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Testing the entanglement of intense beams produced by a non-degenerate optical parametric oscillator

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Abstract

We propose a direct measurement of the quadrature correlations of signal and idler beams in a non-degenerate optical parametric oscillator operating above threshold. We investigate the experimental limits where quantum correlations can be observed, fulfilling an inseparability criterion for defining them as intense entangled beams. The use of optical cavities to access quadrature noise in this situation is studied, and its advantages over homodyne detection are discussed. We also consider the application of this entanglement and the quadrature noise measurement technique to quantum cryptography.

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1. Introduction

Entanglement of two states (e.g., a pair of light beams) is a purely quantum behavior, leading to non-locality, which is in the heart of the quantum information theory [1,2]. When the fields are in an entangled state, the non-classical behavior can be demonstrated by the violation of the Bell inequality [3,4]. For continuous variables, a Bell inequality is no longer directly applicable, but many authors have developed other relations that must be violated to show the inseparability of continuous variable systems [5], such as the proposal of Duan et al. [6], which we will refer to as the DGCZ criterion.

A well known example of entangled states are the twin photons produced by atomic fluorescence [7] or spontaneous parametric down-conversion [8,9]. For continuous variables, it has been demonstrated that squeezed states, combined in a beam splitter, can produce entangled states [10,11]. Experimentally, this source of entanglement has

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already been produced by the combination of pulsed squeezed beams generated by the $\chi^{(3)}$ nonlinearity in optical fibers [12].

Optical parametric oscillators (OPO) or amplifiers (OPA) can be sources of entangled states. Sub-threshold OPO's are shown to produce an entangled pair of fields, either in frequency degenerate OPO's [13] and non-degenerate OPO's [14]. EPR-like beams have also been produced from the combination of the outputs of two degenerate sub-threshold OPO's and used for quantum teleportation [15].

On the other hand, it has already been predicted that the OPO would still produce entangled states above threshold, generating intense entangled beams that would satisfy the DGCZ entanglement criterion. But there are difficulties in performing this kind of measurement. The theory predicts that signal and idler beams will be intensity-correlated and phase-anti-correlated [16,17]. While it is easy to measure intensity correlations leading to sub-shot noise fluctuations in the difference of beam intensities [18], the measurement of phase quadrature fluctuations often requires the use of a local oscillator for an homodyne detection [19].

Typical OPO's will produce non-degenerate signal and idler beams with a frequency separation that is a multiple of the cavity free spectral range [20], but the usual configuration used for measuring quadrature fluctuations can be implemented only if signal and idler fields are frequency degenerate, like in the case of squeezed vacuum [21].

In this article, we investigate the possibility of measuring entanglement between intense signal and idler beams even in the non-degenerate situation, taking into account typical experimental conditions such as a noisy pump beam and a detuned OPO cavity. Our calculations for the OPO indicate that there is an experimentally accessible region of parameters where entanglement can happen. Although excess noise in the phase of the pump field can destroy the entanglement, its intensity noise has little effect.

In order to measure phase fluctuations of signal and idler, we propose a self-homodyne technique that uses external cavities to independently rotate the phase of the fields' fluctuations relative to their mean values [22], leading to a direct conversion of phase fluctuations into intensity fluctuations [23]. By summing or subtracting the measured signal and idler quadrature fluctuations, correlation or anti-correlation between their quadratures can be seen, and the DGCZ criterion can be applied. This implementation allows a generalized quadrature measurement and noise correlation even if the beams are not degenerate, thus overcoming the difficulties associated with homodyne detection in this system.

We finally consider using this implementation for quantum secure key distribution according to a recently proposed protocol [24]. This requires a stronger condition for the correlation between amplitudes and anti-correlation between phases: both of them must be squeezed. We show that this can be the case for the non-degenerate OPO, therefore enabling its use for quantum cryptography.

One direct advantage of this system is that intense beams are easier to manipulate than squeezed vacuum. The beams' mean intensities are another parameter to check any violation of the secure channel. Besides, the use of optical cavities to access quadrature noise has advantages over homodyne detection in this situation. By making Alice and Bob's measurements independent of local oscillators, difficulties like their production and distribution are eliminated, simplifying the experimental setup.

We begin by a brief description of the DGCZ criterion, and then we review the expected results for a type-II OPO above threshold, studying the effect of the pump noise and cavity detuning on entanglement. The conversion of phase noise into intensity noise by a lossy cavity is studied, and the experimental proposal is then described, showing that the use of cavities allows a local selection of the measured quadrature noise of the entangled beams, thus establishing a secure channel for quantum key distribution.

2. Entanglement criterion

As described in [6], a quantum state composed by two modes, 1 and 2, is said to be separable if its density matrix ρ can be described by a statistical mixture of the product of normalized states of the systems 1 and 2 (ρ_{i1} and ρ_{i2}), $\rho = \sum_i p_i \rho_{i1} \otimes \rho_{i2}$ with positive probabilities p_i that satisfy $\sum_i p_i = 1$.

If the system is composed of two modes of the quantized electromagnetic field, we can define annihilation and creation operators for each mode, respectively a_j and $a_j^\dagger$ [25], where $j \in \{1, 2\}$ stands for the mode. These operators follow the usual commutation relation $[a_j, a_{j'}^\dagger] = \delta_{j,j'}$, and since they are not hermitian operators, the mean value will return a complex number that represents the complex field amplitude envelope. From these operators we can define a pair of hermitian operators, giving the quadratures of the field in the Fresnel representation:

$$q_j(t) = \mathrm{i}[\mathrm{e}^{\mathrm{i}\varphi} a^\dagger(t) - \mathrm{e}^{-\mathrm{i}\varphi} a(t)],$$
$$p_j(t) = [\mathrm{e}^{\mathrm{i}\varphi} a^\dagger(t) + \mathrm{e}^{-\mathrm{i}\varphi} a(t)], \qquad (1)$$

where the phase φ rotates the field in the plane of the Fresnel representation. We have then the commutation relation: $[p_j, q_{j'}] = 2\mathrm{i}\delta_{j,j'}$, that implies in the uncertainty relation: $\Delta^2 p_j \Delta^2 q_j \geqslant 1$. For a coherent state, $\Delta^2 p_j = \Delta^2 q_j = 1$, and for a squeezed state [25,26] we have, for instance, $\Delta^2 q_j > 1 > \Delta^2 p_j$. If we study two modes of the field, we can look for a vanishing commutator for a linear combination of the quadrature operators, that are called EPR-like operators of the system. As an example of a pair of EPR-like operators, we have

$$u = \frac{q_1 + q_2}{\sqrt{2}}, \quad v = \frac{p_1 - p_2}{\sqrt{2}}. \qquad (2)$$

In this case, the variables u and v commute, and therefore they can be determined simultaneously with arbitrary precision.

Duan et al. [6] have shown that if a system composed of two modes is separable then there is a lower bound for the variances of the measurements of u and v:

$$\langle \Delta^2 u \rangle + \langle \Delta^2 v \rangle \geqslant 2. \qquad (3)$$

Violation of this inequality is a sufficient condition for a quantum state to be considered inseparable, or, equivalently, for having an entanglement between modes 1 and 2. On the other hand, if we have gaussian states, it can be shown that violation of relation (3) is a sufficient and necessary condition for the inseparability of states.

Therefore one can verify the entanglement of two fields by measuring the variance of the sum of phase quadratures and subtraction of amplitude quadratures. We will now discuss the entanglement of the fields produced by a non-degenerate OPO, and study how its signature is affected by a noisy pump and a detuned cavity.

3. Entanglement of signal and idler fields

Strong intensity correlations between signal and idler beams produced in an above-threshold OPO have already been demonstrated with experimental data [18,27] that agrees with the usual treatment of the fields as classical values with stochastic fluctuations. This is equivalent to the quantum treatment using the density matrix, which is converted to a Wigner representation for the field values [28]. Similarly, there have been studies of phase anti-correlation between the output fields [29]. But, while intensity correlation is shown to be independent of cavity parameters like detuning and pump power and quite insensitive to pump noise, anti-correlation in phase fluctuations were predicted in only a very specific situation: zero cavity detuning and coherent pump [17], which is far from the usual situation observed in the laboratory.

We outline here a full treatment of the problem, showing that, although not completely independent of pump noise as intensity correlation is, phase anti-correlation can be obtained in normal experimental conditions, using the DGCZ criterion to demonstrate entanglement of a pair of macroscopic fields produced by an OPO above threshold.

Consider the complex field amplitudes, represented by the annihilation operators $\{a_0(t), a_1(t), a_2(t)\}$ for pump, signal and idler, respectively. Any field operator can be described in an equivalent way by its mean value and an operator for the field fluctuation taking the general form $a(t) = \alpha + \delta a(t)$, where $\alpha = \langle a(t) \rangle = |\alpha|\, \mathrm{e}^{\mathrm{i}\varphi}$. The intensity of each beam will be given by an average value and a real valued fluctuating term $I(t) = |\alpha|^2 + \alpha\delta a^\dagger(t) + \alpha^*\delta a(t)$, where terms of higher order in the fluctuation are neglected, since all fields are assumed intense ($|\alpha|^2 \gg 1$). Therefore, the intensity will be given by $I(t) = |\alpha|^2 + |\alpha|\delta p(t)$, where $\delta p(t) = [e^{i\varphi}\delta a^\dagger(t) + e^{-i\varphi}\delta a(t)]$ is obtained from Eq. (1). The conjugate operator $\delta q(t)$ defines the phase quadrature fluctuation of the field.

In our procedure, we will begin with the Langevin equations obtained from the Wigner representation of the fields, presented in [28,30], and change it into the equivalent description of fields' quadrature fluctuations $\delta p(t)$, $\delta q(t)$. We will be searching for a DGCZ-like correlation between signal and idler fields, Eq. (3), obtained directly from the sum and subtraction of the fields' quadratures, $p_\pm(t) = [p_1(t) \pm p_2(t)]/\sqrt{2}$ and $q_\pm(t) = [q_1(t) \pm q_2(t)]/\sqrt{2}$.

Changing the density operator representation of the system into the Wigner representation of the fields as complex variables, we can obtain the Langevin equations of the fields, in the form

$$\tau \frac{d}{dt} P = -\mathbf{A}P + \mathbf{B}P_{in}, \qquad (4)$$

where the vector of field fluctuations is $P = [\delta p_-(t), \delta q_-(t), \delta p_+(t), \delta q_+(t), \delta p_0(t), \delta q_0(t)]^T$. Here, τ is the cavity's round trip time, and the drift matrix $\mathbf{A}$ and the damping matrix $\mathbf{B}$ depend on cavity loss and detuning. The external fluctuations P_{in} are coupled to the OPO's cavity through the damping matrix. We will limit ourselves to the study of field fluctuations above oscillation threshold.

From the steady state values of the fields above threshold [20,28] we obtain the drift matrix

$$\mathbf{A} = \begin{bmatrix} 2\gamma & 0 & 0 & 0 & 0 & 0 \\ -2\gamma\Delta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma\Delta & -\sqrt{2}\gamma\eta & \sqrt{2}\gamma\Delta\eta \\ 0 & 0 & 0 & 2\gamma & -2\gamma\Delta\eta & -2\gamma\eta \\ 0 & 0 & \sqrt{2}\gamma\eta & \sqrt{2}\gamma\Delta\eta & \gamma_0 & -\gamma_0\Delta_0 \\ 0 & 0 & -\sqrt{2}\gamma\Delta\eta & \sqrt{2}\gamma\eta & \gamma_0\Delta_0 & \gamma_0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_- & 0 \\ 0 & \mathbf{A}_+ \end{bmatrix}, \qquad (5)$$

where the total cavity losses for pump (γ_0) and signal and idler fields (γ) are related to the coupling mirror transmissivity $T = 2\gamma$. If we have other spurious losses, they can be added to the mirror transmissivity when calculating the output fluctuations, but considering that only the field coming out through the coupling mirror will be detected, resulting in a linear degradation of the squeezing [19]. Δ and Δ_0 are the cavity detunings normalized by the internal loss ($\Delta = \varphi/\gamma$). The ratio η of the mean values of the signal (and idler) and pump fields (α, α_0) is given by

$$\eta = \frac{|\alpha|}{|\alpha_0|} = \sqrt{\frac{\gamma_0}{\gamma}} \left[\frac{\Delta\Delta_0 - 1 + \sqrt{\sigma - (\Delta + \Delta_0)^2}}{1 + \Delta^2} \right]^{1/2}, \qquad (6)$$

where σ is the relative pump power, normalized to threshold power on resonance ($\Delta = \Delta_0 = 0$). Signal and idler detunings are equal as a condition for stable oscillation [20]. Phase matching in the non-linear crystal is implicit in the normalized pump power [20], and will change the threshold power of the OPO and the phase difference of signal and idler to the pump field. The small bistability region close to the oscillation threshold [28] is hardly seen in CW operation, and will not be considered in our calculations.

The incoming fluctuations P_{in}, coupled to the cavity through its input mirror, can be simply considered as vacuum fluctuations for the signal and idler fields, but not for the pump, where the pumping laser fluctuations must be taken into account. The phase difference between the intracavity fields and the incoming fields is considered in the coupling matrix $\mathbf{B}$,

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_- & 0 & 0 \\ 0 & \mathbf{B}_+ & 0 \\ 0 & 0 & \mathbf{B}_0 \end{bmatrix}, \qquad (7)$$

with each submatrix $\mathbf{B}_j$ given by

$$\mathbf{B}_j = \sqrt{2\gamma_j} \begin{bmatrix} \cos(\varphi_j^{in} - \varphi_j) & -\sin(\varphi_j^{in} - \varphi_j) \\ \sin(\varphi_j^{in} - \varphi_j) & \cos(\varphi_j^{in} - \varphi_j) \end{bmatrix}. \qquad (8)$$

The relationship between the phases of intracavity fields φ_j and incident fields φ_j^{in}, for pump, signal and idler modes can be reduced to a simple form if we consider the incoming pump field as a reference and set its phase to zero, $\varphi_0^{in} = 0$. The incoming vacuum fluctuations in the sum and subtraction of signal and idler modes can be arbitrarily set to be in phase with the intracavity fields.

Therefore, we will have for the linear combination of signal and idler beams $\varphi_+^{\rm in} = \varphi_+$ and $\varphi_-^{\rm in} = \varphi_-$. Finally, the phase of the intracavity pump mode is found to be

$$\begin{aligned} e^{2i\varphi_+} &= \frac{\sqrt{\sigma}}{\sqrt{\sigma - (\Delta + \Delta_0)^2} - i(\Delta + \Delta_0)}, \\ e^{i\varphi_0} &= \frac{1 - i\Delta}{\sqrt{1 + \Delta^2}} e^{2i\varphi_+}. \end{aligned} \tag{9}$$

To complete the treatment, and find the output field fluctuations, we need to combine the intracavity field transmitted through the coupling mirror with the incident field. Once again, we have to consider the phase relations between incoming field, intracavity field and output field to maintain our definitions of field quadratures p and q. The output field fluctuations vector will be $P_{\rm out} = \mathbf{B}'P - \mathbf{B}''P_{\rm in}$, where the phase rotated coupling matrices for reflection ($\mathbf{B}''$) and transmission ($\mathbf{B}'$) of the fields are similar to the one defined in Eq. (7), but with the submatrices given by

$$\begin{aligned} \mathbf{B}'_j &= \sqrt{2\gamma_j} \begin{bmatrix} \cos(\varphi_j - \varphi_j^{\rm out}) & -\sin(\varphi_j - \varphi_j^{\rm out}) \\ \sin(\varphi_j - \varphi_j^{\rm out}) & \cos(\varphi_j - \varphi_j^{\rm out}) \end{bmatrix}, \\ \mathbf{B}''_j &= \begin{bmatrix} \cos(\varphi_j^{\rm in} - \varphi_j^{\rm out}) & -\sin(\varphi_j^{\rm in} - \varphi_j^{\rm out}) \\ \sin(\varphi_j^{\rm in} - \varphi_j^{\rm out}) & \cos(\varphi_j^{\rm in} - \varphi_j^{\rm out}) \end{bmatrix}. \end{aligned} \tag{10}$$

From these equations defining the time evolution of the intracavity field and its coupling to the external modes, we can obtain the noise spectrum of each quadrature as a function of the cavity parameters, pump power, pump noise and analysis frequency.

In frequency domain, we have $\tilde{P}(\Omega) = \int P(t) e^{i\Omega t\, dt}$. The matrix of the output noise spectra $\mathbf{V}_{\rm out} = \langle \tilde{P}_{\rm out}(\Omega) \tilde{P}_{\rm out}^{\rm T}(-\Omega) \rangle$ is given by

$$\begin{aligned} \mathbf{V}_{\rm out} &= \left[\mathbf{B}'(\mathbf{A} + i\Omega\mathbf{I})^{-1}\mathbf{B} - \mathbf{B}''\right] \\ &\quad \times \mathbf{V}_{\rm in}\left[\mathbf{B}'(\mathbf{A} - i\Omega\mathbf{I})^{-1}\mathbf{B} - \mathbf{B}''\right]^{\rm T}, \end{aligned} \tag{11}$$

where $\mathbf{V}_{\rm in} = \langle \tilde{P}_{\rm in}(\Omega) \tilde{P}_{\rm in}^{\rm T}(-\Omega) \rangle$.

From the Langevin equations of the fields we can observe that those for the subtraction of the field quadratures are uncoupled from those for the sum of the quadratures and for the pump field. Therefore, we can independently obtain the noise spectra of the subtraction and the sum of the output fields. Considering the input field spectrum normalized to the vacuum fluctuations, we have for the subtraction of the amplitudes and phase quadratures [17]

$$\begin{aligned} S_{p-}(\Omega) &= \langle \delta p_{\rm out-}(\Omega) \delta p_{\rm out-}(-\Omega) \rangle \\ &= 1 - \frac{4\gamma^2}{4\gamma^2 + \tau^2\Omega^2}, \\ S_{q-}(\Omega) &= \langle \delta q_{\rm out-}(\Omega) \delta q_{\rm out-}(-\Omega) \rangle \\ &= \frac{1}{S_{p-}(\Omega)}. \end{aligned} \tag{12}$$

Therefore, the fluctuations of the subtraction of the outcoming fields are in a state of minimum uncertainty, and will show squeezing in the amplitude quadrature. Moreover, they are independent of pump or cavity parameters. For the sum of the fields' fluctuations, the result is not so simple, but can be obtained from Eq. (11), showing their dependence on cavity detuning, pump power and pump noise.

In what follows, we present results for the noise spectrum of the sum of fields quadratures (S_{p+}, S_{q+}), considering that the pump field can be either in a coherent state, or with excess noise either in amplitude or phase (considering that these fluctuations are uncorrelated). While the fluctuations of the difference of fields will depend only on the analysis frequency (Ω) and cavity bandwidth for signal and idler ($\delta\omega = 2\gamma/\tau$), the fluctuation of the sum will also depend on the normalized pump power σ, cavity bandwidth for the pump ($\delta\omega_0 = 2\gamma_0/\tau$), cavity detuning for the pump (Δ_0), signal and idler (Δ) and pump noise (S_{p0}, S_{q0}).

Consider a simple case: resonance ($\Delta = \Delta_0 = 0$) and a coherent pump ($S_{p0} = S_{q0} = 1$), operating at twice the threshold power ($\sigma = 2$), the cavity coupling mirror having a small transmissivity for signal and idler ($T = 2\gamma = 2\%$) and a large one for the pump ($T = 2\gamma = 10\%$). In Fig. 1, the normalized noise for each one of the quadratures of the difference of the field fluctuations are presented.

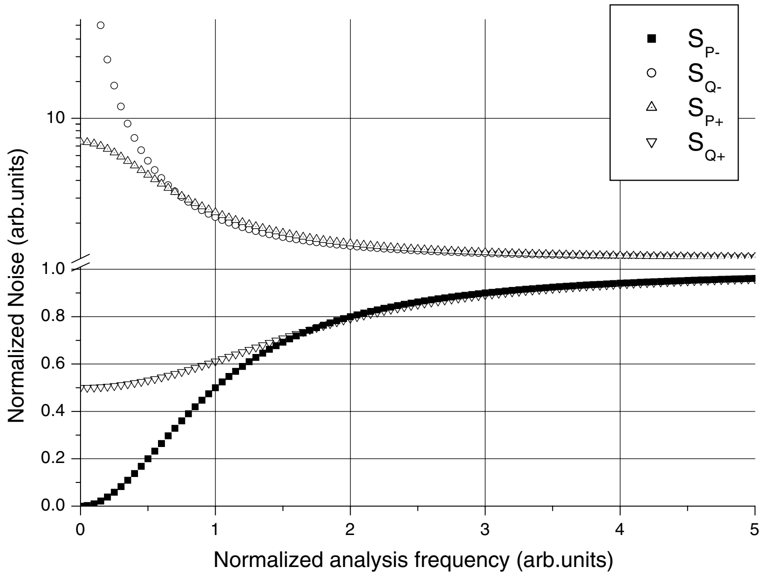

Fig. 1. Noise spectra of the fields correlations in an OPO, as a function of frequency. Analysis frequency normalized to the cavity bandwidth for the signal ($\delta\omega_1 = 2\gamma/\tau$). Parameters are $\gamma = 0.01$, $\gamma_0 = 0.05$, $\Delta = \Delta_0 = 0$, $\sigma = 2$.

As we can see, perfect intensity correlation between the fields can be obtained for a long integration time ($S_{p-} = 0$), and consequently the fluctuations of the conjugate variable (S_{q-}) diverge. This can be seen as a process of phase diffusion, already discussed in [17,31]. Energy conservation and phase matching imply that signal and idler phases are anti-correlated. Hence, the sum of phases is little affected by phase diffusion. The fluctuations in the sum of phases have a lower bound, as a function of analysis frequency, given by $S_{q+} = 0.5$. Lower values could be achieved as the threshold limit is approached ($\sigma \to 1$, $S_{q+} \to S_{p-}$), but the practical stability of the experimental setup is reduced. Of course, these values apply for a cavity without any spurious losses (the only loss is through the coupling mirror) and no losses in the detection process. But from these values we observe that an OPO can act as a source of entangled beams even with intense outputs, according to the DGCZ criterion, Eq. (3). Losses in the beam path and inside the cavity will linearly reduce the level of squeezing [19].

Perfect resonance and a pump laser with small noise are not the usual situation met in a laboratory. Squeezing in S_{p-} has been shown to be insensitive to these non-ideal laboratory conditions. As for the S_{q+} squeezing, we show that the effects of these imperfections are not so drastic, and entanglement can still be obtained in quite fair conditions of operation. In Fig. 2 we present the noise power and the absolute value of the correlation $C_{pq} = \langle \delta p_{\mathrm{out}+}(\Omega)\delta q_{\mathrm{out}+}(-\Omega)\rangle$ of the sum of the quadratures in typical measurement conditions ($\Omega = 2\gamma/\tau$, $\sigma = 2$), as a function of pump and signal detuning, for different pump noise conditions.

The non-zero values are the results for an oscillating OPO operating above threshold. In the first row, we see that although there is some dependence of noise power with detuning, it remains quite flat, with a minimum value of 2 for S_{p+} and a maximum of 0.5 for S_{q+}. Correlation between the quadratures occurs mainly in conditions of detuned cavity for the signal, being almost insensitive to pump detuning. As we will see, the C_{pq} correlation can mask the entanglement characterization of signal and idler fields, and its value should be taken into account when performing a measurement.

In the second row, we consider a pump laser with phase quadrature fluctuations, but shot noise limited intensity fluctuations. That is the typical

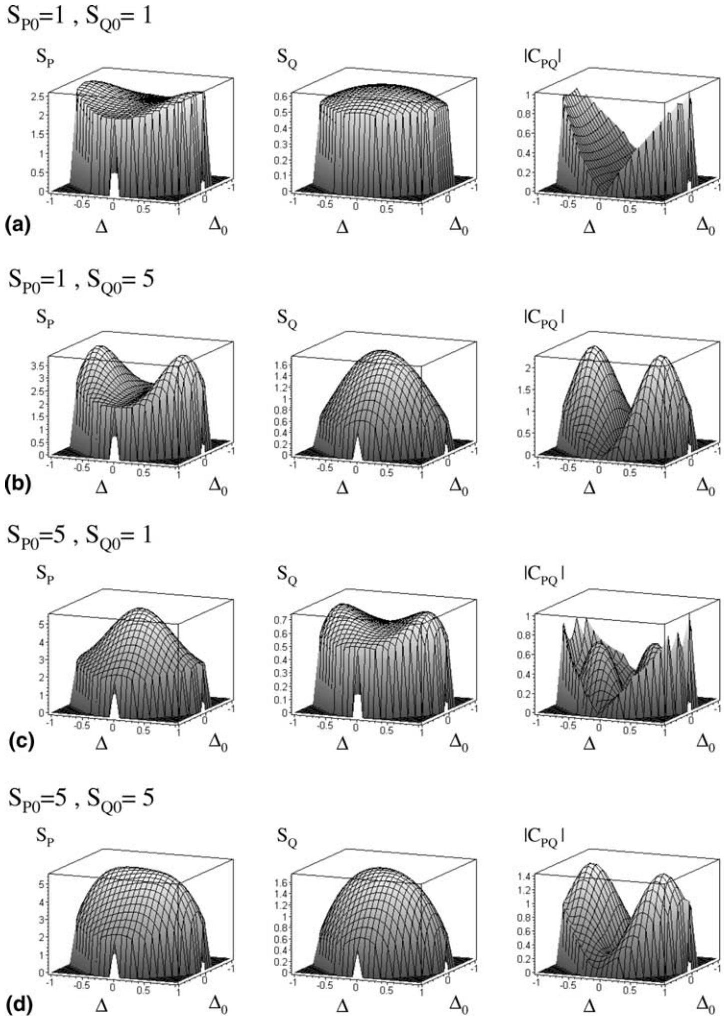

Fig. 2. Normalized noise of the field correlations in an OPO, as a function of the cavity detuning, for different pump noise conditions. S_P: noise of the sum of the amplitude quadratures p_+, S_Q: noise of the sum of the phase-related quadratures q_+, $|C_{pq}|$: absolute value of correlations between the sum of amplitude and phase-related fluctuations. Parameters are $\gamma = 0.01$, $\gamma_0 = 0.05$, $\Omega = 2\gamma/\tau$, $\sigma = 2$.

case of diode lasers [23]. We observe that the fluctuations can destroy the squeezing of S_{q+} at exact resonance; nevertheless, for detuned cavities close to the threshold limit, good squeezing is still achievable, and the DGCZ inequality is still violated. The third row shows a more convenient situation, where the pump has excess noise in amplitude fluctuations, but phase fluctuations are small. The effect in the S_q squeezing is small over the whole range of possible detuning values. Finally, with a noisy beam on both quadratures, the squeezing is much degraded, and the region where the DGCZ inequality is violated is really small, only very close to threshold.

We can conclude that, while the entanglement signature is quite stable under detuned cavity conditions, it is very sensitive to the phase noise of the pump. Therefore, to produce an entangled pair of intense beams in an OPO, we need a low noise pump. In the following section, we will describe how to make an independent measurement of the quadrature fluctuations of each field, allowing a local measurement of the field quadratures that can be applied, for instance, to quantum communications.

4. Phase rotation of noise ellipse

We are interested in accessing the quadrature fluctuations of both signal and idler beams to measure correlation and anti-correlation. Measuring quadrature fluctuations usually requires the use of a local oscillator, but, regarding the difficulties this technique faces when applied to a non-degenerate OPO, the use of an optical cavity can be more interesting. As first showed by Galatola et al. [22], an optical cavity can project phase quadrature fluctuations into amplitude quadrature fluctuations, which is an easily measurable quantity, for some range of cavity detuning and analysis frequency.

In our case, the fields to be analyzed are the signal and idler fields produced by a type-II OPO: they can thus be separated by polarization and each one sent to its own analysis cavity. We treat the physical problem as schematized in Fig. 3. The field to be analyzed (input beam), signal or idler, is injected into a Fabry–Perot cavity, generating reflected and transmitted beams. In this process, owing to imperfections like a residual transmission in the output mirror of the cavity, vacuum leaks inside it and contributes to the noise of reflected and transmitted fields.

In what follows, we treat this problem explicitly relating the reflected beam quadrature fluctuations to the input beam ones, taking into account the input vacuum. All fields (input beam, input vacuum, reflected and transmitted beams) can be described as a stable mean value, with a well defined frequency ω_0, presenting small fluctuations in time with frequencies $\Omega \ll \omega_0$, so that their annihilation operators can be written in the general form

$$a(t) = \alpha(t) + \delta\tilde{a}(t) = [\alpha + \delta a(t)]\mathrm{e}^{-\mathrm{i}\omega_0 t}, \tag{13}$$

which have Fourier components

$$a(\omega) = \int a(t)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t, \qquad a^\dagger(\omega) = \int a^\dagger(t)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t = [a(-\omega)]^\dagger \tag{14}$$

(unless otherwise specified, integrals are made from $-\infty$ to ∞).

In the situation we are considering, the quantum equations relating reflected (a_{out}) and transmitted (b_{out}) beams to input beam (a_{in}) and input vacuum (b_{in}) have the same form as the classical ones. In frequency domain:

$$a_{\mathrm{out}}(\omega) = r(\omega)a_{\mathrm{in}}(\omega) + t(\omega)b_{\mathrm{in}}(\omega), \qquad b_{\mathrm{out}}(\omega) = t(\omega)a_{\mathrm{in}}(\omega) - r'(\omega)b_{\mathrm{in}}(\omega), \tag{15}$$

where $r(\omega)$, $t(\omega)$ are the reflection and transmission coefficients of a Fabry–Perot cavity for the input beam, and $r'(\omega)$ is the reflection coefficient for the input vacuum:

$$r(\omega) = \frac{r_1 - r_2\exp[\mathrm{i}2\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}{1 - r_1 r_2\exp[\mathrm{i}2\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}; \quad t(\omega) = \frac{t_1 t_2\exp[\mathrm{i}\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}{1 - r_1 r_2\exp[\mathrm{i}2\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}; \quad r'(\omega) = \frac{r_2 - r_1\exp[\mathrm{i}2\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}{1 - r_1 r_2\exp[\mathrm{i}2\pi(\omega-\omega_c)/F_{\mathrm{sr}}]}, \tag{16}$$

with F_{sr} being the free spectral range of the cavity, ω_c, its resonance frequency, and r_1, r_2, t_1 and t_2, its mirrors' reflection and transmission coefficients for amplitudes.

As the vacuum mean value β_{in} is null, the average of these equations gives

Fig. 3. Physical situation being considered.

$$\alpha_{\text{out}} = r(\omega_0)\alpha_{\text{in}}, \qquad \beta_{\text{out}} = t(\omega_0)\alpha_{\text{in}}, \tag{17}$$

showing that the transmitted field mean value is rotated through the action of the cavity when compared to the input beam (i.e., it gains a phase). In order to assure the same quadrature is being compared in both input and output fields, this fact must be taken into account when defining the quadrature operators.

Therefore, we define, respectively, $p(t)$ and $q(t)$ as the input beam amplitude and phase quadratures, $x(t)$ and $y(t)$ as the input vacuum quadratures, $P(t)$ as the reflected field amplitude quadrature and $P'(t)$ as the transmitted field one, in the following way: if the input beam mean value is chosen real for simplicity ($\alpha_{\text{in}} = \alpha^*_{\text{in}}$), $r(\omega_0) = |\, r(\omega_0)\,|\, \mathrm{e}^{\mathrm{i}\phi_0}$ and $t(\omega_0) = |\, t(\omega_0)\,|\, \mathrm{e}^{\mathrm{i}\phi'_0}$, we have, in the interaction picture,

$$\begin{aligned}
p(t) &= a_{\text{in}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} + a^\dagger_{\text{in}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t};\\
q(t) &= -\mathrm{i}[a_{\text{in}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} - a^\dagger_{\text{in}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t}];\\
x(t) &= b_{\text{in}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} + b^\dagger_{\text{in}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t};\\
y(t) &= -\mathrm{i}[b_{\text{in}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} - b^\dagger_{\text{in}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t}];\\
P(t) &= \mathrm{e}^{-\mathrm{i}\phi_0}a_{\text{out}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} + \mathrm{e}^{\mathrm{i}\phi_0}a^\dagger_{\text{out}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t};\\
P'(t) &= \mathrm{e}^{-\mathrm{i}\phi'_0}b_{\text{out}}(t)\mathrm{e}^{\mathrm{i}\omega_0 t} + \mathrm{e}^{\mathrm{i}\phi'_0}b^\dagger_{\text{out}}(t)\mathrm{e}^{-\mathrm{i}\omega_0 t}.
\end{aligned} \tag{18}$$

In frequency domain, we define the Fourier transforms of these operators, so that their fluctuations can be written in terms of fluctuations of annihilation and creation operators in the interaction picture. For instance, we have for the transmitted field amplitude quadrature,

$$\begin{aligned}
P(\Omega) = \int P(t)\mathrm{e}^{\mathrm{i}\Omega t}\,\mathrm{d}t \quad &\Rightarrow \delta P(\Omega)\\
&= \mathrm{e}^{-\mathrm{i}\phi_0}\delta a_{\text{out}}(\Omega) + \mathrm{e}^{\mathrm{i}\phi_0}\delta a^\dagger_{\text{out}}(-\Omega),
\end{aligned} \tag{19}$$

where Ω is the analysis frequency.

For the annihilation operators' fluctuations, the first of Eq. (15) becomes

$$\delta\tilde{a}_{\text{out}}(\omega) = r(\omega)\delta\tilde{a}_{\text{in}}(\omega) + t(\omega)\delta\tilde{b}_{\text{in}}(\omega). \tag{20}$$

But, according to our definitions, $\delta a(\Omega) = \delta\tilde{a}(\Omega + \omega_0)$ and $\delta a^\dagger(\Omega) = \delta\tilde{a}^\dagger(\Omega - \omega_0)$, so that, using Eq. (20) in Eq. (18), it is possible to write $\delta P(\Omega)$ and $\delta P'(\Omega)$ in terms of $\delta a_{\text{in}}(\Omega)$, $\delta b_{\text{in}}(\Omega)$ and their adjoint operators. Using the first four equations of Eq. (18), we may express $\delta P(\Omega)$ in terms of input beam's quadratures fluctuations, obtaining

$$\begin{aligned}
\delta P(\Omega) = g_1(\Omega)\delta p(\Omega) &+ \mathrm{i}g_2(\Omega)\delta q(\Omega)\\
&+ g_3(\Omega)\delta x(\Omega) + \mathrm{i}g_4(\Omega)\delta y(\Omega),
\end{aligned} \tag{21}$$

with

$$\begin{aligned}
g_1(\Omega) &= \frac{1}{2}\left[\mathrm{e}^{-\mathrm{i}\phi_0}r(\omega_0+\Omega) + \mathrm{e}^{\mathrm{i}\phi_0}r^*(\omega_0-\Omega)\right],\\
g_2(\Omega) &= \frac{1}{2}\left[\mathrm{e}^{-\mathrm{i}\phi_0}r(\omega_0+\Omega) - \mathrm{e}^{\mathrm{i}\phi_0}r^*(\omega_0-\Omega)\right],\\
g_3(\Omega) &= \frac{1}{2}\left[\mathrm{e}^{-\mathrm{i}\phi_0}t(\omega_0+\Omega) + \mathrm{e}^{\mathrm{i}\phi_0}t^*(\omega_0-\Omega)\right],\\
g_4(\Omega) &= \frac{1}{2}\left[\mathrm{e}^{-\mathrm{i}\phi_0}t(\omega_0+\Omega) - \mathrm{e}^{\mathrm{i}\phi_0}t^*(\omega_0-\Omega)\right].
\end{aligned} \tag{22}$$

In Fig. 4, we present curves of the coefficients appearing in Eq. (21) for various analysis frequencies Ω as a function of analysis cavity detuning. In particular, we notice that it is only possible to completely rotate the noise ellipse (i.e., $|g_1(\Omega)|$ must go to zero for some value of Ω) when

$$\Omega \geqslant \sqrt{2}\,\delta\omega, \tag{23}$$

where $\delta\omega$ is the cavity bandwidth (FWHM), in agreement with [22]. Also noteworthy is the fact that, for common analysis frequencies Ω, the maximum of $|g_2(\Omega)|$ (and, consequently, the minimum of $|g_1(\Omega)|$) occurs approximately at the half maximum of the mean transmitted intensity. In the case of a lossy cavity, we can observe that, as the analysis frequency is increased, the vacuum contribution to the amplitude fluctuations, coupled by g_3 and g_4, will be reduced close to the cavity resonance. This is easily understood when one remembers that the noise term can be seen as a contribution of two sidebands of frequencies $\omega_0 \pm \Omega$. When Ω increases, these sidebands are farther detuned from resonance and therefore do not couple to the cavity field.

An analogous calculation reveals that the transmitted field amplitude fluctuations $\delta P'(\Omega)$ are related to the input fluctuations by a relation similar to Eq. (21). Unfortunately, a careful analysis shows that the noise ellipse is never completely

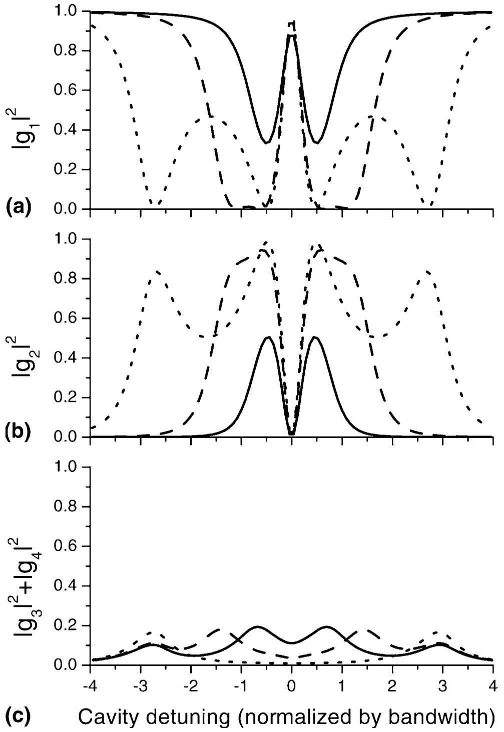

Fig. 4. These curves show the behaviour of the coefficients appearing in Eq. (21) as a function of cavity detuning (in units of cavity bandwidth). A finesse $F = 270$ is assumed. Continuous line: $\Omega = \delta\omega/\sqrt{2}$; dashed line: $\Omega = \sqrt{2}\,\delta\omega$; dotted line: $\Omega = 2\sqrt{2}\,\delta\omega$. (a) $|g_1(\Omega)|^2$, (b) $|g_2(\Omega)|^2$, (c) $|g_3(\Omega)|^2 + |g_4(\Omega)|^2$.

rotated in the transmitted beam, regardless of the analysis frequency or the cavity detuning.

The reflected beam's amplitude quadrature noise spectrum [32,33] $S_P(\Omega) = \langle \delta P(\Omega)\delta P^{\dagger}(\Omega')\rangle$ is given by

$$S_P(\Omega) = |g_1(\Omega)|^2 S_p(\Omega) + |g_2(\Omega)|^2 S_q(\Omega) + 2\mathrm{Im}\{g_1(\Omega)g_2^*(\Omega)C_{pq}(\Omega)\} + |g_3(\Omega)|^2 + |g_4(\Omega)|^2, \quad (24)$$

where $S_p(\Omega)$ and $S_q(\Omega)$ are the input beam p and q quadratures' noise spectra, and $C_{pq}(\Omega)$ is their correlations. $S_x(\Omega)$ and $S_y(\Omega)$ are normalized vacuum fluctuations, therefore equal to 1.

The correlation C_{pq} has the effect of making S_P asymmetric as a function of the analysis cavity detuning, for its coefficient in Eq. (24) is antisymmetric. Nevertheless, we will be interested in a cavity detuning for which $|g_1| \approx 0$ and, consequently, the effect of C_{pq} will be minimized.

5. Experimental setup

We propose the experimental setup as shown in Fig. 5. The two beams created in a type-II OPO, signal and idler, are separated by polarization using a polarizing beam splitter (PBS) just after

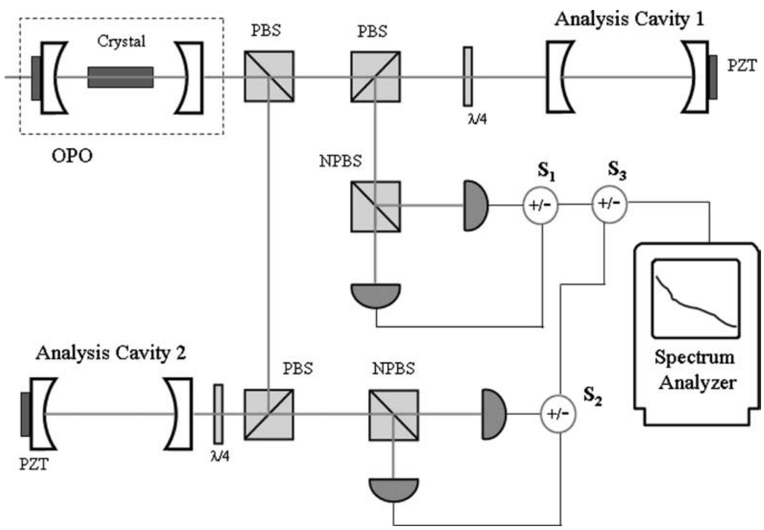

Fig. 5. Experimental setup.

they emerge from the OPO's cavity. Each beam passes through an optical circulator made by a quarter waveplate and a PBS. The field is reflected by a nearly confocal cavity after mode matching by a set of lenses (not shown). The reflected beams are sent to balanced detection setups, made by non-polarizing 50/50 beam splitters (NPBS) and two pairs of balanced photodetectors.

Each balanced detection has a sum/subtraction circuit (S_1 and S_2) for the measured photocurrents. A third circuit (S_3) is used to combine the resulting signals coming from the two detections. Shot noise is measured with S_1 and S_2 in subtraction position. Noise in each beam is measured with both S_1 and S_2 in sum position; these noises can then be summed or subtracted using S_3, where the photocurrent fluctuation will be $\delta P_{\pm}(\Omega) = \delta P_1(\Omega) \pm \delta P_2(\Omega)$, the combination of the signal and idler fluctuations after the transformation through the cavity reflection given by Eq. (21). The noise power is measured in a spectrum analyzer.

When the cavities are far away from resonance, the quadrature being measured is the amplitude. Correlation between amplitude quadrature fluctuations is one of the EPR variables of the pair and, for an OPO, it is well known to be squeezed. The other EPR variable is the anti-correlation between phase quadratures, obtained by a synchronous sweep of both cavities length while measuring the noise of the sum of the photocurrents, looking for a dip of the noise below the shot noise level. This measurement can also be made with stable cavities locked at the side of the transmission peak, making a direct measurement of the anti-correlation of the phase quadratures. Length fluctuations will add some information of the amplitude quadrature, reducing the squeezing level. As far as we can keep the cavity locked in the side of the peak, with usual techniques, this contribution can be neglected.

In order to characterize entanglement of signal and idler while the cavity length is swept, it is experimentally desirable that S_{P+} assumes squeezed values on both sides of cavity resonance, for detunings where $g_1(\Omega) \approx 0$. In this case, even if C_{PQ} causes asymmetries, the result can still be trusted.

Besides the stability of the cavities, it is important that the analysis cavities have very low spurious losses, reducing the amount of (uncorrelated) vacuum fluctuations added in the reflected fluctuations through the coefficients g_3 and g_4 in Eq. (21). The range of frequencies where entanglement can be characterized in this system has, as a lower bound, the bandwidth of the analysis cavity for a full ellipse rotation (according to Eq. (23)), and the diffusion of the phase difference, producing excess noise in the measurement, and as an upper bound, the OPO cavity bandwidth for signal and idler, that will determine the range of values where squeezing is still clearly measurable. With a careful project of the cavities, a range of tens of MHz can be achieved. Higher ranges can be implemented with very short OPO cavities, increasing its bandwidth.

6. Application to quantum cryptography

With the proposed setup, it is easy to imagine an implementation of the quantum communication protocol suggested in [24]. Alice and Bob have a cavity each one, and establish, through a classical channel, the analysis frequency they are going to look at and the synchronicity of the measurement (clock). A third part has the OPO, and can send each one of the entangled beams to Alice and Bob, that will perform, locally, a sequence of quadrature fluctuations measurements. Each one can choose randomly the quadrature to measure, storing the information, and Bob, for instance, sends the values of his photocurrent to Alice. Alice then compares the answers and checks whether they have chosen the same quadrature looking at the noise correlations that are below shot noise. Alice will return to Bob only the information of which events in the sequence were coincident. Establishing a binary relation of 0 and 1 to the quadratures, now Alice and Bob share a randomly generated key they can use to encrypt a message that is shared through the classical channel.

Any attack on the system can be noticed by a reduction in the squeezing level in the correlation measurement, or an increase in the error rate (bad quadrature choices of Alice and Bob), that should be around 50%. In this case, the output beams of an OPO are in the order of a few mW, and can be easily detected. The mean values of the beams are also a tool to improve the quality of the detection, assuring a good mode matching to the analysis cavity, and an extra information on the possibility of an attack, which is an advantage over entanglement with squeezed vacuum.

Finally, using analysis cavities to access quadrature noise has some implementation advantages over homodyne detection schemes in quantum cryptography. Alice and Bob need a local oscillator, that has to be phase-locked to the incoming field, in order to perform the homodyne detection. If they are using entangled intense fields, either the OPO source has to send them the reference, or they will need to tap part of the entangled fields to seed their oscillators, degrading the entanglement characterization. For entangled squeezed vacuum, a distinct channel for sending the local oscillator is needed, making it sensitive to phase perturbations in the beam path.

On the other hand, analysis cavities allow each station to perform its measurement independently of the source. Any non-dispersive phase perturbation in the entangled beam path will affect simultaneously their mean values and their fluctuations, thus not disturbing the entanglement characterization. Besides, the experimental setup is simplified in comparison to the homodyne detection.

CW OPO's operating above threshold are therefore a reliable source of entangled fields, the communication bit rate being only limited by its cavity bandwidth. Frequency degeneracy of signal and idler beams is not an issue since a local self-homodyne technique is used in this implementation.

7. Conclusion

We propose a direct measurement of the entanglement of the intense beams produced by a non-degenerate type-II OPO operating above threshold, showing that entanglement is still preserved in typical experimental conditions, such as a detuned cavity and a noisy pump.

As entanglement is charaterized by the violation of the DCGZ inequality, a measurement of phase quadrature noise of signal and idler fields is needed. The use of analysis cavities to access phase noise allows a non-degenerate operation, and avoids some difficulties related to the use of local oscillators.

This setup can be used in secure key distribution through entanglement. Possible applications of this entanglement can also be expected in quantum teleportation and other implementations of quantum cryptography.

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