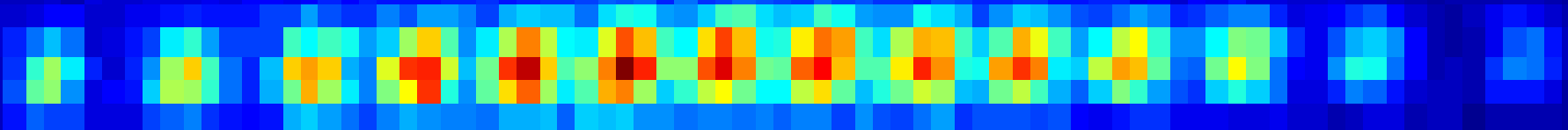


Cold Ions and their Applications for Quantum Computing and Frequency Standards

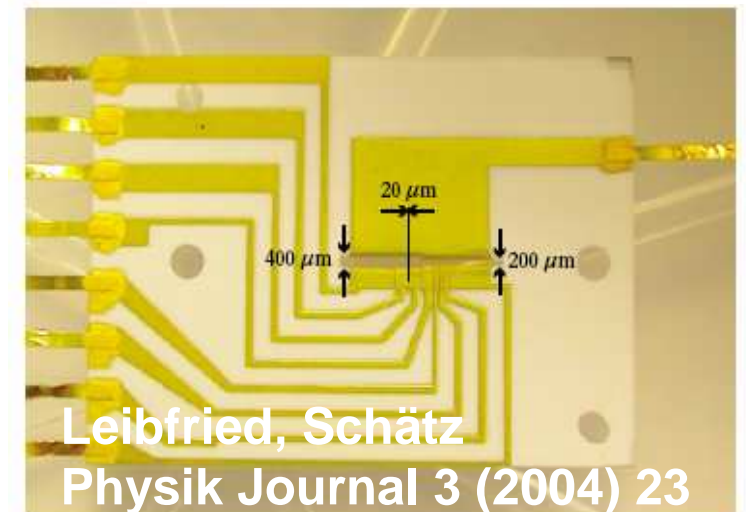
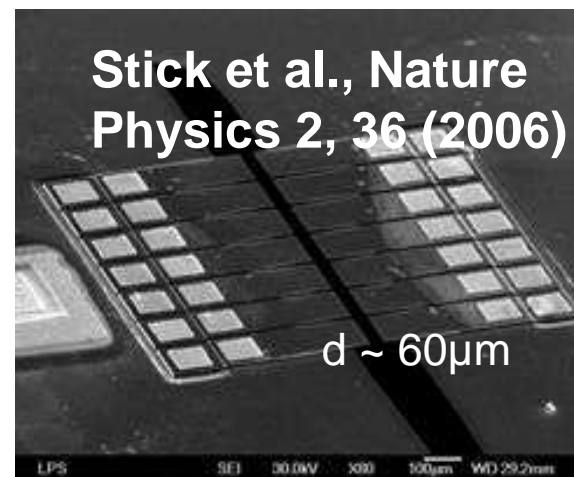
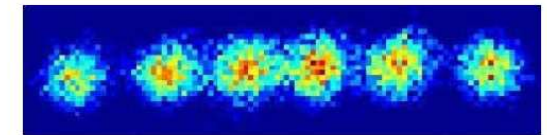
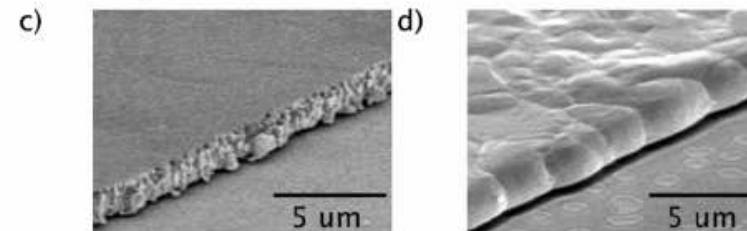
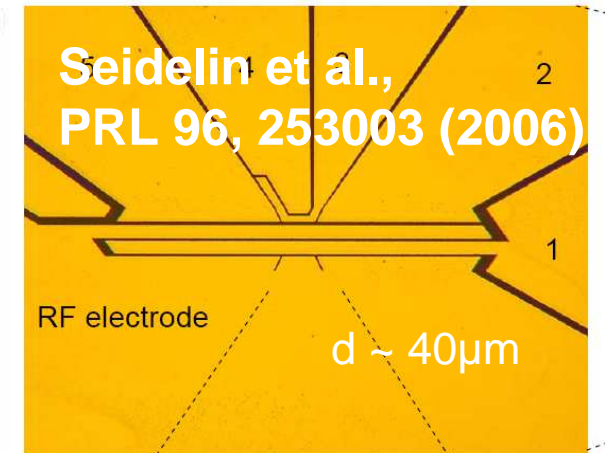
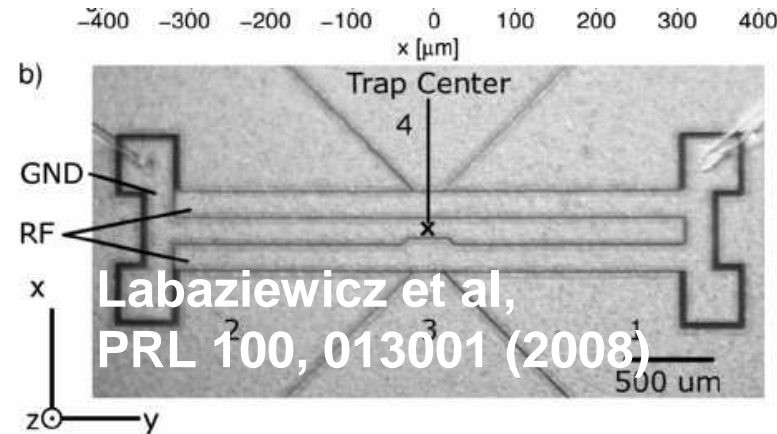
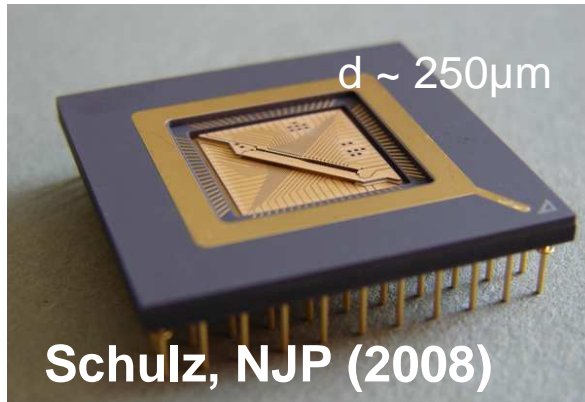
- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- Quantum computer: basics, gates, algorithms, future challenges
- Ion clocks: from Ramsey spectroscopy to quantum techniques

Ferdinand Schmidt-Kaler
Institute for Quantum
Information Processing
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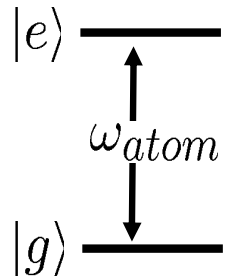
Micro ion traps

*Kielpinski et al.,
Nature 417, 709 (2002)*

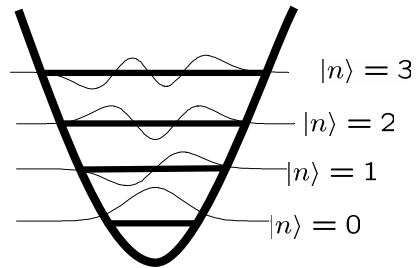


Laser – Ion coupling

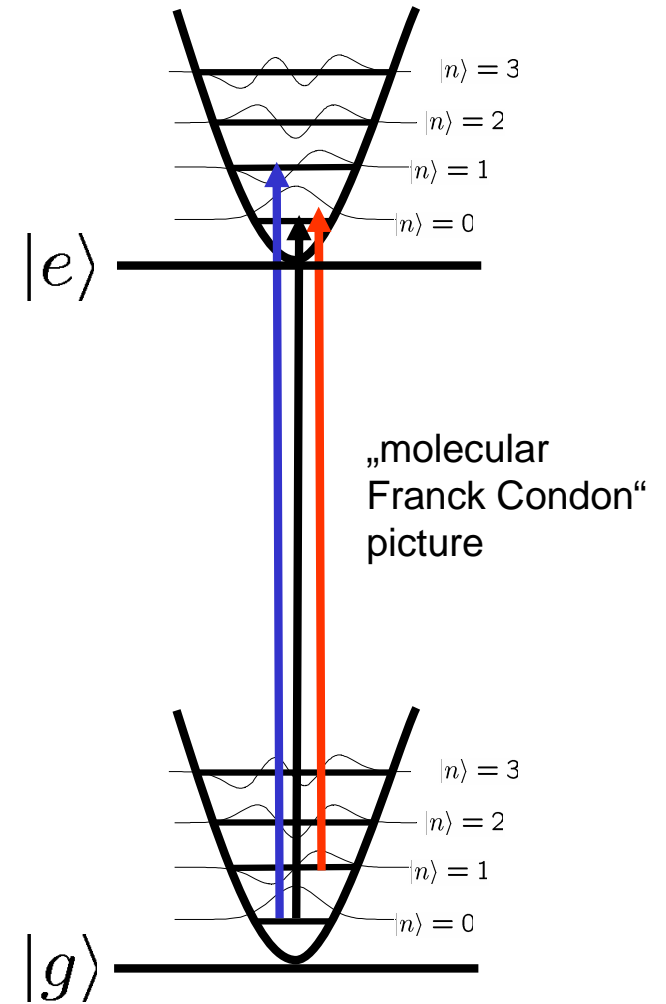
2-level-atom



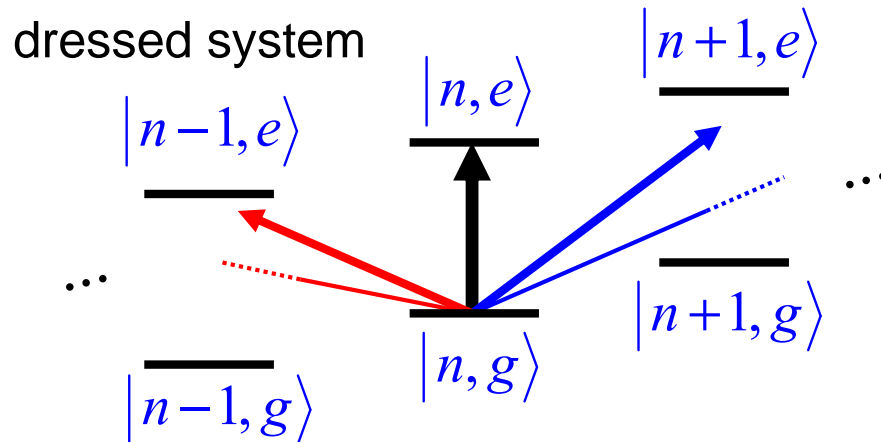
harmonic trap



dressed system



„energy ladder“ picture



Experiments: Resolved sideband spectroscopy

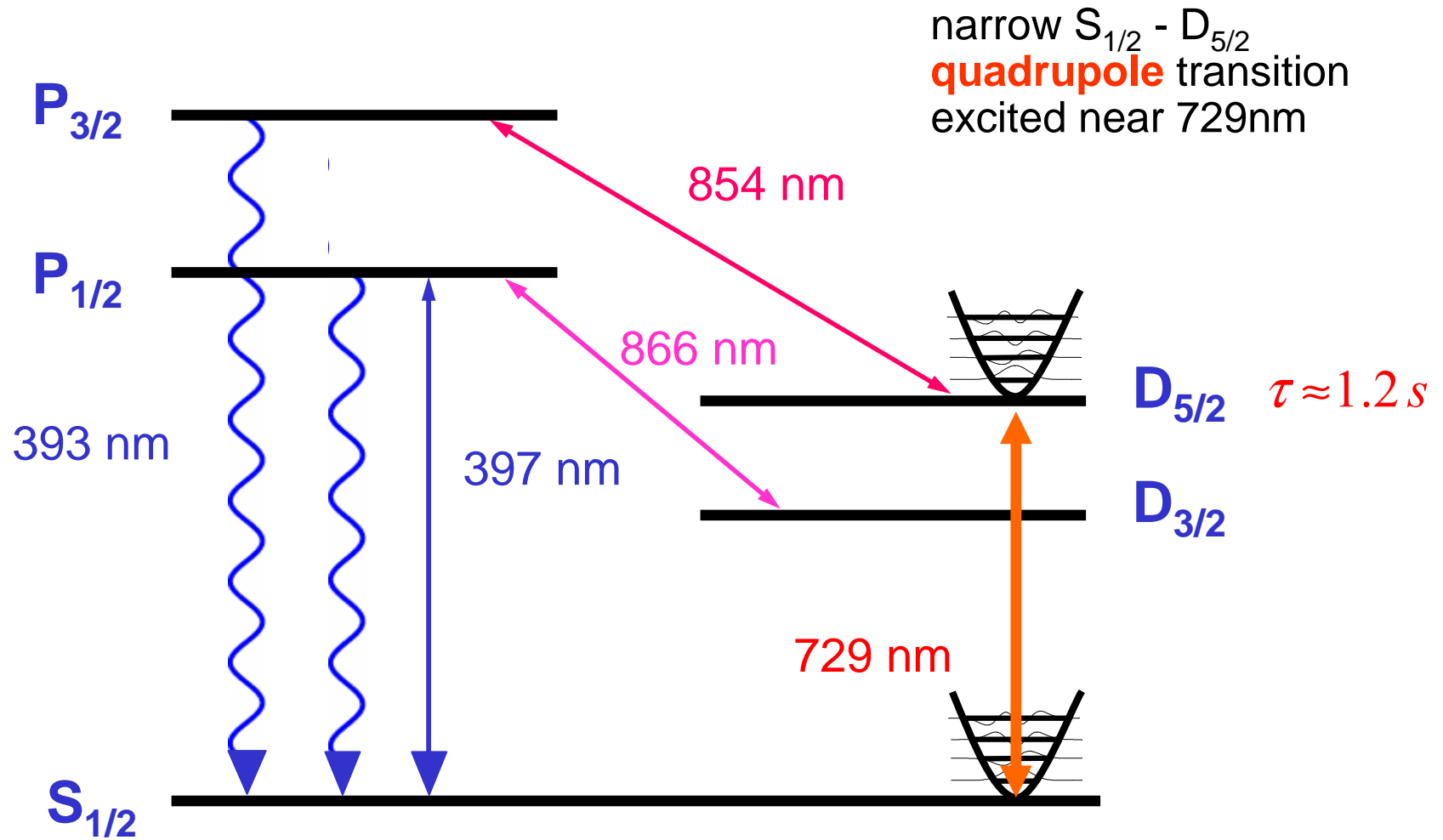
Select narrow optical transition with: $\omega_{trap} \gg \gamma$

- a) Quadrupole transition
- b) Raman transition between Hyperfine ground states
- c) Raman transition between Zeeman ground states
- d) Octopole transition
- e) Intercombination line
- f) RF transition

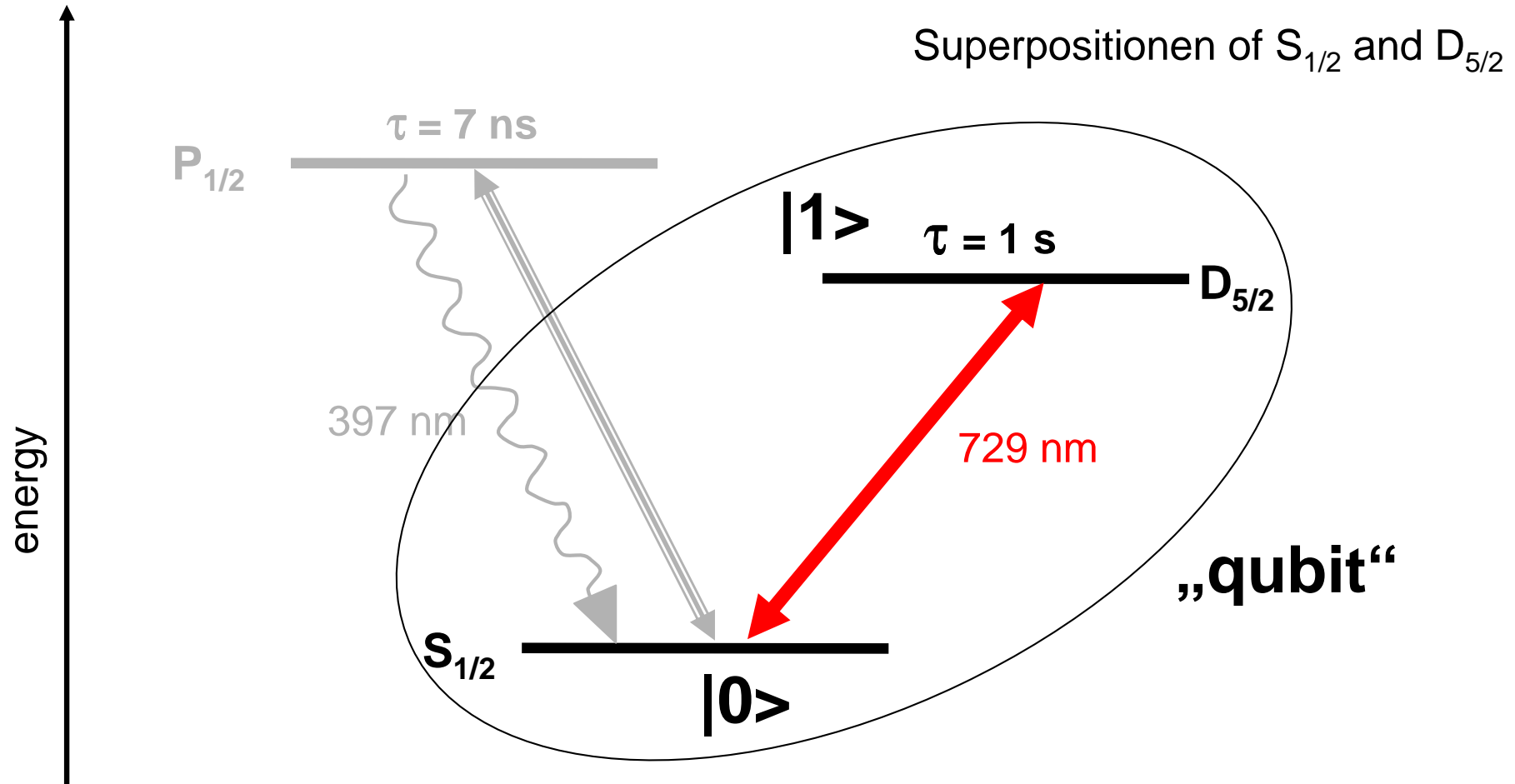
Species and Isotopes:

- for (a) ^{40}Ca , ^{43}Ca , ^{138}Ba , ^{199}Hg , ^{88}Sr ,
- for (b) ^9Be , ^{43}Ca , ^{111}Cd , ^{25}Mg
- for (c) ^{40}Ca , ^{24}Mg ,
- for (d) $^{172/172}\text{Yb}$,
- for (e) ^{115}In , ^{27}Al ,
- for (f) ^{171}Yb ,

Level scheme of $^{40}\text{Ca}^+$



Ion energy levels



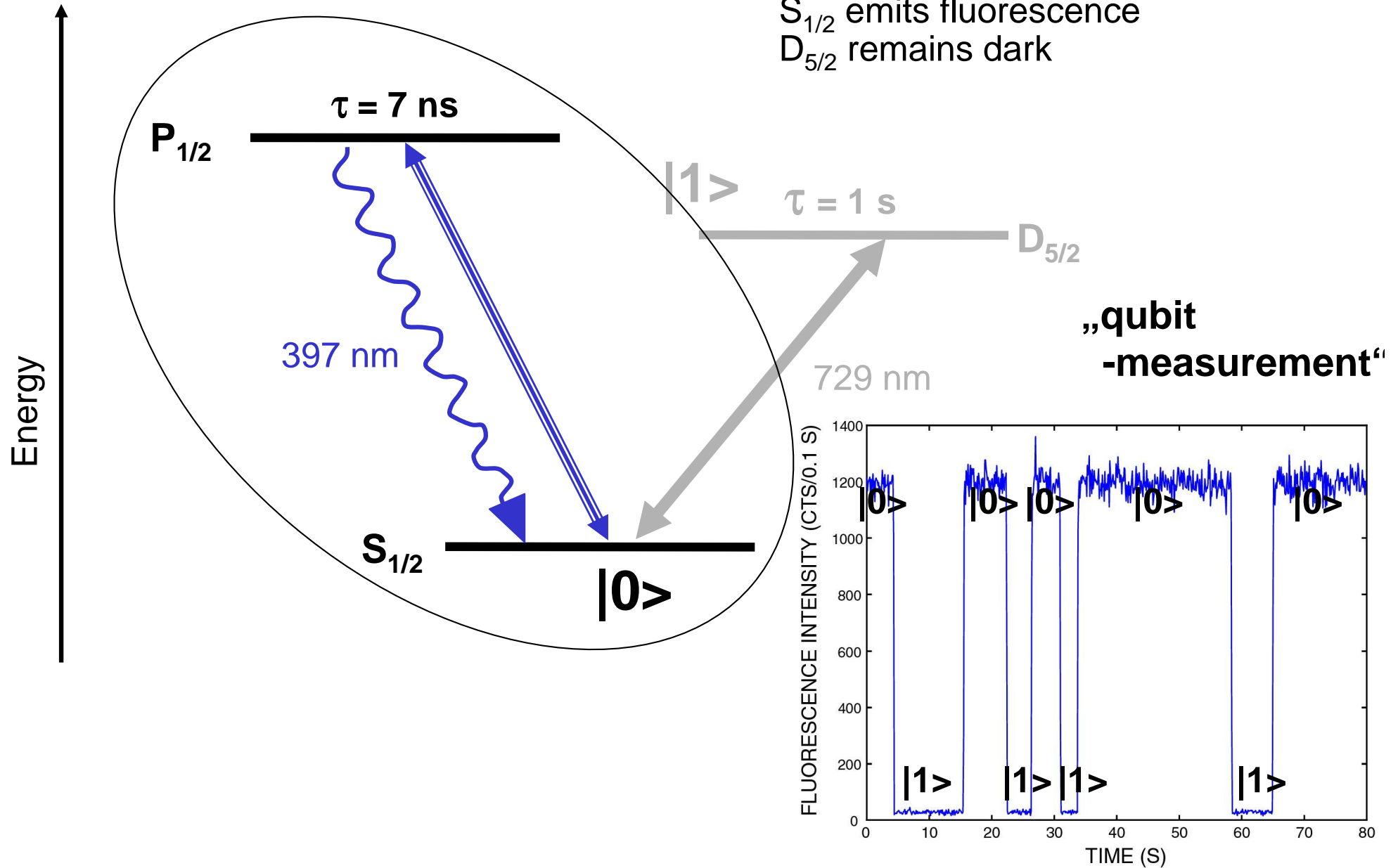
Ion energy levels

Spectroscopy pulse followed by
detection of qubits:

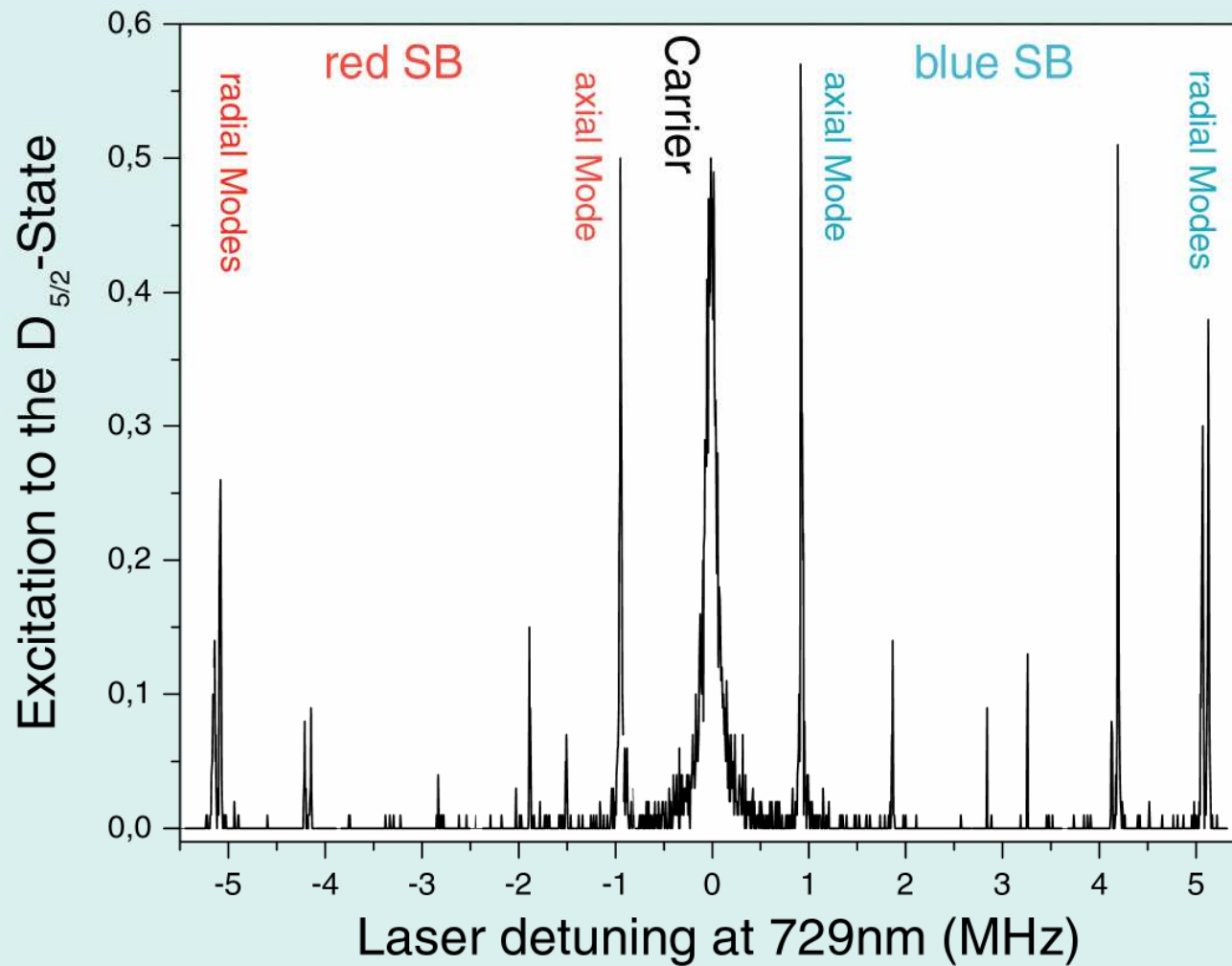
Scatter light near 397nm:

$S_{1/2}$ emits fluorescence

$D_{5/2}$ remains dark



Excitation spectrum of the $S_{1/2} - D_{5/2}$ transition



$\omega_{\text{ax.}} = 1.0 \text{ MHz}$,
 $\omega_{\text{rad.}} = 5.0 \text{ MHz}$

Temperature measurements

different methods

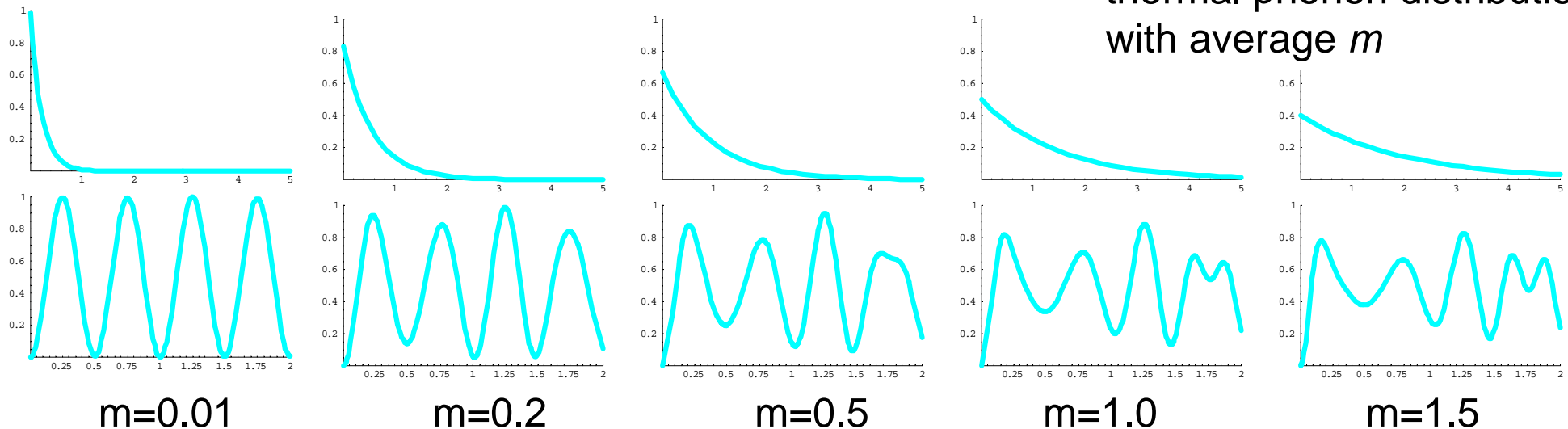
- observe Rabi oscillations on the blue SB
- compare the excitation on the blue SB and the red SB
- compare the excitation on the red SB and the carrier
- observe Rabi oscillations on the carrier
- measure the linewidth of the cooling transition (not recommended)

Experimental: vary the length t of the excitation and record $P_e(t)$
Analysis: Fit, or Fourier transformation

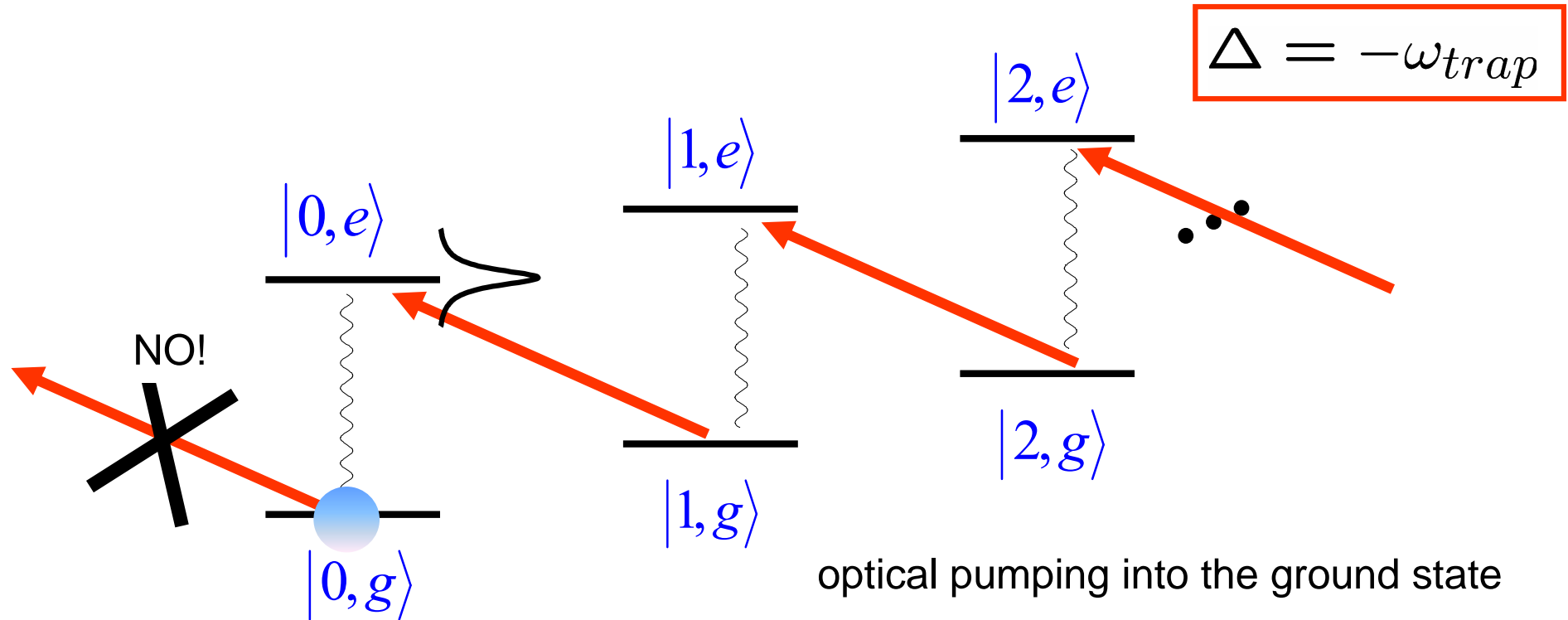
$$P_e^{blue}(t) = \sum n_m \sin^2(2\pi\Omega_{n,n+1}t), \quad n_m = \frac{m^n}{(m+1)^{n+1}}, \quad \langle n \rangle_{ss} = m$$

$$= \sum n_m \sin^2(2\pi\Omega_0\sqrt{n+1}t)$$

thermal phonon distribution
with average m



„Strong confinement“



Signature: no further excitation possible
„dark state“ $|0\rangle$

Temperature measurements

different methods

- observe Rabi oscillations on the blue SB
- compare the excitation on the blue SB and the red SB
- compare the excitation on the red SB and the carrier
- observe Rabi oscillations on the carrier
- measure the linewidth of the cooling transition (not recommended)

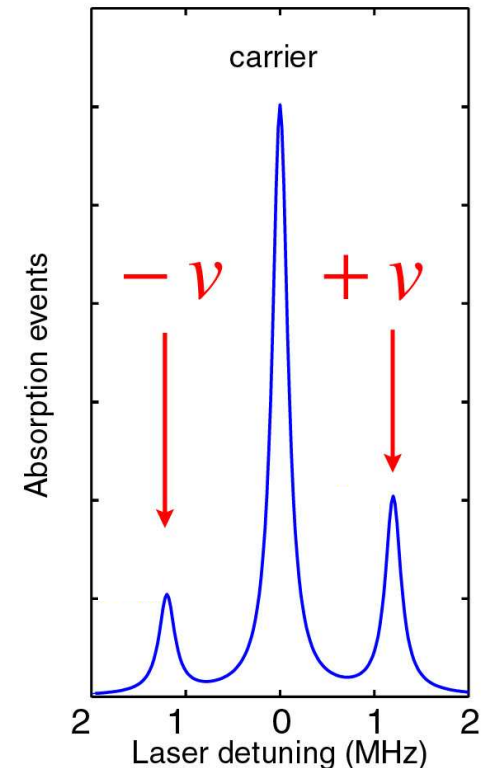
Experimental: test excitation $P_e(t)$ for $\Delta=-\omega$ and $\Delta=+\omega$
Analysis: $P_{red}/P_{blue} = m / (m+1)$

$$\begin{aligned} P_e^{red}(t) &= \sum_{n=1} \frac{m^n}{(m+1)^{n+1}} \sin^2(2\pi\Omega_{n,n-1}t) \\ &= \frac{m}{m+1} \sum_{n=0} \frac{m^n}{(m+1)^{n+1}} \sin^2(2\pi\Omega_{n+1,n}t) \end{aligned}$$

using: $\Omega_{n+1,n} = \Omega_{n,n+1}$

$$\Rightarrow P_e^{red}(t) = \frac{m}{m+1} P_e^{blue}(t)$$

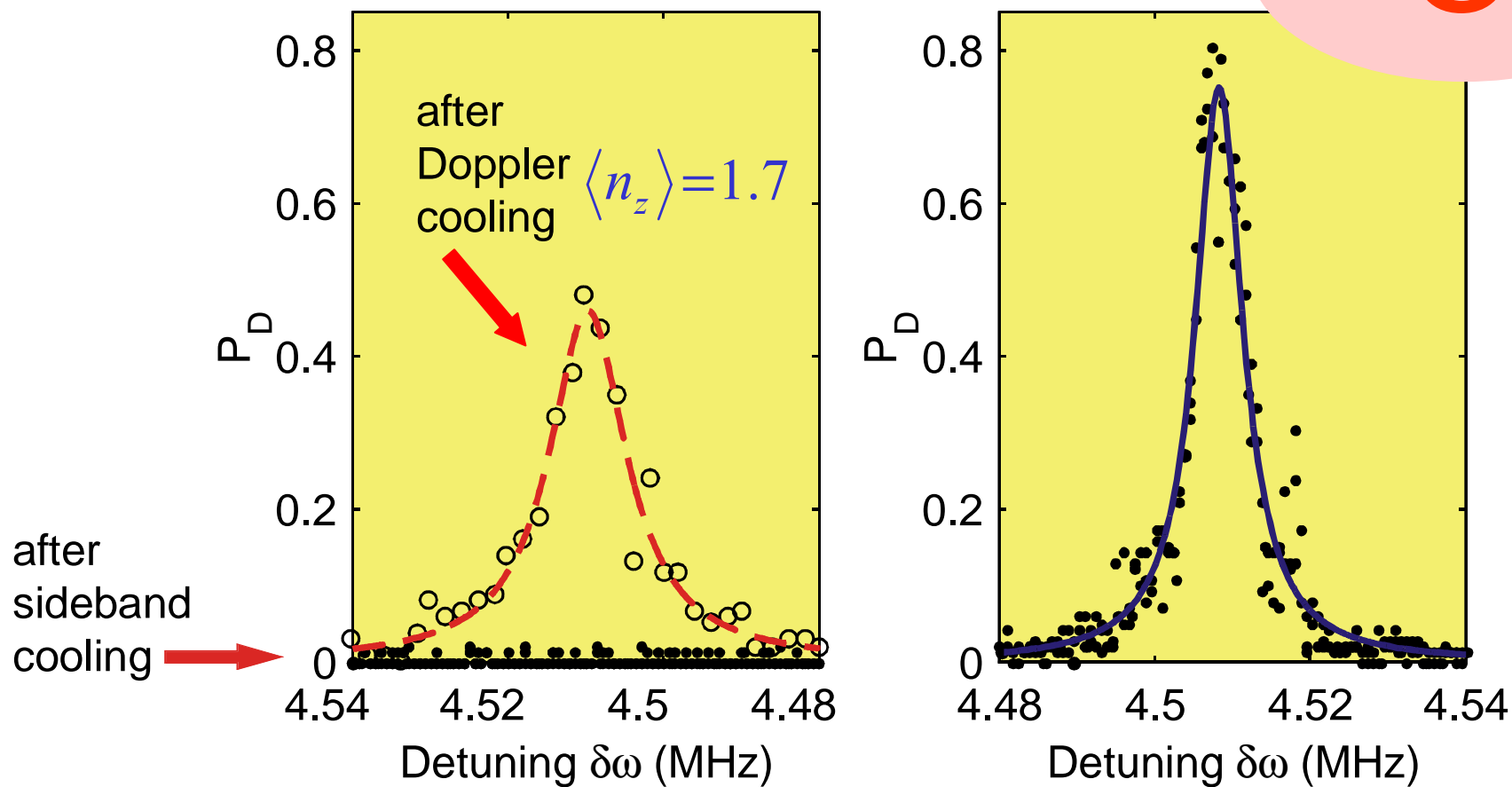
$$m = \frac{R}{1-R}, \quad R = P_e^{red} / P_e^{blue}$$

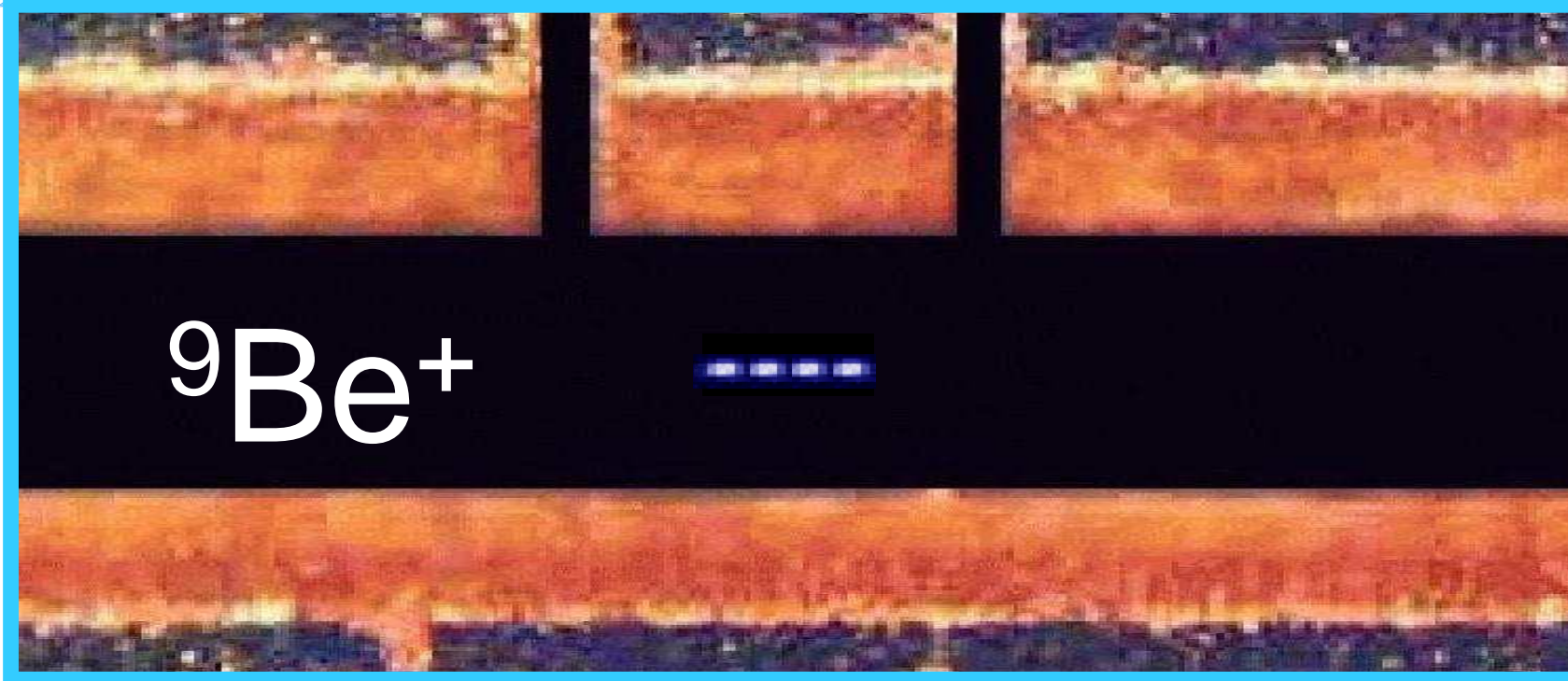


Example: ground state cooling

99.9 % ground state population

$^{40}\text{Ca}^+$

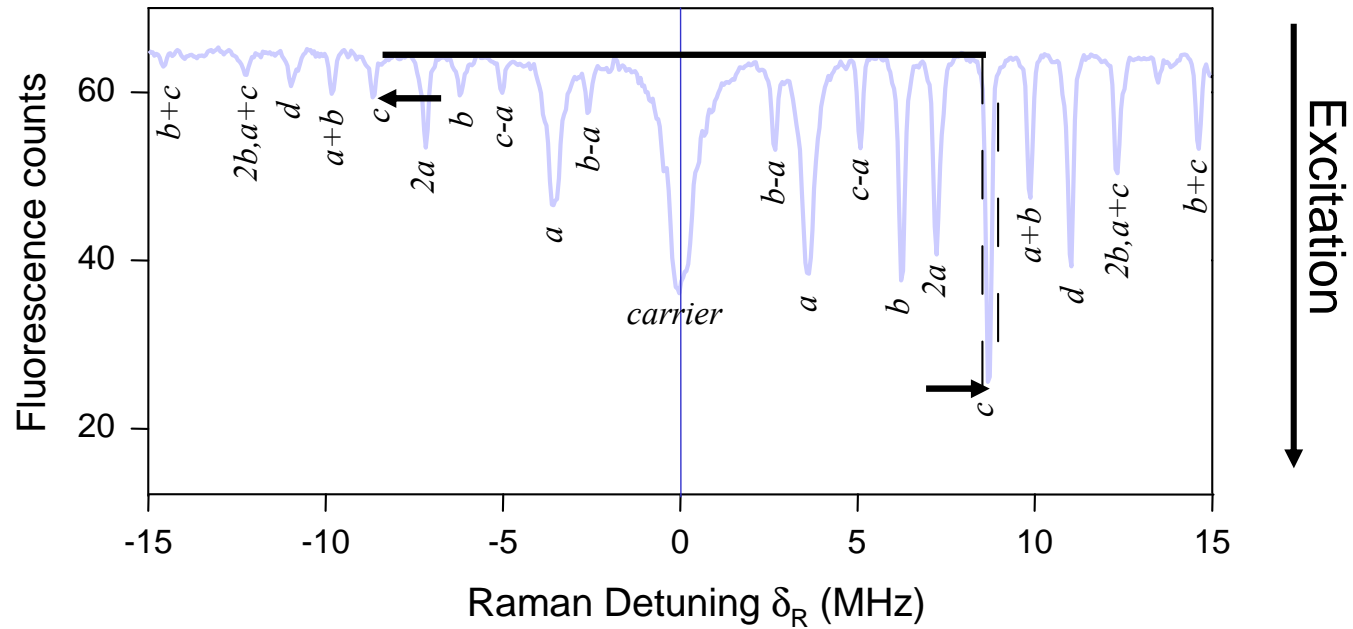




C.A. Sackett, et al., *Nature* **404**, 256 (2000)

D. Wineland, Boulder, USA (2000)

Example: 4-ion axial spectrum

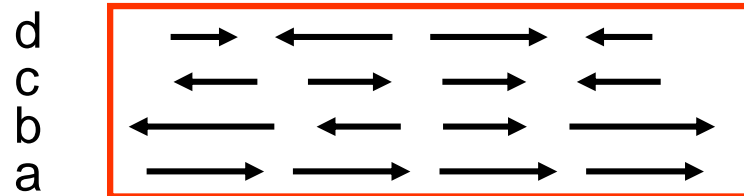


Doppler cooling:

$$\frac{P_{red}}{P_{blue}} \sim 1/7$$

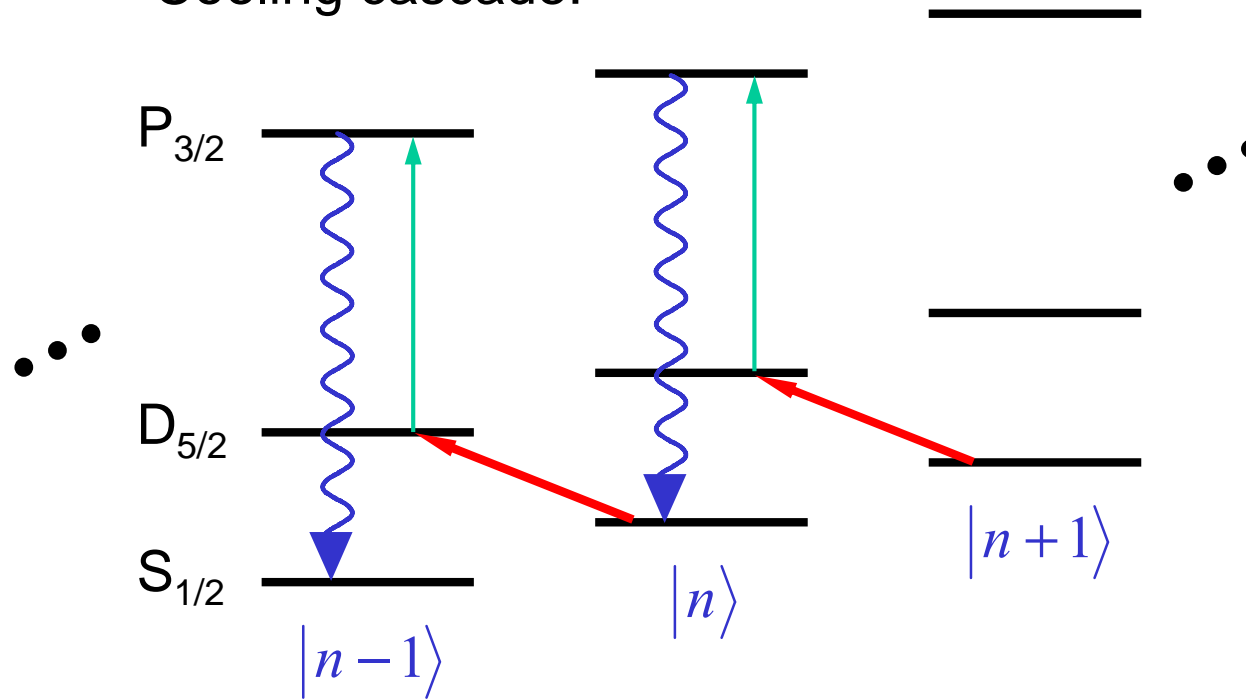
$$m_c = 0.16$$

pictorial eigenmodes



Quadrupole transition: optimize cooling

Cooling cascade:



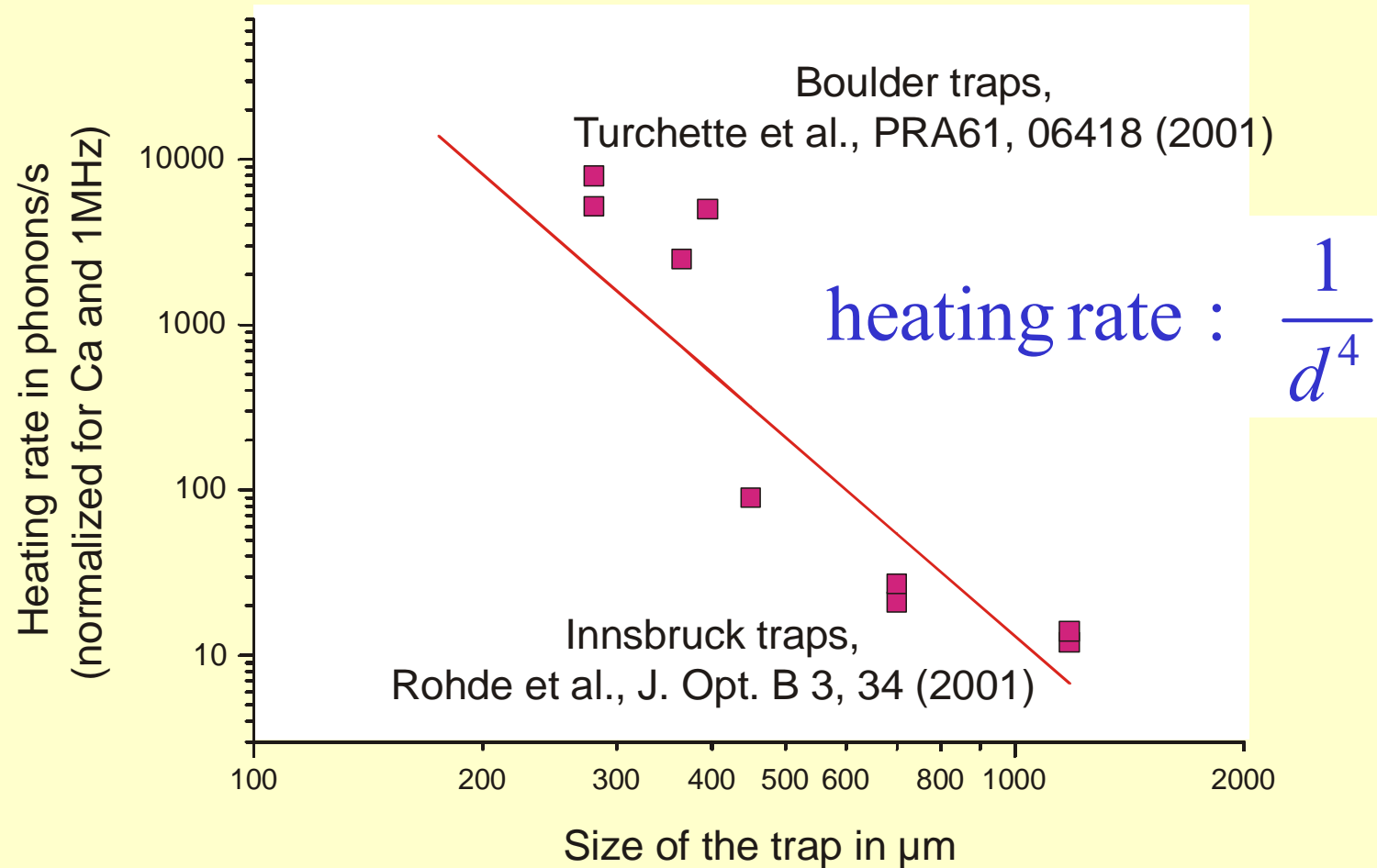
Effective two-level system:

$$\gamma_{\text{eff}} \approx \frac{\Omega_{PD}^2}{\gamma_{SP}^2 + 4\Delta_{PD}^2} \gamma_{SP}$$

⇒ optimize γ_{eff} using the 854nm laser power:
 large γ_{eff} : large cooling rate
 small γ_{eff} : small m

*I. Marzoli et al.,
 Phys. Rev. A 49, 2771 (1994)*

Comparing heating rates



*D. J. Wineland et al.,
J. Res. Nat Inst. Stand. Tech. 103 (259)*

Heating rate ...

PRL 97, 103007 (2006)

PHYSICAL REVIEW LETTERS

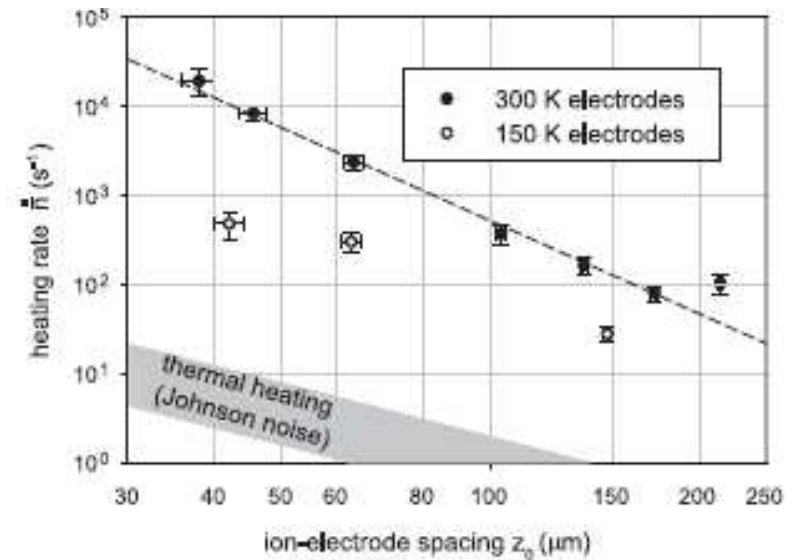
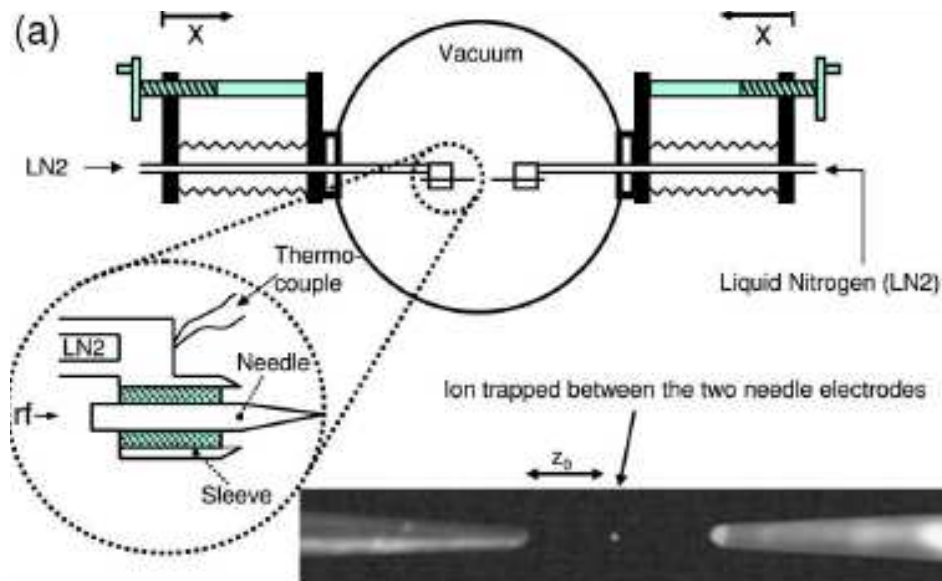
week ending
8 SEPTEMBER 2006

Scaling and Suppression of Anomalous Heating in Ion Traps

L. Deslauriers, S. Olmschenk, D. Stick, W. K. Hensinger, J. Sterk, and C. Monroe*

FOCUS Center and Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA

(Received 31 January 2006; published 8 September 2006)



Measured heating scales with d^{-4}

and is reduced at 150K

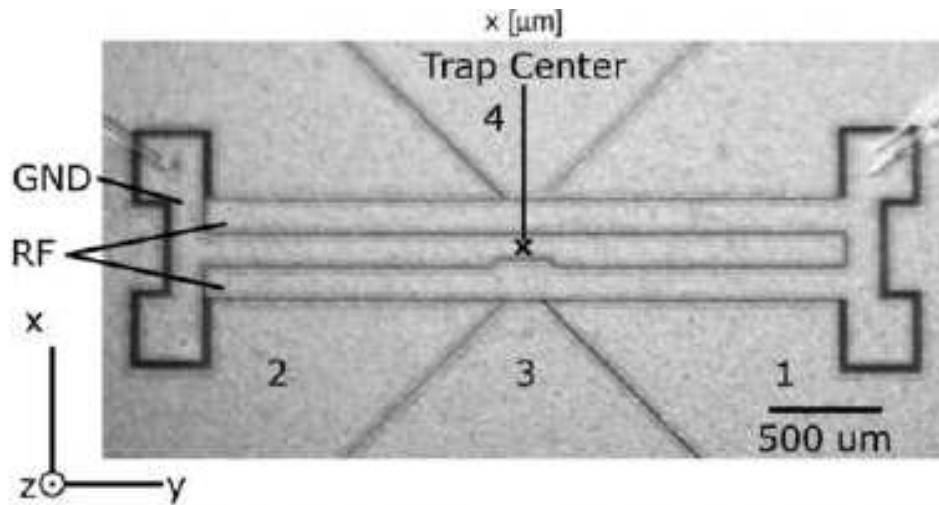
Heating rate in planar cryogenic trap

Suppression of Heating Rates in Cryogenic Surface-Electrode Ion Traps

Jaroslav Labaziewicz,* Yufei Ge, Paul Antohi, David Leibbrandt, Kenneth R. Brown, and Isaac L. Chuang

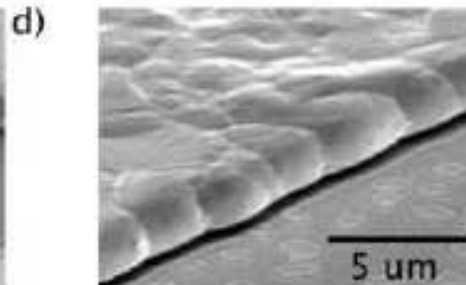
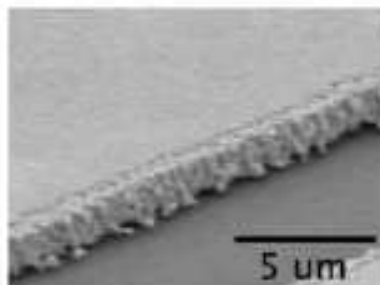
*Massachusetts Institute of Technology, Center for Ultracold Atoms,
Department of Physics, 77 Massachusetts Avenue, Cambridge, MA, 02139, USA*

(Dated: June 26, 2007)



*Labaziewicz et al,
PRL 100, 013001 (2008)*

.....and even much improved at 6K
to ~ two phonons/s which is a 10^7
reduction compared to heating rates
at room temperature



Reminder to Doppler cooling

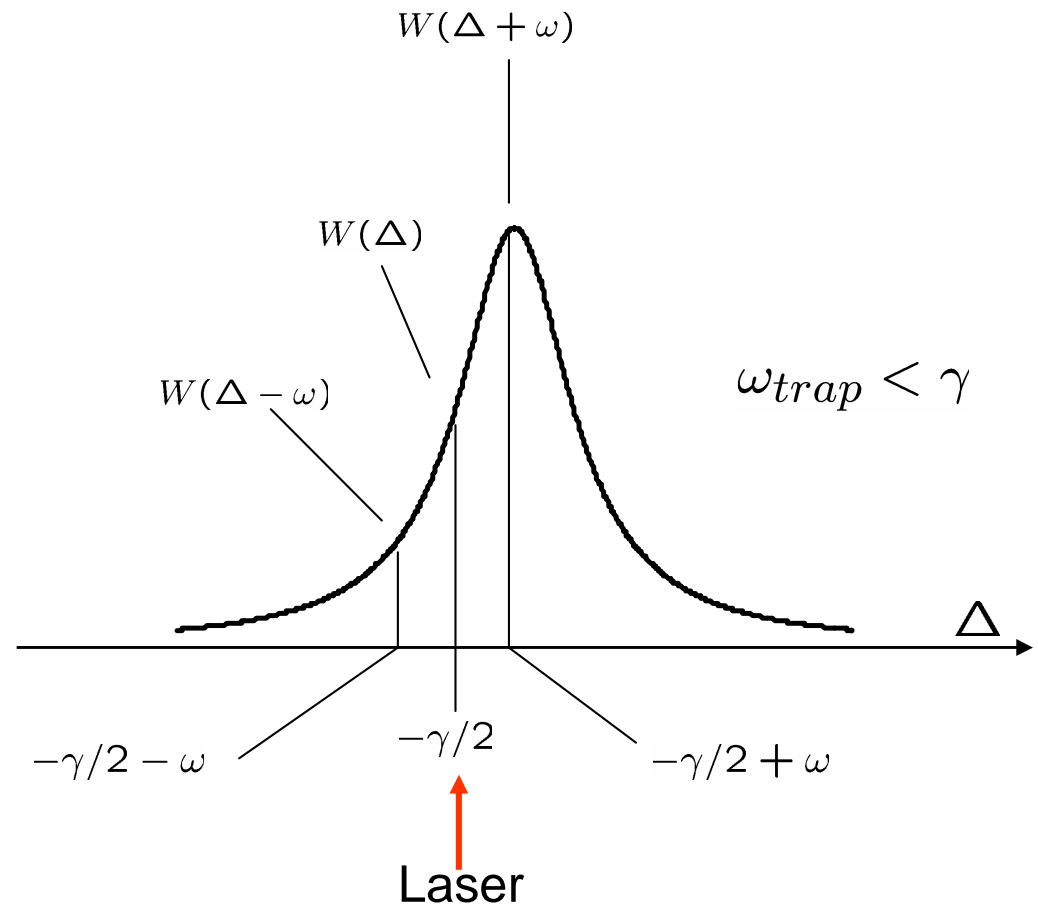
Problems:

a) Sidebands are not resolved on the transition,
 \Rightarrow small differences in

$$W(\Delta \pm \omega), W(\Delta - \omega)$$

b) Carrier excitation leads to diffusion, \Rightarrow heating:

$$W(\Delta = 0) \neq 0$$



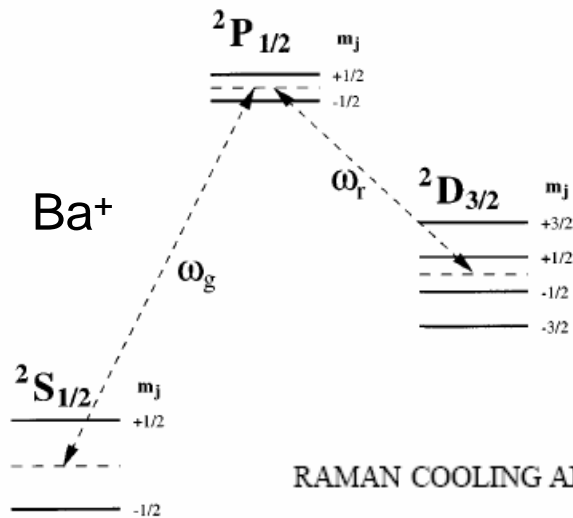
How to shape the atomic resonance line?



Interference

Dark states

Reiß et al., Phys Rev A 65, 053401 (2002)
 Reiß et al., Phys Rev A 54, 5133 (1996)



$$\text{Dark resonances: } |\Psi\rangle = \frac{1}{\sqrt{2}} (|S_{1/2}\rangle - |D_{5/2}\rangle)$$

⇒ spectrally much sharper than Doppler profile

RAMAN COOLING AND HEATING OF TWO TRAPPED Ba⁺ IONS

PHYSICAL REVIEW A 65 053401

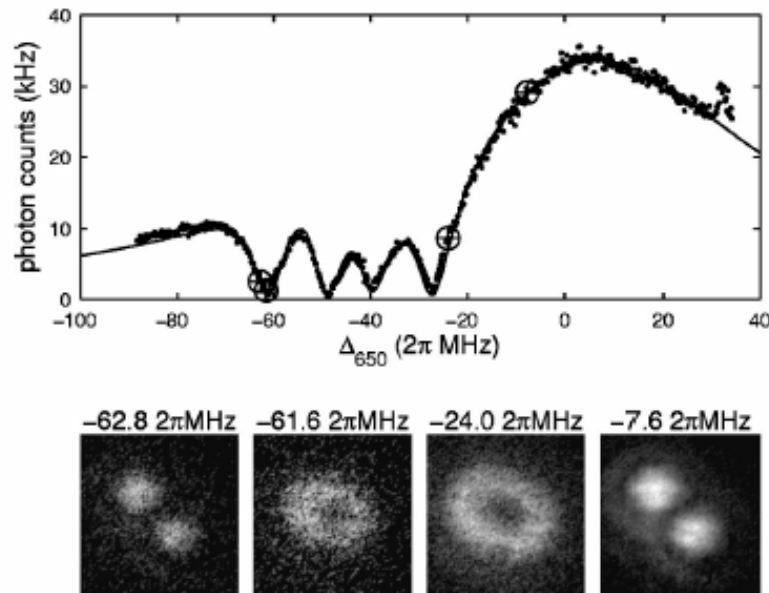


FIG. 2. Two trapped Ba⁺ ions show different motional states depending on laser parameters. Top: fluorescence of two trapped ions as a function of laser detuning, collected in 0.1 s. Bottom: spatial distribution of the two ions at the detunings indicated above.

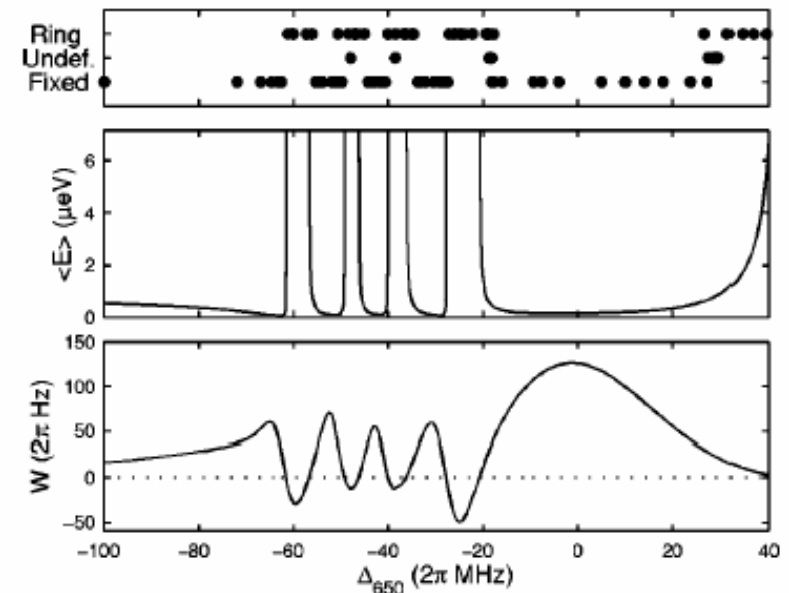
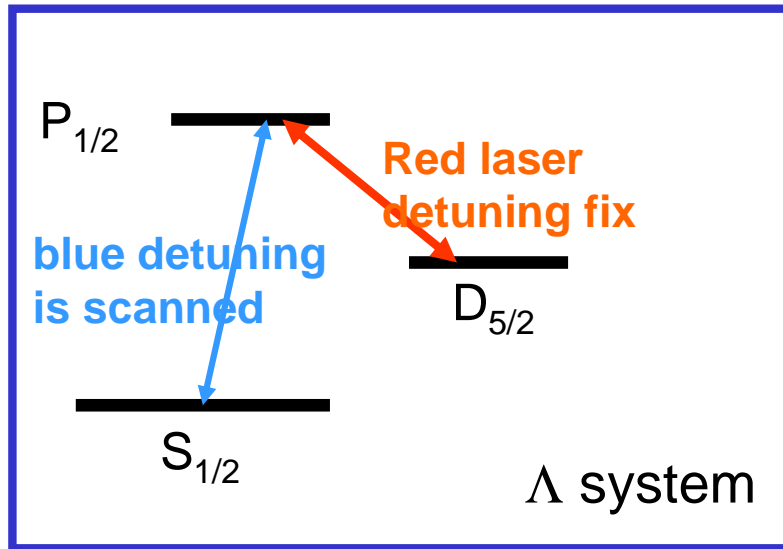
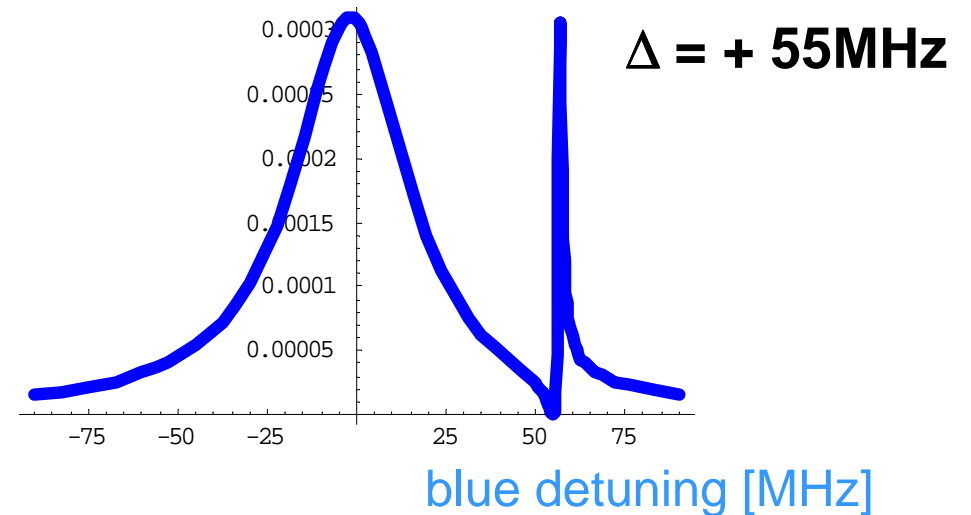
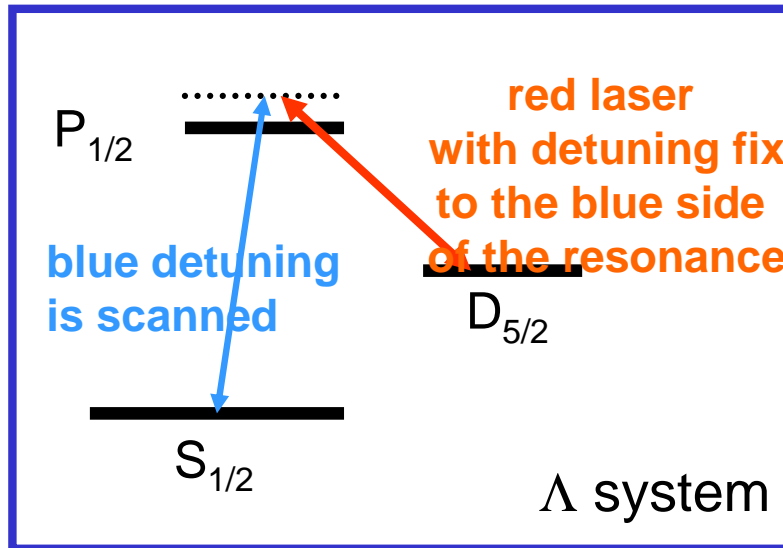
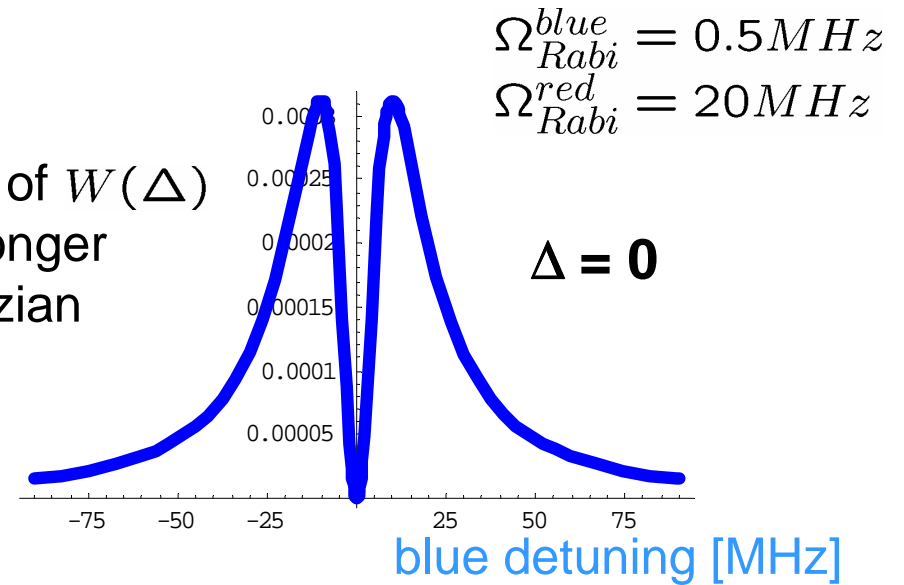


FIG. 3. Top: observed motional states for different detunings of the 650-nm light. The dots correspond to individual observations. Middle: mean motional energy in the \tilde{y} mode calculated from theory. Bottom: cooling rate for the \tilde{y} mode calculated from theory.

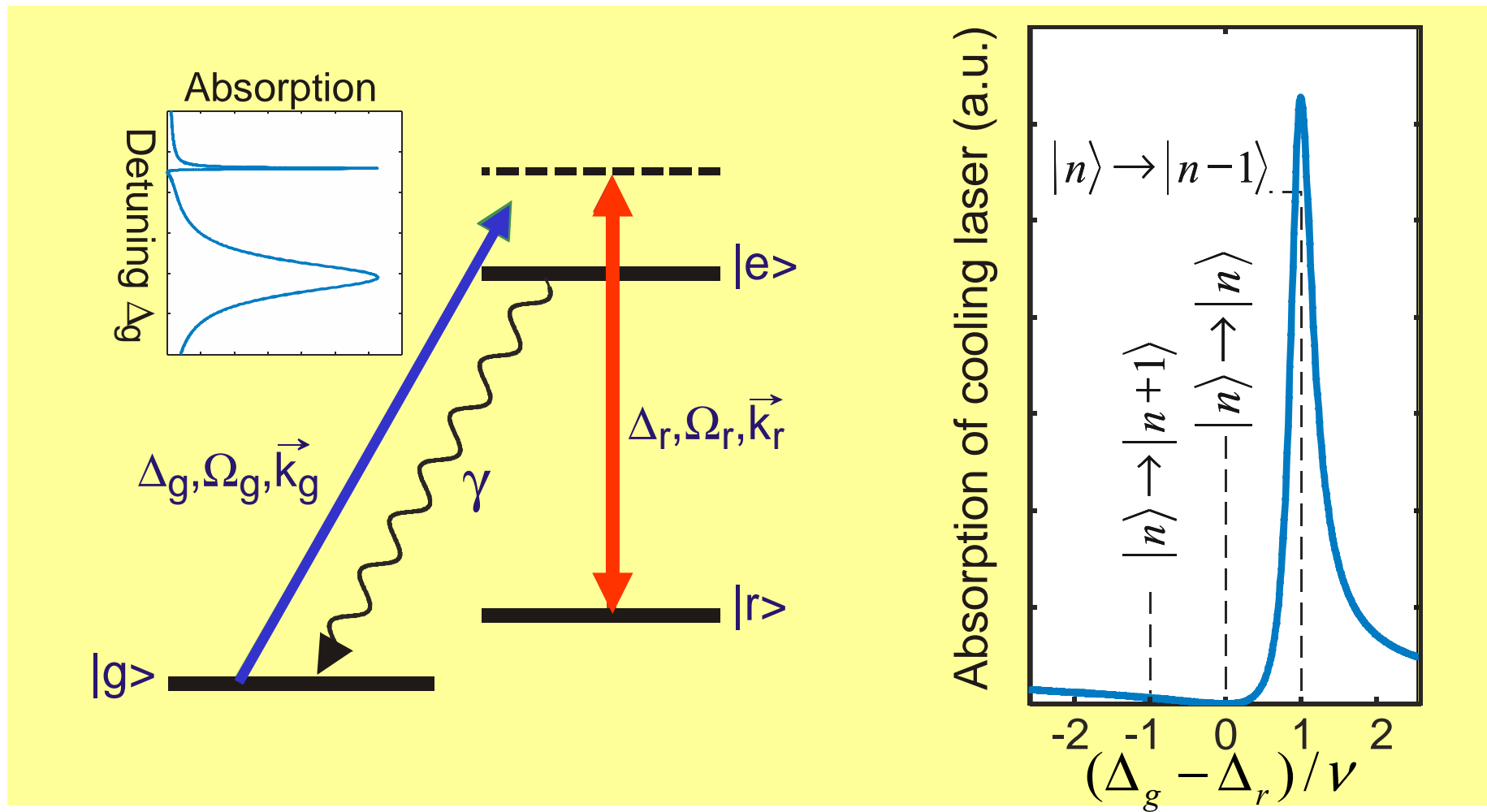
Quantum interference and dark states



Shape of $W(\Delta)$ is no longer Lorentzian



Ground state cooling with quantum interference



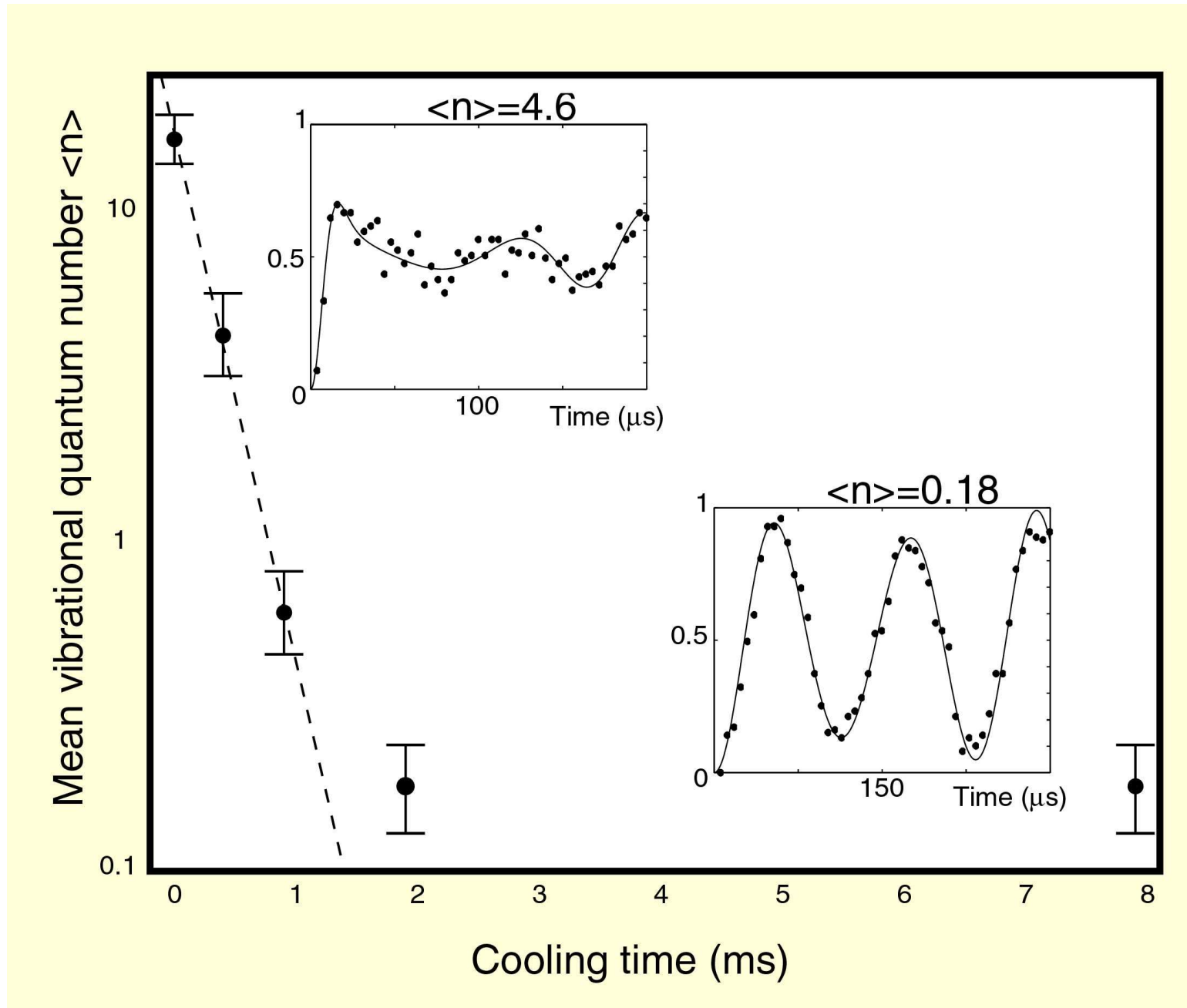
$|n\rangle \rightarrow |n-1\rangle$ transitions are enhanced by bright resonance

$|n\rangle \rightarrow |n\rangle$ transitions are suppressed by quantum interference – no „carrier“ diffusion contribution !

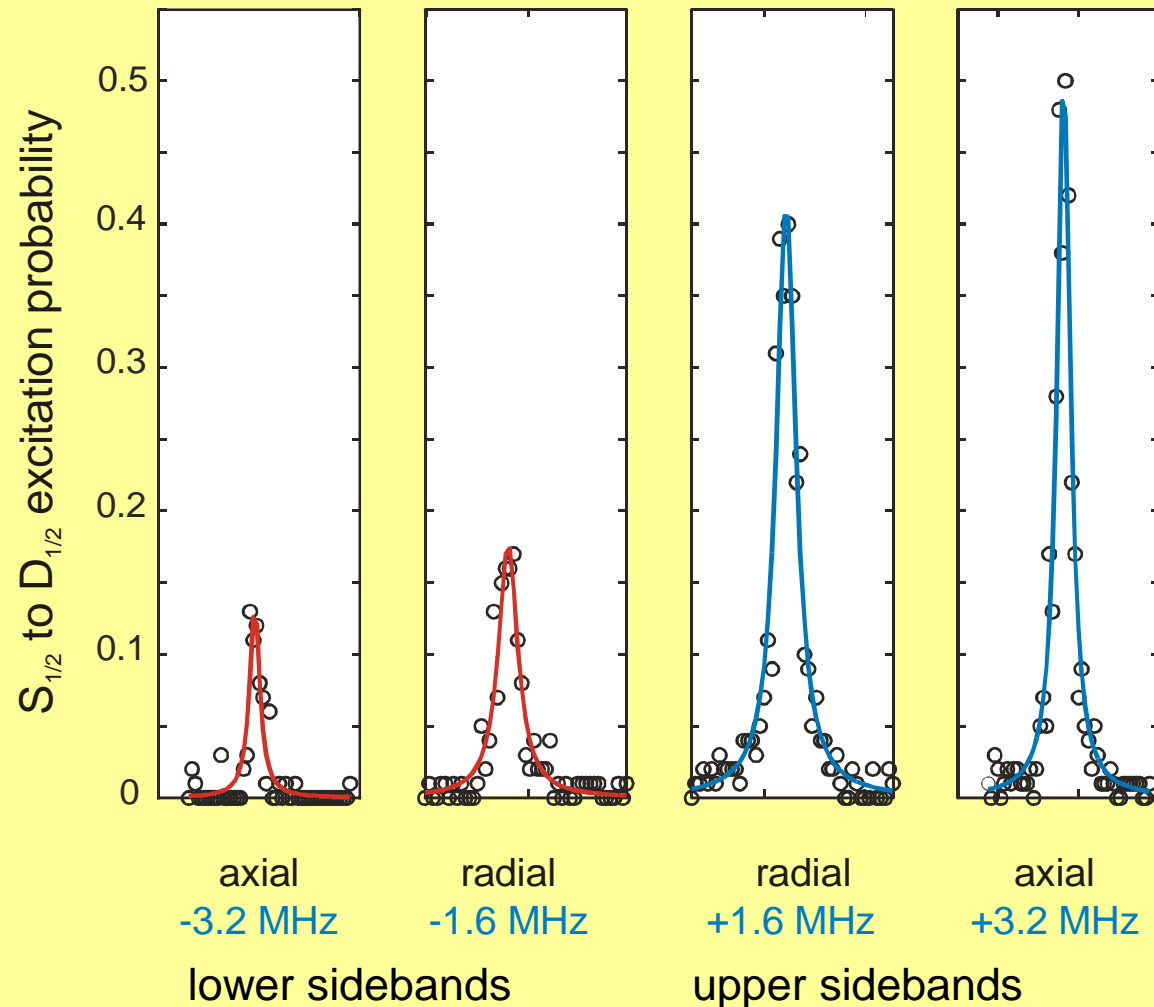
G. Morigi, J. Eschner, C. Keitel, Phys. Rev. Lett. 85, 4458 (2000)

EIT cooling time

C.F. Roos et al., Phys. Rev. Lett. 85, 5547 (2000)



Simultaneous two mode ground state cooling



Simultaneous ground state cooling of axial and radial motion

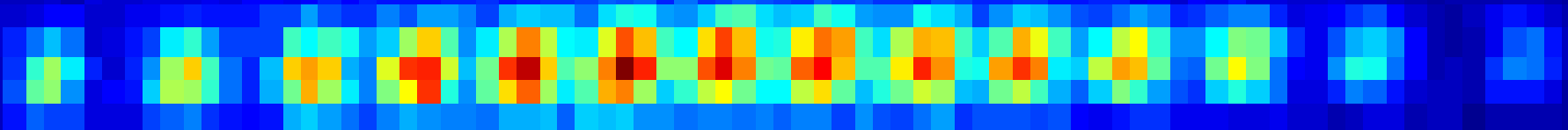
axial:
 $P(0)=73\%$

radial:
 $P(0)=58\%$

Cold Ions and their Applications for Quantum Computing and Frequency Standards

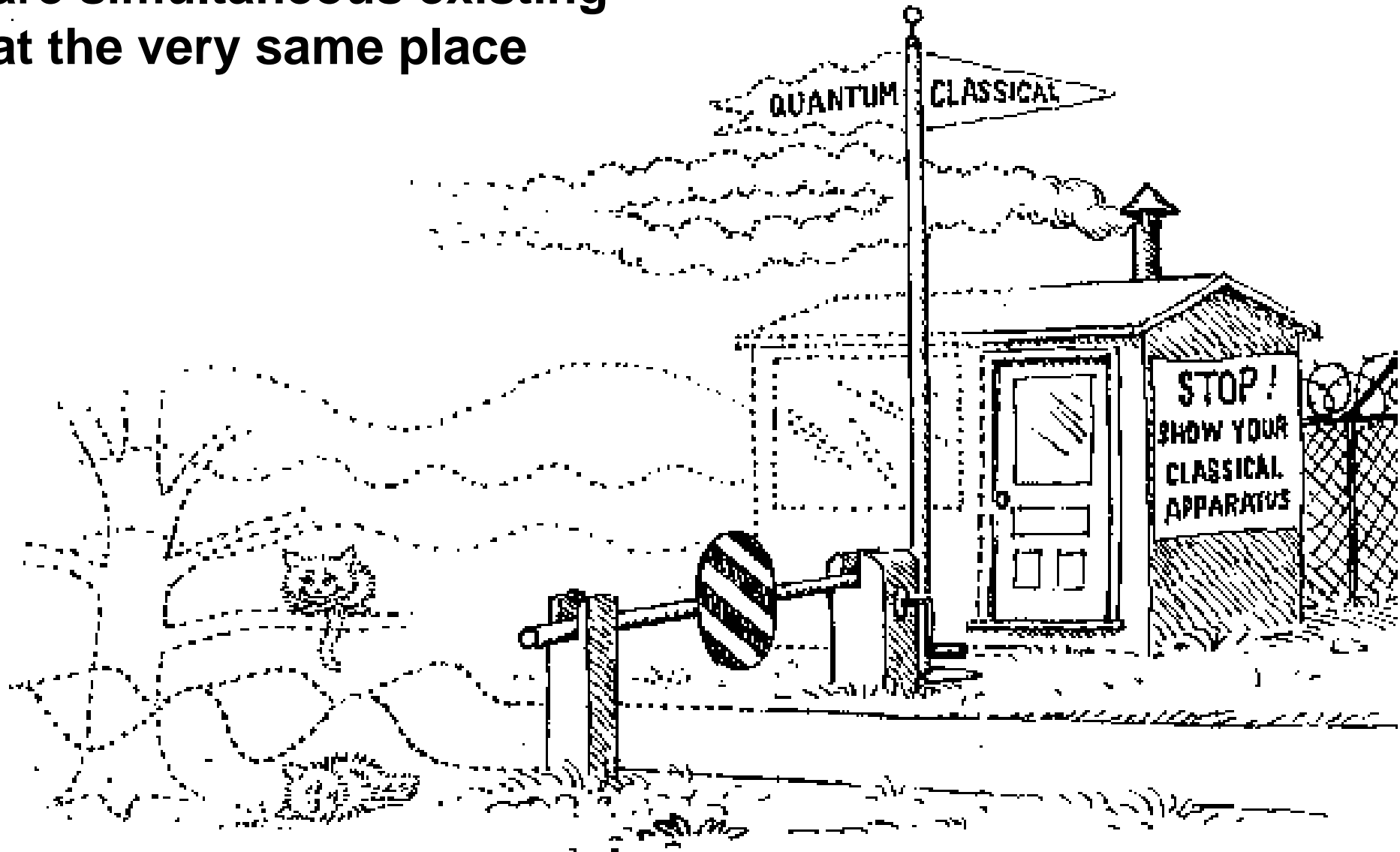
- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- Quantum computer: basics, gates, algorithms, future challenges
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Ferdinand Schmidt-Kaler
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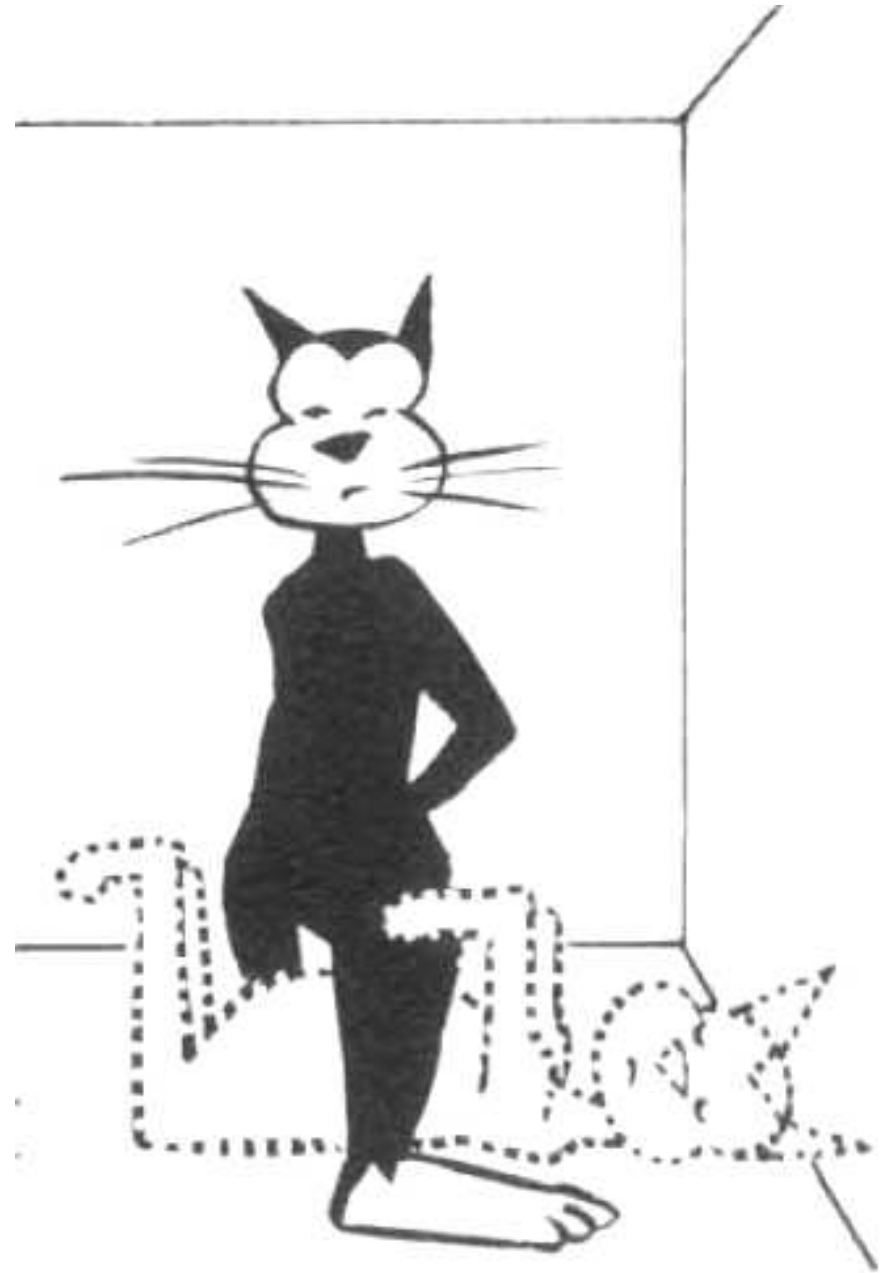
What is a Superposition?

two **different quantum states**
are simultaneous existing
at the very same place



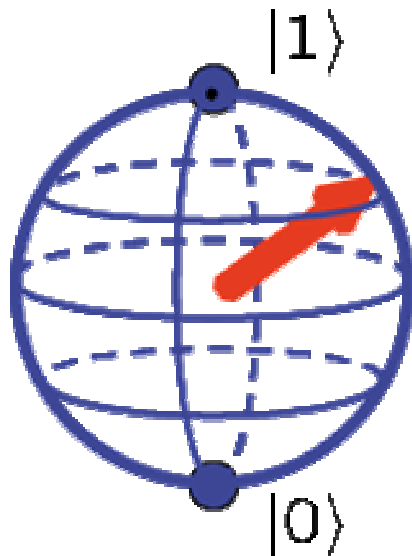
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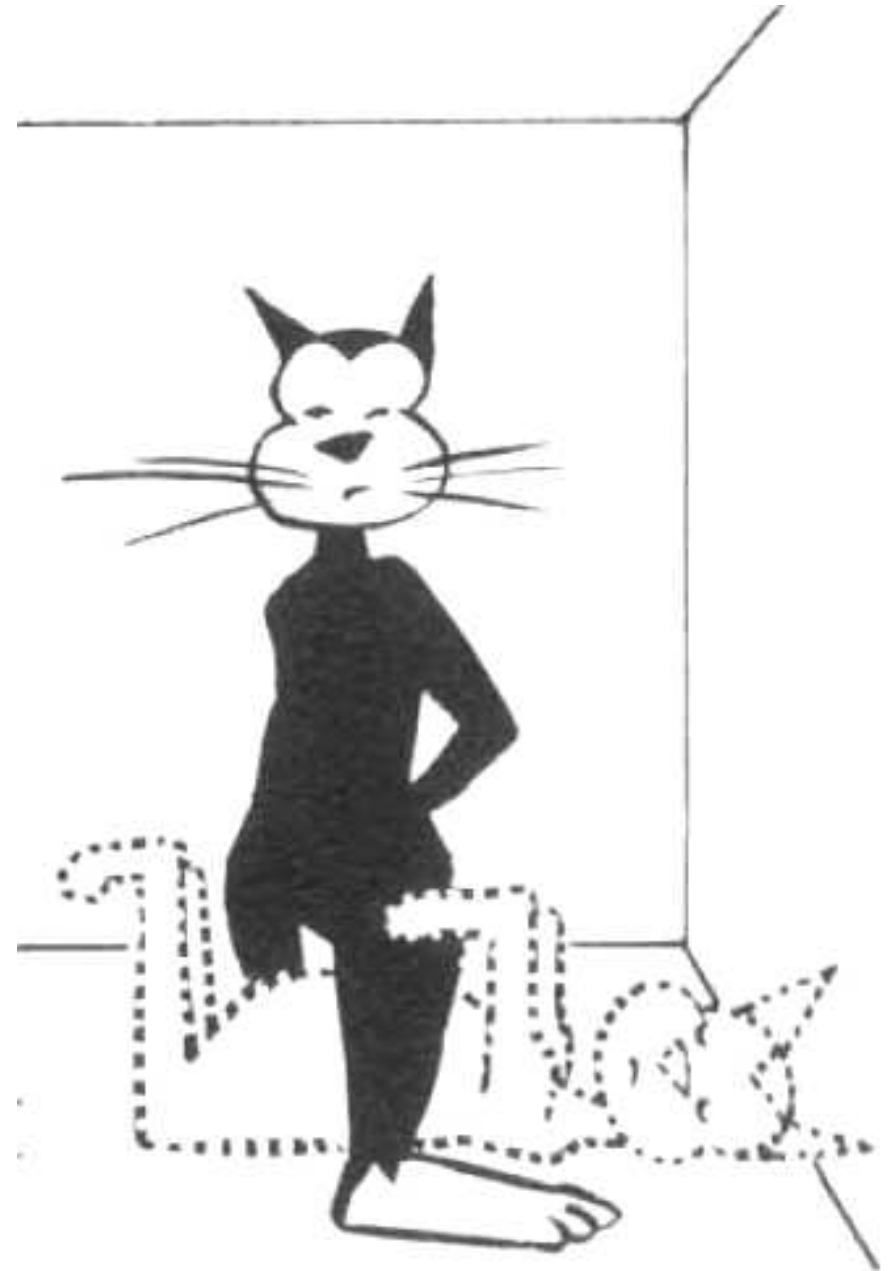


What is a Superposition?

two **different quantum states**
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at the very same place

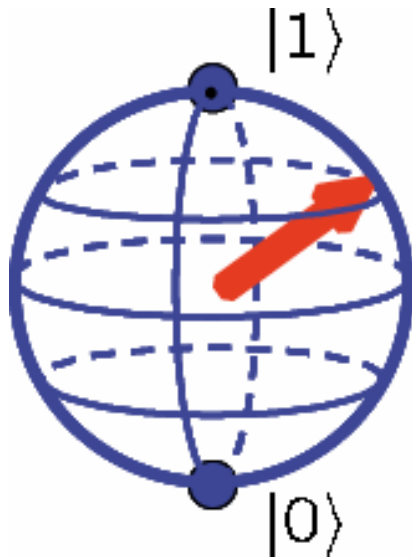


Vector on the Bloch-sphere



What is a Superposition?

two **different quantum states**
are simultaneous existing
at the very same place



A single quantum-bit

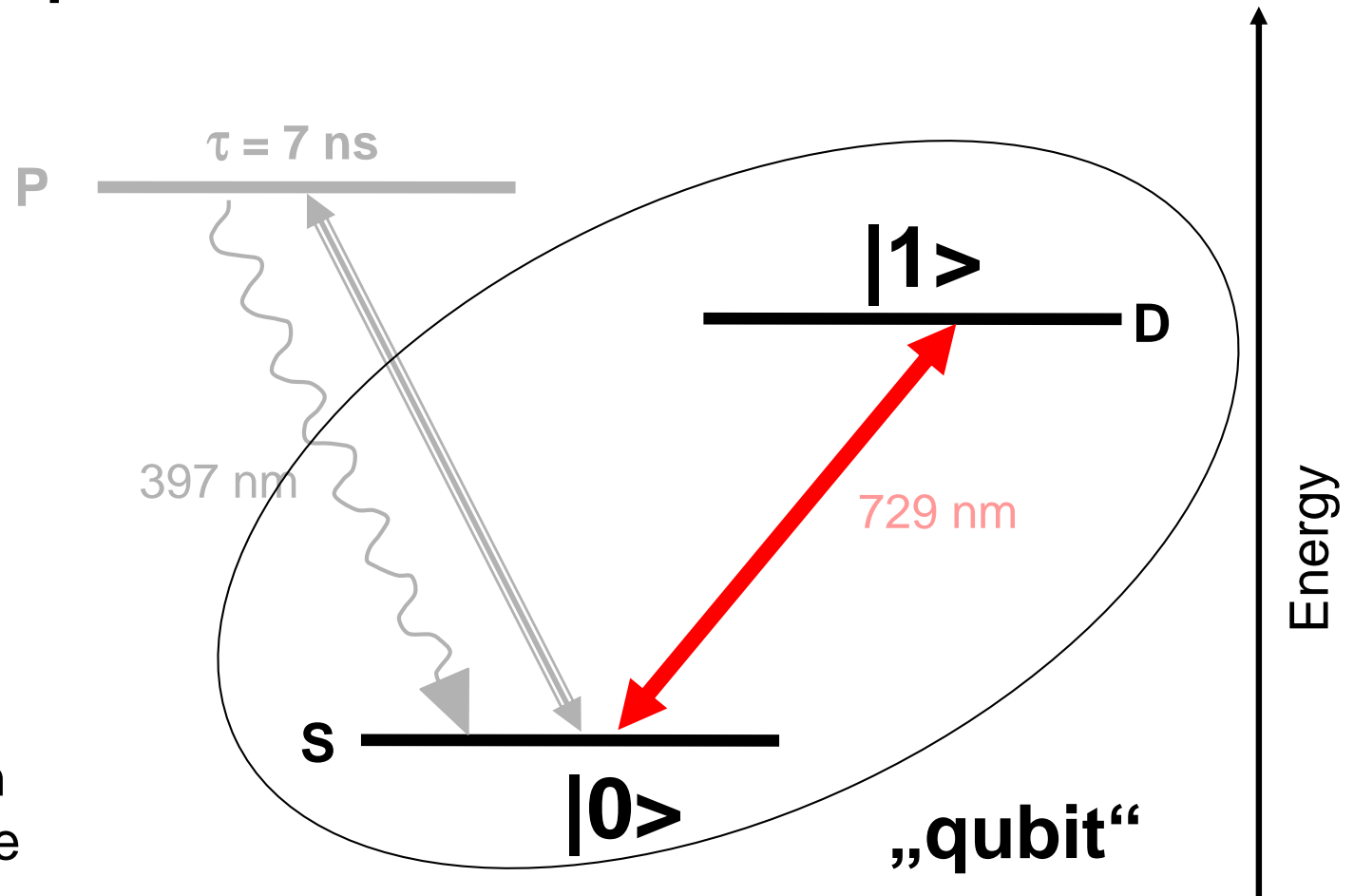
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

in superposition of Zero and One

What is a Superposition?

two **different quantum states**
are simultaneous existing
at the very same place

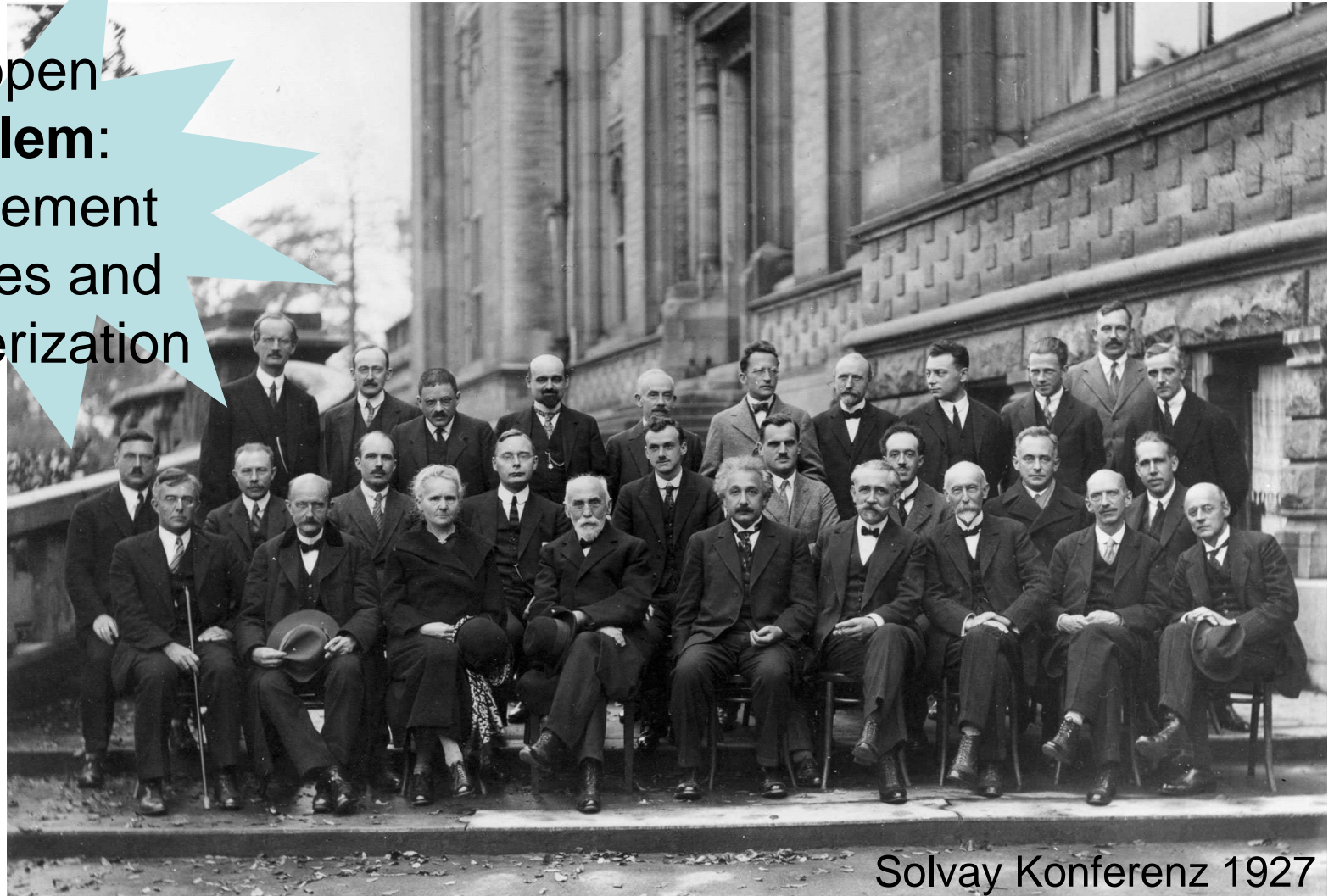
A single **ion**
in superposition
of Zero and One



What is entanglement?

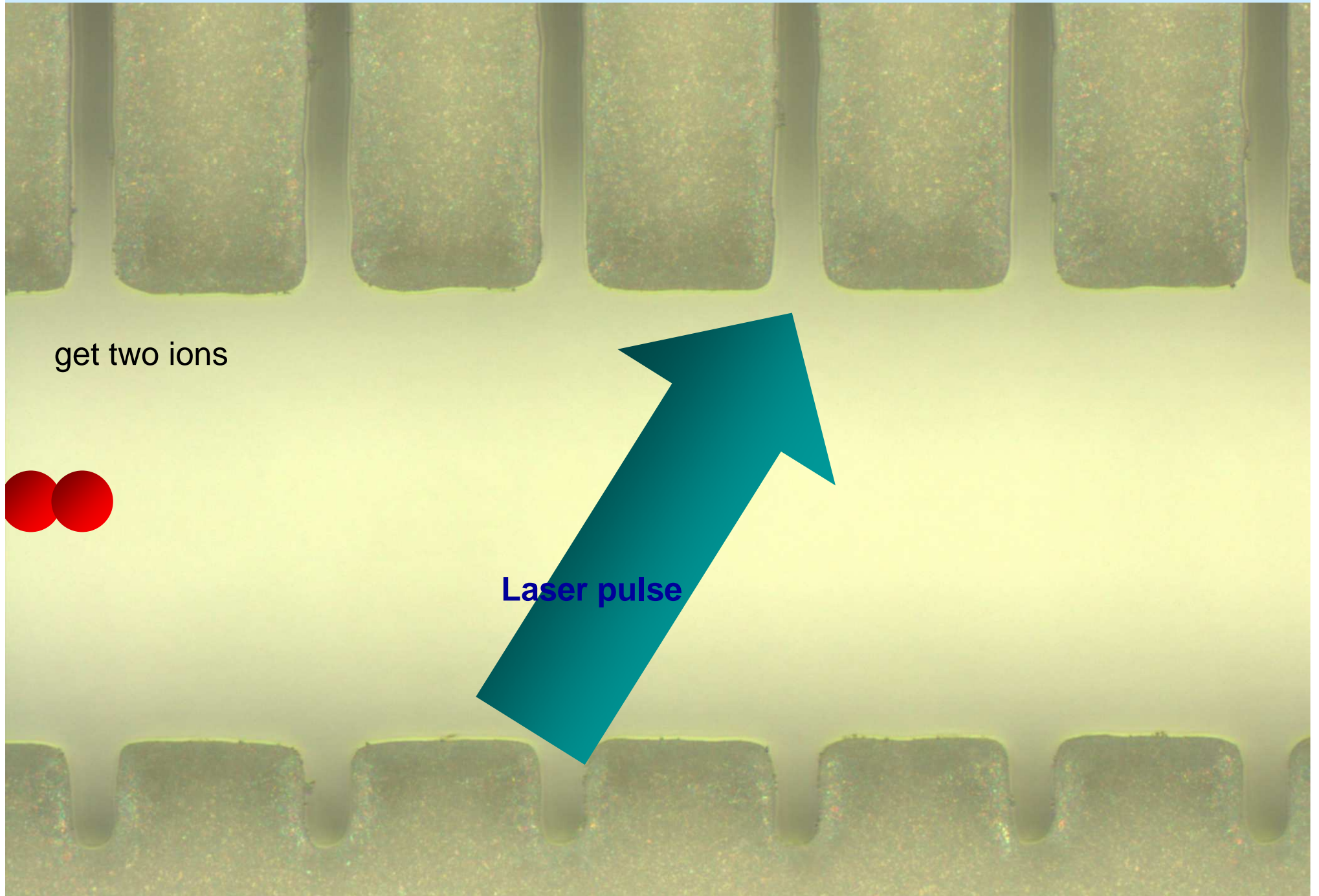
Entangled particles show **correlations** and can only properly described by a **common quantum state**, regardless how **far apart** they are.

Still open
Problem:
Entanglement
measures and
characterization

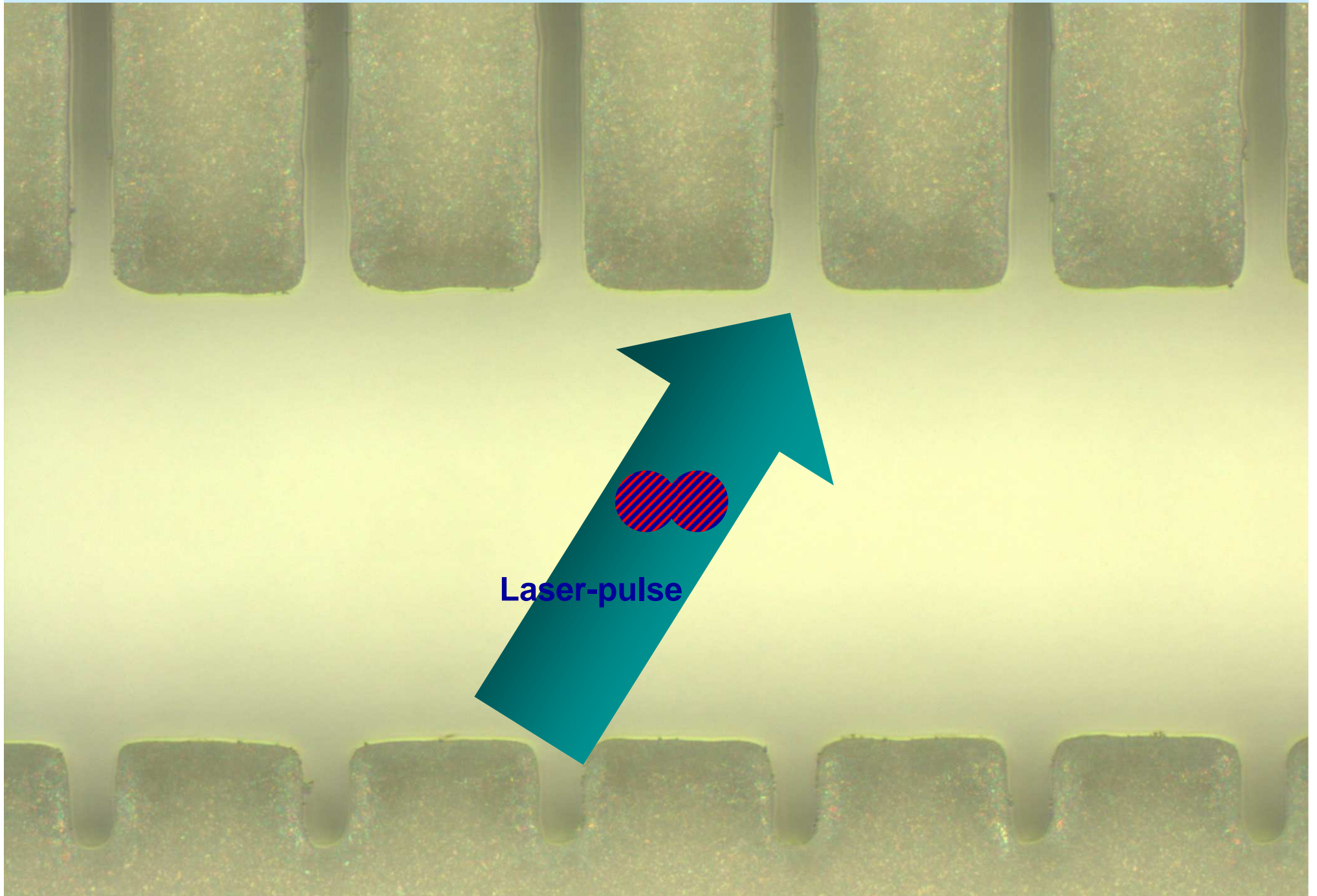


Solvay Konferenz 1927

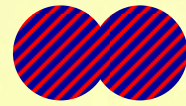
Entanglement by Laser pulses



Entanglement by Laser pulses

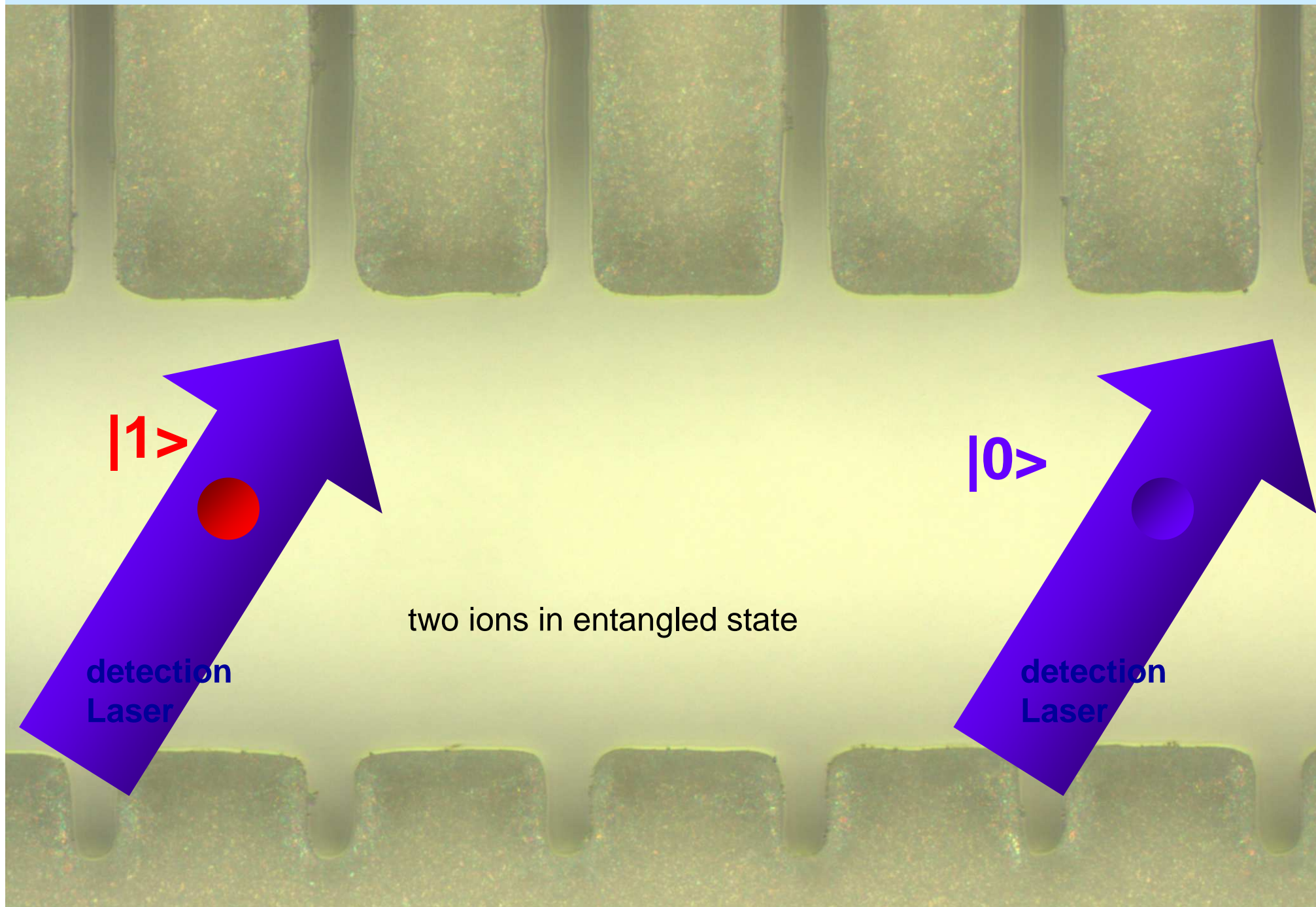


Entanglement by Laser pulses



two ions in entangled state

Measurement



$|1\rangle$

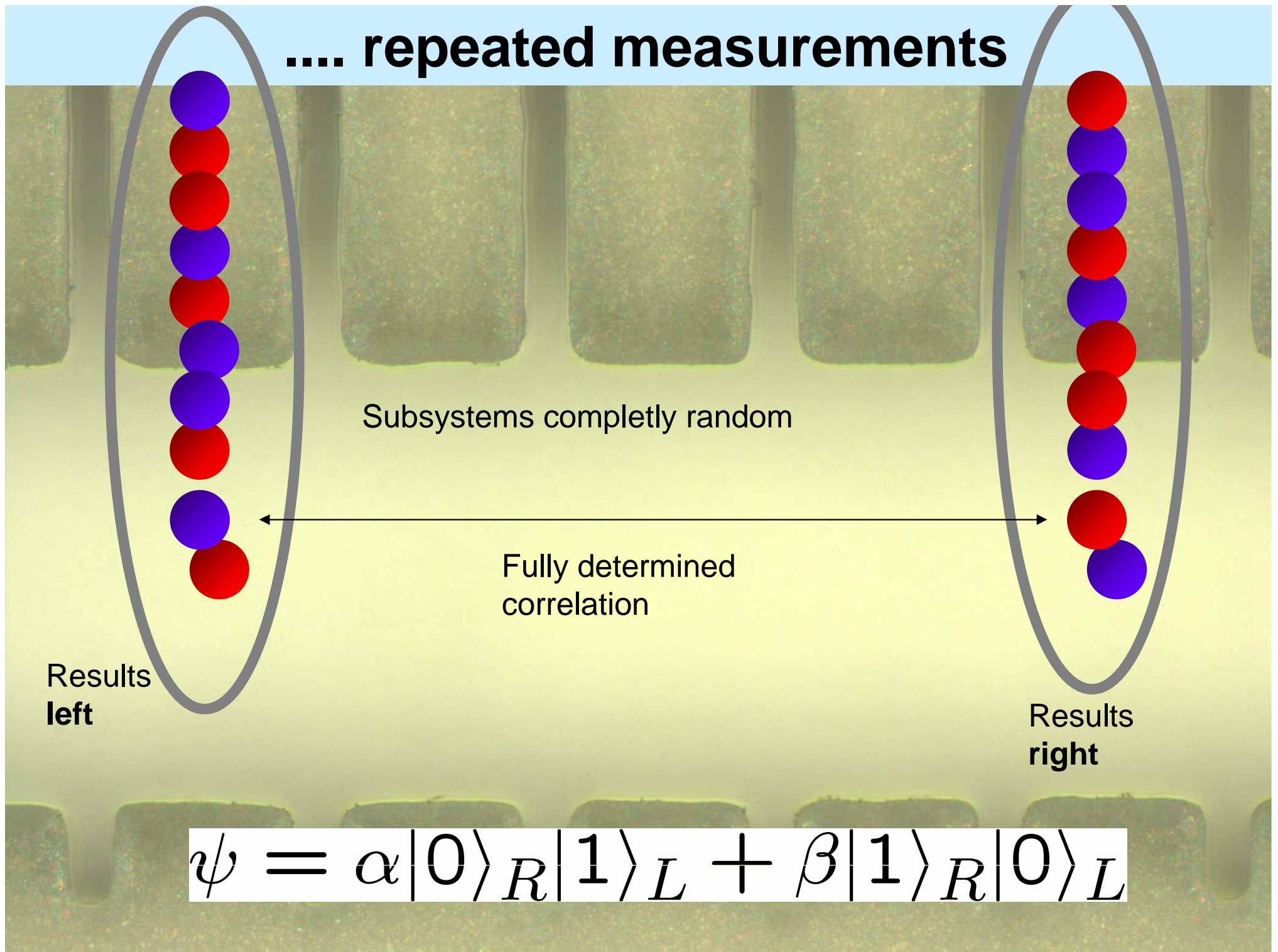
$|0\rangle$

two ions in entangled state

detection
Laser

detection
Laser

.... repeated measurements



Subsystems completely random

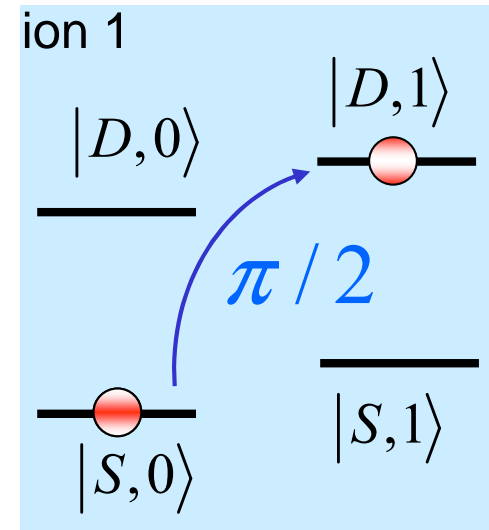
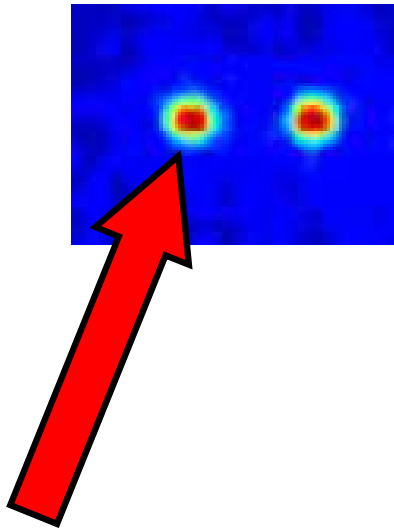
Fully determined correlation

Results
left

Results
right

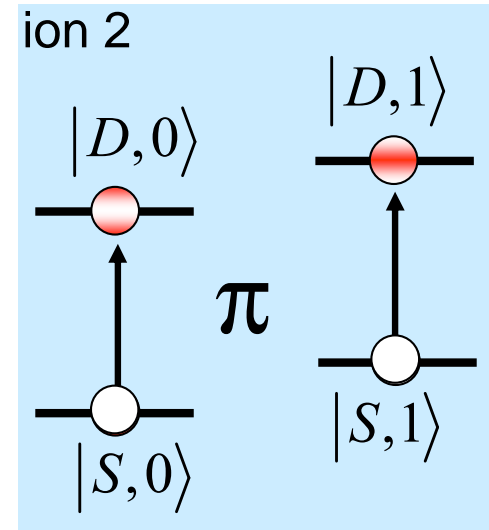
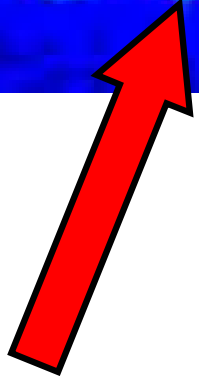
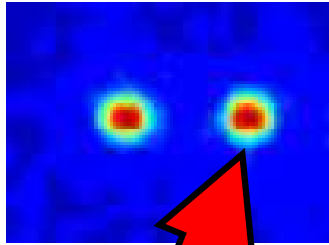
$$\psi = \alpha|0\rangle_R|1\rangle_L + \beta|1\rangle_R|0\rangle_L$$

Deterministic Bell state generation



$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS,0\rangle + |DS,1\rangle$$

Deterministic Bell state generation



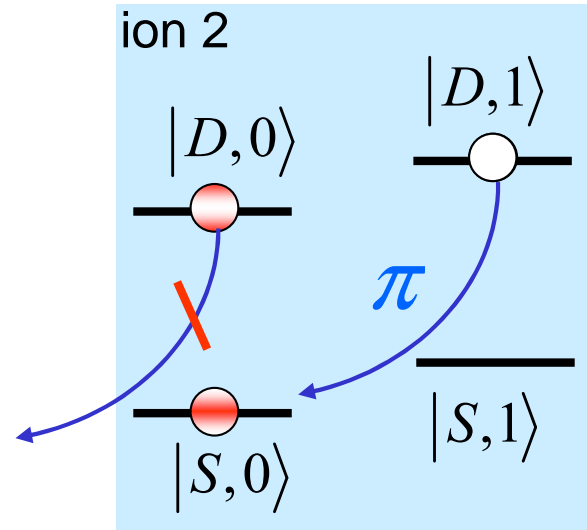
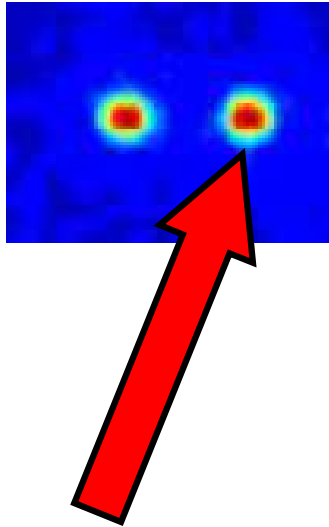
$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}}$$

$$|SS, 0\rangle + |DS, 1\rangle$$

$$\xrightarrow{\text{carrier } \pi \text{ pulse}}$$

$$|SD, 0\rangle + |DD, 1\rangle$$

Deterministic Bell state generation



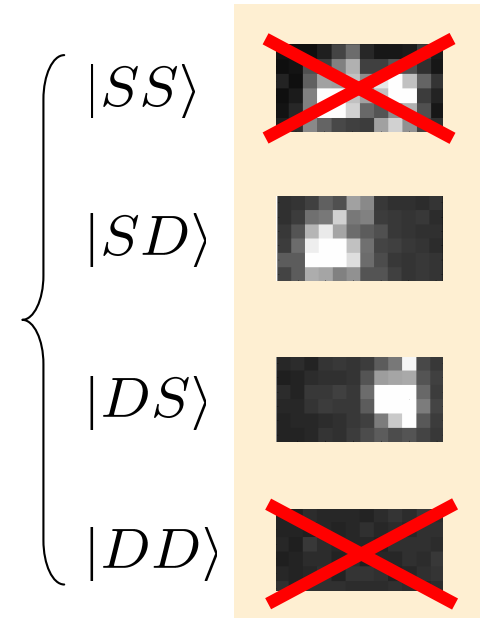
$$\begin{array}{l}
 |SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS,0\rangle + |DS,1\rangle \\
 \xrightarrow{\text{carrier } \pi \text{ pulse}} |SD,0\rangle + |DD,1\rangle \\
 \xrightarrow{\text{blue } \pi \text{ pulse}} |SD\rangle|0\rangle + |DS\rangle|0\rangle
 \end{array}$$

Bell state analysis

$$\begin{aligned} & |SD\rangle|0\rangle + |DS\rangle|0\rangle \\ &= (|SD\rangle + |DS\rangle)|0\rangle \end{aligned}$$

- Coherent superposition of SD and DS states?
- Phase relation between both wave function components?

Fluorescence detection with CCD camera:



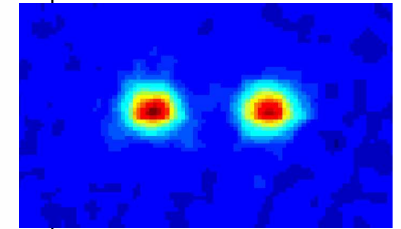
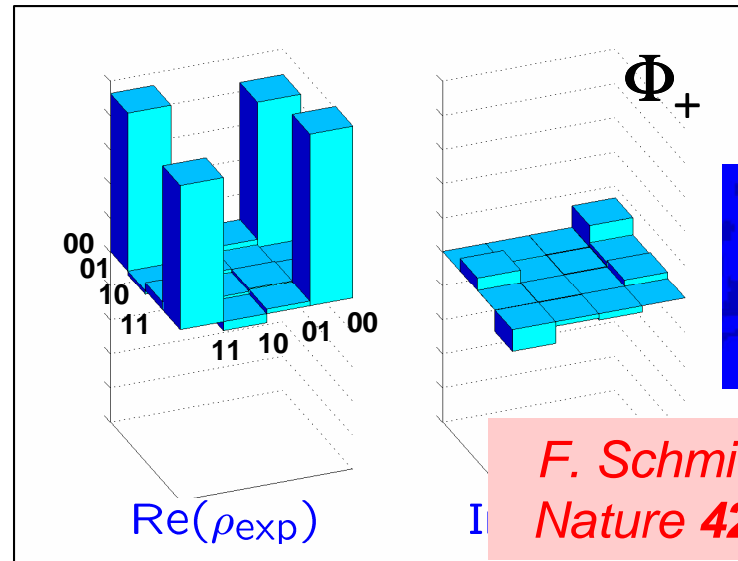
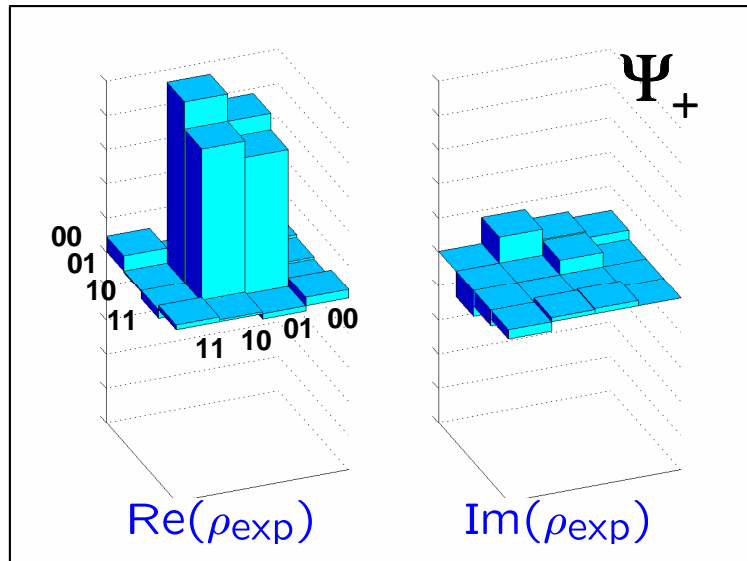
Entire information is contained in the density matrix

→ Measure the density matrix

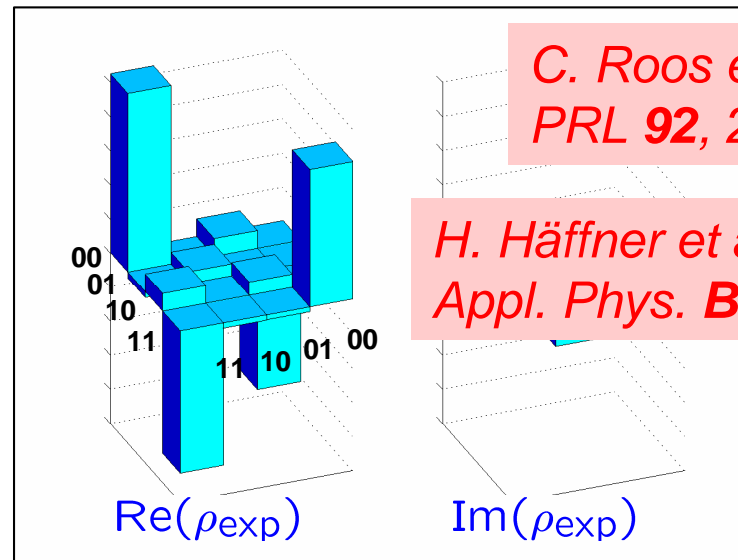
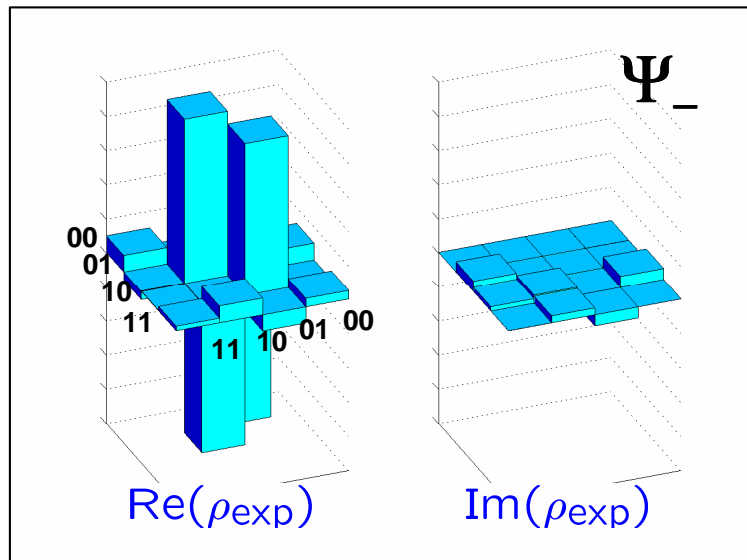
Density matrix for 2 ion entanglement

$$\psi = |0\rangle|1\rangle + e^{i\phi}|1\rangle|0\rangle$$

$$\phi = |1\rangle|1\rangle + e^{i\phi}|0\rangle|0\rangle$$



*F. Schmidt-Kaler et al.,
Nature **422**, 408 (2003)*



*C. Roos et al.,
PRL **92**, 220402 (2004)*

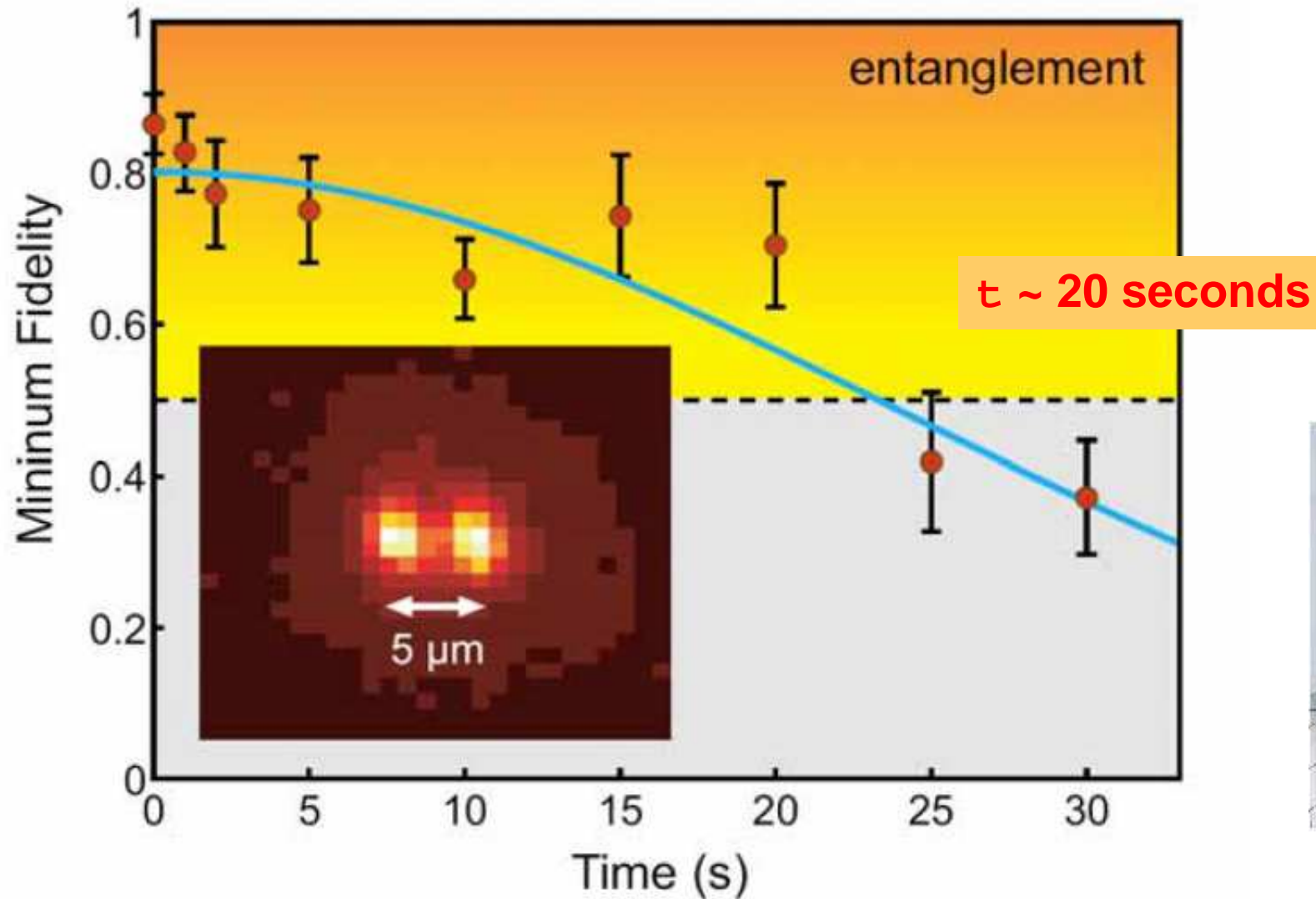
*H. Häffner et al.,
Appl. Phys. **B 81**, 151 (2005)*

Decoherence-free Bell states

*H. Häffner et al,
Appl. Phys. B 81, 151 (2005)*

$$\psi = |S_{-}\rangle|S_{+}\rangle + e^{i\phi}|S_{+}\rangle|S_{-}\rangle$$

entanglement in Zeeman levels $m=\pm 1/2$



John Bell

What is Entanglement?

Properties of the **complete system** are fully determined,

but properties of **sub-systems** completely undetermined

Erwin Schrödinger 1935:

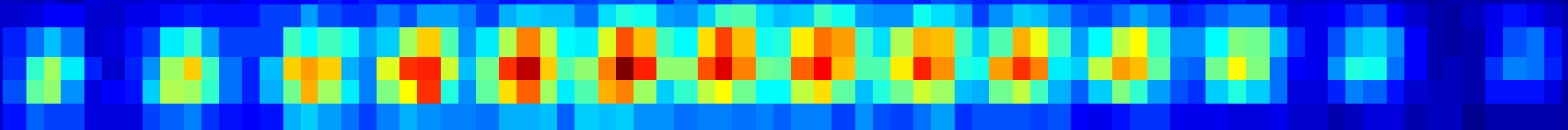
"I would call entanglement not one but rather *the characteristic trait* of quantum mechanics, the one that enforces its entire departure from classical lines of thought."



Cold Ions and their Applications for Quantum Computing and Frequency Standards

- Trapping Ions
- Cooling Ions
- Superposition and Entanglement
- **Quantum computer: basics, gates, algorithms, future challenges**
- Ion clocks: from Ramsey spectroscopy to quantum techniques

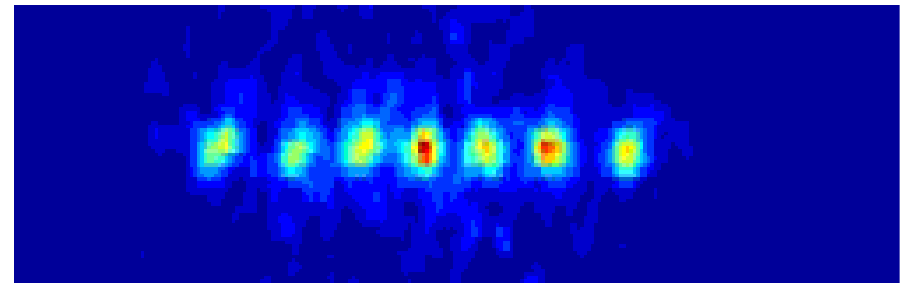
- single qubit gate
- various two qubit gates
... baby-steps shown so far with ion quantum processors
- and how to reach a scalable device in future



III) Quantum computing with trapped ions

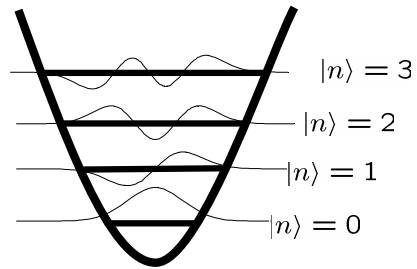
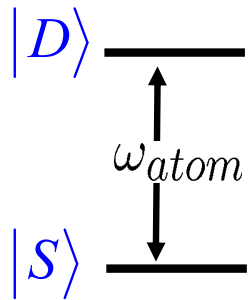
Quantum computation, basic description and single qubit gates,
Cirac-Zoller two qubit gate operation, other two qubit gates
long lived Bell states
GHZ- and W-states
Teleportation with ions

Laser coupling

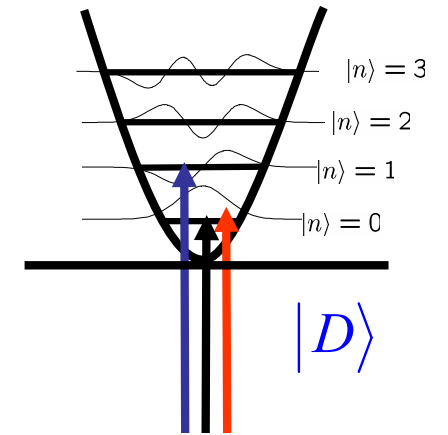


2-level-atom

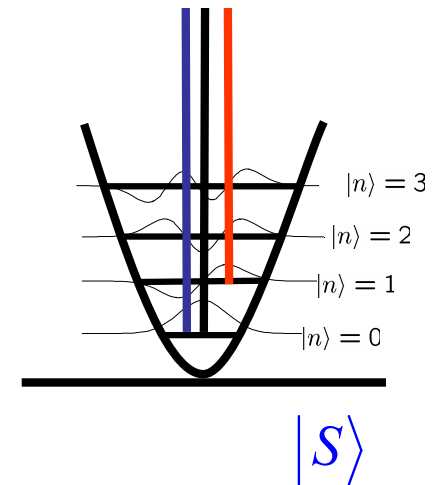
harmonic trap



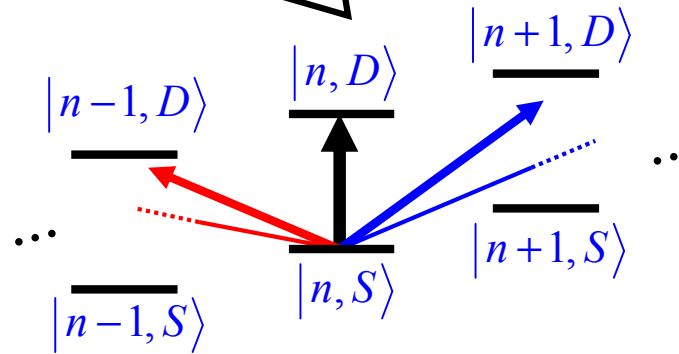
dressed system



„molecular Franck Condon“ picture

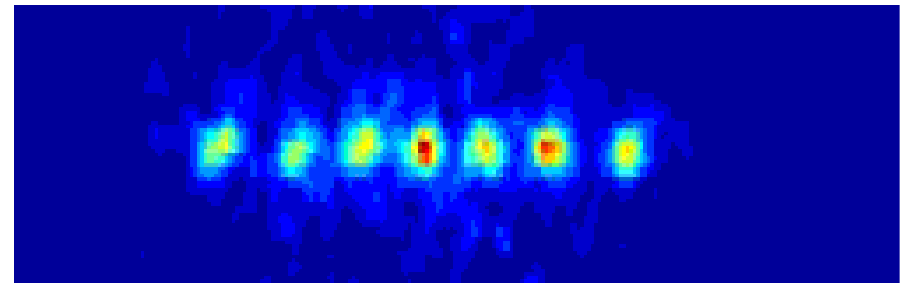


„energy ladder“ picture



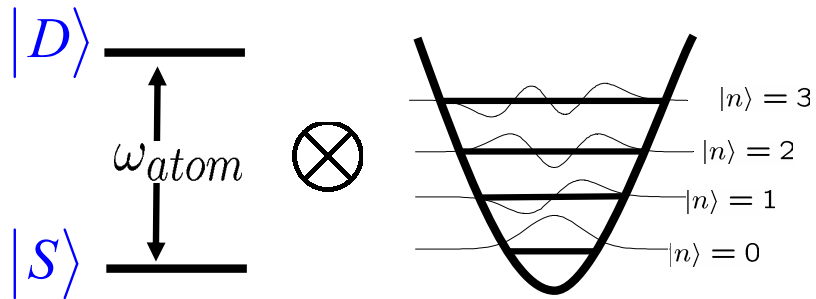
...

Laser coupling

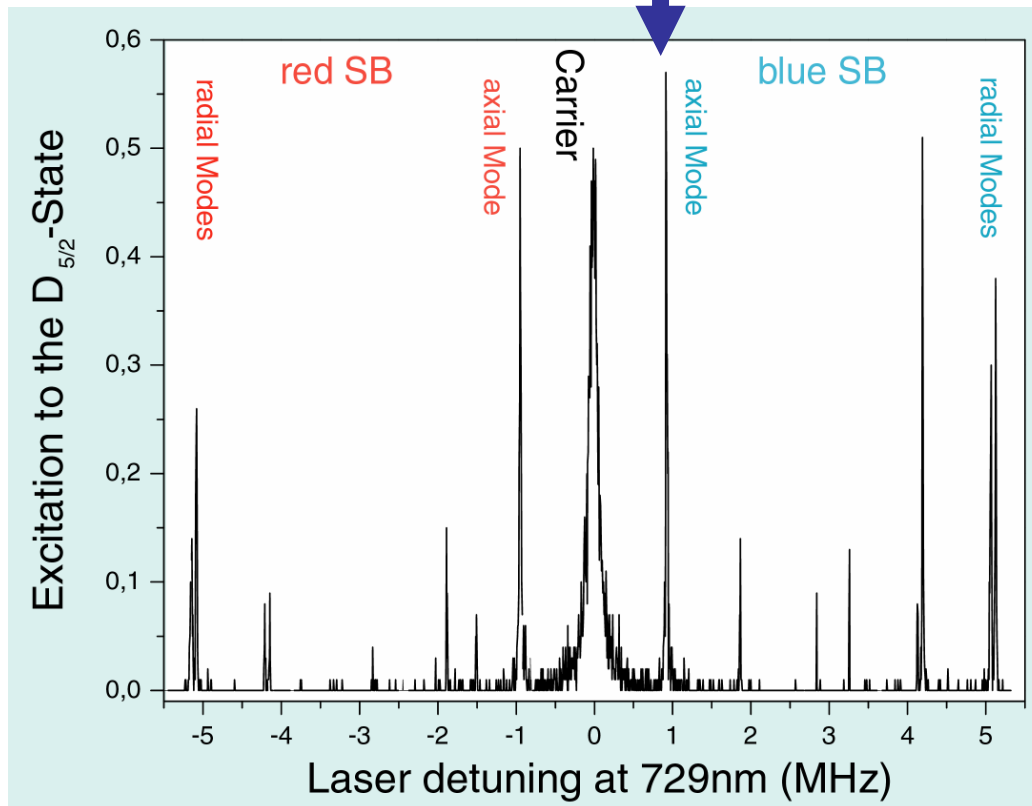
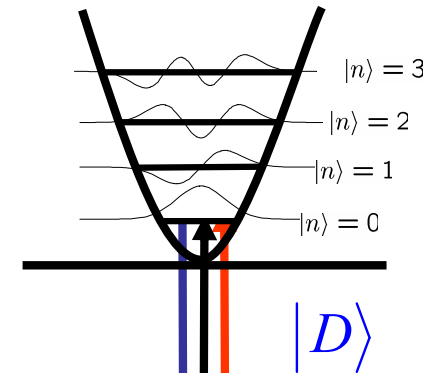


2-level-atom

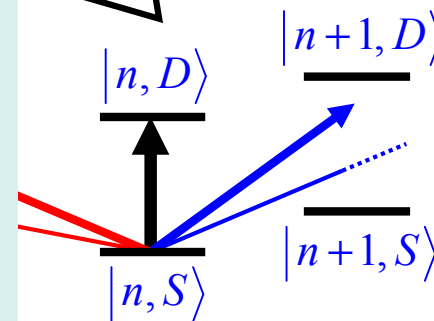
harmonic trap



dressed system

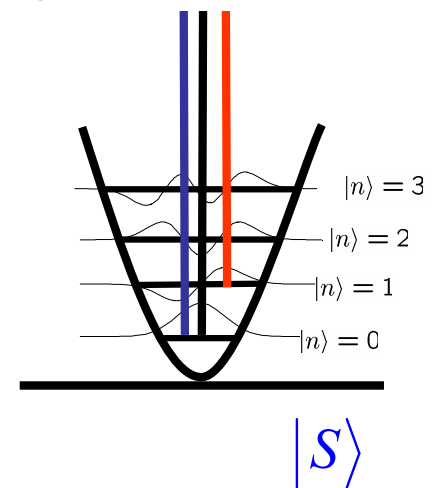


„energy ladder“ picture

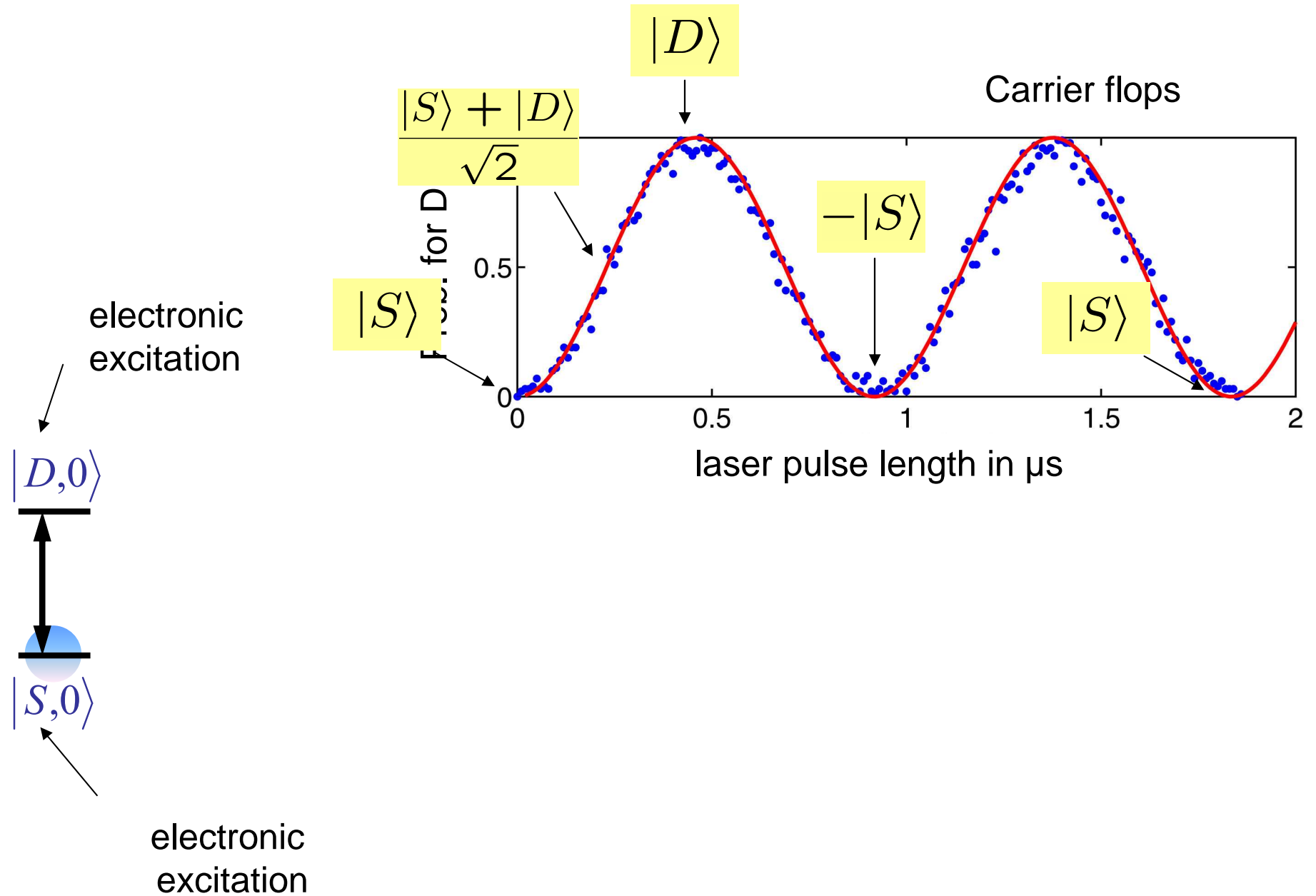


...

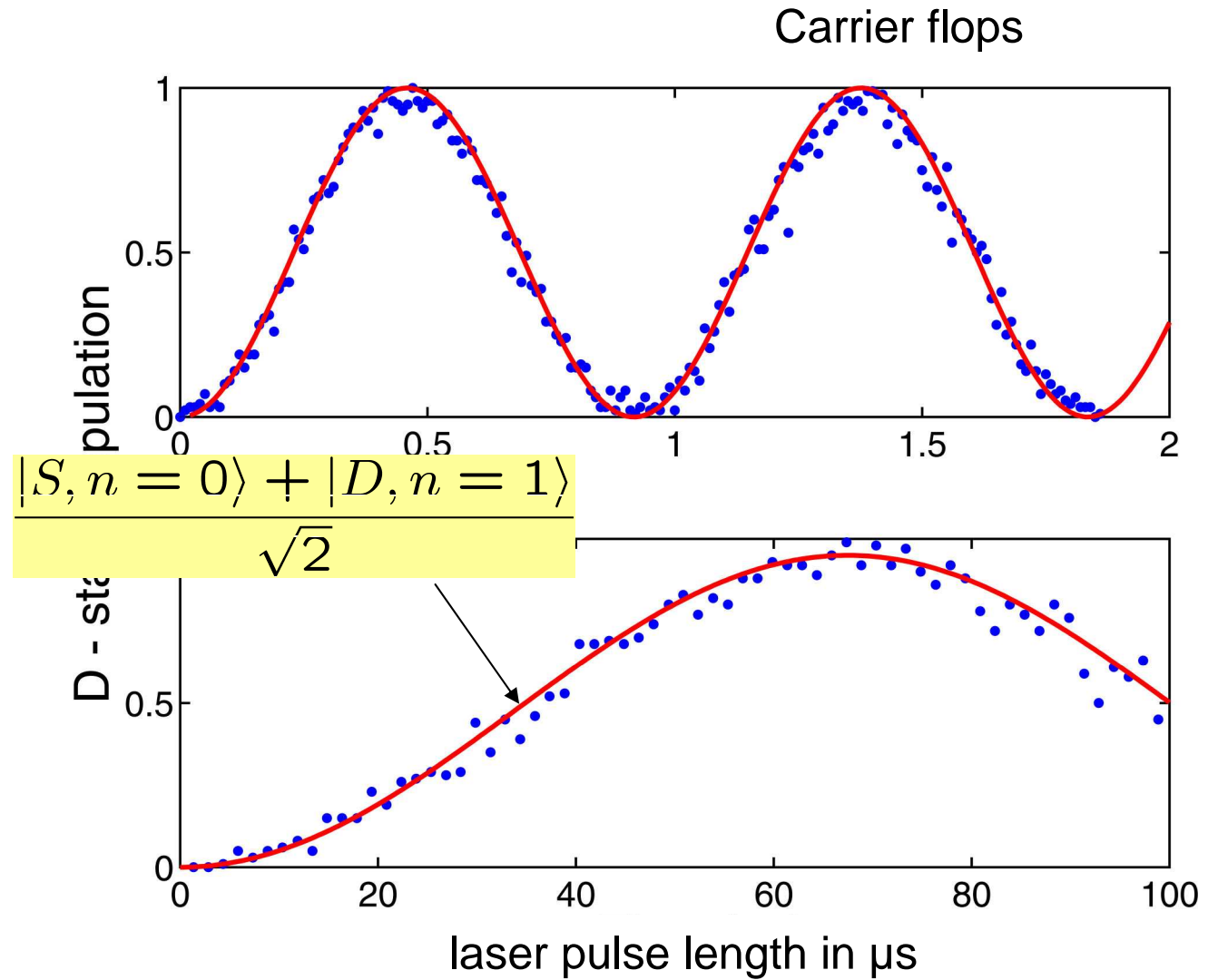
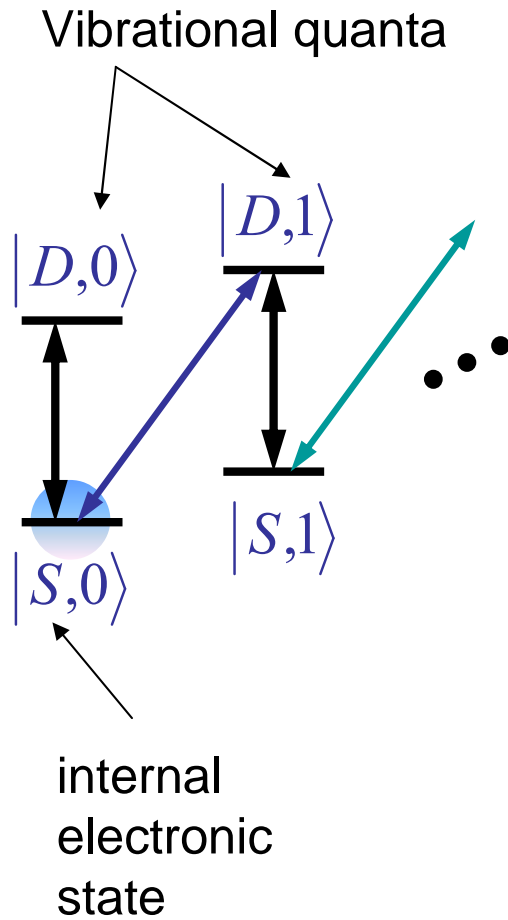
„molecular Franck Condon“ picture



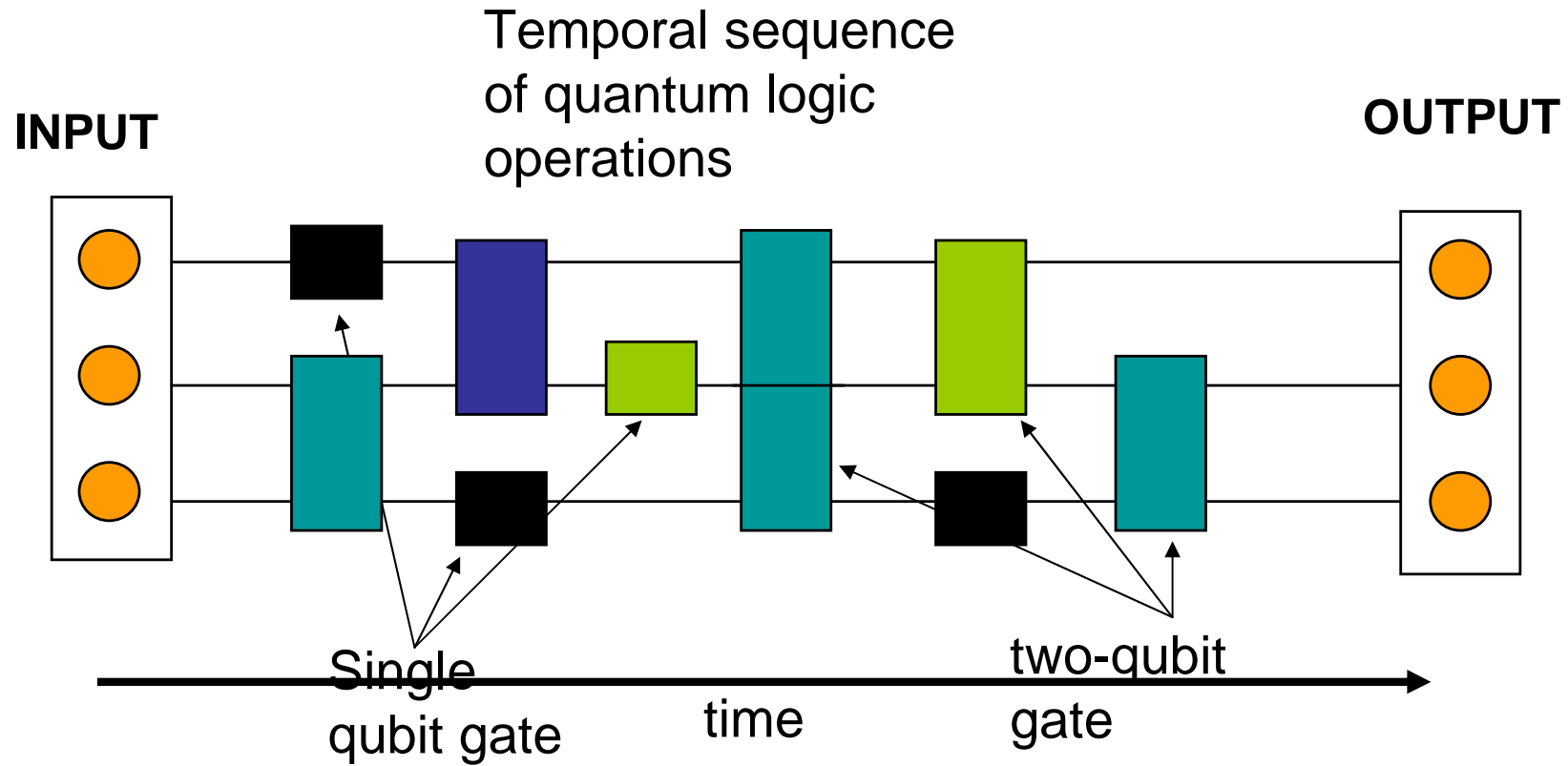
Coherent qubit rotation



Coherent qubit rotation



Basics of a quantum computer



Why?

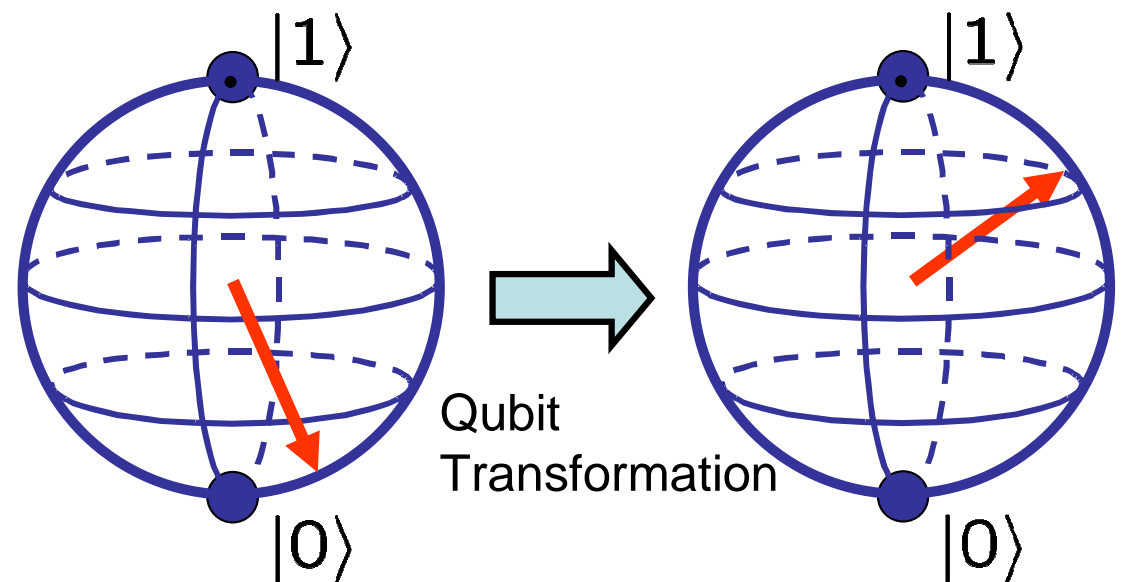


applications in physics and informatics

- P. Shor, 1994: **factorization** of large numbers, L digits, is much more efficient on a quantum computer than with a classical computer:
classical computer: $\sim \exp(L^{1/3})$, quantum computer: $\sim L^2$
- L. Grover, 1997: search data base - quantum computer: $\sim \sqrt{L}$
- **simulation** of Schrödinger equations or any unitary evolution
- spin interactions, **quantum phase transitions**
- quantum **cryptography** / repeaters / quantum links
- improved **atomic clocks**
- understanding the fundamentals of quantum mechanics / **Gedanken-Experimente**
- Experiments with **entangled matter**

The requirements for **experimental** qc

- **Qubits** store superposition information, **scalable** physical system
- Ability to **initialize** the state of the qubits $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- Universal **set of quantum gates**: Single bit and two bit gates
- Long **coherence** times, much longer than gate operation time
- Qubit-specific **measurement** capability



*D. P. DiVincenzo,
Quant. Inf. Comp. 1
(Special), 1 (2001)*

Experimental status

Scalable device?

Table 4.0-1

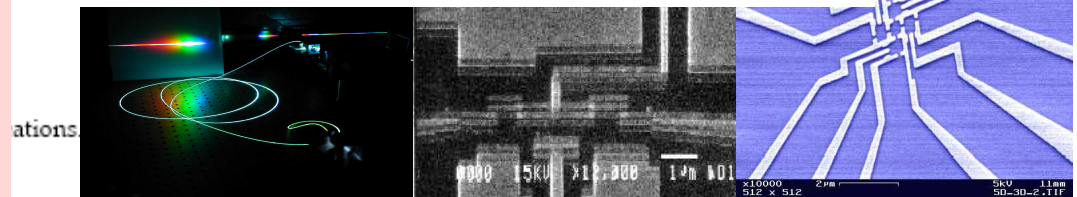
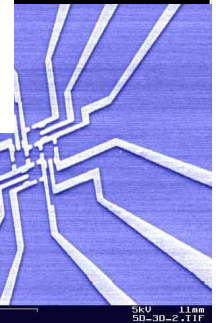
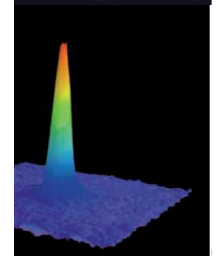
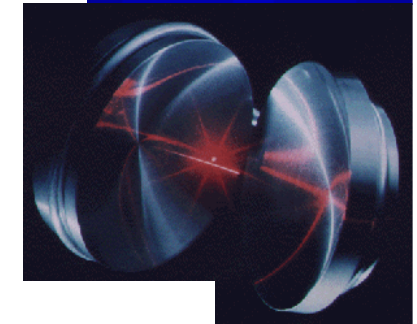
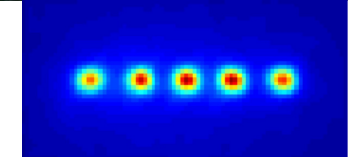
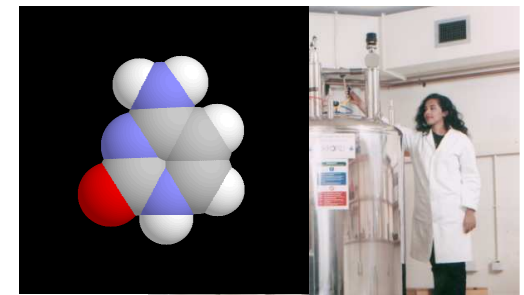
Quantum Computation Roadmap: Promise Criteria

QC Approach	The DiVincenzo Criteria							
	Quantum Computation						QC Networkability	
	#1	#2	#3	#4	#5		#6	#7
NMR								
Trapped Ion								
Neutral Atom								
Optical								
Solid State								
Superconducting								
Unique Qubits								

“... it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway.”

Richard P. Feynman (1985)

Quantum Information Roadmaps
<http://qist.ect.it/>
<http://qist.lanl.gov/>



Quantum gate proposal

74, NUMBER 20 4091

PHYSICAL REVIEW LETTERS

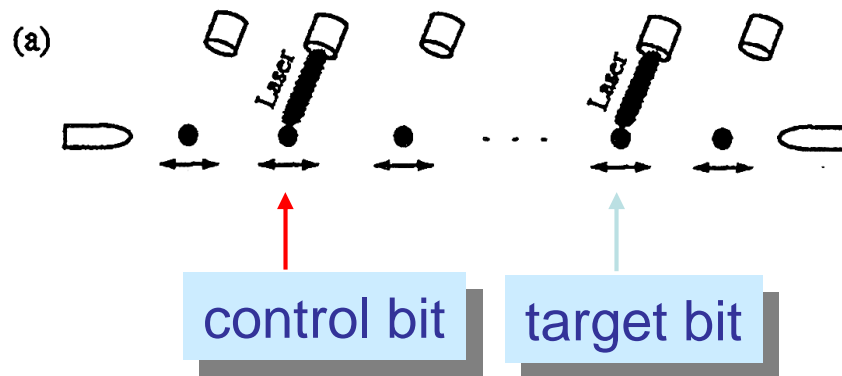
15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.



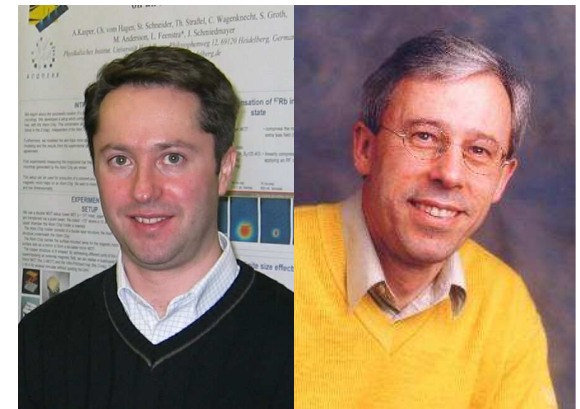
W. Paul

J. I. Cirac

P. Zoller

- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

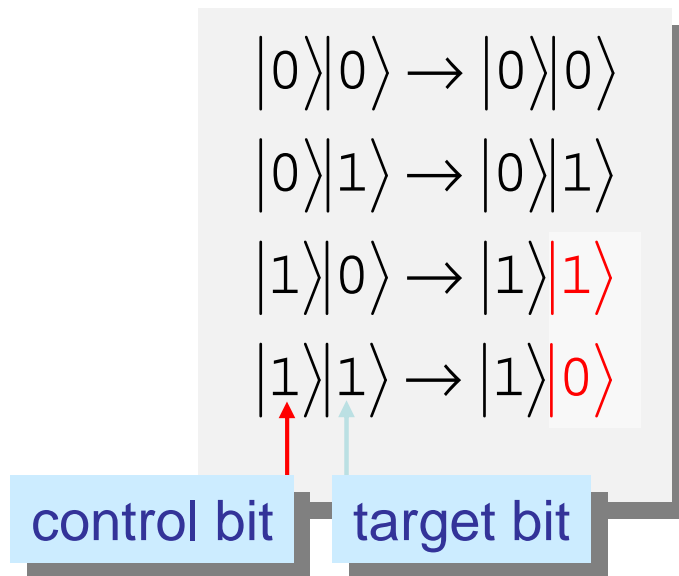
Quantum gate proposal



J. I. Cirac

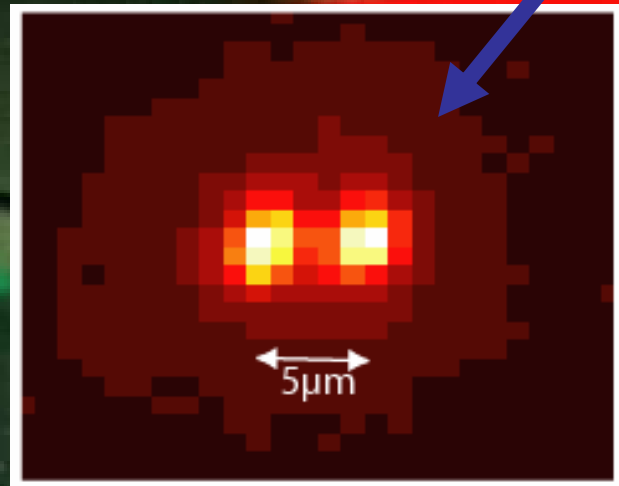
P. Zoller

$$\textit{Controlled - NOT} : |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$



- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

Cirac & Zoller gate
with two ions



Controlled-NOT operation

$|\epsilon_1\rangle$ $|\epsilon_2\rangle$



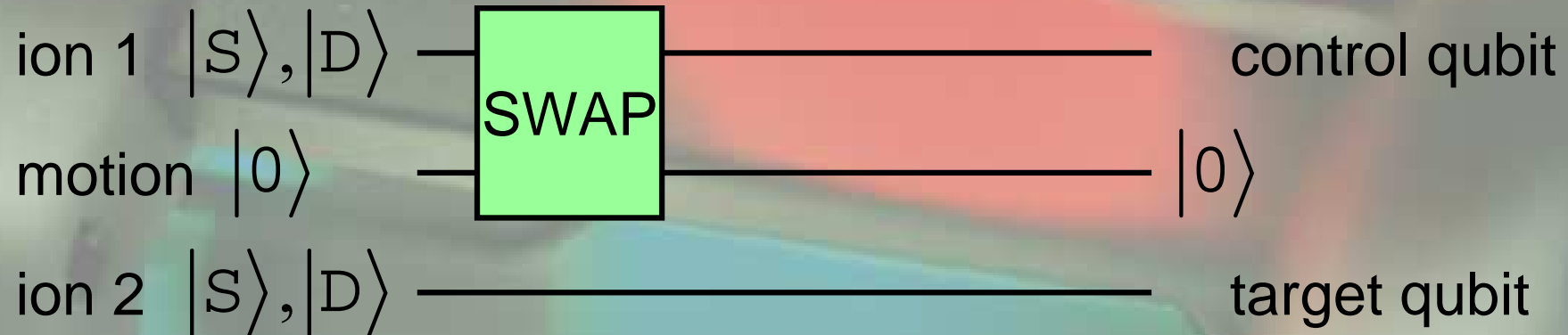
$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

control target



Controlled-NOT operation

$|\epsilon_1\rangle$ $|\epsilon_2\rangle$



$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$



Controlled-NOT operation

$|\epsilon_1\rangle$ $|\epsilon_2\rangle$

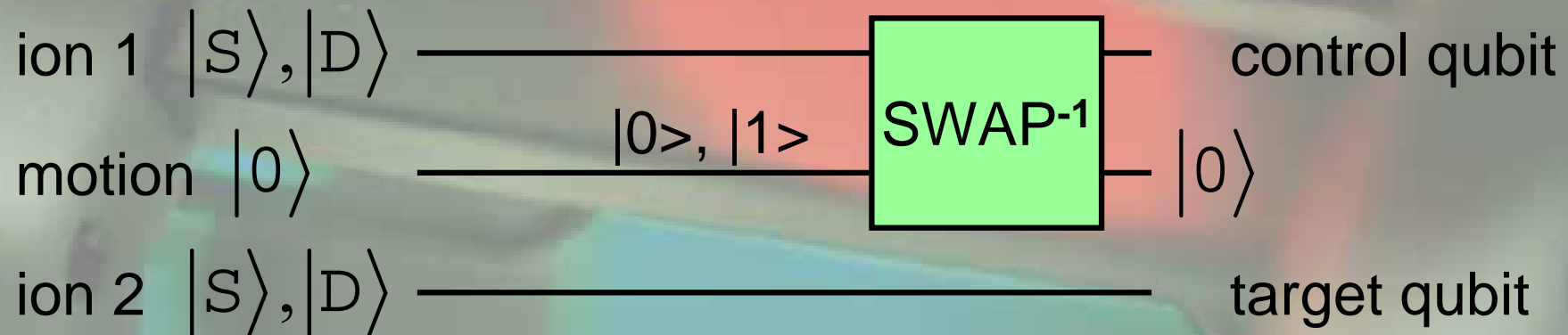


$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$

$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$

$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$

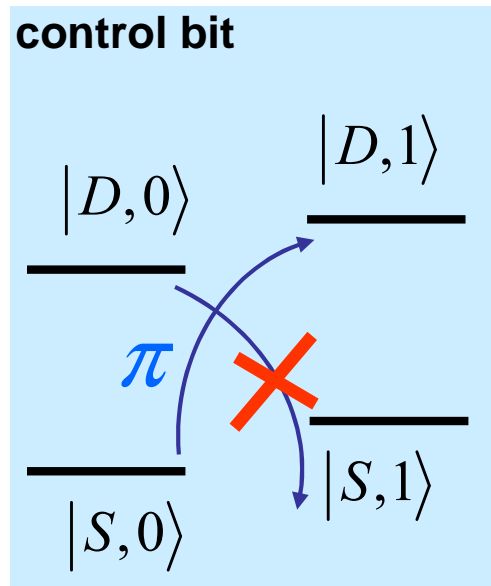
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$



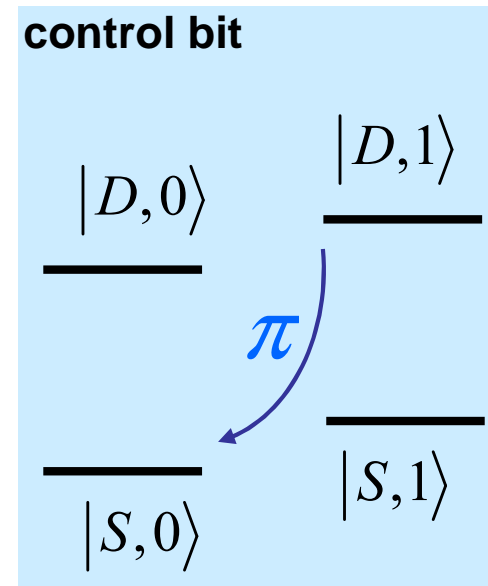
SWAP and SWAP⁻¹

starting with $|n=0\rangle$ phonons,
write into and read from the common vibrational mode

π -pulse on blue SB

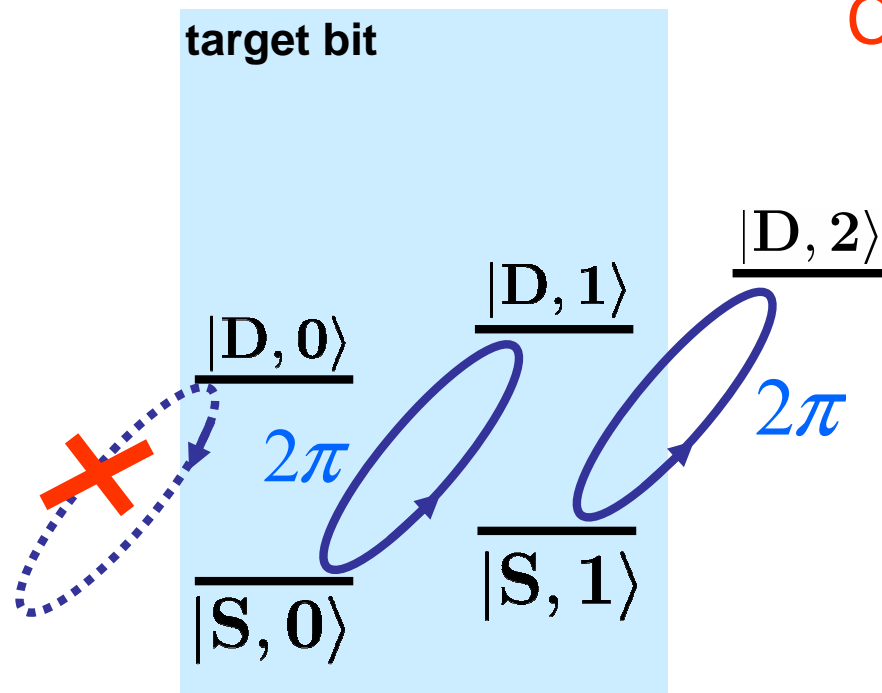


SWAP



SWAP⁻¹

Conditional phase gate



Composite pulse phase gate

*I. Chuang,
MIT Boston*

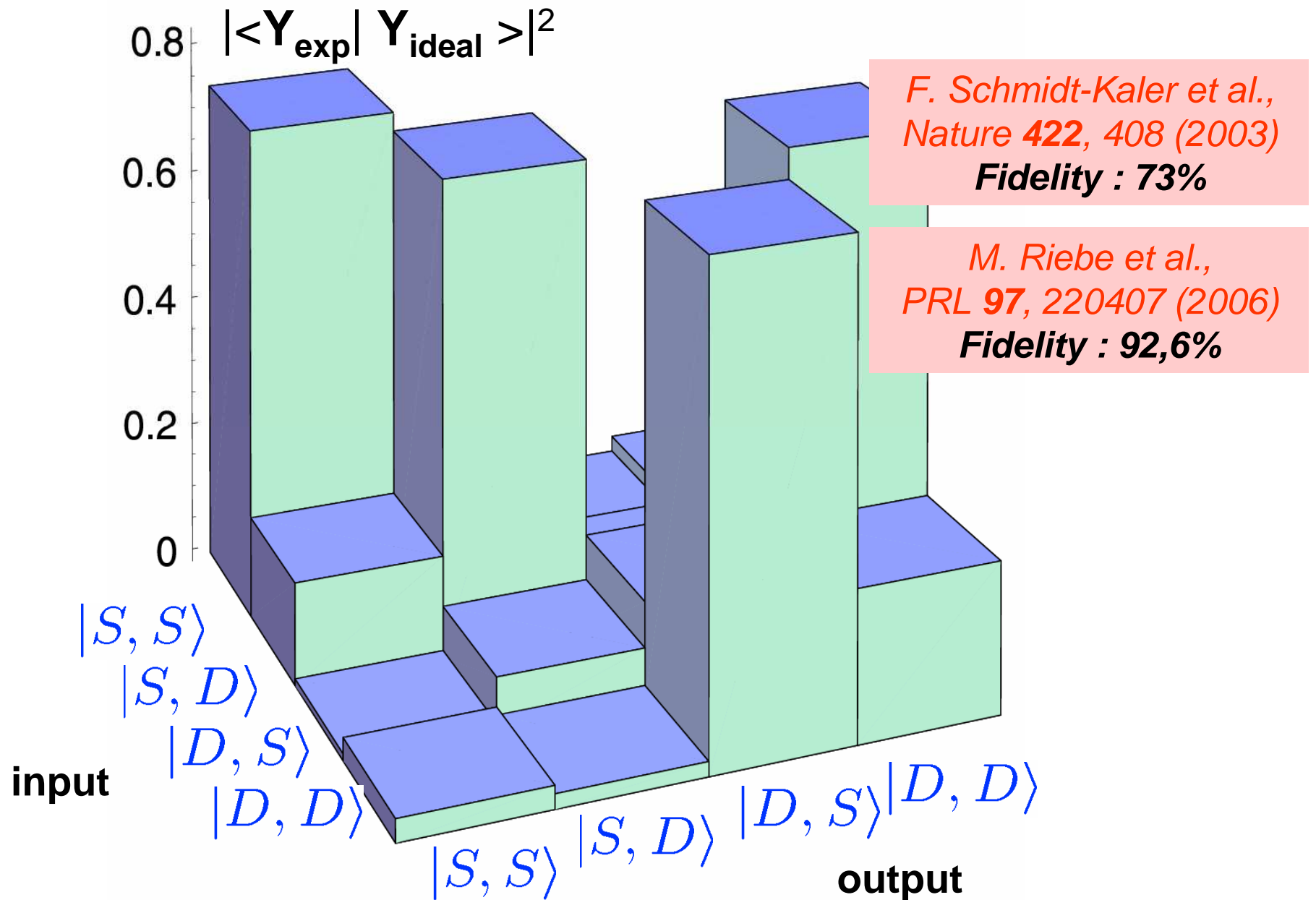
Rabi frequency:

Blue SB: $\Omega \cdot \eta \cdot \sqrt{n+1}$

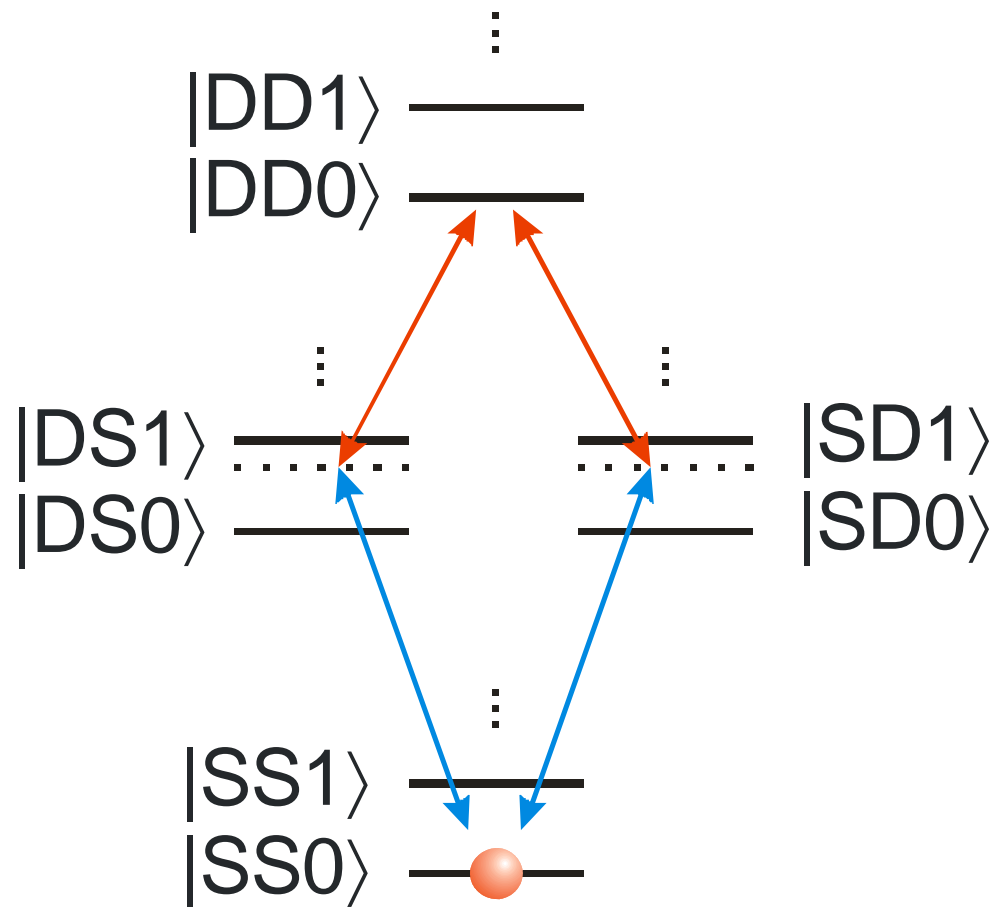
Effect:

phase factor of **-1**
for all, except $|D, 0\rangle$

Fidelity of Cirac-Zoller CNOT



Bichromatic two-qubit gate



The **common** absorption of red and blue detuned light leads to a coherent evolution $|SS\rangle$ to $|DD\rangle$. No excitation of $|DS\rangle$ states.

Requires only Lamb Dicke limit $\eta\sqrt{n_{ther.}} \ll 1$

Milburn, arXiv:quant-ph/9908037.

Milburn, Schneider, and James, *Fortschr. Phys.* **48**, 801 (2000).

Sørensen and Mølmer, *PRL* **82**, 1971 (1999).

Sørensen and Mølmer, *PRA* **62**, 022311 (2000).

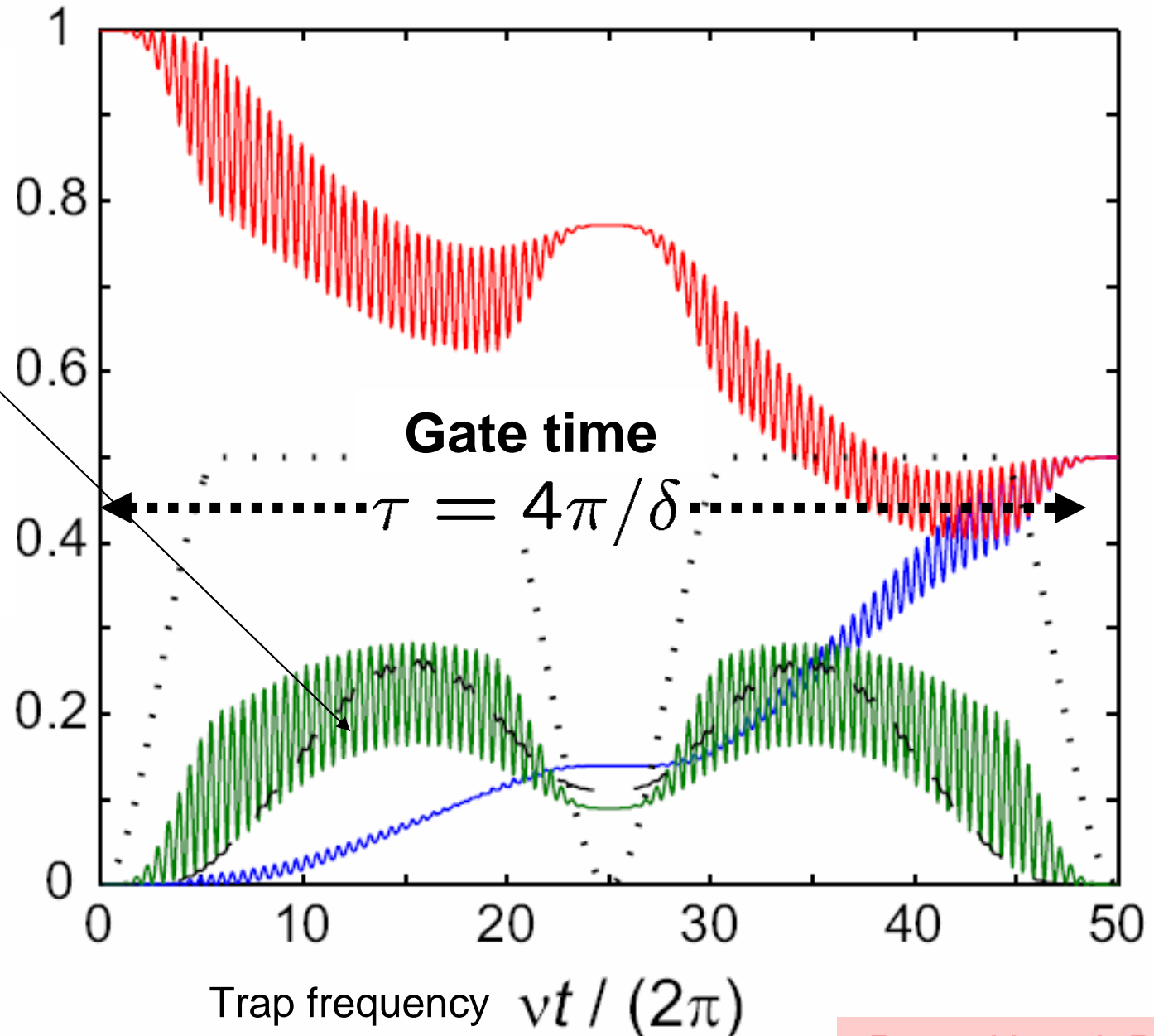
Bell state with $F=83\%$
Sackett et al., *Nature* **406**, 256 (2000)

Bichromatic two-qubit gate

For slow evolution
no excitation of the
states $|SD\rangle$ and $|DS\rangle$

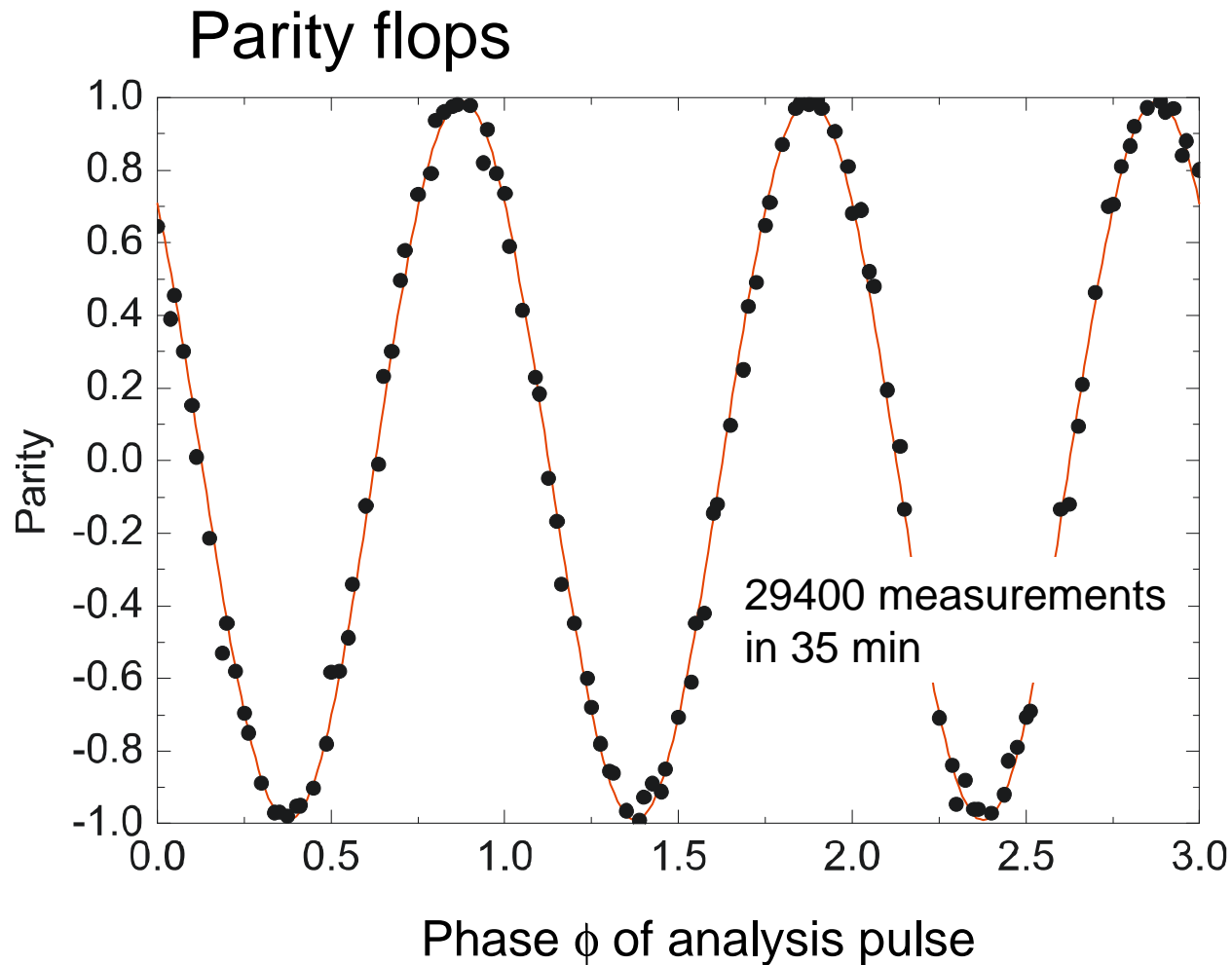
For fast evolution
excitation and de-
excitation

Amplitude shaped
laser pulses help
improving $F > 99.9\%$



Roos, New J. Phys.
10, 013002, (2008)

Fidelity of the created Bell state



Bell state analysis

$|SS\rangle + i|DD\rangle$

→ global $\pi/2$ pulse

with variable phase ϕ

Parity:

$$\Pi \equiv p_0 + p_2 - p_1$$

$$\rightarrow F = 99.3(1)\%$$

$$F_{21} = 80\%$$

$$P_{\text{fit}}(\phi) = A \sin(2\phi + \phi_0)$$

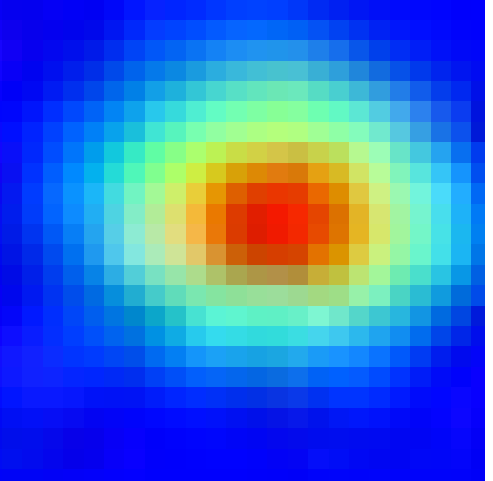
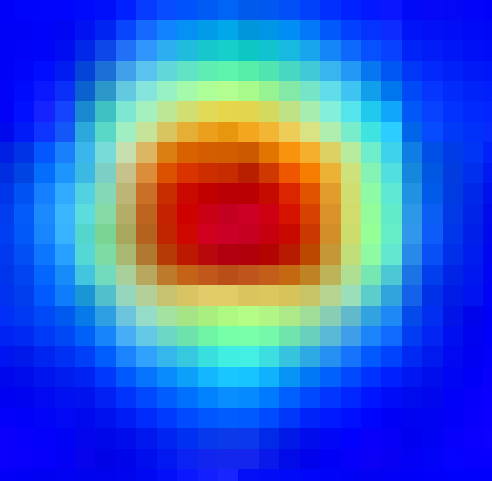
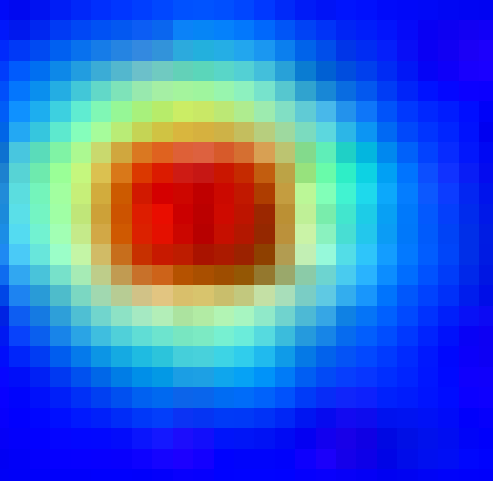
$$A = 0.990(1)$$

$$p_0 + p_2 = 0.9965(4)$$

Roos et al, unpublished data

GHZ state:

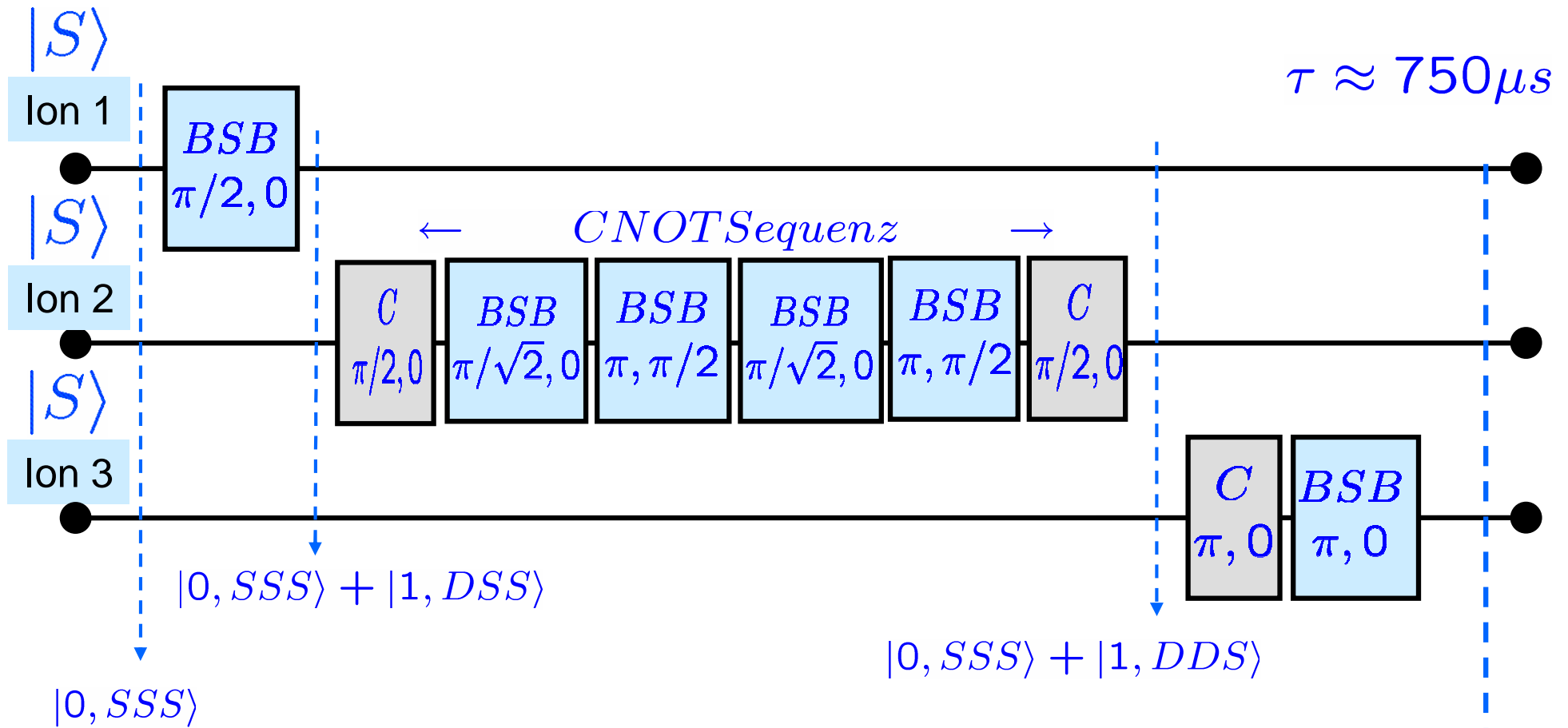
$$|SSS + DDD\rangle$$



W state:

$$|SSD + SDS + DSS\rangle$$

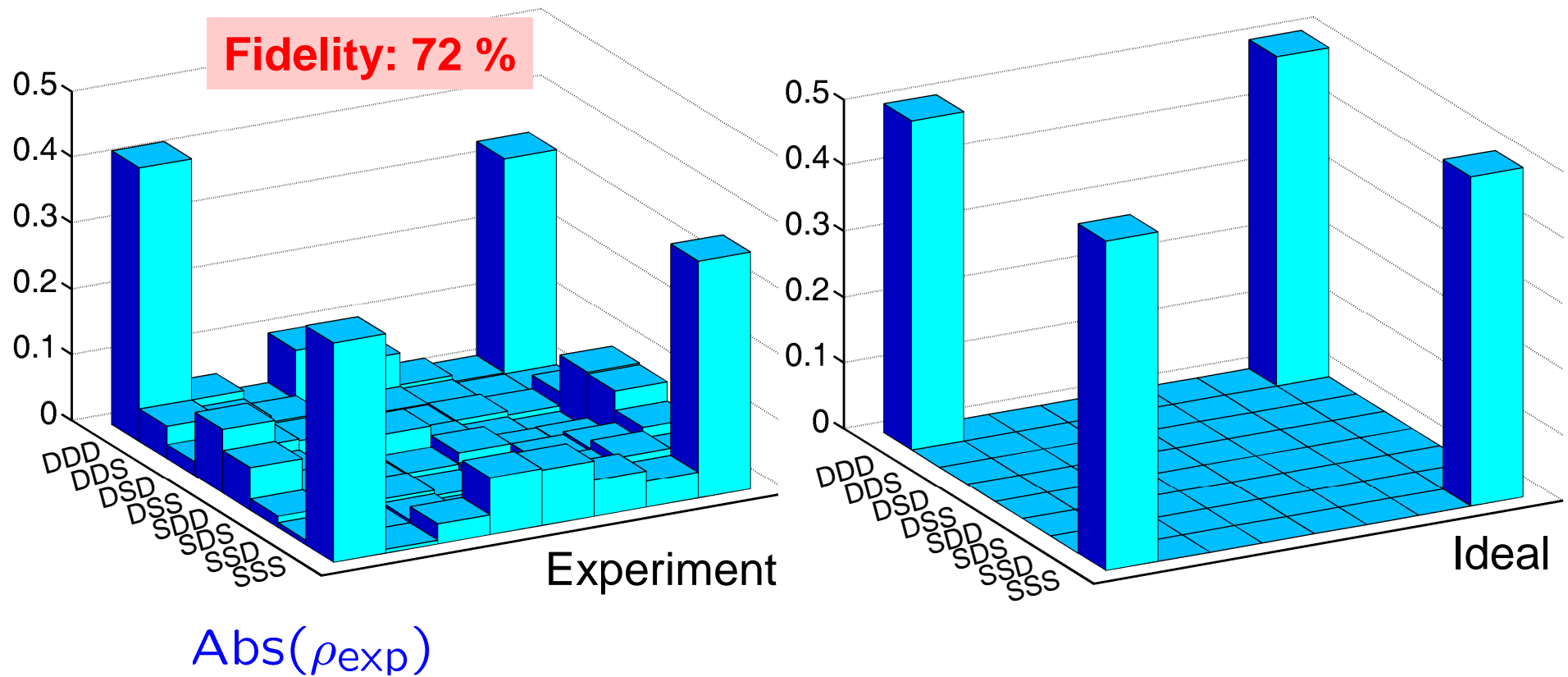
Deterministic generation of GHZ state



$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}(|DDD\rangle + |SSS\rangle)|0\rangle$$

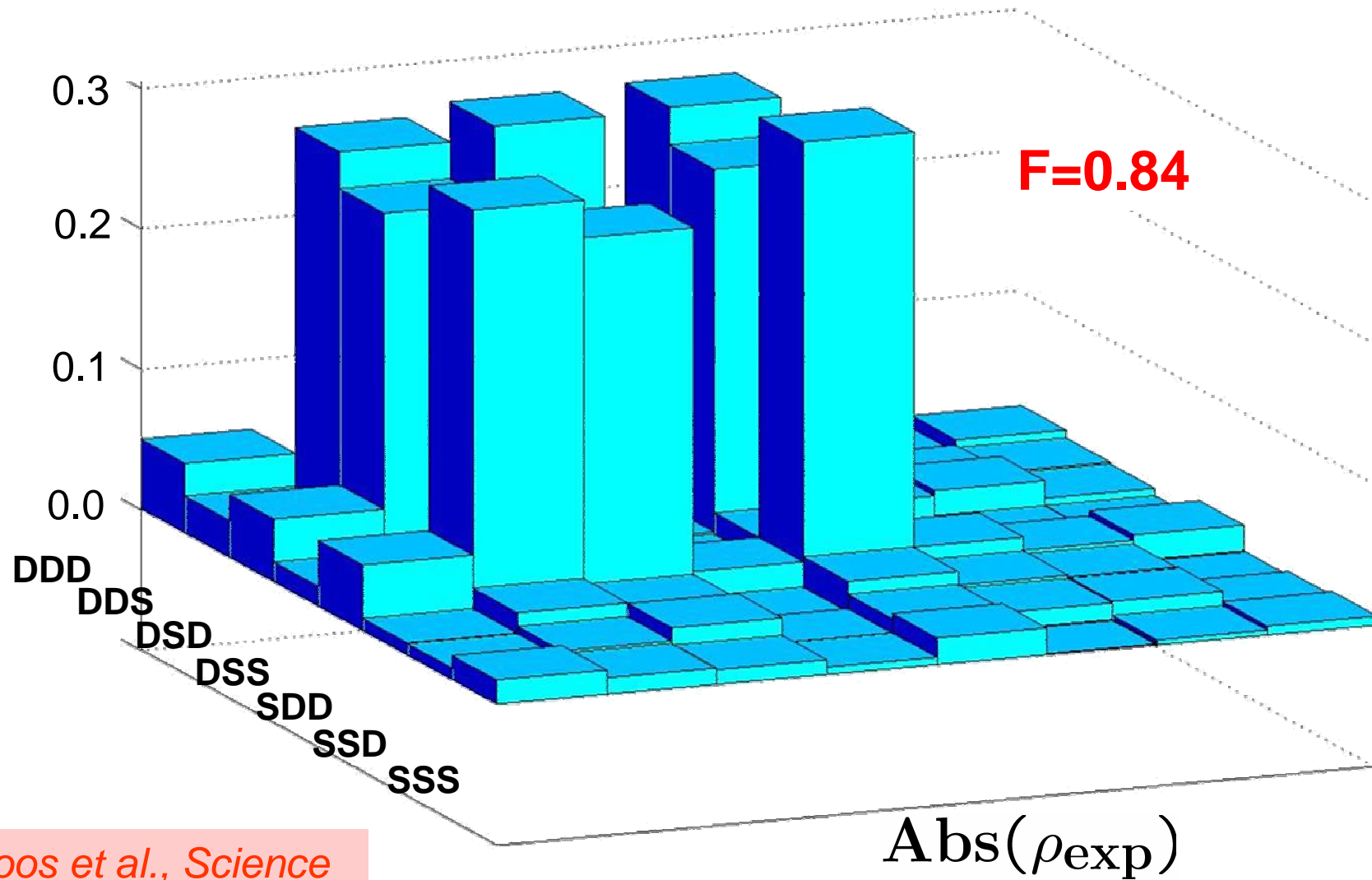
Tomography of the GHZ state

$$|\psi\rangle_{GHZ} = \frac{1}{\sqrt{2}} (|SSS\rangle - |DDD\rangle)$$



*C. Roos et al., Science
304, 1478 (2004)*

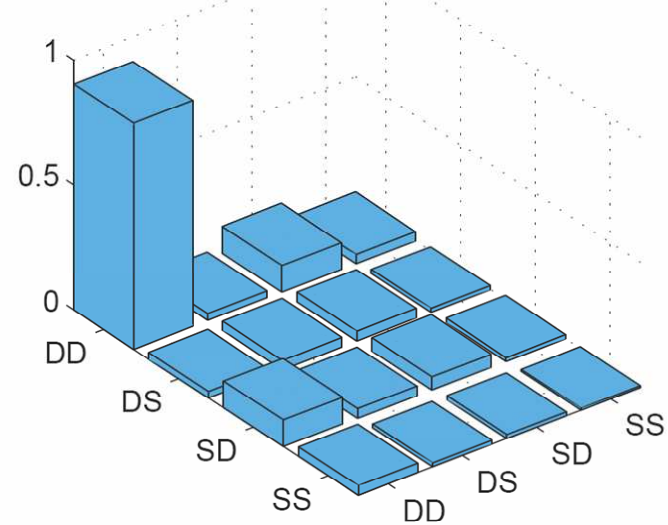
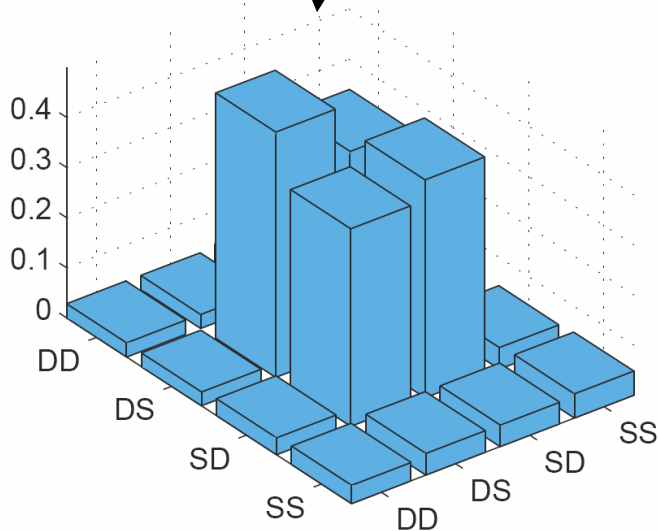
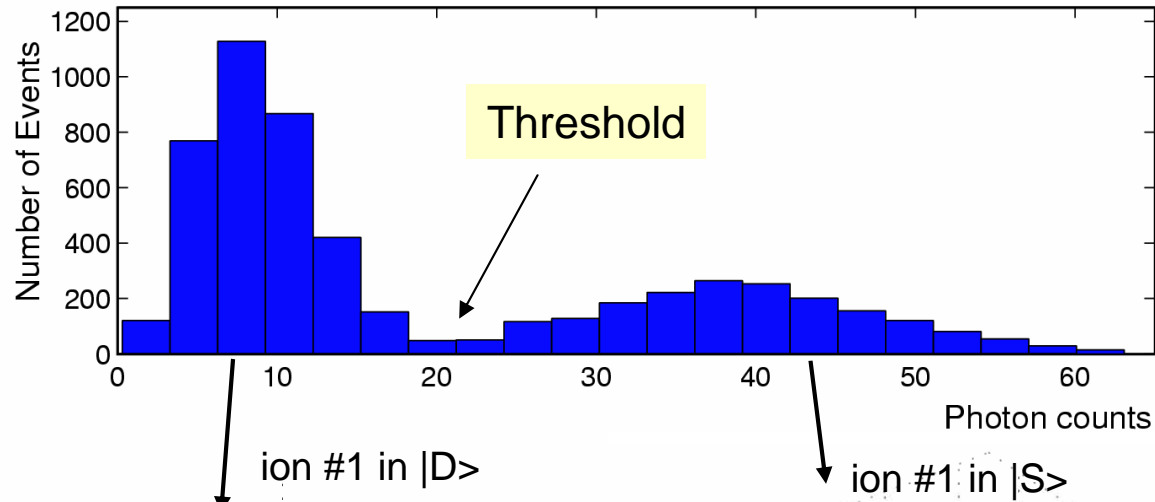
$|\text{SDD}\rangle - |\text{DSD}\rangle - |\text{DDS}\rangle$



*C. Roos et al., Science
304, 1478 (2004)*

Selective measurement

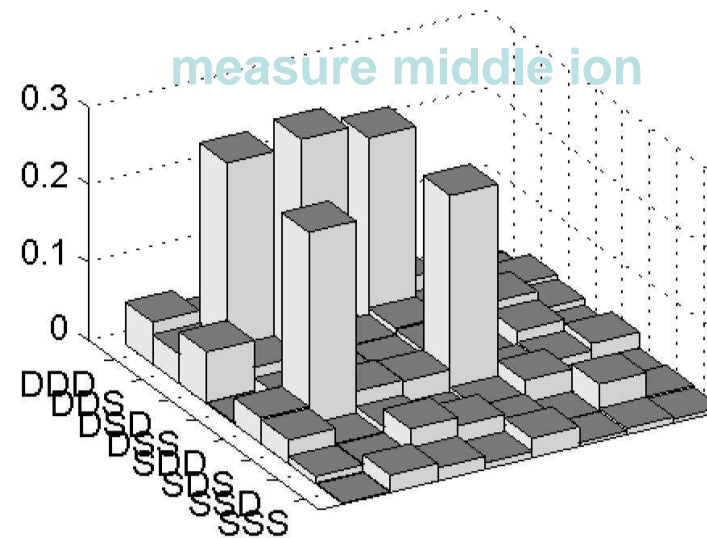
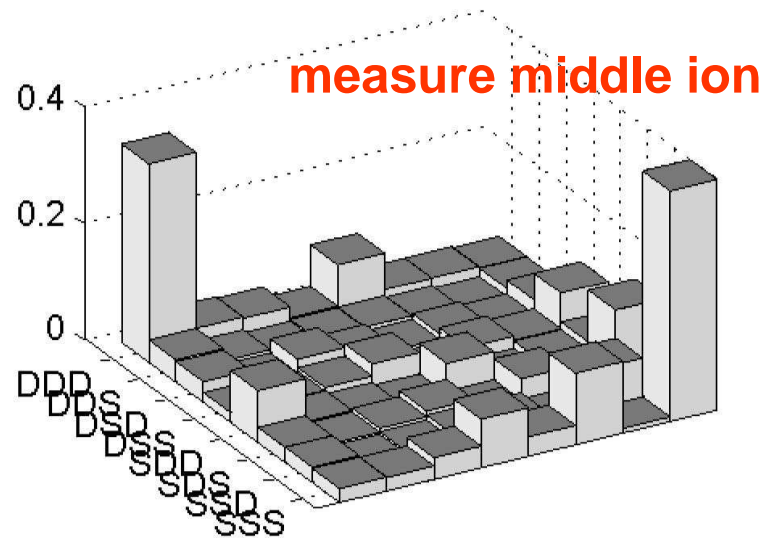
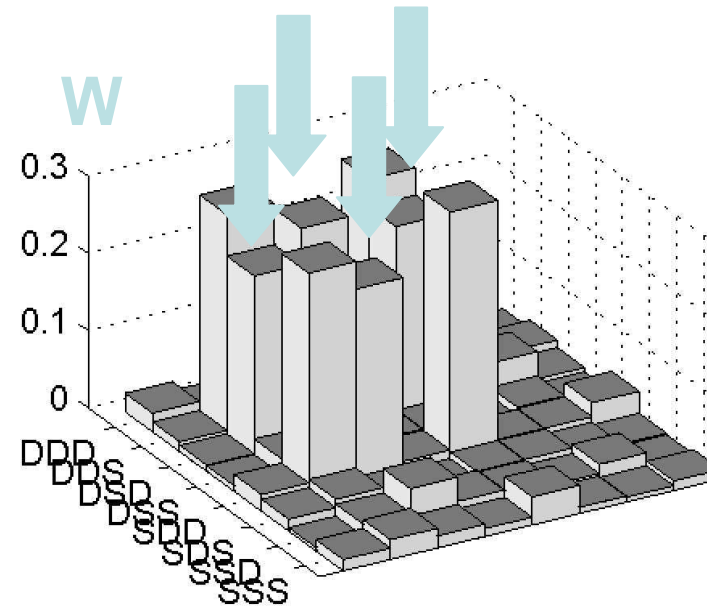
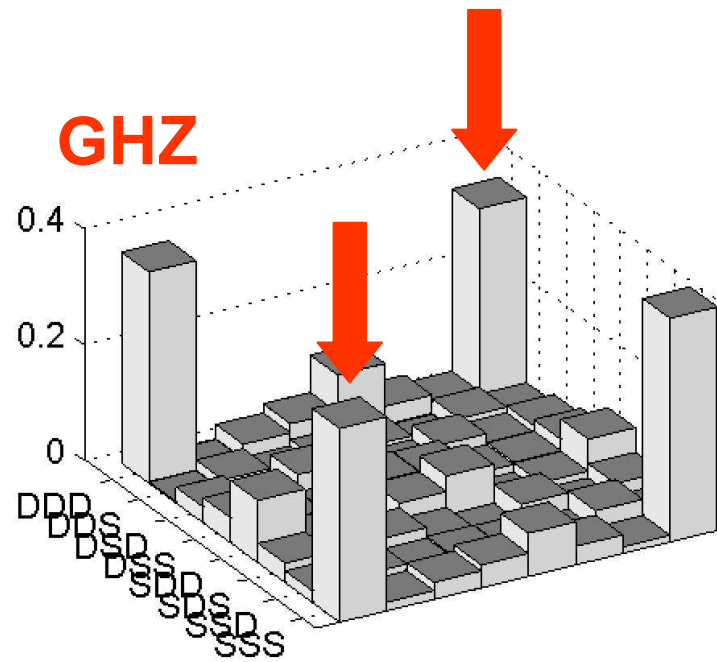
$$|\psi\rangle_W = \frac{1}{\sqrt{3}} (|SDD\rangle + |DSD\rangle + |DDS\rangle)$$



Tomography **after** the measurement result is available!

Selectively projected 3-ion entanglement

Greenberger Horne Zeilinger state



W - state

Teleportation

R. Blatt, H. Häffner, C. Becher,
F. Schmidt-Kaler, J. Benhelm, T. Körber,
G. Lancaster, C. Roos, W. Hänsel, M. Riebe

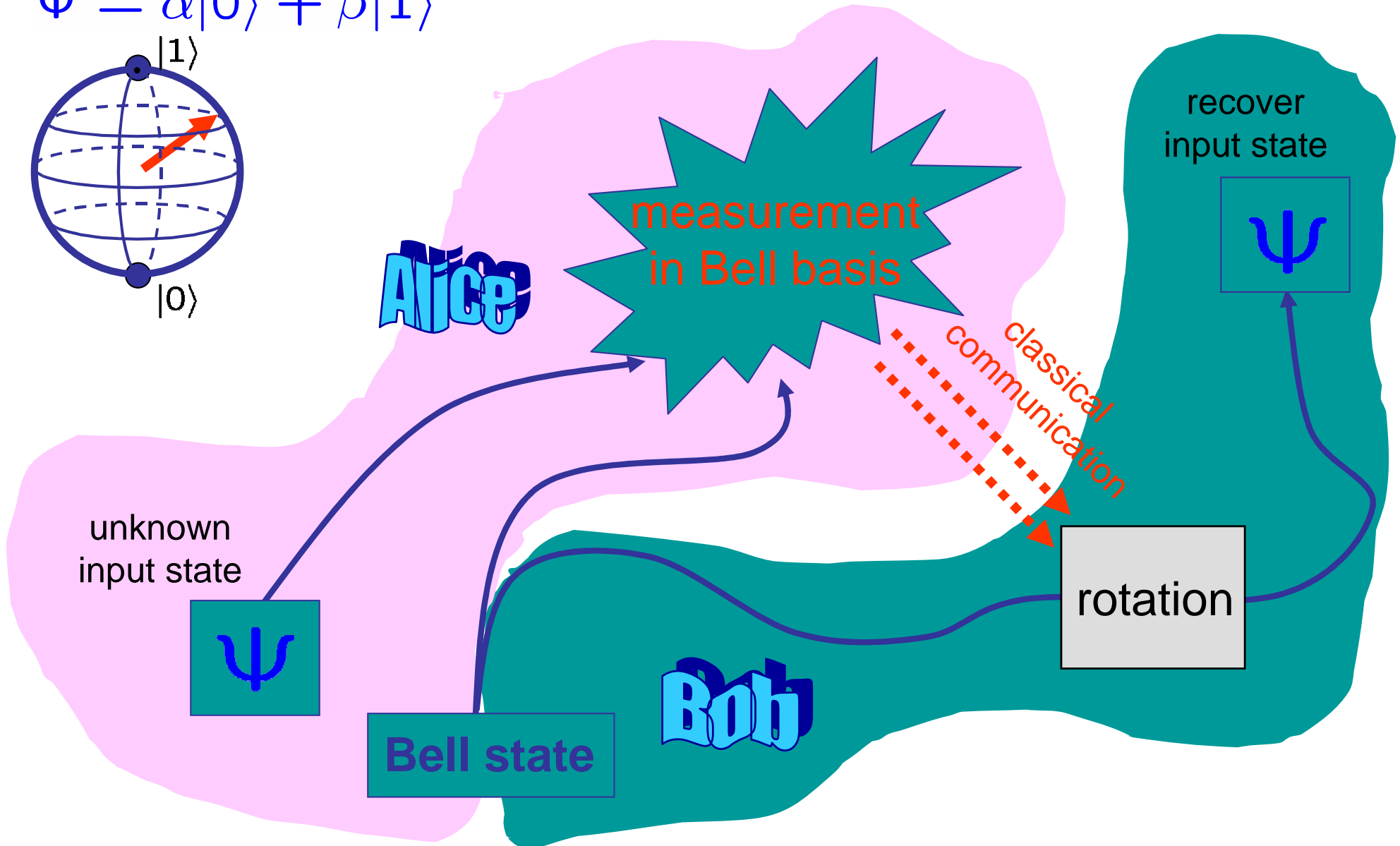
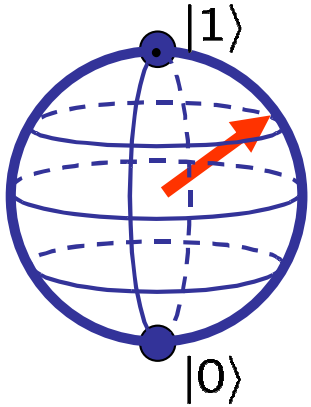
Theorie: D. James, Los Alamos



Teleportation

Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

$$\psi = \alpha|0\rangle + \beta|1\rangle$$



Quantum teleportation: No black magic

Source qubit(#1): pure state $|\chi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$

Target qubit(#3) and ancilla (#2): maximally entangled state

$$|\Psi^+\rangle_{23} = \frac{1}{\sqrt{2}} (|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$$

Combined state

$$|\varphi\rangle = |\chi\rangle_1 \frac{1}{\sqrt{2}} (|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3)$$

Rearrange terms:

$$|\varphi\rangle = \frac{1}{2} (|\Phi^+\rangle_{12} \sigma_x |\chi\rangle_3 + |\Phi^-\rangle_{12} (-i\sigma_y) |\chi\rangle_3 + |\Psi^+\rangle_{12} |\chi\rangle_3 + \boxed{|\Psi^-\rangle_{12} \sigma_z |\chi\rangle_3})$$

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1|0\rangle_2 \pm |1\rangle_1|1\rangle_2)$$

$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2)$$

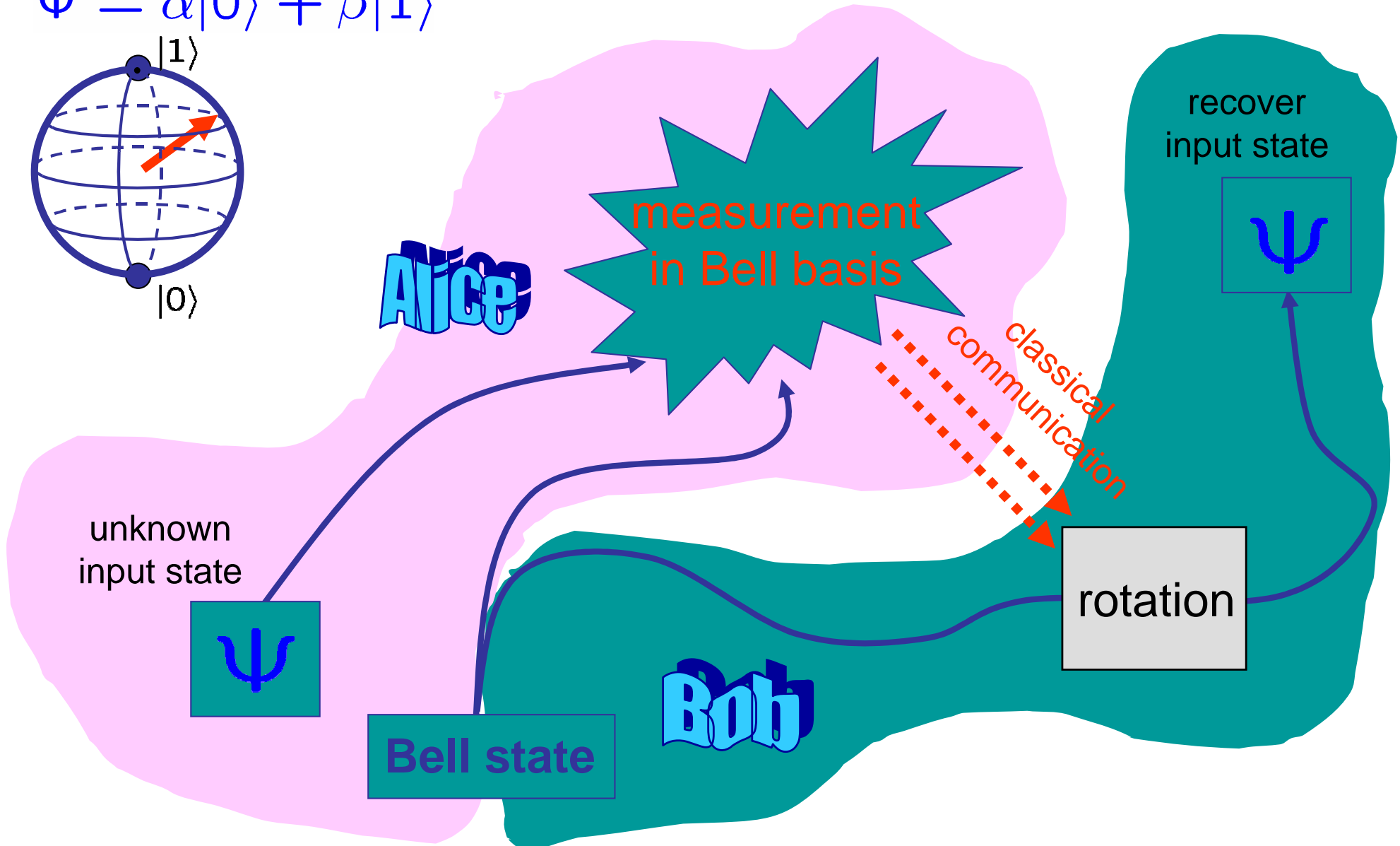
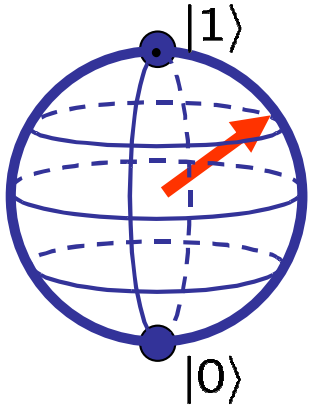
measure #1 and #2 in Bell basis: $|\varphi\rangle$ is projected onto one of 4 pure states

e.g. measure $|\Psi^-\rangle_{12}$:perform $-\sigma_z$ operation on qubit #3 to yield input state back

Teleportation

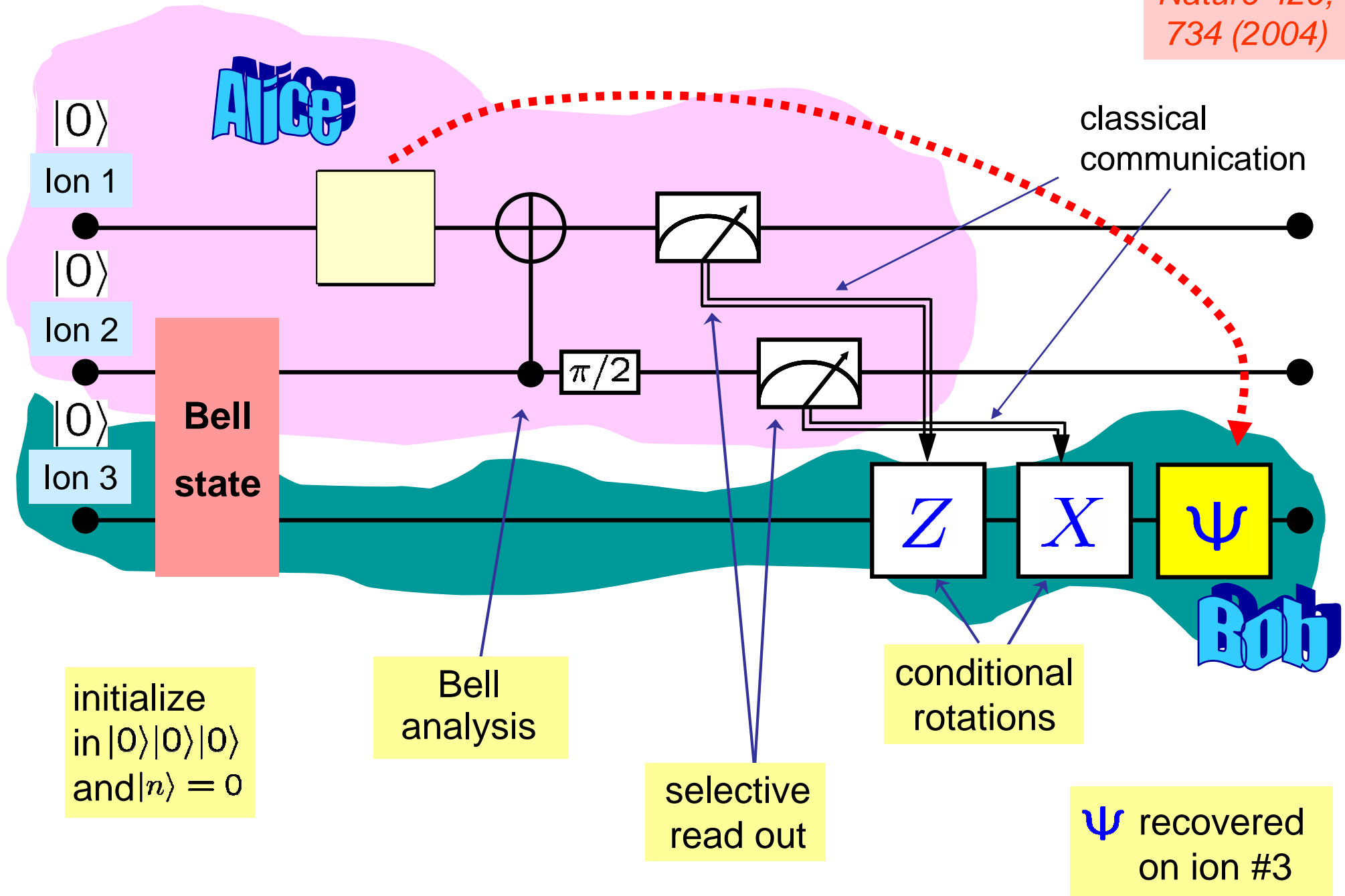
Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

$$\psi = \alpha|0\rangle + \beta|1\rangle$$



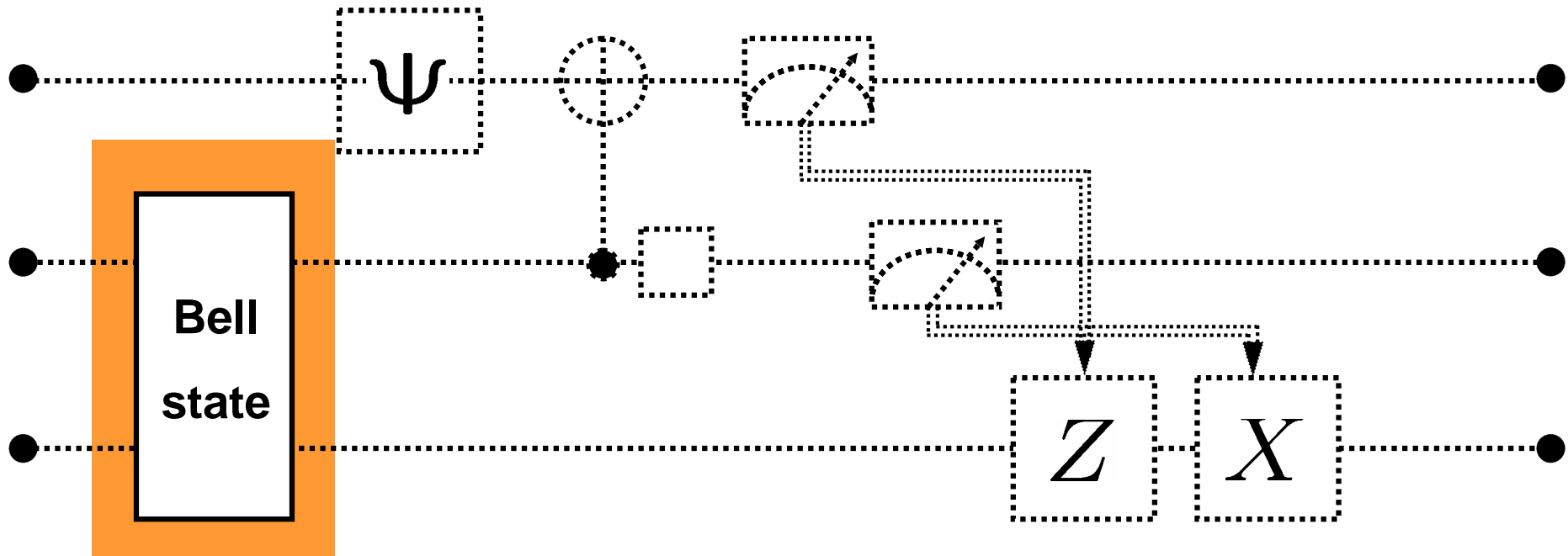
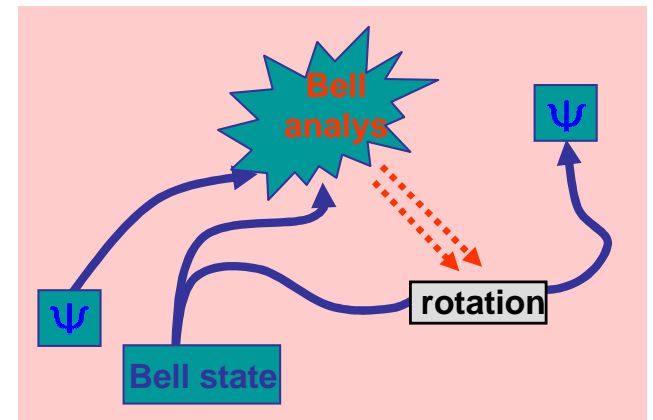
Quantum teleportation protocol

*Riebe et al,
Nature 429,
734 (2004)*



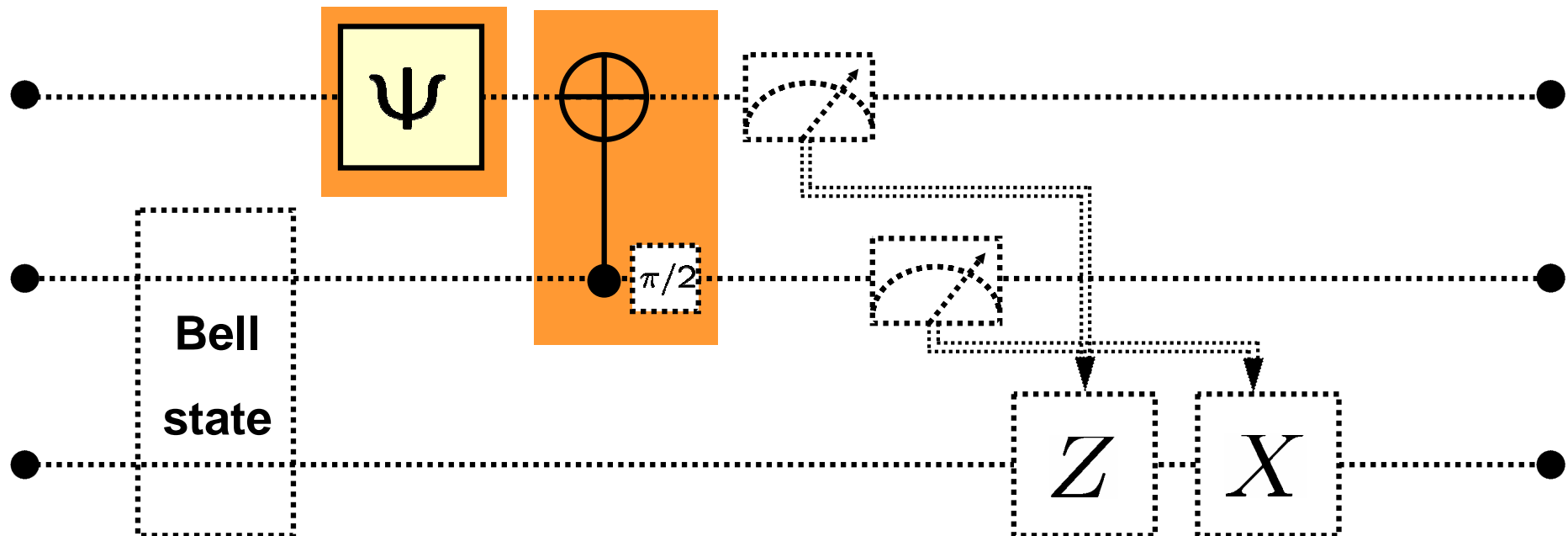
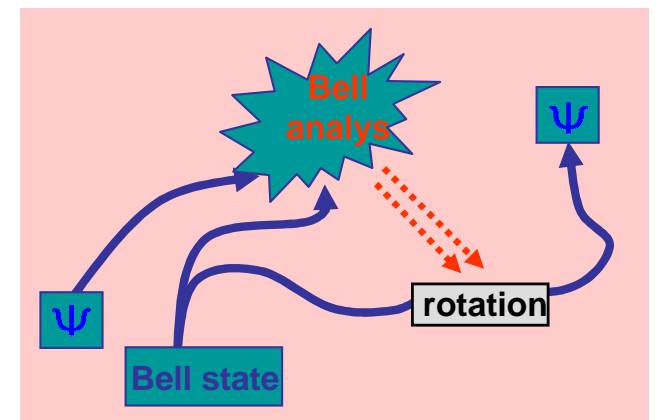
Step by step

1. *Bell state generation*



Step by step

1. *Bell state generation*
2. **Generate ψ**
3. **Bell analysis**



complete Bell analysis

Two qubit
entangled state

CNOT

Superposition state

$\pi/2$

computational
basis state {S,D}

control bit

target bit

$$1/\sqrt{2}\{|S, S\rangle + |D, D\rangle\} \xrightarrow{\text{CNOT}}$$

$$1/\sqrt{2}\{|S + D\rangle|S\rangle\} \xrightarrow{\pi/2}$$

$|S, S\rangle$

complete Bell analysis

Two qubit
entangled state

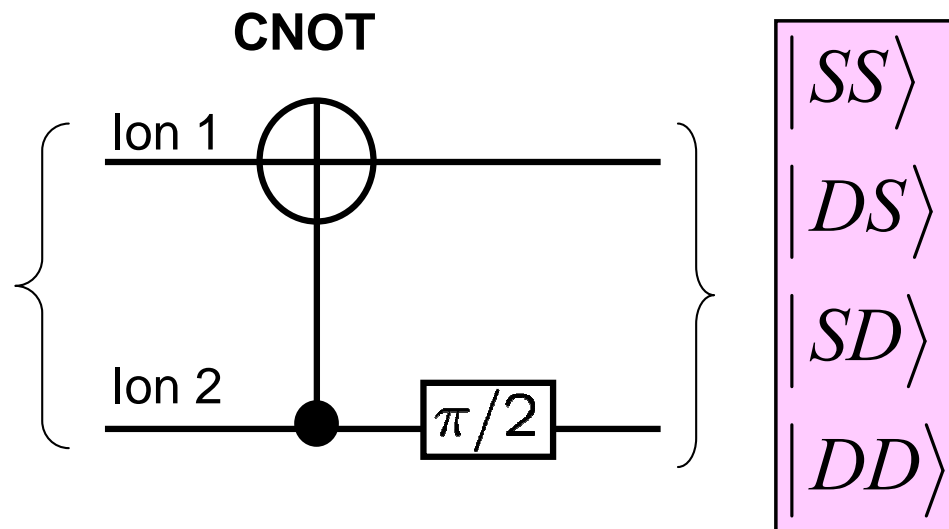
CNOT

Superposition state

$\pi/2$

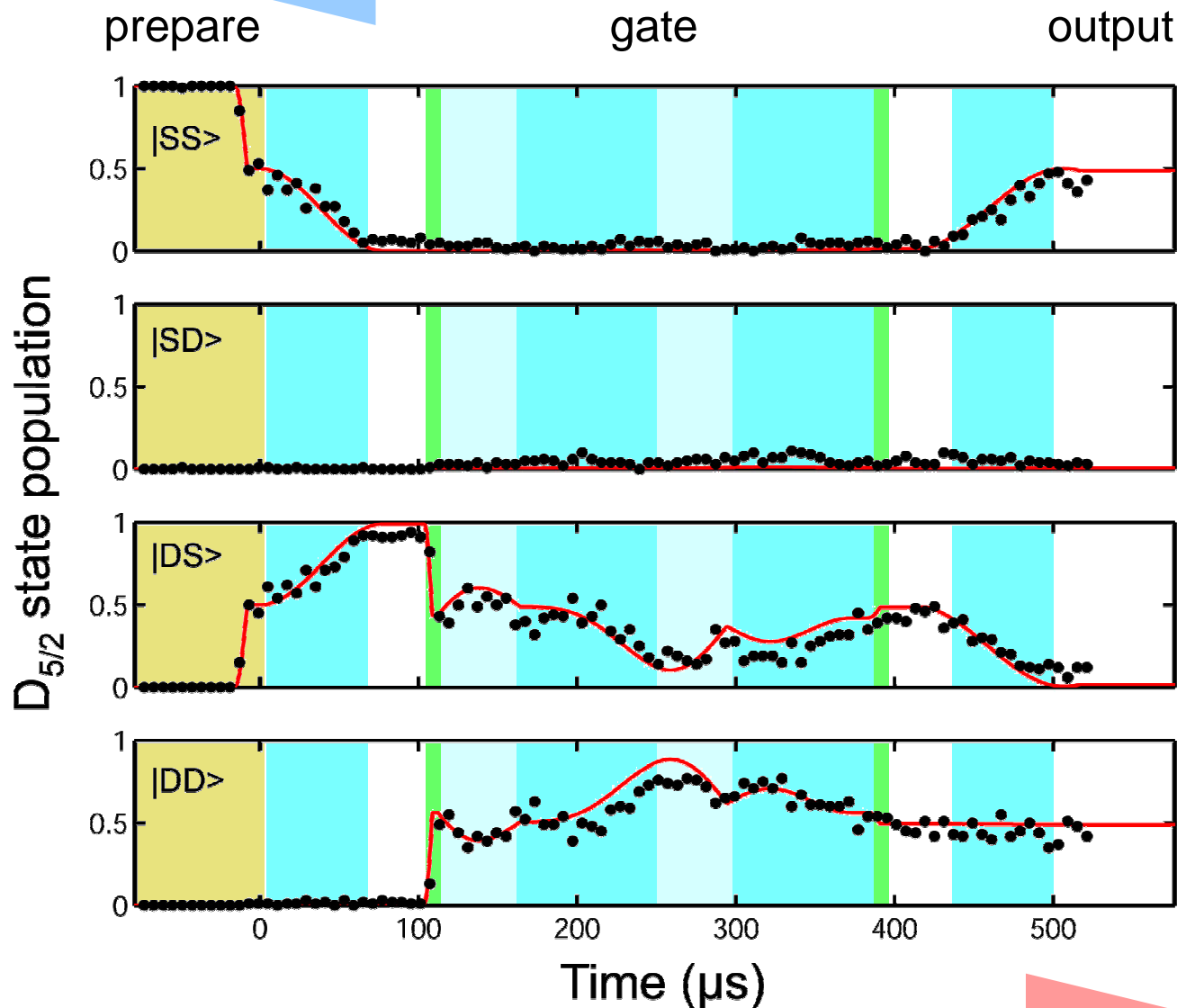
computational
basis state {S,D}

$$\beta_{00} = \frac{1}{\sqrt{2}} (|SS\rangle + |DD\rangle)$$
$$\beta_{10} = \frac{1}{\sqrt{2}} (|SD\rangle + |DS\rangle)$$
$$\beta_{01} = \frac{1}{\sqrt{2}} (|SS\rangle - |DD\rangle)$$
$$\beta_{11} = \frac{1}{\sqrt{2}} (|SD\rangle - |DS\rangle)$$



$$|S+D, S\rangle \xrightarrow{\text{CNOT}} |SS\rangle + |DD\rangle$$

Dis-Entangle two ion Bell states



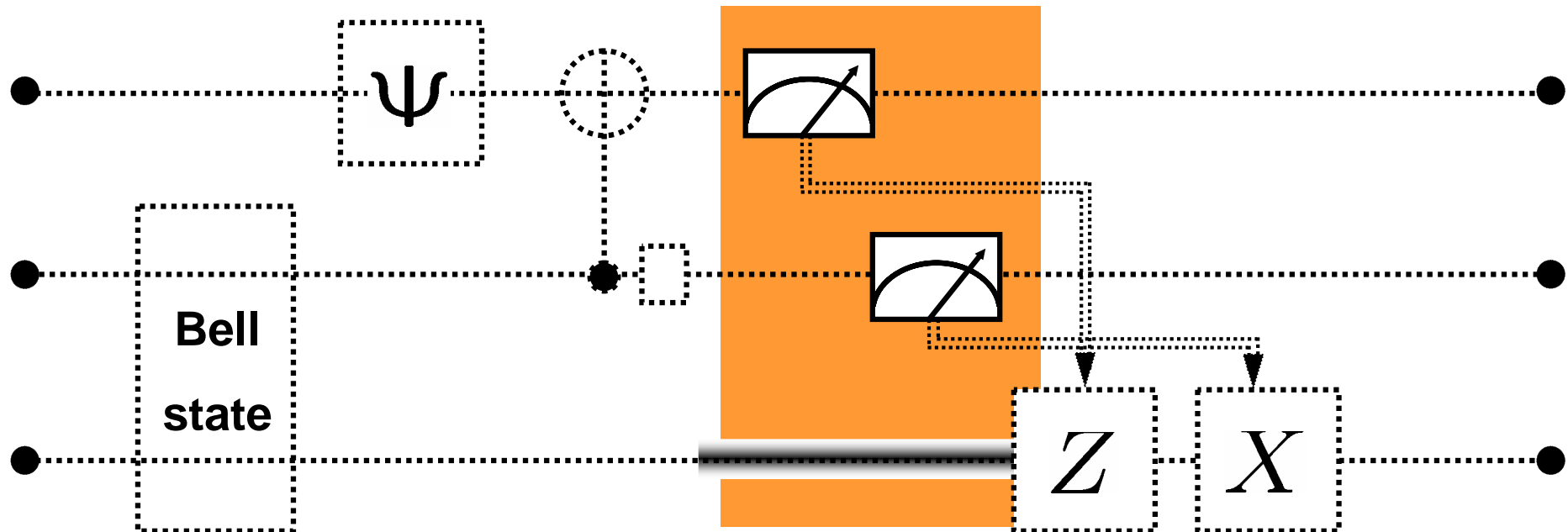
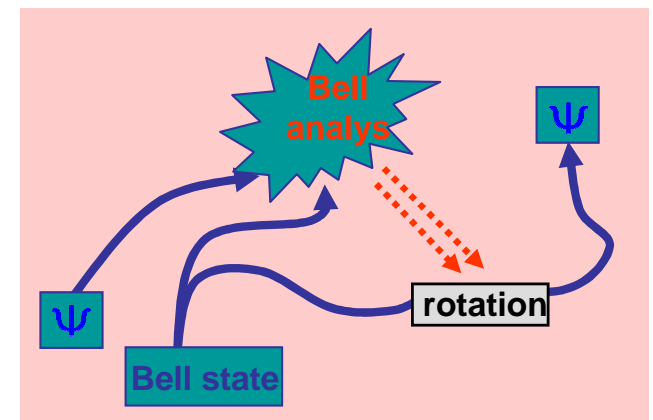
detect



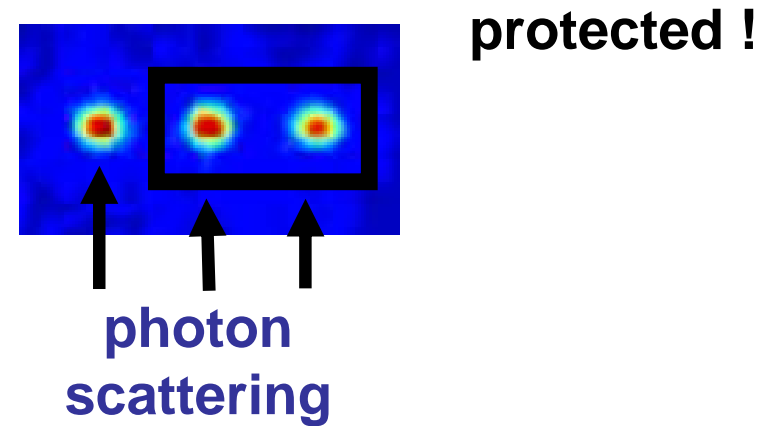
Entangle two ions

Step by step

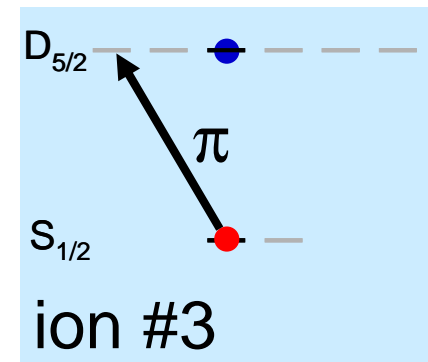
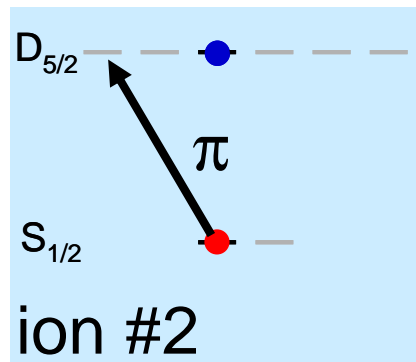
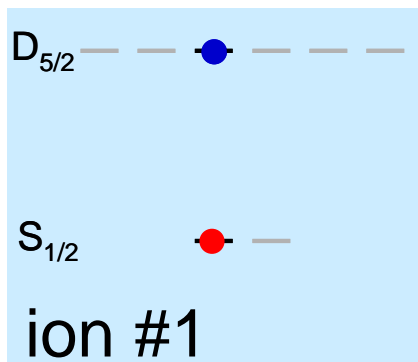
1. *Bell state generation*
2. *Generate ψ*
3. *Bell analysis*
4. ***Selective read-out
(and hiding)***



Hiding a qubit

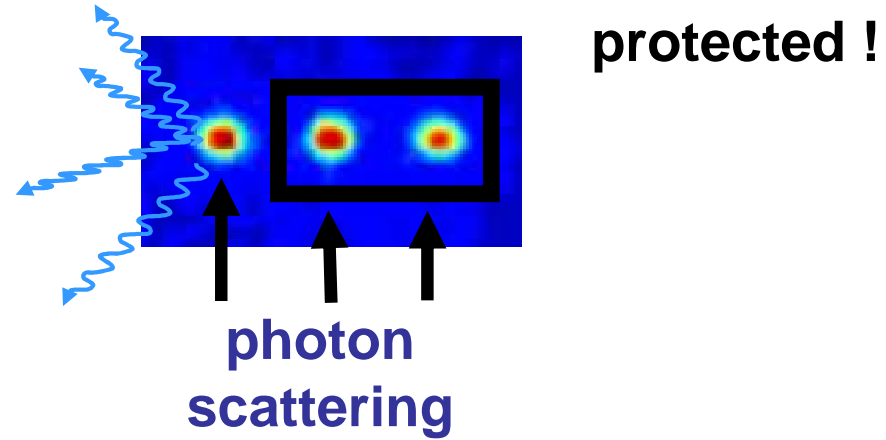


Zeeman levels

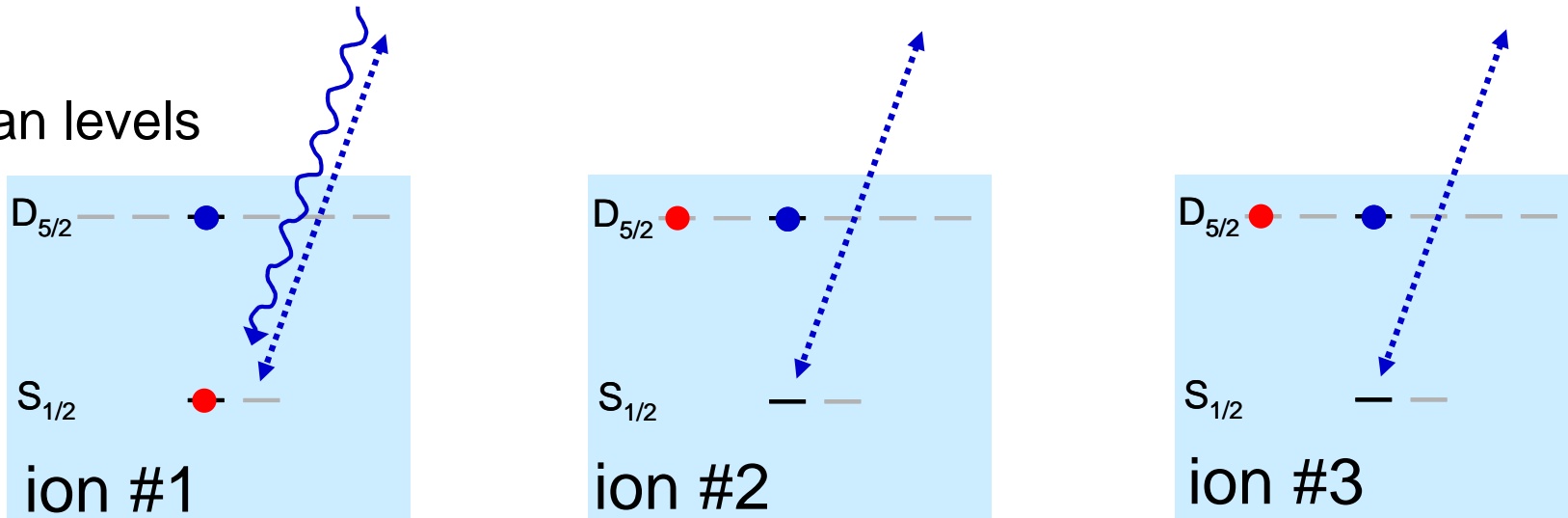


detect quantum state of ion #1 only

Hiding a qubit

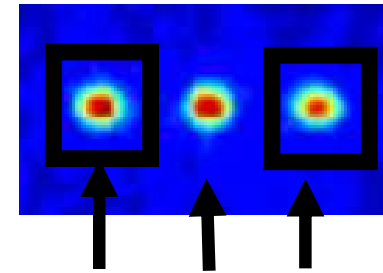


Zeeman levels



detect quantum state of ion #1 only

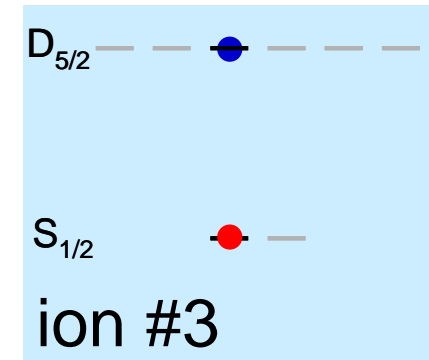
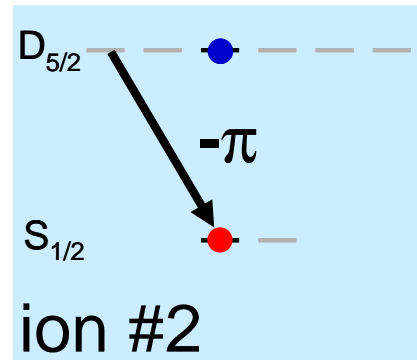
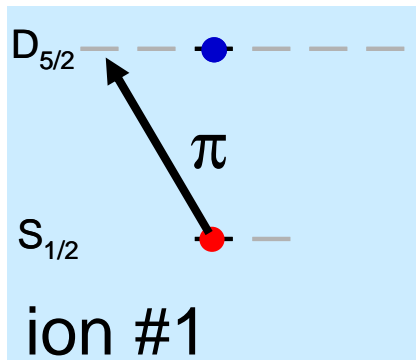
Hiding and unhiding



protected !

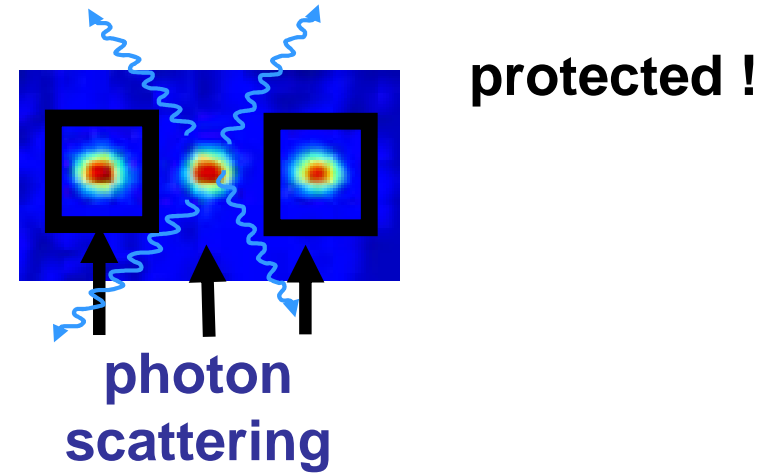
photon
scattering

Zeeman levels

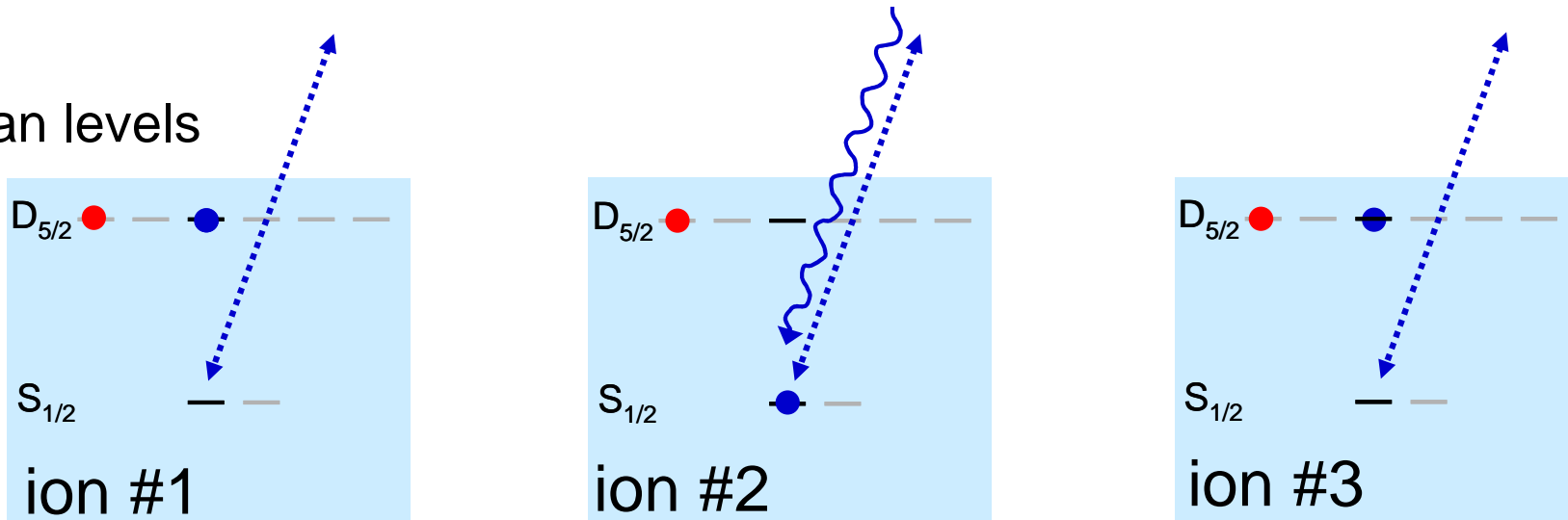


detect quantum state of ion #2 only

Hiding a qubit



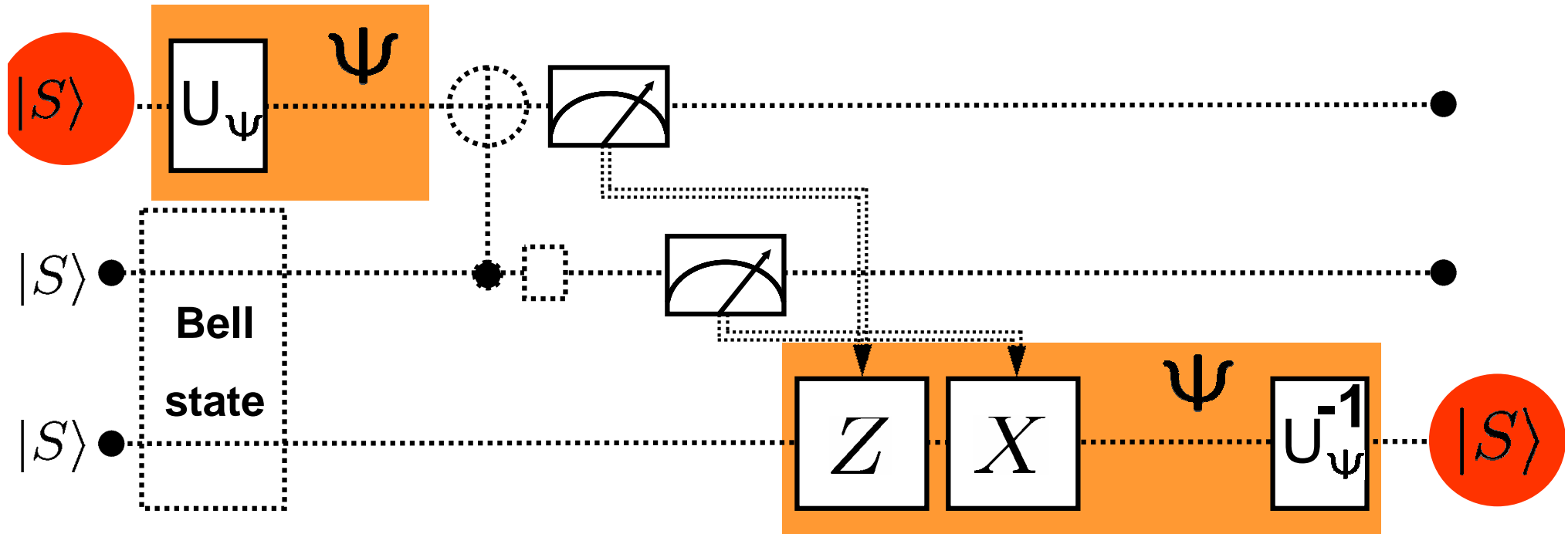
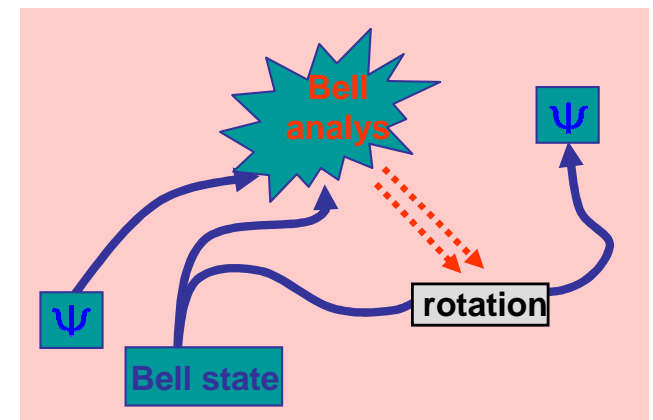
Zeeman levels



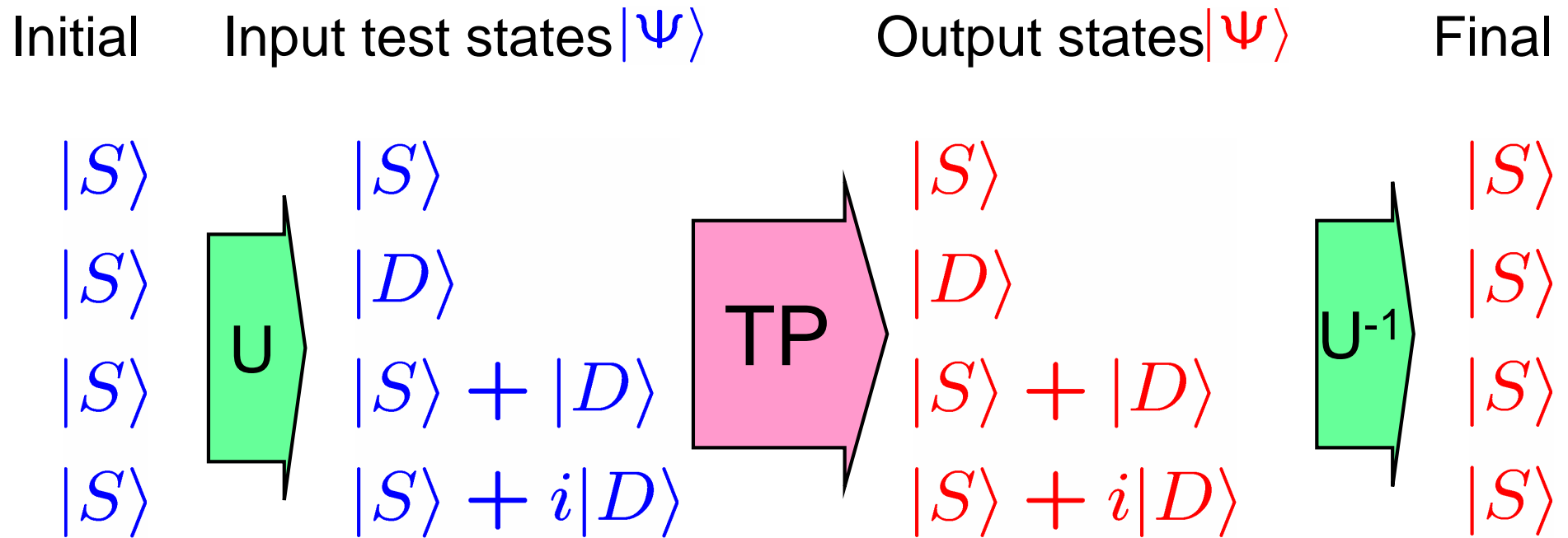
detect quantum state of ion #2 only

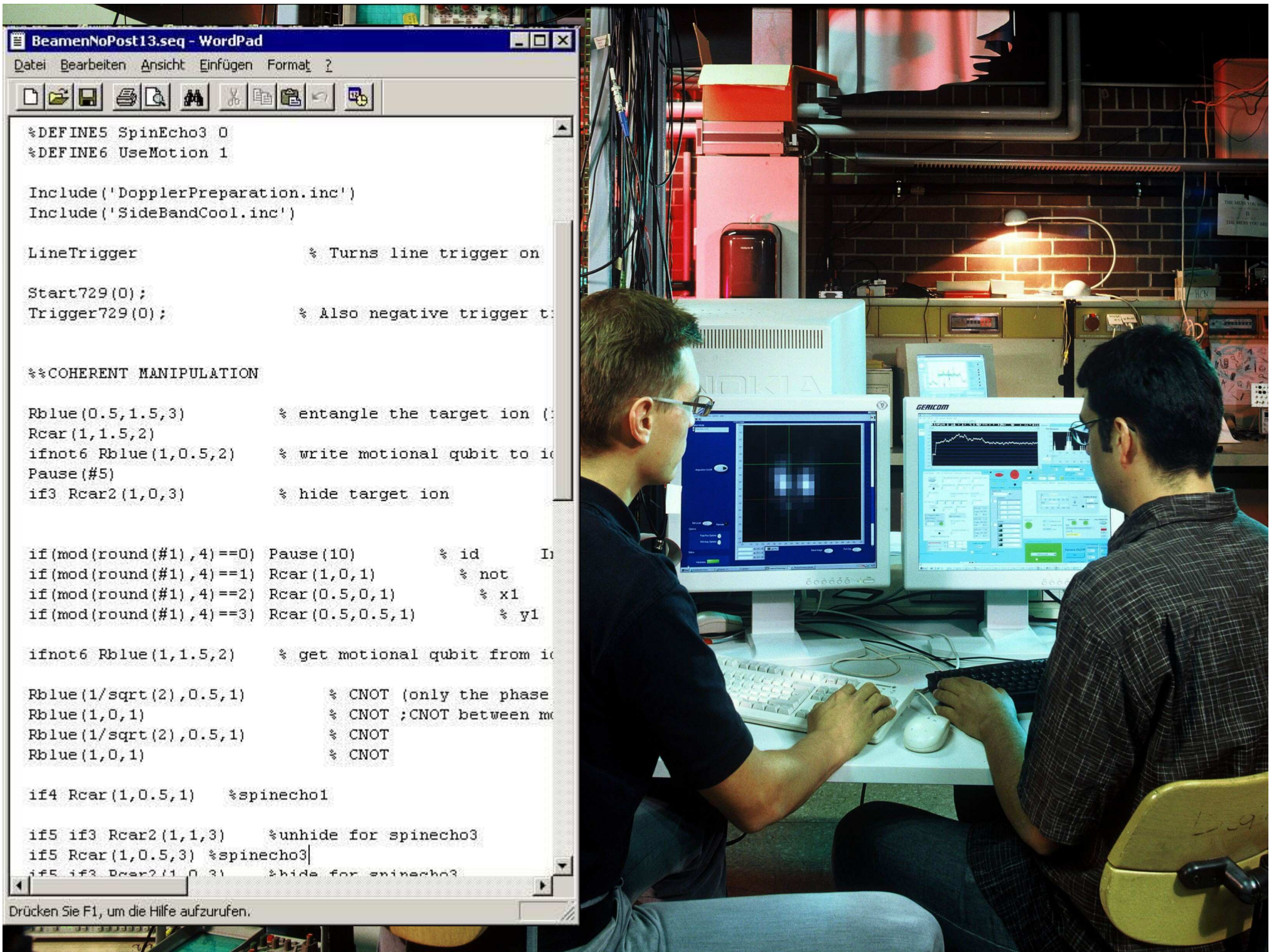
Step by step

1. *Bell state generation*
2. *Generate ψ*
3. *Bell analysis*
4. *Selective read-out*
5. ***Conditional rotations***
6. ***Test performance !***



Analysis of teleportation





```
BeamenNoPost13.seq - WordPad
Datei Bearbeiten Ansicht Einfügen Format ?
[Icons]
%DEFINE5 SpinEcho3 0
%DEFINE6 UseMotion 1

Include('DopplerPreparation.inc')
Include('SideBandCool.inc')

LineTrigger          % Turns line trigger on

Start729(0);
Trigger729(0);      % Also negative trigger t

%%COHERENT MANIPULATION

Rblue(0.5,1.5,3)     % entangle the target ion (
Rcar(1,1.5,2)
ifnot6 Rblue(1,0.5,2) % write motional qubit to ic
Pause(#5)
if3 Rcar2(1,0,3)     % hide target ion

if(mod(round(#1),4)==0) Pause(10)      % id      In
if(mod(round(#1),4)==1) Rcar(1,0,1)     % not
if(mod(round(#1),4)==2) Rcar(0.5,0,1)   % x1
if(mod(round(#1),4)==3) Rcar(0.5,0.5,1) % y1

ifnot6 Rblue(1,1.5,2) % get motional qubit from ic

Rblue(1/sqrt(2),0.5,1) % CNOT (only the phase
Rblue(1,0,1)          % CNOT ;CNOT between m
Rblue(1/sqrt(2),0.5,1) % CNOT
Rblue(1,0,1)          % CNOT

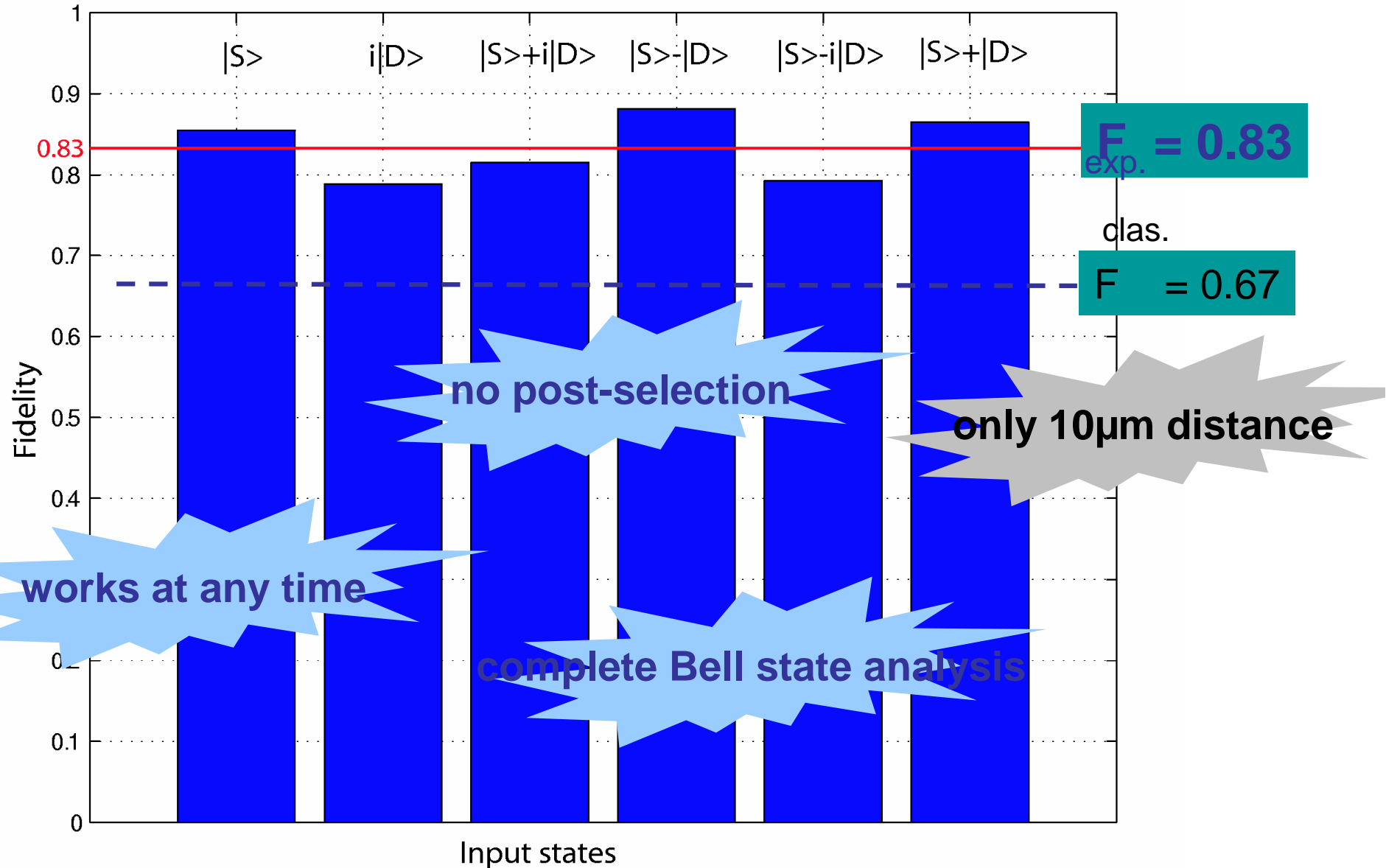
if4 Rcar(1,0.5,1) %spinecho1

if5 if3 Rcar2(1,1,3) %unhide for spinecho3
if5 Rcar(1,0.5,3) %spinecho3
if5 if3 Rcar2(1,0,3) %hide for spinecho3

Drücken Sie F1, um die Hilfe aufzurufen.
```

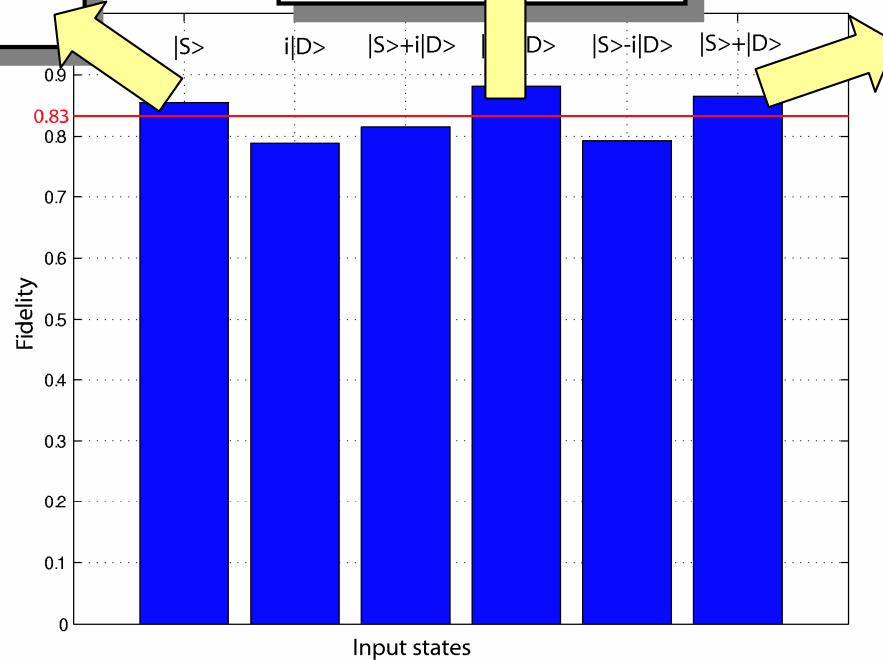
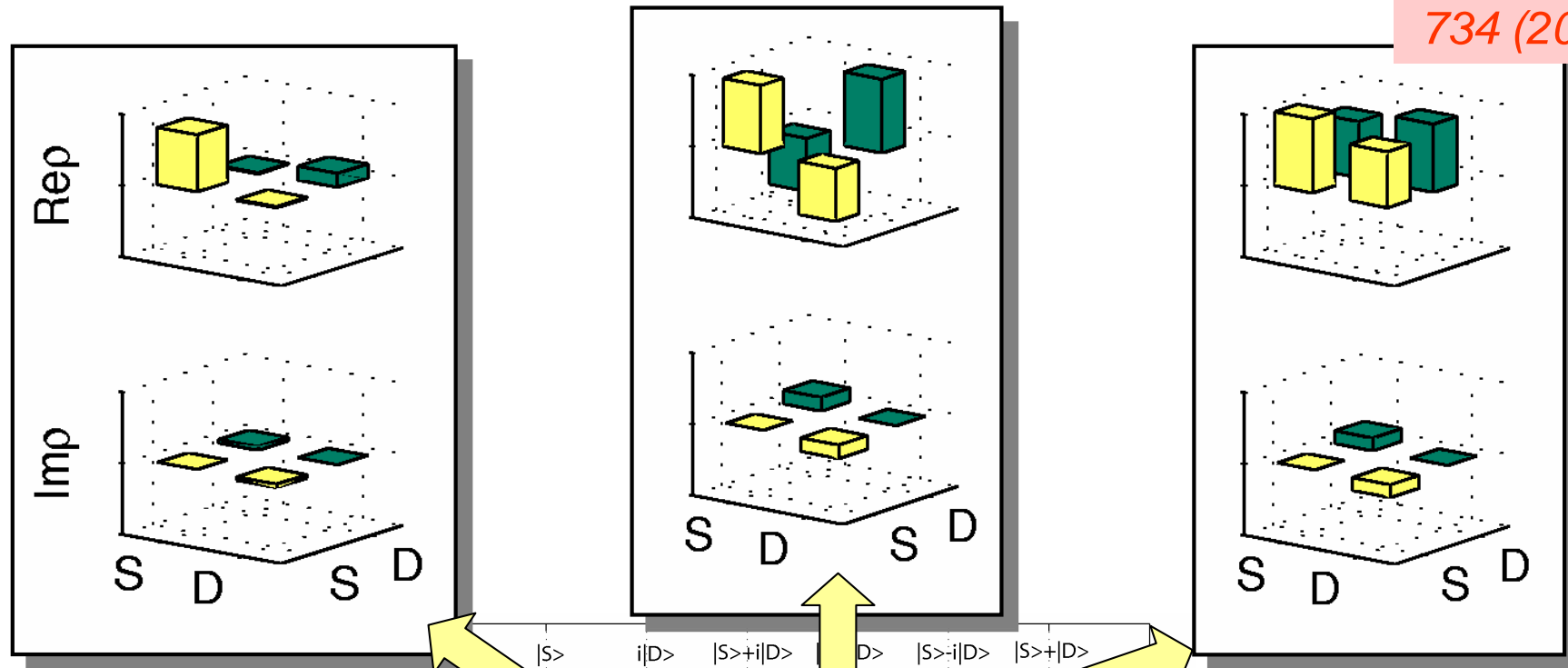
Teleportation „on demand“ : Results

*Riebe et al,
Nature 429,
734 (2004)*



Process tomography of teleportation

Riebe et al,
Nature 429,
734 (2004)



„Trapology“ for Boulder Teleportation

Teleportation / Boulder

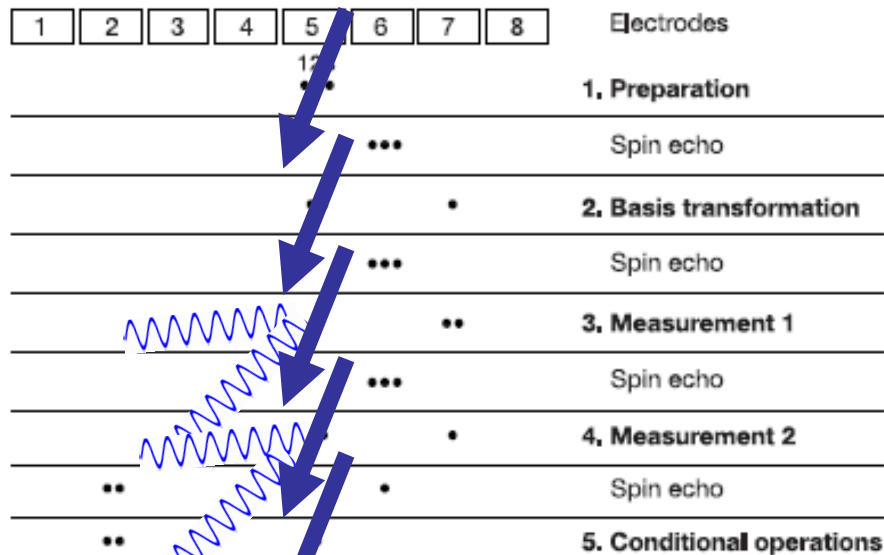
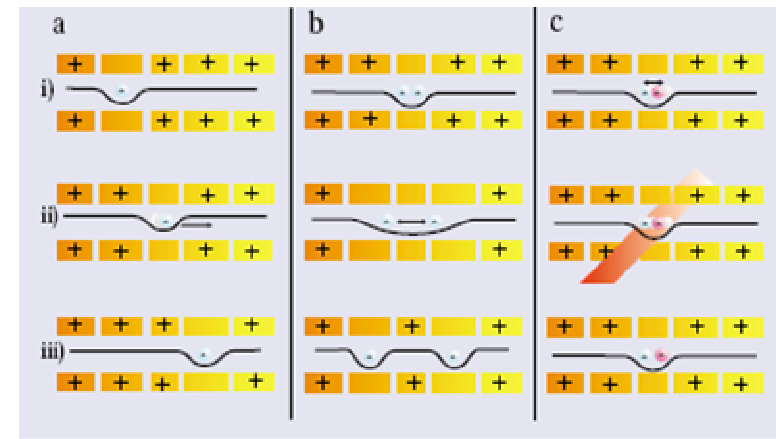
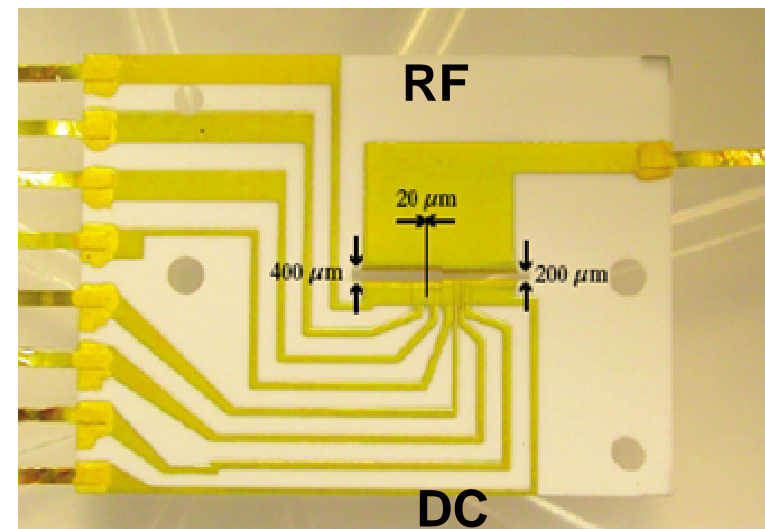


Figure 1 Schematic representation of the teleportation protocol. The ions are numbered left to right, as indicated at the top, and retain their order throughout. Positions, relative to the electrodes, are shown at each step in the protocol. The widths of the electrodes vary, with the width of the separation electrode (6) being the smallest at $100\ \mu\text{m}$. The spacing between ions in the same trap is about $3\ \mu\text{m}$, and laser-beam spot sizes (in traps 5 and 6) at the position of the ions are approximately $30\ \mu\text{m}$. In step 1 we prepare



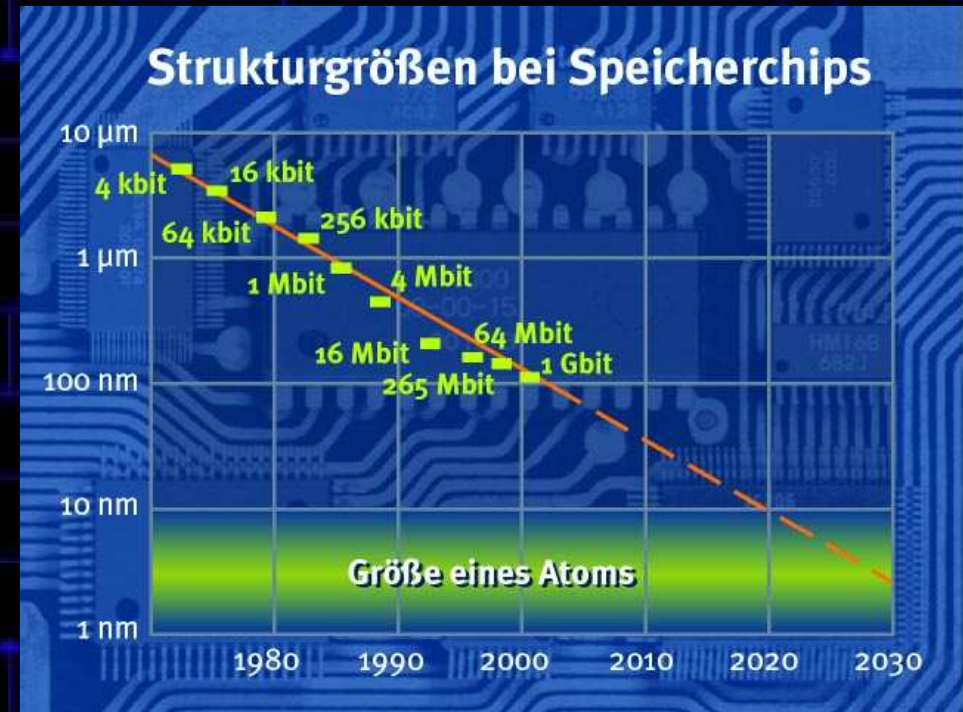
- a) Transport ions from right to left
- b) Separate two ions to right and left side

NIST Boulder



Resultats and **Aims** for QIP

- Quantum-Gates
- Bell states
- Entangled states
of 3 .. 8 –qubits
- Deterministic Teleportation
- **Scalable QC – Micro traps**
- **Error correction, improved gates**
- **Quantum simulation**
- **Improved frequency standards**
- **Atomic Information Processing**



Groups with single ions:
Arhus, Barcelona,
NIST Boulder,
NPL Teddington,
Innsbruck,
Michigan, MPQ, MIT,
Oxford, Siegen,
Southampton, Ulm,

Team of QIV-ULM

K. Singer	S. Schulz	N. Linke
R. Reichle	G. Huber	F. Ziesel
P. Bushev	W. Schnitzler	M. Hettrich
T. Calarco	U. Poschinger	R. Tammer
M. Murpey	M. Hellwig	M. Ferner
J. Baldrusch	J. Eble	M. Bürzle
H. Doerk-Bendig		



Koll. with: R. Kleiner, C. Zimmermann (Tüb.), G. Werth, W. Nörthershäuser (Mainz), J. Wrachtrup (Stuttgart), R. Blatt (Innsbruck), C. Wunderlich (Siegen), P. Gill (Teddington)

<http://www.quantenbit.de>



Interested
to join in?

