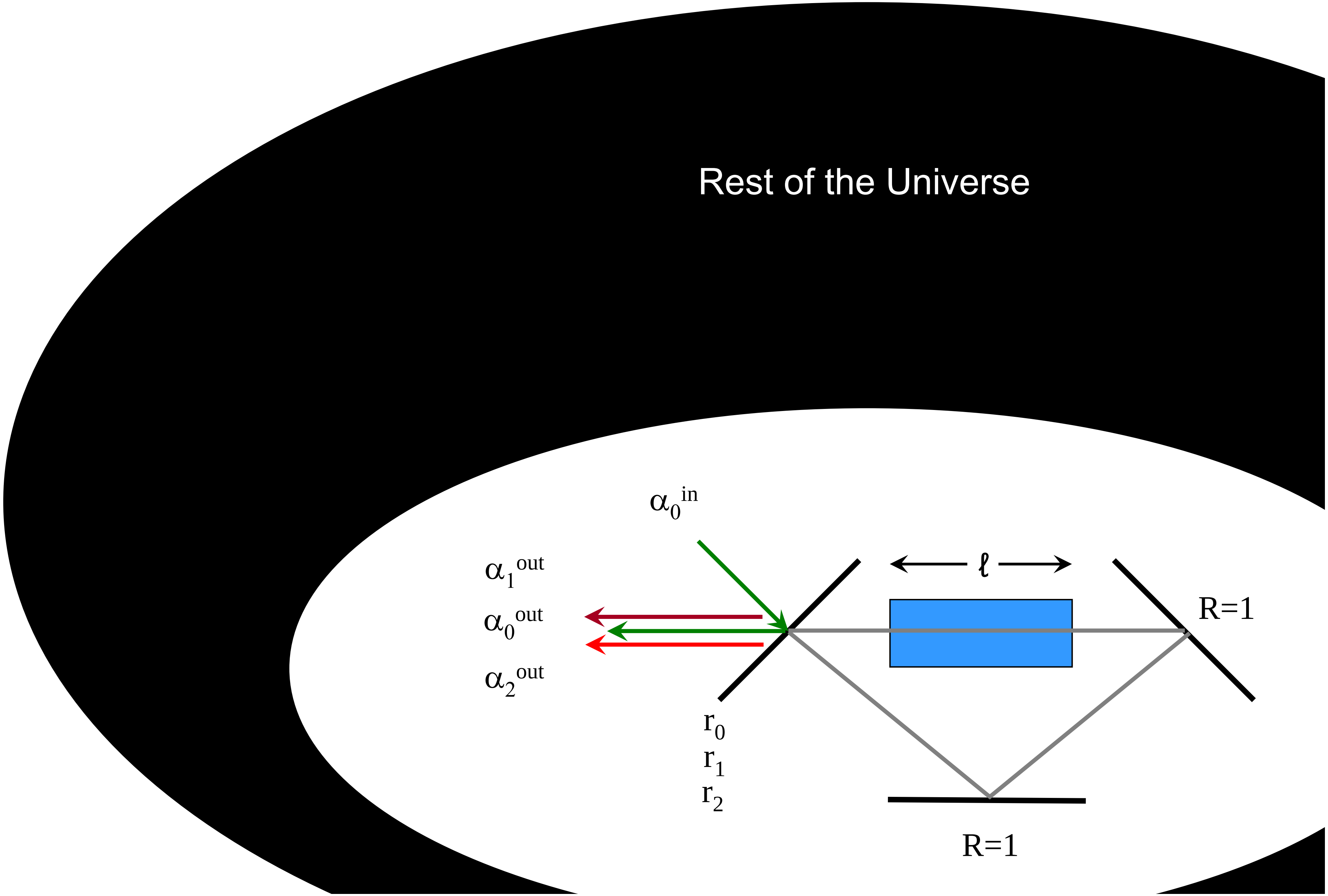
$$P_{out} = \hbar\omega_0 \left[\frac{\gamma\gamma'\gamma'_0}{2|\chi|^2}\left(\sqrt{\sigma}-1\right)\right] = 4\eta_{max}\left(\sqrt{P\cdot P_{th}} - P_{th}\right)$$

Where η_{max} is the maximum efficiency leading to

$$\eta_{max} = \frac{\gamma}{\gamma'}\frac{\gamma_0}{\gamma'_0} = \xi\xi_0 \qquad \xi_j = \gamma_j/\gamma'_j$$

We will see that the parameter ξ determines the maximum squeezing in the above-threshold OPO.

Optical Parametric Oscillator (OPO) - Quantum



Optical Parametric Oscillator (OPO) – Master Equation

Evolution of the density operator $$\frac{d}{dt}\hat{\rho}_{sr} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}_{sr}]$$

$$\hat{H} = \hat{H}_s + \hat{H}_r + \hat{V}$$

System + Reservoir + Interaction

$$\hat{H}_r = \sum_j \hbar\omega_j \hat{b}_j^\dagger \hat{b}_j \qquad \hat{V} = \hbar \sum_j (g_j \hat{a}_j^\dagger \hat{b}_j + g_j^* \hat{b}_j^\dagger \hat{a}_j)$$

Evolution of an operator acting only on the system:

$$\langle \hat{O}(t) \rangle = tr_s\{\hat{O}\, tr_r \hat{\rho}_{sr}(t)\} = tr_s\{\hat{O}\hat{\rho}_s(t)\}$$

Master Equation: Evolution of ρ_s

Quantum Properties of the OPO

Hamiltonian and the master equation:

$$\frac{d}{dt}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H}_f + \hat{H}_i + \hat{H}_{ext}, \hat{\rho}\right] + (\Lambda_0 + \Lambda_1 + \Lambda_2)\,\hat{\rho}$$

$$\hat{H}_f = -\hbar\Delta_0\frac{\gamma_0'}{\tau}\hat{a}_0^\dagger\hat{a}_0 - \hbar\Delta_1\frac{\gamma_1'}{\tau}\hat{a}_1^\dagger\hat{a}_1 - \hbar\Delta_2\frac{\gamma_2'}{\tau}\hat{a}_2^\dagger\hat{a}_2$$

$$\hat{H}_{ext} = i\hbar\frac{\gamma_0}{\tau}\varepsilon\left(\hat{a}_0^\dagger - \hat{a}_0\right)$$

$$\hat{H}_i = i\hbar\frac{2\chi}{\tau}\left(\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_0 - \hat{a}_1\hat{a}_2\hat{a}_0^\dagger\right)$$

$$\Lambda_j\hat{\rho} = \frac{\gamma_j'}{\tau}\left(2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger - \hat{a}_j^\dagger\hat{a}_j\hat{\rho} - \hat{\rho}\hat{a}_j^\dagger\hat{a}_j\right)$$

OK, simpler now?

We can improve this if we change from the density matrix into an equivalent representation: it will replace (ordering sensitive) operators by c-numbers.

But the nonclassicallity makes P representation a tricky choice...

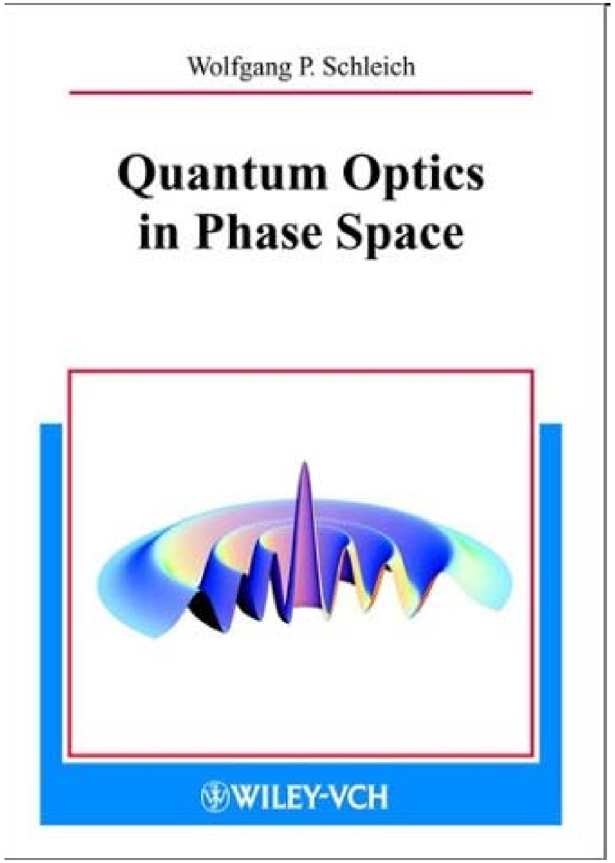
Quasi-Probability Representations

P- Glauber - Sudarshan $\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|\,d^2\alpha$

Wigner $W(\alpha) = \frac{1}{\pi^2}\int e^{\eta^*\alpha - \eta\alpha^*}\chi(\eta)d^2\eta \qquad \chi(\eta) = Tr[\hat{\rho}e^{\eta\hat{a}^\dagger - \eta^*\hat{a}}]$

$$\bar{W}(\bar{x},\bar{p}) = \frac{1}{\pi\hbar}\int dy\langle\bar{x}+y|\rho|\bar{x}-y\rangle\exp(-2iy\bar{p}/\hbar)$$

$$\langle\{a^r(a^\dagger)^s\}_{\mathrm{sym}}\rangle = \int d^2\alpha\,\alpha^r(\alpha^*)^s W(\alpha,\alpha^*).$$



C.W. Gardiner P. Zoller

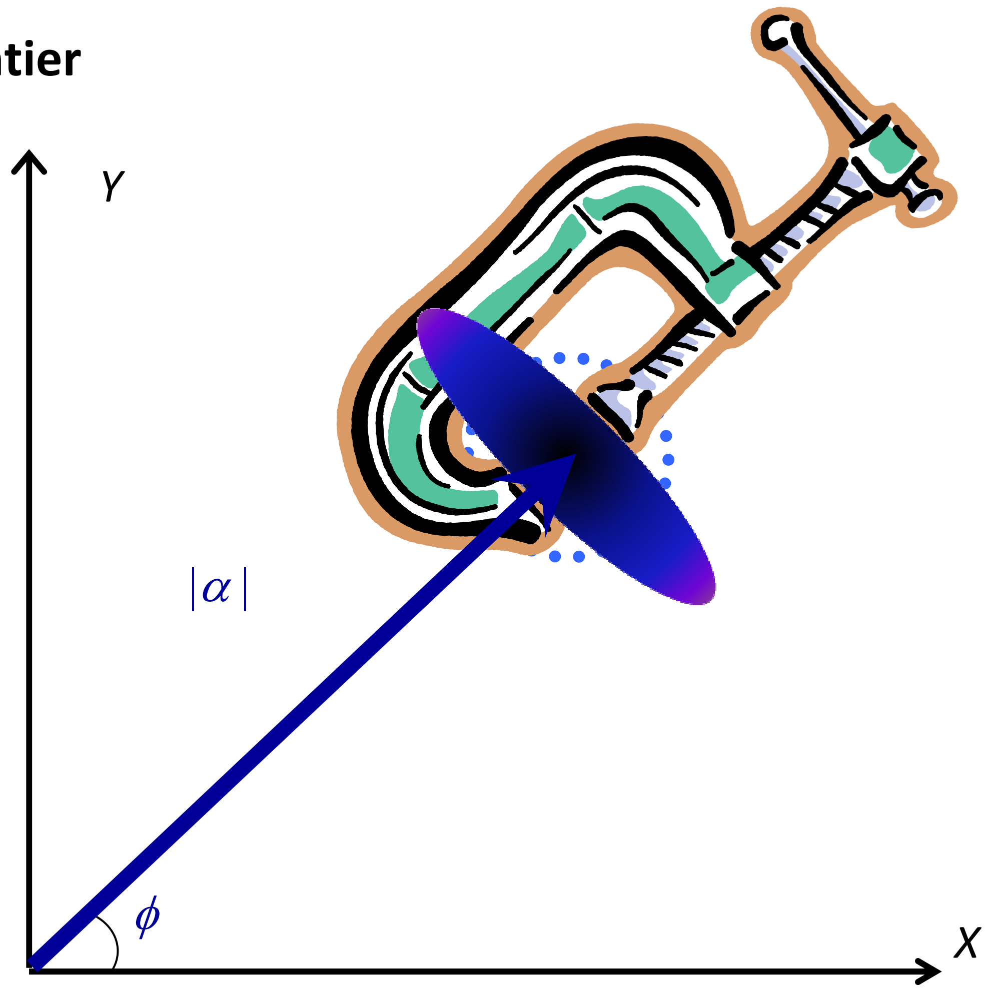
Quantum Noise

A Handbook
of Markovian and Non-Markovian
Quantum Stochastic Methods
with Applications to Quantum Optics

Wigner Representation

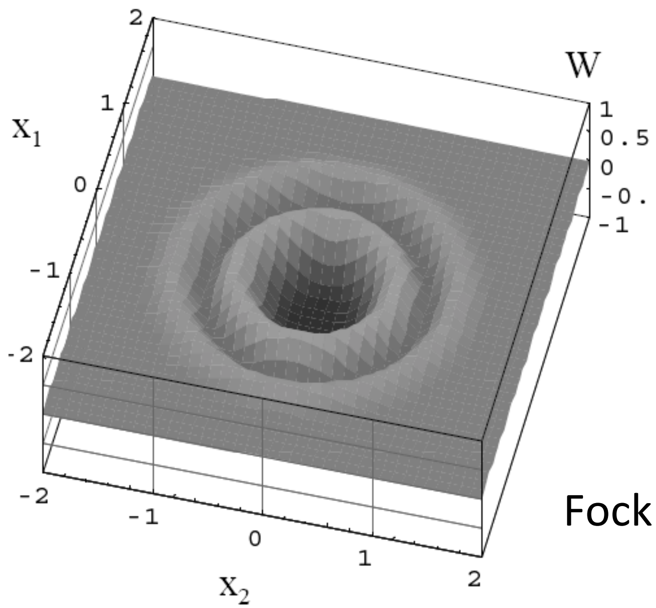
Evident quantum/ classical frontier

Squeezed states



States with W<0

n=3



Fock states $|n\rangle$

Quantum Properties of the OPO

The operators $(\hat{a}^\dagger, \hat{a})$ are replaced by amplitudes (α^*, α)

and the density operator is replaced by $W(\boldsymbol{\alpha})$

$$\boldsymbol{\alpha} = (\alpha_0, \alpha_0^*, \alpha_1, \alpha_1^*, \alpha_2, \alpha_2^*)$$

Using the rules

$$\hat{a}\hat{\rho}_s \Longleftrightarrow \left(\alpha + \frac{1}{2}\frac{\partial}{\partial \alpha^*}\right) W \qquad \hat{a}^\dagger\hat{\rho}_s \Longleftrightarrow \left(\alpha^* - \frac{1}{2}\frac{\partial}{\partial \alpha}\right) W$$

$$\hat{\rho}_s\hat{a} \Longleftrightarrow \left(\alpha - \frac{1}{2}\frac{\partial}{\partial \alpha^*}\right) W \qquad \hat{\rho}_s\hat{a}^\dagger \Longleftrightarrow \left(\alpha^* + \frac{1}{2}\frac{\partial}{\partial \alpha}\right) W.$$

Quantum Properties of the OPO

We obtain

$$\frac{\partial}{\partial t}W(\boldsymbol{\alpha}) = \sum_{j=1}^{3}\frac{\gamma_j'}{\tau}\left[i\Delta_j\left(\frac{\partial}{\partial\alpha_j^*}\alpha_j^* - \frac{\partial}{\partial\alpha_j}\alpha_j\right) + \left(\frac{\partial}{\partial\alpha_j^*}\alpha_j^* + \frac{\partial}{\partial\alpha_j}\alpha_j\right)\right]W(\boldsymbol{\alpha})$$
$$+\frac{2\chi}{\tau}\left(\alpha_1\alpha_2\frac{\partial}{\partial\alpha_0^*} + \alpha_1^*\alpha_2^*\frac{\partial}{\partial\alpha_0}\right)W(\boldsymbol{\alpha}) - \frac{2\chi}{\tau}\left(\alpha_0\alpha_1^*\frac{\partial}{\partial\alpha_2} + \alpha_0^*\alpha_1\frac{\partial}{\partial\alpha_2^*}\right)W(\boldsymbol{\alpha})$$
$$-\frac{2\chi}{\tau}\left(\alpha_0\alpha_2^*\frac{\partial}{\partial\alpha_1} + \alpha_0^*\alpha_2\frac{\partial}{\partial\alpha_1^*}\right)W(\boldsymbol{\alpha}) - \frac{\gamma_0}{\tau}\varepsilon\left(\frac{\partial}{\partial\alpha_0^*} + \frac{\partial}{\partial\alpha_0}\right)W(\boldsymbol{\alpha})$$
$$+\sum_{j=1}^{3}\frac{\gamma_j'}{\tau}\frac{\partial^2}{\partial\alpha_j\partial\alpha_j^*}W(\boldsymbol{\alpha}) - \frac{\chi}{2\tau}\frac{\partial^3}{\partial\alpha_0^*\partial\alpha_1^*\partial\alpha_2^*}W(\boldsymbol{\alpha})\,.$$

Fokker-Planck equation

$$\frac{\partial}{\partial t}W(\boldsymbol{\alpha}) = -\sum_j\frac{\partial}{\partial\alpha_j}A_jW(\boldsymbol{\alpha}) + \frac{1}{2}\sum_{j,k}\frac{\partial}{\partial\alpha_j}\frac{\partial}{\partial\alpha_k}\left[\mathbf{BB}^T\right]_{jk}W(\boldsymbol{\alpha})$$

Quantum Properties of the OPO

Which is equivalent to a set of Langevin equations
(Do you remember the Brownian Motion ?)

$$\frac{d}{dt}\alpha_j = A_j + [\mathbf{B}\boldsymbol{\sigma}(t)]_j$$

$$\langle \sigma_i(t)\sigma_j(t')\rangle = \delta_{ij}\delta(t-t')$$

The mean values in steady state are the same as in the classical treatment.

$$\langle \vec{A}(\vec{X},t)\rangle \simeq \vec{A}(\langle \vec{X}\rangle, t) = 0$$

Since we will (typically) deal with intense fields, we proceed by linearizing the fluctuations, neglecting products of fluctuating terms:

$$\delta\vec{X} = \vec{X} - \langle \vec{X}\rangle \qquad \alpha_j(t) = \bar{\alpha}_j + \delta\alpha_j(t)$$

$$\alpha_j = p_j + iq_j$$

Quantum Properties of the OPO

$$\frac{d}{dt}\delta\alpha_0 = -\frac{\gamma_0'}{\tau}(1 - i\Delta_0)\delta\alpha_0 - \frac{2\chi}{\tau}\alpha_+\delta\alpha_+ + \frac{\sqrt{2\gamma_0'}}{\tau}\sigma_1(t)$$

$$\frac{d}{dt}\delta\alpha_0^* = -\frac{\gamma_0'}{\tau}(1 + i\Delta_0)\delta\alpha_0^* - \frac{2\chi}{\tau}\alpha_+^*\delta\alpha_+^* + \frac{\sqrt{2\gamma_0'}}{\tau}\sigma_2(t)$$

$$\frac{d}{dt}\delta\alpha_+ = \frac{2\chi}{\tau}\alpha_0\delta\alpha_+^* - \frac{2\chi}{\tau}\alpha_+^*\delta\alpha_0 - \frac{\gamma'}{\tau}(1 - i\Delta)\delta\alpha_+ + \frac{\sqrt{2\gamma'}}{\tau}\sigma_3(t)$$

$$\frac{d}{dt}\delta\alpha_+^* = \frac{2\chi}{\tau}\alpha_0^*\delta\alpha_+ - \frac{2\chi}{\tau}\alpha_+\delta\alpha_0^* - \frac{\gamma'}{\tau}(1 + i\Delta)\delta\alpha_+^* + \frac{\sqrt{2\gamma'}}{\tau}\sigma_4(t)$$

$$\frac{d}{dt}\delta\alpha_- = -\frac{2\chi}{\tau}\alpha_0\delta\alpha_-^* - \frac{\gamma'}{\tau}(1 - i\Delta)\delta\alpha_- + \frac{\sqrt{2\gamma'}}{\tau}\sigma_5(t)$$

$$\frac{d}{dt}\delta\alpha_-^* = -\frac{2\chi}{\tau}\alpha_0^*\delta\alpha_- - \frac{\gamma'}{\tau}(1 + i\Delta)\delta\alpha_-^* + \frac{\sqrt{2\gamma'}}{\tau}\sigma_6(t).$$

$$\alpha_+ = \frac{\alpha_1 + \alpha_2}{\sqrt{2}} \qquad \alpha_- = \frac{\alpha_1 - \alpha_2}{\sqrt{2}}$$

Quantum Properties of the OPO

$$
\begin{aligned}
\tau \frac{d}{dt}\,\delta p_- &= -2\gamma'\,\delta p_- + \sqrt{2\gamma}\,\delta u_{p_-} + \sqrt{2\mu}\,\delta v_{p_-}\ ,\\
\tau \frac{d}{dt}\,\delta q_- &= 2\Delta\gamma'\,\delta p_- + \sqrt{2\gamma}\,\delta u_{q_-} + \sqrt{2\mu}\,\delta v_{q_-}\ ,\\
\tau \frac{d}{dt}\,\delta p_+ &= -2\Delta\gamma'\,\delta q_+ + \sqrt{2}\gamma'\beta\,\delta p_0 + \sqrt{2}\Delta\gamma'\beta\,\delta q_0 + \sqrt{2\gamma}\,\delta u_{p_+} + \sqrt{2\mu}\,\delta v_{p_+}\\
\tau \frac{d}{dt}\,\delta q_+ &= -2\gamma'\,\delta q_+ - \sqrt{2}\Delta\gamma'\beta\,\delta p_0 + \sqrt{2}\gamma'\beta\,\delta q_0 + \sqrt{2\gamma}\,\delta u_{q_+} + \sqrt{2\mu}\,\delta v_{q_+}\\
\tau \frac{d}{dt}\,\delta p_0 &= -\sqrt{2}\gamma'\beta\,\delta p_+ + \sqrt{2}\Delta\gamma'\beta\,\delta q_+ - \gamma_0'\,\delta p_0 - \Delta_0\gamma_0'\,\delta q_0 +\\
&\quad +\sqrt{2\gamma_0}\,\cos\varphi_0\,\delta p_0^{\text{in}} + \sqrt{2\gamma_0}\,\text{sen}\,\varphi_0\,\delta q_0^{\text{in}} + \sqrt{2\mu_0}\,\delta v_{p_0}\ ,\\
\tau \frac{d}{dt}\,\delta q_0 &= -\sqrt{2}\Delta\gamma'\beta\,\delta p_+ - \sqrt{2}\gamma'\beta\,\delta q_+ + \Delta_0\gamma_0'\,\delta p_0 - \gamma_0'\,\delta q_0 -\\
&\quad -\sqrt{2\gamma_0}\,\text{sen}\,\varphi_0\,\delta p_0^{\text{in}} + \sqrt{2\gamma_0}\,\cos\varphi_0\,\delta q_0^{\text{in}} + \sqrt{2\mu_0}\,\delta v_{q_0}\ .
\end{aligned}
$$

$$\delta p_\pm = (\delta p_1 \pm \delta p_2)/\sqrt{2}\ , \qquad \delta q_\pm = (\delta q_1 \pm \delta q_2)/\sqrt{2} \qquad \beta = p/p_0$$

Quantum Properties of the OPO

$$\tau \frac{d}{dt} \delta p_- = -2\gamma' \delta p_- + \sqrt{2\mu}\, \delta v_{p_-}$$

$$\tau \frac{d}{dt} \delta q_- = \sqrt{2\mu}\, \delta v_{q_-}$$

The subspace related to the subtraction of the fields decouples from the sum and the pump fluctuations. However, q_- does not have any decay term, thus the solutions are not strictly stable. As a matter of fact, there is *phase diffusion* and the subtraction of the phases is unbounded. Nevertheless, this is a *slow* process and we will be interested in measuring phases with respect to the phase of the mean field (in other words, we will follow "adiabatically" the diffusion).

Instead of solving these equations in the time domain, we look in the frequency domain.