Cold Ions and their Applications for Quantum Computing and Frequency Standards

- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- **Quantum computer: basics, gates, algorithms, future challenges**
- Ion clocks: from Ramsey spectroscopy to quantum techniques

Ferdinand Schmidt-Kaler
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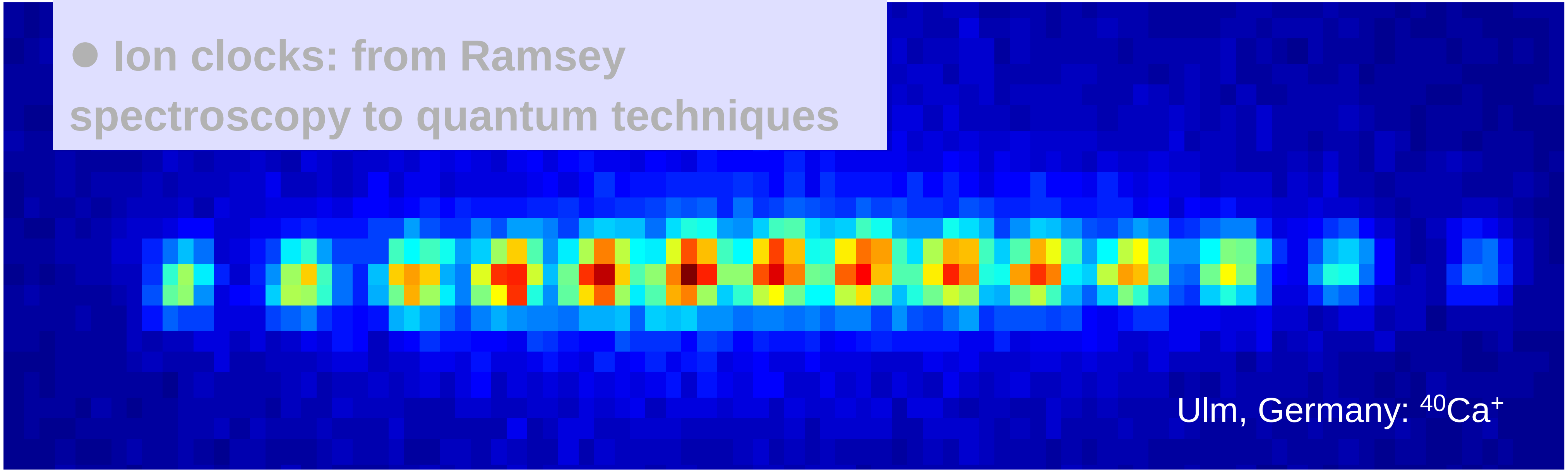
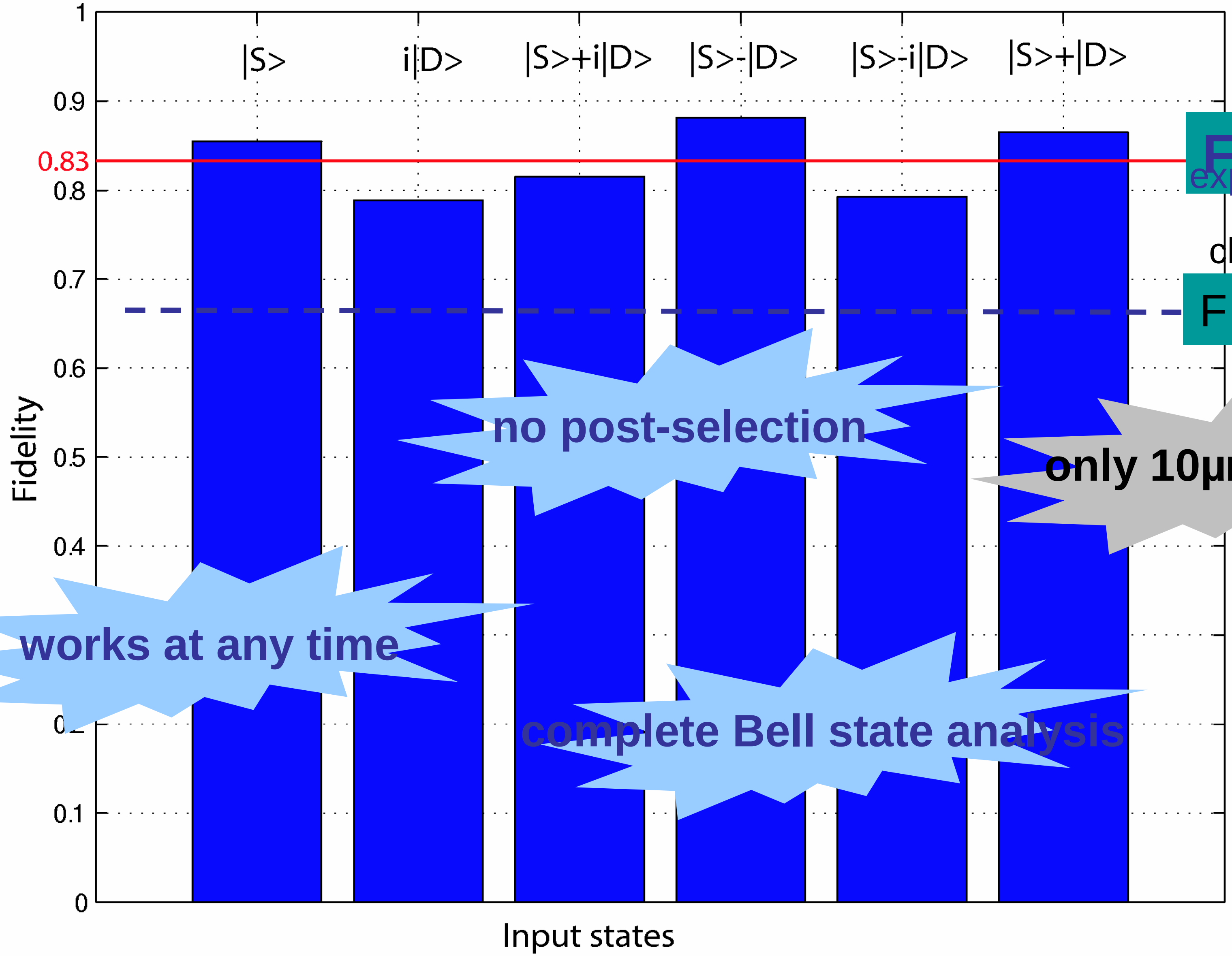
www.quantenbit.de



Ulm, Germany: $^{40}Ca^+$

Teleportation „on demand“ : Results



$F_{exp.} = 0.83$

$F_{clas.} = 0.67$

no post-selection

only 10µm distance

works at any time

complete Bell state analysis

„Trapology“ for Boulder Teleportation

Teleportation / Boulder

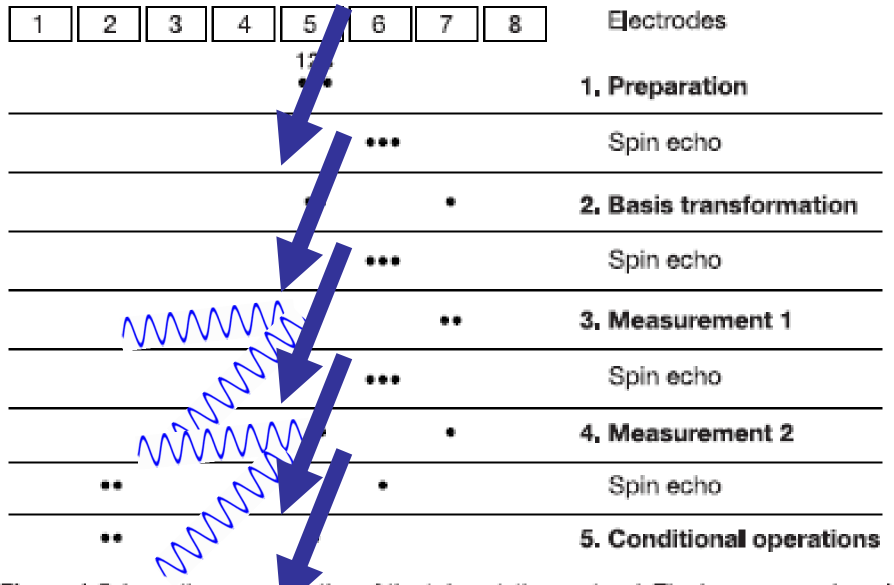
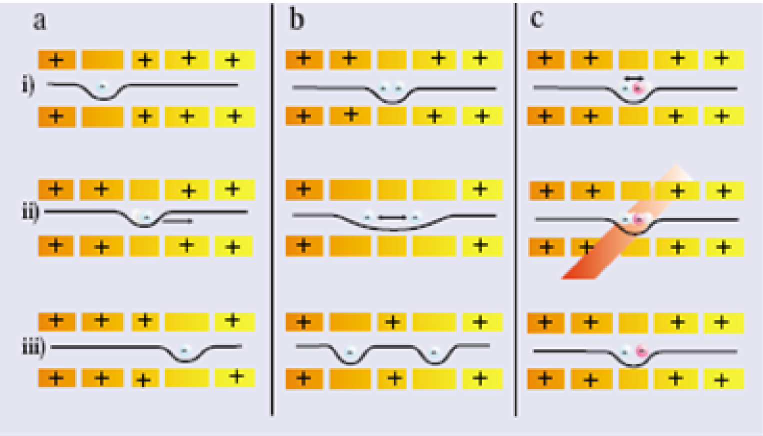


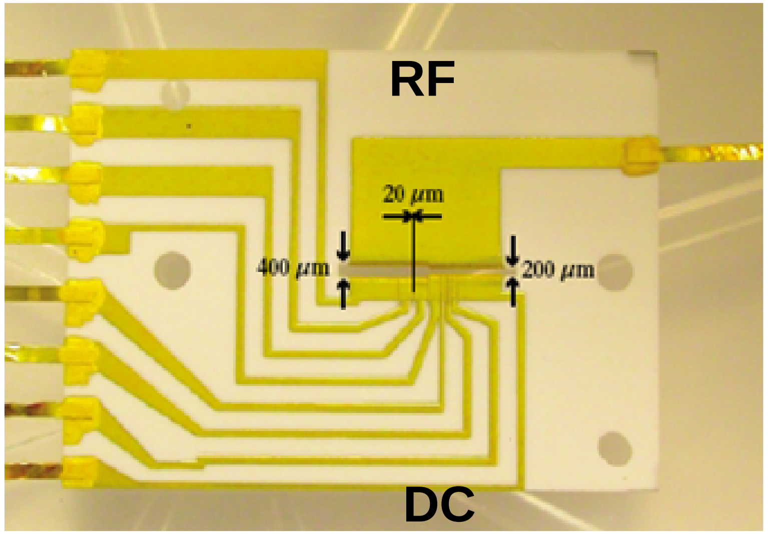
Figure 1 Schematic representation of the teleportation protocol. The ions are numbered left to right, as indicated at the top, and retain their order throughout. Positions, relative to the electrodes, are shown at each step in the protocol. The widths of the electrodes vary, with the width of the separation electrode (6) being the smallest at 100 μm. The spacing between ions in the same trap is about 3 μm, and laser-beam spot sizes (in traps 5 and 6) at the position of the ions are approximately 30 μm. In step 1 we prepare



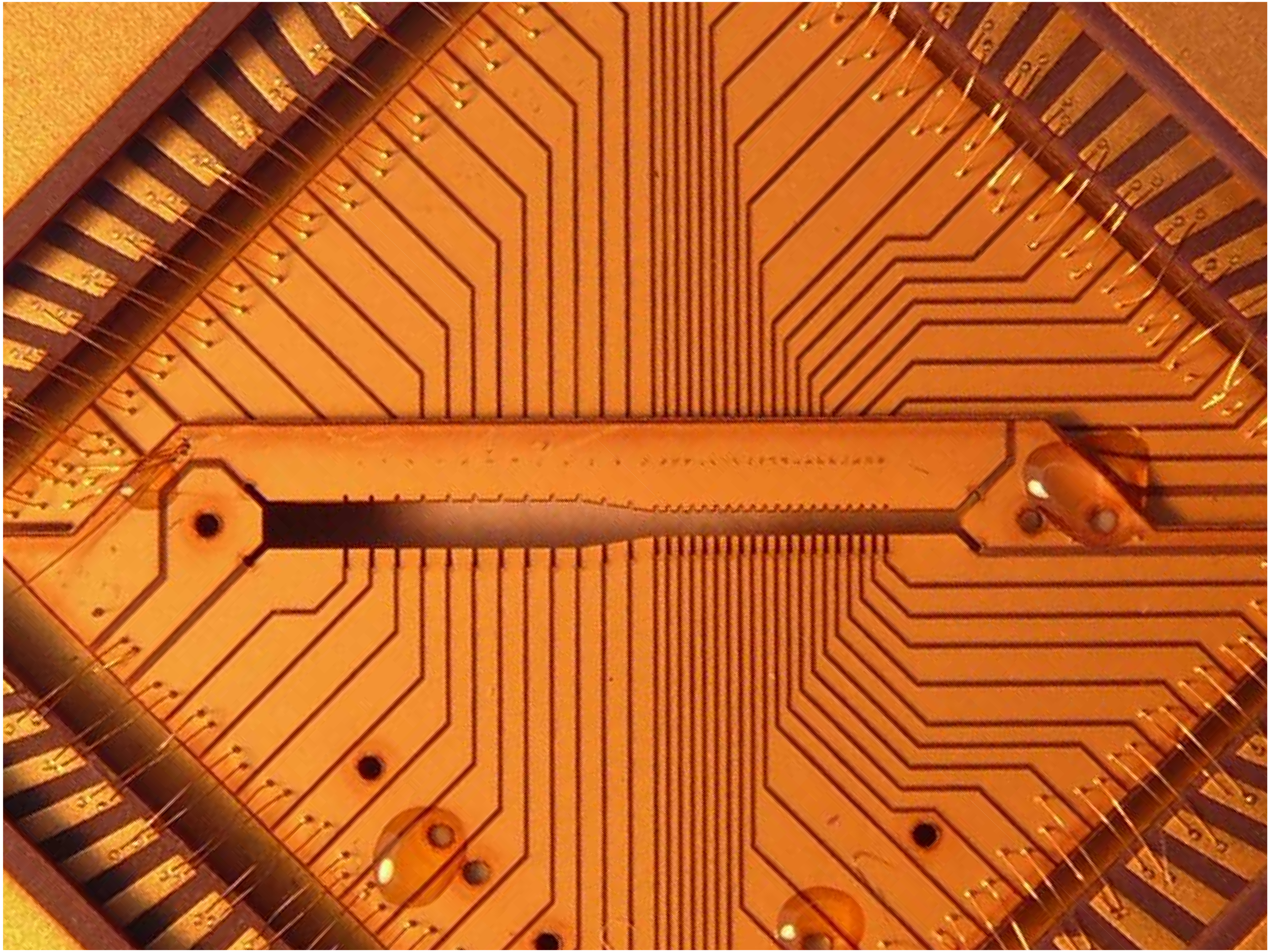
a) Transport ions from right to left

b) Separate two ions to right and left side

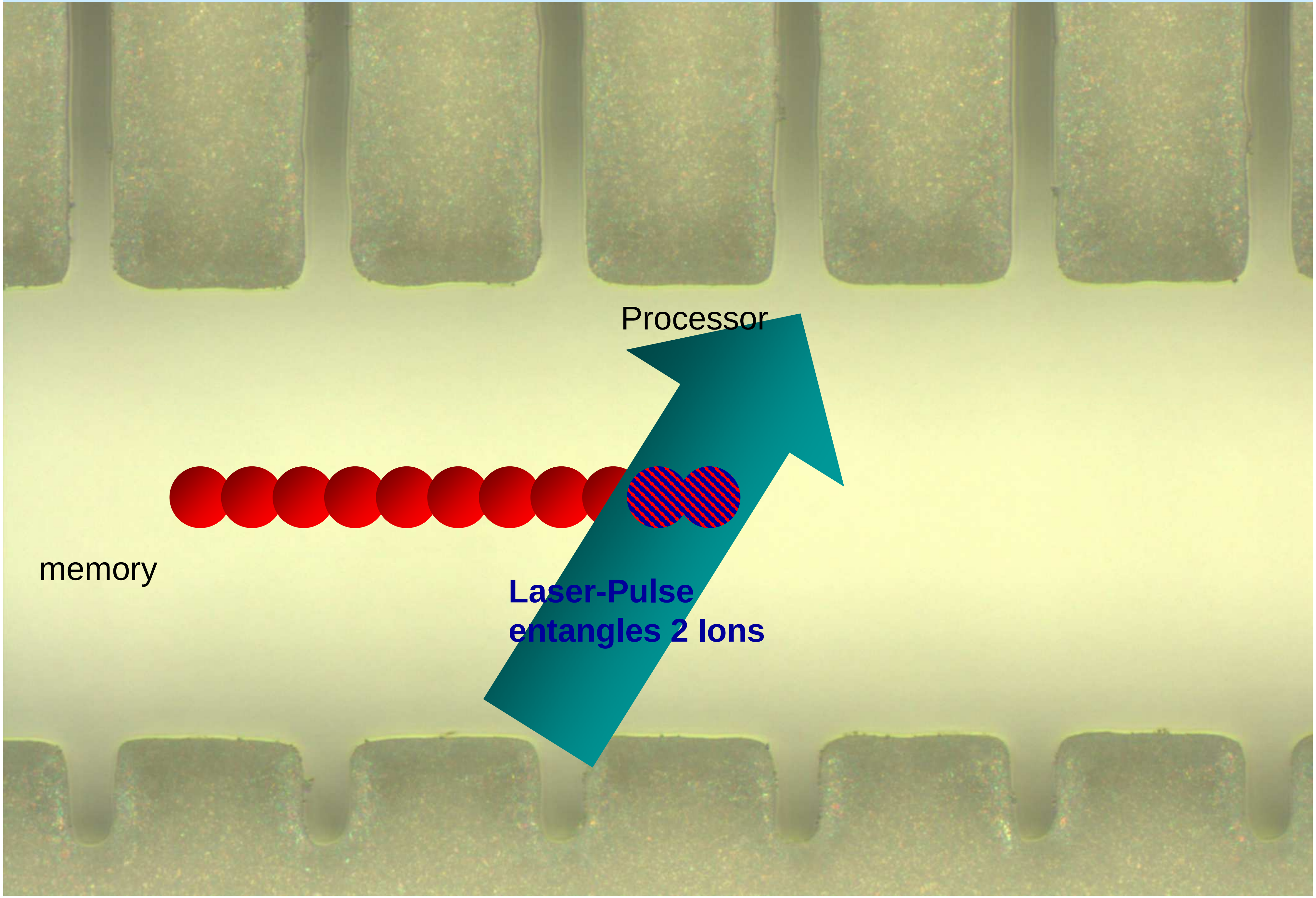
NIST Boulder



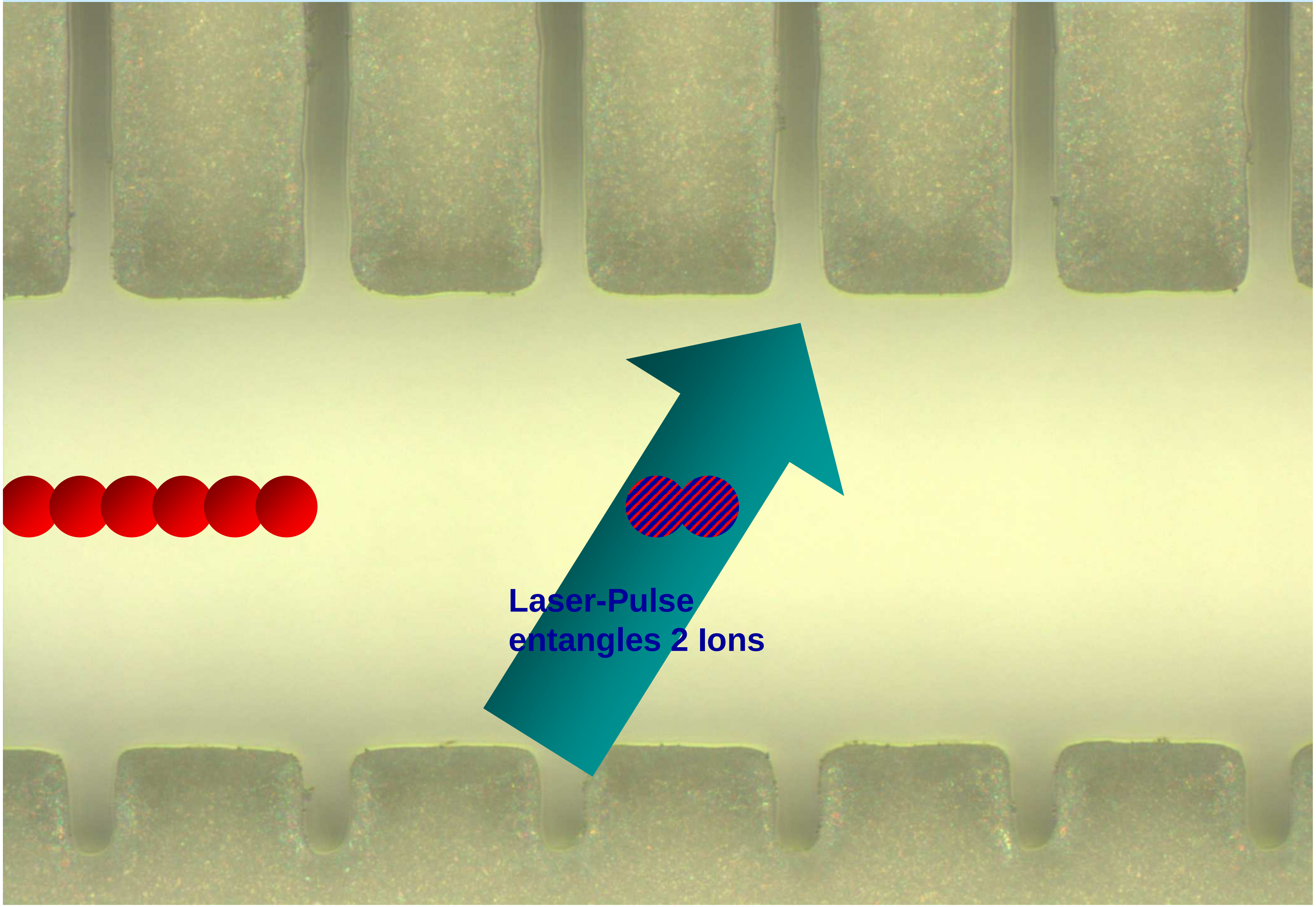
Barrett et al, Nature 429, 727 (2004)



Entanglement: Step by Step

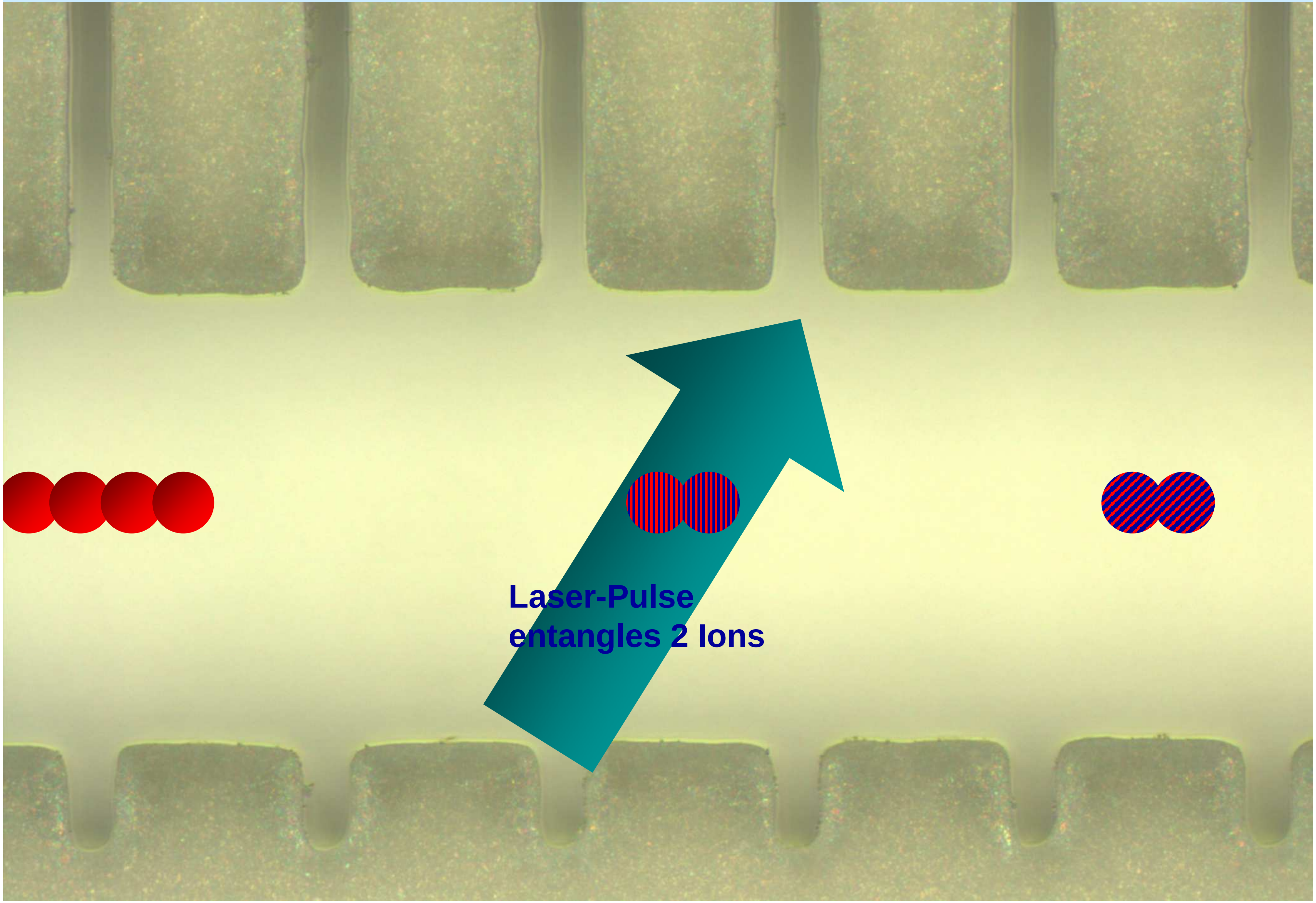


Entanglement: Step by Step



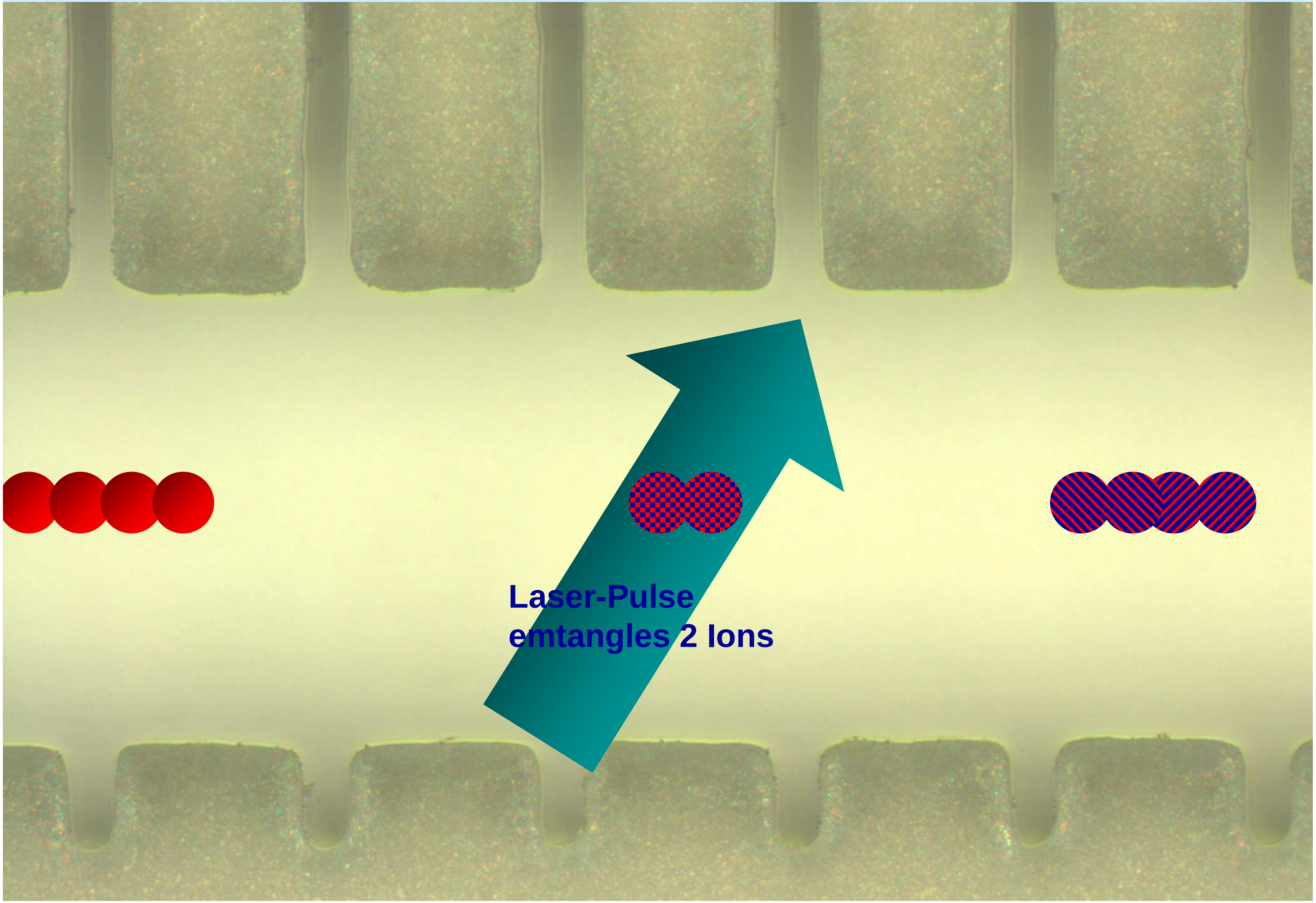
Laser-Pulse
entangles 2 Ions

Entanglement: Step by Step



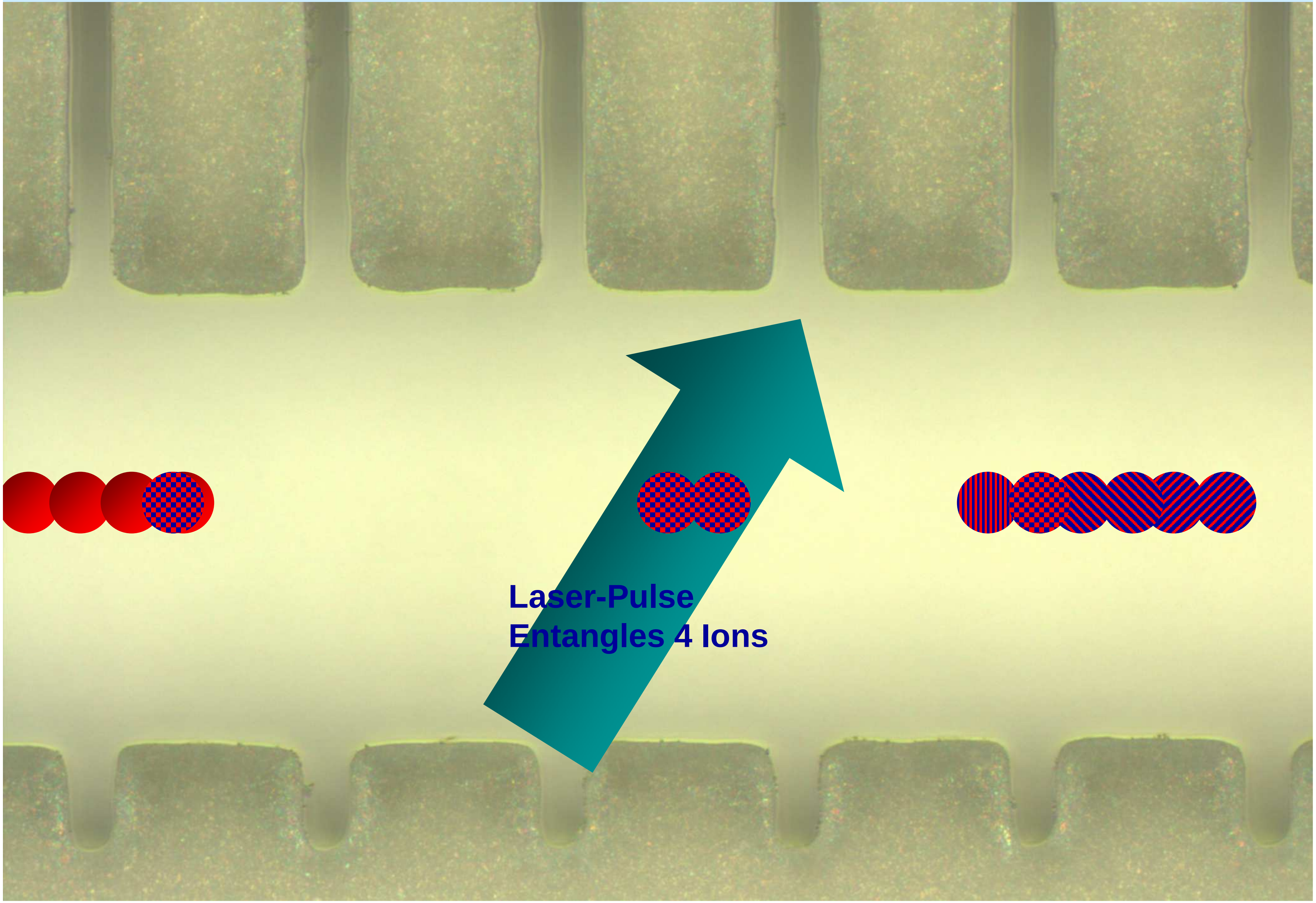
Laser-Pulse
entangles 2 Ions

Entanglement: Step by Step



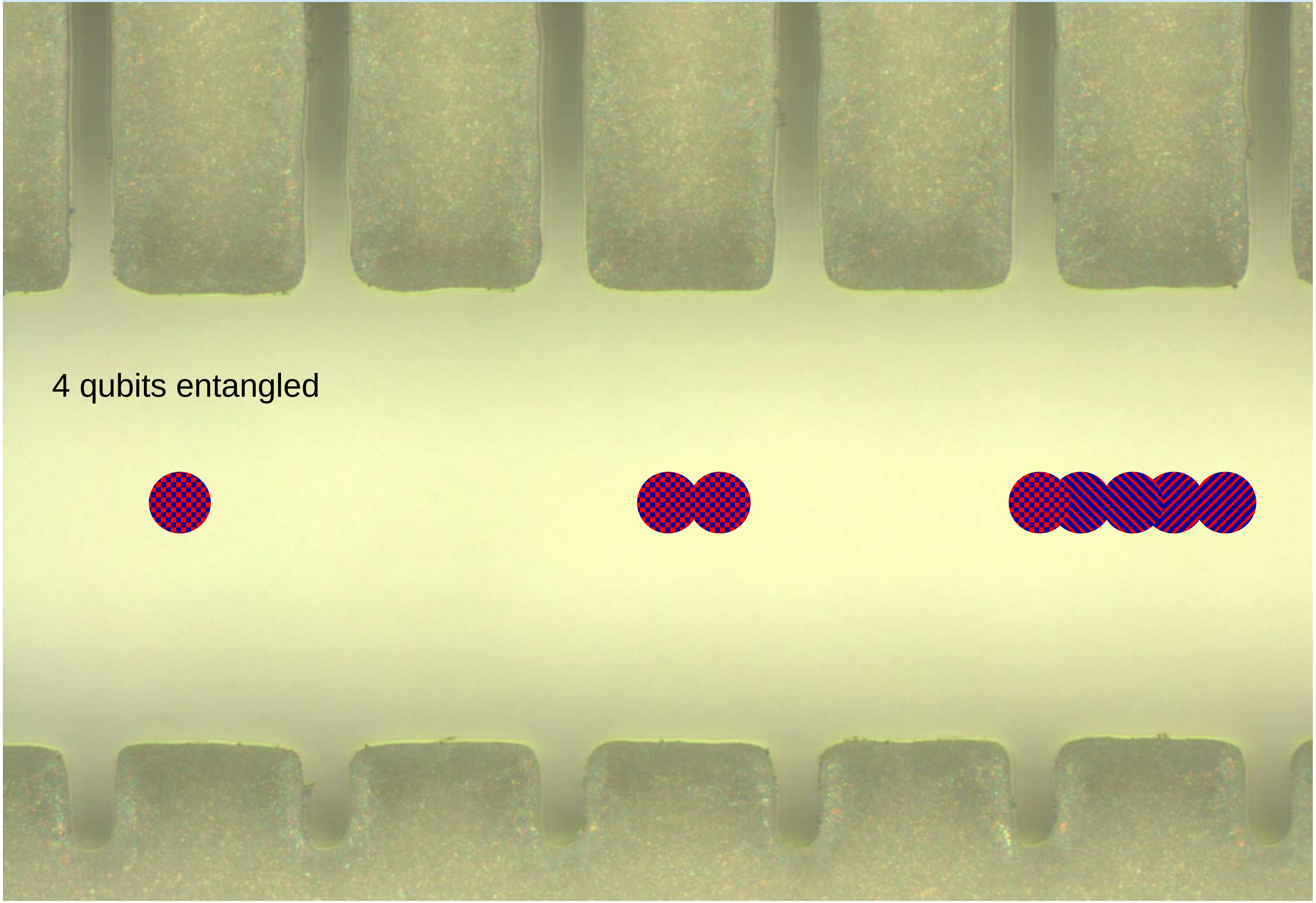
Laser-Pulse
emtangles 2 Ions

Entanglement: Step by Step



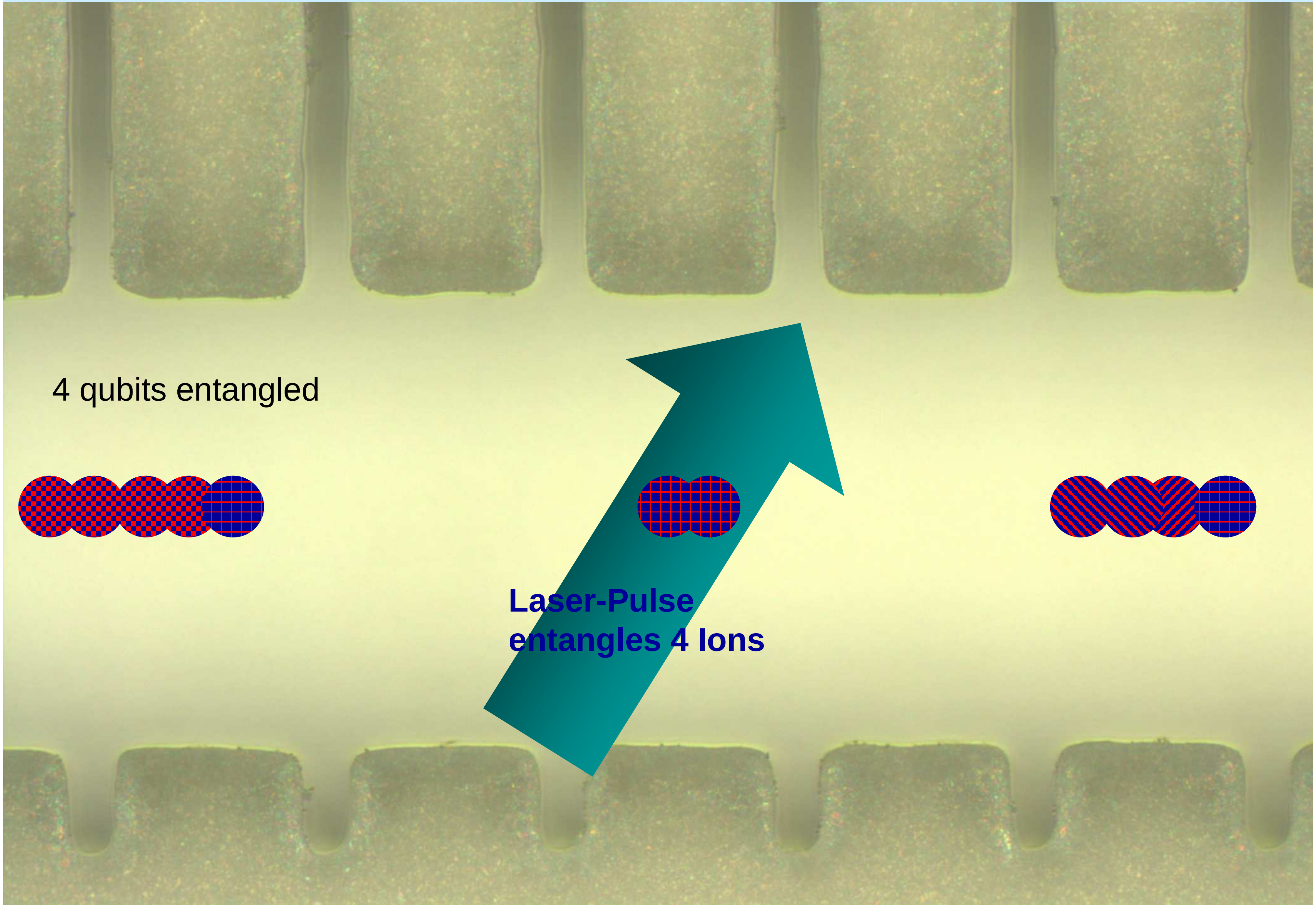
Laser-Pulse
Entangles 4 Ions

Entanglement: Step by Step



4 qubits entangled

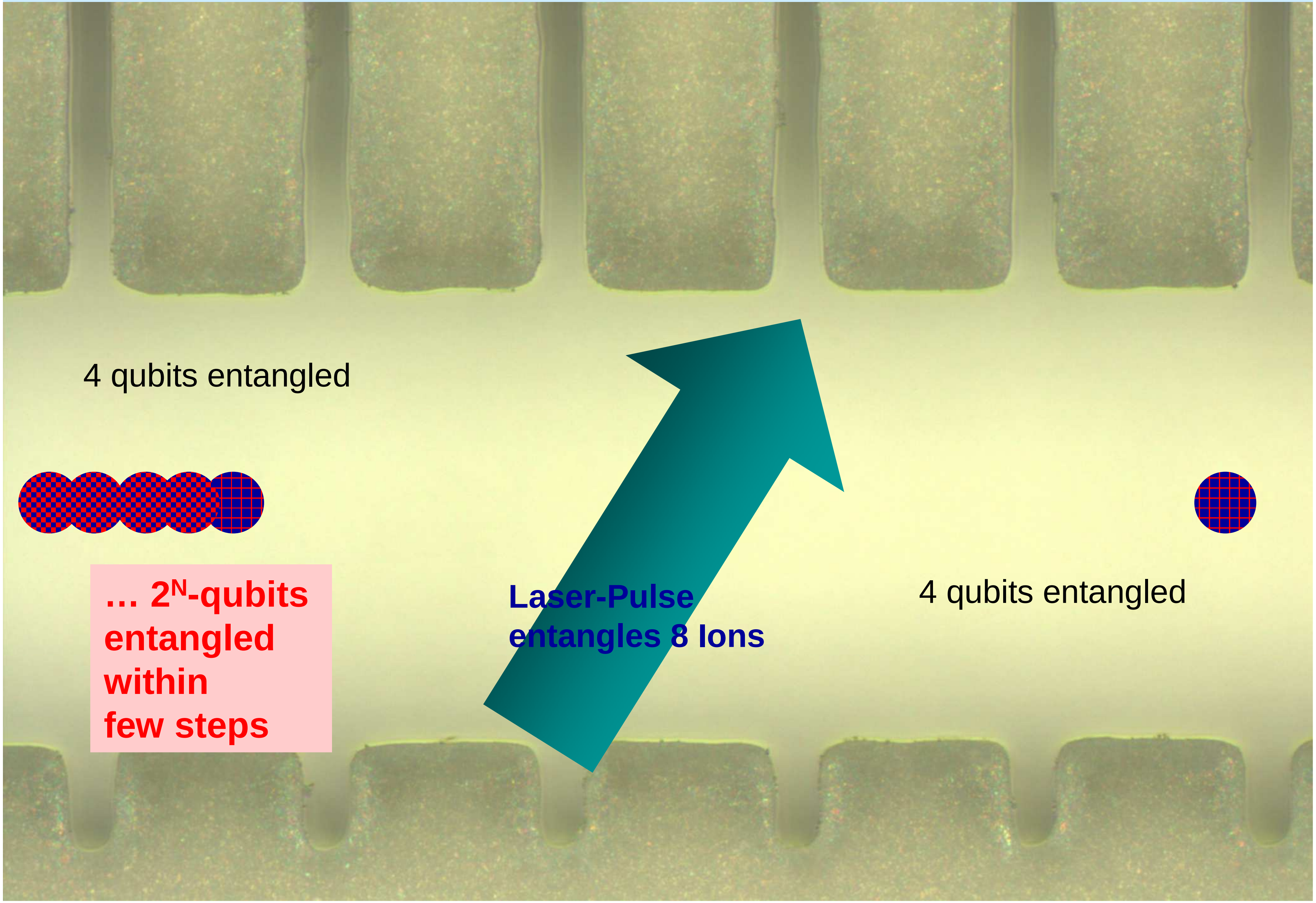
Entanglement: Step by Step



4 qubits entangled

Laser-Pulse entangles 4 Ions

Entanglement: Step by Step



4 qubits entangled

… 2^N-qubits entangled within few steps

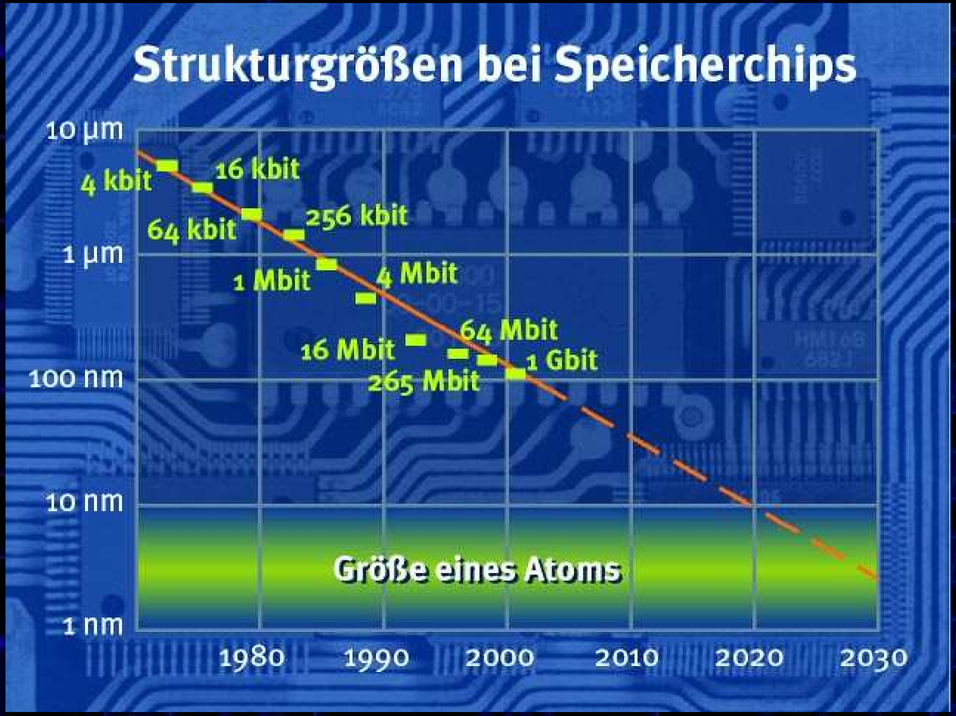
Laser-Pulse entangles 8 Ions

4 qubits entangled

Resultats and Aims for QIP

- Quantum-Gates
- Bell states
- Entangled states

  of 3 .. 8 –qubits
- Deterministic Teleportation
- Scalable QC – Micro traps
- Error correction, improved gates
- Quantum simulation
- Improved frequency standards
- **Atomic Information Processing**



Groups with single ions:
Arhus, Barcelona,
NIST Boulder,
NPL Teddington,
Innsbruck,
Michigan, MPQ, MIT,
Oxford, Siegen,
Southhampton, Ulm, …….

Cold Ions and their Applications for Quantum Computing and Frequency Standards

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- Quantum computer: basics, gates, algorithms, future challenges
- **Ion clocks: from Ramsey spectroscopy to quantum techniques**

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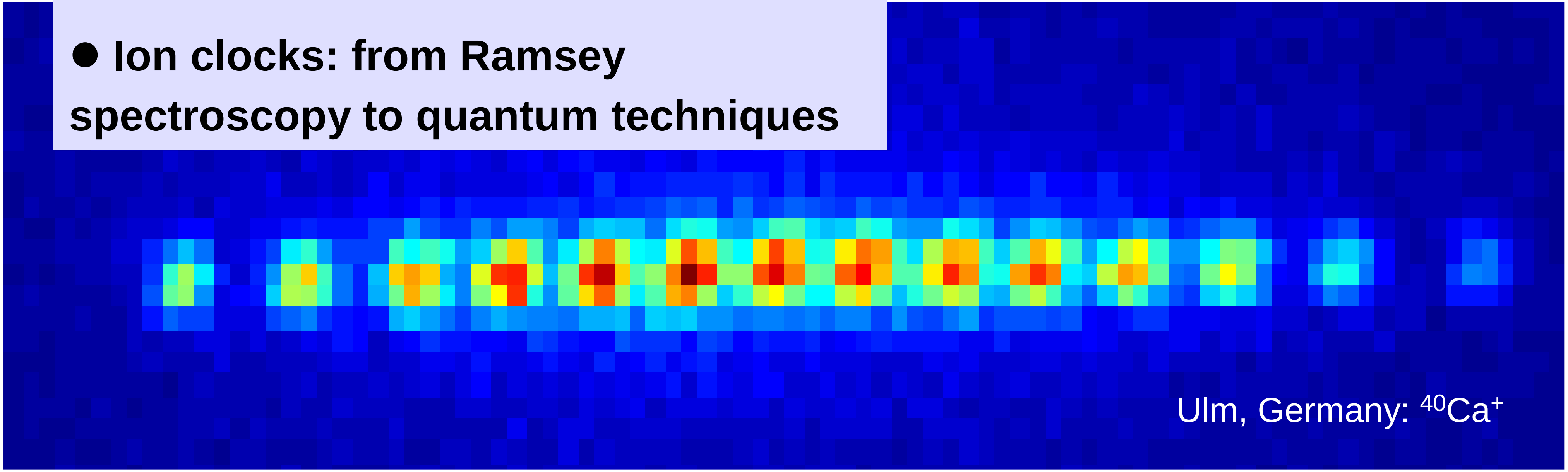

Ulm, Germany: $^{40}Ca^{+}$

Good reasons to use trapped ions for frequency standards

- long interaction time and long coherence
- 100% detection efficiency
- well defined envirment leading to small systematic errors
- but N is small

Allen variance:

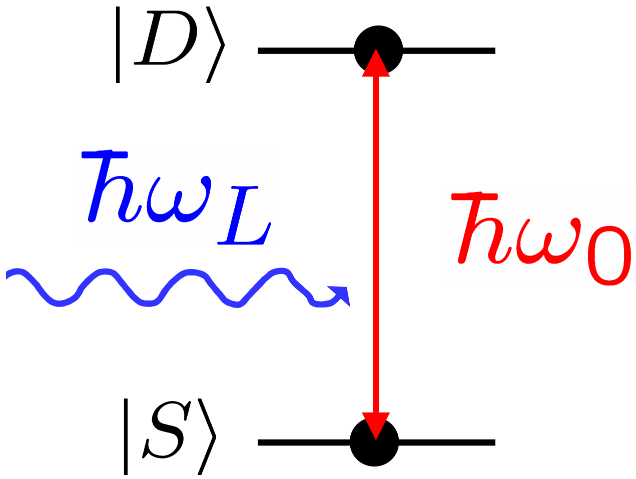
$$\sigma(\tau) = \frac{1}{\pi \cdot Q}\sqrt{\frac{T_c}{\tau}}\sqrt{\frac{1}{N}+\gamma}^{\,*}$$

Q: quality factor
T_c: cycle time,
N: number of atoms,
γ: reference phase noise
τ: integration time

** Santarelli et. al., PRL **82**, 4619 (1999)*

Single ion frequency standards

Ramsey experiment with interrogation time τ



$$|S\rangle + |D\rangle \xrightarrow{\tau} |S\rangle + e^{-i\Delta\tau}|D\rangle,$$

$$\Delta = \omega_L - \omega_0$$

Phase measurement:

$\pi/2$ pulse + state measurement

Transitions better to be insensitive to electric and magnetic fields:

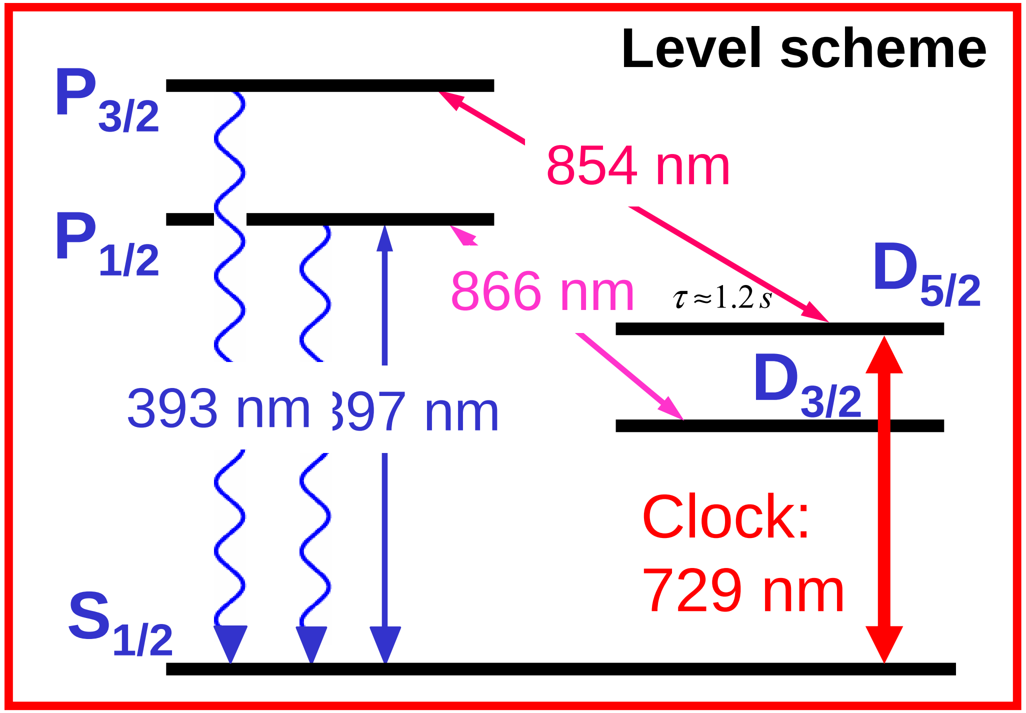
„Clock“ ions:

- half-integer nuclear spins: ^{199}Hg$^+$, ^{171}Yb$^+$, ^{115}In$^+$, ^{27}Al$^+$ ...
  m=0 ⟺ m‘=0 transition (no first-order Zeeman effect)
- „magic“ B-fields

Benhelm et. al., PRA ***75****, 032506 (2006)*

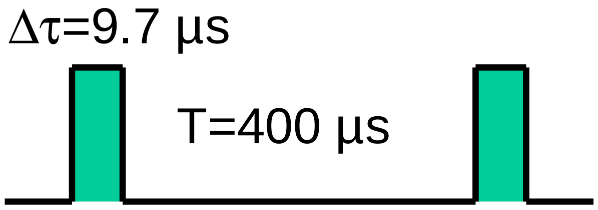
Spectroscopy on the Ca$^+$ $S_{1/2}$ - $D_{5/2}$ transition

Level scheme



Clock:
729 nm

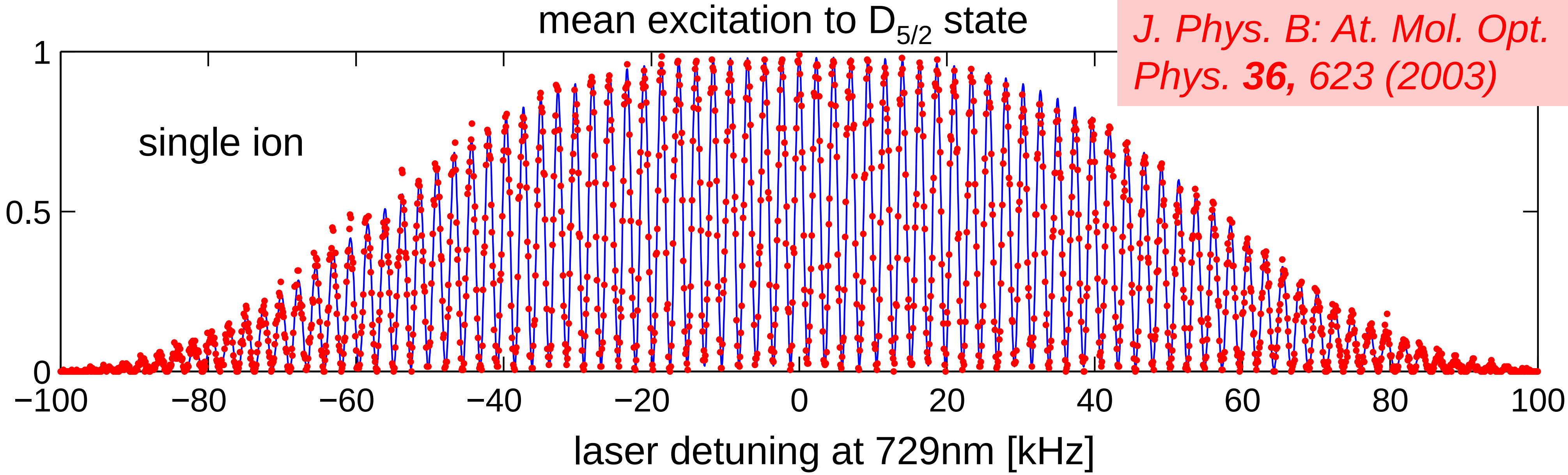
Ramsey scheme



🟥 theoretical curve

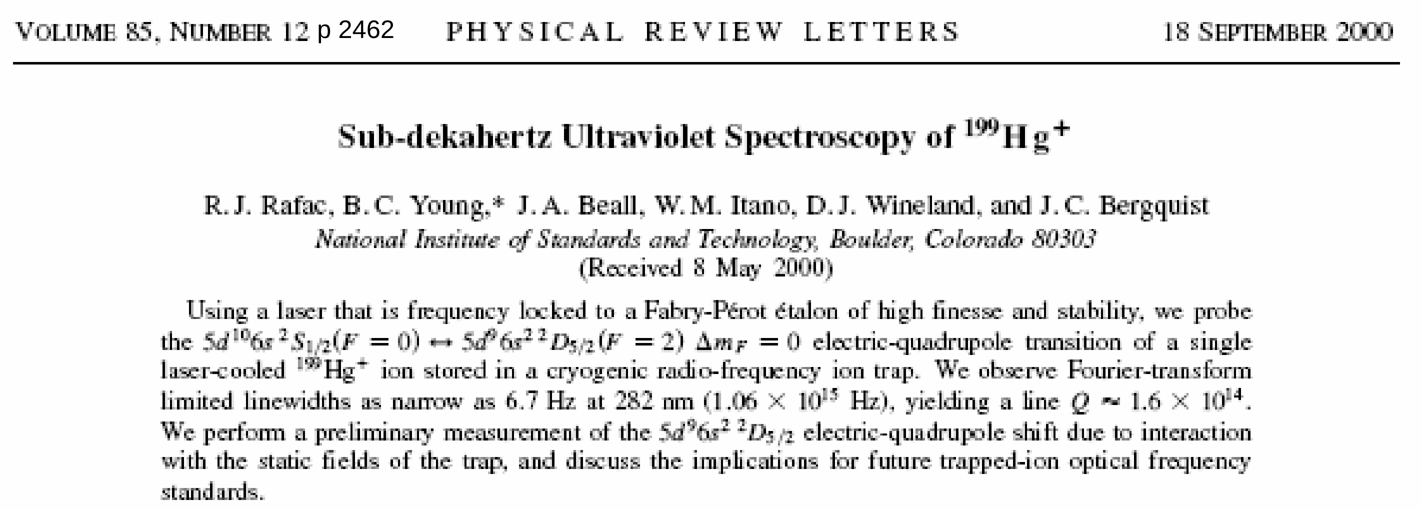
- 200 cycles per experiment

Schmidt-Kaler et al,
J. Phys. B: At. Mol. Opt. Phys. ***36,*** *623 (2003)*



Spectroscopy on a single Hg$^+$

Rafac et al,
*PRL**85**, 2462 (2000)*



VOLUME 85, NUMBER 12 p 2462 PHYSICAL REVIEW LETTERS 18 SEPTEMBER 2000

Sub-dekahertz Ultraviolet Spectroscopy of ^{199}Hg$^+$

R. J. Rafac, B. C. Young,* J. A. Beall, W. M. Itano, D. J. Wineland, and J. C. Bergquist
National Institute of Standards and Technology, Boulder, Colorado 80303

(Received 8 May 2000)

Using a laser that is frequency locked to a Fabry-Pérot étalon of high finesse and stability, we probe the $5d^{10}6s\,^2S_{1/2}(F=0) \leftrightarrow 5d^9 6s^2\,^2D_{5/2}(F=2)$ $\Delta m_F = 0$ electric-quadrupole transition of a single laser-cooled ^{199}Hg$^+$ ion stored in a cryogenic radio-frequency ion trap. We observe Fourier-transform limited linewidths as narrow as 6.7 Hz at 282 nm (1.06×10^{15} Hz), yielding a line $Q \approx 1.6\times10^{14}$. We perform a preliminary measurement of the $5d^9 6s^2\,^2D_{5/2}$ electric-quadrupole shift due to interaction with the static fields of the trap, and discuss the implications for future trapped-ion optical frequency standards.

LOOP:
1) probe clock transition $\tau < 120$ms
2) observe fluorescence / detect quantum jump
3) adjust interrogation laser frequency

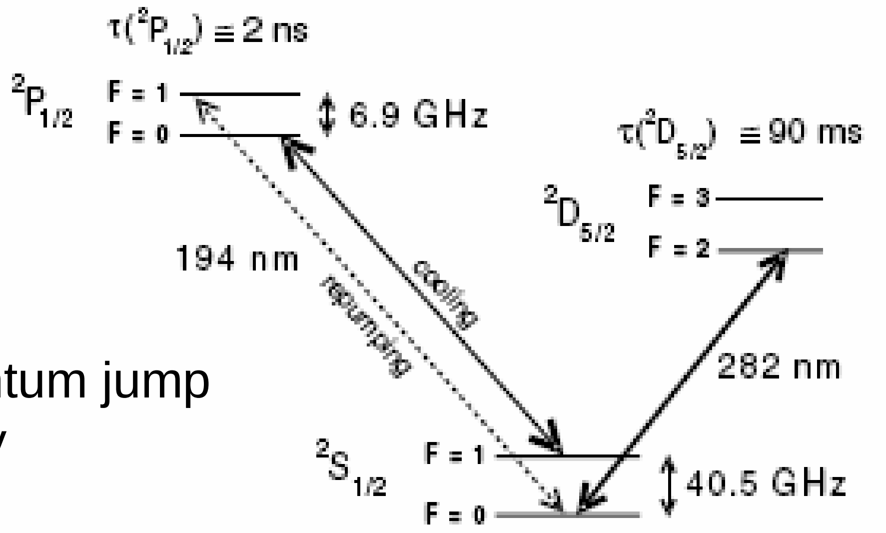
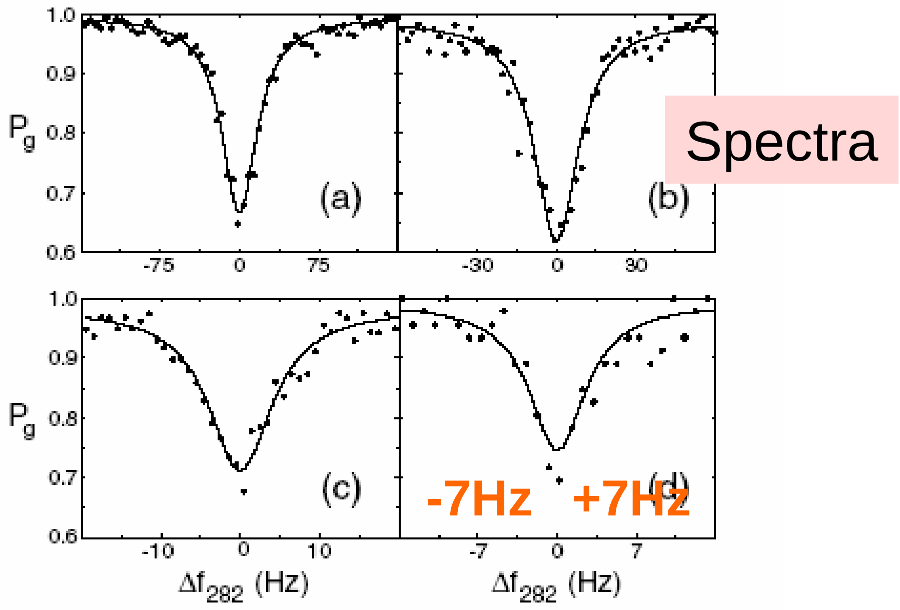


FIG. 1. Partial energy level diagram of ^{199}Hg$^+$ with the transitions of interest indicated.

Spectroscopy on a single Hg^+

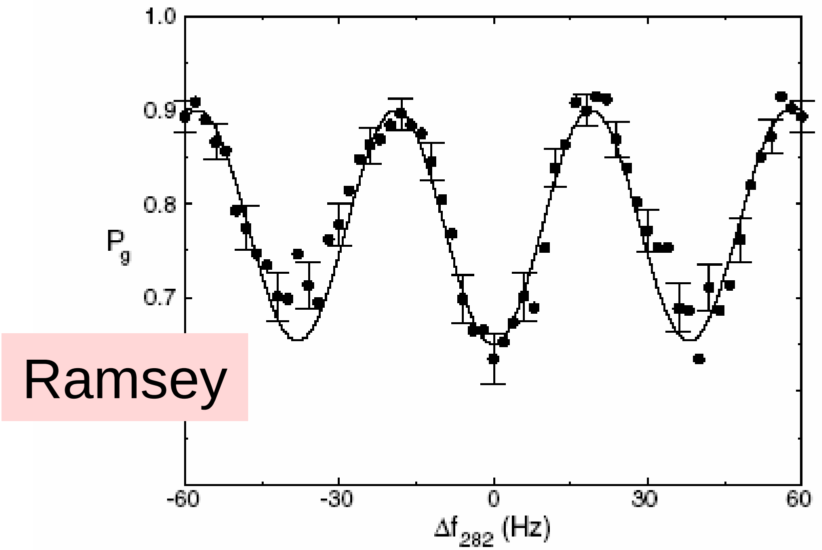
*Rafac et al, PRL**85**, 2462 (2000)*



Spectra

-7Hz +7Hz

FIG. 2. Quantum-jump absorption spectra of the $^2S_{1/2}(F=0) \leftrightarrow {}^2D_{5/2}(F=2)$ $\Delta m_F = 0$ electric-quadrupole transition. Δf_{282} is the frequency of the 282 nm probe laser detuning, and P_g is the probability of finding the atom in the ground state. The four plots correspond to excita n with 282 nm pulses of different lengths: (a) 20 ms (ed over 292 sweeps), (b) 40 ms (158 sw), (c) 80 weeps), and (d) 20 ms (46 sweeps). ew nsistent with the F ransform li , 20 10(1), and



Ramsey

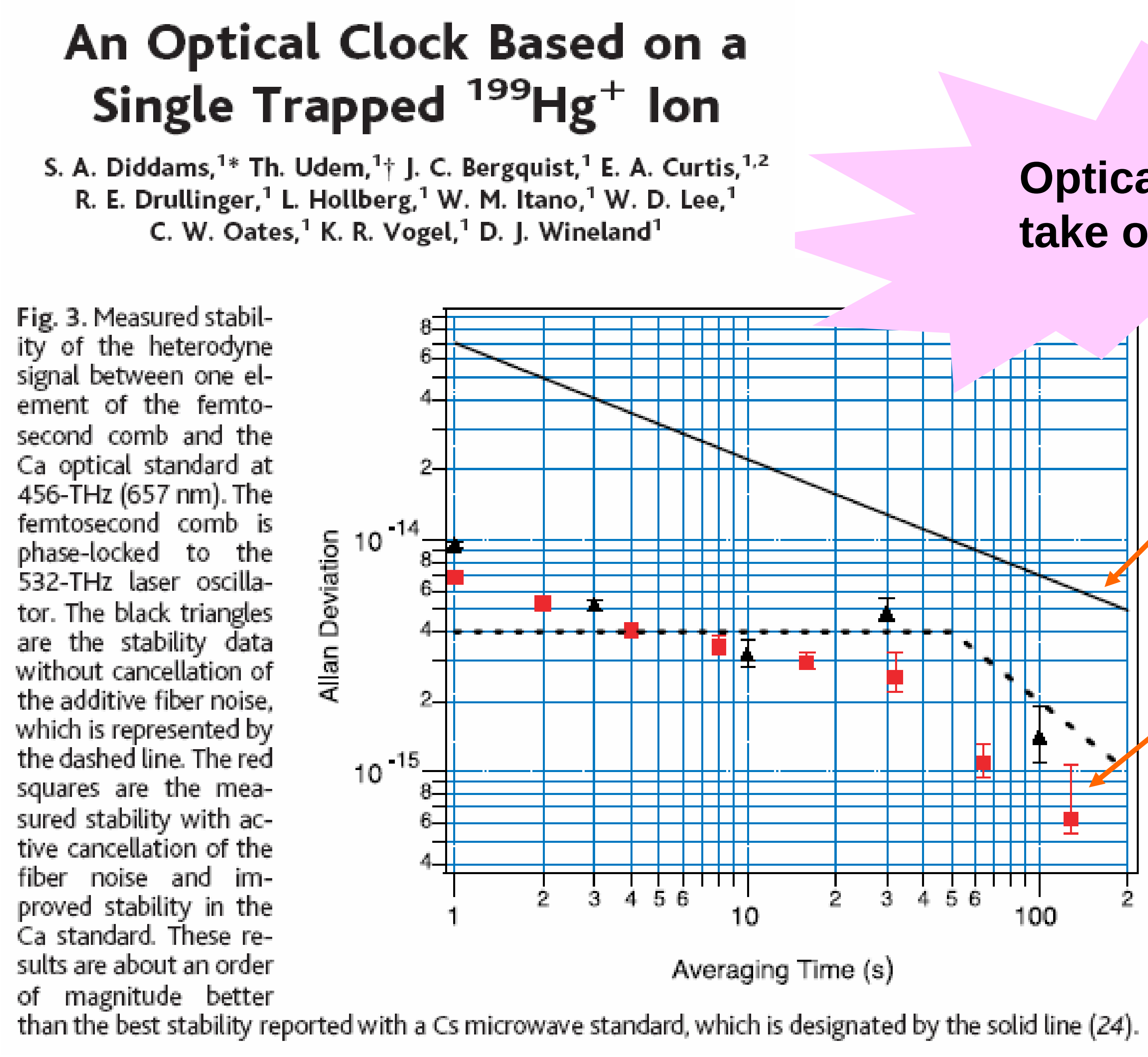
FIG. 4. The central three fringes obtained by time-domain Ramsey interrogation of the electric-quadrupole transition. Δf_{282} is the frequency of the 282 nm probe laser detuning, and P_g is the probability of finding the atom in the ground state. The data are averaged over 328 frequency sweeps with $\tau_{\text{servo}} = 40$ ms. The solid line is the expected Ramsey signal for 5 ms pulses separated by 20 ms of free precession; the ≈10% background arises primarily from suboptimal setting of the discriminator levels in our detection electronics. The quantum-projection-noise-limited uncertai ty is indicated by representative error bars plotted every point.

Systematic
line uncertainty
due to quadrupole shift
~ 10 Hz

Q- factor = 1.6 x 10^{14}

Single ion Hg^+ clock

*Diddams et al, Science **293**, 825 (2001)*



An Optical Clock Based on a Single Trapped $^{199}Hg^+$ Ion

S. A. Diddams,[1*] Th. Udem,[1†] J. C. Bergquist,[1] E. A. Curtis,[1,2]
R. E. Drullinger,[1] L. Hollberg,[1] W. M. Itano,[1] W. D. Lee,[1]
C. W. Oates,[1] K. R. Vogel,[1] D. J. Wineland[1]

Fig. 3. Measured stability of the heterodyne signal between one element of the femtosecond comb and the Ca optical standard at 456-THz (657 nm). The femtosecond comb is phase-locked to the 532-THz laser oscillator. The black triangles are the stability data without cancellation of the additive fiber noise, which is represented by the dashed line. The red squares are the measured stability with active cancellation of the fiber noise and improved stability in the Ca standard. These results are about an order of magnitude better than the best stability reported with a Cs microwave standard, which is designated by the solid line (24).

Optical clocks take over!

best Cs microwave clock

optical clock:

$$\sigma(\tau) = 7 \cdot 10^{-15}/\sqrt{\tau}$$

Improved single ion Hg^+ clock

Oskay et al,
*PRL **97**, 020801 (2006)*

fract. systematic frequency uncertainty $7.2 \ 10^{-17}$

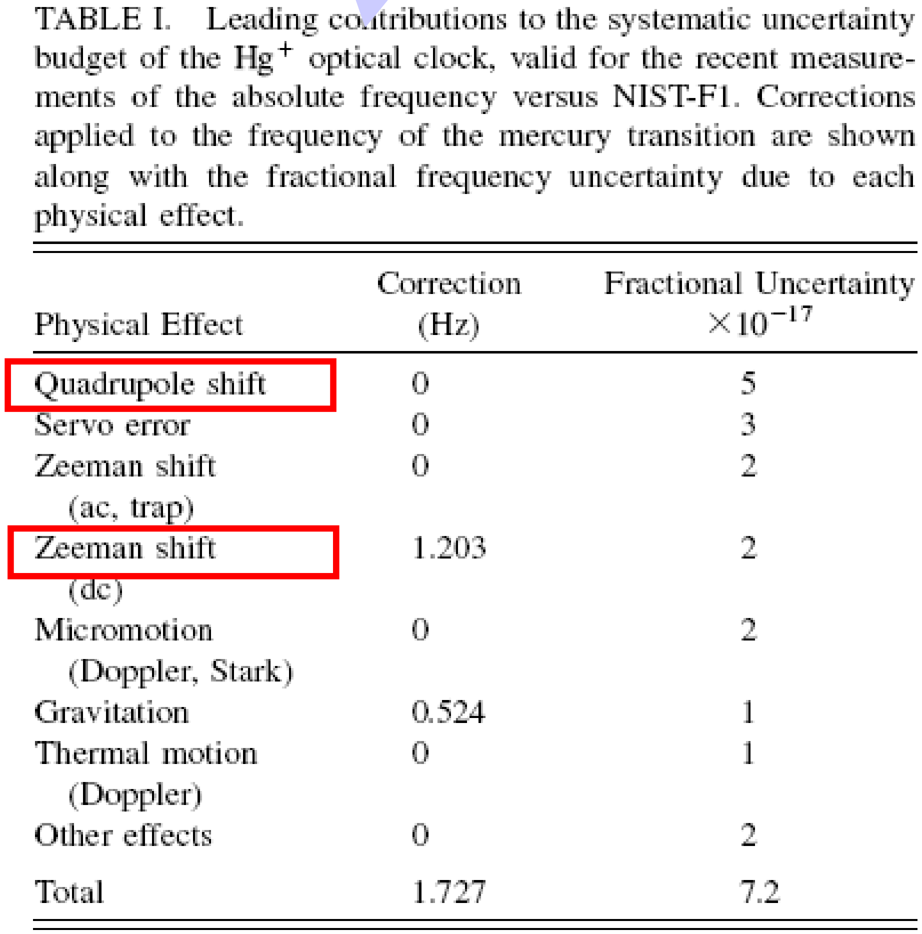
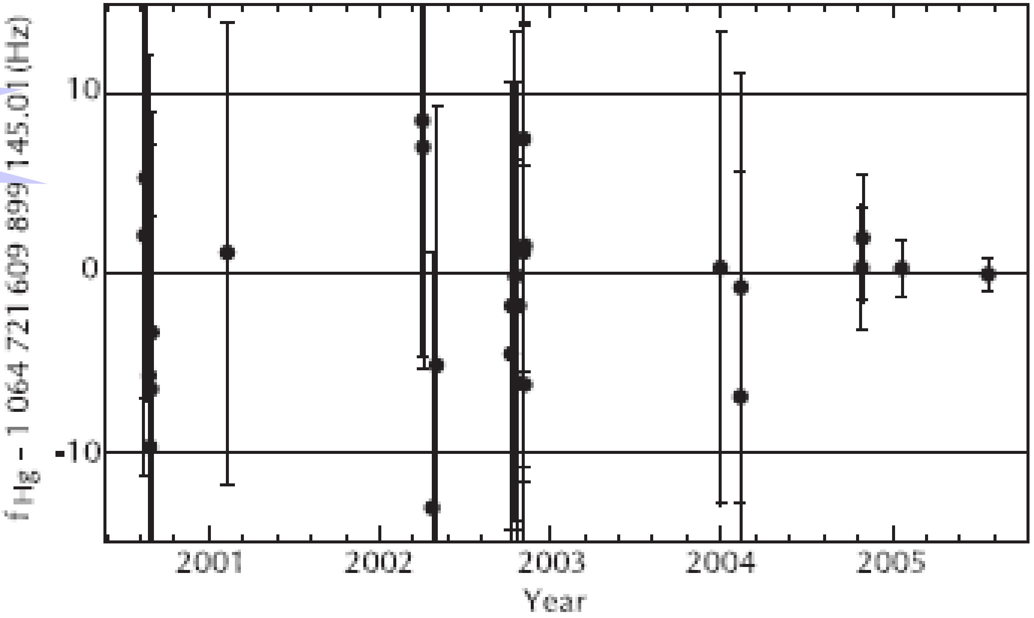


TABLE I. Leading contributions to the systematic uncertainty budget of the Hg^+ optical clock, valid for the recent measurements of the absolute frequency versus NIST-F1. Corrections applied to the frequency of the mercury transition are shown along with the fractional frequency uncertainty due to each physical effect.

Physical Effect	Correction (Hz)	Fractional Uncertainty $\times 10^{-17}$
Quadrupole shift	0	5
Servo error	0	3
Zeeman shift (ac, trap)	0	2
Zeeman shift (dc)	1.203	2
Micromotion (Doppler, Stark)	0	2
Gravitation	0.524	1
Thermal motion (Doppler)	0	1
Other effects	0	2
Total	1.727	7.2



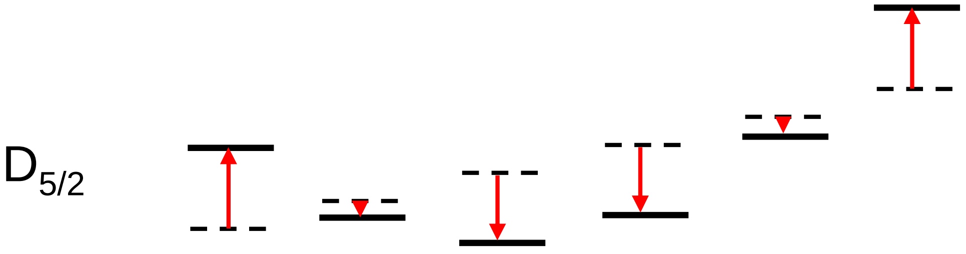
- Improved B-field shielding and stability
- Quadrupole shift eliminated by spatial averaging

Electric quadrupole shift

Quadrupole moment of D level interacts with static trap potential

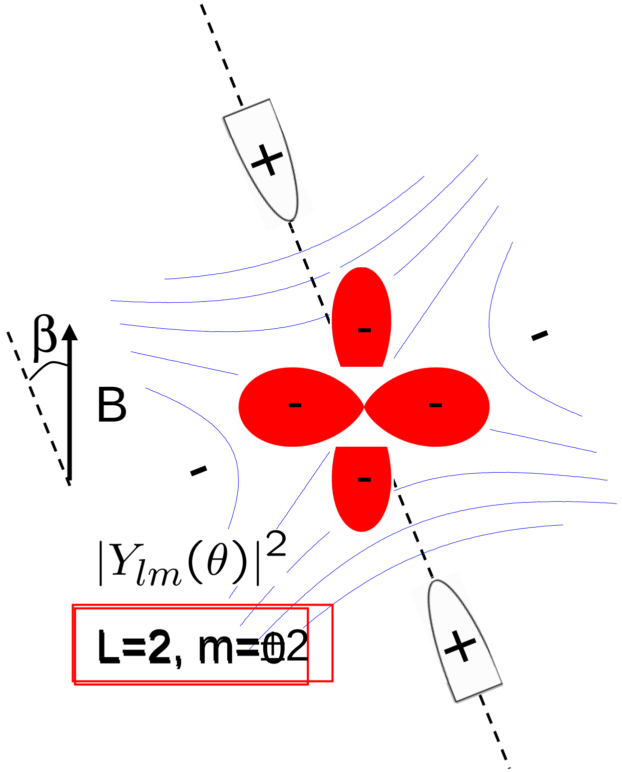
$$H_Q \propto Q\nabla E \longrightarrow \text{level shift } \delta\nu$$

Hg$^+$ ion clock: $\delta\nu$ limited by uncertainty in quadrupole shift



m =	-5/2	-3/2	-1/2	1/2	3/2	5/2
c_{mj} =	1	-1/5	-4/5	-4/5	-1/5	1

$$\delta\nu_{m_j} \propto c_{m_j}(3\cos^2\beta - 1)\,\omega_z^2\langle 3d|r^2|3d\rangle$$



Electric quadrupole shift

Electric quadrupole moment

$$\left.\begin{array}{ll} {}^{199}\mathrm{Hg}^{+} & \langle 5d|r^{2}|5d\rangle = 2.3\,a_0^2 \\ {}^{171}\mathrm{Yb}^{+} & \langle 4f|r^{2}|4f\rangle = 0.72\,a_0^2 \\ {}^{88}\mathrm{Sr}^{+} & \langle 4d|r^{2}|4d\rangle = 16.8\,a_0^2 \end{array}\right\}\ \text{theory}$$

$$ {}^{40}\mathrm{Ca}^{+} \qquad \langle 3d|r^{2}|3d\rangle = 1.917 a_0^2$$

Shift measurements

$$ {}^{199}\mathrm{Hg}^{+} \qquad \delta\nu \approx 10\mathrm{Hz}$$

$$ {}^{40}\mathrm{Ca}^{+} \qquad \langle 3d|r^{2}|3d\rangle = 1.83(1) a_0^2$$

Bergquist, PRL ***85****, 2462 (2000)*

Itano, J. Res. NIST. ***105****, 829 (2001)*

Roos, Nature ***443****, 316 (2006)*

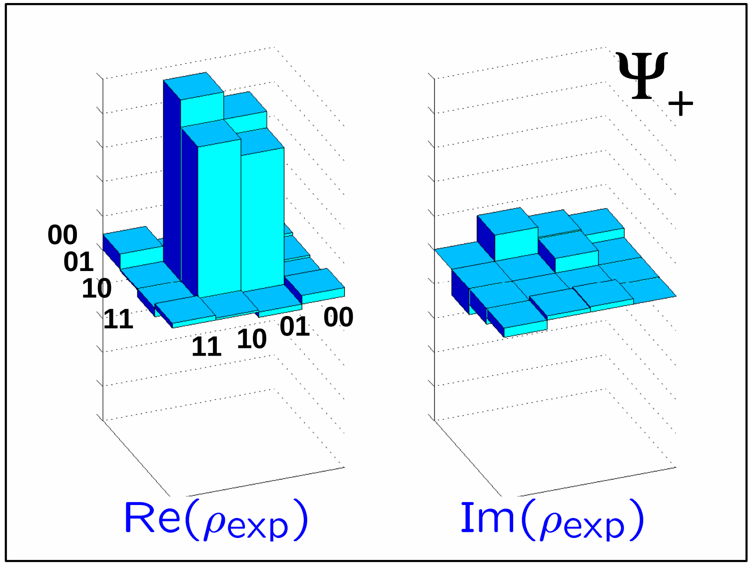
Dube, PRL ***95****, 033001 (2005)*

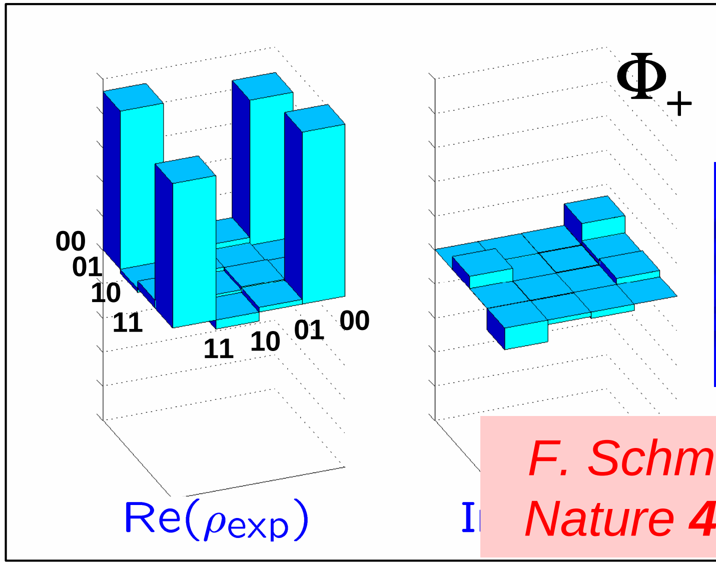
Margolis, Science ***306****, 1355 (2004)*

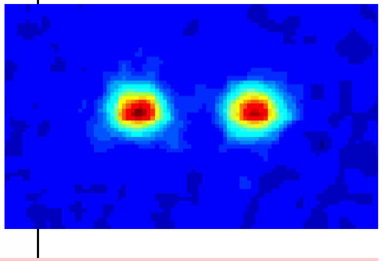
Density matrix for 2 ion entanglement

$\psi = |0\rangle|1\rangle + e^{i\phi}|1\rangle|0\rangle$

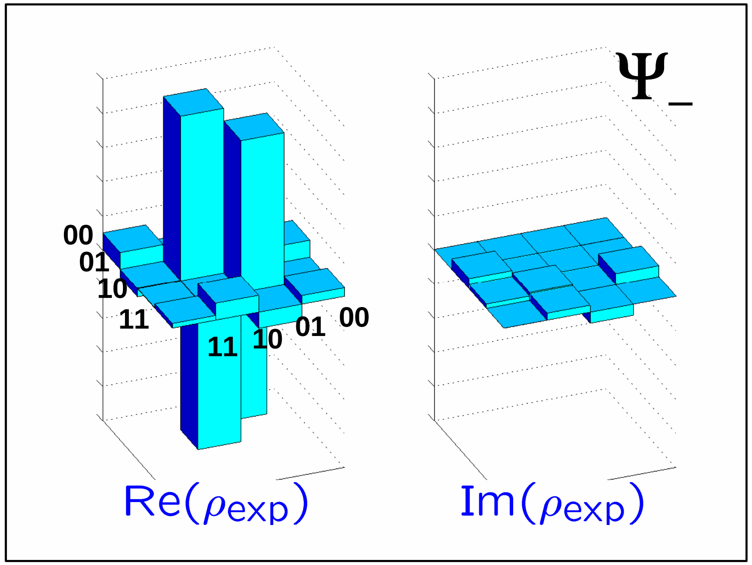
$\phi = |1\rangle|1\rangle + e^{i\phi}|0\rangle|0\rangle$

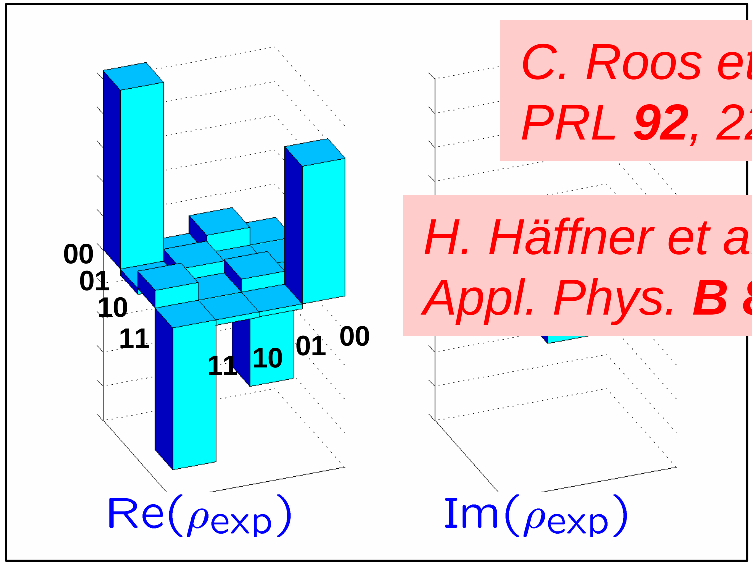






F. Schmidt-Kaler et al.,
Nature ***422****, 408 (2003)*



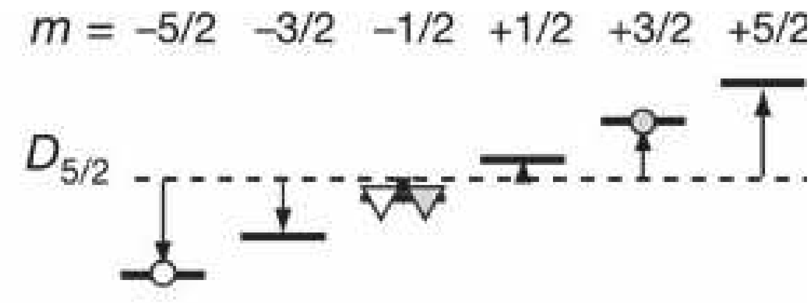


C. Roos et al.,
PRL ***92****, 220402 (2004)*

H. Häffner et al,
Appl. Phys. ***B 81****, 151 (2005)*

„Designer ions“ for spectroscopy

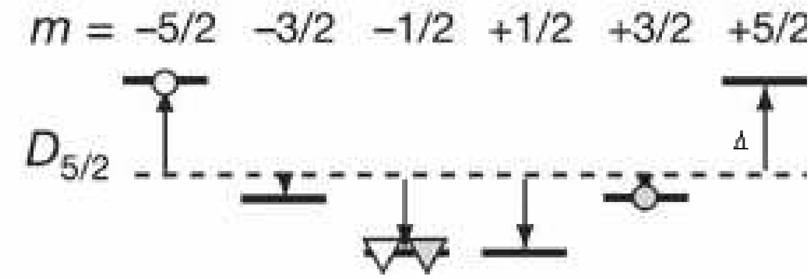
Roos, Nature ***443****, 316 (2006)*



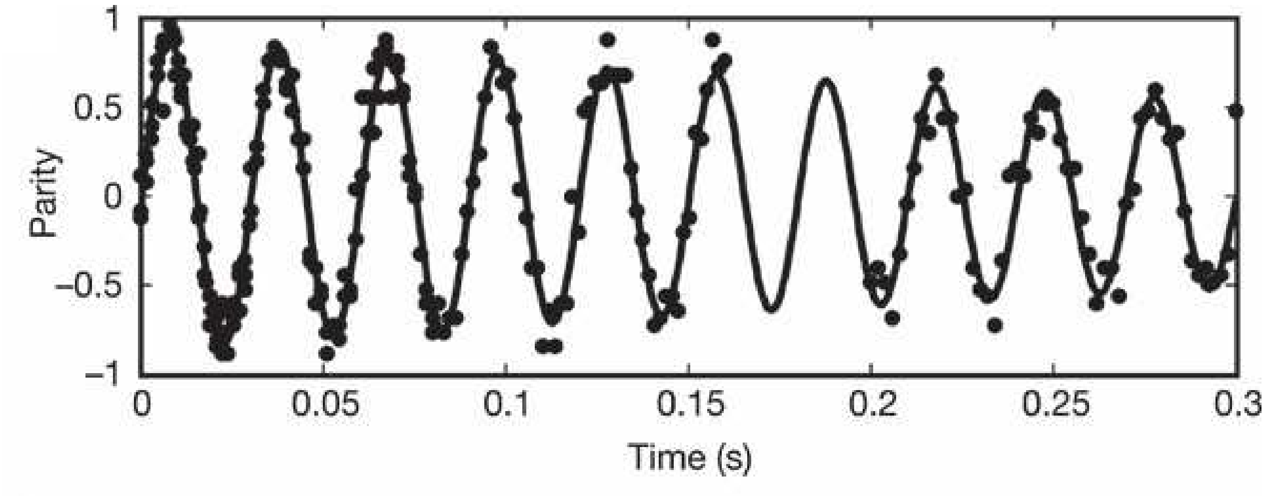
Linear Zeeman
(few MHz)

$$\psi = |-5/2\rangle|+3/2\rangle + e^{i\phi}|-1/2\rangle|-1/2\rangle$$

„magic“ state



Quadrupole shift
(few Hz)



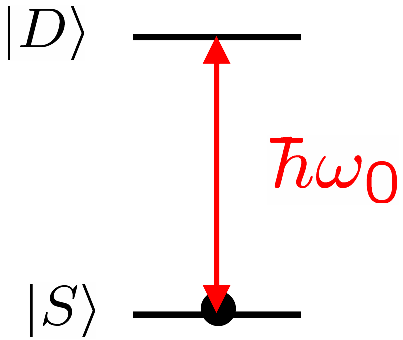
Δ = 33.35(3)Hz

⇩

$$\langle 3d|r^2|3d\rangle = 1.83(1)a_0^2$$

Entangled ion frequency standards

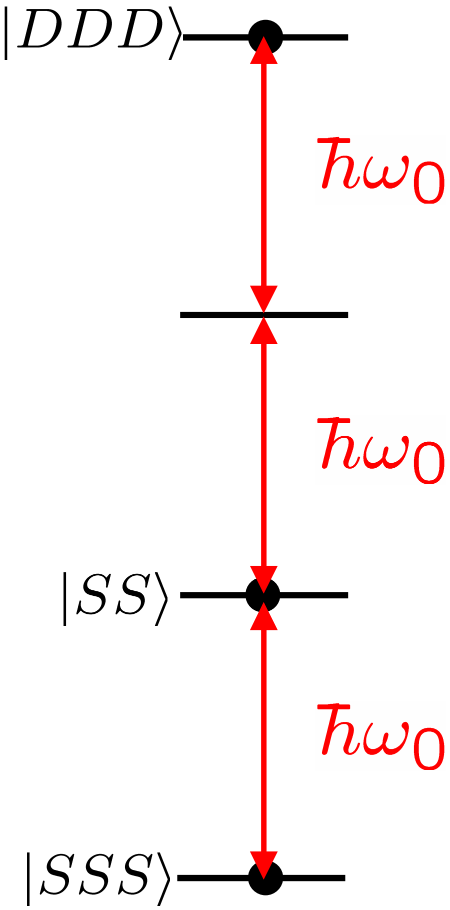
Clock experiments with maximally entangled states



$$|S\rangle + |D\rangle \xrightarrow{\tau} |S\rangle + e^{-i1\Delta\tau}|D\rangle,$$

Entangled ion frequency standards

Clock experiments with maximally entangled states



$$|S\rangle + |D\rangle \xrightarrow{\tau} |S\rangle + e^{-i1\Delta\tau}|D\rangle,$$
$$|SS\rangle + |DD\rangle \xrightarrow{\tau} |SS\rangle + e^{-i2\Delta\tau}|DD\rangle,$$
$$|SSS\rangle + |DDD\rangle \xrightarrow{\tau} |SSS\rangle + e^{-i3\Delta\tau}|DDD\rangle,$$

Measurement uncertainty

$$\boxed{\sigma(\tau) \sim \frac{1}{N}}$$

*Wineland, PRA **46**, R6797 (1992), Bollinger, PRA **54**, R4649 (1996)*

Towards Heisenberg-limited spectroscopy

$$|SSS\rangle + |DDD\rangle \xrightarrow{\tau} |SSS\rangle + e^{-i3\Delta\tau}|DDD\rangle,$$

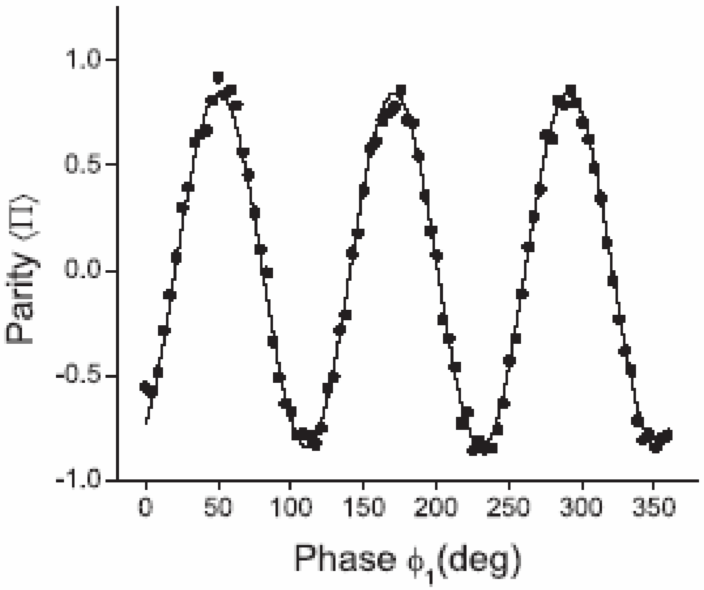


Fig. 1. Experimentally determined parity Π used to determine the fidelity of the three-particle GHZ state, plotted as a function of the phase difference ϕ_1 of the analysis π/2 pulse. The solid line is a fitted sinusoid oscillating with $3\phi_1$ yielding a contrast of 0.84(1). deg, degrees.

factor 3 observed in the phase sensitivity of the „Ramsey“ signal

⇩

spectroscopic sensitivity improved by **1.45(2)**

theoretical (ideal) improvement over 3 individual ions

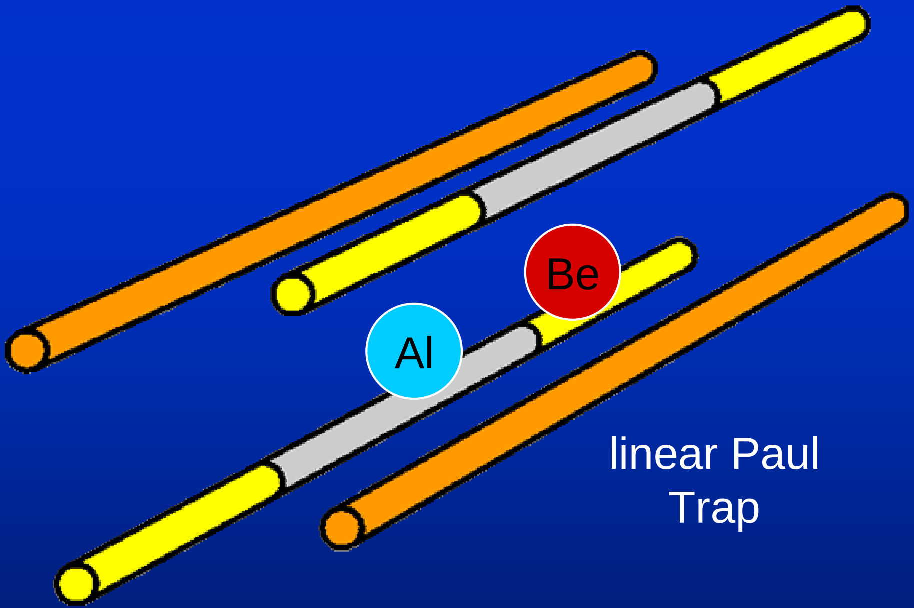
$$\sqrt{N} = \sqrt{3} = 1.732$$

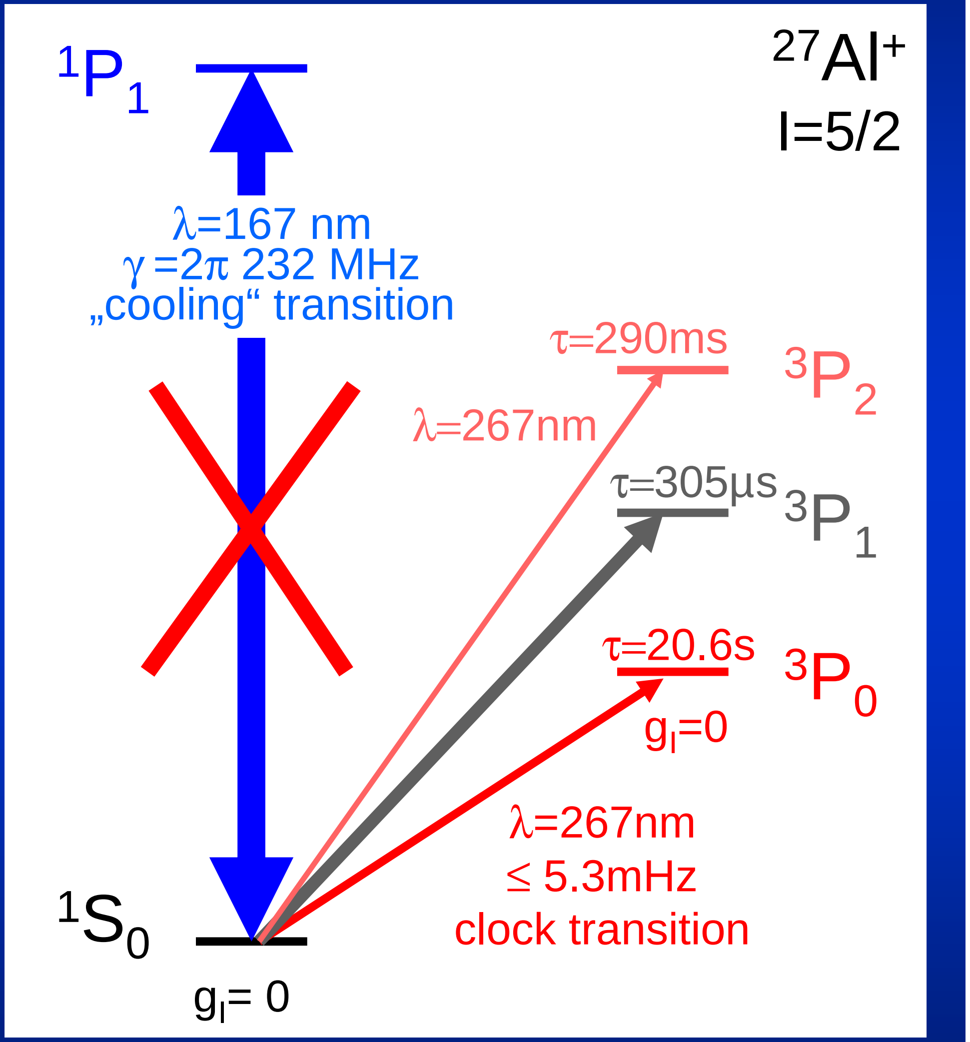
*Leibfried et al., Science **304**, 1476 (2004).*

Scheme for a Single Ion Optical Clock with $^{27}Al^+$

- single $^{27}Al^+$ ion in a linear Paul trap
- <0.5 mHz linewidth clock transition
- absence of static quadrupole shift
- no accessible cooling transition

use $^9Be^+$ for cooling and readout





P. O. Schmidt et al.,
*Science **309**, 749 (2005)*

Experimental Steps

- Load $^{9}\mathrm{Be}^{+}$ with far red-detuned Doppler cooling laser
- Detect via ion fluorescence
- Sympathetically load $^{27}\mathrm{Al}^{+}$ with Doppler-cooled $^{9}\mathrm{Be}^{+}$
- Detect presence of $^{27}\mathrm{Al}^{+}$ via $^{9}\mathrm{Be}^{+}$ displacement and motional sideband spectrum

[begin loop]

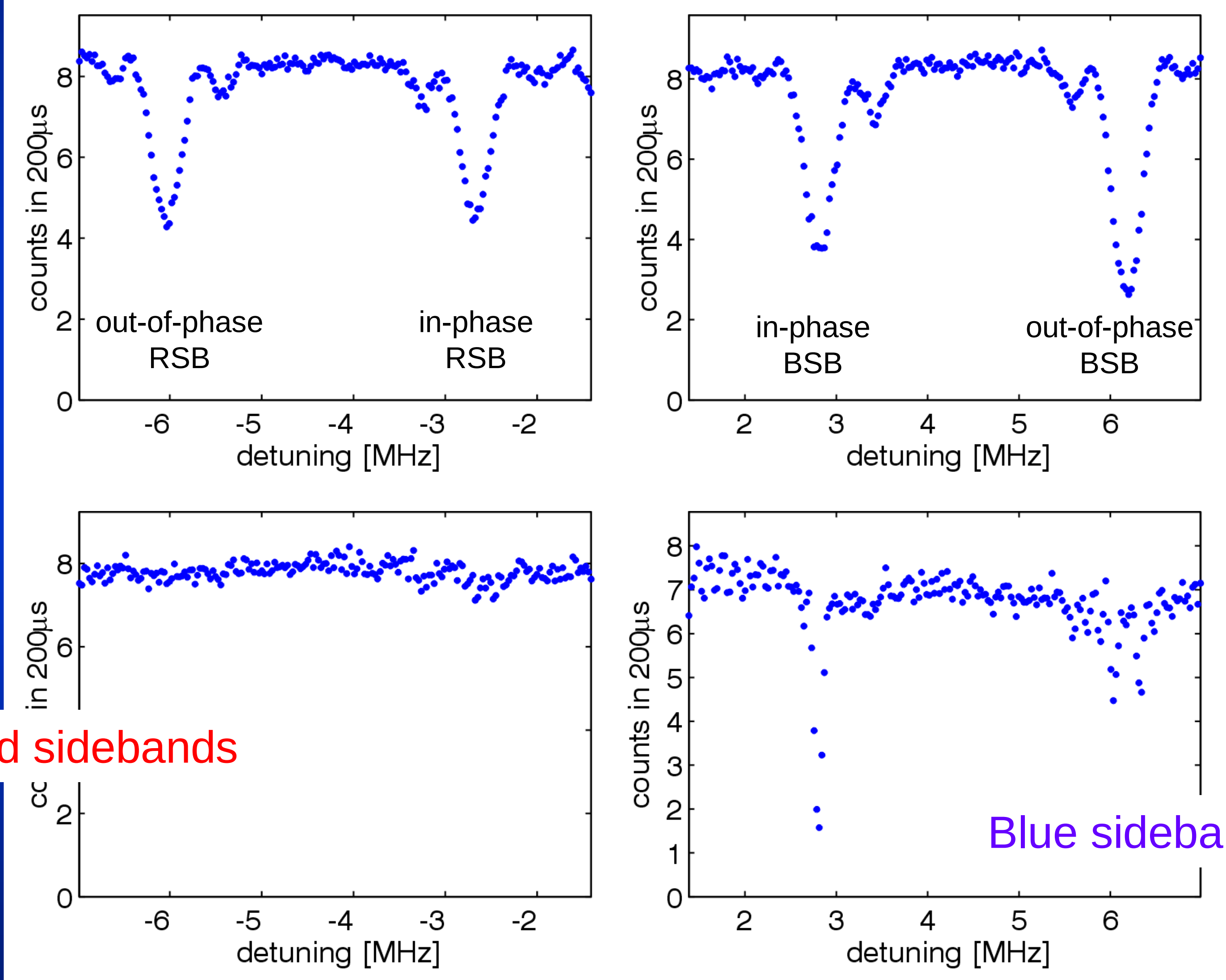
- Sideband cooling ground state cooling of both axial normal modes
- Interrogate $^{27}\mathrm{Al}^{+}$ „clock“ transition
- Transfer clock state amplitudes to $^{9}\mathrm{Be}^{+}$
- State readout on $^{9}\mathrm{Be}^{+}$ using electron shelving
- frequency feedback

[end loop]

- continuously monitor frequency of laser „flywheel“

P. O. Schmidt et al.,
Science ***309****, 749 (2005)*

Sideband cooling of Al^+/Be^+



Doppler cooled

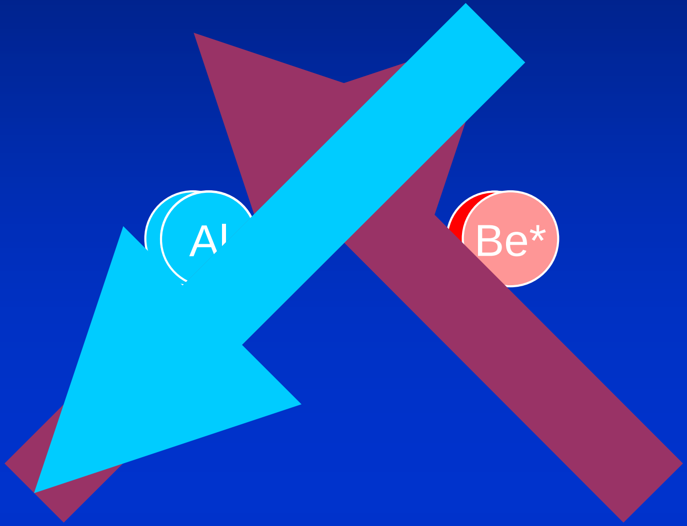
Sideband cooled on Be^+

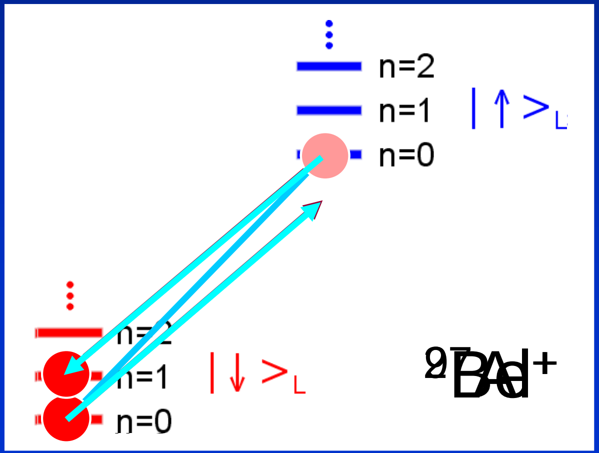
Red sidebands

Blue sidebands

M. Barrett *et. al.*, PRA **68**, 042302 (2003) with $^9Be^+$ and $^{24}Mg^+$

State Transfer and Readout

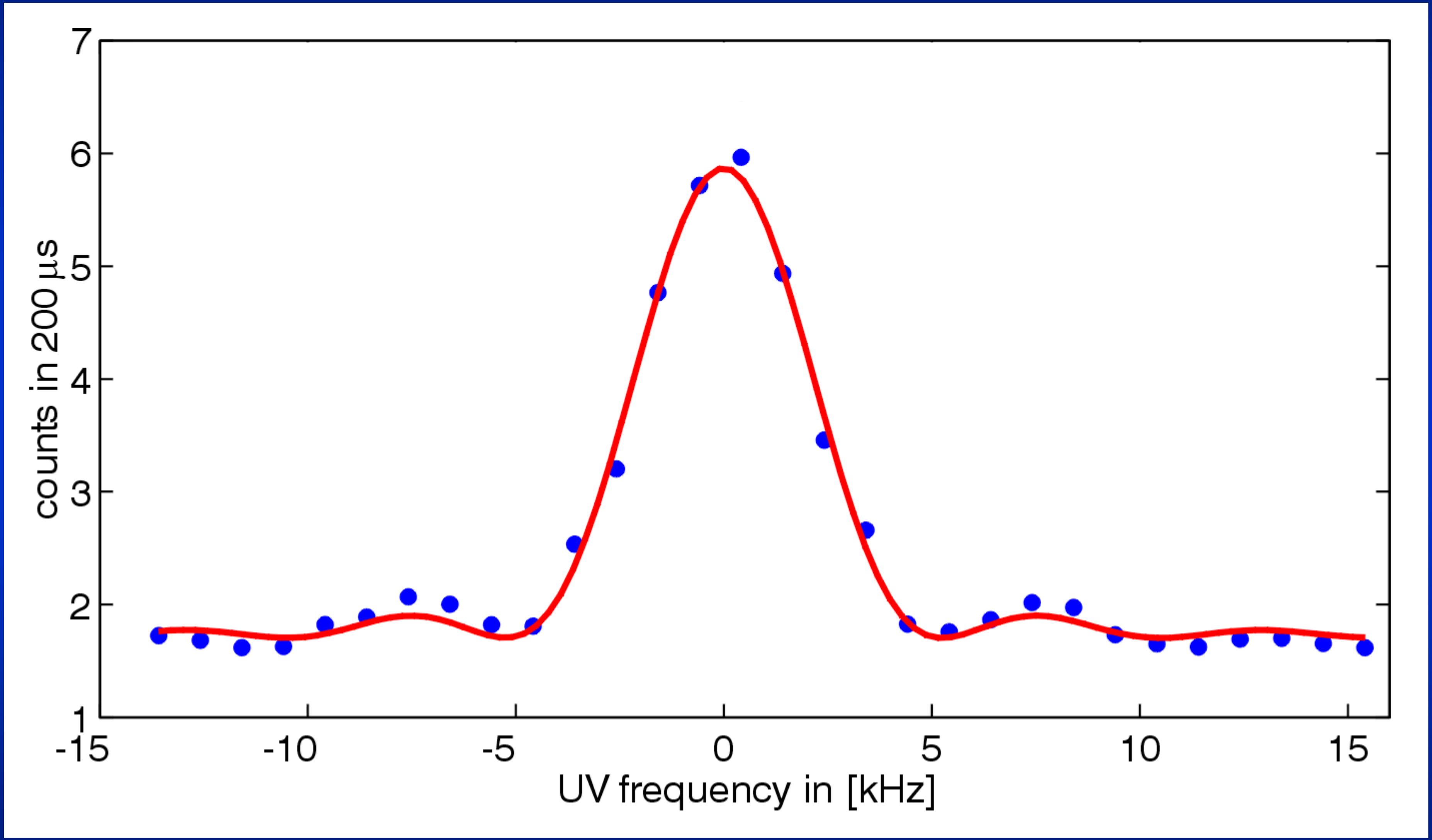




D.J. Wineland *et. al.*,
Proc. 6th Symposium on Frequency Standards and Metrology, edited by P. Gill (World Scientific, Singapore, 2001), pp. 361-368

1. probe clock transition on carrier of clock ion:
   $|g>_L|0>_m|g>_C \;->\; |g>_L|0>_m(\alpha|g>_C+\beta|e>_C)$
2. drive Rabi π-pulse on red sideband of clock ion:
   $|g>_L|0>_m(\alpha|g>_C+\beta|e>_C) \;->\; |g>_L\,(\alpha|g>_C|0>_m+\beta|g>_C|1>_m)$
3. drive Raman π-pulse on red sideband of ^{9}Be^{+} ion:
   $(\alpha|g>_L|0>_m+\beta|g>_L|1>_m)\,|g>_C \;->\; (\alpha|g>_L|0>_m+\beta\,|e>_L|0>_m)\,|e>_C$
4. detect using electron shelving technique on ^{9}Be^{+}

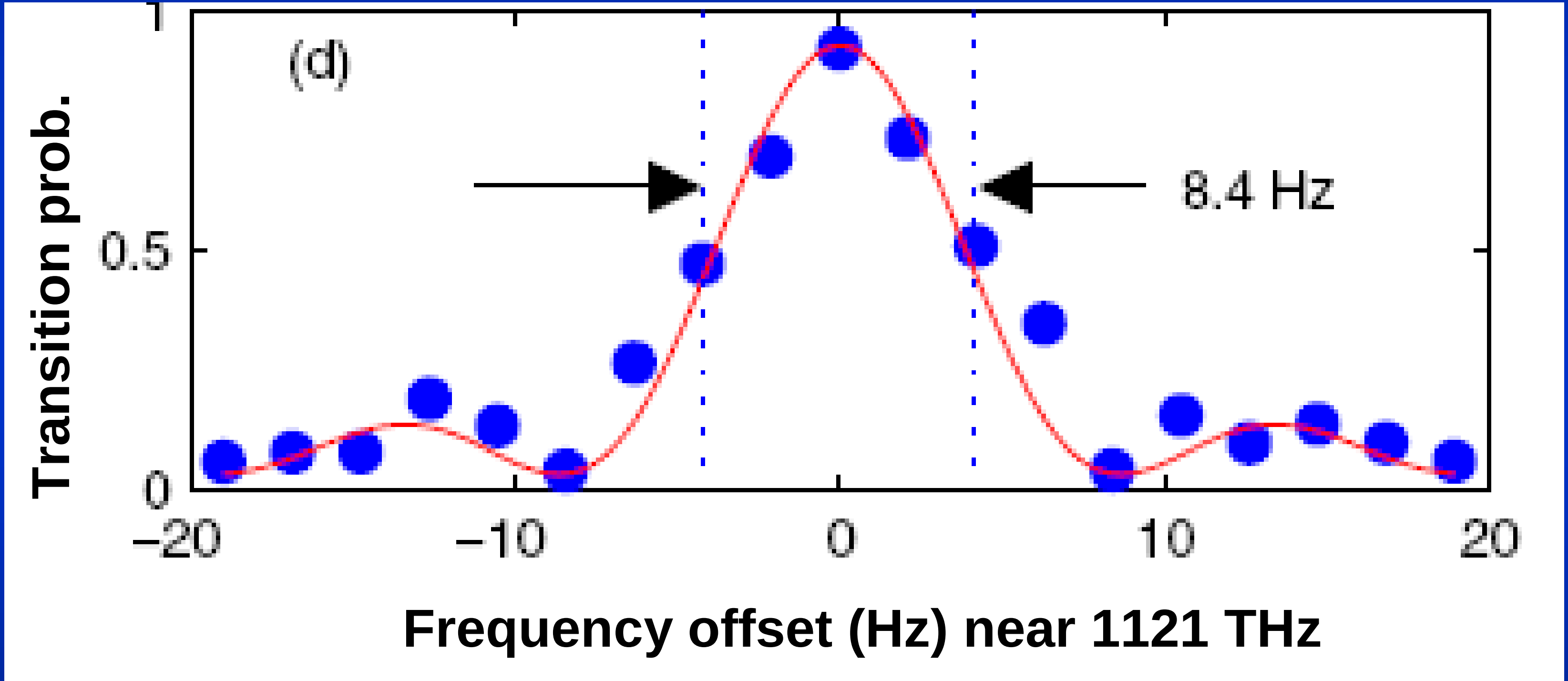
The $^{27}\mathrm{Al}^{+}\ {}^{1}\mathrm{S}_{0}\ \$\ {}^{3}\mathrm{P}_{1}$ transition



P. O. Schmidt et al.,
*Science **309**, 749 (2005)*

The $^{27}\mathrm{Al}^{+}\ {}^{1}S_{0}\ \$\ {}^{3}P_{0}$, $m_F = 5/2\ \$\ 5/2$ transition

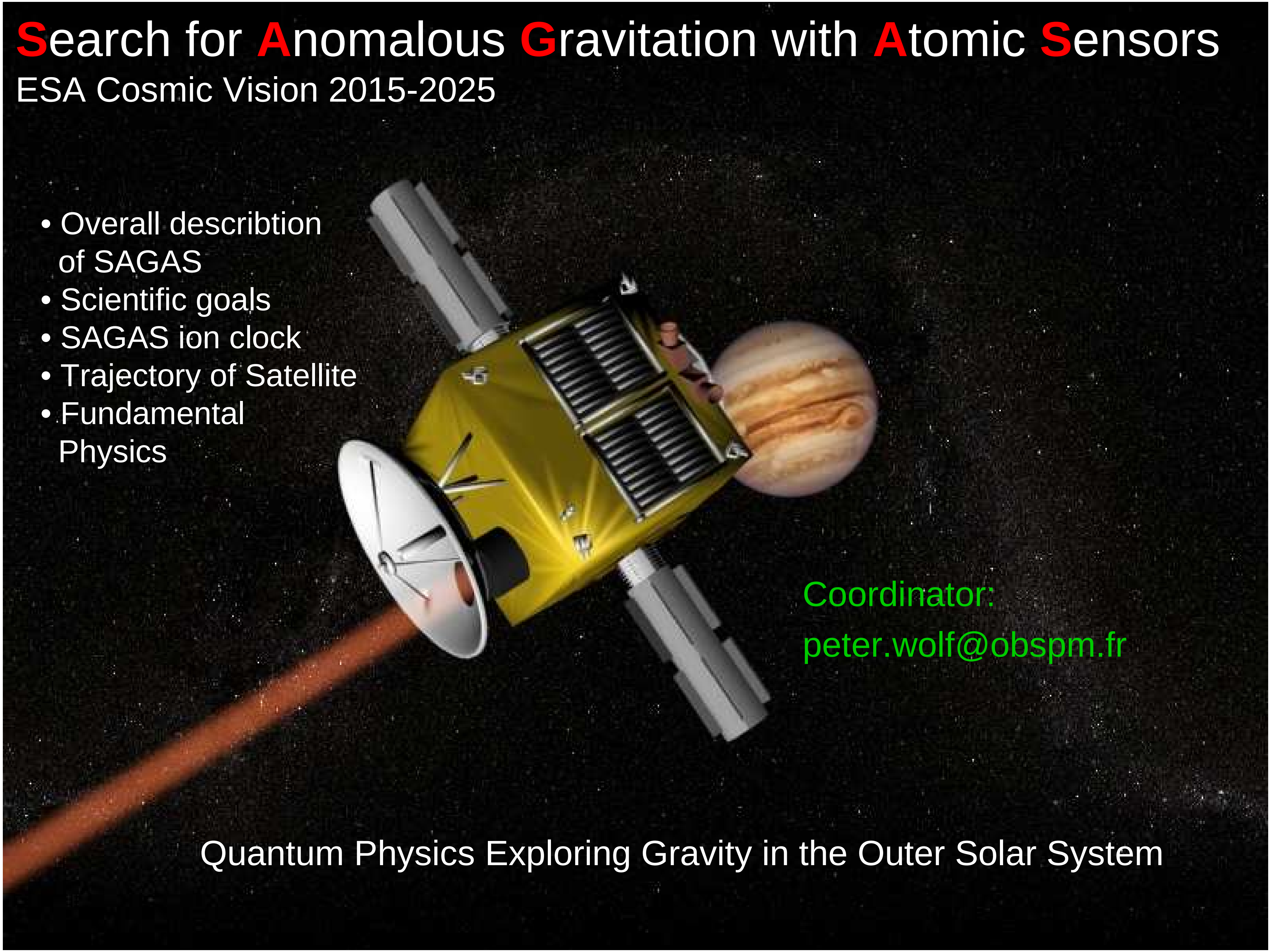
ν = 1 121 015 393 207 851(6) Hz



Rosenband et al,
*PRL **98,** 220801 (2007)*

Search for Anomalous Gravitation with Atomic Sensors

ESA Cosmic Vision 2015-2025



- Overall describtion of SAGAS
- Scientific goals
- SAGAS ion clock
- Trajectory of Satellite
- Fundamental Physics

Coordinator:

peter.wolf@obspm.fr

Quantum Physics Exploring Gravity in the Outer Solar System

\> 70 participants from:

- **France:** SYRTE, IOTA, LKB, ONERA, OCA, LESIA, IMCCE, Univ. Pierre at Marie Curie Paris VI, Univ. Paul Sabatier Toulouse III

- **Germany:** IQO Leibniz Univ. Hannover, ZARM, PTB, MPQ, Astrium, Heine Univ. Düsseldorf, Humboldt Univ. Berlin, Univ. Hamburg, Univ. Ulm, Univ. Erlangen

- **Great Britain:** NPL

- **Italy:** LENS, Univ. of Firenze, INFN, INRIM, Univ. di Pisa, INOA Firenze, Politecnico Milano

- **Portugal:** Instituto Superior Técnico

- **Austria:** Univ. of Innsbruck

- **Canada:** NRC

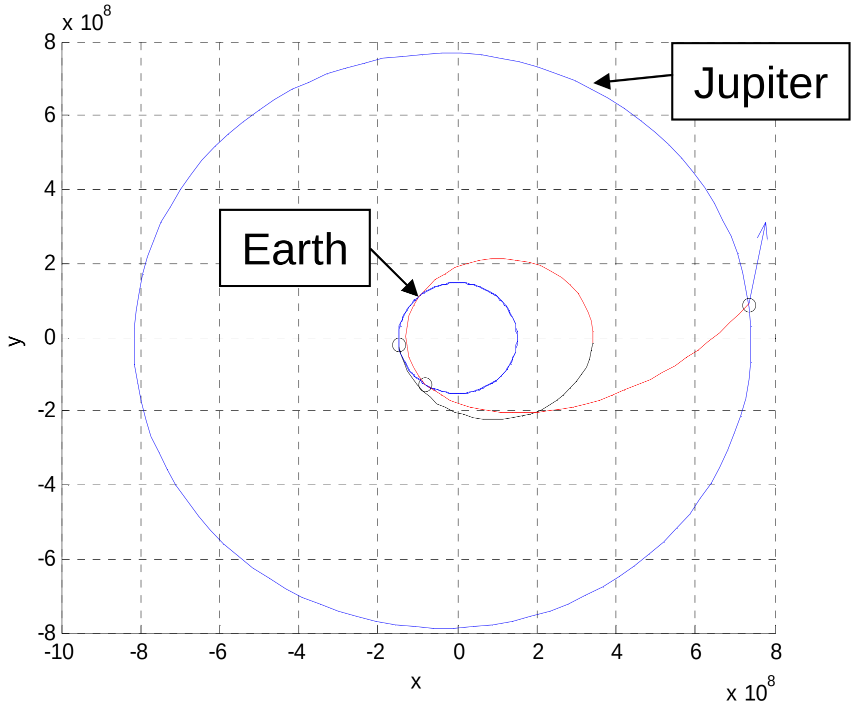
- **USA:** JPL, NIST, JILA, Global Aerospace Corp., Stanford Univ., Harvard Univ.

- **Australia:** Univ. of Western Australia

Schmidt-Kaler, Wolf, Gill, Enrico Fermi Summer School, Course CLXVIII

Wolf et al., arXiv 07110304

SAGAS: Overview



Payload:

1. Cold atom absolute accelerometer, 3-axis measurement of local non-gravitational acceleration
2. Optical ion clock, absolute frequency measurement (local proper time)
3. Laser link (frequency comparison + Doppler for navigation).

Trajectory:

- Jupiter flyby and gravity assist (≈ 3 years after launch).
- Reach distance of ≈39 AU (15 yrs nominal) to ≈53 AU (20 yrs, extended)
- 950 kg, 390 W

Measurements:

- *Gravitational trajectory of test body:* using Doppler ranging and correcting for non-gravitational forces using accelerometer measurements.
- *Gravitational frequency shift of local proper time:* using clock and laser link to ground clocks for frequency comparison

⇒ ***Measure all aspects of gravity !***

Exploring large-scale gravity with SAGAS

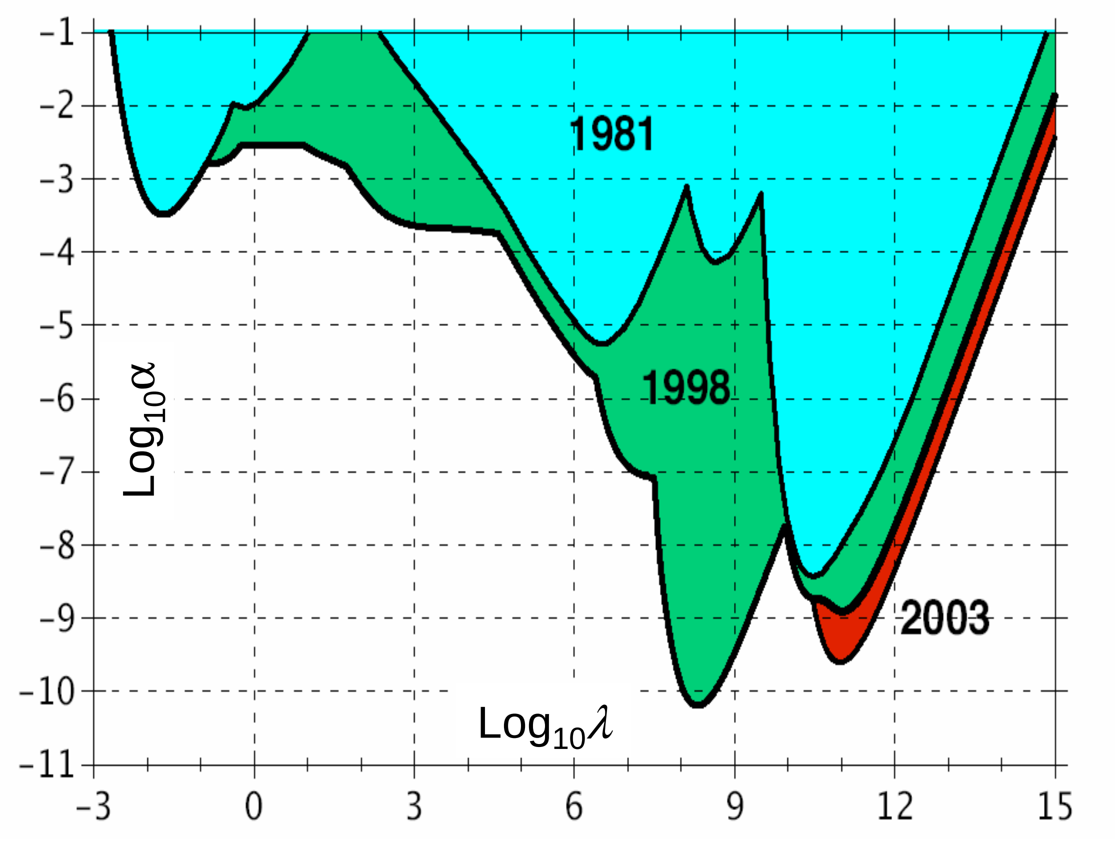
Search for a deviation

$$g_{00} \quad = \quad [g_{00}]_{\mathrm{GR}} + \delta g_{00}$$

For example under the form of a Yukawa correction

$$\delta g_{00}(r) = 2\phi(r)\,\alpha\exp\left(-\frac{r}{\lambda}\right)$$

Pioneer anomaly
at 20 AU …70 AU



Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge, and E. M. Standish (2003)

The Search for Non-Newtonian Gravity, E. Fischbach & C. Talmadge (1998)

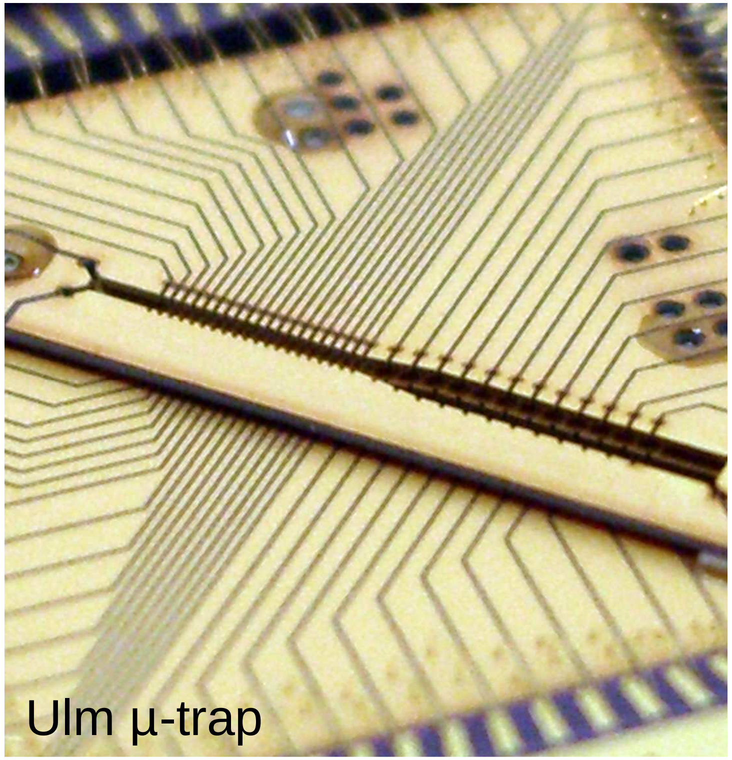
SAGAS optical Clock

- Single trapped ion optical clock, using Sr^+ with 674 nm clock transition.
- Other options kept open (Yb^+, Ca^+,…) subject to development of laser sources.
- Provides narrow and accurate laser:

Stability: $\sigma_y(\tau) = 1\ 10^{-14} / \sqrt{\tau}$ τ = integration time in s
Accuracy: $\delta f/f \ \leq$ few 10^{-17}

- Challenge for SAGAS is not performance but space qualification and reliability


PTB 3D-Paul trap


Ulm µ-trap

Integrated fibre optics
- illumination
- detection

Separted loading and clock zone

General and special Relativity

$$\frac{d\tau_S}{dt} - \frac{d\tau_G}{dt} \simeq \frac{w_G - w_S}{c^2} + \frac{\nu_G^2 - \nu_S^2}{2c^2} + O(c^{-4})$$

velocity

Freq.

grav. Pot.

Gravitational redshift (Gen. Rel.)

2. order Doppler (Sp. Rel.)

10^{-9} GR prediction (10^5 gain on present)

$3 \cdot 10^{-9}$ SR ($10^{2..4}$ gain on present)

Parametrised Post-Newtonian framework

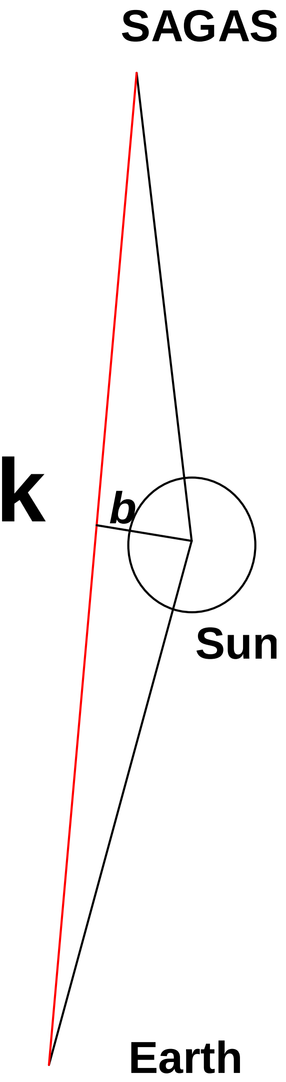
two-way Doppler signal

$$\delta D_\nu(t) = -4(1+\gamma)\frac{GM}{c^3 b}\frac{db}{dt}$$

Cassini, Nature
425, 374 (2003)

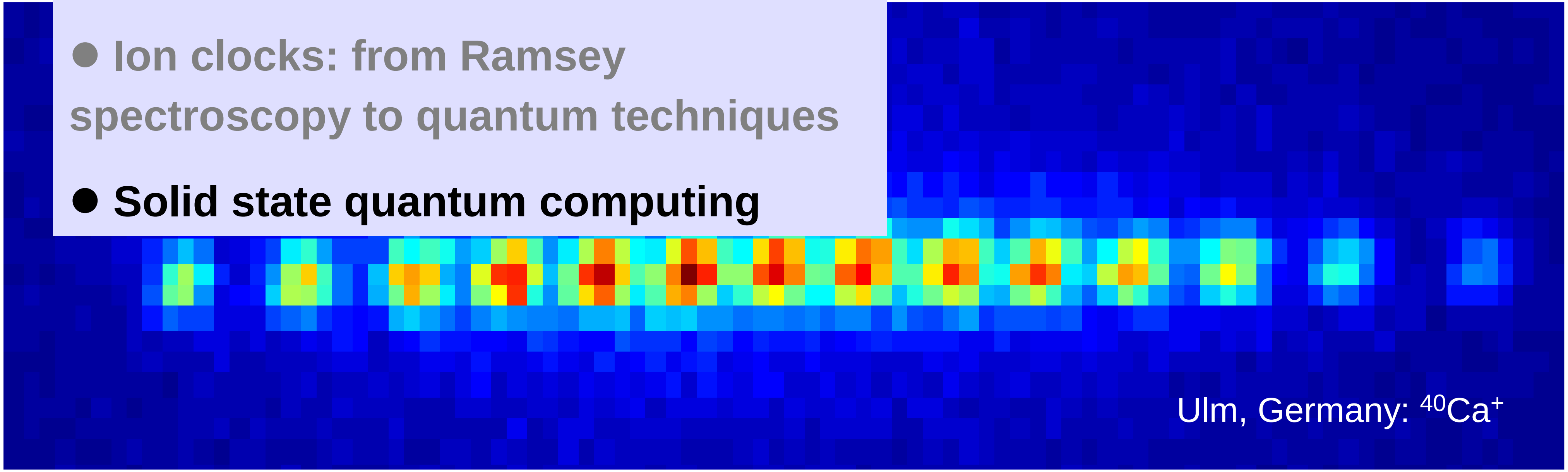
$$\gamma = 1 + (2.1 \pm 2.3)10^{-5}$$

$10^{-7..-9}$ uncertainty on γ (10^2 to 10^4 improvement)



Cold Ions and their Applications for Quantum Computing and Frequency Standards

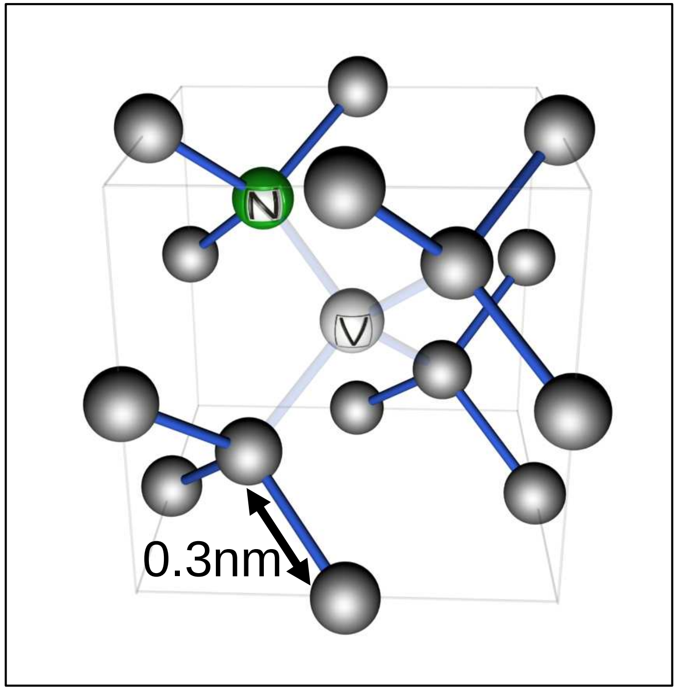
- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- Quantum computer: basics, gates, algorithms, future challenges
- Ion clocks: from Ramsey spectroscopy to quantum techniques
- **Solid state quantum computing**

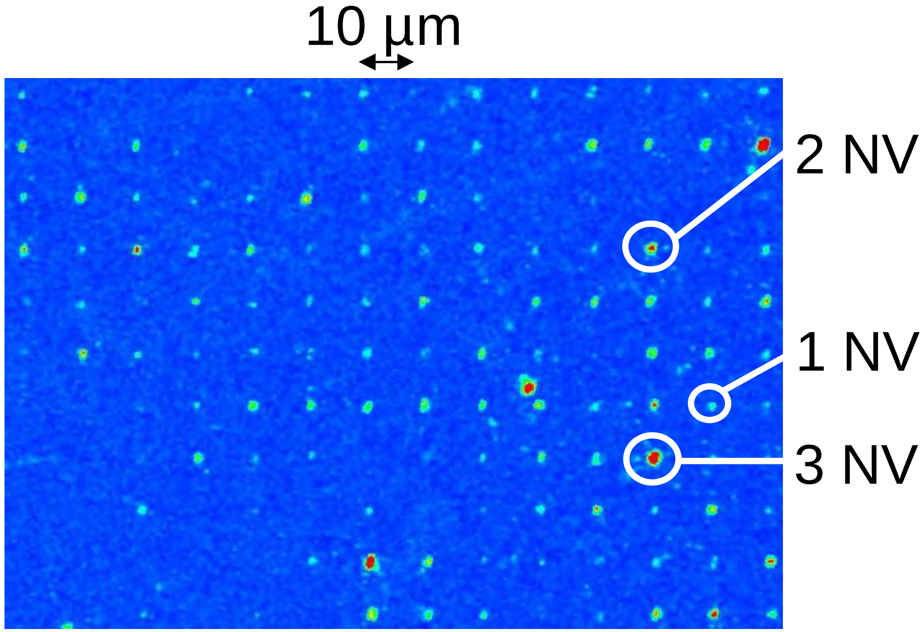


Ulm, Germany: ^{40}Ca$^+$

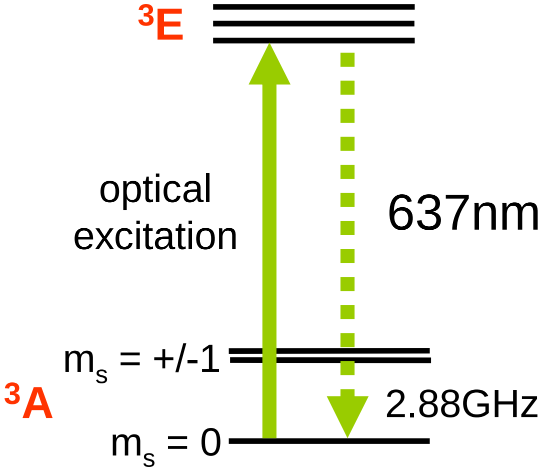
NV colour centers

*F. Jelezko et al., Phys. Rev. Lett. **93**, 13 (2004)*





2 MeV: spot size 300nm

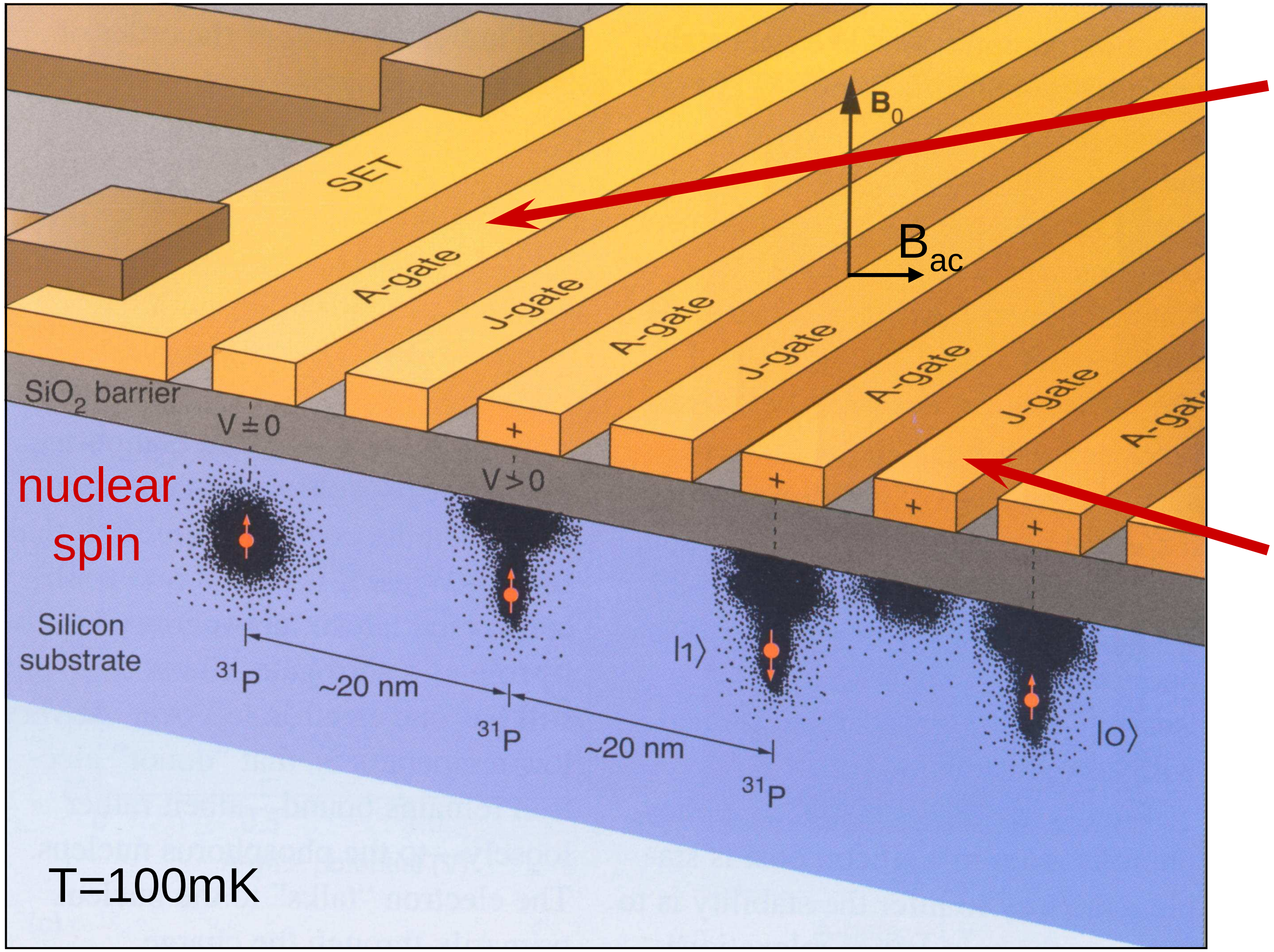


	[NV] color center
Wavelength	637 nm
Linewidth	24 MHz
Dipole Moment	1×10^{-29} Cm

Prof. J. Wrachtrup / F. Jelezko (Universität Stuttgart)

Solid state QC with quantum dots

B. E. Kane,
*Nature **393**, 133 (1998)*



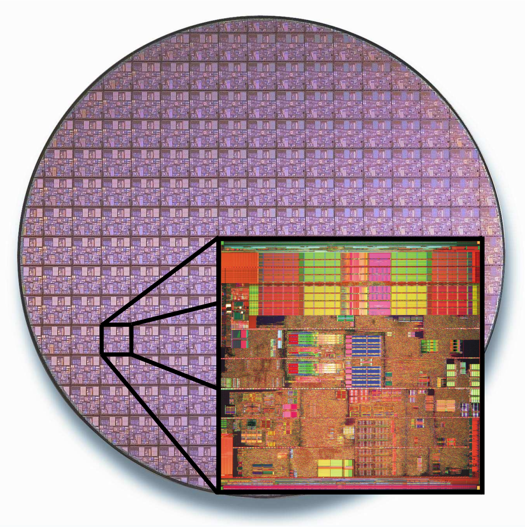
Control of strength
of hyperfine interaction
⇨ "Voltage controlled
oscillator"

B_{ac} flips nuclear
spins at resonance

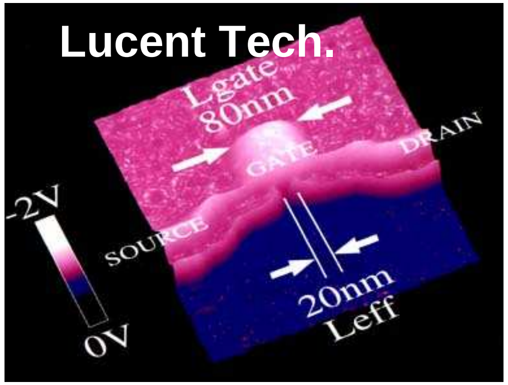
Switch on/off:
Electron mediated
coupling between
nuclear spins

Readout:
nuclear spin
⇨ electron spin
⇨ orbital wavefunction
⇨ capacitance meas.

Enhancing semiconductor device performance

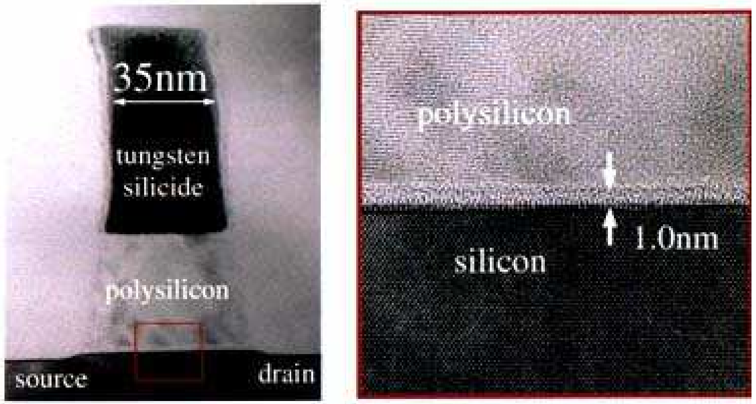


Intel 65nm-Technology



Nano-FET Transistors

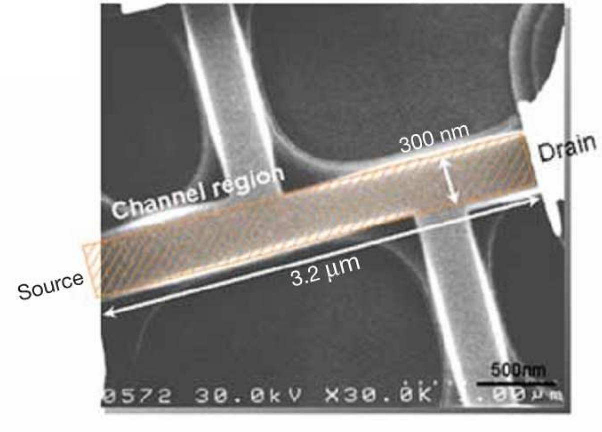
20…35nm gate length

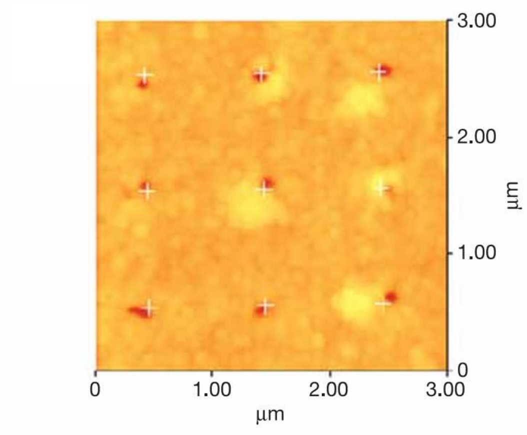


~100 x 10 x 10 lattice places

300nm regions with a typical concentration of 10^{15} cm^{-3}
~ 30 dopants in side (300 nm)^3
statistical fluctuations of about ~20%

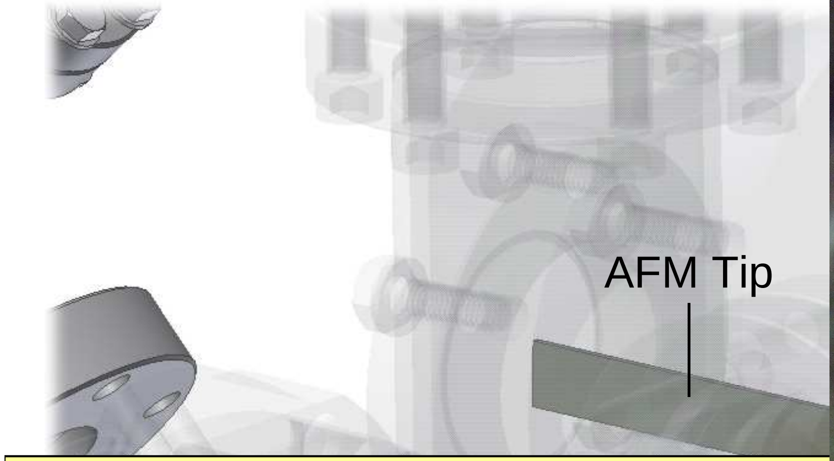
Reduction of the gate threshold voltage by a factor of 2

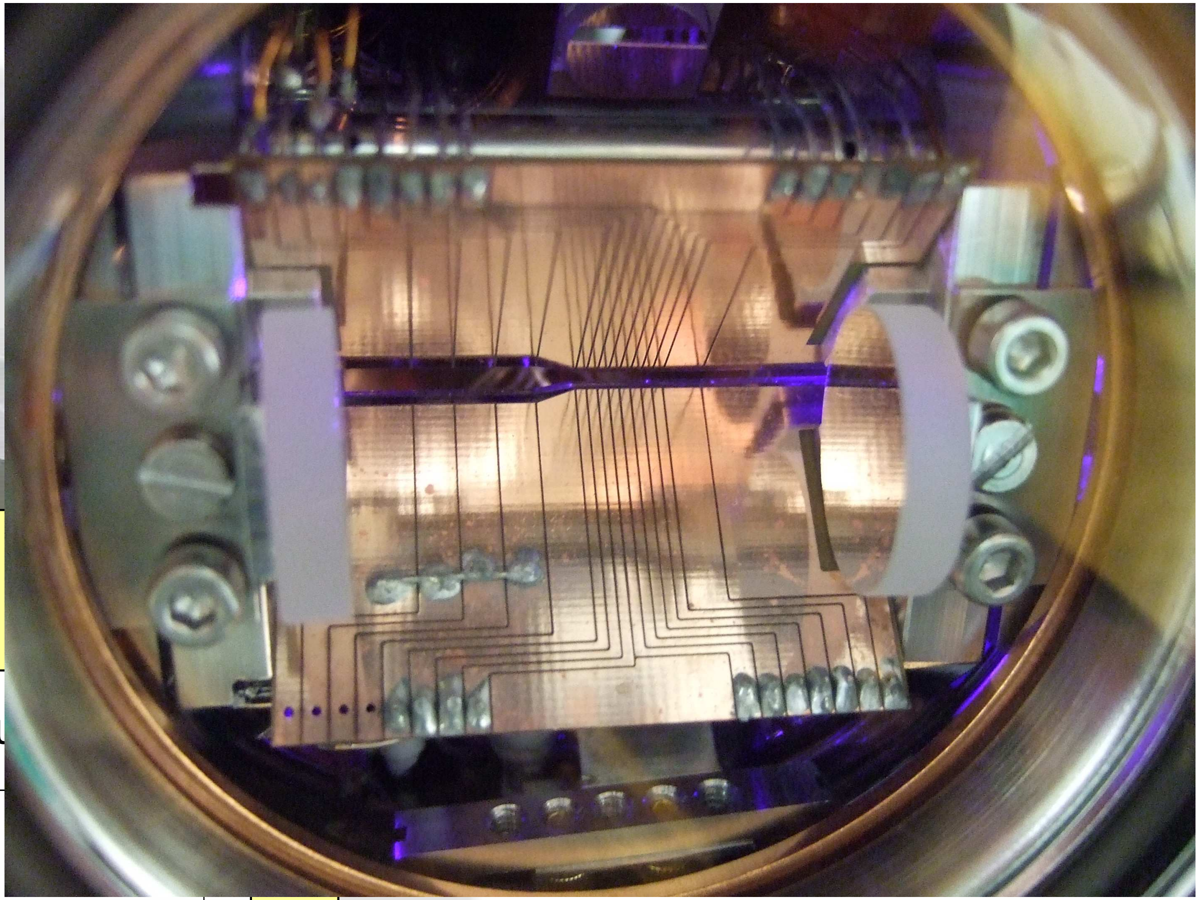




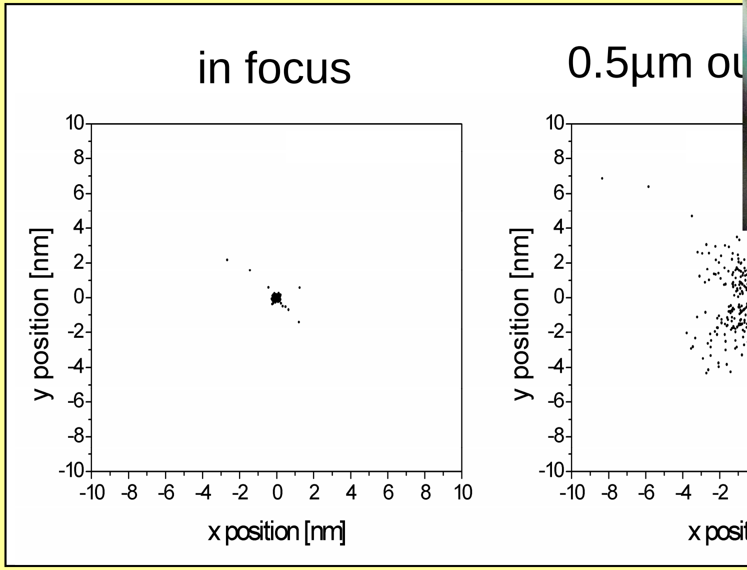
T. Shinada et al.,
J. Vac. Tech. B16, 2489 (1998)

Segmented Paul Trap as perfect point source for laser cooled ions





Focusing



age

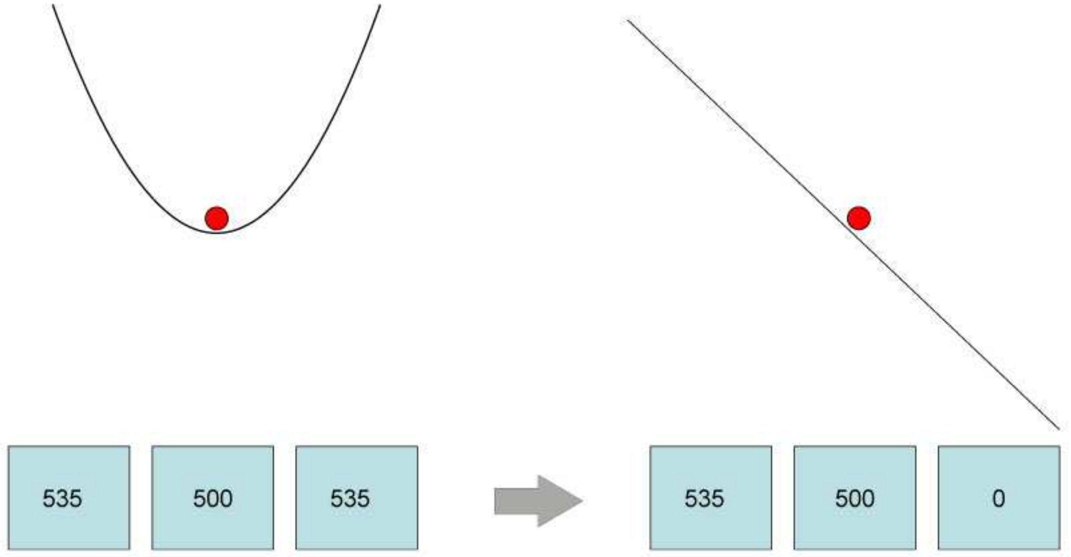
*Meijer, et al, Appl Phys. A **83**, 321 (2006)*

The extraction mechanism

Method A:

Using a potential with an offset to trap the ions

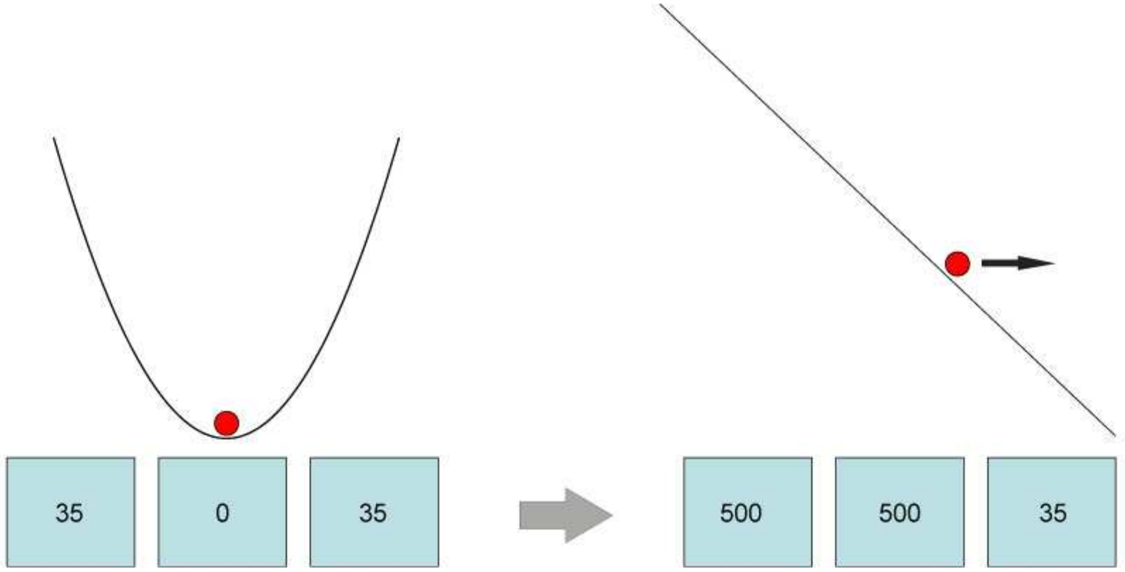
Advantage: Effective energy transfer



Method B (currently in use):

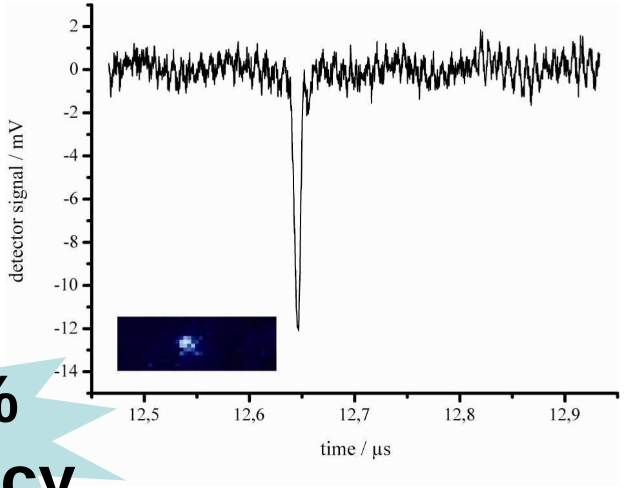
Quick ramp-up of extraction voltage

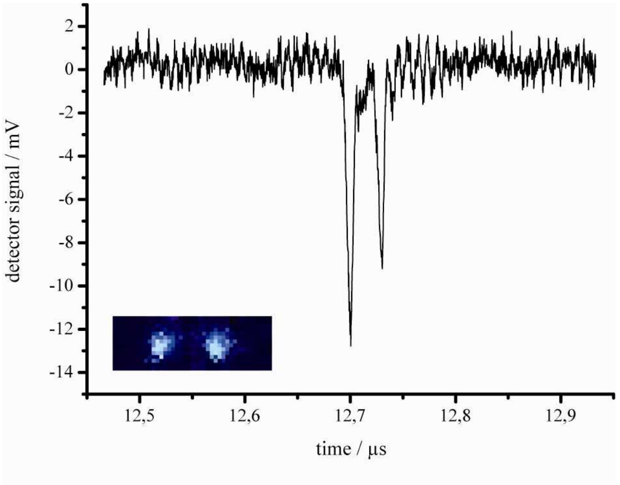
Advantage: Short application of HV helps to maintain trap stability



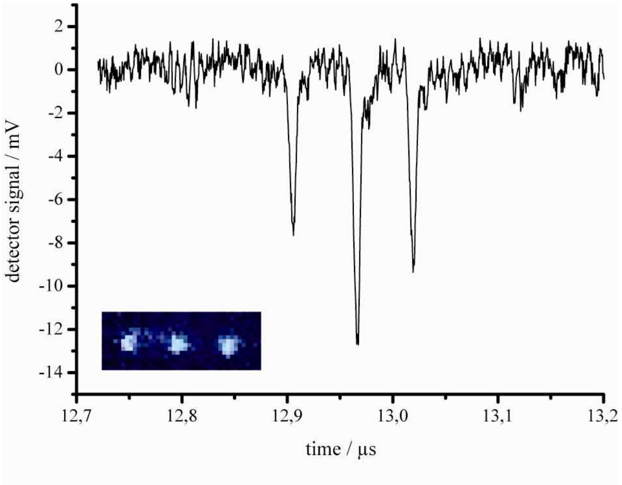
Deterministic extraction of ion crystals

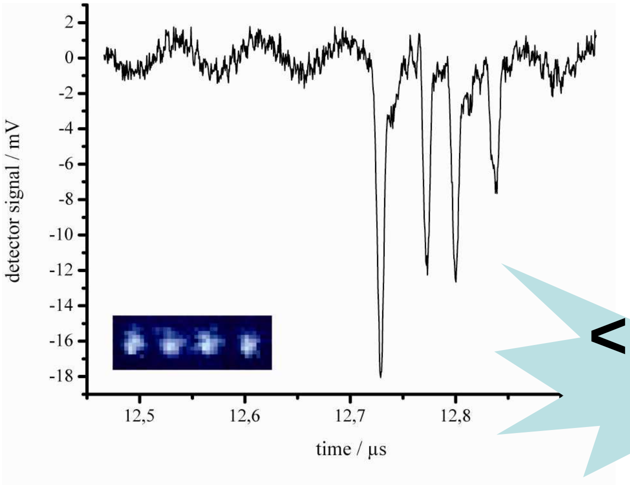
Schnitzler et al,
to be published





> 90% effiziency



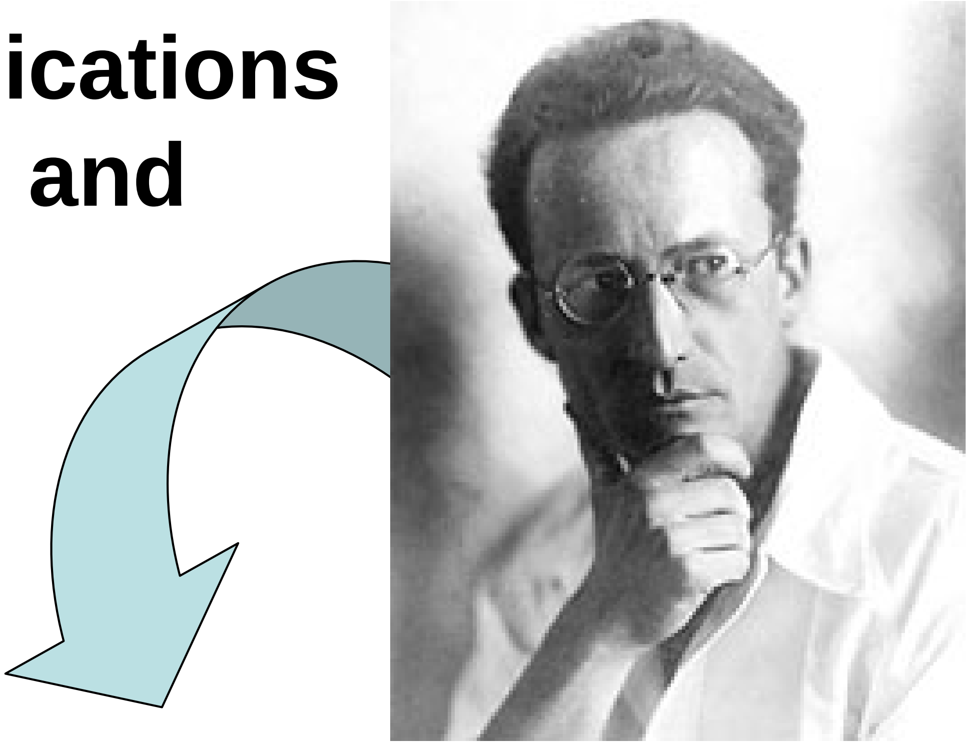


< 0.8% $\delta v/v$

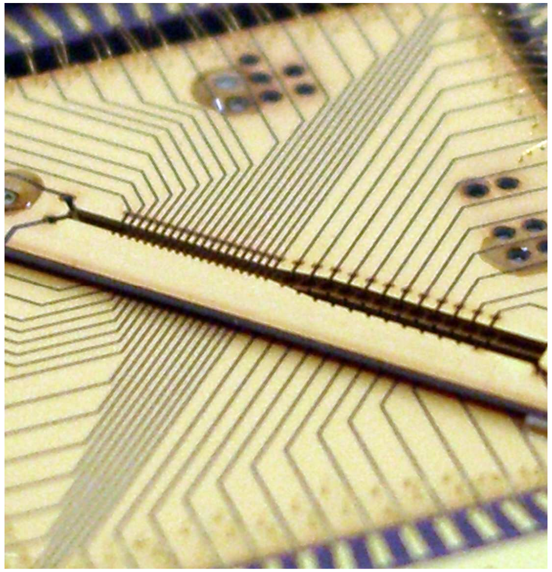
useful for fast implantation

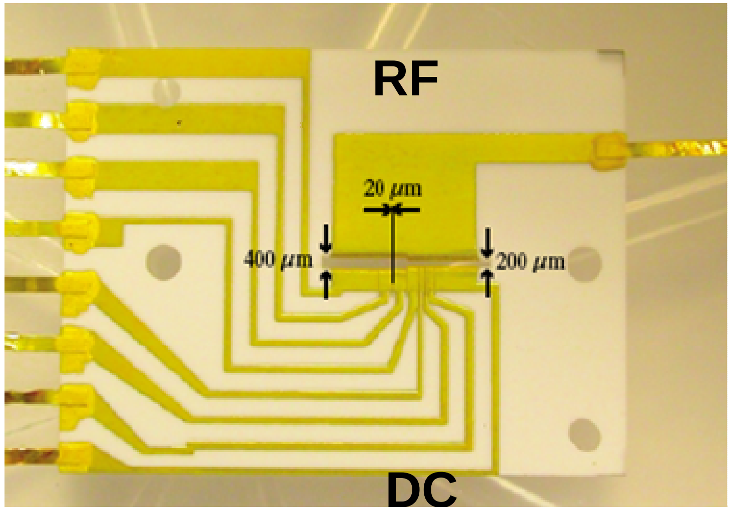
Cold Ions and their Applications for Quantum Computing and Frequency Standards

- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- Quantum computer with Ions
- Ion clocks
- Determinsitic Implantation for Solid state QC



from **Gedanken experiments**
to **modern quantum applications**





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atomics, A8

Bundesministerium für Bildung und Forschung

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http://www.quantenbit.de

Interested to join in?