"Variable Friction Force" Experiment Script - Part I

A) Introduction

We recorded the motion of a coin that is thrown and slides over an inclined panel covered with a checkered paper, which allows measuring the coordinates of its trajectory. In some shots, a transparent acrylic plate covered the grid, in others the coin slides directly onto the paper. The set of images to carry out this experiment is formed by frames from one video of those launches, which allow you to follow the successive positions of the coin and thus determine its displacements. The proposed work consists in reading the position of the coin along time, deducing the physical quantities involved in this movement and interpreting the relations between them.

The analysis of this experiment is divided into two parts. In this first one, the kinematic quantities and the resistive force will be determined at various points of the coin's trajectory. In the second, we will develop a theoretical model of the system that will allow us to calculate the coin trajectory and compare it with the measurement.

B) Analysis procedure

B1. The filmed arrangement is real, therefore friction between the coin and the plane plays a central role in the movement. Watch the video available on the Presentation (*Apresentação*) tab of the experiment page and reflect on the questions below. Make a note of your hypotheses and queries.

i. What is the shape of the coin trajectory?
ii. Is it possible to predict this trajectory?
iii. What physical quantities would you have to know for this prediction? Build your explanations based on Newton's laws and Amontons' laws on dry contact friction.

Also watch the close-up videos (available on the Videos (*Vídeos*) tab, accessible from the Films and Frames (*Filmes e Quadros*) tab of the experiment page) and check if the movement occurred according to your predictions.

B2. Observe the images of the set assigned to you, which might be similar to Figure 1. The paper is divided into large squares, delimited by thick lines every 1 cm, and into small squares, by thinner lines every 0.2 cm. You do not need to use SI base quantities, and in this experiment, it is natural to measure distances in centimeters. Record the adopted unit in your worksheet, as you will be able to use other units in different parts of the analysis.

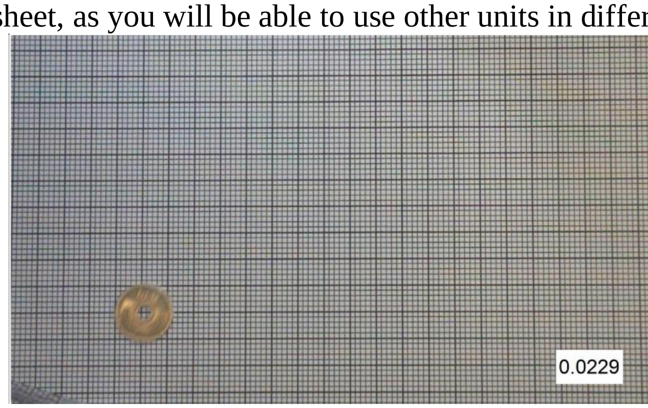

Figure 1. One of the images of the coin over the checkerboard in set A1.5.
The time is marked in the text box on the right, $t_i = 0.0229$ s. See the text for other details.

Choose an xOy reference system to read the coordinates of the center (hole) of the coin at each instant of time. It is practical to choose as the origin of the reference system the crossing point of two thick lines, close to the place where the movement of the coin begins. Henceforth, x identifies the horizontal coordinate oriented to the right, and y, the coordinate perpendicular to Ox that is on the plane, oriented upwards.

Open a spreadsheet on the computer and prepare it to enter the data, a row for each instant of time and a column for each physical quantity, plus one to number the successive images with integers *i* in sequence, starting with 1. Assemble the table with the instants of time t_i and the respective positions $x(t_i)$ and $y(t_i)$ of the center of the coin for each image *i*. In order to facilitate the reading process, the mouse cursor is shaped like a crosshair: center it on the coin and use its cross-shaped lines to determine the coordinates. Take half of the smallest division of the scale that you can read in the grid as the standard deviation of the position measure, which should give $\sigma_x = \sigma_y = \sigma$, with σ in the range 0.02 to 0.04 cm. Ignore the uncertainty in time.

B3. Build the graphs of the coordinates $x(t)$ and $y(t)$ as functions of time t. Remember to include uncertainty bars. Graph the trajectory of the coin, *y* vs. *x*, remembering that, in this case, *x* and *y* must be plotted on the same scale.

B4. In the worksheet built in item **B2**, calculate the velocity components in the O*x* and O*y* directions. As the time interval between successive images of the set is always the same, the horizontal velocity is calculated as

$$v_x(t_i) \cong \bar{v}_x(t_{i-1} \leq t \leq t_{i+1}) = \frac{x(t_{i+1}) - x(t_{i-1})}{t_{i+1} - t_{i-1}} \qquad (1)$$

since the average instant $\bar{t}_i$ of the time interval $t_{i-1} \leq t \leq t_{i+1}$ of this set of images is

$$\bar{t}_i = \frac{t_{i-1} + t_{i+1}}{2} = t_i \qquad (2)$$

This approximation is very good, because the time interval $[t_{i-1}; t_{i+1}]$ is small; see the complementary guide http://www.fep.if.usp.br/~fisfoto/guias/derivada_numerica.pdf for a more detailed explanation. As the calculation of formula (1) needs an image before and another after the instant t_i, it is not possible to calculate the velocity for the first and last images.

Proceed in the same way to calculate $v_y(t_i)$.

B5. Determine the standard deviation of the velocity. Ignoring the uncertainty in time, the rules of propagation of the standard deviation in the position give (see the uncertainty guide for details):

$$\sigma_{vx} = \sigma_{vy} = \sigma_v = \frac{\sqrt{2}\,\sigma_x}{t_{i+1} - t_{i-1}}$$

the same for the two components and all instants, since $t_{i+1} - t_{i-1}$ is constant for all pairs of images.

B6. Determine the magnitude of the coin's velocity at each instant, given by:

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} \qquad (3)$$

B7. Build the graphs of the projections and the velocity module as functions of time, $v_x(t)$, $v_y(t)$ and $v(t)$, respectively. Don't forget to plot the uncertainty bars.

B8. With the accelerations $a_x(t_i)$ and $a_y(t_i)$ calculated by expressions similar to those of eq. (1), use Newton's 2nd Law to get the resultant force projections,

$$\overrightarrow{F_R} = F_x\hat{\imath} + F_y\hat{\jmath} \qquad (4)$$

with $\hat{\imath}$ and $\hat{\jmath}$ the unit vectors in the directions O*x* and O*y*, respectively. Find the mass of the coin in the table "*Massa da Moeda*" from the Films and Frames (*Filmes e Quadros*) tab; clicking on the cell below the legend opens an image of the scale used in the measurement.

B9. Compute the magnitude of the resultant force at each instant, which is given by:

$$\left|\vec{F}_R\right| = \sqrt{F_x^2 + F_y^2} \tag{5}$$

B10. Build the graphs of the magnitude of the resultant force on the coin and its projections O*x* and O*y* as functions of time, $\left|\vec{F}_R\right|(t)$, $F_x(t)$, and $F_y(t)$. Don't forget to plot the uncertainty bars.

B11. Find the angle formed by the plane with the horizontal, θ, in the table *Inclinação do Plano* in the tab *Filmes e Quadros*, which leads to an image of measurements made with a protractor. Pay attention to the protractor, as two pieces of equipment were used that measure the angle in relation to different references. The angle $\theta_{\text{protactor}}$ read on the scale refers to the inclination with respect to the normal to the plane in one of them and, in the other, to the horizontal. In the calculations below, θ is the angle formed between the inclined plane and the horizontal, which is $90° - \theta_{\text{protractor}}$ in a case and $\theta_{\text{protactor}}$, in the other.

B12. Consider the resultant force:

$$\overrightarrow{F_R} = \vec{f} + \vec{N} + \vec{P} \tag{6}$$

where $\vec{f}$, $\vec{N}$ and $\vec{P}$ are the resistive, normal and weight forces, respectively. Once the resultant projections are separated in the directions O*x* and O*y*, determine the projections of the resistive force, which are given by:

$$f_x = F_x \tag{7}$$

$$f_y = F_y + mg \operatorname{sen} \theta \tag{8}$$

where m is the mass of the coin and g the magnitude of the local acceleration of gravity (use $g = 9{,}79$ m/s^2 for São Paulo).

Note that in Equation (8) the sign of $mg \operatorname{sen} \theta$ comes from the orientation of the coordinate axes as described in the item **B2**; if you adopted another orientation, you may get a different signal. When propagating uncertainties, ignore those corresponding m, g and θ.

B13. Determine the magnitude of the resistive force, which is given by:

$$f = \left|\vec{f}\right| = \sqrt{f_x^2 + f_y^2} \tag{9}$$

B14. Build a graph of the resistive force modulus as a function of time, $f(t) = \left|\vec{f}\right|(t)$. Don't forget to plot the uncertainty bars.

You should have built a table similar to Table 1 below, where the standard deviations of the various quantities of are calculated in the last row.

Table 1. Table template for the analysis of the coin trajectory.
No need for another column to $\bar{t}_i$, then $\bar{t}_i = t_i$ for all i.

i	**t** (s)	**x** (cm)	**y** (cm)	$\mathbf{v_x}$ (cm/s)	$\mathbf{v_y}$ (cm/s)	**v** (cm/s)	$\mathbf{a_x}$ (cm/s²)	$\mathbf{a_y}$ (cm/s²)	$\mathbf{F_x}$ (g.cm/s²)	$\mathbf{F_y}$ (g.cm/s²)	$\mathbf{F_R}$ (g.cm/s²)	$\mathbf{f_x}$ (g.cm/s²)	$\mathbf{f_y}$ (g.cm/s²)	**f** (g.cm/s²)
1	###	###	###											
2	###	###	###	###	###	###								
3	###	###	###	###	###	###	###	###	###	###	###	###	###	###
...	###	###	###	###	###	###	###	###	###	###	###	###	###	###
29	###	###	###	###	###	###	###	###	###	###	###	###	###	###
30	###	###	###	###	###	###								
31	###	###	###											
standard deviations	0	0,04	0,04	###	###	###	###	###	###	###	###	###	###	###

C) Synthesis layout

With respect to this first part of the experiment, you must only submit a short description of the experimental results, which will be checked, corrected and returned to you, in order to ensure an adequate basis for the final report. On the tab *Apresentação* from the experiment page, there is a document template for this synthesis, with the requested sections.

C1. *Identification*: list the names of group members and identify the analysed set of images.

C2. *Initial Expectations:* give a brief description of the experiment and record your expectations and predictions after reflecting on the experiment, as suggested by the item **B1**.

C3. *Data Obtained:* present the graphics requested in the item **B3** and their respective interpretation. Check that you have expressed the values of the quantities in appropriate units, as well as that you have inserted uncertainty bars in all graphs.

C4. *Data analysis:* present the table with the data and calculations, which should be similar to the **Table 1** of example above; check the significant digits. Present the graphics requested in the items **B7**, **B10** and **B14** and interpret them. Check that you have expressed the values of the quantities in appropriate units and with the appropriate number of significant figures, as well as that you have inserted uncertainty bars in all graphs.

C5. *Discussion*: in this section, explain *what it looks like and why it happens* – for now, it's not worth saying whether you've found answers compatible with those of theoretical formalism. Use your own words and ideas, rescuing, where necessary, comments in item **C2**.

- **i.** Describe the path of the coin on the inclined plane. Comment if your guesses for the questions on item **B1** were correct; this comment will not be graded, but this was the motivating question, and we hope it engaged you.
- **ii.** List the quantities that influence the trajectory of the coin.
- **iii.** List the quantities that, if modified, change the shape of the trajectory. What would be the change in each case?
- **iv.** List the forces acting on the coin as it moves. Sketch the free-body diagram and explain, in your own words, what the friction force vector along the path would look like.
- **v.** Looking at the forces you cited in the item **iv** above, which one(s) remains constant? Which one(s) varies? Remember that force is a vector.